A Three State Model of Worker Flows in General Equilibrium

Per Krusell† Toshihiko Mukoyama‡ Richard Rogerson§ Ayşegül Şahin¶

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Abstract

We develop a simple model featuring search frictions and a nondegenerate labor supply decision along the extensive margin. The model is a standard version of the neoclassical growth model with indivisible labor and idiosyncratic productivity shocks and frictions characterized by employment loss and employment opportunity arrival shocks. We argue that it is able to account for the key features of observed labor market flows for reasonable parameter values. Persistent idiosyncratic productivity shocks play a key role in allowing the model to match the persistence of the employment and out of the labor force states found in individual labor market histories.

Keywords: Labor Supply, Labor Market Frictions

JEL Classifications: E24, J22, J64

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†IIES, CAERP, CEPR, and NBER
‡University of Virginia and CIREQ
§Arizona State University and NBER
¶Federal Reserve Bank of New York
1 Introduction

Analyses of aggregate employment are dominated by two frameworks. One is the frictionless version of the standard growth model with an endogenous labor leisure choice, as in Kydland and Prescott (1982), but modified as in Hansen (1985) to include the indivisible labor formulation of Rogerson (1988). The other is the class of matching models a la Diamond-Mortensen-Pissarides, as described in Pissarides (2000). Loosely speaking, the former can be viewed as a model of labor force participation, while the latter can be viewed as a model of unemployment conditional on a participation rate. Cross country data reveal that there are significant differences across countries along all three margins: employment, unemployment and non-participation. Moreover, it seems reasonable to think that participation rates, employment rates and unemployment rates are all jointly determined, in the sense that any policy that affects one margin is likely to affect both of the other two margins. This suggests that a comprehensive model of the aggregate labor market should explicitly incorporate all three labor market states.

This paper takes a first step toward the development of a unified model of participation, unemployment and employment. The model can be seen as a hybrid of the two classes of models discussed above, extended to allow for idiosyncratic shocks. Abstracting from labor market frictions, an individual in our model solves a textbook problem of labor supply in a dynamic setting with indivisible labor. That is, the individual must decide what fraction of his or her life to spend in employment, and how to arrange the timing of employment
relative to the idiosyncratic shocks that they experience. A key property of our calibration is
that the solution for lifetime labor supply is interior, i.e., individuals do not want to work in
every period of life. An individual in our model also faces frictions just like a worker in the
textbook Pissarides model: when employed the individual faces a probability of becoming
non-employed, and when not employed, the individual finds an employment opportunity only
with some probability.

A natural criterion for assessing the empirical reasonableness of such a hybrid model of
employment, unemployment and participation is that it be able to account for both the dis-
tribution of workers across the three labor market states and the flows of workers between
them. Although our model is purposefully simplified, we show that empirically reasonable
versions of it satisfy this criterion. Persistent idiosyncratic shocks play a critical role in al-
lowing the model to match the patterns found in the worker flow data. Without idiosyncratic
shocks the model is the same as that studied in Krusell et al (2008), and we show that such
a model is unable to match the flows of workers across states. In particular, it cannot match
the high degree of persistence for the employment and out of the labor force states that is
found in the data. While our benchmark model follows the literature in assuming incomplete
markets for risk sharing and borrowing, we also show that this feature does not play a key
role in allowing the model to match the flows in the data. In addition to capturing the key
features of labor market flows, the model also does a reasonable job of accounting for several
other regularities in the data, such as the distribution of months worked during the year, and
the reasons for separations from employment.

The idiosyncratic shocks in our analysis are intended to capture all of the important shocks faced by individuals that influence the static return to working versus not working. Likely candidates for these shocks are shocks to market opportunities, health shocks, preference shocks, and family shocks. An important finding of our analysis is that the model is able to account for the labor market flows as long as the shock process is fairly persistent and the shocks are sizeable. Both of these seem to be very reasonable assumptions about the nature of idiosyncratic shocks. In this sense we conclude that our model does a good job of accounting for the flows even though we do not have good measures of some of the underlying shocks that our model is seeking to capture.

While we view our benchmark model as being successful in accounting for the key features of aggregate labor market flows, there are some discrepancies between the model and the data along some dimensions. The most significant of these is that the model does not generate enough transitions from unemployment to out of the labor force. Given the simplicity of our model, we believe it presents a natural framework to be used to assess the role of various extensions in resolving these discrepancies. Indeed, we demonstrate that allowing for a purely temporary shock in addition to the persistent shock described above can remove much of the discrepancy.

Our analysis is related to many papers in the literature. In addition to the work cited above, our paper is similar to Merz (1995), Andolfatto (1996), Alvarez and Veracierto (1999),
Gomes et al (2001) and Veracierto (2008) in that these papers all introduce frictions into an otherwise standard version of the growth model. Ljungqvist and Sargent (2006, 2008) consider models that feature indivisible labor and frictions. Merz, Andolfatto, Gomes et al and Ljungqvist and Sargent do not consider unemployment and nonparticipation as distinct states. The other two papers do consider all three labor market states but their model only puts restrictions on the stocks of workers in the three states and does not pin down labor market flows. A further key distinction between our model and those in Merz, Andolfatto and Gomes et al is that these models all have the property that if frictions were removed, the employment rate would be equal to one, with labor supply adjustment occurring only along the intensive margin.

Beginning with Burdett et al (1984), there are also several papers that have extended the simple matching model to allow for nonparticipation.\(^1\) These models assume linear utility and therefore implicitly impose assumptions on the income and substitution effects that govern labor supply that are not consistent with standard specifications of labor supply. Additionally, they cannot address issues in which risk sharing plays a key role. Garibaldi and Wasmer (2005) is a recent general equilibrium model from this class that is most similar to ours. They also assume idiosyncratic shocks and seek to account for labor market flows across the three states. Similar to us, they find that their benchmark model cannot account for the large flow from unemployment to out of the labor force. They develop an extension

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with heterogeneous workers that performs better along this dimension.

Lastly, our work is related to a recent literature that studies labor supply in settings with incomplete markets and idiosyncratic shocks, including papers by Domeij and Floden (2006), Floden and Linde (2001), Low (2005), Chang and Kim (2006, 2007) and Pijoan-Mas (2006). None of these papers allows for trading frictions. Similar to us, Low et al (2010) consider a model with frictions and a nondegenerate labor supply decision. They consider a richer model of frictions and income support programs, but their analysis is partial equilibrium and they address very different issues than we do.

An outline of the paper follows. Section 2 describes the model. Section 3 describes the calibration of the model and presents the implications of the benchmark calibrated model for labor market flows. Section 4 considers an alternative calibration procedure and Section 5 examines sensitivity of the results to changes in the idiosyncratic shock process. Section 6 considers two additional specifications and Section 6 concludes.

2 Model

The economy is populated by a continuum of workers with total mass equal to one. All workers have identical preferences over streams of consumption and time devoted to work given by:

$$\sum_{t=0}^{\infty} \beta^t [\log(c_t) - \alpha e_t]$$
where $c_t \geq 0$ is consumption in period $t$, $e_t \in \{0,1\}$ is time devoted to work in period $t$, $0 < \beta < 1$ is the discount factor and $\alpha > 0$ is the disutility of work. The restriction that $e_t$ is either zero or one reflects the assumption that labor is indivisible, so that all adjustment occurs along the extensive margin. A key feature of the model is that workers are subject to idiosyncratic shocks that affect the relative payoffs to working or not working in a particular period. In reality there are many shocks that serve this role: shocks to market opportunities, shocks to home production opportunities, health shocks, family shocks, preference shocks, etc... To maintain parsimony, we model the net effect of all of these shocks as a single shock, and represent it as a shock to market opportunities. In particular, we assume that workers are subject to idiosyncratic shocks that affect the quantity of labor services that they contribute if working. We denote this value by $s$ and assume that it follows an AR(1) stochastic process in logs:

$$\log s_{t+1} = \rho \log s_t + \varepsilon_{t+1}$$

where $0 < \rho < 1$ is the persistence parameter and the innovation $\varepsilon_t$ is a mean zero normally distributed random variable, with standard deviation $\sigma_\varepsilon$. This process is the same for all workers, but realizations are iid across workers. The reader should keep in mind that these market opportunity shocks are standing in for the combined effect of many shocks that affect the relative value of working versus not working. As a practical matter, we could have alternatively assumed that the single shock affects the disutility of working $\alpha$ instead of the return to working.
We formulate equilibrium recursively. In each period there are competitive markets for output, capital services and labor services, but as in Huggett (1993) and Aiyagari (1994), there are no insurance markets, so individuals will potentially accumulate assets to self-insure.\(^2\) In what follows we will focus on steady state equilibria, so that factor prices will be constant. We normalize the price of output to equal one in all periods, let \(r\) denote the rental price for a unit of capital, and let \(w\) denote the rental price for a unit of labor services.\(^3\) If a worker with productivity \(s\) chooses to work then he or she would contribute \(s\) units of labor services and therefore earn \(ws\) in labor income. We assume that individuals are not allowed to borrow, which is equivalent to assuming that capital holdings must be nonnegative. There is a government that taxes labor income at constant rate \(\tau\) and uses the proceeds to finance a lump-sum transfer payment \(T\) subject to a period-by-period balanced budget constraint. In steady state, the period budget equation for an individual with \(k_t\) units of capital and productivity \(s_t\) is given by:

\[
c_t + k_{t+1} = rk_t + (1 - \tau)ws_t c_t + (1 - \delta)k_t + T.
\]

The production technology is described by a Cobb-Douglas aggregate production function:

\[
Y_t = K_t^\theta L_t^{1-\theta}.
\]

\(^2\)This framework first appears in the undated manuscript by Bewley, entitled “Interest Bearing Money and the Equilibrium Stock of Capital”. We think that the assumption of incomplete markets is a useful benchmark for the study of many applied issues but does not play a key role in allowing the model to account for the behavior of labor market flows. Specifically, in Section 6 we show that a complete markets version of the model can match the flows as well as the incomplete markets model.

\(^3\)As in Lucas and Prescott (1974), we assume that frictions influence a worker’s access to the labor market. But conditional on access, the market operates competitively.
\( K_t \) is aggregate input of capital services and \( L_t \) is aggregate input of labor services:

\[
K_t = \int k_{it} di, \quad L_t = \int e_{it}s_{it} di.
\]

Output can be used either as consumption or investment, and capital depreciates at rate \( \delta \).

We let \( E_t \) represent aggregate employment:

\[
E_t = \int e_{it} di
\]

and let \( S_t \) represent the average productivity of employed workers, i.e.,

\[
S_t = \frac{\int e_{it}s_{it} di}{E_t}.
\]

It follows that \( L_t = S_tE_t \).

To capture frictions in the labor market, we assume that there are two islands, which we refer to as the production island and the leisure island. At the end of period \( t - 1 \) an individual is either on the production island or the leisure island, depending upon whether they worked during the period. That is, as of the end of period \( t - 1 \), a worker who worked in period \( t - 1 \) will be on the production island, and an individual who did not work in period \( t - 1 \) will be on the leisure island. At the beginning of period \( t \) each individual will observe the realizations of several shocks. First, each individual receives a new realization for the value of their idiosyncratic productivity shock. Second, each individual on the production island observes the realization of an \( iid \) separation shock: with probability \( \sigma \) the individual is relocated to the leisure island. Third, each individual on the leisure island, including those that have been relocated on account of the separation shock, observes the realization of an
*iid* employment opportunity shock: with probability $\lambda_w$ an individual is relocated to the production island. Loosely speaking, $\sigma$ is the exogenous job separation rate, and $\lambda_w$ is the exogenous job arrival rate. After all of these shocks have been realized, each individual on the production island decides whether to work and how much to consume. An individual who is on the production island and chooses not to work will then be on the leisure island at the end of period $t$ and will therefore not have the opportunity to return to the production island until receiving a favorable employment opportunity shock. An individual who is on the leisure island after the realization of all of the shocks is not allowed to supply labor, so his or her only choice is how much to consume. Note that this individual still has two sources of income: income from renting out capital services and the transfer payment from the government. This individual will be on the leisure island at the end of period $t$.

A worker’s state consists of his or her location at the time that the labor supply decision needs to be made, the level of asset holdings, and productivity. Let $W(k, s)$ be the maximum value for an individual who works given that they have productivity $s$ and capital holdings $k$, and let $N(k, s)$ denote the maximum value for an individual who does not work given that he or she has productivity $s$ and capital holdings $k$. Define $V(k, s)$ by:

$$V(k, s) = \max\{W(k, s), N(k, s)\}.$$  

The Bellman equations for $W$ and $N$ are given by:

$$W(k, s) = \max_{c, k'}\{\log(c) - \alpha + \beta E_{s'}[(1 - \sigma + \sigma\lambda_w)V(k', s') + \sigma(1 - \lambda_w)N(k', s')]\}$$
\[
s.t. \ c + k' = r k + (1 - \tau) w s + (1 - \delta) k + T
\]

\[
c \geq 0, \ k' \geq 0
\]

and

\[
N(k, s) = \max_{c, k'} \{ \log(c) + \beta E_s [\lambda w V(k', s') + (1 - \lambda w) N(k', s')] \}
\]

\[
s.t. \ c + k' = r k + (1 - \delta) k + T
\]

\[
c \geq 0, \ k' \geq 0.
\]

Let \( \mu(k, s, l) \) denote the measure of individuals over individual states after all of the idiosyncratic shocks have been realized and before any decisions have been taken, where \( l \) indexes location and can take on the two values 0 and 1, with \( l = 1 \) indicating the production island. There are three decision rules: one for \( c \), one for \( k' \), and one for \( e \) (which can only take on the values of 0 or 1).

### 2.1 Properties of Decision Rules

It is useful to discuss some features of the decision rules in order to gain some intuition about the forces that shape individual choices in the steady state equilibrium. It is trivial to show that the value functions are increasing in both assets and the level of the idiosyncratic

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productivity shock. One can then show that the decision rules have some simple reservation properties. Specifically, for a given level of assets, it turns out that the work decision for an individual on the production island is characterized by a reservation rule in terms of the idiosyncratic productivity: work if productivity is above some threshold $s^*(k)$. Similarly, one can show that for a given productivity level, the work decision for an individual on the production island is also characterized by a reservation rule in terms of assets: work if assets are below some threshold $k^*(s)$. It also follows that $s^*(k)$ and $k^*(s)$ are both increasing functions. The fact that $s^*(k)$ is increasing reflects the fact that higher assets lead to a positive wealth effect, effectively lowering labor supply. The fact that $k^*(s)$ is increasing reflects intertemporal substitution effects of optimal labor supply: an individual wants to work when productivity is high and enjoy leisure when productivity is low.

3 Accounting for Labor Market Flows

The model described in the previous section is a relatively simple extension of the standard growth model that serves as the benchmark for many aggregate analyses. In this section we assess the extent to which this extension can account for the salient features of labor market flows in addition to the standard aggregate observations. Given the simplicity of the model there is no presumption that it can account for all the features found in the data. The issue of interest here is to assess the extent to which it can capture the key features of the flows that are found in the data, and isolate those dimensions, if any, along which the model cannot do a good job of replicating the data.
3.1 Measurement of Flows

In this section we describe how we will connect our model with the data on labor market flows. Our model offers a very natural distinction among non-employed workers. In particular, there are some non-employed workers who would like to work but do not have the opportunity, and others who do not want to work even if presented with the opportunity. To us it seems natural to label the first group as unemployed (U) and the second group as non-participants (N). While this notion of unemployment is different than that used by statistical agencies, data gathered by the BLS does allow us to compute the counterpart to this notion of unemployment in the data. That is, the BLS does ask people if they would like to work independently of whether they engaged in active search in the previous four weeks. In what follows, we will use this notion to compute the unemployment rate in the data. This leads to a larger pool of individuals in the unemployment state than does the standard definition, which is based largely on the individual’s level of search effort. For the period 1994-2007, which is the period for which consistent data is available, the standard unemployment rate for the US averages 5.1%, whereas our expanded notion of unemployment averages 8.3%. Given that we provide a different split of the non-employed into the unemployment and out of the labor force states, we also need to correct the data on flows between the states. The appendix details the procedure that we use to construct these flows. Table 1 shows the effects of the adjustment.
Table 1  
Actual and Adjusted Flows in the Data

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
<th>FROM</th>
<th>TO</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>U</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>0.962</td>
</tr>
<tr>
<td></td>
<td></td>
<td>U</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>0.960</td>
</tr>
<tr>
<td></td>
<td></td>
<td>U</td>
<td>0.248</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>0.036</td>
</tr>
</tbody>
</table>

The effect of this adjustment is quite minor. As one would expect, there is a slight decline in the flow rate from $U$ to $E$, since we are expanding the size of $U$ and including those who in general transition into employment with lower probability. Note also that the alternative notion of $U$ has relatively little effect on the flows between $U$ and $N$; in fact, these flows are now somewhat larger.

Given the fact that the flows for the two different definitions are so similar, there is relatively little need to think about which measure is preferable. Nonetheless, we think some discussion about our choice of a non-standard split between unemployment and out of the labor force is worthwhile. While our model offers a sharp distinction between these two groups, in reality the distinction is somewhat less clear. Standard practice among statistical agencies is to use information on the extent of search effort as the key criterion to divide the non-employed between unemployment and not in the labor force. A recent literature has questioned whether the rules by which statistical agencies allocate individuals between these three states is the most useful from the perspective of characterizing economic behavior. Using data from Canada, Jones and Riddell (1999) showed that the active search criterion used by many statistical agencies to determine the allocation of workers between the unem-
ployed state and the out of the labor force state is potentially misleading because it excludes a group of workers (whom they call marginally attached) who say that they would like a job but have not actively searched in the last four weeks. These workers do have somewhat lower transition rates into employment than do active searchers, by about twenty-five percent, though the job finding rates for the marginally attached workers are the same as those of active searchers who report reading job ads or visiting a public employment office as their sole method of active search. In contrast, the marginally attached workers have transition rates into employment that are more than 4 times as high as the other non-participants. We conclude from this that the marginally attached workers (i.e., passive searchers) are more similar to unemployed workers than they are to nonparticipants. These same findings have emerged when this analysis has been repeated for many other countries.4

A second issue has to do with the cost of active search. The relatively new American Time Use Survey that is conducted as part of the Current Population Survey reveals that active searchers devote very little time to search, typically less than one hour per week. This suggests that the cost of active search is best thought of as being quite small. If the cost of active search is actually very small, and given the dynamic nature of search, the decision of whether to engage in active search at a given point in time may not be very meaningful. This would also suggest that allocating individuals to various states on the basis of active search may not be prudent. Consistent with the evidence that the cost of active search is

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very small, our model has not placed any emphasis on search costs and likewise our definition of unemployment also does not stress search effort.

Having raised all of these issues, we note that there is an interpretation of our model that would also fit with standard definitions of unemployment based on active search. Specifically, if we assumed that the search effort decision is a binary decision, and that the cost of search is positive but arbitrarily small, then it would follow that all individuals who prefer working to nonworking given their current state would engage in active search. In this sense one could connect our model with the active search criterion used by the BLS and use the standard measure of unemployment. However, because the flows are not very much affected by the definition of unemployment, this alternative way of connecting with the data has little impact on the results of our analysis. Put somewhat differently, if one views the standard definition of unemployment and our definition of unemployment as two extreme choices, with a continuum of intermediate choices lying in between these two, the basic message is that this choice is not very important for the issues that we address here. Nonetheless, in what follows we will base our comparison on the adjusted numbers in Table 1 since we think those numbers represent the most natural way to connect the model with the data.

### 3.2 Flows for Subgroups

Before proceeding to the calibration of the model we think it is of interest to look at labor market flows for some subgroups in the population. We begin by looking at how labor market flows differ for men and women. If these flows were dramatically different it might suggest
that our approach of using a model with single agent households to account for aggregate flows is questionable. Table 2 presents the flows for men and women.\(^5\)

Table 2  
Flows For Men and Women

<table>
<thead>
<tr>
<th>FROM</th>
<th>E</th>
<th>U</th>
<th>N</th>
<th>FROM</th>
<th>E</th>
<th>U</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>0.961</td>
<td>0.022</td>
<td>0.017</td>
<td>E</td>
<td>0.959</td>
<td>0.021</td>
<td>0.020</td>
</tr>
<tr>
<td>U</td>
<td>0.289</td>
<td>0.539</td>
<td>0.172</td>
<td>U</td>
<td>0.209</td>
<td>0.495</td>
<td>0.295</td>
</tr>
<tr>
<td>N</td>
<td>0.045</td>
<td>0.049</td>
<td>0.906</td>
<td>N</td>
<td>0.030</td>
<td>0.043</td>
<td>0.927</td>
</tr>
</tbody>
</table>

The table shows that most of the flows are very similar. The two exceptions seem to be the flows from \(U\) to \(E\) and from \(U\) to \(N\). Women experience much larger flows from \(U\) to \(N\), with a roughly similar decrease in the rate at which they flow from \(U\) to \(E\). Conditional on not moving into \(N\), the flow rates from \(U\) to \(E\) are more similar, .349 for men and .297 for women. We shall return to a discussion of \(U\) to \(N\) flows later in the paper, as this will prove to be a dimension along which our model does not perform well.

A second dimension of interest is age. The data that we presented above was for all individuals aged 16 and older. One concern might be that the bulk of the action in terms of labor market flows comes from the very young and the very old. Table 3 reports the flows for individuals between the ages of 21 and 65.

\(^5\)In the interest of space we do not present the flows for standard definitions of \(U\). Similar to the case of the aggregate data, there is little difference between the adjusted and unadjusted for the subgroups as well.
Table 3  
Flows By Age and Gender

<table>
<thead>
<tr>
<th>FROM</th>
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<th>FROM</th>
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<th>FROM</th>
<th>TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>U</td>
<td>N</td>
<td></td>
<td>E</td>
<td>U</td>
</tr>
<tr>
<td>0.974</td>
<td>0.017</td>
<td>0.012</td>
<td></td>
<td>0.964</td>
<td>0.018</td>
</tr>
<tr>
<td>0.301</td>
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<td>0.097</td>
<td></td>
<td>0.195</td>
<td>0.554</td>
</tr>
<tr>
<td>0.064</td>
<td>0.060</td>
<td>0.876</td>
<td></td>
<td>0.053</td>
<td>0.049</td>
</tr>
</tbody>
</table>

Eliminating the youngest and oldest workers does not appreciably change the nature of the flows, neither at the aggregate level or disaggregated by gender. For future reference we note again that the flows from $U$ to $N$ are decreased relative to the total population.

Lastly, we consider an even narrower group, examining the flows for individuals aged 25 to 54. The results are in Table 4.

Table 4  
Flows By Gender, Prime Aged Individuals

<table>
<thead>
<tr>
<th>FROM</th>
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<th>FROM</th>
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</thead>
<tbody>
<tr>
<td>E</td>
<td>U</td>
<td>N</td>
<td></td>
<td>E</td>
<td>U</td>
</tr>
<tr>
<td>0.978</td>
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<td>0.065</td>
<td>0.059</td>
</tr>
</tbody>
</table>

The patterns remain very similar to those in the earlier tables. In particular, we want to emphasize that both the magnitude and qualitative properties of the flows seem to be quite similar across the population, suggesting that abstracting from explicit modeling of demographic factors such as age and gender seems a reasonable starting point.
3.3 Flows Between $U$ and $N$

A notable feature of the flows in Table 1 is the relatively large transition rate from $U$ and $N$. Previous work by Abowd and Zellner (1985) and Poterba and Summers (1986) used information from the CPS reinterview survey to argue that a large share of this flow is accounted for by survey response error. In particular, using the standard definition of unemployment, these authors estimated that roughly half of the flows from $U$ to $N$ were caused by survey response error. The reinterview survey has been altered in a way that does not permit one to update these estimates for the period since 2001. Also, the nature of the reinterview survey does not allow us to estimate the importance of survey response error for our definition of unemployment during the earlier period. Nonetheless, we think it is reasonable to think that the extent of misclassification will be similar for our measure of unemployment. This will be relevant in evaluating the model’s performance.

Note that if survey response error leads to errors in classifying workers into the $U$ versus $N$ state, then both the flow from $U$ to $N$ as well as the flow from $N$ to $U$ will be too high. However, this effect is much more pronounced on the flow from $U$ to $N$ for the reason that the size of the group in $N$ is roughly six times as large as the size of the group in $U$.

3.4 Benchmark Calibration

Having described how we will measure flows across states in the data and the model, in this section we consider the issue of how well this model can account for the data in Table 1. The model has nine parameters that need to be assigned: preference parameters $\beta$ and $\alpha$, ...
production parameters $\theta$ and $\delta$, idiosyncratic shock parameters $\rho$ and $\sigma_e$, frictional parameters $\sigma$ and $\lambda_w$, and the tax rate $\tau$. The length of a period will matter for many of the parameter values. We will be interested in the behavior of worker flows between labor market states and since this data is available at a monthly frequency, we set the length of a period equal to one month. Because our model is a variation of the standard growth model, we can choose some of these parameter values using the same procedure that is typically used to calibrate versions of the growth model. Given that our model assumes incomplete markets and uncertainty, our steady state cannot be represented analytically in the same fashion as the standard growth model, and in particular one cannot isolate the connection between certain parameters and target values. Nonetheless, it is still useful and intuitive to associate particular targets and parameter values. Specifically, we set $\theta = .3$ to target a capital share of .3, choose $\delta$ so that the steady state ratio of investment to output is equal to .2, and choose the discount factor $\beta$ to target an annual real rate of return on capital equal to 4%. The other preference parameter $\alpha$, which captures the disutility of working, is set so that the steady state value of employment is equal to .632. This is the value of the employment to population ratio for the population aged 16 and older for the period 1994 – 2007.6

The tax rate is set at $\tau = .30$. Following the work of Mendoza et al (1994) there are several papers which produce estimates of the average effective tax rate on labor income across countries. Examples include Prescott (2004) and McDaniel (2006). There are minor

6We calibrate to values for the period 1994-2007 because this is the period for which we have consistent measures of labor market flows for our definition of unemployment.
variations in methods across these studies, which do produce some small differences in the estimates, and the value .30 is chosen as representative of these estimates.\textsuperscript{7}

The remaining parameters are the two frictional parameters, $\lambda_w$ and $\sigma$, and the two parameters of the shock process, $\rho$ and $\sigma_\varepsilon$. Our basic goal is to assess the extent to which there are values of these four parameters for which our relatively simple model can account for the distribution of workers across the three labor market states and the flows of workers between these states, and if so, to what extent these values seem reasonable. It is important to recall that we want to think of our shock process as capturing a variety of different types of shocks that influence the relative value to working and not working, so that we do not want to limit our attention to a single component of this process, such as wage shocks.

One way to proceed would be to search over parameter vectors $(\lambda_w, \sigma, \rho, \sigma_\varepsilon)$ so as to minimize the difference between a set of moments in the model and the data. It turns out that there are many specifications that do similarly well at accounting for the data, and hence we feel that it is more informative to proceed in a slightly different manner. In particular, we will calibrate the two frictions so as to match two targets in the data, and then ask how the resulting labor market flows are affected by different specifications of the shock process.

Note first that $\lambda_w$ is a key parameter in generating unemployment in our model. Specifically, if $\lambda_w = 1$, then everyone always has the opportunity to work, and as a result there will be no workers unemployed according to the definition that we are using. As $\lambda_w$ is lowered

\textsuperscript{7}Note that Prescott (2004) makes an adjustment to the average labor tax rate to arrive at a marginal tax rate that is roughly 40%. For purposes of computing the effect of changes in taxes this adjustment plays no role.
from one there will be more workers in the situation of wanting to work but not having the
opportunity. Motivated by this, we choose $\lambda_w$ so that the steady state unemployment rate
in our model (i.e., $U/(E + U)$) is equal to .083, which is the average value for our notion of
the unemployment rate in the US data for the period 1994 – 2007.

Next consider $\sigma$. Intuitively, this parameter will play a large role in shaping the transitions
from employment to unemployment. The reason for this is that anyone who transitions from
employment to unemployment must have been hit with a separation shock, since otherwise
they would still have an employment opportunity and could therefore not end up both not
working and wanting to work. Although there is a strong relationship between the incidence
of separation shocks and transitions from $E$ to $U$, they are not identical, for two different
reasons. First, an individual may experience both a separation shock and a negative shock
to idiosyncratic productivity, and hence prefer not to work. Second, our timing convention
allows a worker that experiences a separation shock to also obtain a new employment oppor-
tunity in the same period, so that he or she would not show up as unemployed. Nonetheless,
there is still a close connection between the flow from $E$ to $U$ and the value of $\sigma$, so we will
use this flow to pin down the value of $\sigma$. For our benchmark case we target the flow rate
from employment to unemployment of .021, which is again the average value for this flow for
the period 1994 – 2007. Note that for the two reasons just mentioned, the value of $\sigma$ will be
larger than this value.

The above procedure describes how we set the values of all other parameters for given
choices of $\rho$ and $\sigma_\varepsilon$. By construction we will necessarily generate the same distribution of workers across states as is found in the US data, since our procedure targets both $E$ and $U$. What we seek to assess is the extent to which the model can capture the key features found in the flow data, and if so whether the resulting values of $\rho$ and $\sigma_\varepsilon$ seem to be empirically reasonable. The main result of this exercise is that the flow data generated by the model are quite similar for a very large set of values for $\rho$ and $\sigma_\varepsilon$. As we show below in more detail, the results are quite similar as long as the shocks are relatively persistent, say with an annualized persistence parameter at least equal to .5, and for a wide range of values of $\sigma_\varepsilon$.

In terms of describing the results it is useful to initially focus on one particular specification, even though there are other specifications that seem equally reasonable. For our benchmark we choose values for $\rho$ and $\sigma_\varepsilon$ so that if the annualized data were estimated by an AR(1) it would have persistence parameter .92 and standard deviations of innovations equal to .21. These values correspond to one set of estimates of idiosyncratic wage shocks for prime-aged working males, as reported in Floden and Linde (2001). We emphasize again that we do not consider wage shocks to be the only important source of shocks, and that in addition we want to capture shocks that are relevant for all individuals, not simply prime age employed males. Nonetheless, the available evidence suggests that other shocks, such as health shocks, are also persistent, so we think a persistent shock process seems a reasonable benchmark.

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8 In solving for equilibrium we approximate the AR(1) process by using 20 grids between $-2\sigma_\varepsilon\sqrt{1-\rho^2}$ and $2\sigma_\varepsilon\sqrt{1-\rho^2}$ with Tauchen (1986) method.

9 Other papers that provide similar estimates are Card (1994) and French (2005).
Table 5 shows our calibrated parameter values for our benchmark model.

Table 5
Benchmark Calibrated Parameter Values

<table>
<thead>
<tr>
<th>θ</th>
<th>δ</th>
<th>β</th>
<th>α</th>
<th>ρ</th>
<th>σ_e</th>
<th>λ_w</th>
<th>σ</th>
<th>τ</th>
</tr>
</thead>
<tbody>
<tr>
<td>.30</td>
<td>.0067</td>
<td>.9967</td>
<td>.547</td>
<td>.9931</td>
<td>.1017</td>
<td>.436</td>
<td>.039</td>
<td>.30</td>
</tr>
</tbody>
</table>

3.5 Properties of the Steady State

The only flow rate that we targeted in the calibration was the flow from $E$ into $U$. Next we examine the predictions of the calibrated model for the other flows as well. Table 6 shows the flow rates from the data and from our calibrated model.

Table 6
Flows in the Model and Data

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FROM</td>
<td>TO</td>
</tr>
<tr>
<td>E</td>
<td>$0.960$</td>
</tr>
<tr>
<td>U</td>
<td>$0.248$</td>
</tr>
<tr>
<td>N</td>
<td>$0.036$</td>
</tr>
</tbody>
</table>

Before discussing the results it is useful to briefly consider the issue of how many degrees of freedom there are in this exercise. Given that flows out of each state must sum to one, there are really six independent values in the data. Our calibration procedure targeted the flow rate from $E$ to $U$, leaving five independent flows. Because we target the shares of $E$, $U$ and $N$ in the data we do implicitly impose some additional restrictions on the flows. However, we think it would be misleading to suggest that this removes two degrees of freedom. In particular, when we consider the case of no productivity shocks we will see that these restrictions by themselves do not generate a close match for any of the flows.
Turning to the results in the table, two key features of the data are the very high persistence for both the $E$ and $N$ states. Our calibrated model not only generates a lot of persistence in these two states, but also matches the corresponding values in the data very well. Persistence in the unemployment state is much less than in the other two states. Our model also captures this regularity, though the model generates slightly more persistence in this state than is found in the data. One simple metric to gauge the model’s ability to capture the persistence of the three states is to compute expected spell durations for each state. Measured in months, the duration of $E$, $U$, and $N$ spells in the model are 18.7, 2.1 and 13.0 respectively. The corresponding values in the (adjusted) data are given by 22.5, 2.0 and 14.3. These values are all close.

Given the high persistence of the $E$ and $N$ states and the fact that the model accounts for these values quite well, it follows that the remaining flows out of $E$ and $N$ are also necessarily quite close in the model and the data. The flows for which the discrepancies are largest between the model and the data are the flows involving $U$: the model predicts that the $U$ to $U$ flow is too large, that the $U$ to $N$ flow is too low and that the $U$ to $E$ flow is too large. Several remarks are in order concerning these discrepancies. First, because we calibrate the model to match the level of unemployment and the flow of workers into $U$ from $E$, there is a sense in which a large discrepancy in the $U$ to $N$ flow will necessarily lead to a large discrepancy in the $U$ to $E$ flow. To see this, note that given a flow of workers entering unemployment, the same number must leave unemployment in order to maintain the stock.
So if they are not leaving to \( N \) they must instead by leaving to \( E \). From this we conclude that the large discrepancy concerning the flows out of \( U \) is really one dimensional in nature.

An alternative measure of interest is the probability that a worker moves from \( U \) to \( E \) conditional on not moving to \( N \). In the data this value is .36, while in the model it is .43. Although a discrepancy remains, the difference is much smaller than the discrepancy in flows from \( U \) to \( E \). A second and related issue that is relevant for this comparison is that discrepancies associated with flows out of the \( E \) and \( N \) states are necessarily magnified in terms of consequences for flows out of the \( U \) state. The reason for this is that the stock of workers in the \( U \) state is much less than in the other two states. In relative terms, the stock of employed workers is more than ten times the stock of unemployed workers, and the stock of nonparticipating workers is almost six times the stock of unemployed workers. To see why this matters, note that the flow of workers leaving the employment state in the model is about one percent too high relative to the data. Given that we target the stock of employed workers, it follows that the flow into employment must also be higher in the model than in the data. But a one percent discrepancy relative to the stock of employed workers is roughly a ten percent discrepancy relative to the stock of unemployed workers.

The relatively large flow between \( U \) and \( N \) in the data relative to the model can be interpreted in many ways. First, empirical evidence suggests that this flow is dominated by transitions that are reversed in the following period. Specifically, based on the analysis in Jones and Riddell (2006), one finds that the transition rate from \( U \) to \( N \) at a five month
horizon is only a fraction of what would be predicted on the basis of the monthly numbers.\textsuperscript{10}

This implies that these flows are transitory in nature. As noted earlier, one possibility is that this reflects survey response error, since those individuals who are not employed and are nearly indifferent between working and not working might be expected to have noisy answers to the question of whether they would like to work. A second possibility is that there is a transitory shock which induces a lot of high frequency transitions between the two states. The earlier work of Abowd and Zellner (1985) and Poterba and Summers (1986) suggests that roughly half of the $U$ to $N$ flow is measurement error. In Section 6 we show that the remaining discrepancy can be accounted for without sacrificing the good fit to the other flows if we introduce a purely transitory shock in addition to the persistent shock. One final comment is that if one compares the flows in the benchmark model to those reported earlier for males aged 21-65 or 25-54, the model flows are quite a bit closer, and even a relatively small amount of survey response error makes them very similar.\textsuperscript{11}

One additional statistic implied by the flows that is of interest has to do with the nature of flows into employment. As noted in the introduction, one of the objectives of the current paper is to develop a model in which transitions into employment from both $U$ and $N$ occur in equilibrium as opposed to being all from either one or the other as in the standard frictionless and frictional models. From this perspective it is of interest to compute the mass of workers

\textsuperscript{10}The analysis of Jones and Riddell is based on Canadian data, but given that the patterns in the monthly flow data are similar, there is good reason to think that their findings would also apply to US data.

\textsuperscript{11}One qualification that should be noted concerning this comparison is that the distribution of workers across states for men aged 21-65 or 25-54 is not the same as that for the total population, though the differences are not so large.
that move into \( E \) from each of the other two states. These volumes are given by multiplying the mass of workers in the respective states by the corresponding flow rate into \( E \). In the data the mass of workers moving from \( U \) to \( E \) and from \( N \) to \( E \) are .014 and .011 respectively. In the benchmark calibration displayed above these values are .023 and .011. It follows that our benchmark has relatively too many of the workers that enter employment coming from unemployment. This is related to the fact that the flow from \( U \) to \( E \) is somewhat too high in the model relative to the data, an issue that we discuss more below. However, if we believe that there is response error in how individuals classify themselves between \( U \) and \( N \), then some of the \( U \) to \( E \) flows in our model would presumably show up as \( N \) to \( E \) flows in the data, which would also bring the model’s values closer to those in the data.

To summarize, while our model with a single source of heterogeneity and constant frictions across individuals does not perfectly replicate the flows found in the data, it does capture many of the key features of the flows observed in the data, both qualitatively and quantitatively. The one caveat concerns the flows of workers from \( U \) to \( N \), which is concentrated among certain subgroups of the population. We conclude that the benchmark calibration displayed above represents an empirically reasonable description of the relative importance of standard labor supply considerations and frictions in the overall US economy.

### 3.6 Additional Statistics

While our emphasis in evaluating the performance of the model was on the flows as reported in Table 1, there are many other statistics that are also of possible interest. In this section
we report on some of these additional statistics.

The first statistic that we consider has to do with reasons for transitions out of employment. Specifically, in our model we can isolate three different reasons for a worker to transition out of employment. One is that the worker might receive a lower value of the idiosyncratic shock and choose not to work. A second is that the worker receives a separation shock and does not simultaneously receive a new employment opportunity shock. And third, it is possible that a worker does not receive any shocks but still chooses to transition out of employment. This could result if the worker were to accumulate assets beyond the reservation value \( k^*(s) \) defined earlier. As a practical matter it is possible that a worker receives multiple shocks, thereby complicating this labelling process. We label the reasons for leaving employment as follows. We first ask whether the worker has the opportunity to work. If the answer is no, and the worker would have worked given the current realization of \( s \) and their current assets, then we say that the reason for the transition of out \( E \) is a separation shock. If the individual did have the opportunity to work, then we check whether the individual would have worked this period if instead of their current realization of \( s \) they had the same value for \( s \) this period as last period; i.e., whether they would have worked if there had been no change in the value of their productivity. If the answer to this is yes, then we say that the reason for the transition out of employment is the productivity shock. The residual is what we label ”other”. The results are shown in Table 7.
Table 7

<table>
<thead>
<tr>
<th>Reason for Transitions out of $E$</th>
<th>$\sigma$ Shock</th>
<th>$s$ Shock</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.41</td>
<td>.47</td>
<td>.12</td>
</tr>
</tbody>
</table>

Note that productivity shocks are the dominant source of transitions out of $E$. A related statistic concerns reasons for the flow into unemployment. In our model there are two flows into unemployment: one from employment and the other from not in the labor force. Those that flow from employment into unemployment are necessarily individuals who were hit by the $\sigma$ shock. In this sense it is natural to label them as job losers. In steady state, 38% of the new flows into unemployment represent job losers.

Both of these statistics can be compared to measures in the data. Over the period 1994-2008 the fraction of new flows into unemployment accounted for by job losers is around 49%, so our model is a bit low on this measure. In the JOLTS data set, which covers 2000-2008, 38% of all separations are labelled as involuntary. Some care needs to be taken in comparing this statistic to our model since we do not have any job to job flows in our model, and in reality many voluntary separations are associated with job to job flows. Additionally, the work of Davis et al (2008) shows that due to sampling issues, the JOLTS figures tend to underestimate involuntary separations. Their revised numbers suggest that the fraction of all separations that are involuntary is closer to 45%.

Another statistic that we can look at is the distribution of months worked during a particular year.\textsuperscript{12} Both in the data and in the model the transitions between states are not

\textsuperscript{12}In the data respondents report data on weeks worked during the previous year. We convert this to months by rounding to the nearest integer number of months.
iid over time; that is, there is substantial heterogeneity across workers in terms of expected transition rates, and the persistence of these rates. Looking at how time spent employed accumulates over the year is one way to assess the model’s ability to capture these more complex aspects of the flow data. Figure 1 plots the distributions of weeks worked during the year for both our model and the data.

The data represent averages for the CPS over the period 1994-2008. While the data has slightly more mass in the two tails, the model does a reasonable job of capturing the distribution found in the data. In the next section we will consider sensitivity to changes in the idiosyncratic shock process. Some of the alternative processes that we consider produce slightly more persistence in the $E$ and $N$ states, and yield an even closer match to the months worked distribution.
4 An Alternative Calibration: Matching the $U$ to $E$ Flow

One of the properties of our benchmark calibration is that it generated too large of a flow from $U$ to $E$. We argued that this discrepancy was mitigated somewhat by dealing with the $U$ to $N$ flow. Because the $U$ to $E$ flow seems central in many applications, we think it is important to demonstrate that there is nothing inherent in our model that prevents us from matching the $U$ to $E$ flow as well as the other flows (with the exception of the $U$ to $N$ flow).

To show this, in this section we adopt a different calibration strategy in which we explicitly target the $U$ to $E$ flow instead of the stock of workers in $U$. This would be consistent with a view that many of the $U$ to $N$ flows are spurious transitions for individuals who should be continuously classified as $U$.\footnote{More generally, we could also consider some mismeasurement of $E$, and so not match the employed stock.}

In our benchmark calibration we chose the value of $\lambda_w$ so as to target the level of unemployment in the steady state, having argued that there was intuitively a close link between the two. We begin by displaying the nature of this relationship. To do this we set $\lambda_w$ to an arbitrary value and then calibrate all of the remaining parameters in the same fashion as above, except that we no longer match the steady state unemployment rate in the data. Note that we continue to set the preference parameter $\alpha$ so that the steady state employment rate does match the target value of .632 found in the data. Table 8 gives the results.

<p>| $\lambda_w$ and the Steady State Unemployment Rate |</p>
<table>
<thead>
<tr>
<th>$\lambda_w = 1.0$</th>
<th>$\lambda_w = 0.6$</th>
<th>$\lambda_w = 0.436$</th>
<th>$\lambda_w = 0.4$</th>
<th>$\lambda_w = 0.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.000</td>
<td>.056</td>
<td>.083</td>
<td>.090</td>
<td>.155</td>
</tr>
</tbody>
</table>

Table 8
In the extreme case in which $\lambda_w = 1$, any individual who wants to work at the going wage rate can do so, and hence there are no individuals who would like to work but are unable, implying that the unemployment rate is equal to 0. It is important to note that the elasticity of $U$ with respect to $\lambda_w$ is quite large, in the sense that moving from our calibrated value of .436 to a value of .2 leads to a more than 75% increase in unemployment, while increasing $\lambda_w$ to .6 leads to a drop of more than 25% in unemployment. It follows that our calibration procedure pins down the value of $\lambda_w$ quite precisely.

As we noted earlier, our benchmark equilibrium leads to a flow rate from $U$ to $E$ of .41, whereas in the data this flow is only .25. The equilibrium flow rate from $U$ to $E$ turns out to be just slightly less than $\lambda_w$, so that in order to match the flow rate from the data one would require $\lambda_w = .26$. But as Table 8 shows, this would lead to a steady state unemployment rate equal somewhat above 10%. Table 9 shows the implications for flows for the $\lambda_w = 0.4$ and 0.2.

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
<th>FROM</th>
<th>TO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\lambda_w = 0.4$</td>
<td>$\lambda_w = 0.2$</td>
</tr>
<tr>
<td>$E$</td>
<td>$U$</td>
<td>$N$</td>
<td>$E$</td>
</tr>
<tr>
<td>0.948</td>
<td>0.021</td>
<td>0.031</td>
<td>0.959</td>
</tr>
<tr>
<td>0.374</td>
<td>0.561</td>
<td>0.065</td>
<td>0.189</td>
</tr>
<tr>
<td>0.031</td>
<td>0.046</td>
<td>0.923</td>
<td>$N$</td>
</tr>
</tbody>
</table>

The main message from this table is that one can target the flow rate from $U$ to $E$ rather than the stock of people in $U$ with relatively little effect on the model’s ability to match the flows out of the $E$ and $N$ states. These alternative calibrations continue to have the same
issue regarding $U$ to $N$ transitions.

Changes in $\lambda_w$ also influence the reasons for transitions out of employment. As explained above, the greater the frictions the more individuals want to hold on to employment opportunities once they have them. As a result, separation shocks become more important in accounting for transitions out of employment. Table 10 shows this.

<table>
<thead>
<tr>
<th>$\lambda_w$</th>
<th>$\sigma$ shock</th>
<th>$\rho$ shock</th>
<th>other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.00</td>
<td>0.70</td>
<td>0.30</td>
</tr>
<tr>
<td>0.6</td>
<td>0.38</td>
<td>0.49</td>
<td>0.13</td>
</tr>
<tr>
<td>0.4</td>
<td>0.42</td>
<td>0.47</td>
<td>0.11</td>
</tr>
<tr>
<td>0.2</td>
<td>0.53</td>
<td>0.43</td>
<td>0.04</td>
</tr>
</tbody>
</table>

5 Sensitivity: The Idiosyncratic Shock Process

The previous sections presented results for one particular specification of the idiosyncratic shock process. An important question is to assess the extent to which the model’s implications for worker flows are affected by changes in the values for $\rho$ and $\sigma_\varepsilon$. Is it the case that the model accounts for the flows only if these values lie in very narrow intervals, or do the results look similar for a wide range of these values? This section addresses this question.

As a first step it is instructive to note the importance of the idiosyncratic shocks. To do this we consider a model that abstracts from the idiosyncratic shocks. In particular, we shut down the idiosyncratic productivity shocks ($\rho = 0$, $\sigma_\varepsilon = 0$) in our benchmark model and calibrate the model to the same targets. Table 11 shows the results. The parameters are $\lambda_w = 0.88$, $\sigma = 0.34$, and $\alpha = 0.97$. 
Table 11
Flows Without s Shocks

<table>
<thead>
<tr>
<th>FROM</th>
<th>TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>U</td>
</tr>
<tr>
<td>E</td>
<td>0.488</td>
</tr>
<tr>
<td>U</td>
<td>0.880</td>
</tr>
<tr>
<td>N</td>
<td>0.880</td>
</tr>
</tbody>
</table>

Recall from Table 1 that two of the key features of the actual flow data are the high persistence of the E and N states. Whereas the model with idiosyncratic productivity shocks matched both of these flows very well, this example misses somewhat with regard to the persistence in the E state, and by a huge margin with regard to persistence in the N state.\(^{14}\) Spell durations are now equal to 2.0, 1.1 and 1.0 months respectively for E, U, and N, versus 22.5, 2.0 and 14.3 in the data. The short duration of employment and non-participation spells is striking. The model without idiosyncratic productivity shocks produces far too little persistence at the individual level. Adding a persistent productivity shock adds a source of persistence at the individual level and serves to produce a substantial quantitative improvement in terms of the model’s ability to account for the salient features of the underlying flow data.

Having established that the presence of idiosyncratic shocks is important in matching the flows observed in the data, we next examine how the values of the parameters characterizing this shock process affect the quantitative properties of the flows. Rather than presenting flow tables for a wide range of specifications we have decided to simply report the extent of

\(^{14}\)Because the steady state employment rate is .632, the model must necessarily have a fair amount of persistence in the employment state, in the sense that almost half of those employed this period must also be employed next period.
persistence for each of the three states for different combinations of \( \rho \) and \( \sigma_\varepsilon \).\(^{15}\) For ease of interpretation we have again reported the annualized measures of these parameters. Table 12 displays the results when the persistence parameter \( \rho \) is varied. We note that for each value of \( \rho \) the remaining parameters of the model are recalibrated so as to hit the same targets as before. In particular, the two frictional parameters, \( \sigma \) and \( \lambda_w \), are recalibrated in each case, as is the disutility of work parameter \( \alpha \). This last parameter is particularly important, since changes in the stochastic process for \( s_t \) holding all parameters constant will typically have a large effect on steady state employment.

Table 12

<table>
<thead>
<tr>
<th>Effect of ( \rho ) on Flows, ( \sigma_\varepsilon = .21 )</th>
<th>Data</th>
<th>( \rho = .97 )</th>
<th>( \rho = .92 )</th>
<th>( \rho = .75 )</th>
<th>( \rho = .50 )</th>
<th>( \rho = .00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E \rightarrow E )</td>
<td>.960</td>
<td>.967</td>
<td>.947</td>
<td>.931</td>
<td>.912</td>
<td>.673</td>
</tr>
<tr>
<td>( N \rightarrow N )</td>
<td>.919</td>
<td>.971</td>
<td>.923</td>
<td>.882</td>
<td>.838</td>
<td>.319</td>
</tr>
<tr>
<td>( U \rightarrow U )</td>
<td>.517</td>
<td>.663</td>
<td>.527</td>
<td>.446</td>
<td>.378</td>
<td>.122</td>
</tr>
<tr>
<td>( (U \rightarrow E) \times U )</td>
<td>.014</td>
<td>.018</td>
<td>.023</td>
<td>.025</td>
<td>.027</td>
<td>.033</td>
</tr>
<tr>
<td>( (N \rightarrow E) \times N )</td>
<td>.011</td>
<td>.003</td>
<td>.011</td>
<td>.018</td>
<td>.028</td>
<td>.171</td>
</tr>
</tbody>
</table>

In terms of the persistence of the \( E \) and \( N \) states, we interpret the above results to imply that as long as the shock process is moderately persistent, say with \( \rho \) at least .50, the model does a reasonable job of capturing the persistence of these two states. A similar result holds for the persistence of the \( U \) state, except that as \( \rho \) gets near to unity the persistence in this state becomes quite large relative to the data.

The last two rows report the mass of workers that flow into \( E \) from each of the two other

\(^{15}\)There is a sense in which these statistics almost provide a complete picture of how well the model captures the data. In particular, we know that there are only six independent values to start with, and one of these (the \( E \) to \( U \) flow) is targeted in the calibration. And the \( U \) to \( N \) flow tends to be too low and relatively constant across specifications.
states in each period. They indicate that the flow of workers from $N$ to $E$ varies quite a lot as the persistence varies, with particularly poor matches to the data for low values of $\rho$. As $\rho$ decreases the overall flow of workers into $E$ increases, so the aspect of these final two rows that is of greatest interest is the ratio of the two flows. Our reading of this table is that for either very high or very low persistence the model has trouble matching the observed relative importance of these two flows.

Table 13 repeats this analysis by varying the value of $\sigma_\varepsilon$.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>$\sigma_\varepsilon = .42$</th>
<th>$\sigma_\varepsilon = .21$</th>
<th>$\sigma_\varepsilon = .158$</th>
<th>$\sigma_\varepsilon = .105$</th>
<th>$\sigma_\varepsilon = .021$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E \rightarrow E$</td>
<td>.960</td>
<td>.960</td>
<td>.947</td>
<td>.937</td>
<td>.921</td>
<td>.903</td>
</tr>
<tr>
<td>$N \rightarrow N$</td>
<td>.919</td>
<td>.951</td>
<td>.923</td>
<td>.902</td>
<td>.871</td>
<td>.839</td>
</tr>
<tr>
<td>$U \rightarrow U$</td>
<td>.517</td>
<td>.597</td>
<td>.527</td>
<td>.490</td>
<td>.444</td>
<td>.400</td>
</tr>
<tr>
<td>$(U \rightarrow E) \ast U$</td>
<td>.014</td>
<td>.020</td>
<td>.023</td>
<td>.025</td>
<td>.029</td>
<td>.032</td>
</tr>
<tr>
<td>$(N \rightarrow E) \ast N$</td>
<td>.011</td>
<td>.006</td>
<td>.011</td>
<td>.015</td>
<td>.021</td>
<td>.029</td>
</tr>
</tbody>
</table>

The table considers standard deviations that range from one-tenth to twice that used in the benchmark case. The basic finding of this table is that the basic properties are quite robust to changes in the standard deviation of the shocks. Perhaps somewhat surprisingly, the greater the standard deviation of the shocks, the greater is the persistence in the individual states, though in the case of employment this effect is somewhat small. We again remind the reader that as we change $\sigma_\varepsilon$ we are recalibrating the other parameters (in particular $\alpha$, $\lambda_w$ and $\sigma$) so as to match the same targets as before.
6 Additional Specifications

In this section we consider two alternative specifications. The first one is the same as our benchmark model except that it allows for complete markets. The second one allows for the possibility of a purely transitory shock in addition to the persistent shock.

6.1 Flows in the Complete Markets Model

In this section we consider the flows in a complete markets version of our model. In particular, we solve for the steady state of the solution to the Social Planner’s problem when all individuals are weighted equally. In the complete markets model, we note that the employment decision rule takes a very simple form: work if the individual productivity exceeds a reservation value $s^*$ and the individual has an employment opportunity. We calibrate the model to the same targets as previously. Table 14 shows the results for the same stochastic process on $s$ as in our benchmark model. To facilitate comparison we include the results for the incomplete markets specification as well.

<table>
<thead>
<tr>
<th>Table 14</th>
<th>Flows with Complete and Incomplete Markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Markets</td>
<td>Incomplete Markets</td>
</tr>
<tr>
<td>FROM</td>
<td>TO</td>
</tr>
<tr>
<td>$E$</td>
<td>.960</td>
</tr>
<tr>
<td>$U$</td>
<td>.337</td>
</tr>
<tr>
<td>$N$</td>
<td>.021</td>
</tr>
</tbody>
</table>

The two different market structures lead to quite similar flows, though there are some differences worth noting. First, note that the persistence of each of the three states is somewhat higher in the complete markets setting. This is intuitive. In the incomplete
markets model, there are two reasons for a voluntary transition out of $E$: either the individual receives a sufficiently low value for the idiosyncratic shock or accumulates sufficient assets. Moreover, as the individual accumulates assets, the threshold value for the idiosyncratic shock increases. In the complete markets model this asset accumulation factor is not present. It is intuitive that this increases persistence. However, this effect is quantitatively small. A similar situation is present for the case of transitions out of $N$. In the incomplete markets model, an individual in $N$ will run down his or her assets, but if the assets are sufficiently depleted, the individual will accept an employment opportunity even if they have a low value for the idiosyncratic shock. This leads to somewhat lower persistence relative to the complete markets case where the only factor that can induce movement out of $N$ is an increase in the value of the idiosyncratic productivity. But once again, the magnitude of this effect is relatively small. We note that these increases in persistence could also be obtained in the incomplete markets model by slightly increasing the persistence in the shock process.

Another discrepancy of interest is the flow from $U$ to $N$. While this flow is much smaller in both specifications than it is in the data, it is of interest to note that the flow is higher in the complete markets model. Once again, this is intuitive. In the complete markets model, an individual will move from $U$ to $N$ whenever they receive a draw of $s$ that is below the threshold $s^*$. In the incomplete markets model there is an additional force associated with capital accumulation that works to prevent flows from $U$ to $N$. The reason for this is that in the incomplete markets model, an unemployed individual is running down his or her assets,
and when assets are declining the threshold value for the idiosyncratic shock that determines the desirability of employment also decreases, making it less likely that the individual receives a sufficiently bad shock to cause them to transition to \( N \). In terms of log changes, this effect is significant, increasing the \( U \) to \( N \) flow rate by almost a third.

In summary, while there are some differences between the flows generated in the complete and incomplete markets models, for the most part the two models produce very similar outcomes. We therefore conclude that market structure is not a dominant factor in building a model of labor market flows.

6.2 Matching the \( U \) to \( N \) Flow

As noted earlier, one of the main discrepancies between the flows in our benchmark model and the data is that the model generates far fewer transitions from \( U \) to \( N \). In this section we quantitatively evaluate one extension that serves to reduce the extent of this discrepancy. Before proceeding, we remind the reader that previous work has suggested that roughly half of the \( U \) to \( N \) flow is likely to represent survey error. This suggests that we should not try to match the entire flow. Additionally, we saw in Section 2 that the \( U \) to \( N \) flow is much larger for women than it is for men. To the extent that women are more likely to be secondary earners within households and our model is a single agent household model, this is another reason for not wanting to match the entire flow.

We noted earlier that the flows between \( U \) and \( N \) seemed to be very transitory in nature. Intuitively, transitory shocks will cause individuals who are close to indifference between
working and not working to move back and forth across the boundary. The type of shock that we have in mind are things such as temporary illness or some household situation that requires immediate attention.\textsuperscript{16} Motivated by this, we carry out the following exercise. We extend the benchmark model to allow for a purely transitory shock in addition to the persistent shock. In particular, an individual’s productivity is now given by \( z_s \), where \( s \) evolves as before, and \( z \) is assumed to take on two values: \( \exp(\bar{z}) \) and \( \exp(-\bar{z}) \), each with probability \( .5 \), where \( \bar{z} > 0 \). We calibrate the model to match the same targets as before, but also pin down the value for \( \bar{z} \) by requiring that the model generate a flow from \( U \) to \( N \) equal to \( .118 \), which is one half of the value found in the data. In view of the earlier discussion, we think that this is a reasonable value to target.\textsuperscript{17}

Table 15 shows the flows that result from this exercise. In the interests of space we only report results for the incomplete markets specification.

<table>
<thead>
<tr>
<th>Temporary Shocks</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>FROM</td>
<td>TO</td>
</tr>
<tr>
<td>( E )</td>
<td>.941</td>
</tr>
<tr>
<td>( U )</td>
<td>.406</td>
</tr>
<tr>
<td>( N )</td>
<td>.046</td>
</tr>
</tbody>
</table>

The main message from this exercise is that one can almost double the transition rate from \( U \) to \( N \) by adding transitory shocks without having much impact on the other flows.

\textsuperscript{16}In reality, for many temporary shocks employed individuals can potentially use vacation or sick days and so do not need to contemplate severing the employment match. In this regard our exercise probably overstates the effect of these shocks on the persistence of the employment state.

\textsuperscript{17}The calibrated value of \( \bar{z} \) is \( .0391 \). For the remaining parameters the only changes from the benchmark model are \( \alpha = .54 \), \( \sigma = .0405 \), and \( \lambda_w = .46 \).
As one would expect, adding transitory shocks does lead to a small reduction in persistence in each of the states, though this effect is not very large.

While we do not explore them here, we do note two other extensions that might also help to increase the flow from $U$ to $N$. Specifically, as in Ljungqvist and Sargent (1998) we could assume that productivity experiences a downward drift while not working. As noted earlier in our discussion of the complete and incomplete markets specifications, the incomplete markets model has a force that tends to work against flows from $U$ to $N$. Adding a downward productivity drift for non-employed workers would counteract this force and potentially increase this flow. Second, in a model with multiple workers in a household, the desirability of work for one individual in a household is influenced by the productivity of the other household members. It seems reasonable to conjecture that this would generate more transitions between $U$ and $N$. But we leave a quantitative assessment of these and other extensions to future work.

7 Conclusion

We have built a model that features search frictions and a nondegenerate labor supply decision along the extensive margin. We argue that the steady state equilibrium of our model does a reasonable job of matching labor market flows between the three labor force states of employment, unemployment and out of the labor force as long as idiosyncratic shocks are reasonably persistent. Persistent idiosyncratic shocks play a key role in allowing the model to match the persistence of the employment and out of the labor force states found in individual
labor market histories. Available evidence suggests that the two prime sources of these shocks—wage shocks and health status shocks—are both very persistent. It seems reasonable to posit that family shocks associated with family size and caring for elderly family members are also likely to be persistent. Whereas for some issues it may not be important to identify the exact shocks that individuals face, for some issues this may be important. Understanding how the model behaves in the presence of multiple shocks would also be of interest. Additionally, it is of interest to consider additional extensions to our model that will address some of the discrepancies between the flows in the data and those in the model. Examples might include allowing for human capital accumulation and depreciation.

The fact that the model does a good job of matching worker flows as long as the process is somewhat persistent leads us to believe that this very simple model serves as a reasonable environment for addressing several issues. In Krusell et al (2009), we use this model to evaluate the relative importance of frictions versus labor supply in the determination of aggregate employment. This issue gets at a fundamental distinction between commonly used versions of frictional and frictionless models, which each assume that only one of these factors is important in determining aggregate employment.

A second issue of interest is to carry out a more comprehensive analysis of tax and transfer programs in order to address the debate between Prescott (2004) and Ljungqvist and Sargent (2006, 2008) concerning the role of benefits. Specifically, Ljungqvist and Sargent argue that once one takes into account the generosity of benefit systems in many European countries,
Prescott’s model implies implausibly large responses. But Ljungqvist and Sargent do not distinguish between the unemployed and those out of the labor force, so implicitly assume that all nonemployed individuals can receive unemployment benefits. In reality there is a sharp distinction between the benefits that one has access to and prior labor market history. Understanding the role of these provisions requires a model that can match the heterogeneity in labor market histories that are found in the data. Our model is one such model.\footnote{Mukoyama (2010) takes a step in this direction by looking at some simple UI systems in models that feature labor supply and frictions.}

Third, it would be of interest to examine business cycle fluctuations in our model. Much of the recent literature that emphasizes the role of frictions in understanding aggregate fluctuations implicitly assumes that the labor supply channel is not operative. Standard frictionless models typically get relatively large responses to aggregate shocks because of the labor supply channel. We believe that our framework is the appropriate setting in which to assess the relative importance of frictions and labor supply in accounting for aggregate fluctuations in employment. Related, an important issue for models of aggregate labor market fluctuations is to not only account for aggregate fluctuations in employment or unemployment, but also for the patterns found in the data on the cyclical behavior of labor market flows.

References


Appendix

A.1 Data

The Current Population Survey (CPS) reports the labor market status of the respondents each month that allows the BLS to compute important labor market statistics like the unemployment rate. In particular, in any given month a civilian can be in one of three labor force states: employed \((E)\), unemployed \((U)\), and not in the labor force \((N)\). The BLS definitions for the three labor market states are as follows:

- An individual is counted as employed if he or she did any work at all for pay or profit during the survey month. This includes part-time or temporary work as well as full-time year-round employment.

- An individual is considered unemployed if he or she does not have a job, has actively looked for employment in the past 4 weeks and is currently available to work.

- An individual is classified as not in the labor force if he or she is included in the labor force population universe (older than 16 years old, non-military, noninstitutionalized) but are neither employed nor unemployed.

Households in the CPS are interviewed for several consecutive months. In a given month, approximately 75 percent of the households were interviewed the previous month. This allows the BLS to calculate month by month movements that dictate the changes in employment, unemployment, and not in the labor force. The flows data calculated with the above defini-
tions of the labor market states have recently been made publicly available by the BLS. (See http://www.bls.gov/cps/cpsflows.htm)

However, since our definition of unemployment in our model is not the same as that of the BLS, we cannot use the publicly available labor flows data or any of the other estimates calculated previously (e.g. Shimer (2007), Fujita and Ramey (2009)). We go back to the CPS micro data and redefine the three labor market states consistent with our definition of unemployment and calculate the flow rates. Our definition of $E$ is the same as the BLS while our definition of $U$ is broader. The BLS asks people if they would like to work independently of whether they engaged in active search in the previous four weeks. As a result there are people who are not in the labor force but who actually report that they want a job. We reclassify these people who are not in the labor force according to the BLS but report that they want a job as unemployed. Consequently, the stock of unemployed increases and our alternative unemployment rate becomes considerably higher than the official unemployment rate. For the period 1994-2007 the standard unemployment rate for the US averages 5.1%, whereas our expanded notion of unemployment averages 8.3%.

A.2 Calculation of the Flows

Calculating the flows data has various problems that have been previously reported. (See Frazis et al (2005), Fujita and Ramey (2009) and Shimer (2007)). The first problem is not all the respondents stay in the sample for consecutive months; 75 percent are reinterviewed according to the CPS sampling design. Moreover, many other respondents cannot be found.
in the consecutive month due to various reasons and are reported as missing. The failure to match individuals in consecutive months is known as margin error and it causes biased estimates of the flow rates as discussed by Abowd and Zellner (1985) and Fujita and Ramey (2009).

We apply the procedure that Fujita and Ramey (2009) used to calculate the flow rates for the BLS definition of labor market states. To summarize

- We create the raw flows data using the basic monthly CPS data from Unicon’s CPS Utilities starting (Jan. 1994- Dec. 2007).\(^\text{19}\)

- We retrieve the data by using part of Stata programs written by Robert Shimer.\(^\text{20}\) The Stata program matches individuals by household ID, age, sex and race.

- We use the weights provided by the BLS and aggregate the data by the year, month and specific flow. We create the stock data by aggregating the monthly data by labor force status.

- The flows data and the stock data are turned into flow ratios and stock ratios respectively, according to each monthly total.

- To fill in for the missing observation we use the TRAMO (Time Series Regression with

\(^{19}\)BLS revised the monthly survey in the summer of 1995 and made it impossible to match household IDs from June 1995 - August 1995. Thus, we have have 5 missing observations (3 observations unobservable due to the revision, and 2 observation since we need at least 2 consecutive months worth of data to create a flows series).

\(^{20}\)For additional details, please see Shimer (2007) and his webpage http://robert.shimer.googlepages.com/flows.
ARIMA Noise, Missing Observations and Outliers) program like Fujita and Ramey. TRAMO is a program developed by Gomez and Maravall. It fills in the missing data by interpolating the values and also detects additive and transitory outliers and smooths them out.

- After the missing values are added, we run a nonlinear seemingly unrelated regression on the data series.

- We then compute the annual averages of the margin adjusted data and finally the flow probabilities are calculated for each year.