Correlation Products and Risk Management Issues

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The large, highly publicized losses incurred by some financial institutions in recent years have caused the press and financial regulators to examine the practice of risk management more closely. In particular, institutional losses have raised concerns about the accuracy of the techniques used to assess the risk of an institution's portfolio. While largely effective when applied to traditional financial portfolios, these techniques are not always successful in capturing the complex configurations of risk inherent in today's highly customized derivative products. This article examines *correlation products*, one such class of derivative instruments, which are challenging the traditional measures of price risk.

"Price risk" is defined as the risk that the value of an institution's entire portfolio will change as a result of shifts in market conditions. Market conditions comprise risk factors (also referred to as "state variables") such as foreign exchange rates, equity prices, interest rates, and commodity prices. In traditional products, or "plain vanilla" instruments, price risk is *separable*. In other words, the sensitivity of the traditional portfolio's value to one risk factor is independent of the level of another risk factor. The price risk of these portfolios can be estimated by measuring their sensitivity to individual risk factors and aggregating these sensitivities to arrive at an overall risk profile.

In correlation products, however, price risk is *nonseparable*—that is, a change in one risk factor will affect the price sensitivity of another risk factor. Thus, the pricing, hedging, and risk management of these instruments depend on the *correlations* between the various risk factors rather than on the analysis and aggregation of the individual variables. Because traditional risk management tools do not account for the interdependency of the risk factors, traditional measures of overall price risk may be inaccurate for portfolios that contain correlation products.

This article defines correlation products and explores the problems they raise for risk management systems in financial institutions. It explains the difficulties of analyzing nonseparable risk in one type of correlation product, the differential (diff) swap, and describes the much simpler measurement of separable risks in a standard constant maturity Treasury swap. The article concludes with some general ways nonseparable risk can be managed.

Defining Correlation Products

Financial instruments can be characterized by the legally binding cash flows that they generate. A correlation product is defined by two characteristics of its cash flow. First, the cash flow must be a function of at least two risk factors. Second, at least two of these risk factors must be combined in a non-additive way.[1] The following expressions compare the cash flows of instruments with separable risks to those with nonseparable risks:

$$(1)\quad \begin{array}{rl} \text{separable risk:} & CF(x_1, x_2) = CF(x_1) + CF(x_2) \\ \text{nonseparable risk:} & CF(x_1, x_2) = CF(x_1) \times CF(x_2)\ , \end{array}$$

where *CF*(.) represents the cash flow generated by a security as a function of risk factors x_1 and x_2. The risk factors in the separable risk expression are broken into two separate terms that are summed, while the risk factors in the nonseparable risk expression form a single product and cannot be so separated.[2]

Common forms of correlation products include diff swaps and quanto swaps.[3] (Several other types of correlation products are highlighted in Appendix I.) Both swaps are examples of cross-currency basis trades—that is, trades whose cash flows depend on the difference between the levels of two risk factors. In a diff swap, the risk factors are a floating domestic interest rate and a floating foreign interest rate, but unlike standard cross-currency trades, both payments are made in a single currency. Both payments are also based on the same fixed notional principal value with a set maturity and are made according to the term of the interest rate indexes. For instance, if six-month LIBOR is used for the interest rate index, cash flows would be exchanged every six months. Unlike some standard cross-currency swaps, diff swaps do not require principal payments at the origination and termination of the swap, because all cash flows are denominated in a single currency.

> *In correlation products, . . . price risk is nonseparable—that is, a change in one risk factor will affect the price sensitivity of another risk factor.*

The structure of a quanto equity swap is similar to that of a diff swap. The foreign floating interest rate payment, however, is replaced with a payment based on a foreign equity index return such as the Nikkei index.

In both diff swaps and quanto swaps, the dealer commits to paying a floating foreign rate on a fixed U.S. dollar notional principal amount rather than on a fixed amount in the foreign currency. This commitment exposes the dealer on the foreign leg of the correlation product to a variable notional principal amount that changes whenever the exchange rate or the foreign index fluctuates.

The Demand for Correlation Products

The market for correlation products is small compared with the plain vanilla market, estimated to have notional values of trillions of U.S. dollars (Remolona 1992-93). Nevertheless, the market for correlation products represents a growing portion of the overall market for securities that trade over the counter rather than on organized exchanges. End-user demand appears to be the driving force behind this growth. Why are end users drawn to correlation products? To be sure, some investors are in the market purely as speculators. End users and dealers alike, however, cite several nonspeculative reasons for their interest in correlation products.

First, end-user demand for correlation products can stem from the same type of economic analysis that drives other investment decisions. For example, a U.S. dollar investor who believes that a foreign equity market is undervalued because of some underlying weakness in the country's economy may be reluctant to face the foreign exchange exposure involved in buying the foreign equities directly. In this case, a quanto swap—in which the end user pays U.S. dollar LIBOR in U.S. dollars and receives the foreign index return in U.S. dollars—would allow the investor to express confidence in foreign equities at the same time that it protects him or her from unfavorable changes in foreign exchange rates.

Second, investors may desire to gain the benefits of international equity or bond diversification without being subject to the foreign exchange exposure that would occur if the domestic currency appreciates against the currencies whose assets are being held. This currency risk may be unacceptable if the investor faces large future liabilities in the domestic currency (such as retirement expenses). Of course, the investor would have to weigh the potential benefits of diversification against the costs of these swaps.

Because traditional risk management tools do not account for the interdependency of the risk factors, traditional measures of overall price risk may be inaccurate for portfolios that contain correlation products.

Third, some individuals and institutions use derivative securities to circumvent (sometimes self-imposed) restrictions on holdings. For instance, the investment committee of a pension fund or insurance company may require all investments to be denominated in the domestic currency. While this rule would prohibit direct foreign capital market holdings, the managers of these investments could gain exposure to foreign debt or equity markets through correlation products such as diff swaps or quanto swaps.

Fourth, an end user may negotiate a correlation product with a dealer rather than dynamically create a similar exposure because dealers have a competitive advantage over end users in managing the complex exposures of correlation products. Dealers' advantages include their ability to trade at smaller bid-ask spreads in the cash market and their greater experience in negotiating within the legal environments of foreign economies, particularly in the emerging debt and equity markets. In addition, dealers' risk management systems tend to be more advanced than most end users' systems.

One use for which correlation products are generally not appropriate is the hedging of risks arising from traditional products. Most hedgers have little interest in correlation products because the type of exposure found in them is not available in existing cash or derivative securities. Asset managers are more likely to use these products in an effort to outperform an index or other return measure.

An Example of a Correlation Product: The Diff Swap

The Market for Diff Swaps

One of the first reported diff swaps was negotiated in early 1991 between Credit Suisse First Boston and a Japanese insurance company. Since that time, diff swaps have grown rapidly in popularity, reportedly because of the large differential in short-term interest rates across major currencies. Today, diff swaps make up a significant portion of the exotic instruments market. A recent estimate places the notional principal amount of diff swaps outstanding at $40 billion to $50 billion.[4]

The market for correlation products represents a growing portion of the overall market for securities that trade over the counter.

Through the use of diff swaps, investors in currencies with low yields attempt to enhance their returns by swapping into currencies with higher yields. Diff swaps have been transacted in a wide range of currency pairs, including U.S. dollar LIBOR against LIBOR rates of the deutsche mark, British pound, Swiss franc, and Australian dollar, and LIBOR rates of the deutsche mark and Swiss franc against LIBOR rates of the Italian lira, Spanish peseta, and other high-yielding currencies of the European Exchange Rate Mechanism.

Despite the rapid growth of the diff swap market, it is still controlled by only a handful of dealers. The main barrier to entry for other derivatives dealers is the expertise needed to price, hedge, and manage the nonseparable risks present in these instruments. Unlike traditional instruments, correlation products require risk managers to account for nonseparable risks by making assumptions about the future correlations between risk factors.

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ANALYZING THE PRICE RISK OF A DIFF SWAP

The complex procedures for analyzing the price risk of diff swaps are explained below. Readers may wish to compare these procedures with the relatively simple process of analyzing the price risk of a standard derivative instrument, the constant maturity Treasury (CMT) swap, outlined in Appendix II.

Both the diff swap and CMT swap examples rely on the assumption that markets are competitive. Thus, we determine the price of the instrument by estimating the cost to the dealer of hedging the resulting risk exposures. This does not mean that the dealer will (or should) hedge the resulting exposure. Rather, we determine the price of an instrument by ruling out the only other alternatives. If the cost of replicating the exposures is greater than the price that the counterparty offers to the dealer, the dealer will not enter into the trade. At the same time, if the cost of hedging the exposure is less than the price that the dealer offers to the counterparty, the counterparty will take the business to a dealer with more competitive prices. Therefore, the price must be equal to the cost of hedging. Although this approach does not consider market realities such as transaction costs, liquidity considerations, and model risk, it yields a reasonable approximation to the value of a security.

HEDGING AND PRICING A DIFF SWAP

Suppose a dealer has entered into a diff swap in which for a period of one year it receives six-month U.S. dollar LIBOR in U.S. dollars while it pays six-month deutsche mark LIBOR in U.S. dollars to the end user. The semiannual interest payments are based on a $100 million notional principal amount and are settled in arrears (Exhibit 1).[5]

To value the cash flows of the diff swap, the dealer must determine the level of the cash flows that will take place in the future (in this case, in six months' and in one year's time) and discount these flows to the present.[6] Therefore, the present value of the diff swap can be written as

(2) PV of the diff swap =

$$PV_{6\,mo}(\$100\text{m} \times (r^{@\,t\,=\,today}_{US\$\,LIBOR} - r^{@\,t\,=\,today}_{DM\;\;LIBOR}))$$

$$+\ \text{PV}_{12mo}(\$100\text{m} \times (\tilde{r}^{@\,t\,=\,6mo}_{US\$\,LIBOR} - \tilde{r}^{@\,t\,=\,6mo}_{DM\;\;LIBOR}))$$

where $PV_t(CF)$ indicates the present value of a cash flow, CF, occurring at time t, and r^t_x represents the prevailing interest rate in market x at time t.

The value of the cash flow that will occur in six months' time (the first term in equation 2) is easy to calculate. The parties swap the difference between the current value of U.S. dollar LIBOR and deutsche mark LIBOR paid in U.S. dollars on a notional principal amount of $100 million. The cash flow will not change when interest rates or exchange rates fluctuate, and the cash flows can be discounted at the risk-free U.S. dollar six-month interest rate.[7]

However, the value of the cash flow that will occur in twelve months' time (the second term in equation 2) is more difficult to calculate. The dealer cannot convert the deutsche mark liability embedded in the swap into a U.S. dollar liability, because the level of deutsche mark exposure faced at the swap initiation will be determined by the level of deutsche mark LIBOR and the deutsche mark/U.S. dollar exchange rate in six months' time. Thus, while typical hedging instruments protect against exposure by converting a fixed principal amount from one currency to another,[8] the exposure faced by the dealer in a diff swap involves a floating deutsche mark principal. Ultimately, the lack of a static hedge forces the dealer to make assumptions concerning the future correlation between the deutsche mark/ U.S. dollar exchange rate and deutsche mark LIBOR and to update the hedging position dynamically.

Estimating the Cost of Hedging the Exposures

Once the cash flows of the diff swap are determined, the dealer estimates the cost of hedging the floating interest rate exposures by observing the costs of entering into two plain vanilla interest rate swaps—one in U.S. dollars and one in deutsche marks. These interest rate swaps, which are based on the notional principal amount of the diff swap in U.S. dollars or its dollar equivalent in deutsche marks, will have the same maturity and payment dates as the diff swap (Exhibit 2). Because the market for interest rate swaps is highly competitive, we can assume that these two hedging swaps will be entered into at a net present value of zero. As a result, the overall value of the diff swap will be the same before and after hedging. However, the combination of the diff swap and the two hedging swaps does not eliminate all price risk. The presence of residual risk suggests that the market prices of existing securities alone are not enough to determine the value of the diff swap.

Accounting for Residual Exposures

To account for residual risk, the dealer must assess the joint probability distribution of the deutsche mark/U.S. dollar exchange rate and the deutsche mark LIBOR rate. For the purposes of this example, assume that the U.S. dollar term structure is flat at 6 percent, the deutsche mark term structure is flat at 8 percent, and the current deutsche mark/U.S. dollar exchange rate is 1.6. Exhibit 3 shows the gross cash flows and the net cash flows to and from the dealer.

To determine the value of the residual exposure that occurs in one year, the dealer converts the net cash flows into U.S. dollars at the exchange rate prevailing at

Exhibit 1
Differential Swap: Generic Cash Flows

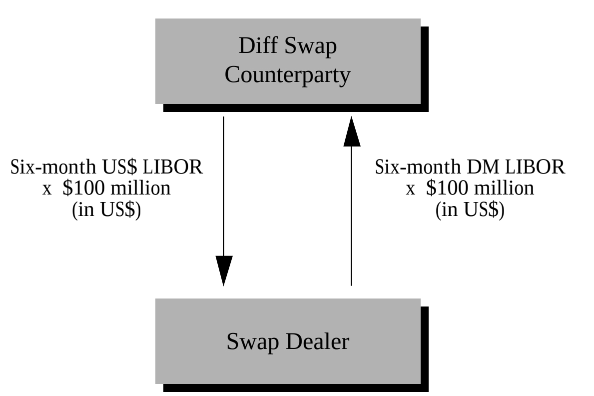
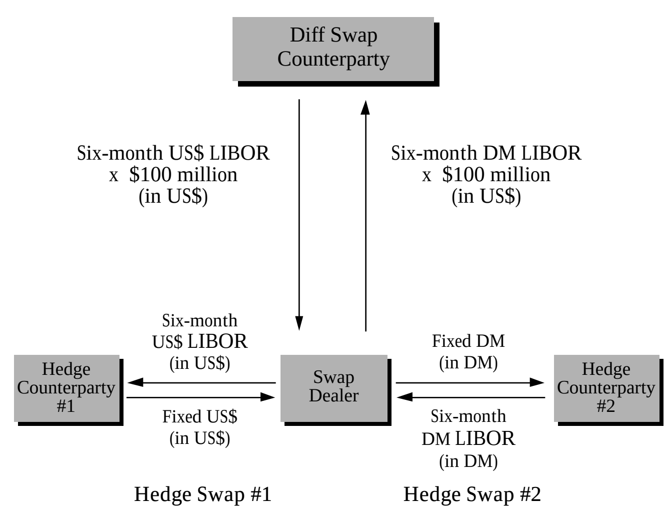


Exhibit 2
Differential Swap: After Dealer Hedges with Interest Rate Swaps



t=6 months, $\tilde{q}_{DM/\$}$:

$$(3) \qquad \$100m \times (6\% - \tilde{r}_{DM\ LIBOR}) + DM160m \times (\tilde{r}_{DM\ LIBOR}/\tilde{q}_{DM/\$} - 8\%/\tilde{q}_{DM/\$}),$$

which can be simplified to:

$$(4) \qquad (\$100m - DM160m / \tilde{q}_{DM/\$}) \times (8\% - \tilde{r}_{DM\ LIBOR}) - \$100m \times 2\%.$$

As shown in expression 4, the residual cash flow contains a risky component (first term) and a fixed component (second term).[9] The cash flow represented by the second term is easy to value: it represents a fixed cash flow on a fixed date in a single currency and therefore can be discounted at the one-year spot rate at time zero. However, the cash flow represented by the first term is difficult to value because two sources of risk are being combined in a single term. This first term fits the definition of *nonseparable risk*: the two random variables, $\tilde{q}_{\$/DM}$ and $\tilde{r}_{DM\ LIBOR}$, are multiplied rather than summed or differenced and therefore cannot be separated into different terms.

Traditional risk management tools properly measure the risk of correlation products only if risk factors do not fluctuate simultaneously. For example, if the exchange rate remains at 1.6 deutsche marks per U.S. dollar, then the first term of expression 4 will equal zero and the resulting cash flow will be zero, regardless of the level of deutsche mark LIBOR. At the same time, if deutsche mark LIBOR remains at the fixed interest rate of 8 percent, the cash flow will be zero regardless of the level of the deutsche mark/U.S. dollar exchange rate (Exhibit 4). These zero cash flows show that the dealer's position is hedged for movements in either deutsche mark LIBOR or deutsche mark/U.S. dollar exchange rates. However, the dealer is not hedged against simultaneous movements.[10]

Simultaneous movements in the foreign index and the exchange rate will determine the sign—positive or negative—of the cash flow. For example, let us assume that the deutsche mark LIBOR decreases and the deutsche mark/U.S. dollar exchange rate increases (the deutsche mark depreciates relative to the U.S. dollar). Because the movements in the deutsche mark LIBOR and the deutsche mark/U.S. dollar exchange rate are negatively correlated, both terms in expression 4 will be positive, and the dealer will receive a positive cash flow. Conversely, if the deutsche mark LIBOR decreases and the deutsche mark/U.S. dollar exchange rate decreases (the deutsche mark appreciates relative to the U.S. dollar), then the cash flow to the dealer will be negative. Therefore, the *correlation* between the risk factors determines whether the cash flow of the diff swap will be positive or negative. Using the data in Exhibit 4, the chart on page 13 offers a graphic representation of the concept of nonseparability.

In summary, while part of the exposure of the diff swap can be hedged with existing securities, residual risk must be evaluated in order to determine the value of the diff swap. An important, and complex, component of the residual risk is the correlation between the risk factors.

Exhibit 3
CASH FLOWS OF A DIFF SWAP TO AND FROM DEALER
All cash flows take place at t=12 months based on rates at t=6 months

Diff swap:	
Inflow:	\$100 million x $\tilde{r}_{US\$\ LIBOR}$
Outflow:	\$100 million x $\tilde{r}_{DM\ LIBOR}$
Hedge swap #1:	
Inflow:	\$100 million x 6%
Outflow:	\$100 million x $\tilde{r}_{US\$\ LIBOR}$
Hedge swap #2:	
Inflow:	DM 160 million x $\tilde{r}_{DM\ LIBOR}$
Outflow:	DM 160 million x 8%
Net cash flows:	
Inflow:	\$100 million x 6% + DM 160 million x $\tilde{r}_{DM\ LIBOR}$
Outflow:	\$100 million x $\tilde{r}_{DM\ LIBOR}$ + DM 160 million x 8%

IMPLICATIONS FOR RISK MANAGEMENT AND SUPERVISORY PRACTICES

The most fundamental problem in estimating the price risk of correlation products occurs at the operational level. The feature of nonseparability means that a dealer cannot break up the price sensitivity of diff swaps or other correlation products into component risks and then assign each risk to its corresponding business function. Instead, an institution's trading desks need to coordinate their activities by establishing formal systems of communication among trading units and between trading units and global risk managers. This level of coordination has not been required in managing traditional instruments, and it may entail substantial changes in an institution's management approach and structure.

Of course, the potential for problems at the operational level does not stop there. The portfolios of large institutions can comprise thousands of individual trading positions across multiple trading desks in several geographic locations. To arrive at a comprehensive estimate of risk, most of these institutions rely on summary statistics of each trading position. They then aggregate these summary statistics to arrive at the risk of the entire firm.[11] Because traditional measures of risk do not accurately reflect the risk of a portfolio that contains correlation products, these summary statistics can misguide corporate decisions. For example, an underestimation of price risk, if large enough, could lead a financial institution to hold less than the optimal amount of capital against potential losses.

Inaccurate estimates can also influence the financial decisions of market participants. Transparency of risks and exposures is an important feature of an institution's financial statements (Bank for International Settlements 1994). If the portfolio of an institution contains significant levels of "hidden" correlation risk, then investors may not efficiently allocate capital to that institution. For instance, a lack of transparency of risk can inhibit the flow of capital to a healthy financial institution that is experiencing a temporary liquidity crisis.

Cash Flow Profile of a Diff Swap

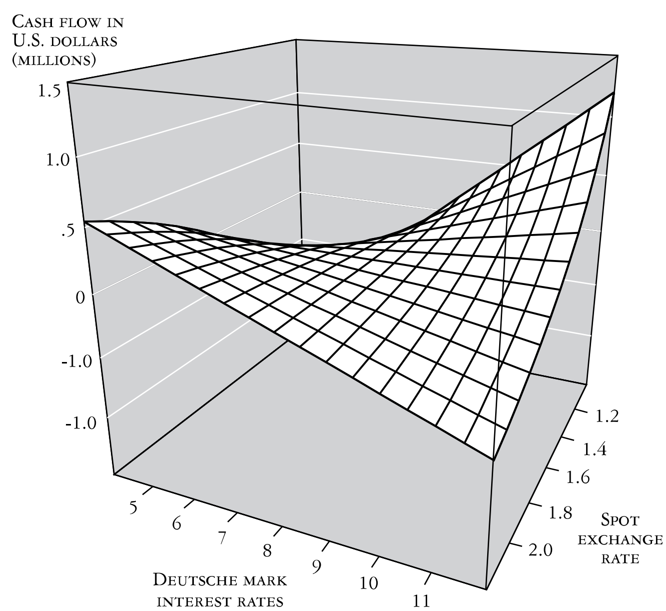


Exhibit 4
CASH FLOW PROFILE FOR DIFF SWAP DEALER

Cash flow occurring in year one for diff swap on $100 million notional principal based on DM LIBOR and DM/U.S. dollar exchange rate in six months:

$$(\$100m - DM\ 160m / \tilde{q}_{DM/\$}) \times (8\% - \tilde{r}_{DM\ LIBOR}).$$

The value of the expression is halved when calculating the cash flows because the diff swap is assumed to have semiannual payments. The following matrix shows the level of cash flow (in millions of dollars) for various possible realizations of the exchange rate and the DM LIBOR rate.

Cash Flow (Millions of Dollars)								
		Exchange Rate (DM/U.S. dollar)						
		1	1.2	1.4	1.6	1.8	2	2.2
DM LIBOR (Percent)	12	1,200,000	666,667	285,714	0	(222,222)	(400,000)	(545,455)
	11	900,000	500,000	214,286	0	(166,667)	(300,000)	(409,091)
	10	600,000	333,333	142,857	0	(111,111)	(200,000)	(272,727)
	9	300,000	166,667	71,429	0	(55,556)	(100,000)	(136,364)
	8	0	0	0	0	0	0	0
	7	(300,000)	(166,667)	(71,429)	0	55,556	100,000	136,364
	6	(600,000)	(333,333)	(142,857)	0	111,111	200,000	272,727
	5	(900,000)	(500,000)	(214,286)	0	166,667	300,000	409,091
	4	(1,200,000)	(666,667)	(285,714)	0	222,222	400,000	545,455

Note: The unshaded regions represent the cash flows of a diff swap resulting from changes in individual risk factors.

From a supervisory perspective, the market for correlation products raises several concerns. First, because the development and execution of correlation products are highly concentrated within the banking community, a shift in market conditions could have potentially adverse consequences for a small number of large institutions. Moreover, some correlation products are structured in the risky, illiquid currencies of emerging markets, where large changes in interest rates and exchange rates can occur overnight or, significantly for correlation products, simultaneously. For example, in 1994, the Mexican peso/U.S. dollar exchange rate, the Mexican equity markets, and Mexican interest rates changed dramatically and concurrently over a short period of time. Although nonseparable structures can provide valuable liquidity to otherwise inaccessible markets, risks may be greatly underestimated in these more volatile environments.

The feature of nonseparability means that a dealer cannot break up the price sensitivity of diff swaps or other correlation products into component risks and then assign each risk to its corresponding business function.

Second, nonseparable risk is one of many factors that may affect the implementation of regulatory capital requirements. The Bank for International Settlements (1995) has recently put forth a proposal that would allow individual financial institutions to use their own internal models to assess risk and to assign regulatory capital requirements. Internal models, if properly constructed, should be able to accurately reflect the effects of nonseparable risks on the institution's portfolio.

Finally, liquidity of the market may be at risk because the exposures of a correlation product may be difficult to reverse if the counterparty is not willing to cancel the contract at a fair value. Unlike the investor in traditional instruments, the end user of a correlation product must find a counterparty who is willing to take on the exact exposure of the original contract in order to counteract the existing contract; otherwise, he or she may be compelled to hedge the exposure dynamically. Therefore, if liquidity for correlation products dries up, end users may be forced into dynamically hedging exposures that they would like to eliminate but cannot because of a lack of counterparty interest. The fact that the market for correlation products is predominantly demand-driven adds to future liquidity concerns. If demand diminishes, financial institutions will have little incentive to maintain an active secondary market.

Managing Nonseparable Risk

As shown by the price risk analysis of a diff swap, traditional risk measures can understate the amount of risk present in correlation products. How can institutions enhance risk management tools to address this potential problem? The first step is to identify the presence of nonseparable risks in a portfolio. Two approaches might be taken:

- Each variable to which a portfolio is exposed may be shocked individually and the sum of these changes in market value compared with the changes brought about when the variables are shocked simultaneously. If the change in value stemming from the simultaneous shock differs from the sum of the effects of the individual shocks, then the portfolio contains nonseparable risks.
- Ex post profits and losses and model-predicted profits and losses may be reconciled, taking into account the realized level of the risk factors. A risk manager could investigate the cause of profits or losses in excess of predictions by analyzing discrepancies between model prices and market prices. Such excess returns could arise if nonseparable risk is being measured by traditional risk management tools.

Once nonseparable risks are identified, the risk manager could then use a simulations-based approach to measure price risk. This type of approach requires a number of time-consuming, expensive steps, as outlined below.

A risk manager first identifies the risk factors to which a portfolio is exposed, collects historical data on these factors, and analyzes and models the volatility of the factors and their relationships to each other. Unfortunately, historical data series do not always exist, particularly for newly developed markets or economies. Alternatively, a risk manager may use current market prices (such as options prices), if available, to derive market-implied estimates of future volatilities. A third option is to rely on the data set for a risk factor similar to that under investigation. For example, a risk manager may estimate a current exposure to an emerging economy by using data from a country whose economy has undergone a similar transformation.

Next, the risk manager generates a large number of future paths for the risk factors through one, or a combination, of two methods—a model-based approach or an empirical-based approach. The former assumes a structure for the data, for example, a multivariate normal distribution or generation by a time-varying volatility process such as an ARCH-type process. The latter uses historical data to create a frequency distribution, or histogram, with which the future distribution of the risk factors is assumed to coincide. The model-based approach has the advantage of simulating an unlimited number of future paths, but the model may be misspecified or incorrect (introducing model risk). The empirical-based approach frees the researcher from a potentially incorrect model, but its use is often limited by the lack of reliable historical data on many risk factors.

Once nonseparable risks are identified, the risk manager could then use a simulations-based approach to measure price risk.

After generating future paths for the risk factors, the manager computes the future value of each security under the various scenarios and estimates the present value of the security as the average discounted value of the simulated future values. This averaging procedure assumes that each of the simulated scenarios is equally likely. Finally, the manager calculates estimates of price sensitivities by "perturbing" each path taken by the risk factors and recalculating the value of the portfolio. The change in the value of the portfolio divided by the perturbation is a measure of the delta (the rate of change of the portfolio to a risk factor). Pair-wise perturbations and revaluations yield estimates of price sensitivities to changes in pairs of risk factors.

Because the process is so involved, a simulations-based approach seems appropriate only for firms that place great emphasis on nonseparable products. Such firms will probably find it useful to develop multiple simulation methodologies (using variations of both the empirical-based and model-based approaches) to ensure that their risk estimates are robust to alternative specifications.

Conclusions

Correlation products, a new class of derivatives instruments, are challenging the effectiveness of existing techniques for measuring price risk. For traditional portfolios, financial institutions evaluate individual risk factors at the trading-unit level and subsequently aggregate the units' estimates to arrive at an accurate risk profile. For correlation products, however, this technique is not effective because the sensitivity of one risk factor is always a function of the level of another risk factor. Thus, because many institutions continue to rely solely on traditional risk management tools, nonseparable risks may go unmeasured.

The potential for understated risk raises several concerns regarding financial institutions' regulatory capital requirements, financial disclosure practices, and supervisory activities. Techniques to capture nonseparable risks—such as the simulations-based approach outlined in the article—can help address these concerns by augmenting traditional risk measures. Given the tremendous rise in financial innovation, new types of risk are likely to prompt an increasing number of financial institutions to reexamine and enhance risk management practices.

APPENDIX I: COMMON FORMS OF CORRELATION PRODUCTS

In addition to diff swaps and quanto swaps, several new types of correlation products have been developed in recent years.

Correlation products include any contract that pays off as a function of the minimum or maximum of two random processes. Specific contracts include the option to trade one asset for another and the outperformance option, which pays some function of the maximum of two indexes, such as stock market indexes. In addition, relative value derivatives, which pay off as a function of the ratio of two variables, appear to be gaining popularity (see, for example, Locke 1995 and Elms 1995).

Correlation effects may also be embedded in more exotic structures. Quanto options—that is, options on a foreign index with the spot and strike prices denominated in a foreign currency but cash flows taking place at a fixed exchange rate in the domestic currency—have become increasingly popular.[12] Also gaining ground are correlation products in the form of a binary option,[13] where the payoff of the option depends on two underlying variables. A hypothetical correlation binary call option would pay a predetermined constant amount, X, if the (constant maturity) three-month U.S. dollar interest rate, r, is above r^* *and* a foreign/U.S. dollar exchange rate, q, is above q^* (that is, the payoff is $\{X$ if $r > r^*$ *and* $q > q^*$; 0 otherwise$\}$). This exotic binary option is simultaneously bullish on the U.S. dollar relative to the foreign currency and bearish on U.S. dollar interest rates. Its value will depend on the anticipated correlation between the three-month U.S. dollar interest rate and the foreign currency/U.S. dollar exchange rate.

Certain yield curve trades also involve nonseparable risk. A call option on a short-term interest rate, with the strike determined by a long-term interest rate, is an example of a nonseparable yield curve trade.

In addition, Asian options with geometric means for the spot price or the strike price fit the definition of correlation products. An example is an option on a stock index for the time period $[0, T]$ with strike price K and a payoff that is a function of the geometric mean of the index level taken at T+1 discrete dates:

$$CF_T = \max\,[0, (S_0 \times S_1 \ldots \times S_T)^{1/(T+1)} - K].$$

The cross partial of the value of this option, $\partial^2 V / \partial S_s \partial S_t$, is not zero for $s \neq t$; therefore the value of this option will be a function of the correlation matrix of S, which is effectively the autocorrelation structure of the process for S. If the option payoff were a geometric average across securities instead of across time, the option on the index would be a function of the entire covariance matrix of stock prices.[14]

Appendix II: Analyzing the Price Risk of a Standard Instrument: The Constant Maturity Treasury Swap

Suppose a securities dealer has entered into a constant maturity Treasury (CMT) swap with a notional value of $100 million. For a term of one year, the dealer pays the current five-year U.S. Treasury rate on a notional value of $100 million and receives the current ten-year Treasury rate on a notional value of $100 million. The dealer obviously benefits if the yield curve steepens (Exhibit A1).

Determining and Valuing the Cash Flows

Exhibit A2 illustrates the cash flows of this simple portfolio as a function of the five-year Treasury rate and the ten-year Treasury rate. This “five-by-ten CMT swap” shows separable risk in the two risk factors: the sensitivity of the flows to changes in the five-year Treasury rate is independent of the level of the ten-year Treasury rate; the sensitivity of the cash flows to changes in the ten-year Treasury rate is independent of the level of the five-year Treasury rate. To value the CMT swap, the dealer breaks the resulting risks into the five-year and ten-year components, then values these components separately and aggregates them.

Because the risks of the CMT swap are separable, the dealer can break up the risks and assign them to two different trading units—for example, the unit responsible for trading in the five-year Treasury sector and the unit responsible for trading in the ten-year Treasury sector. These two trading units would not need to coordinate their efforts.

Estimating the Cost of Hedging the Exposures

Exhibit A3 shows how the dealer may attempt to hedge (and therefore assign a price to) the exposures of the resulting trade. For the five-year Treasury exposure, the trader uses interest rate forward contracts, which require him or her to pay a fixed rate in exchange for the CMT five-year Treasury rate. For the ten-year Treasury exposure, the trader uses an interest rate swap based on the ten-year Treasury rate, which requires him or her to pay the ten-year CMT rate in exchange for a fixed rate. As a result, exposures to the five-year and ten-year Treasuries are eliminated, and the pricing of the CMT swap amounts to the pricing of two riskless fixed flows in the future (Exhibit A4). We can conclude that the price sensitivity of the CMT swap is similar to the price sensitivities of

Exhibit A1
Interest Rate Swap: Generic Cash Flows

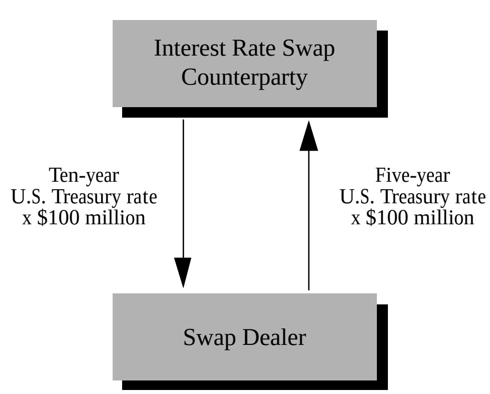


Exhibit A2
Cash Flows of a CMT Swap to and from Dealer
All cash flows take place at t=12 months based on rates at t=6 months

Five-by-ten CMT swap:		
	Inflow:	$100 million x $\tilde{r}_{10}$
	Outflow:	$100 million x $\tilde{r}_{5}$
Hedge swap:		
	Inflow:	$100 million x r_{FIXED1}
	Outflow:	$100 million x $\tilde{r}_{10}$
Hedge forwards:		
	Inflow:	$100 million x $\tilde{r}_{5}$
	Outflow:	$100 million x r_{FIXED2}
Net cash flows:		
	Inflow:	$100 million x r_{FIXED1}
	Outflow:	$100 million x r_{FIXED2}

fixed-for-floating swaps on a five-year Treasury rate and a ten-year Treasury rate.[15] Using the data in Exhibit A4, the chart on this page offers a graphic representation of the concept of separability.

REVIEWING THE LACK OF RESIDUAL EXPOSURES

A lack of residual exposures once the two hedging strategies are implemented indicates that two other instruments—interest rate swaps and interest rate forwards—serve the same economic function as a CMT swap. These instruments can be used as alternate hedging vehicles if the market for CMT swaps becomes illiquid. Lack of residual exposure also indicates that the pricing and hedging of a five-by-ten CMT swap is fully determined by markets for the individual five-year and ten-year risks. In summary, because risk is separable, the pricing and hedging of the CMT swap does not require the dealer to estimate the correlation coefficient between the two risk factors.

Exhibit A4
CASH FLOW PROFILE OF A CMT SWAP

The cash flow of a five-by-ten CMT swap is the notional value of the swap times the difference between the most recently issued ten-year Treasury and the most recently issued five-year Treasury:

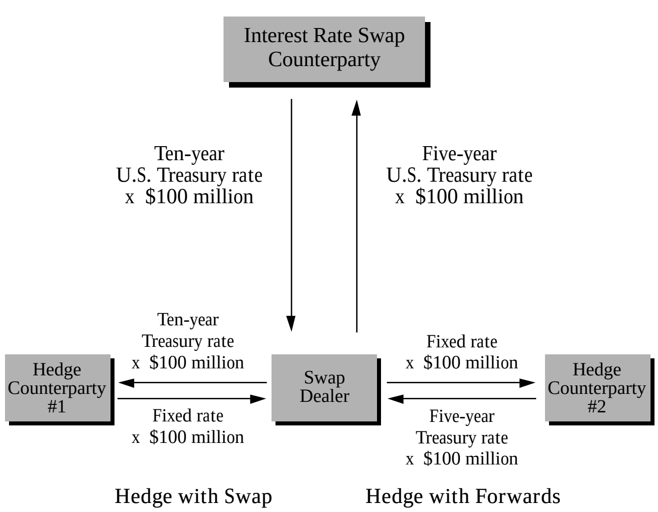
$$CF = \$100m \times (r_{10} - r_5),$$

where the notional principal is assumed to be \$100 million, and r_5 and r_{10} represent the five-year and ten-year Treasury rates, respectively. The following matrix shows the level of cash flow (in millions of dollars) for various possible realizations of the five-year and ten-year Treasury rates at the next payment date.

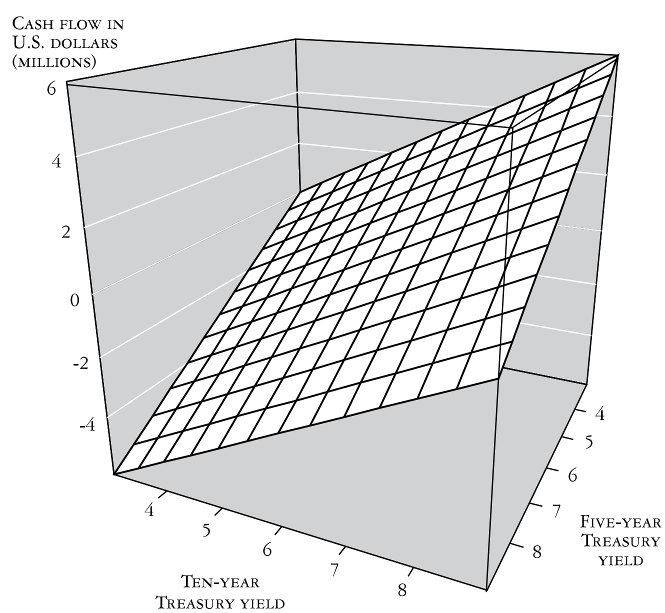
Cash Flow (Millions of Dollars)								
		Five-Year Treasury Rate						
	Percent	3	4	5	6	7	8	9
Ten-Year Treasury Rate	9	6.0	5.0	4.0	3.0	2.0	1.0	0.0
	8	5.0	4.0	3.0	2.0	1.0	0.0	-1.0
	7	4.0	3.0	2.0	1.0	0.0	-1.0	-2.0
	6	3.0	2.0	1.0	0.0	-1.0	-2.0	-3.0
	5	2.0	1.0	0.0	-1.0	-2.0	-3.0	-4.0
	4	1.0	0.0	-1.0	-2.0	-3.0	-4.0	-5.0
	3	0.0	-1.0	-2.0	-3.0	-4.0	-5.0	-6.0

Note: The unshaded regions represent the cash flows of a CMT swap resulting from changes in individual risk factors.

Exhibit A3
INTEREST RATE SWAP: AFTER DEALER HEDGES WITH INTEREST RATE SWAP AND FORWARDS



CASH FLOW PROFILE OF A CMT SWAP



ENDNOTES

1. The term “correlation product” can be misleading because it refers to the structure of the instrument, not to the correlations between the risk factors. If the cash flows of a product cannot be separated into different terms, the instrument is nonseparable and therefore a correlation product.

2. It is not the estimation of the correlations between market risk factors that confounds traditional risk management systems. Indeed, most risk management tools require correlation estimates. Rather, the assumption of separability inherent in most traditional risk management tools leads to the underestimation of risk in correlation products. The nonseparable expression cited in the text shows that the correlation between the risk factors x_1 and x_2, usually denoted $\rho_{x_1 x_2}$, does not enter into the definition of a correlation product.

3. Diff swaps are also referred to in the trade press as quantity-adjusted swaps (quants), guaranteed exchange rate swaps, LIBOR differential swaps, cross index basis (CRIB) swaps, and switch-LIBOR swaps.

4. For a description of the early development of the diff swap market, see Shirreff (1992), Cookson (1992), and Das (1992a, 1992b).

5. Settlement in arrears for a one-year swap with semiannual payments means that the first payment, made in six months’ time, is based on the current values of LIBOR, and the second (and last) payment, made in one year’s time, is based on the values of LIBOR realized in six months’ time.

6. Several authors, including Jamshidian (1993) and Wei (1994), have derived formulas for the present value of a diff swap. These formulas are contingent on the assumed process of the term structure, a complex subject that is not treated in this article.

7. The flows are considered riskless because throughout this paper we assume that there is no counterparty credit risk.

8. For instance, currency futures and forward contracts determine the exchange rate today for a fixed (not a floating) principal exchange from deutsche marks to U.S. dollars in the future.

9. If the market for providing these swaps is competitive, the buyer and seller agree on an additional periodic payment, called “margin,” so that the present value of the swap is zero at swap initiation.

10. When separable risks are present, a dealer hedged against movements in individual risk factors would necessarily be hedged against simultaneous movements in risk factors.

11. Standard summary statistics include the positions’ current market values, deltas (market value sensitivities to underlying risk factors), gammas (sensitivities of the deltas to underlying risk factors), vegas (market value sensitivities to volatility changes), and thetas (market value sensitivities to the passage of time).

12. Quanto Nikkei put warrants, the focus of a study by Dravid, Richardson, and Sun (1993), began trading on the American Stock Exchange in 1992.

13. A plain vanilla binary call option is a derivative security that pays nothing if the underlying asset price or rate, S, finishes at or below the strike price of the option, K, and pays off a predetermined, constant amount, X, if the asset finishes above the strike price (that is, the payoff is $\{X \text{ if } S > K; 0 \text{ if } S \leq K\}$). Binary options are also called all-or-nothing options, bet options, and lottery options.

14. An interesting example of such a contract is the now-defunct Value Line Index Futures contract at the Kansas City Board of Trade (KCBOT) (Thomas 1994). The Value Line Index that was used to determine the delivery price of the contract was a geometric average index, which meant that the appropriate arbitrage model was not the standard cost-of-carry model but rather a dynamic strategy depending on the entire covariance matrix of the stocks in the index. The KCBOT contract failed after other exchanges introduced newer futures contracts based on the arithmetic mean of the components (such as the Standard & Poor’s contracts). The newer futures contracts are much more easily replicated in the cash market because the covariance matrix of their components does not need to be estimated.

15. The reader should note that the important distinctions between diff swaps and swaps with separable risks do not arise because the diff swap involves a foreign currency. The risks of standard cross-currency swaps, for example, can be valued and hedged separately.

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