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by John Y. Campbell and Sydney Ludvigson

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Elasticities of Substitution in Real Business Cycle Models with Home Production

John Y. Campbell* and Sydney Ludvigson**

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Abstract

Recently, there has been considerable interest in modifying the standard real business cycle model to include home production. In this paper, we construct a simple model of home production that demonstrates the connection between the intertemporal elasticity of substitution (IES), and the elasticity of substitution between home and market consumption. Understanding this connection is important because there is much larger body of empirical evidence on the size of the IES than there is on the size of the static home-market substitution elasticity. We use this framework to shed light on the properties of a home production model with empirically plausible (lower) values of the IES. In particular, we find that such a model must display two fundamental properties in order to reproduce certain key aspects of the U.S. aggregate data: first, the steady state growth rate of technology must be the same across sectors. Second, out of steady state, *shocks* to technology must be sufficiently positively correlated across sectors.

*Department of Economics, Littauer Center 213, Harvard University, Cambridge, MA 02138; 617-496-6448; john_campbell@harvard.edu.

**Federal Reserve Bank of New York, Domestic Research Function -3F, 33 Liberty Street, New York, NY 10045; 212.720.6810; sydney.ludvigson@ny.frb.org. The views expressed are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.

1. Introduction

Recently there has been considerable interest in modifying the standard real business cycle model to include home production.[1] This research has focused on documenting the relative importance of the home sector in the U.S. economy, and on illustrating the ways in which introducing home production improves the quantitative performance of the standard model. However, almost without exception, these studies have concentrated their analysis on models of nonseparable utility over home and market consumption. In this paper, we study a simple macroeconomic model which explicitly incorporates a household production sector, but allows utility obtained from consuming this output to enter the objective function separably. We argue that there are several reasons to consider models with household production in which utility is additively separable over market and home consumption.

First, we think it is important to consider additive separability because this approach is quite common in the traditional real business cycle literature when modeling utility over consumption and leisure.[2] The benchmark real business

[1] See for example, Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991) and Greenwood, Rogerson and Wright (1993), and Rupert, Rogerson and Wright (1997). Baxter and Jermann (1994) use a home production model to interpret the excess sensitivity of consumption to current income.

[2] Empirical evidence also suggests no important nonseparability between consumption and

cycle framework to which models with home production have most often been compared assumes that utility is given by the sum of the logarithms of consumption and leisure.[3] If one believes that leisure is not valued for its own sake, but for what individuals can do with it, then it is natural to start with a model in which home consumption enters the utility function in the same way that leisure does. This approach permits a more direct comparison of the home production framework with the standard real business cycle (RBC) model, and avoids confounding the effects of incorporating a new margin of substitution with the effects of moving to a qualitatively different specification of preferences.

Second, considering a framework with additively separable utility across home and market consumption allows us to investigate the standard model with more general preferences than those typically specified in the RBC literature. Specifically, the standard model restricts preferences over consumption to logarithmic utility by imposing both constant market hours along a balanced growth path, and additive separability across consumption and leisure. This restriction is undesirable because it confines the intertemporal elasticity of substitution in con-

labor supply (e.g., see Eichenbaum, Hansen and Singleton,1988; Campbell and Mankiw, 1990; Beaudry and van Wincoop (1996)). For example, Campbell and Mankiw find that even though there is substantial predictable variation in hours, it is not related to predictable changes in consumption growth. Similarly, Eichenbaum, Hansen and Singleton find little evidence against the hypothesis that preferences are logarithmically separable between consumption and leisure.

[3]This is Hansen's 1985 divisible labor specification.

sumption to unity, a value that is inconsistent with a large and growing number of empirical estimates which suggest it is much lower.[4] By introducing steady state technological progress in the household sector, we can free up the curvature of the utility function over consumption and study the effects of introducing empirically plausible degrees of intertemporal substitution into the standard model.

Finally, and perhaps most important, we believe that models with nonseparable utility across home and market consumption obscure the positive relationship between the intertemporal elasticity of substitution in market consumption, and the elasticity of substitution between market and household consumption. In contrast, we construct a simple separable model in which these two elasticities are equal to the same constant. While the intertemporal elasticity in a nonseparable model is a much more complicated function of various preference parameters and of the home-market consumption ratio, it is nevertheless positively related to the static elasticity between market and home consumption over any plausible grid of these variables. A higher elasticity of substitution between market and home consumption goes hand-in-hand with a greater willingness to substitute consumption over time. Compared to the nonseparable framework, where the

[4] E.g., see Attanasio and Weber, 1993; Campbell, Lo, and MacKinlay, 1997; Campbell and Mankiw, 1989; and Hall, 1988. For a dissenting analysis see Beaudry and van Wincoop (1996).

complex intertemporal elasticity is often buried in the computational black box, the separable specification we consider clarifies the positive relationship between these two margins of substitution in a simple and straightforward way.

This relationship is important because most of the documented improvements in the quantitative performance of the standard model rely heavily upon a high degree of substitutability between market and home consumption. Given that there is little independent evidence available to assess whether particular values for the elasticity of substitution between home and market consumption are reasonable, models which obscure the relationship between the two margins of substitution may leave the impression that large values for the static elasticity are, at worst, empirically unrefutable. The goal of this paper is to shed light on the properties of a home production model which permits empirically plausible degrees of intertemporal substitution.

To explore the theoretical properties of a representative agent model with time-separable preferences across home and market consumption, we use the common isoelastic utility specification. Leisure time interacts with a household technological process, and possibly with household capital, to produce home goods and services, and affects utility only through its role as an input to home production. To solve the model, we use the analytical approach of Campbell (1994), focusing on four special cases discussed below. To facilitate comparison with the existing literature, we also use the solution to simulate an artificial time series for the model's variables, and contrast their relative variability and pattern of comovement with those found in the U.S. aggregate data. Several cases are studied, including a benchmark model which assigns a minimal role to the household sector, and a more general model which allows for household technological change and the use of home capital.

Our results suggest two important insights about the underlying structure of a home production model with low IES. First, in steady state, balanced growth requires the household and market sectors to display the same long run growth rate of technology. Second, out of steady state, rectifying certain well-known difficulties with the traditional RBC model requires a sufficient degree of *positive* correlation between the shocks to technology across sectors. In particular, of the four cases we consider, only those which assume that shocks across sectors are the same predict that both market consumption and market hours will be procyclical, consistent with the aggregate data.

Notice that cases which assume that the productivity shock is the same in the market and home sectors imply no *intersectoral* productivity variation resulting from shocks to technology. In contrast, those cases which impose a high degree of relative productivity variation counterfactually move market hours and market consumption inversely in response to an innovation in technology. These findings are in sharp contrast to those of other researchers (e.g., Benhabib et al, (1991)) who have emphasized the importance of intersectoral shocks for improving the quantitative performance of the traditional RBC model. The reason for this disparity is that we permit a much lower value for the IES than has been implicitly assumed elsewhere. By solving the model for a range of parameter values, we show how the cyclical behavior of market hours depends on both the IES, and on the degree to which technology shocks are correlated across sectors.

The rest of this paper is organized as follows. Section 2 presents the model and assumptions. Section 2.1 discusses the steady state, while section 2.2 outlines the solution procedure for studying the economy's response to technology shocks out of steady state. Section 3 presents the approximate analytical solutions, focusing on how technology shocks influence the model economy. Section 4 presents time series simulations of the model, and compares its dynamic properties with those of the standard RBC model, and with the U.S. data. Section 5 contains concluding remarks.

2. The Model

Consider an individual who receives utility from consumption of market goods, C, and home goods, H. This representative agent maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \rho^t U(C_t, H_t), \tag{2.1}$$

where ρ is the discount factor restricted to be between zero and one, and preferences are specified as

$$U(C_t, H_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \theta \frac{H_t^{1-\lambda}}{1-\lambda}. \tag{2.2}$$

The intertemporal elasticity of stubtitution in consumption is given by $1/\gamma \equiv \sigma$. Output is produced in both the home and market sectors according to the following Cobb-Douglas production technologies:

$$Y_t = K_t^{1-\alpha}(A_t N_t)^{\alpha}, \tag{2.3}$$

and

$$H_t = D_t^{1-\beta}(Z_t(1-N_t))^{\beta}, \tag{2.4}$$

where Y_t is market output, K_t is market capital, D_t is household capital, and N_t is the portion of labor's endowed time allocated to market activities. A_t and Z_t are labor augmenting technological shocks to the market and home production sectors, respectively.

Two points about the preferences and technologies specified above deserve mention. First, all nonmarket time is assumed to be devoted to home production rather than dividing it between leisure and home production hours. This designation captures the idea that leisure is not valued for its own sake, but for what can be done with it.[5] Time in prison and time on the golf course, for example, do not have comparable value. Second, equations (2.4) and (2.2) taken together indicate that home production equals home consumption period by period; hence investment can take place only in the market sector. More specifically, if the evolution of each capital stock is denoted by

$$K_{t+1} = (1-\delta)K_t + I_{kt} \tag{2.5}$$

[5] This approach is also taken in Greenwood and Hercowitz (1993).

and

$$D_{t+1} = (1-\delta)D_t + I_{dt}, \tag{2.6}$$

where δ is the common rate of depreciation, I_{kt} is gross business investment and I_{dt} is gross household investment, the resource constraint for market output is given by

$$Y_t = C_t + I_{kt} + I_{dt}. \tag{2.7}$$

The first order conditions are as follows. For market capital accumulation, the standard Euler equation holds:

$$C_t^{-\gamma} = \rho E_t C_{t+1}^{-\gamma}[(1-\alpha)K_{t+1}^{-\alpha}(A_{t+1}N_{t+1})\alpha + (1-\delta)], \tag{2.8}$$

where the quantity in brackets is the gross marginal product of market capital. For household capital accumulation a similar intertemporal condition holds:

$$C_t^{-\gamma} = \rho E_t C_{t+1}^{-\gamma}[(1-\beta)D_{t+1}^{-\beta}(Z_{t+1}N_{t+1})^{\beta}\theta\frac{H_{t+1}^{-\lambda}}{C_{t+1}^{-\gamma}} + (1-\delta)], \tag{2.9}$$

where we define the quantity in brackets analogously as the gross marginal product of home capital. Finally, for the allocation of labor between market and home activities there is a static first order condition:

$$\alpha K_t^{1-\alpha} A_t^{\alpha} N_t^{\alpha-1} C_t^{-\gamma} = \theta H_t^{-\lambda} \beta D_t^{1-\beta} Z_t^{\beta} (1-N_t)^{\beta-1}. \tag{2.10}$$

2.1. The balanced growth path

The driving force of steady state growth is technological progress, and we assume that A_t, Z_t, Y_t, K_t, C_t, D_t, and H_t grow at the common gross rate, G, along a balanced growth path with constant hours, N.

Equation (2.10) illustrates how hours can be constant along the balanced growth path in this model, even if utility over market consumption is not logarithmic. Taking logs and first differences of both sides, the equation implies that the following relationship holds along the balanced growth path:

$$(1-\alpha)g + \alpha g - \gamma g = -\lambda g + (1-\beta)g + \beta g, \tag{2.11}$$

where lowercase letters denote logs of variables, and the approximation $\log(G) \approx 1 + g$ has been used.

Three aspects of equation (2.11) are worth noting. First, no matter what the values of α, β, γ, and λ, balanced growth requires the steady state growth rates of A_t and Z_t to be the same. Second, no matter what the values of α and β, a restriction necessary for balanced growth is $\lambda = \gamma$. We impose this from now on. Third, without steady state technological progress in the home sector, the right hand side of (2.11) would be constant, requiring $\gamma = 1$. This restriction in traditional RBC models pins down the curvature of the utility function over consumption. In contrast, steady state technological progress in the household production sector permits a continuum of values for γ without violating balanced growth.

Along the balanced growth path, (2.8) becomes

$$G^{\gamma} = \rho R, \tag{2.12}$$

where,

$$R_{t+1} \equiv (1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta). \tag{2.13}$$

R_{t+1} is the gross marginal product of market capital, equal to a constant, R, along the balanced growth path. Note that equation (2.12) pins down the value of ρ, given G, R, and γ. By combining (2.12), (2.8), and (2.3), the steady state output to capital ratio can be obtained:

$$\frac{Y}{K} = \left(\frac{A_t N}{K_t}\right)^{\alpha} \approx \frac{r+\delta}{(1-\alpha)}, \tag{2.14}$$

where the approximate equality arises from setting $R \approx 1 + r$. Because the first order condition for market capital accumulation is the same as in the standard RBC model, (2.14) is the standard result for the output-to-capital ratio.

Equations (2.12) and (2.9) can be combined to yield the steady state ratio of home production to home capital:

$$\frac{H}{D} = \left(\frac{Z_t(1-N)}{D_t}\right)^{\beta} \approx \frac{(r+\delta)H_t^{\gamma}}{(1-\beta)\theta C_t^{\gamma}}. \tag{2.15}$$

From (2.10), the steady state ratio of home to market capital is

$$\frac{D}{K} = \frac{(1-N)\alpha(1-\beta)}{N\beta(1-\alpha)}. \tag{2.16}$$

Equation (2.16) implies that the steady state ratio of home to market capital is equal to the steady state ratio of home to market hours, if the share of market capital in market output is the same as the share of home capital in home output.

Finally, equation (2.16), along with (2.5), (2.6), and (2.7) together imply that the steady state market consumption to market capital ratio is:

$$\frac{C}{Y} = 1 - \frac{\alpha}{\beta}(1-\beta)\left(\frac{g+\delta}{r+\delta}\right)\frac{(1-N)}{N} - \frac{(g+\delta)(1-\alpha)}{r+\delta}. \tag{2.17}$$

By combining (2.17), (2.14), and (2.16), an expression for steady state C/D can be obtained. This can be equated with the ratio $C/D = \frac{H/D}{H/C}$, implied from (2.15), which explicitly links A/Z and $(1-N)/N$ *given* θ. It is difficult to know how to calibrate θ. Fortunately, it is much easier to calibrate N, and by considering a number of special cases for A/Z, we can leave the constant θ undefined in modeling fluctuations. We discuss these cases along with calibration assumptions next.

2.2. Fluctuations around steady state

Away from steady state, the model consists of a system of nonlinear expectational equations. To solve this model we use the analytical technique of Campbell (1994), which seeks an approximate solution by transforming nonlinear equations into loglinear difference equations. Each equation is loglinearized around steady state ratios of variables given above, so that variables in logs represent deviations from steady state. Below, we review the procedure only briefly, and refer the reader to Campbell (1994) for details.

Before solving the model, a number of parameter values must be chosen. Two difficult parameters to set are β and the ratio A/Z. Given that there is little evidence available to asses what values for these parameters would be reasonable, we limit our analysis to the following four cases: Case 1: $\beta = 1$, $Z_t = 1$, $\alpha < 1$, A_t varies; Case 2: $\beta = 1$, $\alpha < 1$, $A_t\alpha = Z_t$; Case 3: $\alpha = \beta$, $Z_t = 1$, A_t varies; Case 4: $\alpha = \beta$, $A_t = Z_t$. To close the model, we assume log deviations from steady state technological progress follow a first order autoregressive process, $a_{t+1} = \phi a_t + \varepsilon_{t+1}, 0 \leq \phi \leq 1$.

These four cases cover a range of possibilities. Case 1 minimizes the role of the home production sector by eliminating both home capital and innovations in home technology; thus, it is most similar to the standard RBC setup. The only difference is that home technology grows nonstochastically in steady state. This case with $\sigma \equiv 1/\gamma = 1$ is observationally equivalent to Hansen's (1985) divisible labor model with log utility over consumption and leisure. We will refer to Case 1 with $\sigma \equiv 1/\gamma = 1$ as the *standard model.* Case 2 allows fluctuations in home technology (scaled by the same factor as fluctuations in market technology), but assumes that home capital does not enter the household production function. Case 3 adds in home capital, but assumes only nonstochastic growth in home technology. Cases 1 and 3 deliver the maximum degree of *relative* productivity variation off the balanced growth path. Finally, Case 4 restricts both the technology shock and the share of capital to be the same across production functions. We discuss the model's solution in each of these cases below.

Other parameters in the model are calibrated at quarterly rates as follows. The steady state growth rate g is set to 0.005 (2 percent at an annual rate), the steady state real interest rate, r, is set equal to 0.015 (6 percent at an annual rate); α, labor's share in the market production process, is set to 0.667; the discount rate, δ, is set equal to 0.025 (10 percent at an annual rate), and N, the steady state allocation of hours to market activities is taken to be 1/3. We allow for σ and ϕ to take on a range of values, discussed below.

In cases 3 and 4 we further assume that capital can be re-allocated between the home and market sectors within the period. This assumption implies that the gross marginal products of home and market capital (defined implicitly by the two intertemporal first order conditions (2.8) and (2.9)) are equated within the period, and allows us to define a single summary capital stock state variable, $F_t \equiv K_t + D_t$, rather than having each capital stock enter the model separately. Defining a single capital stock state variable greatly simplifies the analytical solution procedure.[6]

An analytical solution to the system of nonlinear equations is sought by transforming the model into a system of approximate loglinear expectational difference

[6] When each capital stock enters the problem separately, the analytical solution procedure requires solving a pair of quadratic equations for the elasticity of market consumption with respect to each capital stock. This makes the problem intractable since the solution to this highly nonlinear system has at least four roots.

equations. As before, lower case letters denote logs of variables. In cases 1 and 2, this procedure yields a loglinear solution for the log deviation from steady state as a function of the two state variables, k_t, and a_t equal to:

$$v_t = \eta_{vk} k_t + \eta_{va} a_t \tag{2.18}$$

for $v_t = c_t, k_{t+1}, n_t, y_t, h_t$,and where η_{yx} denotes the partial elasticity of y with respect to x, assumed constant. Similarly, for Cases 3 and 4, the procedure yields a solution for the log deviation from steady state in as a function of the two state variables, f_t, and a_t :

$$v_t = \eta_{vf} f_t + \eta_{va} a_t, \tag{2.19}$$

for $v_t = c_t, f_{t+1}, k_t, d_{t,} n_t, y_t$, and h_t. The elasticities are complex functions of the parameters in the model and steady state ratios of variables discussed above. Appendix A gives the complete analytical solutions for each case.

A simplifying feature that the model shares with the standard model is that elasticities with respect to the current period capital stock ($\eta_{.k}$) depend on the IES (and therefore on the elasticity of substitution between home and market consumption), but not on the persistence parameter in the technology process (see Campbell, 1994). This is because elasticities with respect to the capital stock measure the effect on current variables of an increase in capital, holding fixed the level of technology.

3. Interpretation of the Elasticities

In this section, we consider how innovations to market and home technology influence consumption, labor supply, output, and the capital stock. These effects are given by the partial elasticities arising from the loglinear solution. Before considering how technology shocks influence these variables it may be instructive to review several key properties of the general model.

First, the elasticity of substitution between home and market consumption, which we will denote σ_{ch}, is equal to σ, the intertemporal elasticity of substitution in market consumption.[7] This model yields a one-to-one correspondence between willingness to substitute market consumption over time, and willingness to substitute between market and home produced goods. Intuitively, if individuals are relatively unresponsive to technology induced shifts in the expected real interest rate, they will also be relatively unresponsive to technology induced changes in

[7] The elasticity of substitution between home and market consumption is defined as $\frac{\partial \ln(H_t/C_t)}{\partial \ln(P_h/P_c)}$, where P_h/P_c is the shadow price of home goods, equal to $\frac{U_2(C_t,H_t)}{U_1(C_t,H_t)}$.

relative productivity differentials across sectors.

Though the separable specification we consider makes this intuition straightforward, the logic applies more generally to nonseparable specifications. For example, a popular nonseparable specification for preferences in the home production literature is given by $U = u(\tilde{C})v(L) \equiv \tilde{C}^{b(1-r)}L^{(1-b)(1-r)}/(1-r)$, where L is leisure and $\tilde{C}$ is a composite consumption good consisting of market and home consumption equal to $[aC^e + (1-a)H^e]^{1/e}$.[8] Defining the IES in this case as $\sigma \equiv -u_c/(u_{cc}C)$, it is straightforward to show that

$$\sigma = \frac{-\partial\tilde{C}/\partial C}{[(b(1-r)-1)\tilde{C}^{-1}(\partial\tilde{C}/\partial C)^2 + \partial^2\tilde{C}/\partial C^2]C}.$$

Although this quantity depends on the amount of home consumption relative to market consumption, as well as on the parameters a, b, and r, over a grid covering reasonable ranges of these variables and parameters, σ is increasing in e, and therefore increasing in σ_{ch}, equal to $1/(1-e)$ in this model.[9]

Values of e typically assumed in the home production literature are as high as

[8] See for example, Benhabib et al (1993) and McGrattan et al (1993). These papers assume that leisure, L, can be split into home production and non-home production activities, so that nonmarket consumption, H, enters utility separately from leisure.

[9] Over any reasonable range of variables, the value of σ, which does not depend on the scale of C, is at least as large as one, and only approaches one when e is close to zero.

0.8 (e.g., Benhabib, et al, 1991), implying a value for the IES considerably above unity. Values for e that are this sizable are at odds with a large number of empirical estimates cited above on the IES. These estimates are obtained using a number of different data sets and interest rate measures, and using both aggregate and household level data. These studies suggest that the IES is considerably smaller than one.[10]

A second property of the general model concerns the behavior of market hours as market and home consumption become highly complementary. As σ approaches zero, even though the agent is very averse to shifting the ratio of C to H, N can shift, and adjusts passively to insure a fixed ratio of home to market consumption.

3.1. The effects of home and market technology shocks

Elasticities with respect to a_t give the model's predictions for how a technology shock influences the economy. Below, we focus our discussion on these elasticities,

[10] We are aware of only two studies which attempt to estimate the value of e in the home production model specified above. McGratten et al, 1993, use aggregate data, and Rupert et al, 1995, use household level data from the Panel Study of Income Dynamics (PSID). It should be noted that neither of these studies estimate values for e that are nearly as large as 0.8; the former study finds an estimate for e equal to 0.385, while the latter study finds a very small (and imprecisely estimated) value of e for single men, and a statistically significant, albeit again considerably smaller than 0.8, value of e for single women. Moreover, in reconciling these two studies, it is unclear that the degree of substitutability between market and home consumption estimated using household level data can be expected to carry through to the aggregate (i.e., for a representative agent). Deaton (1992) summarizes a large literature which documents the bias that can arise when using aggregate data to estimate individual parameters, or vice versa.

though the capital elasticities are also provided in the tables for reference.

Table 1 gives consumption, capital, employment, and output elasticities for Case 1. The table shows the numerical values of the elasticities, for the benchmark values of the parameters discussed above, and for various values of σ and ϕ. σ is set equal to $0, 0.2, 1, 5$, and ∞. ϕ is set equal to $0, 0.5, 0.95$, and 1.

Case 1 generalizes the standard RBC model by freeing up the curvature in the utility function over consumption. As a result, it is worthwhile elaborating on several features of this case. First, as already noted, when $\sigma = 1$, this case collapses to the standard Hansen RBC model with divisible labor and log utility over consumption and leisure. Hence the elasticities in the middle column of Table 1 are the same as those given in Campbell (1994), which uses the analytical technique employed here to solve the standard model.

Second, the elasticity of consumption with respect to a positive technology shock, η_{ca}, is increasing in persistence for low σ, but decreasing for high σ. When σ is low, substitution effects are weak and the agent responds primarily to income effects which increase with the persistence of a technology shock. When σ is high, substitution effects are important, and a more persistent technology shock increases the interest rate today and in the future, motivating a large substitution into consumption tomorrow; hence consumption elasticities can be very small, or even negative.

Third, in the extreme case when $\sigma = \infty$, a positive technology shock leads to a very large decrease in consumption and a very large increase in next period's capital stock for $\phi > 0$. In this case, the representative agent is risk neutral and consumers are infinitely willing to substitute consumption over time in response to fluctuations in the marginal product of capital. Since risk neutrality fixes the *ex-ante* real interest rate, a positive technology shock produces a very large substitution out of today's consumption, and into tomorrow's consumption.[11] This effect is absent when $\phi = 0$ because a purely transitory technology shock does not directly affect the *ex-ante* real interest rate.

Fourth, when the IES is less than one, the response of labor supply to a positive technology shock (η_{na}) is negative. This implies that consumption and hours are negatively correlated since η_{ca} is positive for small values of σ. This counter-factual prediction illustrates a well known problem in the RBC literature which can be

[11]To see why the consumption response in this case must be so large, recall that tomorrow's consumption effects the expected real interest rate through it's influence on tomorrow's labor supply. An increase in consumption tomorrow lowers the marginal utility of consumption, motivating a decrease in labor supply. As labor supply declines, the marginal product of capital is driven down. Since linear utility fixes the *ex-ante* real interest rate, this decline in the marginal product of capital is needed to offset the increase brought about by a positive shock to technology. However, when $\sigma = \infty (\gamma = 0)$, labor supply is very unresponsive to movements in consumption, because it responds to the *marginal utility* of consumption. Consequently, infinite changes in consumption are required to induce a change in labor supply. In the standard RBC model, the response of consumption to a technology shock is never infinite because the curvature of the utility function over consumption is fixed at $\gamma = 1$.

seen by referring back to (2.10), setting $\beta = Z_t = 1$, and $H_t = (1 - N_t)$: if the real wage is constant, the marginal utility of consumption will be perfectly correlated with the marginal disutility of labor hours. If consumption rises, the marginal utility of consumption falls, requiring a decrease in the marginal disutility of work, or an increase in leisure; hence the model predicts a counter-factual negative correlation between market hours and consumption. This problem can be resolved if the real wage is procyclical, but only if marginal utility does not decline too rapidly, a condition that does not hold when $\sigma < 1$. This explains why hours and consumption are negatively correlated when the intertemporal elasticity of substitution is less than one.

Hence, Case 1, where technology shocks only affect the market sector, illustrates a fundamental difficulty in RBC models with low IES: it predicts that market hours will be countercyclical. Table 2 demonstrates how adding a home production sector to the standard model can remedy this problem. The table shows the elasticities for Case 2, when $\beta = 1$, but $Z_t = A_t^{\alpha}$. In this case, shocks to technology affect the home and market sectors symmetrically. First, note that a shock to technology now leads to an increase in home production ($\eta_{ha} > 0$) since it increases productivity in both sectors. More importantly, the labor supply elasticities are now *decreasing* in σ for all values of ϕ except $\phi = 1$. Hence, when σ

is low, market hours respond positively to an increase in technology, and labor supply is both procyclical and positively correlated with market consumption.

To understand this difference from Case 1, it is easiest to think about how the elasticities vary with ϕ, for a given σ. As previously discussed, the elasticity of consumption with respect to a positive technology shock, η_{ca}, is increasing in persistence for low σ, but decreasing for high σ. Since higher consumption reduces the marginal utility of (market) income, this leads to the opposite pattern for labor supply: the elasticity of market hours with respect to a positive technology shock, η_{na}, is *decreasing* in persistence for low σ, but *increasing* for high σ. This pattern also holds in Case 1 for the same reason. The difference in Case 2 is that higher market consumption (resulting from a positive technology shock) does not reduce the marginal utility of market consumption *relative* to home consumption as much as in Case 1, so that the labor supply elasticities are much larger when σ is low (and income effects are strong) than they are in Case 1.

Intersectoral shocks are unhelpful because they exacerbate the inverse relation between market hours and market consumption that already exists due to the rapidly declining marginal utility that preferences with low σ imply. This suggests that home production models with low IES require a sufficient degree of positive correlation in technology shocks across sectors. Put another way, they require a minimal degree of *relative* productivity variation. This finding contrasts with the results of preexisting home production studies which emphasize the importance of a relatively high degree of productivity variation across sectors in order to improve the quantitative performance of the standard model. A quick inspection of Table 1 (as well as Table 3—discussed in more detail below) displays the reason for the difference: we permit a much lower value for the IES (and therefore a much lower value for σ_{ch}) than has been implicitly assumed elsewhere. As the tables show, when σ is sufficiently high, market hours and market consumption are both procyclical even in cases where there is a large amount of relative productivity variation across sectors.

Table 3 gives the results for Case 3, where there is home capital, but no technology shocks to the household sector. As in Case 1, a shock to a_t creates a large differential in productivity across the home and market sectors.

The value of σ has several notable affects on the elasticities in Case 3. First, elasticities with respect to market consumption, market output, and market hours are generally increasing in σ; the more willing individuals are to substitute both intertemporally and intratemporally, the larger are the effects on the economy of a technology shock to the market sector. Second, when σ is very large, a positive technology shock induces a very large substitution into market consumption and market output, and out of home consumption (home output). Third, when σ is sufficiently large, a positive shock to technology motivates a large increase in market capital and market time ($\eta_{ka} = \eta_{na} = \infty$) which must be offset by a large decrease in home capital and home time ($\eta_{da} = \eta_{1-na} = -\infty$), since capital is reallocated within the period.

Finally, elasticities for Case 4 are shown in Table 4. In this case, both sectors utilize capital and technology in the same proportion, and shocks to technology across sectors are perfectly correlated. This implies that $\eta_{ca} = \eta_{ha}$ for all σ and ϕ. To understand this, it is easiest to consider parameter values for which substitution effects are strong. Combinations of high σ and high ϕ produce strong *intertemporal* substitution effects, but *intratemporal* substitution effects are washed out because a technology shock produces no relative productivity differential between the two sectors. The only way to substitute intertemporally is to increase market output over market consumption; hence market hours rise ($\eta_{na} > 0$) and market consumption falls ($\eta_{ca} < 0$). The relative scarcity of market consumption reduces the benefit of additional home capital, motivating a reallocation to the market sector ($\eta_{da} < 0, \eta_{ka} > 0$), and a decline in home consumption ($\eta_{ca} = \eta_{ha} < 0$).

4. Simulation Results

The elasticities discussed above summarize how the model's properties change with key parameter values. The results indicate that the solution is very sensitive to the assumed values of σ and ϕ. To gain further understanding into the model's predictions at empirically plausible values of σ and ϕ, and to compare them with those of other RBC and home production models, it is useful to undertake simple time-series simulations of the model. We can then carry out the exercise typically performed in the RBC literature of asking how well moments from the simulated data match those from the U.S. data.

We focus on the model's properties when the IES is set equal to empirically plausible levels. A survey of the many studies cited above which estimate this parameter suggests that it is well below one, and in many cases close to zero. Therefore, we consider 0.20 to be a conservative value for this parameter, and we use it in the simulations reported below.

We choose parameters for the technology process that are fairly standard.[12] In particular, we assume that the AR(1) process for the log technology is given

[12] Our parameter choice for the variance of technology shocks coincides with those in Benhabib, Rogerson, Wright, 1993, Greenwood and Hercowitz, 1993 and Greenwood, Rogerson and Wright, 1995, while our choice of ϕ is consistent with Benhabib et al, and Greenwood, Rogerson, and Wright.

by $a_t = 0.95a_{t-1} + \epsilon_t$, where ϵ_t is normally distributed with a standard deviation equal to 0.007. Using allocation rules implied by the elasticities reported above, 100 simulations of 150 periods each are computed.[13] The simulated data are filtered using Hodrick-Prescott technique before computing any statistics, again following the home production literature.

Panel A of Table 6 gives selected moments and comoments from U.S. quarterly data over the period 1959:1-1996:4[14] for the following log real variables: output, y, consumption, c, investment, i, average productivity, w, market capital, k, and market hours, n. For each of these variables, the table gives the percent standard deviation in the variable relative to the percent standard deviation of y, and the cross correlation of the variable with y. Data details are given in Appendix B.

Panel B uses simulated data to summarize the cyclical properties of the standard model (Case 1 with $\sigma = 1$). The panel reveals several well known discrepancies between the model's predictions, and key aspects of the U.S. data. These discrepancies can be summarized as follows: compared to the data, output is not volatile enough; relative to output, consumption and hours are not volatile

[13]Each simulation consists of a random sample of 150 realiztions of ϵ_t, which is then used to compute the values of each of the other variables in the model using the decision rules reported above.

[14]This sample period applies for all series except the capital stock, for which the most recent data runs from 1959:1-1994:1.

enough; relative to output, investment is too volatile, and productivity (w) is too highly correlated with output. Existing home production studies have documented significant improvements in the standard model's performance, along all of these dimensions, as the result of explicitly incorporating a household sector into the standard model (e.g. see Benhabib, et al., 1991). Next, we ask whether those improvements are maintained in our model with low intertemporal elasticity of substitution in consumption.

Panels C-F of Table 6 show statistics computed off the simulated data for cases 1-4. Note that Case 1 is simply the standard model with $\sigma = 0.2$ instead of $\sigma = 1$.

Focusing on the problems discussed above, Table 6 reveals that none of the cases produce results that represent a clear improvement over the standard model's performance. Instead, generalizing the standard model to include household technological progress and a low intertemporal substitution elasticity appears to significantly deteriorate its quantitative performance along several dimensions. For example, in every case, the volatility of investment relative to output is larger, the volatility of consumption relative to output is smaller, and the correlation of productivity with output is higher than in the standard model. Furthermore, Cases 1 and 3 have output less volatile than the standard model, and only Case 4 yields output that is more volatile.

Although the four home production models considered in Table 6 perform worse than the standard model with log utility over consumption and hours, we do not believe the latter is an appropriate benchmark to which models of home production with low IES should be compared. Instead, we argue that Case 1, which minimizes the role of the home production sector but permits a more empirically plausible (lower) IES than the standard model, is the relevant benchmark.

Table 6 shows that, relative to Case 1, Cases 2 and 4, which permit an explicit role for home production with symmetric technology shocks, generally represent an improvement over the low σ benchmark. For example, Case 1 predicts a counter-factual negative correlation between market hours and output. This is consistent with the negative labor supply elasticities (η_{na}) found in Table 1 when $\sigma = 0.2$. And while Case 3 does not produce a negative value for this correlation when $\sigma = 0.2$, a quick inspection of Table 3 indicates that it will produce a negative value for smaller σ (i.e., $\eta_{na} < 0$ when $\sigma = 0$). In contrast, both cases which impose the same technology shocks across sectors (Case 2 and Case 4) yield procyclical market hours. Moreover, unlike Case 3, Cases 2 and 4 also represent a clear improvement in the relative volatility of both hours and wages over the low σ benchmark.

Table 6 nevertheless demonstrates that some problems remain with the low

σ models, even when technology shocks across sectors are the same. The most notable difficulty is that both Cases 2 and 4 continue to predict investment that is too volatile, and consumption that is too smooth, relative to output. While this difficulty may at first seem rather glaring, others (e.g., Baxter and Crucini, 1993) have shown that the problem can be resolved by allowing for adjustment costs in market capital.

In summary, simulation results presented in this section demonstrate that the simple model of home production studied here, with a relatively low value of the IES, does not yield the quantitative improvements over the standard RBC model which has a higher IES. However, if one accepts the that the standard model imposes an implausibly high value for the IES, the introduction of a home production sector that shares a common technology shock with the market sector, does improve the quantitative performance of the RBC model relative to a more appropriate *low σ* benchmark. And, while the home production models we consider have difficulty matching the relative volatility of investment and consumption found in the U.S. data, it seems likely that these problems can be addressed in a richer model with adjustment costs in market capital.

5. Conclusions

Little evidence is available to assess the empirical validity of several key parameters in models with home production. One such parameter is the elasticity of substitution between home and market consumption. Yet theoretical models in the existing household production literature typically assume that home and market consumption are highly substitutable. Our strategy for 'calibrating' the static substitution parameter is to calibrate the intertemporal elasticity of substitution instead, making use of the positive relation that exists between the two. In doing so, we rely on a large body of empirical evidence which suggests that the value of the IES is substantially below unity.

The framework studied in this paper allows us to cover a range of possibilities with respect to generalizing the standard RBC model to include home production. A minimal generalization de-emphasizes the role of the home production sector, but allows us relax the curvature of the utility function over consumption in the standard model. The most general specification incorporates a complete home production function with household capital and stochastic shifts in household technological progress. The most striking finding across the range of specifications we consider is how crucial the value of the intertemporal elasticity of substitution is for determining how well these models match the cyclical properties of the data.

We have utilized a simple time-separable model to illustrate the connection between the intertemporal elasticity of substitution, and the elasticity of substitution between home and market consumption. Many of the previously documented improvements in the quantitative performance of the standard model, which arise from introducing home production, can be achieved in the framework studied here with a much higher value for σ. Hence, the key difference between the model in this paper and those studied elsewhere, is the choice of numerical value for σ.

While preexisting home production studies have focused their analysis on models which allow for a relatively high value for the IES, the goal of this study is to shed light on the underlying structure of a home production model with empirically plausible degrees of intertemporal substitutability. We develop a low IES benchmark by introducing steady state, home sector technological growth into the standard, time-separable RBC model.

Our results suggest two key insights about the underlying structure of a home production model with low IES. First, freeing up the curvature of the utility function while maintaining balanced growth requires that the household and market sectors display the same long run rate of technological progress. Second, we find that the cyclical behavior of market hours is not well captured in a home production model with a high degree of relative productivity variation across sectors. In contrast to models which impose higher values for the IES, intersectoral technology shocks are not helpful because they tend to make hours and consumption move inversely.

Appendix A

This appendix provides complete solutions to the loglinearized model for each of the four cases we consider. We use the method of Campbell (1994). Here we provide only the solutions for the elasticities, and refer the reader to Campbell (1994) for details about the procedure. In each case, the model's equations are made linear in logs by approximating them with first order Taylor expansions around steady state values. We start with the most general case and proceed backwards.

Case 4

Combining (2.6) and (2.5) we get an accumulation equation for $F_t \equiv K_t + D_t$:

$$F_{t+1} = (1-\delta)F_t + Y_t - C_t. \tag{A.1}$$

Taking logs of both sides and linearizing the right hand side yields an equation for f_{t+1} :

$$f_{t+1} = \lambda_1 k_t + \lambda_2(a_t + n_t) + \lambda_3 c_t \lambda_4 f_t, \tag{A.2}$$

where,

$$\lambda_1 \equiv \frac{(r+\delta)N}{1+g}, \quad \lambda_2 \equiv \frac{(r+\delta)N\alpha}{(1+g)(1-\alpha)}, \quad \lambda_3 \equiv \frac{(\delta+g)}{1+g} - \frac{(r+\delta)N}{(1+g)(1-\alpha)}, \quad \lambda_4 \equiv 1 - \frac{\delta+g}{1+g}.$$

Loglinearizing the work-wage first-order condition (2.10) yields an equation for log hours:

$$n_t = \nu_1 k_t + \nu_2 d_t + \nu_3 a_t + \nu_4 c_t, \tag{A.3}$$

where,

$$\nu_1 \equiv \nu^*(1-\alpha), \quad \nu_2 \equiv \nu^*(1/\sigma - 1)(1-\alpha), \quad \nu_3 \equiv \nu^*\alpha/\sigma, \quad \nu_4 \equiv -\nu^*/\sigma,$$

and where,

$$\nu^* \equiv (\nu(1-N)\sigma)/((1-N)\sigma + \nu\alpha N), \quad \nu \equiv (1-N)/(1-\alpha).$$

Equation (2.8) is loglinearized assuming that R_{t+1} and C_{t+1} are jointly log-normal and homoskedastic to obtain:

$$E_t \Delta c_t = E_t[\xi_1 k_{t+1} + \xi_2 d_{t+1} + \xi_3 a_{t+1} + \xi_4 c_{t+1}], \tag{A.4}$$

where,

$$\xi_1 \equiv (\sigma\alpha(r+\delta)(\nu_1 - 1))/(1+r), \quad \xi_2 \equiv (\sigma\alpha(r+\delta)\nu_2)/(1+r)$$

$$\xi_3 \equiv (\sigma\alpha(r+\delta)(\nu_3 + 1))/(1+r), \quad \xi_4 \equiv (\sigma\alpha(r+\delta)\nu_4)/(1+r).$$

We assume that individuals can reallocate capital between the home and market sectors within the period. This allows us to equate the gross marginal products of each type of capital in (2.8) and (2.9) yielding an equation for k_t and d_t in terms of f_t, a_t, and c_t:

$$k_t = \pi_1 f_t + \pi_2 a_t + \pi_3 c_t \tag{A.5}$$

$$d_t = \chi_1 f_t + \chi_2 a_t + \chi_3 c_t, \tag{A.6}$$

where,

$$\chi_1 \equiv \omega_1^* \pi_1, \quad \chi_2 \equiv \omega_1^* \pi_2 + \omega_2^*, \quad \chi_3 \equiv \omega_1^* \pi_3 + \omega_3^*,$$

$$\omega_1^* \equiv (\omega_1 \nu_1 + \omega_3)/(1 - \omega_1 \nu_2), \quad \omega_2^* \equiv (\omega_1 \nu_3 + \omega_2)/(1 - \omega_1 \nu_2)$$

$$\omega_3^* \equiv (\omega_1 \nu_4 + \omega_4)/(1 - \omega_1 \nu_2),$$

and where,

$$\omega_1 \equiv -(N/(1-N)\alpha(1 - 1/\sigma)\sigma/((1-\alpha) + \alpha\sigma) - \alpha\sigma/((1-\alpha) + \alpha\sigma),$$

$$\omega_2 \equiv -\alpha/((1-\alpha) + \alpha\sigma), \quad \omega_3 \equiv \alpha\sigma/((1-\alpha) + \alpha\sigma), \quad \omega_4 \equiv 1/((1-\alpha) + \alpha\sigma),$$

$$\pi_1 \equiv (1/N)/(1 + (1-N)\omega_1^*/N), \quad \pi_2 \equiv -(1-N)\omega_2^*/(N + (1-N)\omega_1^*),$$

$$\pi_3 \equiv -\omega_3^*(1-N)/(N + (1-N)\omega_1^*).$$

The solution proceeds by the method of undetermined coefficients, by making an initial guess that the loglinear solution will be of the form specified in (2.19)-(??).

η_{cf} solves the quadratic equation:

$$Q_2\eta_{cf}^2 + Q_1\eta_{cf} + Q_0 = 0, \tag{A.7}$$

where,

$$Q_2 \equiv (\psi_3\mu_3 - \mu_3), \quad Q_1 \equiv (1 + \psi_1\mu_3 + \psi_3\mu_1 - \mu_1), \quad Q_0 \equiv \psi_1\mu_1.$$

where,

$$\psi_1 \equiv \xi_1\pi_1 + \xi_2\chi_1, \quad \psi_2 \equiv \xi_1\pi_2 + \xi_2\chi_2 + \xi_3, \quad \psi_3 \equiv \xi_1\pi_3 + \xi_2\chi_3 + \xi_4,$$

and where,

$$\mu_1 \equiv \lambda_1^*\pi_1 + \lambda_3^*\chi_1 + \lambda_5^*, \quad \mu_2 \equiv \lambda_1^*\pi_2 + \lambda_3^*\chi_2 + \lambda_2^*, \quad \mu_3 \equiv \lambda_1^*\pi_3 + \lambda_3^*\chi_3 + \lambda_4^*,$$

where,

$$\lambda_1^* \equiv \lambda_1 + \lambda_2\nu_1, \quad \lambda_2^* \equiv \lambda_2 + \lambda_2\nu_3, \quad \lambda_3^* \equiv \lambda_2\nu_2, \quad \lambda_4^* \equiv \lambda_3 + \lambda_2\nu_4, \quad \lambda_5^* \equiv \lambda_4.$$

η_{ca} is given by

$$\eta_{ca} = \frac{-(\psi_1\mu_2 + \psi_3\mu_2\eta_{cf} - \mu_2\eta_{cf} + \psi_2\phi)}{\psi_1\mu_3 + \psi_3\mu_3\eta_{cf} - \mu_3\eta_{cf} + \psi_3\phi + 1 - \phi} \tag{A.8}$$

Elasticities of total capital with respect to last period's total capital and the log technology shock are then found as

$$\eta_{ff} = \mu_1 + \mu_3 \eta_{cf}, \quad \eta_{fa} = \mu_2 + \mu_3 \eta_{ca}.$$

All other elasticities are defined in terms of the elasticities above:

$$\eta_{kf} = \pi_1 + \pi_3 \eta_{cf}, \quad \eta_{ka} = \pi_2 + \pi_3 \eta_{ca}, \quad \eta_{df} = \chi_1 + \chi_3 \eta_{cf}, \quad \eta_{da} = \chi_2 + \chi_3 \eta_{ca},$$

$$\eta_{nf} = \nu_1 \eta_{kf} + \nu_2 \eta_{df} + \nu_4 \eta_{cf}, \quad \eta_{na} = \nu_1 \eta_{ka} + \nu_2 \eta_{da} + \nu_3 + \nu_4 \eta_{ca},$$

$$\eta_{yf} = \alpha \eta_{nf} + (1-\alpha)\eta_{kf}, \quad \eta_{ya} = \alpha(1 + \eta_{na}) + (1-\alpha)\eta_{ka},$$

$$\eta_{hf} = (1-\alpha)\eta_{df} - \alpha N \eta_{nf}/(1-N), \quad \eta_{ha} = (1-\alpha)\eta_{da} + \alpha - \alpha N \eta_{na}/(1-N).$$

Case 3

Parameter definitions for Case 3 are the same as in Case 4, with the following exceptions:

$$\omega_2 \equiv -\alpha\sigma/((1-\alpha) + \alpha\sigma), \quad \nu_3 \equiv \nu^* \alpha, \quad \eta_{ha} = (1-\alpha)\eta_{da} - \alpha N \eta_{na}/(1-N).$$

Case 2

The solutions for the elasticities given in (2.18)-(??) for Case 2 yield a quadratic equation in k for η_{ck},

$$Q_2 \eta_{ck}^2 + Q_1 \eta_{ck} + Q_0 = 0, \tag{A.9}$$

where,

$$Q_2 \equiv (\psi_3\mu_3 - \mu_3), \quad Q_1 \equiv (1 + \psi_1\mu_3 + \psi_3\mu_1 - \mu_1), \quad Q_0 \equiv \psi_1\mu_1.$$

where,

$$\psi_1 \equiv (\nu_1 - 1)\lambda_3\sigma, \quad \psi_2 \equiv (\nu_2 - 1)\lambda_3\sigma, \quad \psi_3 \equiv \nu_3\lambda_3\sigma,$$

and where,

$$\mu_1 \equiv \lambda_2\nu_1 + \lambda_1, \quad \mu_2 \equiv \lambda_2\nu_1 + \lambda_2, \quad \mu_3 \equiv 1 - \lambda_1 - \lambda_2 + \lambda_2\nu_3,$$

$$\nu_1 \equiv \nu^*(1-\alpha), \quad \nu_2 \equiv \nu^*\alpha(1/\sigma), \quad \nu_3 \equiv -\nu^*\sigma,$$

$$\nu^* \equiv \tfrac{(1-N)\sigma}{(1-\alpha)(1-N)\sigma + N},$$

$$\lambda_1 \equiv \tfrac{1+r}{1+g}, \quad \lambda_2 \equiv \tfrac{(r+\delta)\alpha}{(1+g)(1-\alpha)}, \quad \lambda_3 \equiv \tfrac{(r+\delta)\alpha}{1+r},$$

$$\psi_1 \equiv (\nu_1 - 1)\lambda_3\sigma, \quad \psi_2 \equiv (\nu_2 + 1)\lambda_3\sigma, \quad \psi_3 \equiv \nu_3\lambda_3\sigma.$$

The elasticity of consumption with respect to technology is a function of η_{ck}:

$$\eta_{ca} = \frac{-(\psi_1\mu_2 + \psi_3\mu_2\eta_{ck} - \mu_2\eta_{ck} + \psi_2\phi)}{\psi_1\mu_3 + \psi_3\mu_3\eta_{ck} - \mu_3\eta_{ck} + \psi_3\phi + 1 - \phi}, \tag{A.10}$$

and the rest of the elasticities are defined in terms of the consumption elasticities:

$$\eta_{kk} = \mu_1 + \mu_3\eta_{ck}, \quad \eta_{ka} = \mu_2 + \mu_3\eta_{ca},$$

$$\eta_{nk} = \nu_1 + \nu_3\eta_{ck}, \quad \eta_{na} = \nu_2 + \nu_3\eta_{ca},$$

$$\eta_{yk} = 1 - \alpha + \alpha\eta_{nk}, \quad \eta_{ya} = \alpha + \alpha\eta_{na},$$

$$\eta_{hk} = -N\eta_{nk}/(1-N), \quad \eta_{ha} = \alpha - N\eta_{na}/(1-N).$$

Case 1

Parameter definitions for Case 1 are the same as in Case 2, with the following exceptions:

$$\nu_2 \equiv \nu^*\alpha, \quad \eta_{ha} = -N\eta_{na}/(1-N).$$

Appendix B

This appendix reviews the data used to compute the summary statistics in the first panel of Table 6. All series are per capita, measured at quarterly frequency, seasonally adjusted, and chain weighted in 1992 dollars, except where otherwise noted.

Consumption

Consumption is the sum of personal consumption expenditures (PCE) on non-durables and services, excluding expenditure on housing services, 1959:3-1996:4. Source: Bureau of Economic Analysis (BEA).

Investment

Total investment series is defined as residential and non-residential investment plus personal expenditure on consumer durables. Source: Bureau of Economic Analysis (BEA).

Hours

This series is aggregate hours of all wage and salary workers in non-agricultural industries, in millions. These data are monthly and converted to quarterly averages over the period 1959:1-1997:2. Source: Bureau of Labor Statistics.

Capital Stock

This series is the constant-cost net stock of fixed nonresidential structures and equipment, in billions of 1987 dollars from 1959-1994 at annual frequency. The data are linearly interpolated to quarterly frequency. Source: Bureau of Economic Analysis.

Output

The output series is constructed as consumption plus investment, following Benhabib et al.

Productivity

Average productivity (proportional to the real wage with Cobb-Douglas technology) is output divided by hours, defined above.

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Table 1

Elasticities for Model 1: $\alpha < 1, \beta = 1, Z_t = 1, A_t$ varies

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.04	0.21	0.54	1.19	2.00
	1.00	0.97	0.94	0.92	0.91
All ϕ	-0.08	-0.26	-0.24	0.22	1.00
	0.28	0.16	0.17	0.48	1.00
	0.04	0.13	0.12	-0.11	-0.50
	0.00	0.02	.07	0.23	0.44
	0.08	0.09	0.13	0.17	0.20
0.00	-0.01	0.20	0.71	1.43	2.00
	0.66	0.80	1.14	1.62	2.00
	0.00	-0.10	-0.36	-0.72	-1.0
	0.01	0.03	0.10	0.15	$-\infty$
	0.08	0.09	0.13	0.18	∞
0.5	-0.01	0.18	0.68	1.47	2.18
	0.66	0.78	1.12	1.65	2.12
	0.01	-0.09	-0.34	-0.74	-1.09
	0.05	0.17	0.29	-0.36	$-\infty$
	0.07	0.06	0.09	0.25	∞
0.95	-0.11	-0.06	0.45	1.70	3.31
	0.60	0.63	0.97	1.80	2.87
	0.05	0.03	-0.22	-0.85	-1.65
	0.32	0.37	0.46	-7.0	$-\infty$
	0.00	0.01	0.06	0.29	∞
1.0	-0.64	-0.41	0.24	1.86	4.00
	0.24	0.39	0.83	1.91	3.34
	0.32	0.21	-0.12	-0.93	-2.0

Notes: α is the share of labor in market production; β is the share of labor in home production; ϕ is the persistence parameter on the market technology process; σ is the intertemporal elasticity of substitution in consumption. The column of numbers in each cell give the elasticities of market consumption, next period's market capital, market labor supply, market output, and home output, respectively. In the first row are η_{ck}, η_{kk}, η_{nk}, η_{yk}, and η_{hk}, the elasticities with respect to this period's market capital with do not vary with ϕ. The last four rows give η_{ca}, η_{ka}, η_{na}, η_{ya}, and η_{ha}, the elasticities with respect to market technology for selected values of ϕ.

Table 2

Elasticities for Model 2: $\alpha < 1, \beta = 1, Z_t = A_t^{\alpha}$

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.04	0.21	0.54	1.19	2.00
	1.00	0.97	0.94	0.92	0.91
All ϕ	-0.08	-0.26	-0.24	0.22	1.00
	0.28	0.16	0.17	0.48	1.00
	0.04	0.13	0.12	-0.11	-0.50
	0.01	0.04	0.07	0.12	0.15
	0.18	0.17	0.13	0.09	0.07
0.00	1.32	1.11	0.71	0.25	0.00
	1.55	1.41	1.14	0.84	0.67
	0.01	0.11	0.31	0.54	0.67
	0.01	0.06	0.10	0.08	$-\infty$
	0.18	0.16	0.13	0.09	∞
0.5	1.30	1.07	0.68	0.27	0.06
	1.54	1.38	1.12	0.85	0.71
	0.01	0.13	0.33	0.53	0.64
	0.12	0.30	0.29	-0.18	$-\infty$
	0.16	0.11	0.09	0.13	∞
0.95	1.09	0.65	0.45	0.39	0.44
	1.39	1.10	0.97	0.93	0.96
	0.12	0.34	0.76	0.47	0.45
	0.75	0.65	0.46	-0.36	$-\infty$
	0.00	0.02	0.06	0.15	∞
1.0	-0.16	0.04	0.24	0.47	0.67
	0.56	0.69	0.83	0.98	1.11
	0.75	0.65	0.55	0.43	0.33

Notes: See Table 1.

Table 3

Elasticities for Model 3: $\alpha = \beta, Z_t = 1, A_t$ varies

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.00	0.01	.05	0.35	∞
	0.03	0.04	0.08	0.26	22.6
	-0.01	0.24	1.24	5.97	∞
0.00	0.00	-0.12	-0.62	-2.98	$-\infty$
	-0.01	0.24	1.24	5.97	∞
	0.66	0.91	1.91	6.64	∞
	0.00	-0.12	-0.62	-2.98	$-\infty$
	0.01	0.02	0.06	0.52	∞
	0.03	0.04	0.07	0.25	11.8
	-0.01	0.23	1.20	5.62	∞
0.5	0.01	-0.12	-0.60	-2.81	$-\infty$
	-0.01	0.23	1.20	5.62	∞
	0.66	0.90	1.87	6.28	∞
	0.01	-0.12	-0.60	-2.81	$-\infty$
	0.05	0.08	0.23	1.57	∞
	0.02	0.03	0.06	0.15	2.08
	-0.10	0.10	0.88	3.54	∞
0.95	0.05	-0.05	-0.44	-1.77	$-\infty$
	-0.10	0.10	0.88	3.54	∞
	0.57	0.77	1.54	4.20	∞
	0.05	-0.05	-0.44	−1.77	$-\infty$
	0.30	0.22	0.41	2.16	∞
	0.00	0.02	0.04	0.10	1.00
	-0.59	-0.18	0.51	2.34	∞
1.0	0.30	0.09	-0.26	-1.17	$-\infty$
	-0.59	-0.18	0.51	2.34	∞
	0.07	0.49	1.18	3.01	∞
	0.30	0.09	-0.26	-1.17	$-\infty$

Notes: α is the share of labor in market production; β is the share of labor in home production; ϕ is the persistence parameter on the market technology process; σ is the intertemporal elasticity of substitution in consumption. The column of numbers in each cell give the elasticities of market consumption, next period's total capital, this period's market capital, this period's home capital, market labor supply, market output, and home output, respectively. The four rows give η_{ca}, η_{fa}, η_{ka}, η_{da}, η_{na}, η_{ya}, and η_{ha}, the elasticities with respect to market technology for selected values of ϕ.

Table 4

Elasticities for Model 4: $\alpha = \beta, Z_t = A_t$

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.01	0.02	0.05	0.10	0.89
	0.08	0.08	0.08	0.07	0.00
	1.32	1.29	1.24	1.14	-0.44
0.00	-0.66	-0.64	-0.62	-0.57	0.22
	1.32	1.29	1.24	1.14	-0.44
	1.98	1.95	1.91	1.81	0.22
	0.01	0.02	0.05	0.10	0.89
	0.02	0.04	0.06	0.07	-4.69
	0.08	0.08	0.07	0.07	0.50
	1.30	1.25	1.20	1.20	10.7
0.5	-0.65	-0.63	-0.60	-0.60	-5.36
	1.30	1.25	1.20	1.20	10.7
	1.97	1.92	1.87	1.87	11.4
	0.02	0.04	0.06	0.07	-4.69
	0.15	0.25	0.23	-0.11	-9.72
	0.07	0.06	0.06	0.09	0.95
	1.04	0.83	0.88	1.55	20.8
0.95	-0.52	-0.42	-0.44	-0.77	-10.4
	1.04	0.83	0.88	1.55	20.8
	1.70	1.50	1.54	2.21	21.4
	0.15	0.25	0.23	-0.18	-9.72
	0.89	0.70	0.41	-0.21	-10.3
	0.00	0.02	0.04	0.10	1.00
	-0.44	-0.06	0.51	1.75	21.9
1.0	-0.22	0.03	-0.26	-0.87	-10.9
	-0.44	-0.06	0.51	1.75	21.9
	0.22	0.60	1.18	2.41	22.6
	0.89	0.70	0.41	-0.21	-10.3

Notes: See Table 3.

Table 5

Elasticities with respect to capital for Models 3 and 4

	$\sigma =$				
$\phi =$	0	0.2	1	5	∞
	0.11	0.31	0.59	1.21	11.3
	1.00	0.98	0.96	0.90	0.00
	1.44	1.06	0.49	-0.75	-20.9
All ϕ	0.78	0.97	1.26	1.87	11.9
	0.44	0.06	-0.51	-1.75	-21.9
	0.78	0.40	-0.18	-1.14	-21.9
	0.11	0.31	0.59	1.21	11.3

Notes: See Table 4. The column of numbers in each cell give the elasticities of market consumption, next period's total capital, this period's market capital, this period's home capital, market labor supply, market output, and home output, respectively. The four rows give η_{cf}, η_{ff}, η_{kf}, η_{df}, η_{nf}, η_{df}, η_{nf}, η_{yf}, and η_{hf}, the elasticities with respect to total capital.

Table 6

$x =$

	c	i	w	k	n
A. U.S. Data: std(y) = 2.0					
$\frac{std(x)}{std(y)}$	.49	2.44	.65	.25	.76
$corr(x, y)$	.90	.97	.65	.38	.76
B. Standard Model, $\sigma = 1$: std(y) = 1.0					
$\frac{std(x)}{std(y)}$	.35	3.10	.56	.33	.47
$corr(X, Y)$	.90	.99	.98	.04	.98
C. Case 1, $\sigma = 0.2$: std(y) = 0.7					
$\frac{std(x)}{std(y)}$	.28	3.20	1.10	.33	.13
$corr(x, y)$	.98	.99	.99	.01	-.77
D. Case 2, $\sigma = 0.2$: std(y) = 1.0					
$\frac{std(x)}{std(y)}$	.28	3.21	.43	.35	.60
$corr(x, y)$	.98	.99	.96	.02	.98
E. Case 3, $\sigma = 0.2$: std(Y) = .7					
$\frac{std(x)}{std(y)}$	.11	3.71	.88	.19	.15
$corr(x, y)$	.94	.99	.99	.68	.85
F. Case 4, $\sigma = 0.2$: std(Y) = 1.4					
$\frac{std(x)}{std(y)}$	.17	3.58	.45	.56	.55
$corr(x, y)$	.98	.99	.99	.98	.99

Notes: All series are filtered using the Hodrick-Prescott technique. The following variables are in logs: y is output, c is market consumption, i is investment, k is market capital, n is market hours, and w is average productivity. The top of each panel is the percentage standard deviation of output; std(x)/std(y) gives the standard deviation of the series x relative to that of Y, and corr(x,y) gives the correlation of x with y. The numbers are averages over 100 simulations of 150 periods each.

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