RESEARCH PAPER

WHY DO INTEREST RATES PREDICT MACRO OUTCOMES? A UNIFIED THEORY OF INFLATION, OUTPUT, INTEREST AND POLICY

by Arturo Estrella

Federal Reserve Bank of New York
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Abstract

Why Do Interest Rates Predict Macro Outcomes?
A Unified Theory of Inflation, Output, Interest and Policy

Several articles published in the 1990s have identified empirical relationships between the term structure of real and nominal interest rates, on one hand, and future real output and inflation, on the other. Among these are Mishkin (1990a), Estrella and Hardouvelis (1991), Bernanke and Blinder (1992) and Fuhrer and Moore (1995). These articles demonstrate the existence of empirical predictive relationships, but the underlying economic reasons for the empirical regularities remain at least partly as puzzles. This paper presents a theoretical rational expectations model that shows how monetary policy is likely to be a key determinant of these empirical regularities.

1. Introduction

A series of seemingly unrelated articles published in major journals since 1990 has identified empirical relationships between the term structure of interest rates and macroeconomic variables. Although superficially unconnected, these articles share two basic features. First, each paper demonstrates that post-war U.S. data exhibit some empirical predictive relationship between a variable obtained from the term structure of interest rates and either real output or inflation. Second, although there is some discussion of the economics underlying each empirical regularity, the exact reasons for the results remain at least partly a puzzle.

Consider a few examples. Mishkin (1990a) shows that a term structure spread -- the difference between yields on instruments of different maturities -- tends to predict future changes in inflation, especially for longer maturities and predictive horizons. Similarly, Estrella and Hardouvelis (1991) find that the term structure spread is a good predictor of future economic activity (real GNP, recessions, etc.). Bernanke and Blinder (1992) find that the Federal funds rate predicts real output better than the competing alternatives they consider. Finally, Fuhrer and Moore (1995) recast the Bernanke-Blinder results by showing that the long-term real rate, which is generally viewed as an important determinant of future output, moves very closely with the short-term nominal rate.

All of the foregoing papers focus on empirical results. In each, there is some attempt to identify the economic causes of the phenomena observed and to establish a connection with monetary policy, and Fuhrer and Moore provide a suggestive formal model. Nevertheless, the general applicability of the results and the derivation of theoretical implications are limited by the fact all the models are numerical.

This paper develops a theoretical rational expectations model that can be used to explore the sources of the empirical regularities in some generality. The model provides a unified framework from which exact closed-form relationships and policy implications may be obtained. For example, we can address the following questions. Are the stylized facts listed above structural, or do they depend on the monetary policy regime? Can we characterize the forms of monetary policy regimes that are consistent with the individual stylized facts? Are those forms sound and reasonable and do they lead to stable solutions? Are the models that explain the individual stylized facts compatible with one another? Are any of these forms compatible with explicit inflation targeting?

In addition, we can use the model to examine policy questions. For instance, what restrictions does stability impose on the range of policy choices? In particular, what role should output fluctuations play in a policy rule if stability is a concern?

The paper demonstrates that there are no stable monetary policy specifications under which any one of the empirical regularities holds exactly, although some admissible specifications provide clear simultaneous support for all the regularities considered. At the other extreme, there are specifications, such as inflation targeting, that are inconsistent with all of the observed regularities. It is also shown that the degree of persistence in interest rates levels and the policy responsiveness to output fluctuations are important for stability. If interest levels are very persistent, output fluctuations cannot be ignored.

2. A quick review of the stylized facts

Before proceeding with the construction of the model, we review the stylized facts that motivate the model. The facts are drawn from several recent articles that focus on predicting either inflation or real activity using information from the yield curve. These articles focus primarily on the empirical issues rather than on modeling the theoretical relationships.

Mishkin (1990a) uses the difference between an n-month interest rate and an m-month interest rate to predict the difference between average inflation rates over n-months and m-months into the future, where $n,m \leq 12$. This equation is most successful when the values of n and m are large, and in fact Mishkin (1990b) obtains better results with maturities from one to five years. In these papers, the results are traced to a combination of the Fisher equation and the expectations hypothesis of the term structure of interest rates. The strength of the results, however, is seen to hinge on the nature of the variability of the unobserved real interest rates and on their relationship with the observed variables. Schich (1996) obtains similar results for Germany.

Subsequent work by Estrella and Mishkin (1995) abstracted from the constraints of the Fisher equation and the expectations hypothesis and asked simply whether the spread between ten-year and three-month government rates could predict average inflation over horizons ranging up to five years in four European countries and in the United States. In several cases good results were obtained with fairly long horizons -- of at least two years -- which is roughly consistent with the timing found in Mishkin's earlier work.

Estrella and Hardouvelis (1991) use the difference between ten-year and three-month U.S. Treasury rates to predict real variables such as real GNP and NBER-dated recessions. They obtain significant results, particularly for horizons between one and seven quarters. The authors attribute some of the predictive power to the control of the monetary authority over the short end of the yield curve and to private sector expectations of how the influence of current interest rates will play out over the predictive horizon. In a similar equation, Plosser and Rouwenhorst (1994) use U.S., U.K. and German data to match the interest rate maturities and predictive horizons more closely. They find that in the United States and Germany, the long end of the yield curve (two to five years) has significant information about output growth over the corresponding horizon above and beyond the information contained in the short end.

Like the foregoing papers, Bernanke and Blinder (1992) find that term structure spreads are useful in predicting real output. Their main focus, however, is on the strong explanatory power of the Federal funds rate in the context of vector autoregressions that include real output. The result is fairly robust, although it holds most strongly in the pre-1980 period when monetary policy is seen as acting primarily through short-term interest rates. This result may appear somewhat puzzling since it is widely assumed that it is the long-term real rate that has the most impact on aggregate demand. The paper suggests that the banking sector plays a key role in monetary policy and that factors other than long-term interest rates may contribute to the transmission of monetary policy.[1]

Fuhrer and Moore (1995) recast the Bernanke Blinder results purely in terms of interest rates. They argue that if it is the long-term real rate that determines future output and if the short-term nominal rate is highly correlated with the long-term real rate, the short-term rate will tend to be an accurate predictor of future output. Fuhrer and Moore present empirical evidence of the close relationship between the two rates and argue that this relationship is dependent on the conduct of monetary policy. They postulate a monetary policy function in the spirit of Taylor

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[1]For a discussion of the "credit view" of the transmission of monetary policy, see Bernanke (1993).

(1993), whereby the Fed responds to discrepancies between current and desired inflation and to deviations of output from potential. By showing that the similarity between the long- and short-term rates is obtained for small values of the reaction parameters, but not for large values, they provide an explicit connection between monetary policy and the Bernanke-Blinder regularity.

The present paper builds on the Fuhrer-Moore analysis by constructing an analogous, analytically tractable model from which explicit relationships among policy, interest rates and macro variables are derived. In addition, this paper shows that the other empirical regularities, which are more obviously related to the term structure, are in fact also related to the Bernanke-Blinder and Fuhrer-Moore results. A final strand of research that is nested in the present model is the inflation targeting approach of Svensson (forthcoming). The implications of inflation targeting in this case are shown to be at odds with the empirical evidence in the other papers.

The foregoing empirical evidence is summarized in table 1.

Table 1

Reference	Predictor: interest rate or spread	Variable forecasted
Estrella-Hardouvelis (1991)	Long rate minus short rate	Real output
Mishkin (1990a)	Long rate minus short rate	Change in inflation
Estrella-Mishkin (1995)	Long rate minus short rate	Average future inflation
Bernanke-Blinder (1992)	Short rate	Real output
Fuhrer-Moore (1995)	Short rate	Real long-term rate
Svensson (forthcoming)	Short rate (instrument)	Zero expected inflation

3. The model

In order to address simultaneously all the stylized facts, the model contains three main blocks. First, there is a simple dynamic macroeconomic model that relates inflation and output to interest rates with lags that attempt to capture the observed timing of these relationships. Second, there is a model of the monetary policy process, which comprises a reaction function as well as a source of active monetary shocks. Third, a model of the term structure of interest rates connects rates of different maturities and distinguishes real from nominal rates.

The model extends the analysis of Fuhrer-Moore (1995) and Svensson (forthcoming) in several ways. In contrast to Fuhrer-Moore, who rely on numerical results, this model is solved analytically.[2] In contrast to Svensson, the model has a term structure of interest rates, and it is the long-term real rate that determines the level of output. Finally, in contrast to both Fuhrer-Moore and Svensson, the present model has a richer specification for monetary policy that nests the Taylor (1993) rule, the Fuhrer-Moore rule, and Svensson's optimal inflation-targeting rule.

In order to keep the model tractable, short- and long-term interest rates correspond to one- and two-periods, respectively. It is clear that this simplifying assumption does not limit the generality of the results in any substantive way and that it is conceptually easily to relax, but only at the practical cost of substantial computational burden. The basic model is composed of the following equations.

Aggregate supply:

$$\pi_{t+1} = \pi_t + a y_t + \epsilon_{t+1} \tag{1}$$

Aggregate demand:

$$y_{t+1} = b_1 y_t - b_2 \rho_t + \eta_{t+1} \tag{2}$$

Monetary policy reaction function:

$$r_{t+1} = g_1 r_t + g_2 \pi_{t+1} + g_3 y_{t+1} + (1 - g_1 - g_2) z_{t+1} \tag{3}$$

Monetary shock or action function:

$$z_{t+1} = z_t + v_{t+1} \tag{4}$$

Fisher equation:

$$R_t = \rho_t + \frac{1}{2}\left(E_t \pi_{t+1} + E_t \pi_{t+2}\right) \tag{5}$$

Expectations hypothesis:

$$R_t = \frac{1}{2}\left(r_t + E_t r_{t+1}\right) \tag{6}$$

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[2]Fuhrer and Moore (1992) construct a theoretical differential equations model that has some features in common with the model of this paper, as well as similar objectives. Because their model is relatively large, they derive results numerically, based on calibrated parameter values.

where

π_t = inflation in period t

y_t = output gap in period t

r_t = short-term (1 period) nominal interest rate in period t

R_t = long-term (2 period) nominal interest rate in period t

ρ_t = long-term (2 period) real interest rate in period t

z_t = inflation target in period t

ϵ_t, η_t, v_t = i.i.d. shocks (independent over time, but may be contemporaneously correlated)

E_t = expectation operator based on period t information.

The parameters are assumed to have values in the following ranges: $0<a<1$, $0<b_1<1$, $0\le b_2\le 1$, $0\le g_1\le 1$, $g_2, g_3\ge 0$. On occasion, numerical illustrations will be given based on specific parameter values. However, the foregoing assumptions are sufficient to obtain all the theoretical results of the paper.

Equations (1) and (2) constitute a fairly standard skeletal macro model. They are virtually the same as those in Svensson (forthcoming) with one key difference: equation (2) contains the long-term -- as opposed to the one-period -- real interest rate. The model can also be construed as an analogue of Fuhrer-Moore (1995) in which the aggregate supply component has been greatly simplified. Equation (1) is a NAIRU or Phillips curve relationship representing aggregate supply and equation (2) is essentially an IS curve representing aggregate demand.[3]

Equations (3) and (4) comprise the monetary policy sector and together nest a number of specifications suggested in the literature. Equation (3) is a generalization of the Taylor (1993)

[3]Equations (1) and (2) do not contain endogenous right-hand-side variables explicitly, but the model is sufficiently general to accommodate such relationships since the variance-covariance structure of the disturbances has not been specified. The lag structure is an important determinant of the time horizons of the forecasting relationships in the model, but not of the reliability of those relationships, which also depend on policy. At least some of the principal results of this paper seem to be robust to the explicit inclusion of expectations in the macro specification and to the nature of the rational expectations solution. For example, Kerr and King (1996) examine a model consisting of an expectations-based IS equation and a policy rule based on the short-term rate. Using a forward rational expectations solution, Kerr and King obtain restrictions with regard to the inertia of the policy rule that resemble those derived later in this section.

reaction function that contains three policy parameters. Some insight may be gained by rewriting equation (3) as

$$r_{t+1} = g_1 r_t + (1-g_1)\pi_{t+1} + (g_1+g_2-1)(\pi_{t+1}-z_{t+1}) + g_3 y_{t+1}. \tag{3'}$$

The first two terms determine the level of inertia or persistence in the short-term rate. They define the extent to which the short rate is tied to its own previous level or to current inflation, and (3′) simplifies to the Taylor rule,

$$r_{t+1} = \pi_{t+1} + (g_2-1)(\pi_{t+1}-z_{t+1}) + g_3 y_{t+1},$$

when $g_1 = 0$ and to the Fuhrer-Moore version of the Taylor rule,

$$\Delta r_{t+1} = g_2(\pi_{t+1}-z_{t+1}) + g_3 y_{t+1},$$

when $g_1 = 1$. It will be shown that the value of g_1 has nontrivial implications for the stability of the system and for the feasible values of the other parameters.

The other two parameters in the equation, g_2 and g_3, represent the policy reactions per unit of deviation of inflation from the target level and of output from its potential level, respectively. The coefficient of z_t in equation (3) is restricted to make the equation homogeneous in the interest, inflation and target inflation rates, which is convenient in specifying the equilibrium of the system.

Equation (4) describes the movement of the inflation target as a random walk. The shocks v_t are taken to occur very infrequently as, for example, with the appointment of Chairman Volcker in 1979 or with a hypothetical move to a zero inflation target. For most of the analysis, results will be derived conditionally on the value of this target level, which will nevertheless play an important role.

Equations (5) and (6) are the standard Fisher and expectations hypothesis equations. Empirical analysis has cast some doubt on the exact validity of these conceptual relationships. Negative results have been attributed to three factors: time-varying risk premia, the complexities of movements in the real rate, and the influences of monetary policy. Although it might be desirable in principle to model term premia, the points made in this paper are largely independent of them.[4] The other two complicating factors are explicitly modeled in this paper.

The equilibrium point of the model is simply defined. Given a fixed target or equilibrium level of inflation z*, the equilibrium values of the other variables in the system of equations (1)-(6) are $y=\rho=0$ and $R=r=\pi=z*$. In the absence of shocks, these values would be self-perpetuating. However, convergence to these values from any other starting point is not guaranteed and depends in particular on the values of the parameters of the monetary policy reaction function.[5]

4. Basic solution of the model

A solution to the model of equations (1)-(6) is an expression for the stationary process that determines the four state variables π_t, y_t, r_t, z_t in terms of the shocks ϵ_t, η_t, ν_t.[6] The form of the model makes it convenient to express this process as a vector autoregression.

The system is solved by deriving the rational expectations of equations (1)-(6) and solving the resulting linear system for those expectations. Equation (1) implies both

$$E_t\pi_{t+1} = \pi_t + ay_t \tag{7}$$

and

$$E_t\pi_{t+2} = E_t\pi_{t+1} + aE_t y_{t+1}. \tag{8}$$

From equations (2)-(4), we obtain

$$E_t y_{t+1} = b_1 y_t - b_2\rho_t, \tag{9}$$

$$E_t r_{t+1} = g_1 r_t + g_2 E_t\pi_{t+1} + g_3 E_t y_{t+1} + (1-g_1-g_2)E_t z_{t+1} \text{ and} \tag{10}$$

$$E_t z_{t+1} = z_t. \tag{11}$$

Finally, equations (5) and (6) together imply

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[4]Dotsey and Otrok (1995) examine the interaction between policy rules, term premia and rejections of the expectations hypothesis.

[5]The fact that the equilibrium real interest rate is zero may seem artificial, but this rate may be expressed in deviations from a non-zero equilibrium rate with no loss of generality.

[6]See Whiteman (1983) for a general discussion of solutions to rational expectations models.

$$\rho_t = \frac{1}{2}\left(r_t + E_t r_{t+1} - E_t\pi_{t+1} - E_t\pi_{t+2}\right). \tag{12}$$

The six equations (7)-(12) may be solved for the five expectations and ρ_t in terms of the four contemporaneous state variables π_t, y_t, r_t, and z_t.[7]

The solution to the system is given by equations (7) and (11), which represent $E_t\pi_{t+1}$ and $E_t z_{t+1}$ directly, and by

$$\rho_t = \frac{(g_2-2)\pi_t + [(g_2-2)a + (g_3-a)b_1]y_t + (1+g_1)r_t + (1-g_1-g_2)z_t}{2 + (g_3-a)b_2} \tag{13}$$

$$E_t y_{t+1} = \frac{(2-g_2)b_2\pi_t + [(2-g_2)b_2 a + 2b_1]y_t - (g_1+1)b_2 r_t + (g_1+g_2-1)b_2 z_t}{2+(g_3-a)} \tag{14}$$

$$E_t r_{t+1} = \frac{\begin{array}{c}[g_2(2-ab_2)+2g_3b_2]\pi_t + [ag_2(2-ab_2)+2g_3(ab_2+b_1)]y_t \\ +[g_1(2-ab_2)-g_3b_2]r_t + (ab_2-2)(g_1+g_2-1)z_t\end{array}}{2+(g_3-a)b_2} \tag{15}$$

$$E_t\pi_{t+2} = \frac{\begin{array}{c}[2+(a+g_3-ag_2)b_2]\pi_t + a[2+(a+g_3-ag_2)b_2 + 2b_1]y_t \\ -ab_2(1+g_1)r_t + ab_2(g_1+g_2-1)z_t\end{array}}{2+(g_3-a)b_2} \tag{16}$$

In the above solution, note in particular that the period t expectation of the period t+1 value of each of the four state variables is expressed in terms of the period t values of those variables. Thus, if $\tilde{s}_t = (\pi_t, y_t, r_t, z_t)'$ is the vector of state variables, then $E_t\tilde{s}_{t+1} = T\tilde{s}_t$, where T is

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[7]The condition for the system to be solvable is that the six-by-six matrix of coefficients of the unknown expectations must be invertible. The determinant of the matrix is positive if $g_3 > a - 2/b_2$, which follows from the assumptions stated earlier for the parameter ranges.

the transition matrix implied by the equations (7), (11), (14) and (15). Also, referring to the basic equations (1)-(6) to compare the differences between expected and realized values of the state variables, we may write the transition equation

$$\bar{s}_{t+1} = T\bar{s}_t + \bar{\xi}_{t+1}$$

where $\bar{\xi}_t = (\epsilon_t, \eta_t, g_2\epsilon_t + g_3\eta_t + (1-g_1-g_2)v_t, v_t)'$.

As noted earlier, some of the analysis will be performed conditioning on the value of z_t. Therefore, it is convenient to have a three factor version of the transition equation in which the value of z_t is taken as given. Partition the matrix T as

$$T = \begin{bmatrix} T_{11} & T_{12} \\ 0 & 1 \end{bmatrix}$$

Conditional on $z_t = z*$, the system reduces to a three factor specification of the form

$$s_{t+1} = T_{11}s_t + T_{12}z* + \xi_{t+1} \tag{17}$$

with $s_t = (\pi_t, y_t, r_t)'$ and $\xi_t = (\epsilon_t, \eta_t, g_2\epsilon_t + g_3\eta_t)'$.

Figure 1 shows impulse responses of the three state variables to supply, demand and monetary shocks using an illustrative set of parameter values. The upper panel corresponds to a supply shock ($\epsilon_0=1$). The interest rate rises in response to inflation and causes a slowdown in output. The second panel depicts a demand shock ($\eta_0=1$). Again the interest rate rises in response to higher output and to the consequent rise in inflation. The third panel corresponds to a lowering of the inflation target ($v_0=-1$). The interest rate is raised to obtain the lower level of inflation and, in the process, creates a temporary slowdown in output.

The point of figure 1 is that the cyclical movements produced by the model are realistic, even though the model is too parsimoniously parameterized to serve as an accurate empirical model. In particular, the responses to a monetary shock in the third panel may be compared with actual annual data for the nine years following the "Volcker shock" of late 1979, as shown in figure 2. Note that the pattern of rising and then falling interest rates is similar in both figures and that the temporary drop in output and the permanent drop in inflation in figure 1 mimic the consequences of the monetary tightening in figure 2.

A necessary and sufficient condition for the dynamic model to be stable is that all the eigenvalues of the matrix T have modulus less than 1. In the four variable version of the model in which z_t is a random walk, one of the eigenvalues is always 1. However, this unit root problem is benign if we assume that the value of z changes infrequently and that for most purposes we may look at the results conditionally on $z_t = z*$, a constant. In that case, the necessary and sufficient condition for convergence is that the eigenvalues of T_{11} have modulus less than 1. Given the stated ranges for the values of the parameters, an equivalent set of necessary and sufficient conditions may be stated in terms of the basic parameters:

$$1-g_1 < g_2 < B_2 \text{ and } g_3 > B_3,$$

where

$$B_2 = \frac{\begin{gathered}(4b_1-4b_1^2-2ab_1b_2)g_1^2+((-ab_2^2-2b_1b_2-2b_2)g_3-4+4b_1^2+a^2b_2^2+4ab_1b_2)g_1 \\ +2g_3^2b_2^2+(6b_2-2b_1b_2-3ab_2^2)g_3+4-4b_1-4ab_2+a^2b_2^2+2ab_1b_2\end{gathered}}{(2+g_3b_2-ab_2+2b_1g_1)ab_2}$$

and $B_3 = a-(2/b_2)(1-b_1g_1)$.[8]

Using representative values for the parameters of the macro block of the model, Figure 3 shows the bounds for the inflation response parameter for all values of the persistence parameter and for three key values of the output response parameter. The figure illustrates the following results, which are true more generally for all admissible values of the parameters.

First, if $g_3 \geq a$, then for all $0 \leq g_1 < 1$, $g3 > B_3$ and $B_2 > 1-g_1$. That is, if the output response parameter is not less than the aggregate supply parameter, then for each $g_1 < 1$ (less than full persistence in the short rate) there is a range of values of the inflation response parameter g_2 that

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[8]The stability conditions were derived by applying the Schur-Cohn theorem to the characteristic polynomial of T_{11}. See Barnett (1990).

produces stable solutions. When $g_3 = a$, the B_2 curve always passes through the point (1,0) in the graph.

Second, if $g_3 > a$, then even for $g_1 = 1$, $g3 > B_3$ and $B_2 > 0 = 1 - g_1$. That is, if the output response parameter is strictly greater than the aggregate supply parameter, there are always stable choices of the inflation response parameter, even if the short rate is fully persistent.

Third, if $g_3 \le a$, then for $g_1 = 1$ there are no stable solutions, since $B_2 < 0$. That is, if the output response parameter is less than the aggregate supply parameter and the short rate if fully persistent, there are no stable solutions regardless of the inflation response.

Fourth, if $ab_2 \ge 1 - b_1$ (not illustrated), there are no stable solutions when there is no response to output ($g_3 = 0$), regardless of the value of the persistence parameter g_1.

To summarize the foregoing, if the policy response to output is at least commensurate with the aggregate supply parameter, which is typically small but positive, there will be a range of choices for the response to inflation that lead to stability, although that range is bounded above and below. In contrast, if the level of inertia in the setting of the short rate is very high and if there is no policy reaction to output, the system is likely to be unstable regardless of the policy response to inflation.[9]

In figure 3, we see that the upper bound for g_2 increases with the value of g_3. This is true in general and, if $g_2 = g_3$, the two parameters are unbounded above, although they are subject to a second lower bound. This fact is illustrated in figure 4, which presents the usual lower bound for g_2 as well as the additional minimum constraint to which it is subject if the two response parameters are equal.

To close this section, we look at the issue of oscillatory behavior in the model. Cycle-like behavior is observed in the three variable system if the eigenvalues of T_{11} have non-zero imaginary parts or, alternatively, if the discriminant of the characteristic polynomial of T_{11} is negative. The roots of the discriminant may be solved for the values of g_2 that constitute the

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[9]We may also note that the model requires some degree of policy reaction or interest rate responsiveness to current conditions for stability. Purely passive policies such as r_t=*constant* ($g_2 = g_3 = 0$, $g_1 = 0$ *or* 1) are unstable. However, a policy or mechanical response of validating current inflation, $r_t = \pi_t$ ($g_1 = g_3 = 0$, $g_2 = 1$), is arbitrarily close to an infinity of stable solutions in the three-dimensional space defined by the values of the g parameters.

boundary between oscillatory and non-oscillatory solutions to the system.[10] These regions are illustrated in figure 5.

5. An alternative solution in terms of the term structure spread

The solution of the model in section 4 produced an expression for the long-term real interest rate in terms of the state variables. However, some of the empirical regularities of section 2 involve forecasts of inflation and real activity using the term structure spread, and neither the spread nor the nominal long-term rate appear explicitly in the solution. Thus, in order to link the expectations of the state variables with the term structure, it is necessary to solve the model in a different way. Using equation (5) (the Fisher equation), the expectational system (7)-(12) may be restated in terms of the long-term nominal rate R_t instead of the long-term real rate ρ_t. There is then a unique triangular solution such that $R_t = f_1(\tilde{s}_t)$, $E_t y_{t+1} = f_2(R_t, \tilde{s}_t)$, $E_t \pi_{t+2} = f_3(E_t y_{t+1}, R_t, \tilde{s}_t)$, etc.[11] This solution provides the desired relationships between the expectations and the term structure, from which the following equations are obtained.

$$E_t y_{t+1} = \frac{2}{g_3}(R_t - r_t) + \frac{1-g_1}{g_3}(r_t - z_t) + \frac{g_2}{g_3}(z_t - \pi_t - a y_t) \tag{18}$$

$$\frac{1}{2} E_t \Delta \pi_{t+2} = \frac{a}{g_3}(R_t - r_t) + \frac{(1-g_1)a}{2g_3}(r_t - z_t) + \frac{a g_2}{2 g_3}(z_t - \pi_t - a y_t) \tag{19}$$

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[10]The classic theorem of N.H. Abel from Galois theory implies that this equation, which is of degree 4 in g_2, is of maximum degree for a solution by radicals to exist. See Herstein (1964).

[11]This solution is obtained by stating the system in matrix form with the expectations and R_t in reverse order from the listing in the text. If $x_t = (\ldots, E_t \pi_{t+2}, E_t y_{t+1}, R_t)'$, write the system as $A x_t = B s_t$. Gaussian elimination is applied to A to produce a matrix H=MA, where M is composed of elementary operations and H is upper triangular. Then $H x_t = M B s_t$. See Barnett (1990).

$$E_t \Pi_t = \frac{a}{g_3}(R_t - r_t) + \frac{(1-g_1)a}{2g_3}(r_t - z_t) + \left(1 - \frac{ag_2}{2g_3}\right)(\pi_t + ay_t) + \frac{ag_2}{2g_3} z_t \tag{20}$$

In the above,

$$\Pi_t = \tfrac{1}{2}\left(E_t \pi_{t+1} + E_t \pi_{t+2}\right)$$

is average future expected inflation, which is examined by Estrella and Mishkin (1995), and

$$\tfrac{1}{2} E_t \Delta \pi_{t+2} = \Pi_t - E_t \pi_{t+1}$$

is the expected future change in inflation, which is examined by Mishkin (1990a).

Note that R_t enters with a positive sign in the full equilibrium expectation of output, even though its sign is negative in the partial equilibrium expectation of equation (9). Note also that equation (8) may be written $E_t \Delta \pi_{t+2} = a E_t y_{t+1}$. This relationship implies that the results in Mishkin (1990a) regarding inflation acceleration and those in Estrella-Hardouvelis (1991) regarding real output are closely related.

Using equations (18)-(20) and equation (13) for the real long-term rate, we may now proceed to establish direct connections between the empirical regularities of section 2 and the policy parameters of the model.

6. Policy parameters and the special cases

By selecting appropriate values of $g = (g_1, g_2, g_3)$, it is possible to obtain special cases in which the stylized facts of section 2 hold exactly. The analysis of these "extreme" cases helps clarify the influence of monetary policy on the observable phenomena and answers the question whether the special cases may all hold simultaneously. As it turns out, we find that none of the cases are likely to hold exactly, but that a variant of one of the cases has features that coincide with each of the stylized facts.

We examine six particular cases, which are summarized in table 2 and in figure 6. Table 2 restates the stylized facts from table 1 of section 2 in terms of the notation of this paper. In each row, the reference cited has established a predictive relationship between the interest rate or spread noted and the corresponding forecasted variable. Note that M is a special case of EH and that BB and FM are special cases of EM for which, in both cases, $g_3 = a$. In the following analysis of all the special cases, we assume that $g_3 = a$ for simplicity, although cases EH and EM can hold more generally. With this simplifying assumption, we are left with three distinct cases.

Table 2

Special case	Reference	Rate or rate spread	Forecast	g_1	g_2	g_3
EH	Estrella-Hardouvelis (1991)	$R_t - r_t$	$E_t y_{t+1}$	1	0	$g_3 > 0$
M	Mishkin (1990a)	$R_t - r_t$	$\frac{1}{2} E_t \Delta \pi_{t+2}$	1	0	a
EM	Estrella-Mishkin (1995)	$R_t - r_t$	$E_t \Pi_t$	1	$\frac{2g_3}{a}$	$g_3 > 0$
BB	Bernanke-Blinder (1992)	r_t	$E_t y_{t+1}$	1	2	a
FM	Fuhrer-Moore (1995)	r_t	ρ_t	1	2	a
S	Svensson (forthcoming)	r_t	$E_t \pi_{t+2} = 0$	0	$1 + \frac{2(1+b_1)}{ab_2}$	$a + \frac{2(1+2b_1)}{b_2}$

Case EH-M $(g_1 = 1, g_2 = 0, g_3 = a)$

This case is strictly unstable, but there are stable cases arbitrarily close to it in the space defined by the values of g. From equations (18)-(20) and (13), we see that

$$E_t y_{t+1} = \frac{2}{a}(R_t - r_t) \qquad \text{(EH)}$$

$$\frac{1}{2} E_t \Delta \pi_{t+2} = R_t - r_t \qquad \text{(M)}$$

$$E_t \Pi_t = R_t - r_t + \pi_t + a y_t$$

and

$$\rho_t = r_t - \pi_t - a y_t .$$

In this case, the term structure spread $R_t - r_t$ is the optimal predictor of the real output gap one period later and of the change in inflation two periods ahead. These are strong forms of the regression results in Estrella-Hardouvelis (1991) and Mishkin (1990a,b). The intuition is that if the policy rule is concerned with output only, interest rates will affect future output and will be affected only by future output. Therefore, the term structure of interest rates contains the best information about future output and also about the change in inflation, which is closely related in the model. The other expectations are not similarly simplified. In particular, the long-term real rate depends strongly on both inflation and output, contradicting the Fuhrer-Moore (1995) results.

Case BB-FM-EM ($g_1 = 1, g_2 = 2, g_3 = a$) and Case BB′-FM′-EM′ ($g_1 < g_1^*, g_2 = 2, g_3 = a$)

The "true" case BB with $g_1 = 1$ is unstable. However, there is generally a value $g_1^* < 1$ for which any smaller value corresponds to a stable case BB′ whose properties are not too dissimilar from those of the strictly defined case BB, especially with regard to the real long-term rate.[12] To accommodate both BB and BB′, g_1 is left unspecified in the following expressions.

$$E_t y_{t+1} = \frac{2}{a}(R_t - r_t) + \frac{2}{a}(z_t - \pi_t - a y_t) + \frac{1-g_1}{a}(r_t - z_t)$$

[12]See the example in figure 6. There is no such g_1^* only if $a b_2 \geq 2(1-b_1)$, which seems unlikely unless b_1 is very close to one.

$$\frac{1}{2}E_t\Delta\pi_{t+2} = R_t - r_t + z_t - \pi_t - ay_t + \frac{1-g_1}{2}(r_t - z_t)$$

$$E_t\Pi_t = R_t - r_t + z_t + \frac{1-g_1}{2}(r_t - z_t) \tag{EM}$$

$$\rho_t = \left(1 - \frac{1-g_1}{2}\right)(r_t - z_t) \tag{FM}$$

$$y_t = b_1 y_{t-1} - b_2\left(1 - \frac{1-g_1}{2}\right)(r_{t-1} - z_{t-1}) + \eta_t \tag{BB}$$

If $g_1 = 1$ were feasible, all the terms involving $1-g_1$ would be zero. In particular, the term structure spread would be essentially the optimal predictor of long-term average inflation Π_t and the long-term real rate would move one for one with the short-term nominal rate. These are the strong forms of Estrella-Mishkin (1995) and Fuhrer-Moore (1995), respectively. However, a value $g_1<1$ is required for stability. In that case, equation (EM) becomes more complicated, but the relationships (FM) and (BB) are qualitatively unchanged in that the short-term nominal rate is perfectly correlated with the long-term real rate.

Case S ($g_1 = 0,\ g_2 = 1 + \frac{2(1+b_1)}{ab_2},\ g_3 = a + \frac{2(1+2b_1)}{b_2}$)

This case represents a solution to the policy optimization problem of minimizing a weighted sum of expected squared deviations of inflation from the target level. It is obtained by the choice of policy parameters such that

$$\min_{\{g_1, g_2, g_3\}} \quad E_t \sum_{i=1}^{\infty} \delta^i \left(\pi_{t+i} - z*\right)^2$$

where δ is a discount factor between zero and one. The solution is simplified by of the structure of the model, since the interest rate does not affect inflation one period ahead and, when g_1=0, enters linearly in the expression for inflation two periods ahead. The solution is such that $E_t \pi_{t+2} = z*$ For a detailed solution of an analogous case with one-period interest rates only, see Svensson (forthcoming).

This case is always stable and the values of the policy response parameters tend to be much larger than in the other cases. The expectations simplify to the following expressions.

$$E_t y_{t+1} = -\frac{\pi_t + a y_t}{a}$$

$$\frac{1}{2} E_t \Delta \pi_{t+2} = -\frac{\pi_t + a y_t}{2}$$

$$E_t \Pi_t = \frac{\pi_t + a y_t}{2}$$

$$\rho_t = \frac{\pi_t + a(1+b_1) y_t}{a b_2}$$

Note that none of these expressions involve interest rates. The short-term interest rate has no effect on the next period's inflation, but can be set so that expected inflation two periods ahead is identical to the inflation target. Intuitively, there is no further expectational value to the interest rate, once it has been used to hit the inflation target exactly (in expectation). This case represents plausible behavior on the part of the monetary authority, but it is empirically implausible as a representation of past policy in that it contradicts all the stylized facts of section 2.[13] Svensson (1997) characterizes this type of policy rule as "strict inflation targeting", in contrast with possibly more realistic "flexible inflation targeting", which includes output stabilization or other such goals in the objective function. In this paper, we focus on strict targeting because it leads to an interesting extreme case as far as the predictive power of interest rates is concerned.

Discussion of the special cases

How should the results of this section be interpreted in light of the combined evidence from all the stylized facts? First, it seems clear that the strict inflation targeting solution of case S is in no way indicative of actual policy. Strict, or even approximate, adherence to the optimization rule would render interest rates uninformative and contradict all the stylized facts. Second, the pure case EH-M in which policy focuses exclusively on output seems to be ruled out by the evidence from Bernanke-Blinder and Fuhrer-Moore. The behavior of the long-term real rate would be more complicated than their empirical results suggest if the EH-M case were to hold.

This leaves us with cases BB and BB′. BB, which implies that the short nominal rate and the long real rate move one-for-one with each other, can also be ruled out because it is clearly incompatible with stability. Not only is BB unstable, but there is always a neighborhood of this point in the space of values of g that contains only unstable points. Nevertheless, case BB′, under which the short nominal rate and the long real rate are perfectly correlated, does seem to be plausible. It is stable and yields results very analogous to those of Bernanke-Blinder and Fuhrer-Moore. Furthermore, it is not at odds with the Estrella-Hardouvelis, Mishkin and Estrella-Mishkin evidence: the equations corresponding to BB′ clearly show that the term structure spread

[13]The same conclusions are reached when $g_2=g_3$, as in figure 4, and both of these parameters approach infinity.

is an important predictor of both output and inflation, even if it is not the only predictor. Thus, we must conclude that case BB′ is most consistent with the available stylized facts.

We must also infer that the phenomena observed under case BB′ are far from structural. If policy were to move in the direction of case EH-M, the predictive power of the term structure would be enhanced, but the Bernanke-Blinder and Fuhrer-Moore regularities would vanish. Alternatively, if policy were to move in the direction of case S, all the empirical regularities of section 2 would disappear. In the end, the stylized facts, however robust they seem now, are not guaranteed to persist in the future.

7. Conclusions

Empirical regularities

Monetary policy is a key determinant of the precise relationship between the term structure of interest rates and macroeconomic variables such as real output and inflation. In the rational expectations model of this paper, we see that with alternative monetary rules, one or more of the following can occur: the term structure spread is the optimal predictor of real output, the term structure spread is the optimal predictor of changes in inflation, the short-term nominal rate is the best predictor of real output, the short-term nominal rate is perfectly correlated with the long-term real rate, interest rates are uninformative with regard to future output and inflation.

The term structure has at least some predictive power for both real output and inflation under almost all circumstances. This result is consistent with empirical findings such as Mishkin (1990a,b), Estrella and Hardouvelis (1991) and Estrella and Mishkin (1995). It is clear, however, that the empirical relationships are not structural, and that alternative monetary policy regimes could lead to very different outcomes. These exceptions are associated with very large inflation and output responses, either approximating infinity or designed to annihilate expected inflation as quickly as possible.

There are also policy rules that produce a strong correlation between the nominal short-term rate and the real long-term rate, as reported by Fuhrer and Moore (1995), who show that their results constitute a variation on the theme of Bernanke and Blinder (1992). These results require specific values of the output and inflation response parameters and a limited level of inertia. However, like the term structure regularities, these relationships are not structural. The same types of policy regimes that make interest rates uninformative make the relationship between the short-term nominal rate and the long-term real rate much more complicated.

Based on the combined evidence discussed in section 2, the conclusion is that the best characterization of actual policy in the post-war period in the United States is one associated with the Bernanke-Blinder and Fuhrer-Moore regularities. Beyond the obvious consistency with those regularities, the scenario also provides clear support for the empirical evidence related to the term structure.

Policy implications

The model of this paper may be used to derive policy implications with regard to the feasible or advisable forms of the policy rule. These implications may be drawn in connection with the three parameters of the policy rule, which represent short-term rate inertia (g_1) and the strength of the policy responses to inflation (g_2) and to output (g_3).

The analysis shows that "optimality" is not required for convergence to equilibrium. The system converges for an extensive set of values of the policy parameters, and any one convergent point is robust to perturbations. Thus, even if the monetary authority does not have an explicit objective function, its actions may very well lead to convergence to equilibrium, as long as it is stays within certain bounds.

Given the level of inertia in the short rate and the strength of the output responsiveness, the policy response to inflation is subject to both lower and upper bounds for the system to be stable. The lower bound is inversely related to the level of inertia and does not depend on the output response, whereas the upper bound is raised by higher levels of output responsiveness. An arbitrarily large inflation response parameter can be stable as long as the output response parameter is sufficiently large.

Output response plays an important role in the model. Svensson (forthcoming) has shown that a positive level of output responsiveness is required even when inflation is the sole policy target. The model of this paper confirms that result in the context of multi-period interest rates, and yields stronger results regarding the interaction between the output response and inertia parameters.

A low inertia rule, such as the Taylor rule, provides more flexibility with regard to the choice of stable response parameters than a high inertia rule, such as the Fuhrer-Moore rule. When the level of inertia is high, the output responsiveness becomes particularly important. This paper shows that a positive output response may be necessary for convergence regardless of the ultimate policy objectives, and that it is absolutely necessary for convergence if the level of inertia is very high.

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Figure 1

$a=.2,\ b_1=.6,\ b_2=.4 \quad g_1=1,\ g_2=1,\ g_3=1$

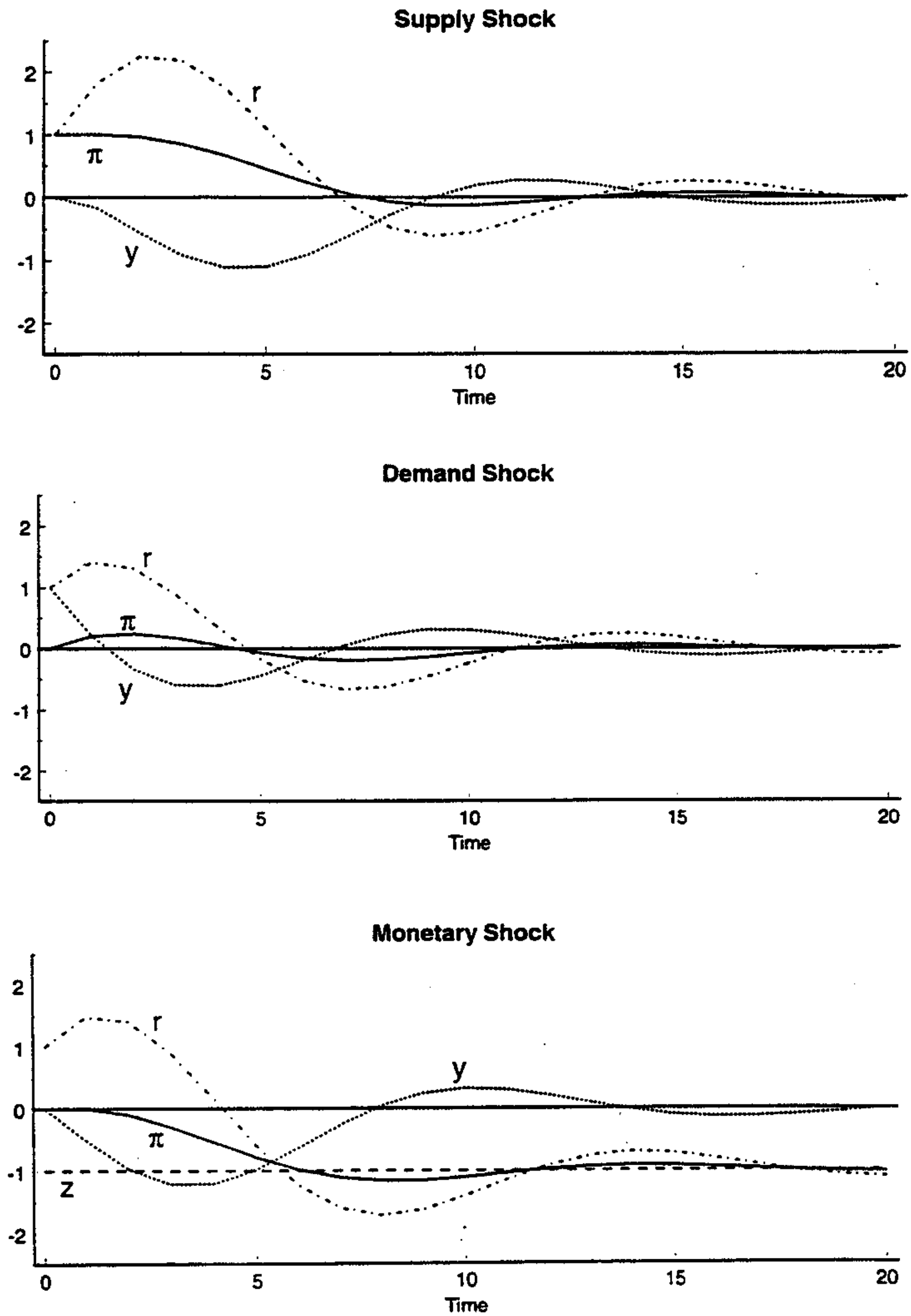
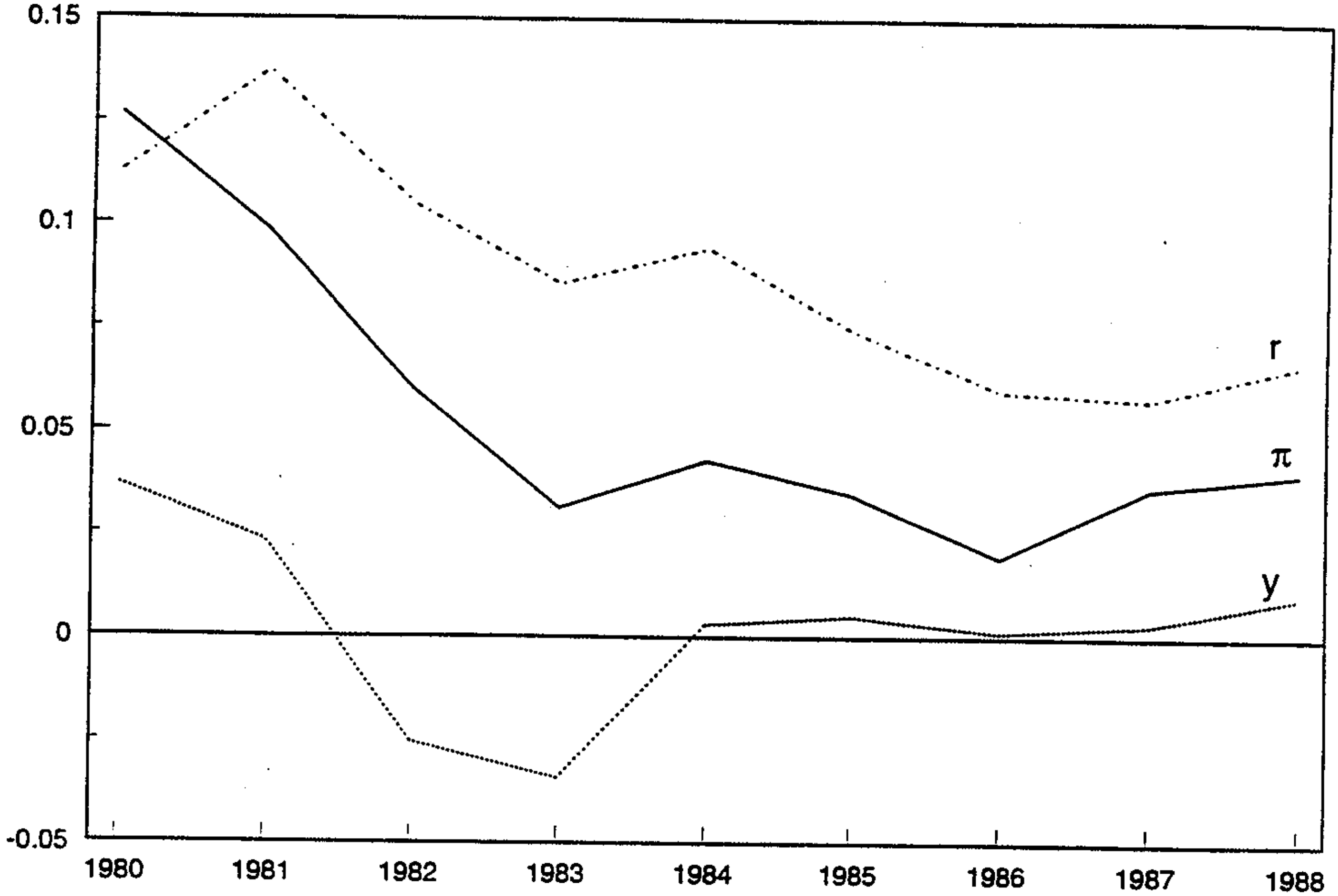


Figure 2
The Volcker Shock



Notes to Figure 2

Series are based on annual averages of monthly data for the consumer price index, the index of coincident indicators, and the one-year Treasury rate. Inflation is the change in the log of the price index from the previous year, the output gap is the difference between the log of the coincident index and its linear trend from 1956 to 1994, and the interest rate is expressed in continuously compounded terms.

Figure 3

Bounds for g_2 with g_3=0, a, and 2a

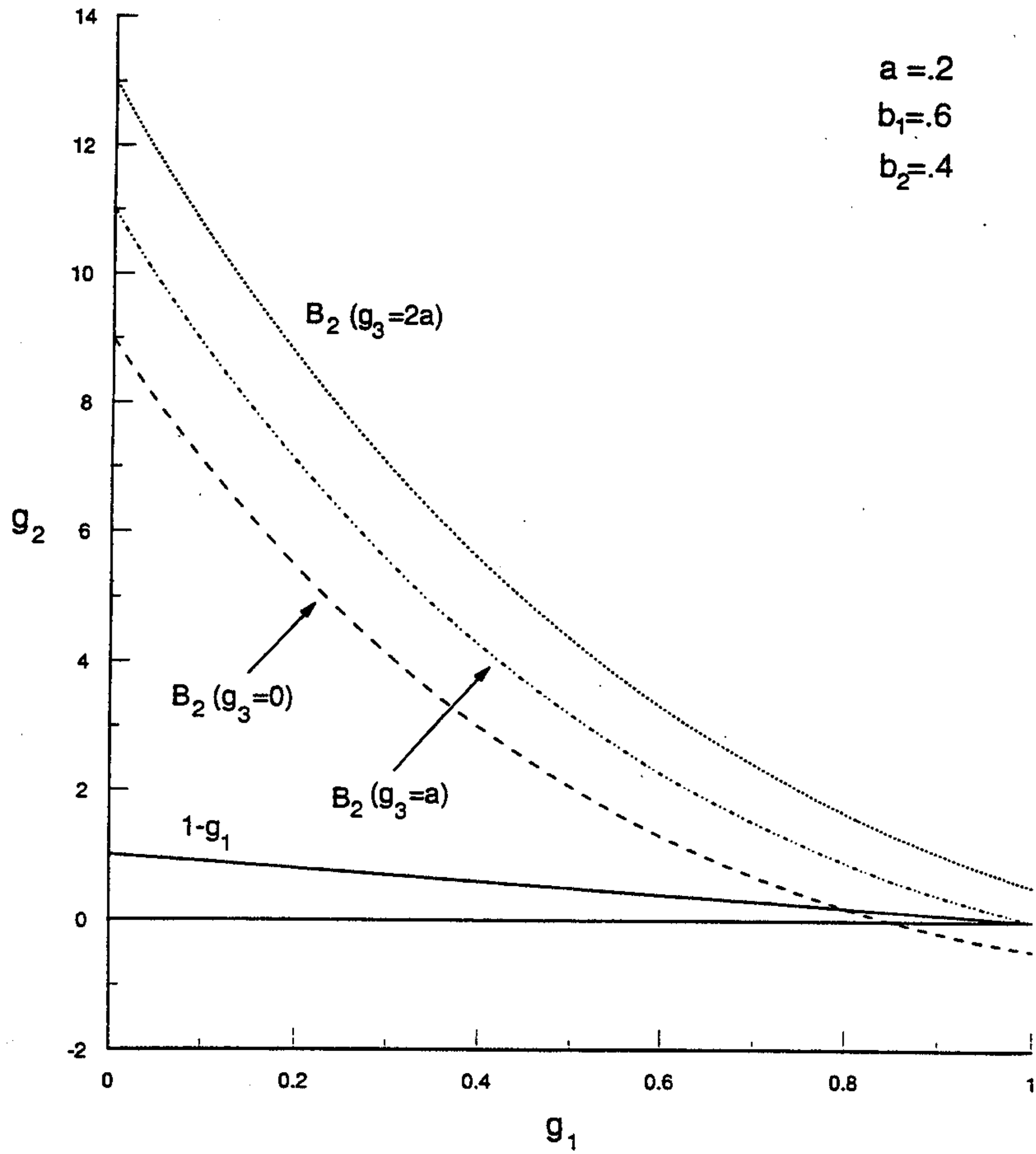


Figure 4
Lower bounds for g_2 with $g_2 = g_3$

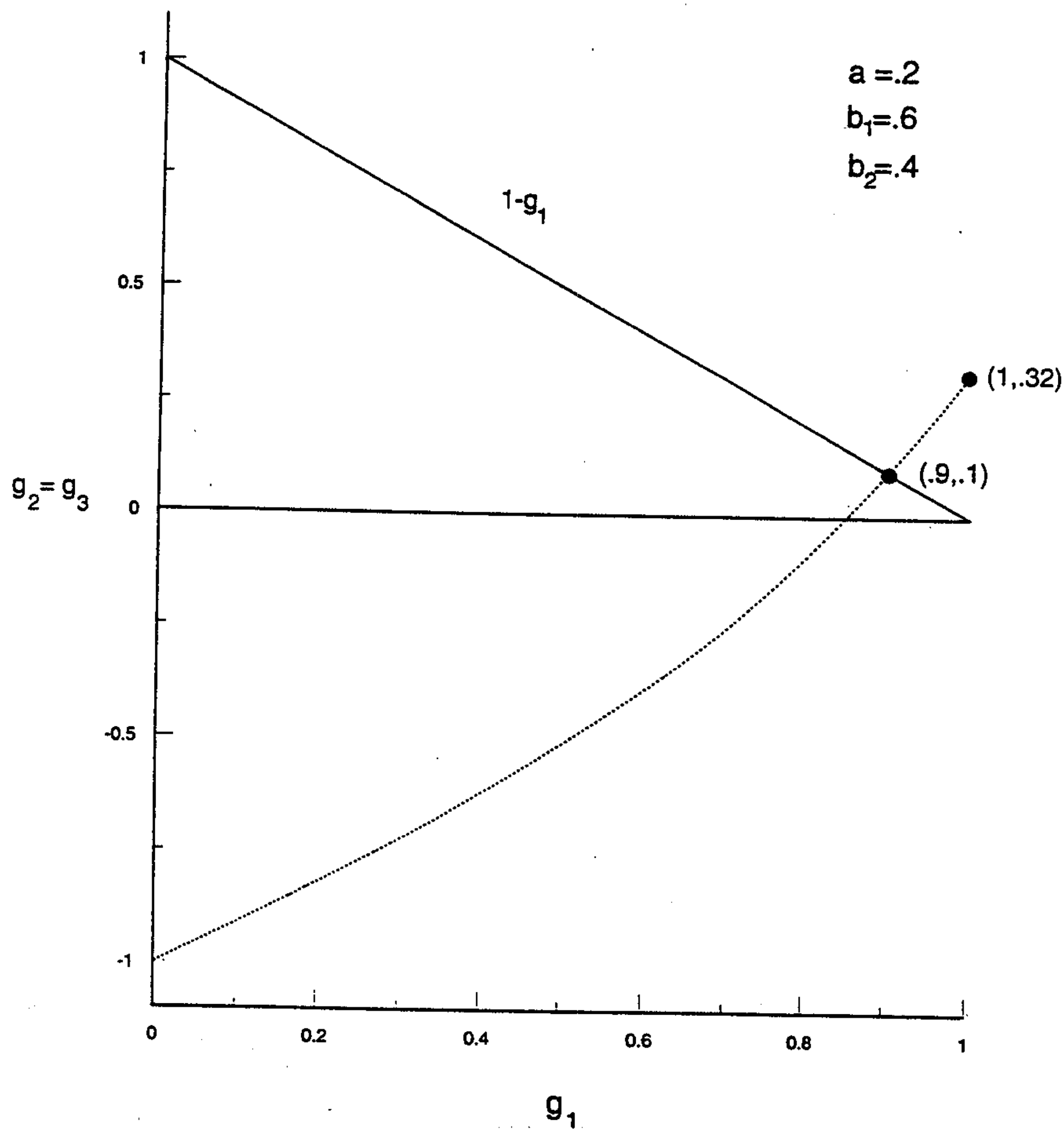
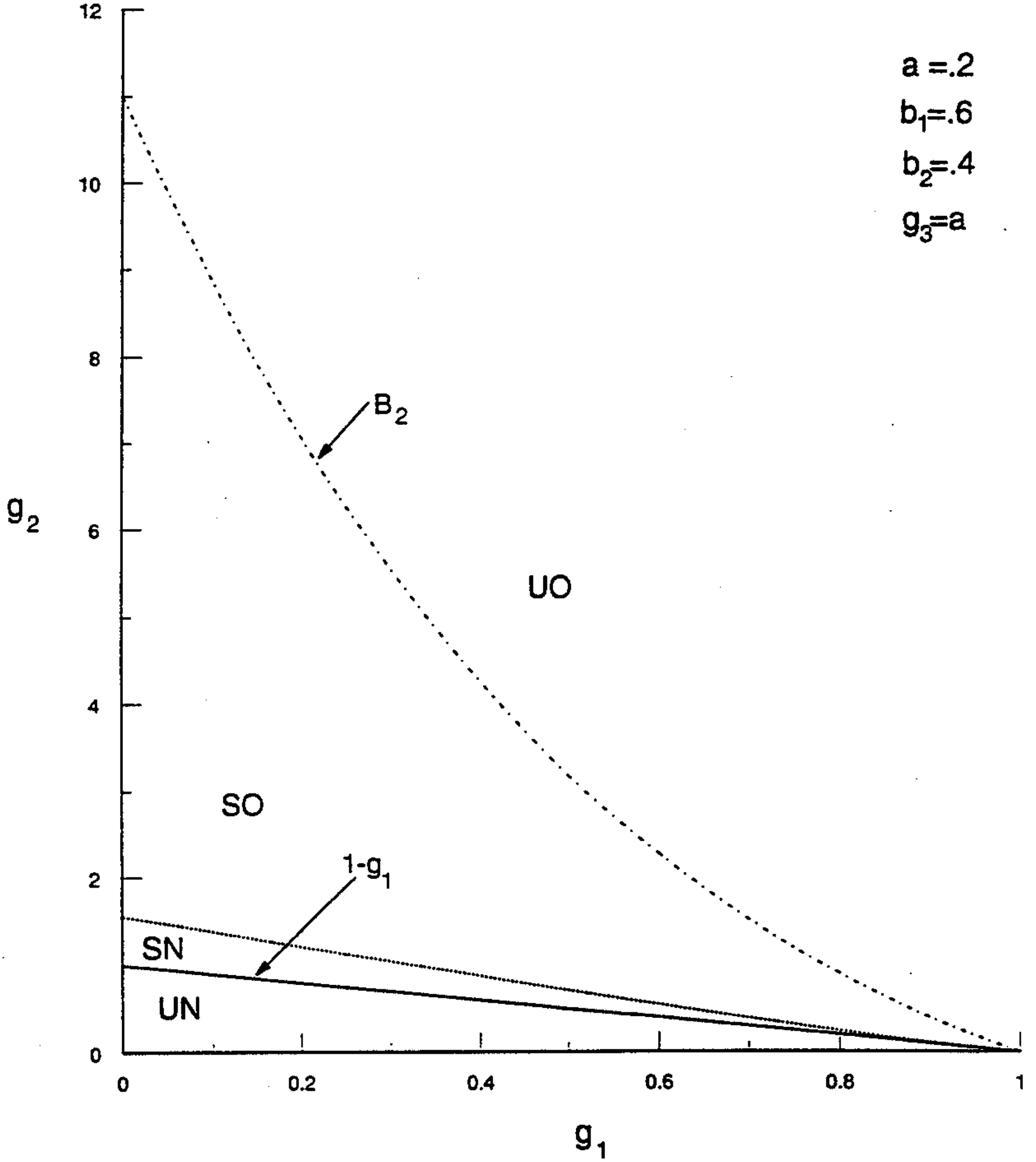
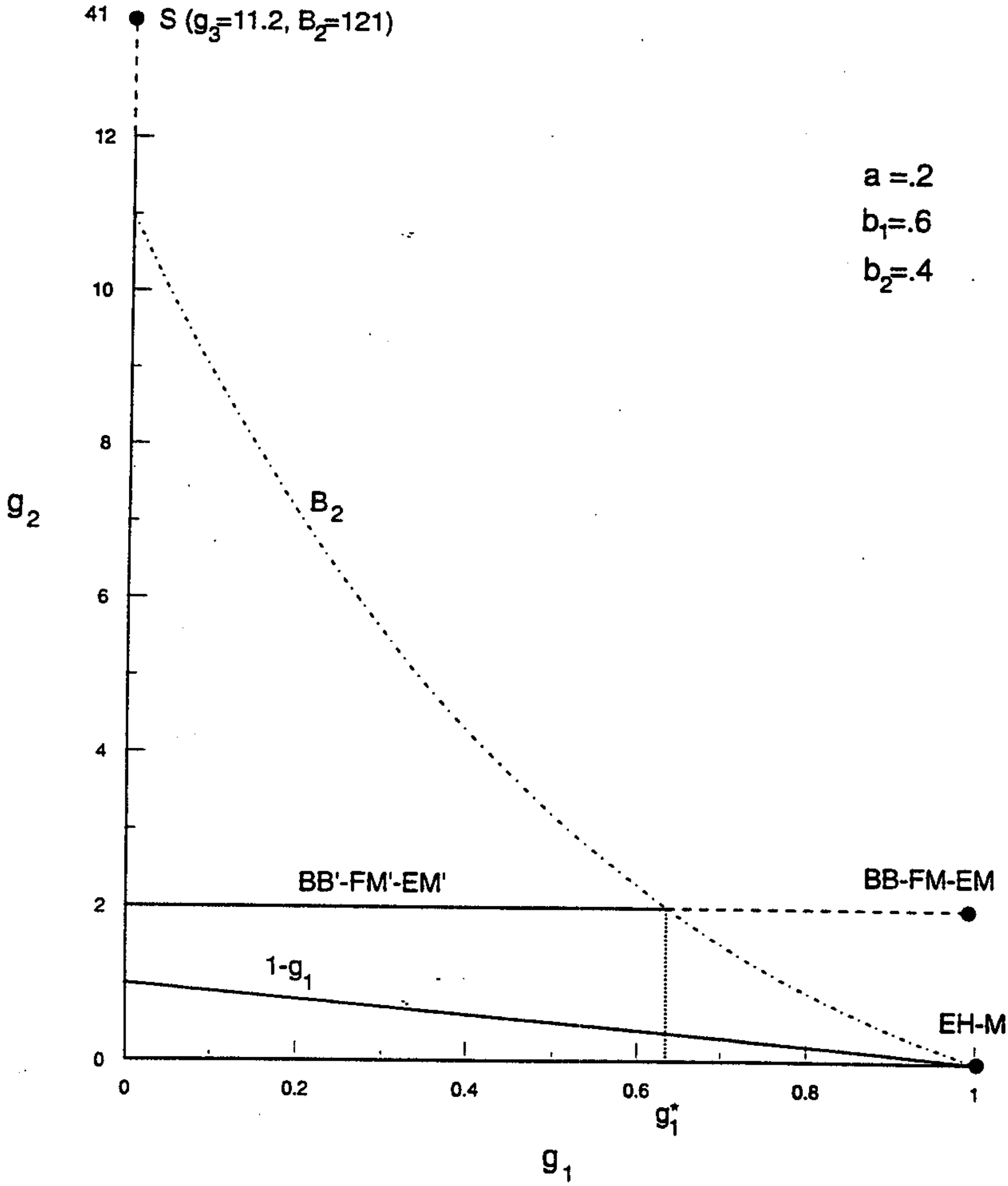


Figure 5
Regions with stable and oscillatory solutions



Notes: S = stable, U = unstable, O = oscillatory, N = non-oscillatory.

Figure 6
Stylized Cases



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