MODELLING THE INSTABILITY OF MORTGAGE-BACKED PREPAYMENTS

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Second draft, March 1998

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ABSTRACT

Prepayment plays a critical role in the performance of mortgage-backed securities. For this reason, market participants have devoted substantial resources to developing formal mathematical models of mortgage prepayment. Despite their considerable efforts, however, the forecasting effectiveness of these propriety models has been unreliable. This paper investigates the structure of the prepayment function. We demonstrate that the prepayment function is nonlinear and heteroskedastic. In particular, we find that prepayments are increasingly more volatile at higher interest rate spreads. Our analysis suggests that these unusual properties of pool prepayments are inherently caused by statistical aggregation.

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1. INTRODUCTION

The U.S. mortgage market is one the largest debt market in the world. As of the end of 1996, banks and other financial institutions owned $3.7 trillion of home mortgage debt. The rapid growth of the mortgage market has been driven by mortgage securitization. Today, almost half of the debt is securitized and resold as mortgage-backed securities (MBS) by governmental, quasi-governmental institutions and private mortgage originators. Mortgage securitization has greatly enhanced liquidity in the mortgage market. The mortgage market has also benefitted from an expanding secondary market facilitated by the increased participation of loan brokers and private insurance mortgage companies. These entities provide a wide variety of services that allow mortgage originators and other investors to trade large portfolios of conforming or nonconforming loans in the secondary market (whole loan market).

In many respects, a mortgage security is similar to an ordinary bond. Like bonds, mortgage securities promise investors a stream of payments over a number of periods. Mortgage passthroughs, however, are different from a typical government bond because the promised cash payments depend on prepayment. Mortgage borrowers in United States are given the right to prepay part or all of the principal without penalty. This embedded option can change drastically the expected cash flows from a mortgage security. The adverse effect of prepayment is particularly exaggerated in more exotic mortgage-backed

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derivative products such as collaterized mortgage obligations (CMOs) and stripped MBSs.

The perils of mortgage prepayment are exemplified by the experience of MBS investors in the early 1990s. Because of the unusually low interest rates during 1992-93, conventional mortgage passthroughs and other MBS derivatives were attractively priced over comparable Treasury securities. Investors needing higher yields to pump up their portfolio return were enticed to take large positions in MBSs. The Federal Reserve's decision to raise interest rates in February 1994, however, sparked an unprecedented collapse in prepayments. Market participants failed to anticipate this sudden slowdown in refinancings, and as a result most MBS holders suffered substantial losses. In some instances, the results were catastrophic. Orange County was forced into bankruptcy in December 1994 after its municipal investment fund lost close to $1.7 billion. This huge loss stemmed primarily from unhedged MBS positions and bad bets on the direction of interests rates (see Jorion (1995)). The losses were even more devastating in the bankruptcy of Askin Capital Management, a group of hedge funds that was exposed to a $2.5 billion leveraged position of CMOs. The damage, however, was not just confined to speculative hedge fund investors. PaineWebber Group, a venerable financial firm, had to inject $33 million of its own capital into its government bond funds to compensate shareholders for MBS losses.

The valuation of mortgage securities is a rather complex process that requires a fairly high level of financial sophistication. Mortgage market participants use option adjusted spread (OAS) models to value MBSs. Most investors do not build their own pricing models, but rely instead on dealers and other independent sources to obtain reliable price quotes. OAS analysis is not an exact process because it is based on subjective assumptions about implied forward rates and interest rate volatility. Moreover, the accuracy of OAS prices depends on the efficacy of prepayment projections. Thus, when investors buy an MBS, they also buy a unique set of assumptions. As the 1994 experience suggests, however, these assumptions can fail badly.

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The inability of forecasters to anticipate the magnitude of the shift in mortgage prepayments is somewhat puzzling given that they experienced similar refinancing episodes in 1986-87 and 1992-93. Some have argued that the propensity to prepay has changed over time. The changing intensity of the prepayment cycles has been attributed to the evolving character of the mortgage market over the last two decades. Bennett et al. (1997) find that 12 percent of the borrowers in the 1990s prepaid their loan after 5 years. In contrast, they estimate that, under similar economic conditions, the prepayment rate in the 1980s would have only been around 7 percent. A study by Lekkas (1994) provides further support to the changing nature of refinancings. The author reports that the 1986-87 experience was dominated by high-rate borrowers who reduced their monthly payments by refinancing into lower-interest loans. By contrast, during 1992-93 borrowers elected to shift into shorter-maturity mortgages.

Although borrowers appear to be more responsive to interest rate movements in the 1990s, we fail to see how this moderate change in refinancing habits would have generated so much uncertainty in mortgage prepayments. Why are MBSs prepayment speeds so erratic and unpredictable? In investigating this question, this paper takes a somewhat unorthodox approach. Most studies in the literature have analyzed prepayment speeds at the aggregate (pool) level. We focus instead on the microstructure of the mortgage-backed bond. The prepayment experience of an MBS is simply the sum of all individual prepayment decisions in the pool. We will argue that this process of aggregation makes the prepayment function inherently unstable.

In Section 2, we briefly describe the instability of mortgage-backed prepayment rates. Section 3 introduces a simple statistical model for individual prepayments. In Section 4, we utilize this specification to establish the effects of aggregation. Section 5 summarizes the model and draws some implications on the pricing of mortgage securities.

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Figure 1. Prepayment Experience of FNMA 6s—12s 30—Year Passthroughs, 1982—94
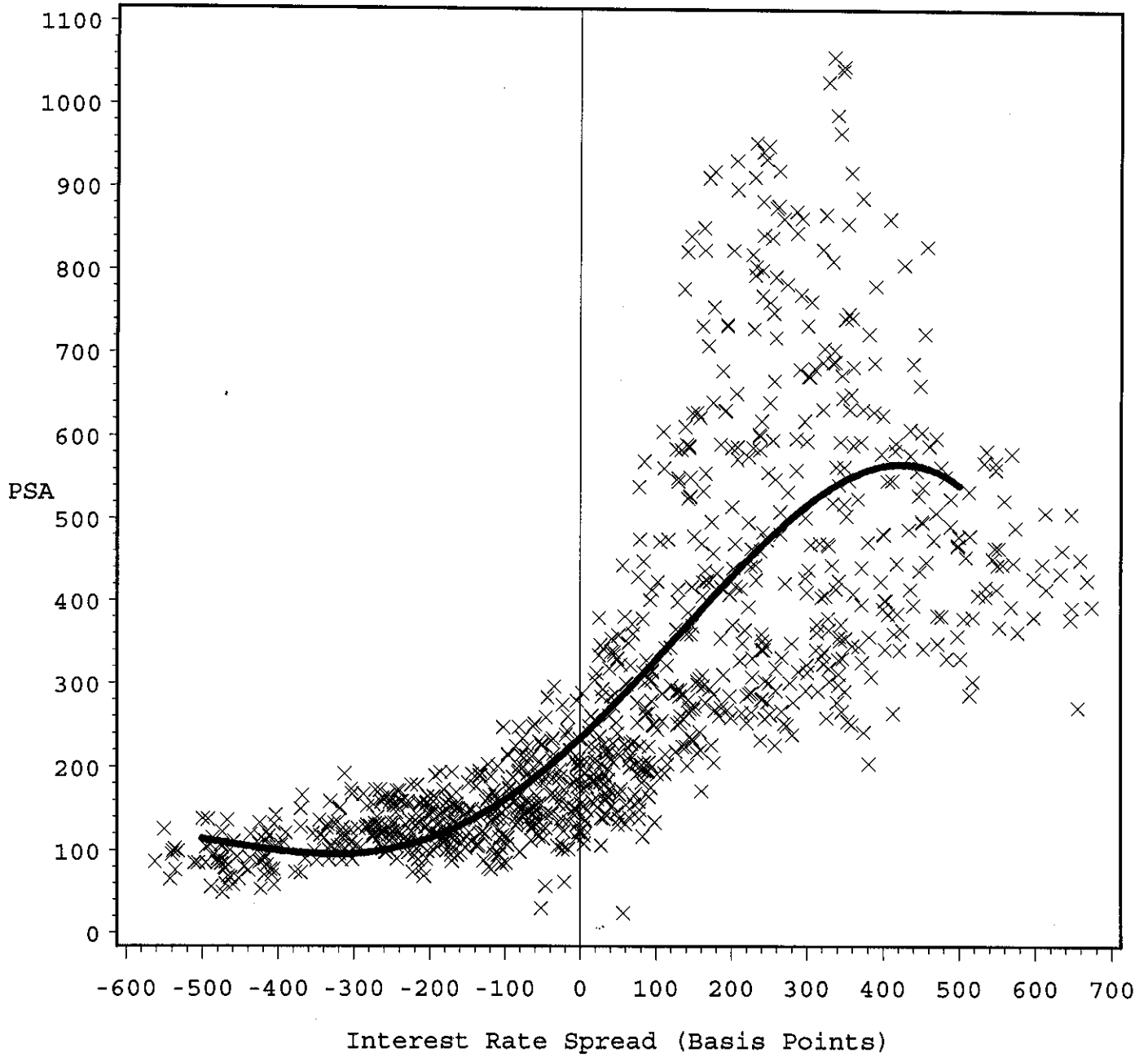
Source: Bloomberg

2. BACKGROUND

To illustrate the instability of MBS prepayment rates, we present a simple scatter diagram of prepayment speeds and the relative interest rate differential between the weighted average coupon (WAC) and the prevailing mortgage rate (Figure 1). The PSA prepayment rates describe the experience of 30-year conventional Federal National Mortgage Association (FNMA) passthroughs with coupon rates ranging between 7 and 12 percent.[1] The solid curve in the scatter plot represents an in-sample forecast of prepayment rates.[2] It is clear from the S-Shape configuration of the in-sample prediction that prepayments are nonlinear. Homeowners are reluctant to refinance when spreads are negative because their mortgage option is out of the money. In this negative range, we observe small residual prepayments, resulting mostly from life events or other idiosyncratic factors.

Prepayment rates begin to accelerate once interest rate spreads widen. The rising incentive to prepay at higher coupon spreads is best seen in the steepening slope of the in-sample forecast. At the same time, however, observe that prepayments become more heteroskedastic. That is, the residuals are more dispersed at higher interest rate spreads. When the spread is equal to zero, FNMA prepayment rates range from 100 PSA to 300 PSA. By contrast, prepayments are more dispersed at a 200 basis points spread, ranging from 200 PSA to 900 PSA. This large disparity in prepayment rates is also found in

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[1] The Public Securities Administration (PSA) convention assumes that pool prepayments rise 0.02 percent per month for the first 30 months of the life of the pool, and then remain constant at 6 percent from the thirtieth month until maturity.

[2] We use a simple polynomial regression model to compute prepayment forecasts. This nonlinear specification is based solely on the coupon spread. Our objective here is to simply illustrate the nonlinear nature of prepayments. In a later section, however, we will demonstrate that the polynomial specification is an excellent approximation of the prepayment function.

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individual FNMA coupon cohorts (for instance, 9 percent FNMA passthroughs) and is even observed in single pools. The presence of heteroskedasticity cannot be therefore attributed to the fact that the scatter plot portrays the prepayment experience of wide class of FNMA securities. Figure 1 also highlights the hazards of investing in MBSs. A relatively small change in interest rates could produce a huge shift in prepayments and generate a significant drop in the price of the security.

Why is the relationship between prepayment rates and coupon spreads heteroskedastic? The conventional view among practitioners relates this characteristic to "path dependency." By construction, an MBS pool is made up of a number of mortgages. When a mortgage is prepaid, the servicer returns the principal to investors and subsequent cash flows of the security are paid out of the remaining mortgages in the pool. Conceptually, prepayment is equivalent to a process of sampling without replacement. This process introduces path dependency because it changes the composition of the pool. Consider, for example, a new mortgage pool that experiences two consecutive interest rate cycles. In the first episode, rate-sensitive homeowners will rush to take advantage of favorable interest rates exiting the pool, pushing prepayments higher. As mortgage rates decline for the second time around, however, prepayment speeds will be slower because the pool now consists of constrained mortgagors who are unable to take advantage of the favorable interest rate environment.

In this paper we offer a somewhat different interpretation to the heteroskedastic traits of prepayment. We will argue that the unusual dispersion in prepayments is not necessarily caused by path dependency. Rather, we will show that this phenomenon is simply a statistical artifact of aggregation. In fact, we will demonstrate that pool-level prepayment rates continue to be heteroskedastic, even though a prepaying mortgage holder is replaced in the pool by an exactly identical individual. To be sure, "burnout" is important. But its role is more critical in shaping the average propensity to prepay, which accounts for the

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nonlinear S-shape of the prepayment function.

2.1 The Role of Prepayments in Pricing MBSs

To price a passthrough security, investors must determine the present value of the expected cash-flows. This task is nontrivial because the cash flows of an MBS are altered by prepayments. In addition, the valuation procedure requires a number of arbitrary assumptions about the dynamic structure of interest rates. Since a mortgage passthrough is similar to a callable bond, several studies have advocated an option-pricing methodology. Rational prepayment models assume that prices and prepayments are endogenously determined by interest rates (Dunn and McConnell (1981) and Stanton (1997)). One implication of this methodology is that borrowers are expected to exercise their option in a homogeneous manner. We know, however, that homeowners often exercise their option in an "irrational" manner. That is, they prepay, although they appear to be out of the money; or fail to take advantage of favorable interest rate differentials.

Typically, mortgage market participants have employed empirical OAS methods to price MBSs. In OAS analysis, prepayments and prices are determined separately. In the first stage, an econometric model is used to derive prepayment forecasts. Subsequently, these prepayment projections are used to determine the cash flows and value of the security. Firms construct econometric models of prepayment for a wide variety of MBS cohorts.[3] A typical prepayment models would control for a number of factors: the interest-rate incentive to refinance, the age of the security (seasoning), seasonal variation in prepayment rates, and burnout.[4]

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[3] Usually, prepayments forecasts are grouped according to issuer (GNMA, FNMA, and FHLMC), maturity (30-year, 20-year, 10-year, and 7-year), and type of loan (single family, balloon, mobile homes, etc).

[4] Most firm models are propriety in nature. Richard and Roll (1988) provide an overview of the Goldman Sachs Model. For a more extensive discussion of prepayment models, see

3. A STATISTICAL MODEL FOR INDIVIDUAL PREPAYMENTS

3.1 Mortgage Mathematics

A traditional mortgage loan is an amortized contract that requires borrowers to pay interest and repay the principal in equal installments. At the same time, the mortgagor is given the right to prepay part or all of the principal before maturity without penalty. Like any contract with standardized payment streams, a mortgage loan obeys a well-developed mathematical framework (for more details, see Hayre and Mohebbi (1992)). Assume that the i-th homeowner takes out a conventional fixed-rate mortgage loan mortgage loan in month ($t = 0$). The mortgage rate is r_i and the loan is amortized over T periods (typically, T equals 360 months). Let B_{ti} represent the remaining balance on the loan in month t (thus, B_{0i} is the original balance). The remaining balance B_{ti} includes all partial (unscheduled) payments. When B_{ti} reaches zero, the loans is assumed to have been fully repaid in month t. In the absence of any prepayment, the remaining balance of a mortgage is given by

$$\bar{B}_{ti} = B_{0i}\frac{(1+r_i)^T - (1-r_i)^t}{(1+r_i)^T - 1} = B_{0i}\alpha_{ti}. \tag{1}$$

The term α_{ti} is known as the amortization factor. It follows that the proportion of the scheduled loan balance outstanding in any month is defined by

$$q_{ti} = \frac{B_{ti}}{\bar{B}_{ti}}. \tag{2}$$

Clearly, a mortgage loan that does not experience any curtailments will always have q_{ti} equal to 100 percent. The variable q_{ti} is useful for defining the standard measures of prepayment. The fraction of the outstanding loan balance that is prepaid each month is simply given by

$$p_{ti} = \frac{\Delta q_{ti}}{q_{t-1,i}}. \tag{3}$$

also Fabozzi (1992), Spahr and Sunderman (1994), and Schwartz and Torous (1989).

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In a monthly context, p_{ti} is known as the single monthly mortality rate (SMM), representing the proportion of the outstanding balance of mortgage i prepaid in month t. Typically, SMM or its annualized version conditional prepayment rate (CPR) are used to measure pool-level prepayments. However, these prepayment measures are also pertinent for a single mortgage, although at this micro level prepayment rates are not continuous.

3.2 A Simple Econometric Model for Mortgage Prepayment

Mortgage prepayments can occur because of three basic reasons: (1) refinancings, (2) property sale, and (3) default. Refinancings represent prepayment by nonmover occupants. The rational prepayment literature stipulates a mortgagor would refinance if the intrinsic value of the loan, defined as the immediate benefit from refinancing measured in present value terms, is greater than the benefit from waiting to refinance in a subsequent period (the "time value" of the option plus transaction costs). The decision to terminate a mortgage by moving or defaulting also depends on the moneyness of the mortgage option. However, these choices are further influenced by personal characteristics (income, education) and other idiosyncratic events (job loss, death, divorce).

Several recent papers have taken a more direct approach to modeling the cross sectional heterogeneity in prepayment behavior. In these studies, the rational prepayment model is replaced by an empirical specification (see Cunningham and Capone (1990), Caplin et al. (1996), and Peristiani et al. (1997)). Using loan-level data on mortgage terminations, these empirical studies find strong evidence that prepayments are driven by two particular factors: post-origination home equity and homeowner creditworthiness.

The empirical methodology is also useful for defining a general stochastic model for individual prepayments. The decision to prepay can be simply expressed as

$$p_{ti}^* = \beta_{0i} + x_t \beta_{1i} + \epsilon_{ti}, \tag{4.1}$$

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where

$$p_{ti} = 100 \qquad if \qquad p_{ti}^* \geq 100; \tag{4.2}$$

$$p_{ti} = p_{ti}^* \qquad if \qquad 0 < p_{ti}^* < 100; \tag{4.3}$$

$$p_{ti} = 0 \qquad if \qquad \text{otherwise.} \tag{4.4}$$

As before, the variable p_{ti} represents a broad measure of actual prepayment (e.g., the SMM rate or the annualized conditional prepayment rate). For simplicity, we assume that p_{ti} is bounded above by 100 percent (full prepayment) and below by zero (no prepayment).[5] The variable p_{ti}^* represents the unobservable notional desire to prepay. In contrast to actual prepayment, the notional desire is a continuous variable that can be negative or exceed 100 percent. If the notional desire to prepay is positive but less than 100 percent, the homeowner will partially prepay the loan.[6]

The willingness to prepay is influenced by a systematic factor x_t, representing market conditions. To make our analysis more intuitive, however, we let x_t be a scalar measure representing the spread between the coupon rate and the prevailing effective market interest rate. The parameters β_{1i} and β_{0i} capture homeowner or loan characteristics. On average, credit- or collateral-constrained borrowers are expected to have small positive

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[5] For PSA prepayments, the appropriate bound is $[\frac{100^2}{6} max\{1, \frac{30}{t}\}]$.

[6] The contribution of partial prepayments (or curtailments) to overall prepayment is generally quite small. For fixed-rate mortgages, partial prepayments contribute, on average, around 0.2 percent to conditional prepayment rates. As with partial prepayments, defaults make up a small portion of total prepayment. Usually, the homeowner default rate on fixed-rate mortgages is less than 0.5 percent per year. Since most passthroughs are insured against credit risk, we do not consider this option in the censored regression model. However, one can explicitly include this outcome by using a more generalized version of the Tobit model.

slope coefficients β_{1i} because they are less sensitive to economic conditions.[7]

The term ϵ_{ti} denotes the random error, which accounts for all unexplained variation in the decision to prepay. The errors are independent (i.e., $\epsilon_{ti}\epsilon_{tj} = 0$ for all $i \neq j$) and identically distributed normal variables with mean zero and variance σ^2. Together equations (4.1)-(4.4) define a two-limit censored regression model (Maddala (1983)). In contrast to the ordinary regression model, the distribution of monthly prepayment in the two-limit Tobit model is determined by a mixture of censored (unobserved) and continuous (observed) variables. The probabilities of the three distinct outcomes of prepayment are given by

$$P(\text{i} - \text{th homeowner prepays fully}) = 1 - \Phi(\lambda_{ti}^u), \tag{5.1}$$

$$P(\text{i} - \text{th homeowner partially prepays}) = \Phi(\lambda_{ti}^u) - \Phi(-\lambda_{ti}), \tag{5.2}$$

$$P(\text{i} - \text{th homeowner does not prepay}) = 1 - \Phi(\lambda_{ti}), \tag{5.3}$$

where $\lambda_{ti} = \dfrac{\beta_{0i} + x_t\beta_{1i}}{\sigma}$ and $\lambda_{ti}^u = \dfrac{100}{\sigma} - \lambda_{ti}$. The function $\Phi(\lambda)$ is the standard normal cumulative distribution integrated between λ and ∞. By definition, all three probabilities defined by (5.1)-(5.3) sum to one.

In the censored regression model, the likelihood of prepayment is still determined by the homeowner's characteristics and interest rate conditions. However, the censored nature of individual prepayments complicates the error structure. We can show that

$$E(\epsilon_{ti}) = \sigma\frac{\varphi(\lambda_{ti}) - \varphi(\lambda_{ti}^u)}{\Phi(\lambda_{ti}^u) - \Phi(\lambda_{ti})} = \sigma h(\beta_{0i}, \beta_{1i}, x_t) = \sigma h(\lambda_{ti}), \tag{6.1}$$

$$Var(\epsilon_{ti}) = \sigma^2[1 - h(\lambda_{ti})^2 + \frac{100\varphi(-\lambda_{ti}^u)}{\Phi(\lambda_{ti}^u) - \Phi(\lambda_{ti})}] = \sigma^2 v(\beta_{0i}, \beta_{1i}, x_t) = \sigma^2 v(\lambda_{ti}). \tag{6.2}$$

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[7] Caplin et al. (1997) and Peristiani et al. (1997) find evidence that strongly supports this premise. Using a large sample of homeowners, these studies estimate a qualitative model for the decision to refinance. Their empirical findings suggest that credit quality and collateral value have a significant effect on the probability of refinancing.

The error in the mortgage prepayment model has therefore a nonzero mean and its variance is heteroskedastic (that is, $Var(\epsilon_{ti})$ is a function of x_t). The significance of these statistical properties of individual prepayments would become more apparent in the next section. Nonetheless, it is not difficult to understand why these unique characteristics of individual prepayments are extremely important. An MBS comprises a finite number of borrowers. If each borrower's decision function is biased and heteroskedastic, then these properties would also transfer to the MBS prepayment function.

4. THE MBS PREPAYMENT FUNCTION

Consider a typical mortgage passthrough security consisting of a number of conventional mortgage loans. At origination $(t = 0)$, the mortgage pool contains n_0 fixed-rate mortgages with maturity T. Because of prepayment, after the MBS is issued the initial number of mortgages in the pool may decline, (that is, $n_{t+\ell} \leq n_t \leq n_0$). The overall size of the pool at origination equals B_0. Each loan in the pool contributes B_{0i}, such that $B_0 = \sum_{i=1}^{n_0} B_{0i}$. Initially, the weighted average coupon (WAC) of the MBS is $\bar{r}_0 = \sum_{i=1}^{n_0} \omega_{0i} r_i$, and the weighted average maturity (WAM) is T months. The scaling factor ω_{0i} represents the relative weight of each mortgage loan at $(t = 0)$ (more generally, $\omega_{ti} = \frac{B_{ti}}{B_t}$).

An MBS is thus assembled by combining the cash flows of n_0 mortgage loans. The cash flows of the security are determined by the prepayment experience of the pool. To complete the model, we assume that the individual prepayment process is defined by equations (4.1)-(4.4). The prepayment experience of the pool at any given month is the sum of all individual prepayments. Algebraically, we can express this aggregate pool prepayment rate as

$$P_t = \beta_{0t} + \beta_{1t} x_t + \epsilon_t, \tag{7}$$

such that $P_t = \sum_{i=1}^{n_t} \omega_{ti} p_{it}^*$, $\beta_{kt} = \sum_{i=1}^{n_t} \omega_{ti} \beta_{ik}$ $k = 0, 1$, and $\epsilon_t = \sum_{i=1}^{n_t} \omega_{ti} \epsilon_{ti}$. Note that

β_{0t} and β_{1t} are time-varying, meaning that the slope and intercept of the prepayment function change over time. Since mortgagors are not replaced when they exit the pool, the composition of the mortgage pool changes over time with prepayment. In our simple framework, in which prepayment rates are determined by a single factor x_t (the coupon spread), the slope of the prepayment function is fairly flat at negative values of x_t. In this range, we anticipate small residual prepayments resulting from idiosyncratic events. The slope of prepayment function would steepen for positive coupon differentials. Large positive spreads trigger rapid refinancings as borrowers with a higher propensity to prepay (those with high positive β_{1i}) are now in the money. Eventually, the slope of prepayment function flattens at very high values of spread because the pool is "burned out," meaning that the pool now consists of mostly constrained borrowers (low-beta homeowners) who are unable to refinance at any rate.[8]

Because individual prepayments are heteroskedastic with a nonzero mean, we expect pool-level prepayments to also have a similar structure. In particular,

$$E(\epsilon_t) = \sigma \sum_{i=1}^{n_t} \omega_{ti} h(\lambda_{ti}), \tag{8}$$

$$Var(\epsilon_t) = \sigma^2 \sum_{i=1}^{n_t} \omega_{ti} v(\lambda_{ti}). \tag{9}$$

The error in pool prepayments is again heteroskedastic in the sense that the variance depends on the level of coupon spread x_t. Equations (8)-(9) can be simplified by linearizing the functions $h^*(\omega_{ti}, \lambda_{ti}) = \omega_{ti} h(\lambda_{ti})$ and $v^*(\omega_{ti}, \lambda_{ti}) = \omega_{ti} v(\lambda_{ti})$. Using a multivariate Taylor approximation rule, we can modify these functions to

$$E(\epsilon_t) \approx \sigma[\alpha_0 + \alpha_1 x_t + \alpha_2 x_t^2 + \ldots + \alpha_k x_t^k] = \sigma h(x_t), \tag{10}$$

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[8] Another way to look at path-dependency in prepayments is by examining the stochastic properties of n_t, the number of mortgagors remaining in the pool at time t. Because loans are not replaced in the pool, the conditional expectation of n_t depends on n_{t-1}. In turn, this means that the conditional expectation of n_t depends on lagged values of x_t.

$$Var(\epsilon_t) \approx \sigma^2[\gamma_0 + \gamma_1 x_t + \gamma_2 x_t^2 + \ldots + \gamma_k x_t^k] = \sigma^2 v(x_t), \qquad (11)$$

where k represents the polynomial order of the Taylor expansion. The error structure of the prepayment function can be approximated by an additive form of heteroskedasticity.

Equations (10) and (11) reveal two important properties of the prepayment function. First, the expected value of the prepayment function error is nonzero. The bias in aggregate prepayments is directly a function of the underlying exogenous factor x_t. Typically, we would expect the error bias to be bigger at wider interest rate spreads. Second, the variance of the prepayment errors is inherently a function of the exogenous factor x_t. This heteroskedastic relationship suggests that the statistical inference from aggregate prepayment models will be more uncertain at larger values of the coupon spread.

We can also apply the Taylor rule to approximate the theoretical structure of the prepayment function. The aggregate prepayment rate can be expressed as

$$E(P_t) \approx \beta_0 + \beta_1 x_t + \beta_2 x_t^2 + \ldots + \beta_k x_t^k. \qquad (12)$$

Assuming that x_t is fully known, the nonlinear prepayment function can be efficiently estimated by a simple polynomial regression model.

These findings can be easily generalized to the case where a borrower's decision to prepay is influenced by several variables represented by the row vector $x_{t\bullet} = (1, x_{t1}, \ldots, x_{tp})$. In this multivariate case, we can show that prepayment errors would still be heteroskedastic, albeit the functional form of additive heteroskedasticity would be more complicated.

4.1 Simulation Examples

Our theoretical analysis suggests that the observed dispersion and prepayments is caused by statistical aggregation. An alternative way of illustrating the effects of aggregation in prepayments is through simulation. The premise of the simulation examples presented in this section is straightforward. We construct artificial pools of mortgages. The

Figure 2. Simulated Pool Prepayments When Homeowners are Identical
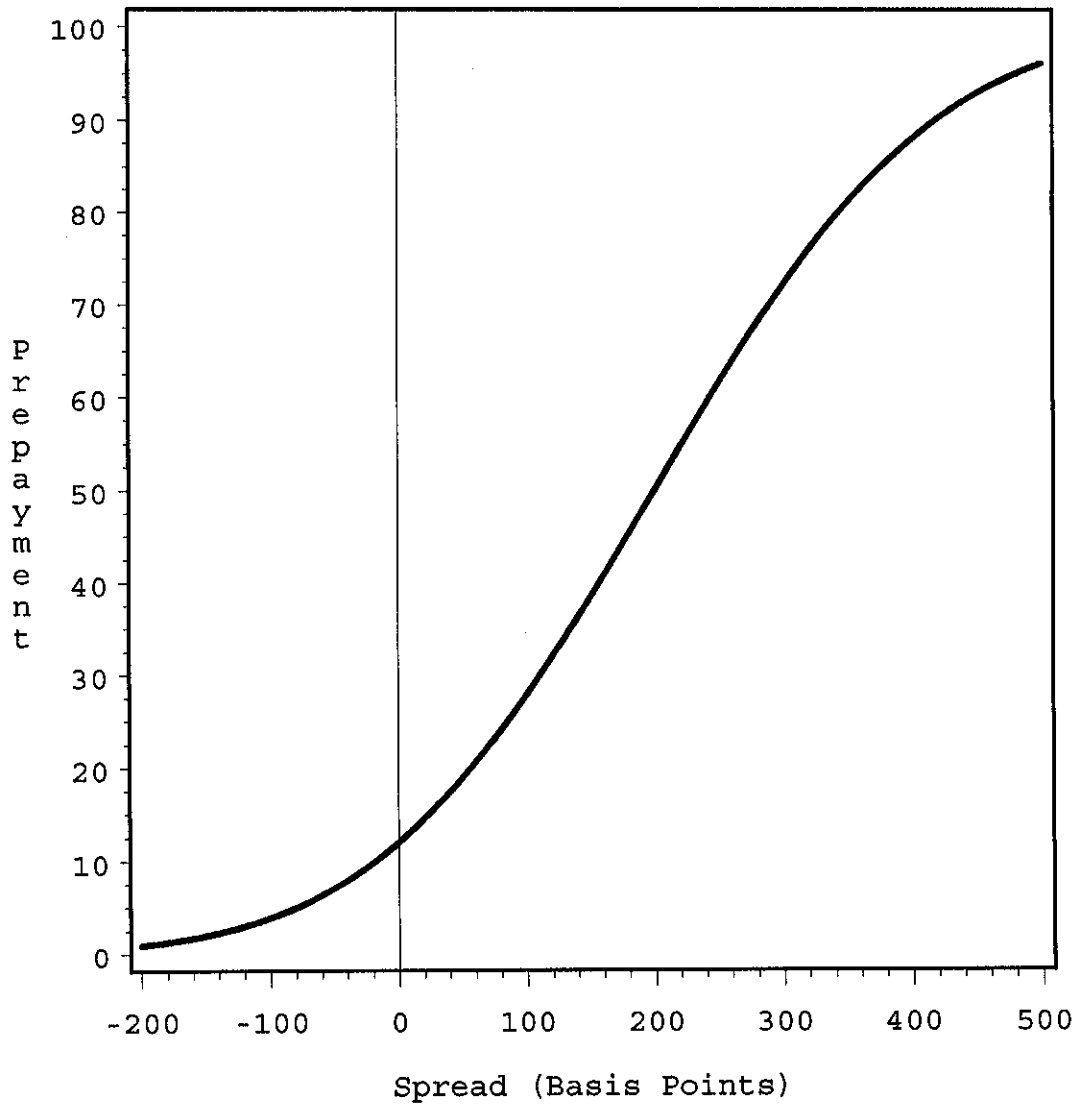
decision to prepay any mortgage in the pool is determined by a stochastic rule given by equations (4.1)-(4.4). In each period, mortgage holders are exposed to a different interest rate spread plus a random shock.

Before we delve into the general prepayment model, consider a special case. Assume that all homeowners in a pool are identical (e.g., $\beta_{0i} = \beta_0$ and $\beta_{1i} = \beta_1$). One benefit of this simplification is that we can now attain a closed-form solution for the expected value of aggregate pool prepayment. In particular, we can show that

$$E(P_t) = 100\Phi(-\lambda_t^u) + (\beta_0 + x_t\beta_1)[\Phi(\lambda_t^u) - \Phi(-\lambda_t)] + \sigma[\phi(-\lambda_t) - \phi(\lambda_t^u)],$$

where $\lambda_t = \dfrac{\beta_0 + x_t\beta_1}{\sigma}$ and $\lambda_t^u = \dfrac{100}{\sigma} - \lambda_t$. To better illustrate the features of this deterministic prepayment function, the model is simulated for a given set of parameter values. Figure 2 presents a graph of the expected prepayments for various levels of the interest rate spread x_t. As seen from the chart, pool prepayments have the characteristic nonlinear S-shape. Although useful, this simulation example is based on the unrealistic assumption that all individuals have identical prepayment decisions. In a way, this limits the degree of burnout as the composition of borrowers in a pool is not altered.

The second simulation example asserts that mortgagors have different propensities to prepay (that is, they have unique β_{0i} and β_{1i}). For simplicity, we also assume that borrowers do not curtail their loans. We perform two distinct simulation experiments. In the first simulation experiment the mortgage holder exits the pool if the willingness to prepay p_{ti}^* is greater than zero; otherwise, the borrower does not prepay. The second simulation experiment stipulates again that a borrower would prepay when $p_{ti}^* > 0$; however, now the prepaying individual is replaced in the pool by an exactly identical borrower. In this way, the pool remains path-independent.[9]

The results of the two simulation examples are graphically presented in Figure 3.

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[9] Borrowers are heterogeneous in the sense that $\beta_{\ell i} = \bar{\beta}_{\ell i}(1 + \rho z_i)$ $\ell = 0, 1$, where $\bar{\beta}_{\ell i}$

Figure 3. Simulated Pool Prepayments With and Without Replacement
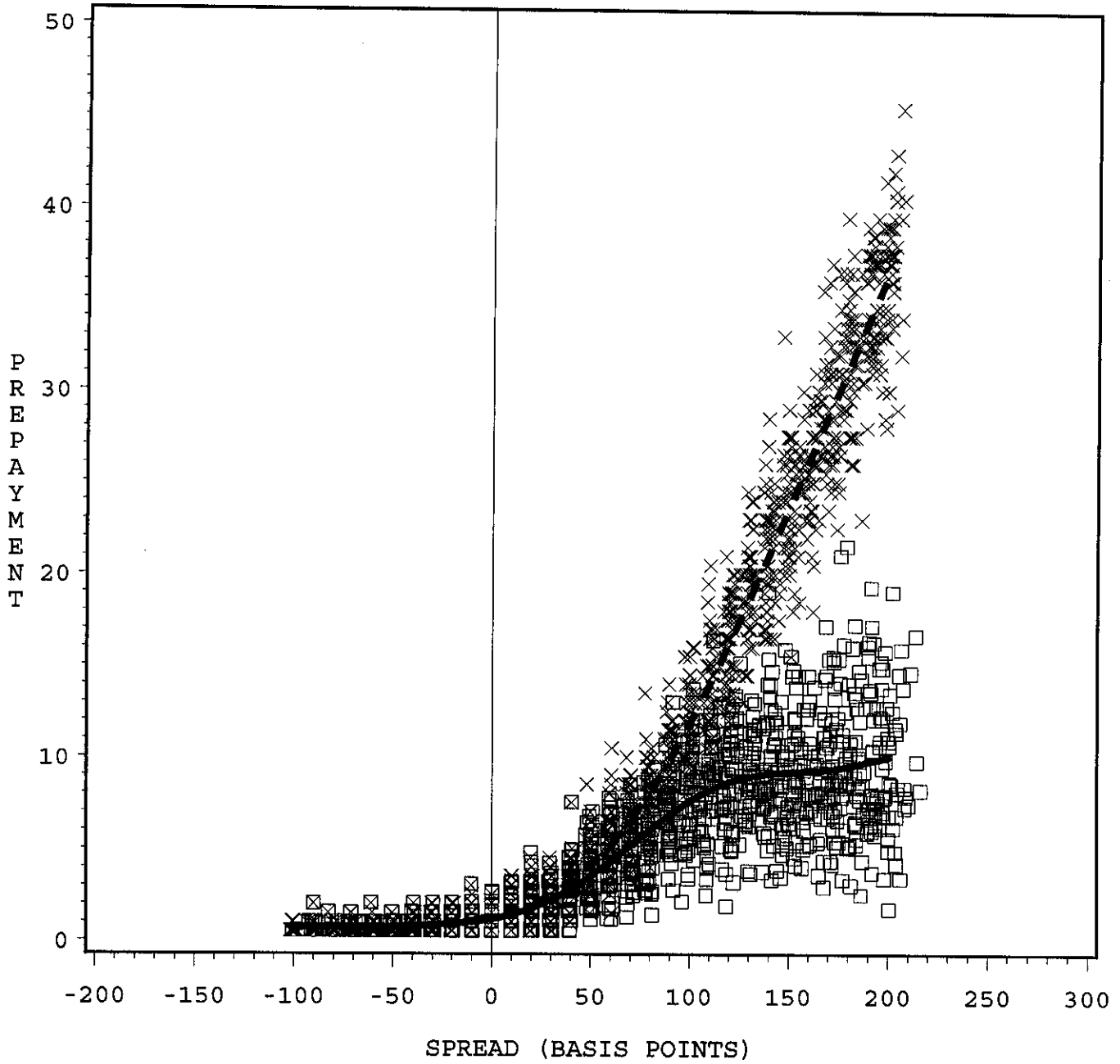
Essentially, we observe two distinct scatter plots in the figure. Observations marked by an ($\times$) represent the prepayment experience of pools that prepay with replacement. The square symbol denotes pools that prepay without replacement. The curves in the figure are again in-sample polynomial regression forecasts of prepayment speeds. Note that prepayments are heteroskedastic in both simulation experiments. This is an important finding because it demonstrates that burnout (prepayment without replacement) is not the cause of heteroskedasticity in prepayments. Even though individuals who prepay are replaced in the pool, the pattern in prepayments is still heteroskedastic.[10]

What clearly distinguishes the two simulation examples is the shape of the average prepayment function. Pool prepayment rates are, on average, much larger when prepaying borrowers are replaced in the pool. This outcome is not surprising because in this case the composition of the pool is unchanged. At negative spreads, only a small fraction of these individuals wish to prepay. But as spreads become positive and widen, an increasing number of mortgage holders are willing to prepay because the pool does not burn out. In contrast, when borrowers are not replaced in the pool, aggregate prepayments tend to level off after a point, giving rise to the distinct S-shape. In summary, our simulation findings suggest that pool burnout does not necessarily account for the phenomenon of heteroskedasticity in MBS prepayment rates. However, burnout is solely responsible for the nonlinear S-shape structure found in most prepayment functions.

5. IMPLICATIONS

Our analysis provides a compelling theoretical argument that the MBS prepayment function is inherently biased and heteroskedastic. As shown above, prepayments are more

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are predetermined values for the intercept and slope, ρ is a small constant (usually, 0.05) and z_i is a random shock generated from a standard normal distribution.

[10] A simple F-test shows that the error sum of squares for the two experiments are not statistically significantly different from each other.

likely to be scattered at large positive coupon spreads. The unusual nature of prepayments raises a number of interesting questions. What is the most effective way of modeling nonlinearity in prepayments? Does the heteroskedastic error structure of prepayments distort valuation?

5.1 Model Specification

At one level, our analysis suggests a simple approach to modeling the prepayment function. Suppose that all factors influencing a mortgage holder's decision to prepay are exogenous and fully represented by the vector $x_{t\bullet}$. We have proved that the aggregate prepayment function can be efficiently approximated by a polynomial functional form (defined by equation (12)). The usefulness of polynomial regression is best illustrated by the second simulation experiment seen in Figure 3. In this case, prepayments depend solely on $x_{t\bullet} = (1, x_t)$ and white noise. Not surprising, the polynomial regression model yields an excellent fit of the average prepayment function. The polynomial model, however, may not be always reliable because in practice $x_{t\bullet}$ is misspecified. Another option would be to estimate the nonlinearity in prepayments using a curve-fitting technique. A recent study by Maxam (1996) proposes a multivariate density estimation approach. Kernel estimation is an appealing alternative because it captures the intricate nonlinear S-shape in prepayments in a fairly parsimonious way.

5.2 The Effect on Pricing

The prepayment function is an indispensable part of any MBS pricing methodology. Prepayment assumptions allow investors to figure out cash flows and determine the price of the security. In theory, the value of an MBS is influenced by interest rate dynamics and prepayment behavior. We can formally define the price of a mortgage security (j) at time (t) as

$$V_{tj} = V[\omega_j, \Re_t, P_t(\beta_j, x_{t\bullet}, \epsilon_t)], \tag{13}$$

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Figure 4. Dealer Prepayment Forecasts for FNMA 8s (as of 12-22-97).
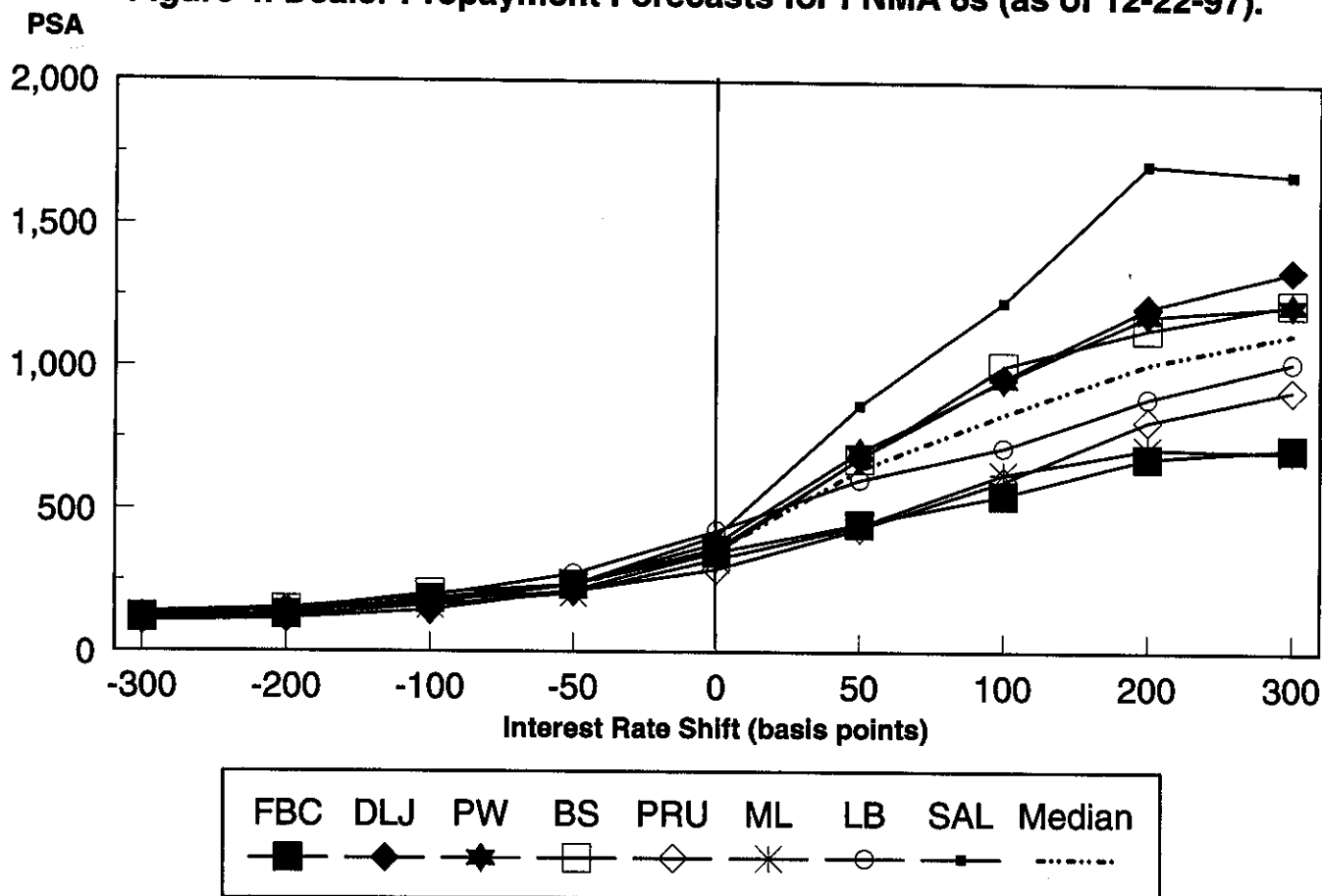
Source: Bloomberg.
Note: FBC = First Boston Corporation, DLJ = Donalson Lufkin Jeanerette,
PW = Paine Webber, BS = Bear Stearns, PRU = Prudential, ML = Merril Lynch
LB = Lehman Brothers, SAL = Salomon.

where $\Re_t$ represents the interest rate process at time (t) and ω_j is a vector of security-specific attributes. Prepayments P_t are influenced by the exogenous vector $x_{t\bullet}$, the individual's characteristics β_j, and a stochastic component ϵ_t. Note that prepayment errors are not identically distributed, rather, as we have shown above, they are heteroskedastic (e.g., $Var(\epsilon_t) = \sigma^2 v(x_{t\bullet})$). The large dispersion in prepayment errors introduces the potential for greater disparity in MBS prices. Thus, two MBSs may end up having very different price realizations, although ex ante they were fundamentally similar.

The importance of heteroskedasticity is quite evident in the wide discrepancy of published forecasts available from Bloomberg. Figure 4 reports the prepayment forecasts made by 8 firms for new FNMA 8s 30-year conventional passthroughs. The figure clearly shows that forecast uncertainty (here measured by the range of the PSA forecasts) is significantly higher for large interest rate shifts. To illustrate the sensitivity of prices to prepayment assumptions, assume that the interest rate jumps by 50 basis. For this shift, prepayment forecasts range from a low of 434 PSA (made by First Boston) to a high of 867 PSA (made by Salomon). Using a Bloomberg pricing algorithm, we can compute the OAS cost of a January-1998 TBA comparable passthrough under the different prepayment scenarios. The median OAS costs for the FNMA 8 percent passthrough is 75 basis points. In light of the large variation in prepayment forecasts, however, OAS values can range from 23 basis points to 112 basis points. Thus, a mere 50 basis points shift in interest rates has produced a huge disparity in prices.

The preceding example makes the rather simplistic assumption that prepayments remain unchanged over the life of the security. In real life, however, the prepayment process is quite dynamic. The prepayment path would depend on the underlying characteristics of the passthrough security, the initial interest rate assumption, and the eventual interest rate realizations. To examine this issue more carefully, we construct a simple OAS simulation experiment for FHLMC 30-year passthroughs with coupons ranging from 6 to 12 percent.

Figure 5. Prepayment Forecasts for FHLMC 6s—12s Passthroughs, 1984—94
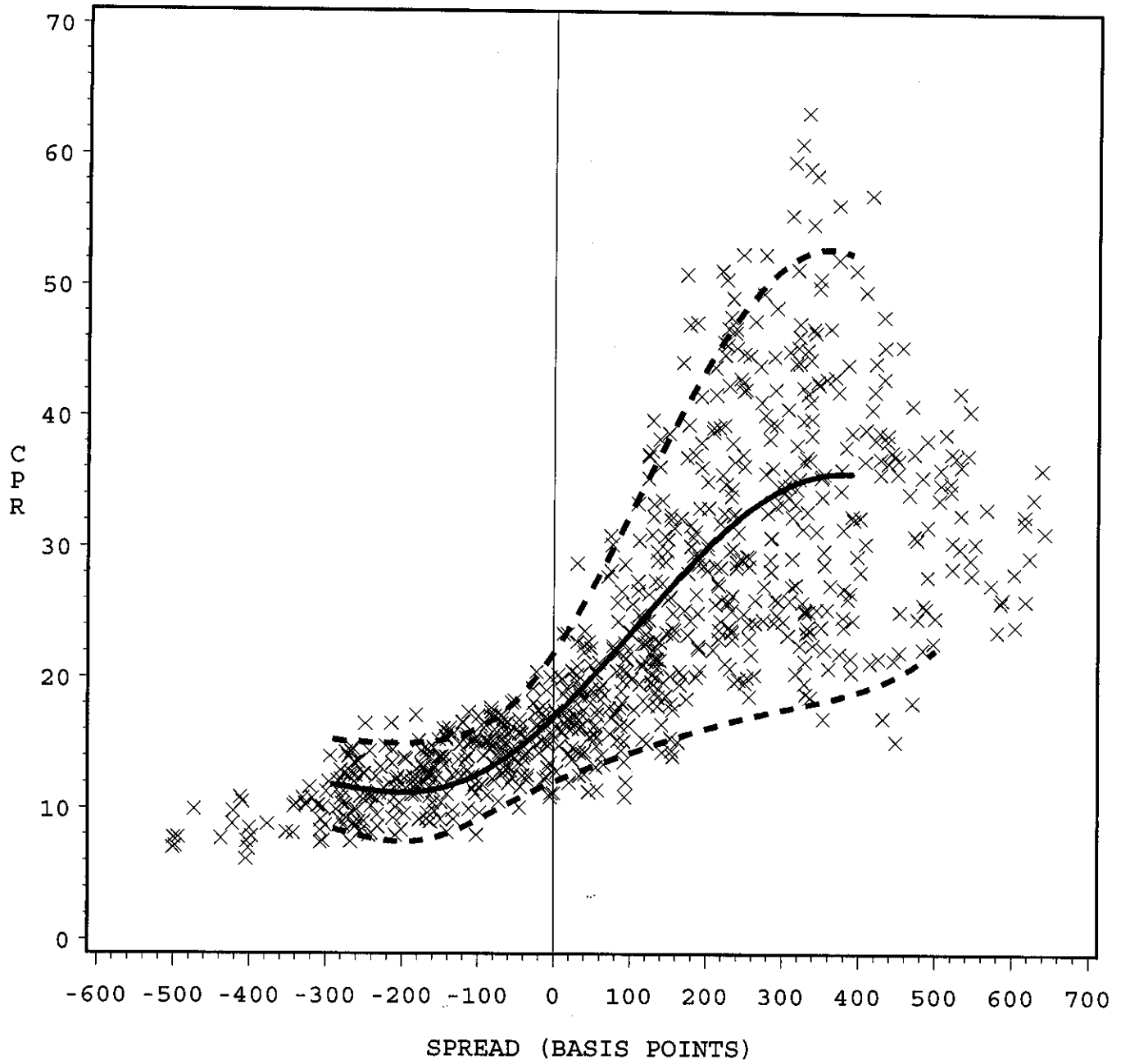
TABLE 1. OAS CONFIDENCE BOUNDS
(number in parentheses represent the average CPR)

Interest Rate Environment	Upper Bound of Prepayment	Midpoint of Prepayment	Lower Bound of Prepayment
Rising	264 (26.2)	206 (14.1)	161 (4.0)
Flat	162 (23.9)	156 (12.8)	151 (4.0)
Declining	59 (28.6)	103 (15.4)	144 (2.4)

NOTES: OAS simulations are based on the prepayment experience of FHLMC passthrough securities (see Figure 5). The upper and lower bound OAS estimates assume a two-standard deviation shift in prepayments. The average CPR measure is computed over the life of the security.

The prepayment experience of these securities is shown in Figure 5. The broken curves in the scatter plot map a two-standard deviation bound for the average prepayment rate (the solid curve). Because of the heteroskedastic nature of the errors, the confidence interval is wider at positive spreads.

Our simple OAS exercise looks at three distinct interest rate path scenarios: rates are assumed to rise, remain flat, or decline over the life the security. Admittedly, this a simplified version of OAS analysis. Nevertheless, this exercise is very useful in illustrating that the adverse effect of heteroskedastic prepayment errors is also path-dependent.[11] Table 1 shows that the OAS confidence bounds are fairly wide when interest rates rise or decline over the life of the security. By contrast, the difference between the midpoint price and its upper or lower bounds is negligible if interest rates remain flat. Note that the underlying conditional prepayment rates are similar in all three interest rate scenarios. Yet, despite the similarity in prepayment rates over the life of the security, the potential for price distortions is much smaller in an economic environment in which interest rates are fairly static.

In a recent paper, Boudoukh et al. (1997) utilize multivariate density estimation to price GNMA passthroughs. When a single factor is used in the kernel estimation (the level of interest rates), they discover a high degree of persistence in the pricing errors. Prepayment errors are found to be related to the long rate (10-year rate) and term structure spread (10-year rate minus 3-month treasury rate). This persistence in pricing errors dissipates when a second factor is introduced in the kernel function (the slope of the term

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[11] The pricing algorithm employs all the prescribed steps of OAS analysis. First, we construct a model for Treasury rates that is consistent with historical behavior. From this model, we generate 2,000 interest rate paths using Monte Carlo simulation. The interest rate realizations are then used to find the appropriate confidence bounds for prepayments (see Figure 5), derive cash flows, and eventually compute present values.

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structure), although it never goes away. Our analysis offers another explanation for these puzzling properties of the pricing errors. Essentially, the observed persistence and large cross-correlation in the pricing errors may be the fallout of the unstable nature of mortgage prepayments.

6. CONCLUSION

This study has demonstrated that mortgage prepayments are extremely unstable if the spread between the weighted average coupon and the effective mortgage rate prevailing in the market is large and positive. The customary view attributes this trait to path dependency or burnout. According to this premise, prepayments are heteroskedastic because often after a few bouts of refinancing the pool will be made up of mostly constrained mortgagors. In this paper, we provide an alternative interpretation. For one, we find that burnout is an important determinant of prepayment. But its effect is more evident in the nonlinear shape of the prepayment function. Further, we illustrate that the large dispersion in prepayments is not necessarily related to burnout, but is caused instead by statistical aggregation.

The findings of this study underscore the riskiness of investing in MBSs. Since the volatility in prepayments is inherently related to interest rate changes, the task of pricing MBSs becomes more arduous in an economic environment marred by unanticipated interest rate movements. Our analysis shows that even a moderate shift in interest rates is capable of creating large mispricing errors in the value of the mortgage security.

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