Argia M. Sbordone, Andrea Tambalotti, Krishna Rao, and Kieran Walsh

Policy Analysis Using DSGE Models: An Introduction

- Dynamic stochastic general equilibrium models are playing an important role in the formulation and communication of monetary policy at many of the world's central banks.
- These models, which emphasize the dependence of current choices on expected future outcomes, have moved from academic circles to the policymaking community—but they are not well known to the general public.
- This study adds to the understanding of the DSGE framework by using a small-scale model to show how to address specific monetary policy questions; the authors focus on the causes of the sudden pickup in inflation in the first half of 2004.
- An important lesson derived from the exercise is that the management of expectations can be a more effective tool for stabilizing inflation than actual movements in the policy rate; this result is consistent with the increasing focus on central bankers' pronouncements of their future actions.

1. Introduction

In recent years, there has been a significant evolution in the formulation and communication of monetary policy at a number of central banks around the world. Many of these banks now present their economic outlook and policy strategies to the public in a more formal way, a process accompanied by the introduction of modern analytical tools and advanced econometric methods in forecasting and policy simulations. Official publications by central banks that formally adopt a monetary policy strategy of inflation targeting—such as the Inflation Report issued by the Bank of England and the monetary policy reports issued by the Riksbank and Norges Bank—have progressively introduced into the policy process the language and methodologies developed in the modern dynamic macroeconomic literature.[1]

The development of medium-scale DSGE (dynamic stochastic general equilibrium) models has played a key role in this process.[2] These models are built on microeconomic foundations and emphasize agents' intertemporal choice. The dependence of current choices on future uncertain

[1] The Bank of England has published a quarterly Inflation Report since 1993. The report sets out the detailed economic analysis and inflation projections on which the Bank's Monetary Policy Committee bases its interest rate decisions. The Riksbank and Norges Bank each publish monetary policy reports three times a year. These reports contain forecasts for the economy and an assessment of the interest rate outlook for the medium term.

[2] A simple exposition of this class of models can be found in Galí and Gertler (2007). Woodford (2003) provides an exhaustive textbook treatment.

Argia M. Sbordone is an assistant vice president and Andrea Tambalotti a senior economist at the Federal Reserve Bank of New York; Krishna Rao is a graduate student at Stanford University; Kieran Walsh is a graduate student at Yale University.
Correspondence: argia.sbordone@ny.frb.org, andrea.tambalotti@ny.frb.org

The authors thank Ariel Zetlin-Jones for his contribution to the early stages of this work. The views expressed are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

outcomes makes the models dynamic and assigns a central role to agents' expectations in the determination of current macroeconomic outcomes. In addition, the models' general equilibrium nature captures the interaction between policy actions and agents' behavior. Furthermore, a more detailed specification of the stochastic shocks that give rise to economic fluctuations allows one to trace more clearly the shocks' transmission to the economy.

The use of DSGE models as a potential tool for policy analysis has contributed to their diffusion from academic to policymaking circles. However, the models remain less well-known to the general public. To broaden the understanding of these models, this article offers a simple illustration of how an estimated model in this class can be used to answer specific monetary policy questions. To that end, we introduce the structure of DSGE models by presenting a simple model, meant to flesh out their distinctive features. Before proceeding to a formal description of the optimization problems solved by firms and consumers, we use a simple diagram to illustrate the interactions among the main agents in the economy. With the theoretical structure in place, we discuss the features of the estimated model and the extent to which it approximates the volatility and comovement of economic time series. We also discuss important outcomes of the estimation—namely, the possibility of recovering the structural shocks that drive economic fluctuations as well as the historical behavior of variables that are relevant for policy but are not directly observable. We conclude by applying the DSGE tool to study the role of monetary policy in a recent episode of an increase in inflation. The lesson we emphasize is that, while they are a very stylized representation of the real economy, DSGE models provide a disciplined way of thinking about the economic outlook and its interaction with policy.[3]

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We work with a small model in order to make the transmission mechanism of monetary policy, whose basic contours our model shares with most DSGE specifications, as transparent as possible. Therefore, the model focuses on the behavior of only three major macroeconomic variables: inflation, GDP growth, and the short-term interest rate. However, the basic framework that we present could easily be enriched to provide more details on the structure of the economy. In fact, a key advantage of DSGE models is that they share core assumptions on the behavior of households and firms, which makes them easily scalable to include details that are relevant to address the question at hand. Indeed, several extensions of the basic framework presented here have been developed in the literature, including the introduction of wage stickiness and frictions in the capital accumulation process (see the popular model of Smets and Wouters [2007]) and a treatment of wage bargaining and labor market search (Gertler, Sala, and Trigari 2008).[4] Recently, the 2008 financial crisis has highlighted one key area where DSGE models must develop: the inclusion of a more sophisticated financial intermediation sector. There is a large body of work under way to model financial frictions within the baseline DSGE framework—work that is very promising for the study of financial intermediation as a source and conduit of shocks as well as for its implications for monetary policy. However, this last generation of models has not yet been subjected to extensive empirical analysis.

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Our study is organized as follows. Section 2 describes the general structure of our model while Section 3 illustrates its construction from microeconomic foundations. Section 4 briefly describes our approach to estimation and presents some of the model's empirical properties. In Section 5, we use the model to analyze the inflationary episode of the first half of 2004. Section 6 concludes.

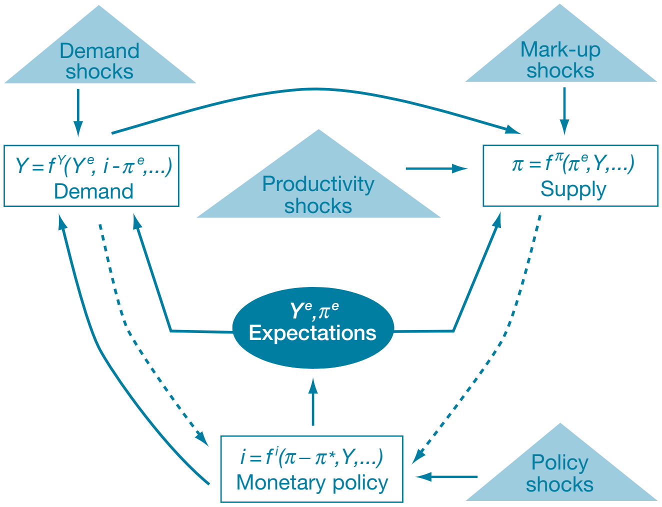
2. DSGE Models and Their Basic Structure

Dynamic stochastic general equilibrium models used for policy analysis share a fairly simple structure, built around three interrelated blocks: a demand block, a supply block, and a

[3]Adolfson et al. (2007) offer a more extended illustration of how DSGE models can be used to address questions that policymakers confront in practice. Erceg, Guerrieri, and Gust (2006) illustrate policy simulations with an open-economy DSGE model.

[4] Some of these larger DSGE models inform policy analysis at central banks around the world: Smets and Wouters (2007) of the European Central Bank; Edge, Kiley, and Laforte (2008) of the Federal Reserve System; and Adolfson et al. (2008) of the Riksbank.

The Basic Structure of DSGE Models



monetary policy equation. Formally, the equations that define these blocks derive from microfoundations: explicit assumptions about the behavior of the main economic actors in the economy—households, firms, and the government. These agents interact in markets that clear every period, which leads to the "general equilibrium" feature of the models. Section 3 presents the microfoundations of a simple DSGE model and derives the equations that define its equilibrium. But first, we begin by introducing the basic components common to most DSGE models with the aid of a diagram.

In the diagram, the three interrelated blocks are depicted as rectangles. The demand block determines real activity (Y) as a function of the ex ante real interest rate—the nominal rate minus expected inflation ($i - \pi^e$)—and of expectations about future real activity (Y^e). This block captures the idea that, when real interest rates are temporarily high, people and firms would rather save than consume or invest. At the same time, people are willing to spend more when future prospects are promising (Y^e is high), regardless of the level of interest rates.

The line connecting the demand block to the supply block shows that the level of activity (Y) emerging from the demand block is a key input in the determination of inflation (π), together with expectations of future inflation (π^e). In prosperous times, when the level of activity is high, firms must increase wages to induce employees to work longer hours. Higher wages increase marginal costs, putting pressure on prices and generating inflation. Moreover, the higher inflation is expected to be in the future, the higher is this increase in prices, thus contributing to a rise in inflation today.

The determination of output and inflation from the demand and supply blocks feeds into the monetary policy block, as indicated by the dashed lines. The equation in that block describes how the central bank sets the nominal interest rate, usually as a function of inflation and real activity. This reflects the tendency of central banks to raise the short-term interest rate when the economy is overheating as well as when inflation rises and to lower it in the presence of economic slack. By adjusting the nominal interest rate, monetary policy in turn affects real activity and through it inflation, as represented by the line flowing from the monetary policy block to the demand block and then to the supply block. The policy rule therefore closes the circle, giving us a complete model of the relationship between three key endogenous variables: output (Y), inflation (π), and the nominal interest rate (i).

While this brief description appears static, one of the fundamental features of DSGE models is the dynamic interaction between the blocks—hence, the "dynamic" aspect of the DSGE label—in the sense that expectations about the future are a crucial determinant of today's outcomes. These expectations are pinned down by the same mechanism that generates outcomes today. Therefore, output and inflation tomorrow, and thus their expectations as of today, depend on monetary policy tomorrow in the same way as they do today—of course, taking into account what will happen from then on into the infinite future.

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The diagram highlights the role of expectations and the dynamic connections between the blocks that they create. The influence of expectations on the economy is represented by the arrows, which flow from monetary policy to the demand and then the supply block, where output and inflation are determined. This is to emphasize that the conduct of monetary policy has a large influence on the formation of expectations. In fact, in DSGE models, expectations are the main channel through which policy affects the economy, a feature that is consistent with the close attention paid by financial markets and the public to the pronouncements of central banks on their likely course of action.

The last component of DSGE models captured in the diagram is their stochastic nature. Every period, random exogenous events perturb the equilibrium conditions in each block, injecting uncertainty in the evolution of the economy and thus generating economic fluctuations. Without these shocks, the economy would evolve along a perfectly predictable path, with neither booms nor recessions. We represent these shocks as triangles, with arrows pointing toward the equilibrium conditions on which they directly impinge. Markup and productivity shocks, for example, affect the pricing and production decisions of firms that underlie the supply block, while demand shocks capture changes in the willingness of households to purchase the goods produced by those firms.

3. Microfoundations of a Simple DSGE Model

We present the microfoundations of a small DSGE model that is simple enough to fit closely into the stylized structure outlined in our diagram. Our objective is to describe the basic components of DSGE models from a more formal perspective, using the mathematical language of economists, but avoiding unnecessary technical details. Despite its simplicity, our model is rich enough to provide a satisfactory empirical account of the evolution of output, inflation, and the nominal interest rate in the United States in the last twenty years, as we discuss in the next section.

Given the constraints we impose on this treatment for the sake of simplicity, our model lacks many features that are standard in the DSGE models that central banks typically use. For example, we ignore the process of capital accumulation,

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which would add another dimension—investment decisions by firms—to the economy's demand block. Nor do we attempt to model the labor market in detail: for example, we make no distinction between the number of hours worked by each employee and the number of people at work, an issue that is hard to overlook in a period with unemployment close to 10 percent. Finally, we exclude any impediment to the smooth functioning of financial markets and assume that the central bank can perfectly control the short-term interest rate—the only relevant rate of return in the economy. The 2008 financial crisis has proved that this set of assumptions can fail miserably in some circumstances and has highlighted the need for a more nuanced view of financial markets within the current generation of DSGE models, as we observe in our introduction.

3.1. The Model Economy

Our model economy is populated by four classes of agents: a representative household, a representative final-good-producing firm (f-firm), a continuum of intermediate firms (i-firms) indexed by $i \in [0, 1]$, and a monetary authority. The household consumes the final good and works for the i-firms. Each of these firms is a monopolist in the production of a particular intermediate good i, for which it is thus able to

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set the price. The f-firm packages the differentiated goods produced by the i-firms and sells the product to households in a competitive market. The monetary authority sets the nominal interest rate.

The remainder of this section describes the problem faced by each economic agent, shows the corresponding optimization conditions, and interprets the shocks that perturb these conditions. These optimization conditions result in dynamic relationships among macroeconomic variables that define the three model blocks described above. Together with market clearing conditions, these relationships completely characterize the equilibrium behavior of the model economy.

3.2. Households and the Aggregate Demand Block

At the core of the demand side of virtually all DSGE models is a negative relationship between the real interest rate and desired spending. In our simple model, the only source of spending is consumption. Therefore, the negative relationship between the interest rate and demand emerges from the consumption decision of households.

We model this decision as stemming from the optimal choice of a very large representative household—the entire U.S. population—which maximizes its expected discounted lifetime utility, looking forward from an arbitrary date t_0

$$\underset{\{B_{t_0+s}, C_{t_0+s}, [H_{t_0+s}(i)]_{i\in[0,1]}\}_{s=0}^{\infty}}{Max} E_{t_0}\sum_{s=0}^{\infty}\beta^s\left\{ b_{t_0+s}\left[\log(C_{t_0+s} - \eta C_{t_0+s-1}) - \int_0^1 v(H_{t_0+s}(i))di\right]\right\}$$

subject to the sequence of budget constraints

$$P_t C_t + \frac{B_t}{R_t} \leq B_{t-1} + \int_0^1 w_t(i)H_t(i)di,$$

for $t = t_0, t_0+1, \ldots, \infty$, and given B_{t_0-1}. The members of this household, we call them "Americans," like consumption, C_t, but dislike the number of hours they spend at work, H_t, to an extent described by the convex function v. The utility flow from consumption depends on current as well as past consumption, with a coefficient η. As a result of this "habit," consumers are unhappy if their current consumption is low, but also if it falls much below the level of their consumption in the recent past. To afford consumption, Americans work a certain amount of hours $H_t(i)$ in each of the i-firms, where they earn an hourly nominal wage $W_t(i)$ which they take as given when deciding how much to work.[5] With the income thus earned, they can purchase the final good at price P_t or save by accumulating one-period discount government bonds B_t, whose gross rate of return between t and $t+1$ is R_t.

From the perspective of time t, the household discounts utility in period $t+1$ by a time-varying factor $\beta b_{t+1}/b_t$, where b_{t+1}/b_t is an exogenous stochastic process. Changes in b_{t+1}/b_t represent shocks to the household's impatience. When b_{t+1} increases relative to b_t, for example, the household cares more about the future and thus wishes to save more and consume less today, everything else equal. In this respect, b_{t+1}/b_t acts as a traditional *demand shock*, which affects desired consumption and saving exogenously. A persistent increase in b_{t+1}/b_t is one way of interpreting the current macroeconomic situation in the United States, in which households have curtailed their consumption—partly to build their savings. Of course, in reality there are many complex reasons behind this observed change in behavior, and an increase in people's concern about the future is surely one of them. For simplicity, the model focuses on this one reason exclusively.

[5] In equilibrium, the wage—and thus the number of hours worked—will settle at the level at which the supply of labor by the household equals the demand of labor by firms. This labor demand in turn is determined by the need of firms to hire enough workers to satisfy the demand for their products, as we describe in the next section.

To find the solution to the optimal problem above, we form the Lagrangian

$$L = E_{t_0}\sum_{s=0}^{\infty}\left\{\beta^s\left[b_{t_0+s}\left(\log(C_{t_0+s} - \eta C_{t_0+s-1}) - \int_0^1 v(H_{t_0+s}(i))di\right) - \Lambda_{t_0+s}\left(P_{t_0+s}C_{t_0+s} + B_{t_0+s}R_{t_0+s}^{-1} - B_{t_0+s-1} - \int_0^1 W_{t_0+s}(i)H_{t_0+s}(i)di\right)\right]\right\},$$

with first-order conditions

(3.1a) $$\frac{\partial L}{\partial B_t} : \Lambda_t = \beta E_t[\Lambda_{t+1}]R_t$$

(3.1b) $$\frac{\partial L}{\partial C_t} : \frac{\Lambda_t}{b_t}P_t = \frac{1}{C_t - \eta C_{t-1}} - \eta E_t\left[\frac{\beta b_{t+1}/b_t}{C_{t+1} - \eta C_t}\right].$$

for $t = t_0, t_{0+1}, \ldots \infty$ and

(3.2) $$\frac{\partial L}{\partial H_t(i)} : \frac{v'(H_t(i))}{\Lambda_t / b_t} = W_t(i)$$

for $t = t_0, t_{0+1}, \ldots \infty$ and $\forall i \in [0,1]$, together with the sequence of budget constraints.

These conditions yield a fully state-contingent plan for the household's choice variables—how much to work, consume, and save in the form of bonds—looking forward from the planning date t_0 and into the foreseeable future. At any point in time, the household is obviously uncertain about the way in which this future will unfold. However, we assume that the household is aware of the kind of random external events, or shocks, that might affect its decisions and, crucially, that it knows the probability with which these shocks might occur. Therefore, the household can form expectations about future outcomes, which are one of the inputs in its current choices. We assume that these expectations are rational, meaning that they are based on the same knowledge of the economy and of the shocks that buffet it as that of the economist constructing the model. We use the notation $E_t[X_{t+s}]$ to denote expectations formed at time t of any future variable X_{t+s}, as in equations 3.1, for example. The optimal plan, then, is a series of instructions on how to behave in response to the realization of each shock, given expectations about the future, rather than a one-time decision on exactly how much to work, consume, and save on each future date.

Together, the optimality conditions in equations 3.1 establish the negative relationship between the interest rate and desired consumption that defines the demand side of the model. The nature of this relationship is more transparent in the special case of no habit in consumption ($\eta = 0$), when we can combine the two equations to obtain

$$(3.3) \qquad \frac{1}{C_t} = E_t\left[\frac{\beta b_{t+1}}{b_t}\frac{1}{C_{t+1}}\frac{R_t}{P_{t+1}/P_t}\right].$$

According to this so-called Euler equation, desired consumption decreases when the (gross) real interest rate $\left(\frac{R_t}{P_{t+1}/P_t}\right)$ increases, when expected future consumption decreases, and when households become more patient (b_{t+1} rises).

A log-linear approximation of the Euler equation (3.3), after some manipulation, gives

$$(3.4) \qquad y_t = E_t y_{t+1} - (i_t - E_t \pi_{t+1}) - \delta_t\,,$$

where $\pi_t \equiv \log P_t / P_{t-1}$ is the quarterly inflation rate, $i_t \equiv \log R_t$ is the continuously compounded nominal interest rate, $\delta_t \equiv E_t \log(\beta b_{t+1}/b_t)$ is a transformation of the demand shock, and $y_t \equiv \log Y_t$ is the logarithm of total output. In this expression, we can substitute consumption of the final good C_t with its output Y_t because in our model consumption is the only source of demand for the final good. Therefore, market clearing implies $Y_t = C_t$.

In this framework, equation 3.4 is similar to a traditional IS equation, since it describes the relationship between aggregate activity y_t and the ex ante real interest rate $i_t - E_t \pi_{t+1}$, which must hold for the final-good market to clear. Unlike a traditional IS relationship, though, this equation is dynamic and forward looking, as it involves current and future expected variables. In particular, it establishes a link between current output and the entire future expected path of real interest rates, as we see by solving the equation forward

$$(3.5) \qquad y_t = -E_t \sum_{s=0}^{\infty} (i_{t+s} - \pi_{t+s+1} - \delta_{t+s}).$$

Through this channel, expectations of future monetary policy directly affect current economic conditions. In fact, this equation shows that future interest rates are just as important to determine today's output as the current level of the short-term rate, as we describe in our discussion of the role of policy expectations.

In our full model, the Euler equation is somewhat more complicated than in equation 3.4 because of the consumption habit ($\eta \neq 0$), which is a source of richer, and more realistic, output dynamics in response to changes in the interest rate. Nevertheless, these more intricate dynamics do not change the qualitative nature of the relationship between real rates and demand.

The third first-order condition of the household optimization problem, equation 3.2, represents the labor supply decision. It says that Americans are willing to work more hours in firms that pay a higher wage and at times when wages are higher, at least for differences in wages modest enough to have no significant effect on their income.[6] Large wage changes, in fact, would trigger an income effect and lead the now richer workers to curtail their labor supply. Mathematically, workers with higher income could afford more consumption, which would lead to a drop in the marginal utility Λ_t and thus to a decrease in labor supply at any given wage level.

We can think of the labor supply schedule (equation 3.2) as a relationship determining the wage that firms must pay to induce Americans to work a certain number of hours. In prosperous times, when demand is high and firms are producing much, firms require their labor force to work long hours and they must correspondingly pay higher hourly wages. This is an important consideration in the production and pricing decisions of firms, as we discuss in the next section.

3.3. Firms and the Aggregate Supply Block

The supply block of a DSGE model describes how firms set their prices as a function of the level of demand they face. Recall that in prosperous times, demand is high and firms must pay their workers higher wages. As a result, their costs increase as do their prices. In the aggregate, this generates a positive relationship between inflation and real activity.

In terms of microfoundations, establishing this relationship requires some work, since firms must have some monopoly power to set prices. This is why our production structure includes a set of monopolistic *i*-firms, which set prices, as well as an *f*-firm, which simply aggregates the output of the *i*-firm into the final consumption good. Because all the pricing action occurs within the *i*-firms, we focus on their problem and omit that of the *f*-firm.

Intermediate firm i hires $H_t(i)$ units of labor of type i on a competitive market to produce $Y_t(i)$ units of intermediate good i with the technology

$$(3.6) \qquad Y_t(i) = A_t H_t(i)\,,$$

where A_t represents the overall efficiency of the production process. We assume that A_t follows an exogenous stochastic process, whose random fluctuations over time capture the unexpected changes in productivity often experienced by modern economies—for example, the productivity boom in the mid-1990s that followed the mass adoption of information technology. We call this process an aggregate *productivity shock*, since it is common to all firms.

[6] Labor supply is upward sloping because ν' is an increasing function, as ν is convex. In other words, people dislike working an extra hour more intensely when they are already working a lot rather than when they are working little.

The market for intermediate goods is monopolistically competitive, as in Dixit and Stiglitz (1977), so firms set prices subject to the requirement that they satisfy the demand for their good. This demand comes from the *f*-firm and takes the form

$$Y_t(i) = Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\theta_t}, \tag{3.7}$$

where $P_t(i)$ is the price of good i and θ_t is the elasticity of demand. When the relative price of good i increases, its demand falls relative to aggregate demand by an amount that depends on θ_t.

Moreover, we assume that firms change their prices only infrequently. The fact that firms do not adjust prices continuously, but leave them unchanged in some cases for long periods of time, should be familiar from everyday experience and is well established in the economic literature (for example, Bils and Klenow [2004]; Nakamura and Steinsson [2008]). To model this fact, we follow Calvo (1983) and assume that in every period only a fraction $1-\alpha$ of firms is free to reset its price while the remaining fraction maintains its old price.[7]

The subset of firms that are able to set an optimal price at t, call it $\Omega_t \subset [0,1]$, maximize the discounted stream of expected future profits, taking into account that s periods from now there is a probability α^s that they will be forced to retain the price chosen today. The objective function of each of these firms is therefore

$$\underset{P_t(i)}{\text{Max}}\; E_t\sum_{s=0}^{\infty}\alpha^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\{P_t(i)Y_{t+s}(i)-W_{t+s}(i)H_{t+s}(i)\}$$

for all $i \in \Omega_t$, subject to the production function 3.6 and to the additional constraint that they must satisfy the demand for their product at every point in time

$$Y_{t+s}(i) = Y_{t+s}\left(\frac{P_t(i)}{P_{t+s}}\right)^{-\theta_{t+s}}, \tag{3.8}$$

for $s = 0,1,\ldots,\infty$. Profits, which are given by total revenue at the price chosen today, $P_t(i)\,Y_{t+s}(i)$ minus total costs $W_{t+s}(i)H_{t+s}(i)$, are discounted by the multiplier $\beta^s\Lambda_{t+s}/\Lambda_t$, sometimes called a stochastic discount factor, which translates dollar profits in the future into a current dollar value.

The first-order condition of this optimization problem is

$$E_t\sum_{s=0}^{\infty}(\alpha\beta)^s\Lambda_{t+s}Y_{t+s}P_{t+s}^{\theta_{t+s}-1}\left[P_t^*(i)-\mu_{t+s}\frac{W_{t+s}(i)}{A_{t+s}}\right]=0, \tag{3.9}$$

for all $i \in \Omega_t$, where $P_t^*(i)$ denotes the optimal price chosen by firm i, $W_{t+s}(i)/A_{t+s}$ is the firm's nominal marginal cost at time $t+s$, and $\mu_{t+s} = \dfrac{\theta_{t+s}-1}{\theta_{t+s}}$ is its desired mark-up—the mark-up it would charge if prices were flexible. As rational monopolists, optimizing firms set their price as a mark-up over their marginal cost. However, this relationship holds in expected present discounted value, rather than every period, since a price chosen at time t will still be in effect with probability α^s in period $t+s$.

We can rewrite the marginal cost of a firm that at time $t+s$ is still forced to retain the price $P_t(i)$ as

$$S_{t+s}(i) \equiv \frac{W_{t+s}(i)}{A_{t+s}} = \frac{v'[H_{t+s}(i)]}{\Lambda_{t+s}/b_{t+s}}\frac{1}{A_{t+s}} = \frac{v'\left(\frac{Y_{t+s}}{A_{t+s}}\left(\frac{P_t(i)}{P_{t+s}}\right)^{-\theta_{t+s}}\right)}{A_{t+s}\Lambda_{t+s}/b_{t+s}}, \tag{3.10}$$

where we use the labor supply relation 3.2 to substitute for the wage as well as the production function 3.6 and the demand function 3.8 to give us an expression for the labor demand $H_{t+s}(i)$.[8] Inserting this expression into the first-order condition 3.9, we see that the solution to the optimal pricing problem is the same for all firms in the set Ω_t, since it depends only on the aggregate variables $\{Y_{t+s}, A_{t+s}, P_{t+s}, \Lambda_{t+s}\}_{s=0}^{\infty}$. We denote this common optimal price as P_t^*.

The equation for the desired mark-up—$\mu_{t+s} = \dfrac{\theta_{t+s}-1}{\theta_{t+s}}$, also known as Lerner's formula—says that monopolists facing a more rigid demand optimally charge a higher mark-up, and thus higher prices, since their clients are less sensitive to changes in the latter. We assume that this sensitivity—the elasticity of demand—and thus the desired mark-up, follows an exogenous stochastic process. Positive realizations of this *desired mark-up shock*, which correspond to a fall in the elasticity of demand, represent an increase in firms' market power, to which they respond by increasing their price.

Equation 3.9, together with the definition of the aggregate price level as a function of newly set prices P_t^* and of the past price index P_{t-1}

$$P_t \equiv [(1-\alpha)P_t^{*\,(1-\theta_t)} + \alpha P_{t-1}^{1-\theta_t}]^{\frac{1}{1-\theta_t}}$$

yields an approximate New Keynesian Phillips curve—a relationship between current inflation, future expected inflation, and real marginal cost—of the form

$$\pi_t = \xi s_t + \beta E_t\pi_{t+1} + u_t, \tag{3.11}$$

where $u_t = \xi \log u_t$ is a transformation of the mark-up shock and $s_t \equiv \log(S_t/P_t)$ is the logarithm of the *real* marginal cost.[9] The sensitivity of inflation to changes in the marginal cost, ξ, depends on the frequency of price adjustment α, as well as on

[7] In our estimated model in Section 5, we actually assume that the α fraction of firms that cannot choose their price freely can in fact adjust it in part to catch up with recent inflation. This assumption improves the model's ability to fit the data on inflation, but it would complicate our presentation of the model's microfoundations without altering its basic message.

[8] These substitutions are equivalent to "solving" for equilibrium in the labor market.

[9] Variables are in logarithms, because equation 3.11, like equation 3.4, is obtained by a log-linear approximation.

other structural parameters, according to $\xi \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}$, where $\omega \equiv \frac{v''H}{v'}$ is the elasticity of the marginal disutility of work, while θ is the average value of the elasticity of demand θ_t.

This Phillips curve, together with the expression for marginal costs (3.10), provides the relationship between inflation and real activity that defines the supply block of the model. In fact, we see from equation 3.10 that marginal cost depends on the level of aggregate activity, among other factors. Higher economic activity leads to higher wages and marginal costs. Thus, firms increase their prices, boosting aggregate inflation.

Another important feature of the Phillips curve is that it is forward looking, just as the Euler equation in the previous section is. As in that case, therefore, we can iterate equation 3.11 forward to obtain

$$\pi_t = E_t \sum_{s=0}^{\infty} \beta^s (\xi s_{t+s} + u_{t+s}),$$

which highlights how inflation today really depends on the entire future expected path of marginal costs, and through those, of real activity. But this path depends in turn on expectations about interest rates, and thus on the entire future course of monetary policy, as equation 3.5 shows. Hence, we have the crucial role of policy expectations for the determination of current economic outcomes in this model, a feature we discuss in Section 2.

Monetary Policy

Recall that when the interest rate—current and expected—is low, people demand more consumption goods (equation 3.5). But if demand is high, firms' marginal costs increase and so do their prices. The end result is inflation. The opposite is true when the interest rate is high. But where does the interest rate come from? In DSGE models, as in the real world, the short-term interest rate is set by monetary policy. In practice, this is a decision made by a committee (the Federal Reserve's Federal Open Market Committee, or FOMC) using various inputs: large data sets, projections from several models, and the judgment of policymakers. Despite the apparent complexity of the process, Taylor (1993) famously demonstrated that it can be reasonably well approximated by assuming that the Federal Reserve raises the federal funds rate when inflation and/or output is "high" with respect to some baseline. This behavior is assumed in almost all variants of DSGE models, although the definition of the appropriate baselines is somewhat controversial.

In our model, we assume that interest rates are set according to the policy rule

$$(3.12) \quad i_t = \rho i_{t-1} + (1-\rho)[r_t^e + \pi_t^* + \phi_\pi(\pi_t^{4Q} - \pi_t^*) + \phi_y(y_t - y_t^e)] + \varepsilon_t^i,$$

where r_t^e, π_t^*, and y_t^e are the baselines for the real interest rate, inflation, and output, respectively, and $\pi_t^{4Q} \equiv (\log P_t / P_{t-4})$ is the rate of inflation over the previous four quarters. The monetary policy shock ε_t^i, a random variable with mean zero, captures any deviation of the observed nominal interest rate from the value suggested by the rule. This rule implies that, if inflation and output rise above their baseline levels, the nominal interest rate is lifted over time above its own baseline, $r_t^e + \pi_t^*$, by amounts dictated by the parameters φ_π and φ_y and at a speed that depends on the coefficient ρ. The higher policy rate, which is expected to persist even after output and inflation have returned to normal, exerts a restraining force on the economy—curbing demand, marginal costs, and inflation. In this respect, π_t^* and y_t^e can be regarded as targets of monetary policy—the levels of inflation and output that the central bank considers consistent with its mandate—and therefore do not elicit either a restrictive or a stimulative policy.

When output is at its efficient level, inflation is not stable, as policymakers would like it to be, but fluctuates because of the presence of mark-up shocks. This is the essence of the monetary policy trade-offs in the economy.

In equation 3.12, we denote the central bank's objective in terms of production as y_t^e, the "efficient" level of output. This unobserved variable can be derived from the microfoundations of the model.[10] It represents the level of output that would prevail in the economy if we could eliminate at once all distortions—namely, force the *i*-firms to behave competitively rather than as monopolists and allow them to change their prices freely. The level of activity that would result from such a situation is ideal from the perspective of the representative household in the model, as its name suggests. This is what makes it a suitable target for monetary policy. However, when output is at its efficient level, inflation is not stable, as policymakers would like it to be, but fluctuates because of the presence of mark-up shocks. This is the essence of the monetary policy trade-offs in the economy. Achieving the

[10] The precise mathematical definition of efficient output in the model is irrelevant for our purposes. We present in Section 4 an estimate of the behavior of this variable over the last twenty years.

efficient level of output requires undesirable movements in inflation. In contrast, a stable inflation implies deviations from the efficient level of output. The two objectives cannot be reconciled, but must be traded off of each other.

Related to the efficient level of output is the efficient real interest rate, r_t^e, which is the rate of return we would observe in the efficient economy described above. This definition implies that, when the actual real interest rate is at its efficient level and is expected to remain there in the future, output will also be at its efficient level. This is why we include r_t^e in our definition

of the baseline interest rate.

The other component of this baseline rate is the inflation target π_t^*. We allow this target to vary slowly over time to accommodate the fact that inflation hovered at about 4 percent for a few years around 1990 before declining to nearly 2 percent after the recession that ended in 1991. Nominal interest rates were correspondingly higher in the first period, thus implying a stable average for the real interest rate. We now present our estimate of the evolution of the inflation target.

4. Empirical Approach and Estimation Results

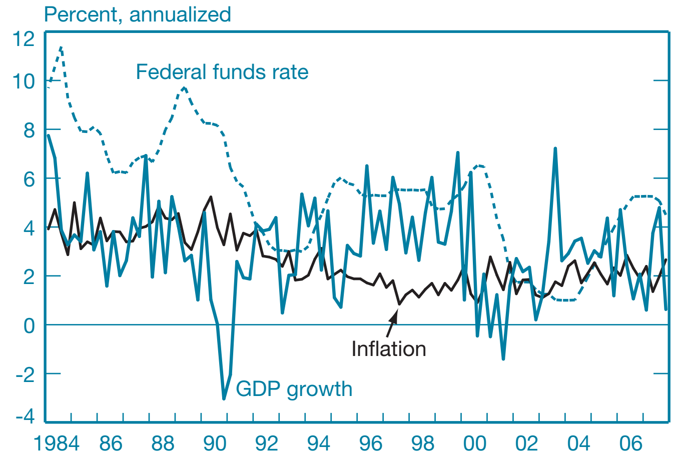
We estimate our model using data on the growth rate of real GDP to measure output growth, Δy_t, the growth rate of the personal consumption expenditures chain price index excluding food and energy (core PCE) to measure inflation, π_t, and the quarterly average of the monthly effective federal funds rate to measure the nominal interest rate, i_t. We measure inflation by core PCE, rather than by a more comprehensive measure, because the monetary policy debate in the United States tends to focus on this index.

Our data span the period 1984:1 to 2007:4 (Chart 1). This is the longest possible data set over which it is reasonable to argue that U.S. monetary policy can be represented by a stable interest rate rule. It follows the period of extremely high interest rates in the early 1980s that brought inflation under control. However, in the first few years of this sample, inflation and the federal funds rate were still relatively high, with a fairly abrupt reduction taking place around the 1991 recession. We capture this low-frequency movement in inflation and the nominal interest rate by including the slow-moving inflation target π_t^* in the policy rule.

We use Bayesian methods to characterize the posterior distribution of the parameters of the model. This distribution combines the model's likelihood function with prior information on the parameters, using techniques surveyed, for

Chart 1

Observable Variables



Sources: Bureau of Economic Analysis; Board of Governors of the Federal Reserve System.

example, by An and Schorfheide (2007).[11] A discussion of these methods is beyond the scope of this article. Instead, we focus on the implications of these estimates for some key properties of the model. Our objective is to show that the estimated model provides a good fit to the data across many dimensions, but also to highlight some of the model's most notable shortcomings.

4.1. Moment Comparisons

We compare the second moments implied by the estimated model with those measured in the data. Table 1 presents the standard deviations of the observable variables, reported as annualized percentages. In the model column, we report the median and 5th and 95th percentiles of the standard deviations across draws from the model's posterior. This interval reflects the uncertainty on the structural parameters—and thus on the model-implied moments—encoded in the parameters' posterior distribution. In the data column, we report the median and 5th and 95th percentiles of the empirical standard deviations in the data. This interval represents the uncertainty on the true empirical moments because of the small sample available for their estimation.

Our model does a very good job replicating the volatilities in the data. It captures the standard deviation of output growth and replicates quite closely the volatility of the nominal interest rate, although it overestimates the standard deviation of

[11] The technical appendix provides information on the priors for the parameters.

Table 1
Model-Implied and Empirical Standard Deviations
Percent

Variable	Model	Data
GDP growth	2.03	2.03
	[1.79, 2.37]	[1.74, 2.27]
Core PCE inflation	1.41	1.15
	[0.98, 2.40]	[0.67, 1.38]
Federal funds rate	2.23	2.46
	[1.61, 3.56]	[1.55, 2.94]

Source: Authors' calculations.

Notes: The table reports the standard deviations of the observable variables. The model-implied standard deviations are medians across draws from the posterior; the 5th and 95th posterior percentiles across those same draws are in brackets. The empirical standard deviations are medians across bootstrap replications of a VAR(4) fit to the data; the 5th and 95th percentiles across those same replications are in brackets. PCE is personal consumption expenditures.

inflation. The ability of the model to accurately reproduce the volatility of the observable variables is not a preordained conclusion, even if we freely estimate the standard deviations of the shocks. The reason is that a likelihood-based estimator tries to match the entire autocovariance function of the data, and thus must strike a balance between matching standard deviations and all the other second moments—namely, autocorrelations and cross-correlations.

These other moments are displayed in Chart 2. The black line represents the model-implied correlation, with the shaded area representing a 90 percent posterior interval. The solid blue line is instead the correlation measured in the data, with a 90 percent bootstrap interval around this estimate represented by the dashed lines. The serial correlation of output growth in the model is very close to its empirical counterpart and well within the data-uncertainty band. For inflation and the interest rate, the model serial correlations are on the high end of the band. This excessive persistence is a result of the low-frequency component in both variables associated with the inflation target, as we can infer from the variance decomposition in Table 2.

Our model does a very good job replicating the volatilities in the data. It captures the standard deviation of output growth and replicates quite closely the volatility of the nominal interest rate, although it overestimates the standard deviation of inflation.

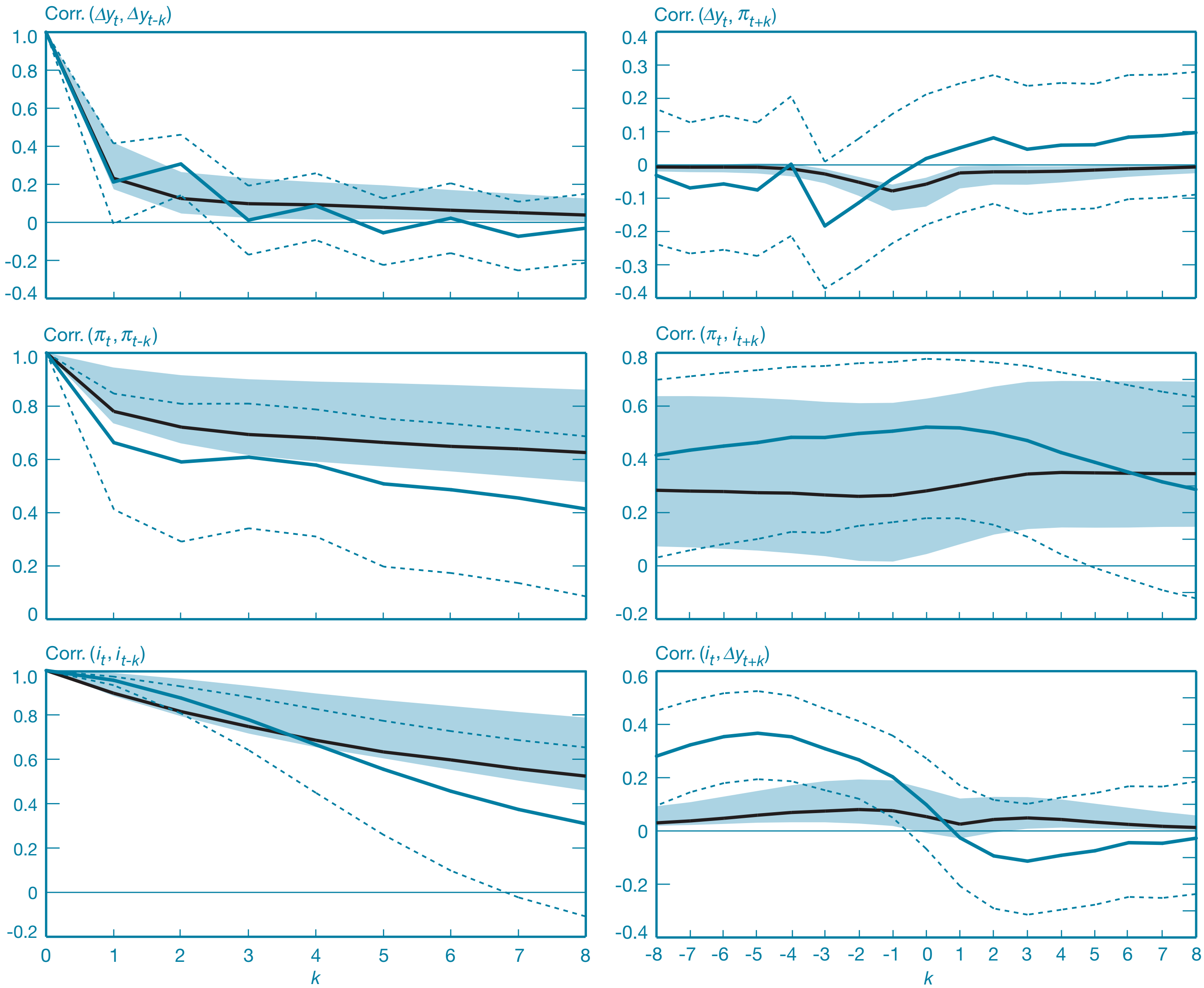
According to the model, shocks to the inflation target account for 85 percent of the unconditional variance of inflation and 38 percent of that of the nominal interest rate. Although we do not calculate a variance decomposition by frequency, we know that the contribution of the inflation target shock is concentrated at low frequencies, since this shock is very persistent (the posterior mean of its autocorrelation coefficient is 0.98). This finding suggests that the model faces a trade-off between accommodating the downward drift in inflation in the first part of our sample and providing a more balanced account of the sources of inflation variability.

The rest of the variance decomposition accords well with conventional wisdom. The productivity shock plays an important role in accounting for the volatility of output growth, although the demand shock and the monetary policy shocks (interest rate plus inflation target) are also non-negligible. Moreover, the demand shock accounts for more than half of the variance of the nominal interest rate. Finally, mark-up shocks play a minor role as sources of volatility. This finding has potentially important policy implications, since in the model mark-up shocks are the only source of the aforementioned policy trade-off between inflation and real activity.

According to the model, shocks to the inflation target account for 85 percent of the unconditional variance of inflation and 38 percent of that of the nominal interest rate. This finding suggests that the model faces a trade-off between accommodating the downward drift in inflation in the first part of our sample and providing a more balanced account of the sources of inflation variability.

Returning now to the cross-correlations in Chart 2, we see that the model is quite successful in capturing the lead-lag relationships in the data. In our sample, there is no statistically significant predictability of future inflation through current output *growth*, a pattern that is reproduced by the model. The

Chart 2
Correlations



Source: Authors' calculations.

Notes: The black line represents the median model-implied correlation across draws from the posterior; the shaded area represents the interval between the 5th and 95th percentiles across those same draws. The solid blue line represents the median autocorrelation across bootstrap replications of a VAR(4) fit to the data; the dashed blue lines represent the interval between the 5th and 95th percentiles across those same replications. Each statistic is calculated at horizons $k = 0, \ldots 8$ for autocorrelations and at horizons $k = -8, \ldots, 0, \ldots 8$ for cross-correlations.

model also reproduces the positive correlation between inflation and the nominal interest rate present in the data both in the leads and in the lags (the middle right panel of the chart). The positive correlation between current interest rates and future inflation might seem puzzling at first. We would expect higher interest rates to bring inflation down over time, which should make the correlation negative. However, over our sample, this negative relationship is confounded by the low-frequency positive comovement between inflation and the nominal interest rate induced by the Fisher effect. Recall that inflation and the nominal interest rate in fact are persistently above their unconditional sample average over the first third of the sample and are persistently below it after about 1992.

The bottom right panel of Chart 2 reports the dynamic

Table 2
Variance Decomposition
Percent

Variable	Shocks: Demand	Productivity	Mark-Up	Interest Rate	Inflation Target
GDP growth	0.20	0.56	0.07	0.04	0.13
	[.13, .28]	[.45, .67]	[.04, .10]	[.02, .06]	[.07, .19]
Core PCE inflation	0.04	0.01	0.10	0.00	0.85
	[.01, .06]	[.00, .02]	[.04, .17]	[.00, .01]	[.76, .94]
Federal funds rate	0.54	0.06	0.01	0.01	0.38
	[.32, .76]	[.00, .13]	[.00, .02]	[.01, .02]	[.16, .61]

Source: Authors' calculations.

Notes: The table reports the share of the unconditional variance of each observable variable contributed by each shock. The point estimates are medians across draws from the posterior; the 5th and 95th posterior percentiles across those same draws are in brackets. PCE is personal consumption expenditures.

correlation between output growth and the nominal interest rate. In the data, high growth rates of output predict high nominal interest rates one to two years ahead, but this predictability is much less pronounced in the model. Moreover, this discrepancy is statistically significant in the sense that the model-implied median autocorrelation lies outside the 90 percent bootstrap interval computed from the data. This failure to match the data highlights the main empirical weakness of our model: its demand-side specification. As in most of the DSGE literature, our demand block consists of the Euler equation of a representative consumer. Standard specifications of a Euler equation of the type adopted here provide an inaccurate description of the observed relationship between the growth rate of consumption (or output, as in our case) and financial returns, including interest rates, as first documented by Hansen and Singleton (1982, 1983) and subsequently confirmed by many others (see Campbell [2003] for a review). Improving the performance of the current generation of DSGE models in this dimension would be an important priority for future research.

We now report our estimates of a few of the variables that play an important role in the model, but that are not directly observable. We focus on the three latent variables that enter the interest rate rule: the inflation target π_t^*, the output gap $y_t - y_t^e$, and the efficient real interest rate r_t^e (Chart 3). As in Charts 1 and 2, the black line is the median estimate across draws from the model's posterior and the shaded area represents a 90 percent posterior probability interval.

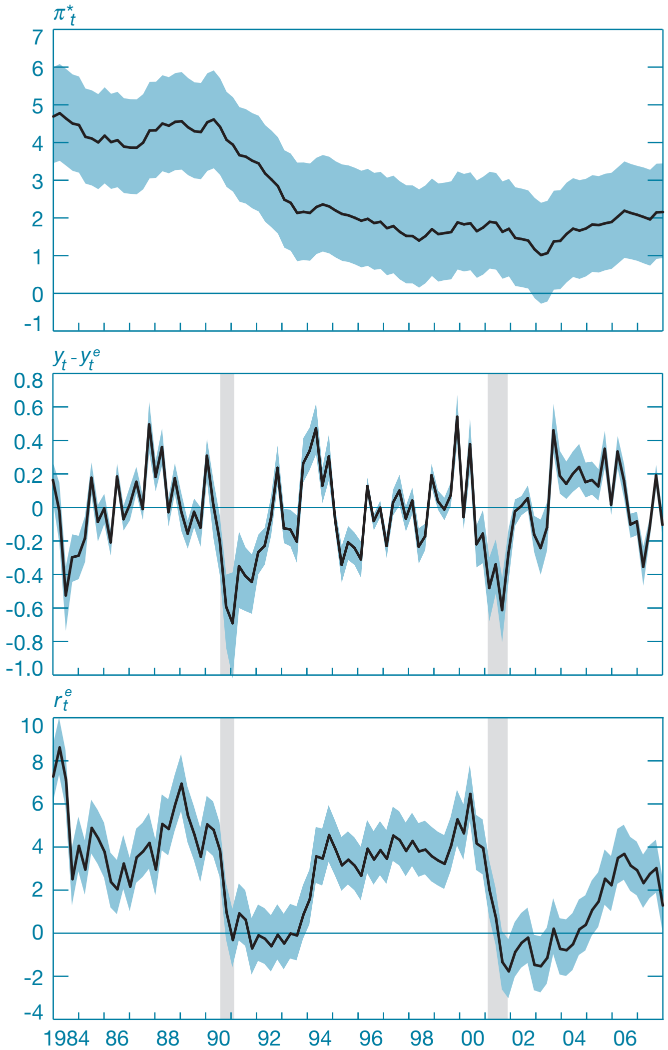
Starting from the top panel, we note that the estimated inflation target captures well the step-down in inflation from a local mean above 4 percent between 1984 and 1991 to an average value of around 2 percent since 1994. This permanent reduction in inflation represents the last stage of the disinflation process initiated by Federal Reserve Chairman Volcker in 1979, which became known as an example of opportunistic disinflation (Orphanides and Wilcox 2002). Needless to say, the estimated target is not completely smooth, but it also displays some higher frequency variation. For example, it reaches a minimum of around 1 percent at the beginning of 2003, but moves closer to 2 percent over 2004. (The next section studies in more detail the implications of these movements.)

The middle and bottom panels of Chart 3 report estimates of the output gap—the percentage deviation of output from its efficient level—and of the efficient real interest rate. Several features of the estimated output gap are noteworthy. First, its two deepest negative troughs correspond to the two recessions in our sample. In this respect, our model-based output gap conforms well with more conventional measures of this variable, such as the one produced by the Congressional Budget Office (CBO). However, the shortfall of output from its efficient level is never larger than 0.7 percent, even in these recessionary episodes. By comparison, the CBO output gap is as low as -3.2 percent in 1991. The amplitude of the business cycle fluctuations in our estimated output gap is small because the efficient level of output is a function of all the shocks in the model and therefore it tracks actual output quite closely. The last notable feature of the efficient output gap is that it displays a very pronounced volatility at frequencies higher than the business cycle. During the 1990s expansion, for example, it crosses the zero line about a dozen times.

Compared with the output gap, the efficient real rate is significantly smoother. Although some high-frequency variation remains, the behavior of the efficient real rate is dominated by swings at the business cycle frequency. The rate spikes and then plunges for some time before the onset of

Chart 3

Kalman Smoother Estimates of Latent Variables



Source: Authors' calculations.

Notes: The black line represents the Kalman smoother estimate of the relevant latent variable conditional on the posterior mean of the parameters; the shaded area represents the interval between the 5th and 95th percentiles of the Kalman smoother estimates across draws from the posterior. The vertical bands indicate NBER recessions.

recessions and recovers a few quarters into the expansions. It is interesting to note that the efficient real rate was negative for the entire period between 2001 and 2004—a time when the FOMC was concerned about the possibility that the U.S. economy would fall into a liquidity trap.[12] A negative efficient real interest rate is a necessary condition for the zero bound on nominal interest rates to become binding, and hence for the liquidity trap to become a problem.

[12] A liquidity trap describes a situation in which nominal interest rates have reached their zero lower bound, as in Japan in the 1990s, and therefore cannot be lowered any further.

5. The Model at Work: The Pickup in Inflation in the First Half of 2004

To show how our model can be used to address specific policy questions, we examine a particular historical episode: the puzzling pickup in inflation in the first half of 2004. This exercise allows us to illustrate how we use the model's forecasts to construct alternative scenarios for counterfactual policy analysis. Moreover, our analysis offers potentially interesting lessons for the current situation—although inflation has recently been quite low, there has been some concern that it might accelerate in the near future.

After approaching levels close to 1 percent between 2002 and 2003, core PCE inflation started moving higher in mid-2003. This pickup accelerated significantly in the first half of 2004, when (year-over-year) core inflation moved from about 1.5 percent to more than 2 percent, where it remained until the end of 2008. We use our DSGE model to analyze the sources of this unusually rapid and persistent step-up in the level of inflation.

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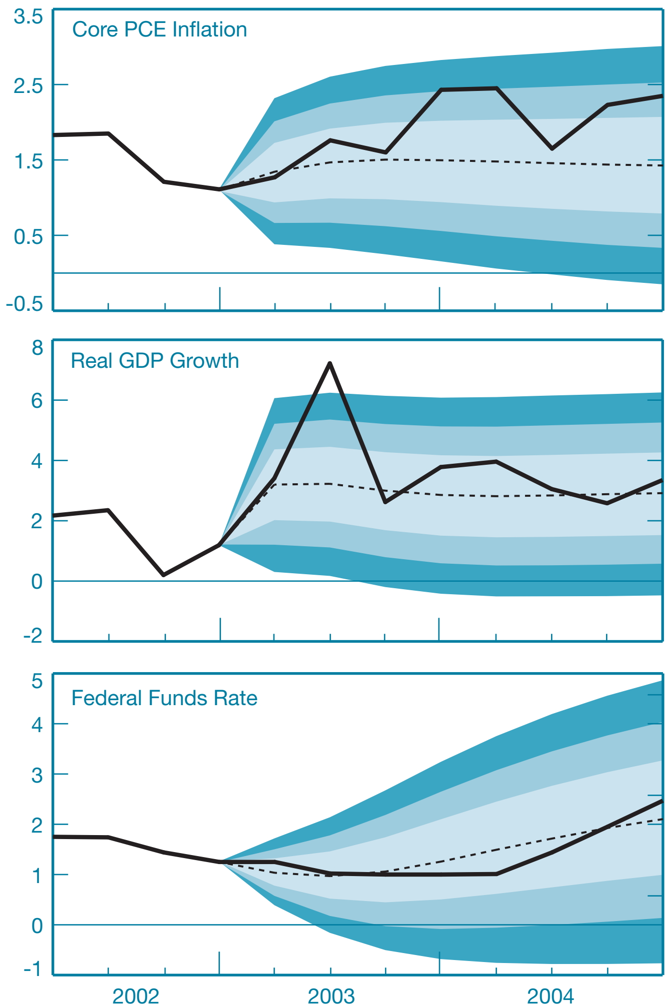
We organize our discussion around three questions. First, was the surge in inflation forecastable? As we will see, the answer to this question is no, at least from the perspective of our model. Second, what accounts for the discrepancy between the model's forecast and the observed paths of inflation, output growth, and the federal funds rate? Third, could monetary policy have achieved a smooth transition to inflation rates below 2 percent and, if so, at what cost in terms of added volatility in output and the interest rate?

Chart 4 presents forecasts of quarterly core PCE inflation, real GDP growth, and the federal funds rate from the DSGE model. The forecast starts in 2003:1, when quarterly inflation reached 1.1 percent (at an annual rate)—its lowest level following the 2001 recession—and extends through the beginning of 2005. In each panel, the dashed line represents

Chart 4
Forecasts of Observable Variables



Source: Authors' calculations.

Notes: The dashed line represents the forecast of the relevant variable conditional on the posterior mean of the parameters; the solid line represents the observed realization. The shaded areas represent 50 (light blue), 75 (medium blue), and 90 percent (dark blue) symmetric probability intervals for the forecast at each horizon. PCE is personal consumption expenditure.

the expected value of the forecast, while the bands show the 50 (light blue), 75 (medium blue), and 90 (dark blue) percent probability intervals. The solid line shows the realized data.

The model performs well in its forecast of output and the federal funds rate, especially in the medium term. Inflation, by comparison, is close to the mean forecast in 2003, but is well above it in 2004 and beyond. Moreover, the probability intervals for the forecast suggest that this realization of inflation was quite unusual, as we see from the fact that the solid line borders the 75 percent probability interval in the first half of 2004. This means that in 2003:1, the model would have assessed only about one in ten chances (12.5 percent) of inflation being as high as it was in that period.

From an economic perspective, it is interesting to note that these sizable forecast errors for inflation roughly correspond to the "considerable period" era that extended from June 2003 to June 2004. At that time, the FOMC kept the federal funds rate constant at 1 percent to guard against the risk of deflation, while indicating in its statement that "policy accommodation can be maintained for a considerable period."[13] According to the model's projection, this path for the federal funds rate represents a deviation from the policy stance historically maintained by the Federal Reserve in similar macroeconomic circumstances. Based on the estimated interest rate rule, in fact,

The DSGE forecast is just a description of what would happen to the variables of interest if we allowed the model economy to "run" from its initial condition, without introducing any innovations. Any observed deviation from the forecast, therefore, must be attributable to the realization of a particular combination of such innovations.

the DSGE predicts a slow rise in the interest rate over 2003 and 2004. Instead, the FOMC maintained the federal funds rate at 1 percent through the first half of 2004.

However, the pickup in inflation over this period is significantly more "unusual" than the deviation of the federal funds rate from the historical norm. Actual inflation in 2004 is mostly outside the 50 percent probability interval of the model forecast (the light blue band), while the actual federal funds rate remains well within it. Moreover, the acceleration in inflation is not accompanied by unexpectedly high real growth, suggesting that it cannot be fully explained by the traditional channel of transmission from an overheated economy to higher inflation.

What else, then, accounts for the unexpected and unlikely deviation of inflation from the model's forecast over 2004? The DSGE framework provides a particularly useful way of addressing this question. As we discuss in Section 2, the economic outcomes predicted by the model—the levels of inflation, output, and the interest rate—are the result of the endogenous responses of the agents in the economy to the

[13] This formulation was maintained in the FOMC statement from August 2003 to December 2003, and was later substituted with "policy accommodation can be removed at a pace that is likely to be measured."

Chart 5

Forecasts of Shocks

Source: Authors' calculations.

Notes: The dashed line represents the forecast of the relevant shock conditional on the posterior mean of the parameters while the solid line represents an estimate of the realization based on the Kalman smoother. The shaded area represents the 75 percent symmetric probability interval for the forecast at each horizon.

realization of a set of exogenous processes, such as productivity or desired mark-ups. The innovations to these driving processes account for the deviations of the data from the model's forecast. In fact, the DSGE forecast is just a description of what would happen to the variables of interest if we allowed the model economy to "run" from its initial condition, without introducing any innovations. Any observed deviation from the forecast, therefore, must be attributable to the realization of a particular combination of such innovations.[14]

Chart 5 depicts the combinations of exogenous driving processes that, according to the estimated DSGE model, are responsible for the observed path of inflation, output, and the interest rate over the period we analyze. In each panel, the dashed line represents the evolution of the shock associated with the mean forecast while the solid line represents the sequence of shocks corresponding to the actual realization of the observable variables. As in Chart 4, the medium blue band denotes the 75 percent probability interval for the forecast.

[14] In this study, we distinguish between exogenous driving processes—shocks, for short—and innovations. Driving processes can be autocorrelated, and thus forecastable, while their innovations are i.i.d.

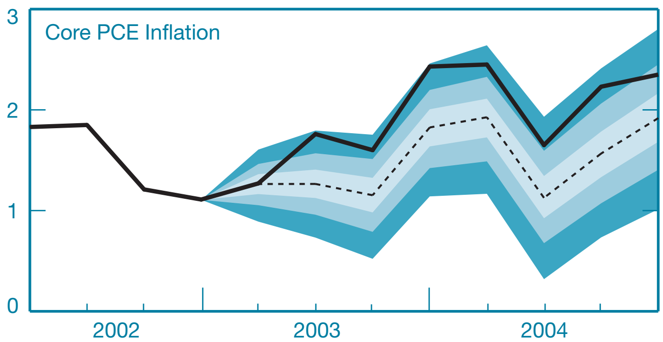
The contribution of three shocks stands out. First, the demand shock recovers from almost -4 percent to around -1 percent. This movement is particularly pronounced during 2004, when inflation was picking up. However, this profile is broadly consistent with the shock's expected evolution, represented by the dashed line. The productivity shock is also broadly in line with expectations, with the exception of 2003:3; this spike in productivity accounts for the corresponding spike in output growth in that quarter.

However, the most significant and direct contribution to the surge in inflation comes from a sizable upward movement in the inflation target, π_t^*. According to our estimates, this target moves by about 1 percentage point, from less than 1 percent to close to 2 percent. Moreover, this movement is at the edge of the 75 percent probability interval for the forecast, suggesting that the realization of this driving process is indeed quite unusual.

To quantify more directly the effect on inflation of the unexpected increase in the implicit inflation target, we depict what would have happened to core PCE inflation in the absence of such an increase (Chart 6). Here, the solid line

Chart 6

Conditional Forecast of Inflation



Source: Authors' calculations.

Notes: The dashed line represents a forecast of inflation conditional on the Kalman smoother estimates of all shocks except for those to the inflation target; the solid line represents the observed realization. The shaded areas represent 50 (light blue), 75 (medium blue), and 90 percent (dark blue) symmetric probability intervals for this conditional forecast. Therefore, they represent uncertainty stemming from future realizations of the inflation target shock alone. PCE is personal consumption expenditure.

is realized inflation. The dashed line represents the counterfactual path of inflation predicted by the model in the absence of shocks to the inflation target. In other words, this is a forecast for inflation, conditional on the estimated path of all but the inflation target shock. The bands therefore represent the usual probability intervals, but in this case they are computed around this conditional forecast.

The chart confirms our conclusion on the role of innovations to the inflation target in accounting for the observed evolution of inflation. According to the model, core inflation would not have increased to above 2 percent, as it did for most of 2004, without the steady increase in the inflation target over the same period. In fact, inflation would have remained within the "comfort zone" of 1 to 2 percent. Moreover, note that the solid line of realized inflation is mostly inside the area associated with the 90 percent probability interval for the conditional forecast. This suggests that the share of the forecast error in inflation accounted for by the innovations in the inflation target in this episode is unusually large compared with the historical average. This is just a more formal way of saying that the increase in the inflation target is disproportionately responsible for the observed increase in inflation that we examine.

The estimated rise in the implicit inflation target provides the missing link for a unified explanation of the pickup in inflation, the "considerable period" monetary policy, and the absence of a concomitant acceleration in output growth. In the model, the inflation target is the main driver of movements in inflation expectations, which are a key determinant of firms' pricing behavior together with the amount of slack in the economy. According to the DSGE model, therefore, a significant fraction of the inflation acceleration in 2003-04 can be attributed to a change in inflation expectations, driven by an increase in the Federal Reserve's implicit inflation target as perceived by the private sector. This increase in the perceived target in turn is consistent with the unusually loose stance of monetary policy maintained by the FOMC during the "considerable period" era.

This brings us to the third question: If the DSGE model is correct, and the pickup in inflation in 2004 is attributable to an increase in the implicit inflation target perceived by the public, could the Federal Reserve have prevented this development?

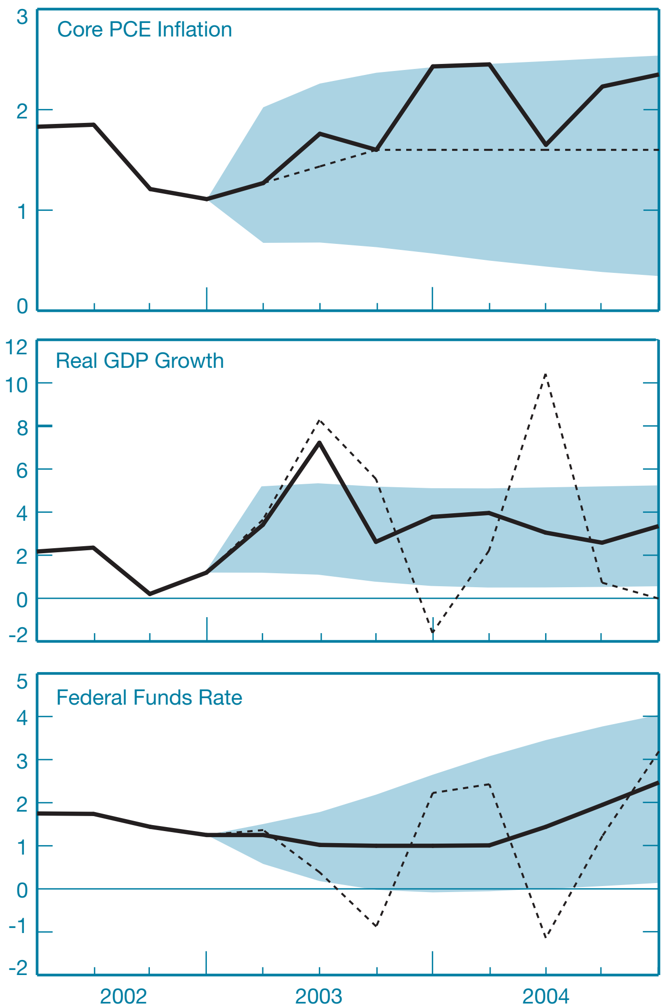
Charts 7 and 8 show the results of this counterfactual analysis. Both charts display the data (solid line) along with the counterfactual outcomes for the economy predicted by the model under a policy consistent with the stabilization of core inflation at 1.6 percent through 2004. The way in which this policy is implemented, however, is different in the two cases. In Chart 7, we present the outcomes associated with what we call a "no-communication" monetary strategy (dashed line) while in Chart 8 we compare these results with those that would emerge under a "full-communication" strategy (blue line). Under the no-communication strategy, the path for the interest rate compatible with the desired evolution of inflation is achieved each period through "surprise" departures from the historical rule. In contrast, under the full-communication strategy, the Federal Reserve implements the same path for inflation by announcing an inflation target that is consistent with it.[15]

According to the DSGE model . . . a significant fraction of the inflation acceleration in 2003-04 can be attributed to a change in inflation expectations, driven by an increase in the Federal Reserve's implicit inflation target as perceived by the private sector.

[15] Technically, in both cases we choose shocks to the monetary policy rule that are compatible with the desired evolution of inflation, conditional on the smoothed value of all other disturbances. Under the no-communication strategy, the shocks we choose are the i.i.d. monetary shocks, ε_t^i. Under the full-communication strategy, our chosen shocks are to the inflation target, $\varepsilon_t^{\pi^*}$.

Chart 7

“No-Communication” Counterfactual

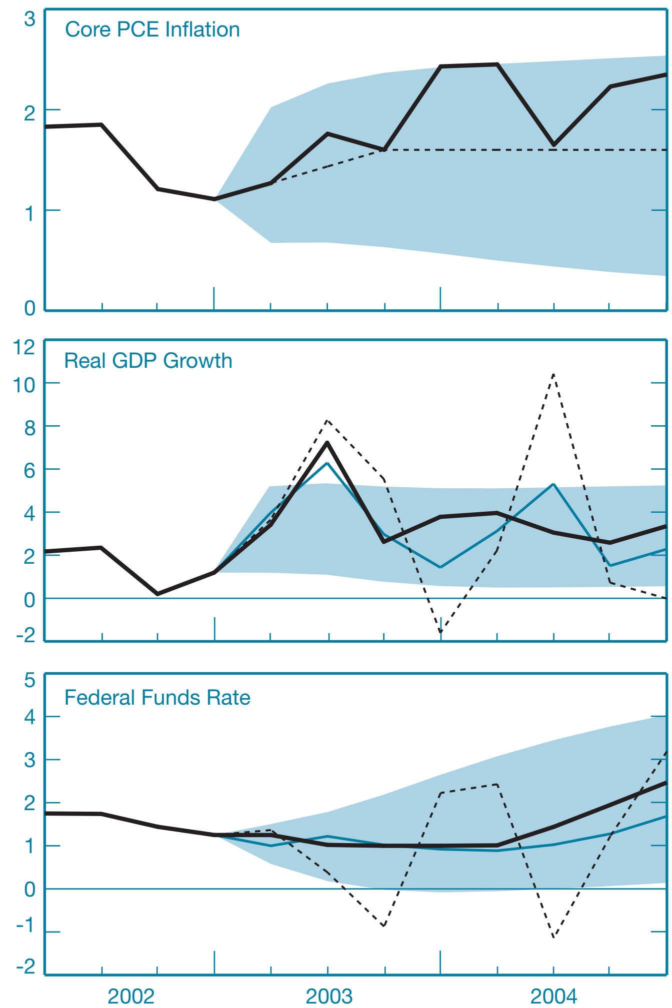


Source: Authors’ calculations.

Notes: The dashed line represents the counterfactual evolution of the economy predicted by our model had monetary policy been set to achieve the path for inflation depicted in the top panel. This counterfactual is conditional on the posterior mean of the parameters. Under the “no-communication” scenario, the desired path for inflation is achieved by the choice of the interest rate shock while all other shocks are set at their Kalman smoother estimate. The shaded area represents the 75 percent symmetric probability interval for the unconditional forecast, which is the same as in Chart 4. The black line represents the observed realization of each series. PCE is personal consumption expenditure.

Chart 8

“Full-Communication” Counterfactual



Source: Authors’ calculations.

Notes: The blue line represents the counterfactual evolution of the economy predicted by our model had monetary policy been set to achieve the path for inflation depicted in the top panel. This counterfactual is conditional on the posterior mean of the parameters. Under the “full-communication” scenario, the desired path for inflation is achieved by the choice of the inflation target while all other shocks are set at their Kalman smoother estimate. The shaded area represents the 75 percent symmetric probability interval for the unconditional forecast, which is the same as in Chart 4. The black line represents the observed realization of each series. The dashed line is the conditional forecast under the “no-communication” scenario. PCE is personal consumption expenditure.

The crucial difference between the results obtained under the two scenarios stems from the key role that expectations play in the DSGE model. Under the full-communication strategy, inflation expectations are immediately affected by the announcement of an inflation target. These expectations in turn have a direct effect on actual inflation without requiring a contraction in real activity to force businesses to contain their price increases. Under the no-communication strategy, inflation expectations remain at their historical level. As a result, inflation can be controlled only by increasing interest rates to contain GDP growth.

The way in which we model the full-communication scenario is quite stark. In practice, expectations would be unlikely to adjust instantaneously, even if the Federal Reserve were completely transparent about its inflation target. Nevertheless, the differences between the results of the two policy strategies are striking. In the no-communication case, inflation can be stabilized only through wild movements in the federal funds rate. As a result, GDP growth is also extremely volatile: it falls below zero in 2004:1, but then recovers to a quarterly (annualized) growth rate of 10 percent and ends the period

> *Our analysis might be interpreted as supportive of the policy stance adopted by the central bank in 2003-04 as part of a successful preemptive strike against a liquidity trap.*

at zero. These movements in output are indeed extreme. They lie well outside the 75 percent forecast probability interval reported in the chart. In fact, the quantitative details of the evolution of output and the interest rate under the counterfactual simulations should not be taken literally, since they depend significantly on the details of the model and on the assumption that the central bank insists on perfectly stabilizing current inflation. However, the qualitative pattern of higher volatility under the no-communication strategy is a robust feature of models in which expectations matter.

Under the full-communication strategy, in contrast, the desired path for inflation can be achieved with much less pronounced fluctuations in real growth and an almost unchanged policy relative to the actual path. Interest rates need not rise and output need not fall significantly because a shift in expectations brought about by clear communication of the Federal Reserve's inflation objective largely brings inflation under control.

Note that the results of these counterfactual exercises should be interpreted with caution. Their objective is not to prescribe an alternative to the policy followed in 2004, but rather to investigate how a different path for inflation might have been achieved. In fact, according to Krugman (1998) and Eggertsson and Woodford (2003), an increase in inflation expectations might be the best monetary strategy to escape a liquidity trap. Many have argued that the main objective of the Federal Reserve around 2003 was to minimize the U.S. economy's likelihood of falling into such a trap.[16] From this perspective, our analysis might be interpreted as supportive of the policy stance adopted by the central bank in 2003-04 as part of a successful preemptive strike against a liquidity trap.

6. Conclusion

This article provides an introduction to dynamic stochastic general equilibrium models and presents an example of their use as tools for monetary policy analysis. Given the mainly educational nature of our presentation, we simplify by using a small-scale model designed to account for the behavior of three key macroeconomic variables: GDP growth, core PCE inflation, and the federal funds rate. Despite its simplicity, our model is rich enough to reproduce some of the salient features of the series of interest. It also allows us to highlight the components common to more articulated and realistic DSGE specifications.

Our model offers insight into the causes of the abrupt pick-up in inflation in the first half of 2004, from levels close to 1 percent at the beginning of 2003 to values steadily above 2 percent through the end of 2008. This exercise highlights the central role of expectations in the transmission of shocks and policy impulses in DSGE models. The main lesson that we derive from the exercise is that the most effective approach to controlling inflation is through the management of expectations, rather than through actual movements of the policy instrument. This lesson seems to be well understood by the public, given the amount of attention and speculation that usually surround the pronouncements of central bankers on their likely future actions. DSGE models have the potential to broaden this understanding by adding a quantitative assessment of the link between current policy, expectations, and economic outcomes—and thus to clarify the effect that different systematic approaches to policy have on those outcomes.

[16] In its August 2003 statement, the FOMC observed that "on balance, the risk of inflation becoming undesirably low is likely to be the predominant concern for the foreseeable future." Very low or negative levels of inflation are one of the most likely triggers of a liquidity trap.

Technical Appendix

The table reports information on the prior distribution for the parameters of the model. Further details on the parameters and the structure of the model are available from the corresponding authors.

Parameter	Distribution	Mean	Standard Deviation
β	Calibrated	0.99	—
ξ	Gamma	0.1	0.05
ω	Gamma	1.0	0.2
η	Beta	0.6	0.2
ζ	Beta	0.6	0.2
ρ	Beta	0.7	0.15
ϕ_π	Normal	1.5	0.25
ϕ_y	Normal	0.5	0.2
π^*	Normal	2.0	1.0
r	Normal	2.0	1.0
γ	Normal	3.0	0.35
ρ_{π^*}	Beta	0.95	0.04
ρ_δ	Beta	0.5	0.2
ρ_γ	Beta	0.5	0.2
ρ_u	Beta	0.5	0.2
σ_δ	InvGamma	0.5	2.0
σ_γ	InvGamma	0.5	2.0
σ_u	InvGamma	0.5	2.0
σ_{π^*}	InvGamma	0.2	1.0
σ_i	InvGamma	0.5	2.0

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