CAN COMPETITION BETWEEN BROKERS MITIGATE AGENCY CONFLICTS WITH THEIR CUSTOMERS?

Sugato Chakravarty and Asani Sarkar

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Sugato Chakravarty

Purdue University West Lafayette, Indiana, 47906

Asani Sarkar

Federal Reserve Bank of New York New York, New York, 10045

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Abstract

We study competitive, but strategic, brokers executing trades for an informed trader in a multi-period setting. The brokers can choose to (a) execute the order, as agents, first, and trade for themselves, as dealers, afterwards; or (b) trade for themselves first and execute the order later. We show that the equilibrium outcome depends on the number of brokers. When the number of brokers exceeds a critical number (greater than one), the informed trader distributes his order (equally) among the available brokers. The brokers, in turn, execute the informed trader's order first and trade personal quantities, as dealers, afterwards. When the number of brokers is below this critical value, the informed trader gives his order to a single broker, who, in turn, trades personal quantities as a dealer first and executes the informed trader's order second. Since the informed trader is hurt in the latter case, he prefers markets with many brokers. Thus, regulators can mitigate trading abuses arising from a conflict of interest between brokers' agency and principal functions (such as front running) by encouraging competition between brokers as an alternative to banning such practices. We empirically show that the critical number of brokers for the favorable competitive equilibrium appears to be satisfied for the futures contracts in our sample.

In his seminal paper, Kyle (1985) addresses the issue of informed trading across multiple periods and how this affects market liquidity and the informativeness of prices. A single risk-neutral informed trader and many uninformed liquidity traders submit orders to a risk-neutral market maker who aggregates orders and clears all trades at a single price. In actual financial markets, however, most traders cannot approach the market maker or be on the trading floor directly. Instead, they have the option of submitting their orders for execution either electronically, such as the SuperDOT at the New York Stock Exchange (NYSE), or go through floor-brokers. For example, Benveniste, Marcus and Wilhelm (1992) estimate that about 75% of the overall trading volume in the NYSE is channeled through brokers, acting as agents on behalf of their customers, on to the market makers.

The presence of floor-brokers leads to new considerations for traders. For example, brokers can, and frequently do, trade for their personal accounts as principals. In this paper we focus on how brokers' personal trading affects their customers and the market structure. The ability to be both an agent and a principal is known as dual trading and is allowed in all major securities' exchanges around the world -- the only restriction being when (relative to the execution of his client's order) a broker can be a principal. The constraint on timing differs by markets. In futures markets, for example, a broker who trades as a principal for his own account, cannot act as an agent for his customer in the same transaction.² In all markets, however, a broker can trade for his customers as an agent and, at other times, trade for his own account as a principal, as long as he does not do both in the same transaction. This practice, known as consecutive dual trading, occurs in futures markets, securities' markets, currency and interest swap markets and fixed income markets.³

Dual trading has been studied by Fishman and Longstaff (1992), Roell (1990) and Sarkar (1995).

But in Fishman and Longstaff (1992), the size of the order flow is fixed, while in Roell (1990) and

¹ The 1995 NYSE Fact Book reports that about 67% of executed NYSE share volume were channeled through floor-brokers.

² This practice, known as simultaneous dual trading, however, is prevalent in securities' markets, currency and interest swap markets, and fixed income markets.

Sarkar (1995), there is no consecutive dual trading. A contribution of the current paper is that we model consecutive dual trading in a multi-period framework, where the order size is also a variable.

In the three papers cited above, there is only a single dual trader. The exception is Chakravarty (1994), who shows, in a single-period framework, that competition among brokers drives individual broker profits from personal trades to zero relatively quickly. In the current paper we have multiple competitive brokers as well as multiple trading periods. This flexibility allows us to address the issue of conflict of interest (for example, the issue of brokers acting as dealers ahead of their customers' orders) and the effect of competition to mitigate such conflicts of interest.

We use an inter-temporal (two-period) framework to examine consecutive dual trading. Specifically, an informed trader observes a private signal about a risky asset payoff and chooses a quantity to buy or sell through brokers as well as the number of brokers among whom this quantity is to be allocated. The brokers, on receiving the order from the informed trader, may choose an action from the following two options: (scenario A) execute the customer's order in the first period as agents and trade on their personal accounts as principals in the second period based on observing their customers' orders; or (scenario B) trade on their personal account as principals in the first period and execute their customer's order as agents in the second period. Additionally, there are noise traders who trade in both periods, and a competitive market maker who observes the aggregate of the noise trades and the net order-flow from the brokers. Conditional on this observation, the market maker sets a price that provides him with zero expected profits.

We show that the equilibrium outcome depends on the number of brokers. When the number of brokers (m) exceeds a critical number of brokers (m' > 1), the informed trader distributes his order (equally) among the available brokers. The brokers, in turn, execute the informed trader's order first and trade personal quantities, as dealers, afterwards. When the number of available brokers is below the

³ See Grossman (1989) for an excellent discussion.

critical value ($m < m^2$), the informed trader gives his order to a single broker, who, in turn, trades personal quantities as a dealer first and executes the informed trader's order second.

In light of the above arguments, we examine the recent controversy surrounding the practice of dual trading in futures exchanges. For example, the Chicago Mercantile Exchange (CME), in June 1991, passed Regulation 552, suspending dual trading privileges in contracts with an average daily trading volume of 10,000 contracts (or more) over the previous six months. The supporters of the ban argue that dual trading enables brokers to cheat their customers through front-running, ⁴ while critics argue that dual trading increases volume and liquidity. Front-running is also an important issue in the New York Stock Exchange (NYSE) as evidenced by a recently proposed widening of NYSE's Rule 92 which allows it to consider all member firms' trades (regardless of whether they occurred at the floor of the NYSE or in some other regional exchange) for evidence of front-running and to prosecute the infractions through an intermarket accord between regulators. In our model, we show that front-running harms customers overall. Compared to a no-front-running benchmark, front-running lowers the informed trader's expected profit, increases the per-period losses by the uninformed traders, and lowers the per-period informativeness of prices. The results are reversed for dual trading, i.e., dual trading is beneficial to customers.

Our results have important policy implications. Specifically, since front-running hurts customers overall, our results suggest that regulators can mitigate trading abuses arising from a conflict of interest between the brokers' agency and principal functions (such as front running) by encouraging competition between brokers as an alternative to banning such practices.

⁴ Front-running occurs when a broker trades personal quantities ahead (and in the same direction) as his customer's order, thereby free-riding off the customer's information for personal profit. Many have argued that front-running results from brokers dealing with the same customers and, over time, recognizing the motives and the information behind specific trades by customers.

⁵ For further details, see Investment Dealers' Digest of July 29, 1996 corresponding to the headline: "Big Board sticks to its guns over front-running rule; wants to regulate member trading off the floor."

Thus, for futures markets regulators, the relevant issues are: What is a reasonable measure of the critical number of brokers m that can lead to the favorable competitive equilibrium in futures markets? And is it likely to be met in actual trading venues? Our simulation results indicate that a "reasonable" upper bound for m is about 11. To answer the second question, we compute the number of dual traders active in selected futures contracts in different trading venues. Specifically, we use Computerized Trade Reconstruction (CRT) data over 30 randomly selected days over a six-month time period starting August 1, 1990 for the following futures' contracts: T-bond futures and soybean oil futures' trading on the Chicago Board of Trade (CBOT); and the 91-day T-bill futures' and live hog futures' trading on the Chicago Mercantile Exchange (CME). In our sample, the average number of dual-traders is 151, 19, 18 and 9, in T-bonds, Soybean oil, Live hogs and T-bill contracts, respectively. Thus, on an average day, there are enough dual trading brokers to create significant competition for front-running profits.

A policy implication of the paper is that dual trading should be allowed, both because it is beneficial to customers overall and because competition among brokers appears to be sufficient to make front running relatively unprofitable for brokers.

In related literature, Madrigal (1996) considers a two period extension of Kyle (1985) and examines the effect of "non-fundamental" speculators on informed trading and related market characteristics. Among other things, he finds that the presence of these speculators can lead to lower market liquidity since they absorb some of the liquidity themselves. In contrast, we have competitive brokers free-riding on the informed trader's private information. And the effect of this free-riding on market liquidity is a function of how the free-riding is done. Holden and Subrahmanyam (1992) consider a multi-period extension of Kyle with multiple (and identical) informed traders (and no brokers). They find that the informed traders trade

^o These contracts are chosen for the diverse range of activity they represent. Specifically, T-bond futures is very active with an average daily trading volume of 408,269 contracts over the chosen time period, while relatively the live hog futures contract is relatively inactive with an average daily trading volume of 12,865 contracts. The two remaining contracts are intermediate in activity.

very aggressively. In contrast, we focus on the competition among brokers and how that can, under certain situations, help the informed trader realize an efficient outcome.

The remainder of the paper is organized as follows. Section I presents the consecutive dual trading model, the two main equilibria and some related results. Section II compares between case A and case B and presents the most significant result of the paper. Section III presents an application of the model, including an empirical verification of an important theoretical bound established by the model. Section IV concludes. All proofs are in the appendix.

I. Consecutive Dual Trading.

Structure and Notation

Consecutive dual trading allows brokers to act as agents on behalf of their client and as principals for their personal trade, but never both in the same transaction. Thus, we are left with two possibilities.

One, where the brokers act as agents first and as principals afterwards; and two, where the brokers act as principals first, based on the information gleaned from observing their fraction of the order received for execution from their client, and as agents afterwards. We analyze these two cases separately.

We consider a two-period market for a single risky asset along the lines of Kyle (1985). The players in the model are: one informed trader, *m* dual-capacity brokers, many noise traders who appear in each of the two trading dates and trade only through the brokers, and a market maker who sees aggregate orders in each period and prices competitively.

There is a single risky asset with random value \widetilde{v} . A continuum of noise traders submit aggregate order flow \widetilde{u}_1 and \widetilde{u}_2 in each period, where \widetilde{u}_i is distributed $N(0, \Sigma_u) \ \forall \ i=1,2$. A single informed trader receives a noisy signal s about the true asset value, where $\widetilde{s}=\widetilde{v}+\widetilde{e}$, and chooses to trade a quantity s through a subset of the s available brokers in the market, where s is known by all. The true asset value, s, is a draw from a Normal distribution with mean 0 and variance s, and the error term s is a draw from a Normal distribution with mean 0 and variance s. Thus, the signal s is

Normally distributed with mean 0 and variance $(\Sigma_n + \Sigma_e)$. Since the brokers observe x, they can invert it and obtain the informed trader's signal. However, each broker is unaware of the true distribution of \mathfrak{F} . Specifically, the brokers (and the market maker) believe that the signal is drawn from a Normal distribution with mean 0 and variance $(\Sigma_n + \Sigma_e + \Sigma_e)$, where $\Sigma_e > 0$, while the informed trader knows that the signal is drawn from a Normal distribution with mean 0 and variance $(\Sigma_0 + \Sigma_e)$. All random variables are independent of one another.

We first solve the informed trader and brokers' equilibrium strategies for case A and case B separately. Then, we compare informed profits in these two cases to establish the unique equilibrium outcome.

Case A. Equilibrium When Brokers Act As Agents In The First Period and Principals In The Second Period.⁸

The sequence of events is as follows: the informed trader observes a perfect signal about the future spot price $\tilde{v} = v$ and chooses an optimal market order quantity x(v) from the following equation. The informed trader knows, when submitting his order, that his order will be executed in the first period. Accordingly,

$$\mathbf{Max} \quad E[\{\widetilde{v} - p_i^{A}(y_i^{A})\}x | \widetilde{s} = s] \tag{1}$$

where p_1^A is the first period market-clearing price of the market maker, which is a function of the aggregate first period order flow y_1^A observed by the market maker and known to brokers.

⁷Kandel and Pearson (1995) use a similar assumption to model different interpretations by traders of the same information signal. In our model, the assumption ensures that the informed trader has an informational advantage over other market participants, and trades in equilibrium.

⁸ All variables and parameters corresponding to the case A are denoted with superscript A.

Next, the brokers, acting as agents, submit the informed trader's order to the market maker for execution. The first period noise trade u_1 is realized and the first period market clearing price $p_1^A(y_1^A)$ is determined, based on the aggregate order flow seen by the market maker: $y_1^A = x + u_1$ (i.e., the informed trader's order and the noise orders only), where

$$p_1^A(y_1^A) = E\left[\widetilde{v} \middle| y_1^A\right] \tag{2}$$

We then move to the second period, when each broker, acting as a principal, chooses his personal trading quantity z from

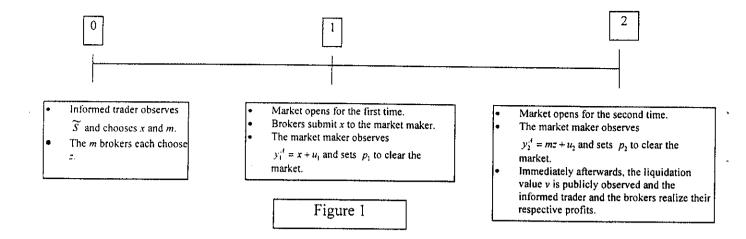
$$\max_{\mathbf{z}} \quad E\left[\left\{\widetilde{\mathbf{v}} - p_{i}^{A}(y_{i}^{A}, y_{i}^{A})\right\} \mathbf{z} \, \middle| \, \mathbf{x} \, \middle| \, \mathbf{m}\right]$$
(3)

where the conditioning is based on each broker observing his portion of the order received from the informed trader, x/m. The m brokers submit their personal trades to the market maker, the second period noise trade u_2 is realized and the second period market clearing price p_2^A is determined, based on y_1^A and the aggregate second period order flow seen by the market maker, $y_2^A = mz + u_2$ (i.e., the brokers' personal orders and noise trades), where

$$p_2^{A}(y_1^{A}, y_2^{A}) = E[\widetilde{v} | y_1^{A}, y_2^{A}]$$
(4)

Thus, the second period pricing function reflects not only what the market maker has learnt after the first-round of trading, but also what he expects to happen in the second-round of trade.

Finally, the liquidation value v is publicly observed and both the informed trader as well as the brokers realize their respective profits (if any). Figure 1 provides a graphical representation.



Formally, an equilibrium is a quintuplet $\{x, m, z, p_i^A, p_i^A\}$ such that equations (1) - (4) hold.

Proposition 1 below solves for the unique linear equilibrium in this market.

Proposition 1: In the case where brokers act as agents in the first period and as principals in the second period, the informed trader distributes his order equally among m brokers, where $m \ge 1$. The resulting equilibrium is unique with:

$$x(s) = \beta^+ s \tag{5}$$

$$z(x) = \gamma^{-1} x \tag{6}$$

$$p_1^A(y_1^A) = \lambda_1^A y_1^A \tag{7}$$

$$p_2^A(y_1^A, y_2^A) = \mu_2^A y_1^A + \lambda_2^A y_2^A$$
(8)

$$\Sigma_{i}^{A} = E\left[\widetilde{v} \middle| p_{i}^{A}\right] \tag{9}$$

$$\Sigma_2^A = E\left[\widetilde{v}\middle|p_1^A, p_2^A\right] \tag{10}$$

where

$$\beta^{A} = \sqrt{\frac{\Sigma_{o}}{\Sigma_{o} + \Sigma_{c} - \Sigma_{b}}} \tag{11}$$

$$\gamma^{A} = \sqrt{\frac{1}{m} \left(\frac{\Sigma_{0} + \Sigma_{c} - \Sigma_{k}}{\Sigma_{0} + \Sigma_{c} + \Sigma_{k}} \right)}$$
 (12)

$$\lambda^{A_{i}} = \frac{\Sigma_{o}}{2(\Sigma_{o} + \Sigma_{c})} \sqrt{\frac{\Sigma_{o} + \Sigma_{c} - \Sigma_{k}}{\Sigma_{w}}}$$
(13)

$$\lambda^{i}_{z} = \frac{\Sigma_{o}(\Sigma_{o} + \Sigma_{e} - \Sigma_{k})}{\left[(\Sigma_{o} + \Sigma_{e} + \Sigma_{k}) + (m+1)(\Sigma_{o} + \Sigma_{e} - \Sigma_{k})\right]} \sqrt{\frac{m}{\Sigma_{u}(\Sigma_{o} + \Sigma_{e} + \Sigma_{k})}}$$
(14)

$$\mu_{z}^{A} = \frac{\Sigma_{o}}{\left(\Sigma_{o} + \Sigma_{e} + \Sigma_{z}\right) + \left(m + 1\right)\left(\Sigma_{o} + \Sigma_{e} - \Sigma_{z}\right)} \sqrt{\frac{\Sigma_{o} + \Sigma_{e} - \Sigma_{z}}{\Sigma_{z}}}$$
(15)

$$\Sigma_{i}^{J} = \frac{\Sigma_{i} \left[\left(\beta^{J} \right)^{2} \left(\Sigma_{i} + \Sigma_{k} \right) + \Sigma_{u} \right]}{\left(\beta^{J} \right)^{2} \left(\Sigma_{i} + \Sigma_{k} + \Sigma_{k} \right) + \Sigma_{u}} \tag{16}$$

$$\Sigma_{z}^{T} = \Sigma_{o} \left\{ 1 - \frac{\left(\beta^{A}\right)^{2} \left(1 + m^{2} \left(\gamma^{A}\right)^{2}\right) \Sigma_{o}}{\left(\beta^{A}\right)^{2} \left(1 + m^{2} \left(\gamma^{A}\right)^{2}\right) \left(\Sigma_{o} + \Sigma_{e} + \Sigma_{k}\right) + \Sigma_{e}} \right\}$$
(17)

Additionally, let π_l^A denote the (unconditional) expected profit to the informed trader and π_b^A denote the (unconditional) expected profit of each of the m brokers. Then,

$$\pi_i' = \frac{\Sigma_n}{2} \sqrt{\frac{\Sigma_n}{\Sigma_n + \Sigma_n - \Sigma_k}} \tag{18}$$

$$\pi'_{\star} = \Sigma_{a} \sqrt{\frac{\Sigma_{a}}{m(\Sigma_{a} + \Sigma_{e} + \Sigma_{k})}} \cdot \left[\frac{(\Sigma_{a} + \Sigma_{e} - \Sigma_{k})}{(\Sigma_{a} + \Sigma_{e} + \Sigma_{k}) + (m+1)(\Sigma_{a} + \Sigma_{e} - \Sigma_{k})} \right]$$
(19)

Proof: See Appendix.

Since, from (18), the informed trader's order size is independent of m, he is indifferent to the number of brokers executing his order. Informed trader profits are independent of the broker's trading intensity because the informed trader knows that his order is executed in the first period and is, therefore, not concerned with what happens afterwards. The unconditional expected profit for each broker's principal trades, on the other hand, is inversely related to the total number of brokers in the market. Competition among brokers drives down the individual broker trading intensity with increasing m.

We now turn to the relative informativeness of prices in the two periods.

Corollary 1: $\Sigma_1^A < \Sigma_1^A$. Also, Σ_1^A is independent of m, and Σ_1^A is increasing in m. Proof: See Appendix.

The information content of time 1 and time 2 prices is measured by (the inverse of) Σ_1^A and Σ_2^A , respectively. Thus, corollary 2 implies that time 2 price is more informative than time 1 price. Interbroker competition in time 2 reveals that portion of the informed trader's information that had not been revealed at time 1. Also, while Σ_1^A is independent of m, Σ_2^A is an increasing function of m. With increasing m, competition among brokers forces each of them to trade at lower trading intensities in the second period. Thus, prices become less informative.

Finally, we examine the market maker's price sensitivity coefficient in the two trading periods. Not surprisingly, these coefficients follow the corresponding informativeness of prices in the two periods (i.e., Σ_1^A and Σ_2^A). That is, greater informativeness is correlated with higher depth and vice versa. Corollary 2: $\lambda_2^A < \lambda_1^A$. Also, λ_1^A is independent of m and λ_2^A increases in m. Proof: See appendix.

Case B. Equilibrium When Brokers Act As Principals In The First Period And As Agents In The Second Period.9

Here the sequence of events is as follows. The informed trader observes a noisy signal s about the future spot price $\tilde{v} = v$ (where, as before, $\tilde{s} = \tilde{v} + \tilde{e}$) and chooses a market order x(s), knowing that the order will be executed in the second period. Accordingly, the informed trader's objective function is given by:

$$\mathbf{Max} \quad E\Big[\Big\{\widetilde{v} - p_2^{B}(y_1^{B}, y_2^{B})\Big\} x \Big| \widetilde{v} = v\Big]$$
(20)

⁹ All variables and parameters corresponding to the case B are denoted with superscript B.

where p_2^B is the second period market clearing price. The informed trader breaks up his order equally among the m available dual traders for execution.

Now each broker chooses his personal trading quantity z, as principal, from the following equation:

$$\max_{\mathbf{z}} \quad E\left[\left\{\widetilde{\mathbf{v}} - p_{i}^{s}(\mathbf{y}_{i}^{s})\right\}\mathbf{z} \mid \mathbf{x} / \mathbf{m}\right]$$
 (21)

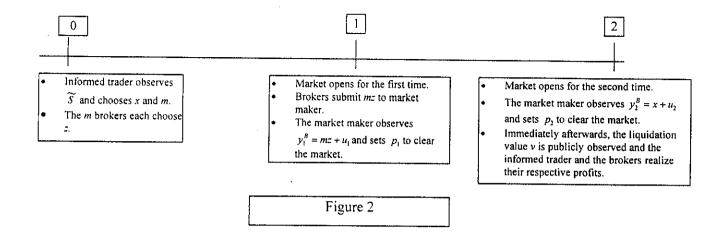
where p_1^B is the first period market clearing price. The *m* brokers submit an aggregate of *mz* to the market maker for execution in the first period. The first period noise trade u_1 is realized and the first period market clearing price p_1^B is determined, based on the aggregate order flow seen by the market maker, $y_1^B = mz + u_1$ (i.e., the aggregate broker orders and noise orders only), where

$$p_i^B(y_i^B) = E[\widetilde{v}|y_i^B] \tag{22}$$

We now move to the second period, where the brokers submit the informed trader's order, as agents, to the market maker for execution. The second period noise trade u_2 is realized, and the second period market clearing price p_2^B is determined, based on y_1^B and the aggregate second period order flow seen by the market maker, $y_2^B = x + u_2$ (i.e., the informed trader's order and noise trades), where

$$p_2^B(y_1^B, y_2^B) = E[\widetilde{v} | y_1^B, y_2^B]$$
 (23)

Finally, the liquidation value v is publicly observed and both the informed trader as well as the brokers realize their respective profits (if any). Figure 2 provides a graphical representation.



An equilibrium in this market is a quintuplet $\{x, m, z, p_1^B, p_2^B\}$ such that equations (20) - (23) hold. Proposition 2 below solves for the unique linear equilibrium in this market.

Proposition 2: In the case where the brokers act as principals in the first period and as agents in the second period, the informed trader trades through a <u>single</u> broker. The resulting equilibrium is unique with:

$$x(s) = \beta^s s \tag{24}$$

$$z(x) = \gamma'' x \tag{25}$$

$$p_1^B(y_1^B) = \lambda_1^B y_1^B \tag{26}$$

$$p_2^B(y_1^B, y_2^B) = \mu_2^B y_1^B + \lambda_2^B y_2^B$$
 (27)

$$\Sigma_1^B = E[\widetilde{v} | p_1^B] \tag{28}$$

$$\Sigma_2^B = E\left[\widetilde{v} \middle| p_1^B, p_2^B\right] \tag{29}$$

where

$$\beta^{n} = \sqrt{\frac{2\Sigma_{k}\Sigma_{k}}{(\Sigma_{n} + \Sigma_{k} - \Sigma_{k})(\Sigma_{n} + \Sigma_{k} + \Sigma_{k})}}$$
(30)

$$\gamma^{B} = \sqrt{\frac{\Sigma_{o} + \Sigma_{c} - \Sigma_{k}}{2\Sigma_{k}}} \tag{31}$$

$$\lambda_{1}^{B} = \frac{\Sigma_{0}}{2} \sqrt{\frac{1}{\Sigma_{x} (\Sigma_{p} + \Sigma_{e} + \Sigma_{x})}}$$
(32)

$$\lambda_{z}^{R} = \frac{\Sigma_{0}}{2\Sigma_{x}} \sqrt{\frac{2\Sigma_{k}(\Sigma_{0} + \Sigma_{x} - \Sigma_{k})}{\Sigma_{x}(\Sigma_{0} + \Sigma_{x} + \Sigma_{k})}}$$
(33)

$$\mu_{z}^{s} = \frac{\sum_{u} \left(\sum_{u} + \sum_{r} - \sum_{k} \right)}{2 \left(\sum_{u} + \sum_{r} \right)} \sqrt{\frac{1}{\sum_{u} \left(\sum_{u} + \sum_{r} + \sum_{k} \right)}}$$
(34)

$$\Sigma_{i}^{B} = \frac{\Sigma_{o} \left[\left\{ \left(\gamma^{B} \beta^{B} \right)^{2} \left(\Sigma_{c} + \Sigma_{k} \right) \right\} + \Sigma_{u} \right]}{\left\{ \left(\gamma^{B} \beta^{B} \right)^{2} \left(\Sigma_{o} + \Sigma_{c} + \Sigma_{k} \right) \right\} + \Sigma_{u}}$$
(35)

$$\Sigma_{2}^{H} = \Sigma_{0} \left\{ 1 - \frac{(\beta^{H})^{2} (1 + (\gamma^{H})^{2}) \Sigma_{0}}{\{ (\beta^{H})^{2} (1 + (\gamma^{H})^{2}) (\Sigma_{0} + \Sigma_{r} + \Sigma_{k}^{T}) \} + \Sigma_{v}} \right\}$$
(36)

Additionally, let π_1^B denote the (unconditional) expected profit to the informed trade and π_b^B denote the (unconditional) expected profit to a broker. Then,

$$\pi_{i}^{"} = \frac{\Sigma_{o}}{2} \sqrt{\frac{2\Sigma_{e}\Sigma_{e}}{(\Sigma_{o} + \Sigma_{e} - \Sigma_{e})(\Sigma_{o} + \Sigma_{e} + \Sigma_{e})}}$$
(37)

$$\pi_b^{"} = \frac{\Sigma_u}{2} \sqrt{\frac{\Sigma_u}{(\Sigma_o + \Sigma_c + \Sigma_k)}}$$
 (38)

Proof: See Appendix.

The informed trader chooses to trade through a single broker because the informed trader's expected payoff function is positive but decreasing in m. The positive expected payoff ensures the informed trader's participation in the trading process. Competition among brokers makes individual broker trades smaller but aggregate broker trades is an increasing function of m. Thus, as m increases, the informed trader is forced to scale back his own trading intensity, lowering his expected profit.

The following corollary compares the relative informativeness of market clearing prices in the two periods.

Corollary 3: $\Sigma_{2}^{B} < \Sigma_{1}^{B}$.

Proof: See Appendix.

As in corollary 2, the second period price is more informative than the first period. More information is revealed in the second period compared to the first as the informed trader's personal order is executed. The market depth follows the informativeness of prices. The second period market depth is greater than the first period market depth.

Corollary 4: $\lambda_1^{\scriptscriptstyle B} < \lambda_1^{\scriptscriptstyle B}$.

Proof: See Appendix.

We now turn to the most important result of the paper.

C. The Informed Trader's Choice of Brokers

So far we have obtained the equilibrium under case A and case B separately. Now we compare informed profits between case A and case B. The first result establishes the fact that, under reasonable parametric conditions, the informed trader makes lower expected profits when brokers trade ahead of this order.

Corollary 5: $\pi_i^{*} < \pi_i^{*}$ as long as $(\Sigma_u + \Sigma_r + \Sigma_k) > 2\Sigma_u$.

Proof: See Appendix.

Corollary 5 states that trading ahead of the informed trader's order is bad for the informed trader in that he makes lower expected profits. This has important consequences that we explore in an application of the model, in the next section.

Now we show how the informed trader allocates his order among available brokers.

Proposition 3: Let
$$m' = \frac{\sum_{n} + \sum_{r} + \sum_{k}}{\sum_{n} + \sum_{r} - \sum_{k}} > 1$$
.

- When m < m, the informed trader gives his order to a single broker who trades ahead of the informed trader.
- When m > m', the informed trader gives his order equally to all m available brokers, all of whom trade after the informed trader.

Proof: See Appendix.

The intuition driving proposition 3 is that when m < m', it is more profitable for the brokers to trade ahead of the informed trades. Since the informed trader's expected profits are decreasing in m, he gives his order to a single broker. When m > m', the aggressiveness of brokers trading ahead of the informed trader drives the expected informed profits below zero. Consequently, equilibrium exists only when brokers trade after the informed customer, and, in this equilibrium, the number of brokers chosen by the informed trader to execute his order strictly exceeds one (because m' itself is greater than one).

The following corollary follows almost immediately from proposition 3 and is stated without a formal proof.

Corollary 6: Brokers are more likely to trade ahead of their customers in a market with few brokers, while they are more likely to trade after their customers in a market with many brokers.

III. An Application Of The Model.

A. Front-running.

We apply the model to analyze the front-running issue. Following the 1989 sting operation at the Chicago Mercantile Exchange (CME) by the Federal Bureau of Investigation, dual trading has been associated with trading abuses in general and with front running in particular. Since 1991, the CME has banned the practice of (consecutive) dual trading The argument given for the ban has been that dual trading enables brokers to cheat their customers through front-running. Case B is particularly suited to examine the front-running and related issues, while case A is suitable for dual trading.

In order to examine the implications of dual trading for markets, we compare the equilibrium in proposition 1 to the Kyle (1985) single-period benchmark where there is no dual trading. We assume $\Sigma_{\star} = \Sigma_{\star} = 0$ for compatibility with the Kyle (1985) setup. $\Sigma_{\star} = 0$ implies that the informed trader observes a pure signal instead of a noisy signal while $\Sigma_{\star} = 0$ implies that brokers and the informed trader have the same interpretation of the information signal.

Proposition 4: Relative to the Kyle (1985) single-period benchmark, in the dual trading equilibrium:

- the informed trader's expected profit per period does not change
- the uninformed traders' expected losses per period are lower
- market clearing price per-period is more informative
- market depth per-period is higher

Proof: See Appendix.

Thus, proposition 4 establishes that the dual trading is "good" for markets and market participants.

Next we show that front-running is "bad" for markets and market participants. Once again, to maintain compatibility with Kyle (1985) we assume that $\Sigma_r = 0$ and take the limit of proposition 2 as $\Sigma_r \to 0.10$

Proposition 5: Relative to the Kyle (1985) single-period benchmark, in the front-running equilibrium:

- the expected informed profit per period is lower
- the uninformed traders' expected losses per period are higher
- market clearing price per-period is less informative
- market depth per-period is higher

Proof: See Appendix.

Regulators would like to encourage the positive effects of dual trading while protecting customers from negative side effects, such as front-running. Our earlier results suggest that if the number

of brokers is high enough, such an outcome can occur on its own, without specific action by regulators. But exactly how "high" must m be?

A numerical simulation of m for different values of Σ_{ϵ} and Σ_{ϵ} relative to Σ_{ϵ} reveals that the maximum value of m over the solution space defined jointly by Σ_{ϵ} ranging from 1% to 40% of Σ_{ϵ} and Σ_{ϵ} ranging from 1% to 40% of Σ_{ϵ} is approximately 11. More frequently, though, the maximum m is around 5 for the solution space defined jointly by Σ_{ϵ} ranging from 10% to 40% of Σ_{ϵ} and Σ_{ϵ} also ranging from 10% to 40% of Σ_{ϵ} . Thus, the simulation implies that with about 5-11 (or less) brokers in a particular contract, front-running becomes an insignificant issue at best and any measure banning dual trading, in order to prevent front-running, is both unnecessary and harmful, since it robs the market of much needed liquidity. If we interpret the profitability through front-running as a front-running option that is "in the money" when m is very low, but having value at all times, then to resolve the issue of how often the option is in the money, we need to compare this theoretical bound with empirical data on actual numbers of competing dual traders in futures contracts. We now proceed with our empirical test.

B. Number Of Competing Dual Traders In Selected Futures Contracts.

The sample period covers 30 randomly selected trading days for the 6 month time period starting August 1, 1990 for the following futures contracts: T-bond futures and soybean oil futures trading on the Chicago Board of Trade (CBOT); the 91 day T-bill futures and the live hog futures trading on the Chicago Mercantile Exchange (CME). They represent very active contracts such as T-bond futures with an average daily trading volume of 408,269 contracts over the chosen time period, to moderately active contracts such as live hog futures with an average daily trading volume of 12,865 contracts. The two remaining contracts, namely the 91 day T-bill futures and the soybean oil futures fall in between in activity and have an average daily trading volume of 14,484 contracts and 17,823 contracts, respectively.

 $^{^{10}}$ The limiting assumption of $\Sigma_{_k} \to 0$ is necessary because there is no equilibrium at $\Sigma_{_k} = 0$.

The data, known as the Computerized Trade Reconstruction (CRT) data, is from the Commodity Futures Trading Commission (CFTC) and includes the following variables, dated by a 15 minute time bracket: trade quantity (number of contracts), a Customer Type Indicator (CTI) code indicating whether the trade was made for an outside customer (CTI 4) or a floor trader's personal account (CTI 1)11, and a code for the floor trader executing the trade.

To identify dual and other traders, we first calculate a trading ratio for each floor trader for each day she is active. Specifically, define d = (personal trading volume)/(personal trading volume + customer trading volume), the proportion that personal trading volume is of a floor trader's total trading volume on a day. We calculate <math>d for each floor trader each day. For a particular day, we categorize a floor trader as a dual trader if d lies on the closed interval [0.02, 0.98]. To eliminate infrequent floor traders, we also create an active dual trader sample. For this sample, we exclude dual traders with less than three dual trading days in the sample.

Table I reports the daily distribution of the number of dual traders for the two samples. Panel A reports results for all dual traders. The mean number of dual traders on a day varies substantially across the four contracts, ranging from 8.54 for T-bills to 151 for T-bonds. Overall, the results show that, on an average day, there are enough dual trading brokers to create significant competition for front-running profits. Panel B reports results for active dual traders. The numbers do not change much from their values in panel A. The general conclusion remains the same.

Earlier, we appealed to a trigger-strategy mechanism to sustain the dual trading equilibrium (see the proof of proposition 1 in the appendix). For such a mechanism to be feasible, it is necessary fordual

¹¹The other indicators are CTI 2 (trades executed for a clearing member's house account) and CTI 3 (trades for another member present on the exchange floor).

¹²The 2% filter is used to allow for the possibility of error trading. As Chang, Locke and Mann (1994) state, "when a broker makes a mistake in executing a customer order, the trade is placed into an error account as a trade for the broker's personal account. A value of 2% for this error seems reasonable from conversations with CFTC and exchange staff."

traders to interact with one another repeatedly over time. The brokers, knowing that they will interact frequently, can effectively threaten deviating brokers who act on their own for short term gains. We provide empirical support for this, by showing that, in our sample, the average number of *active* dual traders is almost identical to the average number of *total* dual traders. For example, from table 1, the average number of active dual traders in the Soybean oil futures contract is 16.83 while the average number of all dual traders in the same contract over the sample period is 17.5. Other futures contracts follow similarly.

Thus, a policy implication of the paper is that dual trading should be allowed, both because it is beneficial to the market and because encouraging competition among brokers can, and does, make front-running an insignificant issue at best.

IV. Conclusion.

In this article, we use a two period model to study the effect of competitive, but strategic, brokers standing between an informed trader and the market maker, on informed profits, market liquidity and the informational efficiency of market clearing prices.

We show that the equilibrium outcome depends on the number of brokers. When the number of brokers exceeds a critical number of brokers m', the informed trader distributes his order (equally) among the available brokers. The brokers, in turn, execute the informed trader's order first and trade personal quantities, as dealers, afterwards. When the number of available brokers is below the critical value, the informed trader gives his order to a single broker, who, in turn, trades personal quantities as a dealer first and executes the informed trader's order second.

We show that front-running reduces informed profits and increases uninformed losses, while dual trading lowers uninformed losses and has no effect on informed profits. Our results suggest that regulators can mitigate trading abuses arising from a conflict of interest between the brokers' agency and principal functions (such as front running) by encouraging competition between brokers as an alternative

to banning dual trading. Through simulation, we show that trading abuses can be mitigated if the number of brokers exceeds 10. We empirically show that the number of dual traders exceeds 10 in three out of the four futures contracts in our sample. Thus, for the three futures contracts, competition among brokers appears to be sufficient to make front-running relatively unprofitable for brokers.

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Table I

Number of Dual Traders in Selected Futures Pits

The table gives the distribution of the number of (consecutive) dual traders each day. Dual trading occurs when a floor trader trades both for his own account and for customers on the same day. In panel A, a dual trader is defined as a floor trader who dual trades at least one day during the sample period. In panel B, a dual trader is defined as a floor trader who dual trades at least three days during the sample period. The sample period is 30 randomly selected trading days between August 1, 1990 and January 31, 1991 for four futures contracts: live hogs and 91 day T-bills, trading on the Chicago Mercantile Exchange; T-bonds and soybean oil, trading on the Chicago Board of Trade.

	Live hogs	Soybean oil	T-bills	T-bonds
Average daily trading volume	12,865	17,823	14,484	408,269
	Par	nel A. Number of all dual trade	ers	
Mean	17.5	18.93	8.54	151.1
Standard deviation	3.14	3.21	1.93	12.31
Minimum	9	14	5	117
1st Quartile	16	16	7	143
Median	18	18	9	153
3rd Quartile	19	21	10	161
Maximum	23	27	13	168
	Panel	B. Number of active dual trad	lers	*** *
Mean	16.83	17.77	8.1	142.77
Standard deviation	2.94	3.09	1.77	11.03
Minimum	9	13	5	111
1st Quartile	16	15	7	138
Median	17	17	8.5	145
3rd Quartile	19	20	9	151
Maximum	22	25	12	158

Appendix

Proof of Proposition 1:

Note that we suppress the superscript A throughout the proof.

The Broker's problem

The informed trader observes $\mathfrak{F} = s$ and chooses x and the number of brokers to have his order executed with. For the moment we will assume that the informed trader, after choosing x, divides this order equally among the m available brokers, such that each broker gets x/m. Hence, a broker's problem reduces to choosing z so as to maximize expected profit conditional on the fact that all m brokers will trade in the second period. Note that in solving an individual broker's problem we assume that given the remaining (m-1) brokers trade in period two, this broker also trades in period two. Later, we justify the use of this assumption. The broker's problem can be written as:

$$\max_{\mathbf{z}} \quad E\Big[\Big(\widetilde{\mathbf{v}} - \mathbf{p}_{2}\Big) \mathbf{z} \, \big| \, \mathbf{x} \, / \, \mathbf{m}\Big] \tag{1}$$

or,
$$Max \quad E\left[\left(\widetilde{v} - \mu_2 y_1 - \lambda_2 y_2\right)z \mid x \mid m\right]$$
 (2)

$$Max \quad E\left[\left(\widetilde{v} - \mu_2\left(x + \widetilde{u}_1\right) - \lambda_2\left((m-1)\overline{z} + z + \widetilde{u}_2\right)\right)z \mid x/m\right]$$
z (3)

We define

$$E[v \mid x] = \xi x \tag{4}$$

where
$$\xi = \frac{\Sigma_0}{\left(\Sigma_0 + \Sigma_1 + \Sigma_2\right)} \frac{1}{\beta}$$
 (5)

Now going back to the broker's maximization problem:

$$Max \left[\xi x z - \mu_2 x z - \lambda_2 (m-1) \overline{z} z - \lambda_2 z^2 \right]$$
 (6)

from first order condition and setting $\overline{z} = z$ in equilibrium we obtain

$$z = \left\{ \frac{\left(\frac{\Sigma_{n}}{\Sigma_{n} + \Sigma_{r} + \Sigma_{r}} \frac{1}{\beta}\right) - \mu_{2}}{\lambda_{2}(m+1)} \right\} x$$

$$= \gamma x$$
(7)

The Informed Trader's problem.

The informed trader observes $\tilde{s} = s$ and chooses x so as to maximize expected profits knowing that his order will be executed in the first period. The informed trader's problem can be written as:

$$\begin{aligned}
Max & E\left[\left(\widetilde{v} - p_{1}\right)x \mid s\right] \\
&= Max & E\left[\left\{v - \lambda_{1}\left(x + \widetilde{u}_{1}\right)\right\}x \mid s\right] \\
&x
\end{aligned} \tag{8}$$

From first order condition, after simplification, we obtain:

$$\beta = \left(\frac{\Sigma_a}{\Sigma_a + \Sigma_c}\right) \frac{1}{2\lambda_1} \tag{9}$$

Pricing coefficients.

Now we solve for the pricing coefficients.

$$p_{1}(y_{1}) = \lambda_{1} y_{1} = E \left[\tilde{v} \middle| y_{1} = x + u_{1} \right]$$
(10)

$$p_{2}(y_{1}, y_{2}) = \mu_{2}\dot{y}_{1} + \lambda_{2}y_{2} = E\left[\tilde{v} \middle| y_{1} = x + u_{1} \middle| y_{2} = mz + u_{2}\right]$$
(11)

Using the same statistical identities as in proposition 1, we obtain:

$$\lambda_1 = \frac{\Sigma_{01}}{\Sigma_{11}}, \quad \mu_2 = \frac{\Sigma_{01}\Sigma_{22} - \Sigma_{02}\Sigma_{12}}{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2}, \quad \lambda_2 = \frac{\Sigma_{02}\Sigma_{11} - \Sigma_{01}\Sigma_{12}}{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^2}$$

where $\Sigma_{01} = \text{Cov}(v, y_1)$, $\Sigma_{02} = \text{Cov}(v, y_2)$, $\Sigma_{11} = \text{Var}(y_1)$ and $\Sigma_{22} = \text{Var}(y_2)$ and $\Sigma_{12} = \text{Cov}(y_1, y_2)$.

Noting that

$$y_1 = \beta \, \tilde{v} + \tilde{u}_1 \tag{12}$$

$$y_2 = m\gamma \beta \widetilde{v} + \widetilde{u}_2 \tag{13}$$

and defining

$$t = \frac{\sum_{0}^{\infty}}{\sum_{0}^{\infty} + \sum_{0}^{\infty}}$$

$$t_{k} = \frac{\Sigma_{0}}{\Sigma_{0} + \Sigma_{c} + \Sigma_{k}}$$

we can express the pricing coefficients formally as

$$\lambda_{1} = \frac{\beta}{\frac{\beta^{2}}{t_{*}} + \frac{\Sigma_{*}}{\Sigma_{*}}} \tag{14}$$

$$\mu_2 = \frac{\beta}{\left(1 + m^2 \gamma^2\right) \frac{\beta^2}{t_*} + \frac{\Sigma_*}{\Sigma_0}} \tag{15}$$

$$\lambda_2 = \frac{m\gamma\beta}{\left(1 + m^2\gamma^2\right)\frac{\beta^2}{t_k} + \frac{\Sigma_u}{\Sigma_0}} \tag{16}$$

Also, from (15) and (16)

$$\frac{\lambda_2}{\mu_2} = m\gamma \tag{17}$$

From (9) and (14)

$$\beta = \sqrt{\frac{\Sigma_{\star}}{\Sigma_{a} + \Sigma_{\star} - \Sigma_{\star}}}$$
 (18)

$$\lambda_{1} = \frac{1}{2} \left(\frac{\Sigma_{o}}{\Sigma_{o} + \Sigma_{c}} \right) \sqrt{\frac{\Sigma_{o} + \Sigma_{c} - \Sigma_{c}}{\Sigma_{o}}}$$
(19)

From (7) and (18), after simplification

$$\gamma = \sqrt{\frac{1}{m} \left(\frac{\Sigma_n + \Sigma_r - \Sigma_s}{\Sigma_n + \Sigma_s + \Sigma_s} \right)} \tag{20}$$

The remaining coefficients are similarly solved as

$$\mu_{z} = \frac{\Sigma_{o}}{\left(\Sigma_{o} + \Sigma_{e} + \Sigma_{k}\right) + \left(m + 1\right)\left(\Sigma_{o} + \Sigma_{e} - \Sigma_{k}\right)} \sqrt{\frac{\Sigma_{o} + \Sigma_{e} - \Sigma_{k}}{\Sigma_{e}}}$$
(21)

$$\lambda_{z} = \frac{\Sigma_{o}(\Sigma_{o} + \Sigma_{e} - \Sigma_{e})}{(\Sigma_{o} + \Sigma_{e} + \Sigma_{e}) + (m+1)(\Sigma_{o} + \Sigma_{e} - \Sigma_{e})} \sqrt{\frac{m}{\Sigma_{e}(\Sigma_{o} + \Sigma_{e} + \Sigma_{e})}}$$
(22)

Finally, the informativeness of price in the two periods is given by

$$\Sigma_{i}^{A} = Var(v) - \frac{\left\{Cov(v, p_{i})\right\}^{2}}{Var(p_{i})} \quad \forall i = 1, 2$$
(23)

where p_1 are given by equations (10) and (11). After substitution and simplification, we obtain

$$\Sigma_{1} = \frac{\Sigma_{0} \left[\beta^{2} \left(\Sigma_{c} + \Sigma_{k} \right) + \Sigma_{k} \right]}{\beta^{2} \left(\Sigma_{0} + \Sigma_{c} + \Sigma_{k} \right) + \Sigma_{k}}$$
(24)

Similarly, p_1 is given by equation (11). Once again, from (23) we obtain

$$\Sigma_{2} = \Sigma_{0} \left\{ 1 - \frac{\beta^{2} (1 + m^{2} \gamma^{2}) \Sigma_{0}}{\beta^{2} (1 + m^{2} \gamma^{2}) (\Sigma_{0} + \Sigma_{s} + \Sigma_{k}) + \Sigma_{u}} \right\}$$

$$(25)$$

The expected informed profit can be expressed as

$$\pi_{i}^{A} = E[(\widetilde{v} - p_{i})x \mid s]$$

$$= E[(\widetilde{v} - \lambda_{i}(\beta \widetilde{s} + \widetilde{u}_{i}))\beta \widetilde{s} \mid s]$$

$$= \frac{\beta \Sigma_{o}}{2}$$

where β is given by

$$\beta = \sqrt{\frac{\Sigma_{b}}{\Sigma_{b} + \Sigma_{c} - \Sigma_{b}}}$$

Similarly, the expected broker profit is expressed as

$$\pi_{k}^{t} = E[(v - p_{1})z \mid s]$$

$$= E[(\widetilde{v} - \mu_{1}(\beta \widetilde{s} + \widetilde{u}_{1}) - \lambda_{2}(m\gamma\beta \widetilde{s} + \widetilde{u}_{2}))\gamma\beta \widetilde{s} \mid s]$$

$$= \sum_{0} \left[\frac{\sum_{n} + \sum_{r} - \sum_{k}}{(\sum_{0} + \sum_{r} + \sum_{k}) + (m+1)(\sum_{n} + \sum_{r} - \sum_{k})} \right] \sqrt{\frac{\sum_{u}}{m(\sum_{0} + \sum_{r} + \sum_{k})}}$$

Note from (18) that β is independent of m. Consistent with this, the first period pricing coefficient λ_1 obtained in (19) is also independent of m. Thus, the informed trader is indifferent between the number of brokers to give his order to and we assume that he divides his order equally between all m available (and identical) brokers.

But does a single broker have an incentive to deviate from his equilibrium strategy, given that the remaining (m-1) brokers all follow the equilibrium strategy? Lemma 1 provides the answer.

Lemma 1: It is more profitable for a single broker to deviate and trade as a principal in the first period, given that the remaining (m-1) brokers all trade as principals in the second period.

Proof: Follows immediately after the proof of proposition 1.

Thus, given that an incentive to deviate does exist among the brokers, we ensure equilibrium in the following way. We assume (without explicitly modeling) that brokers can implicitly collude in equilibrium and punish the deviant. This implicit collusion on the part of the brokers can be sustained, for example, by a trigger-strategy described in Fudenberg and Tirole (1986, pp.52-53).

Suppose that the two-stage game (with the informed trader submitting market orders in stage one, and brokers trading in stage two) is repeated an infinite number of times (alternatively, for a finite number of periods, but where the exact number is not known). Let Π^A and Z^A be each broker's profit and personal trading quantity, respectively, when all brokers trade personal quantities in the second period, after executing the informed trader's order in the first period (case A). Similarly, let Π^B and Z^B be a broker's profit and personal trading quantity, respectively, when all brokers trade personal quantities in the second period, after executing the informed trader's order in the first period (case B).

The trigger-strategy works as follows. Initially, all brokers follow scenario A and each make Π' , and continue doing so until a broker deviates to scenario B and trades $z \neq z'$. Once a deviation occurs, all brokers switch to scenario B (each trading z'' in their personal accounts) and stay with it for a fixed number of periods T' before reverting back to scenario A. Observe that this is a credible threat because (as we formally show in proposition 2) z'' is an equilibrium strategy in the appropriate subgame. A broker's gain from deviation is equal to the one-period increase in profits from front-running minus T' times the discounted value of $\left[\Pi'' - \Pi'''\right]$. Therefore, for sufficiently high values of the discount rate and T', the broker will have no incentive from the equilibrium strategy (of following case A). Note, further, that for the trigger-strategy equilibrium to work, it is important that all brokers receive exactly the same order from the informed trader, which is true in our model.

(QED)

Proof of Lemma 1:

Suppose the deviant broker trades

$$z^{n} = \frac{x}{m} \tag{1}$$

in the first period in his personal account. Since the first period order flow is still

$$y_{i}^{p} = x + u_{i}$$

the market maker's first period pricing coefficient λ_1 remains the same as in equilibrium. Thus,

$$\lambda_{i} = \frac{1}{2} \left(\frac{\Sigma_{o}}{\Sigma_{o} + \Sigma_{e}} \right) \sqrt{\frac{\Sigma_{o} + \Sigma_{e} - \Sigma_{k}}{\Sigma_{u}}}$$
 (2)

The expected profit for the deviant broker can, thus, be expressed as:

$$\pi^{p} = E[(v - p_{1})z^{p}]$$

$$= \frac{1}{m} \left[\Sigma_{o}\beta - \lambda_{1}\beta^{2} \left(\Sigma_{o} + \Sigma_{\epsilon} + \Sigma_{\epsilon} \right) \right]$$
(3)

after simplification and noting that β above is the equilibrium β given by

$$\beta = \sqrt{\frac{\Sigma_{s}}{\Sigma_{n} + \Sigma_{c} - \Sigma_{s}}}$$
 (4)

Substituting (4) in (3) and simplifying we obtain:

$$\pi^{D} = \frac{\Sigma_{o}}{2m(\Sigma_{o} + \Sigma_{c})} \sqrt{\Sigma_{c}(\Sigma_{o} + \Sigma_{c} - \Sigma_{k})}$$
 (5)

It is now fairly easy to show that $\pi^p > \pi^A_b$, where π^A_b is the equilibrium per-broker profit when all m brokers trade personal quantities in the second period after executing the informed trader's order in the first period and is given by

$$\pi_{b}^{A} = \Sigma_{0} \sqrt{\frac{\Sigma_{c}}{m(\Sigma_{0} + \Sigma_{c} + \Sigma_{k})}} \cdot \left[\frac{(\Sigma_{0} + \Sigma_{c} - \Sigma_{k})}{(\Sigma_{0} + \Sigma_{c} + \Sigma_{k}) + (m+1)(\Sigma_{0} + \Sigma_{c} - \Sigma_{k})} \right]$$
 (6)

Hence, it is more profitable for a broker to deviate and trade as a principal in the first period, given that the remaining (m-1) brokers all trade as principals in the second period.

(QED)

Proof of Corollary 1:

The basic equations Σ_1^4 and Σ_2^4 are given by equations (16) and (17) in proposition 1. It is easy to substitute β and γ from (11) and (12) in proposition 1 in (16) and (17), simplifying and comparing. The result follows immediately.

That $\Sigma_i' \perp m$ follows directly from observing Σ_i' and noting the fact that both this expression as well as that for β are independent of m.

Finally, after the necessary substitutions discussed above

$$\Sigma_{2}^{A} = \Sigma_{0} \left[1 - \frac{\Sigma_{0}}{\left(\Sigma_{0} + \Sigma_{e} + \Sigma_{k}\right)} \cdot \frac{\left(\Sigma_{0} + \Sigma_{e} + \Sigma_{k}\right) + m\left(\Sigma_{0} + \Sigma_{e} - \Sigma_{k}\right)}{\left(\Sigma_{0} + \Sigma_{e} + \Sigma_{k}\right) + \left(m + 1\right)\left(\Sigma_{0} + \Sigma_{e} - \Sigma_{k}\right)} \right]$$

It is a matter of simple calculus to show that $\partial \Sigma_i^A / \partial m > 0$.

(QED)

Proof of Corollary 2:

This follows directly from a comparison of (13) with (14) in proposition 1.

(QED)

Proof of Proposition 2:

Here we solve the overall game for the case where the brokers trade ahead of the informed trader (scenario B). For ease of exposition, we suppress the superscript B from all notations.

The Broker's problem.

The informed trader observes $\tilde{s} = s$ and chooses x and the number of available brokers, m, to execute his order. For the moment we will assume that the informed trader, after choosing x, divides this order equally among the m available brokers, such that each broker gets x/m. Thus, a broker's problem reduces to choosing z so as to maximize expected profit conditional on the fact that all m brokers will trade in the first period. We will later show that, given (m-1) brokers trade in the first period, it is optimal for a broker to also trade in the first period (lemma 2). Thus, the broker's problem can be written as:

$$\frac{M\alpha x}{z} E[(\widetilde{v} - p_1)z \mid x / m] \tag{1}$$

or,
$$\frac{Max}{z} E \left[\left\{ \widetilde{v} - \lambda_1 \left((m-1)\overline{z} + z + \widetilde{u}_1 \right) \right\} z \mid x/m \right]$$
 (2)

Now, we let
$$E[\widetilde{v} \mid x] = \xi x$$
 (3)

From projection rule, under the assumption of Normality, we know

$$\xi = \left(\frac{\Sigma_{u}}{\Sigma_{u} + \Sigma_{e} + \Sigma_{k}}\right) \frac{1}{\beta} \tag{4}$$

Substituting (4) and (3) in (2), taking the first order condition of (2) with respect to z and setting the derivative equal to zero, we obtain

$$z = \frac{(t, /\beta)}{\lambda_1(m+1)} x$$

$$= \gamma x$$
(5)

where γ is the trading intensity of each broker and, as in proposition 1,

$$t_{k} = \frac{\Sigma_{0}}{\Sigma_{0} + \Sigma_{r} + \Sigma_{k}}$$

$$t = \frac{\sum_{n}}{\sum_{n} + \sum_{n}}$$

The Informed Trader's problem.

The informed trader observes v and chooses x as discussed above. He also knows that his order will be executed in the second period. His problem can be written as

$$\frac{Max}{x} E\left[\left(\widetilde{v} - p_2\right) x \mid \widetilde{s} = s\right]$$
 (6)

or,
$$\frac{Max}{x} E \left[\left\{ v - \mu_2 \left(mz + \widetilde{u}_1 \right) - \lambda_2 \left(x + \widetilde{u}_2 \right) \right\} x \right]$$
 (7)

From first order condition, after simplification, we obtain

$$x = t \cdot \frac{1}{2\left(\lambda_2 + \mu_2 m\gamma\right)} s$$

$$= \beta s$$
(8)

Solving for the market clearing prices.

Now we solve for the pricing coefficients, given that

$$p_{1}(y_{1}) = \lambda_{1}y_{1} = E\left[\widetilde{v}\middle|y_{1} = mz + u_{1}\right]$$

$$\tag{9}$$

$$p_{2}(y_{1}, y_{2}) = \mu_{2}y_{1} + \lambda_{2}y_{2} = E\left[\widetilde{v} \middle| \begin{aligned} y_{1} &= mz + u_{1} \\ y_{2} &= x + u_{2} \end{aligned}\right]$$
 (10)

From standard statistics

$$\lambda_{1} = \frac{\Sigma_{01}}{\Sigma_{11}}, \quad \mu_{2} = \frac{\Sigma_{01}\Sigma_{22} - \Sigma_{02}\Sigma_{12}}{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^{2}}, \quad \lambda_{2} = \frac{\Sigma_{02}\Sigma_{11} - \Sigma_{01}\Sigma_{12}}{\Sigma_{11}\Sigma_{22} - \Sigma_{12}^{2}}$$
(11)

where Σ_{01} =Cov(v,y₁), Σ_{02} =Cov(v,y₂), Σ_{11} =Var(y₁) and Σ_{22} =Var(y₂) and Σ_{12} =Cov(y₁,y₂). Thus,

$$\lambda_{1} = \frac{m \gamma \beta}{\frac{m^{2} \gamma^{2} \beta^{2}}{t_{k}} + \frac{\Sigma_{k}}{\Sigma_{0}}}$$
(12)

$$\lambda_{2} = \frac{\beta}{\frac{\left(1 + m^{2} \gamma^{2}\right)\beta^{2}}{t_{k}} + \frac{\Sigma_{u}}{\Sigma_{0}}}, \quad \mu_{2} = \frac{m\gamma \beta}{\frac{\left(1 + m^{2} \gamma^{2}\right)\beta^{2}}{t_{k}} + \frac{\Sigma_{u}}{\Sigma_{0}}}$$

$$(13)$$

Also notice from (13) that

$$\frac{\lambda_z}{\mu_z} = \frac{1}{m\gamma} \tag{14}$$

From (8), (13) and (14) we obtain

$$\beta^{2} \left(1 + m^{2} \gamma^{2} \right) = \frac{\Sigma_{x}}{\Sigma_{x} + \Sigma_{x} - \Sigma_{k}} \tag{15}$$

Equating the expression for λ , in (13) and (15), we obtain

$$\lambda_{z} = \frac{\beta \Sigma_{o} (\Sigma_{o} + \Sigma_{c} - \Sigma_{z})}{2 \Sigma_{z} (\Sigma_{o} + \Sigma_{c})}$$
(16)

Rearranging (5) we obtain

$$\lambda_1 = \frac{t_*}{(m+1)\beta\gamma} \tag{17}$$

Equating λ_1 in (12) and (17) we obtain, after simplification, an expression in $\gamma^2 \beta^2$ which is then combined with the $\gamma \beta$ term in (17) to obtain

$$\lambda_1 = \frac{\Sigma_0}{\Sigma_1(m+1)} \sqrt{\frac{m}{(\Sigma_0 + \Sigma_1 + \Sigma_2)}}$$
 (18)

The remaining coefficients are similarly obtained as

$$\beta = \sqrt{\left[\frac{(\Sigma_{n} + \Sigma_{r} + \Sigma_{k}) - m(\Sigma_{n} + \Sigma_{r} - \Sigma_{k})}{(\Sigma_{n} + \Sigma_{r} - \Sigma_{k})(\Sigma_{n} + \Sigma_{r} + \Sigma_{k})}\right] \Sigma_{k}}$$
(19)

It is easy to verify that β is positive as long as

$$m < m^{2} = \frac{\sum_{o} + \sum_{e} + \sum_{e}}{\sum_{o} + \sum_{e} - \sum_{e}}$$
 (20)

where m' > 1.

Similarly,

$$\gamma = \sqrt{\frac{\sum_{n} + \sum_{e} - \sum_{k}}{m\left\{\left(\sum_{n} + \sum_{e} + \sum_{k}\right) - m\left(\sum_{n} + \sum_{e} - \sum_{k}\right)\right\}}}$$
(21)

$$\lambda_{z} = \frac{\Sigma_{o}}{2(\Sigma_{o} + \Sigma_{e})} \sqrt{\frac{\left\{ \left(\Sigma_{o} + \Sigma_{e} + \Sigma_{k}\right) - m\left(\Sigma_{o} + \Sigma_{e} - \Sigma_{z}\right)\right\} \left(\Sigma_{o} + \Sigma_{e} - \Sigma_{k}\right)}{\Sigma_{z}\left(\Sigma_{o} + \Sigma_{e} + \Sigma_{k}\right)}}$$
(22)

$$\mu_{1} = \frac{\sum_{a} \left(\sum_{o} + \sum_{e} - \sum_{k} \right)}{2 \left(\sum_{o} + \sum_{e} \right)} \sqrt{\frac{m}{\sum_{u} \left(\sum_{o} + \sum_{e} + \sum_{k} \right)}}$$
(23)

Now, from (19) it is clear that β is a decreasing function of m. Thus, β is at its maximum when the broker gives his order to a single broker for execution. Hence, in equilibrium, γ is given by substituting m=1 in (21). This gives us:

$$\gamma = \sqrt{\frac{\sum_{i} + \sum_{c} - \sum_{k}}{2\sum_{k}}}$$
 (24)

The corresponding equilibrium β is recalculated as:

$$\beta = \sqrt{\frac{2\Sigma_{k}\Sigma_{u}}{(\Sigma_{u} + \Sigma_{c} - \Sigma_{k})(\Sigma_{u} + \Sigma_{c} + \Sigma_{k})}}$$
(25)

Finally, similar to proposition 1, the informativeness of prices (in the sense of the error variance of price) in the two periods can be expressed as

$$\Sigma_{i} = \frac{\Sigma_{i} \left[\left\{ \left(\gamma \beta \right)^{2} \left(\Sigma_{i} + \Sigma_{k} \right) \right\} + \Sigma_{i} \right]}{\left\{ \left(\gamma \beta \right)^{2} \left(\Sigma_{i} + \Sigma_{k} + \Sigma_{k} \right) \right\} + \Sigma_{i}}$$
(26)

$$\Sigma_{2} = \Sigma_{0} \left\{ 1 - \frac{\beta^{2} (1 + \gamma^{2}) \Sigma_{0}}{\left\{ \beta^{2} (1 + \gamma^{2}) (\Sigma_{0} + \Sigma_{e} + \Sigma_{e}) \right\} + \Sigma_{e}} \right\}$$
(27)

The expected informed profit can be expressed as

$$\pi_{i}^{"} = E[(\widetilde{v} - p_{1})x \mid s]$$

$$= E[(\widetilde{v} - \mu_{1}(\gamma \beta \widetilde{s}) + \lambda_{1}(\beta \widetilde{s} + \widetilde{u}_{1}))\beta \widetilde{s} \mid s]$$

$$= \frac{\beta \Sigma_{0}}{2}$$

where β is given by

$$\beta = \sqrt{\frac{2\Sigma_{k}\Sigma_{k}}{(\Sigma_{n} + \Sigma_{k} - \Sigma_{k})(\Sigma_{n} + \Sigma_{k} + \Sigma_{k})}}$$

Similarly, the expected broker profit is expressed as

$$\pi_{s}^{B} = E[(v - p_{1})z \mid s]$$

$$= E[(\widetilde{v} - \lambda_{1}(m\gamma\beta\widetilde{s} + \widetilde{u}_{1}))\gamma\beta\widetilde{s} \mid s]$$

$$= \frac{\sum_{o}}{2} \sqrt{\frac{\sum_{o}}{\sum_{o} + \sum_{s} + \sum_{s}}}$$

In this Nash equilibrium there is no reason for the broker to deviate. The following lemma explains why.

Lemma 2: It is less profitable for a single broker to deviate and trade as a principal in the second period, given that the remaining m-1 brokers all trade as principals in the first period.

Proof of Lemma 2:

If the broker deviates, he submits the informed order in the first period and trades personally in the second period. Thus, the net order flow seen by the market maker in the two periods under this deviation is given as:

$$y_1 = x + u_1$$

$$y_2 = z^0 + u_2$$
(1)

where z^n stands for the personal trading quantity of the deviating broker. Of course, we need to compute z^n . This is done as follows:

$$\max_{\mathbf{z}^{\mathsf{D}}} E[(\widetilde{v} - p_{z})z^{\mathsf{D}} \mid x]$$
 (2)

$$\max_{\mathbf{Z}_{\mathbf{D}}} E\Big[\Big\{\widetilde{\mathbf{v}} - \mu_{1}^{n}(\mathbf{x} + \widetilde{\mathbf{u}}_{1}) - \lambda_{2}^{n}(\mathbf{z}^{n} + \widetilde{\mathbf{u}}_{1})\Big\} z^{n} \mid \mathbf{x}\Big]$$
(3)

Taking the FOC of the above with respect to z^{p} , setting the derivative equal to zero and simplifying, we obtain:

$$z^{n} = \left(\frac{\frac{\Sigma_{n}}{\beta(\Sigma_{n} + \Sigma_{r} + \Sigma_{r})} - \mu_{2}^{n}}{2\lambda_{2}^{n}}\right) x$$

$$= \chi^{n} x$$
(4)

Note that in (4), the β is the equilibrium value obtained in proposition 2, while the pricing coefficients λ_1^n , μ_1^n and λ_1^n are each recomputed to account for the deviation. Following similar techniques as in propositions 1 and 2, and using (1) above, we obtain:

$$\lambda_1^p = \frac{\beta \Sigma_0}{\beta^1 (\Sigma_0 + \Sigma_s + \Sigma_k) + \Sigma_w} \tag{5}$$

$$\lambda_{2}^{D} = \frac{\gamma^{D} \beta \Sigma_{0}}{\left(1 + \left(\gamma^{D}\right)^{2}\right) \beta^{2} \left(\Sigma_{0} + \Sigma_{c} + \Sigma_{k}\right) + \Sigma_{k}}$$

$$(6)$$

$$\mu_{1}^{D} = \frac{\beta \Sigma_{0}}{\left(1 + \left(\gamma^{D}\right)^{2}\right) \beta^{2} \left(\Sigma_{0} + \Sigma_{c} + \Sigma_{k}\right) + \Sigma_{c}}$$

$$(7)$$

Using (4), (5), (6), (7) and the equilibrium expression for β , we obtain

$$\gamma^{p} = \frac{1}{2} \sqrt{\frac{\Sigma_{o} + \Sigma_{c} + \Sigma_{k}}{\Sigma_{k}}}$$
 (8)

$$\mu_{z}^{D} = \frac{2}{3} \frac{\Sigma_{o}}{\left(\Sigma_{o} + \Sigma_{e} + \Sigma_{z}\right)} \sqrt{\frac{2(\Sigma_{o} + \Sigma_{e} - \Sigma_{z})\Sigma_{z}}{\Sigma_{z}(\Sigma_{o} + \Sigma_{e} + \Sigma_{z})}}$$
(9)

$$\lambda_{2}^{o} = \frac{1}{3} \frac{\Sigma_{o}}{\left(\Sigma_{o} + \Sigma_{e} + \Sigma_{z}\right)} \sqrt{\frac{2\left(\Sigma_{o} + \Sigma_{e} - \Sigma_{z}\right)}{\Sigma_{u}}} \tag{10}$$

Now, we can compute the expected profit for the deviant broker as:

$$\pi_{s}^{n} = E[(\widetilde{v} - p_{i})z^{n} \mid s]$$

$$= E[\{\widetilde{v} - \mu_{i}^{n}(\beta \widetilde{s} + \widetilde{u}_{i}) + \lambda_{i}^{n}(z^{n} + \widetilde{u}_{i})\}z^{n} \mid s]$$
(11)

After considerable simplification, we obtain:

$$\pi_{\star}^{D} = \frac{2}{3} \Sigma_{0} \left\{ \frac{1}{\sqrt{2}} - \frac{\Sigma_{\star}}{\Sigma_{0} + \Sigma_{\star} + \Sigma_{\star}} \right\} \tag{12}$$

Now the broker profit in the non-deviation case π_b^B is given by (38) in the text. We need to show that $\pi_b^B > \pi_b^D$. This implies showing that

$$\frac{1}{2} \frac{1}{\sqrt{\Sigma_n + \Sigma_r + \Sigma_k}} > \frac{2}{3} \frac{1}{\sqrt{\Sigma_n + \Sigma_r - \Sigma_k}} \left\{ \frac{1}{\sqrt{2}} - \frac{\Sigma_k}{\Sigma_n + \Sigma_r + \Sigma_k} \right\}$$
(13)

Clearly, when $\Sigma_k = 0$, the right hand side has its maximum value. If we can show that LHS > Max(RHS) in (13), then we have proved our case. And this is indeed verified with a little algebra. Thus, $\pi_b^B > \pi_b^D$, and it does not pay for the broker to deviate from his equilibrium strategy.

(QED)

Proof of Corollary 3:

This result follows from a simple substitution of (30) and (31) in (35) and (36) in proposition 2 and simplifying. (QED)

Proof of Corollary 4:

The result follows from a direct comparison of (32) and (33) in proposition 2. (QED)

Proof of Proposition 3:

Given m < m' (where m' > 1), from lemma 2 we know that the equilibrium under proposition 2 dominates. In this equilibrium, the informed trader's expected profit is positive but decreasing in m. Thus, the informed trader's optimal strategy is to give his order to a single broker for execution. The chosen broker, in turn, does not deviate from his equilibrium strategy of trading first.

When m > m', the equilibrium under proposition 2 does not exist and the only viable equilibrium is proposition 1. Since the informed trader's expected profit function under this assumption is independent of m, we assume that he chooses to divide his order equally among the m (>1) available brokers. Now a single broker does indeed have an incentive to deviate from the equilibrium strategy of trading second, given that the remaining (m-1) brokers all follow the equilibrium strategy (lemma 1), but this deviation is precluded with a trigger strategy mechanism as in Fudenberg and Tirole (1986, pp.52-53).

(QED)

Proof of Corollary 5:

Follows directly from (18) in proposition 1 and (37) in proposition 2.

(QED)

Proof of Proposition 4:

- (a) The informed expected profit in the Kyle (1985) single-period benchmark case is given by $\beta \Sigma_{\circ} / 2$, where β is the trading intensity of the informed trader and Σ_{\circ} is the unconditional variance of the risky asset. It is easily verified that the Kyle β and our β are identical. Hence the conclusion follows.
- (b) The expected profits of the informed trader and the m brokers is given by

$$\Pi_{\tau} = \left(\frac{1}{2} + \frac{1}{(m+2)\sqrt{m}}\right) \sqrt{\Sigma_{n} \Sigma_{v}}$$

The above follows from equations (18) and (19) in the text, after substituting the assumptions $\Sigma_{i} = \Sigma_{i} = 0$, and simplifying. Correspondingly, the expected informed profit in the Kyle benchmark is given by

$$\Pi_{\kappa_{k'k'}} = \frac{1}{2} \sqrt{\Sigma_n \Sigma_u}$$

Clearly, $\Pi_r < \Pi_{\text{\tiny KMe}}$, and the conclusion follows.

(c) From proposition 1, a substitution of (11) in (16) in the text and some simplification gives us $\sum_{i}^{J} < \frac{\sum_{i}}{2}$

A similar substitution of (11) and (12) in (17) in the text and some simplification gives us $\Sigma_{3}^{A} < \Sigma_{1}^{A}$

Thus, the per-period informativeness of the market clearing price is given by

$$\Sigma = \frac{1}{2} \left(\Sigma_1^{A} + \Sigma_2^{A} \right) < \frac{\Sigma_0}{2}$$

In contrast, the informativeness of price in the Kyle (1985) single period benchmark model is $\Sigma^{\text{Kyle}} = \frac{\Sigma_n}{2}$

Thus, $\Sigma < \Sigma^{\kappa n}$, and the conclusion follows.

(d) The (inverse of) depth in Kyle is given by

$$\lambda^{\textit{Kvie}} = \frac{1}{2} \sqrt{\frac{\Sigma_{n}}{\Sigma_{n}}}$$

 λ_1^4 and λ_2^4 are given by (13) and (14) in the text. After substituting $\Sigma_z = \Sigma_k = 0$ in these equations and simplifying we obtain $(\lambda_1^4 + \lambda_2^4) = \left(\frac{1}{2} + \frac{\sqrt{m}}{m+2}\right)\sqrt{\frac{\Sigma_0}{\Sigma_k}}$. This implies that the per-period price

sensitivity coefficient (inverse of depth), given by $\frac{1}{2}(\lambda_1^A + \lambda_2^A) < \lambda^{Kyle}$. Thus, the conclusion follows. (QED)

Proof of Proposition 5:

(a) The informed expected profit in the Kyle (1985) single-period benchmark case is given by $\beta^{\text{Kyle}} \Sigma_{1}/2$, where β^{Kyle} is the trading intensity of the informed trader and Σ_{0} is the unconditional variance of the risky asset and is given by

$$eta^{ extit{Kule}} = \sqrt{rac{\Sigma_u}{\Sigma_u}}$$

In contrast, from (30) in the text, $\beta \to 0$ under the assumption of $\Sigma_{\star} = 0$ and $\Sigma_{\star} \to 0$. Thus, the conclusion follows.

(b) The expected profits of the informed trader and the single broker, after substituting $\Sigma_{\epsilon} = 0$ and $\Sigma_{\epsilon} \to 0$ in $(\pi_{i}^{B} + \pi_{k}^{B})$, where π_{i}^{B} is given by (37) and π_{k}^{B} is given by (38), is given by $(\pi_{i}^{B} + \pi_{k}^{B}) \to \frac{1}{4} \sqrt{\Sigma_{0} \Sigma_{k}}$

In contrast, the expected informed profit in the Kyle benchmark is given by $\Pi_{\text{Kyle}} = \frac{1}{2} \sqrt{\Sigma_{\text{o}} \Sigma_{\text{w}}}$. The conclusion follows immediately.

(c) From proposition 2, a substitution of (30) and (31) in (35) in the text and some simplification gives us

$$\Sigma_1'' > \frac{\Sigma_0}{2}$$

A similar substitution of (30) and (31) in (36) in the text and some simplification gives us

$$\frac{\Sigma_{u}}{2} < \Sigma_{z}^{B} < \Sigma_{1}^{B}$$

Thus, the per-period informativeness of the market clearing price is given by

$$\Sigma = \frac{1}{2} \left(\Sigma_1^B + \Sigma_2^B \right) > \frac{\Sigma_0}{2}$$

In contrast, the informativeness of price in the Kyle (1985) single period benchmark model is

$$\sum_{k''}^{k''e} = \frac{\sum_{0}}{2}$$

Thus, $\Sigma > \Sigma^{Kite}$, and the conclusion follows.

(d) The (inverse of) depth in Kyle is given by

$$\lambda^{Kote} = \frac{1}{2} \sqrt{\frac{\sum_{n}}{\sum_{n}}}$$

 λ_1^n and λ_2^n are given by (32) and (33) in the text. After substituting $\Sigma_k = 0$ and $\Sigma_k \to 0$ in these

equations and simplifying it is easy to show that $\lambda_1^B = \frac{1}{2} \sqrt{\frac{\Sigma_0}{\Sigma_u}}$ and $\lambda_2^B \to 0$, which implies that the per-

period price sensitivity coefficient $\frac{1}{2}(\lambda_1^4 + \lambda_2^4) \rightarrow \frac{1}{4}\sqrt{\frac{\Sigma_o}{\Sigma_u}} < \frac{1}{2}\sqrt{\frac{\Sigma_o}{\Sigma_u}}$. The conclusion follows

immediately.

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