Thomas Eisenbach, Todd Keister, James McAndrews, and Tanju Yorulmazer

Stability of Funding Models: An Analytical Framework

- During the recent financial crisis, many institutions and some market-based intermediation arrangements experienced strains owing to declining asset values and a drying-up of funding sources.
- Although these stress events led several institutions to fail and others to require extraordinary public support, a full understanding of their causes still proves elusive.
- This study clarifies that understanding by providing a rigorous, yet easily applicable, framework for analyzing the sources of the stress events and the effect of various funding structure characteristics on financial stability.
- The framework can potentially help policymakers form views on regulatory reform and evaluate the ways that policy options may affect financial stability.

1. Introduction

The recent financial crisis highlighted the fragility of many financial intermediaries. A large number of commercial banks, investment banks, and money market mutual funds (MMFs) experienced strains created by declining asset values and a loss of funding sources, as did some market-based intermediation arrangements such as asset-backed commercial paper (ABCP). These strains were severe enough to cause several institutions to fail and others to require extraordinary public support. In reviewing these events, one notices that some arrangements appear to have been more stable—that is, better able to withstand shocks to their asset values and/or funding sources—than others.[1] The precise determinants of this stability are not well understood. Gaining a better understanding of these determinants is a critical task for both market participants and policymakers as they try to design more resilient arrangements and improve financial regulation.

In this article, we use a simple analytical framework to illustrate how the characteristics of an arrangement for financial intermediation (a funding model) affect its ability to survive stress events. There is a large and growing literature on this issue; see Yorulmazer (2014b) for a detailed review. Our

[1] See Yorulmazer (2014a) for a detailed discussion of the experiences of several distinct types of intermediation arrangements during the crisis.

Thomas Eisenbach is an economist at the Federal Reserve Bank of New York; Todd Keister is a professor of economics at Rutgers University; James McAndrews is an executive vice president and the director of research at the Federal Reserve Bank of New York; Tanju Yorulmazer is a research officer at the Bank.

Correspondence: tanju.yorulmazer@ny.frb.org

aim here is to present an approach that is sufficiently general to encompass a wide range of intermediation arrangements, but sufficiently simple to illustrate the economic forces at work in a transparent and intuitive way. Our hope is that this analysis will provide policymakers with a useful starting point for more detailed evaluations of alternative arrangements and for the analysis of regulatory proposals.

Our framework begins with the simplified balance sheet of a representative financial intermediary. The intermediary holds two types of assets: safe and risky. Safe assets are always liquid, but risky assets may be illiquid in the short run. On the liability side of its balance sheet, the intermediary has short-term debt, long-term debt, and equity. This intermediary faces two types of risk: The value of its assets may decline and/or its short-term creditors may decide not to roll over their debt. We measure the stability of the intermediary by looking at what stress events it can survive, that is, what combinations of shocks to the value of its assets and to its funding it can experience while remaining solvent.

An important issue in any such analysis lies in determining the conditions under which short-term creditors will and will not choose to roll over their debt. We do not try to explain creditor behavior in our framework; instead, we treat this behavior as exogenous. This approach greatly simplifies the model and allows us to present an intuitive analysis of the determinants of stability. Again, a way to think of our analysis is that it subjects banks to different types of stress events. In most of our applications, we hold fixed the balance sheet of the bank, and ask whether the bank is stable for different sizes of short-term creditor runs and declines in the value of its assets. The creditor behavior in our framework is used as a parameter that generates a certain size of run on the bank. The insights from our analysis are likely to carry over to more complex models where creditor behavior is endogenous; developing such models is a promising area for future research.[2]

We study how the stability of this intermediary depends on various balance-sheet characteristics, such as its leverage, the maturity structure of its debt, and the liquidity and riskiness of its asset portfolio. Some of the results we derive are straightforward, such as the effect of higher leverage and a higher liquidation value of the risky asset. Higher leverage increases the debt burden of the financial intermediary, makes it more susceptible to creditor runs, and decreases the buffer provided by equity capital. As a result, higher leverage always makes the intermediary more vulnerable to shocks. As the liquidation value of the risky asset increases, the intermediary needs to liquidate a smaller portion of the risky asset in its portfolio to make the payments to the short-term creditors that choose not to roll over. As a result, a higher liquidation value of the risky asset always makes the intermediary more resilient to creditor runs.

Other results, however, demonstrate that the determinants of stability can be subtle. For example, lengthening the maturity structure of the intermediary's debt tends to make it more resilient to funding shocks by decreasing reliance on short-term debt that can be withdrawn. However, since long-term debt can be a more costly way of finance compared with short-term debt, lengthening the maturity structure can increase the debt burden and make the intermediary more vulnerable to shocks to the value of its assets. Similarly, holding a safer asset portfolio can make the intermediary either more or less vulnerable to shocks, depending on the other characteristics of its balance sheet. Some of these effects are dependent on the characteristics of both the asset and liability side of the bank's balance sheet, and one advantage of our framework is that it allows us to consider the influence of both sides of the balance sheet simultaneously.

We then show how our framework can be applied to study various policy issues. While capital requirements have traditionally been a tool for regulators, recently there have been attempts at introducing liquidity requirements. First, we analyze how liquidity holdings and equity capital interact in achieving bank stability. Again, the results can be quite subtle. As one would expect, liquidity and capital can be substitutes but they can also be complements. If the risky asset pays more than cash in expectation, higher liquidity holdings can decrease the return on the bank's portfolio and therefore would result in the bank requiring more equity capital to achieve the same level of stability.

In the wake of the crisis, a number of policies related to financial intermediation are being reconsidered and new regulations are being designed. We show how our framework can help illustrate the effects of the Basel III Liquidity Coverage Ratio. We show that liquidity requirements can have competing effects on stability, making a bank more resilient to funding shocks but less resilient to shocks to the value of its risky assets.

We also show how the framework can be used to study discount window (DW) policy, where the bank can borrow from the window rather than liquidating the risky asset at a cost. We show that a lenient DW policy that has a lower haircut and a lower interest rate can allow the bank to withstand higher shocks ex post. However, we should mention that any

[2] Within the growing literature on this topic, our paper is most closely related to that of Morris and Shin (2009), who also study the stability of an intermediary. They define the illiquidity component of credit risk to be the probability that the intermediary will fail because it is unable to roll over its short-term debt, even though it would have been solvent had the debt been rolled over. Morris and Shin (2009) use techniques from the theory of global games to determine creditors' behavior as part of the equilibrium of their model.

such ex post benefit should be weighed against the effect on bank behavior ex ante.

Since the crisis, the difference between collateralized and uncollateralized funding and asset encumbrance has received attention. We use our framework to study the effect of asset encumbrance on bank stability. We show that asset encumbrance can increase insolvency risk when the fraction of encumbered assets is sufficiently high.

Money market mutual funds were at the heart of many important debates since the Reserve Primary Fund "broke the buck" after the failure of Lehman Brothers. We use our framework to analyze different approaches to reforming money market mutual funds. In particular, we analyze the effect of the minimum-balance-at-risk proposal, where creditors can only redeem up to a fraction of their claims early while the remaining fraction becomes a long-term junior debt claim. This increases the resilience of the fund to funding shocks and mitigates the fragility created by the requirement to sustain a net asset value of 1.

In the next two sections, we present our baseline model and examine the determinants of stability within this framework. In section 4, we adapt the model in order to apply it to a collection of current policy issues, including the effects of liquidity regulation, discount window policies, and approaches to reforming money market mutual funds. We offer some concluding remarks in section 5.

2. A Simple Model

There are three dates, labeled $t = 0, 1, 2$, and a single, representative financial institution. We refer to this institution as a bank for simplicity but, as we discuss below, it can be thought of as representing a variety of different arrangements for financial intermediation. We begin by specifying the elements of this bank's balance sheet.

2.1 The Balance Sheet

At $t = 0$, the bank holds m units of a safe, liquid asset, which we call cash, and y units of a risky, long-term asset. Cash earns a gross return r_1 between periods 0 and 1 and a gross return r_s between periods 1 and 2. The risky asset yields a random gross return θ if held until $t = 2$, but a smaller return $\tau\theta$ if liquidated at $t = 1$. The realized value of θ is observed by all agents at the beginning of $t = 1$.

The bank has issued s units of short-term debt that matures at $t = 1$ and ℓ units of long-term debt that matures at $t = 2$. To simplify the analysis, we assume that the promised return on the bank's short-term debt is the same as the return it earns on the liquid asset, that is, r_1 between periods 0 and 1 and r_s between periods 1 and 2.[3] The long-term debt ℓ promises a gross interest rate $r_\ell > r_s$ between periods 0 and 2. In addition, the bank has an amount e of equity. We normalize $r_1 = 1$ throughout the analysis.[4] The bank's balance sheet thus has the following form:

Assets	Liabilities
m	s
y	ℓ
	e

.

Short-term debtholders decide whether to roll over their claims at $t = 1$ after observing the realized value of θ. If the bank is able to meet its obligations to all debtholders, any remaining funds at $t = 2$ are paid to equityholders. If the bank is unable to meet its obligations, it enters bankruptcy and a fraction ϕ of its assets is lost to bankruptcy costs. The remaining assets are then distributed to debtholders on a pro-rata basis.

We make the following assumptions on parameter values:

Assumption 1: $r_s < r_\ell < \frac{1}{\tau}$.
This assumption ensures that neither form of financing—long-term or short-term debt—strictly dominates the other. As will become clear below, $1/\tau$ is the cost of repaying short-term debtholders that withdraw early and force asset liquidation, while r_s is the cost of repaying short-term debtholders that roll over. Since r_ℓ is the cost of repaying a long-term debtholder, Assumption 1 states that short-term debt is cheaper than long-term debt ex post if and only if it is rolled over or does not force early liquidation.

Assumption 2: $\theta\tau \leq 1$.
This second assumption implies that paying an early withdrawal with cash is always cheaper than liquidating the risky asset.

[3] The framework can be easily generalized by allowing these returns to differ.

[4] Alternatively, we can interpret s, ℓ, and m as the $t = 1$ values of each variable, including all interest accrued between $t = 0$ and $t = 1$.

2.2 Solvency

The bank is solvent if it is able to meet all of its contractual obligations in both periods. The solvency of the bank will depend on the realized return on its assets as well as the rollover decisions of the short-term debtholders. Let α denote the fraction of short-term debtholders who decide *not* to roll over—that is, to withdraw funding from the bank—at $t = 1$. If $\alpha s \leq m$, the bank can pay all of these claims from its cash holdings. If $\alpha s > m$, however, the bank does not have enough cash to make the required payments and must liquidate some of the long-term asset.

The matured value of the bank's remaining assets at $t = 2$ when $\alpha s \leq m$ holds is given by

$$\theta y + r_s(m - \alpha s).$$

In this case, paying out an additional dollar at $t = 1$ would reduce the bank's cash holdings by one unit, lowering the $t = 2$ value of assets by r_s. When $\alpha s \geq m$, however, paying out an additional dollar at $t = 1$ requires liquidating $1/(\tau\theta)$ units of the long-term asset, which lowers the $t = 2$ value of the bank's assets by $1/\tau$. In this case, the matured value of the bank's remaining assets at $t = 2$ can be written as

$$\theta\Big(y - \frac{\alpha s - m}{\tau\theta}\Big).$$

We can combine these two expressions by defining $\chi(\alpha)$ to be the marginal cost at $t = 2$ of funds used to make $t = 1$ payments, that is,

1) $$\chi(\alpha) \equiv \begin{cases} r_s \text{ for } \alpha \leq \frac{m}{s} \\ 1/\tau \text{ for } \alpha > \frac{m}{s} \end{cases}.$$

The matured value of the bank's remaining assets at $t = 2$ can then be written for any value of α as

2) $$\theta y + \chi(\alpha)(m - \alpha s).$$

Note that if expression 2 is negative, the bank is actually insolvent at $t = 1$, as it is unable to meet its immediate obligations even after liquidating all of its assets. In this case, short-term debtholders that withdraw funding at $t = 1$ in expectation receive a pro-rata share of the liquidation value of the bank's assets while all other debtholders receive zero.[5] When expression 2 is positive, short-term debtholders that withdraw funding at $t = 1$ receive full payment and the bank is solvent at $t = 2$ if and only if the matured value of its remaining assets is larger than its remaining debts, that is,

3) $$\theta y + \chi(\alpha)(m - \alpha s) \geq (1 - \alpha)s r_s + \ell r_\ell.$$

Note that solvency of the bank at $t = 2$ implies that it is also solvent at $t = 1$. We can rewrite condition 3 as

4) $$\theta \geq \frac{s r_s + \ell r_\ell + [\chi(\alpha) - r_s]\alpha s - \chi(\alpha)m}{y} \equiv \theta(\alpha).$$

The variable $\theta(\alpha)$ identifies the minimum return on the risky asset that is needed for the bank to be solvent, conditional on a fraction α of short-term debtholders withdrawing funding and the remaining $(1 - \alpha)$ rolling over their claims. For $\alpha s \leq m$, this cutoff value simplifies to

5) $$\theta(\alpha) = \frac{s r_s + \ell r_\ell - m r_s}{y} \equiv \underline{\theta} \text{ for all } \alpha \leq \frac{m}{s}.$$

When none of the long-term asset is liquidated at $t = 1$, solvency of the bank depends only on the $t = 2$ values of its assets and debts. Within this range, the value of α does not matter because additional withdrawals at $t = 1$ reduce the value of the bank's assets and liabilities by exactly the same amount.

For $\alpha s > m$, the cutoff becomes

6) $$\theta(\alpha) = \frac{s r_s + \ell r_\ell + [1/\tau - r_s]\alpha s - (1/\tau)m}{y} \equiv \theta^*(\alpha)$$

$$\text{for all } \alpha > \frac{m}{s}.$$

In this case, Assumption 1 implies that $\theta^*(\alpha)$ is increasing in α. Additional withdrawals at $t = 1$ now force liquidation of the long-term asset and thus reduce the value of the bank's assets more than they reduce the value of its liabilities. As a result, a higher return on the long-term asset is required to maintain solvency. If all short-term creditors withdraw funding, we have

7) $$\theta(1) = \frac{s + \tau\ell r_\ell - m}{\tau y} \equiv \overline{\theta}.$$

If the realized return θ is greater than $\overline{\theta}$, the bank will be solvent at $t = 2$ regardless of the actions short-term debtholders take at $t = 1$.

[5] We assume that the bank cannot suspend convertibility, so that the bank pays in full the promised amount to short-term debtholders that withdraw at $t = 1$ until it runs out of funds. We assume that the position of the short-term debtholders that decide to withdraw at $t = 1$ in the line is randomly assigned from a uniform distribution. Thus, short-term debtholders that withdraw at $t = 1$ in expectation receive a pro-rata share of the liquidation value of the bank's assets while all other debtholders receive zero.

2.3 Stability

We measure the stability of the bank by asking for what combinations of α and θ it remains solvent. In other words, what stress events, in terms of both asset values and funding conditions, will the bank survive? Exhibit 1 illustrates the answer by dividing the space of pairs (α, θ) into four regions. When θ is below $\underline{\theta}$, the return on the risky asset is so low that the bank will be insolvent regardless of how many short-term debtholders roll over their claims. In this case, we say the bank is *fundamentally insolvent*. When θ is between $\underline{\theta}$ and $\bar{\theta}$, the bank will survive if sufficiently many short-term debtholders roll over their claims, but will fail if too few do. In the former case, we say the bank is *conditionally solvent*, meaning that the fact that it remains solvent depends on the realized rollover decisions of the short-term debtholders. In the latter case, when (α, θ) fall in the triangular region below the blue line in the exhibit, we say the bank is *conditionally insolvent*. Finally, when θ is larger than $\bar{\theta}$, the bank will be solvent regardless of the actions of short-term debtholders. In this case, we say the bank is *fundamentally solvent*.

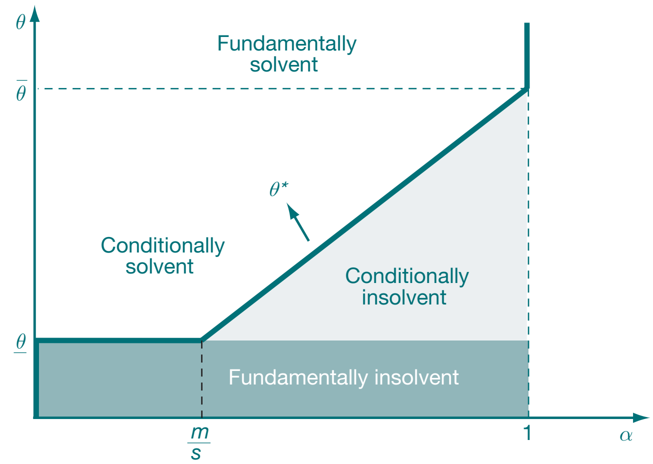
In the sections that follow, we ask how the characteristics of the bank's balance sheet determine the size of the four regions in the diagram in Exhibit 1. We then use this diagram to study how various changes and policy reforms would affect the bank's ability to survive these stress events.

2.4 Discussion

Our goal is to present an analysis of bank stability that can be largely understood graphically, using diagrams like that in Exhibit 1. This approach requires keeping the model simple, so that the relevant information can be conveyed clearly. One of our key simplifying assumptions is that the behavior of short-term debtholders is exogenous to the model. In particular, we assume that the joint probability distribution over the random variables (α, θ) is independent of the bank's balance sheet. It is worth noting, however, that short-term debtholders' incentives are perfectly aligned with the regions in this diagram. Specifically, we show in the appendix that individual short-term debtholders would prefer to roll over their claims at $t = 1$ if and only if the realization of (α, θ) places the bank in one of the two solvency regions in Exhibit 1. In this sense, our analysis is at least broadly consistent with optimizing behavior by debtholders.

There is a large literature that uses equilibrium analysis to study the determinants of creditor behavior in settings similar to the one we study here. The seminal paper by Diamond and Dybvig (1983), for example, shows how multiple equilibria

Exhibit 1
Solvency Regions



can arise in the game played by a bank's depositors—one in which they leave their funds deposited and the bank survives, and another in which they withdraw their funds and the bank fails. The subsequent literature has debated the extent to which historical banking panics were driven by this type of self-fulfilling belief or by real shocks that made banks fundamentally insolvent.[6] Other papers have aimed to uniquely determine creditor behavior within the model in order to pin down the set of states in which insolvency occurs.[7] We do not attempt to contribute to either of these debates here. Instead, we take an intentionally agnostic view of creditor behavior: The fraction of short-term creditors that withdraw funding is random and is determined by factors outside of our simple model. Doing so allows us to focus on our question of interest—the determinants of a bank's ability to survive stress events—with minimal technical complication.

3. Determinants of Bank Stability

In this section, we investigate how the stability of the bank depends on the parameters of the model. We begin by examining how the solvency regions in Exhibit 1 depend on two characteristics of the bank's liabilities: its leverage and the maturity structure of its debt. We then evaluate

[6] See, for example, Gorton (1988), Saunders and Wilson (1996), Allen and Gale (1998), and Ennis (2003).

[7] Contributions on this front include Postlewaite and Vives (1987), Chari and Jagannathan (1988), and Goldstein and Pauzner (2005).

the effects of changing two asset-side characteristics: the liquidation value of the risky asset and the composition of the bank's asset portfolio.

3.1 Leverage

Let $d \equiv s + \ell$ denote the bank's total amount of debt and let

$$\sigma \equiv \frac{s}{s+\ell} \tag{8}$$

denote the fraction of this debt that is short term. We normalize the total size of the bank's balance sheet to 1, so that the amount of equity is given by $e = 1 - d$. We can then write the quantities of short-term and long-term debt, respectively, as

$$s = \sigma(1-e) \text{ and } \ell = (1-\sigma)(1-e).$$

To examine the effect of leverage, we hold the maturity structure σ of the bank's debt fixed and vary the amount of equity e.

Using this modified notation, the cutoff value $\underline{\theta}$ below which the bank is fundamentally insolvent, as defined in equation 5, can be written as.

$$\underline{\theta} = \frac{[\sigma r_s + (1-\sigma)r_\ell](1-e) - r_s m}{y}. \tag{9}$$

This cutoff is strictly decreasing in e: More equity (that is, lower leverage) reduces the size of the fundamental insolvency region because there is less total debt that must be repaid. In the region where $\alpha s > m$ and the bank must liquidate assets at $t = 1$, the critical value separating conditional solvency and insolvency defined in equation 6 can be written as

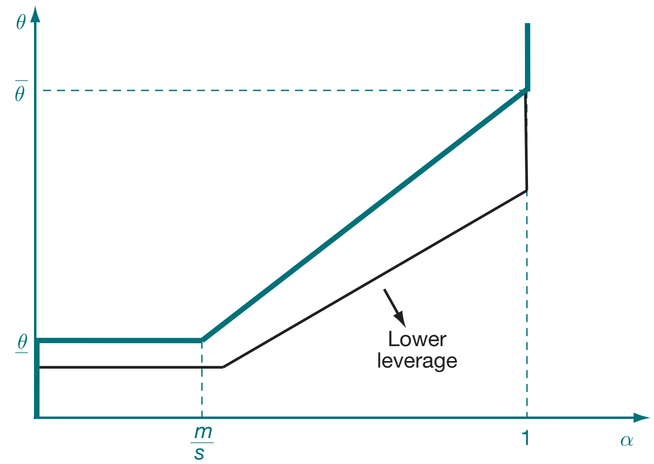
$$\theta^*(\alpha) = \frac{[\sigma(\alpha\frac{1}{\tau} + (1-\alpha)r_s) + (1-\sigma)r_\ell](1-e) - \frac{1}{\tau}m}{y}. \tag{10}$$

This cutoff is also strictly decreasing in e, for exactly the same reason. The changes in these two solvency boundaries are depicted in Exhibit 2, where an increase in equity (that is, a decrease in leverage) corresponds to a move from the blue curve to the black one. The exhibit demonstrates that lower leverage strictly reduces the bank's insolvency risk by making it better able to withstand shocks to both its asset values and its funding. In other words, lower leverage is associated with unambiguously greater stability.

The sensitivity of the solvency threshold $\theta^*(\alpha)$ to additional withdrawals is given by the derivative

$$\frac{d\theta^*(\alpha)}{d\alpha} = \frac{\sigma(\frac{1}{\tau} - r_s)(1-e)}{y}. \tag{11}$$

Exhibit 2
Effect of Leverage



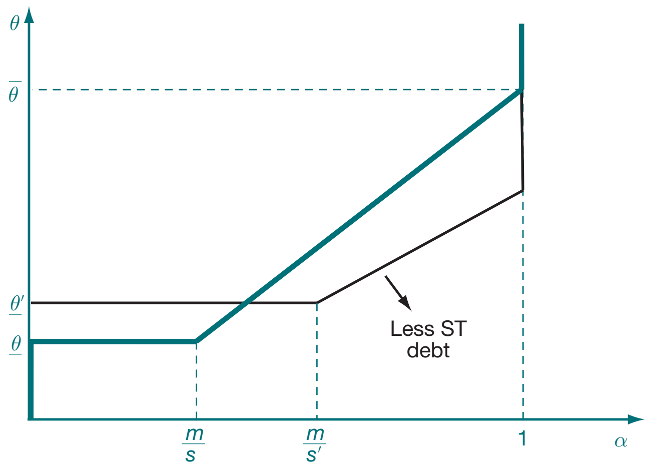
This derivative corresponds to the slope of the line separating the conditionally solvent and conditionally insolvent regions in the exhibit. The slope is positive because additional withdrawals reduce the value of the bank's remaining assets by more than they reduce the value of its remaining liabilities, effectively increasing the debt burden at $t = 2$. Notice, however, that the slope is decreasing in e. Holding more equity (and less debt) reduces the sensitivity of the debt burden to withdrawals and thus also reduces the sensitivity of the conditional solvency threshold to withdrawals. In other words, lower leverage makes the slope of the solvency boundary flatter, as depicted in Exhibit 2.

3.2 Maturity Structure of Debt

Next, we study the effects of changing the maturity structure of the bank's debt. Recall from equation 8 that σ measures the fraction of the bank's debt that is short term. Our interest is in how changing σ, while holding equity e and total debt d fixed, affects the bank's ability to survive stress events.

The cutoff value $\underline{\theta}$ below which the bank is fundamentally insolvent was given in equation 9. Assumption 1 states that $r_\ell > r_s$ and hence this cutoff is strictly decreasing in σ. In other words, lengthening the average maturity of the bank's debt (by shifting some from short term to long term) makes the bank more likely to become fundamentally insolvent. Intuitively, long-term debt is more costly than short-term debt and therefore lengthening the average maturity increases the bank's total debt burden at $t = 2$. The higher debt burden,

Exhibit 3
Effect of Maturity Structure of Debt



in turn, implies that a higher return θ on the risky asset is required to avoid insolvency. This change is illustrated in Exhibit 3, which shows the effect of lowering the quantity of short-term debt from s to s' while increasing the quantity of long-term debt by the same amount. For returns in the interval $(\underline{\theta}, \underline{\theta}')$, the bank will now be fundamentally insolvent, whereas it would have potentially been solvent with the higher level of short-term debt s.

Exhibit 3 also highlights two countervailing effects of decreasing short-term debt. First, the cutoff point m/s increases, meaning that the bank can withstand a larger funding shock (α) without having to liquidate any of its long-term assets. In addition, the slope of the solvency boundary in the region where $\alpha > m/s$ becomes flatter. This slope was given in equation 11 and—because $1/\tau > r_s$—is easily seen to be increasing in σ. Taken together, these two changes imply that decreasing the bank's short-term debt shrinks the conditional insolvency region in the diagram. For any given funding shock α, a bank with less short-term debt will have less need to liquidate assets at $t = 1$ and is thus less likely to become insolvent due to the loss of funding.

Our framework thus demonstrates how changing the maturity structure of a bank's debt has two competing effects on its ability to survive stress events. Having less short-term debt makes the bank less vulnerable to funding shocks by decreasing its dependence on the actions of short-term debtholders. At the same time, however, it also increases the bank's total debt burden at $t = 2$ and therefore increases the likelihood that the return on the bank's assets will be insufficient to cover these debts. Put differently, a bank financed largely by long-term debt and equity is protected from the conditional insolvency caused by a loss of funding from short-term debtholders. However, it is also clear that long-term debt is not equivalent to equity and increasing the long-term debt burden can raise the likelihood of fundamental insolvency.

A key takeaway from our analysis therefore is that having banks lengthen the maturity structure of their liabilities does not make them unambiguously more stable or less likely to become insolvent. Instead, the benefits of having lower rollover risk must be balanced against the costs associated with a higher debt burden.

3.3 Liquidation Value

We now turn to the characteristics of the bank's asset holdings and ask how the solvency and insolvency regions in Exhibit 1 depend on the liquidation value τ. Equation 5 demonstrates that the bound for fundamental insolvency, $\underline{\theta}$, is independent of τ. This lower bound represents a scenario in which the bank has enough cash to pay short-term debtholders that do not roll over at $t = 1$, so that no liquidation is needed and the value of τ has no effect on the bound.

Looking next at the threshold for conditional solvency in equation 6, we have

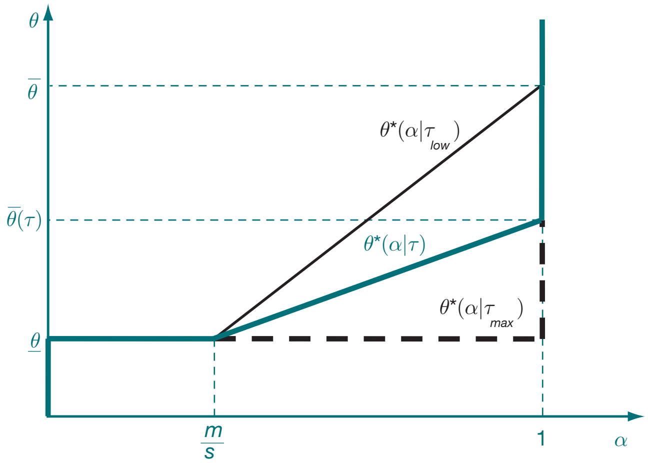
$$\frac{d\theta^*(\alpha)}{d\tau} = -\frac{\alpha s - m}{\tau^2 y} < 0.$$

We know this expression is negative because $\theta^*(\alpha)$ applies only in the region where $\alpha s > m$. This result demonstrates that for all such values of α, the threshold value θ^* is strictly decreasing in τ.

Exhibit 4 illustrates this result. The blue curve corresponds to the baseline value of τ used in the previous exhibits. If the liquidation value is lower, such as at τ_{low}, the curve shifts to that depicted in black. For values of α smaller than m/s, there is no change in the threshold value θ^* because no liquidation takes place; insolvency in this case is determined solely by the period-2 value of assets and liabilities. For higher values of α, however, the threshold value θ^* becomes larger (shifts up in the exhibit) because payments made to short-term creditors are now more expensive in terms of period-2 resources. As the exhibit shows, shifting to τ_{low}, shrinks the region of conditional solvency and expands the region of conditional insolvency.

If the liquidation value rises, however, the threshold value of θ^* falls (shifts down in the exhibit) and the solvency region becomes larger. The extreme case is where $\tau = 1/r_s$, which means that liquidating the long-term asset is not more costly than using cash to pay investors at $t = 1$. In this case, the

Exhibit 4
Effect of Asset Liquidation Value τ



threshold value θ^* is equal to $\underline{\theta}$ for all values of α. The curve separating the solvency and insolvency regions in this case corresponds to the dashed black line in Exhibit 4—the bank is solvent for values of θ above $\underline{\theta}$ and insolvent for values below $\underline{\theta}$, regardless of the value of α.

3.4 Liquidity Holdings

We now study the effect of changing the composition of the bank's asset holdings. We again normalize the size of the bank's balance sheet to 1, so that we have $m + y = 1$. Both the critical value $\underline{\theta}$ for fundamental insolvency and the critical value $\theta^*(\alpha)$ for conditional insolvency depend on the composition of the bank's assets. Substituting $y = 1 - m$ into equations 5 and 6, these two critical values become

12) $$\underline{\theta} = \frac{sr_s + \ell r_\ell - r_s m}{1 - m}$$

and

13) $$\theta^*(\alpha) = \frac{s(\alpha\frac{1}{\tau} + (1 - \alpha)r_s) + \ell r_\ell - \frac{1}{\tau}m}{1-m}.$$

Looking first at the critical value for fundamental insolvency, the effect of increasing cash and decreasing risky asset holdings by the same amount is given by

$$\frac{d\underline{\theta}}{dm} = \frac{\underline{\theta} - r_s}{1 - m}.$$

This expression is negative, and hence the risk of fundamental insolvency is reduced by a more liquid asset portfolio, if and only if $\underline{\theta} < r_s$. Intuitively, if $\underline{\theta}$ is less than r_s, then at the insolvency boundary, the return on the risky asset is lower than the return on cash, which means that having more cash raises the bank's total return on assets. In this case, insolvency risk is decreasing in liquidity holdings. However, if $\underline{\theta} > r_s$, then the risky asset pays off *more* than cash at the insolvency boundary and holding more cash lowers the bank's total return on assets. In this case, insolvency risk is *increasing* in liquidity holdings.

To see when this latter case of "harmful liquidity" applies, we can use the expression for $\underline{\theta}$ in equation 12 to show that $\underline{\theta} > r_s$ if and only if

$$sr_s + \ell r_\ell > r_s.$$

This condition is more likely to be satisfied, first, when total debt $s + \ell$ is large and second, when ℓ is large relative to s for given total debt. Since we have fixed the size of the balance sheet to $s + \ell + e = 1$, this means situations with high leverage and/or long debt maturity, respectively. The intuition for this result is as follows: Cash has return r_s, which is less than the interest rate on long-term debt r_ℓ. The only way to repay long-term debt is with assets that pay a higher return than cash. A bank with little equity and a large amount of long-term debt therefore increases its risk of fundamental insolvency if it shifts to a more liquid asset portfolio. These two possibilities are illustrated in Exhibit 5 for a bank with more long-term debt and higher leverage (thin black line) and a bank with less long-term debt and lower leverage (thick black line). Both banks share the same initial insolvency boundary (blue) but respond differently to an increase in their cash holdings from m to m'.

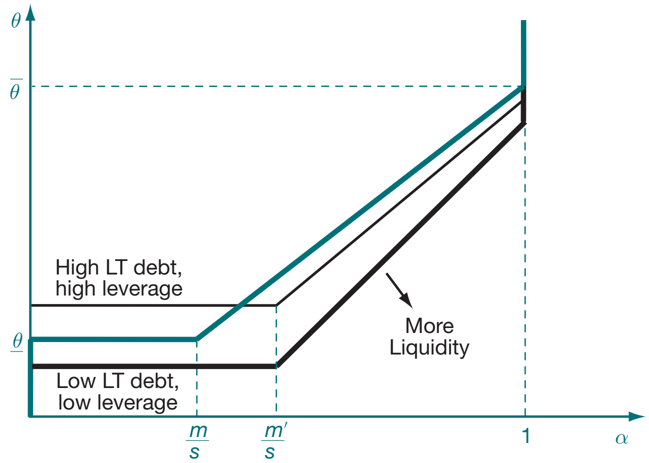
We now turn to the effect of asset composition on the risk of conditional insolvency. Using equation 13, we have

14) $$\frac{d\theta^*(\alpha)}{dm} = \frac{\theta^*(\alpha) - \frac{1}{\tau}}{1 - m}.$$

Similar to above, the effect of liquidity on conditional insolvency risk depends on the relative returns of risky assets and cash at the insolvency boundary. However, now the effective return to holding an extra unit of cash is $\frac{1}{\tau} > r_s$ because it saves on the liquidation of long-term assets at $t = 1$. Using the expression for $\theta^*(\alpha)$ in equation 13, we can show that the derivative in equation 14 is always negative. First, equation 13 implies that $\theta^*(\alpha) < \frac{1}{\tau}$ holds if and only if

15) $$s\left(\alpha\frac{1}{\tau} + (1 - \alpha)r_s\right) + \ell r_\ell < \frac{1}{\tau}.$$

EXHIBIT 5
Effect of Liquidity Holdings



Note that the left-hand side of condition 15 is increasing in α, meaning that the condition is harder to satisfy with higher values of α. Setting $\alpha = 1$ and using the fact that $s + \ell + e = 1$, the condition simplifies to

$$\left(\tfrac{1}{\tau} - r_\ell\right)\ell + \tfrac{1}{\tau}e > 0,$$

which holds because Assumption 1 states that $r_\ell < 1/\tau$. Since condition 15 is satisfied for $\alpha = 1$, it is also satisfied for any $\alpha < 1$. We can therefore conclude that $d\theta^\star(\alpha)/\,dm < 0$, that is, extra liquidity unambiguously reduces the risk of conditional insolvency.

Looking at how liquidity holdings affect the slope of the conditional solvency threshold, we have

$$\frac{d\theta^\star(\alpha)}{d\alpha} = \frac{\sigma(\frac{1}{\tau} - r_s)(1 - e)}{1 - m}.$$

Recall that the slope $d\theta^\star(\alpha)/d\alpha$ represents the sensitivity of the solvency threshold $\theta^\star(\alpha)$ to additional withdrawals. Because we are in the region where some long-term assets must be liquidated at $t = 1$, additional withdrawals reduce the value of the bank's remaining assets by more than they reduce the value of its remaining liabilities, increasing its debt burden at $t = 2$. Meeting this higher debt burden requires a higher total return on assets $(\theta(1 - m) + \frac{1}{\tau}m)$. Holding a more liquid asset portfolio reduces the sensitivity of this total payoff to the asset payoff θ, meaning that for a given increase in α, a larger increase in θ is required to maintain conditional solvency: The slope gets steeper.

These different effects of liquidity on bank stability are all present in Exhibit 5. Where insolvency is conditional—that is, the boundary has a positive slope—the curve shifts down and becomes steeper for both banks (thin black and thick black lines): More liquidity reduces insolvency risk but increases the sensitivity to withdrawals. Where insolvency is fundamental—and the boundary is horizontal—the line can shift up or down: More liquidity can reduce the risk of fundamental insolvency (thick black line), but it can also increase it if leverage is high and/or debt maturity is long (thin black line).

3.5 Discussion

The results in this section have shown how the determinants of a bank's ability to survive stress events are often intuitive, but can sometimes be rather subtle. Decreasing leverage, for example, clearly improves stability, since it decreases both the probability of fundamental insolvency and the probability of conditional insolvency. Having a higher liquidation value for assets also unambiguously improves stability. While this change has no effect on the likelihood of a bank becoming fundamentally insolvent, it always reduces the likelihood of conditional insolvency.

For other changes in balance-sheet characteristics, however, a trade-off can arise in which improving stability in one dimension tends to undermine it in the other. Lengthening the average maturity of a bank's debt lowers the probability of conditional insolvency, for example, but raises the probability of fundamental insolvency. In other words, this change tends to make the bank better able to withstand shocks to its short-term funding sources, but less able to withstand shocks to the value of its assets. Shifting the composition of the bank's assets toward safe, liquid assets also tends to lower the probability of conditional insolvency, but can either raise or lower the probability of fundamental insolvency. In cases like this where the results are ambiguous, our framework helps illustrate the sources of this ambiguity and when a trade-off is most likely to arise. Increasing the bank's liquid asset holdings is most likely to raise the probability of fundamental insolvency when the bank is highly leveraged or has a large amount of long-term debt.

In the next section, we build on the results presented so far to study a range of current policy issues. In each case, we study how a particular change or policy proposal would affect the balance-sheet characteristics of the relevant financial intermediaries. We then derive the corresponding changes in the solvency regions of our diagram and interpret the results.

4. Applications

In this section, we utilize our framework to analyze a series of current policy issues. First, we analyze the effect of liquidity and capital on stability and the trade-off between the two. We then study the effects of policy tools such as the Liquidity Coverage Ratio and discount window lending. Another issue we analyze is the effect of encumbered assets on bank stability. As a specific intermediation structure, we study asset-backed commercial paper structures, which illustrate an interesting case with their asset structure and heavy reliance on short-term debt. Finally, we analyze money market mutual funds and various policy proposals to make them more stable.

4.1 Liquidity versus Capital

Traditionally, capital requirements have been the main tool of bank regulators. Since the financial crisis, liquidity requirements have received increased attention. Further below we analyze a specific liquidity requirement, the Liquidity Coverage Ratio. But first, we study more generally how liquidity holdings (on the asset side) and equity capital (on the liability side) interact in our framework and whether they are substitutes or complements.

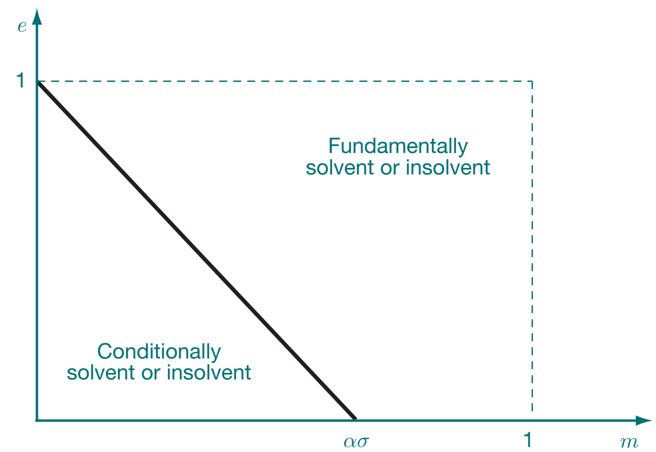
As in sections 3.1 and 3.4, we normalize the size of the bank's balance sheet to 1 so that $y + m = 1$ on the asset side and $s + \ell + e = 1$ on the liability side, and then denote the fraction of short-term debt by $\sigma \equiv s/(s + \ell)$. We now take α and θ as given and study bank solvency for different combinations of m and e. Note the difference from the analysis before, where we took m and e as given and studied bank solvency for different combinations of α and θ.

As before, one of two solvency conditions will be relevant, depending on whether the bank is facing fundamental insolvency or conditional insolvency. The distinction is whether the bank has to liquidate assets to satisfy withdrawals or not, that is, $\alpha\sigma(1 - e) \gtrless m$. This condition divides the m-e space into two regions with the dividing line given by:

$$e = 1 - \frac{1}{\alpha\sigma} m.$$

Exhibit 6 illustrates the two regions. For combinations (m, e) above and to the right of the line, the bank has enough cash to pay all withdrawing creditors so it is either fundamentally solvent or fundamentally insolvent. For combinations (m, e) below and to the left of the line, the bank is forced to liquidate assets so it is either conditionally solvent or conditionally insolvent.

Exhibit 6
Different Regions in *m*-*e* Space



We start with the region of conditional solvency/insolvency where the solvency constraint is given by:

$$\theta(1 - m) + \tfrac{1}{\tau}m \geq \left[\sigma\left(\alpha\tfrac{1}{\tau} + (1 - \alpha)r_s\right) + (1 - \sigma)r_\ell\right](1 - e).$$

To depict this solvency threshold in the m-e space, we solve for e:

$$e = 1 - \frac{\theta(1 - m) + \frac{1}{\tau}m}{\sigma(\alpha\frac{1}{\tau} + (1 - \alpha)r_s) + (1 - \sigma)r_\ell}.$$

For a given level of withdrawals α and a given asset payoff θ, this line is the solvency threshold in terms of liquidity m and capital e. Therefore, it represents the trade-off between different combinations of liquidity and capital that keep the bank on the solvency threshold. To illustrate this trade-off, we note that the slope of the line is:

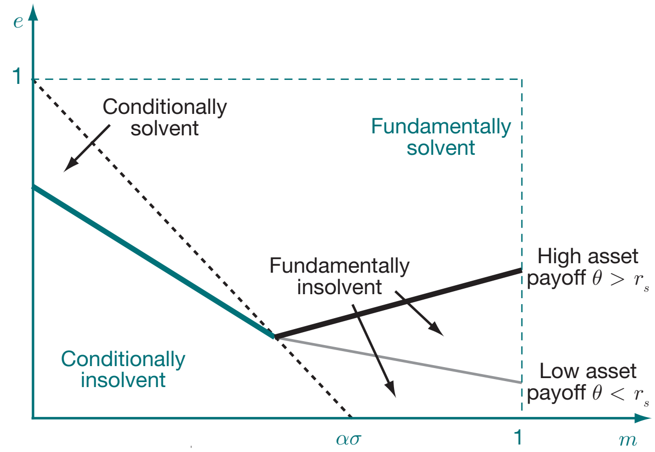
$$\frac{de}{dm} = \frac{\theta - \frac{1}{\tau}}{\sigma(\alpha\frac{1}{\tau} + (1 - \alpha)r_s) + (1 - \sigma)r_\ell} < 0.$$

The slope is negative since $\theta\tau < 1$ by Assumption 2. This implies that liquidity and capital are substitutes: An increase in liquidity holdings can compensate for a decrease in capital while maintaining the same level of bank stability. The blue line in Exhibit 7 represents this threshold between conditional solvency and conditional insolvency.

We now turn to the region of fundamental solvency or insolvency. Here the solvency constraint is given by:

$$\theta(1 - m) + r_s m \geq [\sigma r_s + (1 - \sigma)r_\ell](1 - e).$$

Exhibit 7
Trade-Off Liquidity versus Capital



Again, we solve for e to depict the solvency threshold in the m-e space:

$$e = 1 - \frac{\theta(1-m) + r_s m}{\sigma r_s + (1-\sigma) r_\ell}.$$

To illustrate the trade-off between liquidity and capital, we derive the slope of the solvency threshold:

$$\frac{de}{dm} = \frac{\theta - r_s}{\sigma r_s + (1-\sigma) r_\ell}.$$

The sign of this slope depends on the relative size of θ and r_s. For low asset payoffs $\theta < r_s$, the slope is negative so that liquidity and capital are substitutes as in the region of conditional solvency/insolvency. The thin black line in Exhibit 7 illustrates this trade-off.

For any asset payoff $\theta > r_s$, however, the slope is positive as illustrated by the thick black line in Exhibit 7. This implies that liquidity and capital are complements, so an increase in liquidity holdings requires an increase in capital for the bank to maintain the same level of stability. The intuition for this case is similar to the situation of "harmful liquidity" in section 3.4. If the assets pay off more than cash, higher liquidity holdings reduce the bank's total payoff and therefore weaken its solvency position. To compensate, the bank has to hold more capital.

4.2 Liquidity Coverage Ratio

The new regulatory framework proposed by Basel III introduces new liquidity requirements for banks. In particular, the Liquidity Coverage Ratio requires banks to hold sufficient high-quality liquid assets to cover their total net cash outflows over thirty days under a stress scenario.

In this section, we analyze the potential effects of the Liquidity Coverage Ratio on bank stability. In particular, we focus on a liquidity requirement where banks are required to hold high-quality liquid assets equal to at least a fraction γ of their short-term liabilities s, that is, $m \geq \gamma s$. Since holding liquid assets entails opportunity costs in terms of forgone investment opportunities in the risky asset, we assume that this requirement will be binding, that is, banks will hold $m = \gamma s$ on their balance sheet. We analyze the effect of making the liquidity requirement more strict, that is, increasing γ. This would qualitatively have a similar effect as increasing liquidity holdings m, as analyzed in section 3.4. In particular, we obtain for the fundamental insolvency threshold:

$$\underline{\theta} = \frac{s r_s (1-\gamma) + \ell r_\ell}{1 - \gamma s},$$

which implies

$$\frac{d\underline{\theta}}{d\gamma} = \frac{s(\underline{\theta} - r_s)}{1 - \gamma s}.$$

Analogous to section 3.4, if the critical value $\underline{\theta}$ is less than the return on cash r_s, then the risk of fundamental insolvency is decreasing in the liquidity requirement; at the insolvency boundary, the assets pay off less than cash, so having more cash is better than having more assets. However, if instead $\underline{\theta} \geq r_s$, then fundamental insolvency risk is *increasing* in the liquidity requirement; the assets pay off *more* than cash at the insolvency boundary, so having more cash is *worse* than having more assets. As discussed in section 3.4, this possibility of liquidity regulation being harmful is more likely for institutions with high leverage and/or long debt maturity.

In the case of insufficient cash to pay for withdrawals and therefore liquidation ($\alpha s \geq m$), the critical value for conditional solvency is

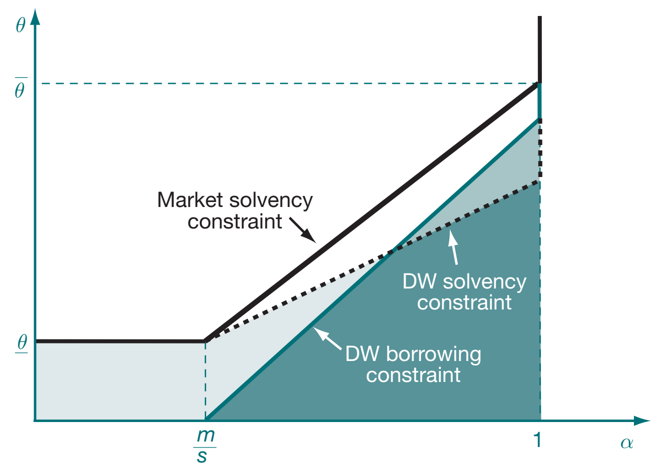
$$\theta^*(\alpha) = \frac{s(\alpha \frac{1}{\tau} + (1-\alpha) r_s) + \ell r_\ell - \frac{1}{\tau}\gamma s}{1 - \gamma s}.$$

The overall effect of γ on $\theta^*(\alpha)$ is again most clearly illustrated by the following:

$$\frac{d\theta^*(\alpha)}{d\gamma} = \frac{s(\theta^*(\alpha) - \frac{1}{\tau})}{1 - \gamma s}.$$

As in section 3.4, we can show that $\theta^*(\alpha) < \frac{1}{\tau}$, and therefore the risk of conditional insolvency is unambiguously reduced by stricter liquidity requirements. Finally, looking at the slope of $\theta^*(\alpha)$,

Exhibit 8
Discount Window Constraints



$$\frac{d\theta^*(\alpha)}{d\alpha} = \frac{s(\frac{1}{\tau} - r_s)}{1 - \gamma s},$$

we see that making the liquidity requirement more strict (increasing γ) strictly *increases* the sensitivity of the critical value to withdrawals. The effects are analogous to the effects of increasing m, which are illustrated in Exhibit 5.

4.3 Discount Window

Traditionally, central banks have attempted to address banks' liquidity problems with discount window lending, where the principle of lending to banks that are solvent but illiquid is set out by Bagehot (1873). In our model, this corresponds to banks in the conditional insolvency region that would be solvent if fewer of their creditors demanded liquidity. An interesting question is whether discount window lending can eliminate the entire conditional insolvency region.

We assume that in period 1 a bank can borrow from the central bank's discount window at an interest rate $r_d \geq r_s$, but has to pledge assets as collateral subject to a haircut h_d. Since the DW does not address issues of fundamental insolvency, the threshold $\underline{\theta}$ remains unchanged from the benchmark setting:

$$\underline{\theta} = \frac{sr_s + \ell r_\ell - mr_s}{y}.$$

When facing conditional insolvency, that is, once the bank runs out of cash ($\alpha s > m$), it can access the DW to borrow the shortfall $d = \alpha s - m$. However, due to the haircut h_d, DW borrowing is constrained:

$$d \leq (1 - h_d)\theta y.$$

Substituting in for d, this is a constraint on α and θ:

16) $$\alpha s - m \leq (1 - h_d)\theta y.$$

As long as the shortfall is not too large, the bank can use the DW loan to pay all withdrawals in period 1.

In period 2, the bank receives back the assets it pledged but has to pay off the DW loan in addition to the long-term creditors and the remaining short-term creditors. The solvency condition in period 2 is therefore:

$$\theta y \geq (1 - \alpha)\, sr_s + \ell r_\ell + dr_d.$$

Substituting in for d, this condition becomes:

17) $$\theta y \geq sr_s + \ell r_\ell - mr_d + (r_d - r_s)\alpha s.$$

Hence, the DW entails two constraints on the rate of withdrawals α and the asset return θ. Constraint 16 is a period-1 constraint since it limits the DW borrowing capacity in period 1 when the bank has to meet withdrawals. If α is too high or θ is too low so that constraint 16 is violated, the bank cannot survive period 1 even if it pledges all its assets to the DW. This borrowing constraint is represented by the blue line in Exhibit 8. Only for combinations (α, θ) above and to the left of the blue line can the bank meet all withdrawals in period 1 with cash and DW borrowing.

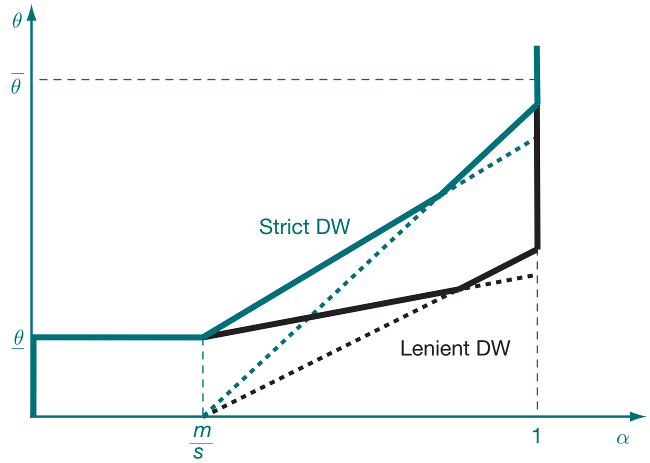
Constraint 17 is a period-2 constraint since it gives the solvency condition in period 2 which is similar to the standard case. The key difference is that with DW access the bank regains the assets it pledged as collateral but has to pay off an additional loan. This solvency constraint is represented by the dashed line in Exhibit 8. The DW solvency constraint is very similar to the market solvency constraint in the benchmark case. The difference is that using the DW, the bank does not have to sell assets but incurs an additional liability. The solvency constraint imposed by the DW is flatter than the one imposed by the market—implying a larger solvency region—as long as $r_d < \frac{1}{\tau}$, that is, as long as the DW interest rate is small relative to the liquidation discount.

The combination of both DW constraints separates the solvency from the insolvency region with the stricter constraint forming the boundary at every point. To the left of the intersection of the two constraints the solvency constraint is binding while to the right of the intersection the borrowing constraint is binding.

Exhibit 9 compares two different DW policies (h_d, r_d) and (h_d', r_d'); the first policy is stricter while the second policy is

Exhibit 9

Comparison of Discount Window Policies



more lenient: $h_d > h_d'$ and $r_d > r_d'$. The lower haircut and lower interest rate of the more lenient policy imply flatter slopes for both the borrowing constraint and the solvency constraint. The solvency region is therefore strictly larger for the more lenient policy.

4.4 Asset Encumbrance

Since the financial crisis, the difference between collateralized and uncollateralized funding has received increased attention.[8] We can use our framework to study the effect of asset encumbrance on bank stability. For simplicity, we assume that the bank has only short-term debt, some of which is collateralized debt c, the remainder is uncollateralized debt u. Both have the same interest rate $r_1 = 1$ between $t = 0$ and $t = 1$ and potentially different interest rates r_c and r_u, respectively, between $t = 1$ and $t = 2$. On the asset side, we assume that the bank only holds long-term assets, $y = 1$, a fraction $x \in [0, 1]$ of which is encumbered as collateral for the debt c. The bank's balance sheet therefore has the following form:

Assets	Liabilities
x	c
$1-x$	u
	e

[8] See Perotti (2010) for a discussion of the risks originating in collateralized funding. For theories on the use of collateral see, for example, Bester (1985), Geanakoplos (2003), or Hart and Moore (1994).

For a given haircut h, the fraction x of assets that is encumbered is determined by the following condition:

$$E[\theta](1-h)x = cr_c, \tag{18}$$

so that the expected value of the collateral in period 0 net of the haircut has to be sufficient to cover the secured creditors' claim. As the key feature of encumbered assets, we assume that they are held by the collateralized creditors and can therefore not be used by the bank to satisfy payouts to uncollateralized creditors.

Denoting the fraction of uncollateralized lenders that withdraw at $t = 1$ by α, the bank's solvency constraint in $t = 2$ becomes:

$$\theta(1-x) - \alpha u \frac{1}{\tau} \geq (1-\alpha)ur_u.$$

This condition states that the payoff of the unencumbered assets net of $t = 1$ liquidations has to be sufficient to repay the remaining uncollateralized creditors at $t = 2$.

Substituting in for x using equation 18, we can solve for the critical value:

$$\theta^*(\alpha) = \frac{\alpha u \frac{1}{\tau} + (1-\alpha)ur_u}{1 - \frac{cr_c}{\mathrm{E}[\theta](1-h)}}.$$

We see that the critical value $\theta^*(\alpha)$ is increasing in the haircut h: With a higher haircut, more of the bank's assets are encumbered. Effectively, there is less "implicit collateral" for the unsecured creditors, which increases the risk of bank failure.

Keeping in mind that $u = 1 - e - c$, we can differentiate the critical value $\theta^*(\alpha)$ with respect to the amount of collateralized debt to get:

$$\frac{d\theta^*(\alpha)}{dc} = \frac{-(\alpha\frac{1}{\tau} + (1-\alpha)r_u)\left(1 - \frac{(c+u)r_c}{E[\theta](1-h)}\right)}{\left(1 - \frac{cr_c}{E[\theta](1-h)}\right)^2}.$$

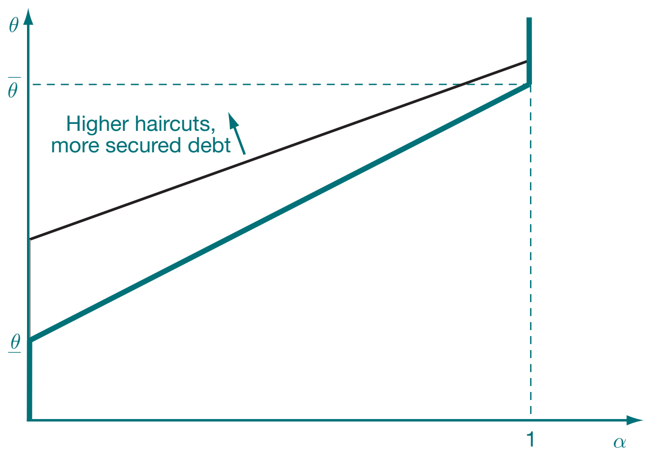
Substituting in $E[\theta](1-h) = cr_c/x$ from equation 18, we can simplify the expression and arrive at:

$$\frac{d\theta^*(\alpha)}{dc} = \frac{(\alpha\frac{1}{\tau} + (1-\alpha)r_u)}{(1-x)^2}\left(x\frac{u}{c} - (1-x)\right)$$

$$> 0 \Leftrightarrow \frac{x}{1-x} > \frac{c}{u}.$$

This implies that replacing uncollateralized funding with collateralized funding increases the critical value and therefore insolvency risk if and only if the ratio of encumbered to unencumbered assets is greater than the ratio of collateralized to uncollateralized funding. The reason is that

EXHIBIT 10
Effect of Asset Encumbrance



the explicit overcollateralization of secured funding due to haircuts reduces the implicit collateral for unsecured funding. Exhibit 10 illustrates the effects of secured funding for bank stability; for higher haircuts and/or greater reliance on secured funding, the solvency region shrinks (the curve shifts up).

4.5 Asset-Backed Commercial Paper Structures

Asset-backed commercial paper is a form of secured, short-term borrowing. Prior to the crisis, ABCP was widely issued by off-balance-sheet conduits of large financial institutions. These conduits increasingly held long-term assets, thus becoming significant vehicles of maturity transformation. In order to enhance their attractiveness, they relied on both credit and liquidity guarantees, typically provided by the sponsoring institutions. The ABCP market experienced significant distress starting in August 2007 as a result of increasing uncertainty about the quality of assets backing commercial paper issuance. This enhanced uncertainty, coupled with the pronounced maturity mismatch of conduits' balance sheets, triggered a run on their liabilities (Covitz, Liang, and Suarez 2013).

Here we use our framework to illustrate the insolvency risk associated with ABCP structures. The structures typically have long-term (risky) assets backing their short-term funding. Hence, the balance sheet of an ABCP conduit would look like:

Assets	Liabilities
$y = 1$	$s = 1$
	$\ell = 0$
	$e = 0$

Furthermore, the ABCP conduit would have a credit and/or liquidity enhancement from a sponsoring institution. First, we focus on the ABCP conduit solely, leaving aside the effect of the credit and liquidity enhancements.

Note that the ABCP conduit does not hold any cash, so that all early claims should be paid by liquidating the risky asset. Using our framework, we can show that the ABCP conduit is solvent at $t = 2$ if and only if $\theta(1 - \frac{\alpha}{\theta\tau}) \geq (1 - \alpha)r_s$, which gives us

$$\theta \geq (1 - \alpha)r_s + \frac{\alpha}{\tau} \equiv \theta^*(\alpha).$$

If all creditors roll over their debt at $t = 1$, that is, when $\alpha = 0$, the ABCP conduit is solvent when $\theta \geq r_s \equiv \underline{\theta}$. If no creditor rolls over its debt at $t = 1$, that is, when $\alpha = 1$, we obtain $\theta^*(1) = \frac{1}{\tau} \equiv \bar{\theta}$. Note that the ABCP structure does not hold any cash ($m = 0$). Hence, we do not observe a flat region, as in the case of an intermediary that holds some cash, where $\theta = \underline{\theta}$ for $\alpha \in [0, m]$. This is all illustrated in Exhibit 11.

As argued, ABCP conduits would typically have credit and/or liquidity enhancements from sponsoring institutions, which would make the liquidation of the assets less costly. For example, in a case where the sponsor guarantee is strong, the costs associated with liquidations can be completely eliminated, that is, $\tau = \frac{1}{r_s}$, so that there is only the risk of fundamental insolvency. Hence, the strength of the guarantee affects τ, which has already been analyzed in section 3.3.

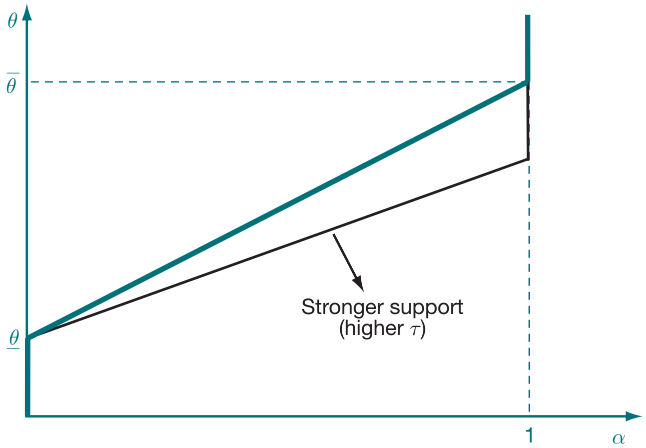
4.6 Money Market Mutual Funds

Money market mutual funds typically attract highly risk-averse investors. Their liabilities are mostly short term that can be claimed at short notice, so that $s = 1$. On the asset side, they have mostly safe assets, that is, the asset side of the balance sheet would have a high value for m and a relatively small value for y. An important feature of an MMF is that when it states a share price lower than \$1.00, the fund "breaks the buck." Hence, our analysis focuses on when an MMF breaks the buck, which would be analogous to a bank being insolvent in the benchmark case.[9]

Using our benchmark framework, we can find the threshold values for θ as follows. Suppose that a fraction α of creditors redeem at $t = 1$, whereas the remaining $1 - \alpha$ wait until $t = 2$. The fund can pay all creditors one unit and it does not break the buck when $\theta y + \chi(\alpha)(m - \alpha) \geq 1 - \alpha$.

[9] In a recent paper, Parlatore Siritto (2012) develops a general equilibrium model of MMFs and analyzes the effect of recently proposed regulations on liquidity provided by these funds and their fragility.

Exhibit 11
Stability of Asset-Backed Commercial Paper Structures



Note the difference between this case and an intermediary's solvency constraint, where the MMF does not break the buck when it can pay all creditors a minimum gross return of 1, whereas the intermediary has to pay the promised interest to the creditors to be solvent. This gives us

$$\theta \geq \frac{1-\alpha-\chi(\alpha)(m-\alpha)}{y} \equiv \theta^*(\alpha).$$

If $\alpha \leq m$, the fund can pay all early claims from its cash holdings so that $\chi(\alpha) = r_s$. For $\alpha > m$, the fund does not have enough cash for all early claims and needs to liquidate some of the risky asset so that $\chi(\alpha) = 1/\tau$. Hence, we obtain

$$19) \qquad \theta^*(\alpha) \equiv \begin{cases} \dfrac{1-\alpha-r_s(m-\alpha)}{y} & \text{for } \alpha \in [0,m] \\ \\ \dfrac{1-\alpha-(1/\tau)(m-\alpha)}{y} & \text{for } \alpha \in (m,1] \end{cases},$$

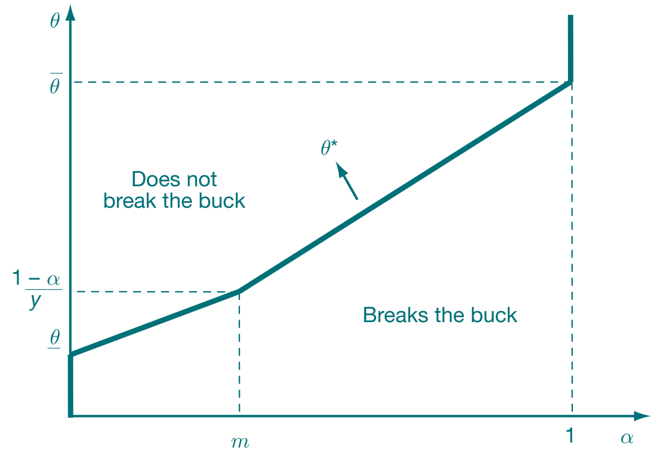
which is illustrated in Exhibit 12.[10]

Note that if all creditors redeem at $t = 1$, that is, for $\alpha = 1$we have $\theta^*(1) = \frac{1}{\tau} \equiv \bar{\theta}$. If the realized return from the risky asset is high enough, that is, for $\theta \geq \bar{\theta}$, the fund never breaks the buck at $t = 2$ regardless of the actions creditors take at $t = 1$.

Reform Proposals. While MMFs have performed well historically and are appreciated by investors for their stability, during the recent crisis the Reserve Primary Fund broke

[10] Exhibit 12 illustrates the case where $mr_s > 1$.

Exhibit 12
Money Market Mutual Fund Regions for "Breaking the Buck"



the buck after the failure of Lehman Brothers. This, in turn, affected financial markets significantly. Since then, there has been some debate about and reform proposals to increase the stability of MMFs. McCabe et al. (2012) develop a reform proposal for MMFs called "minimum balance at risk." The proposal implies that a creditor can only redeem up to a fraction $1 - \mu$ of the claims early and the remaining fraction μ becomes a junior debt claim at $t = 2$ (or an equity claim, as we analyze in this section). In that case, the balance sheet of the fund effectively looks as follows:

Assets	Liabilities
m	$s = 1 - \mu$
y	$\ell = \mu$
	$e = 0$

At $t = 1$, the realization of withdrawals is $\alpha(1-\mu)$. At $t = 2$, the creditors that redeemed at $t = 1$ are owed $\ell_J = \alpha\mu$, where ℓ_J represents junior debt. The creditors that did not redeem at $t = 1$ are owed $\ell_S = 1 - \alpha$, where ℓ_S represents senior debt. The balance sheet of the fund looks as follows after the withdrawal decisions at $t = 1$:

Assets	Liabilities
m	$s = \alpha(1-\mu)$
y	$\ell_S = 1 - \alpha$
	$\ell_J = \alpha\mu$
	$e = 0$

The fund does not break the buck at $t = 2$ if and only if it can pay a return of 1 to all creditors, that is, when $\theta y + \chi(\alpha)(m - \alpha(1-\mu)) \geq \alpha\mu + 1 - \alpha$, which gives us

$$\theta^{\star}_{R}(\alpha) = \frac{\alpha\mu + 1 - \alpha - \chi(\alpha)(m - \alpha(1-\mu))}{y}.$$

If $\alpha(1 - \mu) \leq m$, the fund can pay all of the early claims from its cash holdings so that $\chi(\alpha) = r_s$. When $\alpha(1 - \mu) > m$, the fund needs to liquidate some of its risky assets so that $\chi(\alpha) = 1/\tau$. Hence, we obtain

$$20) \qquad \theta^{\star}_{R}(\alpha) \equiv \begin{cases} \dfrac{\alpha\mu + 1 - \alpha - r_s(m - \alpha(1-\mu))}{y} & \text{for } \alpha \in [0, \frac{m}{1-\mu}] \\ \dfrac{\alpha\mu + 1 - \alpha - (1/\tau)(m - \alpha(1-\mu))}{y} & \text{for } \alpha \in (\frac{m}{1-\mu}, 1] \end{cases},$$

which is illustrated by the black boundary in Exhibit 13, along with the blue original boundary for the MMFs characterized in equation 19.

Next, we analyze the effect of the reform proposal on the stability of MMFs. Note that the region over which the fund can use its cash holdings for the early withdrawals is larger in this case since $m < \frac{m}{1-\mu}$. We can also show that

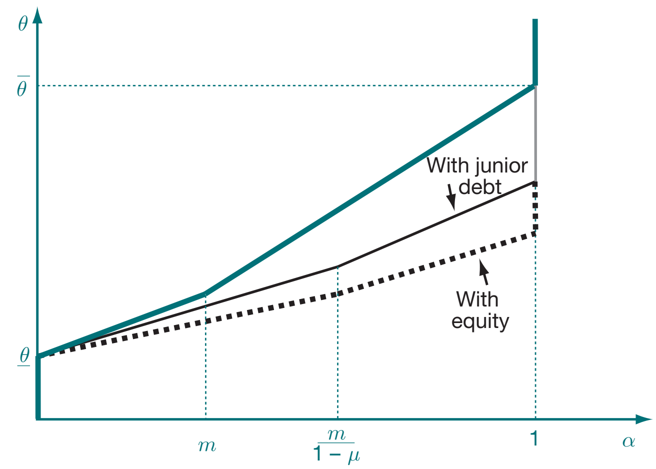
$$\theta^{*}_{R}(\alpha) = \theta^{*}(\alpha) - \frac{\alpha\mu(\chi(\alpha) - 1)}{y} < \theta^{*}(\alpha).$$

In the region where $\alpha \in (m, \frac{m}{1-\mu}]$, the slope of θ^{*}, which is $1/\tau$, is larger than the slope of θ^{*}_{R}, which is r_s. Hence, with the reform proposal the region in which the MMF breaks the buck shrinks, as illustrated in Exhibit 13. The reason for this is that the reform proposal limits the amount that can be redeemed early and hence mitigates the adverse effect of early withdrawals by lowering the amount of the risky asset the fund has to liquidate. This, in turn, makes it less likely that the fund breaks the buck.

Equity versus Junior Debt. A variant of the proposal is that the creditors that redeem at $t = 1$ become equityholders, rather than junior debtholders, at $t = 2$. In that case, the balance sheet looks as follows after the withdrawal decisions:

Assets	Liabilities
m	$s = \alpha(1-\mu)$
y	$\ell = 1 - \alpha$
	$e = \alpha\mu$

EXHIBIT 13
Money Market Mutual Fund Reform Proposals with Junior Debt and Equity



Hence, the withdrawals at $t = 1$ help create an equity buffer, which makes it harder for the fund to break the buck. The fund does not break the buck at $t = 2$ if and only if

$$\theta \geq \frac{1 - \alpha - \chi(\alpha)(m - \alpha(1-\mu))}{y} \equiv \theta^{\star}_{E}(\alpha).$$

Note that if $\alpha(1 - \mu) \leq m$, the fund can pay all of its early claims from its cash holdings so that $\chi(\alpha) = r_s$. When $\alpha(1 - \mu) > m$, the fund needs to liquidate some of its risky assets so that $\chi(\alpha) = 1/\tau$. Hence, we obtain

$$21) \qquad \theta^{\star}_{E}(\alpha) \equiv \begin{cases} \dfrac{1 - \alpha - r_s(m - \alpha(1-\mu))}{y} & \text{for } \alpha \in [0, \frac{m}{1-\mu}] \\ \dfrac{1 - \alpha - (1/\tau)(m - \alpha(1-\mu))}{y} & \text{for } \alpha \in (\frac{m}{1-\mu}, 1] \end{cases},$$

which is illustrated by the dashed boundary in Exhibit 13. The important difference between this proposal and the first proposal, where the creditors that redeem at $t = 1$ become junior debtholders at $t = 2$, is that in this case early withdrawals generate an equity cushion so that the region over which the fund does not break the buck widens. In particular, we have

$$\theta^{\star}_{E}(\alpha) = \theta^{\star}_{R}(\alpha) - \frac{\alpha\mu}{y}.$$

Hence, the region over which the fund breaks the buck shrinks further under the second proposal.

5. Conclusion

During the recent financial crisis, we observed disruptions and the near disappearance of important markets, record-high borrowing rates, haircuts almost reaching 100 percent, significant shortening of maturities, and institutions almost unable to borrow even against high-quality collateral. We are yet to fully understand the exact determinants of these disruptions. In this article, we present a simple analytical framework to tackle this important question. The framework provides an analytical and rigorous, yet easily applicable, tool to analyze the sources of fragility and the effect of various characteristics of funding structures on financial stability. Hence, it can be used to illustrate the trade-offs that may assist policymakers in forming their views about appropriate ways to approach regulatory reform and to evaluate various policy options in terms of their consequences for financial stability.

Appendix

We examine the rollover decision of an individual short-term debtholder. At $t = 1$, each agent observes the realized value of θ and anticipates some behavior of other short-term creditors, as summarized by the value of α.[11] The agent then decides whether or not to roll over its debt; the payoffs associated with each decision are:

	Roll Over	Not Roll Over
Solvent	r_s	1
Insolvent at $t = 2$	$(1 - \phi)\left[\frac{\theta y + \chi(\alpha)(m - \alpha s)}{(1 - \alpha)sr_s + \ell r_\ell}\right]r_s$	1
Insolvent at $t = 1$	0	$\frac{m + \tau\theta y}{\alpha s}$

If the bank is solvent, the agent would clearly prefer to roll over its claim and earn the return $r_s > 1$. If the bank is insolvent at $t = 1$, the agent would receive nothing if it rolled over its debt, so the agent would clearly prefer to redeem its claim at $t = 1$ and receive in expectation a pro-rata share of the bank's liquidated assets. Things are slightly more subtle in the intermediate case, where the bank survives at $t = 1$ but is insolvent at $t = 2$. In this case, the agent would receive the face value of its claim at $t = 1$ if the agent does not roll over. If the agent does roll over, it receives a pro-rata share of the bank's matured assets at $t = 2$, after the bankruptcy costs have been paid. If we assume that $\phi > 1 - \frac{1}{r_s}$, then this return is always smaller than 1, which gives us the following result:

Proposition 1: For $\phi > 1 - \frac{1}{r_s}$, *a short-term debtholder will choose to roll over its claim if and only if* (α, θ) *is such that the bank is solvent in all periods.*

[11] To keep things simple, we assume that an agent anticipates a particular value of α rather than having a belief represented by a probability distribution over different values of α.

References

Allen, F., and D. Gale. 1998. "Optimal Financial Crises." Journal of Finance 53: 1245-84.

Bagehot, W. 1873. Lombard Street: A Description of the Money Market. London: H. S. King.

Bester, H. 1985. "Screening vs. Rationing in Credit Markets with Imperfect Information." American Economic Review 75, no. 4: 850-5.

Chari, V. V. and R. Jagannathan. 1988. "Banking Panics, Information, and Rational Expectations Equilibrium." Journal of Finance 43: 749-60.

Covitz, D., N. Liang, and G. Suarez. 2013. "The Evolution of a Financial Crisis: Collapse of the Asset-Backed Commercial Paper Market." Journal of Finance 68, no. 3: 815-48.

Diamond, D., and P. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." Journal of Political Economy 91: 401-19.

Ennis, H. M. 2003. "Economic Fundamentals and Bank Runs." Federal Reserve Bank of Richmond Economic Quarterly 89, no. 2: 55-71.

Geanakoplos, J. 2003. "Liquidity, Defaults, and Crashes: Endogenous Contracts in General Equilibrium." Advances in Economics and Econometrics: Theory and Applications. Eighth World Congress, Volume 2. Cambridge University Press.

Goldstein, I., and A. Pauzner. 2005. "Demand Deposit Contracts and the Probability of Bank Runs." Journal of Finance 60, no. 3: 1293-1328.

Gorton, G. 1988. "Bank Panics and Business Cycles." Oxford Economic Papers 40: 751-81.

Hart, O. D., and *J. Moore.* 1994. "A Theory of Debt Based on the Inalienability of Human Capital." Quarterly Journal of Economics 109, no. 4: 841-79.

McCabe, P. E., M. Cipriani, M. Holscher, *and A. Martin*. 2012 "The Minimum Balance at Risk: A Proposal to Mitigate the Systemic Risks Posed by Money Market Funds." Federal Reserve Bank of New York Staff Reports, no. 564, July.

Morris, S., and *H. S. Shin*. 2009. "Illiquidity Component of Credit Risk." Princeton University working paper.

Parlatore Siritto, C. 2012. "The Regulation of Money Market Funds: Adding Discipline to the Policy Debate." New York University working paper.

Perotti, E. 2010. "Systemic Liquidity Risk and Bankruptcy Exceptions." Duisenberg School of Finance Policy Papers, no. 8.

Postlewaite, A., and X. Vives. 1987. "Bank Runs as an Equilibrium Phenomenon." Journal of Political Economy 95, no. 3: 485-91.

Saunders, A., and B. Wilson. 1996. "Contagious Bank Runs: Evidence from the 1929-1933 Period." Journal of Financial Intermediation 5, no. 4: 409-23.

Yorulmazer, T. 2014a. "Case Studies on Disruptions during the Crisis." Federal Reserve Bank of New York Economic Policy Review 20, no. 1 (February): 17-28.

———. 2014b. "Literature Review on the Stability of Funding Models." Federal Reserve Bank of New York Economic Policy Review 20, no. 1 (February): 3-16.