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Informational Easing: Improving Credit Conditions through the Release of Information

1. Introduction

To ensure repayment of borrowed funds, lenders require that borrowers undergo costly credit evaluations. In the financial sector, credit often flows along chains of borrowers and lenders who are already familiar with each other's creditworthiness—a process that minimizes the cost of credit evaluations. However, if the creditworthiness of key participants along a chain is called into question, the chain can break and cut off the flow of credit to final borrowers. If enough chains in the economy break, a financial crisis can ensue, investment by final borrowers can dry up, and output can decline.

The flow of credit can stop because a lender believes a borrower has a high default probability or because a lender is uncertain about whether a borrower has a high default probability. The latter may often be the more likely scenario. For example, in a classic bank run, it is unlikely that depositors know the probability that their bank will become insolvent, but it is likely that they worry about the possibility that their bank has high default probability and withdraw their deposits as a precaution.[1]

More generally, a decision maker faces risk if the outcomes in his decision problem are random; he faces uncertainty if the outcomes are random and he does not know the probabilities of the outcomes.[2] For example, when lenders are uncertain, they cannot assign a single figure to a borrower's default probability, so they instead assign a range. During economic expansions, this range may be small, such as 1/4 to 1/2 percent; however, during economic downturns, the range may be 2 to 5 percent. If a lender is uncertainty-averse in the sense of Gilboa and Schmeidler (1989), it will charge spreads based on the high end of its range. This decision will be unimportant during expansions, when the range is narrow, but during downturns the required spread may be so high that a borrower cannot afford a loan—and the flow of credit from that borrower to any borrowers farther along the lending chain will be cut off.[3]

This paper addresses how central banks can resuscitate lending chains by providing information that reduces

[1] Easley and O'Hara (2009) argue that deposit insurance was instituted to eliminate bank runs motivated by uncertainty among small depositors because it allays the worries of small depositors that their bank will become insolvent. In a similar vein, Caballero and Krishnamurthy (2008) model an excessive flight to quality and flight to liquid assets that can occur when there is uncertainty over the timing of liquidity shocks—and argue for government intervention aimed at reversing the flight.

[2] For examples of different methods of modeling decision making under uncertainty in nondynamic settings, see the discussion and approach in Rigotti and Shannon (2005) as well as the approaches in Klibanoff, Marinacci, and Mukerji (2005) and Easley and O'Hara (2009). For an overview of uncertainty in dynamic settings, see Hansen and Sargent (2007) and their references.

[3] In this paper, the terms "lending chain" and "credit chain" are used interchangeably.

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uncertainty about participants along the chains. This action has been taken before: the Bank Holiday of 1933, declared by President Franklin Delano Roosevelt, resolved uncertainty about the health of individual banks by using bank inspections to publicly identify which banks were sound. This event restored the flow of funds to the banking sector and facilitated bank lending. During the 2007-09 financial crisis, the Federal Reserve used "stress tests" to measure and report on the health of large banks in the U.S. banking system and to identify those banks that required shoring up through capital injections.

In addition to providing information to the financial sector, central banks have other tools at their disposal to revive lending. When credit chains involve financial intermediaries such as banks, central banks can lower their target rates to reduce intermediaries' costs of borrowing, accept a wider range of collateral, guarantee interbank loans, or shore up banks' health through capital injections. Alternatively, they can bypass intermediaries altogether and lend directly to final borrowers in credit chains.

Each of these tools has merit in some situations—but none is perfect. Monetary easing may lower target rates to 0, but if credit spreads remain too high, lending along credit chains may still cease. Broadening the range of acceptable collateral, loan guarantees, and government-sponsored capital injections increases lending, but it can also increase the central bank's exposure to credit and market risk. Direct lending outside the financial sector may reduce lending efficiency, because such intermediation is not a central bank's usual function.

Under conditions of less uncertainty, many of these efforts would be less costly and more effective. This statement is intuitive, as it is easier to convince potential lenders that a solvency problem has been fixed if they have better information about the scope of the problem. It follows that during a crisis, steps to reduce uncertainty through information provision should be taken as soon as possible.

In theory, information designed to reduce uncertainty could be provided privately by borrowers. However, because borrowers may have an incentive to exaggerate their financial strength during economic downturns, private information provision may lack credibility. Moreover, uncertainty reduction by borrowers upstream in a credit chain may generate external benefits to borrowers downstream that are not internalized by private information providers. As a result, the private sector may provide less than the socially optimal level of uncertainty reduction. For both these reasons, situations may arise in which government information provision to reduce uncertainty may be warranted.

The remainder of this paper is divided into two sections. In Section 2, we provide a model of credit chain lending that illustrates how uncertainty can cause credit chains to break and how government policies that reduce uncertainty can restore the flow of credit. Section 3 considers potential future uses of uncertainty reduction policies.

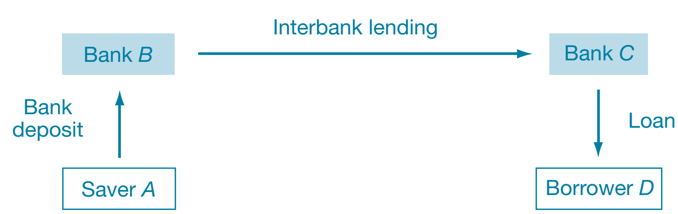
2. The Model

Our stylized model of a credit chain has four participants: *A*, *B*, *C*, and *D*, and three dates: 0, 1, and 2. Participant *A* is a short-term depositor who has excess funds at date 1 that he wants to lend until date 2. Participants *B* and *C* are banks that make long-term loans at date 0 and short-term loans at date 1. Both loan types mature at date 2. Participant *D* is a short-term borrower who unexpectedly needs a loan at date 1 that matures at date 2.

We assume that some participants are familiar with each other's credit risk based on a previous bilateral lending relationship, while others are not. In particular, *A* has previously loaned funds to *B*, *B* to *C*, and *C* to *D*. These relationships suggest a natural basis for a credit chain to form at date 1. *D* could borrow from *A*, *B*, or *C*. Since *A* and *B* are unfamiliar with *D*, a costly credit evaluation would be needed before either would extend a loan to *D*. Instead, *C* is the logical lender to *D*; but if *C* does not have the funds, then *C* will need to turn to *A* or *B* for funding. Because of previous relationships, *B* is the logical lender to *C*, and if *B* needs funds then *A* is the logical source. Thus, a short-term loan from saver *A* is intermediated to borrower *D* along a credit chain in which bank *B* makes a loan to bank *C* through the interbank market (Exhibit 1).

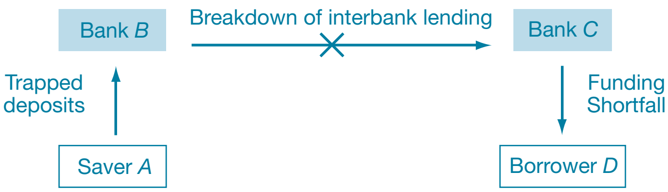
Because many loans are intermediated through the interbank market, the functioning of the market is important for credit extension. *C* can lend to *D* only if the maximum rate that *D* can afford to pay *C* for a loan, denoted $\overline{R_D}$, is less than *C*'s cost of funds. When *C* borrows from *B*, its cost of funds is equal to the risk-free rate R_f plus a spread S_C that reflects its credit risk. Therefore, *D* will be able to borrow from *C* only if:

Exhibit 1
Short-Term Lending Chain



Note: Borrower *D* needs a short-term loan, and saver *A* has excess short-term funds. Because *A* and *B*, *B* and *C*, and *C* and *D* have had previous borrowing relationships, a lending chain from *A* to *B* to *C* to *D* is the least expensive way to fund *D*'s loan since there is no need for costly credit evaluations.

Exhibit 2
Broken Lending Chain



Note: If there is a breakdown of lending from B to C in the interbank market, the funds are trapped with bank B, and borrower D is short of funds.

$$R_f + S_C < \overline{R_D}\,. \tag{1}$$

Under normal economic conditions, the spreads that banks charge each other for loans are small and would not typically be an impediment to D's borrowing. However, during the financial crisis of 2007-09, interbank spreads increased markedly, and lending through the interbank market declined. A consequence of high interbank spreads is that funds can become trapped at the wrong place, such as with bank B instead of borrower D (Exhibit 2). Whether interbank spreads increase at date 1 depends on B's assessment of C's default risk as of date 1. This in turn depends on C's long-run asset portfolio and capital structure, both chosen at date 0.

At date 0, banks B and C both choose their long-run asset portfolios and capital structures. Since the main concern is B's willingness to lend to C, we focus only on C's portfolio choices hereafter. For simplicity, C's long-run asset portfolio consists only of loans to wheat farmers (w) and oat farmers (o). The loans generate gross returns R_w and R_o at date 2 per dollar invested at date 0. The return on the loans is assumed to be multivariate normal.[4] Bank C's portfolio weights are ω_w and ω_o and its portfolio generates return R_p:[5]

$$R_p = \omega_w R_w + \omega_o R_o\,. \tag{2}$$

To finance its long-run portfolio, C is endowed with equity capital E and insured certificates of deposit with face value F that mature at date 2 and pay gross interest $R^C_{0,2}$ at maturity.

At date 1, information I_1 about the return on the long-term loans arrives. Conditional on this information, the returns on the loan portfolio are distributed normally with mean μ_1 and variance σ_1^2:

$$R_p | I_1 \sim N(\mu_1, \sigma_1^2)\,, \tag{3}$$

where the parameters μ_1 and σ_1^2 depend on the portfolio weights as well as the means, standard deviations, and correlation of the assets' returns, given the information available at date 1 (see Appendix A).

Additionally, recall that at date 1 bank C has the opportunity to extend a short-term loan to D that matures at date 2, which it needs to fund in the interbank market by borrowing from B.[6]

The spread that bank C pays on its interbank loans depends on bank B's perception of the probability that C will default on its debt at date 2. We assume that bank C's long-term loan portfolio is so much larger than its short-run lending opportunities that the performance of its short-run loans and their funding does not affect whether C will default. Under this assumption, C will default only if the value of its long-term loan portfolio at date 2 is less than what is owed on its deposits:

$$(F + E)R_p < FR^C_{0,2}\,. \tag{4}$$

From this expression, we show that bank C's probability of default—and therefore the loan spread that B charges C—depends on C's portfolio weights, financial leverage L ($L = F/E$), and the parameters of the return distribution of C's loan portfolio.

We assume that the risk inherent in both types of loans is known by bank B, as is C's leverage, since leverage information is usually readily available. However, B does not know C's portfolio weights. There are two cases to consider: The first is that B has beliefs about C's portfolio that are sufficiently well formed as to be described by a unique prior probability distribution, which means that for each portfolio that C could hold, B assigns a single probability number to the likelihood that C could hold that portfolio. In this first case, B's assessment of C's probability of default is just a single number given by the sum of C's default probability for each portfolio it could hold multiplied by B's belief about the probability that C will hold that portfolio.[7] Because B's assessment of C's default probability is a single figure, B is not uncertain about C's default probability.

The second case is that B does not know enough about C's portfolio weights, and B's beliefs cannot be described by a unique prior probability distribution. Instead, B may be uncertain about the portfolio weights and thus may hold

[4] Pritsker (2009) illustrates conditions under which the average return on loans to a diversified group of borrowers can be approximately normally distributed even if the returns to individual borrowers are not.

[5] The portfolio weights are each assumed to be greater than or equal to 0 and to sum to 1.

[6] Bank C may fund some of its short-term loans in the interbank market because it did not fully anticipate the short-term loan demand or because the interbank market is usually an inexpensive funding source.

[7] For example, suppose B believes C holds only one of two portfolios, 1 or 2, and the probability that C holds 1 or 2 is 0.3 and 0.7, respectively. Also suppose the probability that portfolio 1 defaults is .01 and the probability that portfolio 2 defaults is .02. Then, B believes the probability that C defaults is given by $PD = 0.3 \times 0.01 + 0.7 \times 0.02$.

multiple priors over the weights. Thus, *B* assigns a range of probabilities to some or all of the portfolio holdings that *C* may have. For example, if bank *B* is asked about the probability that *C* holds a portfolio with a weight of 0.4 in loans to oat farmers and 0.6 in loans to wheat farmers, *B* might respond that it is unsure, but it believes the probability ranges from 10 to 20 percent.[8]

There are many reasons why *B* might be uncertain about *C*'s portfolio composition. For example, *C* may have a very complex portfolio, and thus researching *C*'s holdings in extensive detail may be very expensive. This may be true for *C*'s portfolio because it consists of loans to farmers, and it may be very difficult for *B* to verify which loans are to oat or wheat farmers because this information may not be readily available, and it may be costly to obtain.[9] Information costs are important because many of the most active banks in the U.S. interbank market have more than $1 trillion of assets on their balance sheets, and ascertaining the loan composition, or even learning enough to form a unique prior probability distribution about the balance-sheet composition, can be very expensive.

A simple and parsimonious way to model multiple priors is to assume that bank *B* knows *C* makes only long-term loans to oat and wheat farmers, and that *B* knows *C* has risk concentration limits that prevent it from making more than 60 percent of its loans to one type of farmer—and that is all *B* knows about *C*'s portfolio. Given its information, bank *B* knows that *C* could have a set of possible portfolios, and that the weight on wheat is a number *t* between 0.4 and 0.6 and that the weight on oats is $1 - t$. Given bank *B*'s information, it does not know the probability that *C* will hold any particular portfolio, but it does know the probability that *C* will default on each portfolio that it could hold. From this information, bank *B* can compute a range of possible default probabilities for bank *C*. The range can be written as

$$PD \in [\underline{PD}, \overline{PD}] \ ,$$

meaning that based on bank *B*'s information about bank *C*, bank *B* believes *C*'s default probability lies within a range between a lower bound $\underline{PD}$ and an upper bound $\overline{PD}$.

The fact that *B* assigns a range of possible default probabilities to *C* is precisely the type of situation described in the introduction to this paper. The above logic, formally derived in Appendix A, shows that the result of *B*'s uncertainty about *C*'s portfolio weights is that *B* assigns a range of possible values to *C*'s probability of default. The spread that *B* charges *C* will depend on the range of uncertainty that *B* has about *C* and on *B*'s preferences. In particular, if bank *B* sets spreads in an uncertainty-averse fashion, as in Gilboa and Schmeidler (1989), then *B* will set *C*'s spread as if it believes *C*'s default probability is equal to $\overline{PD}$, the upper end of its range. Other decision rules for setting spreads in the face of uncertainty are plausible. It seems reasonable to believe that for many rules, all else equal, *B* would charge a higher spread when the upper end of the range of possible default probabilities increases.

For illustrative purposes, we assume that in the face of uncertainty, there are many banks like *B* that set spreads in an uncertainty-averse fashion. As a consequence, banks like *C* will pay a premium for uncertainty. More specifically, let PD^* denote *C*'s true default probability, and for simplicity assume that bank *B* is risk-neutral and uncertainty-averse. In this circumstance, if at date 1 bank *B* can invest at the risk-free rate between dates 1 and 2, or bank *B* can lend to bank *C* at interbank rate $R_{1,2}^C$, then for *B* to be indifferent between the two, $R_f = R_{1,2}^C(1 - \overline{PD})$, which implies $S_C = R_{1,2}^C - R_f$ is given by:

$$S_C = R_f \frac{\overline{PD}}{1 - \overline{PD}} .$$

Suppose *C*'s true *PD* at time 1 based on all information is PD^*. Then if PD^* was known by *B*, *C*'s spread based on risk alone but not uncertainty would be

$$S_C^* = R_f \frac{PD^*}{1 - PD^*} .$$

Because of uncertainty and uncertainty aversion, bank *C*'s spread will consist of the risk premium S_C^* plus an additional uncertainty premium given by:

$$S_C - S_C^* = R_f \left[\frac{\overline{PD}}{1 - \overline{PD}} - \frac{PD^*}{1 - PD^*} \right] .$$

If *B* sets its spread based on its worst-case-scenario beliefs about *C*'s default probability, then the uncertainty premium will always be positive. The size of the uncertainty premium paid by bank *C* depends on *C*'s capital structure as well as the conditional expected return and volatility of its loan portfolio. To analyze the uncertainty premium, we compute the premium when *C*'s loan portfolio is split evenly between oats and wheat. Our analysis shows that the uncertainty premium can be very low when leverage is low, but it can also be low when leverage is high, provided that economic conditions are favorable enough. In particular, all else equal, for reasonable parameter values, uncertainty premia are lower when the volatility of the returns on both types of loans is low, when the expected returns on both types of loans is high, or in both circumstances (Charts 1 and 2).[10] This explains how banks can

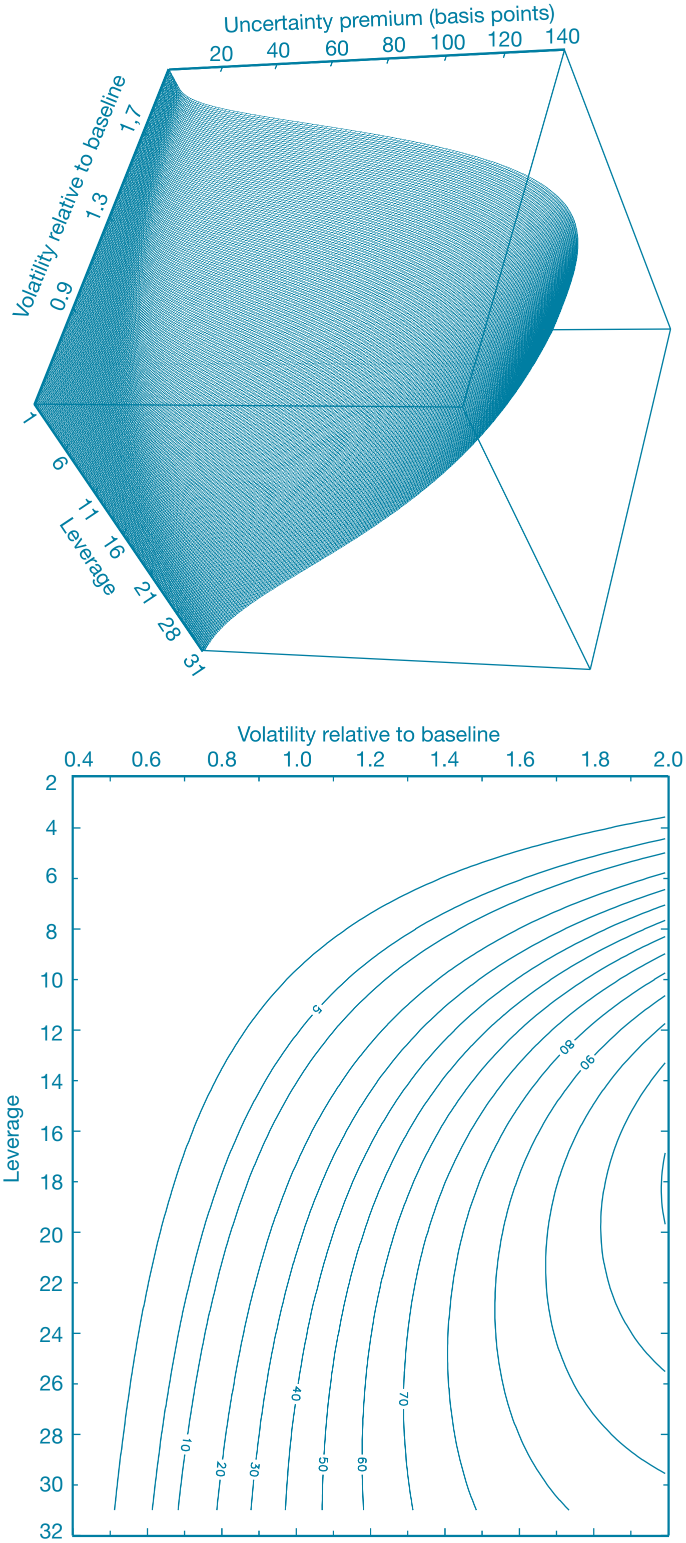
[8] Knowledge of bank *B*'s portfolio weight in one of the risky assets is sufficient to describe its portfolio because its weight in the other risky asset is 1 minus the weight of the first asset.

[9] Gorton (2008, 2009) argues that uncertainty about the types of assets collateralizing asset-backed securities was an important factor behind the 2007-09 credit crisis.

[10] The simulations are for illustrative purposes. Details are available from the author upon request.

Chart 1

Uncertainty Premium as a Function of Leverage and Loan Volatility

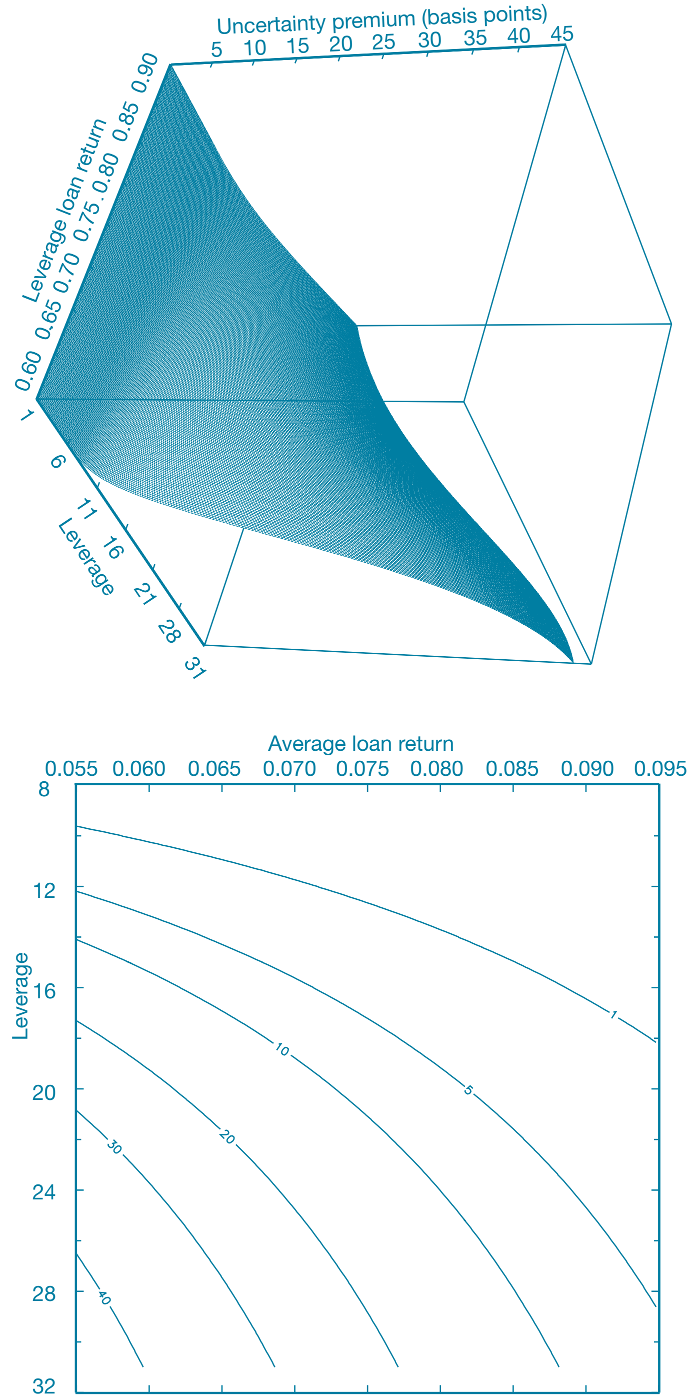


Source: Author's calculations.

Note: For the stylized risky loan portfolio held by bank *C*, the chart presents surface and contour plots of the uncertainty premium that bank *C* pays for its short-term unsecured interbank borrowing as a function of *C*'s leverage and as a function of the volatility (standard deviation) of *C*'s assets relative to their baseline volatility.

Chart 2

Uncertainty Premium as a Function of Leverage and Average Loan Return

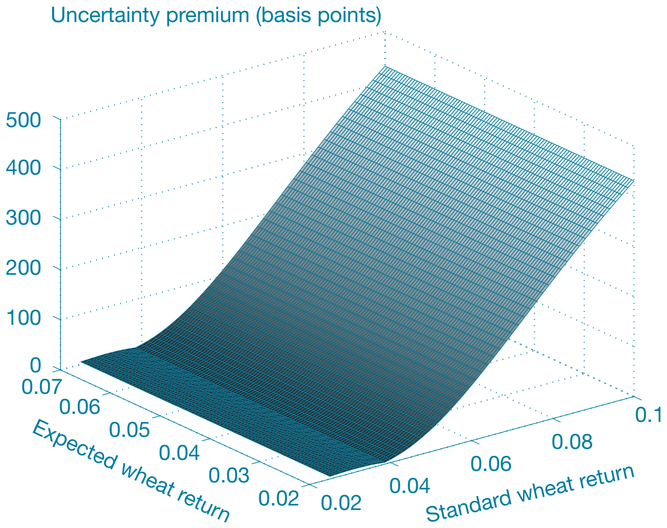


Source: Author's calculations.

Note: For the stylized risky loan portfolio held by bank *C*, the chart presents surface and contour plots of the uncertainty premium that bank *C* pays for its short-term unsecured interbank borrowing as a function of *C*'s leverage and as a function of the average return on its loans when the average return on each loan in its portfolio is increased or decreased by the same amount.

Chart 3

Uncertainty Premium as a Function of Sector Performance



Source: Author's calculations.

Note: For the stylized loan portfolio held by bank *C*, the chart presents *C*'s uncertainty premium as a function of the performance of loans to wheat farmers, one of two types of long-term loans extended by *C*. The chart shows that the uncertainty premium grows when loans to wheat farmers become more risky, and when the expected return on loans to wheat farmers decreases.

often be uncertain about each other's portfolio composition, and yet because of their choice of capital structure they can usually lend and borrow from each other while charging low spreads. The analysis also shows that banks may be able to take on very significant leverage during very prosperous times, and still pay only a small uncertainty premium. In fact, this roughly describes the situation prior to the global financial crisis of 2007-09, because before that time volatility was considered very low by historical standards, the spread paid by banks was low, and yet bank leverage was fairly high (Chart 1, bottom panel).

During the crisis, the bursting of the housing bubble heralded the arrival of bad news about the housing sector. Interbank spreads increased appreciably because of uncertainty over which banks were exposed to housing—and especially uncertainty over which banks were exposed to subprime loans. To understand the same effects for bank *C*, suppose the bad news is a wheat blight that increases the likelihood that wheat farmers default on their loans, and thereby increases the volatility and decreases the expected returns on loans to wheat farmers. For given leverage, these changes can have a dramatic effect on the uncertainty premium paid by bank *C*. As illustrated in Chart 3, the bank's uncertainty premium ranges from near 0 when volatility is low and expected returns are high to several hundred basis points when expected returns are low and volatility is high. The result of the elevated premium is high interbank spreads that cause borrowers such as *D* to lose access to their funding.

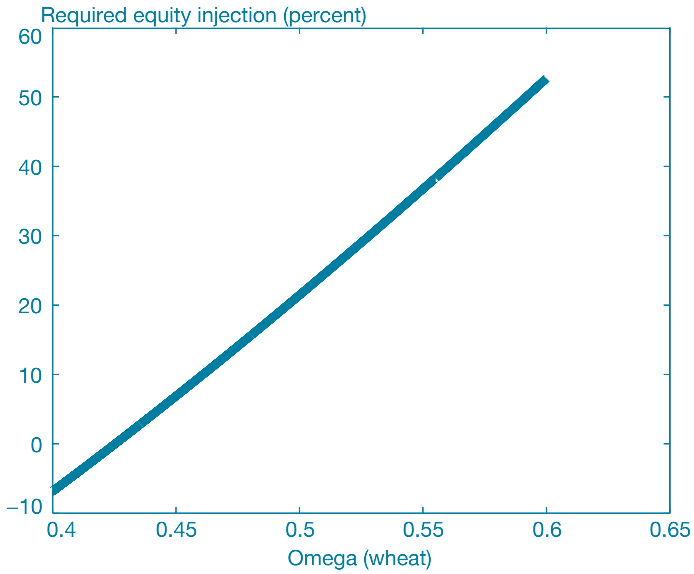
A government-sponsored stress test would reveal information on bank *C*'s solvency, through a publicly released assessment of *C*'s financial health, the release of summary information on *C*'s risk exposures, or a combination of the two. There is a strong case for doing both. For example, recall that government action may be needed to reduce uncertainty when the private costs of providing information to reduce uncertainty are too high. There are two sources of costs: The first is the cost of compiling and disclosing the information on risk exposures at the finer level of detail that is required during economic downturns. This is a nontrivial cost for very large banks. The second is the cost of processing the information on risk exposures to make inferences about the bank's solvency risk. If the second cost is high enough, then some potential lenders to *C* will not be able to process the information on exposures, and thus would be unwilling to lend to *C*. For this reason, the government may have to intervene to provide processed information on the bank's health, which it did as part of the recent Supervisory Capital Assessment Program stress testing in the United States. In that case, the information provided was the amount of capital injection required by banks to ensure capital adequacy during a particular stress scenario that was common across banks. The case for releasing better information on exposures is that the information provides more detail on bank portfolios that further reduces the uncertainty premia charged by lenders that can process the exposure information.

Under ideal circumstances, *C*'s true condition would be revealed by the stress tests, all uncertainty about its risk exposures would be eliminated, and its uncertainty spread would decrease to 0. More realistically, stress tests will reduce, but not eliminate, uncertainty spreads because although they may eliminate uncertainty over risk exposures, other sources of uncertainty may remain (such as uncertainty over the correct form of pricing models for some assets).

If the information revealed by the stress test about *C* is favorable enough, then *C* will be able to borrow from *B* to lend to *D* and the chain of credit will be restored. If instead it is learned that *C*'s balance sheet is weak, or its loans are not performing, then additional steps, such as bank equity injections or temporary government-sponsored guarantees on interbank lending, may be warranted.

Equity injections and government-sponsored loan guarantees can both be implemented without a stress test. The value-added benefit of the stress test is its ability to make these other steps more cost-effective if lenders are uncertainty-averse.

Chart 4
Size of Equity Injection Required to Restore Lending



Source: Author's calculations.

Note: For the set of different long-term loan portfolios that bank *C* could possibly hold, indexed by omega—the fraction of long-term loans extended to wheat farmers—the chart presents the percentage increase in *C*'s equity (the required equity injection) that would be needed to restore C's ability to acquire a short-term loan from bank *B* to finance a loan to borrower *D*. If bank *B* is uncertainty-averse, and does not know *C*'s portfolio, it will require a conservative equity injection of 50 percent before lending. If *B* becomes familiar with *C*'s portfolio, the required equity injection will be smaller, and could be negative.

Consider first an equity injection into bank *C*. If bank *C* is to restart lending to borrowers such as *D*, a sufficient amount of equity must be injected to bring bank *C*'s spread down to the level

$$S_C = \bar{R}_D - R_f.$$

If the equity injection occurs before the stress test, then *B* remains uncertain about *C*'s portfolio, and consequently a large amount of equity will be required to bring *C*'s loan spread down. This scenario is depicted in Chart 4, with details provided in Appendix B. In the chart, *C* needs to inject enough equity to bring its perceived probability of default down to 2 percent. If *B* is uncertainty-averse, it will charge spreads based on the most pessimistic beliefs about *C*'s portfolio, which correspond to a portfolio invested 60 percent in wheat, attributable to the wheat blight. In this case, *C* will need to increase the equity in the bank by about 50 percent to drive down *B*'s lending rate sufficiently so that *C* can lend to *D*.

If the stress test was instead conducted before the equity injection, then *B* would discover *C*'s portfolio holdings, eliminating the uncertainty. If *B* is uncertainty-averse, then because *C*'s holdings can be no worse than the worst case, the amount of equity it will need to inject is smaller. For example, if *C*'s true portfolio is split evenly in each type of loan, the size of the required equity injection would be only about 20 percent, and in some cases no equity injection would be required.[11]

For similar reasons, stress tests reduce the costs and increase the effectiveness of government programs that guarantee interbank loans. To illustrate, we note that interbank loan guarantees are very expensive because they transfer credit risk from the banking system to the government. Therefore, in the United States the guarantees offered by the Federal Deposit Insurance Corporation were limited as to the amount of new interbank lending that was guaranteed, and banks that participated in the program were charged a fee based on the amount borrowed. While the fees and limitations on the amount of new loans that are covered reduce the government's exposure as a guarantor of interbank loans, they also limit banks' ability to borrow under these programs.

If a stress test is conducted before the loan guarantee program is put in place, then the test may help the market distinguish low- from high-risk banks. The banks that are identified as low risk may then be able to borrow more at better rates than the loan guarantee program could provide; thus, they could potentially increase lending while saving money.

Finally, stress tests and other programs to restart lending may work better in combination than alone. For example, in equation 1, lowering R_f to 0 may be insufficient to restart lending, and eliminating the uncertainty spread without lowering R_f may also be insufficient—but both actions together may be sufficient.

3. Conclusion

When credit is provided along chains of borrowers and lenders, uncertainty over borrowers' economic conditions can sometimes cause the flow of credit to break down. However, when a breakdown occurs, a central bank can take action to restart the flow of credit. One such action is to reduce uncertainty through government provision of information on financial intermediaries, such as banks, that are key links in lending chains. Information provision works by reducing those components of borrowers' credit spreads attributable to uncertainty over their economic conditions. Because information provision can reduce the interest rates paid by borrowers, it can be viewed as a substitute for easing interest

[11] For details, see Appendix B.

rates by other means, such as lowering central bank target rates, and may prove especially useful when central bank target rates are at their lower bounds.

Although government-sponsored information provision may improve the flow of credit ex post, its use has been—and probably should be—relatively infrequent for two reasons. First, gathering information is costly, and the benefits of providing it, in terms of lower spreads, will probably not exceed the costs in many circumstances. Second, government provision of information is a two-edged sword: It may be needed to reduce uncertainty ex post because private incentives to do the same are inadequate ex ante. However, government information provision ex post may further worsen private incentives to choose capital structures and transparent portfolio holdings that reduce uncertainty spreads. Thus, in the future, perhaps central banks should be concerned with uncertainty reduction ex post and with efforts to improve private incentives to reduce uncertainty ex ante.

Appendix A: Details of Model Derivation

We show how bank B calculates a range of possible default probabilities for bank C when bank B is uncertain about C's portfolio holdings.

As we discuss in the text, the returns on two types of loans, to wheat farmers (w) and to oat farmers (o), conditional on the information known at date 0, are multivariate normal. At date 1, news arrives. Conditional on I_1, the information that is known at date 1, the return on bank C's assets is multivariate normal with means μ_w and μ_o, standard deviations σ_w and σ_o, and correlation parameter $\rho_{w,o}$. Therefore, the conditional distribution of the return on the long-term loan portfolio is given in equation 2, with parameters μ_1 and variance σ_1^2 as follows:

(A1) $$\mu_1 = \omega_w \mu_w + \omega_o \mu_o \ ,$$

(A2) $$\sigma_1^2 = \omega_w^2 \sigma_w^2 + \omega_o^2 \sigma_o^2 + 2\omega_w \omega_o \sigma_o \sigma_w \rho_{w,o} \ .$$

Bank C will default at date 2 under the condition in equation 4. The bank's probability of default conditional on the information known at date 1 is given by:

(A3) $$PD(\omega_w, \omega_o, L, 1) = \Phi\left(\frac{\frac{L}{1+L}R_{0,2}^C - \mu_1}{\sqrt{\sigma_1^2}}\right),$$

where ω_w and ω_o are bank C's portfolio weights, μ_1 and σ_1^2 are the mean and variance of the portfolio's return distribution given the portfolio weights (equations A1 and A2), and $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.

To model uncertainty about C's portfolio weights, we assume that B knows that C could have a set of possible portfolios, and that the weight on wheat is some number t between 0.4 and 0.6 and the weight on oats is $1 - t$. More formally, C's possible portfolios can be written as

$$\omega_w = t \ , \ \omega_o = 1 - t \ , \ t \in [0.4, 0.6] \ .$$

Given the available information, bank B does not know the probability that C will hold any particular portfolio; however, from equation A3 bank B does know the probability that C will default on each portfolio that it could hold. The set of default probabilities is given by the probability of default in the equation below for different choices of t:

(A4) $$PD(t) = PD(\omega_w = t, \omega_o = 1 - t, L, 1) \ , \ t \in [0.4, 0.6] \ .$$

Therefore, given the set of possible portfolios, we have a range of possible default probabilities that bank C could have.

Appendix B: Solving for the Size of Bank *C*'s Required Equity Injection

We solve for the size of the equity injection needed to sustain interbank lending from bank *B* to bank *C* when there is uncertainty about bank *C*'s portfolio holdings and when bank *B* knows *C*'s portfolio composition because it has been revealed as part of a stress test.

When there is uncertainty about bank *C*'s portfolio holdings, an uncertainty-averse lender will assess the default risk as equal to $\overline{PD}$, which is the highest default probability that bank *C* could have, given its possible portfolio holdings:

$$\overline{PD} = \max_{t \in [0.4,\, 0.6]} PD(\omega_w = t,\, \omega_o = 1 - t,\, L,\, 1),$$

where *PD(.)* is defined in equation A3. Provided that $\overline{PD} < 0.5$, which is very plausible, Pritsker (2009) shows that *PD(.)* is a convex function of the portfolio weights. Therefore, the problem of solving for $\overline{PD}$ maximizes a convex objective function over a convex set. It follows that the solution is on the boundary, at either $t = 0.4$ or $t = 0.6$.

Using the expression for *PD(.)*, $\overline{PD}$ can be expressed as

$$\overline{PD} = \Phi\left(\frac{\frac{L}{1+L}R^C_{0,2} - \overline{\mu}_1}{\sqrt{\overline{\sigma}^2_1}}\right),$$

where $L = F / E$; $\overline{\omega}_w$ and $\overline{\omega}_o$ are the portfolio weights for the portfolio that generates the maximum probability of default; and $\overline{\mu}_1$ and $\overline{\sigma}_1$ are the mean and standard deviation, respectively, of the return on the portfolio that maximizes *C*'s default probability.

Solving the above equation for *E*, it then follows that the original amount of equity capital in bank *C*, denoted E_0, is:

$$E_0 = \frac{F[R^C_{0,2} - (\overline{PD}\,\overline{\sigma}_1 + \overline{\mu}_1)]}{\overline{PD}\,\overline{\sigma}_1 + \overline{\mu}_1}.$$

Let PD^T, "the *PD* target," denote the required maximum level of *PD* for which it is possible to support an interbank loan between banks *B* and *C* when *B* is uncertainty-averse. From the above equation it follows that, holding *F* constant, the amount of equity in *C*'s capital structure needed to reduce its maximum level of *PD* to PD^T is

$$E_{T,NI} = \frac{F[R^C_{0,2} - (PD^T\overline{\sigma}_1 + \overline{\mu}_1)]}{PD^T\overline{\sigma}_1 + \overline{\mu}_1}$$

when information to reduce uncertainty is not provided (*NI* is no information).

When information is provided to reduce uncertainty, revealing *C*'s portfolio weights, then the amount of equity needed in *C*'s capital structure is

$$E_{T,I} = \frac{F[R^C_{0,2} - (PD^T\sigma_1 + \mu_1)]}{(PD^T\sigma_1 + \mu_1)},$$

where *I* is information.

When uncertainty is unresolved, the percentage equity injection that is required is $100 \times (E_{T,NI} / E_0 - 1)$; when information is provided that resolves uncertainty, it is $100 \times (E_{T,I} / E_0 - 1)$. The percentage equity injections are reported in Chart 4 in the text for different initial portfolios $\omega(t)$.

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