REGIME-SWITCHING MONETARY POLICY AND REAL BUSINESS CYCLE FLUCTUATIONS

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ABSTRACT

This paper investigates the implications of a regime switching monetary policy on real business cycle fluctuations. In a Cash-in-Advance model, a regime switching monetary policy with the typical observed business cycle durations could cause sizable fluctuations in real variables such as consumption, and to a lesser extent, investment. The correlations of these real variables with output matched those in the data very well. It is also found that the expected durations of the monetary policy in each regime have a significant effect on the fluctuation of real variables such as consumption and investment. In the longer duration case, the agents would supply more hours and invest less (thus consume more) in the low inflation regime than in the high inflation regime. However, if the monetary policy has a very short expected duration in each regime and switches a lot between the states, the agents' decision rules in different regimes will be close, and contrary to the long duration case, hours are a little lower and investment a little higher in the lower money growth regime than in the higher growth regime. The findings are consistent with the agents' behavior with rational expectations. Adding monetary shock in real business cycle models helps to explain the fluctuations not only in monetary and price variables, but also in real variables. Compared to a non-monetary model, the variations in the model economy are closer to what we see in the data. The implication is that, if there are different policy regimes and people are uncertain about the timing of policy changes, then the expected duration of monetary policy could affect the size of business cycle fluctuations even in a world where agents are assumed to behave rationally and there are no "confusions" or "rigidities".

Key Words: Real Business Cycle, Cash-in-Advance, Monetary Policy, Markov Regime Switching, Calibration, Simulation.

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1 Introduction

Hamilton (1989) finds that business cycle fluctuations can be characterized as a Markov regime switching between two states. Could a regime switching monetary policy cause the swing in real variables as observed by Hamilton (1989)? There is much literature on the causality relationship between money and real business cycle variables. This paper does not intend to solve the causality problem. The question addressed here is: could a regime switching monetary policy matter in a real business model with rational expectations and flexible prices?

The Fed usually switches between periods of expansionary policy and periods of tight policy. Rudebusch (1995) finds that a change in the fed funds target is likely to be followed by another change in the same direction. Dziwura, Pedraza and Remolona (1995) also find evidence for different regimes in Fed's monetary stance. Discrete policy regimes could cause additional fluctuations in the real business cycle since the agents do not have perfect information about how long they could can in one particular regime; they only know the expected duration of each state.

This paper is also motivated by Cooley and Hansen (1993). In their model, the technology shock causes most of the business cycle fluctuations in real variables such as output, hours, consumption, and investment. But introducing monetary shocks "causes little change in the behavior of the real business cycle economy". It is not surprising that money does not matter very much in a rational expectations model with flexible prices and agents have no uncertainty about the monetary process except current shocks. For money to matter in a rational expectation's world, one has either to assume some informational uncertainty, as Lucas's (1972, 1975) island economy, or price rigidities, as Gray (1976), Fischer (1977), Taylor (1979), Mankiw (1985), Parkin (1986), and the third model in Cooley and Hansen (1993).

This paper investigates the effects of the duration of monetary policy on real business cycle fluctuations. Could money matter in a world where there are different monetary policy regimes and agents don't exactly know how long each regime will last? The small monetary effects in Cooley and Hansen (1989, 1993) could be due to the assumption that the monetary process is linear and the policy regime is permanent. Prescott (1993) finds that the duration of regime change is important when accessing the quantitative effects of money.

To formally model the regime-changing monetary process, it is assumed that money supply follows a Markov regime switching process. According to Hamilton (1989) and others, evidence suggests that the business cycle behavior for many aggregate time series could be characterized by a discontinuous nonlinear Markov chain process. If there is strong co-movement between monetary aggregates and the aggregate output, then it is plausible to assume that the monetary growth also follows a Markov two-regime process, no matter the direction of the causality.

To compare the effects of a regime-switching monetary policy on real business cycle fluctuations with the findings of Cooley and Hansen (1993), the monetary process is assumed to be exogenous in the model, and money is valued in equilibrium in a Cash-in-Advance (CIA) setting, as in Cooley and Hansen (1993). A regime switching monetary policy with the typical business cycle durations would cause sizable fluctuations in real variables such as consumption, and to a lesser extent, investment. The expected durations of the monetary policy in each regime have a sizable effect on the fluctuations of real variables.

In the long duration case, agents would supply more hours and invest less (thus consume more) in the low inflation regime than in the high inflation regime. However, if the monetary policy has a very short expected duration in each regime and switches a lot between the states, then agents' decision rules in different regimes would be close. Contrary to the long duration case, hours would be a little lower and investment a little higher in the lower money growth regime than in the higher growth regime. Agents' behaviors here are consistent with rational expectations.

Adding monetary shock in real business cycle models could help to explain not only the fluctuations in monetary and price variables, but also the fluctuation in real variables such as consumption and investment. The fluctuations in the model are closer to what we see in the data. The implication is that, if there are different policy regimes and people are uncertain about the exact timing of policy shifts, then the expected duration of monetary policy could affect the size of business cycle fluctuations even in a world where agents are assumed to behave rationally and there are no "confusions" or "rigidities".

The method used to solve the equilibria of the model follows Hansen and Prescott (1990), except that, here we have a Markov regime switching transition process, and the dynamic equilibrium is solved simultaneously in the two regimes. The model is set up in Section II. Section III defines the recursive competitive equilibrium. Section IV discusses the model calibration and the linear quadratic approximation for the model. Section V presents the solution and simulation findings. The last section discusses the limitations of the model and future research.

2 The Model

Our analysis is based on a CIA model in which money is valued in equilibrium because fiat money is required when purchasing a subset of consumption goods. The law of motion for the growth rate of money supply is parametized in two special ways: One as a Markov chain process with two states; the other as a Markov switching autoregressive process with two states.

In this economy, aggregate output comes from a Cobb-Douglas production function as the following:

$$Y_t = e^{z_t} K_t^{\theta} H_t^{1-\theta},$$

where K_t is the aggregate capital stock, H_t is the aggregate hours of labor input, and z_t represents a multiplicative technology shock that evolves by the following AR process:

$$\begin{aligned} z_{t+1} &= \rho z_t + \epsilon_{t+1}, \qquad 0 < \rho < 1. \\ \epsilon_{t+1} &\sim N(0, \sigma_\epsilon) \end{aligned}$$

z_t is revealed at the beginning of period t. Because of the constant return-to-scale property of the production technology, we can set all the aggregate variables in per capita terms. Let I_t denote aggregate per capita investment, the law of motion for the aggregate capital stock is given by

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad 0 < \delta < 1.$$

Production sector of this economy chooses to maximize profit. Real wage rate and interest rate are given by the first order conditions:

$$w(z_t, K_t, H_t) = (1-\theta)e^{z_t}(\frac{K_t}{H_t})^{\theta} \tag{1}$$

$$r(z_t, K_t, H_t) = \theta e^{z_t}(\frac{H_t}{K_t})^{1-\theta} \tag{2}$$

There is a large number of identical households who are price takers and they derive their utility from consumption and leisure according to

$$E_0 \sum_{t=1}^{\infty} \beta^t (\alpha log c_{1t} + (1-\alpha) log c_{2t} - \gamma h_t),$$

where c_{1t} and c_{2t} represent two types of consumption goods: c_1 is a "cash good," previously accumulated cash balances are required to purchase units of c_1; c_2 is a "credit good.". Utility is a linear function of Hours, which comes from the indivisible labor assumption by Hansen (1985) and employment lotteries by Rogerson (1988).

Consumer holds money balance m_t from period (t-1) to t in order to purchase cash goods c_{1t} in period t. The total money balance in period t is $m_t + T_t$, where T_t is the money transfer from the government. The CIA constraint is

$$P_t c_{1t} \leq m_t + T_t$$

Household budget constraint is

$$c_{1t} + c_{2t} + x_t + \frac{m_{t+1}}{P_t} \leq w_t h_t + r_t k_t + \frac{m_t + T_t}{P_t}$$

Household chooses c_{1t}, c_{2t}, x_t, and m_{t+1} at each period t to maximize its utility. Where w_t and r_t are given by equations (1) and (2).

Suppose from time $t-1$ to t the per capita aggregate money supply is M_t. We have $M_{t+1} = T_t + M_t$. Define

$$u_t = log M_{t+1} - log M_t$$

Set

$$g_t = e^{u_t},$$

so $T_t = (g_t - 1)M_t$.

Assume u_t evolves by

$$\begin{aligned} u_{t+1}(s_{t+1}) &= \bar{u}(s_{t+1}) + \mu_{t+1} \\ &= \tau_0 + \tau_1 s_{t+1} + \mu_{t+1}, \end{aligned} \tag{3}$$

where μ_{t+1} is a zero mean AR process

$$\mu_{t+1} = \eta(L)\mu_t + \xi_{t+1},$$

and $\eta(L)$ is the lag operator

$$\eta(L) = \eta_0 + \eta_1 L^{-1} + \ldots \qquad 0 < \eta_i < 1, i = 0, 1, \ldots$$

The random variable ξ_t is normally distributed with mean 0 and standard deviation σ_ξ. The $\tau s'$ are positive constants. Thus, in this setup, $E_t u_{t+1} > 0$ and $E_t g_{t+1} > 1$, the CIA constraint is always binding (see Appendix A).

We know u_t at the beginning of period t. s_t is revealed also at the beginning of period t. Suppose the state variable s_t is a Markov chain process which takes only two values:

$$\text{if } s_t = 0 \qquad s_{t+1} = \begin{cases} 0 & \text{with prob. p} \\ 1 & \text{with prob. 1-p} \end{cases}$$

$$\text{if } s_t = 1 \qquad s_{t+1} = \begin{cases} 0 & \text{with prob. 1-q} \\ 1 & \text{with prob. q} \end{cases}$$

We define

$$\pi \equiv \begin{vmatrix} p & 1-q \\ 1-p & q \end{vmatrix} \tag{4}$$

In the following sections, I will solve the model for two monetary processes.

Case I:

u_t only switches between two deterministic levels, that is, $(\eta(L) \equiv 0)$ and $var(\xi) \equiv 0$. So $g = e^u$ only switch between two deterministic levels $g_0 > 1$ and $g_1 > 1$:

$$\text{if } g_t = g_0 \qquad g_{t+1} = \begin{cases} g_0 & \text{with prob. p} \\ g_1 & \text{with prob 1-p} \end{cases}$$

$$\text{if } g_t = g_1 \qquad g_{t+1} = \begin{cases} g_0 & \text{with prob. 1-q} \\ g_1 & \text{with prob q} \end{cases}$$

Case II:

As stated in equation (3) but the autoregressive part is $AR(1)$, that is, $\eta(L) \equiv \eta$.

3 The Recursive Competitive Equilibrium

To apply the solution method of *Hansen* and *Prescott* (1990), we first transform variables so that all variables in the deterministic version of the household problem converge to a steady state, or an invariant state. Let's define

$$\hat{m}_t \equiv \frac{m_t}{M_t} \qquad \hat{P}_t \equiv \frac{P_t}{M_{t+1}}$$

We write the household problem in a recursive way by *Bellman's* equation:

Case I:

$$v_i(z, K, k, \hat{m}) = \max\{\alpha log c_1 + (1-\alpha) log c_2 - \gamma h + \beta \sum_j \pi_{ji} E v_j(z', K', k', \hat{m}')\}, \tag{5}$$

where $i, j = 1, 2$, coresponding to state $s = 0, 1$ respectively. The constraints are:

$$z' = \rho z + \epsilon \tag{6}$$

$$K' = (1-\delta)K + X \tag{7}$$

$$k' = (1-\delta)k + x \tag{8}$$

$$
\begin{aligned}
c_1 &= \frac{\hat{m} + g_i}{(g_i + 1)\hat{P}} && (9)\\
c_2 + x + \frac{\hat{m}'}{\hat{P}} &= w(z, K, H)h + r(z, K, H)k && (10)\\
X_i &= X_i(z, K) && (11)\\
H_i &= H_i(z, K) && (12)\\
\hat{P}_i &= \hat{P}_i(z, K), && (13)
\end{aligned}
$$

where $w(z, K, H)$ and $r(z, K, H)$ are given by equations (1) and (2).

Substitute c_1 and c_2 into the objective function (5), we have

$$
\begin{aligned}
v_i(z, K, k, \hat{m}) &= \max_{h,x,\hat{m}'} \{U^i(z, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}') + \beta \sum_j \pi_{ji} E v_j(z', K', k', \hat{m}')\} \\
& \text{subject to (6) (7) (8),} \qquad (14)
\end{aligned}
$$

and (11),(12),(13) are the associated equilibrium aggregate decision rules.

Case II:

$$
\begin{aligned}
v_i(z, \mu, K, k, \hat{m}) &= \max\{\alpha log c_1 + (1-\alpha) log c_2 - \gamma h + \beta \sum_j \pi_{ji} E v_j(z', \mu', K', k', \hat{m}')\} \\
i, j &= 1, 2 && (15)
\end{aligned}
$$

subject to

$$
\begin{aligned}
z' &= \rho z + \epsilon && (16)\\
\mu' &= \eta \mu + \xi && (17)\\
K' &= (1-\delta)K + X && (18)\\
k' &= (1-\delta)k + x && (19)\\
c_1 &= \frac{\hat{m} + e^{\bar{u}(s)+\mu} - 1}{e^{\bar{u}(s)+\mu}\hat{P}} \\
s &= \begin{cases} 0 & \text{if } i = 1 \\ 1 & \text{if } i = 2 \end{cases} && (20)\\
c_2 + x + \frac{\hat{m}'}{\hat{P}} &= w(z, K, H)h + r(z, K, H)k && (21)\\
X_i &= X_i(z, \mu, K) && (22)
\end{aligned}
$$

$$H_i = H_i(z, \mu, K) \tag{23}$$
$$\hat{P}_i = \hat{P}_i(z, \mu, K), \tag{24}$$

where $w(z, K, H)$ and $r(z, K, H)$ are given by equations (1) and (2).

Substitute c_1 and c_2 into the objective function (15), we have

$$v_i(z, \mu, K, k, \hat{m}) = \max_{h,x,\hat{m}'} \{U(z, \mu, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}', \bar{u}(s)) + \beta \sum_j \pi_{ji} E v_j(z', \mu', K', k', \hat{m}')\}$$

$$\text{subject to (16), (17), (18) and (19),} \tag{25}$$

where $i, j = 1, 2$, and (22), (23), (24) are the associated equilibrium decision rules.

Formally, **a Recursive Competitive Equilibrium** consists of a set of decision rules for the household, $c_1^s(S), c_2^s(S), x_s(S), h_s(S)$, and $\hat{m}_s'(S)$, where $S = (z, K, k, \hat{m})$ for Case I, and $S = (z, \mu, K, k, \hat{m})$ for Case II, $(s = 0, 1)$; a set of per-capita aggregate decision rules, $X_s(z, \mu, K)$ and $H_s(z, \mu, K)$; pricing function $P_s(z, \mu, K)$, $w(z, K_s, H_s), r(z, K_s, H_s)$; and a value function $v^s(S)$ such that:

(1) The Household optimizes: Given the pricing functions and per capita decision rules, $v^s(S)$ solves the functional equation (5) or (15) and $c_1^s, c_2^s, x_s, h_s, \hat{m}_s'$ are the associated decision rules.

(2) The firm optimizes: The functions w and r are given by equations (1) and (2).

(3) Individual decisions are consistent with aggregate outcomes:

For Case I:

$$\begin{aligned} x_s(z, K, K, 1) &= X_s(z, K) \\ h_s(z, K, K, 1) &= H_s(z, K) \\ \hat{m}_s'(z, K, K, 1) &= 1 \end{aligned}$$

for all (z, K)

For Case II:

$$
\begin{aligned}
x_s(z,\mu,K,K,1) &= X_s(z,\mu,K) \\
h_s(z,\mu,K,K,1) &= H_s(z,\mu,K) \\
\hat{m}'_s(z,\mu,K,K,1) &= 1 \\
\text{for all } (z,\mu,K) &
\end{aligned}
$$

In the following section, I will first calibrate the model so that a steady state, or **an invariance version** of the model economy, matches post Korean war actual US economic time series data.[2] Then I will solve for the per-capita aggregate decision rules through a linear-quadratic approximation of the model economy, a method described in Hansen and Prescott (1990).

4 Model Calibration and the Linear Quadratic Approximation

4.1 Model Calibration

Before resorting to the numerical method of the linear quadratic approximation as described in Hansen and Prescott (1990), we have to calibrate the model. As in standard business cycle models, such as Cooley and Prescott (1993) and Cooley and Hansen (1993), the parameters are chosen so that certain characteristics of the steady state of the model economy match the average values of the post-Korean war US time series data. More specifically, average values of capital's share of income, the investment-to-output ratio, the capital-to-output ratio, and the time fraction households spend at work in the market are used to calibrate parameters $\theta, \delta, \beta, \gamma$, as follows:

For Case I

To calibrate $\theta, \delta, \beta, \gamma$, and α, we may set $\sigma_\epsilon = 0$, and $\epsilon_t = E\epsilon_t = 0$, so we have a

[2]Calibration is done here so that findings are comparable to existing models in the literature, such as Cooley and Hansen (1993)

"certainty" version of the model economy (5):

$$v^i(K,k,\hat{m}) = \max\{U(c_1,c_2,h) + \beta \sum_j \pi_{ji} v^j(K',k',\hat{m}')\}$$

Subject to (7) (8) (9) (10). Since g_i switches between two levels, the value function, thus the decision rules will switch between two regimes. We still need an *invariant version* of the model economy so that we can use the average values of actual US time series data. To do that, we have to find the invariant state $\bar{s}$ of the Markov chain. Note that $\bar{s}$ is the eigenstate of the transition matrix π corresponding to the eigenvalue 1. (We know that '1' must be an eigenvalue for π.) $\bar{s}$ can be found by setting:

$$\bar{s} = \pi \bar{s},$$

thus

$$\bar{s} = \begin{bmatrix} \bar{s_0} \\ \bar{s_1} \end{bmatrix} = \begin{bmatrix} \frac{1-q}{2-p-q} \\ \frac{1-p}{2-p-q} \end{bmatrix}$$

Note that if $p = q$ then $\bar{s_0} = \bar{s_1} = \frac{1}{2}$.

Using the invariant state $\bar{s}$, we have the following problem:

$$v^0 = \max\{U^0 + \beta(\bar{s_0}v^0 + \bar{s_1}v^1)\} \tag{26}$$

$$v^1 = \max\{U^1 + \beta(\bar{s_0}v^0 + \bar{s_1}v^1)\}, \tag{27}$$

subject to the law of motion (7) and (8). Where U^0 and U^1 denote the utility when c_1 and c_2 are substituted out by (9) and (10), and $g = g_i, i = 0, 1$.

It is easy to show the following lemma:

Lemma 1 *The value functions $v^0 = v^1 = v$ are the solution to the simultaneous problem (26) and (27) if we set $g = \bar{g} \equiv \bar{s_0} g_0 + \bar{s_1} g_1$, so that in (26) and (27) $U^0 = U^1 = U$.*

Thus, to calibrate $\theta, \delta, \beta, \gamma$ and α, we only need to solve the following problem and find out the steady state:

$$v(K,k,\hat{m}) = \max_{h,x,\hat{m}'}\{U(c_1,c_2,h) + \beta v(K',k',\hat{m}')\} \tag{28}$$

subject to (7) (8) (9) (10) with $g_i = \bar{g}$

Appendix B shows the procedure of calibrating model (28) to a quarterly version of post-Korean war US economy. The Markov monetary process is calibrated in two ways: one as a symmetric Markov chain (p=q) and the other has the typical observed business cycle durations. The Results are presented in Tables 1 and 2.

θ	δ	β	γ	α	ρ	σ_ϵ	$\bar{u}$	u_1	u_0	$p = q = x$
0.401	0.021	0.987	2.51	0.841	0.95	0.007	0.014	0.024	0.005	0.578

Table 1: Calibrated Model Parameters in Case I, Symmetric Markov Process

θ	δ	β	γ	α	ρ	σ_ϵ	$\bar{u}$	u_1	u_0	p	q
0.401	0.021	0.987	2.51	0.841	0.95	0.007	0.014	0.029	0.008	0.75	0.90

Table 2: Calibrated Model Parameters in Case I, Markov Process with Business Cycle Durations

To compare with the calibration results in the CIA model of Cooley and Hansen (1993), we show their calibrated parameters in Table 3.

θ	δ	β	γ	α	ρ	σ_ϵ	$\bar{u}$	u_1	u_0	$p = q = x$
0.40	0.019	0.989	2.53	0.84	0.95	0.007	0.015	–	–	–

Table 3: Calibrated Parameters In Cooley and Hansen (1993)

For Case II

First we have to calibrate the monetary process as outlined in Equation (3). Since the money process is assumed to be exogenous and non-model specific, we can estimate the process separately. I adopt the filtering algorithm outlined in Hamilton (1989). The maximum likelihood estimates of the parameters using sample 59:1-91:3 are reported in table

(4),[3] and the estimates using sample 74:1-91:3 are reported in table (5). We see that the implied durations by the (p,q) estimates in table (5) are closer to typical observed business cycle durations with $p = 0.7 - 0.8$, $q = 0.9 - 0.95$. While the estimates in table (4) show that one of the regimes has a relatively longer duration, the whole sample smoother shows that during the entire 60s the monetary growth seemed to have stayed in one regime. It was during late 70s and the 80s money growth seemed to have switched regimes frequently. Note that, since M1 is only an empirical proxy for the monetary process in this model, I don't expect the estimated monetary process capture the business cycle duration characteristics as we see in the data. In the actual solution of the model, I will also use the values of (p,q) that reflect typical business cycle durations, such as $(p,q) = (0.75, 0.9)$, found by Hamilton (1989). In this way, we can ask whether a regime switching monetary policy with typical business cycle durations has a significant effect on real business cycle fluctuations.

$Parameters$	$\bar{u_0}$	$\bar{u_1}$	p	q	η	σ_ξ
$Estimates$	0.033	0.013	0.66	0.97	0.57	0.007
$StdError$	0.004	0.002	0.21	0.02	0.10	0.002

Table 4: Estimated Parameters in the Monetary Model (3), SMPL 59:1-91:3

$Parameters$	$\bar{u_0}$	$\bar{u_1}$	p	q	η	σ_ξ
$Estimates$	0.032	0.014	0.72	0.94	0.36	0.008
$StdError$	0.004	0.002	0.16	0.04	0.18	0.003

Table 5: Estimated Parameters in the Monetary Model (3), SMPL 74:1-91:3

Other parameters in this model were similarly calibrated as in Case I; that is, we calibrate

[3]The parameter estimates of the regime switching money process reported in table 4 are sensitive to what M1 data we use–seasonally adjusted or unadjusted. If we use the seasonally adjusted data, then estimating an AR(4) process is appropriate. But adding regime switching to the AR(4) process makes the coefficients of the first and second lags insignificantly different from zero, only the third and forth lags are significant. As Hamilton (1989) pointed out, this could be the spurious effects created by the way the data are adjusted. So the estimates in table 4 are obtained by using seasonally unadjusted M1.

the parameters according to the steady state of the nonstochastic and invariant version of the model economy. We set the technology shock $z = 0$, and also set the monetary shock $\mu = 0$, and get the mean of the monetary growth u in the invariant state as follows:

$$\text{Invariant state} \quad \bar{s} = \begin{bmatrix} \bar{s_0} \\ \bar{s_1} \end{bmatrix} = \begin{bmatrix} \frac{1-q}{2-p-q} \\ \frac{1-p}{2-p-q} \end{bmatrix},$$

and

$$\bar{u} = \bar{s_0} u_0 + \bar{s_1} u_1$$

4.2 The Linear Quadratic Approximation

For Case I

We first do a Taylor expansion up to the second order to the utility function

$$U(z, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}', u) \tag{29}$$

then we approximate the utility function by a quadratic approximation:

$$\begin{aligned} U^i(z, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}') &\approx y^T Q^i y \\ \text{where y is a } 11 \times 1 \text{ vector } \; y^T &= (1, z, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}') \end{aligned}$$

Q^i is a 11×11 matrix resulting from the Taylor expansion. The index i denote the case when $u = u_i, i = 0, 1$. In the Taylor expansion, we expand $U(z, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}', u)$ around the steady state value of y, and around $u = \bar{u}$, where $\bar{u}$ is the expected value of u in the **invariant state** $\bar{s}$.

After we find the quadratic Taylor expansion, we put in $u = u_i$ in the quadratic form

$$(y^T, u) Q \begin{pmatrix} y \\ u \end{pmatrix}$$

and get the reduced 11×11 Q^i. Note Q is a 12×12 matrix.

The steady state values for y are shown in Appendix B.

Define

$$
\begin{aligned}
x^T &= (1,\text{z},\text{K},\text{k},\hat{m}) \\
(x')^T &= (1,z',K',k',\hat{m}')
\end{aligned}
$$

Because the quadratic nature of the problem, the $FOCs$ are linear functions of the state and control variables, and the solutions are independent of the variance of these variables. Thus we have the certainty equivalence version of the objective function (5):

$$x^T v^0 x = \max_{x,h,\hat{m}'}\{y^T Q^0 y + (x')^T \beta(pv^0 + (1-p)v^1)(x')\} \tag{30}$$

$$x^T v^1 x = \max_{x,h,\hat{m}'}\{y^T Q^1 y + (x')^T \beta((1-q)v^0 + qv^1)(x')\} \tag{31}$$

We solve the above simultaneous functional equations by an initial guess for the negative semi-definite 5×5 matrices v^0, v^1, and using the following law-of-motion to solve for v^0, v^1.

$$
\begin{bmatrix} 1 \\ z' \\ K' \\ k' \\ \hat{m}' \end{bmatrix}
=
\begin{vmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1-\delta & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1-\delta & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}
\begin{bmatrix} 1 \\ \text{z} \\ \text{K} \\ \text{k} \\ \hat{m} \\ \text{H} \\ \text{h} \\ \text{X} \\ \text{x} \\ \hat{P} \\ \hat{m}' \end{bmatrix}
\tag{32}
$$

For Case II

As shown in equation (25), the utility function after substituted in c_1 and c_2 is:

$$U(z, \mu, K, k, \hat{m}, H, h, X, x, \hat{P}, \hat{m}', \bar{u}(s))$$

Let

$$y^T = (1,\text{z},\mu,\text{K},\text{k},\hat{m},\text{H},\text{h},\text{X},\text{x},\hat{P},\hat{m}',\bar{u}(s))$$

We first do a quadratic approximation around $z = 0, \mu = 0, \bar{u}(s) = \bar{u}$, (where $\bar{u}$ is the expected money growth in the invariant state,) and around the steady state values of the other variables,

$$U(z,\mu,K,k,\hat{m},H,h,X,x,\hat{P},\hat{m}',\bar{u}(s)) \approx y^T Q y$$

Where Q is a 13×13 matrix. Then we substitute $\bar{u}(s) = u_0, u_1$ in

$$y^T = (y_0^T, u_i), i = 0,1$$

and get the reduced quadratic approximation corresponding to each state $i = 0,1$.

$$U^i(z,\mu,K,k,\hat{m},H,h,X,x,\hat{P},\hat{m}') \approx (y_0^T, u_i) Q \begin{pmatrix} y_0 \\ u_i \end{pmatrix} = y_0^T Q^i y_0$$

where Q^i is a 12×12 matrix, $i = 0,1$.

Now Let

$$\begin{aligned} x^T &= (1,z,\mu,K,k,\hat{m}) \\ (x')^T &= (1,z',\mu',K',k',\hat{m}') \end{aligned}$$

As stated before, because of the quadratic nature of the problem, the $FOCs$ are linear functions of the state and control variables, and the solutions are independent of the variance of these variables. Thus we have the certainty equivalence version of the objective function (15):

$$x^T v^0 x = \max_{x,h,\hat{m}'} \{y^T Q^0 y + (x')^T \beta (p v^0 + (1-p) v^1)(x')\} \tag{33}$$

$$x^T v^1 x = \max_{x,h,\hat{m}'} \{y^T Q^1 y + (x')^T \beta ((1-q) v^0 + q v^1)(x')\} \tag{34}$$

We solve the above simultaneous functional equations by an initial guess for the negative semi-definite 6×6 matrices v^0, v^1, and using the following law-of-motion to find v^0, v^1.

$$\begin{bmatrix} 1 \\ z' \\ \mu' \\ K' \\ k' \\ \hat{m}' \end{bmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\delta & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-\delta & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} 1 \\ \text{z} \\ \mu \\ \text{K} \\ \text{k} \\ \hat{m} \\ \text{H} \\ \text{h} \\ \text{X} \\ \text{x} \\ \hat{P} \\ \hat{m}' \end{bmatrix} \tag{35}$$

5 Solutions, Simulation Results, and Some Experiments

5.1 Solution and Simulation Results

For Case I

This section presents the solution and simulation results, which are calculated by using a calibrated symmetric Markov process and a Markov process with $(p, q) = (0.75, 0.9)$ for the monetary growth rate.

Table 6 reports the summary statistics for the cyclical behavior of the U.S. economy during 1954:1-1991:2. These statistics come from Table 1 of Cooley and Hansen (1993).

For comparison purpose, Table 7 presents the summary statistics of real variables in a non-monetary model with only technology shocks. The data are from table 2 of Cooley and Hansen (1993).

Table 8 reports the summary statistics for the model economy, in which the monetary growth rate is constant ($\bar{u} = 0.014$) and other parameters have values as shown in Table 1. The decision rules in this case are:

$$\begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.452 & 0.445 & -0.0076 \\ 0.906 & 2.461 & -0.0270 \\ 1.561 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

Table 9 reports summary statistics for the model economy, in which the monetary growth rate u switches between two levels u_0 and u_1 as a symmetric Markov chain process, and the other parameters have values as shown in Table 1. The decision rules in this case are[4]

$$\text{If } S_t = 1 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4520 & 0.445 & -0.0076 \\ 0.9070 & 2.481 & -0.0270 \\ 1.5630 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

$$\text{If } S_t = 0 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4521 & 0.445 & -0.0076 \\ 0.9043 & 2.481 & -0.0270 \\ 1.5601 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

Table 10 reports summary statistics for the model economy, in which the monetary growth rate u switches between two levels u_0 and u_1 as a non-symmetric Markov chain process with $(p, q) = (0.75, 0.9)$ (which implies a policy cycle of 14 quarters.) The calibration process is shown in Appendix B and other parameters have values shown in Table 1. The decision rules in this case are:

$$\text{If } S_t = 1 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4516 & 0.445 & -0.0076 \\ 0.9132 & 2.481 & -0.0270 \\ 1.5702 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

$$\text{If } S_t = 0 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4524 & 0.445 & -0.0076 \\ 0.9031 & 2.481 & -0.0270 \\ 1.5578 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

[4]The statistics are calculated from 100 simulations of 150 periods in length, taking logarithms of each series and using the **HP filter** to get the detrended time series. This procedure is always followed in all the following tables.

Compare the values in Tables 6-10, we see that, with respect to the actual economy, the non-monetary model captures most of the variations in real variables. But the variations in consumption, investment, and hours are substantially lower in the non-monetary model than those in the data, and the correlations of consumption and hours with output are higher than those in the data. In the model economy with constant growth rate in money supply, the fluctuations in consumption, investment, and hours are only marginally higher than those in a non-monetary model. However, adding money into the model economy reduces the correlation between hours and output substantially. In terms of capturing the variations in consumption, investment, and hours, the economy with a symmetric Markov chain money growth rate performs better than the economy with constant money growth rate, but not significantly so. It is very interesting to see that, as shown in Table 10, if we model the money growth rate as a non-linear Markov regime switching process and calibrate it so that it has the typical observed business cycle duration, then the fluctuations of consumption in the monetary model is significantly higher than those of the non-monetary model. The variations of the real variables in this model economy are closer to those in the data, and the correlations of consumption, investment, and hours with output match the values in the data very well, much better than the non-monetary economy with only technology shocks. Another interesting finding is that, the regime switching monetary process does not seem to have caused any significant additional fluctuations in output.

For Case II

Table 11 reports the summary statistics from Monte Carlo experiments for the model economy with both technology and monetary shocks. The money growth follows an AR(1) process with regime switching in means. The decision rules in this case are:

$$\text{If } S_t = 1 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4514 & 0.4448 & -0.0285 & -0.0076 \\ 0.9147 & 2.4808 & 0.44958 & -0.0270 \\ 1.5725 & -0.4173 & 0.52884 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ \mu \\ K \end{pmatrix}$$

$$\text{If } S_t = 0 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4521 & 0.4448 & -0.0285 & -0.0076 \\ 0.9049 & 2.4808 & 0.44958 & -0.0270 \\ 1.5605 & -0.4173 & 0.52884 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ \mu \\ K \end{pmatrix}$$

We see that both hours and investment respond positively to technology shocks, and hours respond negatively to monetary shocks, while investment reacts positively to monetary shocks. People supply slightly more hours in the higher money growth regime than in the low growth regime. In the higher money growth regime people would invest less than whate they would do in the low growth regime. Thus, the stochastic switching between the two regimes adds volatility to real variables.

Compare tables 11-13 with tables 6-8, we see that adding monetary shocks in the model economy helps to explain not only the fluctuations of monetary and price variables, but also the fluctuations of real variables, specially for consumption. The time variations of real variables in the model economy are closer to the variations we see in the data. The correlations of consumption, investment, and hours with output in the model economy are remarkably close to the correlations we see in the data. Another interesting finding worth to mention is that, adding monetary shocks with regime switches does not add any significant fluctuations to output, because the fluctuations in total hours and capital stock do not change much.

5.2 Two Experiments

We see in previous sections that the regime switching monetary policy with asymmetric business cycle durations have significant effects on the time variations of real variables. What would happen if monetary policy is very instable, in the sense that it switches between different regimes a lot? We will carry out such experiments for Case I and Case II by setting $(p, q) = (0.1, 0.1)$.

Table 14 reports the results of the instable monetary policy experiment for Case I, and table 15 contains the results for Case II. Compare the results in these two tables with the results in previous tables, we see that, real variables such as consumption and investment fluctuate much more, and the correlations of these variables with output is closer to what we see in the data. These suggest that money matters in real business cycle fluctuations. Note that the added variations in real variables come from a model where no "confusion" and "rigidity" stories are introduced.

The decision rules are,

For Case I:

$$\text{If } S_t = 1 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4521 & 0.445 & -0.0076 \\ 0.8984 & 2.481 & -0.0270 \\ 1.5542 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

$$\text{If } S_t = 0 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4519 & 0.445 & -0.0076 \\ 0.9129 & 2.481 & -0.0270 \\ 1.5689 & -0.417 & -0.0305 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ K \end{pmatrix}$$

For Case II:

$$\text{If } S_t = 1 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4489 & 0.4415 & -0.0282 & -0.0076 \\ 0.8914 & 2.4622 & 0.4456 & -0.0270 \\ 1.5675 & -0.4211 & 0.5337 & -0.0310 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ \mu \\ K \end{pmatrix}$$

$$\text{If } S_t = 0 \text{ then} \quad \begin{pmatrix} H \\ X \\ \hat{P}' \end{pmatrix} = \begin{bmatrix} 0.4487 & 0.4415 & -0.0282 & -0.0076 \\ 0.9070 & 2.4622 & 0.4456 & -0.0270 \\ 1.5835 & -0.4211 & 0.5337 & -0.0310 \end{bmatrix} \begin{pmatrix} 1 \\ z \\ \mu \\ K \end{pmatrix}$$

As we can see, a noticeable difference here in the short duration monetary policy case is that, as opposite to the cases before, people will work a little more (hours is higher), invest a little less (thus consume more) in the higher monetary growth regime. The difference is due to people's expectation of inflation in next period. In this case, when you are in a higher inflation regime, you expect next period's inflation to be lower, thus you want to work a little more in this period to make more money so that you can consume more cash good next period.

6 Conclusions, Limitations and Future Research

This paper uses a Cash-in-Advance model to investigate the implications of a Markov regime-switching monetary process on real business cycle fluctuations. A regime switching monetary policy with the typical observed business cycle durations would cause sizable fluctuations in real variables such as consumption, and to a lesser extent, investment. The correlations of these real variables with output matched those in the data very well. It is also found that the expected durations of the monetary policy in each regime have a significant effect on the fluctuations of real variables such as consumption and investment. If the regime switching monetary policy has a relatively longer expected durations, then agents would supply more hours and invest less (thus consume more) in the low inflation regime than in the high inflation regime. However, if the monetary policy has very short expected duration in each regime and switches a lot between the states, then agents' decision rules in different regimes will be close, and contrary to the long duration case, hours are a little lower and investment a little higher in the lower money growth regime than in the other regime. Agents' behaviors here are consistent with rational expectations. Adding monetary shock in real business cycle models could help to explain not only the fluctuations in monetary and price variables, but also the fluctuations in real variables. The fluctuations in the model economy are closer to what we see in the data than those of a non-monetary model. The implication is that, if there are different policy regimes and people are uncertain about the timing of policy changes, then the expected duration of monetary policy can affect the size of business cycle fluctuations even in a world where agents are assumed to behave rationally and there are no "confusions" or "rigidities".

There are limitations of CIA-type models. In a CIA model, when inflation (money growth rate) is higher and expected to be higher, people will subsitute away from cash-needed activities such as consumption, and enjoy more leisure; real interest rate will rise and output will fall. Thus the correlations between inflation and nominal interest rate will be positive. However, one of the stylized facts of the post WWII US economy is that, the correlation between money growth and nominal interest rate is negative. This suggests that, in order to explain how the money actually matters in the post WWII US economy, other type models have to be explored, such as Christiano and Eichenbaum's (1992) liquidity effects model.

Appendix A

To show that under the specification of equation (3), the CIA constraint is always binding, we need only to show the equilibrium interest rate $R_t > 0$. We know that, under the given monetary model, the equilibrium nominal interest rate is:

$$R_t = [\beta E_t e^{-u_{t+1}}]^{-1} - 1.$$

Since $e^{-u_{t+1}}$ is a convex function, we have

$$E_t e^{-u_{t+1}} < e^{-E_t u_{t+1}} < 1$$

This is because given the specification in (3), $E_t u_{t+1} > 0$. So we have $R_t > 0$.

Appendix B

A Model Calibration

1) Calibrate θ, the share of capital income:

Let Y_{kb}, Y_{gb}, and Y_{hb} denote the income of business capital, government capital, and household capital respectively. Then:

$$Y_{kb} = \theta_b GNP = \text{Rental Income(RI)} + \text{Corporate Profits(CP)}$$

$$+\text{Net Interest(I)} + \text{Dep} + \theta_b(\text{Proprietors Income(PPIC)}$$

$$+\text{Net National Product(NNP)} - \text{National Income(NI))}$$

$\Longrightarrow$

$$\theta_b = \frac{RI + CP + I + Dep}{GNP - PPIC - NNP - NI}$$

Data for $GNP, RI, CP, I, Dep, PPIC, NNP, NI$ are from CITIBASE. The calibrated value (the average of the above ratio) is $\theta_b = 0.296$. The implied interest rate

$$i = \frac{Y_{kb} - Dep}{K_b}$$

Where

$$Y_{kb} = \theta_b GNP$$

K_b includes the fixed reproducible private capital from Musgrave (1992), the stock of consumer durable is not included. The stock of inventories is from CITIBASE and the stock of land is from the Flow of Funds. The resulted value for i is 0.0196 for quarterly data.

We also get government capital stock K_g and household capital stock K_h from Musgrave(1992), and calculate government capital income and household capital income in the following way:

$$\begin{aligned} Y_{kg} &= (i + \delta_g)K_g \\ Y_{kh} &= (i + \delta_h)K_h \end{aligned}$$

Where δ_g, δ_h are calibrated from the following:

$$\begin{aligned} K_{g,t+1} &= (1 - \delta_g)K_{g,t} + I_{g,t} \\ K_{h,t+1} &= (1 - \delta_h)K_{h,t} + I_{h,t} \end{aligned}$$

Divide the two equations by GNP_t and assume balanced growth path, we have

$$\begin{aligned} \delta_g &= \frac{\overline{I_g/GNP}}{\overline{K_g/GNP}} - \overline{g_{gnp}} \\ \delta_h &= \frac{\overline{I_h/GNP}}{\overline{K_h/GNP}} - \overline{g_{gnp}} \end{aligned}$$

where g_{gnp} is the real growth rate of GNP. Now we are ready to calibrate θ. The total capital income is just:

$$Y_k = Y_{kb} + Y_{kg} + Y_{kh}$$

and the total income is

$$Y = GNP + Y_{kg} + Y_{kh}$$

So

$$\theta = \overline{Y_k/Y}$$

The resulted θ is 0.401.

2. Calibrate δ

In this model, we can calibrate δ according to the steady state investment-capital stock ratio.

$$\delta = \overline{X/K} = \frac{\overline{X/Y}}{\overline{K/Y}}$$

The value is 0.021 for quarterly data.

3. Calibrate the Preference Parameters α, β, γ

In the model (28), substitute out c_1, c_2 and x by the law of motion for k', and by the CIA and budget constraint; set $z = 0$, find the FOCs for h, k' and $\hat{m}'$, we then get the following in the steady state:

$$\beta = \frac{1}{r + 1 - \delta} = \frac{1}{\theta \frac{Y}{k} + 1 - \delta}$$

Use the values we already got above, we have $\beta = 0.987$.

To calibrate α, note that in steady state the FOCs also give us:

$$\begin{aligned}
c_2 &= \frac{1-\alpha}{\gamma} W \\
c_1 &= \frac{\alpha}{\gamma} \frac{\beta}{\bar{g}} W \\
c &= c_1 + c_2 \\
&= \frac{W}{\gamma}(1 - \alpha + \alpha \frac{\beta}{\bar{g}}) \\
\Longrightarrow & \\
v &= \frac{c_1}{c} \\
&= \frac{\alpha \beta \bar{g}}{\bar{g}(1-\alpha) + \alpha\beta} \\
\Longrightarrow & \\
\alpha &= \frac{\bar{g} v}{\beta(\bar{g} - v) + \bar{g} v},
\end{aligned}$$

where W is the wage rate, v is the ratio of cash good consumption over the total consumption of nondurables and services. In data, $v = 0.84$, as shown in Cooley and Hansen (1993), who computed this ratio from Federal Reserve's Commissioned surveys of consumer transactions (Avery 1986, 1987). Using this ratio and the calibrated β above, and $g = 1$ we get $\alpha = 0.841$.

Having got α, we use the equation for c to calibrate γ from the following

$$\gamma = \frac{Y}{C}\frac{1}{H}(1-\theta)(1-\alpha+\alpha\frac{\beta}{g})$$

We have the average investment-output ratio from part 1, using $\frac{C}{Y} = 1 - \frac{I}{Y}$, $H = 0.31$, and the above calibrated parameters, we get $\gamma = 2.51$.

4. Calibrate u_0, u_1, p, q

4.1 Calibrate a symmetric Markov process

To calibrate u_0, u_1, p, q in Case I, we choose $M1$ as the empirical counter part of the money in this model. The $M1$ data is from CITIBASE. We calculated the mean(mdlnm), standard deviation (stdlnm) and first order autocovariance (autovlnm) of the log-difference of $M1$, the values are:

$$\begin{aligned} mdlnm &= 0.0141 \\ stdlnm &= 0.0096 \\ autovlnm &= 5.324 \times 10^{-5} \end{aligned}$$

We use the above data to calibrate a symmetric Markov process as in Case I; that is, we set $p = q = x$, so that the invariant states:

$$\bar{s} = \begin{bmatrix} \bar{s_0} \\ \bar{s_1} \end{bmatrix} = \begin{bmatrix} \frac{1-q}{2-p-q} \\ \frac{1-p}{2-p-q} \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

And we solve the following equations for μ_0, μ_1, x:

$$\bar{u} = 0.5u_0 + 0.5u_1 = 0.0141$$

$$0.5(u_1 - \bar{u})^2 + 0.5(u_0 - \bar{u})^2 = 0.0096^2$$

$$0.5(x(u_1 - \bar{u})^2 + (1-x)(u_1 - \bar{u})(u_0 - \bar{u}) + x(u_0 - \bar{u})^2) = 5.324 \times 10^{-5}$$

The solutions are $u_1 = 0.0237, u_0 = 0.0045, x = 0.58$.

4.2 Calibrate an asymmetric Markov process with business cycle durations $(p, q) = (0.75, 0.9)$

We have

$$\bar{s} = \begin{bmatrix} \bar{s}_0 \\ \bar{s}_1 \end{bmatrix} = \begin{bmatrix} \frac{1-q}{2-p-q} \\ \frac{1-p}{2-p-q} \end{bmatrix} = \begin{bmatrix} 0.286 \\ 0.714 \end{bmatrix}$$

And we solve the following equations for μ_0, μ_1:

$$\bar{u} = 0.286u_0 + 0.714u_1 = 0.0141$$

$$0.286(u_1 - \bar{u})^2 + 0.714(u_0 - \bar{u})^2 = 0.0096^2$$

The solutions are $u_0 = 0.0291, u_1 = 0.0081$.

5. The technology shock

The time series of technology shock can be generated by

$$z_t = lnY_t - \theta lnK_t - (1-\theta)lnH_t$$

As stated in Cooley and Prescott (1993), a quarterly measure of the technology shock is highly persistent. In fact we can't reject a random walk process in z_t. Here, in order to compare with the previous results, I assume a value of $\rho = 0.95$, and $\sigma_\epsilon = 0.007$, as in Cooley and Prescott (1993).

B Steady State Values

Given the parameters in this model, and using equilibrium conditions, the steady state value of y is calculated by the following formula from the FOCs and the law of motion:

1) Capital-Output, Investment-output, and Consumption-output Ratios:

$$\text{Rental rate} = r = \frac{1}{\beta} + \delta - 1$$

$$
\begin{aligned}
\frac{Y}{K} &= \frac{r}{\theta} \\
\frac{X}{Y} &= \frac{X}{K}\frac{K}{Y} = \frac{\delta\theta}{r} \\
\frac{C}{Y} &= 1 - \frac{X}{Y}
\end{aligned}
$$

2) K, H, Y, X, and $\hat{P}$:

$$
\begin{aligned}
H &= \frac{Y}{C}\frac{(1-\theta)(\bar{g}(1-\alpha)+\alpha\beta)}{\gamma\bar{g}} \\
K &= (\frac{r}{\theta H^{1-\theta}})^{\frac{1}{\theta-1}} \\
Y &= K^{\theta}H^{1-\theta} \\
X &= \delta K
\end{aligned}
\tag{36}
$$

$\hat{P}$ can be calculated from the FOCs and the CIA constraint:

$$
c_1 = \frac{\hat{m} + (\bar{g}-1)}{\bar{g}\hat{P}} = \frac{\alpha\beta}{\gamma\bar{g}}W = \frac{\alpha\beta(1-\theta)}{\gamma}\frac{Y}{H}
$$

Using the equilibrium condition $\hat{m} = 1$, we have

$$
\hat{P} = \frac{\bar{g}\gamma}{\alpha\beta W}
$$

In steady state we set $z = 0$, and using the equilibrium condition we have $H = h, K = k, X = x, \hat{m}' = \hat{m} = 1$.

Now we have the steady state values for all the variables in vector y. We then can carry out the quadratic approximation to solve the models.

Stat.	Output	C	INV	HS	GNPDEF	CPI	INFL	MB	M1
Std(%)	1.72	0.86	8.24	1.59	0.88	1.43	0.57	0.84	1.52
Cor.w.Y	1.00	0.77	0.91	0.86	-0.57	-0.52	0.34	0.30	0.33

Table 6: Cyclical Behavior of the U.S. Economy

Stat.	Output	C	INV	K	HS
Std(%)	1.62	0.41	5.02	0.36	1.25
Cor.w.Y	1.00	0.85	0.96	-0.05	0.99

Table 7: Cyclical behavior of the non-monetary economy with only technology shocks

Stat.	Y	C	INV	HS	P	INFL	K	M
Std(%)	1.66	0.45	5.70	1.29	0.45	0.27	0.41	0
Cor.w.Y	1.00	0.83	0.92	0.83	0.001	-0.45	0.61	0

Table 8: Economy with Tech.Shock and Constant Money Growth

Stat.	Y	C	INV	HS	P	INFL	K	M
Std(%)	1.69	0.48	5.80	1.32	1.45	1.07	0.42	1.33
Cor.w.Y	1.00	0.81	0.92	0.82	-0.10	-0.112	0.57	-0.08

Table 9: Economy with Tech.Shock and Symmetric Markov Chain Money Growth

Stat.	Y	C	INV	HS	P	INFL	K	M
Std(%)	1.68	0.61	5.78	1.31	2.06	1.13	0.42	1.86
Cor.w.Y	1.00	0.79	0.91	0.82	-0.01	-0.125	0.56	0.02

Table 10: Model Economy with Tech.Shock, Markov Chain Money Growth With Business Cycle Durations $(p, q) = (0.75, 0.8)$

Stat.	Y	C	INV	HS	P	INFL	K	M
Std(%)	1.67	0.62	5.79	1.30	2.13	1.27	0.42	1.95
Cor.w.Y	1.00	0.80	0.92	0.83	0.073	-0.120	0.59	0.09

Table 11: Economy with Tech.Shock and Markov AR(1) Regime Switching Money Growth as in table 4

Stat.	Y	C	INV	HS	P	INFL	K	M
Std(%)	1.64	0.68	5.76	1.28	2.65	1.51	0.40	2.44
Cor.w.Y	1.00	0.79	0.90	0.81	-0.07	-0.104	0.60	-0.058

Table 12: Economy with Tech.Shock and Markov AR(1) Regime Switching Money Growth with $(p, q) = (0.75, 0.9)$

Stat.	Y	C	INV	HS	P	INFL	K	M
Std(%)	1.66	0.69	5.83	1.30	2.71	1.51	0.41	2.50
Cor.w.Y	1.00	0.78	0.91	0.82	0.03	-0.09	0.57	0.05

Table 13: Economy with Tech.Shock and Markov AR(1) Regime Switching Money Growth with $(p, q) = (0.8, 0.9)$

Stat.	*Y*	*C*	*INV*	*HS*	*P*	*INFL*	*K*	*M*
Std(%)	1.68	0.79	6.08	1.31	0.67	0.67	0.42	0.61
Cor.w.Y	1.00	0.78	0.90	0.80	-0.03	-0.199	0.60	-0.01 -0.01

Table 14: Economy with Tech.Shock, Markov Chain Money Growth as in Case I with $p = q = 0.1$

Stat.	*Y*	*C*	*INV*	*HS*	*P*	*INFL*	*K*	*M*
Std(%)	1.67	0.95	6.16	1.30	1.63	1.32	0.41	1.55
Cor.w.Y	1.00	0.76	0.90	0.83	-0.006	-0.11	0.58	0.01

Table 15: Economy with Tech.Shocks and Monetary Shocks, Markov Regime Switching AR(1) Money Growth with $p = q = 0.1$

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Appendix A

Dataset Description

All data in the paper are from the April 1993 Current Population Survey (CPS). Survey of Employee Benefits, except where otherwise noted. The survey was conducted as a supplement to that month's CPS for all persons employed for pay in one-half of the CPS sample. A subset of questions was asked for unemployed persons who had previously worked for pay. The Survey of Employee Benefits data were matched with labor force data from the April and May 1993 CPS and with income data from the March 1993 income supplement to the CPS.

Weights provided with the data to represent the nation as a whole are used in the descriptive analysis but not in the regression models.[a] No adjustment to the weights is made for missing observations, except where noted. Each step of the analysis in the paper is carried out on the largest sample possible (for example, a worker with a missing value for job tenure is not excluded from analyses where job tenure is not a variable).

The sample used for the participation and contribution analysis meet the following criteria:

1. in Survey of Employee Benefits (employed),
2. not self-employed,
3. 18-64 years old,
4. work at least 20 hours per week,
5. know if participate in 401(k) plan.

Nineteen thousand two hundred (19,200) individuals meet the above criteria. Most of the participation and contribution analysis (Tables 3, 5 - 7, Charts 2 - 3) is based on a smaller sample of 8,129 individuals meeting the additional criterion:

6. offered 401(k) plan by employer.

For the withdrawal analysis, individuals must meet the following criteria:

W1. in Survey of Employee Benefits subsection (employed or once employed),
W2. received lump-sum distribution from pension plan on previous job,
W3. 18-54 years old at time of distribution,
W4. received distribution after 1979.

Two thousand one hundred twenty (2,120) individuals meet the above criteria. Tables 8 - 10 focus on the subset of 1,237 individuals who received distributions between 1988 and 1992.

[a] Results are similar when the weights are used in the regression analysis.