

**A GENERAL MODEL OF BROKERS' TRADING, WITH APPLICATIONS TO  
ORDER FLOW INTERNALIZATION, INSIDER TRADING AND OFF-  
EXCHANGE BLOCK SALES**

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# **A General Model of Brokers' Trading, With Applications to Order Flow Internalization, Insider Trading and Off-exchange Block Sales**

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## **Abstract**

Multiple informed traders and noise traders pay fees to trade through multiple brokers. Brokers may trade with their customers in the same transaction (simultaneous dual trading) or trade after their customers in a separate transaction (consecutive dual trading). Brokers' expected profits from fees and trading, net of brokerage costs, are zero and so brokers are indifferent between simultaneous and consecutive dual trading. Market depth and price informativeness are higher with consecutive dual trading, compared to both simultaneous dual trading and the no-dual-trading benchmark. If the number of brokers exceeds the number of informed traders, then both noise traders and informed traders prefer consecutive dual trading. If the number of brokers is less than the number of informed traders, however, net informed profits and net uninformed losses may be higher with simultaneous dual trading in markets with many brokers; net informed profits and net uninformed losses are higher with consecutive dual trading in markets with relatively few brokers. We use our model to study order flow internalization by broker-dealers. Finally, we interpret brokers as "follow-on traders" whose trading is derived from the order flows of preceding traders. With this broader definition, we (1) explain certain puzzling empirical results related to the effect of insider trading on non-insider trading volume and market liquidity; and (2) discuss large block sales conducted off the exchange floor by insiders to avoid "follow-on" trading.

Personal trading by brokers is pervasive throughout U.S. securities and futures markets, and in financial markets in the rest of the world (Grossman (1989)). Yet, academics and policy makers are unable to conclude if brokers' trading is beneficial for customers and markets. For example, the U.S. Congress passed the Futures Trading Practice Act in 1992, which directed the Commodity Futures Trading Corporation (CFTC) to pass regulations prohibiting dual trading<sup>1/</sup> in high volume contracts. However, although dual trading is currently banned on the Chicago Mercantile Exchange (CME) futures markets, it is practiced freely on other futures exchanges.<sup>2/</sup>

In the U.S. equity markets, by contrast, regulators have not sought to restrict personal trading by brokers. Yet, even in equity markets, serious concerns remain regarding certain aspects of brokers' trading. One such concern is the internalization of order flow by brokers, which occurs when a broker-dealer executes customer orders as a market maker or, more generally, when a broker-dealer directs order flow to an affiliated specialist. The Securities and Exchange Commission (SEC), in its recent study concludes that preferencing (which is one way for broker-dealers to internalize customer order flow) does not harm market quality, but it also notes that "such programs ...raise significant agency-principal concerns."<sup>3/</sup> Other market participants have also spoken out against the internalization of order flow.<sup>4/</sup>

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<sup>1/</sup>Dual trading in futures markets is the practice whereby futures floor traders execute trades for their own and customers' accounts on the same day.

<sup>2/</sup>The regulations allow affected exchanges to petition for relief based on 1) an acceptable audit trail, or ability to track a floor traders' activities, or 2) a threat to the hedging utility and price discovery function of futures markets, should the practice of dual trading be prohibited. All affected exchanges have petitioned for relief, although the CFTC has yet to act on these petitions.

<sup>3/</sup>See "Report on the practice of preferencing," the SEC, April, 1997.

<sup>4/</sup>See, for example, the comments of Robert Murphy, president of RPM Specialist Corp., as reported in *Securities Week*, March 11, 1996.

The lack of a consensus on brokers' trading at the policy level is also reflected in the academic literature. Previous empirical studies do not agree on the effect of dual trading on market liquidity. Depending on the market studied, dual trading may increase, decrease or have no effect on liquidity (Chang and Locke (1996), Chang, Locke and Mann (1994), Fishman and Longstaff (1992), Smith and Whaley (1994)). The theoretical literature also does not agree on the relationship between dual trading and liquidity. Whereas Grossman (1989) argues in favor of a positive effect of dual trading on liquidity, Roell (1990) and Sarkar (1995) conclude that dual trading reduces liquidity.<sup>2/</sup> In Fishman and Longstaff (1992), the effect of dual trading on liquidity is ambiguous.

To provide clearer guidance to policy makers, we attempt to provide, in this paper, a fuller examination of brokers' trading than is available in the literature, by modeling differences in the way brokers trade in different markets. Our approach draws on Grossman's (1989) distinction between **simultaneous** dual trading (where a broker trades for himself and a customer in the same transaction) and **consecutive** dual trading (where a broker trades for customers as an agent and for himself at other times, but not in the same transaction). The constraint on timing differs by markets. Simultaneous dual trading occurs in securities markets, currency and interest rate swap markets, and the fixed income market, but not in futures markets. Consecutive dual trading occurs in futures markets, and in all markets with simultaneous dual trading (Grossman (1989)).

While our model initially deals with dual trading, we later apply it to examine order flow internalization by brokers. We further extend the model by reinterpreting brokers as "follow-on

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<sup>2/</sup>In Roell (1990), although market depth is lower, uninformed traders whose trades are observed by the dual trader have higher profits with dual trading.

traders"--i.e., traders whose trading decisions are related to the order flow of preceding traders.<sup>6f</sup> We show that our model, with this broader definition, explains certain previous empirical results on the effect of insider trading on non-insider volume and liquidity--results that are difficult to explain with current adverse selection models of trading. Finally, we use the extended model to study off-exchange sales of large blocks of stocks, which are motivated by the desire of large block sellers to hide their trades from "follow-on traders" (see below for a fuller description).

In our model, based on Kyle (1985), multiple informed traders and noise traders trade by dividing their orders equally among multiple dual traders. Later, we endogenize the number of dual traders chosen by informed traders. Informed traders observe private signals about the payoff of a single risky asset and choose a quantity to buy or sell through brokers. The brokers, on receiving orders from informed traders and noise traders, may choose an action from the following two options: (A) dual trade consecutively, by executing customers' orders in the first period as agents and trading on their personal accounts in the second period as principals; or (B) dual trade simultaneously, by acting as agents for customers and trading for themselves as principals in the same transaction.

If brokers dual trade simultaneously, each broker submits the net order (his personal trades plus his customers' trades) to the market maker for execution, and the game ends in a single period. If brokers trade consecutively, trading occurs over two periods. In period one, each broker submits the sum of informed and period one noise trades to the market maker for

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<sup>6f</sup>We assume, in our model, that traders' orders are observed perfectly after one period. In practice, legal insider trades are probably observed after a lag of days. For example, Disclosure Inc. has a newswire that provides updates on each day's most important insider filings with the SEC, combined with analysis on the potential impact of the transaction on the stock (*Securities Week*, April 28, 1997). It is likely, however, that at least some traders observe noisy realizations of *illegal* insider trades in the same day the trades occur by way of leaks, rumor, tips, etc. Thus, our model should be considered illustrative of the general idea that trading decisions are often derived from actual prior trading.

execution. In period two, each broker submits his personal trade to the market maker. The market maker batches brokers' trades and period two noise trades and executes them at a single price. In all cases, the market maker prices the asset to earn zero expected profits, conditional on the history of order flows realized so far.

Customers pay a fee to transact through brokers. We assume that the fee is fixed-- i.e., it is independent of the order size.<sup>27</sup> Following Fishman and Longstaff (1992), the broker's fee is competitively determined--i.e., the fee is set at a level that equates a broker's expected fee income plus expected trading revenues to the expected costs of brokerage. Thus, we allow for the possibility that practices such as internalization of order flow, which are profitable for broker-dealers, may benefit customers through lower brokers' fees.<sup>28</sup>

Given that brokers make zero expected profits from trading and brokerage, they are indifferent between consecutive and simultaneous dual trading. However, informed profits and noise trader losses depend on whether brokers trade in the same or in a separate transaction as customers.

Compared to the no-dual-trading benchmark, and ignoring brokers' fees, informed traders' period one trading is unaffected with **consecutive** dual trading, while noise traders' period two losses to brokers are lower than their losses to insiders in period one. With brokers' fees, informed profits are higher and uninformed losses are lower with consecutive dual trading since the competitive fee is lower by an amount equal to the broker's trading revenues per

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<sup>27</sup>Ideally, we want the broker's fee to be related to customers' order sizes. If we do so, however, informed traders' trading rule, given the price function, is non-linear (since there is no trade for very imprecise information). Given a non-linear trading rule, the equilibrium price function is no longer determinate.

<sup>28</sup>Such a possibility has been suggested by the SEC in its Market 2000 report and by the NASD economic research staff. The NASD research staff argues that many brokers who use preferencing are discount-commission brokers. See "The introduction of Naqcess into the Nasdaq stock market: Intent and expectation," June, 1996.

customer trade. Also, price informativeness and market depth are higher with consecutive dual trading. Thus, consecutive dual trading is unambiguously beneficial for customers and the market, compared to the no-dual-trading benchmark.

Ignoring brokers' fees, **simultaneous** dual trading is bad for customers and the market relative to the no-dual-trading benchmark. The reason is that brokers mimic informed trades by trading with insiders in the same direction, causing insiders to trade less and at a higher price (in absolute value), and lowering gross informed profits. Uninformed losses are higher since brokers' aggregate trading revenues exceed the reduction in aggregate informed profits. In addition, brokers offset a portion of noise trades, reducing market depth.

When brokers' fees are included, informed profits (and uninformed losses) may be higher with simultaneous dual trading when there are many brokers and informed traders and the precision of information is high. Conversely, informed profits and uninformed losses may be lower with simultaneous dual trading in markets with few brokers and insiders, and low information precision. The reason is that, with many brokers, aggregate broker profits are high, which greatly reduces commissions paid by customers. Also, with many insiders and very precise information, the adverse effect of brokers' mimicking trades on an individual insider is minimized. Thus, net informed profits are higher. Since aggregate broker profits and informed profits are high, uninformed losses are higher even with reduced commissions.

Next, we endogenize informed traders' choice of the number of brokers and the type of dual trading.<sup>97</sup> We find that if the number of insiders is less than the number of brokers, then

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<sup>97</sup>We can formally solve for noise traders' choice of the number of brokers and the type of dual trading, too. We do not do so because, first, in practice broker-dealers often automate retail order flows. For example, broker-dealers may automatically direct retail order flows to an affiliated specialist, which amounts to simultaneous dual trading in our model (see the discussion on page six). Second, noise traders and insiders may often disagree on their decisions, in which case equilibrium does not exist in our model.



informed traders prefer consecutive dual trading, and allocate orders to a single broker. The reason is that, when the number of brokers is large relative to the number of insiders, the adverse effect of brokers' mimicking trades on an individual insider is very strong, and the market with simultaneous dual trading breaks down. In addition, with consecutive dual trading, brokers' profits and the reduction in brokers' fees are maximized with one broker. Our result is consistent with the institutional fact that, in futures markets, where typically many dual traders are present in each contract<sup>107</sup>, simultaneous dual trading is not permitted.

If the number of informed traders exceeds the number of brokers, then informed traders' choice between consecutive and simultaneous dual trading depends on the market structure. In competitive markets, with many brokers and informed traders, informed traders prefer simultaneous dual trading and allocate orders to all participating brokers. They prefer consecutive dual trading in markets with relatively few brokers and informed traders. However, relative to simultaneous dual trading, market depth and price informativeness are always higher with consecutive dual trading. Thus, informed traders' strategic need to protect the value of their information from brokers sometimes leads them to choose the lower quality market.

In the simultaneous dual trading model, brokers act as market makers to noise traders by offsetting a portion of their order flow in the same transaction. Thus, the model is appropriate for studying order flow internalization, which primarily affects retail order flow.<sup>117</sup> To further enhance the applicability of the model, we choose parameter values such that, for moderate numbers of brokers, most of brokers' trading involves dealing to noise traders rather than

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<sup>107</sup>For example, Fishman and Longstaff (1992) found an average of 116 dual traders per day in the soybean oil futures contract. Chakravarty and Sarkar (1997a) estimated that the number of dual traders in four futures contracts varied, on average, between 9 and 151 per day.

<sup>117</sup>See "Report on the practice of preferencing," the SEC, April, 1997.

mimicking informed trades. We find that commissions are lower with internalization, as predicted by the SEC and the NASD research staff (see footnote eight). However, noise traders' losses, net of fees, are higher with internalization and market depth is lower. These adverse effects worsen as the degree of internalization increases.

Some recent studies of insider trading events have uncovered two intriguing results. One, there appears to be a surge of non-insider volume around insider trading events. For example, Meulbroek (1992), in a study analyzing 320 illegal insider trading events, shows that non-insider trading is 64% higher than average on insider trading days. Cornell and Sirri (1992) note the unusual increase in uninformed volume around the period of insider trading in Campbell Taggart stock. Chakravarty and McConnell (1997) report that average daily volume was twice as high in Carnation Company stock on days when Ivan Boesky traded illegally in that stock on the basis of "alleged" inside information. Two, liquidity at the time of insider trading increases (Cornell and Sirri (1992)) or is unchanged (Chakravarty and McConnell (1997))—a result apparently inconsistent with adverse selection models of trading, which generally predict that liquidity is negatively related to asymmetric information.<sup>12/</sup>

We explain these results by conjecturing that the reported volume surrounding insider trading events are due to market watchers who follow trading by insiders. Reinterpreting brokers as "follow-on traders," we show that the expected non-insider trading volume following insider trading events increases and market depth decreases if the number of "follow-on traders" is large relative to the number of insiders.

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<sup>12/</sup>The relation between adverse selection and market liquidity may be non-monotonic if informed traders are risk averse (Subrahmanyam, 1991) or uninformed traders are risk averse (Spiegel and Subrahmanyam, 1992) or if there are multiple dual traders (Chakravarty and Sarkar, 1997b).

Using the Chakravarty and McConnell (1997) insider trading data, we obtain the following empirical results. One, the average trading volume surrounding Boesky's trades is significantly higher than the average volume in the hours preceding Boesky's trades. Two, market depth in the hour of Boesky's trades is significantly negatively related to trading volume surrounding Boesky's trades but is unrelated to trading volume in the hours preceding Boesky's trades. These results are consistent with our starting premise that trading by "insider watchers" explain the non-insider volume and liquidity effects surrounding insider trading events.

Next, we consider the issue of off-exchange block sales, which refer to the sale of very large blocks of stocks by big investors or institutional holders to broker-dealers away from U.S. exchange floors. In a typical off-exchange sale, a dealer offers to buy the block of stock at a discount to the closing price on the exchange, acting as a principal. Recently, such sales have become popular.<sup>12/</sup> One reason for the popularity is that, if the sale were to be made on an exchange in piecemeal fashion (as predicted by Kyle (1985)), the first sale may alert other traders, who may then drive the price down. For example, Carl Icahn, who sold off-exchange 19.9 million shares in RJR Nabisco Holding Corp. to Godman, Sachs & Co, is quoted as saying that with a piecemeal sale on the exchange floor, "word would have spread, giving short-sellers a chance to drive the price down at his expense."<sup>13/</sup> Proponents say off-exchange sales provide liquidity for block trading, while opponents claim that it reduces transparency (since the trades occur away from the floor of the exchange).

We use a two-period version of Holden and Subrahmanyam's (1992) sequential auctions model for the off-exchange transaction. We use our consecutive dual trading model to describe

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<sup>12/</sup>See "More big stocks are handled 'off-Board'," the *Wall Street Journal*, Wednesday, April 9, 1997, page C1.

<sup>13/</sup>See "More big stocks are handled 'off-Board'," the *Wall Street Journal*, Wednesday, April 9, 1997, page C1.

the exchange transaction, reinterpreting brokers as follow-on traders (such as short-sellers).

When insiders sell a second time on the exchange floor, we assume they have to sell at the period two price determined by follow-on traders. Our results demonstrate that, provided the number of insiders selling stocks is relatively small, and the number of follow-on traders relatively large, then compared to the exchange market, the off-exchange transaction is indeed more liquid but the off-exchange price is less informative. Thus, whenever off-exchange sales are beneficial for insiders, they will be bad for the exchange (since the exchange price would be more informative if insiders had traded on the floor).

In related literature, Fishman and Longstaff (1992) study consecutive dual trading in a model with a single broker and fixed order size. As in our consecutive dual trading model, uninformed traders in their model benefit from dual trading on an after-commission basis. Unlike our model, dual trading hurts informed traders both on a pre-commission and post-commission basis. This difference occurs because, in our model, insiders only trade once whereas, in Fishman and Longstaff (1992), the second trade is by a broker only if the probability of the information event is high enough. When an insider makes the second trade, he may receive a worse price relative to the no-dual-trading benchmark if the market maker, thinking the trade is by the broker, attributes a higher probability to the information event. In our model, also, the period two price following brokers' trades is worse than the period one price. Hence, if we allowed our insiders to trade a second time, they too would be worse off.

Chakravarty and Sarkar (1997a) also study consecutive dual trading in a model with multiple brokers, but with a single informed trader and without brokers' fees. The informed trader in their model is indifferent between dual trading and no-dual-trading since his initial trading decision is unaffected by dual trading.

Roell (1990) and Sarkar (1995) study simultaneous dual trading. Our simultaneous dual trading model is closest to Sarkar (1995), except that we have multiple brokers. This is an important difference because, with a single broker, the savings in brokers' fees are not large enough to offset the insiders' loss to dual trading, and so the insiders in Sarkar's model are always worse off with simultaneous dual trading.

Chakravarty (1994) shows, in a single-period framework, that competition among brokers drives individual broker profits from personal trades to zero relatively quickly. Madrigal (1996) considers a two period extension of Kyle (1985) and examines the effect of "non-fundamental" speculators on informed trading and related market characteristics. Holden and Subrahmanyam (1992) study a multi-period extension of Kyle with multiple (and identical) informed traders (and no brokers).

The remainder of the paper is organized as follows. Section I presents a trading model with multiple customers and brokers. Sections II and III solve the consecutive and simultaneous dual trading models, respectively. Section IV compares the two dual trading equilibria. Section V analyzes order flow internalization, while section VI studies insider trading and off-exchange stock sales. Section VII concludes. All proofs are in the appendix.

## **I. A Model of Trading With Multiple Customers and Multiple Brokers.**

We consider an asset market structured along the lines of Kyle (1985). There is a single risky asset with random value  $v$ , drawn from a normal distribution with mean 0 and variance  $\Sigma_v$ . There are  $n$  informed traders, each of whom receives a signal about the true asset value and submit market orders. For an informed trader  $i$ ,  $i=1, \dots, n$ , the signal is  $s^i = v + e^i$ , where  $e^i$  is drawn from a normal distribution with mean 0 and variance  $\Sigma_e$ . A continuum of noise traders also

submit aggregate market orders  $u$ , where  $u$  is normally distributed with mean zero and variance  $\Sigma_u$ . All random variables are independent of one another.

All customers, informed and uninformed, must trade through brokers. There are  $m$  brokers in the market, who submit customer orders to the market maker. By observing informed orders, brokers can infer the informed traders' signals. By observing orders of noise traders, they are aware of the size of uninformed trades. Thus, brokers have an incentive to trade based on their customers' orders. However, brokers are not allowed to trade ahead of (i.e., front run) their customers.

Brokers may trade in two possible ways. They may execute their customers' orders first, and trade for their own accounts after in a separate transaction--i.e., engage in *consecutive* dual trading. Alternatively, they may trade with their customers in the same transaction--i.e., engage in *simultaneous* dual trading.

Each broker charges a fee of  $Sc$  per trade, which is independent of the order size. Also, the fee is paid irrespective of whether the customer makes a trade, as with market access fees. We treat the aggregate of noise trades as one trade,<sup>13'</sup> so that each broker handles a total of  $(n+1)$  trades. In addition, brokers face a fixed cost of  $k_0$ , with  $k_0 > 0$ , and a variable cost of  $k_1$  per trade, with  $k_1 > 0$ , of conducting brokerage. Following Fishman and Longstaff (1992), we assume that the brokerage business is competitive. Thus, each broker chooses  $c$  so that broker's expected revenues from trading plus expected fee income equals the expected cost of brokerage.

Since brokers earn zero expected profits, they are indifferent, in equilibrium, between consecutive and simultaneous dual trading. On the other hand, informed customers' net trading profits (net losses for noise traders) depends on whether brokers dual trade simultaneously or

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<sup>13'</sup>This assumption is without loss of generality, as long as there are a finite number of noise trades.

consecutively, as well as on the number of brokers. We allow informed traders to choose the number of brokers to allocate their orders to, and instruct these brokers whether to trade simultaneously or consecutively. However, we assume that, if more than one broker is chosen, informed and uninformed orders are divided equally among the brokers.

The sequence of events (illustrated in figure 1) is as follows: in stage one, informed trader  $i, i=1, \dots, n$  observes  $s^i$  and chooses a trading quantity  $x^i$ . The informed traders choose the number of brokers  $m$ , and whether brokers should trade simultaneously or consecutively. In stage two, the chosen brokers trade consecutively or simultaneously, and each place orders of an amount  $z$ . In the subsequent sections, we describe in more detail how simultaneous and consecutive dual trading differ. Finally, all trades (including brokers' personal trades) are submitted to a market maker, who sets a price that earns him zero expected profits conditional on the history of net order flows realized.

## II. Consecutive Dual Trading.

In this section, we solve for the equilibrium, assuming that informed traders instruct brokers to trade consecutively. We also assume that brokers do not trade with their customers in the same transaction. Further, informed and noise traders allocate their orders equally to a fixed number of brokers  $m$ . In section IV, we endogenize informed traders' choice of  $m$ .

### A. *The Consecutive Dual Trading Model With Broker's Fees*

Trading occurs in two periods. In period one, brokers receive market orders from  $n$  informed traders and the noise traders, which they then submit to the market maker in exchange for a fee of  $Sc_2$  per customer. In period two, brokers trade for themselves, along with period two

noise traders. Each period, a market maker observes the history of net order flow realized so far and sets a price such to earn zero expected profits, conditional on the order flow history.

The sequence of events is as follows: in period one, each broker chooses the fee  $c_d$  to charge each customer. Informed trader  $i$ ,  $i=1, \dots, n$  observes  $s^i$  and chooses  $x^{i,d}$ , knowing that his order will be executed in the first period. Accordingly, informed trader  $i$ ,  $i=1, \dots, n$ , chooses  $x^{i,d}$  to maximize conditional expected net profits  $E[(v-p_1)x^{i,d} | s^i] - mc_d$ , where the period one price is  $p_1 = \lambda_1 y_1$ , the period one net order flow is  $y_1 = x_d + u_1$ , the aggregate informed trade is  $x_d = \sum_i x^{i,d}$  and  $u_1$  is the period one noise trade.

In period two, brokers choose their personal trading quantity after observing the  $n$ -vector of informed trades  $\{x^{1,d}, \dots, x^{n,d}\}$  and  $u_1$ . Thus, broker  $j$ ,  $j=1, \dots, m$ , chooses  $z^j$  to maximize conditional expected profits  $E[(v-p_2)z^j | \{x^{1,d}/m, \dots, x^{n,d}/m\}, u_1/m]$ , where the conditioning is based on each broker observing his portion of the informed and uninformed orders received.

The  $m$  brokers submit the net of their personal trades to the market maker, who sets  $p_2 = \lambda_2 y_2 + \mu_2 y_1$  where  $y_2 = \sum_j z^j + u_2$ ,  $\sum_j z^j$  is the aggregate trade of all brokers and  $u_2$  is the noise trade in period two. Finally, the liquidation value  $v$  is publicly observed and both informed traders as well as brokers realize their respective profits (if any).

Define  $t = \Sigma_i / \Sigma_s$ , where  $t$  is the unconditional precision of  $s^i$ ,  $i=1, \dots, n$ . Note that  $0 \leq t \leq 1$ . Further, define  $Q = 1 + t(n-1)$ , where  $(Q-1)s^i$  represents informed trader  $i$ 's conjecture (conditional on  $s^i$ ) of the remaining  $(n-1)$  informed traders' signals. Since informed traders have different information realizations, they also have different conjectures about the information of other informed traders.  $t$  also measures the correlation between insider signals. For example, if  $t=1$  (perfect information), informed signals are perfectly correlated,  $Q=n$ , and informed trader  $i$



conjectures that other informed traders know  $(n-1)s^i$ --i.e., the informed trader believes other informed traders have the same information as he.

Proposition 1 below solves for the unique linear equilibrium in this market.

**Proposition 1:** In the consecutive dual trading case, there is a unique linear equilibrium for  $t > 0$ . In period one, informed trader  $i$ ,  $i=1, \dots, n$ , trades  $x^{i,d} = A_d s^i$ , and the price is  $p_1 = \lambda_1 y_1$ . In

period two, each broker  $j$ ,  $j=1, \dots, m$  trades  $z = B_1 x_d + B_2 u_1$ , where  $x_d = \sum_i x^{i,d}$

and the price is  $p_2 = \lambda_2 y_2 + \mu_2 y_1$ , where:

$$A_d = \frac{\sqrt{t \Sigma_u}}{\sqrt{n \Sigma_v}} \quad (1)$$

$$\lambda_1 = \frac{\sqrt{nt \Sigma_v}}{\sqrt{\Sigma_u} (1+Q)} \quad (2)$$

$$B_1 = \frac{1}{\sqrt{mQ} (1+Q)} \quad (3)$$

$$B_2 = -\frac{\sqrt{Q}}{\sqrt{m} (1+Q)} \quad (4)$$

$$\lambda_2 = \frac{\sqrt{nt \Sigma_v} \sqrt{m}}{\sqrt{Q(1+Q) \Sigma_u}^{1+m}} \quad (5)$$

$$\mu_2 = \frac{\sqrt{nt \Sigma_v}}{(1+Q) \sqrt{\Sigma_u}} \quad (6)$$

Each broker charges a fee of  $c_d$  to traders, where  $c_d$  satisfies:

$$c_d = k_1 + \frac{k_0 - W_d}{n+1} \quad (7)$$

and  $W_d$  is the expected trading revenue per broker, as given by:

$$W_d = \frac{\sqrt{nr\Sigma_u\Sigma_v}}{\sqrt{Q(1+Q)}} \frac{1}{\sqrt{m(1+m)}} \quad (8)$$

The period one solution is identical to Kyle's (1985) single period equilibrium, extended to include multiple informed traders and noisy signals.<sup>16/</sup> Thus, informed trades are not affected by dual trading since informed traders trade only in period one and are unaffected by brokers' trading in period two.<sup>17/</sup> In period two, dual traders piggyback on period one informed trades ( $B_1 > 0$ ) and offset noise trades ( $B_2 < 0$ ). They charge a fee that equates their profit margin per trade ( $c_d - k_f$ ) to the fixed cost of brokerage per trade, net of dual trading profits (see equation 7).

**Corollary 1:** (1)  $\delta B_1 / \delta n < 0$ , and  $\delta B_1 / \delta t < 0$  for  $n > 1$ .  $\delta |B_2| / \delta n > 0$  and  $\delta |B_2| / \delta t > 0$  for  $n > 1$ .

(2)  $\delta B_1 / \delta m < 0$ ,  $\delta |B_2| / \delta m < 0$ .  $\delta(mB_1) / \delta m > 0$ ,  $\delta(m|B_2|) / \delta m > 0$ .

The extent of piggybacking  $B_1$  is decreasing in the number of informed traders  $n$  and the information precision  $t$ . Competition between informed traders leads insiders to use their information less, making piggybacking less valuable. An increase in  $t$  increases the correlation between insiders' signals, reducing (for  $n > 1$ ) the value of observing multiple informed orders. Since the marginal value of observing noise trades is higher when the marginal value of observing informed trades is lower,  $|B_2|$ , the extent to which noise trades are offset, is increasing in  $n$  and  $t$ . Competition between brokers reduces the extent to which each broker exploits customer trades. However, brokers in the aggregate exploit customer trades more.

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<sup>16/</sup>See, for example, lemma one in Admati and Pfleiderer (1988) and lemma two in Sarkar (1995).

<sup>17/</sup>This result is analogous to the dual trading equilibrium in Fishman and Longstaff (1992), where the first trade is not affected by dual trading.

## *B. The Impact of Consecutive Dual Trading on Traders' Profits and the Market*

Since the period one equilibrium is identical to one without dual trading, we can compare it with the period two equilibrium to obtain the impact of consecutive dual trading.

**Corollary 2.** (1)  $\mu_2 = \lambda_1$  and  $\lambda_2 < \lambda_1$ .

(2) **Price informativeness is higher with consecutive dual trading.**

$\mu_2$  is that part of the market maker's period two adverse selection costs which is related to the period one order flow,  $y_1$ . The law of iterated linear projections implies:

$$E(v|y_1, y_2) = \lambda_2 y_2 + [\lambda_1 - \lambda_2 E(y_2|y_1)] = \lambda_2 y_2 + \mu_2 y_1$$

$\mu_2 = \lambda_1$  because  $\text{covariance}(y_1, y_2) = 0$ , implying that  $E(y_2|y_1) = 0$ . Thus, to the market maker, the period one order flow is not informative about the period two order flow. The reason is that dual traders, by piggybacking on informed trades, induce a positive serial correlation between the order flows. But, by offsetting noise trades, dual traders also induce a negative serial correlation between the order flows. These two effects are exactly offsetting. Also,  $\lambda_2 < \lambda_1$  implies that the marginal increase in the market maker's adverse selection costs due to brokers' trading in period two is less than the adverse selection cost from insider trading in period one.

Since the covariance between the order flows of the two periods is zero, information revealed by the period two price is simply the information revealed by the period one price plus the additional information revealed by brokers' trading.

**Corollary 3:** (1) **Ignoring broker's fees, uninformed losses per period are lower while informed profits are unaffected with consecutive dual trading.**

(2) **With broker's fees, informed profits are higher and uninformed losses per period are lower with dual trading.**

Informed trades and, therefore, informed gross profits, are not affected by dual trading.

Since  $\mu_2 = \lambda_1$ , it follows that the period two price  $p_2 = p_1 + \lambda_2 y_2$ : the dual trader trades at worse

prices than the informed trader. Thus, dual trading profits (gross of brokers' fees) are lower than informed profits in the first period and, consequently, uninformed losses (gross of brokers' fees) to dual trading are lower than losses to informed traders.

Brokers' fees without dual trading, denoted  $c_n$ , are given by:

$$c_n = k_1 + \frac{k_0}{n + 1} \quad (9)$$

Comparing (7) and (9), brokers' fees are lower with dual trading by an amount equal to expected dual trading profits per trade. Therefore, net of fees, informed profits are higher and uninformed losses are lower.

### III. Simultaneous Dual Trading.

We now solve for the equilibrium, assuming that informed traders instruct brokers to trade with their customers in the same transaction. As in the previous section, informed and noise traders allocate their orders equally to a fixed number of brokers  $m$ .

#### A. The Simultaneous Dual Trading Model With Broker's Fees

Simultaneous dual trading is modeled in a single period Kyle (1985) framework. The notations are the same as in section II. All variables and parameters related to simultaneous dual trading are denoted either with superscript  $s$  or subscript  $s$ .

Each broker charges a fee of  $c_s$ . A group of  $n$  informed traders receive signals  $s^i$  about the unknown value  $v$ , and choose quantities  $x^{i,s}$  knowing that his order will be executed along with the orders of brokers and noise traders in the same transaction. Accordingly, informed trader  $i$ ,  $i=1, \dots, n$ , chooses  $x^{i,s}$  to maximize conditional expected net profits  $E[(v-p_s)x^{i,s} | s^i] - mc_s$ , where the

price is  $p_s = \lambda_s y_s$ , the net order flow is  $y_s = x_s + mz + u$ , the aggregate informed trade is  $x_s = \sum_i x^{i,s}$ ,  $z$  is the amount each broker trades and  $u$  is the noise trade.

Upon receiving the orders of their informed and uninformed customers, brokers choose their personal trading quantity after observing the  $n$ -vector of informed trades  $\{x^{1,s}, \dots, x^{n,s}\}$  and  $u$ . Thus, broker  $j, j=1, \dots, m$ , chooses  $z^{j,s}$  to maximize conditional expected profits  $E[(v - p_s)z^{j,s} | \{x^{1,s}/m, \dots, x^{n,s}/m\}, u/m]$ , where the conditioning is based on each broker observing his portion of the informed and uninformed orders received.

The  $m$  brokers submit the net of their customer trades and personal trades to the market maker, who sets the price that earns him zero expected profits conditional on the net order flow realized. Finally, the liquidation value  $v$  is publicly observed and both informed traders as well as brokers realize their respective profits (if any).

Proposition 2 below solves for the unique linear equilibrium in this market.

**Proposition 2:** In the simultaneous dual trading case, there is a unique linear equilibrium for  $t > 0$  and  $Q > m$ . Informed trader  $i, i=1, \dots, n$ , trades  $x^{i,s} = A_s s^i$ , broker  $j, j=1, \dots, m$  trades  $z = B_{1,s} x_s + B_{2,s} u$ , where  $x_s = \sum_i x^{i,s}$ , and the price is  $p_s = \lambda_s y_s$ , where:

$$A_s = \frac{m}{1+m} \frac{(Q-m) \sqrt{t \Sigma_u}}{Q \sqrt{n \Sigma_v}} \quad (10)$$

$$\lambda_s = \frac{(1+m) \sqrt{nt \Sigma_v}}{m \sqrt{\Sigma_u} (1+Q)} \quad (11)$$

$$B_{1,s} = \frac{1}{(Q-m)} \quad (12)$$

$$B_{2,s} = -\frac{1}{(1+m)} \quad (13)$$

each broker charges a fee of  $c_s$  to traders, where  $c_s$  satisfies:

$$c_s = k_1 + \frac{k_0 - W_s}{n + 1} \quad (14)$$

and  $W_s$ , the expected trading revenue per broker, satisfies:

$$W_s = \frac{\sqrt{nt \Sigma_u \Sigma_v} (Q+m)}{Q(1+Q) (1+m)} \quad (15)$$

**Corollary 4:** (1)  $\delta B_{1s}/\delta n < 0$ , and  $\delta B_{1s}/\delta t < 0$  for  $n > 1$ .  $\delta |B_{2s}|/\delta n = 0$  and  $\delta |B_{2s}|/\delta t = 0$ .

(2)  $\delta B_{1s}/\delta m > 0$ ,  $\delta |B_{2s}|/\delta m < 0$ .  $\delta(mB_{1s})/\delta m > 0$ ,  $\delta(m|B_{2s}|)/\delta m > 0$ .

Equilibrium exists if  $Q > m$ . Since  $n \geq Q$ , existence implies  $n > m$ : the number of informed traders must exceed the number of brokers. The intuition behind this result is as follows. Suppose an insider is buying. Dual traders also buy in the same transaction, piggybacking on the insider trade (i.e.  $B_1 > 0$ ), and increasing the price paid by the insider for his purchase. From corollary four, the order of an individual insider is exploited less as the number of insiders increases, and is exploited more as the number of brokers increases. If the number of brokers is too large relative to the number of insiders, the adverse price effect makes it too costly for insiders to trade.

Unlike the consecutive dual trading case, the extent to which noise trades are offset depends inversely on the number of brokers but is independent of the number of informed traders  $n$ . In the consecutive dual trading model, knowledge of period one noise trades was relevant to brokers because it influenced the way market makers priced the asset in period two. The market maker's pricing problem, in turn, depended on the extent of private information. This interaction between information and noise trades is absent in the case of simultaneous dual trading.

## *B. The Impact of Simultaneous Dual Trading on Traders' Profits and the Market*

In this section, we compare the simultaneous dual trading equilibrium with one where dual trading is banned (i.e., the period one solution for the consecutive dual trading equilibrium).

**Corollary 5. Compared to the no-dual-trading benchmark,**

**(1) Market depth is lower with dual trading.**

**(2) Price informativeness is the same with or without dual trading.**

Market depth is lower with dual trading because brokers reduce the net order flow of noise traders, increasing the market maker's adverse selection costs. Price informativeness depends positively on the variance of the net order flow and inversely on market depth. Dual trading reduces depth and decreases the variance of the net order flow in the same proportion. Hence, price informativeness is invariant to dual trading.

**Corollary 6: (1) Ignoring broker's fees, uninformed losses are higher and informed profits are lower with simultaneous dual trading.**

**(2) With broker's fees, informed profits and uninformed losses are higher (lower) with simultaneous dual trading if  $n$ ,  $t$  and  $m$  are large (small).**

Gross informed profits are lower with simultaneous dual trading because the dual traders' piggybacking forces insiders to reduce the size of their orders (for the same level of information). However, dual trading profits exceed the reduction in informed profits, and, consequently, uninformed losses to dual traders are also higher.

Dual traders' profits are passed on to customers in the form of lower commission fees. When  $m$  is large, aggregate dual trading profits and the consequent reduction in commission are also large. If, in addition,  $n$  and  $t$  are large, the effect of piggybacking on an individual insider is relatively small and thus gross informed profits with dual trading is relatively large (although still less than gross informed profits without dual trading). The combination of these two factors

serves to market net informed profits higher with dual trading. Uninformed losses are also higher since aggregate broker and informed profits with dual trading are large.

#### IV. Comparison Between Simultaneous And Consecutive Dual Trading

Since brokers' earn zero expected net profits, they are indifferent between trading consecutively or simultaneously. However, expected net profits of insiders are a function of whether brokers trade simultaneously or consecutively, as well as on the number of brokers executing trades. In this section, we study informed traders choice between the two forms of dual trading, contingent on the number of brokers they optimally give their orders to. Thus, we first consider how expected net profits of insiders vary with the number of brokers.

**Corollary 7. (1) When brokers trade consecutively, an informed trader's net profits are decreasing in  $m$ .**

**(2) Suppose  $Q > m$ . If brokerage costs are small, then an informed trader's net profits with simultaneous dual trading are increasing in  $m$ .**

Insiders' gross profits are independent of  $m$  when brokers trade consecutively. However, the amount they pay in brokers' fees is negatively related to aggregate brokers' profits, which is decreasing in  $m$  due to increased competition among brokers.

With simultaneous dual trading, ignoring brokerage costs, an increase in  $m$  has two opposite effects on informed profits. The piggybacking effect increases (see corollary four), which reduces informed gross profits for sufficiently large values of  $m$ . Aggregate broker profits also increase, which reduces the amount informed traders pay in brokers' fees. Given  $Q > m$  (the existence condition), the second effect dominates and net informed profits increase with  $m$ . However, total brokerage costs are also increasing with  $m$ , and so  $[k_b + k_f/(1+n)]$  must be small



for net informed profits to increase in  $m$ . For example, when  $t=1$ ,  $\Sigma_u = \Sigma_v = 256$  (standard deviation = 16%),  $m=4$ ,  $n=5$ , we need  $[k_o + k_i/6] < 0.05$  for net informed profits to increase in  $m$ .

**Proposition 3:** If  $Q \leq m$ , or if  $m \leq 3$ , then informed traders opt for consecutive dual trading and allocate their orders to a single broker.

When there are too few brokers or too few insiders relative to the number of brokers (i.e.,  $Q \leq m$ ), the effect of piggybacking on an individual insider is very strong. In these cases, either a market with simultaneous dual trading breaks down (when  $Q \leq m$ ) or insiders make very small profits. By contrast, the market with consecutive dual trading is robust to the number of brokers since insider's trading strategies are unaffected by consecutive dual trading. From corollary seven, informed net profits are maximized with a single broker.

**Proposition 4.** Suppose  $Q > m > 3$ . When  $m$ ,  $n$  and  $t$  are small, informed traders may prefer consecutive dual trading and allocate their orders to a single broker. When  $m$ ,  $n$  and  $t$  are large, informed traders may prefer simultaneous dual trading and allocate their orders to all available brokers.

**Corollary 8.** Relative to simultaneous dual trading, with consecutive dual trading

- 1) price is more informative,
- 2) market depth is higher, and
- 3) brokers' trading revenues are lower and their fee is higher.

In markets where both simultaneous and consecutive dual trading are feasible, informed traders prefer consecutive dual trading in markets with few brokers and few informed traders with relatively noisy information. By contrast, informed traders prefer simultaneous dual trading

when there are many brokers<sup>18/</sup> and many informed traders with relatively precise information. The reason is as follows. Relative to consecutive dual trading, gross informed profits and the brokers' fee are lower (see corollary eight) with simultaneous dual trading. With many insiders, the difference in gross informed profits is small. If, in addition, the number of brokers is large, then brokers' profits and, thus, the fee reduction are large enough for net informed profits to be larger with simultaneous dual trading.

An implication of proposition 4 and corollary 8 is that informed traders do not necessarily prefer trading in deeper markets if they are also concerned with the strategic issue of protecting their information.

## **V. Internalization of order flow by broker-dealers**

Since broker-dealers can internalize order flow in several ways,<sup>19/</sup> for concreteness we focus on off-exchange internalization, which has become a major source of profits for broker-dealers.<sup>20/</sup> Such transactions are also called 19c-3 trading, where the name refers to Rule 19c-3 allowing New York Stock Exchange (NYSE) stocks listed after April 26, 1979 to be traded off-exchange. In a typical 19c-3 internalization, broker-dealers sell to small investors out of their own inventory, profiting from the transaction by selling to investors at a price higher than the current market price for the stock on the exchange.

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<sup>18/</sup>This result is consistent with Chakravarty and Sarkar (1997a), who find that informed traders prefer markets with many brokers because there is a smaller likelihood of front running.

<sup>19/</sup>In addition to off-exchange internalization, discussed here, internalization may occur through preferencing, by third market makers in listed stocks, and in the over-the-counter market. See "Report on the practice of preferencing," the SEC, April, 1997.

<sup>20/</sup>19c-3 trading was rumored to have earned over \$500 million in 1994. See "'In-House' trades can be costly for small investors," the *Wall Street Journal*, December 20, 1994, page C1.

We use our simultaneous dual trading model to analyze order flow internalization. Since brokers typically sell only to small investors, we specialize the model to ensure that most of brokers' trading involves selling to noise traders instead of piggybacking on informed trades. We do so by assuming that  $n=m^2$  (the number of insiders varies as the square of the number of brokers). Given this assumption, and from (12) and (13), each broker trades:

$$z = \sqrt{\frac{\sum_u s}{\sum_v m^2(1+m)}} - \frac{u}{(1+m)} \quad (16)$$

Since the piggybacking component (the first term of (16)) is inversely related to the cube of  $m$ , it becomes relatively small for moderate values of  $m$ , and most of brokers' trading reflects selling  $u$  out of inventory (the second term of (16)). Note that  $m$  can be interpreted as an **inverse** measure of the degree of internalization of order flow by the broker.

We compare uninformed losses, brokers' commissions and the market quality between the simultaneous dual trading model (given the above assumption) and a model with no brokers' trading. For simplicity, we assume  $\tau=1$ . The following proposition shows the effect of order flow internalization on noise trader losses and the market.

**Proposition 5.** Let  $n=m^2$ .

- (1) With internalization of order flow, uninformed losses are higher for  $m \geq 2$ , commissions and market depth are lower and price informativeness is unchanged.**
- (2) As the degree of internalization increases, net uninformed losses increase, while commissions, market depth and price informativeness decrease.**

With at least two brokers, noise traders' losses to brokers exceed the reduction in informed profits from piggybacking. Brokers make positive profits, reducing the competitive commissions. The results on market depth and price informativeness follow from corollary 5. As

the degree of internalization increases ( $m$  decreases), brokers sell more to noise traders, and their profits increase, increasing gross uninformed losses. The increase in brokers' profits reduce the competitive commissions, as conjectured by the SEC and the NASD (see footnote eight), but this is not enough to offset the increase in noise traders' gross losses.

## VI. Follow-on Trading.

In this section, we re-interpret the brokers in our model as follow-on traders--i.e., traders whose trading is derived from observing the order flow of preceding traders. In the context of this broader definition, we study two issues: volume and liquidity around insider trading events; and off-exchange sales of large blocks of stocks.

### A. Trading Volume and Market Depth Around Insider Trading Events

We seek to explain two results: an unusual increase in non-insider volume around insider trading events; and increased or unchanged liquidity following insider trading. We interpret brokers in our consecutive dual trading model as "follow-on traders" and present some theoretical results on the effect of insider trading on volume and market depth. Then, we derive testable hypotheses regarding volume following insider trading events and the relationship between "follow-on volume" and liquidity. Finally, we test our hypotheses using data from a particular insider trading episode (to be described later).

The trading variables in our model are net order flows, not volume. Thus, following Admati and Pfleiderer (1988), we define trading volume as follows. The expected trading volume per broker is  $E(|z|) = \sqrt{\Sigma_z} \sqrt{2II}$ . Similarly, the expected noise trading volume is  $E(|u|) = \sqrt{\Sigma_u} \sqrt{2II}$  and the expected trading volume per informed trader  $I$  is  $E(|x'|) = \sqrt{\Sigma_x} \sqrt{2II}$ .

**Proposition 6:** The expected trading volume is higher (lower) with "follow-on" trading if  $m$  is greater (less) than  $n$ .

We show in the appendix (within the proof of proposition six) that aggregate expected "follow-on" trading volume is proportional to the number of "follow-on" traders times the expected noise trading volume. Similarly, expected informed trading volume is proportional to the number of informed traders times the expected noise trading volume. Thus, if the number of "follow-on" traders exceeds the number of insiders, then expected trading volume is higher following trading by insiders.

Define the liquidity in period two (i.e., the period with follow-on trading) as the (inverse of) the period two price per unit of period two order flow. Thus:

$$\frac{p_2}{y_2} = \lambda_2 + \mu_2 \frac{y_1}{y_2} \quad (17)$$

**Proposition 7:** Suppose  $m = [Q(1+Q)]^{-1}$ . If  $m$  is large (small), then  $(p_2/y_2)$  is less (greater) than  $\lambda_1$ .

The proposition explains why market depth in period two, following insider trading, may be less than market liquidity in period one, when insiders trade. Since  $Q$  is positively related to  $n$ , the condition  $m = [Q(1+Q)]^{-1}$ , with  $m$  large, implies that the number of "follow-on" traders is relatively large and the number of insiders relatively small. In this case,  $(y_1/y_2)$  is relatively small and  $(p_2/y_2)$  is approximated closely by  $\lambda_2$ . From corollary two, we know  $\lambda_2 < \lambda_1$ . Intuitively, the market maker's adverse selection costs in period two are derived primarily from brokers' trading in period two, which has lower information content than insider trading in period one.

We test the empirical implications emanating from Propositions 6 and 7 using insider trading data surrounding the acquisition of Carnation Company by Nestle S.A. in 1984. Between June 1, 1984, and August 31, 1984, prior to the first public announcement by Carnation or Nestle of merger discussions, Ivan Boesky acquired 1,711,200 shares of Carnation stock which constituted just under 5% of Carnation's outstanding shares. The Securities and Exchange Commission (SEC) later charged Boesky with illegally trading Carnation stock on the basis of illegally obtained information. Boesky acknowledged that he had received material non-public information regarding the Nestle take over of Carnation from Martin Siegel, and investment banker at Kidder, Peabody & Co.

Our database includes information on Boesky's transactions in Carnation stock, which includes dates, times, quantities and prices. Specifically, Boesky's purchases of Carnation stock span 366 separate transactions over 23 trading days between June and August of 1984. The transaction details were compiled from trading records maintained at Boesky's offices. Other details of this data, including a variety of summary statistics, are given in Chakravarty and McConnell (1996, 1997) and are not reported here for reasons of brevity.

Proposition 6 shows that the expected trading volume is higher after insider trading if the number of follow-on traders is greater than  $n$ . Chakravarty and McConnell (1996) verify that there is only insider on record who traded Carnation stock during the summer of 1984 -- Ivan Boesky. While the presence of other (unknown) insiders cannot be ruled out, it is safe to assume that  $m > n$  in the data set.

**Hypothesis 1. Expected trading volume following Boesky's trades is higher than Boesky's trading volume or trading volume preceding Boesky's trades.**

Proposition 7 states that if the number of follow-on traders is large compared to the number of insiders, then market depth is lower than when insiders trade.

**Hypothesis 2. The price impact (price/order flow) following Boesky's trades is lower than when Boesky trades.**

We use the Institute for the Study of Security Markets (ISSM) data set to calculate relevant market variables on an hourly basis.<sup>21/</sup> The Boesky trading hour (BTH) is defined as any hour in which Boesky traded. The follow-on hour (FOH) is the hour after Boesky trades, except when Boesky trades in consecutive hours. In the latter case, an FOH is defined as the hour after the last time Boesky traded on a day. A pre-Boesky hour (PBH) is the hour immediately before hour Boesky trades. When Boesky trades in consecutive hours, a PBH is hour preceding the first time Boesky traded on a day (if there is one). Finally, an all-other-hour (AOH) is any hour other than a BTH, FOH, or PBH. All hours are within the same trading day.

To test hypothesis one, we define follow-on trading volume as all non-Boesky trading in BTH plus all trading volume in FOH. We divide this sum by two to obtain the follow-on volume per hour and compare this with the average trading volume during an AOH.

To test hypothesis two, we define the price impact over an hour as follows. For every hour, the average price of Carnation stock (STKPR) is divided by the average hourly trading volume (SIZE), where STKPR is the simple mean of trade prices and SIZE is the total hourly volume divided by the number of trades for the hour. Then we run the following regression:

$$\begin{aligned} \text{STKPR}/\text{SIZE}_t = & a_0 + a_1\text{STDPR}_t + a_2\text{BKVOL}_t + a_3\text{NONBKVOL}_t \\ & + a_4\text{PREI}_t + a_5\text{CURL}_t + a_6\text{POSTI}_t + e_t \end{aligned} \quad (17)$$

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<sup>21/</sup>See Chakravarty and McConnell (1996) as to why an hour is chosen for a basis of analysis.

The independent variables are: (1) the hourly standard deviation of Carnation's stock price (STDPR), which is included as a measure of risk; (2) hourly Boesky volume (BKVOL); (3) the hourly non-Boesky volume (NONBKVOL) computed as the difference of the hourly Boesky volume from the hourly total volume; (4) an indicator variable (PREI) taking the value 1 if the hour is a PBH and zero otherwise; (5) an indicator variable (CURI) taking the value 1 if the hour is a BTH and zero otherwise; and (6) an indicator variable (POSTI) taking the value 1 if the hour is a FOH and 0 otherwise.

The coefficients  $a_2$ ,  $a_3$  and  $a_6$  are the important variables from our standpoint. Consistent with our theory, we expect  $a_3 < 0$ ,  $a_6 < 0$  and  $a_2$  to be insignificantly different from zero. Further, we expect  $a_3 > a_6$  in magnitude.

The mean trading volume in Carnation for the two FOH hours is 57,682 shares per hour while the trading volume for the AOHs is 29,003 shares per hour. We perform a two-tailed T-test of equality of means of total trading volume across the FOHs and the AOHs. The null hypothesis of equality of means is rejected at the 1% level of significance (p-value = 0.0004). Thus, hypothesis one appears to be supported by the data.

Table 1 presents the regression results. The p-values of coefficient significance are in parentheses under the coefficient estimates. Interestingly, the coefficient on PREI is negative but statistically insignificant from zero (at the 10% level of significance). In contrast, the coefficient of CURI is negative (-6.857580) with a p-value = 0.0074 (significant at the 1% level), while the coefficient of POSTI is negative (-3.147740) with a p-value = 0.0436 (significant at the 5% level). Notice that the coefficient for POSTI (the follow-on period) is less than half the magnitude of the coefficient for CURI (the insider trading period), implying that, after controlling for volume and a



risk-measure, the price impact in the follow-on hour is less than the half that in the insider-trading hour. Thus, hypothesis two also appears to be supported by the data.

### *B. Off-exchange stock sales by broker-dealers*

In a typical off-exchange sale, a dealer offers to buy a block of stock at a discount to the closing price on the exchange, acting as a principal. To model the off-exchange transaction, we use a simplified version of Holden and Subrahmanyam (1992).  $n$  insiders commit to sell their block of stock off-exchange, and over two periods. In deciding how much stock to sell, insiders maximize their total expected profits in the current and future period. The market is structured along the lines of Kyle's (1985) sequential auctions model.

We use our consecutive dual trading model to describe the insiders' alternative to selling off-exchange. In period one, insiders sell stock on an exchange. Other traders (equivalent to the consecutive dual traders),  $m$  in number, observe the stock sale in period one, and engage in "follow-on" trading in period two. When insiders sell a second time, we assume they have to sell at the price  $p_2$  determined by follow-on traders in period two.

To determine whether off-exchange trading benefits insiders, we compare market depth and price informativeness in period two for the two cases. We use the subscript "o" to denote "off-exchange" trading. Let  $p_{2,o}$  be the period two price change,  $y_{2,o}$  the contemporaneous change in order flow,  $\Sigma_{2,o}$  the inverse of price informativeness in the off-exchange market, where  $p_{2,o} = \lambda_{2,o} y_{2,o}$ .

To maintain comparability between the off-exchange and exchange transactions, we assume  $t=1$  (perfect information) in the consecutive dual trading model and let the time between two auctions in the Holden and Subrahmanyam (1992) model ( $\Delta t_n$  in their notation) equal one. Let  $q$  be

the unique root of equation (22) in Holden and Subrahmanyam (1992) when the number of periods  $N=2$  in their model. Specifically,  $q$  is the unique root of:

$$2nq^3 - (1+n)q^2 - \frac{2q}{(1+n)^2} + \frac{1}{(1+n)^2} = 0 \quad (18)$$

The following proposition compares the market quality in period two for off-exchange and exchange trading by insiders.

**Proposition 8.** If  $n(1-2q)$  is less (greater) than  $m$ , then price informativeness is lower (higher) but market depth may be higher (is lower) with off-exchange trading.

$n(1-2q)$  is less than  $m$  for relatively high values of  $m$  and  $q$  and low values of  $n$ . If the number of insiders is relatively small and the number of "follow-on" traders is relatively large, then off-exchange trading may provide more liquidity to insiders. A large value of  $m$  implies a large amount of "follow-on" trading, which drives up period two prices, making it costly for insiders when they trade a second time on the exchange. A relatively small number of insiders reduces the adverse selection problem for market makers in period two. This latter effect favors off-exchange trading, since insiders spread their orders over both periods.

However, greater liquidity is obtained at the expense of the price efficiency of the exchange market. This is because  $n(1-2q) < m$  implies a high value of  $q$ , which is positively related to market depth and negatively related to informed trading intensity in period one. If informed intensity is low and markets are deep in period one, this tends to lower price informativeness in both periods one and two.

The above intuition is illustrated in the following numerical example.

**Example 1.** If  $n=1$ ,  $q=0.28$ , and market depth and price informativeness are **higher** with off-exchange trading for all values of  $m$ .

If  $n=2$ ,  $q=0.18$ , and market depth and price informativeness are **lower** with off-exchange trading for  $m=1$ .

If  $n=3$ ,  $q=0.12$ , and market depth and price informativeness are **lower** with off-exchange trading for  $m \leq 2$ .

## VII. Conclusion.

In this article, we study a general model of brokers' trading and use the model to study a wide variety of issues related to brokers' trading. Multiple informed traders and noise traders trade through multiple brokers, who have the choice of trading in the same transaction as their customers (simultaneous dual trading) or in a separate transaction (consecutive dual trading) setting. Customers pay fees to brokers for their services. The fee is competitively determined--i.e., it is set at a level that assures zero expected profits to brokers. Therefore, brokers are indifferent between trading simultaneously or consecutively.

While the consecutive dual trading equilibrium always exists, the simultaneous dual trading equilibrium fails when the number of available brokers is greater than the number of informed traders. The reason is that brokers trade with informed traders in the same direction, thus worsening informed traders' terms of trade. This effect is magnified with many brokers, leading informed traders to stop trading. When both equilibria exist, net informed profits and net uninformed losses are higher with simultaneous (consecutive) dual trading in markets with many (few) brokers and insiders. However, market depth and price informativeness are higher with consecutive dual trading, compared both to no-dual-trading and simultaneous dual trading.

In the simultaneous dual trading model, brokers internalize the uninformed order flow, by selling to noise traders as dealers, out of inventory. We find that although internalization results in lower brokers' fees, noise traders are still worse off and market depth is lower.

We reinterpret brokers as "follow-on traders", whose trading is derived from the order flows of preceding traders. We conjecture that such follow-on trading is common around insider trading events. In particular, we predict that non-insider volume may be higher than average around insider trading events, and consequently, that market liquidity may decrease. These predictions are consistent with earlier insider trading studies. We verify these predictions with our own empirical analysis of illegal insider trading data.

Finally, we apply our model to explaining large block sales conducted off the exchange floor to avoid "follow-on" trading. We find that such off-exchange transactions may provide additional liquidity for large block sales, but at the expense of reducing the informativeness of the exchange market.

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Table 1

Trading Volume and Price Impact Around Boesky's Trading in Carnation Stock

We estimate the following regression:

$$STKPR/SIZE_t = a_0 + a_1STDPR_t + a_2BKVOL_t + a_3NONBKVOL_t + a_4PREI_t + a_5CURI_t + a_6POSTI_t$$

where STKPR is the hourly average of Carnation stock trading price, SIZE is the total hourly volume divided by the number of (hourly) trades, STDPR is the hourly standard deviation of Carnation's stock price, BKVOL is the hourly Boesky volume, NONBKVOL is the difference between the hourly Boesky volume and the hourly total volume, PREI equals 1 in the hour immediately before the hour Boesky trades on a day and zero otherwise, CURI equals 1 in the hour when Boesky trades and zero otherwise and POSTI equals 1 in the hour immediately following the Boesky trading hour and 0 otherwise.

The p-values of the coefficient estimates are in parentheses below the coefficients. The number of observations are 336. The sample period is June 1, 1984 through August 31, 1984.

Independent variables	Dependent variable: STKPR/ SIZE
Intercept	12.0050 (0.0001)
STDPR	-5.2618 (0.1103)
BKVOL	0.000082 (0.9895)
NONBKVOL	-0.010043 (0.0001)
PREI	-1.3124 (0.4233)
CURI	-6.8576 (0.0074)
POSTI	-3.1477 (0.0436)
R-Squared	0.22

## Appendix

### Proof of Proposition 1:

#### **Consecutive dual trading equilibrium.**

There are  $n$  insiders and  $m$  brokers

#### Period 1:

Dual trader submits order to market maker. The informed trades are given by

$\{x^{1,d}, x^{2,d}, \dots, x^{n,d}\}$ . Let  $x_j = \sum_{i=1}^n x^{i,d}$  be the aggregate informed trade. Thus, the net order flow to the market maker:  $y_1 = x_j + u_1$ . The corresponding market clearing price is given by  $p_1 = \lambda_1 y_1$ .

#### The insider $i$ 's problem:

$$\text{Max}_{x^{i,d}} E\left[(v - \lambda_1 x^{1,d} - \lambda_1 x^{i,d} - \lambda_1 u_1) x^{i,d} | s^i\right] \quad (1.0)$$

where  $x^{i,d} = \sum_{j=1}^n x^{j,d}$ . Also,  $s^i = v + e^i$ ,  $i = 1, \dots, n$ , where  $e^i \perp e^j$ , for all  $i \neq j$ . The maximization program (1.0) can now be rewritten as

$$\text{Max}_{x^{i,d}} \left[ (ts^i - \lambda_1 x^{i,d} - \lambda_1 E[x^{i,d} | s^i]) x^{i,d} \right]$$

where  $t = \Sigma_j / \Sigma$ , and  $E(x^{i,d} | s^i) = \sum_{j=1}^n E(x^{j,d} | s^i) = A_j \sum_{j=1}^n E(s^j | s^i) = (n-1) A_j t s^i$ . Thus, the above can be rewritten as

$$\text{Max}_{x^{i,d}} \left[ (ts^i - \lambda_1 x^{i,d} - \lambda_1 A_j t (n-1) s^i) x^{i,d} \right]$$

From above, both the first and second order conditions are given by:

$$\text{Foc: } ts^i [1 - \lambda_1 A_j (n-1)] - 2\lambda_1 x^{i,d} = 0 \quad (1.1)$$

$$\text{Soc: } -2\lambda_1 < 0, \text{ or } \lambda_1 > 0 \quad (1.2)$$

$$\text{From (1.1), } x^{i,d} = \frac{ts^i}{2\lambda_1} [1 - \lambda_1 A_j (n-1)]$$



Comparing coefficients:

$$A_j = \frac{t}{2\lambda_1} - \frac{t}{2} A_j (n-1)$$

$$\therefore A_j = \frac{t}{\lambda_1 \frac{2+t(n-1)}{t}} \quad (1.3)$$

$$\text{or, } A_j = \frac{t}{\lambda_1(1+Q)}$$

$$\text{where } Q = 1+t(n-1) \quad (1.4)$$

Now,

$$y_i = x_j + u_i$$

$$= \sum_{i=1}^n x^{i,j} + u_i \quad (1.5)$$

$$= A_j \sum_{i=1}^n s^i + u_i$$

$$\text{or, } y_i = A_j s + u_i, \text{ where } s = \sum_{i=1}^n s^i.$$

$$\lambda_1 = \frac{\text{Cov}(v, y_i)}{\text{Var}(y_i)} = \frac{A_j(n-1)\Sigma_s}{(A_j)^2 \text{Var}(s) + \Sigma_u} \quad (1.6)$$

Now,

$$\begin{aligned} \text{Var}(s) &= \text{Var}\left(\sum_{i=1}^n s^i\right) \\ &= n\Sigma_s + n(n-1)\Sigma_s \\ &= n\Sigma_s[1+(n-1)t] \\ &= nQ\Sigma_s \end{aligned} \quad (1.7)$$

$$\text{Therefore, from (1.6) and (1.7), } \lambda_1 = \frac{nA_j\Sigma_s}{(A_j)^2 n[\Sigma_s + (n-1)\Sigma_s] + \Sigma_u} \quad (1.8)$$

$$\text{After simplification, } \lambda_1 = \frac{1}{\sqrt{\Sigma_s}} \frac{1}{1+Q} \sqrt{nt\Sigma_s} \quad (1.9)$$

$$\text{Also, from (1.3), } A_j = \frac{t}{\lambda_1(1+Q)} = \frac{t\sqrt{\Sigma_s}}{\sqrt{nt\Sigma_s}} = \frac{\sqrt{t\Sigma_s}}{\sqrt{n\Sigma_s}} \quad (1.10)$$

### Period 2:

Each dual trader  $j$  observes  $x_j/m$  and  $u_j/m$  and trades  $z^j$ . The net order flow  $y_j$  is given

$$\text{by } y_j = \sum_{i=1}^m z^i + u_j = mz + u_j \quad (1.11)$$

Note that we will show that  $z^j = z, \forall j$ , since each  $z^j$  is a function of  $x_j$ . Now, the second period market-clearing price  $p_2$  is given by

$$p_2 = \mu_1 y_1 + \lambda_1 y_2 \quad (1.12)$$

The  $j$ th dual trader's problem:

$$\text{Max}_j E \left[ (v - p_j) z^j \left| \frac{x^{1,d}}{m}, \frac{x^{2,d}}{m}, \dots, \frac{x^{n,d}}{m}, \frac{u_1}{m} \right. \right] \quad (1.13)$$

$$\text{or, Max}_j E \left[ (v - \lambda_j z^j - \lambda_j \bar{z}^j - \lambda_j \mu_j - \mu_j A_j s - \mu_j u_1) z^j \left| \frac{x^{1,d}}{m}, \frac{x^{2,d}}{m}, \dots, \frac{x^{n,d}}{m}, \frac{u_1}{m} \right. \right] \quad (1.14)$$

where  $\bar{z}^j = \sum_{j \neq k} z^k$ . From above, the first and second order conditions are:

$$\text{Foc: } E \left[ v \left| \frac{x^{1,d}}{m}, \frac{x^{2,d}}{m}, \dots, \frac{x^{n,d}}{m} \right. \right] - 2\lambda_j z^j - \lambda_j \bar{z}^j - \mu_j A_j s - \mu_j u_1 = 0 \quad (1.14)$$

$$\text{Soc: } -2\lambda_j < 0, \Rightarrow \lambda_j > 0 \quad (1.15)$$

$$E \left[ v \left| \frac{x^{1,d}}{m}, \frac{x^{2,d}}{m}, \dots, \frac{x^{n,d}}{m} \right. \right] = \left( \frac{x_j}{m} \right) \frac{\frac{A_j}{m} n \Sigma_s}{\left( \frac{A_j}{m} \right)^2 \text{Var}(s)} = \left( \frac{x_j}{m} \right) \left( \frac{m}{A_j} \right) \frac{n \Sigma_s}{n Q \Sigma_s} = \frac{t}{Q} s \quad (1.16)$$

Substituting (1.16) in (1.14) and solving for  $z = z^j$ , we get

$$z = \frac{ts}{\lambda_j Q (1+Q)} \cdot \frac{T_u}{\lambda_j (1+m)} - \frac{\mu_j}{\lambda_j (1+m)} u_1 \quad (1.17)$$

$$\text{where, } T_u = \lambda_j (1+Q) - \mu_j Q \quad (1.18)$$

Now,

$$\begin{aligned} y_j &= m z + u_1 \\ &= \frac{ts}{\lambda_j Q (1+Q)} T_u \frac{m}{1+m} \frac{1}{\lambda_j} - \frac{m}{1+m} \frac{1}{\lambda_j} \mu_j u_1 + u_1 \\ &= \frac{ts}{\lambda_j Q (1+Q)} T_u T_1 - T_1 \mu_j u_1 + u_1 \end{aligned} \quad (1.19)$$

$$\text{where, } T_1 = \frac{m}{1+m} \frac{1}{\lambda_j} \quad (1.20)$$

Now, we know from before

$$\begin{aligned} y_1 &= A_j s + u_1 \\ &= \frac{ts}{\lambda_j (1+Q)} + u_1 \end{aligned} \quad (1.21)$$

$$\Sigma_{11} = \text{Cov}(v, y_1) = \frac{nt \Sigma_s}{\lambda_j (1+Q)} = L \Sigma_s \quad (1.22)$$

$$\text{where, } L = \frac{nt}{\lambda_j (1+Q)} \quad (1.23)$$

$$\begin{aligned}\Sigma_{02} &= \text{Cov}(v, y_2) = \frac{nt\Sigma_s}{\lambda_1 Q(1+Q)} T_2 T_1 \\ &= \frac{L\Sigma_s}{Q} T_2 T_1\end{aligned}\quad (1.24)$$

$$\begin{aligned}\Sigma_{12} &= \text{Cov}(y_1, y_2) \\ &= \frac{t^2 T_2 T_1}{Q[\lambda_1(1+Q)]^2} \text{Var}(s) - \mu_1 T_1 \Sigma_s \\ &= \frac{t^2 T_2 T_1}{Q[\lambda_1(1+Q)]^2} nQ\Sigma_s - \mu_1 T_1 \Sigma_s \\ &= \frac{nt\Sigma_s}{[\lambda_1(1+Q)]^2} T_2 T_1 - \mu_1 T_1 \Sigma_s \\ &= \frac{L^2 \Sigma_s}{nt} T_2 T_1 - \mu_1 T_1 \Sigma_s\end{aligned}\quad (1.25)$$

$$\begin{aligned}\Sigma_{11} &= \text{Var}(y_1) \\ &= \frac{t^2}{[\lambda_1(1+Q)]^2} \text{Var}(s) + \Sigma_s \\ &= \frac{t^2}{[\lambda_1(1+Q)]^2} nQ\Sigma_s + \Sigma_s \\ &= \frac{nt\Sigma_s Q}{[\lambda_1(1+Q)]^2} + \Sigma_s \\ &= \frac{L^2 \Sigma_s Q}{nt} + \Sigma_s\end{aligned}\quad (1.26)$$

$$\begin{aligned}\Sigma_{22} &= \text{Var}(y_2) \\ &= \frac{t^2 \text{Var}(s)}{[\lambda_1 Q(1+Q)]^2} (T_2 T_1)^2 + (T_2 \mu_1)^2 \Sigma_s + \Sigma_s \\ &= \frac{t^2 nQ\Sigma_s}{[\lambda_1 Q(1+Q)]^2} (T_2 T_1)^2 + [(T_2 \mu_1)^2 + 1] \Sigma_s \\ &= \frac{L^2 \Sigma_s}{Qnt} (T_2 T_1)^2 + [(T_2 \mu_1)^2 + 1] \Sigma_s\end{aligned}\quad (1.27)$$

$$\text{Let } D = \Sigma_{11} \Sigma_{22} - (\Sigma_{12})^2 \quad (1.28)$$

Substituting (1.25), (1.26) and (1.27) in (1.28) and after considerable simplification

$$D = (\Sigma_s)^2 (1+Q) + \frac{(\Sigma_s)^2 (T_1)^2 (\lambda_1)^2 (1+Q)^2}{Q} \quad (1.29)$$

Let

$$\begin{aligned}
 N_x &= \Sigma_{01} \Sigma_{11} - \Sigma_{02} \Sigma_{12} \\
 &= \left[ \frac{L\Sigma_x}{Q} T_x T_x \right] \left[ \frac{L^2 \Sigma_x Q}{nt} + \Sigma_x \right] - (L\Sigma_x) \left[ \frac{L^2 \Sigma_x T_x T_x}{nt} - \mu_x T_x \Sigma_x \right] \\
 &= \frac{L^2 \Sigma_x}{Q} T_x T_x + L\Sigma_x \Sigma_x \mu_x T_x \\
 &= L\Sigma_x \Sigma_x T_x \left[ \frac{T_x}{Q} + \mu_x \right] \\
 &= \frac{L\Sigma_x \Sigma_x T_x}{Q} \lambda_x (1+Q)
 \end{aligned} \tag{1.30}$$

$$\begin{aligned}
 \lambda_x &= \frac{N_x}{D} \\
 &= \frac{L\Sigma_x \Sigma_x T_x \lambda_x \{(1+Q)/Q\}}{(\Sigma_x)^2 \left[ (1+Q) + \frac{(T_x)^2 (\lambda_x)^2 (1+Q)^2}{Q} \right]} \\
 &= \frac{L\Sigma_x T_x \lambda_x (1+Q)}{Q(1+Q) + (T_x)^2 (\lambda_x)^2 (1+Q)^2}
 \end{aligned} \tag{1.31}$$

$$\begin{aligned}
 \Sigma_x \lambda_x &= \frac{\left( \frac{1+m}{m} \right) \lambda_x L\Sigma_x \lambda_x (1+Q)}{\left( \frac{1+m}{m} \right)^2 (\lambda_x)^2 Q(1+Q) + (\lambda_x)^2 Q(1+Q)} \\
 &= \frac{\left( \frac{1+m}{m} \right) \lambda_x L\Sigma_x \lambda_x (1+Q)}{\left( \frac{1+m}{m} \right)^2 (\lambda_x)^2 Q(1+Q) + (\lambda_x)^2 Q(1+Q)}
 \end{aligned}$$

From above, after simplification, we obtain

$$\lambda_x = \frac{\sqrt{nt\Sigma_x}}{\sqrt{\Sigma_x} \sqrt{Q(1+Q)}} \frac{\sqrt{m}}{1+m} \tag{1.32}$$

$$\text{Now, } N_{x_2} = \Sigma_{01} \Sigma_{21} - \Sigma_{02} \Sigma_{12} \tag{1.33}$$

Substituting from above and simplifying, we obtain

$$\begin{aligned}
 N_{x_2} &= (L\Sigma_x) \left[ \frac{L^2 \Sigma_x}{Qnt} (T_x T_x)^2 + \Sigma_x \{1 + (T_x \mu_x)^2\} \right] - \frac{L\Sigma_x}{Q} (T_x T_x) \left[ \frac{L^2 \Sigma_x}{nt} T_x T_x - \mu_x T_x \Sigma_x \right] \\
 &= (L\Sigma_x \Sigma_x) \{1 + (T_x \mu_x)^2\} + \frac{L^2 \Sigma_x}{Q} (T_x T_x) T_x \mu_x \Sigma_x \\
 &= L\Sigma_x \Sigma_x + \frac{L^2 \Sigma_x (T_x)^2 \mu_x}{Q} [\mu_x Q + T_x] \\
 &= L\Sigma_x \Sigma_x + \frac{L^2 \Sigma_x (T_x)^2 \mu_x}{Q} \lambda_x (1+Q)
 \end{aligned} \tag{1.34}$$

$$\begin{aligned}
 \mu_x &= \frac{N_{x_2}}{D} = \frac{L\Sigma_x \Sigma_x \left[ 1 + \frac{(T_x)^2 \mu_x \lambda_x (1+Q)}{Q} \right]}{(\Sigma_x)^2 \left[ (1+Q) + \frac{(T_x)^2 \{\lambda_x (1+Q)\}^2}{Q} \right]}
 \end{aligned} \tag{1.35}$$

Simplifying the above

$$\begin{aligned}
\mu_1 &= \frac{L\Sigma_s}{\Sigma_s(1+Q)} = \frac{nt\Sigma_s}{(\Sigma_s)\lambda_1(1+Q)^2} \\
&= \frac{nt\Sigma_s}{(\Sigma_s)(1+Q)} \frac{1}{\sqrt{nt\Sigma_s}} \\
&= \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \frac{1}{1+Q}
\end{aligned} \tag{1.36}$$

$$\begin{aligned}
z &= \frac{ts}{Q} \frac{1}{\lambda_1(1+Q)} \frac{T_u}{\lambda_2(1+m)} - \frac{\mu_2}{\lambda_2} \frac{u_1}{1+m} \\
&= \frac{1}{\lambda_2(1+m)} \left[ \frac{ts}{Q} \frac{1}{\lambda_1(1+Q)} \{ \lambda_1(1+Q) - \mu_2 Q \} - \mu_2 u_1 \right] \\
&= \frac{1}{\lambda_2(1+m)} \left[ \frac{ts}{Q} - \frac{ts}{\lambda_1(1+Q)} \mu_2 \right] - \frac{\mu_2 u_1}{\lambda_2(1+m)} \\
&= \frac{1}{\lambda_2(1+m)} \left[ \frac{ts}{Q} - \frac{ts}{\sqrt{nt\Sigma_s}} \sqrt{\Sigma_s} \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \frac{1}{1+Q} - \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \frac{u_1}{1+Q} \right] \\
&= \frac{1}{\lambda_2(1+m)} \left[ \frac{ts}{Q(1+Q)} - \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \frac{u_1}{1+Q} \right] \\
&= \frac{\sqrt{\Sigma_s} \sqrt{Q(1+Q)}}{\sqrt{m} \sqrt{nt\Sigma_s}} \left[ \frac{ts}{Q(1+Q)} - \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \frac{u_1}{1+Q} \right] \\
\therefore z &= \left( \frac{1}{\sqrt{m}} \frac{1}{\sqrt{Q(1+Q)}} \right) x - \left( \frac{1}{\sqrt{m}} \frac{\sqrt{Q}}{\sqrt{(1+Q)}} \right) u_1
\end{aligned} \tag{1.37}$$

Therefore, comparing coefficients:

$$\begin{aligned}
B_1 &= \frac{1}{\sqrt{m}} \frac{1}{\sqrt{Q(1+Q)}} \\
B_2 &= \frac{1}{\sqrt{m}} \frac{\sqrt{Q}}{\sqrt{(1+Q)}}
\end{aligned} \tag{1.38}$$

The jth broker profits:

$$\begin{aligned}
W &= E[(v - p_j)z] \\
&= E[(v - \lambda_2 y_2 - \mu_2 y_1)z] \\
&= E[(v - \lambda_2(mz + u_2) - \mu_2(x_d + u_1))z]
\end{aligned}$$

Substituting for  $z$  from above, taking expectations and simplifying we obtain:

$$W = \frac{\sqrt{nt\Sigma_v\Sigma_y}}{\sqrt{Q(1+Q)}} \cdot \frac{1}{\sqrt{m(1+m)}} \quad (1.39)$$

**Proof of Corollary 1:**

(1) Since  $\partial Q / \partial n > 0$  and  $\partial Q / \partial \alpha > 0$  for  $n > 1$  and  $Q$  is independent of  $m$ ,  $\partial B_1 / \partial n < 0$  and  $\partial B_1 / \partial \alpha < 0$  for  $n > 1$ . Further,

$$\frac{\partial |B_1|}{\partial \sqrt{Q}} = \frac{1 + \sqrt{Q}[\sqrt{Q} - 1]}{(1+Q)^{3/2}} > 0 \text{ since } Q \geq 1.$$

(2) It is clear that  $B_1$  is decreasing in  $m$  from its definition. The same definition also

shows that  $\partial(mB_1) / \partial m > 0$ . Now,  $|B_1| = \frac{1}{\sqrt{m}} \frac{\sqrt{Q}}{\sqrt{1+Q}}$  is decreasing in  $m$  since  $Q$  is

independent of  $m$ . Therefore,  $m|B_1| = \frac{\sqrt{m}\sqrt{Q}}{\sqrt{1+Q}}$  is increasing in  $m$ .

**Proof of Corollary 2:**

(1)  $\mu_2 = \lambda_1$  follows from (2) and (6).

$$\lambda_2 = \lambda_1 \frac{\sqrt{1+Q}}{\sqrt{Q}} \frac{\sqrt{m}}{(1+m)}$$

Note that  $Q \geq 1$  and  $m \geq 1$ . For  $Q = 1$  and  $m = 1$ ,  $\lambda_2 = \frac{\lambda_1}{\sqrt{2}} < \lambda_1$ .

Further,  $\frac{\sqrt{1+Q}}{\sqrt{Q}}$  is decreasing in  $Q$  and  $\frac{1+m}{\sqrt{m}}$  is increasing in  $m$ . Thus,  $\lambda_2 < \lambda_1$  for  $Q > 1$  and  $m > 1$ .

(2) Define period one price informativeness as:  $PI_1 = \Sigma_v - \text{Var}(v|p_1) = 2\lambda_1 \Sigma_{v1} - (\lambda_1)^2 \Sigma_{11}$ ,

where  $\Sigma_{v1} = \text{Cov}(v, y_1)$  and  $\Sigma_{11} = \text{Var}(y_1)$ . Thus,  $PI_1 = \frac{nt\Sigma_v}{1+Q}$ .

Similarly, period two price informativeness is defined as:

$$\begin{aligned} PI_2 &= \Sigma_v - \text{Var}(v|p_2) = 2\mu_2 \Sigma_{v2} - (\mu_2)^2 \Sigma_{22} + 2\lambda_2 \Sigma_{v2} - (\lambda_2)^2 \Sigma_{22} - 2\lambda_2 \mu_2 \Sigma_{22} \\ &= PI_1 + 2\lambda_2 \Sigma_{v2} - (\lambda_2)^2 \Sigma_{22} \end{aligned}$$

since  $\mu_2 = \lambda_1$  and  $\Sigma_{12} = 0$ . Hence,  $PI_2 = PI_1 + \frac{nt\Sigma_v}{Q(1+Q)} \frac{m}{1+m} > PI_1$ .

**Proof of Corollary 3:**

(1) Ignoring commissions, the expected informed profit for insider  $i$  is

$$E[(v - p_i)x^{i,d}] = \frac{\sqrt{t\Sigma_s\Sigma_s}}{\sqrt{n(1+Q)}}$$

which is independent of brokers' trading since  $x^{i,d}$  is independent of brokers' trading.

$$\text{Now, the aggregate insider profits } \Pi_i = \frac{n\sqrt{t\Sigma_s\Sigma_s}}{\sqrt{n(1+Q)}} = \frac{\sqrt{nt\Sigma_s\Sigma_s}}{(1+Q)}. \quad (1)$$

Additionally, aggregate broker profits are given by

$$\Pi_b = mE[(v - p_s)\varepsilon] = \frac{\sqrt{nt\Sigma_s\Sigma_s}}{\sqrt{1+Q}\sqrt{Q}} \frac{\sqrt{m}}{1+m} \quad (2)$$

Hence, from (1) and (2), the noise traders' per period losses are:  $U = \frac{1}{2}(\Pi_i + \Pi_b) < \Pi_i =$

noise traders' losses without dual trading if:  $\Pi_b < \Pi_i$ , or,  $\frac{\sqrt{m}}{1+m} \cdot \frac{\sqrt{1+Q}}{\sqrt{Q}} < 1$ . The last

inequality holds since  $\lambda_s < \lambda_i$ ,

(2) With consecutive dual trading, the  $i$ th insider's net profit is given by

$$I_i = \frac{\sqrt{t\Sigma_s\Sigma_s}}{\sqrt{n(1+Q)}} - mc_i \quad (4)$$

while, without dual trading, the  $i$ th insider's net profit is given by

$$I_s = \frac{\sqrt{t\Sigma_s\Sigma_s}}{\sqrt{n(1+Q)}} - mc_s \quad (5)$$

Now,  $c_d < c_s \Leftrightarrow I_s < I_i$

Finally, since noise traders' losses are lower without commissions, they are also lower with commissions since  $c_d < c_s$ .

**Proof of Proposition 2:**

**Simultaneous dual trading equilibrium**

There are  $m$  dual traders and  $n$  informed traders. The net order flow  $y_s$  is given by

$$y_s = \sum_{i=1}^m z^i + \sum_{i=1}^n x^{i,s} + u \quad (2.1)$$

where  $\{x^{1,s}, x^{2,s}, \dots, x^{n,s}\}$  denotes the informed trades,  $x_s = \sum_{i=1}^n x^{i,s}$  and

$\{z^1, z^2, \dots, z^m\}$  denotes the personal trades by the  $m$  dual traders. Finally, the market maker's single market clearing price is given by  $p_s = \lambda_s y_s$ .

**The dual trader  $j$ 's problem:**

$$\text{Max}_j E \left[ (v - p_s) z^j \left| \frac{x^{1,s}}{m}, \frac{x^{2,s}}{m}, \dots, \frac{x^{n,s}}{m}, \frac{u}{m} \right. \right] \quad (2.2)$$

$$\text{or, Max}_j E \left[ (v - \lambda_s z^j - \lambda_s z^{-j} - \lambda_s x_s - \lambda_s u) z^j \left| \frac{x^{1,s}}{m}, \frac{x^{2,s}}{m}, \dots, \frac{x^{n,s}}{m}, \frac{u}{m} \right. \right]$$

The first order condition gives us

$$E \left[ v \left| \frac{x^{1,s}}{m}, \frac{x^{2,s}}{m}, \dots, \frac{x^{n,s}}{m}, \frac{u}{m} \right. \right] - 2\lambda_s z^j - \lambda_s z^{-j} - \lambda_s x_s - \lambda_s u = 0 \quad (2.3)$$

$$\text{where } E \left[ v \left| \frac{x^{1,s}}{m}, \frac{x^{2,s}}{m}, \dots, \frac{x^{n,s}}{m}, \frac{u}{m} \right. \right] = \frac{t}{Q} \hat{s}; \quad \hat{s} = \frac{\sum_{i=1}^n x^{i,s}}{A_s} = \frac{x_s}{A_s} \text{ and } Q = 1 + t(n-1)$$

Substituting  $z = z^j = z^{-j}$ , and solving for  $z$ , we obtain

$$z = \frac{x_s}{\lambda_s(1+m)} \left[ \frac{t}{QA_s} - \lambda_s \right] - \frac{u}{1+m} \quad (2.4)$$

**Insider  $i$ 's problem:**

$$\text{Max}_i E \left[ (v - \lambda_s m z - \lambda_s x^{i,s} - \lambda_s x^{-i,s} - \lambda_s u) x^{i,s} | s' \right] \quad (2.5)$$

$$\text{or, Max}_i E \left[ \left( v - \frac{x_s m}{1+m} \left[ \frac{t}{QA_s} - \lambda_s \right] - \lambda_s x^{i,s} - \lambda_s x^{-i,s} - \frac{\lambda_s u}{1+m} \right) x^{i,s} | s' \right]$$

$$\text{Max}_i \left[ x^{i,s} \left( t s' - x^{i,s} \left[ \lambda_s + \frac{m}{1+m} \left( \frac{t - \lambda_s QA_s}{QA_s} \right) \right] \right) - E[x^{-i,s} | s'] \left[ \lambda_s + \frac{m}{1+m} \left( \frac{t - \lambda_s QA_s}{QA_s} \right) \right] \right]$$



From above, the first order condition gives us

$$\begin{aligned}
 2x^{i,s} \left[ \frac{\lambda_i}{1+m} + \frac{m}{1+m} \cdot \frac{t}{QA_i} \right] &= - \frac{(n-1)A_i t s^i}{1+m} \left[ \lambda_i + \frac{mt}{QA_i} \right] + t s^i \\
 &= \frac{t s^i}{Q(1+m)} [Q(1+m) - \lambda_i A_i (n-1)Q - mt(n-1)] \\
 &= \frac{t s^i}{Q(1+m)} [Q + m - \lambda_i A_i (n-1)Q]
 \end{aligned} \tag{2.6}$$

The second order condition is:

$$\lambda_i + \frac{mt}{QA_i} > 0.$$

Thus, given  $\lambda_i > 0$ ,  $A_i$  has to be greater than 0 to satisfy the second order condition.

Simplifying (2.6), we get

$$x^{i,s} = \frac{t s^i}{Q(1+m)} \cdot \frac{[m + Q\{1 - \lambda_i A_i (n-1)\}](1+m)QA_i}{2[tm + \lambda_i QA_i]} \tag{2.7}$$

$$A_i = \frac{A_i t [m + Q\{1 - \lambda_i A_i (n-1)\}]}{2[tm + \lambda_i QA_i]} \tag{2.8}$$

Simplifying the above,

$$A_i = \frac{t(Q-m)}{\lambda_i Q(1+Q)} > 0 \text{ if } Q > m \text{ and } \lambda_i > 0 \tag{2.9}$$

Note that for  $t=1$ ,  $Q=1+t(n-1)=n$ . Thus,  $A_i > 0$  iff  $n > m$ .

$$\begin{aligned}
 \varepsilon &= \frac{t}{\lambda_i Q(1+m)} \cdot \frac{x_i}{A_i} - \frac{x_i}{1+m} - \frac{u}{1+m} \\
 &= \frac{x_i}{1+m} \left[ \frac{t}{\lambda_i A_i Q} - 1 \right] - \frac{u}{m+1} \\
 &= \frac{x_i}{1+m} \left[ \frac{1+Q}{Q-m} - 1 \right] - \frac{u}{m+1} \quad \text{by substituting for } A_i \\
 &= \frac{x_i}{Q-m} - \frac{u}{m+1}
 \end{aligned} \tag{2.10}$$

Therefore,  $B_{i,s} = \frac{1}{(Q-m)}$  and  $B_{i,s} = \frac{1}{(1+m)}$

Now,

$$y_j = x_j + mz + u$$

$$= x_j + \frac{x_j m}{Q-m} + \frac{m}{1+m} u \quad (2.11)$$

$$= x_j \left[ \frac{Q}{Q-m} \right] + \frac{m}{1+m} u$$

$$\lambda_j = \frac{\left( \frac{Q}{Q-m} \right) n A_j \Sigma_j}{\frac{Q^2}{(Q-m)^2} (A_j)^2 n Q \Sigma_j + \left( \frac{m}{1+m} \right)^2 \Sigma_j}$$

$$= \frac{\left( \frac{Q}{Q-m} \right) \cdot \frac{t(Q-m)}{\lambda_j Q(1+Q)} \cdot n \Sigma_j}{\frac{Q^2}{(Q-m)^2} \cdot \frac{t^2 (Q-m)^2}{\lambda_j^2 Q^2 (1+Q)^2} n Q \Sigma_j + \left( \frac{m}{1+m} \right)^2 \Sigma_j} \quad (2.12)$$

$$= \frac{n t \Sigma_j \lambda_j (1+Q)}{n t Q \Sigma_j + \left( \frac{m}{1+m} \right)^2 \Sigma_j \lambda_j^2 (1+Q)^2}$$

After further simplification, we obtain

$$\lambda_j = \frac{1+m}{m} \cdot \frac{\sqrt{n t \Sigma_j}}{\sqrt{\Sigma_j} (1+Q)} \quad (2.13)$$

$$A_j = \frac{t(Q-m)}{\lambda_j Q(1+Q)}$$

$$= \frac{t(Q-m)}{Q(1+Q)} \cdot \frac{m(1+Q)\sqrt{\Sigma_j}}{\sqrt{n t \Sigma_j} (1+m)} \quad (2.14)$$

$$= \frac{m \sqrt{t(Q-m)} \sqrt{\Sigma_j}}{Q(1+m) \sqrt{n \Sigma_j}} > 0, \text{ for } Q > m$$

Finally, the  $j$ th broker profits

$$\begin{aligned}
W_t &= E[(v - p_t)z] \\
&= E\left[\left(v - \frac{\lambda_t Q}{Q - m} x, -\frac{\lambda_t um}{1 + m}\right)z\right] \\
&= \frac{nA_t \Sigma_t}{Q - m} - \frac{\lambda_t Q}{Q - m} \cdot \frac{(A_t)^2 nQ \Sigma_t}{(Q - m)} + \frac{\lambda_t m \Sigma_t}{(1 + m)^2} \\
&= \frac{nA_t \Sigma_t}{Q - m} \left[1 - \lambda_t A_t \frac{Q}{Q + m} \cdot \frac{Q}{t}\right] + \frac{\lambda_t m \Sigma_t}{(1 + m)^2} \\
&= \frac{nA_t \Sigma_t}{Q - m} \left[1 - \frac{t(Q - m)}{Q(1 + Q)} \cdot \frac{Q^2}{t(Q - m)}\right] + \frac{\sqrt{nt \Sigma_t}}{\sqrt{\Sigma_t}} \cdot \frac{(1 + m)}{m(1 + Q)} \cdot \frac{m \sqrt{\Sigma_t}}{(1 + m)^2} \\
&= \frac{nA_t \Sigma_t}{Q - m} \cdot \frac{1}{1 + Q} + \frac{\sqrt{nt \Sigma_t} \Sigma_t}{(1 + Q)(1 + m)} \\
&= \frac{\sqrt{n \Sigma_t}}{(Q - m)} \cdot \frac{Q(1 + m) \sqrt{t \Sigma_t}}{1 + Q} + \frac{\sqrt{nt \Sigma_t} \Sigma_t}{(1 + Q)(1 + m)} \\
&= \frac{\sqrt{nt \Sigma_t} \Sigma_t}{(1 + Q)(1 + m)} \cdot \frac{Q + m}{Q}
\end{aligned} \tag{2.15}$$

**Proof of Corollary 4:**

Follows immediately from the definitions of  $B_1$  and  $|B_2|$  from (12) and (13) in the text, and from  $\partial Q / \partial n > 0$  and  $\partial Q / \partial \alpha > 0$  for  $n > 1$ .

**Proof of Corollary 5:**

$$(1) \quad \lambda_t = \frac{\sqrt{nt \Sigma_t}}{\sqrt{\Sigma_t}} \cdot \frac{1}{1 + Q} \left(\frac{1 + m}{m}\right) = \lambda_1 \left(\frac{1 + m}{m}\right)$$

Thus,  $\lambda_t > \lambda_1$ . The above implies that market depth is lower with simultaneous dual trading compared to the no-dual trading benchmark (i.e., the inverse of  $\lambda_1$ ).

(2)

$$\begin{aligned}
PI_t &= \lambda_t \text{Cov}(v, y_t) \\
&= \lambda_t A_t n \Sigma_t \cdot \frac{Q}{Q - m} \\
&= \frac{t(Q - m)}{Q(1 + Q)} n \Sigma_t \cdot \frac{Q}{Q - m} \\
&= \frac{nt \Sigma_t}{1 + Q}
\end{aligned}$$

which is the same without dual trading.

Proof of Corollary 6:

(1)

Individual insider profit without commission fees =  $\frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} \frac{m}{1+m} \frac{Q-m}{Q} < \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}}$ ,

where the term to the right of the inequality is the *i*th insider's net profit without dual trading.

The inequality follows since  $\left(\frac{m}{1+m}\right) < 1$  and  $\left(\frac{Q-m}{Q}\right) < 1$ .

Ignoring fees, uninformed losses with simultaneous dual trading equals the sum of aggregate insider and broker profits, or

$$\frac{\sqrt{nt\Sigma_i\Sigma_i}}{(1+Q)} \frac{2m}{1+m}$$

By contrast, uninformed losses with no dual trading equals

$$\frac{\sqrt{nt\Sigma_i\Sigma_i}}{(1+Q)}$$

The final result follows directly.

(2) With commissions, net profit for the *i*th insider (with simultaneous dual trading) is given by

$$I_i = \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} \frac{m}{1+m} \frac{Q-m}{Q} - mc,$$

The net profit for the *i*th insider without dual trading is given by

$$I_i = \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} - mc,$$

From above, the net insider profits  $I_i - I_i$  is given by

$$I_i - I_i = \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} \frac{m}{1+m} \frac{Q-m}{Q} - mc, - \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} + mc,$$

The above can be expressed as

$$\begin{aligned} I_i - I_i &= \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} \left[ \frac{m}{1+m} \frac{Q-m}{Q} - 1 \right] + \frac{nW_i}{1+n} \\ &= \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}} \left[ \frac{m}{1+m} \frac{Q-m}{Q} - 1 + \frac{m}{1+m} \frac{n}{1+n} \frac{m+Q}{Q} \right] \\ &= \frac{\sqrt{t\Sigma_i\Sigma_i}}{\sqrt{n(1+Q)}Q(1+m)(1+n)} [mQn - m^2 - Q(1+n)] \end{aligned}$$

For large values of  $m$  and  $Q$  (and, hence, of  $n$  and  $t$ ), the positive term  $mQn$  dominates, and  $I_s > I_u$ . For relatively small values of  $m$  and  $Q$ , the negative terms dominate.

Net uninformed losses with simultaneous dual trading is given by:

$$U_s = \frac{2m}{1+m} \frac{\sqrt{nt\Sigma_u\Sigma_s}}{1+Q} + mc_s$$

Net uninformed losses without dual trading are:

$$U_u = \frac{\sqrt{nt\Sigma_u\Sigma_s}}{1+Q} + mc_s$$

Therefore,

$$\begin{aligned} U_s - U_u &= \frac{\sqrt{nt\Sigma_u\Sigma_s}}{1+Q} \left[ \frac{2m}{1+m} - 1 \right] - \frac{mW_s}{1+n} \\ &= \frac{\sqrt{nt\Sigma_u\Sigma_s}}{(1+Q)Q(1+m)(1+n)} [mQn - m^2 - Q(1+n)] \end{aligned}$$

The expression in square brackets is the same as that for  $(I_s - I_u)$ . Hence, whenever net informed profits increase (decrease) with simultaneous dual trading, uninformed losses are also higher (lower).

### Proof of Corollary 7:

(1) Under consecutive dual trading, net informed profits ( $I_d$ ) is

$$I_d = \frac{\sqrt{t\Sigma_u\Sigma_s}}{\sqrt{n(1+Q)}} + \frac{\sqrt{m}}{1+m} \frac{\sqrt{nt\Sigma_u\Sigma_s}}{\sqrt{Q}\sqrt{1+Q}} \cdot \frac{1}{1+n} - m \left[ k_i + \frac{k_o}{1+n} \right]$$

$$\frac{\partial I_d}{\partial m} = \frac{(1-m)}{2\sqrt{m}(1+m)^2} \cdot \frac{\sqrt{nt\Sigma_u\Sigma_s}}{\sqrt{Q}\sqrt{1+Q}} \cdot \frac{1}{1+n} - \left[ k_i + \frac{k_o}{1+n} \right] < 0 \quad \text{for } m \geq 1$$

(2) Net informed profits with simultaneous dual trading

$$I_s = \frac{\sqrt{t\Sigma_u\Sigma_s}}{\sqrt{nQ(1+Q)}} \cdot \frac{1}{1+n} \left[ \frac{m}{1+m} \{Q(1+2n) - m\} \right] - m \left[ k_i + \frac{k_o}{1+n} \right]$$

Consider the term in the first square bracket above

$$\begin{aligned} \frac{\partial}{\partial m} \left[ \frac{m}{1+m} \{Q(1+2n) - m\} \right] &= \frac{Q(1+2n) - 2m - m^2}{(1+m)^2} \\ &= \frac{(Q-m) + (nQ-m) + (nQ-m^2)}{(1+m)^2} \\ &> 0, \quad \text{since } n \geq Q > m \end{aligned}$$

The second term is decreasing in  $m$  and so  $\partial I_s / \partial m > 0$  only for small values of  $k_1$  and  $\frac{k_0}{1+n}$ . For

$\Sigma_u = \Sigma_v = 256, \tau=1, n=5$  and  $m=4$ , we need  $\left[ k_1 + \frac{k_0}{1+n} \right] < 0.05$  approximately, for  $\partial I_s / \partial m > 0$ .

**Proof of Proposition 3:**

When  $Q \leq m$ , there is no equilibrium with simultaneous dual trading. Informed traders instruct brokers to trade consecutively and, from corollary 7, choose  $m=1$ .

At  $m=1$ , net informed profits with consecutive dual trading are:

$$I_d = \frac{\sqrt{t\Sigma_u\Sigma_v}}{\sqrt{nQ(1+Q)}} \left[ Q + \frac{n\sqrt{Q(1+Q)}}{2(1+n)} \right] - \left[ k_0 + \frac{k_1}{1+n} \right] \quad (3.1)$$

Similarly, net informed profits with simultaneous dual trading (recall that the insiders use all  $m$  brokers) is:

$$I_s = \frac{\sqrt{t\Sigma_u\Sigma_v}}{\sqrt{nQ(1+Q)}} \cdot \frac{m}{1+m} \left[ \frac{Q(1+2n)-m}{1+n} \right] - m \left[ k_0 + \frac{k_1}{1+n} \right] \quad (3.2)$$

For small values of  $m, k_0$  and  $k_1$ , we can ignore the last term in the right hand side of (3.2), and

$$I_d - I_s = \frac{\sqrt{t\Sigma_u\Sigma_v}}{\sqrt{nQ(1+Q)}} \cdot \frac{1}{1+n} \left[ Q(1+n) + \frac{n\sqrt{Q(1+Q)}}{2} - \frac{m}{1+m} Q(1+2n) + \frac{m^2}{1+m} \right] \quad (3.3)$$

$$\frac{\partial(I_d - I_s)}{\partial m} = \frac{\sqrt{t\Sigma_u\Sigma_v}}{\sqrt{nQ(1+Q)}} \cdot \frac{1}{1+n} \left( \frac{1}{1+m} \right)^2 \left[ (m-Q) + (m-Qn) + (m^2 - Qn) \right] < 0$$

since  $n \geq Q > m$ .

Evaluate  $I_s$  at  $m=3$ , and consider the difference in net profits (terms in square brackets only)

$$\begin{aligned} (I_d - I_s)|_{m=3} &= \left[ Q(1+n) + \frac{n\sqrt{Q(1+Q)}}{2} - \frac{3}{4} Q(1+2n) + \frac{9}{4} \right] \\ &= \frac{Q}{4} - \frac{nQ}{2} + \frac{n\sqrt{Q(1+Q)}}{2} + \frac{9}{4} \\ &> 0 \end{aligned}$$

Since  $\frac{\partial(I_d - I_s)}{\partial m} < 0$ ,  $I_d > I_s$  for values of  $m \leq 3$ .

**Proof of Proposition 4:**

Continuing with the proof from proposition 3, evaluate  $I_s$  at  $m=4$ , and consider again the term in square brackets from equation (3.3) in proposition 3 above.

$$I_d - I_s \leq \frac{Q}{5} - \frac{3Qn}{5} + \frac{n\sqrt{Q(1+Q)}}{2} + \frac{16}{5}$$

where the equality holds for  $k_0 = k_1 = 0$ . Let  $t=1$  and  $n=10$ . Then  $I_d - I_s < 0$ . Clearly, if  $t=1$  and  $n=10$ ,  $I_d - I_s < 0$  for  $m > 4$ . More generally, large values of  $m$ ,  $n$  and  $t$  are more likely to make the negative term in the square brackets dominant. In addition, with  $k_0 > 0$  and  $k_1 > 0$ , the term  $m \left[ k_0 + \frac{k_1}{1+n} \right]$  is positive and non-negligible, making  $I_d < I_s$  more likely. By the same token,  $I_d > I_s$  is more likely for small values of  $m, n$  and  $t$ .

### Proof of Corollary 8:

(1) Price informativeness with simultaneous dual trading

$$PI_s = \frac{nt\Sigma_s}{1+Q}$$

Price informativeness with consecutive dual trading

$$PI_d = \frac{nt\Sigma_s}{1+Q} + \frac{nt\Sigma_s}{(1+Q)Q} \cdot \frac{m}{1+m}$$

Thus,  $PI_d > PI_s$ .

(2) Inverse of market depth with simultaneous dual trading

$$\lambda_s = \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \cdot \frac{1}{1+Q} \cdot \left( \frac{1+m}{m} \right)$$

Inverse of market depth with consecutive dual trading

$$\lambda_d = \frac{\sqrt{nt\Sigma_s}}{\sqrt{\Sigma_s}} \cdot \frac{1}{\sqrt{Q(1+Q)}} \cdot \frac{\sqrt{m}}{1+m}$$

It is easy to see from above that  $\lambda_d < \lambda_s$ .

(3)

$$mW_d - W_s = \frac{\sqrt{nt\Sigma_s \Sigma_s}}{Q(1+Q)(1+m)} \left[ Qm + m^2 - \frac{(1+m)}{2} \sqrt{Q(1+Q)} \right]$$

where  $W_d$  is evaluated at  $m=1$ . From above,  $mW_d > W_s$  if and only if

$$4m^2(Q+m)^2 > (1+m)^2 Q(1+Q)$$

$$\text{or if, } Q^2(3m^2 - 2m - 1) + Q(8m^2 - 1 - m^2 - 2m) + 4m^2 > 0$$

For  $m \geq 1$ , each of the terms in parentheses sum to a positive number. Hence,  $mW_d > W_s$ .

By definition,  $c_d < c_s$ .

**Proof of Proposition 5:**

(1) With internalization of order flow and  $n = m^2$ , the inverse of market depth in the simultaneous dual trading model is given by:

$$\begin{aligned}\lambda_i &= \sqrt{\frac{\Sigma_v}{\Sigma_s}} \cdot \frac{(1+m)}{(1+m^2)} \\ &> \sqrt{\frac{\Sigma_v}{\Sigma_s}} \cdot \frac{m}{1+m^2} \\ &= (\text{inverse of}) \text{ market depth with no internalization}\end{aligned}$$

Price informativeness in the simultaneous dual trading model (given  $n = m^2$ ) is:

$$PI_i = \Sigma_v \left( \frac{m^2}{1+m^2} \right)$$

which is the same as that without internalization. The increase in net uninformed losses due to internalization is equal to the increase in net uninformed losses from simultaneous dual trading relative to no dual trading. The increase in net uninformed losses from internalization is given by:

$$\begin{aligned}\Delta U_i &= \frac{\sqrt{\Sigma_v \Sigma_s}}{m^2(1+m^2)} \cdot \frac{m^2}{1+m} \cdot \left( \frac{1}{1+m^2} \right)^2 [m^2 - m^2 - 2] \\ &> 0 \text{ for } m \geq 2\end{aligned}$$

(2) Note that  $m$  is an inverse measure of internalization.

$$\frac{d\lambda_i}{dm} = \frac{\Sigma_v}{\Sigma_s} \cdot \frac{(1-2m)}{(1+m^2)^2} < 0$$

$$\frac{dPI_i}{dm} = \Sigma_v \cdot \frac{2m}{(1+m^2)^2} > 0$$

$$\frac{d\Delta U_i}{dm} > 0$$

$$c_i = k_i + \frac{k_v - W_i}{1+n}$$

where, for  $n = m^2$ ,

$$W_i = \frac{\sqrt{\Sigma_v \Sigma_s}}{1+m^2}, \text{ which is decreasing in } m.$$

**Proof of Proposition 6:**

Consider the consecutive dual trading solution. Let  $sd(\tilde{r})$  denote the standard deviation of  $\tilde{r}$ . The expected trading volume of informed trader  $i$  is

$$E(|x^i|) = \frac{1}{\sqrt{2\pi}} sd(x^i) = \frac{A}{\sqrt{2\pi}} sd(s^i) = \frac{\sqrt{\Sigma_s}}{\sqrt{2\pi}\sqrt{n}}.$$



Therefore, the expected trading volume in period one =  $\frac{n\sqrt{\Sigma_v}}{2\sqrt{2\pi m}} + \frac{\sqrt{\Sigma_v}}{\sqrt{2\pi}} = \frac{\sqrt{\Sigma_v}}{\sqrt{2\pi}}(1 + \sqrt{n})$ .

The expected trading volume per broker =

$$\frac{1}{\sqrt{2\pi}}sd(z) = \frac{1}{\sqrt{2\pi}}[AB_1sd(s) + B_2\sqrt{\Sigma_v}] = \frac{1}{\sqrt{2\pi}}\left[\frac{\sqrt{\Sigma_v}}{\sqrt{m}}\right].$$

Aggregate expected trading volume in period two =  $\frac{m}{\sqrt{2\pi}}\frac{\sqrt{\Sigma_v}}{\sqrt{m}} + \frac{\sqrt{\Sigma_v}}{\sqrt{2\pi}} = \frac{\sqrt{\Sigma_v}}{\sqrt{2\pi}}(1 + \sqrt{m})$

Thus, the aggregate expected trading volume (or the aggregate expected "follow-on" volume) is proportional to the number of "follow-on" traders times the expected noise trader volume given by

$$\frac{\sqrt{\Sigma_v}}{\sqrt{2\pi}}$$

The statement of the proposition follows directly from above.

### Proof of Proposition 7:

Given  $m = [Q(1+Q)^{-1}]$  in the consecutive dual trading equilibrium, we have

$$\lambda_1 = \frac{\lambda_1}{Q(1+m)}$$

$$B_1 = +1$$

$$B_2 = -Q$$

$$\text{Now, } \frac{y_1}{y_2} = \frac{x + u_1}{mx + [u_2 - mQu_1]}$$

$$\frac{p_2}{y_2} = \lambda_1 \left[ \frac{1}{Q(1+m)} + \frac{x + u_1}{m(x - Qu_1) + u_2} \right]$$

$< \lambda_1$  for large  $m$

### Proof of Proposition 8:

Consider the Holden-Subrahmanyam (1992) model, specialized to two-periods and  $\Delta t_n \equiv 1$ . From propositions 1 and 2 in their paper, the period two depth and price informativeness are, respectively:

$$\lambda_{2,0} = \frac{\sqrt{n\Sigma_v}}{\sqrt{\Sigma_v}} \cdot \frac{1}{1+n} \cdot \sqrt{\frac{1}{1+n-2nq}}$$

$$\Sigma_{2,0} = \frac{\Sigma_v}{1+n} \cdot \frac{1}{1+n-2nq}$$

where  $q$  is defined by (18) in the text and  $\Sigma_{2,0} = \text{Var}(v|y_1, y_2)$ . From our consecutive dual trading model, the period two depth and price informativeness are:

$$\lambda_2 = \frac{\sqrt{\Sigma_2}}{\sqrt{\Sigma_1}} \cdot \frac{1}{\sqrt{1+n}} \cdot \frac{\sqrt{m}}{1+m}$$

$$\Sigma_2 = \frac{\Sigma_1}{1+n} \cdot \frac{1}{1+m}$$

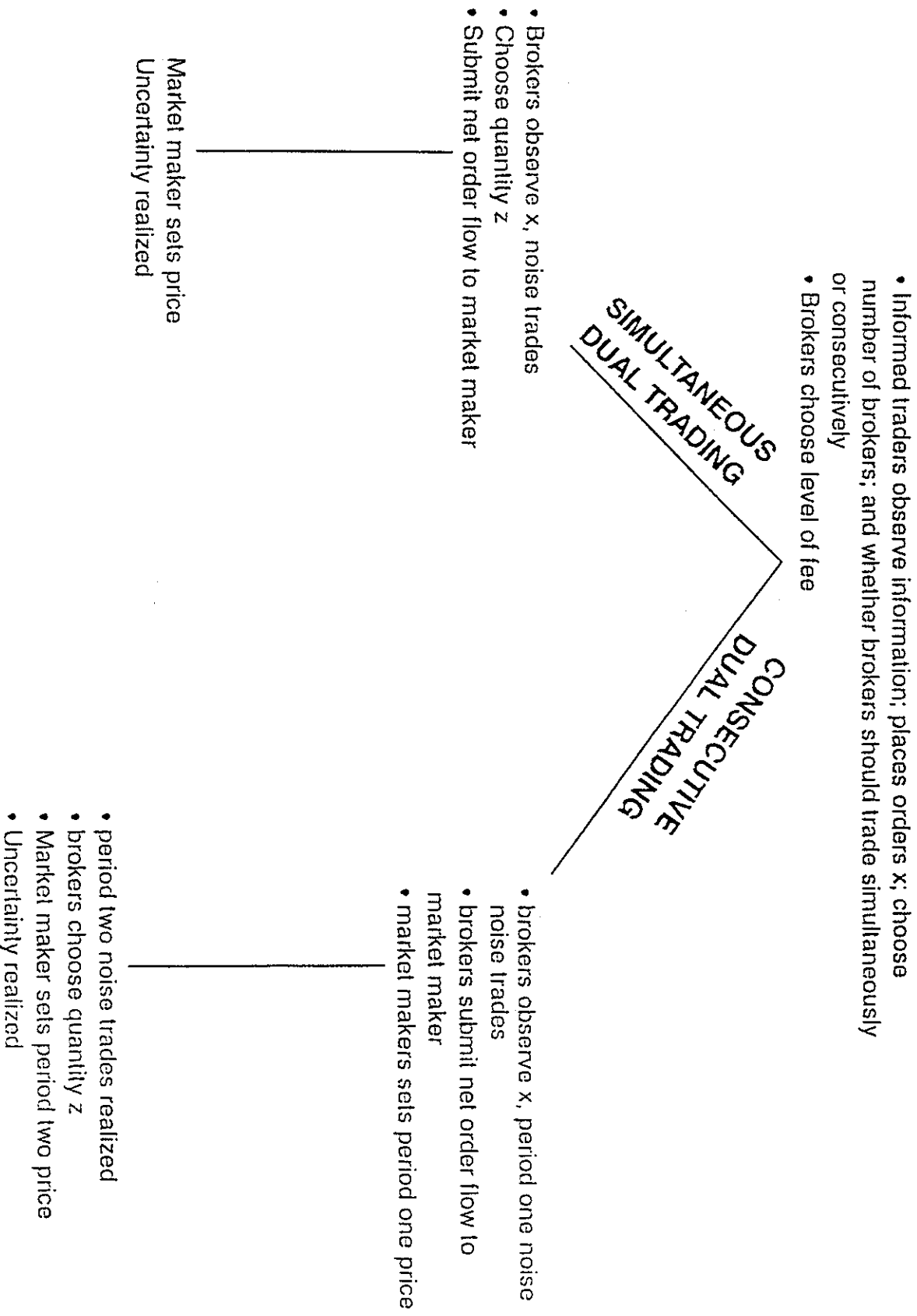
Comparing,  $\Sigma_{2,o} < \Sigma_2$  iff  $n(1-2q) > m$ .

Since,  $\Sigma_{2,o}$  and  $\Sigma_2$  are the inverse of price informativeness, if  $n(1-2q) > m$ , then price informativeness is higher with off-board trading. Further,

$$\begin{aligned} \lambda_{2,o} &= \sqrt{\frac{\Sigma_1}{\Sigma_{2,o}}} \cdot \frac{1}{\sqrt{1+n}} \cdot \left[ \frac{\sqrt{n}}{\sqrt{1+n}} \cdot \frac{1}{\sqrt{1+n-2nq}} \right] \\ &> \sqrt{\frac{\Sigma_1}{\Sigma_2}} \cdot \frac{1}{\sqrt{1+n}} \cdot \left[ \frac{\sqrt{n}}{1+n} \right] \\ &> \sqrt{\frac{\Sigma_1}{\Sigma_2}} \cdot \frac{1}{\sqrt{1+n}} \cdot \left[ \frac{\sqrt{m}}{1+n} \right] \quad (\text{since } n > m) \\ &= \lambda_2 \end{aligned}$$

If  $n(1-2q) < m$ , then  $\Sigma_{2,o} > \Sigma_2$ , but if  $n < m$ , then it is possible that  $\lambda_{2,o} < \lambda_2$ .

Figure 1: Sequence of moves



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