NON-LINEAR CONSUMPTION DYNAMICS

Angelos A. Antzoulatos

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Public Information Department
Federal Reserve Bank of New York
New York, NY 10045
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Angelos A. Antzoulatos
International Research Department
Federal Reserve Bank of New York
33 Liberty Street
New York, N.Y. 10045
(212) 720-6813
aa.antzoulatos@ny.frb.org

ABSTRACT.

Taking explicitly into account the forward-looking nature of consumption, this paper derives a non-linear equation for consumption growth in which the coefficient of contemporaneous expected income growth is an increasing (decreasing) function of lagged variables positively (negatively) correlated with future income growth. Estimating it with aggregate data, the paper finds statistically and economically significant non-linear consumption dynamics for three major industrial countries. These dynamics imply, among other things, that monetary policy may have a more immediate and profound effect on consumption, and through it on real economic activity, than commonly thought.

J.E.L. Classification Numbers: E21, E32

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1 INTRODUCTION

Modeling non-linear consumption dynamics has not been very successful so far, despite the substantial progress over the past fifteen years in modeling such dynamics for GNP and several other macroeconomic variables (for a comprehensive account of the estimated models, see Brunner [1997], and French and Sichel [1993]). In theory, this relative failure may merely reflect the consumption-smoothing behavior of forward-looking individuals, postulated by the permanent income hypothesis (PIH), that precludes non-linear dynamics. In practice, however, this explanation seems unlikely, because the PIH has been rejected in tests with aggregate data for all countries for which relevant data is available. See, for example, Campbell and Mankiw (1990 and 1991) for the G-7 countries, and Agell and Berg (1996) for Sweden. In fact, the very few studies which have had some success in documenting non-linear consumption dynamics start from a stylized fact associated with PIH’s empirical rejection. That is, consumption change (growth in log form), far from being unpredictable as the PIH postulates, is strongly correlated with contemporaneous expected income change (growth in log form).

Specifically, Kim (1994) and Shea (1995) argue that under borrowing constraints, a leading explanation for the PIH’s rejection, consumption should respond asymmetrically to predictable income increases and declines. Both authors recover expected income growth from the projection of actual growth on own, consumption growth and interest rate lags, and use it to document that U.S. consumption of non-durable and services responds more strongly to expected income increases than declines. In a slightly different framework, Caballero (1995) shows theoretically that, if individuals adjust their consumption infrequently, when its deviation from the PIH-determined optimal one reaches a trigger level, aggregate consumption will respond more strongly to positive (negative) permanent income shocks in good (bad) times. Good (bad) times are identified as those that follow a series of positive (negative) shocks. Caballero recovers permanent income shocks from a bi-variate VAR of income growth and savings, and uses them to identify good and bad times. He then documents that U.S. non-durables consumption is consistent with his theoretical results.

This paper, in its exploration of non-linear consumption dynamics, improves both on modeling technique and geographical coverage over the aforementioned studies. Briefly, the paper uses in section 2 the PIH under borrowing constraints to derive an equation for consumption growth, $\Delta c_t$, in which the coefficient of contemporaneous expected income growth, $E_{t-1}\Delta y_t$, is a continuous and increasing function of expected future income growth, $E_{t-1}\Delta y_{t+k}$ ($k \geq 1$). ($\Delta$ and $E$ are the usual difference and expectations operators, $t$ the time subscript, while small letters denote logs). This testable equation has two conceptual advantages over those in the aforementioned
studies. First, it spares the econometrician from the challenging task of identifying periods of positive/negative expected income growth or good/bad times. More importantly, it takes explicitly into account that expected consumption, $E_{t-1}c_t$, is an increasing function not only of contemporaneous, $E_{t-1}y_t$, but of future expected income, $E_{t-1}y_{t+k}$ ($k \geq 1$), as well. (In the analysis below, expected and realized values for consumption and income are used interchangeably for the reason that, under rational expectations, the two values differ by an unpredictable stochastic term.)

Another advantage of the paper's testable equation is that it affords a great deal of flexibility in accounting for the effect of expected future income. In principle, any lagged variable that is correlated with $\Delta y_{t+k}$ ($k \geq 1$) can be used to test for non-linear dynamics. This flexibility helps overcome the well-documented difficulty of finding good instruments (predictors) for income growth. To reduce, however, the magnitude of the task to manageable levels, the paper considers only three variables: the real interest rate and the growth rate of the composite leading indicator, when they are correlated with future income growth; plus $E_{t-1}\Delta y_{t+1}$, instrumented with variables dated at $t-1$ and before.

The value of this flexibility in accounting for the effect of expected future income, and of the modeling technique in general, is demonstrated in section 3. This section documents strong, and consistent with the paper's hypotheses, non-linear consumption dynamics for three major industrial countries. In particular, it documents that the regression coefficient of Japan's consumption growth on contemporaneous expected income growth is increasing in $E_{t-1}\Delta y_{t+1}$, and the lagged growth rate of the composite leading indicator, $\Delta LEAD_{t-1}$. The latter is positively correlated with $\Delta y_{t+k}$ ($k = 1, 4$). For the U.S., this coefficient is increasing in $\Delta LEAD_{t-1}$, which is positively correlated with $\Delta y_{t+k}$ ($k = 1, 4$), while for France it is decreasing in the real interest rate, $r_{t-1}$, which is negatively correlated with $\Delta y_{t+k}$ ($k = 1, 4$).

Non-linear dynamics are not documented with the three variables under consideration for Canada, Germany and the U.K.. But, far from diminishing the paper's contribution, this rather illustrates the diversity of economic dynamics across countries, the natural consequence of differences in economic organization, financial markets and people's preferences. It may also reflect the loss of efficiency in estimation resulting from the use of proxies for the unobserved income expectations. As for Italy, the last G-7 country, no tests were carried out for the lack of appropriate income series.

In any event, the paper's contribution is further highlighted in section 4, which touches upon some pertinent policy implications.
2 THEORY

2.1 Conceptual Framework

This section derives a non-linear-equation-linking consumption growth with contemporaneous expected income growth, using the model of a representative agent who faces a borrowing constraint. This model summarizes neatly the forward-looking nature of consumption in a way consistent with the empirical rejection of the PIH. Nevertheless, as briefly discussed at the end of the section, the derived non-linear equation can also be justified using a slight variation of Caballero’s model (Caballero [1995]).

The section begins with Antzoulatos’ observation (Antzoulatos [1994]) that, under borrowing constraints, consumption growth should be a non-decreasing function of contemporaneous and expected future income, as described by equation (1). In it, \( \alpha_0 \) is a constant, \( \phi(.) \) a function non-decreasing in its arguments, and \( u_t \) is a stochastic term unrelated to variables known at \( t-1 \). Following much of the literature, which is motivated by the finding that the real interest rate is uncorrelated with contemporaneous consumption growth, the interest rate is assumed constant and subsumed into \( \alpha_0 \).

\[
\Delta c_t = \alpha_0 + \phi_t(E_{t-1}y_{t+k}, k \geq 0) + u_t
\]

(1)

Intuitively, if the borrowing constraint does not bind at \( t-1 \), both \( c_{t-1} \) and \( E_{t-1}c_t \) are increasing in \( E_{t-1}y_{t+k} \) \( (k \geq 0) \) thus rendering expected consumption growth, \( E_{t-1}c_t - c_{t-1} \), unpredictable with variables known at \( t-1 \) and the function \( \phi_t(.) \) equal to zero. But, if the constraint does bind at \( t-1 \), \( c_{t-1} \) cannot increase in anticipation of higher future income. Still, however, \( E_{t-1}c_t \) is increasing in \( E_{t-1}y_{t+k} \) \( (k \geq 0) \), thus rendering \( \phi_t(.) \) increasing in \( E_{t-1}y_{t+k} \) \( (k \geq 0) \) as well (for a more formal analysis, see Antzoulatos [1994]).

A first-order Taylor expansion of equation (1) gives equation (2), in which the coefficient of contemporaneous expected income growth, \( \beta_{t-1} \), is zero when the constraint does not bind at \( t-1 \), and an increasing-function of expected future income, \( E_{t-1}y_{t+k} \) \( (k \geq 1) \), when it does. In the limiting case when the constraint binds at \( t-1 \) and is expected to bind at \( t \), \( \beta_{t-1} \) will approximately be equal to one.

\[
\Delta c_t = \alpha_1 + \beta_{t-1}(E_{t-1}y_{t+k}, k \geq 1)E_{t-1}\Delta y_t + u_t
\]

(2)

Before proceeding, it is instructive to compare the implications of equation (2) with those
of previous studies. Campbell and Mankiw's "rule-of-thumb" model postulates that $\beta_{t-1}$ should be constant (Campbell and Mankiw [1990 and 1991]). In essence, this model focuses only on the effect of contemporaneous income growth on consumption growth, and thus does not take fully into account the forward-looking nature of the consumer's decision problem (here captured by $\beta_{t-1}$'s dependence on $E_{t-1}\Delta y_{t+k}$ ($k \geq 1$)). Kim's (1994) and Shea's (1995) models, which recognize the possibility of an asymmetric response to predictable income increases ($E_{t-1}\Delta y_t > 0$) and declines ($E_{t-1}\Delta y_t < 0$), represent a major step to the right direction. Yet, they leave much to be desired in accounting for the effect of expected future income, as, controlling for $E_{t-1}\Delta y_t$, they cannot readily account for the possibility that $\Delta c_t$ could be different if, say, $E_{t-1}\Delta y_{t+1}$ is $-5\%$ or $5\%$.

2.2 Empirical Implementation

Since $\beta_{t-1}(.)$ is a non-linear function of unknown form, it is approximated with $\beta_0 + \gamma_1 E_{t-1}\Delta y_{t+1} + \gamma_2 E_{t-1}\Delta y_{t+2} + \ldots$, in which the stationary (log) income change replaces the non-stationary levels. Thus, equation (2) becomes:

$$\Delta c_t = \alpha_1 + (\beta_0 + \gamma_1 E_{t-1}\Delta y_{t+1} + \gamma_2 E_{t-1}\Delta y_{t+2} + \ldots) E_{t-1}\Delta y_t + u_t \quad (3)$$

In the representative agent framework, non-linear dynamics will be denoted by positive and statistically significant coefficients $\gamma_k$ ($k \geq 1$) for the periods which follow periods of a binding constraint. For the remaining periods, $\gamma_k$ ($k \geq 1$), as well as $\beta_0$, should be zero. Yet, estimating equation (3) over the whole sample can yield statistically significant $\gamma$'s even when the constraint binds for only part of it. This allows testing for non-linear dynamics without worrying about the identification of likely periods of a binding/non-binding constraint –the analogue of Kim's and Shea's periods of predictable income increases/declines and Caballero's good/bad times. By the way, the "rule-of-thumb" model postulates that $\gamma_k = 0$ for all $k \geq 1$.

Taking also into account the difficulty of finding good instruments for $\Delta y_{t+k}$ ($k \geq 1$), a difficulty that increases as the forecast horizon $t+k$ gets longer, the paper proposes to substitute the terms $E_{t-1}\Delta y_{t+k}$ ($k \geq 1$) with stationary variables known at $t-1$ and correlated with $\Delta y_{t+k}$ ($k \geq 1$). These variables are denoted by the vector $X_{t-1}$. With this substitution, equation (3) becomes:

$$\Delta c_t = \alpha + (\beta + \gamma X_{t-1}) E_{t-1}\Delta y_t + u_t \quad (4)$$

The virtue of this substitution can be illustrated by comparing the coefficients $\gamma_k$ ($k \geq 1$) in equation (3) with $\gamma$ in (4). For simplicity, but without loss of generality, this will be done for the
case $X_{t-1}$ contains only one variable. Differentiating the two equations with respect to $X_{t-1}$ gives:

$$\frac{\partial \Delta c_t}{\partial X_{t-1}} = \left( \sum_{k \geq 1} \gamma_k \frac{\partial E_{t-1} \Delta y_{t+k}}{\partial X_{t-1}} \right) E_{t-1} \Delta y_t + (\beta_0 + \sum_{k \geq 1} \gamma_k E_{t-1} \Delta y_{t+k}) \frac{\partial E_{t-1} \Delta y_t}{\partial X_{t-1}}$$

and

$$\frac{\partial \Delta c_t}{\partial X_{t-1}} = \gamma E_{t-1} \Delta y_t + (\beta + \gamma X_{t-1}) \frac{\partial E_{t-1} \Delta y_t}{\partial X_{t-1}}$$

Next, equating the coefficients of $E_{t-1} \Delta y_t$ — since the two derivatives must hold for all values of $E_{t-1} \Delta y_t$, the coefficients of $E_{t-1} \Delta y_t$ and $\frac{\partial E_{t-1} \Delta y_t}{\partial X_{t-1}}$ must be the same in both — gives:

$$\gamma = \sum_{k \geq 1} \gamma_k \frac{\partial E_{t-1} \Delta y_{t+k}}{\partial X_{t-1}}$$  \hspace{1cm} (5)$$

The last equation indicates that if significant non-linear dynamics exist, i.e., the coefficients $\gamma_k \ (k \geq 1)$ are high enough, and $X_{t-1}$ is correlated with future income growth, i.e., $\frac{\partial E_{t-1} \Delta y_{t+k}}{\partial X_{t-1}} \neq 0$, $X_{t-1}$ can be statistically significant in equation (4). As a result, one can test for non-linear consumption dynamics using this equation, even when the coefficients $\gamma_k \ (k \geq 1)$ in (3) cannot be estimated precisely for the lack of good instruments for income growth. Moreover, if $\frac{\partial E_{t-1} \Delta y_{t+k}}{\partial X_{t-1}} > 0$ for all $k \geq 1$, i.e., $X_{t-1}$ is positively correlated with future income growth, $\gamma$ should be positive, and vice-versa.

But the flexibility equation (4) affords in accounting for the effect of expected future income comes at some cost. Namely, the substitution of $X_{t-1}$ for the unobserved $E_{t-1} \Delta y_{t+k} \ (k \geq 1)$ decreases the signal-to-noise ratio. The resultant loss in efficiency compounds that of the linear approximation of the function $\beta_{t-1}()$, thus making it more difficult to uncover statistical evidence of non-linear dynamics, provided, of course, such dynamics exist. On the positive side, if the estimated $\gamma$-coefficients are statistically significant and have the correct sign, one can reasonably expect that the non-linear relationship between $\Delta c_t$ and $E_{t-1} \Delta y_t$ is actually stronger. Conversely, a statistically insignificant $\gamma$ may reflect either the loss in efficiency or the inexistence of non-linear dynamics.

From the potentially large number of candidates for inclusion in the $X_{t-1}$ vector, only three are considered, in order to reduce the magnitude of the task to manageable levels and also avoid the ‘data mining’ trap. These are the real interest rate and the growth rate of the composite leading indicator; plus the expected income growth rates for $t + k \ (k \geq 1)$, instrumented with variables known at $t - 1$. The difficulty, however, of finding good instruments for $\Delta y_{t+k} \ (k \geq 1)$ restricts the last candidate to $\Delta y_{t+1}$ only.
Some econometric issues are in order. Following a common practice in tests of rational expectations models with unobserved expectations, realized income growth is used as a proxy for expected income growth. As a consequence, equation (4) must be estimated with instrumental variables. Moreover, the instruments must be lagged at least two periods, to account for the AR(1) term in $u_t$ induced by the time aggregation of consumption. Since, however, two lags would exacerbate the problem of finding good instruments for income growth, non-linear least squares is used in which the error term is assumed to follow the process $u_t = \rho u_{t-1} + \epsilon_t$ (see Carroll et al. [1994] for details). This estimation approach renders variables dated $t - 1$ as valid instruments.

At this point, it should be stressed that the testable equations (3) and (4) can be justified by other forward-looking models besides the one with borrowing constraints. Consider, for example, a slight variation of Caballero’s model (Caballero [1995]), in which desired consumption as of $t - 1$ is determined not by permanent income as Caballero assumes, but by expected future income for all $t + k \ (k \geq 0)$. (The difference between the two definitions of desired consumption is a matter more of semantics than of substance, especially in the case non-human wealth is low and, thus, permanent income is mainly a function of expected future income.) Everything else in the model remains the same. Notably, although individuals adjust desired consumption continually in response to changes in expected future income, they adjust actual consumption only when it deviates from the desired one by more than a trigger level. Consider, next, a $t - 1$ increase in expected future income for periods $t + k \ (k \geq 1)$, while holding $E_{t-1}\Delta y_t$ constant. As individuals adjust upwards their desired consumption, some hit their trigger levels and adjust actual consumption as well. But as time goes by, the deviation of actual from desired consumption becomes more pronounced, and more individuals hit their trigger levels. As a result, the $t - 1$ increase in expected future income will manifest itself in a consumption surge next period. Thus, controlling for $E_{t-1}\Delta y_t, c_t$ and $\Delta c_t$ will be increasing in $E_{t-1}\Delta y_{t+k} \ (k \geq 1)$.

3 EVIDENCE

The empirical analysis documents statistically significant non-linear consumption dynamics for Japan, the U.S., and France. In particular, and in line with the paper’s hypotheses, it documents that the regression coefficient of consumption growth on contemporaneous expected income growth is an increasing (decreasing) function of variables positively (negatively) correlated with future income growth. Further, based on the size of the estimated $\gamma$-coefficients, the analysis indicates that the non-linear dynamics are economically significant as well.
Table 1 summarizes the estimation results for the variables that produce statistically significant coefficients $\gamma$. For the remaining variables the results are briefly discussed in the text. Starting from the left, Table 1 reports the name of the country; the $X_{t-1}$ variable; the estimated coefficients $\alpha$, $\beta$, $\gamma$ and $\rho$, and the corresponding $R^2$ and D.W. statistics; the instruments; the $\tilde{R}^2$ of the first-stage regression of $E_{t-1}\Delta y_t$ and $X_{t-1}$ on the instruments; and the sample period. Below the estimated coefficients their $t$-statistics appear in parentheses, while one, two and three asterisks denote significance at the ten, five, and one percent level. Table 1 reports the results with the instrument set which produced the highest first-stage $\tilde{R}^2$. Nevertheless, the results are robust to the instrument set used. Finally, the notes at the end of the table describe the data and their sources in detail. Here it suffices to note that personal disposable income is converted to “real” by dividing with the consumer price index.

The country-by-country analysis below focuses on $\gamma$, the coefficient that captures the strength of non-linear dynamics. Nonetheless, as Table 1 documents, $\alpha$ and $\beta$ are positive and significant at the one percent level in all cases.

For Japan, $\gamma$ is positive and significant at the one percent level when the lagged growth of the composite leading indicator, $\Delta LEAD_{t-1}$, and expected income growth for $t + 1$, $E_{t-1}\Delta y_{t+1}$, are used in place of $X_{t-1}$. It is insignificant though when the real interest rate, $r_{t-1}$, is used. These results are consistent with the paper’s hypotheses, because $\Delta y_{t+k}$ ($k = 1, 4$) is positively correlated with $\Delta LEAD_{t-1}$ but virtually uncorrelated with $r_{t-1}$. The results are also robust to the instrument set used. For example, four or eight lags of the instruments produce essentially the same coefficient estimates and $t$-statistics for both $\beta$ and $\gamma$. Six lags, i.e., those reported in the table, produce the highest first-stage $\tilde{R}^2$ (= 0.318) for $\Delta y_t$.

Japan’s non-linear consumption dynamics are economically significant as well. This can be illustrated with a simple example using the second estimated equation. For $E_{t-1}\Delta y_t = 0.02$ (two percent) and $E_{t-1}\Delta y_{t+1} = 0.0$, and abstracting from the constant term, consumption growth is $(0.439 + 7.695 \times 0.0) \times 0.02 \approx 0.00878$ (0.878 percent). If $E_{t-1}\Delta y_{t+1}$ increases by one percent, consumption growth rises by $(7.695 \times 0.01) \times 0.02 \approx 0.0015$. The latter represents an increase of almost twenty percent over the base figure of 0.00878.

For the U.S., $\gamma$ is positive and significant at the one percent level when $\Delta LEAD_{t-1}$ is used in place of $X_{t-1}$. $\Delta LEAD_{t-1}$ is positively correlated with $\Delta y_{t+k}$ ($k = 1, 4$). Also, the real interest rate, $r_{t-1}$, which is negatively correlated with future income growth, and $E_{t-1}\Delta y_{t+1}$ were significant and had the expected sign for some instrument sets. But for most sets they were insignificant.
U.S.'s non-linear dynamics are economically significant, too. Every one percent increase in \( \Delta \text{LEAD}_{t-1} \) increases the coefficient of \( E_{t-1} \Delta y_t \) by \( 5.395 \times 0.01 = 0.05395 \), which amounts to almost twenty five percent of \( \beta \) \((= 0.216)\).

For France, \( \gamma \) is negative and significant at the five percent level when \( r_{t-1} \), which is negatively correlated with \( \Delta y_{t+k} (k = 1, 4) \), is used in place of \( X_{t-1} \). It should be noted though that \( \gamma \)'s significance level is sensitive to the instrument set. For example, with four or six lags for all instruments, \( \gamma \) is significant at the six percent level. But with these two sets the \( R^2 \) of the first stage regression of \( \Delta y_t \) were respectively 0.050 and 0.028, considerably below the 0.066 with the instrument set reported in Table 1. \( \Delta \text{LEAD}_{t-1} \), which is uncorrelated with future income growth, produces an insignificant \( \gamma \) – as expected. So does \( \Delta y_{t+1} \) which, by the way, is difficult to predict. Further, the estimated coefficients indicate that every one percent increase in the real interest rate reduces the coefficient of \( E_{t-1} \Delta y_t \) by 0.048, almost ten percent of \( \beta \) \((= 0.545)\).

The main results for the remaining major industrial countries are as follows. For Germany and Canada, \( \gamma \) was negative when \( r_{t-1} \), which is negatively correlated with \( \Delta y_{t+k} (k = 1, 4) \), was used in place of \( X_{t-1} \). Its significance though was very sensitive to the instrument set. But arguing against the existence of non-linear dynamics, for both countries \( \gamma \) was insignificant when \( \Delta \text{LEAD}_{t-1} \) was used, despite that the latter is correlated with future income growth. For the U.K., \( \Delta \text{LEAD}_{t-1} \) was significantly positive at the five percent level. This result, however, cannot be readily explained by the paper's forward-looking framework, because \( \Delta \text{LEAD}_{t-1} \) is weakly correlated with future income growth. For none of the three countries was \( \gamma \) significant when \( \Delta y_{t+1} \) was used in place of \( X_{t-1} \). As for Italy, no tests were carried out for the lack of appropriate income series.

4 CONCLUDING REMARKS

This paper, in conclusion, fills an important gap in the literature by documenting significant non-linear consumption dynamics for three major industrial countries. Instrumental in its success is the explicit consideration of the forward-looking nature of consumption. This consideration not only led to a particular non-linear relationship to test for, but in addition guided the selection of the variables called upon to overcome the difficulty of predicting future income. Based on the paper's results, it is tempting to conjecture that the gap in the literature persisted for so long because the non-linear structures primarily used in earlier studies, such as, Markov-switching models and ARCH-type processes, do not readily account for consumption's forward-looking nature.
Yet, the paper's contribution is better highlighted by its policy implications. To begin with, controlling for contemporaneous income growth, consumption growth will be faster in times of optimistic income expectations than in times of pessimistic expectations. Policy-makers concerned about the inflationary effect of fast growth, as well as macroeconomic forecasters and financial-market participants, should bear in mind this cyclical variation in the coefficient of expected income growth and the potential for consumption surges during good times it implies.

Most importantly though, monetary policy may have a more immediate and profound impact on real economic activity than commonly thought. This is more evident for France, for which the regression coefficient of consumption growth on contemporaneous expected income growth is decreasing in the lagged real interest rate. This indicates that a monetary easing at \( t - 1 \), by causing the real interest rate to drop, can lead to faster consumption growth next period even after controlling for the easing's effect on expected income growth for \( t \). For Japan and the U.S., the monetary easing can lead to faster consumption growth next period through its capacity to improve future income expectations. For these three countries, the above challenge the prevailing view that monetary policy affects the real economy with long (and unpredictable) lags.
REFERENCES


\[ \Delta c_t = \alpha + (\beta + \gamma X_{t-1})E_{t-1}\Delta y_{t-1} + \epsilon_t \]
\[ u_t = \rho u_{t-1} + \epsilon_t \]

<table>
<thead>
<tr>
<th>Country</th>
<th>(X_{t-1})</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\rho)</th>
<th>(R^2)</th>
<th>D.W.</th>
<th>Instruments</th>
<th>(R^2) of 1st Stage Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>(\Delta LEAD_{t-1})</td>
<td>0.0032</td>
<td>0.356</td>
<td>1.910</td>
<td>-0.249</td>
<td>0.473</td>
<td>1.93</td>
<td>(\Delta c_{t-k}, \Delta y_{t-k}), (\Delta LEAD_{t-k}, k = 1, 6)</td>
<td>0.318</td>
</tr>
<tr>
<td></td>
<td>(E_{t-1}\Delta y_{t-1})</td>
<td>0.0037</td>
<td>0.439</td>
<td>7.695</td>
<td>-0.273</td>
<td>0.268</td>
<td>1.77</td>
<td>(\Delta c_{t-k}, \Delta y_{t-k}), (\Delta LEAD_{t-k}, k = 1, 6)</td>
<td>0.318</td>
</tr>
<tr>
<td>U.S.</td>
<td>(\Delta LEAD_{t-1})</td>
<td>0.0033</td>
<td>0.216</td>
<td>5.395</td>
<td>0.176</td>
<td>0.144</td>
<td>2.03</td>
<td>(\Delta c_{t-k}, \Delta y_{t-k}), (X_{t-k}, r_{t-k}, k = 1, 6)</td>
<td>0.165</td>
</tr>
<tr>
<td>France</td>
<td>(r_{t-1})</td>
<td>0.0034</td>
<td>0.545</td>
<td>-0.048</td>
<td>-0.252</td>
<td>0.193</td>
<td>2.24</td>
<td>(\Delta c_{t-k}, \Delta y_{t-k}, k = 1, 4); (r_{t-m}, m = 1, 6)</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Notes:
1. All instrument sets include a constant.
2. Significance levels: * : 10%, ** : 5%, *** : 1%.
3. The sample periods are determined by data availability.
4. Variable definitions:
   - \(c\) and \(y\): log real, per capita consumption and personal disposable income. For the U.S., consumption corresponds to non-durables and services; for all other countries to total consumption. Because Japan's personal disposable income is non-seasonally adjusted, the seasonally adjusted employee compensation is used instead.
   - \(\Delta LEAD_{t-1}\): log-change of the non-seasonally adjusted composite leading indicator, between the third month of the \(t-1\) quarter and the corresponding month from a year ago.
   - \(r\): real interest rate, per cent, per annum, calculated as the quarterly average of the nominal rate minus contemporaneous consumer-price inflation. The nominal interest rate corresponds to the three-month Treasury bill rate for the U.S. and France, and the short-term loans rate for Japan.
5. Data Sources:
   - Composite Leading Indicator: OECD Main Economic Indicators.
6. Japan's and France's monthly population is converted to quarterly by averaging. U.K.'s annual population is converted to quarterly with a linear approximation.