Consistent Margin Requirements: Are They Feasible?

With the development of a wide variety of markets in equity-related financial instruments, investors have at their disposal numerous ways of investing in stock "exposure." That is, they may invest in various instruments whose returns are determined primarily by the returns on individual stocks or portfolios of stocks. For every position in each of these equity-related instruments, there are minimum margin requirements that compel the investor to maintain a specified level of equity in a margin account. This article examines the issues surrounding the consistency of margin requirements across equity-related markets, suggests methods for reducing inconsistencies, and identifies the inherent problems.

The equity-related instruments currently available are summarized in Table 1. There are three basic constructs through which new instruments are created: indexes, futures contracts, and options contracts. In addition, these techniques may be combined to produce other derivative securities, such as index futures or options on index futures. The available combinations allow investors to obtain equivalent returns in various markets and to choose the market that is most suitable for their particular needs (as regards transaction costs and the timing of the transactions, for example). The derivative markets also permit reallocations of risk-bearing among investors over time with a flexibility that would be difficult to achieve with the underlying instruments alone.

Because of the relationship between the returns on derivative assets and those of the corresponding stocks, each derivative instrument is priced in a way that is closely related to that of the underlying equity position. Otherwise, arbitrage profits would be available on an almost riskless basis to investors who assume positions in pairs of instruments that are mispriced according to the basic implicit relationships.

Two questions are examined here. First, should margin requirements be made consistent across all equity-related markets? Margin requirements serve more than one objective, and they are set by numerous institutions with different backgrounds in different markets, so they exhibit little apparent consistency across markets.[1] Second, if it is deemed advisable to make margin requirements more consistent, how does one go about this task? The analysis that follows concludes that it is desirable to have a degree of consistency across markets, but not necessarily identical requirements. On the other hand, the opportunities for fine tuning are limited by the uncertainty that prevails as to the exact results of applying margin requirements. The lesson is that a healthy dose of good judgment is essential in the setting of margin requirements.

Why impose margin requirements?

Before proceeding to the questions regarding the consistency of margin requirements, it is necessary to consider the ultimate objectives of such requirements. Only in that context will the appropriate criteria for consistency become clear.

At no point in time has there been a clear consensus about the rationale for imposing margin requirements.

[1]George Sofianos, "Margin Requirements on Equity Instruments," this issue of the *Quarterly Review*, provides a detailed summary of the margin requirements on equity-related instruments for various investor categories

As expertise has developed in this area, some of the proposed motivations have lost most of their support For example, the argument has been advanced that margin requirements reduce the diversion of funds from productive uses (such as physical investment) to speculative uses Real resources, however, are not in general used up by margin loans, which represent the insertion of an additional instrument in the chain of financial intermediation channeling savings into investment It is possible, however, that margin requirements could be used to deal with market imperfections

Another formerly popular claim is that margin requirements protect unwise small investors from themselves by limiting the amount of risk they can incur Margin requirements apply to broad classes of investors and thus are, at best, a blunt instrument for weeding out these problem cases In addition, they only restrict the credit that may be obtained directly by using the securities purchased as collateral and take no account of the investor's overall leverage

Only two motivations seem to have withstood the test of time, although the issue of their validity is by no means completely settled. The first of these is the protection of the integrity of the markets In practice, this involves limiting the degree of credit risk to which market participants are exposed so that, in a period of adverse events, defaults do not cumulate to cause a breakdown in the market as whole. In the absence of dictated margin requirements, creditors would be expected to protect their own interests by requiring prudent margin levels But they would focus on their own perception of their own risks—not the risks to the system—and they might be subject to competitive pressures Thus, the proximate objective of margin requirements may be to protect the creditors from the risk of default, but this objective serves the ultimate goal of protecting the system.

The other side of the tradeoff in setting margin levels under this criterion involves the liquidity of the market It is generally possible to reduce credit risk to arbitrarily low levels by imposing very strict margin requirements A side effect of this strategy, however, is to exclude from the market certain investors who, given sufficient potential for borrowing, would take positions that would enhance the liquidity of the market. With extreme margin requirements, the whole market might be stifled

Initial margins are usually emphasized in considering the effects of high margin requirements on liquidity Large margin calls, however, which might result from strict maintenance or variation margin requirements, could be just as disruptive to the markets as strict initial requirements A significant cushion between initial and maintenance margins, such as exists for individual stocks, allows for the possibility of major price changes without an accompanying unexpected strain on the demand for short-term liquidity

A second motivation behind the establishment of margin requirements may be the control of excessively speculative activity, which could exacerbate the deviations of actual stock prices from the values implicit in the fundamental information on the corporations issuing the securities These deviations may be in the form of increased volatility in stock price movements or they may involve persistent discrepancies between the actual and fundamental stock prices, as in the phenomena known as "bubbles" and "fads"[2] Once again, the drawback in setting higher margin requirements is the possible loss of liquidity

Table 1

Menu of Available Equity Instruments

Instrument	Representative Exchanges	Underlying Security
Individual stocks	NYSE, AMEX, NASDAQ	—
Futures on stocks	NA	—
Options on stocks	CBOE, AMEX, PHLX, PSE, NYSE	Individual Stocks
Options on futures	NA	—
Indexes	NA (proposed for AMEX, NYSE, PHLX)	—
Index futures	CME	S&P 500
	NYFE	NYSE Composite
	CBT	Major Market Index
	KC	Value Line
Index options	CBOE	S&P 100, S&P 500
	AMEX	Major Market Index
	NYSE	NYSE Composite
	PHLX	Value Line, OTC Composite
	NASDAQ	NASDAQ 100
Options on index futures	CME	S&P 500
	NYFE	NYSE Composite

Key
AMEX American Stock Exchange
CBOE Chicago Board Options Exchange
CBT Chicago Board of Trade
CME Chicago Mercantile Exchange
KC Kansas City Board of Trade
NYFE New York Futures Exchange
NYSE New York Stock Exchange
NASDAQ National Association of Securities Dealers Automated Quotation System
PSE Pacific Stock Exchange
PHLX Philadelphia Stock Exchange
NA Not available

[2]See Gikas Hardouvelis, "Margin Requirements and Stock Market Volatility," in this issue of the *Quarterly Review* Earlier empirical work had not found persuasive evidence that margin requirements curb speculative activity Using different statistical methods, Hardouvelis

This second motivation for margin requirements is not altogether distinct from the first. If margin requirements are effective in reducing excessive price volatility by controlling speculation, some of the price uncertainty that contributes to credit risk will be eliminated. Indeed, experience has shown that the most significant threats of credit disturbances to the stock markets occur during episodes of excessive volatility. Thus, reducing the likelihood of such volatility may be an important channel through which margin requirements protect the integrity of the markets.

Running parallel to the two basic motivations for imposing margin requirements is the notion that the stock market is special in that it involves the trading of claims in the ownership of the productive resources of the economy. In a market-oriented democracy, broad involvement in such activities on the part of individual investors is usually considered a desirable objective. Any development that would tend to chase these investors away from the market (such as unwarranted volatility, systemic risk, or manipulation) should, in this view, be vigorously avoided.

Perhaps because few instances of severely destabilizing volatility have been experienced in the U.S. stock markets, the empirical evidence supporting the use of margin requirements either for protecting market integrity or for curbing excessive speculation is technically not very strong. Conversely, the results of the technical studies have not rejected the usefulness of margin requirements as an instrument for protecting the markets or guarding against excessive speculation.

General considerations in the setting of margin requirements

The general principles to be followed in setting margin requirements, as well as the final results, will differ according to the particular goals pursued by regulators. In this section, a basic course of action is laid out for each of the two major objectives identified earlier. Most of the issues raised here are examined in greater detail in subsequent sections.

In the case of the market integrity motivation, there are three questions to investigate in trying to determine what level of margin requirements would provide a given level of systemic protection. The first concerns the accuracy of knowledge about the probability distribution of future price movements in the underlying security. Simply looking at the past or making some theoretical assumption may not be sufficient to obtain a precise representation of future price movements. This is particularly true for worst case scenarios, which may not seem plausible or even conceivable until after the fact.

The second question concerns the relationship between the movements in the prices of the underlying equity security and the price of a particular derivative instrument. Important strides have been made in the last two decades in working out the mathematics of the appropriate pricing of derivative securities, such as options and futures, under given conditions. The pricing relationships developed, however, apply only to some types of instruments, involve substantial complications, and may produce results that differ consistently from observed prices. The difficulties vary from instrument to instrument but are most severe in the case of options.

The third question is probably the hardest. Once the credit risk in an individual transaction or position has been analyzed, what are the implications for the market as a whole? A liquid market may be able to absorb a number of delinquencies, but how many defaults would cause a serious market failure? Are several small defaults worse than a large one? What is the interaction among different market participants in the event of defaults? Does this interaction tend to accelerate the collapse of a market, and by how much? In view of these uncertainties, the setting of margin requirements to protect the integrity of the markets can hardly be approached as a simple academic exercise in measuring the credit risk associated with a range of potential price movements.

The use of margins to control speculation raises equally daunting questions. One must ask whether it is desirable to control speculation at all, and whether margin requirements are an adequate means of achieving that objective. Even if they do contain speculation in one market, high margin requirements may drive speculators to other markets. The empirical evidence in this respect is far from clear cut, but there are reasons to believe that the control of some types of speculative activity is a valid concern of regulators and that margins may be useful for that purpose.[3]

Finally, the consistency of the two regulatory objectives poses a potential problem. If the objectives of protecting financial integrity and limiting speculation have different implications for the level of margin requirements, what relative weight should be assigned to each objective? All of the foregoing difficulties are encountered for each individual instrument even before considerations of consistency across different instruments are entertained.

Footnote 2 continued
presents evidence that when margin requirements are higher, stock price volatility is lower For an examination of the possibility of "bubbles" in stock prices, see Hardouvelis, "Evidence on Stock Market Speculative Bubbles Japan, United States, and Great Britain," in this issue of the *Quarterly Review*

[3]For a discussion and some evidence, see Hardouvelis, "Margin Requirements"

Margin requirements and the integrity of the markets

This section outlines the analytical process by which margin requirements may be used to control credit risk as a means of protecting the integrity of the markets. The process is described separately for stocks, futures, and options.[4] The same kind of analysis is performed in the next section for the alternative objective of controlling speculative activity.

In general, the procedure involves determining the probability of an exposure to credit loss that is associated with each margin level, and choosing the level that produces an acceptable amount of risk for the creditor. The first step is to identify the potential credit risks. In the case of stocks bought on margin, a loan to the investor is collateralized by the stocks purchased. The danger to the creditor is that the value of the security may fall to levels that would be insufficient to cover the amount of the loan. Even though the debtor would still have a legal obligation to repay the loan in full, the practical likelihood of a default is clearly greater if some or all of the loan is unsecured by the assets in the margin account.

Suppose a margin of 25 percent is required on the purchase. Equivalently, the amount of the loan may not exceed 75 percent of the initial value of the security. If no further margin calls are made, the stock price may fall by as much as 25 percent before the lender is exposed to any actual credit risk.

Given a set of precise—though probabilistic—assumptions about the future behavior of the price of the stock, the probability of developing an exposure to credit risk during the period allowed for the posting of margin may be computed. The same may be done for other proposed margin levels from zero to 100 percent. The level selected would then be the lowest that would keep the probability of credit exposure within acceptable limits. It should be noted, however, that there is no objective way of selecting the acceptable level of risk, so that ultimately judgment is the only available guide.

The foregoing example applies to a long position in stocks. Similar principles apply to the short sale of stocks, but with short sales, the risk is related to a rise, as opposed to a decline, in the price of the securities.

In the case of futures contracts, credit risk exists whenever the futures price, which is determined at the outset of the contract, differs from the value of the underlying stock portfolio at maturity. Either side may show a deficiency at that time, depending on who is long and short and on the realized price of the security. Thus, each side is a potential credit risk and margin must be required from both sides. In general, the long side profits from upward movements in the stock price and loses if the price declines, while the opposite is true for the short side of the contract. The relevant probability is that of the event that the amount at risk over the margin-posting period—the current shortfall for a given party, if any—exceeds the total margin that has been collected from that party, either as initial or variation margin.

Once the probability is calculated for each set of margin requirements,[5] one proceeds as before to choose a combination of margin requirements that keeps the probability within acceptable bounds.

With options contracts, the fundamental asymmetry of returns relative to those of the underlying asset means that the level of credit risk is dramatically different for the buyer and the writer. The buyer of an option—be it a put or a call—obtains exposure to each share of the underlying stock for a premium that is generally considerably less than the stock's price per share. Unlike a futures contract, the long side of an option poses no credit risk once the premium is paid in full, since no further payments are ever required from that party. If the option expires (or is exercised) in the money, a credit accrues to the long side. If on the other hand the option expires out of the money, there is no obligation to exercise it and, hence, no further loss. For this reason, there is no need to require margin from the buyer beyond the premium itself.

For the writer, quite the opposite is true. The writer makes no initial payment other than the posting of margin and is subject to adverse changes in the price of the underlying security that may cause the option to move far into the money. Traditionally, margins on written options have implicitly included components reflecting expected movements in stock prices as well as the volatility or uncertainty of future price movements. In the margin formulas, these parameters are represented by proxies—for example, the amount by which an option is in or out of the money, or a percentage of the current market price of the underlying asset. The requirements are marked to market daily, and additional margin calls or withdrawals are made accordingly.

As with futures, the margin regulator starts with a given structure of margin requirements and, using a model of stock price movements, calculates the probability of any remaining credit risk. This is then done for other conceivable levels of margin requirements, whereupon a structure with an acceptable probability level is selected.

[4]The technical analysis that underlies the procedures described in this section is illustrated in Appendix B

[5]Margins in the futures markets have traditionally included variation margin as well as initial margin. The variation margin requirement for stock index futures is 100 percent, but there is no conceptual difficulty in having a variation rate other than zero or 100 percent

Traditionally, margin requirements have been applied separately to the positions held by an investor with different brokers, in different markets, or through different clearing corporations With cooperation and coordination among brokers, exchanges, and clearing houses, it would be possible to apply margin requirements to an individual's consolidated overall position. Such "cross margining" currently exists to a limited degree, and further initiatives are in progress among several exchanges and clearing corporations.

For each instrument or combination of instruments considered above, the setting of margin requirements to control credit risk requires precise knowledge about the instruments, the markets, and the probability distributions of price movements. The structure of the market is taken as given, and it is assumed that changes in margin requirements do not affect the fundamental pricing of the underlying securities.

Margin requirements and speculative activity

A widely accepted principle of financial theory states that stock prices should reflect all the available and relevant information about the issuing firm's fundamentals. This is not a statement of the "efficient markets hypothesis,"[6] but a prescriptive statement to the effect that an investor should be able to determine and pay a fair price for a share in the ownership of a corporation. Most would agree that the stock market should be used by corporations to raise capital, but not by speculators to place uninformed bets that could drive prices away from their fundamental values.

If making stock prices conform to fundamentals is the ultimate objective, why use margin requirements, which control the degree of leverage available to investors? One argument might be that speculators tend to be risk-takers and find leveraged positions, which involve greater risk than their unleveraged counterparts, attractive. In addition, leverage allows an investor to control a greater amount of shares with a given dollar amount of initial capital. To the extent that long-term fundamental investors are not heavily leveraged, the relative influence of speculators may increase as the maximum permitted leverage increases

As an illustration, consider a speculator who tends to overreact to news. With $100,000 of capital and a 100 percent margin requirement, he would be able to purchase only $100,000 worth of stocks. If the required margin were only 10 percent, however, he would be able to buy $1,000,000 of stocks, a purchase that would have a much greater effect on stock prices. Other investors with longer-term objectives would be subject to excessive price volatility resulting from the greater purchasing power of the overreacting speculator

The statistical relationships between margin requirements, speculation, and volatility are not well established. Hence, there is no precise way of determining the degree of allowable leverage that would reduce the volatility associated with speculation to acceptable proportions, just as there is some fuzziness in the relationship between individual credit risk and the integrity of the market. In this case, it is necessary to identify the parties whose activities should be controlled and to understand the nature and magnitude of their operations. Because many important market participants (such as pension funds) typically want to hold long unleveraged positions on balance, they need not be significantly affected by stricter margin requirements.

How could margin requirements be used to limit leverage in the futures market? A futures position in stocks is economically equivalent to a fully leveraged position in the underlying securities over the term of the futures contract. In both cases, there is no initial cash outflow. At the futures maturity date, the long party in the futures has the value of the stock minus the initial futures price, while the long leveraged position has the stock minus the amount owed on the loan. The similarity of these positions causes the market to set the futures price at inception to the amount owed on the loan, including interest, at the futures maturity date. Otherwise, arbitrageurs could obtain riskless profits by shorting the position with the higher price (futures or loan price) and buying the other. The difference between the spot and futures prices thus tends to be the interest cost of the loan.[7]

Thus, in terms of the amount of leverage permitted, a zero initial margin requirement on the futures would be equivalent to a zero initial margin requirement on the underlying stock in the cash market. Similarly, an initial margin requirement on the futures of any magnitude between zero and 100 percent is equivalent in terms of leverage to an initial margin requirement of the same magnitude on the underlying stocks

The implicit leverage in an options position is more difficult to determine because of the complexity of options pricing. For options on relatively simple assets, fairly accurate pricing formulas have been developed.[8]

[6]One form of the efficient markets hypothesis states that security prices reflect all available information This is a (perhaps) testable empirical proposition, as opposed to a normative statement such as the one in the text

[7]The futures prices predicted by these arbitrage relationships do not in general coincide with observed prices, in large measure due to institutional factors These factors include transactions costs, dividend payments, different settlement practices in the spot and futures markets, and the fact that the futures apply to an index, whereas only individual stocks are traded in the spot market

[8]As in the case of a non-dividend-paying stock studied by Fischer Black and Myron Scholes in "The Pricing of Options and Corporate

In these cases, it is possible to construct a "hedge portfolio" that consists of time-varying proportions of cash and the underlying stocks and that replicates the option returns The resulting hedge portfolio for a call option normally consists of a long position in the stock and a short cash position (a loan), so that an implicit leverage ratio may be easily computed from the pricing formula This implicit leverage is the ratio of the value of the loan to the value of the stocks in the hedge portfolio

Table 2 presents the implicit leverage ratios for long positions in various call options, as calculated using the Black-Scholes option pricing formula The implicit leverage is quite substantial, particularly for options that are around the money or out of the money The additional margin that would be necessary to bring the leverage down to 50 percent, as required for stocks in the spot market, is also shown in Table 2 This amount may be several times the option premium A similar exercise may be performed for a short call or a put option, with similar results

It should be clear from the foregoing discussion that using margin requirements to control the leverage obtained with options is a difficult task. The rules based on implicit leverage are complex, even in the basic Black-Scholes case discussed above. For some options, there are no explicit pricing formulas on which to rely

Another complication that arises in the context of margins on options is the wide discrepancy that may result from applying the two objectives for margin requirements It has been previously argued that if credit risk is the major concern, there is no need for margin beyond the option premium for a long call, since the credit risk posed by the long side is then zero A glance at Table 2, however, shows that implicit leverage in excess of 80 or 90 percent is possible with option positions even if they involve no credit risk. A speculator who prefers the return patterns arising from a highly leveraged stock position could bypass the requirements of the spot market by investing in an option position with very high implicit leverage. Under most circumstances, this strategy would produce essentially the same investment results as investing in the corresponding leveraged position in the spot market Through arbitrage, such activities would ultimately affect pricing in the spot market

Footnote 8 continued
Liabilities," *Journal of Political Economy,* May-June 1973

Table 2

Implicit Leverage for a Long Call Option

Volatility (Percent Per Annum)	Exercise Price (In Dollars)	Implicit Leverage (Percent)*	Call Premium (In Dollars)†	Additional Margin (In Dollars)‡
20	70	68	32 42	17 47
20	100	88	7 43	23 81
20	130	94	0 36	2 75
40	70	64	33 28	13 54
40	100	79	12 86	17 37
40	130	85	3 76	8 93
60	70	59	35 72	7 88
60	100	70	18 29	12 51
60	130	76	8 78	9 87

Assumptions
Underlying stock price = $100
Maturity = 6 months
Interest rate = 7 percent per annum

*Black and Scholes (see footnote 8 in text) have shown that a call option may be replicated with a continuously rebalanced portfolio consisting of a long stock position and a short cash position (a loan) At time t, the values of these two positions should be
A_t = stocks = $S_t N(h_t)$,
L_t = loan = $e^{rT} K N(h_t - \sigma T)$,
where S_t is the value and σ is the volatility of the underlying stocks, K is the exercise price, T is the time to maturity of the option, r is the risk-free interest rate, N( ) is the standard Gaussian cumulative distribution function, and
$h_t = [\log(S_t/K) + (r + 5\sigma)T)]/\sigma T$
The implicit leverage is L_t/A_t

†The Black-Scholes premium is $A_t - L_t$

‡Margin required to bring implicit equity proportion to 50 percent, that is, $\max[0, L_t - 5A_t]$

Why make margins consistent?

The question whether margins should be consistent across markets must be considered within the context of the basic objectives for margin requirements It is not clear a priori that the two objectives would produce the same results. In some cases, similar margin requirements may be used to satisfy both goals at once For some instruments, notably options, the solution is dramatically different depending on which of the two objectives is given priority.

Furthermore, the structure of each of the various equity-related markets is so unique in ways that are fundamental to the problem at hand that, in addition to the margin rates, a whole series of other parameters must be considered in the context of margin requirements Before going into the consistency question in detail, it is useful to list the parameters that are potentially under the regulators' control Not all of the following have been explicitly utilized in all markets

Initial margin. Initial margin requirements for individual stocks are set by the Federal Reserve Board (they are currently 50 percent) The Board also controls initial margins on stock options and stock index options but has left the details to the appropriate exchanges subject to the approval of the Securities and Exchange

Commission. For stock index futures and options on index futures, initial margins are set by the exchanges and by the self-regulatory organizations.

Maintenance margin. Some form of maintenance margin requirement is found in virtually all markets. In the markets for individual stocks, there is a large gap between the initial (50 percent) and the maintenance (25 percent) requirements, and there is no obligation to issue margin calls before the maintenance level is hit. However, whenever the equity in a margin account falls below the maintenance level, new cash or securities must be deposited to bring it back up to that level (or to the initial level, as in the index futures market). The level of the requirement is generally set by the exchange. With options, maintenance margins are based on current market premiums and are used as a means of marking positions to market.

Variation margin. The concept of variation margin is used primarily in the futures markets. Investors are required to mark their positions to market (on a daily or intraday basis for stock index futures) and to post an amount corresponding to any adverse change in the futures price. While variation margin has traditionally been set at 100 percent in the futures markets and not required in the cash markets, it is conceptually possible to set this requirement at fractional values of the change resulting from marking to market.

Posting period. The length of the period allowed for the posting of margin calls is of the utmost importance for the credit risk control objective. Risk and uncertainty are clearly greater if investors are allowed up to 15 business days to post margin (as in the spot market, in principle) than if they are allowed no more than one day (as in the index futures market). The length of the posting period bears a direct relationship to the clearing and settlement practices of the individual markets.

Form of margin. The types of securities accepted to cover margin requirements differ from market to market. In the various markets, these may include cash, Treasury securities, and nonpublic instruments, including credit lines.

Explicit exemptions. Different types of investors have different margin requirements in each market. A broker-dealer, for example, will generally have more flexibility than a customer. The same applies with greater force to a market maker in the security. In some cases, customers are classified as hedgers or speculators for the purpose of applying different margin requirements.

Degree of discretion. In some cases, the regulatory authority may grant specific exemptions to investors on a discretionary basis. For example, the Options Clearing Corporation may reduce overall margin require-

Table 3

Margin Simulation Statistics for Spot and Futures Markets over One Year

	Stocks (Five-Day Periods)		Futures (One-Day Periods)	
Volatility (Percent Per Annum)	60	40	40	20
Equity				
Average	40 2 percent	42 1 percent	15 2 percent	14 7 percent
Probability of negative equity	0 002 percent	0 percent	0 01 percent	0 percent
Margin Calls				
Minimum	−$53 44	−$23 04	−$28 31	−$7 60
Maximum	$40 76	$11 69	$28 35	$7 51
Average positive call	$ 0 26	$ 0 08	$ 1 05	$0 51
Probability of				
positive call	8 4 percent	3 9 percent	49 7 percent	49 2 percent
call of at least $1	6 4 percent	2 6 percent	34 1 percent	21 4 percent

Assumptions
- Initial value of security = $100
- Instantaneous expected return = 7 percent per annum
- Margins on stocks
  - Initial margin = 50 percent
  - Maintenance margin = 25 percent
  - Posting period = 5 days
- Margins on futures
  - Initial margin = $15
  - Variation margin = 100 percent
  - Posting period = 1 day
- Number of iterations = 2000

ments for its members on a discretionary basis when options held long by the members are substantially in the money.

The implications—in terms of the likelihood of negative equity and of margin calls—of recent choices as to margin parameters in the stock market and in the stock index futures market are illustrated by the simulation statistics presented in Table 3 The basic assumptions for the simulations are intended to be generally representative of conditions in the New York Stock Exchange for the stock market and in the Chicago Mercantile Exchange for the futures market Price movements are represented by a mathematical formulation (the Wiener process) widely used in the context of stock prices and derivative instruments.

Two different volatility assumptions are examined in each market In general, a diversified portfolio of stocks will experience lower price volatility than an individual issue Since index values correspond to the prices of such a diversified portfolio, a relatively low volatility is assumed for index futures (20 percent) For the stock market, higher values are used (40 and 60 percent) The case of index futures with a volatility of 40 percent is included for the purpose of comparison with the stock market

The results in Table 3 indicate that the requirements in the spot and futures markets are roughly equivalent in terms of the probability of exposure to credit risk (probability of negative equity) A range of volatilities has to be considered for the spot market, but the probabilities tend to be quite low for most reasonable values, as they are for the index futures Nevertheless, other statistics vary markedly across markets.

In the stock market, initial margins are relatively high, and there is a built-in buffer against margin calls provided by the difference between initial and maintenance margins. In the futures market, initial margins are lower, but additional margin is required any time prices change The effects are noticeable in the relationship between equity levels and margin calls

Equity in the spot market is on average between two and three times higher than in the futures market. A large portion of this difference is attributable to the buffer against calls As a result, both the incidence and magnitude of positive margin calls (in contrast to negative calls, or allowable margin withdrawals) are much lower in the stock market. The chances of a margin call are about even in the futures market, and a call of 1 percent or more of the original stock price occurs about one-fifth of the time. The dollar value of the average call in the futures market is about twice that corresponding to a stock whose volatility (60 percent) is three times that of the index. These figures provide a clear illustration of the tradeoff between high initial margins and frequent large margin calls[9]

Consistent margins and the integrity of the markets

All of the parameters identified in the previous section affect the expectations and probability distributions associated with credit risk for each of the equity-related instruments A simple rule of thumb to make margins consistent is to set the parameters so that the probability of an equity deficiency in an investor's position is the same for all instruments. This ignores the distinct possibility that the relationship between individual default and the overall integrity of the market may vary from one market to another. A system with many essentially independent intermediaries is more resilient than one in which intermediation takes place in several steps with the potential of a chain reaction of defaults Alternatively, the netting out of positions may be different from market to market. A large volume of open positions on either side is not necessarily risky if the holdings of individual investors are hedged for the most part. In any case, the rule of thumb described above is a useful first step

More specifically, the regulator would proceed with the analysis described earlier for controlling credit risk in each particular instrument Consistency would require that the acceptable probability level selected be the same for each instrument

Conceptually, the application of this method is not

[9]Note that the figures reported in Table 3 are based on a mathematical simulation and not on historical data The mathematical techniques have been used elsewhere to calculate the probability of negative equity during a single posting period, starting from the maintenance level (for example, *Interim Report of the Working Group on Financial Markets*, Washington, D C , May 1988) The simulation in Table 3 is more general in that it covers all the events that may develop over the course of a year, incorporating initial, maintenance, and variation margin requirements, as well as an explicit posting period

Table 4

S&P Composite Index: Frequency of Extreme Monthly Returns

(Percent of Observations within Period)

Period	Loss of More Than 8 5 Percent	Gain of More Than 8 2 Percent
1930-39	18 3	15 8
1940-49	2 5	0 8
1950-59	0 0	1 7
1960-69	0 8	0 8
1970-79	4 2	4 2
1978-87	4 2	5 0
1926-87	5 0	5 0

Source Ibbotson Associates, SBBI/PC data base

difficult in the context of the spot and futures markets where, because of arbitrage pricing, the relevant events are essentially the same.[10] A simple way to impose consistent margins would be to make them *uniformly equivalent,* that is, to set every parameter—initial, maintenance, and variation rates, posting period; exemptions; and so forth—at the same level in each market. While theoretically attractive, this requires very fundamental changes in the way these markets presently operate. Virtually every one of the parameters described above varies significantly from market to market. Since some of these differences—such as the margin posting period—arise from operational features of the markets, regulators contemplating a change must consider the potential disruption.[11]

Another less disruptive way to deal with the problem is to make the requirements *dynamically equivalent,* that is, to allow for the possibility of setting the parameters at different levels in the spot and futures markets, but in such a way that the resulting probabilities of equity deficiencies are the same across markets. For example, if the initial margin requirement were lowered in the spot market, the probability of deficiencies would increase. To lower the probability to the original level, some fractional variation margin requirement might be imposed. Alternatively, the posting period might be shortened, and so on.

While the calculation of these tradeoffs is theoretically feasible, it is by no means an easy task in practice. It requires detailed knowledge of the probability distribution of movements in the price of the underlying security, as well as a clear representation of the relationship between the pricing of futures and the pricing of the underlying security.

To illustrate the problems, Table 4 presents the frequency of unusually large positive or negative price movements in the S&P Composite index for the period from 1926 to 1987 and for a series of 10-year periods within those years. The results indicate that the assumption that future volatility will resemble past volatility is highly suspect, even though some stability is imposed by the substantial length of the periods considered.

Option returns bear a complicated relationship to those of the underlying asset, and the problems they create in the context of consistent margins are even greater. The use of uniform equivalence is out of the question. It is still possible to impose dynamic equivalence, though the complexity of the pricing relationship makes this even harder than in the case of futures.

In general, theoretical analysis along these lines may provide regulators with some guidelines for the establishment of consistent margin requirements. It is clearly not an exact science, however, and substantial judgment is required.

Consistent margins and speculation

Initial margin is the most important parameter in the setting of margin requirements if the control of speculation *qua* leverage is the objective. The goal is to make it harder for pure speculators to borrow a large proportion of the amount that they invest in equity securities, and thus lower their chances of affecting trading volume and market prices.

Once again, the equivalence between spot and futures markets is not conceptually difficult because of the close relationship between their returns. The practical problem is that in each market, initial margin has been set in conjunction with all the other parameters. If the markets have dynamically consistent margin requirements (that is, if the exposure to credit risk is the same in each market), it may be inadvisable to change the initial margin requirement without making offsetting changes in at least some of the other parameters.

Options again present a greater challenge, since an implicit leverage level must be computed as in Table 2, and it is quite difficult to come up with precise values, especially if no theoretical representation exists for the price of a particular option.

The natural tendency is that speculators will shift to markets where initial margin requirements are effectively lower. That is, they will move to markets where a position with a large degree of actual or implicit leverage is permitted. Because of strong interconnections among markets, however, those markets with high margin requirements are not immune to the actions of speculators in other derivative markets. Excessive volatility, as well as nonfundamental pricing, may be transmitted from one market to another. Thus, if speculation is a real issue, the consistency of initial margins should be seriously considered.

Conclusion

The results of this article are perforce not a neat set of rules, but a series of guidelines to be considered by regulators. Making margins consistent across markets demands some serious thought about why there are margin requirements at all. It also confronts regulators

[10] Margin requirements in the spot and futures markets are analyzed graphically in Appendix A.

[11] As markets evolve in response to generally available technical advances, their operational features may converge and thus simplify the establishment of cross-market consistency. For example, the New York Stock Exchange and the American Stock Exchange are currently moving towards a one-day clearing system for all listed equity trades. This development would facilitate the use of shorter margin-posting periods (closer to those in the futures markets), if such a move seems desirable.

with difficult technical problems. Since the mathematical accuracy of the available methods is limited, it is necessary for those regulators to exercise a great deal of judgment in the process.

Even if the technical problems are adequately handled, there are still significant difficulties in bringing together markets that have developed operationally in dramatically different ways. Massive changes would be necessary to equalize each parameter across all markets, even if that were mechanically feasible.

Nevertheless, the concerns about the integrity of the markets and about the dangers of destabilizing speculation are genuine. Dealing with them in only some markets, or in a piecemeal fashion, does not adequately confront the issue. In seeking to adjust margin requirements to meet these objectives, regulators can look to technical studies for guidance but must rely on their good judgment as the ultimate tool.

Arturo Estrella

Appendix A: Graphical Analysis of the Spot and Futures Markets*

The basic diagram

The elements of margin requirements may be compared graphically across markets in the cases of individual stocks and index futures Because of the simplicity of the arbitrage pricing relationship between spot and futures markets, margin requirements apply in much the same way in the two markets As argued in the text, options present a greater challenge in terms of comparative analysis and do not easily lend themselves to this type of graphical exposition

Chart 1 illustrates the three basic types of margin requirements in a single diagram The investor's equity in a stock position is graphed on the vertical axis against the price of the stock on the horizontal axis The 45-degree line, OG, shows the equity that would exist in an unmargined account, namely, 100 percent of the stock value If the initial price of the shares is S_0, then the unmargined investor has initial equity of exactly S_0 In the absence of margin calls, account equity increases or decreases by a dollar for every dollar change in the price of the stock

If the stock is subject to an initial margin requirement of m_I, then account equity must at least equal $m_I S_0$ at the time the stock is purchased (point A) The line OB, which has slope m_I, demonstrates this constraint By choosing to borrow less than the maximum allowable amount, the investor could initially lie anywhere between A and G

A maintenance margin requirement of m_M restricts the position equity to be at all times in excess of $m_M S_t$, or above the line OD, whose slope is m_M As long as the maintenance margin is less than the initial margin, the line OD will lie everywhere beneath OB

If the variation margin requirement is m_V, the one-for-one change in the account equity given a change in the stock price is offset by the amount m_V Thus, equity will change by $1 - m_V$ for each dollar change in the price of the stock Consequently, a line such as AF, passing through point A with slope $1 - m_V$, demonstrates this type of margin requirement In contrast to the lines demonstrating the other two types of margin requirement, the variation margin line may shift as the stock price moves if the upper and lower bounds for required margin are binding This phenomenon is illustrated below in the discussion of spot market requirements

Two extreme cases help to illustrate the effects of variation margin If there is a 100 percent variation margin, the variation margin line will be horizontal In other words, account equity is restricted to remain constant—each dollar change in the underlying price will be fully passed through to the investor By contrast, if the variation margin is zero, the slope of the line will be unity because account equity changes dollar-for-dollar with every change in the underlying price

An interesting case arises if the sum of the initial and variation margin requirements is exactly 100 percent This is equivalent to setting the margin requirement to be at all times a constant proportion of the current stock value Under these circumstances, the line AF in Chart 1 coincides with AB They intersect the schedule AD of maintenance requirements only at the origin, so that the concept of maintenance margin is essentially irrelevant

Both the stock and futures markets in the United States have, in some form, initial, maintenance, and variation margins, although variation margin is somewhat disguised in the stock market and prominent in the futures market

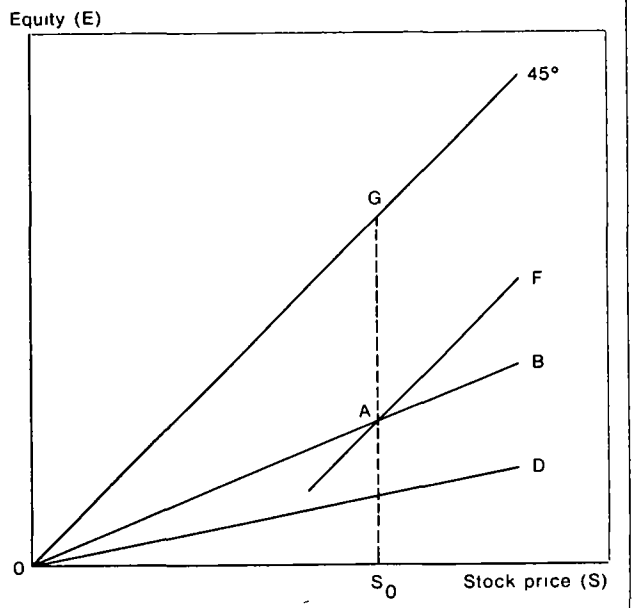
Current institutional framework: a stylized summary

The stock market (New York Stock Exchange). To purchase stock on margin at the New York Stock Exchange (NYSE), a retail investor must put down cash for at least 50 percent of the value of the stock at the time of the purchase † This minimum initial margin requirement

†In addition, a margin account must be opened with at least $2,000

Chart 1

Initial, Maintenance, and Variation Margins



*Stephen R King made valuable contributions to the writing of this appendix

Appendix A: Graphical Analysis of the Spot and Futures Markets *(continued)*

is set by the Federal Reserve Board's Regulation T In addition, the NYSE requires that a retail customer's equity must at all times exceed 25 percent of the current value of the stock (the so-called 25 percent maintenance margin) ‡ The equity in the stock position may only be reduced from 50 to 25 percent as a result of declines in the stock price, not by additional borrowing On the other hand, if the price of the stock were to rise, the investor would be entitled to increase the size of the margin loan to 50 percent of the current stock price §

The margin requirements in the stock market are illustrated diagrammatically in Chart 2, which follows the same basic construction as Chart 1 Once again, the investor starts at point A, which represents a margin of m_0S_0 on a position worth S_0 In this case, equity must exceed the line OGAB, where the slopes of the line segments OG, GA, and AB are 0 25, 1, and 0 5, respectively The segment OG is simply the maintenance margin requirement GA is determined by a variation margin requirement of zero—position losses can be fully subtracted from account equity. Although AB is defined by the initial margin requirement, it also performs the role of a variation margin applied at the initial rate, because it specifies that the investor can withdraw 50 cents for each dollar by which the stock price rises above its initial value

As a numerical example, consider a customer who buys 100 shares for $1 each, financing the purchase by borrowing $50 from a broker If the price of the shares rises to $1 50, the customer's equity rises to $100, or two-thirds of the current value of the investment The margin requirements would allow borrowing of up to $75 (50 percent of the current share value), so the customer would be entitled to withdraw $25 from the broker Note that this is also 50 percent of the rise in value

If, instead of rising, the price had fallen from its initial $1.00 to $0 50, the customer's equity would have evaporated (the value of the stock would exactly equal the $50 00 debt to the broker) The NYSE maintenance requirements demand that the customer's equity be at least 25 percent of the current value of the stock ($12 50, in this case), so the customer would have to post this amount to avoid being sold out

Since margin may be removed from the account if it exceeds 50 percent, and since there are margin calls whenever equity drops below 25 percent, the line segment AG in Chart 2 may shift as stock prices move through time For example, if the stock price rises to S_1, then the investor will be faced with a new variation margin line, A'G', showing the allowable decline in account equity should the stock price subsequently decline Similarly, if equity drops to the line segment OG following a price decline, any subsequent increases would be along a line parallel to AG, but not necessarily along AG itself

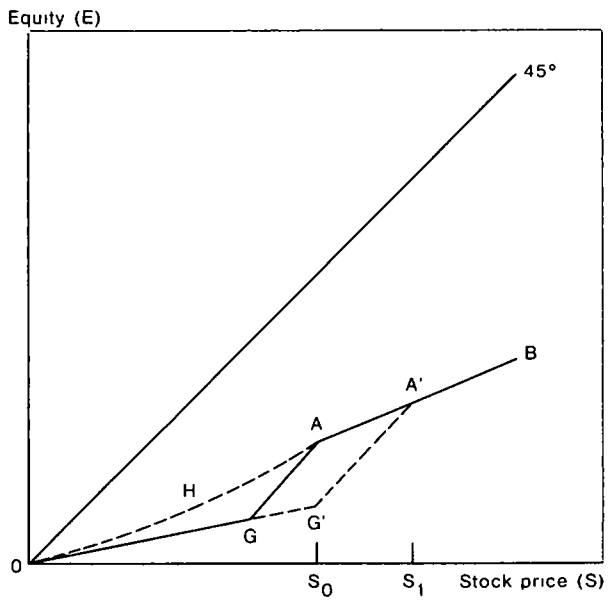
This shifting makes it difficult to anticipate the exact relationship between the uncertain stock prices and the minimum required equity Point G', for instance, which represents a level of equity lower than the initial amount at A, is attainable only if prices and equity first move up to point A' In general, knowing the value of the stock at the end of a given period (or equivalently, the average return over the period) is insufficient to determine the required equity at that time because the whole path of stock prices over the period must be taken into consideration

This phenomenon may be illustrated using the numerical examples given earlier Suppose that the stock price goes from $1 00 to $1 50, and then back to $1.00 As shown above, the margin requirement after the first price movement is $75 After the price drops back to $1 00, the value of the portfolio is $100 once more, but equity is allowed to fall by the full price drop of $50 to the maintenance level of $25

Alternatively, suppose that the price first falls to 50

Chart 2

Stock Market



‡The same regulations do not necessarily apply to specialists or some other professional organizations Cf *The Report of the Presidential Task Force on Market Mechanisms*, January 1988, p VI-15

§In other words, to withdraw equity from the account

Appendix A: Graphical Analysis of the Spot and Futures Markets *(continued)*

cents and then rebounds to $1.00 The maintenance margin requirement, as calculated above, would be binding at $12 50 after the initial drop When the price rises again to $1 00, equity increases to $62 50, but the excess over 50 percent may be withdrawn so that the required level is $50 Thus, we have two situations in which the value of the portfolio starts and ends at $100, but the margin requirement at the end of the period is either at the minimum or at the maximum rate (25 and 50 percent, respectively)

Another complication arises from the length of the period allowed for the posting of margin calls In the stock market, margin calls may be satisfied by a deposit of cash into an investor's margin account, typically within five days. In the intervening time, the stock price might move adversely, lowering the customer's equity Partly as a response to the delayed payment, brokers generally make margin calls before the customer reaches the margin limit. Diagrammatically, this would imply that the path OGA would contain some curvature If price moves are gradual, then a curve such as OHA might capture the effective requirement However, if prices were to drop very sharply, OHA could actually dip below OGA before margin payments were made

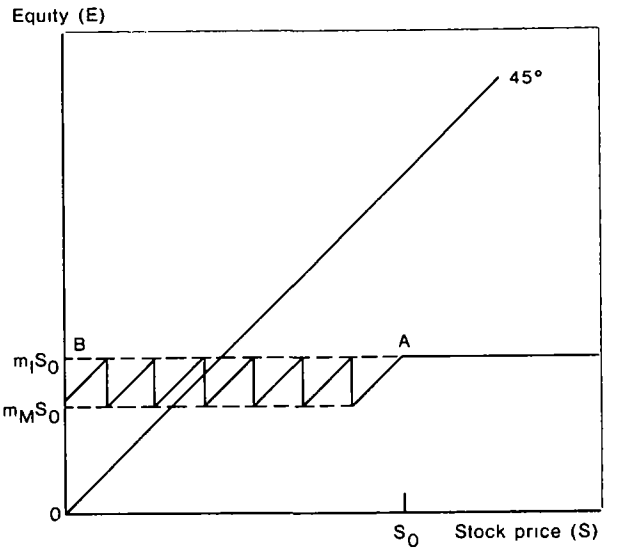
The Futures Market (Chicago Mercantile Exchange) Customer margins in the futures markets perform essentially the same function as margins in the cash market, but they do differ in some important institutional respects. As in the spot market, futures market customers are constrained by both initial and maintenance margins. At the end of 1987, initial margins for a speculator on an S&P 500 futures contract were $20,000, or about 16 percent of the price of the contract Maintenance margins were $15,000, or 12 percent ‖ For a hedger, margins are considerably lower In contrast to the spot market, variation margins are 100 percent of price movements They must be posted by the beginning of the following trading day, and in some instances there may be intraday margin calls

Futures margins are diagramed in Chart 3 ¶ This formulation is particularly simple if the futures price rises. In this case, the 100 percent variation margin allows the investor to withdraw all equity in excess of the initial margin. If the price falls, then the equity in the account may be reduced by that amount to pay for the variation call, unless the balance in the equity falls below the maintenance level. If that occurs, then equity must be raised to its initial level. Consequently, the constraint on the investor may exhibit a sawtooth shape to the left of the initial price In practice, additional margin may be required from the customer at the broker's discretion so that the actual minimum equity may be closer to the horizontal line BA

For comparison with the numerical example in the previous section, we can consider the situation of an investor purchasing a hypothetical $100 futures contract with initial margin of $16 and maintenance margin of $12 Before undertaking the transaction, the investor will be required to have $16 in a margin account At no stage is credit actually extended in a futures transaction, but the investor's initial equity is the $16 down payment If the contract rises in value to $150, the investor will have an equity of $66 (the initial $16 plus the increase of $50 in the value of the contract) Because the contracts are marked to market each business day, the investor would receive the increase in the value of the contract ($50) at that time and could withdraw the full amount of this increase in value as cash. However, the investor can never withdraw an amount that would reduce the position's equity beneath its initial margin amount

Chart 3

Futures Market



‖All institutional details on futures in this appendix relate to contracts on the S&P 500 on the Chicago Mercantile Exchange These contracts are for $500 multiplied by the value of the S&P 500 index, or about $125,000 per contract at year-end 1987 prices Initial margins have been reduced somewhat since that time, to $15,000

¶No distinction between spot and futures prices is made here or in Appendix B It is assumed that the futures price is adjusted for interest costs (which are known contemporaneously) and dividend payouts (which are highly predictable)

Appendix A: Graphical Analysis of the Spot and Futures Markets *(continued)*

If, instead of rising, the value of the contract had fallen to $50, the investor's equity would drop from $16 to −$34 (a capital loss of $50 on the contract) The exchange requires that if the margin account drops below its maintenance value ($12), it must be increased by the start of the next day's trading to the full initial amount Consequently, the investor would be required to put $50 into the account

Had the price decline been less severe—for example from $100 to $97—the situation would be somewhat different In this case, account equity would have shrunk from $16 to $13 Since the margin account would still exceed the minimum maintenance amount ($12), the broker would not be required to demand a margin payment from the customer Instead, the broker could simply forward the $3 of variation margin to the clearing house, debiting the customer's margin account by the same amount

Differences between the two markets Chart 4 combines the analysis from Charts 2 and 3 to show the relationship between margins in the stock and futures markets in a single figure For clarity, it is assumed that customers in the futures market are required to keep their equity at the minimum maintenance level ** The diagram immediately reveals the high initial burden placed on an investor purchasing an instrument on the cash market rather than the futures market. However, it also reveals that in a severe market decline, when prices fall by more than one-half of their initial levels, the minimum equity in the futures market would exceed that in the cash market The reason for this difference is that the futures margins are specified in absolute dollar terms, whereas cash market maintenance margin is stated as a percentage of the current stock price As prices decline, the required margin rate on the futures market investor increases, unless the requirements are modified on an ad hoc basis ††

The same information can be displayed in terms of marginal and average margin requirements in the cash and futures markets, as in Charts 5 and 6 The marginal rate (Chart 5) is simplest in the futures market since it is constant at 100 percent—the investor's margin calls increase dollar-for-dollar with a decline in the price of

**As they would be permitted to do in principle if they kept their balance one cent above the minimum maintenance level While in practice the minimum may be closer to the initial dollar level, the maintenance requirement represents the lowest possible—if not typical—level

††The futures exchanges can and do adjust margin levels on current and existing contracts in response to changed market conditions, principally to variations in volatility If prices move downwards sharply, with an apparent increase in volatility, then the exchanges would likely increase margin requirements If they fell gradually without an increase in volatility, then it is uncertain whether margin levels would be reduced

Chart 4

Comparison of Stock and Futures Markets

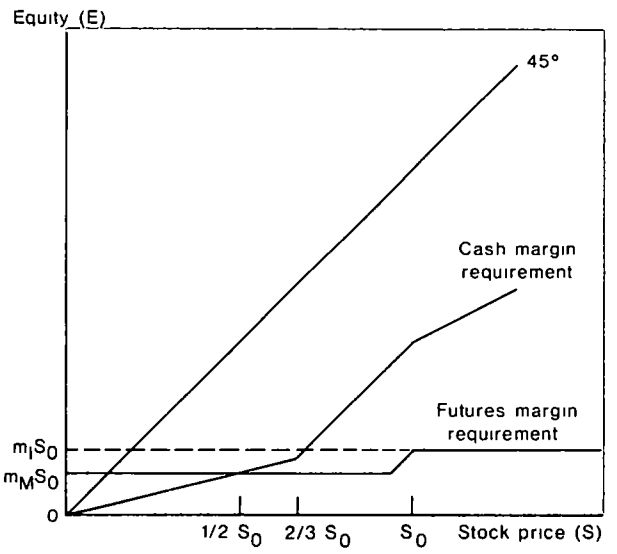
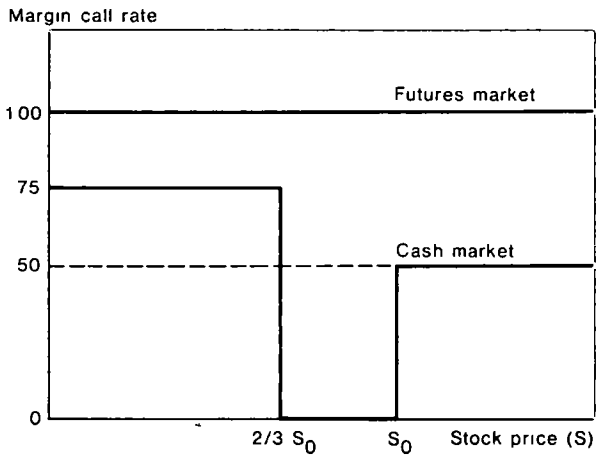


Chart 5

Marginal Margins



Appendix A: Graphical Analysis of the Spot and Futures Markets *(continued)*

the contract.

There are three different marginal margin rates in the cash market, depending on the relationship between the initial price and the current price of the stocks. If the price of the stock rises from its initial value, the investor may withdraw 50 cents for each dollar of price change. If the price falls from its initial value, no additional equity need be added until the 25 percent maintenance level is hit. In this range, therefore, the marginal margin is zero. Once the maintenance level is hit, however, the investor must deposit 75 cents for each dollar by which the price falls. As indicated earlier, the position of the middle range over which there are no margin calls may change if the initial or maintenance margin rates become binding. This corresponds to the shifting of line segment AG in Chart 2.

The average margin rate is computed by dividing total required equity by the price of the underlying investment. The average rates for the cash and futures markets are plotted in Chart 6. In the cash market, the average rate is 50 percent above the initial price and 25 percent once the maintenance level is hit. The futures margin rate is always decreasing because the requirement is fixed in dollar terms. As the contract price rises, the average margin drops towards zero, and as the price falls, the average margin increases indefinitely. If the value of the contract falls beneath the initial margin, the average margin rate can exceed 100 percent.

An important difference between the spot and futures

Chart 6

Average Margins

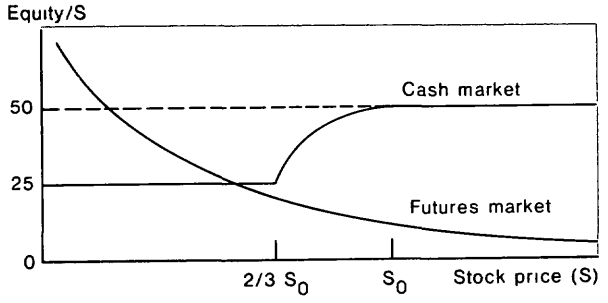


Chart 7

Sample Realization (Bull Market)

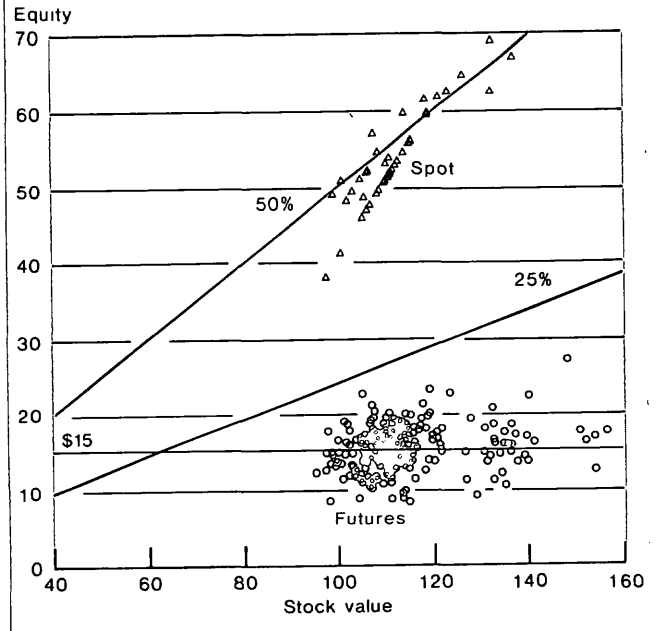
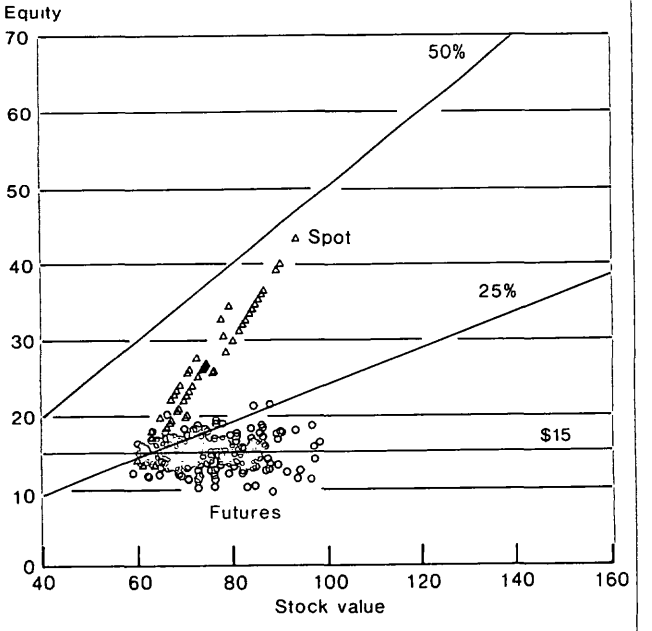


Chart 8

Sample Realization (Bear Market)



Appendix A: Graphical Analysis of the Spot and Futures Markets *(continued)*

markets concerns the length of time that customers have to post margin calls with their brokers This is technically 15 days in the spot market, as compared with at most one trading day in the futures market These numbers overstate the actual difference, however, since brokers in the spot market have the right to be more demanding, and usually are

Simulation analysis

To provide a more specific illustration of how required margins in the spot and futures markets vary over time in relation to the value of the underlying stocks, Charts 7 and 8 present the results of a simulation of margin requirements for a stock or futures portfolio over a period of a year The underlying stock prices are drawn randomly from a distribution with a mean return of 15 percent and a volatility (standard deviation) of 40 percent ‡‡ For the spot market, the requirements are those described earlier, with initial margin of 50 percent and maintenance margin of 25 percent For the futures market, it is assumed that required margin is always 15 percent of the original value of the stocks

The value of each point on the vertical axis represents the dollar value of equity in a customer's margin account just before a margin call is posted, with the corresponding value of the stocks on the horizontal axis Because of the different periods allowed for posting margin calls in the two markets, it is assumed that the time between two consecutive observations is one trading day in the futures market and five trading days in the spot market Hence, there are 250 and 50 points, respectively, for the futures and spot markets

Each chart starts with a stock value of $100 and presents a particular realization (series of randomly generated values) of the stock value process over the course of a year The same realization is used in each chart for both the spot and futures markets In Chart 7, daily returns were generally positive over the course of the year and a wide discrepancy developed between the margin levels in the two markets Some equity was removed from the spot market account when the level exceeded 50 percent, but the maintenance level was not tested The realization of Chart 8 is essentially a bear market, and the margin levels are much more comparable across markets, especially when stock prices fell to 60 percent or less of their original levels

Broadly speaking, margin requirements in both markets perform a similar role, restricting the investor's exposure in the instruments and the creditor's risk Because of the daily and intraday marking to market for futures positions, futures exchanges set their initial and maintenance margin requirements considerably lower than those set in the cash market This represents a rational response to the lower risk exposure that results from frequent marking to market

‡‡The S&P 500 index, which corresponds to a well-diversified stock portfolio, has a historical volatility of about 15 to 20 percent Volatilities for individual stocks vary substantially, but most would be greater than that of the index as a whole, some significantly so

Appendix B: Calculation of Credit Risk for Equity-Related Instruments

This appendix provides specific illustrations of the procedures described heuristically in the text for calculating the likelihood of an equity deficiency in a margin account A model of margin requirements and position equity is developed along the lines of the graphical analysis of Appendix A For stocks, options, and index futures, the events that correspond to negative equity positions within a margin-posting period are identified Numerical examples are also provided, based on a theoretical Wiener process distribution for stock price movements

The model

The following basic definitions (consistent with those of Appendix A) are used in the subsequent discussion

S_t = value of the underlying stocks at time t,
x_t = $\log(S_t/S_o)$ = return from time 0 to time t,
E_t^r = required equity at time t,
m_I = initial margin requirement (proportion),
m_M = maintenance requirement (proportion)

For long *stocks*, the initial required equity is given by

$$E_t^r = m_I S_o$$

Thereafter, equity is allowed to change by any movement in stock prices,

$$E_t = E_{t-1} + S_t - S_{t-1},$$

except that E_t^r is constrained above and below by

$$m_M S_t \leq E_t^r \leq m_I S_t$$

Thus,

$$E_t^r = \min [\max[E_{t-1}^r + S_t - S_{t-1}, m_M S_t], m_I S_t]$$

Appendix B: Calculation of Credit Risk for Equity-Related Instruments *(continued)*

In the NYSE, the current requirements are $m_I = .5$ and $m_M = .25$. The maximum period for posting margin calls is officially 15 business days, but in practice brokers rarely allow more than 5 days, usually just 1 or 2 days.* Since position equity must in principle always be above m_M, the key question from a credit risk point of view is whether, starting from m_M, equity will become negative at any time during the posting period. This event may be represented as:

$$\min_{T \leq t \leq T+H} m_M S_T + S_t - S_T < 0,$$

where H is the length of the posting period This condition may be restated in terms of returns (using the earlier definition for x_t) as:

$$\min_{T \leq t \leq T+H} (x_t - x_T) < \log(1 - m_M).$$

If the distribution of price movements is stationary, as is the case for a Wiener process, the last condition is equivalent to:

$$\min_{0 \leq t \leq H} x_t < \log(1 - m_M) \tag{1}$$

In the market for *index futures,* initial margin is stated in dollar terms, so in proportional terms m_I varies inversely with the level of the index:

$$E_0^r = D,$$
$$m_I = D/S_0,$$

where D is the required dollar amount.† There is a variation margin requirement of 100 percent of movements in the futures price, that is, positions must be marked to market. Any additional margin must be posted within 1 business day, but when large sudden price movements occur, there may be intraday margin calls. Thus, at the start of every business day, the position equity should equal D. A further complication is that brokers are allowed to let their clients' equity positions fall to a maintenance level that is about 75 percent of the initial requirement. In practice, however, the effective requirement is probably closer to D.‡ Thus,

$$m_M = pD/S_t,$$

where $.75 \leq p \leq 1$. Negative equity is observed within the posting period if

$$\min_{T \leq t \leq T+H} pD + S_t - S_T < 0.$$

In terms of returns, this is

$$\min_{T \leq t \leq T+H} (x_t - x_T) < \log(1 - pD/S_T)$$

or, if stationarity holds,

$$\min_{0 \leq t \leq H} x_t < \log(1 - pD/S_T). \tag{2}$$

For a written *call option* on an individual stock, the NYSE margin requirement is

$$E_T^r = \pi_T + \max\,[.15\ S_T - \max[K - S_T, 0],\ .05\ S_T],$$

where π_T is the current call premium and K is the exercise price. This formula applies to both initial and maintenance requirements with the π_T and S_T marked to market daily.§

Here, negative equity results within the posting period (the buyer of the call is exposed to credit risk) if the intrinsic value of the option exceeds the margin, that is, if

$$\max_{T \leq t \leq T+H} S_t - K > E_t^r$$

Based on returns, this expression becomes

$$\max_{T \leq t \leq T+H} (x_t - x_T) > \log((E_t^r + K)/S_T)$$

or, if stationary,

$$\max_{0 \leq t \leq H} x_t > \log((E_T^r + K)/S_T). \tag{3}$$

An illustrative probability distribution: the Wiener process

Once the types of events that concern creditors and regulators are identified, the likelihood of those events can be evaluated. In this section, a Wiener process is used to represent the distribution of future price movements, as is the case in much of the theoretical stock market literature. The parameters of the process (instantaneous mean and variance) are chosen on the basis of empirical evidence, but the shape of the probability distribution is constrained to a Gaussian or normal form. An alternative is to use actual empirical distributions from the past. Such distributions, however, vary substantially over time and do not lend themselves to accurate measurement and prediction.‖ The Wiener formulation, while certainly imperfect, is roughly representative of actual movements and is useful for sensitivity analysis.

The basic definition of a Wiener process for logarithmic changes in stock prices, that is, for

*Details about the rules and practices regarding margin posting periods in the stock market are found in *New York Stock Exchange Guide,* Rule 431, Paragraph (6), Robert P Rittereiser and John P Geelan, *Margin Regulations and Practices,* 2d ed (New York Institute of Finance, 1983), and Richard J Teweles and Edward S Bradley, *The Stock Market,* 5th ed (Wiley, 1987)

†As explained in Appendix A, no distinction is made between spot and futures prices

‡Margins on index futures are discussed in John L Maginn and Donald L Tuttle, eds , *Managing Investment Portfolios A Dynamic Process,* 1985-1986 Update, chap 16

§See Sofianos, "Margin Requirements "

‖See Table 4 in the text, as well as the discussion there

Appendix B: Calculation of Credit Risk for Equity-Related Instruments *(continued)*

$$x_t = \log(S_t/S_o),$$

is given by the stochastic differential equation

$$dx = \mu dt + \sigma dz,$$

where dz represents driftless unit-variance Brownian motion. Given this process, it may be calculated that

$$P[x_H<a] = N\left(\frac{a}{\sigma\sqrt{H}} - \frac{\mu\sqrt{H}}{\sigma}\right),$$

where N(•) is the standard Gaussian distribution, and, more importantly for present purposes,

(4)
$$P\left[\min_{0\le t\le H} x_t<a\right] = N\left(\frac{a}{\sigma\sqrt{H}} - \frac{\mu\sqrt{H}}{\sigma}\right) + e^{2\mu a/\sigma^2}N\left(\frac{a}{\sigma\sqrt{H}} + \frac{\mu\sqrt{H}}{\sigma}\right)$$

Since the Wiener process is stationary,

$$P\left[\min_{T\le t\le T+H} (x_t - x_T)<a\right]$$

has the same value as (4) above.¶ Also,

$$P[\max x > a] = P[\min -x < -a],$$

which leads to expression (4) with the signs reversed for the arguments of the function N (since μ becomes $-\mu$ and a becomes $-a$ in equation (4)).

The right hand side of equation (4) is a function of four parameters.

$$a,\ H,\ \mu,\ \sigma$$

Only the first two of these parameters depend on the particular type of instrument, the last two being determined by the characteristics of the underlying asset. In order to apply expression (4) to the events defined in

¶For a discussion of Wiener processes, including the calculation of these expressions, see D R Cox and H D Miller, *The Theory of Stochastic Processes* (Chapman and Hall, 1980), chap 5

Probability of Negative Equity within Posting Period

Instrument	Exercise Price – Value (Percent)	Maintenance Margin (Percent)	Posting Period (Days)	Volatility 20 Percent	Volatility 40 Percent	Volatility 60 Percent
Stocks	—	25	2	—	0	0
			3		0	000011
			5		0	000634
			15		002672	045657
Options	70	(18 3, 20 7)	2	—	000003	000485
			3		000142	004475
			5		003367	028321
			15		097719	213326
	100	(27 9, 33 3)	2	—	0	0
			3		0	000013
			5		000016	000776
			15		014426	055525
	130	(38 8, 43 8)	2	—	0	0
			3		0	0
			5		0	000021
			15		001045	015167
Index futures	—	7 5	1	0	—	—
		6 0	1	000001		
		4 5	1	000237		
		7 5	2	000010		
		6 0	2	000450		
		4 5	2	008747		

Notes

(1) It is assumed that the underlying stocks follow a Wiener process with an expected return of 12 percent per annum and a volatility as indicated in the table There are 250 trading days per year

(2) An entry of "0" denotes a probability of less than 0000005

(3) For stocks and options, some parameter values are based on NYSE rules and praxis, for index futures, on the CME Further values are included to illustrate the sensitivity of the results to these parameters and to aid in intermarket comparisons

(4) Options are priced using the Black-Scholes formula with no dividends and a riskless interest rate of 7 percent per annum The maintenance margins given are based on the NYSE rules for options on individual stocks and correspond to volatilities of 40 and 60 percent, respectively

Appendix B: Calculation of Credit Risk for Equity-Related Instruments *(continued)*

(1)-(3) of the preceding section, values of μ, σ, and H must be determined, and the specific form of parameter a must be obtained from the appropriate expression in (1)-(3). The parameter a is

$$\log(1-m_M)$$

for stocks,

$$\log(1 - pD/S_T)$$

for futures, and

$$\log((E_t' + K)/S_T)$$

in the case of options.

Numerical examples based on the Wiener process

The accompanying table provides numerical estimates of the probability of negative equity based on the Wiener process. These figures illustrate the range of probabilities that correspond to parameter values roughly representative of those currently observed in the markets. Stocks and options are assumed to correspond to individual securities, while index futures are based on a broad index such as the S&P Composite. For this reason, the volatility of the latter is taken to be lower than those of the individual instruments.

Almost all the probabilities based on realistic parameters are less than 1 percent, in most cases significantly so. An exception is the in-the-money option (K=70) on a stock with a volatility of 60 percent, for which the probability of an equity deficiency within five days is 2.8 percent Creditors would presumably be aware of the reduced margin protection on options that are well into the money and would accordingly reduce the posting period for margin calls. The probabilities in the table seem in general to be quite low. Any such appraisal, however, is of necessity subjective.