Comments welcome

Towards New Open Economy Macroeconometrics [*]

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Abstract

I develop a model that improves upon the recent literature in open economy macroeconomics in that it lends itself more directly to empirical investigation. I solve the stationarity problem that characterizes many existing models by adopting an overlapping generations structure a' la Weil (1989). I model nominal rigidity by assuming that firms face explicit costs of output price inflation volatility. The specification generates an endogenous markup that fluctuates over the business cycle. I identify the two economies in my model with Canada—a small open economy—and the United States—taken as an approximation of the rest-of-the-world economy. In the second part of the paper, I present a plausible strategy for estimating the structural parameters of the Canadian economy. I do so by using non-linear least squares at the single-equation level. Estimates of most parameters are characterized by small standard errors and are in line with the findings of other studies. I also develop a plausible way of constructing measures for non-observable variables. To verify if multiple-equation regressions yield significantly different estimates, I run full information maximum likelihood, system-wide regressions. The results of the two procedures are similar. Finally, I illustrate a practical application of the model, showing how a shock to the U.S. economy is transmitted to Canada under an inflation targeting monetary regime.

Keywords: Markup; Nominal rigidity; Open economy macroeconometrics; Stationarity
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1. Introduction

After a long-lasting predominance of non-microfounded Keynesian models, the publication of Obstfeld and Rogoff's "Exchange Rate Dynamics Redux" in 1995 opened the way to a new generation of models of macroeconomic interdependence. These models combine a rigorously microfounded approach with analytical tractability. The literature following Obstfeld and Rogoff's work has been mainly theoretical. Papers in this literature are said to belong to the so-called "new open economy macroeconomics."[1] However, empirical performance will ultimately decide whether this new generation of models will supplant the time-honored Mundell-Fleming-Dornbusch framework as the main tool for understanding interdependence and for formulating policy advice. This paper is a contribution in that direction. It presents a model that is more suitable for empirical investigation than those presented thus far, and it provides—to the best of my knowledge—the first comprehensive attempt at estimating an open-economy model in line with the recent developments in the theoretical literature. For this reason, the paper can be thought of as an initial contribution to "new open economy macroeconometrics."

Two main strands have been developing in the theoretical literature. On one side, followers of the original Obstfeld-Rogoff approach have built models in which a country's current account reacts to changes in economic policy and plays a relevant role in the transmission of disturbances. On the other side, Corsetti and Pesenti (1998) have proposed a model in which the importance of the current account is de-emphasized. They achieve this by assuming unitary intratemporal elasticity of substitution between domestic and foreign goods in consumption. Under this assumption, the current account does not react to shocks, and thus plays no role in their transmission. The justification for claiming that this is not a bad approximation of reality when the purpose is providing normative conclusions is that, even when the current account does move, the difference its movements make for a country's welfare is only second order.

Both the Obstfeld-Rogoff and the Corsetti-Pesenti models—and all models that follow their approach—share an important problem that makes their conclusions questionable from a theoretical and empirical perspective, namely the absence of a well-defined endogenously determined steady state. In these models, the position of the domestic and foreign economies that is taken to be the steady state in the absence of shocks is a point to which the economies never return following a disturbance. The consumption differential between countries follows a random walk. So do an economy's net foreign assets in the Obstfeld-Rogoff framework. Whatever level of asset holdings materializes in the period immediately following a shock becomes the new long-run position of the current account, until a new shock happens. Stationarity fails because the average rate of growth of the economies' consumption in the models does not depend on average holdings of net foreign assets. Hence, setting consumption to be constant is not sufficient to pin down a steady-state distribution of asset holdings. This makes the choice of the economy's initial position for the purpose of analyzing the consequences of a shock arbitrary. When the model is log-linearized, one is actually approximating its dynamics around a "moving steady state." The results of comparative statics exercises are thus particularly questionable. The reliability of the log-linear approximation is low in this setting, especially for analyses whose time horizon is longer than the one-period exercises of the original literature, because variables wander away from the initial steady state. *De facto*, one cannot perform any stochastic analysis in this framework without first shutting off the current account channel of international interdependence, as Corsetti

[1] For a survey, see Lane (1999).

and Pesenti do. The inherent unit root problem complicates empirical testing. The long-run non-neutrality of money that characterizes the results can be attacked on empirical grounds.

The failure of stationarity is not the only problem of the existing models. The assumption of one-period price rigidity that characterizes them is not appealing for empirical purposes. The absence of investment and capital accumulation from the models limits their appropriateness for thorough empirical investigations of current-account behavior and of the consequences of alternative policy rules for medium to long-run dynamics. Different policy rules can cause different dynamics in asset prices, whose features are often of interest to policymakers and analysts but cannot be studied in models that do not incorporate investment.[2]

In this paper, I propose a perfect-foresight, two-country, general equilibrium model that offers solutions to these issues. As do Obstfeld and Rogoff (1995, 1996 Ch. 10), I assume that a continuum of goods is produced in the world by monopolistically competitive firms, each of which produces a single differentiated good. Preferences for consumption goods are identical in the two countries, and the law of one price and consumption-based Purchasing Power Parity (PPP) hold.[3]

The demographic structure is one of the innovative features of my model. I depart from the basic representative agent framework. Rather, I follow Weil (1989) in assuming that the world economy consists of distinct infinitely lived households that come into being on different dates and are born owning no assets. The demographic structure, combined with the assumption that newly born agents have no financial wealth, allows the model to be characterized by a steady state to which the world economy returns over time following temporary shocks.[4] Agents consume; hold money balances, bonds, and shares in firms; and supply labor. I thus extend the Weil (1989) framework to allow for endogenous labor supply and differences in income across agents of different generations at each point in time.

My model does not rely on the Corsetti-Pesenti simplifying assumption that removes current-account effects. The fact that the elasticity of substitution between domestic and foreign goods may easily differ from one in reality induces me to prefer a framework in which movements of the current account are not removed *a priori*. The justification they give for their key assumption itself deserves empirical investigation.[5]

Firms produce using labor and physical capital. Capital is accumulated via investment, and new capital is costly to install as in a familiar Tobin's *q* model. The presence of monopoly power

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[2] Bergin (1997) extends the Obstfeld-Rogoff framework to allow for investment and capital accumulation and performs calibration exercises. Kollmann (1999) analyzes the implications of nominal rigidity for the behavior of asset prices. Nonetheless, the arbitrariness of the point around which to log-linearize is not resolved in their models.

[3] Engel and Rogers (1996) provide evidence of deviations from the law of one price between the U.S. and Canada—the economies on which I focus in my estimation exercise. This notwithstanding, I limit myself to the simpler case here, to focus on other directions along which the original Obstfeld-Rogoff framework can be extended. Allowing for deviations from the law of one price is left for future work.

[4] Mendoza (1991) and Correia, Neves, and Rebelo (1995) provide different solutions to the non-stationarity issue, which still rely on a representative agent model. Their approaches are discussed below.

[5] The assumption of complete markets would remove current-account effects without the need for the Corsetti-Pesenti simplification. I do not make that assumption either. As argued by Obstfeld and Rogoff (1995, 1996 Ch. 10), it seems at odds with the presence of real effects of unanticipated monetary shocks in a world in which prices are sticky. See G. Benigno (1999) and Gali and Monacelli (1999) for models that rely on the complete markets assumption.

has consequences for the dynamics of employment, by introducing a wedge between the real wage index and the marginal product of labor.

I assume that firms face costs of adjusting the price of their outputs. I choose a quadratic specification for these costs, as in Rotemberg (1982). This specification produces aggregate dynamics similar to those induced by staggered price setting a' la Calvo (1982). It also generates a markup endogenous to the conditions of the economy as long as the latter is not in steady state.[6] The dynamics of the markup play an important role in business cycle fluctuations, consistent with the analysis of Rotemberg and Woodford (1990). The dynamics of the real wage are not tied to those of the marginal product of labor.

Although the model is potentially a tool for analyzing bilateral interdependence between countries, in its first empirical implementation, I focus on a small open economy case in order to make use of a set of simplifying exogeneity assumptions. The home economy is identified with Canada, which is small and open when compared to the rest-of-the-world economy, approximated by the United States. For this reason, when presenting the model, I assume that the home economy is much smaller than the foreign one. The small open economy assumption implies that foreign variables and world aggregates are given from the perspective of the domestic economy. The actual situation is one of unilateral dependence of the domestic economy on the rest of the world rather than of explicitly bilateral interdependence. Exogeneity of foreign variables with respect to home's provides a set of restrictions I use in my empirical analysis.

In the second part of the paper, I estimate the structural parameters of the Canadian economy by making use of non-linear least squares at the single-equation level. Calibration is used only when the regressions do not yield sensible estimates. The sensibility of the parameter values obtained in this way is verified by comparing them to the findings of a large empirical literature. Estimates of most parameters turn out to be characterized by small standard errors and are in line with the findings of other studies. Illustrating a plausible way of constructing measures for non-observable variables is a contribution of this part of the paper. To verify if multiple-equation regressions yield significantly different estimates, I also run full information maximum likelihood system-wide regressions taking the estimates from the single-equation procedure as initial values. The results of the two procedures are similar.

Finally, I illustrate the functioning of the model by using the parameter estimates to calibrate it and analyze the transmission of a shock to U.S. GDP to the Canadian economy under inflation targeting, the monetary rule currently followed by the Bank of Canada. When doing this exercise, I combine the theoretical model of the Canadian economy with a simple VAR that traces the comovements of U.S. variables affecting Canada directly. The exercise illustrates the role of markup and relative price dynamics in the model. The latter does a better job than the flexible-price frameworks used by Schmitt-Grohé (1998) at explaining the transmission of U.S. cycles to Canada.[7]

The structure of the paper is as follows. Section 2 presents the model. Section 3 analyzes the characteristics of the steady state for the accumulation of financial assets. In Section 4, I log-

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[6] Carré and Collard (1997) adopt a similar approach, as well as Ireland (1997, 1999) in a closed economy setting. G. Benigno (1999) and P. Benigno (1999) rely on Calvo-type mechanisms.

[7] In Ghironi (1999*b*), I evaluate the performance of the Canadian economy under alternative monetary rules when Canada is subject to different sources of volatility. Because the exercise relies on estimates of the structural parameters of the model, the bearing of the Lucas critique on the results is weakened.

linearize the main equations and illustrate the estimation procedure. Section 5 illustrates the example. Section 6 concludes.

2. The Model

2.a. The Setup

The model is a perfect-foresight general equilibrium model. The world is assumed to consist of two countries, home and foreign. Home is identified with Canada and foreign with the U.S. Variables referring to the foreign economy are denoted by an asterisk. World variables are denoted with a superscript *W*. In each period *t*, the world economy is populated by a continuum of distinct infinitely lived households between 0 and N_t^W. Each of these households consumes, supplies labor, and holds money balances, bonds, and shares in firms. Following Weil (1989), I assume that households come into being on different dates and are born owning no financial assets or cash balances. N_t —the number of households in the home economy—grows over time at the exogenous rate n, so that $N_{t+1} = (1+n)N_t$. I normalize the size of a household—or dynasty—to 1, so that the number of dynasties alive at each point in time is also the economy's population. Foreign population grows at the same rate as home, and I assume that the ratio N_t / N_t^* is sufficiently small that home's population is small relative to the rest-of-the-world's. World population at time 0, when the economy starts, is normalized to the continuum between 0 and 1, so that $N_0^W = 1$.

At time 0, the number of households in the world economy is equal to the number of goods that are supplied. As in Obstfeld and Rogoff (1995, 1996 Ch. 10), a continuum of goods $z \in [0, 1]$ is produced in the world by monopolistically competitive infinitely lived firms, each of whom produces a single differentiated good. Over time, the number of households grows, but the commodity space remains unchanged. The ownership of the firms is thus spread among a larger number of households as time goes by.[8] I assume that the domestic economy produces goods in the interval [0, *a*]—which is also the size of the home population at time 0—whereas the foreign economy produces goods in the range (*a*, 1]. Because the ratio N_t / N_t^* is constant, it is always equal to $a/(1-a)$, and the assumption that N_t / N_t^* is small is sufficient to ensure that home produces a small share of the goods available for consumption in each period.

Consumers have identical preferences over a consumption index, leisure, and real money balances. At time t_0, the representative home consumer *j* born in period $\upsilon \in [0,\ t_0]$ maximizes the intertemporal utility function:

$$U_{t_0}^{\upsilon j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\left({C_t^{\upsilon j}}^{\rho} {LE_t^{\upsilon j}}^{1-\rho} \right)^{1-\frac{1}{\sigma}} \Big/ \left(1-\frac{1}{\sigma}\right) + \chi \left(M_t^{\upsilon j} / P_t \right)^{1-\frac{1}{\mu}} \Big/ \left(1-\frac{1}{\mu}\right) \right], \qquad (2.1)$$

where $0 < \rho < 1$ and $\chi,\ \mu,$ and σ are all strictly positive. Consumers in the foreign economy have a similar objective. The functional form of their period utility function, as well as the parameters

[8] Firms' profits are distributed to consumers via dividends. The structure of the market for each good is taken as given. It is possible to extend the model to allow for growth in the number of goods available in the world economy.

that characterize it, and the foreign consumers' discount factor are allowed to differ from the corresponding features of the domestic consumers' utility. The variable *C* is a real consumption index, *LE* denotes leisure, *M* is nominal money, and *P* is the price deflator.

Preferences for consumption goods are assumed to be identical in the two countries. At time *t*, the consumption index for the representative domestic consumer born in period υ is:

$$C_t^{\upsilon j} = \left[\int_0^a \left(c_t^{\upsilon j}(z) \right)^{\frac{\theta-1}{\theta}} dz + \int_a^1 \left(c_{t\,*}^{\upsilon j}(z) \right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \tag{2.2}$$

with θ > 1.[9] $c_{t\,*}^{\upsilon j}(z)$ is time *t* consumption by the representative home resident born in period υ of good *z* produced in the foreign country. *a* captures both the geographic location of production and the allocation of spending. Since *a* is small, the relative share of domestic goods in consumption is small.

The assumptions that the domestic population is small relative to the rest-of-the-world's, that the number of goods produced in the home economy is small, and that the relative weight of foreign goods in the consumption basket is large—combined with that of free capital mobility made below—are equivalent to the assumption that home is a small open economy, whose actions have a negligible impact on the rest of the world.

Consumers in the foreign economy are assumed to have identical preferences for consumption goods as those in the domestic country and with the same elasticity of substitution.

I assume that workers supply labor in competitive labor markets. The total amount of time available in each period is normalized to 1, so that:

$$LE_t^{\upsilon j} = 1 - L_t^{\upsilon j}, \tag{2.4}$$

and a similar constraint holds in the foreign economy.[10]

The price deflator for nominal money balances is the consumption-based money price index. Letting $p_t(z)$ ($p_t^*(z)$) be the home (foreign) currency price of good *z*, the money price levels in home and foreign are, respectively:

$$P_t = \left[\int_0^1 \left(p_t(z) \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_t^* = \left[\int_0^1 \left(p_t^*(z) \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

I assume that there are no impediments to trade, so that the law of one price holds for each individual good. Letting ε denote the domestic currency price of one unit of the foreign currency, this implies $p_t(z) = \varepsilon_t p_t^*(z)$.[11]

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[9] The parameter θ will turn out to be the price elasticity of demand faced by each producer. $\theta > 1$ is required to ensure an interior equilibrium with a positive level of output. I assume that the degree of substitutability is the same across all goods. Tille (1998*b*) obtains interesting results about the consequences of monetary shocks in the presence of differences in the degree of substitutability between goods inside each country and between the baskets of goods that each country produces.

[10] The model can be extended to allow for workers' monopoly power. However, that makes the algebra more complicated. In Corsetti and Pesenti (1998), population is constant and workers have monopoly power.

[11] Formally, the price index for each country solves the problem of minimizing total spending evaluated in units of the country's currency subject to the constraint that the real consumption index be equal to 1. The assumption that consumers born at different points in time all share the same characteristics ensures that firms have no incentives to price discriminate across consumers of different ages. In addition, the assumption that substitutability across goods is identical in the two countries ensures that firms have no incentives to price discriminate across markets.

Using the law of one price and recalling that the home economy produces goods in the range between 0 and *a*, makes it possible to show that $P_t = \varepsilon_t P_t^*$. Consumption-based PPP holds because consumption baskets are identical across countries and there are no departures from the law of one price.

Private agents are not the only consumers of goods. Governments also consume goods. To keep things simple and avoid composition effects, I assume that government spending is purely dissipative and that the government's real consumption index takes the same form as the private sector's in each country and with the same elasticity of substitution, θ.[12]

To keep the model relatively simple, I assume that each economy's firms are owned only by domestic residents. Thus, home agents buy shares of domestic firms' future profits in the stock market, whereas foreign agents hold shares of foreign firms.[13] The only internationally traded assets are bonds issued by the two countries. Each country issues bonds denominated in units of the country's currency. These bonds are regarded as perfect substitutes and an arbitrage condition—uncovered interest parity—holds in equilibrium:[14]

$$1+i_{t+1} = \left(1+i_{t+1}^*\right)\varepsilon_{t+1}/\varepsilon_t \quad . \tag{2.7}$$

i_{t+1} is the date *t* nominal interest rate on bonds denominated in home's currency. Letting r_{t+1} denote home's consumption-based real interest rate between *t* and *t* + 1, the familiar Fisher parity condition ensures that it is:

$$1+i_{t+1} = \left(1+r_{t+1}\right)P_{t+1}/P_t \ , \ 1+i_{t+1}^* = \left(1+r_{t+1}^*\right)P_{t+1}^*/P_t^* \ .$$

Perfect capital mobility and consumption-based PPP imply real interest rate equalization, so that $r_{t+1} = r_{t+1}^*$. Because home is small compared to the rest of the world, home agents take the foreign nominal interest rate and the world real interest rate r_{t+1} as exogenous.

2.b. Consumers' Behavior

Dropping the *j* superscript, because symmetric agents make identical equilibrium choices, optimal supply of labor is determined by the labor-leisure tradeoff equation:[15]

$$L_t^\upsilon = 1 - LE_t^\upsilon = 1 - (1-\rho)C_t^\upsilon / \left[\rho\left(1-\tau_t^L\right)W_t \ / P_t \ \right]. \tag{2.8}$$

When agents are optimizing, the marginal cost of supplying more labor equals the marginal utility of the additional consumption that the increase in labor income allows to afford. W is the nominal wage paid for one unit of labor, taken as given by workers. τ^L is the rate of distortionary taxation of labor income, which I assume constant across generations. *Ceteris paribus*, higher taxation causes workers to supply less labor. The choices of consumption and leisure—and real money holdings—are related to one another. This has important consequences. As noted by

G. Benigno (1999), Betts and Devereux (1996), Devereux and Engel (1998), and Tille (1998*a*) propose models of interdependence that allow for pricing to market.

[12] $G_t^{\upsilon j}$ is to be interpreted as time *t* government consumption of goods per each consumer *j* born in period υ. The presence of government consumption does not affect the expressions for the consumption-based price indexes.

[13] My assumption is consistent with the evidence in favor of a home bias in international markets for equities, although I provide no reason for the bias in my model. The model can be extended easily to allow for the ownership of firms to be spread between the two countries.

[14] This condition can be derived from the first-order conditions governing the consumer's optimal choice of bond holdings once indifference on the margin between domestic and foreign bonds is imposed.

[15] Details omitted here and elsewhere can be found in Ghironi (1999*a*).

Obstfeld and Rogoff (1996 Ch. 2.5), it implies that investment in the economy cannot be decoupled from the behavior of consumption. Because changes in consumption typically affect the marginal utility of leisure, they alter the amount of labor that residents are willing to supply at every wage. As a result, the marginal product of capital changes, with effects on investment that depend on the technology for installing capital.

Making use of the previous equation, the first-order conditions for the optimal holdings of domestic and foreign bonds reduce to the Euler equation:

$$C_{t+1}^{\upsilon} = \beta^{\sigma}\left(1+r_{t+1}\right)^{\sigma} C_t^{\upsilon}\left\{\left[\left(1-\tau_t^L\right)W_t \,/ P_t\right] / \left[\left(1-\tau_{t+1}^L\right)W_{t+1}/P_{t+1}\right]\right\}^{-(1-\rho)(1-\sigma)}, \qquad \upsilon \le t. \tag{2.9}$$

Unless $\sigma = 1$, in which case period utility is additively separable in consumption and leisure, the rate of consumption growth depends on the rate of growth of the net real wage. Depending on whether σ is smaller or larger than 1, even in the special case $\beta\left(1+r_{t+1}\right) = \beta\left(1+r\right) = 1$, net real wages that grow over time will introduce an upward or downward tilt in the path of consumption, respectively.

Demand for real balances is given by:

$$M_t^{\upsilon}/P_t = \left\{\left(\chi/\rho\right)\left(C_t^{\upsilon}\right)^{\frac{1}{\sigma}}\left[\left(1+i_{t+1}\right)/i_{t+1}\right]\left[\left(1-\rho\right)/\left[\rho\left(1-\tau_t^L\right)W_t \,/ P_t\right]\right]^{(1-\rho)\left(\frac{1-\sigma}{\sigma}\right)}\right\}^{\mu}. \tag{2.10}$$

Real balances increase with consumption and decrease with the opportunity cost of holding money. The impact of a higher real wage depends on σ. If $\sigma < 1$, a higher real wage will cause demand for real balances to decrease for any given level of consumption.[16]

2.c. Firms' Behavior

2.c.1. Output Supply

I assume that production requires labor and physical capital. Capital is a composite good, whose composition is the same as the consumption bundle, with the same elasticity of substitution in order to avoid composition effects. Output supplied at time *t* by the representative domestic firm *i* is:

$$Y_t^{Si} = Z_t\left(K_t^i\right)^{\gamma}\left(E_t\, L_t^i\right)^{1-\gamma}. \tag{2.11}$$

Because all firms in the world economy are born in period 0, after which no new good appears, it is not necessary to index output production and factor demands by the firms' date of birth. K_t^i is the firm's capital stock, and L_t^i is labor employed by the firm. Z_t measures economy-wide exogenous shocks to productivity. E_t is exogenous worldwide labor-augmenting technological progress. I assume that $E_t = \left(1+g\right)E_{t-1}$, where g will turn out to be the steady-state rate of growth of aggregate output per capita. I also assume that $1+r > \left(1+n\right)\left(1+g\right)$, where r is the steady-state world real interest rate.

Rotemberg and Woodford (1993) argue that, when competition is not perfect, it is important to consider materials explicitly as an input distinct from capital. In their model, material inputs are a basket of all goods in the economy, with the same composition as the consumption

[16] As usual, first-order conditions and the period budget constraint need to be combined with an appropriate transversality condition to ensure optimality. The optimal behavior by consumers in the foreign economy is governed by similar conditions.

bundle. I choose not to consider materials as a distinct input for two reasons. First is my desire to keep the model simple. Second, I will assume below that purchases of goods are necessary to install new capital and make it operational and for marketing reasons. This will provide a channel through which materials affect firms' costs, and thus production, even if they do not enter the production function directly. Another difference between my model and Rotemberg and Woodford's is that I do not allow for the possibility of increasing returns and stick to a constant returns Cobb-Douglas technology. Again, reasons of simplicity motivate my choice. In the context of my model, increasing returns would pose problems of aggregation that would unnecessarily cloud the analysis.

2.c.2. Output Demand and Price Stickiness

Demand for the firms' output comes from several sources. The demands of goods produced in the two countries by the representative home consumer born in period υ are:[17]

$$c_t^{\upsilon}(z) = \left(p_t(z)/P_t\right)^{-\theta} C_t^{\upsilon}, \;\; c_{t\,*}^{\upsilon}(z) = \left(p_t(z)/P_t\right)^{-\theta} C_t^{\upsilon}.$$

Given identity of preferences, expressions for the foreign consumers' demands are analogous.

Governments are assumed to act as price takers and their demand functions for individual goods have the same form as the private sector's.

At time *t*, total demand for home good *z* coming from domestic consumers is:[18]

$$c_t(z) = a\left[c_t^0(z) + nc_t^1(z) + n(1+n)c_t^2(z) + \ldots + n(1+n)^{t-1}c_t^t(z)\right] =$$
$$= \left(p_t(z)/P_t\right)^{-\theta}\left\{a\left[C_t^0 + nC_t^1 + n(1+n)C_t^2 + \ldots + n(1+n)^{t-1}C_t^t\right]\right\} = \left(p_t(z)/P_t\right)^{-\theta}\left[a(1+n)^t C_t\right],$$

where C_t is aggregate private home consumption per capita, defined by:

$$C_t \equiv a\left[C_t^0 + nC_t^1 + n(1+n)C_t^2 + \ldots + n(1+n)^{t-1}C_t^t\right] / \left[a(1+n)^t\right].$$

Similarly, total demand for the same good by foreign consumers is:

$$c_t^*(z) = \left(p_t(z)/P_t\right)^{-\theta}\left[(1-a)(1+n)^t C_t^*\right],$$

where $C_t^* \equiv (1-a)\left[C_t^{0^*} + nC_t^{1^*} + n(1+n)C_t^{2^*} + \ldots + n(1+n)^{t-1}C_t^{t^*}\right] / \left[(1-a)(1+n)^t\right].$

Aggregate consumption per capita and total demand for each good by the two governments are defined similarly.

Investment is modeled as in the familiar Tobin's *q* framework. Capital accumulation by firm *i* obeys the familiar equation:

$$K_{t+1}^i - K_t^i = I_t^i - \delta\, K_t^i, \tag{2.12}$$

where I is investment and δ is the rate of depreciation. Investment is a composite index of all the goods produced in the world economy, defined as the private and government consumption indexes *C* and *G*.

Adjusting the capital stock is costly. I assume that, in order to install new capital and make it operational, the firm needs to purchase materials in the amount $CAC_t^i = \eta I_t^{i^2} / \left(2K_t^i\right)$.

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[17] Demand functions are obtained by maximizing C subject to a spending constraint.

[18] Vintage $\upsilon = 0$ of home consumers, born at time 0, has size a. Home population at time 1 is $a(1+n)$, of which an individuals are born at time 1. Similarly, time 2 population contains $N_2 - N_1 = an(1+n)$ individuals born in period 2. Continuing with this reasoning, we see that $an(1+n)^{\upsilon-1}$ individuals are born in each period $\upsilon > 0$.

This quantity—which is measured in units of the composite consumption good—represents the real cost of adjusting the firm's capital stock. The cost is convex in the amount of investment. Faster changes in the capital stock are accompanied by more than proportional increases in installation costs. A larger amount of capital in place reduces adjustment costs because larger firms can absorb a given amount of new capital at a lower cost. Note the different interpretation of the adjustment cost relative to more traditional versions of the *q* model. There, the adjustment cost is measured as a reduction in the firm's output due to the investment activity. Here, because the cost is measured in units of the composite good, it seems natural to think of investment as causing costs due to the need of purchasing a set of goods that are required to make the installation possible and new capital operational.

Changing the output price is another source of costs. This assumption makes it possible to introduce nominal rigidities in the model, consistent with the strong evidence in favor of sluggish adjustment of prices. Specifically, I assume that the real cost of output-price inflation volatility around a steady-state level denoted by $\bar{\pi}_0$ is $PAC_t^i = \phi\left[\left(p_t(i)/p_{t-1}(i)\right)-1-\bar{\pi}_0\right]^2 K_t^i/2$. This cost is measured in units of the composite good. The intuition is that, when the firm changes the price of its output, a set of material goods—new catalogs, price tags, etc.—need to be purchased. PAC_t^i can be thought of as the amount of marketing materials that the firm needs to purchase when a change in price is implemented. Because the amount of these materials is likely to increase with the size of the firm, the cost of adjusting the price increases with the firm's capital stock, which is taken as a proxy for size. The cost is convex in inflation. Faster price movements are more costly to the firm. More marketing activity is likely to be required to preserve demand from falling too much as a consequence of a large price increase. Symmetrically, a large price cut gives the firm incentives to do more marketing as a way of letting a larger fraction of the public know about the lower price. The quadratic specification for the cost of adjusting prices yields dynamics for the economy that are similar to those resulting from staggered price setting a' la Calvo (1982), as pointed out by Blanchard and Fischer (1989 Ch. 8.2). Thus, the specification adopted in this paper can also be thought of as an approximation for a mechanism of price staggering.[19]

Total demand for good *i* produced in the home country is obtained by adding the demands for that good originating in the two countries. Making use of the results above, it is:

$$Y_t^{Di} = \left(p_t(i)/P_t\right)^{-\theta} \hat{Y}_t^{DW}, \tag{2.13}$$

where a hat on a variable denotes the aggregate level of the variable and $\hat{Y}_t^{DW}$ is aggregate world demand of the composite good, defined by $\hat{Y}_t^{DW} \equiv \hat{C}_t^W + \hat{G}_t^W + \hat{I}_t^W + C\hat{A}C_t^W + P\hat{A}C_t^W$.

2.c.3. Optimality Conditions

The representative firm chooses the price of its product, labor, investment, and capital in order to maximize the present discounted value of current and future profits subject to the constraints (2.11), (2.12), (2.13), and the market clearing condition $Y_t^i = Y_t^{Si} = Y_t^{Di}$. The firm takes the aggregate wage and price indexes, *Z*, *E*, world aggregates, and the rate of taxation of its revenues as given.

The first-order condition with respect to $p_t(i)$ returns the pricing equation:

[19] Rotemberg (1982) has first explored the consequences of quadratic costs of adjusting prices.

$$p_t(i) = \Psi_t^i P_t \lambda_t^i. \tag{2.14}$$

In each period, firms charge a price which is equal to the product of the (nominal) shadow value of one extra unit of output—the (nominal) marginal cost, $P_t \lambda_t^i$—times a markup. The latter depends on output demand as well as on the impact of today's pricing decision on today's and tomorrow's costs of adjusting the output price.[20] If $\phi = 0$, *i.e.* if prices are fully flexible, Ψ_t^i reduces to $\theta / [(\theta - 1)(1 - \tau_t^F)]$, which is a familiar constant-elasticity markup that increases if taxation of revenues—τ^F—is higher.

Introducing price rigidity generates endogenous fluctuations of the markup even in the presence of a constant elasticity specification of demand that is consistent with the assumptions of Rotemberg and Woodford's (1993) basic model, where the markup is constant. Because Ψ_t^i depends on $p_{t+1}(i)$, $p_t(i)$, and $p_{t-1}(i)$—as well as on Y_t^{Di}—if $\phi \neq 0$, equation (2.14) defines $p_t(i)$ implicitly as solution to a second-order non-linear difference equation. In Ghironi (1999*a*), I show that, if it is optimal to have relatively higher price inflation today than tomorrow, the firm will find it optimal to react to an increase in demand today by raising its markup. If instead today's weighted optimal inflation is lower than tomorrow's, an increase in demand will be accompanied by a decrease in the markup. I also argue that the introduction of nominal price rigidity in a constant-elasticity framework that would otherwise be characterized by a constant markup makes some of the predictions of the model resemble those of the implicit collusion model put forth by Rotemberg and Woodford (1990), thus providing an alternative explanation for the evidence they gather on the U.S. economy.

If θ approaches infinity, firms have no monopoly power, and the markup reduces to the competitive level $\Psi_t^{PC^i} = 1/(1 - \tau_t^F)$ regardless of the (finite) value of ϕ.[21] Under perfect competition, the presence of a cost of adjusting the price level is *de facto* irrelevant for the firm's decisions. Some degree of monopoly power is necessary for the nominal rigidity to matter. When the elasticity of substitution across goods is finite, $\Psi_t^i > \Psi_t^{PC^i}$ as long as the real net revenue from output sale is larger that the real marginal cost of a change in price, condition that must be satisfied for the firm to be optimizing.

The first-order condition for the optimal choice of L_t^i yields:

$$\lambda_t^i (1-\gamma) Y_t^{Si} / L_t^i = W_t / P_t \ . \tag{2.15}$$

At an optimum, the real wage index must be equal to the shadow value of the extra output produced by an additional unit of labor.

In a model in which firms have no market power and they take the price as given, labor demand is determined by the familiar equality between the real wage index—W_t / P_t—and the (net) marginal product of labor measured in units of the composite good—$(1 - \tau_t^F)(p_t(i)/P_t)(1-\gamma) Y_t^{Si} / L_t^i$. Here, the combination of pricing and labor demand decisions yields $W_t / P_t = [p_t(i) / (P_t \Psi_t^i)](1-\gamma) Y_t^{Si} / L_t^i$, which reduces to the familiar condition that

[20] The exact expression of the markup can be found in Ghironi (1999*a*).

[21] I am implicitly assuming that all variables in the definition of Ψ_t^i have finite limit values as θ approaches infinity.

determines labor demand in a competitive framework when θ approaches infinity. The presence of monopoly power introduces a wedge between the real wage index and the marginal product of labor. Because $\Psi_t^i > 1/\left(1-\tau_t^F\right)$ when θ is finite, monopoly power causes firms to raise the marginal product of labor above the real wage, *i.e.*, to demand less labor than they would under perfect competition, as in Rotemberg and Woodford (1993). The wedge between the real wage index and the marginal product of labor reflects also the presence of costs of adjusting the price level. A finite value of θ causes price stickiness to have a direct effect on labor demand, which would disappear if firms had no monopoly power.

The cyclical behavior of the markup in my model, and its impact on labor demand, are consistent with Rotemberg and Woodford's (1990) claim that markup variations play an important role in business cycle fluctuations. When goods markets are competitive, increases in the demand for goods cannot shift the labor demand curve, because capital is predetermined in the short run, and demand shocks have no impact on technology. Hence, changes in demand can affect output only insofar as the short-run labor supply is affected, and firms move along their labor demand schedules in response to adjustments in the real wage. Because empirical evidence shows that real wages fail to behave countercyclically—as this theory of output fluctuations requires—one is left with productivity shocks as the only potentially relevant source of business cycles under perfect competition. For demand shocks to be relevant for output behavior, a firm's willingness to hire additional workers at a given real wage must change with output demand. This happens if the markup reacts to the conditions of demand, as it does in my model and in the framework proposed by Rotemberg and Woodford (1990).

The first-order condition for the optimal choice of I_t^i implies that firm i's investment is positive if and only if the shadow value of one extra unit of capital in place at the end of period t—q_t^i—is larger than 1:

$$I_t^i = K_t^i\left(q_t^i - 1\right)/\eta . \tag{2.16}$$

q_t^i obeys the difference equation:

$$q_t^i = \left(\frac{1}{1+r_{t+1}}\right)\left[q_{t+1}^i(1-\delta) + \frac{W_{t+1}}{P_{t+1}}\frac{\gamma}{1-\gamma}\frac{L_{t+1}^i}{K_{t+1}^i} + \frac{\eta}{2}\left(\frac{I_{t+1}^i}{K_{t+1}^i}\right)^2 - \frac{\phi}{2}\left(\frac{p_{t+1}(i)}{p_t(i)} - 1 - \overline{\pi}_0\right)^2\right]. \tag{2.17}$$

The shadow price of one unit of capital in place at the end of period t is the discounted sum of the shadow price of capital at time t + 1 net of depreciation, of the shadow value of the incremental output generated by capital at t + 1, and of the marginal contribution of capital in place at the end of period t to the costs of installing capital and changing the price of the firm's output at time t + 1. Solving this equation under the assumption of no speculative bubbles yields:

$$q_t^i = \sum_{s=t+1}^{\infty} R_{t,s}(1-\delta)^{s-(t+1)}\left[\frac{W_s}{P_s}\frac{\gamma}{1-\gamma}\frac{L_s^i}{K_s^i} + \frac{\eta}{2}\left(\frac{I_s^i}{K_s^i}\right)^2 - \frac{\phi}{2}\left(\frac{p_s(i)}{p_{s-1}(i)} - 1 - \overline{\pi}_0\right)^2\right].$$

The shadow value of an additional unit of capital installed during period t is equal to the present discounted value of its marginal contributions to production and to the firms' costs. [22]

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[22] The discount factor $R_{t,s}$ is defined by: $R_{t,s} \equiv 1\Big/\prod_{u=t+1}^{s}\left(1+r_u\right)$. $R_{t,t}$ is interpreted as 1.

2.c.4. Average and Marginal q

Equation (2.17) yields an expression for the so-called marginal *q*. It is possible to show that an alternative expression is given by:

$$q_t^i = \left\{ \left(V_t^i / P_t\right) + \sum_{s=t+1}^{\infty} R_{t,s} \left[\left(1/\Psi_s^i\right) - \left(1-\tau_s^F\right) \right] p_s(i) Y_s^{Si} / P_s \right\} \Big/ K_{t+1}^i , \tag{2.18}$$

where V_t^i is the price of a claim to firm *i*'s future profits. This equation shows the result first obtained by Hayashi (1982). The ratio of the firms' equity to the capital stock—$\left(V_t^i / P_t\right) \big/ K_{t+1}^i$—is the so-called average *q*—q^{AVG^i}. Under perfect competition (when θ is infinite), the markup reduces to $1 \big/ \left(1-\tau^F\right)$, and marginal and average *q* coincide. When firms have monopoly power, Ψ is higher than $1 \big/ \left(1-\tau^F\right)$, and marginal *q* is smaller than average *q*. The shadow value of an additional unit of capital installed at the end of period *t* is smaller under monopolistic competition because a larger capital stock causes production to increase and the output price to decrease. This conflicts with a monopolist's incentive to keep the price higher and supply less output than it would be optimal in the absence of monopoly power. Fluctuations in the markup during the business cycle affect investment and capital accumulation by generating fluctuations in the difference between q^{AVG^i} and q^i.[23]

3. A Steady State for Home's Net Foreign Assets

The behavior of the world economy is determined by the optimality conditions for consumers and firms in the two countries and by the constraints facing private agents and governments. In this section, I explore some characteristics of the steady state equilibrium for the home economy, holding the features of the foreign country exogenous.

One of the main problems in the models proposed by Obstfeld and Rogoff (1995, 1996 Ch. 10) and by others is the absence of a well-defined endogenously determined steady state. The choice of the initial point for the purpose of performing comparative statics exercises is arbitrary, and all shocks have permanent consequences via redistribution of wealth across countries, a result that is debatable on empirical grounds. From a technical point of view, the models are linearized around an initial steady state to which the economy *never* returns no matter the nature of the disturbances that affect it. This raises suspicions on the reliability of the log-linear approximation.

De facto, indeterminacy of the steady state and non-stationarity preclude any stochastic application of the framework, unless the current-account channel of international interdependence is shut off. This can be accomplished either by assuming unitary intratemporal elasticity of substitution across domestic and foreign goods—along the lines of Corsetti and Pesenti (1998)—or by assuming that financial markets are complete. Obstfeld and Rogoff (1998) rely on the Corsetti-Pesenti simplification when extending their model to the stochastic case. G. Benigno (1999) and Gali and Monacelli (1999) resolve the stationarity issue by assuming that markets are complete. Under both approaches, the current account does not react to shocks. The realism of the Corsetti-Pesenti hypothesis is an empirical issue. Complete markets appear at odds with the presence of real effects of monetary policy in the presence of nominal rigidity. Regardless of these

[23] The model is closed by the governments' budget constraints and current-account equilibrium.

observations, the current account is an important channel of international interdependence for several economies. Hence, one would want to be able to work with models that do not shut it off a priori, possibly performing stochastic simulations of the models themselves. In order to do so, it is first necessary to understand why the indeterminacy arises and stationarity fails and to provide a solution to the problem.

Stationarity fails for an open economy whenever the equilibrium rate of aggregate per capita consumption growth is independent of the economy's aggregate per capita net foreign assets. In that case, the requirement that consumption be constant in steady state does not determine a unique steady state for net foreign assets. The model presented here has the advantage that its steady state is entirely determined by the structural parameters—and by the steady-state levels of some policy instruments—and is stable, if appropriate conditions are satisfied. In the model, aggregate per capita consumption growth does depend on aggregate per capita net foreign assets due to the discrepancy between the financial wealth of the newly born—zero—and the aggregate per capita financial wealth of those already alive. The solution for steady-state asset accumulation is described in detail in this section.

To keep things simple, I assume that all the government is doing in the analysis of this section is rebating seignorage revenues to the public. There is neither taxation, nor spending, nor government debt. I focus on the determination of the constant steady-state level of detrended aggregate per capita real net foreign asset holdings in the home economy, $a_t \equiv Q_t / E_t P_{t-1}$.[24]

The analysis clarifies how the demographic structure of the model and the assumption that newly born households have no financial wealth play a crucial role for the existence of a steady state. Correia, Neves, and Rebelo (1995) develop a representative agent model of a small open economy in which a stable steady state exists for *L*, *k*/*L*, *c*/*k*, and *a*/*k*. My strategy has the advantage of producing a steady state for variables that are directly relevant for the purpose of performing normative analysis, rather than for ratios of variables.[25] Mendoza (1991) solves the stationarity problem by assuming that the rate of time preference depends on net foreign assets, an approach originally proposed by Uzawa (1968). Smets and Wouters (1999) assume that agents derive utility from asset holdings. These assumptions ensure that the equilibrium rate of consumption growth depends on asset holdings, so that setting consumption to be constant pins down a steady-state distribution of net foreign assets. My approach is less subject to arbitrariness in the choice of the functional form of the discount factor and/or period utility function.

The derivation of a law of motion for aggregate per capita assets that takes the optimal path of consumption into account is more complicated in this model—in which agents' labor supply is governed by a labor-leisure tradeoff equation—than in Weil's (1989) or Obstfeld and Rogoff's (1996 Ch. 3.7), where labor income is exogenous.

If income is exogenous, one can assume that agents of different generations have identical income at each point in time.[26] Under this hypothesis, aggregate per capita income at each point in time is equal to the representative household's income. But assuming identical incomes for agents of different ages would be wrong here. Given that all agents face the same wage rate, the

[24] Steady-state levels of other variables are determined in Ghironi (1999*a*). I assume that money balances are endogenous and that monetary policy is conducted by choosing the nominal interest rate *i*. The process for this rate is assumed to converge to a steady state. Similarly, I assume that shocks to productivity—Z_t—are distributed around a steady-state value of 1. Q_{t+1}^{v} denotes the value of a dynasty's asset holdings entering period $t + 1$.

[25] See Ghironi (1999*b*) for an example.

[26] This is Weil's (1989) and Blanchard's (1985) assumption, as well as Obstfeld and Rogoff's (1996 Ch. 3.7).

assumption would imply that agents of different ages are supplying the same amount of labor. By the labor-leisure tradeoff condition, this would require agents born at different dates to have identical consumption levels, which cannot be true, given that agents of different generations will have accumulated different amounts of assets. The impossibility of constant labor income across generations complicates the solution of the model. This notwithstanding, it turns out that the complications can be dealt with by making use of the consumption-Euler equation and the labor-leisure tradeoff condition.

From the Euler equation for consumption,

$$C_s^{\upsilon} = \beta^{\sigma(s-t)}\left(R_{t,s}\right)^{-\sigma}\left[\left(W_t/P_t\right)/\left(W_s/P_s\right)\right]^{(1-\rho)(\sigma-1)}C_t^{\upsilon}, \qquad s>t. \tag{3.1}$$

Combining the Euler equation for consumption with the labor-leisure tradeoff yields an Euler equation for labor supply:

$$L_{t+1}^{\upsilon} = \beta^{\sigma}\left(1+r_{t+1}\right)^{\sigma}\left(\frac{W_t}{P_t}\frac{P_{t+1}}{W_{t+1}}\right)^{(1-\rho)(\sigma-1)+1} L_t^{\upsilon} + 1 - \beta^{\sigma}\left(1+r_{t+1}\right)^{\sigma}\left(\frac{W_t}{P_t}\frac{P_{t+1}}{W_{t+1}}\right)^{(1-\rho)(\sigma-1)+1} \tag{3.2}$$

Hence,

$$L_s^{\upsilon} = \beta^{\sigma(s-t)}\left(R_{t,s}\right)^{-\sigma}\left(\frac{W_t}{P_t}\frac{P_s}{W_s}\right)^{(1-\rho)(\sigma-1)+1} L_t^{\upsilon} + 1 - \beta^{\sigma(s-t)}\left(R_{t,s}\right)^{-\sigma}\left(\frac{W_t}{P_t}\frac{P_s}{W_s}\right)^{(1-\rho)(\sigma-1)+1}, \qquad s>t. \tag{3.3}$$

In real terms, the (equilibrium) budget constraint for the representative home agent is:

$$Q_{t+1}^{\upsilon}/P_t = \left(1+r_t\right)Q_t^{\upsilon}/P_{t-1} + W_t L_t^{\upsilon}/P_t - C_t^{\upsilon}. \tag{3.4}$$

Assuming that transversality and no-Ponzi-game conditions are satisfied, the intertemporal budget constraint is thus:

$$\sum_{s=t}^{\infty} R_{t,s}C_s^{\upsilon} = \left(1+r_t\right)Q_t^{\upsilon}/P_{t-1} + \sum_{s=t}^{\infty} R_{t,s}W_s L_s^{\upsilon}/P_s \ . \tag{3.5}$$

I now define:

$$\Theta_t \equiv \sum_{s=t}^{\infty}\beta^{\sigma(s-t)}\left(R_{t,s}\right)^{1-\sigma}\left[\left(W_t/P_t\right)/\left(W_s/P_s\right)\right]^{(1-\rho)(\sigma-1)}. \tag{3.6}$$

Substituting (3.1) and (3.3) into (3.5) and making use of (3.6) yields:

$$C_t^{\upsilon} = \Theta_t^{-1}\left[\left(1+r_t\right)Q_t^{\upsilon}/P_{t-1} + \sum_{s=t}^{\infty} R_{t,s}W_s/P_s\right] - \left(1-L_t^{\upsilon}\right)W_t/P_t \ . \tag{3.7}$$

Once the Euler equations for consumption and labor supply are taken into account, today's consumption depends positively on today's assets and on the present discounted value of the household's lifetime endowment of time in terms of the real wage. It depends negatively on the real value of today's leisure. Θ_t^{-1} is the (time-varying) slope of the consumption function that relates consumption to asset holdings and the present discounted value of the path of the real wage. Slightly rearranging equation (3.7) gives:

$$C_t^{\upsilon} + \left(1-L_t^{\upsilon}\right)W_t/P_t = \Theta_t^{-1}\left[\left(1+r_t\right)Q_t^{\upsilon}/P_{t-1} + \sum_{s=t}^{\infty} R_{t,s}W_s/P_s\right].$$

This equation shows that Θ_t^{-1} can be interpreted as a generalized propensity to consume goods or leisure out of the expected path of the agent's resources.

Substitution of (3.7) into (3.4) yields an important result:

$$Q_{t+1}^{\upsilon}/P_t = (1+r_t)\left[1-\Theta_t^{-1}\right]Q_t^{\upsilon}/P_{t-1} + W_t/P_t - \Theta_t^{-1}\sum_{s=t}^{\infty} R_{t,s}W_s/P_s. \tag{3.8}$$

This equation expresses asset accumulation by the representative dynasty born in vintage υ as a function of the time paths of the real wage index and of the real interest rate, which do not depend on the household's date of birth. Equation (3.8) allows me to provide a relatively straightforward generalization of the results in Weil (1989) and Obstfeld and Rogoff (1996 Ch. 3.7)—as well as of Blanchard's (1985) findings—to the case of endogenous labor income.

Applying the familiar aggregation procedure to equation (3.8), and recalling that dynasties are born holding no assets, yields the following law of motion for aggregate per capita real assets in the home economy:

$$\frac{Q_{t+1}}{P_t} = \frac{(1+r_t)\left[1-\Theta_t^{-1}\right]}{1+n}\frac{Q_t}{P_{t-1}} + \frac{W_t/P_t - \Theta_t^{-1}\sum_{s=t}^{\infty} R_{t,s}W_s/P_s}{1+n}. \tag{3.9}$$

Dividing both sides by trend productivity growth E_t, equation (3.9) can be rewritten as:

$$a_{t+1} = \frac{(1+r_t)\left[1-\Theta_t^{-1}\right]}{(1+g)(1+n)}a_t + \frac{w_t - \Theta_t^{-1}\sum_{s=t}^{\infty} R_{t,s}(1+g)^{s-t}w_s}{(1+g)(1+n)},$$

where Θ_t has been re-defined as:

$$\Theta_t \equiv \sum_{s=t}^{\infty} \beta^{\sigma(s-t)}\left(R_{t,s}\right)^{1-\sigma}(1+g)^{(1-\rho)(1-\sigma)(s-t)}\left(w_t/w_s\right)^{(1-\rho)(\sigma-1)},$$

and $w_t \equiv W_t/(E_tP_t)$. As long as the (time-varying) slope coefficient is smaller than 1 and the forcing function—which depends on the path of interest rate and real wage—converges to a finite value, home's detrended real net foreign assets converge to a steady-state level starting from any initial position.[27]

The steady state of the home economy is characterized by a constant detrended real wage—which is determined by labor market clearing—and by a constant real interest rate r—determined abroad. In particular, assuming that $[\beta(1+r)]^{\sigma}(1+g)^{(1-\rho)(1-\sigma)} < (1+n)(1+g)$, the steady-state level of detrended aggregate per capita net foreign assets accumulated by home agents as a function of the steady-state real wage rate is given by:

$$\overline{a}_0 = \left\{\frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)} - (1+g)}{(r-g)\left[(1+n)(1+g) - \beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}\right]}\right\}\overline{w}_0, \tag{3.10}$$

where I use overbars to denote the steady-state levels of variables, and the subscript 0 denotes variables that are *constant* in steady state and refers to the fact that the steady state will be taken to be the position of the home economy in the absence of shocks.

The home economy is a net creditor if $[\beta(1+r)]^{\sigma}(1+g)^{(1-\rho)(1-\sigma)} > 1+g$. It is a net debtor if $[\beta(1+r)]^{\sigma}(1+g)^{(1-\rho)(1-\sigma)} < 1+g$. To gain intuition on this result, consider the case in which $\sigma = 1$ and $g = 0$. Suppose also that the rest-of-the-world economy has already completed the transition to a steady-state position when the situation at home is taken into consideration, *i.e.*,

[27] I assume that the conditions ensuring convergence are satisfied. The steady-state level of the detrended real wage is obtained in Ghironi (1999*a*).

the world real interest rate is constant and equal to r along the path to home's steady state.[28] Finally, to further simplify the argument, suppose that the real wage rate is already constant at its steady-state level. The law of motion of home's aggregate per capita assets reduces to:

$$\frac{Q_{t+1}}{P_t} = \frac{\beta(1+r)}{1+n}\frac{Q_t}{P_{t-1}} + \left\{\frac{\beta(1+r)-1}{r(1+n)}\right\}\overline{w}_0 .$$

If $\beta(1+r)/(1+n) < 1$, a steady state level of real aggregate per capita assets exists and is stable. For this steady state level to be positive, the intercept of the linear relation between Q_{t+1}/P_t and Q_t/P_{t-1} must be positive. Under the assumptions of the special case we are considering, $\beta(1+r)$ is the slope of the time path of individual consumption. When $\beta(1+r) > 1$, individual consumption is increasing over time. Now, if income were exogenous—as in Blanchard (1985), Weil (1989), and Obstfeld and Rogoff (1996 Ch. 3.7)—one could assume that agents of different generations have the same income at each point in time. Under the assumption of constant individual labor income, for agents' consumption to be increasing over time, it must be the case that households are accumulating financial assets. Hence, the steady state—whose existence is ensured by population growth—must be characterized by positive aggregate per capita assets, since no individual has negative asset holdings.

In a framework in which labor income is endogenous, because agents of different generations supply different amounts of labor, individual labor income is *not* constant even when aggregate per capita income is. When income is not constant, one can think of situations in which individual consumption increases over time while assets are being decumulated, for example, depending on the agent's age. However, this is not the case in the steady state of the model. In fact, taking the Euler equation for labor supply into account removes the (direct) dependence of an agent's accumulation of assets on the quantity of labor supplied (which depends on the individual's date of birth) and shows that equilibrium asset accumulation is a function of the wage rate alone (which does not depend on the individual's age). When the economy is in steady state, individual asset accumulation follows:

$$\frac{\overline{Q}^{\upsilon}_{t+1}}{\overline{P}_t} = \beta(1+r)\frac{\overline{Q}^{\upsilon}_t}{\overline{P}_{t-1}} + \left[\frac{\beta(1+r)-1}{r}\right]\overline{w}_0 ,$$

which shows that $\beta(1+r) > 1$ is sufficient to ensure that the household's assets are increasing over time regardless of the household's date of birth. The intuition is clear if we look at the Euler equation for labor supply. The rate of change of an individual's supply of labor between any two periods during which the economy is in steady state is:

$$\frac{\overline{L}^{\upsilon}_{t+1} - \overline{L}^{\upsilon}_t}{\overline{L}^{\upsilon}_t} = -\left[\beta(1+r)-1\right]\frac{1-\overline{L}^{\upsilon}_t}{\overline{L}^{\upsilon}_t} ,$$

which is *negative* if $\beta(1+r) > 1$. Because labor income is declining over time in steady state, the household accumulates assets in order to sustain an increasing consumption. The individual

[28] r is determined by the structural characteristics of the foreign economy. The assumption that the latter is already in steady state, whereas home is not, is not innocuous in general. It can be made here because the disparity in the economies' sizes ensures that changes in domestic variables over time have no impact on the foreign ones. If the economies were of comparable size, it would be necessary to analyze the simultaneous convergence of the two economies to the steady state, because the evolution of home variables would affect foreign ones.

consumption-tilt factor $\beta(1+r)-1$ determines whether or not the country is a creditor or a debtor in steady state. If individual consumption is increasing over time, the country is a net creditor in the long run. Else, home runs a debt.

The result is robust to the adoption of a more general isoelastic utility function, in which σ is different from 1, and to the introduction of productivity growth. $\dfrac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+g)}$ is the slope coefficient of a household's Euler equation under the assumption that the real wage and the real interest rate be constant. As in the simpler case, this expression determines also the tilt of labor supply. Again, a household's consumption can increase over time in steady state only if the household is accumulating assets. $\dfrac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}$ is the slope coefficient of the law of motion for detrended aggregate per capita real asset holdings in this situation. If this coefficient is smaller than 1, *i.e.*, if population growth is sufficiently fast, new households with no inherited assets are entering the economy sufficiently quickly that detrended aggregate per capita assets reach a stable steady state. This involves a positive level of asset holdings because the assumption $[\beta(1+r)]^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}>1+g$ implies that there are no households with negative asset holdings. If it were $[\beta(1+r)]^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}<1+g$, all households would be dissaving, and steady state aggregate per capita assets would be negative. The existence/stability condition $\dfrac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}<1$ determines the sign of the denominator of $\overline{a}_0$, while the individual consumption-tilt factor determines the sign of the numerator.

Given steady-state asset holdings, aggregate per capita consumption and labor supply can be obtained easily. Steady-state aggregate per capita labor supply turns out to be vertical in the $\left(\overline{L}_0,\ \overline{w}_0\right)$ space. In steady state, employment is determined by the amount of labor that is supplied, and the real wage adjusts to clear the market.

Steady-state levels endogenous variables other than asset holdings are in Ghironi (1999*a*). Once the steady state is determined, the model can be log-linearized around it knowing that the transition dynamics following temporary or nominal shocks will bring the economy back to the original position over time—provided proper stability conditions are satisfied. There are several advantages to that, leaving aside the long-run neutrality of money, which some may argue is still an issue for empirical investigation. The arbitrariness of the starting point for the purpose of analyzing the consequences of a shock is removed. The reliability of the log-linear approximation is greatly increased, with positive implications for the confidence in the results of the model's dynamics. The presence of a well-defined steady state to which the economy returns over time makes it possible to perform sensible stochastic simulations without shutting off the current account channel, as I do in Ghironi (1999*b*). Finally, a stationary model significantly facilitates econometric work, to which I turn in the next section.

4. Estimating the Log-Linear Economy

In this section, I present the log-linear equations that govern the dynamics of the home economy following small perturbations to the steady state and illustrate a plausible approach to the estimation of the structural parameters of the domestic economy. In the empirical exercise, the latter is identified with Canada. The United States is taken as an approximation of the rest-of-the-world economy. For consistency with Ghironi (1999*b*), where I focus on monetary issues for Canada, I assume that fiscal policy variables are kept at their steady-state levels in what follows, and drop them from the log-linearized equations. Unobservable variables appear in some of these equations. Hence, an important part of this section deals with my empirical approach to the issue of measuring these variables.

The strategy used to estimate the structural parameters of the model consists of two steps. I first run single equation regressions. These—and the use of calibration when the regressions fail to yield sensible results—provide me with a set of baseline estimates. To verify the reliability of these estimates, I use them as starting values for full information maximum likelihood regressions of the systems of the firms' and consumers' first-order conditions.

Rotemberg and Woodford (1997) pursue an alternative approach to the issue of estimating a dynamic microfounded model. They run a three-variate recursive VAR for the U.S. economy and estimate the structural parameters of their model by calibrating them so that the model's impulse responses match the VAR's. I did not follow this strategy for two reasons. First, I find it ad hoc. Second, Rotemberg and Woodford's model is significantly simpler than mine, and it involves a smaller number of parameters. If I had followed their strategy, I would have faced the problem that a possibly large number of combinations of sensible parameter values are likely to allow the model to match the VAR's responses.

Cushman and Zha (1997) use the small open economy assumption to estimate a structural VAR of the U.S. and Canadian economies. Block exogeneity helps identify Canadian monetary shocks. However, no underlying model is estimated. Working along their lines, I could have run a large scale identified VAR a' la Leeper, Sims, and Zha (1996), but this would have left me with the problem of mapping the estimates of the VAR coefficients into estimates of the structural parameters of my model.

For these reasons, the strategy I decided to follow seems to be a reasonable way to let the data reveal something about parameter values. As it turns out, the strategy is fairly successful. As the reader will find out, I have to rely on pure calibration only in one case in the single-equation procedure, and the system-wide regressions yield estimates that are similar to those obtained in the first step. In addition, the estimates are in line with the findings of a large body of empirical literature.

4.a. Investment, Pricing, and Labor Demand

The production function in detrended aggregate per capita terms can be written as:
$y_t^S = \left(p_t(i)/P_t\right) Z_t k_t^{\gamma} L_t^{1-\gamma}$.
In steady state, both PPI and CPI inflation are constant at the level $\bar{\pi}_0$. Hence, the price of the representative domestic good in terms of the consumption bundle— $p_t(i)/P_t$ —has a constant steady-state level. Let $RP_t \equiv p_t(i)/P_t$. Log-linearizing the aggregate production function yields:

$$y_t = RP_t + \gamma\ k_t + (1-\gamma)L_t + Z_t, \tag{4.1}$$

where arial variables denote percentage deviations from the initial steady-state level of the original variables.[29]

For reasons that will be clear below, the elasticity of Canadian output with respect to hours worked—$1-\gamma$—is the first structural parameter that I need to estimate. The data come from the CANSIM database and from the IMF International Financial Statistics. I use quarterly data, consistent with the purpose of working at business cycle frequencies. Availability problems for some crucial series force me to restrict attention to the sample period 1980:1-1997:4. I construct the trend series E_t by assuming that g is the average rate of growth of Canadian real GDP per capita during the sample period and letting $E_{1980:2} = 1$. Steady-state levels of variables are calculated as the unconditional means of the series over the sample period. Variables in the regressions are defined as percentage deviations from the steady state, to match the concepts in the model.

Regressing Canadian GDP on the ratio of the industrial price index to the CPI, on the capital stock, on hours, and on a set of seasonal dummies, yields an estimate for $1-\gamma$ of approximately .9.[30] This is a high value for the elasticity of output to hours. However, it seems reasonable that GDP be much more sensitive to hours than capital on a quarterly basis. For this reason, I will take .9 to be the baseline value of $1-\gamma$ in what follows.

The log-linearized equation that determines detrended investment at each point in time is:

$$inv_t - k_t = \frac{1+\eta\left[(1+n)(1+g)-(1-\delta)\right]}{\eta\left[(1+n)(1+g)-(1-\delta)\right]} q_t. \tag{4.2}$$

From the law of motion for detrended aggregate per capita capital, it follows that:

$$k_{t+1} - k_t = \frac{(1+n)(1+g)-(1-\delta)}{(1+n)(1+g)}(inv_t - k_t). \tag{4.3}$$

The change in aggregate per capita capital between t and $t + 1$ is faster the larger investment and the smaller the stock of capital at time t.

Log-linearizing the Euler equation for capital accumulation in detrended aggregate per capita terms gives:[31]

$$\mathrm{q}_t = -\tilde{\mathrm{r}}_{t+1} + \frac{1-\delta}{1+r}\mathrm{q}_{t+1} + \frac{\gamma}{1-\gamma}\frac{\overline{w}_0\overline{L}_0}{\overline{q}_0\overline{k}_0(1+r)}(\mathrm{w}_{t+1} + \mathrm{L}_{t+1} - \mathrm{k}_{t+1}) + \frac{\eta\left[(1+n)(1+g)-(1-\delta)\right]^2}{\overline{q}_0(1+r)}(\mathrm{inv}_{t+1} - \mathrm{k}_{t+1}). \tag{4.4}$$

Tobin's q is one of the variables in my model that pose significant measurement problems. I construct a measure for the economy-wide q for Canada as follows.[32]

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[29] I have dropped the superscript S because of equality of demand and supply in equilibrium.

[30] The standard error of the estimate is .067, so that the estimated coefficient is highly significant. The estimated coefficient on capital in an unrestricted regression is .06, and it is hardly significant. The R^2 of the regression is .82, and the Durbin-Watson statistic is 1.3. Alternative specifications of the regression improved the significance of capital and raised the Durbin-Watson statistic somewhat, yielding estimates for $1-\gamma$ in the range .86 to .93.

[31] A tilde over an arial rate denotes the percentage deviation of the *gross* rate from its steady-state level.

[32] Schaller (1993) investigates the empirical performance of the q model for the Canadian economy using data from a panel of firms. He uses these data to construct series for q and shows that informational asymmetries cause firms' cash flows to have a significant impact on investment. Notwithstanding this result, I stick to the standard q

Given the expression for the individual firm's q in equation (2.18), it is possible to show that $q_t^i = \hat{q}_t = q_t$, where $\hat{q}_t$ is measured using aggregate data, and q_t is defined in terms of detrended aggregate per capita variables:

$$q_t = \left\{ v_t + \sum_{s=t+1}^{\infty} R_{t,s} \left[(1+n)(1+g) \right]^{s-t} \left[\frac{1}{\Psi_s} - \left(1-\tau_s^F\right) \right] y_s \right\} \Big/ \left[(1+n)(1+g) k_{t+1} \right],$$

$v_t \equiv V_t / E_t P_t$. I now define average q and the "adjustment for monopoly power" as:

$$q_t^{AVG} \equiv \frac{v_t}{(1+n)(1+g)k_{t+1}}, \quad adj_t \equiv \frac{\sum_{s=t+1}^{\infty} R_{t,s} \left[(1+n)(1+g) \right]^{s-t} \left[\frac{1}{\Psi_s} - \left(1-\tau_s^F\right) \right] y_s}{(1+n)(1+g)k_{t+1}}. \tag{4.5}$$

Measuring average q does not pose significant problems. Quarterly data for aggregate equity and firms' net capital assets since 1980:1 are available for the Canadian economy, and—together with the series for the CPI—they make it possible to obtain the desired measure. n is calculated as the average rate of growth of the Canadian population over the sample.

Measuring the adjustment term is harder. First, in reality agents do not have perfect foresight. adj_t is actually an expectation conditional on the information set available to firms at time t of the present discounted value of the "monopoly effect" from $t + 1$ on. Second, this effect depends on the behavior of the markup, which itself needs to be estimated.

The equilibrium value of the markup can be written as:

$$\Psi_t = (1-\gamma)\left(Y_t / L_t\right) / \left(W_t / P_t\right) = (1-\gamma) y_t / \left(w_t L_t\right).$$

Given the estimate of the elasticity of GDP with respect to hours obtained above, I can calculate a baseline series of the markup using this expression. The results suggest that the Canadian economy is characterized by a fairly small degree of monopoly power at the aggregate level. The average level of the markup implied by the data is 1.11. Figure 1 shows the series of the growth rate of the markup over the sample period and the series of the growth rate of hours. As expected, and consistent with the evidence for the U.S. economy in Rotemberg and Woodford (1990), the markup is strongly countercyclical. This is a feature of the model that is important in the analysis of the model's dynamics performed in Ghironi (1999*b*) and in the example of Section 5. Because the steady-state markup is given by $\overline{\Psi}_0 = \theta \Big/ \left[(\theta - 1)\left(1 - \overline{\tau}_0^F\right) \right]$, where $\overline{\tau}_0^F$ is the unconditional mean of the series of the tax rate on firms' revenues, it is possible to use this result to obtain an initial estimate of θ around 12.08.[33]

I now define the variable $\Omega_{t,s} \equiv R_{t,s} \left[(1+n)(1+g) \right]^{s-t} \left[(1/\Psi_s) - \left(1-\tau_s^F\right) \right] y_s$. I measure the relevant real interest rate with a series of the *ex post* real rate on Canadian T-Bills, calculated by

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model as an initial way to bring investment into the scene. Allowing for a role of cash flow in investment decisions is another direction along which the model can be extended.

[33] Although $\overline{\tau}_0^F$ is assumed to be zero in most of the discussion, I do make use of the data on taxation of firms' revenues in constructing a measure for *adj*. Schmitt-Grohé (1998) calibrates the steady-state markup to 1.4 in her analysis of the transmission of U.S. business cycles to the Canadian economy. This yields an estimate for θ of 3.68. To allow an easier comparison of my results with Schmitt-Grohé's, I use this as baseline value of θ in Ghironi (1999*b*) and in the example of Section 5.

deflating the nominal rate with CPI inflation.[34] The rate of taxation of firms revenues is proxied by a series of the ratio of corporate income taxes to sales. As Figure 2 shows, if the series of the variable $\Omega_{t,s}$ is plotted, the diagram suggests that the process for $\Omega_{t,s}$ is non-stationary: the variance of the series drops to zero as time goes by.[35] When discounted back to the initial date by making use of the real interest rate, the value of the variable $X_s \equiv [(1+n)(1+g)]^{s-t}[(1/\Psi_s)-(1-\tau_s^F)]y_s$ decays towards zero, *i.e.*, discounting by the real interest rate introduces a trend in $\Omega_{t,s}$.

It is possible to show that $(1+r_s)\Omega_{t,s} = (X_s/X_{s-1})\Omega_{t,s-1} = (1+g_s^X)\Omega_{t,s-1}$, $\forall s \geq t+1$, where $g_s^X \equiv (X_s/X_{s-1})-1$. Averaging the ratio X_s/X_{s-1} over the sample, and letting *r* denote the steady-state real interest rate, it can be argued that the following process is a reasonable approximation for the behavior of $\Omega_{t,s}$ in a stochastic setting:

$$\Omega_{t,s} = [(1+g^X)/(1+r)]\omega\, \Omega_{t,s-1} + z_s,$$

where *z* is a series of unanticipated disturbances. Writing the process for $\Omega_{t,s}$ in this form makes it possible to remove the trending effect of the real interest rate when estimating the coefficient ω. Running the autoregression and controlling for seasonal effects yields a highly significant estimate for ω around .66.[36] The implied value of $[(1+g^X)/(1+r)]\omega$ is .73. Because the effects of *g* and *n* are already taken into account when detrending GDP in the definition of $\Omega_{t,s}$, one can also run the regression:

$$\Omega_{t,s} = [\omega'/(1+r)]\Omega_{t,s-1} + z_s. \tag{4.6}$$

The estimate for ω' when seasonal effects are accounted for is around .79. The implied value of $\omega'/(1+r)$ is .73, as expected.

Under the assumption that (4.6) is a reasonable approximation of the process for $\Omega_{t,s}$, the expectation of the realization of the process at any future date *t* + *s* is given by $\tilde{E}_t\Omega_{t,s} = [\omega'/(1+r)]^{s-t}\Omega_{t,t}$, where I differentiate the rational expectation operator—which I had not introduced so far—from the trend labor efficiency by use of a tilde. Thus:

$$adj_t = \frac{\Omega_{t,t}\sum_{s=t+1}^{\infty}[\omega'/(1+r)]^{s-t}}{(1+n)(1+g)k_{t+1}} = \frac{\omega'}{(1+r-\omega')(1+n)(1+g)}\left[\frac{w_t L_t}{(1-\gamma)k_{t+1}} - (1-\tau_t^F)\frac{y_t}{k_{t+1}}\right]. \tag{4.7}$$

If the (exact) expression for *adj* in (4.5) is used to calculate its steady-state level in the absence of unexpected shocks,

$$\overline{adj}_0 = \frac{1}{1+r-(1+n)(1+g)}\left[\frac{\overline{w}_0\overline{L}_0}{(1-\gamma)\overline{k}_0} - (1-\overline{\tau}_0^F)\frac{\overline{y}_0}{\overline{k}_0}\right].$$

[34] The series of the U.S. and Canadian real interest rates show an average differential of about 2 percentage points over the sample period—with the Canadian rate being higher than the U.S. on average. This contradicts real interest rate equalization not only in the short run but also over a fairly long horizon. For this reason, I use the Canadian real rate to calculate the steady-state real rate in Canada.

[35] s = 1980:1 in the diagram.

[36] The standard error of the estimate is .047.

The value of this expression should be close to what would be obtained from the approximation in (4.7):

$$a\bar{d}j_0 = \frac{\omega'}{(1+r-\omega')(1+n)(1+g)}\left[\frac{\bar{w}_0 \bar{L}_0}{(1-\gamma)\bar{k}_0} - \left(1-\bar{\tau}_0^F\right)\frac{\bar{y}_0}{\bar{k}_0}\right].$$

Thus, the model yields a theoretical value of ω' approximately equal to $(1+n)(1+g)$. The data imply $(1+n)(1+g) \cong 1.004$. Hence, the estimate of ω' falls short of the value that the theory would dictate by approximately .25. This notwithstanding, I will use (4.7) as my measure of the adjustment for monopoly power in the expression for marginal *q*, setting $\omega'/(1+r-\omega') = 2.7$. Because the value of $\omega'/[(1+r-\omega')(1+n)(1+g)]$ is but a normalization of the variable $[(1/\Psi_t) - (1-\tau_t^F)]y_t/k_{t+1}$, my choice does not affect the results of regressions in which average *q* and the adjustment for monopoly power are treated as separate variables.[37]

The series for average *q* and the adjustment effect calculated following the procedure described above suggest a fairly strong relation for the larger part of the sample between average *q* and investment if inventories are not included in the definition of investment and capital. The relation is somewhat weaker when inventories are included (see Figure 3).[38] The series for the monopoly effect (not shown) is consistently negative—as suggested by the theory—and shows a much larger volatility.

In order to gain a sense of the empirical performance of my measure for the economy-wide marginal *q*, I ran an initial regression of $inv_t - k_t$ on q_t. This yielded a small and negative coefficient, in contrast with the theory. The values of R^2 and the Durbin-Watson statistic signaled a very poor performance of the regression.[39] I separated the effects of average *q* and the monopoly adjustment factor on the investment-capital ratio by running a regression of $inv_t - k_t$ on the series of the percentage deviations of average *q* and the monopoly adjustment factor from their steady-state levels. Because *adj* is negative, a positive deviation from the average signals a smaller monopolistic distortion, which increases marginal *q* and should cause larger investment.[40]

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[37] Results obtained by selecting a much higher value for $\omega'/(1+r-\omega')$ and using marginal *q* as a regressor were not significantly different. Note that I am implicitly assuming that the same process dictates the behavior of $\Omega_{t+1,s}, \Omega_{t+2,s}, \ldots$ and so on when firms are looking forward to formulate expectations about the behavior of output, taxation, and the markup at time *t* + 1, *t* + 2, ... and so on. This is a strong assumption—which I will make again below—although it seems a reasonable one under normal economic conditions.

[38] Because I did not differentiate between capital accumulation in the form of fixed capital and accumulation of inventories, I define investment as the sum of the change in the fixed capital stock and in inventories. Analogously, my measure of the capital stock includes fixed capital and inventories. The data on stocks come from quarterly balance sheets for all industries. They are interpreted here as measures of end-of-period stocks. Thus, the data on k_t is the actual empirical correspondent of k_{t+1} in the model. Arguably, the behavior of inventories is ruled by a different model than that of fixed capital. Ramey and West (1997) survey the research on inventories. They argue that this variable should receive independent attention in analyses of business cycle fluctuations. They also provide arguments for the importance of inventories as a factor of production. Treating inventories as a productive factor in the empirical implementation of my model in the same way as I treat fixed capital seems consistent with Rotemberg and Woodford's (1993) argument about the importance of materials in production.

[39] $R^2 = .027$, DW = .21.

[40] When the average of a series is negative, I calculate the percentage deviation of the series from the steady state as $x_t = (x_t - \bar{x}_0)/|\bar{x}_0|$.

The results of this regression were not encouraging either. The most likely explanation for the poor result of both regressions appeared to be serial correlation of the residuals. A regression in first differences proved more successful. The coefficient on average *q* was positive and significant. The coefficient on the monopoly factor turned out to be negative, but insignificantly different from zero. This suggested that the monopolistic distortion may not be a very relevant factor in driving the behavior of Canadian investment. It prompted me to continue treating average *q* and the adjustment for monopoly power as separate variables in the regressions calculated later. Log-linear equations for these variables can be obtained as follows.

Log-linearizing equation (4.7) yields:[41]

$$\mathrm{adj}_t = \left|\left[\frac{\overline{w}_0\overline{L}_0}{(1-\gamma)\overline{k}_0} - \left(1-\overline{\tau}_0^F\right)\frac{\overline{y}_0}{\overline{k}_0}\right]\right|^{-1}\left[\frac{\overline{w}_0\overline{L}_0}{(1-\gamma)\overline{k}_0}\left(\mathrm{w}_t + \mathrm{L}_t - \mathrm{k}_{t+1}\right) - \left(1-\overline{\tau}_0^F\right)\frac{\overline{y}_0}{\overline{k}_0}\left(\mathrm{y}_t - \mathrm{k}_{t+1}\right)\right]. \qquad (4.8)$$

Because *q* is the sum of q^{AVG} and *adj*, $q_t^{AVG} = \left(\overline{q}_0/\overline{q}_0^{AVG}\right)q_t - \left(\left|a\overline{d}j_0\right|/\overline{q}_0^{AVG}\right)adj_t$. Thus, making use of the log-linear equation for marginal *q*,

$$q_t^{AVG} = -\frac{\overline{q}_0}{\overline{q}_0^{AVG}}\tilde{r}_{t+1} + \frac{1-\delta}{1+r}q_{t+1}^{AVG} + \frac{1-\delta}{1+r}\frac{\left|a\overline{d}j_0\right|}{\overline{q}_0^{AVG}}\left(adj_{t+1} - adj_t\right) - \frac{r+\delta}{1+r}\frac{\left|a\overline{d}j_0\right|}{\overline{q}_0^{AVG}}adj_t +$$
$$+\frac{\gamma}{1-\gamma}\frac{\overline{w}_0\overline{L}_0}{\overline{q}_0^{AVG}\overline{k}_0(1+r)}\left(w_{t+1} + L_{t+1} - k_{t+1}\right) + \frac{\eta\left[(1+n)(1+g)-(1-\delta)\right]^2}{\overline{q}_0^{AVG}(1+r)}\left(inv_{t+1} - k_{t+1}\right). \qquad (4.9)$$

As noted above, a positive value of adj_t signals a reduction in the absolute value of the monopoly effect on marginal *q* relative to its mean. This increases the value of capital for the firm and has a negative impact on today's average *q* in equilibrium by inducing the firm to accumulate a larger capital stock by the end of period *t*. However, a positive value of $adj_{t+1} - adj_t$ tends to decrease the value of a unit of capital installed at the end of the current period relative to that of capital installed in the future. If the extent of the monopolistic distortion is expected to decrease in the future more than it does today, the firm has an incentive to postpone capital accumulation to the next period. Hence, today's average *q* is higher.

Given log-linear equations for the investment part of the economy, I turn to estimating the rate of depreciation δ and the parameter that dictates the size of the cost of adjusting the capital stock, η. I used non-linear least squares to estimate δ from a regression based on equation (4.3):

$$k_t = Ak_{t-1} + \frac{(1+n)(1+g)-(1-\delta)}{(1+n)(1+g)}\left(inv_{t-1} - k_{t-1}\right).$$

Restricting *A* to be equal to 1 and omitting seasonal dummies yielded an estimate of δ of .031.[42] Allowing *A* to differ from 1 and controlling for seasonal effects raised the estimate of the rate of depreciation to approximately .04.[43] I will use $\delta = .035$ as baseline value in what follows.

Estimating η is more troublesome. Non-linear least squares regressions based on (4.2) ran into singularity problems, regardless of the separation of average *q* from the monopolistic distortion effect, and even when the latter was dropped. An alternative log-linearization of the

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[41] In Ghironi (1999*b*) and in the example of Section 5, an exact forward-looking log-linear equation for *adj* is used, as opposed to the log-linearization of the empirical counterpart used in the estimation procedure.

[42] The standard error was .01, $R^2 = .89$.

[43] The estimate of *A* was close to .88, with a standard error around .04. The standard error for the estimate of δ was .098 ($R^2 = .9$).

investment equation, in which the steady-state relation between investment, capital, and q was not imposed, did not help.[44] Thus, in order to obtain a baseline value for η, I used the following approach. I ran an OLS regression in first differences of the investment-capital ratio on capital, average q, the monopoly adjustment factor, and a set of seasonal dummies. The coefficient on average q is approximately equal to $\left(\bar{k}_0 \bar{q}_0^{AVG}\right) / \left(\eta \ i\bar{n}v_0\right)$. Given the estimated coefficient on average q in the regression, it is thus possible to obtain an approximate estimate for η. The procedure suggested that values of η as high as 2 were consistent with the estimated coefficient on average q in equations that included lagged capital as a regressor. This estimate implies that the cost of adjusting the capital stock is approximately equal to the ratio I^2/K—a very large amount. Mendoza (1991) uses annual data between 1946 and 1985 to calibrate a model of the Canadian economy. The cost of adjusting capital in his paper is $(\eta/2)(I - \delta\ K)^2$. He finds that values of η between .023 and .028 allow the model to mimic the observed percentage standard deviation of investment. The expression of adjustment costs in this paper allows the costs to decrease with firms' size and accounts for the fact that replacing depreciated capital can be as costly as a net addition to the capital stock. Leaving other reasons aside, the much larger value of η that is produced by my procedure can be at least partially explained by the different data frequency. A much larger adjustment cost would explain the very small changes in the capital stock that are observed on a quarterly basis.[45]

From the labor demand equation,

$$L_t - L_{t-1} = k_t - k_{t-1} + (1/\gamma)\left[l_t - l_{t-1} + Z_t - Z_{t-1} - (w_t - w_{t-1})\right], \tag{4.10}$$

where $l_t \equiv d\lambda_t / \bar{\lambda}_0$, $Z_t \equiv dZ_t / \bar{Z}_0 = dZ_t$. Labor demand decreases with the real wage index; it increases with the shadow value of output. From the firms' pricing equation, it is apparent that increases in the markup that produce a decrease in the shadow value of output lead firms to demand less labor.

Market clearing for domestic GDP implies:

$$Z_t - Z_{t-1} + \gamma(k_t - k_{t-1}) + (1-\gamma)(L_t - L_{t-1}) = -\theta\left(\tilde{\pi}_t^{PPI} - \tilde{\pi}_t^{CPI}\right) + y_t^{SW} - y_{t-1}^{SW}, \tag{4.11}$$

with $y_t^{SW} \equiv dy_t^{SW} / \bar{y}_0^{SW}$ and $\tilde{\pi}_t^{PPI} \equiv d\left(1 + \pi_t^{PPI}\right) / \left(1 + \bar{\pi}_0\right)$. Combining (4.10) and (4.11) gives:

$$L_t - L_{t-1} = l_t - l_{t-1} - (w_t - w_{t-1}) - \theta\left(\tilde{\pi}_t^{PPI} - \tilde{\pi}_t^{CPI}\right) + y_t^{SW} - y_{t-1}^{SW}, \tag{4.12}$$

An increase in world output raises labor demand in the home economy by increasing the demand for domestic output. Instead, a higher producer price inflation lowers labor demand because of its negative impact on equilibrium production.

From the pricing equation,

$$\tilde{\pi}_t^{PPI} = \psi_t - \psi_{t-1} + \tilde{\pi}_t^{CPI} + l_t - l_{t-1}, \tag{4.13}$$

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[44] It is: $inv_t - k_t = \left\{\left[(\bar{q}_0 - 1)\bar{k}_0 / \left(\eta\ i\bar{n}v_0\right)\right] - 1\right\}k_t + \left[\bar{q}_0 \bar{k}_0 / \left(\eta\ i\bar{n}v_0\right)\right]q_t$.

The theory predicts that the steady-state levels of q, k, and inv should be such that the first term in this equation is zero. However, the sample means of these variables suggest that this is not the case for realistic values of η.

[45] Bergin (1997) uses a model of investment similar to mine. He argues that a value of η as high as 20 would be required in a calibration of the model for it to generate results that are consistent with the empirical evidence on adjustment costs for Japanese firms reported by Hayashi and Inoue (1991). If compared to such value, my choice of 2 for the calibration exercise of Ghironi (1999*b*) and Section 5 appears conservative!

where $\psi_t \equiv d\Psi_t / \overline{\Psi}_0$.

Log-differentiating the expression of the markup and substituting in (4.13) yields:

$$\tilde{\pi}_t^{PPI} = \tilde{\pi}_t^{CPI} + l_t - l_{t-1} - \frac{\phi(1+\overline{\pi}_0)^2}{\theta - 1} \frac{\overline{k}_0}{\overline{y}_0^S} \left[\tilde{\pi}_t^{PPI} - \tilde{\pi}_{t-1}^{PPI} - \frac{(1+n)(1+g)}{1+r} \left(\tilde{\pi}_{t+1}^{PPI} - \tilde{\pi}_t^{PPI} \right) \right].$$

The markup reacts endogenously to the behavior of PPI inflation over time. Markup growth is smaller if the current growth in PPI inflation is larger. Faster PPI inflation growth has an unfavorable effect on output demand via its impact on relative prices. Firms sustain their profitability by slowing down growth in the markup component of prices. However, the change in the markup is larger if the future change in PPI inflation is expected to be large. This reflects firms' incentives to smooth the behavior of output price over time.

If (4.12) is taken into account, we have an equation for the dynamics of PPI inflation:

$$\tilde{\pi}_t^{PPI} - \tilde{\pi}_{t-1}^{PPI} = \frac{(1+n)(1+g)}{1+r} \left(\tilde{\pi}_{t+1}^{PPI} - \tilde{\pi}_t^{PPI} \right) + \frac{\theta - 1}{\phi(1+\overline{\pi}_0)^2} \frac{\overline{y}_0^S}{\overline{k}_0} \begin{bmatrix} \mathrm{w}_t - \mathrm{w}_{t-1} + \mathrm{L}_t - \mathrm{L}_{t-1} + \\ + (\theta - 1)\left(\tilde{\pi}_t^{PPI} - \tilde{\pi}_t^{CPI} \right) - \left(\mathrm{y}_t^{SW} - \mathrm{y}_{t-1}^{SW} \right) \end{bmatrix}. \tag{4.14}$$

Today's acceleration in producer price inflation is faster the faster the future expected change in the inflation rate, the larger the real wage bill, and the larger the current deviation of the PPI from the CPI. Instead, an increase in world output causes PPI inflation to slow down.

I tried to estimate ϕ by running non-linear regressions based on equation (4.14). When the regressions did not run into singularity problems, the estimates of ϕ turned out to be characterized by extremely large standard errors. Hence, I decided to limit myself to calibration of this parameter. If (4.14) is used, together with values of θ between 3.68 and 12.08, ϕ as high as 200 is required to generate a pattern of deviations of PPI inflation from the steady state that matches the behavior of the observed series. Would $\phi = 200$ be an absurd value? Recall that the cost of adjusting prices is $(\phi/2)\left(\pi^{PPI} - \overline{\pi}_0\right)^2 K$. If steady-state quarterly inflation is about 1 percent per quarter, this says that increasing inflation by 10 percent—to 1.1 percent—would require the representative firm to purchase materials in an amount equal to .01 percent of its capital stock! Although the value of ϕ is very high, the actual cost borne by the firm for a substantial acceleration in its output price inflation does not seem unrealistic.

Aggregate labor demand per capita can be written as:

$$L_t = -w_t - (\theta - 1) RP_t - \psi_t + y_t^W, \tag{4.15}$$

Labor demand is larger if world output is larger. It is lower the higher the real wage and the markup. A higher real wage implies that the cost of labor is higher. A larger markup means that firms are exploiting their monopoly power more significantly. As a consequence, they supply less output and demand less labor. This is consistent with the empirical evidence on markup behavior in Rotemberg and Woodford (1990) and with the results obtained above. Larger deviations of the PPI from the CPI depress output demand more than they raise firms' profits for given output. Hence, they cause labor demand to decrease.

Because the equilibrium markup can be written as $\Psi_t = (1-\gamma) y_t / w_t L_t$, log-linearizing this expression and making use of the production function yields:

$$L_t = -[\theta/(1-\gamma)] RP_t - [\gamma/(1-\gamma)] k_t + [1/(1-\gamma)] y_t^W - [1/(1-\gamma)] Z_t. \tag{4.16}$$

This is the levels counterpart of equation (4.11): the goods' market clearing condition ultimately determines labor demand in a small open economy. Given output demand, labor demand will be smaller if the capital stock is larger or if firms are experiencing favorable shocks to productivity.

Using the production function and the log-linear expression for the equilibrium markup yields the following equation for the relative price:

$$\mathrm{RP}_t = \psi_t + \mathrm{w}_t + \gamma(\mathrm{L}_t - \mathrm{k}_t) - \mathrm{Z}_t .$$

Increases in the markup, the real wage, and/or the labor-capital ratio cause the PPI to increase relative to the CPI.

This equation can be combined with the definition of *RP* to obtain an alternative equation for PPI inflation that shows the direct linkage between the behavior of the PPI and that of the CPI in the model:

$$\tilde{\pi}_t^{PPI} = \tilde{\pi}_t^{CPI} + \psi_t - \psi_{t-1} + \mathrm{w}_t - \mathrm{w}_{t-1} + \gamma\left[\mathrm{L}_t - \mathrm{L}_{t-1} - (\mathrm{k}_t - \mathrm{k}_{t-1})\right] - (\mathrm{Z}_t - \mathrm{Z}_{t-1}). \tag{4.17}$$

Equation (4.17) is at the core of the results in Ghironi (1999*b*). Different monetary rules generate different CPI inflation dynamics. These cause different PPI dynamics, and thus different paths for the markup and the relative price of the representative domestic good. In turns, different markup and relative price dynamics affect the real side of the economy by changing firms' labor demand and investment decisions.

To verify the reliability of the estimates of the structural parameters obtained in this sub-section, I ran full information maximum likelihood regressions of the system of equations that govern the production side of the economy. I took the following as starting values for the procedure: $\delta = .035$, $\phi = 200$, $\gamma = .1$, $\eta = 2$, $\theta = 3.68$. The estimated parameters differed somewhat, but the results generally supported my choice of these values as baseline for the empirical evaluation of alternative monetary rules for Canada in Ghironi (1999*b*).[46]

4.b. Consumption, Labor Supply, and Money Demand

Applying the aggregation formula to the Euler equation for consumption and dividing both sides by trend productivity growth yields the following law of motion for detrended aggregate per capita consumption in the home economy:

$$c_{t+1} = \frac{\beta^{\sigma}(1+r_{t+1})^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} c_t \left[\frac{w_{t+1}(1-\tau_{t+1}^L)}{w_t(1-\tau_t^L)}\right]^{(1-\rho)(1-\sigma)} + \frac{n}{1+n} c_{t+1}^{t+1}, \tag{4.18}$$

where $c_{t+1}^{t+1} \equiv C_{t+1}^{t+1} / E_{t+1}$ is detrended consumption by the representative dynasty born at time $t + 1$ in the first period of its life, the steady-level of which is determined in Ghironi (1999*a*).

Log-linearizing (4.18) yields:

$$\mathrm{c}_{t+1} = \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\left[\mathrm{c}_t + \sigma\tilde{\mathrm{r}}_{t+1} + (1-\rho)(1-\sigma)(\mathrm{w}_{t+1} - \mathrm{w}_t)\right] + \left[1 - \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\right]\mathrm{c}_{t+1}^{t+1}. \tag{4.19}$$

Ceteris paribus, a positive change in the real interest rate causes future consumption to increase relative to current. An increase in the real wage between t and $t + 1$ has a positive impact on aggregate per capita consumption at time $t + 1$ relative to its level in period t if σ is smaller than 1.

[46] Given that the estimates did not differ too much, but were characterized by larger standard errors, I chose to use the values obtained from the single-equation procedure.

The existence/stability condition of a steady state with constant real wage and interest rate ensures that an increase in the time t + 1-newborn household's consumption causes aggregate per capita consumption at t + 1 to increase.

In order to analyze the response of aggregate per capita variables to shocks, it is necessary to determine c_{t+1}^{t+1}—the response of a newborn dynasty's consumption to the path of the shocks—as a function of variables that are *not* indexed by the dynasty's date of birth. A newborn household's consumption is a forward-looking variable that depends on the present discounted value of the entire stream of the dynasty's resources. Making use of the individual budget constraint and of the optimality conditions for newborn dynasties at time t + 1, it is possible to show that $c_{t+1}^{t+1} = \rho\ \Theta_{t+1}{}^{-1} inc_{t+1}$, where $inc_{t+1} \equiv \sum_{s=t+1}^{\infty} R_{t+1,s}(1+g)^{s-(t+1)}\left[\left(1-\tau_s^L\right)w_s - t_s\right]$, $t_s \equiv T_s/E_s$ under the assumption that lump-sum taxes and transfers do not differ across vintages, and $\Theta_{t+1} \equiv \sum_{s=t+1}^{\infty} \beta^{\sigma[s-(t+1)]}\left(R_{t+1,s}\right)^{1-\sigma}(1+g)^{(1-\rho)(1-\sigma)[s-(t+1)]}\left\{\left(1-\tau_{t+1}^L\right)w_{t+1}/\left[\left(1-\tau_s^L\right)w_s\right]\right\}^{(1-\rho)(\sigma-1)}$. *inc* is the net present discounted value of the representative dynasty's endowment of time in terms of the real wage. As mentioned above, Θ^{-1} can be interpreted as a time-varying propensity to consume out of the available resources. In terms of percentage deviations from the steady state,

$$c_{t+1}^{t+1} = -\tilde{\Theta}_{t+1} + inc_{t+1}.$$

Assuming that ρ is sufficiently high, a persistent increase in the real wage rate that lasts beyond t + 1 and causes the present discounted value of a newborn household's resources to be higher has a positive impact on aggregate per capita consumption at t + 1 by inducing the newborn household to consume more in the initial period of its life.

The present discounted value of a household's lifetime endowment of time in terms of the real wage rate is another variable for which a proxy needs to be found. Because an agent's endowment of time does not change across periods, I will retain the assumption that this is normalized to 1 in each period also in the empirical implementation of the model. In a stochastic setting, inc_{t+1} is actually defined by the rational expectation conditional on information available at time t + 1 of the stream of net real wage rates. Following the procedure used to find a measure for marginal q, I define $\Gamma_{t+1,s} \equiv R_{t+1,s}(1+g)^{s-(t+1)}\left[\left(1-\tau_s^L\right)w_s - t_s\right]$. The behavior of $\Gamma_{t+1,s}$ is similar to that of $\Omega_{t,s}$. Thus, one can reasonably approximate the expectation of $\Gamma_{t+1,s}$ at future dates with $\tilde{E}_{t+1}\Gamma_{t+1,s} = [\nu/(1+r)]^{s-(t+1)}\Gamma_{t+1,t+1}$, where $\nu/(1+r)$ is the coefficient in the process for $\Gamma_{t+1,s}$: $\Gamma_{t+1,s} = [\nu/(1+r)]\Gamma_{t+1,s-1} + \hat{z}_s, \forall s > t+1$. When this approximation is used, $inc_{t+1} = [(1+r)/(1+r-\nu)]\left[\left(1-\tau_{t+1}^L\right)w_{t+1} - t_{t+1}\right]$, and log-linearization around a steady state with no taxes yields $inc_{t+1} = w_{t+1}$.

This result suggests that, if the elements of the summation that defines *inc* decay towards zero as the time-horizon becomes longer, the percentage deviation of the current level of the real wage rate from its average is a good measure of the deviation of the present discounted value of the lifetime stream of real wage rates from its steady-state level. This result has implications for the findings of the literature on the sensitivity of aggregate consumption to current income. In an overlapping generations framework such as that explored in this paper, the aggregate Euler equation for consumption requires an adjustment that reflects the impact of a newborn generation's consumption on aggregate consumption. Because aggregate consumption at $t-1$ does not reflect this, time *t* income may contain information that is relevant for the behavior of aggregate consumption at time *t*, in conflict with the basic random walk result of Hall (1978). This is true regardless of the presence of liquidity constraints or other imperfections in financial markets.[47]

The last variable for which a proxy needs to be found is the time-varying propensity to consume Θ^{-1}. I will follow again the now familiar strategy. Given the expression for Θ, I define $\Sigma_{t+1,s} \equiv \beta^{\sigma[s-(t+1)]}\left(R_{t+1,s}\right)^{1-\sigma}(1+g)^{(1-\rho)(1-\sigma)[s-(t+1)]}\left\{\left(1-\tau_{t+1}^{L}\right)w_{t+1}/\left[\left(1-\tau_{s}^{L}\right)w_{s}\right]\right\}^{(1-\rho)(\sigma-1)}$. Under the assumption that the behavior of $\Sigma_{t+1,s}$ can be reasonably approximated by a non-stationary process analogous to those used above, Θ_{t+1} reduces to:

$$\Theta_{t+1} = \left[(1+r)/(1+r-\xi)\right]\Sigma_{t+1,t+1} = (1+r)/(1+r-\xi),$$

where ξ would be the parameter to be estimated in the process. Because this is a constant, its impact is lost when the equation for c_{t+1}^{t+1} is log-linearized, leaving the deviation of this variable from its steady-state level depending only on inc_{t+1}.

Log-linearization of the aggregate per capita labor-leisure tradeoff around a steady state with no taxes yields:

$$L_t = \frac{1-\rho}{\rho}\frac{\bar{c}_0}{\bar{w}_0\bar{L}_0}\left(w_t - c_t\right). \tag{4.20}$$

Labor supply is an increasing function of the current real wage and a decreasing function of consumption. The latter is higher the higher the present discounted value of wage income. If agents expect to receive high wages in the future, their incentive to supply labor today is correspondingly weaker.

Equation (4.19) governs the intertemporal dynamics of aggregate consumption per capita. The intratemporal tradeoff between consumption and leisure at each point in time obeys equation (4.20). The representative dynasty's consumption Euler equation and labor-leisure tradeoff can be combined to obtain an Euler equation for labor supply. My approach to the estimation of σ and ρ relies on (4.19) and (4.20) as well as on the Euler equation for labor supply. In aggregate per capita terms, this can be written as:

$$1-L_{t+1} = \frac{\beta^{\sigma}\left(1+r_{t+1}\right)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\left(1-L_t\right)\left[\frac{w_{t+1}\left(1-\tau_{t+1}^{L}\right)}{w_t\left(1-\tau_{t}^{L}\right)}\right]^{-[1-(1-\rho)(1-\sigma)]} + \frac{n}{1+n}\left(1-L_{t+1}^{t+1}\right). \tag{4.21}$$

If $\sigma > 1-\left[1/(1-\rho)\right]$, an increase in the detrended real wage between *t* and *t* + 1 causes future aggregate per capita leisure to decrease, *i.e.*, it causes labor supply to increase, for any given level of the *t* + 1-newborn dynasties' leisure.

Log-linearizing equation (4.21) yields:

$$L_{t+1} = \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\left\{L_t - \frac{1-\rho}{\rho}\frac{\bar{c}_0}{\bar{w}_0\bar{L}_0}\left\{\sigma\tilde{r}_{t+1} - \left[1-(1-\rho)(1-\sigma)\right]\left(w_{t+1}-w_t\right)\right\}\right\}. \tag{4.22}$$

Because $c_{t+1}^{t+1} \cong w_{t+1}$ under my assumptions, the independent effect of a newborn dynasty's labor-leisure choice on the aggregate supply of labor at *t* + 1 washes out.

[47] Of course, the importance of this phenomenon will be limited by the rate at which population is growing.

From the aggregate per capita money demand equation, it is possible to obtain an equation for the rate of growth of detrended real money balances. In log-linear terms:

$$\tilde{g}_t^m = (\mu/\sigma)(c_t - c_{t-1}) - (\mu/\bar{i}_0)(\tilde{i}_{t+1} - \tilde{i}_t) - \mu(1-\rho)[(1-\sigma)/\sigma](w_t - w_{t-1}). \tag{4.23}$$

Faster consumption growth causes faster growth in real balances. Aggregate per capita money balances grow more slowly if the growth in the opportunity cost of holding money is faster. *Ceteris paribus,* if $\sigma < 1$, faster growth in the wage rate causes growth in real balances to slow down.

Given the log-linear first-order conditions for consumption, labor supply, and money demand, it is possible to estimate the intertemporal elasticity of substitution in utility of consumption and leisure—σ, the relative importance of consumption and leisure in utility—ρ, and the intertemporal elasticity of substitution in demand for real money balances—μ.

I begin with the estimation of ρ. A simple non-linear least squares regression of hours on the difference between the percentage deviations of the real wage and consumption from the steady state as in equation (4.20) yielded an estimate of ρ of 2.93.[48] One reason for the failure to obtain a value of ρ between 0 and 1 could be the nature of the data. I use a series of actual hours worked in the Canadian economy. This is more likely to reflect labor *demand* than labor *supply* in an economy where unemployment is an issue. A strong negative effect of the real wage on hours due to labor demand dynamics may cause the estimate of ρ to be larger than 1. To explore this possibility, I ran a simple unrestricted regression of hours on the real wage and consumption. The estimated coefficients were both positive. The coefficient on the real wage was small and hardly significant, but the coefficient on consumption was very significantly different from zero.[49] The result was thus at odds with the initial conjecture, and the reason appeared to be a *positive* impact of consumption on hours, rather than negative as predicted by the theory. I thus ran the following regression:[50]

$$L_t = \frac{1-\rho}{\rho}\frac{\bar{c}_0}{\bar{w}_0\bar{L}_0}w_t - \left(\frac{1-\rho}{\rho}\frac{\bar{c}_0}{\bar{w}_0\bar{L}_0} + A\right)c_t,$$

assuming an initial value of zero for the parameter A. This yielded an estimate for ρ of .79, with a standard error of .228. The estimate for A was -.94, with a standard error of .36. Allowing the coefficient of consumption in the labor-leisure tradeoff equation to differ from the prediction of the theory yielded a fairly high estimate of ρ—though smaller than 1, consistent with the expectation of a small coefficient for the real wage.[51]

To take care of the serial correlation in the residuals signaled by the low Durbin-Watson statistic, I ran:

$$L_t = A_1 L_{t-1} + \frac{1-\rho}{\rho}\frac{\bar{c}_0}{\bar{w}_0\bar{L}_0}w_t - \left(\frac{1-\rho}{\rho}\frac{\bar{c}_0}{\bar{w}_0\bar{L}_0} + A_2\right)c_t,$$

[48] Standard error = .86.

[49] The coefficient on the real wage was .19 (standard error = .26, t-statistic = .74); the coefficient on consumption was .74 (.12, 5.98). R^2 = .43, DW = 1.59.

[50] The error term is omitted.

[51] If I included seasonal dummies in the regression, the estimate of ρ dropped to .57 and that of A became -1.39. All coefficients were highly significant.

with an initial value of zero for A_1. When only the significant seasonal dummies were included[52], the estimates (standard errors) of ρ, A_1, and A_2 were, respectively, .62 (.081), .46 (.072), and -.96 (.2).[53]

The results of the regressions based on the labor leisure tradeoff thus suggested a range of values between .57 and .79 for ρ. By including lagged hours as an explanatory variable, the last regression somewhat shifted the focus to Euler-equation type considerations. I thus ran a second set of single-equation regressions based on equation (4.22) to verify if this yielded similar results. The first was an exploratory regression of hours on lagged hours, the real interest rate, and real wage growth. The coefficient on the real interest rate was positive but hardly significant. Hence, I tried to separate the effects of the nominal interest rate and inflation and ran:

$$L_t = A_1 L_{t-1} + A_2 \tilde{i}_t + A_3 \tilde{\pi}_t^{CPI} + A_4 \left(w_t - w_{t-1} \right) .$$

The estimates were as follows (standard errors in parenthesis):
A_1: .499 (.096); A_2: .4 (.21); A_3: -1.59 (.86); A_4: -1.52 (.47); R^2 = .40, DW = 2.14.
The nominal interest rate and inflation had comparable levels of significance. When seasonal dummies were included, they were significant, and the estimated coefficients changed to:
A_1: .83 (.069); A_2: .21 (.13); A_3: -1.13 (.53); A_4: -.177 (.43); R^2 = .8, DW = 2.42.
These results suggested some preliminary observations. Contrary to the predictions of the theory, the impact of the real interest rate on the supply of hours appeared positive. Separating the nominal interest rate from inflation did not seem to change this result. Because higher inflation is consistent with a lower real interest rate, the theory would suggest that higher inflation causes labor supply to be higher, but this is not consistent with the finding of the exploratory regression. Once seasonal dummies were included, the effect of real wage growth was not significantly different from zero.

I then imposed the parameter restrictions dictated by the model and ran the non-linear least squares regression:

$$L_t = \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} \left\{ L_{t-1} - \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\overline{w}_0 \bar{L}_0} \left\{ \sigma \tilde{r}_t - \left[1-(1-\rho)(1-\sigma)\right](w_t - w_{t-1}) \right\} \right\}.$$

I calibrated β to .99—a fairly safe choice for the discount factor at quarterly frequency. I did a grid search over a range of values of σ between .01 and .31 (this choice will be motivated below) and found an estimate of ρ consistently above 1. This result seemed to support the observation that the effect of the real interest rate can be positive—as suggested by the regressions above—only if ρ is larger than 1. However, the result vanished once I controlled for seasonal effects. For σ = .16, the estimated value of ρ turned out to be .695 (.317). The estimate raised to .74 for σ = .21, with a slight decrease in the value of the likelihood function (from 173.657 to 173.510).[54]

I tried GMM and IV estimation to control for correlation of variables with the error term and endogeneity effects, always including seasonal dummies in the regressions. I used lagged hours, real interest rate, and wage as instruments. Non-linear IV yielded estimates of ρ significantly above 1 for very low values of σ, but the estimates were close to those obtained with the non-linear least squares regression for σ between .11 and .31, although with larger standard errors. GMM estimation with the same set of instruments and with starting values via non-linear

[52] The dummy for the third season turned out to be only marginally significant.
[53] R^2 = .84, DW = 2.17.
[54] Standard error = .286.

two-stage least squares yielded values of ρ greater than 1 over a larger range of values of σ. For σ = .21, the estimate of ρ was .55 (.16). The estimated ρ was somewhat lower when the starting value was not chosen via non-linear 2SLS.

To summarize, under the assumption that the series of actual hours worked does contain information on labor supply behavior, the results of the single-equation regressions based on both the intratemporal tradeoff equation and the intertemporal optimality condition for labor supply suggest a range of values for ρ between .55 and .8. .55 seems too low a weight for consumption in agents' utility. The regressions below will actually suggest that values of ρ as high as .99 cannot be dismissed.

The consumption Euler equation can be used to obtain an estimate of σ. I initially tried a non-linear least squares regression based on the following equation, doing grid searches over a range of values of ρ:

$$\mathrm{c}_t = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\left[\mathrm{c}_{t-1} + \sigma \tilde{\mathrm{r}}_t + (1-\rho)(1-\sigma)(\mathrm{w}_t - \mathrm{w}_{t-1})\right] + \left[1 - \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\right]\mathrm{w}_t . \tag{4.24}$$

The regression yielded *negative* and significant estimates for σ, with the likelihood function increasing for higher values of ρ. This result appeared puzzling.[55] In order to gain an understanding of what could motivate it, I ran an unrestricted OLS regression of the type:

$$c_t = A_1 c_{t-1} + A_2 \tilde{r}_t + A_3 w_t + A_4 (w_t - w_{t-1}) + \sum_{i=5}^{8} A_i D_i \ ,$$

where the D_is are seasonal dummies. The estimates were as follows:
A_1: 1.04 (.038); A_2: -.18 (.047); A_3: .28 (.098); A_4: -.099 (.17); R^2 = .96, DW = 2.17.
Contrary to what the theory would suggest, the real interest rate has a negative and significant impact on consumption. This result resembles the findings of a previous exploration of this type of models for the Canadian economy by Altonji and Ham (1990). The coefficient on real wage growth appears insignificantly different from zero. This is consistent with a value of ρ close to 1 in equation (4.24). The current real wage has a positive and significant effect on consumption.[56] The fact that the real interest rate has a negative impact on consumption explains the negative estimate of σ in the initial non-linear least squares regression. The coefficient on the real interest rate is equal to σ times the coefficient on lagged consumption. Because the latter is positive, the negative effect of the real interest rate on consumption translates into a negative estimate of σ.

Following Altonji and Ham (1990), I separated the impact of the nominal interest rate and inflation and ran the exploratory regression:

$$c_t = A_1 c_{t-1} + A_2 \tilde{i}_t + A_3 \tilde{\pi}_t^{CPI} + A_4 w_t \ .$$

Based on the previous results, I dropped the real wage growth term. The results were:
A_1: .99 (.036); A_2: -.07 (.056); A_3: -.35 (.21); A_4: .19 (.087); R^2 = .96, DW = 1.94.

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[55] The result was robust to alternative estimation techniques (GMM, IV) and to the use of U.S. variables rather than lagged Canadian ones as instruments.

[56] As suggested above, this result—which conflicts with the basic random walk hypothesis—is consistent with the dynamics of population in the economy and needs no imperfection in capital markets to be explained. However, the low rate of population growth in Canada implies that imperfections in financial markets are likely to be a more relevant empirical motivation of the result.

The impact of both the nominal interest rate and inflation on consumption is only marginally significant, though the effect of inflation is larger. This motivates the negative effect of the real interest rate.[57]

The result of the previous regression induced me to drop the nominal interest rate from the regressions used to estimate σ and to focus on the effect of inflation. To get an initial estimate of σ, I ran:

$$c_t = Ac_{t-1} - \sigma \; A\tilde{\pi}_t^{CPI} + (1-A)w_t .$$

I found A = .95 (.02) and σ = .55 (.14).[58] This was a more encouraging result, although the value of the elasticity was significantly higher than that found by Altonji and Ham (1990). I then went back to the log-linear Euler equation and ran the non-linear least squares regression:

$$c_t = \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\left(c_{t-1} - \sigma\tilde{\pi}_t^{CPI}\right) + \left[1 - \frac{\beta^{\sigma}(1+r)^{\sigma}(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)}\right]w_t .$$

β and ρ were set to .99. I chose such a high value of ρ for consistency with the statistical insignificance of the coefficient on real wage growth.[59] The estimated intertemporal elasticity of substitution was .23, with a standard error of .1 (R^2 = .95, DW = 1.64). Reintroducing the real wage growth term in the regression did not affect the results significantly. The coefficients on seasonal dummies turned out to be insignificantly different from zero.

In order to take care of problems of correlation between the error term and the regressors and of issues of endogeneity, I re-estimated σ using non-linear IV and GMM. Following the suggestion of Altonji and Ham (1990), I tried alternative sets of instruments—lagged Canadian variables and U.S. variables. A GMM estimation with starting values via non-linear 2SLS and lagged Canadian consumption, CPI inflation, and real wage as instruments yielded a value for σ of .14 (.11). Setting σ = 1 as starting value raised the estimate to .18, with approximately the same standard error.[60] When I used U.S. GDP and inflation as instruments, I found a higher value of σ, around .21. Overall, these single-equation regressions seem to support a range of values between .14 and .25 as plausible for the intertemporal elasticity of substitution in the Canadian economy. Altonji and Ham (1990) concluded in favor of a range between .1 and .2 using annual data between 1951 and 1984. My results seem fairly consistent with their findings.

The next structural parameter to be estimated is the intertemporal elasticity of substitution in utility from real money balances—μ. I initially ran the exploratory regression:

$$\tilde{g}_t^m = A_1(c_t - c_{t-1}) + A_2\left(\tilde{i}_{t+1} - \tilde{i}_t\right) + A_3(w_t - w_{t-1}).$$

Because i_{t+1} is the nominal interest rate between t and t + 1, it is known by agents at time t. The estimated coefficients were:

A_1: 2.01 (.63); A_2: - 1.09 (.51); A_3: .6 (.66); R^2 = .19, DW = 3.24.

When taken into account, seasonal effects were small but significantly different from zero. The estimate of A_1 dropped to 1.39 and that of A_2 to - .93. R^2 increased to .74 and—more

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57 Letting the sample start in 1981:1, the results suggest more strongly that inflation has a negative and significant impact, while the effect of the nominal interest rate is insignificantly different from zero.

58 R^2 = .96, DW = 1.67. When I added seasonal dummies, they were not significant.

59 The results of the regressions below suggest that the coefficient is not statistically insignificant because of a value of σ close to 1.

60 The instruments were consumption and the real wage at dates t - 1 and t - 2 and inflation at t -1. Using consumption and the real wage at t - 2 and t - 3 and inflation at t - 2 as instruments yielded σ = .146 (.11).

importantly—DW dropped to 2.2. The estimate of A_3 was hardly significant. This could be interpreted as a further signal that a high value of ρ may be consistent with the data.

Given these results, I ran the following non-linear least squares regression:

$$\tilde{g}_t^m = (\mu/\sigma)(c_t - c_{t-1}) - (\mu/\bar{i}_0)(\tilde{i}_{t+1} - \tilde{i}_t) + \sum_{i=1}^{4} A_i D_i \ ,$$

doing a grid search over a range of values for σ. $\bar{i}_0$ is the unconditional mean of the series of the T-Bills interest rate over the sample period. For σ as low as .01, the procedure yielded an estimate of μ of .015 with a t-statistic of 3.8 (R^2 = .71, DW = 2.11). For a value of σ in the Altonji-Ham range (.11), the estimated value of μ was .106, with a standard error of .024. The value of the log-likelihood was 151.089. When σ = .31, the estimated μ was .099 (.029). The log-likelihood was 147.710. Reintroducing the real wage growth term in the regression did not affect the results significantly.

To summarize, results from the single equation regressions suggest the following ranges for the relevant structural parameters: $\mu \in [.09,\ .12]$, $\rho \in [.8,\ .99]$, $\sigma \in [.1,\ .25]$. As I had done for the production side of the economy, I used these estimates as starting values for FIML regressions of the system of the consumers' first-order conditions, to verify if the procedure yielded similar estimates.

The results of the FIML estimation were more reliable in this case: estimates came with small standard errors. The results confirmed the ranges above for μ, ρ, and σ. They suggested that values of σ smaller than .1 could be plausible, and that μ could be as low as .07.

To summarize the results of this section, Table 1 displays a set of parameter values that were found to be plausible for the Canadian economy and are used in the policy rule evaluation exercise of Ghironi (1999*b*) and in the example below.[61]

Given the sample mean of the Canadian T-Bills rate, μ = .08 ensures that the elasticity of growth in demand for real balances to the interest rate is not too large. I choose σ = .1 for reasons of convergence of aggregate consumption to the steady state after a shock.[62] In order to speed up convergence, I actually set β = .95 rather than .99 in the calibration exercise. This change in the value of β does not affect the ranges of estimates of the other parameters in any significant way and is useful for exposition purposes. ρ is probably a more controversial choice, though .8 seems a sensible starting value.

5. A Recession in the United States

In this section, I illustrate the functioning of the model by analyzing the transmission of a recession in the U.S. to Canada under inflation targeting—the monetary rule currently followed by the Bank of Canada.

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[61] Steady-state levels of variables are set to the corresponding sample means.

[62] Higher values of σ yield slower convergence to the steady state. In addition, because—with one exception—I calibrate the exact equations of the model in the exercise, but the data suggest that the nominal interest rate may have a very small impact on consumption, I choose a value of σ that ensures such small effect. The exception is that I use the equation in footnote 44 for investment. If I did not do that, I would have an investment equation whose coefficients are absolutely out of line with what suggested by the regressions. In that case, η = 20 would actually be required to yield sensible coefficients, but this would cause absurdly high adjustments costs (in the order of 70 percent of GDP during the sample period).

Shocks to U.S. variables cannot be taken in isolation. I have not modeled the structure of the U.S. economy as explicitly as Canada's but—at a minimum—one must recognize that four variables that appear in the equations for Canadian variables will be affected by shocks to U.S. GDP or interest rate: besides these, the U.S. CPI inflation rate and the real interest rate will change. One cannot analyze the consequences of a shock to U.S. output or the interest rate for Canada without explicitly accounting for the comovements in all relevant variables that are triggered by the initial disturbance.

In the exercise, I impose a minimal amount of structure on the U.S. economy. I take the Federal Funds Rate to be the relevant nominal interest rate. The Federal Reserve is assumed to set this rate as its policy instrument. Following Rotemberg and Woodford (1997), I assume that the Fed sets the nominal interest rate based on a reaction function that depends on past levels of the rate and on the current and past levels of CPI inflation and GDP. In terms of deviations from the steady state:[63]

$$\tilde{i}_{t+1}^{*} = \sum_{k=1}^{n_{\tilde{i}^*}} \varphi_{1k} \tilde{i}_{t+1-k}^{*} + \sum_{k=0}^{n_{\tilde{\pi}^*}} \varphi_{2k} \tilde{\pi}_{t-k}^{CPI*} + \sum_{k=0}^{n_{y^W}} \varphi_{3k} y_{t-k}^{W} \,. \tag{5.1}$$

Shocks to this equation are exogenous shocks to monetary policy. Because Canada is small relative to the U.S., the Fed's reaction function does not incorporate any Canadian variable. The negligible impact of Canadian GDP on world aggregates allows me to identify U.S. GDP with y^W in the model.

I model the U.S. economy as a recursive structural VAR that includes equation (5.1) and equations for GDP and inflation. The state vector is $\left[\tilde{\pi}_t^{CPI*}, y_t^W, \tilde{i}_{t+1}^*\right]'$, and the causal ordering of variables is the order in which they are listed. I follow Rotemberg and Woodford (1997) in assuming that the interest rate affects output and inflation only with a lag, but I do not include future inflation and GDP in the time-*t* state vector, because I do not believe that future consumption and inflation levels are entirely predetermined.

I estimate the VAR with three lags using full information maximum likelihood. I use data between 1980:1 and 1997:4. The estimated coefficients for the three equations and the standard errors are in the columns of Table 2. Seasonal dummies were not significant, as well as further lags. The estimated coefficients for the Fed's reaction function suggest behavior in line with a generalized Taylor rule, consistent with the findings of Rotemberg and Woodford (1997).

Figure 4 shows the responses of GDP, inflation, and the Federal Funds Rate to a 1% decrease in U.S. GDP.[64] [65] The deviation of GDP from the steady state increases in the first two quarters. Inflation reacts with a lag, and subsequently drops. The Fed reacts immediately by

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[63] Remember that i_{t+1}^* is the time *t* nominal interest rate.

[64] Because markets clear in the model, an exogenous decrease in U.S. GDP can be interpreted both as the consequence of a generalized decline in world demand for goods and as the outcome of a negative supply shock. I interpret the shock as an exogenous contraction in demand. The interpretation is consistent with the fact that U.S. inflation declines following the disturbance.

[65] In the impulse responses, the level of the interest rate at each point in time is the value chosen by the monetary authority at that date.

lowering the Federal Funds Rate to sustain GDP.[66] Over time, all variables go back to the steady state.

The paths of U.S. variables generated by the shock constitute the paths of the exogenous world-economy variables following the initial impulse in my model of the Canadian economy. The estimated VAR equations are included in the system of equations that govern the dynamics of the world economy following the initial shock, along with the model equations for Canada and the monetary rule followed by the Bank of Canada. The system is then solved using routines that follow Uhlig (1997).

At this point, it is important to recall that the monetary rule followed by the Bank of Canada is perfectly neutral as far as the real interest rate is concerned in all periods in which no shock happens. Under all rules the Canadian real interest rate is equal to the U.S. real interest rate—except in the case of short-run deviations from uncovered interest parity due to unexpected shocks. Different rules can make a difference for the dynamics of the Canadian economy via real interest rate effects only in the very short run. However, alternative policy rules can produce different dynamics by causing differences in the behavior of the relative price, $p(i)/P$, which can be taken as a measure of the terms of trade for the Canadian economy.[67]

In this illustrative example, I focus on inflation targeting, the rule currently followed by the Bank of Canada. Under this rule, I assume that the Bank of Canada sets the Canadian nominal interest rate to keep CPI inflation at its steady-state level in *all* periods, including when an unexpected shock happens: $\tilde{\pi}_t^{CPI} = 0 \;\; \forall t \geq t_0$.[68] [69]

An operational rule that is consistent with this target can be obtained from the money demand equation. Inflation will be constant at its steady-state level in all periods if the Bank of Canada sets its interest rate as:

$$\tilde{i}_{t+1} = \tilde{i}_t - (\bar{i}_0/\mu)\tilde{g}_t^M + (\bar{i}_0/\sigma)(c_t - c_{t-1}) - \bar{i}_0(1-\rho)[(1-\sigma)/\sigma](w_t - w_{t-1}).$$

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[66] A measure of the U.S. real interest rate can be obtained by using the response of the inflation rate to deflate that of the Federal Funds Rate. The real interest rate reacts with a lag. It falls below the steady state in the first quarter after the shock and remains lower than its long-run level until it eventually returns to it.

[67] The terms of trade are actually given by $p(i)/\varepsilon \; p^*(f)$, where $p^*(f)$ is the U.S. PPI. Under my assumptions, the fraction of Canadian goods in the U.S. consumption bundle is negligible. Hence, $p^*(f)$ is only marginally different from P^*. Because of purchasing power parity, $p(i)/\varepsilon \; P^* = p(i)/P$.

[68] Svensson (1998) distinguishes between strict and flexible inflation targeting depending on the weights attached to different targets in the policymaker's loss function. In both cases, the central bank is minimizing a loss function that attaches weight to inflation variability and interest-rate changes. Under flexible inflation targeting, the policymaker cares about output volatility too. In reality, the Bank of Canada has an interval target for CPI inflation.

[69] An important dimension of inflation targeting in the real world is the choice of the steady-state inflation rate, *i.e.*, the actual target, given by $\overline{\pi}_0$ in my model. When evaluating the consequences of inflation targeting, one would want to discuss the implications of different levels of the target. My model is not appropriate to perform such analysis. Because firms face costs of output price inflation volatility around $\overline{\pi}_0$, the choice of the steady-state level of inflation has no consequence for welfare in the economy, because it does not affect the steady-state level of the markup or the dynamics of relative prices.

Strict inflation targeting requires the Bank of Canada to raise its interest rate if it did so in the previous period and if consumption is growing, and to lower it if money growth and/or the real wage are rising.[70]

Impulse responses under this rule are in Figure 5. The Canadian dollar depreciates after the initial period, consistent with the decline in U.S. inflation.[71] The Bank of Canada lowers its interest rate by less than the Fed, which generates expectation of depreciation in the periods following the shock. PPI inflation falls slightly on impact, but climbs above the steady state by even less within a year. The relative price of the representative Canadian good falls, and the markup rises. According to the model, firms absorb part of the contractionary consequences of the shock by accepting a lower real price for their output. They preserve profitability by raising the markup component of the price level. Consistent with theory and data, the markup is countercyclical. Labor demand falls, and so does the real wage, though both are above the steady state six quarters after the shock. Tobin's q rises. Firms invest and substitute capital for labor.[72] This causes an initial increase in share price inflation, which is absorbed rapidly. GDP drops, but it bounces above the steady state in two years. The recession is not as pronounced as in the U.S. Consumption falls and, save for a brief recovery in the third and fourth quarters after the shock, it continues to fall for eight years. It goes back to the steady state only in the very long run.[73] Money balances initially rise, but fall below the steady state in the third quarter. Canada runs a fairly persistent current-account deficit (not shown).

Schmitt-Grohé (1998) uses a VAR approach to study how shocks to U.S. GDP are transmitted to the Canadian economy. She compares the predictions of the estimated VAR to those of alternative flexible price models. She finds that the models can explain the observed responses of output, hours, and capital only if larger-than-realistic movements in relative prices are allowed. She raises the question of whether relaxing the assumption of price flexibility might help remove the puzzle. My exercise provides a positive answer to the question. The impulse responses for output, hours, and capital in the example above are consistent with the size and duration of the recession in the U.S. At the same time, fairly small movements in the relative price of Canadian goods and other variables are observed.

In Ghironi (1999*b*), I compare inflation targeting to several other rules, including a fixed exchange rate regime with the U.S. and the Taylor rule. I assume that Canada is subject to three sources of volatility: shocks to U.S. GDP, exogenous movements in U.S. monetary policy, and technology shocks. Welfare comparisons support the choice of a constant inflation rate as the optimal monetary rule among those considered for the Canadian economy. Markup and relative price dynamics under that rule ensure the smallest volatility of consumption. Abandoning inflation targeting for a fixed exchange rate regime with the U.S. would not be welfare improving, although the performance of the two regimes is fairly similar. A forward-looking version of the Taylor rule dominates the traditional version.

[70] The latter as long as $\sigma < 1$.

[71] In what follows, I use the word depreciation to refer to an increase of the rate of depreciation above its steady-state level. The rate of depreciation does not change on impact because U.S. inflation reacts with a lag.

[72] In the impulse responses, capital at each point in time is capital at the end of the corresponding period rather than at the beginning.

[73] Slow population growth motivates slow convergence of consumption to the steady state.

6. Conclusions

This paper has developed a model of macroeconomic interdependence that is more suitable for empirical investigation that those presented thus far in the so-called "new open economy macroeconomics." One of the main problems in these models is the absence of a well-defined endogenously determined steady state. The choice of the initial point for the purpose of analyzing the consequences of shocks is arbitrary, and all disturbances have permanent consequences via redistribution of wealth across countries, a result that is debatable on empirical grounds. From a technical point of view, the models are linearized around an initial steady state to which the economy *never* returns no matter the nature of the disturbance that affects it. This raises suspects on the reliability of the log-linear approximation. *De facto*, it precludes stochastic applications of the framework that do not shut off current account dynamics. The model presented here has the advantage that its steady state is entirely determined by the structural parameters—and by the steady-state levels of some policy instruments—and is stable, if appropriate conditions are satisfied. Existence of the steady state is achieved by changing the demographic structure from the usual representative agent framework to an overlapping generations structure a' la Weil (1989), in which new infinitely lived households enter the economy at each point in time and are born owning no financial assets. The existence of an endogenously determined steady state removes the arbitrariness of the choice of the initial point. It makes it possible to calculate reliable log-linear approximations. It guarantees that temporary shocks do not have permanent consequences and, in particular, that monetary policy is neutral in the long run. One can use the model to perform stochastic simulations without having to shut off the current account. All this appears to be a significant improvement upon the existing theoretical literature.

I also modeled nominal rigidity more explicitly than in previous contributions to the literature. The presence of costs of adjusting output prices generates an endogenously variable markup. This unties the dynamics of the real wage and the marginal product of labor. The markup plays an important role in business cycle fluctuations, and the real wage can be procyclical, consistent with the empirical evidence.

Finally, I brought investment and capital accumulation into the scene, adopting a standard q model of investment. Notwithstanding the limitations of this approach to investment decisions, attention to investment makes it possible to perform more thorough analyses of current account dynamics and of the consequences of policy actions and rules. It also provides tools for studying the effect of these actions and rules on asset prices, by focusing on Tobin's q and the dynamics of share prices. Markup dynamics play an important role in investment decisions.

In the second part of the paper, I have presented my approach to the issue of estimating the structural parameters of a small open economy, identifying the home economy in my model with Canada, and the foreign economy with the United States. This allows me to make use of a set of simplifying exogeneity assumptions, which would not be warranted were I studying interdependence between economies of comparable size. I have also illustrated a plausible strategy for constructing measures for unobservable variables. I estimated the parameters mainly by making use of non-linear least squares at the single equation level. I then verified whether multiple equation regressions yielded significantly different estimates by running full information maximum likelihood regressions based on the systems of the consumers' first-order conditions and the firms' first-order conditions. The approach was fairly successful. Non-linear least squares and full information maximum likelihood yielded fairly similar estimates. Most parameter estimates were characterized by small standard errors and were in line with the findings of other studies. Perhaps not surprisingly, given the use of aggregate data for the estimation procedure, the size of the nominal rigidity and the size of the cost of adjusting the capital stock were the only troublesome parameters. More work on these parameters is to be done in the future, along with extending the model in directions mentioned throughout the paper.

In Section 5, I illustrated the functioning of the model by analyzing the transmission of a recession in the U.S. to Canada under inflation targeting, the rule currently followed by the Bank of Canada. Consistent with the goal of combining theoretical rigor with an empirical approach, I used a simple VAR to trace the impact of comovements in U.S. variables on the Canadian economy. The estimated VAR equations for the U.S. were combined with the model equations for Canada to determine the response of the Canadian economy to the initial shock. The example illustrated the role of markup and relative price dynamics in the model.

In Ghironi (1999*b*), I use the model and the parameter estimates to evaluate the performance of alternative monetary rules for the Canadian economy. A constant path of inflation yields the smallest volatility of consumption and is thus the optimal rule among those I consider. The exercise provides interesting insights on the pros and cons of different rules, pointing to the importance of transmission channels that are not featured in other studies of monetary rules for open economies. The findings of this paper—which I see as an initial contribution to "new open economy macroeconometrics"—and of its companion suggest an empirical "case-by-case" approach as a profitable way to understand macroeconomic interdependence and address relevant policy issues.

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Table 1. Structural parameters

β = .95	μ = .08	ρ = .8	σ = .1	δ = .035	ϕ = 200
γ = .1	η = 2	θ = 3.68	n = .00308	g = .00134	r = .07988

Table 2. The U.S. economy

	$\tilde{\pi}_t^{CPI*}$	y_t^W	$\tilde{i}_{t+1}^*$
y_t^W			.397 (.187)
$\tilde{\pi}_t^{CPI*}$		-.446 (.537)	1.090 (.592)
$\tilde{\pi}_{t-1}^{CPI*}$	.370 (.196)	.244 (.544)	.063 (.484)
y_{t-1}^W	.047 (.134)	1.386 (.240)	-.057 (.340)
$\tilde{i}_t^*$	.100 (.082)	-.037 (.283)	.511 (.313)
$\tilde{\pi}_{t-2}^{CPI*}$	-.022 (.206)	.186 (.362)	.971 (.473)
y_{t-2}^W	.030 (.211)	-.243 (.300)	.028 (.393)
$\tilde{i}_{t-1}^*$	-.106 (.114)	-.293 (.399)	.090 (.472)
$\tilde{\pi}_{t-3}^{CPI*}$	.373 (.176)	.111 (.325)	.266 (.563)
y_{t-3}^W	-.053 (.133)	-.180 (.232)	-.337 (.273)
$\tilde{i}_{t-2}^*$	.018 (.101)	.274 (.212)	.108 (.223)

Figure 1. Markup and hours

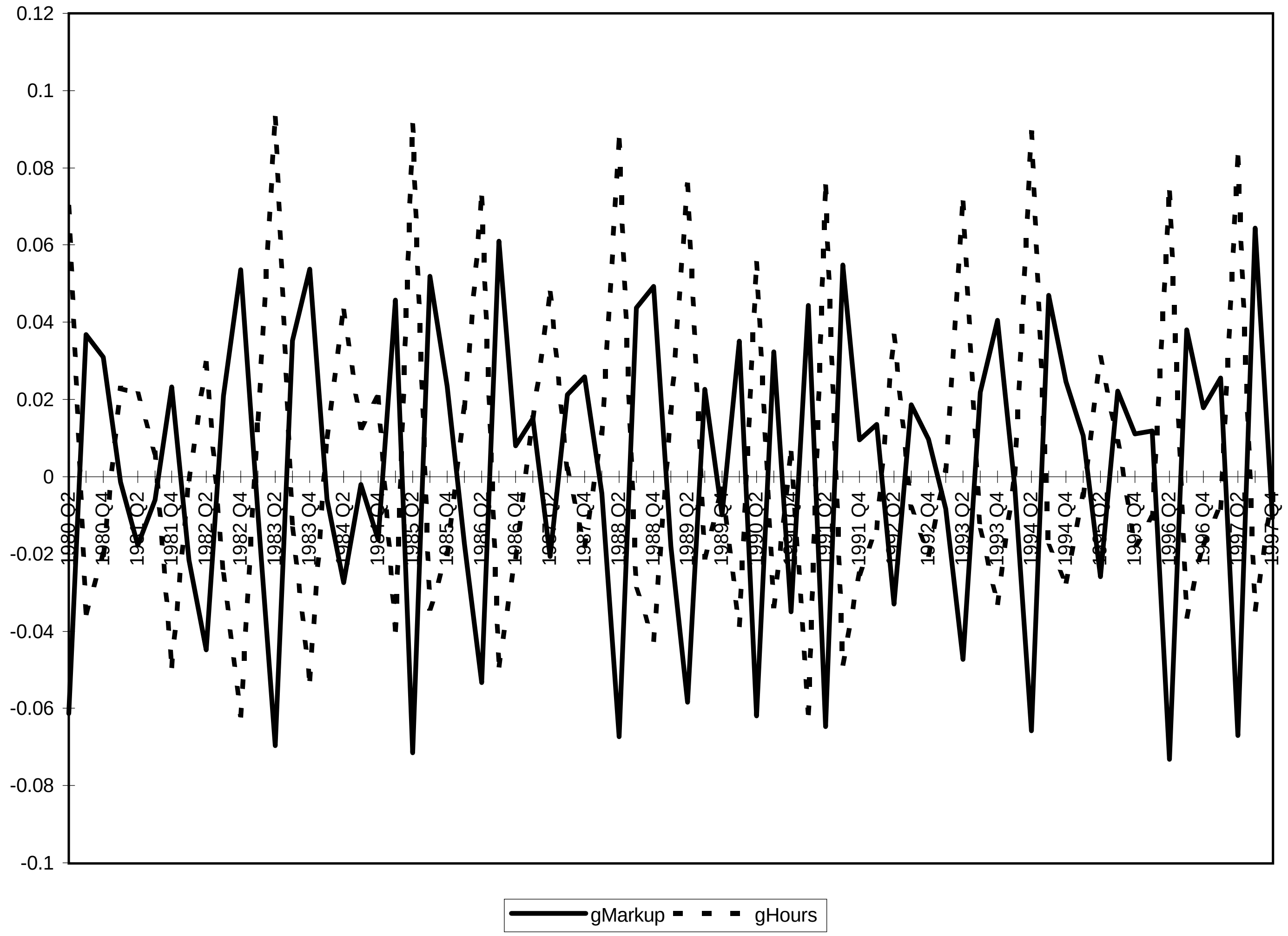

Figure 2. OMEGA

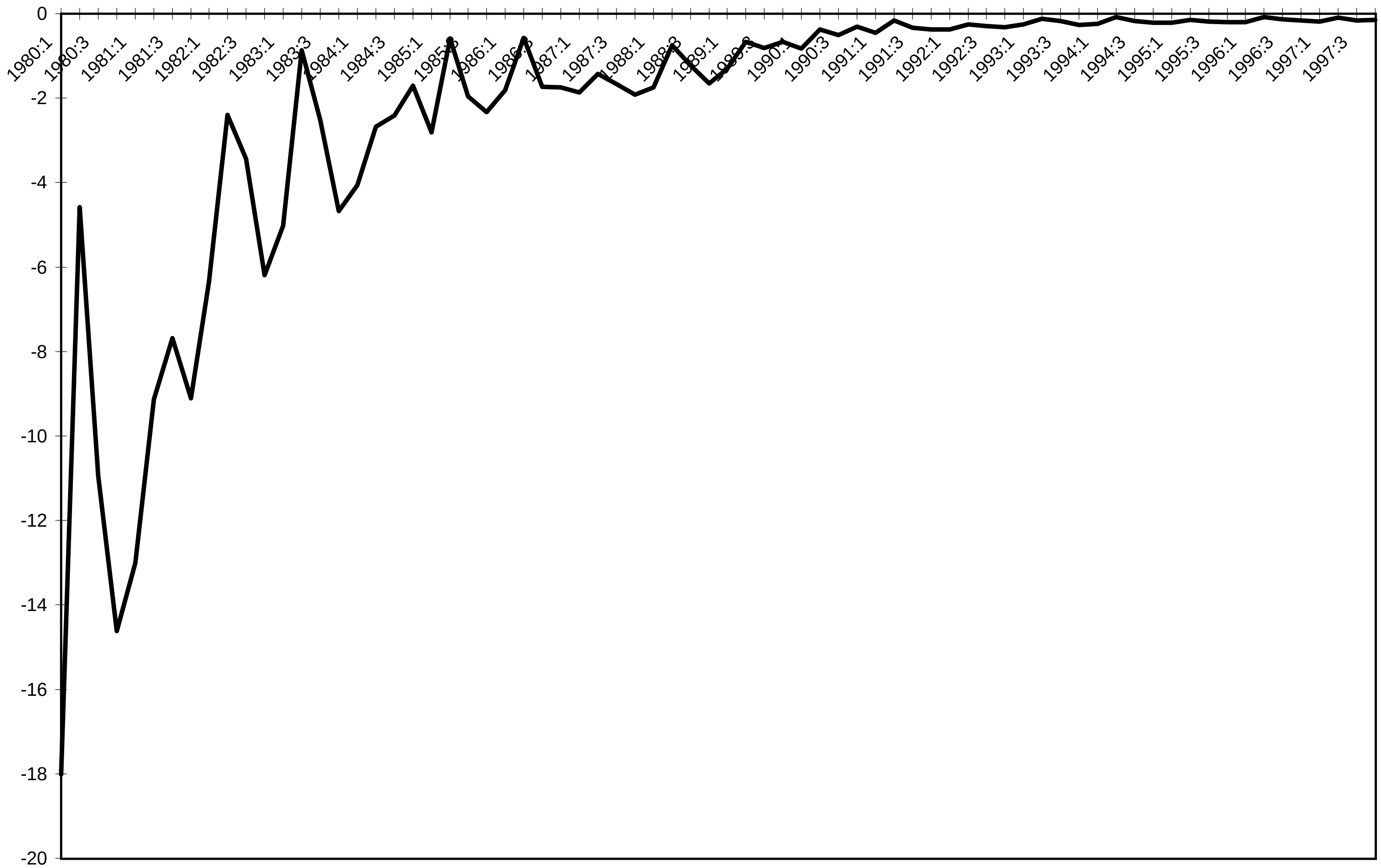

Figure 3.a. Investment and average *q*

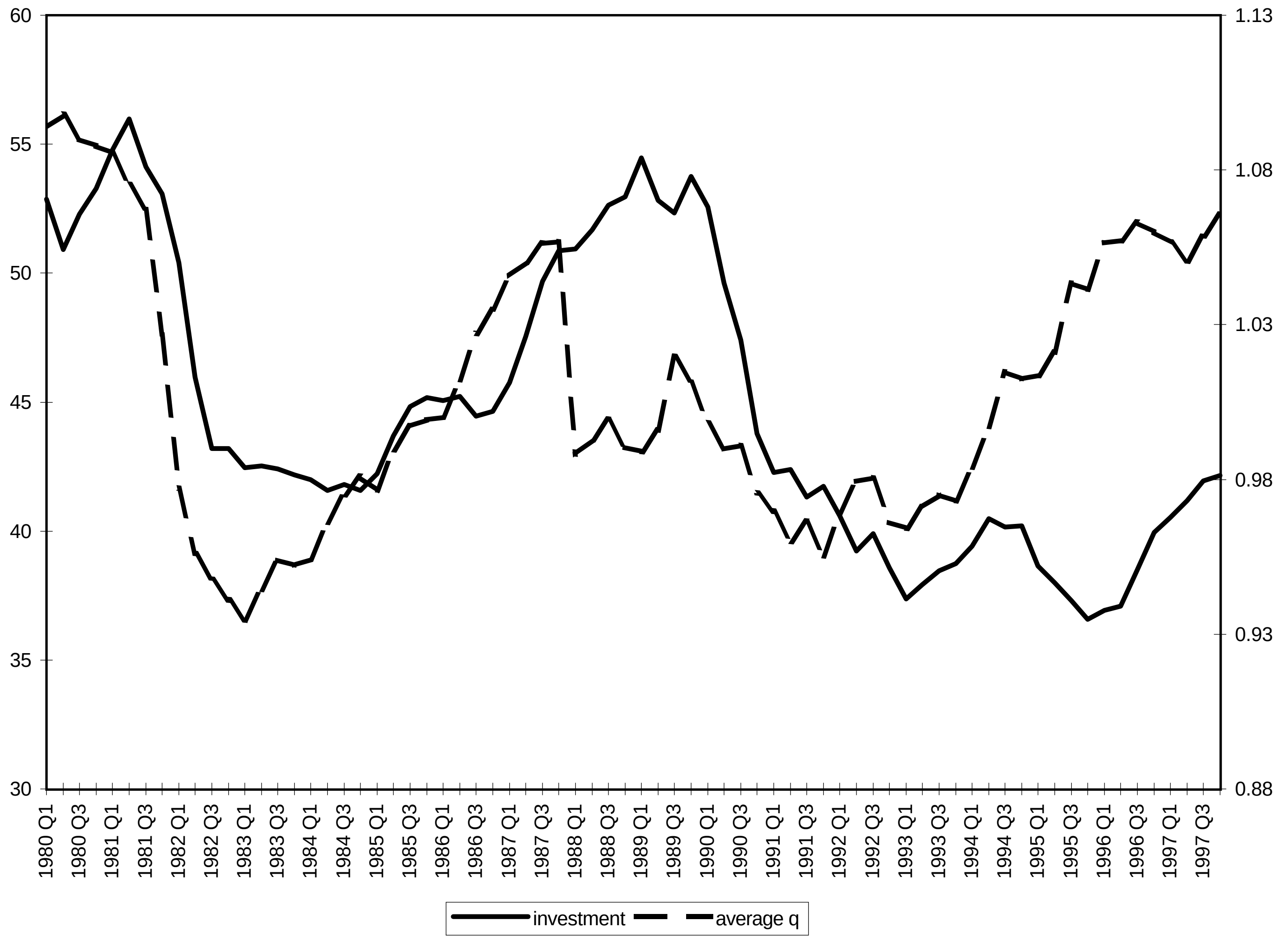

Figure 3.b. Investment and average *q*, adjusted for inventories

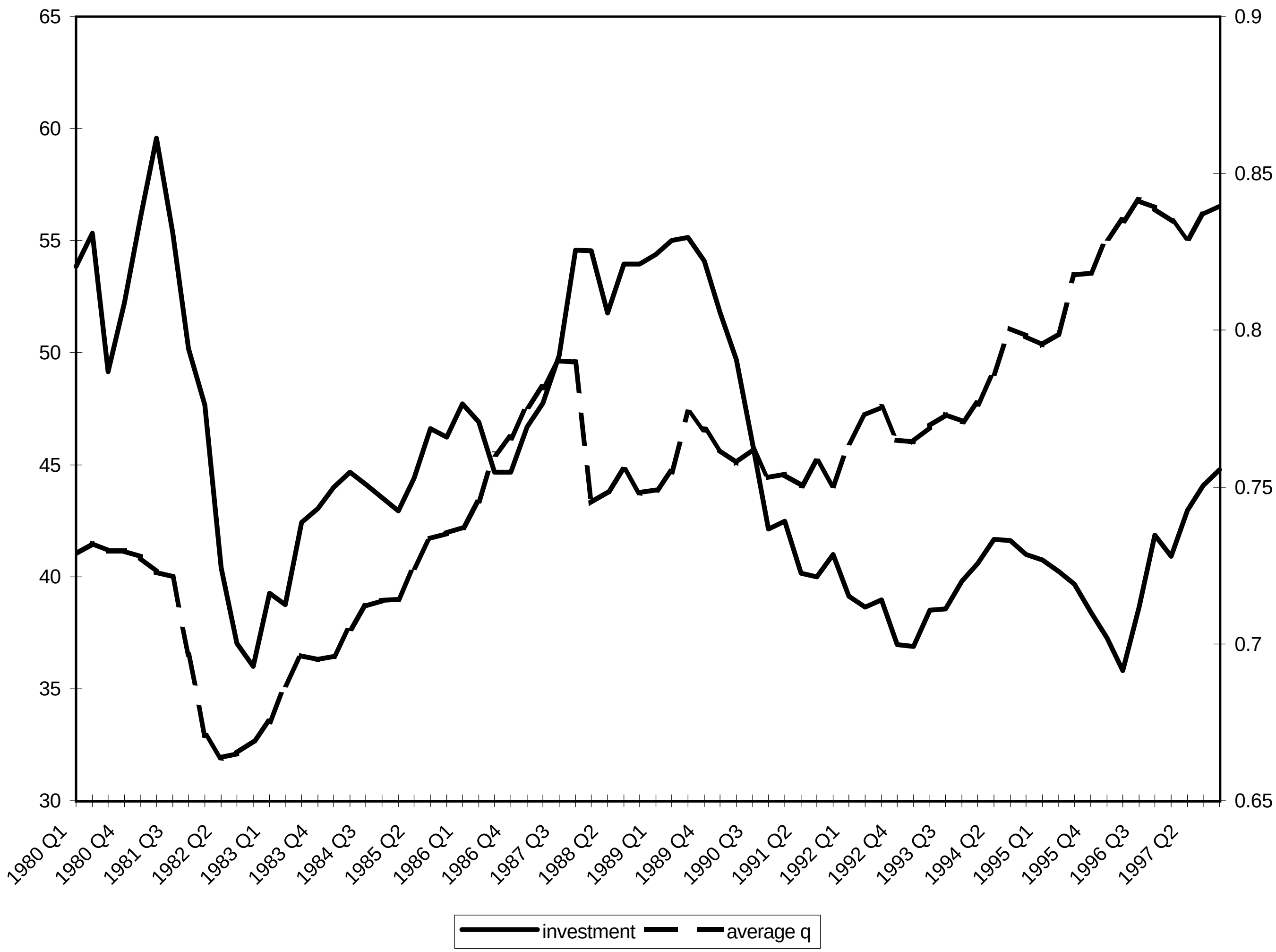

Figure 4. Impulse responses, U.S. economy, shock to U.S. GDP

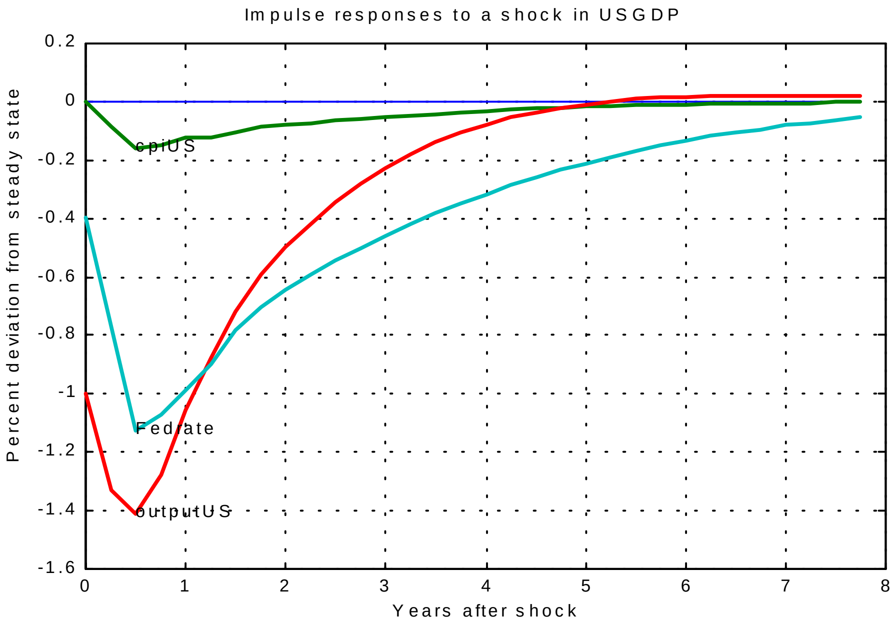

Figure 5. Impulse responses, strict inflation targeting, shock to U.S. GDP