STAFF
REPORTS

NO. 1006
FEBRUARY 2022

REVISED
JUNE 2025

Buy Big or Buy Small? Procurement Policies, Firms' Financing, and the Macroeconomy

Julian di Giovanni | Manuel García-Santana | Priit Jeenas | Enrique Moral-Benito | Josep Pijoan-Mas

FEDERAL RESERVE BANK of NEW YORK

Buy Big or Buy Small? Procurement Policies, Firms' Financing, and the Macroeconomy
Julian di Giovanni, Manuel García-Santana, Priit Jeenas, Enrique Moral-Benito, and Josep Pijoan-Mas
Federal Reserve Bank of New York Staff Reports, no. 1006
February 2022; revised June 2025
JEL classification: E22, E23, E62, G32

Abstract

This paper examines the macroeconomic effects of public procurement. We exploit novel data to show that procurement eases firms' borrowing constraints and has persistent effects on firm growth. Using a macroeconomic model with heterogeneous firms, asset- and earnings-based borrowing frictions, and government purchasing, we simulate revenue-neutral reforms that increase the share of small firms in procurement. We find that, despite helping financially constrained firms grow, these policies lead to non-trivial unintended negative effects. On net, the policies lead to a modest decline in GDP. The findings highlight how procurement design influences aggregate outcomes through firm-level financial frictions and reallocation dynamics.

Key words: government procurement, financial frictions, capital accumulation, productivity

---

Di Giovanni: Federal Reserve Bank of New York, CEPR (email: juliandigiovanni@gmail.com). García-Santana: World Bank, UPF, CREi, CEPR (email: mgarciasantana@worldbank.org). Jeenas: Universitat Pompeu Fabra, Barcelona School of Economics, CREI (email: priit.jeenas@upf.edu). Moral-Benito: Bank of Spain (email: enrique.moral@bde.es). Pijoan-Mas: CEMFI, CEPR (email: pijoan@cemfi.es).
This paper was previously circulated under the title "Government Procurement and Access to Credit: Firm Dynamics and Aggregate Implications." The authors thank their discussants Francesco Decarolis, Joachim Jungherr, Eduardo Morales, Veronika Penciakova, Timo Reinelt, and Burak Uras, as well as Manuel Amador, Marco Bellifemine, Wyatt Brooks, Paco Buera, Laura Castillo-Martínez, Thomas Drechsel, Pablo Fajgelbaum, Cosmin Ilut, Loukas Karabarbounis, Fernando Leibovici, Huiyu Li, Ezra Oberfield, Yongseok Shin, Kjetil Storesletten, Gustavo Ventura, Gianluca Violante, Daniel Xu, and David Zeke for very useful comments. They are also grateful to conference and seminar participants at the Minneapolis Fed, Georgetown, Atlanta Fed, St. Louis Fed, San Francisco Fed, Richmond Fed, USC, Princeton, New York Fed, Duke, ASU, ND, CREi, Cornell, UB, and the Bank of Chile for valuable comments. Julian di Giovanni and Manuel García-Santana acknowledge financial support from Fundacíon BBVA. Priit Jeenas and Manuel García-Santana acknowledge financial support from the Spanish Ministry of Economy and Competitiveness, Severo Ochoa Programme for Centres of Excellence in R&D (CEX2019-000915-S). Priit Jeenas acknowledges financial support from the Juan de la Cierva - Formacíon Grant by the Spanish Ministry of Science and Innovation. Josep Pijoan-Mas acknowledges financial support by the Spanish Ministry of Science and Innovation.

To view the authors' disclosure statements, visit
https://www.newyorkfed.org/research/staff_reports/sr1006.html.

1 Introduction

Governments play a key role in economic activity. They are large employers, set taxes and transfers, and purchase goods and services from the private sector. These purchases are made by awarding public procurement contracts to private firms. The size of public procurement varies over time and across countries, but it consistently represents a large fraction of GDP —12.8% in OECD countries, 14% in EU countries, and 9.3% in the United States.[1] Because of its large size and high level of discretion, many governments use the public procurement system as an industrial policy tool to allocate resources to specific types of firms. In this sense, fiscal policies are not only characterized by the *level* of spending but also by the *composition* of the set of firms from which the government sources its goods and services.

This paper provides a novel framework to study the long-term macroeconomic effects of public procurement policies. In particular, we focus on how public procurement contracts affect the credit access and growth prospects of heterogeneous firms. Our research question is the following: How would the level of aggregate output, TFP, and provision of public goods be affected if governments changed the composition of firms from which they buy while keeping total expenditure fixed?

In order to address this question, we carry out an analysis that integrates a novel firm-level dataset with a macroeconomic model of firm dynamics. Our dataset merges administrative panel data on firm outcomes with public procurement data and credit allocation data at the bank-firm level for the Spanish economy over the 2000-2013 period. Our model builds on the canonical framework of firm dynamics with financial frictions (e.g., Midrigan and Xu, 2014) and incorporates two novel elements. First, there is a government that purchases goods and services from a subset of firms. Firms that are willing to sell to the government must make a risky investment in advance, which reflects the costs of preparing a good proposal and increases the chances of winning the contract. This costly investment governs the *selection* of heterogeneous firms into procurement. Second, we allow for earnings-based borrowing constraints where the pledgeability of firms' earnings from procurement may differ from the pledgeability of earnings from selling to the private sector. This provides flexibility to capture firm dynamics after obtaining a procurement contract (i.e., *treatment*).

Regarding selection, we document that firms that participate in procurement are 115 log points larger (in terms of employment) *before winning a procurement contract*, but they also display a marginal revenue product of capital that is 15 log points higher. This suggests that procurement firms are more productive, but that they face financial frictions preventing them from accumulating their optimal level of capital. Our model generates this pattern of

[1] See the EU Commission's website and the OECD's website for details.

selection through two firm-level state variables: productivity and net worth. As is standard in heterogeneous firm models, the value of participating in a given market —the procurement market in this case— depends on firms' ability to deliver large projects (e.g., Melitz, 2003, in the context of international trade). In our model, this ability uniquely depends on firms' productivity in the case of financially unconstrained firms. However, for constrained firms, it also depends on their financing capacity, which itself depends on firms' net worth (e.g., Chaney, 2016, also in the context of international trade). In our calibrated model, firms that participate in procurement are ex-ante 43 log points more productive and hold 83 log points more net worth.

Regarding the treatment effect of procurement, the model reproduces several facts related to the evolution of firms' credit and sales, which we document by estimating local projections with our micro data.

First, we show a positive and persistent effect of participating in procurement on firms' credit. Importantly, the effect on impact is completely explained by an increase in loans for which no tangible collateral is posted. This finding highlights the importance of earnings-based constraints and supports the results of Hebous and Zimmermann (2021), who show that procurement alleviates firms' financial constraints. The subsequent effects over time, however, are partly explained by loans that require capital as collateral. This finding points towards the presence of asset-based constraints playing an important role, and the fact that firms' ability to accumulate capital increases as a result of participating in procurement. To identify the strength of the earnings-based constraints, we reproduce a structural regression in which the change in firms' leverage depends on the change in total earnings and the change in earnings from procurement. We find that firms can pledge 24% of their annual earnings from selling to the private sector and 54% of their annual earnings from procurement, on top of the pledgeability of their tangible capital.

Second, we also show that sales to the private sector decline upon obtaining a procurement contract (a "crowding-out" effect) and increase afterward (a "crowding-in" effect) leading to firm growth in the medium run. In our model, this crowding-out effect of the procurement shock on impact occurs because financially constrained firms have to split their scarce collateral to serve both procurement and private sector operations. The fact that government sales can be collateralized partly alleviates but does not eliminate this problem. Importantly, we show that the crowding-out effect is particularly stronger for firms that are more likely to be financially constrained, which suggests that the financial frictions at play in the model are also relevant in the data.

Our economy with heterogeneous firms can be aggregated into a two-sector model with two final goods —the private sector good and the government sector good. Financial frictions distort this economy. First, selection into procurement is not only based on firm productivity but also on firm net worth, which reduces aggregate productivity in the government sector, increases its relative price, and ultimately damages aggregate productivity. Second, among financially constrained firms active in procurement, there is within-firm misallocation: production is relatively shifted towards procurement due to its higher collateral value. This further reduces aggregate productivity. And third, as is common in this type of models, capital accumulation is suboptimal and leads to between-firm misallocation.

To assess the interplay between procurement policies and the macroeconomy, we use our calibrated model to perform some counterfactual experiments that consist of reallocating procurement contracts across firms while *keeping government expenditure unchanged*. In particular, we consider counterfactual economies where a higher share of procurement contracts is allocated to small firms. The motivation for this exercise follows from the widespread use of policies promoting small firms' participation in public procurement.

Our main exercise, aimed to reproduce the "set-aside" policies for small businesses implemented by the U.S. Small Business Administration, increases the probability of small firms winning procurement projects.[2] In practice, we implement this policy counterfactual by targeting a 10 percentage point increase in the share of government suppliers that are small (20 employees or less) while keeping constant the fraction of firms from which the government buys. We find that promoting the participation of small firms in government procurement generates a real GDP loss of 0.8%, which is the result of several economic effects, some positive and some negative.

The main positive effect is that the policy does help small firms grow and overcome their financial constraints. The high pledgeability of procurement contracts, together with the extra profits they generate, reinforces the self-financing channel previously emphasized in the literature (Moll, 2014), leading to aggregate capital accumulation and a decline in misallocation.

However, the policy also generates two important unintended negative effects on the private sector's production. First, when financially constrained firms obtain a procurement contract, there is a contemporaneous negative crowding-out of private sector sales, as firms own limited collateral and the pledgeability of public sector revenues only covers part of the increase in credit demand. This channel leads to TFP losses from misallocation because the crowding-out effect is larger for more financially constrained firms, i.e., the ones with higher MRPK. Second, there is an unintended consequence related to the change in capital accumulation incentives for relatively big firms (those for which the expected probability

[2] In the U.S. the Small Business Act aims to "ensure that a fair proportion of federal contracts is awarded to small business" (see Blackford, 2024).

of receiving procurement contracts decreases). In particular, one of the reasons firms accumulate financial wealth in our model is the possibility of obtaining a public procurement contract in the future. A procurement contract represents a large demand shock in response to which firms want to expand their capital stock. Intuitively, productive firms want to have enough net worth to build capacity in case an opportunity for a big procurement contract is realized. A procurement policy that aggressively targets smaller firms will hence remove savings incentives for medium-sized and large firms, whose chances of obtaining a procurement contract are diminished.

Overall, combining the gains and losses, aggregate production in the private sector expands by 0.4%. However, the economy suffers aggregate losses related to the production of the public sector good. By buying relatively more from the pool of small firms, the government also buys from firms that are fundamentally less productive and thus charge higher prices. Under a constant government expenditure constraint, this phenomenon inevitably translates into a lower provision of public goods, a 9.5% decline in our exercise. When accounting for this reduction in the amount of final good provided by the government, the overall effect of the "buy small" policy is to shrink real GDP by the aforementioned 0.8%.

We conduct our main policy experiment in several different economies, with the goal of highlighting that the same policy may have different quantitative implications depending on the characteristics of the environment. Our analysis shows that our baseline "buy small" policy generates better macroeconomic outcomes when financial frictions are more severe or governments can choose procurement firms based on their productivity. Conversely, the policy leads to worse macroeconomic outcomes when government contracts are equally as pledgeable as private sector sales.

Finally, we conduct two alternative policy experiments that consist of promoting small firms' participation in procurement by reducing the average size of contracts or by explicitly targeting young firms. Our main finding is that these alternative policies would not be able to expand output either, but would generate a smaller reductions in aggregate GDP.

1.1 Related literature

There is practically no literature analyzing how the microeconomic aspects of public procurement can affect the macroeconomy. One recent exception is Cox, Müller, Pastén, Schoenle and Weber (2024), who emphasize the fact that government spending is concentrated in sectors with stickier prices, which can affect the short-run fiscal transmission mechanism in a New Keynesian model. Our interest differs in that we focus on quantifying the long-run macroeconomic effects of different procurement allocation systems through their impact on firm dynamics.

Several papers have studied how governments may harm the long-run economic performance of countries through the implementation of policies that distort the allocation of resources across firms. Some examples are credit subsidies to state-owned-enterprises (Song, Storesletten and Zilibotti, 2011), size-dependent policies (Guner, Ventura and Xu, 2008; García-Santana and Pijoan-Mas, 2014), labor market regulations (Garicano, Lelarge and Reenen, 2016), tariffs (Berthou, Chung, Manova and Sandoz, 2019), or capital markets regulation (Bau and Matray, 2023). However, one of the central roles that governments play in modern economies, i.e., being buyers of goods and services from private sector firms, has been overlooked.

Our focus on firm-level financial frictions as a channel through which public procurement can affect the macroeconomy builds on work that quantifies the effects of financial constraints on aggregate output and productivity. Some examples are Buera, Kaboski and Shin (2011), Midrigan and Xu (2014), Gopinath, Kalemli-Özcan, Karabarbounis and Villegas-Sanchez (2017), Leibovici (2021), David and Venkateswaran (2019), David, Schmid and Zeke (2022), and Catherine, Chaney, Huang, Sraer and Thesmar (2022).[3] A few papers in this literature have studied the interplay of financial frictions with different forms of taxation (Erosa and González, 2019; Itskhoki and Moll, 2019; Guvenen, Kuruşçu, Kambourov, Ocampo and Chen, 2023) but none has focused on the expenditure side of government policies.

Our finding that the type of financial frictions matters in understanding the effects of procurement on the macroeconomy is also related to recent papers that show that the type of financial frictions, i.e., earnings- vs. asset-based, and not only their severity, plays a crucial role in explaining important economic outcomes: the gains from trade liberalization (Brooks and Dovis, 2020), aggregate productivity (Li, 2022), macroeconomic fluctuations (Drechsel, 2023; Drechsel and Kim, 2024), and the transmission of monetary policy (Caglio, Darst and Kalemli-Özcan, 2021).

Finally, our results on the treatment effects of winning procurement contracts on firms are related to the recent literature analyzing the relationship between public procurement and firm dynamics. Ferraz, Finan and Szerman (2016) and Lee (2021) use quasi-experimental designs for Brazil and South Korea, respectively, to show that firms winning procurement contracts have a positive and permanent effect on firms' performance. Hebous and Zimmermann (2021) document for the U.S. a positive relationship between winning a procurement contract and firm investment. Using credit registry data for Portugal, Gabriel (2024) shows that obtaining a procurement contract allows firms to increase their credit at a lower interest rate.[4] Our results are consistent with this body of research. We further provide novel

[3]See Buera, Kaboski and Shin (2015) for a survey of this literature.

[4]Cappelletti et al. (2024) use data for Italy to show that firms that receive public procurement contracts

evidence on loan acceptances and on the fact that only non-collateralized credit increases at impact, which along with the other empirical facts that we document, can be taken as evidence of earnings-based financial constraints that are alleviated with procurement projects. Additionally, our results on the short-run crowding-out of sales to the private sector by procurement sales are related to recent papers that investigate within-firm spillover effects across markets, like Almunia, Antràs, López-Rodríguez and Morales (2021) and Kohn, Leibovici and Szkup (2016) with domestic versus foreign markets and Alfaro-Ureña, Manelici and Vasquez (2022) with multinational corporations versus other buyers.[5]

The rest of the paper is structured as follows. Section 2 describes the construction of the dataset and provides summary statistics. Section 3 provides our empirical evidence organized in three main stylized facts. Section 4 presents the model of firm dynamics with procurement. Section 5 discusses how we parameterize the model. Section 6 describes our benchmark economy. Section 7 provides the main quantitative results. Section 8 concludes.

2 Data and Summary Statistics

Our empirical work is based on merging three large datasets at the firm level. First, we construct a novel dataset on Spanish public procurement contracts published by the official bulletin of the Spanish Central Government (*Boletín Oficial del Estado*, BOE) over the 2000-2013 period. We have information on the type of good or service provided, the institution awarding the contract, the initial bidding and final price of the contract, the type of procedure used to allocate the contract, and the firm(s) that won the contract. Second, we use more standard firm-level data on balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Bank of Spain and taken from the Spanish Commercial Registry. And third, we use credit register data for Spain, which contains detailed information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain (including whether or not non-personal collateral was posted on a particular loan). Additionally, the credit register data set contains rich information on loan applications. Appendix A provides details about the different data sources and samples that we use. We merge the three data sets to build a panel of firms at the annual frequency (which is the frequency of observation of our firm-level data) with detailed information on credit and procurement activity.

survive longer.

[5] Our paper also relates to the vast micro/IO literature that analyzes the factors determining the efficiency of procurement outcomes. Some examples are Bandiera, Prat and Valletti (2009), Decarolis (2018), Decarolis, Giuffrida, Iossa, Mollisi and Spagnolo (2020), and Lotti, Muço, Spagnolo and Valletti (Forthcoming).

Some facts. We start presenting some summary statistics comparing procurement and non-procurement firms for a given year. We find that firms participating in procurement are significantly larger and older on average, but there is considerable overlap in the support of the size and age distribution for procurement and non-procurement firms (see Table A.1 in the Appendix). For example, the average number of employees of a procurement firm is around 6 times larger than for the rest of the firms (73.6 vs. 12.8), total sales are 7 times larger (8.9 million euros vs. around 1.2 million), and procurement firms are 9 years older (20 vs 11 years). Yet, around 25% of procurement firms have less than 16 employees, have revenues that are lower than 1.1 million euros, and are 12 or fewer years old. We also find that, conditional on having at least one procurement project, there is a lot of variation in the importance of these projects as a fraction of firms' total revenue. The ratio of procurement value to total revenues is 0.20 on average, with the 25th, 50th, and 75th percentiles being 0.01, 0.03, and 0.10, respectively. Finally, we observe large differences between procurement and non-procurement firms in terms of their composition of credit. In particular, procurement firms seem to rely more on non-collateralized credit (86% vs 71% on average) despite holding higher levels of assets.

Main sample. For our empirical work below, we only keep those firms that get at least one procurement project over the sample period.[6] This leaves 12,000 unique firms observed for an average of 12 years. Alternatively, in the Appendix, we also build a sample at a quarterly frequency and a smaller sample for projects with data on the full ranking of bidders. These two samples deliver a tighter identification of the effects of procurement on credit growth, but cannot be used to examine the effects of procurement on firm sales.

3 Empirical Evidence

We document three key facts related to the effects of procurement on firms' outcomes. First, we show a positive relationship between obtaining a procurement contract and firms' credit growth, which in the short run comes entirely from credit for which no tangible collateral is posted. Second, obtaining a procurement contract is related to a persistent increase in firm revenues, with a short-run *decline* in sales to the private sector (crowding-out) followed by an increase in private sector sales above the level before the procurement shock (crowding-in). Third, the initial crowding-out of private sector sales is stronger for firms more likely to be financially constrained.

[6] All our empirical work uses firm-level fixed effects; therefore, the relevant variation is within-firm and only for those firms that get at least one procurement project in the sample period.

3.1 Main facts

To study the relationship between participation in procurement and firm dynamics, we estimate local projection panel regressions (Jordà, 2005). We regress the cumulative difference of variable x, $\Delta_h \log(x_{it+h}) \equiv \log(x_{it+h}) - \log(x_{it-1})$ on the regressor PROC_{it}, firm fixed effects α_i, sector×year fixed effects α_{st}, and lagged x:

$$\Delta_h \log(x_{it+h}) = \alpha_i + \alpha_{st} + \beta_1^h \text{PROC}_{it} + \beta_2^h \log\ x_{it-1} + \varepsilon_{it+h} \tag{1}$$

where i denotes a firm and $h = 0, 1, ..., H$ denotes the horizon at which the impact of procurement is estimated.[7] The regressor PROC_{it} is a dummy variable that takes the value 1 if the firm obtains a procurement contract in year t. The inclusion of firm fixed effects implies that we are comparing the differential within-firm growth of credit across periods with and without procurement.

Fact 1. Credit growth. Figure 1 shows the results from running the regressions given by equation (1) at an annual frequency, with x referring to credit. We plot the estimated β_1^h coefficients and their associated confidence intervals. Our main result is that winning a procurement contract is associated with a persistent effect on firms' credit. On impact ($h = 0$), winning a procurement contract is associated with an increase in firms' total credit of around 5.5 percentage points (see panel (a) of Figure 1). This effect is persistent, which implies that firms increase their credit beyond the duration of the procurement contract.

We also find that the effect on impact is entirely driven by non-collateralized credit: the effect on non-collateralized and collateralized credit is around 8 and 0 percentage points, respectively, when $h = 0$ (see panels (b) and (c) of Figure 1). Finally, we find that, although absent on impact, the effect of procurement on collateralized credit increases over time. For example, the 5 years ($h = 4$) cumulative effect is around 6 percentage points.

This final result is consistent with the idea that firms use their extra profits earned in procurement to increase their net worth and therefore increase their borrowing capacity in the future. One potential concern with this interpretation is that the estimated "persistent" effect of procurement on credit may result from either long contracts or repeated contracts after the first one. Regarding the first potential explanation, our data does not contain information about the duration of each contract. However, we can use a different dataset,

[7] In practice, when we calculate $\Delta_h \log(x_{it+h})$, observations with $x_{it+h} = 0$ are dropped, and hence the results capture the intensive margin of credit growth between periods. This issue is irrelevant for the case of total and non-collateralized credit, where the fraction of firm-year observations with zeros are 0% and 3.6%, respectively. For the case of collateralized credit, where the fraction of observations with zeros is around 48%, we re-run our results using the commonly-used transformation *growth rate* $= [x_t - x_{t-1}]/[(x_t + x_{t-1})/2]$ (Davis et al., 2006) and find that results are robust.

Figure 1. Procurement effect on credit

(a) total credit **(b)** non-collateralized credit **(c)** collateralized credit

Notes: This figure shows the cumulative impact of the estimate of β_1^h from regression (1) for different time horizons $h = 0, 1, 2, 3, 4$, as well as its 5% confidence intervals. Panel (a) shows the results for the case of x being firms' total credit. Panel (b) shows the results for the case of x being firms' non-collateralized credit. Panel (c) shows the results for the case of x being firms' collateralized credit. As mentioned above, we have run these regressions with around 12,000 firms that have at least one contract during the sample period. Standard errors clustered at the firm level.

Tenders Electronic Daily (TED), to document the short duration of contracts.[8] For the years 2018-2019, for example, around 75% of the contracts have a duration of one year or less, 89% have a duration of two years or less, and 93% have a duration of three years or less.[9] Regarding the second potential explanation, we find that the persistent effect of procurement on credit is very similar for a sub-sample of firms that only obtain one contract during our sample period.

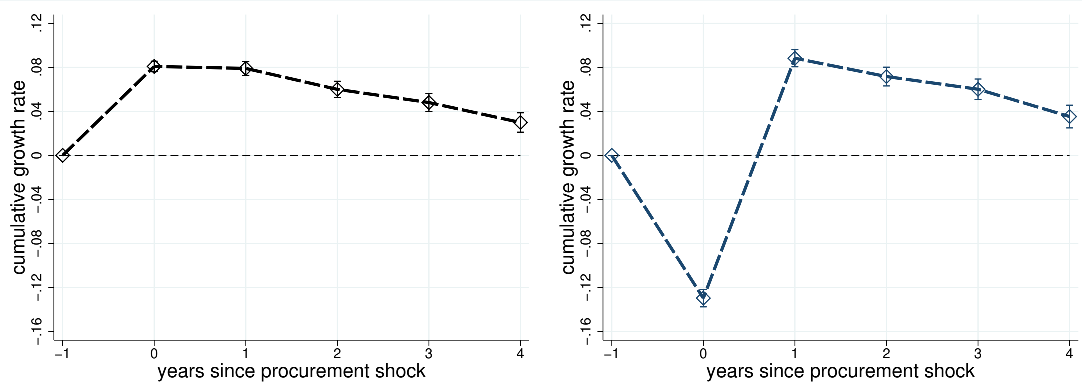
Fact 2. Sales composition and sales growth. We next look at the evolution of firms' total sales and their composition. We run regression (1) for the case of x = total sales (see panel (a) of Figure 2) and x = sales to the private sector (see panel (b) of Figure 2). Several important results emerge from running these regressions.

First, we find that firms' total sales increase after winning a procurement contract by around 8 percentage points. Second, we find that this effect is persistent, similar to the previous result on credit. Third, we find that sales to the private sector fall right after a firm wins a procurement contract, which means that procurement generates a short-run crowding-out of sales to the private sector. In the model presented below, this crowding-out occurs because financial constraints limit firms' production capacity, leading firms to decrease the amount of output sold to the private sector in order to deliver their procurement contracts. Finally, we find that this crowding-out effect disappears and actually reverses over time, implying that procurement crowds in sales to the private sector in the medium run. One

[8] TED is the online version of the supplement to the Official Journal of the EU dedicated to European public procurement. See García-Santana and Santamaría (2022) for details.

[9] These numbers likely represent an upper bound, given that very large contracts, which tend to last longer, are over-represented in TED. As a reference for a different country, Cox et al. (2024) find that the median contract in the U.S. has a duration of 31 days and 90% of contracts last less than one year.

Figure 2. Procurement effect on sales



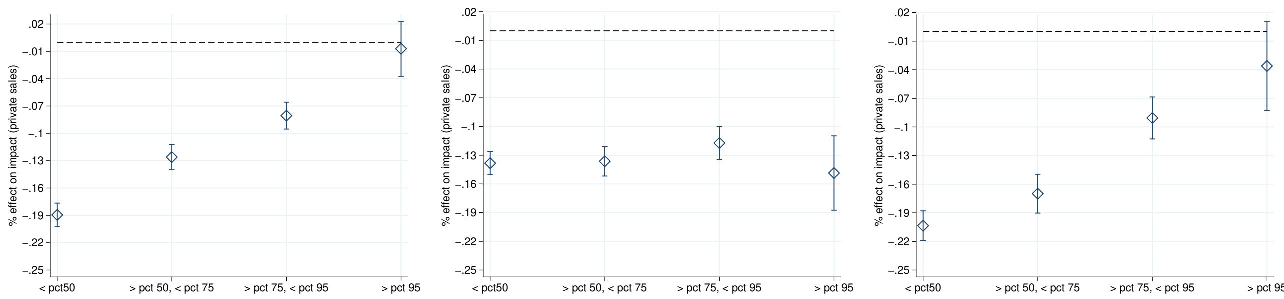
(a) total sales　　**(b)** sales to the private sector

Notes: This figure shows the cumulative impact of the estimate of β_1^h from regression (1) for different time horizons $h = 0, 1, 2, 3, 4$, as well as its 5% confidence intervals. Panel (a) shows the results for the case of x being firms' total sales. Panel (b) shows the results for the case of x being firms' sales to the private sector. Standard errors clustered at the firm level.

interpretation of this fact is that the extra earnings of the procurement contract can be used by the financially constrained firms to accumulate wealth and finance larger operations in the future.

Fact 3. Heterogeneous effects of crowding-out of private sales. The initial crowding-out of sales to the private sector may arise due to firms' lack of borrowing capacity to serve the private and public sectors at the same time, which the extra credit obtained with the procurement contract only partially offsets. However, it could also arise due to capacity constraints or decreasing returns to scale at the firm level. If the effect is at least partly explained by financial frictions, we should find that firms more likely to be financially constrained exhibit a larger crowding-out. Indeed, our simple static production problem in Section 4.3.2 shows this to be the case (see Appendix E.4). In the data, it is hard to identify which firms are more financially constrained. However, three conventional proxies for how financially constrained a firm is, are size (measured by assets), leverage (measured by credit/assets), and age. We test the hypothesis that the crowding-out effect of private-sector sales is greater for more financially constrained firms by interacting the effect of procurement on dummy variables for several percentiles of the fixed assets, leverage, and age distribution. We present these results in Figure 3. In terms of assets and age, we find a negative monotonic relationship between them and the extent of the crowding-out effect on private sales. In terms of leverage, the relationship is not so clear, but we find that the crowding-out effect is strongest for firms whose leverage is above the 95th percentile.

Figure 3. Heterogeneous effects on sales to the private sector



(a) distribution of fixed assets **(b)** distribution of leverage **(c)** distribution of age

Notes: This figure shows the effect on impact, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on sales to the private sector for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). Standard errors clustered at the firm level.

Heterogeneous effects of procurement on credit growth. In Appendix B, we provide evidence of the heterogeneous effects of procurement on firms' credit growth. We find that the empirical relationship between the change in firms' credit after procurement and proxies for financial constraints at the firm level is non-monotonic. However, as we discuss in Appendix B, this evidence is not very informative. Models of financial constraints would have conflicting predictions on which firms should increase credit more. Crucially, it is key whether proxies for financial frictions in the data represent the likelihood of being financially constrained or the severity of the financial constraints, conditional on these being binding.

3.2 Threats to identification and robustness

Given the lack of quasi-experimental variation in our empirical setup, selection, reverse causation, and other endogeneity issues may contaminate our baseline reduced-form estimates. Appendix C provides evidence to help assuage these concerns, along with further evidence on the relationship between firms' participation in procurement and credit growth. We summarize the evidence here.

Selection and reverse causality. We follow five strategies to address selection and reverse causality issues. First, we exploit quarterly data, which allows us to include firm-year fixed effects in our baseline regressions. Second, we exploit information on how firms rank in procurement contests (also using quarterly data), along with scores for a subset of procurement contests, to see whether there is a differential impact of winning a contract on credit growth also for close competitions (where the winner won by a small margin). Results in Appendix C.1 show this to be the case along with evidence supporting our baseline results for the whole sample of firms when including firm-year fixed effects. Third, we exploit information on firms' credit ratings to test whether low-credit rated firms tend to select into procurement. Appendix C.2 provides evidence to rule out this concern. Fourth, in the spirit of Hebous and Zimmermann (2021) we estimate a propensity score matching (PSM) model. Appendix C.3 presents the regression results from the PSM estimation, which are in line with our baseline OLS results. Finally, we exploit information on new loan applications to show in Appendix C.4 that the probability of receiving a bank loan *conditional on having applied for it* increases by approximately 2 percentage points in the quarter that a firm wins a procurement project.

Omitted variable bias. While we control for firm-level fixed effects and other time-varying characteristics at the sector-level, it is still possible that unobserved variables bias our results. One particular concern is the existence of political connections at the firm-level that facilitate both the success in winning a procurement project and the access to credit. We address this concern using two empirical strategies. First, we exploit information on contest type and in particular whether results differ for competitive vs. non-competitive procurement processes. Appendix C.5 shows that our baseline results hold, and that the point estimates are in fact larger for the competitive contests. Second, we examine whether estimates differ across bank types. The concern of political connections may be particularly acute for the "cajas" (savings banks), as these banks are often tightly connected to firms via joint-board memberships and are known to interact with local politicians. Indeed, there are several examples of political connections and corruption cases that led cajas to be over-leveraged and go bankrupt during the 2008-09 Spanish banking crisis. Appendix C.6 presents the regression estimates that utilize bank-firm level data. The baseline results hold across all bank types.

Heterogeneous effects and additional robustness. Finally, we provide further cuts of the data to examine the robustness of our main results on the impact of procurement on credit growth. First, we examine heterogeneity across firms and sectors. Results in Appendix C.7 show that the estimated impact of procurement on credit growth is positive and significant across samples split by firm-size quartiles. Appendix C.8 shows that while there is some variation in estimates at the sector-level, the mean and median of estimates are similar and of the same magnitude as the average effect that we estimate in the pooled sample. Further, estimates that fall in the interquartile range of the sectoral distribution are all positive. Finally, we examine the impact of the intensive margin of procurement on credit growth in Appendix C.9. To do this, we replace the procurement dummy variable with the ratio of awarded procurement value to a firm's total sales in a given year. The regression results are qualitatively similar as in our baseline regressions that use the procurement dummy as the main explanatory variable: the coefficient is positive and significant for the sample of all loans and non-collaterilized loans, while being positive but insignificant for the collateralized loan sub-sample.

3.3 Taking stock

The arrival of a procurement project may represent a *credit demand shock* at the firm level if firms need to borrow in advance to increase production. But it may also represent a *credit supply shock* if procurement contracts can be pledged for extra borrowing and the firm is credit-constrained. While both interpretations are plausible and potentially relevant for different sub-groups of firms in the data, our results point to the *credit supply shock* interpretation playing a relevant role in the population because (i) the short-run increase in credit is limited to non-collateral credit, (ii) the short-run crowding-out of private sales is larger for firms more likely to be constrained, and (iii) the increase in credit goes hand in hand with a fall in the rejection of loan applications. The *credit supply shock* interpretation is also consistent with firms using the extra revenues of procurement to build up their wealth, which allows collateral credit and sales to the private sector to increase in the medium run, as documented. In Section 4 we present a model incorporating these mechanisms.

4 The Model

We set up a model of privately-held heterogeneous firms. We build on standard models of firm dynamics with collateral constraints —as Midrigan and Xu (2014), Moll (2014), or Buera and Moll (2015)— and extend this setting to allow for (a) a public sector demanding goods from private firms, (b) a firm's intertemporal choice to compete for procurement projects, and (c) earnings-based borrowing constraints, with possibly different pleadgeability of public and private sector earnings. In the model, financial frictions distort the economy through two different channels. First, for a given selection of firms into procurement, financial frictions lead to lower aggregate capital, misallocation of capital across firms, and misallocation of capital within firms across sectors. Second, financial frictions distort the selection of firms into procurement, giving more weight to firm net worth at the expense of firm productivity.

4.1 Demographics and Technology

Time t is discrete. The economy is populated by a representative household, a unit measure continuum of heterogeneous entrepreneurs, two representative final good producers, and a government. The representative household consumes and saves and supplies labor inelastically as in the standard neo-classical growth model. Each entrepreneur $i \in [0, 1]$ runs a firm that produces a differentiated intermediate good y_{it}. The representative final good producers assemble the two final goods in the economy: the "private sector" good Y_{pt} and the "public sector" good Y_{gt}. Only the government purchases the "public sector" good, which is paid for with lump sum taxes raised from the representative household.

Final goods. The two final goods are assembled by combining the differentiated intermediate goods y_{it} through the following CES aggregators:

$$Y_{pt} = \left(\int_{[0,1]} y_{ipt}^{\frac{\sigma_p - 1}{\sigma_p}} \, di \right)^{\frac{\sigma_p}{\sigma_p - 1}} \quad \text{and} \quad Y_{gt} = m_g^{\frac{1}{1-\sigma_g}} \left(\int_{\mathrm{I}_{gt}} y_{igt}^{\frac{\sigma_g - 1}{\sigma_g}} \, di \right)^{\frac{\sigma_g}{\sigma_g - 1}} \quad \text{with } \sigma_p, \sigma_g > 1 \tag{2}$$

where I_{gt} is the subset of goods purchased by the public sector and m_{gt} is the measure of this set. Note that Y_{gt} is adjusted by m_{gt} to prevent a love-for-variety effect.[10] We also note that m_{gt} is a policy variable and the identity of firms in the set I_{gt} is the result of the procurement process discussed below. The final goods producers are perfectly competitive and choose the optimal demand of intermediate goods y_{ipt} and y_{igt}, respectively, to maximize profits taking intermediate good prices p_{ipt} and p_{igt}, final good prices P_{pt} and P_{gt}, and the set I_{gt} as given. This leads to the standard downward-sloping demands, $p_{ipt} = B_{pt} y_{ipt}^{-1/\sigma_p}$ and $p_{igt} = B_{gt} y_{igt}^{-1/\sigma_g}$, where for convenience we have defined $B_{pt} \equiv P_{pt} Y_{pt}^{1/\sigma_p}$ and $B_{gt} \equiv m_{gt}^{-1/\sigma_g} P_{gt} Y_{gt}^{1/\sigma_g}$. In equilibrium, Y_{pt} is determined by the demand of the private final good from the households and entrepreneurs. Instead, Y_{gt} is the demand of the public final good from the government and it is a policy variable in the model.[11] The aggregate prices P_{pt} and P_{gt} of the private and public goods are given by the usual aggregators:

$$P_{pt} = \left(\int_{[0,1]} p_{ipt}^{1-\sigma_p} \mathrm{d}i \right)^{\frac{1}{1-\sigma_p}} \quad \text{and} \quad P_{gt} = \left(\int_{\mathrm{I}_{gt}} \frac{1}{m_{gt}} p_{igt}^{1-\sigma_g} \mathrm{d}i \right)^{\frac{1}{1-\sigma_g}} \tag{3}$$

We will use the final private good as the numeraire, so we set $P_{pt} = 1$ in what follows.

Intermediate goods. Each intermediate input $i \in [0,1]$ is produced by a different entrepreneur i. Entrepreneurs are infinitely-lived, and at any period in time, they are characterized by their idiosyncratic stochastic productivity s_{it}, their capital stock k_{it} (which depreciates at rate δ), their debt level l_{it}, and whether they currently have access to procurement demand $d_{it} \in \{0,1\}$. Output y_{it} is given by a simple CRS production function, $y_{it} = s_{it} k_{it}^{\alpha} n_{it}^{1-\alpha}$, that depends on capital k_{it}, labor n_{it}, and managerial productivity s_{it}. The firm-specific s_{it} follows a stochastic first-order Markov process. Entrepreneurs make production decisions as well as dynamic consumption vs. saving decisions under financial constraints. If access

[10] Governments purchase only a fraction of goods and services provided by the private economy mainly because their needs are different than the needs of private households and firms. By removing 'love-for-variety' we eliminate this trivial effect from the analysis of the effects of the number of contracts offered.

[11] We model the government demand for each variety $i \in [0,1]$ as an isoelastic downward-sloping function for tractability. See Appendix D for a microfoundation of such variety-specific government demand curves.

to procurement is active ($d_{it} = 1$), the firm must choose how to split the production factors across the two markets. The prices p_{ipt} and p_{igt} of input i may differ because the intermediate good i producer has monopoly power over its variety and may be selling different quantities to each market.

4.2 Competition for public contracts

The government chooses the overall demand Y_{gt}, the measure of varieties m_{gt}, and the composition of the subset $\mathrm{I}_{gt} \subset [0,1]$. In order to introduce structure in this last choice, we assume that firms that wish to sell to the government next period ($d_{it+1} = 1$) must first invest today an amount of private sector good $b_{it} > 0$. This quantity b_{it} may reflect the costs of learning how the process works, the costs of establishing connections with government officials, or the actual costs of preparing a successful proposal. We will favor this last interpretation.

For firms, given their choice of b_{it}, there is always uncertainty in the outcome of the application. This can be because they do not know who are the other applicants competing for the same contracts. But also, because in practice, the government criteria to decide the final ranking of firms may be based on a judgment call over the quality of the proposal or over the degree of compliance with several technical requirements. We prefer this latter interpretation, and model the competition for contracts in reduced form assuming that firms face a probability $\Pr(d_{it+1} = 1 \mid b_{it}, d_{it}, s_{it})$ of winning access to procurement demand next period. In order to have a well-defined interior choice of b_{it}, we assume that this probability is increasing and concave in b_{it}, with standard Inada conditions for its derivative with respect to b_{it}. Note that we consider that the government can also assess the quality of the firm, based on its experience d_{it} and (perhaps a proxy of) its productivity s_{it}. The set of firms i which win access to procurement demand (obtain $d_{it+1} = 1$) form the set I_{gt+1} next period.

We will specify a flexible functional form for this probability function in the calibration Section 5, but at this stage it is useful to anticipate that the probability function must contain a level constant $\bar{p}_t$ ensuring that, in equilibrium, the probability $\Pr(d_{it+1} = 1 \mid b_{it}, d_{it}, s_{it})$ is consistent with the government's desired measure m_g of varieties.[12]

[12]In Appendix D we provide a possible microfoundation for our procurement process. The main idea is that quality and price competition are separated into two stages. Quality competition is an intertemporal problem decribed by the probability function $\Pr(d_{it+1} = 1 \mid b_{it}, d_{it}, s_{it})$, while price competition is a static problem described by the isoelastic demand function $p_{igt} = B_{gt} y_{igt}^{-1/\sigma_g}$. In this microfoundation, the level constant $\bar{p}_t$ is the inverse of the quality threshold for the government to deem an application successful.

4.3 Entrepreneurs

Entrepreneurs have CRRA preferences over consumption flows c_{it} with curvature μ, and their objective is to maximize the sum of utilities discounted with β.[13] They obtain income from running their firm, so their budget constraint is given by:

$$c_{it} + b_{it} + k_{it+1} - l_{it+1} \leq p_{ipt}y_{ipt} + p_{igt}y_{igt} + (1-\delta)k_{it} - (1+r_t)l_{it} - w_t n_{it} \tag{4}$$

We only allow for one-period debt contracts l_{it} that pay a risk-free interest rate r_t. The amount of debt is limited by the repayment capacity of the firm through a combination of earnings-based and asset-based collateral constraints. In particular, the amount of debt of a firm coming into $t+1$ is limited by:

$$l_{it+1} \leq \varphi_a \, k_{it+1} + \varphi_p \left(p_{ipt+1}y_{ipt+1} - w_{t+1}n_{ipt+1}\right) + \varphi_g \left(p_{igt+1}y_{igt+1} - w_{t+1}n_{igt+1}\right) \tag{5}$$

If $\varphi_a = 0$, $\varphi_p = 0$, and $\varphi_g = 0$ no external finance is available and all production needs to be self-financed. With $\varphi_a > 0$ the firm can leverage up against fixed capital. With $\varphi_p > 0$ and $\varphi_g = \varphi_p$ firms can borrow against their earnings net of labor costs (as in Drechsel (2023), for instance).[14] There are no reasons to assume that φ_p and φ_g should be equal to each other. Through the lens of our model, we will later find that $\varphi_g > \varphi_p$, that is, revenues from procurement offer better collateral. This may happen for several reasons that we do not explicitly model. For instance, it may indicate that the government pays its contract with a higher probability than the average purchase in the private sector. Hence, a government contract could reduce uncertainty about a firm's near-future earnings, potentially lowering its risk of default.

A fraction $1-\theta$ of entrepreneurs die every period and are replaced by the same number of new entrepreneurs, with the entrants producing the varieties left vacant by the exiting entrepreneurs. Dying entrepreneurs leave accidental bequests that, for simplicity, are used to provide the entrant entrepreneurs' (exogenously fixed) start-up financial wealth, with the remainder distributed to the representative household. Entrant entrepreneurs start with a joint distribution Γ_0 of financial wealth and productivity and with no procurement projects.

[13] We model firms as run by entrepreneurs with curvature in preferences over consumption because, in Spain, 96% of firms are (and 73% of total employment is in) partnerships and privately-owned limited liability companies, whose entrepreneurs are scarcely diversified, see Boar et al. (2025). This assumption could also be justified by firms' dividend-smoothing motives, as empirically documented, e.g. Leary and Michaely (2011).

[14] An alternative and more structural borrowing constraint would limit repayment $(1+r_{t+1})l_{it+1}$ explicitly by a fraction of undepreciated capital $(1-\delta)k_{it+1}$ plus earnings: $(1+r_{t+1})l_{it+1} \leq \tilde{\varphi}_a (1-\delta)k_{it+1} + \tilde{\varphi}_p \left(p_{ipt+1}y_{ipt+1} - w_{t+1}n_{ipt+1}\right) + \tilde{\varphi}_g \left(p_{igt+1}y_{igt+1} - w_{t+1}n_{igt+1}\right)$. In steady states with constant r this specification is identical to (5) with the redefinitions: $\varphi_a \equiv \frac{(1-\delta)\tilde{\varphi}_a}{1+r}$, $\varphi_p \equiv \frac{\tilde{\varphi}_p}{1+r}$, and $\varphi_g \equiv \frac{\tilde{\varphi}_g}{1+r}$. In our counterfactual exercises r remains constant, so these details are immaterial for our purposes.

4.3.1 Timing and state space simplification

We follow the timing convention commonly used in the firm dynamics literature. First, we assume that resources devoted to consumption are spent at the beginning of each period t, that production in $t+1$ is carried out using capital installed at the end of period t, and that the entrepreneurs' survival shock and the $t+1$ productivity shock are revealed (in this order) before firms decide how much capital to install for the next period, k_{it+1}, and how much debt to issue for next period, l_{it+1}. Regarding the variables related to procurement, we follow a similar logic. The amount of resources b_{it} devoted to increasing the probability of being active in procurement in $t+1$ is spent at the beginning of each period t. Whether or not the firm is successful and becomes active in procurement in $t+1$ is revealed at the same time as $t+1$ productivity and right after the survival shock. This means that procurement applications of dying entrepreneurs are ignored by the government and hence dying entrepreneurs are not awarded a procurement project that cannot be delivered.

As shown by Moll (2014) or Buera and Moll (2015), these assumptions on timing simplify the state-space dimensionality of the problem. In particular, let $a_{it+1} \equiv k_{it+1} - l_{it+1}$ be the firm's net worth to be carried to the next period in units of private good today. Then we can redefine the budget constraint as

$$c_{it} + b_{it} + a_{it+1} \leq (1 + r_t)\, a_{it} + p_{ipt} y_{ipt} + p_{igt} y_{igt} - (r_t + \delta) k_{it} - w_t n_{it} \tag{6}$$

The collateral constraint becomes

$$k_{it} \leq \phi_a a_{it} + \phi_p \left(p_{ipt} y_{ipt} - w_t n_{ipt}\right) + \phi_g \left(p_{igt} y_{igt} - w_t n_{igt}\right) \tag{7}$$

where the parameters in the borrowing constraint are redefined as:

$$\phi_a \equiv \frac{1}{1-\varphi_a} \in [1, \infty), \quad \phi_p \equiv \frac{\varphi_p}{1-\varphi_a} \in [0, \infty), \quad \phi_g \equiv \frac{\varphi_g}{1-\varphi_a} \in [0, \infty) \tag{8}$$

Hence, the production decisions are intratemporal, while the accumulation of net worth and the investment in procurement are intertemporal. This allows us to split the firm's problem in two: a *static production* problem and a *dynamic consumption-saving* problem. In the following, we describe them in turn.

4.3.2 The static production problem

The intratemporal production problem is characterized by firm productivity $s \in \mathbf{S} \equiv \{s_1, ..., s_n\}$, firm net worth $a \in \mathbf{A} \equiv [0, \infty)$, and access to procurement demand $d \in \{0, 1\}$. For simplicity, we drop the firm and time subindices i and t. Firms with $d = 0$ only have to choose their optimal capital k and labor n subject to the borrowing constraint, while firms with $d = 1$

also decide on the allocation of capital and labor across sectors. We can write the formal maximization problem for the firm of type $(s, a, d = 1)$ as:

$$\pi(s, a, 1) = \max_{k_p, k_g, n_p, n_g} \left\{ p_p y_p + p_g y_g - wn - (r + \delta) k \right\}$$

$$\text{subject to:}$$

$$p_p y_p = B_p \left[s k_p^{\alpha} n_p^{1-\alpha} \right]^{\frac{\sigma_p - 1}{\sigma_p}}; \quad p_g y_g = B_g \left[s k_g^{\alpha} n_g^{1-\alpha} \right]^{\frac{\sigma_g - 1}{\sigma_g}}$$

$$k \in \left[0, \phi_a a + \phi_p \left(p_p y_p - w n_p\right) + \phi_p \left(p_g y_g - w n_g\right)\right]$$

For the firm of type $(s, a, d = 0)$ all the terms $p_g y_g$, k_g, and n_g trivially disappear. Let λ be the multiplier of the intratemporal borrowing constraint and let's consider the general case with $d = 1$. The optimal choices are described by the following FOCs:

$$\frac{\partial p_p y_p}{\partial n_p} = w \quad \text{and} \quad \frac{\partial p_g y_g}{\partial n_g} = w \tag{9}$$

$$(1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial k_p} = r + \delta + \lambda \quad \text{and} \quad (1 + \lambda \phi_g) \frac{\partial p_g y_g}{\partial k_g} = r + \delta + \lambda \tag{10}$$

$$\lambda \geq 0, \ \phi_a a + \phi_p \left(p_p y_p - w n_p\right) + \phi_p \left(p_g y_g - w n_g\right) - k \geq 0 \tag{11}$$

where $k \equiv k_p + k_g$ and $n \equiv n_p + n_g$. Equations (9) determine the optimal choices of labor, which require equalizing the marginal revenue product of labor in each sector to the common wage rate. The FOC for labor are not distorted by the financial frictions because labor costs are deducted from revenues in the financial constraint. Because of the concavity of the revenue function and the complementarity of capital and labor in production, these equations imply that labor demand increases with capital in each sector (see Lemma 5 in Appendix E).

Equations (10) determine the optimal capital choices, which for the unconstrained firm $(\lambda = 0)$ require equalizing the marginal revenue product of capital in each sector to its cost, which is just $r + \delta$. With binding financial constraints $(\lambda > 0)$, however, the effective cost of capital is $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ for sales to the private sector and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ for sales to the public sector. The multiplier of the financial constraint λ has two opposite effects on the cost of capital: on the one hand it increases the cost of capital as in standard asset-based financial constraints (numerator), but on the other hand, it decreases the cost of capital because a fraction of the generated output can also be collateralized (denominator). We will restrict ϕ_p and ϕ_g as indicated in Assumption 1 below to ensure that the earnings-based constraints cannot finance the optimal level of capital, that is, to ensure that the financial constraint is binding for at least the entrepreneurs with zero net worth (see Proposition 1 in Appendix E).

Assumption 1 *The model parameters satisfy the constraints* $\phi_p < \frac{\sigma_p - 1}{\sigma_p} \frac{\alpha}{r+\delta}$ *and* $\phi_g < \frac{\sigma_g - 1}{\sigma_g} \frac{\alpha}{r+\delta}$.

Because $\sigma_p, \sigma_g > 1$ and $\alpha < 1$, Assumption 1 implies $\phi_p < (r+\delta)^{-1}$ and $\phi_g < (r+\delta)^{-1}$, which in turn implies that the effective costs of capital for the private and public sector, $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$, are monotonically increasing in λ (Lemma 1 in Appendix E), consistent with financially constrained firms operating with less capital and less labor (Proposition 5 in Appendix E). This has two consequences. First, to the extent that a significant fraction of firms are financially constrained, aggregate capital will be inefficiently low. Second, because λ depends on the firm's state variables (s, a, d), there will be heterogeneity in the cost of capital across firms, which will generate misallocation of capital *across firms* and lower aggregate TFP in both the private sector and the procurement sector.

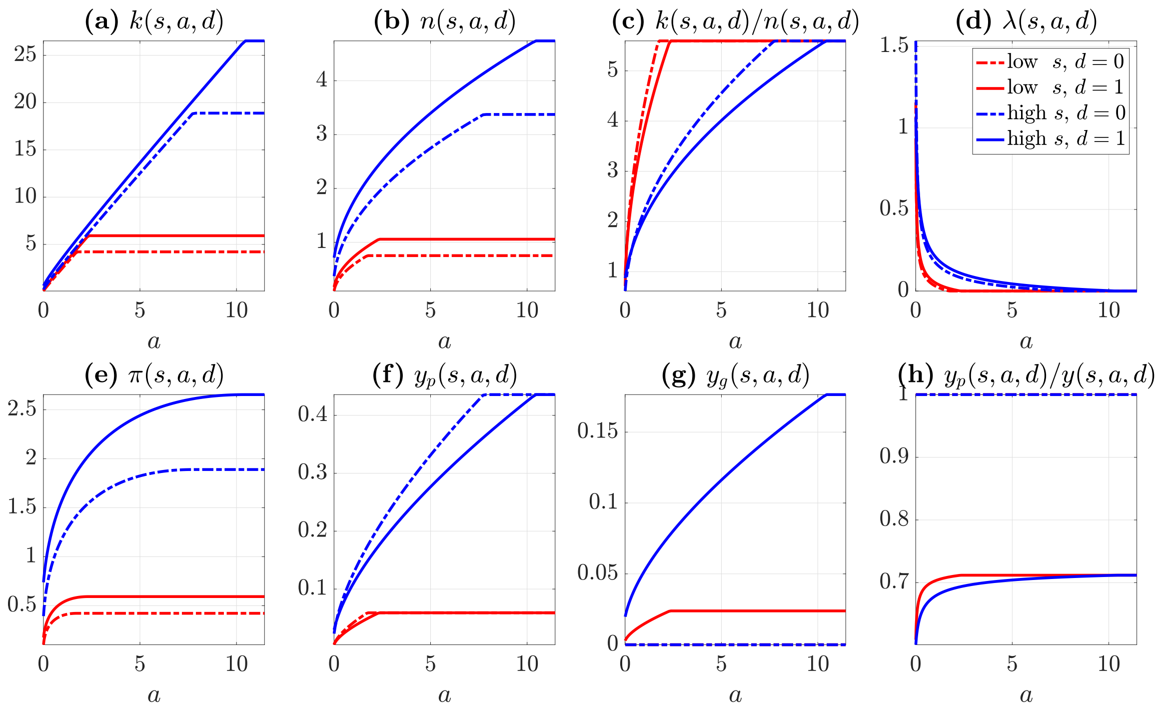
Finally, note that combining the two FOCs for capital yields,

$$(1+\lambda\phi_p)\frac{\partial p_p y_p}{\partial k_p} = (1+\lambda\phi_g)\frac{\partial p_g y_g}{\partial k_g} \tag{12}$$

Equation (12) characterizes the allocation of capital across sectors. For unconstrained firms $(\lambda = 0)$, the optimal choice requires the equalization of the marginal revenue product of capital obtained from each sector. With binding financial constraints $(\lambda > 0)$, capital is shifted towards the sector whose output can be better collateralized. For instance, if procurement contracts offer better collateral value than sales to the private sector $(\phi_g > \phi_p)$ the optimal choice requires a lower marginal revenue product of capital from public procurement relative to the private sector, which happens when production is shifted towards the public sector and away from the private sector. This generates misallocation of capital and labor *within firms* and sectoral misallocation in the aggregate.

Static policy functions. In Appendix E we provide a detailed characterization of the analytical solution to the static production problem (for the special case $\sigma_g = \sigma_p$). In Figure 4, we illustrate the numerical solution for one particular parameterization (the calibration discussed in Section 5). First, as is common in standard models of firm dynamics with collateral constraints, financially constrained firms see their capital, employment, capital to labor ratio, and profits increase with net worth, while the shadow value of the borrowing constraint declines. This happens up to the point where the financial constraint stops binding and net worth no longer plays a role, see the dashed blue lines in Panels (a)-(e). Second, and different from models with only asset-based constraints, financially constrained firms increase capital, labor, and profits when productivity increases (compare the dashed red and blue lines). This happens through the earnings-based constraint, which allows more productive firms to generate more earnings at the same level of net worth and, hence, expand production. Third, more productive firms are more financially constrained at any level of net worth (λ is higher) because the expansion of borrowing possibilities with s is lower than the increase in the optimal size (thanks to Assumption 1). Finally, looking only at firms with procurement,

Figure 4. Solution of the static profit maximization problem



Notes: This figure shows the solution to the static production problem (for the parameterization discussed in Section 5) plotted against firm's net worth a, for two different levels of productivity s, and for the cases $d = 0$ and $d = 1$. Panels (a) and (b) show total capital $k(s, a, d)$ an employment $n(s, a, d)$, while Panel (c) shows their ratio; Panel (d) shows the multiplier of the financial constraint $\lambda(s, a, d)$; Panel (e) shows the profits $\pi(s, a, d)$; and Panel (f) and (g) show the output sold to the private and public sector respectively, while Panel (e) shows the share of output sold to the private sector (the last two for $d = 1$ firms only).

the fraction of output sold to the private sector by constrained firms is increasing in net worth a and decreasing in productivity s, see Panel (h). This result is true under $\phi_g > \phi_p$ (it would be the opposite if $\phi_g < \phi_p$), as more constrained firms switch their sales towards the sector that offers better collateral.

A procurement shock for unconstrained firms. In Figure 4 we analyze the static effect of a procurement shock by comparing the solutions of the $d = 1$ and $d = 0$ cases at any value of the state variables s and a (details in Appendix E.4). For unconstrained firms, the production decisions for the public and private sectors are unrelated (thanks to the constant returns to scale technology). A procurement shock leaves operations in the private sector unchanged, increases firm capital and labor to serve the public demand, and increases profits, with the profit gain from procurement being independent from a and increasing with s (as more productive firms can deliver larger projects).

A procurement shock for constrained firms. For constrained firms, things are more complex. If $\phi_a > 0$ and $\phi_g = \phi_p = 0$, a procurement shock makes the firm more financially constrained because the firm with $d = 1$ has two demands to serve and has the same scarce collateral a to finance capital. This makes the firm reallocate some of its capital from the private sector to the public sector, which generates a negative private sector spillover (see Proposition 9). Instead, with $\phi_a = 0$ and $\phi_g > \phi_p > 0$, procurement makes the firm less financially constrained because the revenues from the public sector are more pledgeable. This leads the firm with procurement to expand its size and use part of the extra financing to increase production in the private sector as well, generating a positive private sector spillover (see Proposition 10). The strength of this mechanism increases with the size of the gap $\phi_g - \phi_p$. In the empirically relevant case with $\phi_a > 0$ and $\phi_g > \phi_p > 0$ both forces operate at the same time, with the former (latter) dominating for firms with high (low) wealth (see Proposition 11). For the particular case of our calibration, a procurement shock leads most firms to experience a tightening of their financial constraints —see Panel (d)— and a negative spillover of private sector operations —see Panel (f)— with only firms with zero or nearly zero wealth reacting in the opposite direction.[15] Finally, a procurement shock also increases profits among constrained firms —see Panel (e). For the empirically relevant case with $\phi_a > 0$ and $\phi_g > \phi_p$, this is more so for the more productive and the wealthier firms, because these two variables determine the capacity to deliver large projects.[16]

4.3.3 The dynamic problem

We can write the dynamic consumption-saving problem in recursive form. For simplicity, we already impose the steady-state condition that policy parameters, prices, and the aggregate distribution are constant, and use primes to denote next period variables:

$$
\begin{aligned}
V(s,a,d) &= \max_{c,b,a'} \left\{ u(c) + \beta\theta\, \mathbb{E}_{s',d'|b,d,s}\left[V(s',a',d')\right] \right\} \\
&\quad \text{subject to:} \\
&\quad \mathbb{E}_{s',d'|s,d,b}\left[V(s',a',d')\right] = \Pr(d'=1 \mid b,d,s)\,\mathbb{E}_{s'|s}\left[V(s',a',1)\right] + \Pr(d'=0 \mid b,d,s)\,\mathbb{E}_{s'|s}V(s',a',0) \\
&\quad c + b + a' = a + \left[ra + \pi(s,a,d)\right] \quad \text{and} \quad a' \geq 0
\end{aligned}
$$

The first constraint says that the entrepreneur's expected value for tomorrow is an average of the value under procurement and no procurement, weighted by the endogenous probability of procurement. The non-negativity constraint on net worth a' follows from our timing assumptions and the fact that entrepreneurs are not allowed to die in debt. The FOCs for

[15]Indeed, in our quantitative application, the share of all procurement awardings that lead to a relaxation of the awardees' financial constraints is just 0.4%.

[16]The only exception is for firms with nearly zero wealth, in which case the increase of profits with procurement actually falls with net worth (Proposition 12 in Appendix E).

the choices of a' and b are:

$$u_c(c) \geq \beta\theta\,\mathbb{E}_{s',d'|d,s,b}\left[\left(1+r+\frac{\partial\pi(s',a',d')}{\partial a'}\right)u_c(c')\right] \tag{13}$$

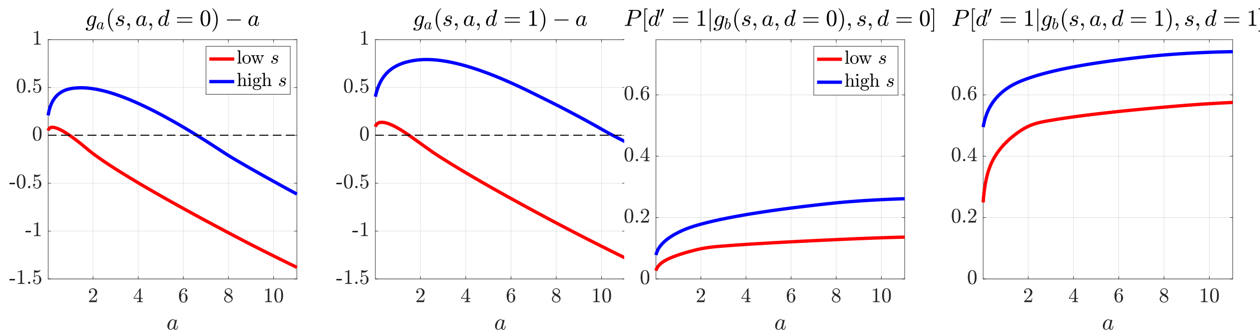
$$u_c(c) = \beta\theta\frac{\partial\Pr(d'=1 \mid b,d,s)}{\partial b}\mathbb{E}_{s'|s}\left[V(s',a',1)-V(s',a',0)\right] \tag{14}$$

The first equation is the standard Euler equation that emerges in models of heterogeneous entrepreneurs with financial constraints. In future states where the firm is not constrained in the static production problem $\partial\pi(s',a',d')/\partial a' = 0$ and the return to accumulating wealth a is just r. Instead, in states where the firm is financially constrained in the static production problem, there is an extra return $\partial\pi(s',a',d')/\partial a' = \phi_a\lambda(s',a',d') > 0$, which is given by the increase in profits due to relaxing the firm's collateral constraint (see Appendix E). The second equation determines the optimal spending on b: the entrepreneur will equalize its marginal utility of consumption to the marginal return of b, which is given by the expected increase of the firm's value coming from the possibility of selling to the government.

Decision rules. Figure 5 illustrates the net saving decision $a'-a$ of entrepreneurs with and without procurement (first and second panels, respectively). At low levels of net worth there is a hump-shaped relationship between net savings and net worth that is driven by the tradeoff between smoothing consumption vs. relaxing future borrowing constraints, a feature present in similar models like Midrigan and Xu (2014). At larger levels of wealth, the saving behavior follows the logic in Aiyagari (1994): net savings decrease monotonically with net worth and there is a target level of wealth that is larger for larger productivity s. These figures also show large differences between procurement and non-procurement firms. In particular, procurement firms save more conditionally on their current net worth a and productivity shock s. This difference is driven by the facts that (i) profits are higher for firms that are active in procurement (which allows them to accumulate more net worth) and (ii) the probability of procurement in the next period is larger for procurement firms, which raises the expected return of saving.

Figure 5 also shows the probability of $d'=1$ evaluated at the actual choice of b for firms with different levels of net worth and productivity, both for non-procurement (third panel) and procurement firms (fourth panel). The first thing to notice is that high-net-worth firms invest more resources in increasing their probability of being able to sell to the government. This emerges as a result of an interesting trade-off: in the dynamic problem, there are two competing uses for current resources to increase future income. On the one hand, entrepreneurs can accumulate net worth a', which relaxes the asset-based constraint and increases profits next period (right-hand side of equation (13)). On the other hand, they can invest in applications for procurement projects, which increases the probability of

Figure 5. Decision rules



Notes: This figure shows the solution to the dynamic problem of the entrepreneurs (for the parameterization discussed in Section 5) plotted against firm's net worth a, for two different levels of productivity s, and for the cases $d = 0$ and $d = 1$. The first and second panels show the net saving rules) for non-procurement and procurement firms respectively. The third and fourth panels show the endogenous probability of obtaining procurement contracts evaluated at the optimal rules for b for non-procurement and procurement firms respectively. $g_a(s, a, d)$ and $g_b(s, a, d)$ are the policy functions for a' and b.

winning a public contract and hence it increases expected profits next period (right-hand side of (14)). For constrained firms, the return of accumulating net worth declines with net worth (see Corollary 5) and the profit premium of a procurement project $\pi(s, a, 1) - \pi(s, a, 0)$ generally increases with net worth (see Proposition 12) and so does $V(s, a, 1) - V(s, a, 0)$.[17] Therefore, we obtain the result that procurement investment b increases monotonically with firm net worth. Second, the probability of obtaining procurement also increases with firm productivity s. This is because of the direct effect of s in the probability of procurement, but also as a result of higher investment b because higher s allows a firm to obtain larger profits from procurement sales (again, see Proposition 12). Finally, we see how current procurement firms have a higher probability of obtaining procurement next period, mainly thanks to the direct effect of experience d in the application process, but also thanks to currently higher profits allowing them to spend more on b.

4.4 Closing the model

The infinitely-lived representative household has the same preferences as the entrepreneurs, i.e., same CRRA parameter μ and discount factor β. She chooses consumption and savings by maximizing the discounted sum of period utilities and supplies one unit of labor inelastically.

[17] The higher profit premium for firms with larger a arises because selling to the government does not relax the borrowing constraints completely (see Proposition 11), which means that firms still rely on their own assets for determining the size of their procurement supply. This reflects a "size effect": the bigger the procurement project the firm expects to be able to deliver, the higher the expected profits that participating in procurement generates. This argument can be proved whenever $\phi_g \leq \phi_p$ and for firms with net worth that is not too small whenever $\phi_g > \phi_p$. In this last case, a procurement shock makes firms less constrained whenever $a \simeq 0$ as procurement revenue can be pledged to a larger extent than private sector sales.

The budget constraint is given by $C_t + A_{t+1} = (1 + r_t) A_t + w_t - \tau_t + T_t$, where C_t is consumption, A_t are wealth holdings, T_t are net transfers from entrepreneurs' accidental bequests, and τ_t is a lump sum tax. This leads to the standard Euler equation $C_{t+1}/C_t = [\beta(1 + r_{t+1})]^{1/\mu}$, which in steady state pins down the equilibrium interest rate r. A precise definition of the steady state equilibrium can be found in Appendix F.

5 Calibration

The model period is one year. We classify the model parameters into four different blocks. The first block contains parameters related to preferences, technology, and productivity, which we set externally to the model's equilibrium. Instead, most of the parameters in the other three blocks are calibrated in equilibrium such that the model matches a variety of moments measured in the Spanish economy in 2006 unless we specify otherwise. Panel A in Table 1 shows the definition of the parameters as well as their inferred values. Panel B shows the description of moments and their value in the data and in the model.

Block #1: Preferences and technology. We set the relative risk aversion coefficient μ equal to 1 (as Azariadis et al., 2016) and the discount factor β to 1/1.04 (which gives an interest rate $r = 0.04$ in the steady state equilibrium). We set the CES elasticities σ_p and σ_g both equal to 4, a standard value in the literature and between the two values used by Hsieh and Klenow (2009). The capital share α and the annual depreciation rate δ are set to 1/3 and 0.06, both within the range of standard values in the literature (e.g., Midrigan and Xu (2014), Gopinath et al. (2017)). The survival probability θ is set to 0.92 such that the firm entry and exit rate in the model equals the average of these rates in Spain as reported by Eurostat, which are around 10% and 6%, respectively.[18] Finally, we assume that the log of a firm's productivity process s evolves over time according to an AR(1) process with Gaussian shocks. We set the autocorrelation coefficient ρ_s equal to 0.80 and the standard deviation of the innovations σ_s equal to 0.30, as estimated by Ruiz-García (2020) using the same dataset of firms. We discretize the process following the Rouwenhorst method, with $N_s = 5$ states, normalizing average unconditional productivity to $\bar{s} = \mathbb{E}[s] = 1/20$.[19]

Block #2: New entrants. We assume all entrants start with no procurement project ($d = 0$), the same initial wealth level a_0, and draw their productivity s from a distribution shifted to the left compared to the ergodic productivity distribution in the economy. In particular,

[18]See Eurostat Webpage for details.

[19]Our model also matches the moments that are usually used in the literature to pin down these two parameters. In our sample the one-year autocorrelation of firms' log sales is 0.85 (which compares to 0.89 implied by our model) and the standard deviation of firms' sales growth is 0.67 (which compares to 0.58 in our model), see Appendix H.

Table 1. Calibration

Panel A: Parameters		(1) Baseline	Panel B: Moments	(2) Data	(3) Model
Block 1					
μ	CRRA coefficient	1.00	*Standard*		
σ_p	CES private sector	4.00	*Standard*		
σ_g	CES government	4.00	*Standard*		
β	Discount factor	1/1.04	Interest rate	0.04	0.04
δ	Depreciation rate	0.06	*Standard*		
θ	Survival probability	0.92	Avge exit, entry rates	0.06	0.06
ρ_s	AR(1) correlation	0.80	Ruiz-García (2020)		
σ_s	AR(1) variance	0.30	Ruiz-García (2020)		
$\bar{s}$	Productivity (average)	1/20	Normalization		
Block 2					
Δs_0	Entrants' productivity shift	-0.18	Rel. firm size young firms	0.54	0.54
a_0	Entrants' net worth	0.01	Leverage premium young firms	0.24	0.23
Block 3					
ϕ_a	Borrowing const. (a)	2.18	Credit/K	0.55	0.55
ϕ_p	Borrowing const. ($p_p y_p$)	0.52	Leverage reg. coefficient (β_1)	0.27	0.27
ϕ_g	Borrowing const. ($p_g y_g$)	1.18	Leverage reg. coefficient (β_2)	0.38	0.38
Block 4					
Y_g	Demand shifter	0.37	Share of procurement in GDP	0.12	0.12
m_g	measure of procurement goods	0.13	Share of procurement firms	0.13	0.13
$\bar{p}$	Prob. function (level)	0.91	Equilibrium equation (F.1)		
η_1	Prob. function (b-slope)	0.46	Procurement premium, empl	1.15	1.14
η_2	Prob. function (d-premium)	3.58	$\mathbb{P}(d_{it+1} = 1 \mid d_{it} = 1)$	0.60	0.59
η_3	Prob. function (s-slope)	0.00	Procurement premium, MRPK	0.15	0.15

Notes: This table summarizes our baseline calibration. For the choices in Block 1 labeled as "*standard*", see details in the main text; the targets for β and θ can be obtained without solving the model. All parameters in Blocks 2 to 4 are calibrated through SMM, except for m_g (whose target can be obtained without solving the model) and $\bar{p}$ (which satisfies an equilibrium condition).

we assume that new entrants draw their initial productivity from $\log(s) \sim N(\mathbb{E}[\log(s)] + \Delta s_0, \sigma_s/\sqrt{1-\rho_s^2})$, where $\mathbb{E}[\log(s)]$ is the unconditional mean of $\log(s)$, and σ_s, and ρ_s are the parameters described above, and Δs_0 is the entrants' productivity shifter. To calibrate a_0 and Δs_0 we focus on very young firms, which we define as firms up to 5 years of age both in the data and the model. To identify a_0 we target a leverage premium of young firms of 24 pp, which requires $a_0 = 0.01$.[20] To identify Δs_0, we target the average firm size of young firms relative to the average firm size in the economy, which is 0.54. The model requires a productivity shifter of $\Delta s_0 = -0.18$.[21]

[20] Our definition of young and old firms follows Midrigan and Xu (2014). The leverage premium is the estimated β in the following regression: $\log \text{LEV}_i = \beta \mathbb{1}[\text{YEARS}_i \leq 5] + \gamma \mathbb{1}[\text{YEARS}_i \in (6,10]] + \text{FE}_{j(i)} + u_i$, where "old" is the omitted category; and $\text{FE}_{j(i)}$ is an industry fixed effect.

[21] Given the initial net worth and productivity shifter, the model is consistent with the untargeted yearly average growth rates of value added for young and old firms, which equal 32 and 3 log points in the data,

Block #3: Financial constraints. Our model contains three parameters governing firms' financial constraints: φ_k, φ_p, and φ_g. We choose φ_k such that the model matches the aggregate credit-to-fixed assets ratio observed in our micro-level data in 2006, equal to 0.55. Regarding φ_p and φ_g, we proceed as follows. We start by rewriting the credit constraint (5) in first differences and relative to capital k, such that the change in leverage for financially constrained firms is given by:

$$\Delta\left(\frac{l_t}{k_t}\right) = \beta_1\,\Delta\left(\frac{p_{pt}y_{pt} + p_{gt}y_{gt} - w_t n_t}{k_t}\right) + \beta_2\,\Delta\left(\frac{p_{gt}y_{gt} - w_t n_{gt}}{k_t}\right) \tag{15}$$

where $\beta_1 = \varphi_p$, $\beta_2 = (\varphi_g - \varphi_p)$, and l_t/k_t is the firms' leverage (total credit divided by fixed assets). We can use equation (15) to recover φ_g and φ_p by indirect inference.[22] Specifically, we require the OLS estimation of equation (15) to deliver the same regression coefficients in the model and in the data. In our firm-level data we directly observe firms' total value added "VA_t", capital stock k_t, and wage bill "wage bill_t", and hence we can construct the first term in the right-hand-side of equation (15) as $(\text{VA}_t - \text{wage bill}_t)/k_t$. Instead, while we observe the split of total sales across sectors, we do not observe the split of the value added and the wage bill, so the second term on the right-hand side is not directly observed. However, our model implies that, whenever $\sigma_p = \sigma_g$, the labor share (wage bill over value added) is the same in all sectors (see Appendix E.1). Hence, we can obtain the value added net of the wage bill in procurement as $(\text{VA}_{gt} - \text{wage bill}_{gt}) \simeq (\text{sales}_{gt}/\text{sales}_t) \times (\text{VA}_t - \text{wage bill}_t)$.

Table 2 presents the results from running the empirical counterpart of equation (15) for firms with at least one procurement contract in our sample, i.e., the same sample that we use in Section 3. Columns (1) and (2) show the estimated coefficients when including and not including firm FEs, respectively. Because equation (15) should hold with equality only for firms whose financial constraint is binding, in columns (3) and (4), we run the same regression using a sample of firms that are likely to be financially constrained. In particular, we run the regressions for firms with leverage ratios above the median, which, through the lens of our model, is a good proxy for whether the firm is financially constrained. We find the same two qualitative results across all the specifications. First, the estimated coefficient β_1 is positive, which is evidence of earnings-based borrowing constraints ($\varphi_p > 0$). Second, the estimated coefficient β_2 is also positive, which is evidence that earnings from the public sector can be pledged to a higher extent than earnings from the private sector ($\varphi_g > \varphi_p$).

We take column (3) as our preferred specification. This delivers estimates of $\beta_1 = 0.267$ and $\beta_2 = 0.378$. For the model to generate these regression coefficients and a credit-to-fixed

and 30 and 2 log points in the model, respectively, see Appendix H.

[22]The identification of the parameters in the earnings-based constraint is similar to the one used by Li (2022) for the case of private sector earnings only.

Table 2. Change in Leverage and Procurement

	All firms		> Median leverage	
$\Delta\left(\text{VA}_t - \text{wage bill}_t\right)/k_{it}$	0.177***	0.176***	0.267***	0.255***
	(0.005)	(0.005)	(0.010)	(0.010)
$\Delta\left(\text{VA}_{gt} - \text{wage bill}_{gt}\right)/k_{it}$	0.261***	0.301***	0.378***	0.460***
	(0.063)	(0.060)	(0.121)	(0.109)
Observations	58,769	59,840	26,320	27,898
R-squared	0.245	0.077	0.314	0.115
Sector×year FE	Yes	Yes	Yes	Yes
Firm FE	Yes	No	Yes	No

Notes: This table presents results from estimating the relationship between the change in firm's leverage and the change in its average product of capital and the change in its earnings coming from selling to the government divided by the firm's total stock of capital. Regression (15) is estimated with firms obtaining at least one procurement project over 2000-13 using annual data. Standard errors are clustered at the firm level; *** indicates significance at the 1% level. We deal with outliers by trimming the bottom and top 2% of all the variables used in the regression.

assets ratio of 0.55 we need $\varphi_p = 0.24$, $\varphi_g = 0.54$, and $\varphi_k = 0.54$. Hence, firms can pledge 24% of their annual earnings from selling to the private sector, 54% of their annual earnings from selling to the government, and 54% of their capital stock. Using equation (8), these numbers translate into $\phi_p = 0.52$, $\phi_g = 1.17$, and $\phi_a = 2.17$, which means that the maximum affordable capital stock increases by 0.52 units per 1 unit increase in annual private sector earnings and by 1.17 units per 1 unit increase in annual earnings from selling to the government.[23] This is the result of a multiplier effect: firms can borrow against their earnings, allowing them to buy more capital, which can be partly collateralized to obtain further credit. This is an important interaction and captures how earnings-based constraints affect a firm's ability to grow also depends on the value of φ_a.

Block #4: Participation and size of procurement. We start by giving a flexible functional form to the probability of winning a procurement project:

$$\Pr\left(d' = 1 \mid b, d, s\right) = 1 - e^{-\bar{p}\, b^{\eta_1}\, \eta_2^{d}\, s^{\eta_3}} \tag{16}$$

The parameter $\bar{p} > 0$ is a level constant. The elasticity parameter η_1 controls the extent to which the quality of proposals matters, and it drives the degree of decreasing returns to investment in b, shaping the degree of self-selection of firms into procurement. We restrict $\eta_1 \in (0,1)$ to ensure concavity and Inada conditions. The parameter $\eta_2 \geq 0$ reflects the value of procurement experience in obtaining new contracts, as firms serving the public sector today ($d = 1$) have a higher chance of being government suppliers tomorrow whenever $\eta_2 > 1$.

[23] We note that both ϕ_p and ϕ_g satisfy Assumption 1 given the values for α, σ_p, σ_g, δ, and r, which means that capital cannot be self-financed through the earnings-based constraints, see Lemma 2 in Appendix E.

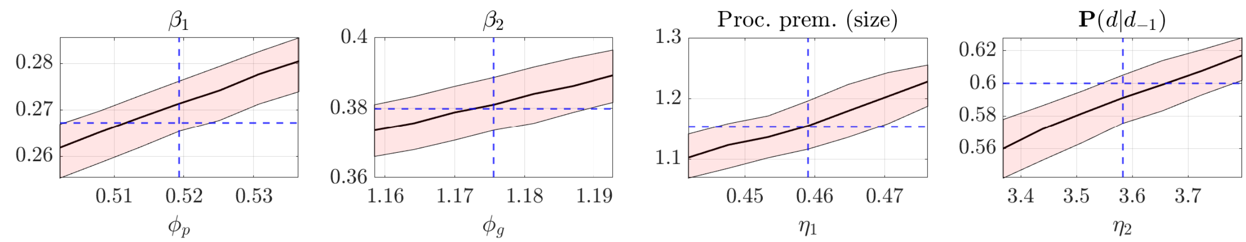
The parameter $\eta_3 \geq 0$ reflects the weight of firm productivity in the government's decision, which captures its potential productivity-based screening ability.[24]

Given this functional form, there are 6 parameters shaping the size and composition of procurement: Y_g, m_g, $\bar{p}$, η_1, η_2, and η_3. The parameters Y_g and m_g reflect the government's policy choices governing the relative size of procurement in the economy and the fraction of varieties purchased by the government. To identify the former, we set Y_g to match the share of procurement in GDP equal to 12.1%, while the latter directly represents the fraction of firms doing procurement in equilibrium. To have a comprehensive estimate of the actual number of firms active in procurement, we make use of a recently available dataset provided by the Spanish Government, *Plataforma de Contratación del Sector Público (PLACSP)*, where all types of public agencies in Spain are required to upload their procurement activity. Although some information is available since 2013, compliance was only strong starting in March 2018, when introducing information on all contracts became compulsory by law. We use information for the years 2019, 2021, and 2022, when the coverage is already high and firms' participation is not directly affected by COVID. In each of those years, we compute the number of firms with at least one procurement contract in the platform and divide it by the number of active firms in the micro data. We obtain an average of 12.6% across the three years. In our baseline calibration, we thus employ this value as the target for the equilibrium share of firms participating in procurement. We choose the level parameter $\bar{p}$ to ensure that the fraction of firms doing procurement equals the fraction of goods bought by the government, m_g, which is the equilibrium equation (F.1) (see Appendix F). This condition requires that $\bar{p}$ equals 0.91.

We identify the parameters η_1, η_2, and η_3 by matching the selection pattern of firms into procurement observed in the data. We proceed as follows. First, we choose η_2 to match persistence in procurement as measured by firms' probability of selling to the government in $t+1$ conditional on doing so in t. In the data, this fraction is equal to 60%, requiring an η_2 equal to 3.58. Second, we choose η_1 and η_3 to match the ex-ante size and MRPK premia of procurement firms, which capture firms' selection into procurement both through productivity s (driven by η_3 and η_1) and net worth a (driven by η_1). In the data, we select firms with no procurement contracts between 2001 and 2004. Then, we classify procurement firms as those firms that obtained at least one contract in 2005. We define the "ex-ante procurement premium" as the relative difference in log employment or log MRPK (measured by value added divided by fixed assets) between procurement and non-procurement firms in

[24] In Appendix D, we provide a possible microfoundation of this probability function together with a microfoundation of the isoelastic public demand for all firms with $d = 1$. We think of $\bar{p}$ as an equilibrium object, whose value we adjust in all model solutions, ensuring that the share of firms selling to the government m_g remains as determined by government policy, see equation (F.1).

Figure 6. Identification of the main parameters



Notes: This figure plots the relationship between percentiles of a given moment (vertical axis) and a specific parameter (horizontal axis). The thick black line refers to the median, and the thinner black lines above and below refer to the 85th and 15th percentiles, respectively. The horizontal dashed blue line represents the targeted value of the moment, while the vertical one represents the actual calibrated parameter value. These lines have been generated by solving the steady-state equilibrium of the model using a large number of combinations of parameters drawn from a 9-dimensional hypercube.

2004 (only exploiting variation across firms within the same 4-digit industry).[25] We measure the procurement premium to be 1.15 for employment and 0.15 for MRPK.

Identification of the main parameters. Our calibration strategy consists of choosing values for the 9 parameters $(\Delta s_0, a_0, \phi_a, \phi_p, \phi_g, Y_g, \eta_1, \eta_2, \eta_3)$ to match 9 moments from the data in a simulated method of moments algorithm.[26] In the previous sub-sections we have discussed the intuition for the identification of each parameter. In this section, we provide a more formal argument about the *global* identification of our parameters. To do so, we proceed as follows. First, we draw a large random sample of parameter combinations based on a 9-dimensional hypercube. Second, for each vector of parameters, we solve for the model's steady-state equilibrium and calculate the relevant moments. Third, we plot how the 15th percentile, the 50th, and the 85th percentile of a given moment change as we move along the values of any given parameter. This figure shows how a particular moment is affected by a specific parameter, letting other parameters vary randomly. The steeper the slope of the relationship between the parameter values and percentiles of the moment, the stronger the identification.[27]

In Figure 6, we show these relationships for some of the most important parameters of our model, ϕ_p, ϕ_g, η_1, and η_2, together with the value of the targeted moment in the data (horizontal dashed blue line). The figure shows that the two estimated coefficients in the leverage regression, equation (15), the procurement size premium, and the persistence in

[25] In practice, we run the following regression: $\log X_{i,2004} = \beta \text{PROC}_{i,2005} + \text{FE}_{j(i)} + u_i$, where $X_{i,2004}$ refers to either employment or MRPK of firm i in 2004, $\text{FE}_{j(i)}$ is an industry fixed effect, and $\text{PROC}_{i,2005}$ is a dummy variable taking value one if firm i has at least one procurement contract in 2005.

[26] In addition to the iteration over these 9 parameters, in an inner loop we iterate over w, $\bar{p}$, and Y_p, to ensure that the equilibrium conditions 4, 5, and 9 are satisfied.

[27] Note that this is different from the matrix with the derivatives of the model-generated moments with respect to the calibrated parameters at some given point in the parameter space, which gives information about *local* identification.

procurement are highly informative about their associated parameters. In Appendix I, we show these relationships for all possible combinations of moments and parameters.

6 The Benchmark Economy

This section describes three relevant dimensions of our benchmark economy: the selection pattern of firms into procurement, the treatment effect of a procurement shock on firm dynamics, and the macroeconomic consequences of procurement.

6.1 Selection

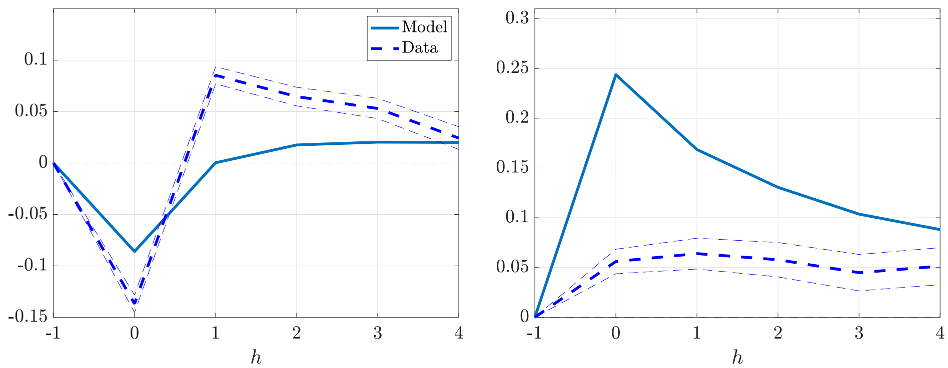
In our calibration, we use η_1, η_2, and η_3 to target three moments related to firm selection into procurement: the "ex-ante procurement premium" in employment of 115 log points, the "ex-ante procurement premium" in marginal revenue product of capital of 15 log points, and the probability for a firm to repeat procurement next year of 60%. To match these patterns of selection, the model generates procurement firms that are ex-ante 43 log points more productive and hold ex-ante 83 log points more net worth. Panel (a) in Table H.1 shows that the model does reasonably well in accounting for a series of non-targeted statistics related to firm selection into procurement, including productivity, net worth, leverage, value added, and persistence beyond the first lag.

6.2 Treatment

In our calibration, we only target one moment related to the treatment effect of becoming a government supplier: the change in leverage resulting from increasing sales to the government for firms likely to be financially constrained. We next examine the model-predicted dynamic responses of private sector sales and total credit after a procurement shock, and compare them to the evidence in Section 3. To do so, we simulate a large unbalanced panel of firms and estimate local projection regressions similar to the ones run in the data.

Sales to the private sector. In Panel (a) of Figure 7, we show the estimated cumulative effects of a procurement shock on firms' sales to the private sector. Similar to the data, the model predicts a "crowding-out" effect on impact and a "crowding-in" effect from $h = 1$ onwards. As discussed in Section 4.3.2, this is because, at impact, constrained firms have to split their scarce collateral between the two sectors. However, the new profits generated from procurement allow the firm to accumulate more net worth over time. This higher level of net worth will ease the firm's financial constraint (lowering λ) and hence allow it to increase output in the private sector in the subsequent periods. While in principle, firms' net worth

Figure 7. Procurement effect on credit and private sales



(a) Sales to the private sector

(b) Credit

Notes: This figure shows the cumulative impact of the estimate of β_1^h (and its associated 10% confidence bands) from regression (1) for different time horizons $h = 0, 1, 2, 3, 4$. We simulate an unbalanced panel of approximately 100,000 firms and 50 periods. Panel (a) shows the results for the case of x being firms' sales to the private sector. Panel (b) shows the results for the case of x being firms' total credit. The lines "Data" show the estimated coefficients using the real data, i.e., the same results as in Panel (b) of Figure 2 and Panel (a) of Figure 1 respectively. The lines "Model" show the results when using all firms in our simulated sample.

and production capacity are higher immediately at $h = 1$, an immediate boost to private sector sales might not materialize because winning access to procurement demand in $h = 0$ significantly increases the chances of doing so again in $h = 1$, due to the calibrated $\eta_2 > 1$. This effect adds a degree of persistence to the crowding-out effect. Quantitatively, our model slightly underpredicts the crowding-out effect at impact (8 log points in the model vs 13 in the data), and shows a slower crowding-in than in the data. However, 4 years after the shock, the accumulated increase in private sector sales due to procurement is the same in the model as in the data (2 log points).

Credit. In Panel (b) of Figure 7, we show the effects of a procurement shock on credit. The first thing to note is that, qualitatively, the model generates a persistent increase in credit as found in the data. At impact, a procurement shock represents a credit demand shock for all firms. This is because firms need to increase their productive capacity and, by the timing assumptions in our model, their net worth is fixed and hence credit must go up. For constrained firms, procurement also represents a credit supply shock because, with $\phi_g > 0$, the earnings arising from the newly awarded procurement demand allow them to increase credit through the earnings-based part of the financial constraint. In the periods after the shock, constrained firms use their higher earnings to accumulate net worth, which allows them to increase credit by relaxing the asset-based part of the financial constraint. Among unconstrained firms, those that do not get a procurement contract next period decrease their capital stock back to their original level, and credit declines for them. Instead, those that keep selling more to the government in the following years keep their higher capital stock and, hence, their higher level of credit. The quantitative increase in credit predicted by the model, however, is substantially larger than measured in the data: 24.5 versus 5.5 log points. In part, this could be because in reality firms' balance sheets are more complex than in our model, and a fraction of new investment may be financed by a reduction of other assets (e.g., cash holdings) or increases in other liabilities (e.g., equity injections). That being said, the cumulative effect of procurement on credit predicted by the model gets closer to the one in the data as we look at a longer horizon.

6.3 The macroeconomy

Aggregating output across firms we get that $Y_p = \text{TFP}_p K_p^\alpha N_p^{1-\alpha}$ and $Y_g = \text{TFP}_g K_g^\alpha N_g^{1-\alpha}$, where K_p refers to aggregate capital used by firms in private sector production (analogously for the other inputs), and TFP_p and TFP_g can be written as weighted averages of firm-level productivities s and distortions λ (see Appendix G for details). We define GDP in units of the private sector good as $Y \equiv Y_p + P_g Y_g$, which implies $Y = \text{TFP} K^\alpha N^{1-\alpha}$, where aggregate capital and labor are given by $K \equiv K_p + K_g$ and $N \equiv N_p + N_g$ and aggregate productivity is defined as $\text{TFP} \equiv \left(\frac{K_p}{K}\right)^\alpha \left(\frac{N_p}{N}\right)^{1-\alpha} \text{TFP}_p + P_g \left(\frac{K_g}{K}\right)^\alpha \left(\frac{N_g}{N}\right)^{1-\alpha} \text{TFP}_g$. Column (1) of Table 3 reports all these objects alongside a variety of other aggregate outcomes for our benchmark economy.

Productivity. We find significant differences in aggregate productivity across the two sectors: TFP in the procurement sector is 34% higher than in the private sector (1.997 vs. 1.492). The difference in sectoral TFP can arise for two reasons: (a) selection of firms into procurement based on s and (b) different misallocation of capital across firms in the two sectors (see Appendix G for details). In our calibrated economy, the main factor is the selection of more productive firms into procurement, which accounts for around 80% of the higher productivity in the procurement sector. But there is also lower misallocation of capital among firms in the public sector, with potential TFP gains of capital reallocation of 4.5% in the public sector, compared to the TFP gains of 9% in the private sector.

The selection of more productive firms into procurement is due to their ability to deliver larger projects. In particular, the average s is around 40% higher among procurement firms. The lower misallocation in the procurement sector arises because firms selling to the government are *ex-post* less financially constrained (the average λ for procurement firms is 11.5 percentage points lower than for non-procurment firms). This is because, despite these firms facing a higher demand, they also have higher levels of net worth (the average $\log(a)$ of procurement firms is 224 log points higher) and enjoy the financial advantage of $\phi_g > \phi_p$.

Table 3. Counterfactuals

	(1) **Bench.** (levels)	(2) **Small** (changes)	(3) $\phi_p = \phi_g$ (changes)	(4) **Low** ϕ_p (changes)	(5) **High** η_3 (changes)	(6) **Size** (changes)	(7) **Age** (changes)
Output							
Y_p	1.971	0.41	0.35	0.60	0.34	0.19	0.60
Y_g	0.372	-9.55	-10.15	-10.12	-8.01	-5.84	-8.29
Real GDP	2.240	-0.79	-0.90	-0.69	-0.66	-0.53	-0.47
Capital and Labor							
K_p	2.977	1.16	1.16	1.53	0.86	0.50	1.80
K_g	0.449	-4.29	-5.12	-5.30	-2.66	-0.75	-9.33
$K_p + K_g$	3.426	0.45	0.35	0.62	0.40	0.34	0.34
N_p	0.880	0.06	0.05	0.07	0.04	0.04	0.08
N_g	0.120	-0.35	-0.31	-0.53	-0.30	-0.16	-0.52
Productivity							
TFP_p	1.492	-0.02	-0.06	0.04	0.03	-0.00	-0.06
TFP_g	1.997	-8.01	-8.37	-8.14	-7.00	-5.50	-4.92
Real TFP	1.486	-0.94	-1.03	-0.88	-0.79	-0.66	-0.59
$\overline{\text{MRPK}}_p$	0.166	-0.75	-0.80	-0.92	-0.51	-0.31	-1.18
$\overline{\text{MRPK}}_g$	0.149	4.48	5.39	5.59	2.73	0.75	10.28
TFP_p gain	0.090	0.19	0.78	-0.33	-0.32	0.04	0.67
TFP_g gain	0.045	11.04	17.06	10.71	9.96	4.28	22.79
w	1.120	0.35	0.30	0.52	0.30	0.15	0.51
Procurement							
P_g/P_p	0.722	10.56	11.28	11.25	8.71	6.20	9.03
Share proc. firms	0.127	0.00	0.00	0.00	0.00	0.13	0.00
P_gY_g/GDP	0.120	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
$\bar{p}$	0.912	-0.42	-0.44	-0.47	-1.92	0.88	-0.41
η_1	0.459	-0.16	-0.16	-0.17	-0.18	0.00	0.00
Selection							
Δ mean $\log(s)$	0.399	-0.14	-0.14	-0.14	-0.12	-0.03	-0.08
Δ mean $\log(a)$	2.244	-0.65	-0.66	-0.70	-0.58	-0.02	-0.93
Δ mean λ	-0.115	0.03	0.03	0.04	0.02	-0.02	0.07

Notes: Column (1) shows the most relevant variables measured in our calibrated benchmark economy. Column (2) reports the % changes in these variables resulting from our main policy experiment (Section 7.1). Columns (3)-(5) report the changes resulting from the same policy experiment under different scenarios (Section 7.2). In particular, they refer to the scenarios discussed in Sections 7.2.1, 7.2.3, and 7.2.2, respectively. Columns (6) and (7) report the changes resulting from the alternative policy experiments of diminishing contract size (Section 7.3.1) and targeting younger firms (Section 7.3.2), respectively. The last 7 rows of columns (2)-(7) report the difference, not the relative change for variables that are already in shares and log-differences, i.e., the share of procurement firms, the share of procurement in GDP, as well as $\bar{p}$ and η_1, and the differences in mean $\log(s)$, $\log(a)$ and λ across procurement and non-procurement firms.

Relative prices. We next look at the price of goods produced by the government relative to private goods, which will be important to understand some of the results of the policy counterfactuals. The relative price can be written as:

$$P_g = \left(\frac{\overline{\text{MRPK}}_g}{\overline{\text{MRPK}}_p}\right)^{\alpha} \frac{\text{TFP}_p}{\text{TFP}_g} \tag{17}$$

where TFP_g and TFP_p are described above and $\overline{\text{MRPK}}_g$ and $\overline{\text{MRPK}}_p$ are the (weighted) sectoral averages of marginal revenue products of capital in each sector, see Appendix G for details. As in standard multi-sector models, the ratio of relative prices is inversely related to sectoral TFPs. However, equation (17) shows that the relative price is also positively related to the ratio of average marginal revenue products in each sector. That is, a suboptimal allocation of capital across sectors generates a wedge that also affects the relative price. Because firms active in procurement are on average less financially constrained, $\overline{\text{MRPK}}_g$ is lower than $\overline{\text{MRPK}}_p$ by around 9%. Together with TFP_g being 34% higher than TFP_p, the relative price of public goods is $P_g/P_p = 0.722$.

7 Procurement Policy

In addition to changing the overall level of spending P_gY_g, governments can change how procurement contracts are allocated to firms. In this section we address the question whether, by changing the allocation procedure of procurement contracts, the government can expand economic activity without incurring any extra expenditure.

7.1 Main policy experiment: "buying small"

Our main policy experiment aims to increase the share of small firms becoming government suppliers. Policies directing purchases explicitly towards smaller firms exist in many countries —like the United States, Indonesia, Dominican Republic, or Peru— and are typically implemented by reserving a portion of government contracts for small firms or by giving them preferential treatment in the bidding process. Promoting small firms' participation in procurement is likely to increase the allocation of contracts to financially constrained firms (as they tend to be smaller despite high productivity) but also to less efficient firms (which are small because of low productivity, not because of lack of credit). Furthermore, our empirical evidence in Section 3 and the quantitative results in Section 6 show that, while procurement contracts help financially constrained firms save their way out of financial constraints, in the short run they generate a crowding-out of private sector sales. Therefore, assessing the economic consequences of such a policy reform requires a quantitative evaluation.

To implement a "buy small" policy we reduce the parameter η_1 in the probability function $\Pr(d' = 1 \mid b, d, s)$ in equation (16). This reduces the extra benefits of making large investments b relative to small ones, making it easier for small firms to compete.[28] At the same time, we adjust Y_g and keep m_g unchanged to ensure that in the new steady-state equilibrium the policy is "expenditure-neutral" ($P_g Y_g$ does not change) and that the number of government suppliers is the same. Finally note that, to satisfy the equilibrium equation (F.1), $\bar{p}$ must also adjust because, in addition to η_1, in the new steady-state equilibrium the policy function $b(s, a, d)$ and the equilibrium distribution Γ also change.

To choose the actual value for η_1 we target a 10 percentage points increase in the share of government suppliers that are small (defined as firms with $n_p + n_g \leq 20$).[29] In our economy, the share of government suppliers that are small is 67%, so we reduce η_1 such that this share increases to 77%.[30] We present the main results from this exercise in column (2) of Table 3, which shows the relative changes of some relevant variables compared to their counterparts in the benchmark economy.

Aggregate output. First, we find that the reform increases output Y_p in the private sector by 0.41%. As a reference, this output increase is 4.5% of the output gains that would be generated by eliminating all the misallocation of capital in the private sector. Therefore, targeting small firms is not a particularly effective way to lessen misallocation in the private sector due to financial frictions. At the same time, we find that the output in the government sector, Y_g, falls by 9.55%. The combination of the changes in Y_p and Y_g leads to a reduction in GDP of 0.79% in real terms.[31]

[28]In terms of the microfoundation of the government demand in Appendix D, the change in η_1 represents a change in the mapping between the firm's chosen application quality and the required investment to achieve it. This mapping can be changed by the government by modifying the trade-offs between different aspects of a proposal or by making it easier for firms to reach an acceptable level of quality albeit more costly to guarantee success.

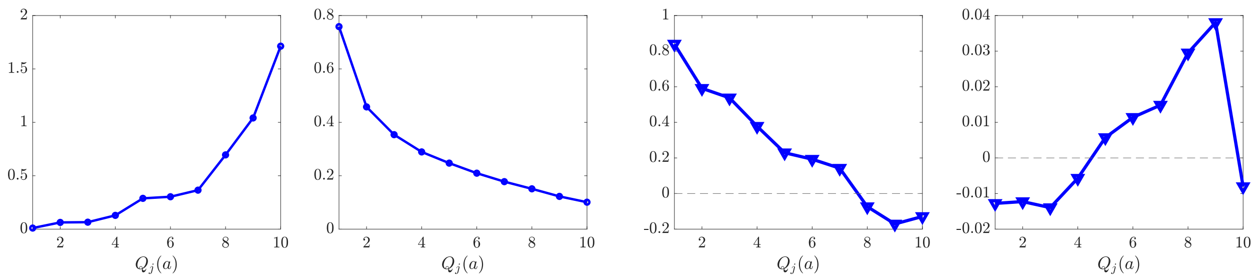
[29]Average firm size in our model is given by the ratio of the measure of the representative household to the measure of firms, which are both normalized to 1. Hence, we map the model labor units n to the empirically observed number of workers in Spanish firms via the percentiles of the firm size distribution. That is, we identify the percentile that corresponds to firms with 20 employees in the population of Spanish firms, and label the employment level of the firm at the corresponding employment percentile in the benchmark model population as corresponding to 20 employees.

[30]In the data the share of government suppliers with 20 or fewer employees is 76% not too far from the 67% in the model. This is computed using information for the universe of contracts and taking the average for the years 2019, 2021, and 2022, when the coverage is already high and firms' participation is not directly affected by COVID.

[31]Recall that aggregate GDP (in units of the private good) is defined as $\text{GDP} \equiv Y_p + P_g Y_g$. To compute changes in GDP across two different economies (e.g., the baseline versus the counterfactual), we mimic national accounting practices in constructing quantity indices and focus on the relative change implied by:

$$\frac{\text{GDP}^1}{\text{GDP}^0} = \frac{Y_p^1 + P_g^0 Y_g^1}{Y_p^0 + P_g^0 Y_g^0} \tag{18}$$

Figure 8. Understanding the effects of the procurement policy



(a) $p_g y_g$ (baseline) **(b)** MRPK_p (baseline) **(c)** $p_g y_g$ (log change) **(d)** k_p (log change)

Notes: Panels (a) and (b) show the group-specific average levels of $p_g y_g$ and MRPK_p in the baseline economy for firms of the same productivity (s_4) classified into groups based on the deciles of the conditional net worth distribution among these firms. Panels (c) and (d) plot the log differences of these groups' mean $p_g y_g$ and k_p levels between the "buy small" counterfactual economy and the baseline economy.

The change in Y_p. We decompose the change in private sector output as follows:

$$\underbrace{\frac{\Delta Y_p}{Y_p}}_{+0.41\%} = \underbrace{\frac{\Delta \text{TFP}_p}{\text{TFP}_p}}_{-0.02\%} + \frac{1}{3}\underbrace{\frac{\Delta K_p}{K_p}}_{+1.16\%} + \frac{2}{3}\underbrace{\frac{\Delta N_p}{N_p}}_{+0.06\%} \tag{19}$$

The increase in Y_p is almost entirely explained by the 1.16% increase in capital K_p. The amount of labor N_p allocated to the production of the private sector good also increases, but by a much smaller amount (0.06%), while the aggregate productity TFP_p barely changes (-0.02%).

We illustrate some of the mechanisms driving these results in Figure 8, which is constructed using the distribution of firms in the model's steady state equilibrium. Along the x-axis, we classify firms with a fixed and relatively high productivity draw (s_4) into deciles of the distribution of net worth conditional on the given productivity shock. Panels (a) and (b) plot the mean levels of $p_g y_g$ and MRPK_p for each group in the baseline economy. These two panels show how firms with more net worth (a) sell more to the government (because they are more likely to participate in procurement and they sell relatively more when doing so) and (b) are less financially constrained. Panels (c) and (d) plot the log differences in the groups' $p_g y_g$ and k_p means between the counterfactual "buy small" economy and the baseline economy.

where X^1 and X^0 refer to the variable's value in the counterfactual and baseline economy, respectively. By fixing the relative price P_g to its initial value, this change in GDP allows us to illustrate the impact of reforms on private production Y_p and the provision of public goods Y_g. One can think of the implied change in GDP as a particular discrete time approximation to a Divisia index. In particular, we can rewrite (18) as $\left[\frac{\text{GDP}^1}{\text{GDP}^0} - 1\right] = \left(\frac{Y_p^0}{\text{GDP}^0}\right) \times \left[\frac{Y_p^1}{Y_p^0} - 1\right] + \left(\frac{P_g^0 Y_g^0}{\text{GDP}^0}\right) \times \left[\frac{Y_g^1}{Y_g^0} - 1\right]$. That is, we effectively use a quantity index with weights equal to each sector's GDP share in the benchmark economy.

The first thing to notice is that, as a result of the size-dependent policy, firms with relatively lower net worth increase their sales to the government compared to the benchmark economy (for instance, for the case of s_4, this is so for the bottom 70 percent of the wealth distribution). This increases their profits, which helps them accumulate net worth and save their way out of financial constraints, which leads to the increase in K_p.

This positive direct effect on capital accumulation, however, is partly offset by three unintended consequences of the reform. First, the long-run benefit of net worth accumulation is preceded by the short-run crowding-out effect of production y_p for the private sector, as illustrated by Panel (f) in Figure 7 above. Therefore, at any given point in time, there are some firms with more capital k_p than in the benchmark economy (those firms that got a procurement contract some periods ago and had time to accumulate net worth) and some other firms with less capital k_p than in the benchmark economy (those firms that just got a procurement contract and decreased their private sector activity). In particular, for shock s_4, Panel (d) in Figure 8 shows how the bottom 40% of firms with lower net worth operate with lower private sector capital than in the benchmark economy.

The second unintended consequence is given by the reduction in the probability of obtaining a contract for the largest firms. One of the reasons why firms accumulate net worth is the option value of obtaining a public procurement contract in the future. Obtaining a contract represents a demand shock in response to which firms want to expand their capital stock. This expectation increases the returns to net worth accumulation (see equation (14)), incentivizing net worth accumulation even among relatively large firms. The expected return to net worth accumulation diminishes when it is less likely to obtain a procurement contract, inducing a decline in these firms' accumulation of wealth in the counterfactual economy.

Finally, the third unintended consequence is given by the general equilibrium effects of the "buy small" policy. Since the policy has an overall expansionary effect on the production of constrained firms, their demand for labor increases, resulting in a higher equilibrium wage w and hence a smaller optimal size for unconstrained firms. The effects of the latter two unintended consequences can be seen in the fall of k_p by the top decile of firms in Panel (d) of Figure 8.

The previous discussion also helps in understanding the small effect of the reform on TFP_p. On the one hand, the fact that the constrained firms in wealth deciles 5 through 9 end up having relatively higher levels of k_p in the counterfactual economy (see panel (d) of Figure 8) increases TFP_p, since these firms have a higher MRPK (e.g. they are more constrained) than many other firms in the benchmark economy. On the other hand, the crowding out effect experienced on impact by procurement firms operates precisely at the time when the MRPK of these firms is the highest, which works to decrease TFP_p in the counterfactual economy. Our results show that the latter effect slightly dominates, explaining the small negative effect on TFP_p.

The change in Y_g. We decompose the change in the procurement sector output as follows:

$$\underbrace{\frac{\Delta Y_g}{Y_g}}_{-9.55\%} = \underbrace{\frac{\Delta \text{TFP}_g}{\text{TFP}_g}}_{-8.01\%} + \frac{1}{3}\underbrace{\frac{\Delta K_g}{K_g}}_{-4.29\%} + \frac{2}{3}\underbrace{\frac{\Delta N_g}{N_g}}_{-0.35\%}$$

The reduction in Y_g is mainly driven by a reduction of aggregate productivity in the public sector (-8.01%) but also by the reduction in capital (-4.29%). The reduction in TFP_g is explained by a worsening in the selection of firms into procurement. In particular, as a result of the policy targeting smaller firms, procurement firms in this counterfactual economy have a lower productivity s (14 log points less), lower net worth a (65 log points less), and a larger dispersion in TFPR_g than in the benchmark. As a result, new procurement firms are relatively less productive and relatively more financially constrained (as reflected by larger λ). This leads to the decline in TFP_g due to both lower s (lower first-best productivity) and more misallocation. At the same time, because the new procurement firms have lower productivity and net worth, they also use less capital K_g and slightly less labor N_g.

A reduction in the efficiency of public goods provision. Given that the policy exercise is "expenditure neutral" ($P_g Y_g$ constant) the fall in Y_g happens because of the increase in the equilibrium average price P_g that the government needs to pay for procurement. As explained in Section 6.3, the relative price of the public good depends on the sectoral wedge times the inverse of relative sectoral TFPs, see equation (17). The first thing to note is that the ratio $\text{TFP}_g/\text{TFP}_p$ decreases substantially in the counterfactual economy as discussed above. Moreover, because the counterfactual procurement system also selects firms that have relatively lower net worth a, and are thus more financially constrained, all else equal, it leads to an increase in the relative sectoral wedge, as captured by $\overline{\text{MRPK}}_g/\overline{\text{MRPK}}_p$. Both of these forces raise P_g. This is an important result that highlights a key tradeoff for the government: targeting smaller firms may increase output in the private sector, but it also implies that government ends up buying from firms that are more financially constrained and fundamentally less productive, which increases the cost of public procurement.

7.2 Robustness: "buying small" in alternative economies

Next, we implement the same policy counterfactual in economies that are different from our baseline economy along various dimensions. The goal is to highlight how the same policy may have different quantitative implications depending on the characteristics of the economic environment. The different scenarios of alternative economies we consider are perturbations of the baseline economy, changing the values of certain key parameters of interest. In each case, we solve for the steady-state equilibrium in the new "reference" economy, adjusting Y_g and keeping m_g constant so that the importance of the public procurement sector, measured by the share of procurement expenditures in GDP and the fraction of procurement firms, remains unchanged compared to the baseline calibration. This also requires adjusting $\bar{p}$ such that the equilibrium equation (F.1) holds. Next, we repeat our "buy small" policy experiment by lowering η_1 by whatever is needed so that the share of procurement suppliers with ≤ 20 workers increases by 10 percentage points relative to the corresponding reference economy, also again adjusting $\bar{p}$ and Y_g exactly as explained at the beginning of Section 7.1.

7.2.1 Symmetry of earnings-based constraints ($\phi_g = \phi_p$)

Our firm-level panel regressions show that leverage increases with firm earnings, and more so when earnings come from the public sector (see Table 2). Through the lens of our model, this evidence implies that earnings from the public sector can be pledged to obtain credit to a larger extent than earnings from the private sector ($\phi_g > \phi_p$). In this section we assess the quantitative role played by this asymmetry. To do so, we analyze the macroeconomic effects of the policy reform in a world where $\phi_g = \phi_p$. In particular, we decrease ϕ_g such that $\phi_g = \phi_p = 0.52$ (the value of ϕ_p in our baseline) and run our "buy small" policy counterfactual described above.

Column (3) in Table 3 reports the results. We find that the "buy small" policy leads to relatively worse macroeconomic outcomes when government contracts are equally pledgeable as sales to the private sector. In particular, output in the private sector Y_p would increase less (0.35% instead of 0.41%) than in the exercise with the benchmark economy. This result is mostly driven by a larger reduction in TFP_p under this scenario (0.06% fall versus 0.02%). The reason for the larger drop in TFP_p is that, when allocating procurement to more financially constrained firms, the short-run crowding out of private sector production is stronger when $\phi_g = \phi_p$, as there is no extra financing through public earnings to alleviate the problem of scarce collateral (see Proposition 11 in Appendix E). Moreover, the lower ϕ_g means that, with the "buy small" policy, procurement firms are more financially constrained than with the benchmark ϕ_g, inducing larger efficiency losses in the procurement sector. All this results in a larger drop of Y_g (10.15% drop instead of 9.55%). Overall, there is a larger loss of GDP under this scenario: -0.90% versus -0.79%.

7.2.2 Stronger financial frictions ($\phi_p = 0$)

The motivation for the "buy small" policies is the fact that, in a context of financial frictions, reallocating procurement expenditure towards more financially constrained firms may alleviate the negative effects of those frictions. So, in principle, the impact of our "buy small" policy could be larger in an economy where firms are more financially constrained. To explore this, we implement our "buy small" policy in a scenario where firms cannot borrow against their revenues in the private sector, i.e., $\phi_p = 0$.

We report the results in Column (4) of Table 3. Our main finding is that the fall in GDP would be smaller than in the baseline exercise, 0.69% as compared to 0.79%. This result is explained by a larger increase in K_p (1.53% versus 1.16%), which leads to a larger increase in Y_p (0.60% versus 0.41%), and despite the larger decline in the efficiency of the provision of the public good, which leads to a higher fall in Y_g (10.12% vs 9.55%). The larger increase in K_p is consistent with the idea that targeting procurement contracts to small firms helps those firms grow toward optimal size and that the policy is more effective whenever financial frictions are more binding. In contrast, the larger inefficiency in the provision of public good (whose price increases by 11.25% as compared to 10.56% in the benchmark exercise) comes from a worse selection of firms into procurement, as the lack of credit excludes some productive firms that cannot finance either the project investments or the actual production in case of getting a project.

7.2.3 Better screening of firm productivity (higher η_3)

As shown above, an unintended consequence of the "buy small" policy is the fact that governments may end up buying from relatively unproductive firms, hence suffering an increase in the overall cost of procurement. The parameter η_3 in the probability function captures the government's ability to select procurement firms based on their productivity s. We next implement the same "buy small' policy as before, but assuming we live in an economy where the government has a higher ability to screen firm-level productivity, and we do so by considering $\eta_3 = 0.5$ (versus $\eta_3 = 0.0$ in our baseline calibration).

We report the results in Column (5) of Table 3. We find that the negative effects of introducing our "buy small" policy would be smaller, with a reduction in GDP of about 0.66% versus 0.79% in the benchmark. The main reason for the smaller reduction in GDP is the notably lower reduction in Y_g. In particular, TFP_g falls by 7 percent as compared to the 8 percent decline in the benchmark exercise. The reason is that when the procurement allocation system puts more emphasis on firms' fundamental productivity, the "buy small" policy generates a weaker selection of low productivity firms into procurement (the average productivity of procurement firms falls by 12 log points instead of 14).

7.3 Alternative policy experiments

Our policy experiment in the previous section is only one possible way in which governments could promote the participation of small firms in procurement. In this Section, we explore two other possibilities.

7.3.1 Smaller contracts

One alternative policy would be to decrease the size of procurement contracts by splitting big contracts into smaller ones, which is often mentioned in the European Commission's agenda for public procurement regulation. Given that the contract size is closer to their production capacity, this may induce smaller firms to self-select into procurement.

Motivated by this, our second policy experiment consists of reducing the average size of contracts ($P_g Y_g / m_g$) while keeping the same level of public expenditure $P_g Y_g$. To implement this policy, we need to increase m_g and adjust Y_g until we obtain a 10 percentage point increase in the share of government suppliers that are small, as we did in the main policy exercise of Section 7.1. As always, we need to adjust $\bar{p}$ for the equilibrium equation (F.1) to hold. This results in an increase in the share of firms selling to the government from 12.7% to around 26% and a decline in average contract size of around 1/2.[32]

We report our results in Column (6) in Table 3. We find that contract size reduction leads to less negative aggregate effects than the "buy small" policy, but it still generates output losses: a 0.53% GDP drop versus 0.79% for the "buy small" policy. This is explained by a smaller reduction in the provision of the public good (Y_g drops by 5.85% instead of 9.55%), which offsets a weaker increase in private sector output (Y_p increases by 0.19% instead of 0.41%).

A key aspect of this policy is that it is less effective than the "buy small" policy in shifting procurement demand towards very small firms. To have a sense on how the two policy counterfactuals differ in this dimension, we compute the share of government suppliers that are "micro firms" (i.e., firms with ≤ 10 employees) in our baseline economy and in the two counterfactual economies. In our baseline economy, around 44% of government suppliers are "micro firms". This number increases to around 58% in the "buy small" policy exercise but to only 53% in the alternative one. The implications of this policy reaching out to very

[32]The European Commission is not explicit about by how much governments should decrease the size of the contracts: "[...] Such division could be done on a quantitative basis, making the size of the individual contracts better correspond to the capacity of SMEs [...]." (see the Public Sector Directive 2014/24/EU for details.)

small firms less strongly are as follows. On the one hand, because the most constrained firms tend to be the smallest, the reduction in contract size generates smaller positive effects on K_p and Y_p resulting from the strengthening of the self-financing channel. On the other hand, selection into procurement along s and a does not worsen as much as in the "buy small" policy counterfactual (see last rows of column (6) in Table 3). Both $\overline{\text{MRPK}}_g$ and TFP_g decrease significantly less than in the main counterfactual (0.75 versus 4.48% in the case of the former and 8 versus 5.5% in the case of the latter). In that respect, the efficiency cost in the production of public goods resulting from the alternative policy is lower.

7.3.2 Targeting young firms

As a final experiment, we consider the possibility that the government targets younger firms. The interest of this experiment is that, by targeting the pool of young instead of small firms, the policy may better reach highly productive, financially constrained firms.

To implement this policy, we need to add age to the state space of the dynamic optimization problem of entrepreneurs in Section 4.3.3. We do so by considering stochastic aging with two categories, *young* and *old*. Firms are born young, and we choose the constant probability of aging to be 0.2 such that the average duration of the young age is 5 years. The role of age in the model will only be to increase the probability of procurement, so in the benchmark economy, nothing changes by adding age. Next, in the policy counterfactual we add a term η_4 for young firms (analogously to how η_2 operates for current suppliers) in the probability function (16), and we choose $\eta_4 > 1$ such that the share of government suppliers that are small increases by 10 percentage points as in the previous policy exercises.

Overall, we find that this policy would be better than targeting small firms or reducing the size of contracts, see Column (7) in Table 3: targeting younger firms generates a GDP decline of 0.47%, which is lower than the 0.79% and 0.53% GDP declines in the two size-dependent policy exercises.

Looking at the different components of GDP, we see that private sector output Y_p increases more than in the "buy small" or the contract size reduction policies (0.60% as compared to 0.41% and 0.19% respectively). This is mainly driven by the increase in K_p, which happens as a result of better reaching the population of interest: more productive and more constrained firms. This also helps generate a smaller decline in TFP_g (4.92% loss as compared to the 8.01% and 5.50% losses in the size dependent policies) but a larger increase in $\overline{\text{MRPK}}_g$ (10.28% increase as compared to the 4.48% and 0.75% increases in the size dependent policies). Overall, and as a result of these changes, the relative price of the public good increases less than in the "buy small" policy (9.03% vs 10.56%) but more than in the policy reducing contract sizes (9.03% vs 6.20%), which means that the final effect on public sector output Y_g is better than in the "buy small" policy (8.29% loss vs 9.55% loss) but worse than in the policy reducing contract size (8.29% loss vs 5.84% loss).

8 Conclusion

This paper quantifies the macroeconomic impact of changes in the public procurement allocation system. We find that while financially constrained firms benefit in the medium run —using additional procurement earnings to boost internal savings— there are also significant unintended negative effects that lead to overall output losses. Specifically, allocating contracts to small firms results in a short-run decline in production by financially constrained firms, reduced wealth accumulation among large firms, and poorer supplier selection by the government. The overall macroeconomic impact of such policies critically depends on the nature and severity of financial frictions, as well as the specific design of the reform.

Our study contributes to a broader, and still underdeveloped, research agenda on the macroeconomic effects of government procurement. We focus here on the long-run consequences of expenditure-neutral changes in procurement allocation, leaving short-run dynamics and implications for fiscal policy effectiveness for future work. Advancing this agenda promises important insights for policymaking.

References

Aiyagari, S. R., "Uninsured Idiosyncratic Risk, and Aggregate Saving," *Quarterly Journal of Economics*, 1994, *109* (3), 659–684.

Alfaro-Ureña, Alonso, Isabela Manelici, and José P. Vasquez, "The effects of joining multinational supply chains: New evidence from Firm-to-firm Linkages," *Quarterly Journal of Economics*, 2022, *137* (3), 1495–1552.

Almunia, Miguel, David López-Rodríguez, and Enrique Moral-Benito, "Evaluating the Macro-Representativeness of a Firm-Level Database: An Application for the Spanish Economy," 2018. Banco de España Documento Ocasional 1802.

_ , Pol Antràs, David López-Rodríguez, and Eduardo Morales, "Venting Out: Exports during a Domestic Slump," *American Economic Review*, 2021, *111* (1), 3611–3662.

Anderson, Simon P., Andre de Palma, and Jacques-Francois Thisse, *Discrete Choice Theory of Product Differentiation*, The MIT Press, 1992.

Azariadis, Costas, Leo Kaas, and Yi Wen, "Self-Fulfilling Credit Cycles," *Review of Economic Studies*, 2016, *83*, 1364–1405.

Bandiera, Oriana, Andrea Prat, and Tommaso Valletti, "Active and Passive Waste in Government Spending: Evidence from a Policy Experiment," *American Economic Review*, September 2009, *99* (4), 1278–1308.

Bau, Natalie and Adrien Matray, "Misallocation and Capital Market Integration: Evidence From India," *Econometrica*, 2023, *91* (1), 67–106.

Berthou, Antoine, Jong Hyun Chung, Kalina Manova, and Charlotte Sandoz, "Trade, Productivity, and (Mis)allocation," *Working Paper*, 2019.

Blackford, R. Corinne, "An Overview of Small Business Contracting," May 2024. Congressional Research Service R45576.

Boar, Corina, Denis Gorea, and Virgiliu Midrigan, "Why Are Returns to Private Business Wealth So Dispersed?," 2025. Mimeo New York University.

Brooks, Wyatt and Alessandro Dovis, "Credit market frictions and trade liberalizations," *Journal of Monetary Economics*, 2020, *11*, 32–47.

Buera, Francisco and Benjamin Moll, "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity," *American Economic Journal: Macroeconomics*, 2015, *7* (3), 1–42.

—, Joe Kaboski, and Yongseok Shin, "Entrepreneurship and Financial Frictions: A Macro-Development Perspective," *Annual Review of Economics*, 2015, *7*, 409–436.

—, Joseph Kaboski, and Yongseok Shin, "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 2011, *101*, 1964–2002.

Caglio, Cecilia, R. Matthew Darst, and Şebnem Kalemli-Özcan, "Risk-Taking and Monetary Policy Transmission: Evidence from Loans to SMEs and Large Firms," 2021. NBER Working Paper 28685.

Cappelletti, Matilde, Leonardo Giuffrida, and Gabriele Rovigatti, "Procuring Survival," *The Journal of Industrial Economics*, 2024, *72* (4), 1451–1506.

Catherine, Sylvain, Thomas Chaney, Zongbo Huang, David Sraer, and David Thesmar, "Quantifying Reduced-Form Evidence on Collateral Constraints," *Journal of Finance*, 2022, *77* (4), 2143–81.

Chaney, Thomas, "Liquidity Constrained Exporters," *Journal of Economic Dynamics and Control*, 2016, *72*, 141–54.

Cox, Lydia, Gernot J. Müller, Ernesto Pastén, Raphael Schoenle, and Michael Weber, "Big G," *Journal of Political Economy*, 2024, *132* (10), 3260–3297.

David, Joel M. and Venky Venkateswaran, "The Sources of Capital Misallocation," *American Economic Review*, 2019, *109* (7), 2531–2567.

— , Lukas Schmid, and David Zeke, “Risk-adjusted Capital Allocation and Misallocation,” *Journal of Financial Economics*, 2022, *145*, 684–705.

Davis, Steven J., John Haltiwanger, Ron Jarmin, and Javier Miranda, “Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms,” *NBER Macroeconomics Annual*, 2006, (107-180).

Decarolis, Francesco, “Comparing Public Procurement Auctions,” *International Economic Review*, 2018, *59* (2), 391–419.

— , Leonardo Giuffrida, Elisabetta Iossa, Vinzenzo Mollisi, and Giancarlo Spagnolo, “Bureaucratic Competence and Procurement Outcomes,” *The Journal of Law, Economics, and Organization*, 2020, *36* (3), 537–597.

Delgado, Javier, Vicente Salas, and Jesus Saurina, “Joint Size and Ownership Specialization in Bank Lending,” *Journal of Banking and Finance*, 2007, *12* (31), 3563–83.

Drechsel, Thomas, “Earning-Based Borrowing Constraints and Macroeconomic Fluctuations,” *American Economic Journal: Macroeconomics*, April 2023, *15* (2), 1–34.

— and Seho Kim, “Macroprudential Policy with Earning-Based Borrowing Constraints,” *Journal of Monetary Economics*, 2024, *147*, 103595.

Erosa, Andrés and Beatriz González, “Taxation and the Life Cycle of Firms,” *Journal of Monetary Economics*, 2019, *105* (4), 114–130.

Ferraz, Claudio, Frederico Finan, and Dimitri Szerman, “Procuring Firm Growth: The Effects of Government Purchases on Firm Dynamics,” 2016. Mimeo, PUC-RIO and U.C. Berkeley.

Gabriel, Ricardo Duque, “The Credit Channel of Public Procurement,” *Journal of Monetary Economics*, 2024, *147* (103601).

García-Santana, Manuel and Josep Pijoan-Mas, “The Reservation Laws in India and the Misallocation of Production Factors,” *Journal of Monetary Economics*, 2014, *66* (193-209).

García-Santana, Manuel and Marta Santamaría, “Understanding Home Bias in Procurement: Evidence from National and Subnational governments,” *Working Paper*, 2022.

Garicano, Luis, L. Lelarge, and J. Van Reenen, “Firm Size Distortions and the Productivity Distribution: Evidence from France,” *American Economic Review*, 2016, *106* (11).

Gavia, Sergio, Alfredo Maldonado, and Antonio Marcelo, “The Banco de España in-house credit assessment system,” *Financial Stability Review, Banco de España*, 2020, *38*, 95–122.

Gopinath, Gita, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez, “Capital Allocation and Productivity in South Europe,” *Quarterly Journal of Economics*, 2017, *132* (4), 1915–1967.

Guner, Nezih, Gustavo Ventura, and Yi Xu, "Macroeconomic Implications of Size-dependent Policies," *Review of Economic Dynamics*, 2008, *11*, 721–744.

Guvenen, Fatih, Burhanettin Kuruşçu, Gueorgui Kambourov, Sergio Ocampo, and Daphne Chen, "Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation," *Quarterly Journal of Economics*, 2023, *138* (2), 835–894.

Hebous, Shafik and Tom Zimmermann, "Can government demand stimulate private investment? Evidence from U.S. federal procurement," *Journal of Monetary Economics*, 2021, *118* (1), 178–194.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.

Itskhoki, Oleg and Benjamin Moll, "Optimal Development Policies With Financial Frictions," *Econometrica*, 2019, *87* (1), 139–173.

Ivashina, Victoria, Luc Laeven, and Enrique Moral-Benito, "Loan types and the bank lending channel," *Journal of Monetary Economics*, 2022, *126*, 171–187.

Jordà, Oscar, "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 2005, *95* (1), 161–182.

Kohn, David, Fernando Leibovici, and Michal Szkup, "Financial Frictions and New Exporter Dynamics," *International Economic Review*, 2016, *57* (2).

Leary, Mark T. and Roni Michaely, "Determinants of dividend smoothing: Empirical evidence," *The Review of Financial Studies*, 2011, *24* (10), 3197–3249.

Lee, Munseob, "Government Purchases and Firm Growth," *Working Paper*, 2021.

Leibovici, Fernando, "Financial Development and International Trade," *Journal of Political Economy*, 2021, *129* (12).

Li, Huiyu, "Leverage and Productivity," *Journal of Development Economics*, 2022, *154*.

Lotti, Clarissa, Arieda Muço, Giancarlo Spagnolo, and Tommaso Valletti, "Indirect Savings from Public Procurement Centralization," *American Economic Journal: Economic Policy*, Forthcoming.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

Midrigan, Virgiliu and Daniel Xu, "Finance and Misallocation: Evidence from Plant Level Data," *American Economic Review*, 2014, *2* (104), 422–58.

Moll, Benjamin, "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, 2014, *104* (10), 3186–3221.

Ruiz-García, Juan Carlos, "Financial Frictions, Firm Dynamics and the Aggregate Economy: Insights from Richer Productivity Processes," *Working Paper*, 2020.

Santos, Tano, "Antes del diluvio: The Spanish banking system in the first decade of the euro," 2017. In E. L. Glaeser, T. Santos, and E. G. Weyl (Eds.), After the Flood: How the Great Recession Changed Economic Thought, University of Chicago Press.

Song, S., S. Storesletten, and F. Zilibotti, "Growing like China," *American Economic Review*, 2011, *101*, 196–233.

Trybus, Martin, "The Promotion of Small and Medium Sized Enterprises in Public Procurement: A Strategic Objective of the New Public Sector Directive," *SSN Working Paper*, 2014.

On-line Appendix

Buy Big or Buy Small? Procurement Policies, Firms' Financing, and the Macroeconomy

Julian di Giovanni   Manuel García-Santana   Priit Jeenas
Enrique Moral-Benito   Josep Pijoan-Mas

June 10, 2025

Table of Contents

A Data details

This section provides details on the main and complementary datasets used in the paper.

A.1 Main datasets

The main datasets we use are the following:

1. **Public procurement data scraped from the *Agencia Estatal Boletín Oficial del Estado*(BOE)**. Time coverage: 2000-2013. Relevant variables used (at the annual frequency):

    (a) Name of the firm to which the contract has been awarded.

    (b) Date when the contract has been awarded.

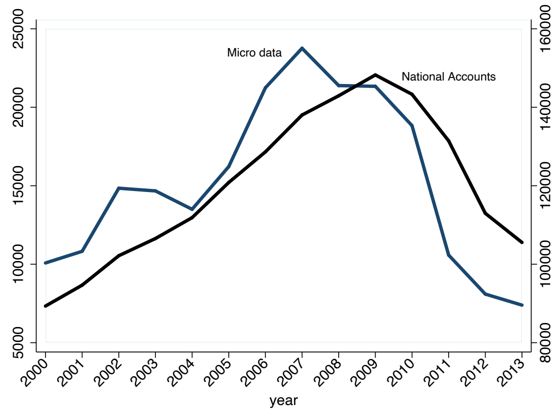
    (c) Value of the contract awarded.

    (d) Type of procedure used to award the contract.

According to Spanish law, all procurement contracts above a certain threshold awarded by public institutions must be published in official bulletins.[33] If the contract is awarded by the central government, the information on this contract must be published in the *Agencia Estatal Boletín Oficial del Estado* (BOE), which is the official bulletin of the central government of Spain. In contrast, if the entity that awards the contract is a regional government or a municipality, the information about this contract can alternatively be published at their respective regional or local bulletin. We construct a novel dataset on Spanish public procurement contracts by scraping the BOE website over the 2000-2013 period. Each contract provides information on the type of contract (kind of good or service provided), the awarding institution, the type of procedure used to allocate the contract, and the firm(s) that won the contract. In total, we scraped more than 150,000 projects over 2000-2013, which we assign to the month that the project was awarded. Of these, 130,633 projects have a value assigned to them that we were able to recover. The sum of all these projects totals around 220 billion euros. On average, our micro data account for around 13% of total public procurement as measured in National Accounts. Despite the level differences, our micro data are able to capture the overall evolution of public procurement over time, which increased from 9.9 to 13.8 percent between 2000 and 2009 and decreased from 13.8 to 10.0 percent between 2010 and 2013; see Figure A.1.

[33] The thresholds above which the contract must be advertised in official bulletins depend on the type of contract. In the case of supplies and services, for example, the threshold is 60,000 euros.

Figure A.1. Evolution of Public Procurement in Spain, 2000-13



Notes: This figure shows the evolution of public procurement in Spain over 2000-13 (in millions of Euros). The blue line ("Micro data", left y-axis) is computed by aggregating the individual projects scraped from the BOE, https://www.boe.es/. The black line ("National accounts", right y-axis) is measured from Spanish national accounts. To calculate government procurement in national accounts, we follow the OECD definition of government procurement and compute it as *government intermediate consumption + government gross fixed capital formation + social transfers in kind.*

2. **Firms' balance sheet and income statement data from the Bank of Spain**. Time coverage: 2000-2013. Relevant variables used (at the annual frequency):

   (a) Firm's total sales.

   (b) Firm's value added.

   (c) Firm's fixed assets.

   (d) Firm's number of employees.

   (e) Firm's age.

   (f) Firm's sector of activity (4-digit NACE Rev. 2 code).

We use the balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2013, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. For each firm and year, this dataset includes information on the firm's name, fiscal identifier, sector of activity (4-digit NACE Rev. 2 code), age, net operating revenue, material expenditures, number of employees, labor expenditures, total fixed assets, and total assets. The final sample covers around 85-90% of non-financial firms for all size categories in terms of both turnover and number of employees. See Almunia et al. (2018) for a detailed description of this dataset and its coverage.

3. **Credit registry from the Bank of Spain**. Time coverage: 2000-2013. Relevant variables used (at the annual frequency):

Table A.1. Descriptive evidence from the final merged dataset, year 2006

	mean		25th pctile		50th pctile		75th pctile	
	Proc	NoProc	Proc	NoProc	Proc	NoProc	Proc	NoProc
Age	20.42	10.95	12.00	5.00	17.00	10.00	24.00	15.00
Employment	73.56	12.75	16.00	3.00	45.00	6.00	155.0	12.00
Sales	8.96	1.19	1.14	0.10	4.22	0.28	16.89	0.86
Procurement/Sales	0.20	0.00	0.01	0.00	0.03	0.00	0.10	0.00
Fixed Assets	3.80	0.85	0.21	0.03	0.82	0.14	3.58	0.50
Credit	2.51	0.57	0.11	0.03	0.48	0.08	2.32	0.30
Coll. Credit (share)	0.14	0.29	0.00	0.00	0.00	0.00	0.14	0.74

Notes: This table presents summary statistics from our merged dataset for the year 2006, separately for firms with at least one procurement contract (n = 2,411) vs. the rest of the firms (n = 406,261). The variable Employment measures the number of full-time workers employed by the firm; the variable Sales is just firm's revenue measured in millions of euro; Procurement/Sales measures the value of all the procurement projects awarded to a firm in a given year divided by total revenue in that year. Assets measures the value of fixed assets; Credit measures the value of all firm's outstanding loans in millions of euro; Coll. Credit (share) is the share of Credit collateralized against firm's assets; Def. Credit (share) is the share of defaulted credit over total Credit; age measures the age of the firm. We winsorize the 1% tails of all variables.

(a) Firm's outstanding total *commercial and industrial (C&I)* loans.

(b) Firm's outstanding *commercial and industrial (C&I)* loans collateralized by hard assets.

(c) Firm's outstanding *commercial and industrial (C&I)* loans not collateralized by hard assets.

(d) Firm's loan applications.

The *Central de Información de Riesgos* (CIR) is maintained by the Banco de España in its role as primary banking supervisory agency, and contains detailed monthly information on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain since 1984. Given the low reporting threshold, virtually all firms with outstanding bank debt appear in the CIR. There are three main types of loans classified by "class": *commercial and industrial loans (C&I)*, *trade finance*, and *leasing*. Throughout our paper, we only consider the first, and hence abstract from any change in firms' overall credit coming from changes in trade credit. We believe that considering *commercial and industrial loans (C&I)* only is appropriate for two reasons. First, they account for more than 90% of the overall loan value in Spain. And second, we can decompose them into the two types of borrowing we have in our model. In particular, for each loan within that category, we have information on whether the loan is collateralized by hard assets (e.g., real state, deposits) or not. We classify the former ones as "collateral credit" and the latter ones as "non-collateral credit". Ivashina et al. (2022) provide more details on the different loan types and their characteristics.

Loan applications. Besides the information on outstanding loans, we also have information about loan applications at the firm-bank level. The construction of this dataset is as follows. Spanish banks can request information about a firm whenever this firm "seriously" approaches them to obtain credit.[34] Because banks already have information about the firms with which they have a credit relationship, banks only request information on firms that have never received a loan from them or that ended the credit relationship before the current request. By matching the loan applications with the information on outstanding loans from CIR, we can infer whether the loan was granted or not.

Banks versus Cajas. Public savings banks (cajas) represented a large share of overall credit in Spain over our sample period, especially before the 2008-09 Spanish banking crisis. However, as discussed in Santos (2017), cajas operated under a different institutional framework than "regular" commercial banks, and were often controlled by local politicians. Delgado et al. (2007)) explain the main features of the Spanish banking system, focusing on the differences in behavior of commercial banks and public savings banks.

A.2 Additional datasets

We also use a number of additional datasets throughout the paper:

4. **Small sample of projects with information on bidders**. The BOE website does not provide the identity of the firms that competed for the project but did not win. This is a limitation of our dataset because it does not allow us to construct a well-defined control group. To overcome this limitation, we construct a sample of procurement projects for which we have detailed information about the awarding process. Although we did not find any government agency that provided information about the awarding process during our main sample period (2000-2013), we could identify around 50 agencies that started providing detailed information about their projects starting in 2013. Putting all these agencies together, we were able to uncover the identity of the firms competing for the same projects as well as their final rankings for around 1,000 contracts over the 2013-2016 period. We were able to scrape information on the specific bidders' score in the auction for around half of the contests.

5. **Firms' credit scores**. This is information is based on internal credit assessments of private Spanish non-financial corporations conducted by Bank of Spain. These assessments estimate the probability of default using the most recent available financial

[34] The Law stipulates that a bank can not request information about the firm without its consent, which indicates the seriousness of the approach

statements and assign each firm a score accordingly. The main purpose of these assessments is to provide Spanish banks with information about firms' financial reliability. Gavia et al. (2020) provide a detailed description of the so-called Banco de España in-house credit assessment system.

6. **Universe of procurement contracts in Spain**. Time coverage: 2018-2022. To have a comprehensive estimate of the actual number of firms active in procurement, we make use of a recently available dataset provided by the Spanish Government, *Plataforma de Contratación del Sector Público (PLACSP)*, where all types of public agencies in Spain are required to upload their procurement activity. Although some information is available since 2013, compliance was only strong starting in March 2018, when introducing information on all contracts became compulsory by law.[35] We use information for the years 2018, 2019, 2021, and 2022, when the coverage is already high and firms' participation not directly affected by COVID.

B Heterogeneous effects of credit growth

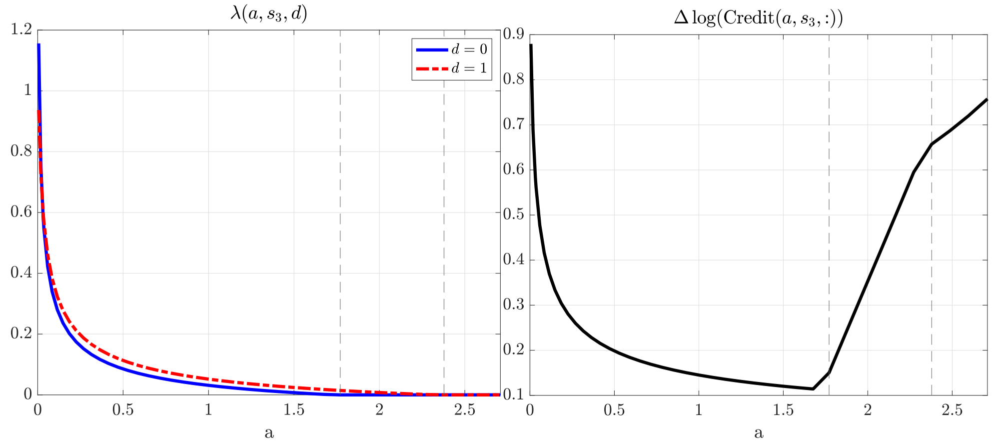
We investigate the heterogeneous effects of procurement on firms' credit growth. Before looking at the data, we first shed light on this relationship using our model's calibrated version. In panel (a) of Figure B.1, we show how the Lagrange multiplier associated with the financial constraint, $\lambda(s, a, d)$, changes across firms with different levels of net-worth, a.[36] We show this relationship for firms with procurement, i.e., $d = 1$, and without procurement, i.e., $d = 0$. As discussed in Section 4.3.2 of the main text, higher net-worth (a) firms are less constrained, and, conditional on a, procurement firms ($d = 1$) are more likely to be constrained because of their higher demand. The dashed lines divide the graph into three regions. The region to the left is one in which firms, with and without procurement, are financially constrained. In the middle region, only firms with procurement are constrained. In the region to the right, all firms are unconstrained.

In panel (b), we show the change in credit on impact ($h = 0$), of a firm becoming active in procurement ($d = 0 \rightarrow d = 1$), as a function of net worth a, i.e., the inverse of financial constraints all else equal. The main takeaway from this graph is that the model predicts a non-monotonic relationship between firms' financial constraints and the effect of procurement on credit. When the financial constraint is binding both before and after the procurement shock (left region), the model predicts that less financially constrained firms (higher a and

[35] In particular, that law ("Ley 9/2017, de 8 de noviembre, de Contratos del Sector Público") was aimed to adopt the European Parliament directives 2014/23/UE and 2014/24/UE.

[36] We produce this graph fixing the level of firms' productivity at the middle point in our productivity grid, i.e., s_3. For other levels of s, the graph would simply be an identical, scaled, version of Figure B.1.

Figure B.1. Heterogeneous effects of credit (calibrated model)



(a) Lagrange multiplier λ **(b)** change in log credit

Notes: Panel (a) shows the Lagrangian multiplier associated to the financial constraint, λ, for firms with different levels of net-worth in our model, both for firms with and without procurement ($d = 0$ and $d = 1$). The dotted lines organize the graph in three regions. In Panel (b) the effect on impact, i.e., $h = 0$, for different levels of firms' financial constraints as proxied by a (and implicitly given by λ).

hence lower λ) exhibit a smaller increase in credit when becoming active in procurement. However, the impact for firms in the other two groups (when transitioning from unconstrained to constrained or remaining unconstrained) increases with net worth. As a result, the model exhibits the u-shaped relationship between net credit growth and net worth.

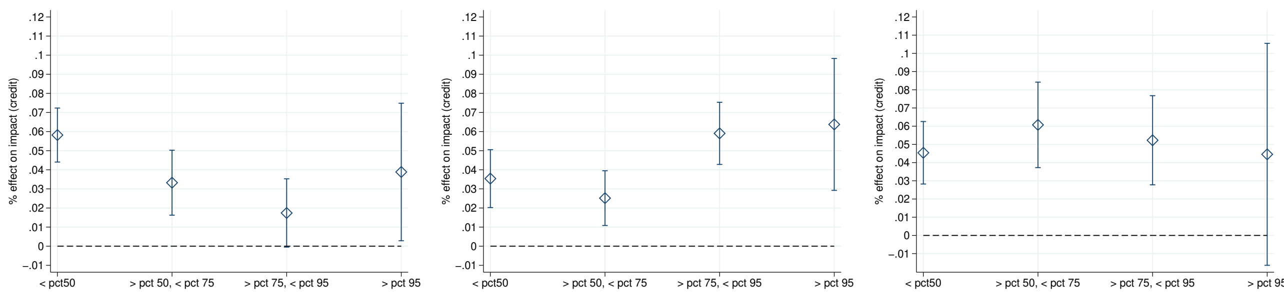
In Section 6.2 we discuss the intuition for why unconstrained firms exhibit a larger increase in credit due to the procurement shock. The idea is that these firms, precisely because they are unconstrained, can expand freely by increasing credit. On the contrary, even if procurement allows them to borrow more, constrained firms remain constrained, limiting the amount of credit they can raise after the demand shock implied by procurement.

We next look at the heterogeneous effects of procurement on firms' credit in the data. Similarly to Section 3.1 in the main text, we show the impact effect, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). See Figure B.2. As in the model, the relationship between proxies for firms' financial constraints and the change in credit is non-monotonic. With fixed assets and leverage as proxies, the empirical evidence is reasonably consistent with the u-shaped relationship predicted by the model.

C Additional empirical evidence

This Appendix provides additional evidence to support our baseline results that measure the impact of a firm obtaining a procurement contract on its credit growth. To simplify the

Figure B.2. Heterogeneous effects of procurement on credit (data)



(a) distribution of fixed assets **(b)** distribution of leverage **(c)** distribution of age

Notes: This figure shows the effect on impact, i.e., $h = 0$ (as well as its 10% confidence intervals), of public procurement on firms' credit for different percentiles of the distribution of assets (panel a), leverage (panel b), and age (panel c). Standard errors clustered at the firm level.

analysis and given that the largest impact that we find in our local projection regressions is in the initial period, we focus on static regressions of the following form:

$$\Delta \log l_{it} = \alpha_i + \alpha_{st} + \beta_1 \mathrm{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it}, \tag{C.1}$$

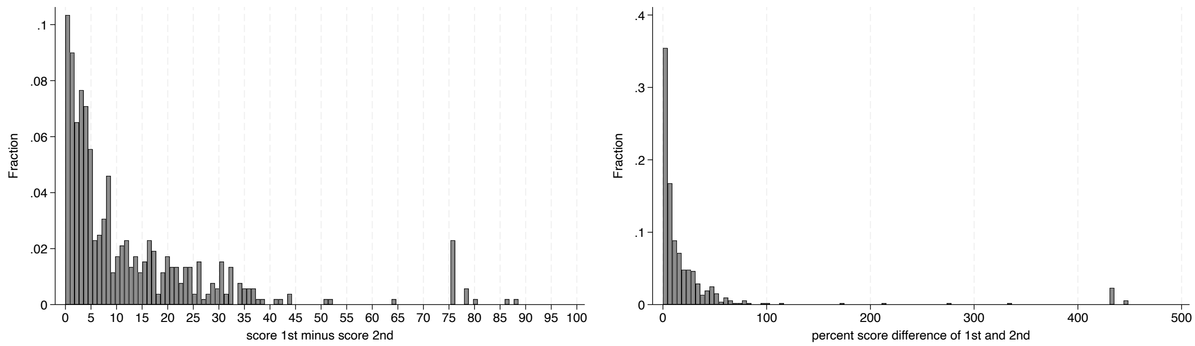
where t may represent a year or a year-quarter depending on the frequency of the data we use. As such, the dependent variable $\Delta \log l_{it}$ is either the annual or annualized quarterly growth of credit (loans) of firm i between $t-1$ and t defined as $\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}$. The regressor PROC_{it} is a dummy variable that takes value one if the firm obtained a procurement contract in t. We include the firm's lagged credit at $t-1$ to control for the fact that firms with large outstanding loan volumes may mechanically have less room for credit growth than firms with smaller outstanding loan levels.[37] The baseline annual regressions control for firm-level and sector×time fixed effects, but given the different datasets that we exploit, we will be able to control for additional fixed effects, such as firm×time fixed effects when using quarterly data, additional control variables, as well as moving beyond simple OLS as described below.

C.1 Effects on impact using bidders information

We use the sample of procurement projects where we have information on all bidders as well as the final ranking. We were able to scrape this information for around half of these contests. Doing so allows us to run regressions analogous to equation (C.1), except that we can identify the association between a firm's ranking in a given auction and its ensuing credit growth. To be more precise, we run two regressions similar to specification equation

[37] The estimation results without lagged credit are similar and are available upon request.

Figure C.1. Distribution of differences in 1st and 2nd place procurement contest scores



(a) Score differential

(b) Percent differential

Notes: This figure plots the distributions of the difference in scores of 1st and 2nd place firms in procurement contests. Scores are out of a possible 100. Panel (a) presents the difference between 1st and 2nd place, while panel (b) plots the percentage difference of scores between 1st and 2nd place.

(C.1) at the auction level.

Figure C.1 panel (a) presents a histogram of the difference in scores for winning and runner-up firms in these contests. The distribution is skewed with a few outliers with large score differentials, but otherwise a large fraction of the sample has score differentials less than 10 out of 100 points, with the majority of these observations with a score differential of 5 points or less. Panel (b) next shows the distribution of the percentage change differential between winners and runner-ups, where we normalize the absolute difference in score by that of the 2nd place firm. The distribution is quite dispersed, but with roughly twenty five percent of the contests decided by a percentage difference less than or equal to 3%.

Table C.1 presents credit growth regressions for a variety of sub-samples based on procurement contest characteristics, where we only report the coefficient on the procurement dummy for regressions using total credit (A), non-collateralized credit (B), and collateralized credit (C). The only difference in specifications is how we constrain the sample as we keep the set of fixed effects as stringent as possible across all regressions as well as including the lagged value of log credit (not reported).

Column (1) presents estimates from our baseline specification that includes the whole sample of firms with quarterly data. We then present regressions including all firms with information on auctions in column (2). We then zoom into characteristics of the sample based on the placement and scores in the procurement contest, as well as whether the auction was close or not. Column (3) next runs regressions including only winners and runners-ups in the sample. The number of observations does not drop dramatically relative to sample (2) given that most of the contests for which we have bidder information only include two

Table C.1. Quarterly credit growth regressions: whole sample and auctions

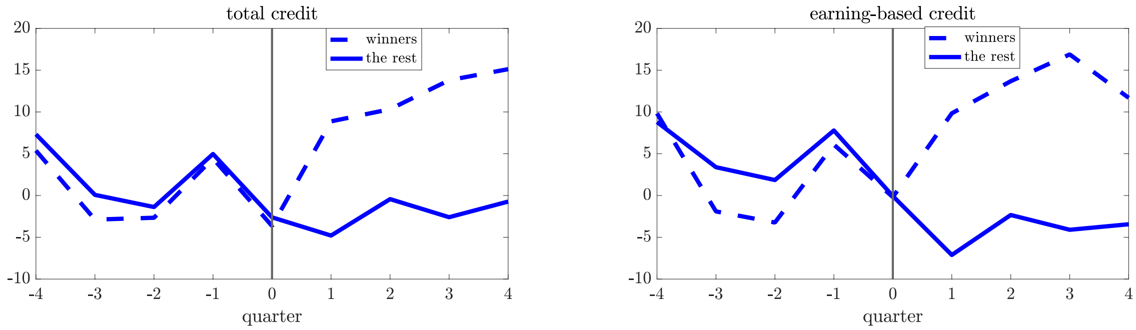
	(1)	(2)	(3)	(4)	(5)
(A) Total credit	0.055***	0.073***	0.073***	0.115***	0.124*
	(0.004)	(0.028)	(0.028)	(0.042)	(0.059)
Observations	700,780	8,310	6,264	6,555	715
R-squared	0.786	0.360	0.335	0.389	0.292
(B) Non-collateralized credit	0.070***	0.080**	0.081***	0.120**	0.127**
	(0.005)	(0.031)	(0.031)	(0.047)	(0.052)
Observations	557,873	8,110	6,128	6,357	678
R-squared	0.764	0.368	0.350	0.395	0.348
(C) Collateralized credit	0.001	-0.011	-0.012	0.026	0.195
	(0.006)	(0.029)	(0.028)	(0.021)	(0.139)
Observations	224,011	2,690	2,037	2,321	260
R-squared	0.791	0.357	0.367	0.358	0.315
Firm-Year FE	Yes	Yes	Yes	Yes	Yes
Sector-Quarter FE	Yes	No	No	No	No
Year-Quarter FE	No	Yes	Yes	Yes	Yes
Auction FE	No	Yes	Yes	Yes	Yes
Sample	All	Auction	1^{st} & 2^{nd}	Scores	1^{st} & 2^{nd}
Close auction	No	No	No	No	Yes (< 3%)

Notes: This table presents firm-level regression estimates using quarterly data for all firms as well as a sub-sample of observations with information on the procurement auctions. Columns (1)-(5) present the estimated coefficient on the procurement dummy. Column (1) presents baseline results using quarterly data for all possible firms in the dataset. Column (2) includes a subset of all firms with auction information, column (3) includes only 1^{st} and 2^{nd} place firms, column (4) includes all firms with contests reporting scores, and column (5) includes only 1^{st} and 2^{nd} place firms in "close" contests, where the percentage difference in scores is less than 3%. Standard errors are double clustered at the firm and sector×year levels, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

bidder firms. Still, it is reassuring that the coefficient on the procurement dummy does not differ substantially from the baseline sample's. Column (4) constrains the samples to all bidder firms that are in procurement contests with recorded scores for all participants. This constraint reduces the sample by roughly one quarter, but the procurement dummy coefficient is relatively similar to the baseline for all types of loans. Finally, column (5) considers "close" auction outcomes, defined as those located in the first quartile of the score percent differential, which implies a percentage difference in the scores between the first and second place firms of at most 3%. Looking at column (5), we see that the estimated procurement coefficient is approximately the same magnitude as our baseline estimates and is statistically significant, both for the all-loans and collateralized-loans regressions.

Pre-trends for winners vs. the rest. We also check to see if the evolution of credit growth differs for winners of procurement contracts relative to the rest of the sample. Graphically, the right panel in Figure C.2 shows the average growth of credit without collateral of firms that win a procurement project in quarter 0 before and after winning the project, and compares it to the rest of firms. Again, there is a similar evolution of credit growth before procurement (parallel trends) and a clear (and persistent) divergence after that.

Figure C.2. Credit Growth: bidders sample



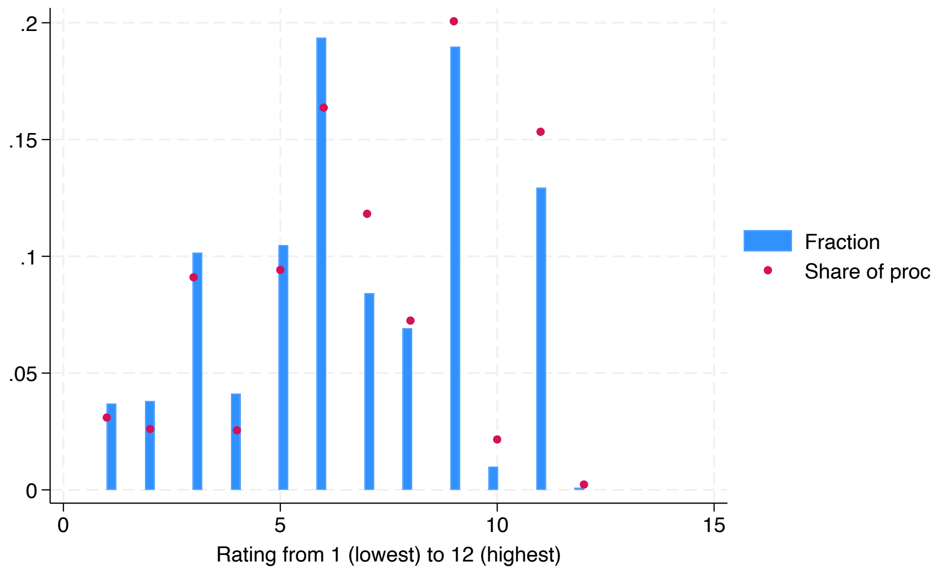
Notes: These graphs plot the evolution of the average change in credit for winning vs. non-winning firms, before and after the quarter in which the auction takes place (Quarter=0). The left panel is for all credit. The right panel is for non-collateral credit only.

C.2 Effects on impact using credit ratings data

We next turn to using information on firm credit scores and consider the possibility that a firm's credit quality may impact their procurement status. To that end, we use internal internal credit assessments of private Spanish non-financial corporations calculated by the Bank of Spain. These assessments estimate the probability of default using the most recent available financial statements and assign each firm a score accordingly. One of the main purposes of these assessments is to provide Spanish banks with information about firms' financial reliability. We merged these credit assessments with our main dataset.

After merging this information with our main dataset, we first examine the distribution of procurement winners conditional on credit ratings in Figure C.3. The blue bars plot the fraction of firms with a given rating, which ranges from 1 to 12, with 12 being the best credit rating, across all firms that have at least one contract of procurement over the sample period. Note that a firm's rating can change over time. The red circles then plot the share of procurement contracts won by firms in a given credit rating bin over the sample period. We would expect red circles to fall exactly at the top of the blue bars if procurements were won uniformly across firms regardless of their credit score. Instead, if we think that firms that have lower credit ratings, thus a lower chance of being able to borrow from a bank (ignoring the potential lending rate that might price the additional risk), are the ones that make more

Figure C.3. Distribution of procurement contracts and credit ratings



Notes: This figure presents the distribution of credit ratings over all firms in the sample (the blue bars), along with the share of procurements that firms with a given credit rating receive over the whole sample period ('Share of proc,' pink dots).

of an effort to win a procurement contract, we would expect to see the red circles above the blue bars for lower credit rating bins. Figure C.3 actually shows the opposite, where it is the higher rated firms that have a greater proportion of procurement contracts over the sample period. In this respect, the potential for a selection problem seems to be working in the opposite direction to your concern based on credit ratings.

We explore the impact of taking ratings into account by comparing subsets of firms based on their credit ratings, as you suggested above. Given sample-size considerations for the regressions, we divide the sample based on four groups of rating: 1-3 (Group 1), 4-6 (Group 2), 7-9 (Group 3), and 10-12 (Group 4). Therefore, we would be evaluating the impact of procurement on firms with very low credit ratings in the first group of ratings 1-3, for example. Table C.2 presents the regression for total credit growth based on the four sub-samples, including both firm and sector×year fixed effects. The coefficient on the procurement dummy is positive for all groups and significant for Groups 1 through 3. Significance disappears for the last group of highest rated firms, though it should be noted that this sub-sample has the smallest number of observations. It is interesting to note that the size of the coefficient on the procurement dummy falls monotonically as credit ratings increase. Therefore, obtaining a procurement contract increases the credit growth of high-risk firms relative to low-risk firms, but there remains a procurement effect for higher-rated firms.

Table C.2. Credit growth regressions for sub-sample of firms based on credit ratings

	(1) Group 1	(2) Group 2	(3) Group 3	(4) Group 4
Procurement_t	0.062***	0.043***	0.033**	0.021
	(0.018)	(0.013)	(0.013)	(0.026)
$\text{Log}(\text{Credit}_{t-1})$	-0.402***	-0.387***	-0.368***	-0.412***
	(0.026)	(0.025)	(0.025)	(0.031)
Observations	11,465	22,639	22,998	8,101
R-squared	0.602	0.521	0.497	0.538
Firm FE	Yes	Yes	Yes	Yes
Sector×Year FE	Yes	Yes	Yes	Yes

Notes: This table presents firm-level regression estimates splitting observations based on firm credit ratings in a given year, with Group 1 representing the bottom quartile of ratings, Group 2 is the 25th-50th quantile, Group 3 is the 50th-75th quantile, and Group 4 is the top quartile of ratings. Standard errors are double clustered at the firm and year levels, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

C.3 Effects on impact using a propensity score matching model

Our next approach to handle the selection issue is to follow a version of the empirical setup suggested by Hebous and Zimmermann (2021), who estimate a propensity score matching model (PSM) to allow the possibility of selection in the procurement contract. Similarly to our work, their procurement measure is based on the realization of a contract winner with no quasi-experimental design to exploit, so their methodology is a "second-best" approach to measure the impact of winning the contract for a set of procurement and non-procurement firms that are similar based on observables.

Specifically, we estimate PSM regressions guided by the predictions of our theoretical model. Our revised model predicts that the probability that a firm gets a contract depends on (i) its investment in the preparation of contracts, (ii) lagged participation, and (iii) the firm's productivity. To proxy for firms' investment in obtaining contracts and productivity, we use a measure of firm size, i.e., log assets, which in our model are highly correlated (we also experimented with more direct measures of productivity but results did not differ from what we report below). We can directly control for lagged participation. The first stage of the PSM runs a logit model on these variables along with sector×year effects to create control and treatment groups of firms that are comparable within a sector in a given period. Note that given data and estimation constraints, we define the fixed effects at the two-digit sector level. We then use the matched groups of firms to run a second-stage panel regression, where the key dependent variable is a dummy (obtaining a contract or not) or the procurement value divided by firm sales. We further control for sector×year effects, as in our baseline regressions with sectors at the four-digit level. We further control for the lagged value of log credit. We run the second-stage regression using either the growth rate of (i) total credit, (ii) non-collaterilized credit, or (iii) collaterilized credit based on the same control/treatment group of firms, and double cluster standard errors at the firm and sector×year levels.

Table C.3 presents the second-stage results of the PSM regressions. We report the first-stage logit regression results in the table's notes, but it is also worth comparing the predicted probability of obtaining a contract for the control and treatment groups of firms, which we plot in Figure C.4 for selection model 1 (includes) and selection model 2 (includes log assets + lagged procurement). Overall, the distributions of propensities match quite closely in both selection models. The pseudo-R^2 of the first-stage logit model increases substantially when including the lagged procurement variable in selection model 2 (from 0.059 to 0.132), but controlling for this variable does not impact the second-stage regression results. The second-stage results for selection model 1 are presented in columns (1) and (2), and for selection model 2 in columns (3) and (4) of Table C.3. Columns (1) and (3) present regressions using the procurement dummy as the main regressor, while columns (2) and (4) use the procurement/sales ratio instead. We present results for credit growth for firms' total credit, non-collateralized loans, and collateralized loans in panels A, B, and C, respectively. The estimated coefficients in all panels are in line with the baseline results we present in the main text. Namely, procurement (measured as a dummy or ratio) is positively correlated with credit growth for total and non-collateralized credit, while having zero correlation with collaterlized credit growth.

C.4 Loan applications

In this Section we ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de España's credit register allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms' borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

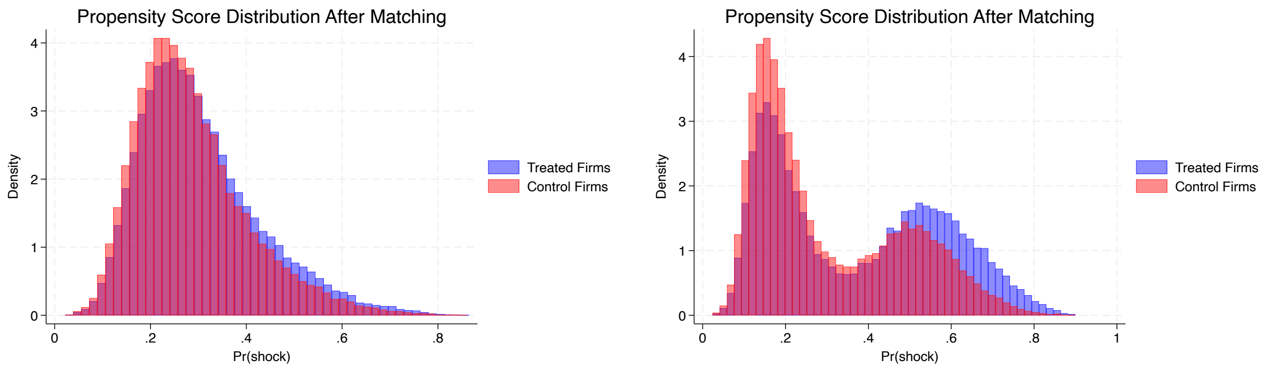
$$\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt} \tag{C.2}$$

Table C.3. Credit growth propensity score model regressions

	I. Selection model 1		*II. Selection model 2*	
	(1)	(2)	(3)	(4)
		Panel A. Total credit		
Proc. Dummy$_t$	0.045***		0.035***	
	(0.006)		(0.009)	
(Proc Value/Sales)$_t$		0.103**		0.098***
		(0.039)		(0.032)
Observations	43,255	34,367	41,145	32,248
R-squared	0.411	0.443	0.423	0.458
		Panel B. Non-collateralized credit		
Proc. Dummy$_t$	0.043***		0.037***	
	(0.009)		(0.008)	
(Proc Value/Sales)$_t$		0.127**		0.116***
		(0.048)		(0.038)
Observations	41,507	32,921	39,465	30,883
R-squared	0.408	0.441	0.421	0.457
		Panel C. Non-collateralized credit		
Proc. Dummy$_t$	0.015		0.011	
	(0.016)		(0.014)	
(Proc Value/Sales)$_t$		-0.007		0.011
		(0.068)		(0.079)
Observations	11,670	8,769	10,879	7,975
R-squared	0.477	0.510	0.482	0.518

Notes: This table presents firm-level regressions based on propensity score matching. All specifications include lagged log credit, and firm and sector×year fixed effects. *Selection model 1* is a logit of a procurement shock on lagged firm log assets and 2-digit sector×year fixed effects. The coefficient on log assets is 0.289 with a standard error of 0.004, and the pseudo R-squared of the regression is 0.059. *Selection model 2* is a logit of a procurement shock on lagged firm log assets, lagged procurement shock and 2-digit sector×year fixed effects. The coefficient on log assets is 0.224 with standard errors 0.005, the coefficient on the lagged procurement shock is 1.565 with a standard error of 0.017, and the pseudo R-squared of the regression is 0.132. Both logit models use 109,445 observations. Standard errors are double clustered at the firm and sector×year levels, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

Figure C.4. Distribution of predicted procurement probabilities for control and treatment groups: Selection models 1 and 2



(a) Selection model 1 **(b)** Selection model 2

Notes: This figure plots the propensity scores for firm-level procurement. *Selection model 1* is logit of a procurement shock on lagged firm log assets and 2-digit sector×year fixed effects. The coefficient on log assets is 0.289 with a standard error of 0.004, and the pseudo R-squared of the regression is 0.059. *Selection model 2* is a logit of a procurement shock on lagged firm log assets, lagged procurement shock and 2-digit sector×year fixed effects. The coefficient on log assets is 0.224 with standard errors 0.005, the coefficient on the lagged procurement shock is 1.565 with a standard error of 0.017, and the pseudo R-squared of the regression is 0.132. Both logit models use 109,445 observations.

where the variable 'Loan granted' is a dummy variable equal to 1 when firm i receives a loan from bank b in quarter t conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, α_{ib}, which implies that we are identifying the coefficient β on the procurement variable via the variation within a firm-bank relationship over time. We further control for overall bank credit supply in a given period with bank×quarter fixed effect α_{bt}, and for macroeconomic events with sector×quarter fixed effects α_{st}.

Table C.4 shows the results from running this regression. We include only firm×bank fixed effects in column (1), and add the time-varying bank and sector fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percent in the quarter that a firm wins a procurement project.

C.5 Effects by contest type

We recover information on the type of procurement contest for our main sample of procurement contracts. Using that information, we run our baseline regression based on the types of procurement contests in which a firm participates. Table C.5 presents the estimated coefficient for the procurement dummy in our baseline regression. We divided the samples into firm-year observations for firms that participate at least once in a type of contest. Columns

Table C.4. Probability of a New Loan and Procurement

	All firms	
	(1)	(2)
PROC_{it}	0.024***	0.023**
	(0.008)	(0.011)
Observations	36,857	26,924
R-squares	0.395	0.628
Firm×bank FE	Yes	Yes
Bank×quarter FE	No	Yes
Sector×quarter FE	No	Yes

Notes: Results from estimating the relationship between loan participation and procurement participation (PROC) by regression (C.2) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors clustered at the firm level; *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level.

(1) and (2) presents results for firms that participate in "Abierto" and "Concurso" contests, respectively. These procurement contests are competitive open-bid competitions. Column (3), "Negociado", represents procurement contracts where the governmental institution approaches targeted firms and negotiates bilaterally.

Table C.5 reveals several interesting findings. First, the majority of contests that underlie our baseline regressions are competitive. Second, the procurement dummy coefficient is positive and significant for all types of contests, but is larger in magnitude for competitive competitions (columns 1 and 2). Finally, these patterns hold for non-collaterilzed credit growth, though the coefficient is marginally significant for the 'Negociado' sub-sample of firms. Therefore, our baseline results appear to be driven by competitive procurement competitions.

C.6 Effects by bank type

We exploit *bank-firm loan-level* data, as is commonly used in the credit register literature, to examine whether the procurement shock has a differential impact across loan growth across types of banks. The idea is to make sure that our results are not driven by cajas. These regressions results are reported in Table C.6. Given the granularity of these data, we are able to control for several additional levels of fixed effects relative to our baseline firm-level regressions, including (1) *bank*×*firm* FE that control for (non-time varying) bank-firm relationships, (2) *bank*×*year* FE, and *sector*×*year* FE. Therefore, we are capturing variation in loan growth within a given bank-firm pair over time. The results in the three panels

Table C.5. Credit growth regression by contest type

	(1)	(2)	(3)
(A) Total credit	0.049***	0.066**	0.035***
	(0.005)	(0.027)	(0.011)
Observations	92,116	7,889	24,446
R-squared	0.273	0.326	0.286
(B) Non-collateralized credit	0.052***	0.071**	0.022*
	(0.006)	(0.029)	(0.012)
Observations	88,493	7,721	23,730
R-squared	0.218	0.298	0.248
(C) Collateralized credit	0.005	0.019	0.039*
	(0.009)	(0.044)	(0.018)
Observations	29,441	1,903	6,791
R-squared	0.209	0.319	0.257
Firm FE	Yes	Yes	Yes
Sector-Year FE	Yes	Yes	Yes
Contest	Abierto	Concurso	Negociado
Contest	(competitive)	(competitive)	(not competitive)

Notes: This table presents firm-level regression estimates splitting observations based on the type of procurement contest that a firm participates. Columns (1) and (2), 'Abierto' and 'Concurso' are competitive open-bid competitions, while column (3), 'Negociado,' represents procurement contracts where the governmental institution approaches targeted firms and negotiates bilaterally. Standard errors are double clustered at the firm and year levels, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

of Table C.6 reinforce our main reduced-form finding: the procurement shock is positively correlated to credit growth, and particularly for non-collateralized credit. Furthermore, in examining the coefficients for the three types of banks included in the sample for estimation across the three panels, we see that the estimated procurement coefficient does not vary dramatically for the "All," "Commercial" or "Caja" bank samples.

C.7 Firm-level heterogeneity of estimates

We have accessed more recent procurement data that are available electronically and which covers the universe of Spanish procurement contracts and are available **here**. Although the platform includes information since 2013, compliance was only strong starting in March 2018, when introducing information on all contracts became compulsory by law.[38] Although

[38] In particular, that law ("Ley 9/2017, de 8 de noviembre, de Contratos del Sector Público") was aimed to adopt the European Parliament directives 2014/23/UE and 2014/24/UE.

Table C.6. Bank-firm level credit growth regressions

	(1) All	(2) Commercial	(3) Cajas
Panel A. Total credit			
Procurement_t	0.019***	0.014***	0.026***
	(0.004)	(0.003)	(0.007)
$\text{Log}(\text{Credit}_{t-1})$	-0.326***	-0.333***	-0.319***
	(0.020)	(0.019)	(0.020)
R-squared	0.369	0.369	0.371
Observations	7,740,651	3,736,605	4,003,834
Panel B. Non-collateralized credit			
Procurement_t	0.017***	0.014***	0.021***
	(0.003)	(0.003)	(0.006)
$\text{Log}(\text{Credit}_{t-1})$	-0.195***	-0.226***	-0.164***
	(0.011)	(0.009)	(0.008)
R-squared	0.225	0.239	0.216
Observations	7,740,651	3,736,605	4,003,834
Panel C. Collateralized credit			
Procurement_t	0.002	-0.002	0.008*
	(0.003)	(0.002)	(0.005)
$\text{Log}(\text{Credit}_{t-1})$	-0.141***	-0.111***	-0.172***
	(0.015)	(0.012)	(0.016)
R-squared	0.216	0.197	0.240
Observations	7,740,651	3,736,605	4,003,834

Notes: This table presents bank-firm level regression estimates for all loans and two sub-samples of lenders: commercial banks and saving banks ('cajas'). All regressions include bank×firm, bank×year, and sector× year fixed effects. Standard errors are double clustered at the bank, firm and year levels, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

the Bank of Spain's credit register has undergone some changes since last time we used it, we were able to obtain the main information we use in our benchmark regressions (i.e., total credit, collateralized credit, and non-collateralized credit) for the years 2018 and 2019. Therefore, we can use credit growth by type of credit 2018–19 as our dependent variable after merging it with the universe of procurement contracts.

Table C.7 presents cross-sectional regressions of credit growth regressed on a procurement dummy (panel A) or the procurement-to-sales ratio (panel B), along with lagged log credit and sector-level fixed effects. Column (1) presents results for all firms and total credit, while columns (2) and (3) present the estimates for collateral and non-collateral credit, respectively. Finally, columns (4)-(7) cut the sample into quartiles based on firm size (log assets) in the case of non-collateral credit regressions in column (3). Both sets of regressions reveal similar

Table C.7. Credit growth cross-sectional regression using the new 2018–19 sample

			Panel A. Procurement dummy				
Credit	Total	collateral	non-coll	non-coll	non-coll	non-coll	non-coll
Firms	All	All	All	Q1	Q2	Q3	Q4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Procurement_t	0.067***	0.004	0.124***	0.067***	0.095***	0.084***	0.083***
	(0.003)	(0.006)	(0.005)	(0.010)	(0.010)	(0.010)	(0.010)
$\text{Log}(\text{Credit}_{t-1})$	-0.057***	-0.098***	-0.113***	-0.150***	-0.135***	-0.133***	-0.122***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Av. dep var.	-0.026	-0.049	0.042	0.008	0.052	0.064	0.046
Observations	410,604	410,604	410,604	102,124	102,119	102,123	102,122
R-squared	0.028	0.056	0.057	0.080	0.068	0.067	0.063
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

			Panel B. (Procurement value)/Sales				
Credit	Total	collateral	non-coll	non-coll	non-coll	non-coll	non-coll
Firms	All	All	All	Q1	Q2	Q3	Q4
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Procurement_t	0.181***	0.025	0.291***	0.106**	0.307***	0.231***	0.226***
	(0.016)	(0.026)	(0.023)	(0.042)	(0.047)	(0.047)	(0.044)
$\text{Log}(\text{Credit}_{t-1})$	-0.058***	-0.100***	-0.115***	-0.159***	-0.143***	-0.138***	-0.121***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Av. dep var.	-0.021	-0.046	0.048	0.014	0.057	0.069	0.052
Observations	387,561	387,561	387,651	96,749	96,766	96,755	96,758
R-squared	0.029	0.058	0.058	0.085	0.073	0.070	0.063
Sector FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

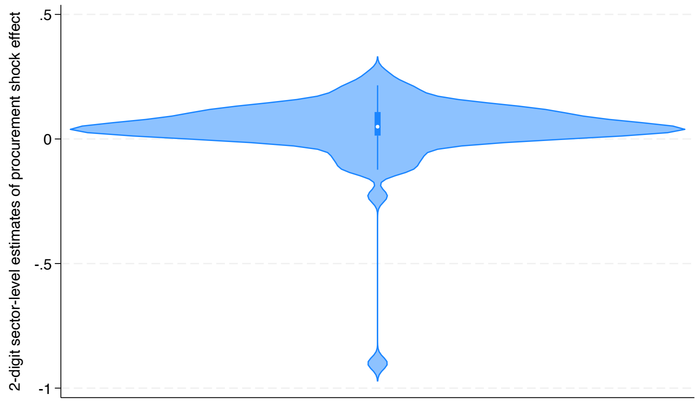
Notes: This table presents firm-level regression estimates using the procurement dummy and procurement-to-sales ratio. Panel A present results using the procurement dummy as a regressor, while panel B uses the procurement value-to-sales ratio. Standard errors are robust, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

results. The effect of procurement on firm's credit is systematically positive and significant across the entire firm size distribution, both when using the dummy and the continuous.

C.8 Sector-level heterogeneity of estimates

We run regressions sector-by-sector, at the 2-digit sectoral level to insure sufficient number of firm-level observations, to estimate the procurement effect. In total, this yields 82 betas, which we summarize in a violin plot in Figure C.5. While there certainly is variation in the estimated effects across sectors, the mean (median) beta is 0.040 (0.049) and the interquartile range lies between 0.013 and 0.108. So, we are comfortable that the average effect that we estimate in our full panel regression is representative of the sample.

Figure C.5. Violin plot of the estimated effect of a procurement shock on credit growth across 2-digit sectors



Notes: This figure plots the cross-sector distribution of estimated coefficients of the procurement dummy for our baseline credit growth regressions run at the 2-digit sector level. We plot a "violin" of the overall distribution along with an interquartile range of estimates (dark blue bar), the median sector estimate (white dot), and cross-sectional standard deviation tails (dark blue lines).

C.9 The intensive margin of procurement

In this Section, we ask how the value of procurement relative to a firm's total sales impact credit growth. That is, we examine the impact of the intensive margin of procurement on credit growth. To do so, we replace the procurement shock variable in (C.1) with the procurement value-to-sales ratio, where we sum the values of all available procurement contracts that a firm wins in a given year. We also include firms' credit ratings as additional controls. Table C.8 presents the regression results by loan type. The estimated coefficient on the procurement ratio is qualitatively the same as our baseline regressions using the procurement shock variable: the coefficient is positive and significant for the sample of all loans (column 1) and non-collaterilized loans (column 2), while being positive but insignificant for the collateralized loan sub-sample (column 3).

D A microfoundation for government demand

In this Appendix, we present a microfoundation for our stylized model of firm competition for public sector demand. Competition for procurement projects is a multidimensional process, where public agencies assess the offered price, different measures of project quality, and compliance with technical requirements. Our microfoundation incorporates these aspects of reality but leverages a key simplification: quality and price competition are separated into two different stages, with the former being an intertemporal dynamic choice and the latter a static problem. This is what makes the model tractable.

Table C.8. The intensive margin of procurement

	Type of credit		
	Total	Non-collateral	Collateral
	(1)	(2)	(3)
(Proc. value/Sales)$_t$	0.0914***	0.110***	0.0361
	(0.0248)	(0.0312)	(0.0464)
Log(Credit$_{t-1}$)	-0.353***	-0.301***	-0.233***
	(0.0281)	(0.0222)	(0.0254)
Observations	70,419	66,304	21,388
R-squared	0.370	0.315	0.345
Firm FE	Yes	Yes	Yes
Sector-Year FE	Yes	Yes	Yes
Rating FE	Yes	Yes	Yes

Notes: This table presents firm-level regression estimates using the procurement-to-sales ratio as an explanatory variable. Standard errors are double clustered at the firm and sector×year levels, with *** significant at the 1% level, ** at the 5% level, and * at the 10% level.

In the economy there is a continuum of varieties indexed by $i \in [0,1]$, each produced by a different firm. A government has a budget e and a continuum of size M of identical and independent agencies, which want to procure one variety each. The direct conditional utility or valuation of an agency $j \in [0, M]$ buying quantity x_{jig} of any variety i is given by:

$$\tilde{U}_{ji} = \mathbf{1}_{[q_i \geq \bar{q}]} \, x_{jig} \, \epsilon_{ji} \tag{D.1}$$

where $\mathbf{1}_{[q_i \geq \bar{q}]}$ is an indicator of whether the perceived quality of the firm's project q_i is above some required level $\bar{q}$ identical across agencies and ϵ_{ji} is a random taste shock for variety i in agency j, reflecting heterogeneous preferences for varieties across agencies. We can think of the quality q_i as the assessment by public officials that the offered good aligns with the technical requirements of the government. Each agency has an (identical) budget e/M and the price of variety i offered to the public agencies is p_{ig}. Hence, the agency j conditional demand for variety i is $x_{ig} = e/Mp_{ig}$ (it does not depend on j because of symmetry across agencies), which can be used to obtain the indirect utility function:

$$\tilde{V}_{ji} = \mathbf{1}_{[q_i \geq \bar{q}]} \, (e/Mp_{ig}) \, \epsilon_{ji} \tag{D.2}$$

Firms compete for contracts in two stages. In the first stage (see Section D.1), they make risky investments b_i into improving the perceived quality q_i of their variety. The varieties that succeed in getting at least a level of quality $\bar{q}$ form the set $\mathrm{I}_g \subset [0,1]$ of measure $m_g < 1$ available to government agencies. Next period (see Section D.2), the producers of varieties in the set I_g compete for all agencies' demands along the price margin. We think of the budget e, the measure m_g, and the quality threshold $\bar{q}$ as policy variables, while the set I_g is the result of these policies plus the actions by firms in their competition for contracts.

D.1 First stage: quality competition

The quality threshold $\bar{q}$ is mandated by the government, and it internalizes the competition for contracts because in equilibrium the fraction of firms whose quality q_i is above the threshold $\bar{q}$ must equal the measure m_g of the set I_g.

We assume that the investment technology to improve perceived quality is given by $q_i = \tilde{q}_i/u_i$, where $\tilde{q}_i$ is chosen by the firm and u_i is a shock capturing the riskiness in this technology. The shock u_i may account for true uncertain returns to improving project quality or the idiosyncratic subjectivity of public officials in assessing each project. If we assume that u_i follows an exponential distribution with parameter λ_U, the probability of success for firm i with quality choice $\tilde{q}_i$ is given by:

$$\mathbf{P}[q_i \geq \bar{q}] = \mathbf{P}\left[u_i \leq \tilde{q}_i/\bar{q}\right] = 1 - e^{-\lambda_U \frac{\tilde{q}_i}{\bar{q}}} \tag{D.3}$$

Let's assume that to produce quality $\tilde{q}_i$ requires an investment $b_i = \left[\tilde{q}_i/\eta_0\eta_2^{d_i}s_i^{\eta_3}\right]^{1/\eta_1}$, with $\eta_0 > 0$, $\eta_1 \in [0,1)$, $\eta_2 \geq 0$, and $\eta_3 \geq 0$, which potentially allows for current procurement suppliers ($d_i = 1$) and more productive firms (if $\eta_3 > 0$) to fulfill the government's requirements at a lower cost. Inverting the cost function, the probability for producer i to pass the threshold $\bar{q}$ is given by,

$$\mathbf{P}[q_i \geq \bar{q}] = 1 - e^{-\frac{1}{\bar{q}}\lambda_U \eta_0 b_i^{\eta_1}\eta_2^{d_i}s_i^{\eta_3}} \tag{D.4}$$

Defining $\bar{p} \equiv \lambda_U\eta_0/\bar{q}$ we get the probability function (16) that we use in the quantitative exercise of the paper.

This microfoundation treats η_1, η_2, and η_3 as technology parameters, but in the paper we perform counterfactual exercises where these parameters are treated as policy parameters. There are two different ways of seeing η_1, η_2, and η_3 as policy parameters. First, the government can change the quality requirements for procurement goods, the trade-offs between different aspects of the application, or the complexity of the whole application process, which will change the parameters of the cost function. Alternatively, and focusing on η_2 and η_3, the government may set a lower quality threshold $\bar{q}$ for firms with procurement experience $d_i = 1$ and higher firm productivity s_i. In that case, one can define $\bar{q}(d_i, s_i) \equiv \nu_2^{d_i}s_i^{\nu_3}\bar{q}$, and properly redefining the parameters in (D.4) means that η_2 and η_3 are the product of underlying cost and policy parameters.

D.2 Second stage: price competition

The second stage follows standard steps in discrete choices with extreme value shocks, see for instance Anderson et al. (1992). Given the set I_g (with the varieties fulfilling $q_i \geq \bar{q}$) and the indirect utility (D.2), the agency j buys the variety $i \in \mathrm{I}_g$ that yields the highest value $\tilde{V}_{ji}$, with the choice depending on the prices of all varieties in the set I_g as well as on the idiosyncratic preference shocks ϵ_{ji}. Let's assume that $\ln \epsilon_{ji}$ are $i.i.d$ and drawn from a Gumbel distribution with dispersion parameter η. Then, the probability for an agency j of purchasing variety i is given by:

$$\mathrm{Pr}_i = \frac{e^{\left[\frac{1}{\eta}(\ln e/M - \ln p_{ig})\right]}}{\int_{i' \in \mathrm{I}_g} e^{\left[\frac{1}{\eta}\ln e/M - \ln p_{i'g}\right]} \mathrm{d}i'} = \frac{1}{m_g} \frac{p_{ig}^{-1/\eta}}{P_g^{-1/\eta}} \tag{D.5}$$

where $P_g \equiv \left[\frac{1}{m_g} \int_{i' \in \mathrm{I}_g} p_{i'g}^{-1/\eta} \mathrm{d}i'\right]^{-\eta}$ is an average price index of all varieties and Pr_i does not depend on j as all agencies are identical.

Hence, the total demand y_{ig} of variety i faced by its producer is given by:

$$y_{ig} = M \, \mathrm{Pr}_i \, x_{ig} \tag{D.6}$$

That is, the demand y_{ig} is given by the mass M of agencies times the fraction Pr_i of them selecting variety i times the amount x_{ig} each agency purchases when selecting variety i. Using the expressions for x_{ig} and Pr_i above gives:

$$y_{ig} = \frac{1}{m_g} \frac{e}{p_{ig}} \left(\frac{p_{ig}}{P_g}\right)^{-1/\eta} \tag{D.7}$$

which is a downward-sloping isoelastic demand for each variety i. To map this formulation into the one in Section 4.1, we can redefine $P_g Y_g \equiv e$ as the government budget and $\sigma_g \equiv 1 + \alpha/\eta$ as the price elasticity such that:

$$y_{ig} = \left(\frac{p_{ig}}{P_g}\right)^{-\sigma_g} \frac{Y_g}{m_g} \qquad \text{or} \qquad p_{ig} = B_g y_{ig}^{-1/\sigma_g} \tag{D.8}$$

where $B_g \equiv m_g^{-1/\sigma_g} P_g Y_g^{1/\sigma_g}$. Note also that the price index P_g defined above is as in equation (3) and that, due to constant returns to scale, the aggregator of Y_g defined in equation (2) is consistent with total expenditure in varieties e being equal to $P_g Y_g$. The intuition for this equivalence is straightforward. If the different agencies have very homogeneous preferences across varieties (low η), that shows up as a very high elasticity of substitution across varieties (high σ_g) and price increases generate a large drop in demand.

E Details on the static production problem

In this Appendix we characterize analytically the solution of the static production problem. First, in Section E.1 we rewrite the FOC of the problem using the actual functional form for the revenue functions. In Section E.2 we derive the results that serve to restrict the parameters ϕ_p and ϕ_g such that the problem is well-behaved. Then, focusing on the case $\sigma_p = \sigma_g$, we characterize the solutions of the production problem in Section E.3, while in Section E.4 we study the static effect of a procurement shock, that is, the differences in allocations and profits between a firm with $(s, a, d = 1)$ and a firm with $(s, a, d = 0)$.

E.1 The FOC

We start rewriting the FOC of the static production problem as follows. Adding up equations (10), defining $k \equiv k_p + k_g$, and using $\frac{\partial p_p y_p}{\partial k} = \frac{\partial p_p y_p}{\partial k_p}\frac{k_p}{k}$ and $\frac{\partial p_g y_g}{\partial k} = \frac{\partial p_g y_g}{\partial k_g}\frac{k_g}{k}$ we obtain

$$\text{MRPK} \equiv \frac{\partial(p_p y_p + p_g y_g)}{\partial k} = u\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) + (1-u)\left(\frac{r+\delta+\lambda}{1+\lambda\phi_g}\right) \tag{E.1}$$

where we have defined $u \equiv \frac{k_p}{k}$ as the share of capital in the private sector. That is, the revenue marginal product of capital for the whole firm (MRPK) is a weighted average of the capital costs in the two sectors, with the weights given by the shares of capital used in each sector. Likewise, and defining $n \equiv n_p + n_g$, we can obtain a similar expressions for labor,

$$\text{MRPN} \equiv \frac{\partial(p_p y_p + p_g y_g)}{\partial n} = w$$

which tells us that the marginal revenue product of labor at the firm level must equal the wage. It will be useful to substitute the actual revenue functions in equations (9)-(10) to obtain the model FOC as,

$$(1-\alpha)\left(\frac{\sigma_p-1}{\sigma_p}\right)\frac{p_p y_p}{n_p} = w \tag{E.2}$$

$$(1-\alpha)\left(\frac{\sigma_g-1}{\sigma_g}\right)\frac{p_g y_g}{n_g} = w \tag{E.3}$$

$$\alpha\left(\frac{\sigma_p-1}{\sigma_p}\right)\frac{p_p y_p}{k_p} = \frac{r+\delta+\lambda}{1+\phi_p\lambda} \tag{E.4}$$

$$\alpha\left(\frac{\sigma_g-1}{\sigma_g}\right)\frac{p_g y_g}{k_g} = \frac{r+\delta+\lambda}{1+\phi_g\lambda} \tag{E.5}$$

Finally, using equations (E.2) and (E.3), it is easy to show that the labor shares are constant in each sector and given by $wn_p/p_p y_p = (1-\alpha)(\sigma_p-1)/\sigma_p$ and $wn_g/p_g y_g = (1-\alpha)(\sigma_g-1)/\sigma_g$.

Hence, we can write revenues net of wage expenses as constant fractions of revenues:

$$p_p y_p - w n_p = [1-(1-\alpha)(\sigma_p-1)/\sigma_p]\, p_p y_p \tag{E.6}$$

$$p_g y_g - w n_g = [1-(1-\alpha)(\sigma_g-1)/\sigma_g]\, p_g y_g \tag{E.7}$$

where $[1-(1-\alpha)(\sigma_p-1)/\sigma_p] \in (0,1)$ and $[1-(1-\alpha)(\sigma_g-1)/\sigma_g] \in (0,1)$ as long as $\sigma_p > 1$ and $\sigma_g > 1$ respectively.

E.2 Some preliminary results

Lemma 1 *The terms $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$ describing the cost of capital for the production of the private sector and the public sector goods respectively, are (a) strictly below $1/\phi_p$ and $1/\phi_g$ respectively, (b) increasing in λ, and (c) strictly above $r+\delta$ when $\lambda > 0$, if and only if $\phi_p < (\delta+r)^{-1}$ and $\phi_g < (\delta+r)^{-1}$ respectively.*

Proof: Part (a) is straightforward:

$$\frac{r+\delta+\lambda}{1+\lambda\phi_p} < \frac{1}{\phi_p} \Leftrightarrow \phi_p(r+\delta+\lambda) < (1+\lambda\phi_p) \Leftrightarrow \phi_p(r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

For part (b) note that

$$\frac{d}{d\lambda}\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) \propto (1+\lambda\phi_p) - \phi_p(r+\delta+\lambda) > 0 \Leftrightarrow \phi_p(r+\delta) < 1 \Leftrightarrow \phi_p < (r+\delta)^{-1}$$

Finally, part (c) is proved by noting that $\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ equals $r+\delta$ whenever $\lambda = 0$ and its derivative w.r.t. λ is positive, see part (b). The same arguments apply for $\frac{r+\delta+\lambda}{1+\lambda\phi_g}$. ■

Lemma 2 *The optimal unconstrained capital for the private and the public sector respectively cannot be self-financed through its own revenues if $\phi_p \frac{\sigma_p}{\sigma_p-1}\frac{r+\delta}{\alpha} < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g-1}\frac{r+\delta}{\alpha} < 1$ respectively.*

Proof: The optimal unconstrained solution for the private sector capital is given by equation (E.4) when $\lambda = 0$, which implies $\frac{p_p y_p}{k_p} = \frac{\sigma_p}{\sigma_p-1}\frac{r+\delta}{\alpha}$. When $\phi_p \frac{\sigma_p}{\sigma_p-1}\frac{r+\delta}{\alpha} < 1 \Leftrightarrow \frac{\sigma_p}{\sigma_p-1}\frac{r+\delta}{\alpha} < \phi_p^{-1}$, which means that $\frac{p_p y_p}{k_p} < \phi_p^{-1} \Leftrightarrow k_p > \phi_p p_p y_p \Rightarrow k_p > \phi_p(p_p y_p - w n_p)$, that is, the optimal unconstrained capital for the private sector, k_p, cannot be self-financed through its own revenues. The proof for the public sector capital is analogous by use of the FOC (E.5). ■

Proposition 1 *Entrepreneurs with zero net worth are financially constrained if both $\phi_p \frac{\sigma_p}{\sigma_p-1}\frac{r+\delta}{\alpha} < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g-1}\frac{r+\delta}{\alpha} < 1$.*

Proof: Note that if both $\phi_p \frac{\sigma_p}{\sigma_p-1}\frac{r+\delta}{\alpha} < 1$ and $\phi_g \frac{\sigma_g}{\sigma_g-1}\frac{r+\delta}{\alpha} < 1$, then following Lemma 2 both $\phi_p(p_p y_p - w n_p) < k_p$ and $\phi_g(p_g y_g - w n_g) < k_g$. Adding them up leads to $\phi_p(p_p y_p - w n_p) +$

$\phi_g(p_g y_g - w n_g) < k$, which implies that the capital of the unconstrained solution cannot be financed through revenue based constraints and hence entrepreneurs with zero net worth are constrained. ■

Lemma 3 *The terms* $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ *and* $\phi_g \frac{\partial p_g y_g}{\partial k_g}$ *are positive and strictly smaller than one for all firms if and only if* $\phi_p < (r+\delta)^{-1}$ *and* $\phi_g < (r+\delta)^{-1}$ *respectively.*

Proof: Given the revenue function, $\phi_p \frac{\partial p_p y_p}{\partial k_p} > 0$ is straightforward. To show $\phi_p \frac{\partial p_p y_p}{\partial k_p} < 1$ just note that the FOC for k_p in equation (10) plus Part (a) in Lemma 1 tell us that $\frac{\partial p_p y_p}{\partial k_p} < \phi_p^{-1}$ iff $\phi_p < (r+\delta)^{-1}$. The proof for y_g is identical. ■

Lemma 4 *The terms* $\phi_p \frac{\partial p_p y_p}{\partial k_p}$ *and* $\phi_g \frac{\partial p_g y_g}{\partial k_g}$ *increase with* λ *if and only if* $\phi_p < (r+\delta)^{-1}$ *and* $\phi_g < (r+\delta)^{-1}$ *respectively.*

Proof: The FOC for k_p establishes that $\partial p_p y_p / \partial k_p = (r+\delta+\lambda)/(1+\phi_p\lambda)$, see equation (10). Multiplying both sides by ϕ_p we obtain $\phi_p \partial p_p y_p / \partial k_p = (r+\delta+\lambda)/(\phi_p^{-1}+\lambda)$, and the derivative of $(r+\delta+\lambda)/(\phi_p^{-1}+\lambda)$ with respect to λ has ther same aign as $(\phi_p^{-1}-(r+\delta))$, which is positive whenever $\phi_p < (r+\delta)^{-1}$. The proof for y_g is identical. ■

Lemma 5 *Labor choices in each sector increase monotonically with the capital choices in that same sector*

Proof: Equations (9) determine the optimal labor demand in each sector. Totally differentiating them with respect to capital and labor gives

$$\frac{dn_p}{dk_p} = -\frac{\partial^2(p_p y_p)/\partial n_p^2}{\partial^2(p_p y_p)/\partial n_p \partial k_p} > 0 \quad \text{and} \quad \frac{dn_g}{dk_g} = -\frac{\partial^2(p_g y_g)/\partial n_g^2}{\partial^2(p_g y_g)/\partial n_g \partial k_g} > 0$$

as the revenue functions are concave in labor (negative numerator), and capital and labor are complements in the Cobb-Douglas production function (positive denominator). ■

E.3 Characterzing the solutions

We now analyze the production problem for the case $\sigma_p = \sigma_g = \sigma$. We do it for firms with procurement ($d = 1$), with the case ($d = 0$) easy to derive from it.

E.3.1 Unconstrained firms

We can obtain closed-form solutions to all objects of interest. We specify how these objects vary with the state variables a and s in Proposition 2 below.

Policy functions. With $\lambda = 0$ the FOC for k_p and n_p become $\frac{\partial p_p y_p}{\partial k_p} = r + \delta$ and $\frac{\partial p_p y_p}{\partial n_p} = w$. These two equations define the optimal factor demands for the private sector $k_p^*(s, a, d)$ and $n_p^*(s, a, d)$. In particular, one gets $\frac{\sigma-1}{\sigma}\alpha\frac{p_p y_p}{k_p} = r + \delta$ and $\frac{\sigma-1}{\sigma}(1-\alpha)\frac{p_p y_p}{n_p} = w$ and substituting for the revenue function yields the optimal demands for capital and labor in the private sector

$$k_p^*(s, a, d) = \left(\frac{\sigma_p - 1}{\sigma_p} B_p\right)^{\sigma}\left(\frac{\alpha}{r+\delta}\right)\left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.8}$$

$$n_p^*(s, a, d) = \left(\frac{\sigma_p - 1}{\sigma_p} B_p\right)^{\sigma}\left(\frac{1-\alpha}{w}\right)\left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.9}$$

where $c(r, w) \equiv \left[\left(\frac{r+\delta}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\right]$ is the optimal marginal cost of producing one unit of output for a firm with $s = 1$. The factor demands for the public sector are identical, just replacing B_p by B_g. Hence, adding up the sector-specific factor demands we obtain the aggregate factor demands,

$$k^*(s, a, d) = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma}\left(B_p^{\sigma} + B_g^{\sigma}\right)\left(\frac{\alpha}{r+\delta}\right)\left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.10}$$

$$n^*(s, a, d) = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma}\left(B_p^{\sigma} + B_g^{\sigma}\right)\left(\frac{1-\alpha}{w}\right)\left(\frac{s}{c(r,w)}\right)^{\sigma-1} \tag{E.11}$$

The ratio of both equations gives us that

$$\frac{k^*(s, a, d)}{n^*(s, a, d)} = \frac{\alpha}{1-\alpha}\frac{w}{r+\delta} \tag{E.12}$$

which states that the optimal capital to labor ratio is the same for all firms. This is also true for the capital to labor ratio in each sector, which equals the aggregate one. Finally, let's define $u^*(s, a, d) \equiv k_p^*(s, a, d)/k^*(s, a, d)$ as the optimal share of capital into the private sector. Then, we can write

$$u^*(s, a, d) = \frac{B_p^{\sigma}}{B_p^{\sigma} + B_g^{\sigma}} = \left(1 + \left(\frac{B_g}{B_p}\right)^{\sigma}\right)^{-1} \tag{E.13}$$

with $n_p^*(s, a, 1)/n^*(s, a, 1)$ and $y_p^*(s, a, 1)/y^*(s, a, 1)$ being also equal to $u^*(s, a, 1)$.

Profit and revenue functions. Next, note that profits are given by $\pi = p_p y_p + p_g y_g - (r+\delta)k - wn$, which given the condition for the optimal choice of capital can be written as $\pi^* = \frac{1}{\sigma}\left(p_p^* y_p^* + p_g^* y_g^*\right)$ or $\pi^* = \frac{1}{\sigma-1}\frac{r+\delta}{\alpha}k^*$. Substituting the optimal capital demand in equation

(E.10) we obtain

$$\pi^*(s,a,d) = \left(\frac{B_p^\sigma + B_g^\sigma}{\sigma}\right)\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{s}{c(r,w)}\right]^{\sigma-1} \tag{E.14}$$

Finally, it can also be shown that $k_p^* = \frac{\alpha}{r+\delta}\frac{\sigma-1}{\sigma}\left(p_p^* y_p^* + p_g^* y_g^*\right)$, $n_p^* = \frac{1-\alpha}{w}\frac{\sigma-1}{\sigma}\left(p_p^* y_p^* + p_g^* y_g^*\right)$.

Proposition 2 *For the unconstrained problem, the factor demands $k^*(s,a,d)$ and $n^*(s,a,d)$, firm size $y^*(s,a,d)$, and profits $\pi^*(s,a,1)$ all increase monotonically with productivity s and are invariant with the net worth a. The same is true for the sector-specific factor demands and output. Finally, the relative factor and output shares across sectors are independent of both s and a and are only determined by the relative sectoral demands B_p/B_g.*

Proof: Equations (E.8)-(E.14) plus the production function show this trivially. ■

E.3.2 Constrained firms

The problem for constrained firms does not deliver closed-form solutions for the policy and profit functions. However, one can still characterize how they vary with the state variables a and s.

Policy functions. The optimal solution is fully described by the FOC in equations (9)-(10) plus the financial constraint (11). To characterize the policy functions, we totally differentiate the binding borrowing constraint (11) with respect to the endogenous choices k_p, k_g, n_p, n_g and the exogenous variables a and s in turn, which gives,

$$\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)dk_p + \left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)dk_g = \phi_a da \tag{E.15}$$

$$\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)dk_p + \left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)dk_g = \left(\phi_p\frac{\partial p_p y_p}{\partial s} + \phi_g\frac{\partial p_g y_g}{\partial s}\right)ds \tag{E.16}$$

where all the terms dn_p and dn_g disappear due to FOC (9). If $\phi_p = 0$ and $\phi_g = 0$ we are in the case without earnings-based collateral constraints where $dk/da = \phi_a$ and $dk/ds = 0$, as higher net worth allows the firm to operate with more capital but higher productivity does not. With at least one of $\phi_p > 0$ or $\phi_g > 0$, we have $dk/ds > 0$, that is, constrained firms with higher productivity operate with more capital. Also, with at least one of $\phi_p > 0$ or $\phi_g > 0$, the derivative $dk/da > \phi_a$ because of the multiplier effect: the extra capital obtained by an increase in net worth a allows for an increase in revenues, which allows further borrowing. We show this in the next proposition.

Proposition 3 *The derivative of $k(a,s,d)$ w.r.t. net worth a is positive, while the derivative of $k(a,s,d)$ w.r.t. productivity s is positive as long as at least one of ϕ_p or ϕ_g is non-zero (and zero otherwise). Furthermore, these derivatives are bounded below by $\partial k(a,s,d)/\partial a > \phi_a$ and $\partial k(a,s,d)/\partial s > (\phi_p \partial p_p y_p/\partial s + \phi_g \partial p_g y_g/\partial s)\, ds$ as long as at least one of ϕ_p or ϕ_g is non-zero.*

Proof: The derivatives of k_p and k_g with respect to a and s are characterized by equations (E.15) and (E.16). Lemma 3 states that the terms multiplying dk_p and dk_g on the left-hand-side are positive and smaller than one as long as ϕ_p and ϕ_g are non-zero. Hence, equation (E.15) states that $dk_p + dk_g \geq \phi_a da$, with strict inequality if at least one of ϕ_p or ϕ_g is non-zero, which leads to $dk/da \geq \phi_a > 0$. The proof for $dk/ds \geq 0$ is identical. ■

Proposition 4 *More constrained firms (firms with higher λ) have lower capital to labor ratio on aggregate and in each sector.*

Proof: Dividing equations (E.2) and (E.4) or (E.3) and (E.5) we obtain

$$\frac{k_p}{n_p} = \frac{\alpha}{1-\alpha} \frac{w}{(r+\delta+\lambda)/(1+\lambda\phi_p)} \tag{E.17}$$

and

$$\frac{k_g}{n_g} = \frac{\alpha}{1-\alpha} \frac{w}{(r+\delta+\lambda)/(1+\lambda\phi_g)} \tag{E.18}$$

Given Lemma 1, the capital to labor ratio declines with λ in both sectors and hence it is lower for more financially constrained firms. The same applies for k/n. ■

Proposition 5 *Holding s constant, more constrained firms (firms with higher λ) produce less output $y(s,a,d)$ and use less capital $k(s,a,d)$ and less labor $n(s,a,d)$. Furthermore, their output and their factor demands are lower in both sectors.*

Proof: Let's combine the FOC (E.2) with the demand equation and equation (E.17) to produce the expression,

$$\frac{\sigma-1}{\sigma} B_p y_p^{-1/\sigma} = \frac{1}{s} \left[\frac{(r+\delta+\lambda)/(1+\lambda\phi_p)}{\alpha} \right]^{\alpha} \left[\frac{w}{1-\alpha} \right]^{1-\alpha} \tag{E.19}$$

which requires equalizing the marginal revenue of y_p (left-hand-side) to its marginal cost (right-hand-side). By virtue of Lemma 1 the marginal cost increases with λ whenever $\phi_p < (\delta + r)^{-1}$ and then y_p must fall with λ. Finally, Lemma 5 states that capital and labor demand move in the same direction. Hence, if $y_p = s k_p^{\alpha} n_p^{1-\alpha}$ falls with λ so must both k_p and n_p. The case for y_g is analogous, and the cases for y, k, and n follow. ■

Proposition 6 *More constrained firms (firms with higher λ) have (a) higher capital to labor ratio in the sector with more pledgeable revenues and (b) relative factor use larger in the sector with more pledgeable revenues. Instead, if $\phi_g = \phi_p$, relative factor use is as in the case with non-binding financial constraints.*

Proof: To prove (a) divide equations (E.17) and (E.18) to obtain

$$\frac{k_p/n_p}{k_g/n_g} = \frac{1+\lambda\phi_p}{1+\lambda\phi_g} \tag{E.20}$$

such that the relative capital to labor ratio increase with λ in the sector with higher ϕ. To prove (b), use the set of FOC in (E.2)-(E.5) and the explicit functional form for the revenue functions to write

$$\frac{u}{1-u} = \frac{k_p}{k_g} = \left(\frac{B_p}{B_g}\right)^{\sigma}\left(\frac{1+\lambda\phi_p}{1+\lambda\phi_g}\right)^{\alpha(\sigma-1)+1} \tag{E.21}$$

that is, the allocation of capital across sectors depends on the relationship between ϕ_g and ϕ_p as well as on the relative demand B_p/B_g. In particular, the relative use of capital increases with λ in the sector with higher ϕ. Combining equations (E.20) and (E.21) we see that the same is true for labor. Finally, with $\phi_g = \phi_p$ we see that equation (E.21) is equivalent to equation (E.13). ■

Proposition 7 *The derivative of $\lambda(s,a,d)$ with respect to net worth a is negative, while the derivative of $\lambda(s,a,d)$ with respect to productivity s is positive.*

Proof: The FOC for k_p in equation (10) relates capital k_p, the capital to labor ratio k_p/n_p, the multiplier λ, and productivity s. Indeed, using the revenue function, it can be written as,

$$\frac{\partial p_p y_p}{\partial k_p} = B_p\left(\frac{\sigma-1}{\sigma}\right)\alpha\left[s^{\frac{\sigma-1}{\sigma}}k_p^{-\frac{1}{\sigma}}\left(\frac{k_p}{n_p}\right)^{-(1-\alpha)\frac{\sigma-1}{\sigma}}\right] = \frac{r+\delta+\lambda}{1+\phi_p\lambda} \tag{E.22}$$

In turn, equation (E.17) shows that the capital to labor ratio k_p/n_p decreases with the cost of capital $(r+\delta+\lambda)/(1+\phi_p\lambda)$, so we can rewrite

$$B_p\left(\frac{\sigma-1}{\sigma}\right)\alpha\left[s^{\frac{\sigma-1}{\sigma}}k_p^{-\frac{1}{\sigma}}\left(\frac{\alpha}{1-\alpha}w\right)^{-(1-\alpha)\frac{\sigma-1}{\sigma}}\right] = \left(\frac{r+\delta+\lambda}{1+\phi_p\lambda}\right)^{1-(1-\alpha)\frac{\sigma-1}{\sigma}} \tag{E.23}$$

and a similar expression obtains for k_g. Let's start with a. Equation (E.23) implies that $\partial k_p/\partial a = (\partial k_p/\partial\lambda)(\partial\lambda/\partial a)$ and we will also have $\partial k_g/\partial a = (\partial k_g/\partial\lambda)(\partial\lambda/\partial a)$. Given that $k = k_p + k_g$, we can also write $\partial k/\partial a = (\partial k/\partial\lambda)(\partial\lambda/\partial a)$. The financial constraint implies that

$\partial k/\partial a > 0$ (Proposition 3), while the FOC for capital imply that $\partial k/\partial\lambda < 0$ (Proposition 5), hence it must the case that $\partial\lambda/\partial a < 0$. Next, let's look at the case of s. Taking the derivative with respect to s in both sides of equation (E.23) we get

$$\left[\frac{d}{d\lambda}\left(\frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)\right]\frac{d\lambda}{ds} \propto \left[(\sigma-1)-\frac{s}{k_p}\frac{dk_p}{ds}\right]$$

Hence, given that the cost of capital increaes with λ (Lemma 1) we have:

$$\frac{d\lambda}{ds} > 0 \quad \Leftrightarrow \quad \sigma - 1 > \frac{dk_p}{ds}\frac{s}{k_p} \tag{E.24}$$

That is, λ increases with s if the marginal revenue product of k_p increases with s, which will happen whenever the positive direct effect of s, $(\sigma-1)/\sigma$ is larger than the negative indirect effect of s through the increase in capital $\frac{1}{\sigma}\frac{dk_p}{ds}\frac{s}{k_p}$ obtained through the extra financing allowed by the earnings-based financial constraint. Following the same steps we can also see that,

$$\frac{d\lambda}{ds} > 0 \quad \Leftrightarrow \quad \sigma - 1 > \frac{dk_g}{ds}\frac{s}{k_g} \tag{E.25}$$

Next, using $\frac{\partial p_p y_p}{\partial s} = \frac{1}{\alpha}\frac{k_p}{s}\frac{\partial p_p y_p}{\partial k_p}$, equation (E.16) gives

$$\frac{dk_p}{ds}\frac{s}{k_p}u\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)+\frac{dk_g}{ds}\frac{s}{k_g}(1-u)\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)=\frac{1}{\alpha}\left[\phi_p u\frac{\partial p_p y_p}{\partial k_p}+\phi_g(1-u)\frac{\partial p_g y_g}{\partial k_g}\right]$$

where recall that $u \equiv k_p/k$. Then, given the inequalities in (E.24) and (E.25), $\frac{d\lambda}{ds} > 0$ requires

$$(\sigma-1)u\left(1-\phi_p\frac{\partial p_p y_p}{\partial k_p}\right)+(\sigma-1)(1-u)\left(1-\phi_g\frac{\partial p_g y_g}{\partial k_g}\right)>\frac{1}{\alpha}\left[\phi_p u\frac{\partial p_p y_p}{\partial k_p}+\phi_g(1-u)\frac{\partial p_g y_g}{\partial k_g}\right]$$

which implies,

$$u\phi_p\frac{\partial p_p y_p}{\partial k_p}+(1-u)\phi_g\frac{\partial p_g y_g}{\partial k_g} < \frac{\alpha(\sigma_p-1)}{1+\alpha(\sigma_p-1)} < 1$$

And Lemma 3 says that both $\phi_p \partial p_p y_p/\partial k_p < 1$ and $\phi_g \partial p_g y_g/\partial k_g < 1$, so the inequality is verified and it must be that $d\lambda/ds < 0$. ■

Corollary 1 *The derivatives with respect to net worth a of capital demands $k_p(s,a,d)$, $k_g(s,a,d)$, labor demands $n_p(s,a,d)$, $n_g(s,a,d)$, $n(s,a,d)$, and output supplies $y_p(s,a,d)$, $y_g(s,a,d)$, $y(s,a,d)$ are all positive.*

Proof: By the chain rule (see proof of Proposition 7), we write $\partial k_p/\partial a = (\partial k_p/\partial\lambda)(\partial\lambda/\partial a)$. The FOC for capital imply that $\partial k_p/\partial\lambda < 0$ (Proposition 5), and Proposition 7 tells that

$\partial\lambda/\partial a < 0$, so it must be that $\partial k_p/\partial a > 0$. The proof for $\partial k_g/\partial a > 0$ is identical. Given this, Lemma 5 proves the case for $n_p(s,a,d)$ and $n_g(s,a,d)$, and $n(s,a,d)$ follows. Given that both production factors in each sector increase with a so must output $y_p(s,a,d)$ and $y_g(s,a,d)$ in each sector, and so must total output $y(s,a,d)$. ■

Corollary 2 *If $\phi_p = \phi_g$, the derivatives with respect to productivity s of capital demands $k_p(s,a,d)$, $k_g(s,a,d)$, labor demands $n_p(s,a,d)$, $n_g(s,a,d)$, $n(s,a,d)$, and output supplies $y_p(s,a,d)$, $y_g(s,a,d)$, $y(s,a,d)$ are all positive.*

Proof: Equation (E.21) tells us that k_p/k_g is constant whenever $\phi_p = \phi_g$, which means that k_p and k_g comove with k. Hence, given that $dk/ds > 0$ (Proposition 3), it must also be that $dk_p/ds > 0$ and $dk_g/ds > 0$. The marginal revenue product of labor in each sector is equal to the wage, see equation (9). Given that whenever s increases so do k_p and k_s, equation (9) require $dn_p/ds > 0$ and $dn_g/ds > 0$ and hence $dn/ds > 0$. Finally, given that all production factors increase with s in all sectors, so do output in each sector and total output. ■

The results in Corollary 2 cannot be proved for $\phi_p \neq \phi_g$. The reason is that changes in s increase λ (Proposition 7), which moves the ratio k_p/k_g towards the sector with higher ϕ (Proposition 6). There is nothing preventing $dk_p/ds < 0$ while $dk/ds > 0$ whenever $\phi_g > \phi_p$.

Corollary 3 *The derivative of $k(s,a,d)$ with respect to net worth a increases with λ.*

Proof: With $\phi_g = \phi_p$ this is straightforward to show. Using the FOC in (10), equation (E.15) becomes:

$$\frac{\partial k(s,a,d)}{\partial a} = \phi_a \left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right)^{-1} \tag{E.26}$$

where the cost of capital $(r+\delta+\lambda)/(1+\lambda\phi_p)$ increases with λ (Lemma 1) and so does $\partial k(s,a,d)/\partial a$. For the case $\phi_g \neq \phi_p$, let's start noting that the terms multiplying dk_p and dk_g in equation (E.15) decline with λ (see Lemma 4). Then, at least one of dk_p or dk_g must increase with λ. Let's prove the corollary by contradiction by assuming that $dk/d\lambda \leq 0$. Rewrite equation (E.15) as,

$$\left(1 - \phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p}\right) dk + \left(\phi_p \frac{r+\delta+\lambda}{1+\lambda\phi_p} - \phi_g \frac{r+\delta+\lambda}{1+\lambda\phi_g}\right) dk_g = \phi_a da \tag{E.27}$$

where we have used again (10) and $dk = dk_g + dk_p$. Now, if $\phi_g > \phi_p$ the term in parenthesis multiplying dk_g is negative. With $dk \leq 0$ the first term in the left-hand-side is either zero or negative, which requires $dk_g < 0$ for equation (E.27) to hold. However, this is not possible as we need at least one of dk_p or dk_g to be positive and part (b) in Proposition 6 states that

k_g/k_p increases with λ when $\phi_g > \phi_p$ and therefore $dk_g > 0$, which generates the contradiction. Therefore, it must by that $dk/d\lambda > 0$. The case $\phi_g < \phi_p$ is analogous. ∎

Corollary 4 *The derivative of $\partial k(s,a,d)/\partial a$ is negative w.r.t. a and positive w.r.t. s.*

Proof: By the chain rule we can write

$$\begin{aligned}\frac{\partial^2 k(s,a,d)}{\partial a^2} &= \frac{\partial^2 k(s,a,d)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,d)}{\partial a} \\ \frac{\partial^2 k(s,a,d)}{\partial a \partial s} &= \frac{\partial^2 k(s,a,d)}{\partial a \partial \lambda}\frac{\partial \lambda(s,a,d)}{\partial s}\end{aligned}$$

The first derivative in the r.h.s. of these expressions is positive by Corollary 3. Hence, the sign of the derivatives $\frac{\partial^2 k(s,a,d)}{\partial a^2}$ and $\frac{\partial^2 k(s,a,d)}{\partial a \partial s}$ is the same as the sign of the derivatives $\frac{\partial \lambda(s,a,d)}{\partial a}$ and $\frac{\partial \lambda(s,a,d)}{\partial s}$ described in Proposition 7. ∎

Profit function. We can obtain the derivatives of the profit function $\pi(s,a,d)$ as

$$\frac{\partial \pi(s,a,d)}{\partial a} = \left[\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)\right]\frac{\partial k_p(s,a,d)}{\partial a} + \left[\frac{\partial p_g y_g}{\partial k_g} - (r+\delta)\right]\frac{\partial k_g(s,a,d)}{\partial a} \tag{E.28}$$

$$\begin{aligned}\frac{\partial \pi(s,a,d)}{\partial s} &= \left[\frac{\partial p_p y_p}{\partial k_p} - (r+\delta)\right]\frac{\partial k_p(s,a,d)}{\partial s} + \left[\frac{\partial p_g y_g}{\partial k_g} - (r+\delta)\right]\frac{\partial k_g(s,a,d)}{\partial s} \\ &+ \frac{\partial p_p y_p}{\partial s} + \frac{\partial p_g y_g}{\partial s}\end{aligned} \tag{E.29}$$

Using the FOC for k_p and k_g given by equations (10) to substitute out the terms in square brackets of equation (E.28), and next using equation (E.15) we obtain

$$\frac{\partial \pi(s,a,d)}{\partial a} = \phi_a \lambda(s,a,d) \tag{E.30}$$

likewise, doing the same thing with equations (10), (E.29), and (E.16) we obtain

$$\frac{\partial \pi(s,a,d)}{\partial s} = (1+\phi_p\lambda(s,a,d))\frac{\partial p_p y_p}{\partial s} + (1+\phi_g\lambda(s,a,d))\frac{\partial p_g y_g}{\partial s} \tag{E.31}$$

Proposition 8 *The derivatives of $\pi(s,a,d)$ w.r.t. net worth a and productivity s are positive.*

Proof: The derivatives of the profit function with respect to a and s are given by (E.30) and (E.31). These derivatives are positive because $\lambda(s,a,d) > 0$ for constrained agents and $\frac{\partial p_p y_p}{\partial s} > 0$ and $\frac{\partial p_g y_g}{\partial s} > 0$ (see the revenue functions). ∎

Profits increase with a because more net worth allows to increase capital and hence profits. Profits increase with s for two reasons. First, there is the direct increase of revenues with s for given capital. Second, if $\phi_p > 0$ and/or $\phi_g > 0$ the increase in revenues with s allows to increase capital, which in turn increases profits. We can also show, Corollary 5 below, that the marginal value of net worth decreases with net worth a and increases with productivity s.

Corollary 5 *The derivative of $\partial\pi(s,a,d)/\partial a$ is negative with respect to a and positive with respect to s.*

Proof: Using (E.30) we can write the second derivatives as, $\frac{\partial^2\pi(s,a,d)}{\partial a^2} = \phi_a \frac{\partial\lambda(s,a,d)}{\partial a}$ and $\frac{\partial^2\pi(s,a,d)}{\partial a \partial s} = \phi_a \frac{\partial\lambda(s,a,d)}{\partial s}$, with $\partial\lambda/\partial a < 0$ and $\partial\lambda/\partial s > 0$ (Proposition 7). ■

E.4 A procurement shock

Finally, in this Section we analyze how firm choices change upon arrival of a procurement project for the case $\sigma_p = \sigma_g = \sigma$. To do so, we compare the choices of firms in the $(s,a,1)$ state with firms in the $(s,a,0)$ state.

E.4.1 Unconstrained firms

For unconstrained firms, the increase in total capital is given by $\frac{k^*(s,a,1)}{k^*(s,a,0)} = 1 + \left(\frac{B_g}{B_p}\right)^\sigma$, as $k^*(s,a,0) = k_p^*(s,a,1)$, that is, the amount of capital used in the private sector for the unconstrained firm with a procurement project equals the capital stock it was using without procurement. This means that unconstrained firms do not change their private sector operations and increase their capital stock to meet the extra demand. The increase in capital $k^*(s,a,1) - k^*(s,a,0)$ is given by $\left(\frac{B_g}{B_p}\right)^\sigma k^*(s,a,0)$. Because $k^*(s,a,0)$ increases with s and is independent from a, so does the capital increase with procurement. We can also see that the value of a procurement contract increases with firm productivity s and is independent from firm net worth a. This can be seen by use of the expression $\pi = \frac{1}{\sigma-1}\frac{r+\delta}{\alpha}k$, which implies that $\pi^*(s,a,1) - \pi^*(s,a,0)$ is proportional to the capital increase $k^*(s,a,1) - k^*(s,a,0)$. This could have also be seen by using equation (E.14) for $d = 1$ and $d = 0$, which allows to express $\pi^*(s,a,1) - \pi^*(s,a,0) = \left(\frac{B_g^\sigma}{\sigma}\right)\left[\left(\frac{\sigma-1}{\sigma}\right)\frac{s}{c(r,w)}\right]^{\sigma-1}$.

E.4.2 Constrained firms

For financially constrained firms, the effects of a procurement shock are more intricate, depending on the size of ϕ_g relative to ϕ_p and the net worth of the firm. Let's start by stating, in Lemma 6 and Lemma 7, two results that relate the private sector spillovers of a procurement shock to the change in the severity of the financial constraints.

Lemma 6 *A procurement shock generates a negative private sector spillover if and only if the procurement shock makes the firm more constrained, that is,* $y_p(s,a,1) < y_p(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$.

Proof: The FOC for optimal k_p is given by equation (E.4), whose right-hand side increases with λ (see Lemma 1), so more constrained firms have a higher marginal revenue product of capital k_p. Proposition 5 establishes that this happens with lower k_p, lower n_p, and lower y_p. Hence, $y_p(s,a,1) < y_p(s,a,0) \Leftrightarrow \lambda(s,a,1) > \lambda(s,a,0)$. ■

Lemma 7 *A procurement shock generates a negative private sector spillover for constrained firms if and only if the chosen production for the public sector cannot be self-financed, that is, if and only if* $\phi_g[p_g(s,a,1)\,y_g(s,a,1) - wn_g(s,a,1)] < k_g(s,a,1)$

Proof: A private sector negative spillover of procurement means that $y_p(s,a,1) < y_p(s,a,0)$. Given that capital and labor in each sector co-move together (Lemma 5), we have that a private sector negative spillover requires both $k_p(s,a,1) < k_p(s,a,0)$ and $n_p(s,a,1) < n_p(s,a,0)$. Let's define $R_p(s,a,d) \equiv p_p(s,a,d)\,y_p(s,a,d)$ and $R_g(s,a,d) \equiv p_g(s,a,d)\,y_g(s,a,d)$. The choices of capital for constrained firms, with and without procurement, are given by

$$\begin{aligned} k_p(s,a,0) - \phi_p(R_p(s,a,0) - wn_p(s,a,0)) &= \phi_a a \\ k_p(s,a,1) - \phi_p(R_p(s,a,1) - wn_p(s,a,1)) &= \phi_a a - [k_g(s,a,1) - \phi_g(R_g(s,a,1) - wn_g(s,a,1))] \end{aligned}$$

Importantly, the left-hand side of these equations increases with k_p. To see how, first note that the derivative of this term w.r.t. k_p is equal to $1 - \phi_p \frac{\partial p_p y_p}{\partial k_p}$, as the terms with n cancel out due to the FOC in equation (9). Now, $\phi_p \frac{\partial p_p y_p}{\partial k_p} < 1$ according to Lemma 3, so the derivative is positive. Hence, $k_p(s,a,1) < k_p(s,a,0)$ (a private sector negative spillover of procurement) implies

$$k_p(s,a,1) - \phi_p[R_p(s,a,1) - wn_p(s,a,1)] < k_p(s,a,0) - \phi_p[R_p(s,a,0) - wn_p(s,a,0)]$$

which requires $k_g(s,a,1) > \phi_g[R_g(s,a,1) - wn_g(s,a,1)]$. ■

Given this, we can now analyze the effect of a procurement shock in two special cases. First, when there are no earnings-based financial constraints, a procurement shock always generates a negative private sector spillover, as any given collateral a has to be split between the public and private sector markets, making the firm more financially constrained.

Proposition 9 *When* $\phi_g = 0$ *and* $\phi_p = 0$*, if* $\phi_a > 0$ *and* $a > 0$ *a procurement shock does not change firm size, that is,* $k(s,a,1) = k(s,a,0)$*, makes the firm more constrained, that is,* $\lambda(s,a,1) > \lambda(s,a,0)$*, and generates a negative private sector spillover, that is,* $y_p(s,a,1) <$

$y_p(s,a,0)$.

Proof: We can write the borrowing constraint for $d=0$ and $d=1$ firms as

$$\begin{aligned} k_p(s,a,0) &= \phi_a a \\ k_p(s,a,1)+k_g(s,a,1) &= \phi_a a \end{aligned}$$

so it must be that $k_p(s,a,1)+k_g(s,a,1)=k_p(s,a,0)$ and hence firm size does not change with procurement. The FOC for k_g, equation (E.5), states that $k_g(s,a,1)>0$, hence it must be $k_p(s,a,1)<k_p(s,a,0)$. Lemma 6 then implies that $\lambda(s,a,1)>\lambda(s,a,0)$. ■

Things are different in the other simple case, when there are earnings-based financial constraints but no collateral constraints (or, in the general case for firms with $a=0$). In this case, a procurement shock makes the firm less financially constrained if $\phi_g>\phi_p$, allowing it to grow and generating a positive private sector spillover. However, whenever $\phi_g<\phi_p$ the firm becomes more financially constrained, there is a negative private sector spillover, and the firm may even shrink in size.

Proposition 10 *When $\phi_a=0$ (or $\phi_a>0$ but $a=0$) and $\phi_g>\phi_p$, a procurement shock increases firm size, that is, $k(s,a,1)>k(s,a,0)$, makes the firm less constrained, that is, $\lambda(s,a,1)<\lambda(s,a,0)$, and generates a positive private sector spillover, that is, $y_p(s,a,1)>y_p(s,a,0)$. Instead, if $\phi_g<\phi_p$, a procurement shock makes the firm more constrained, generates a negative private sector spillover, and might reduce firm size if ϕ_g is low enough. Whenever $\phi_g=\phi_p$, a procurement shock changes neither λ nor y_p and allows the firm to increase its size.*

Proof: Let's define $R_p(s,a,d)\equiv p_p(s,a,d)\,y_p(s,a,d)$ and $R_g(s,a,d)\equiv p_g(s,a,d)\,y_g(s,a,d)$. We can take the difference of the borrowing constraint for $d=1$ and $d=0$ firms to write,

$$\tilde{\phi}_p\frac{R_p(s,a,0)}{k_p(s,a,0)} = u(s,a,1)\,\tilde{\phi}_p\frac{R_p(s,a,1)}{k_p(s,a,1)}+(1-u(s,a,1))\,\tilde{\phi}_g\frac{R_g(s,a,1)}{k_g(s,a,1)} \tag{E.32}$$

where $u\equiv k_p/k$, and we have used equations (E.6) and (E.7) to replace revenues minus labor costs in the financial constraint and have defined $\tilde{\phi}_p\equiv\phi_p[1-(1-\alpha)(\sigma-1)/\sigma]$ and $\tilde{\phi}_g\equiv\phi_g[1-(1-\alpha)(\sigma-1)/\sigma]$. If $\phi_g=\phi_p$ (and hence $\tilde{\phi}_g=\tilde{\phi}_p$), firms with $d=1$ equalize the average product in the public and private sectors, see equations (E.4) and (E.5), and hence equation (E.32) becomes $R_p(s,a,0)/k_p(s,a,0)=R_p(s,a,1)/k_p(s,a,1)$, which implies $k_p(s,a,1)=k_p(s,a,0)$ and hence there is no private sector spillover. Because the average product? in the public sector goes to infinity when $k_g=0$, it must be that $k_g(s,a,1)>0$ and hence $k(s,a,1)\equiv k_p(s,a,1)+k_g(s,a,1)>k(s,a,0)$, that is, a procurement shock makes the firm grow. If $\phi_g>\phi_p$, then the second term in the weighted average of the right-hand-side of equation (E.32) is larger than the first one. This can be easily seen by multiplying both sides of equation (E.4) by $\tilde{\phi}_p$ and both sides of equation (E.5) by $\tilde{\phi}_g$. Then equation (E.32) implies that $R_p(s,a,1)/k_p(s,a,1) < R_p(s,a,0)/k_p(s,a,0)$ and hence $k_p(s,a,1) > k_p(s,a,0)$, that is, there is a positive private sector spillover, and because $k_g(s,a,1) > 0$ it must be that $k(s,a,1) > k(s,a,0)$. If $\phi_g < \phi_p$, then the second term in the weighted average of the right-hand-side of equation (E.32) is smaller than the first one. Then $R_p(s,a,1)/k_p(s,a,1) > R_p(s,a,0)/k_p(s,a,0)$ and $k_p(s,a,1) < k_p(s,a,0)$, that is, there is a negative private sector spillover. The sign of $k(s,a,1) - k(s,a,0) = k_p(s,a,1) - k_p(s,a,0) + k_g(s,a,1)$ is underermined. However, note that when $\phi_g = 0$, the whole second term in the right-hand-side of equation (E.32) vanishes and we have that $R_p(s,a,0)/k_p(s,a,0) = R_p(s,a,1)/k(s,a,1)$ or $k(s,a,1)/k(s,a,0) = R_p(s,a,1)/R_p(s,a,0)$. Because of the negative private sector spillover, this ratio is less than one and hence $k(s,a,1) < k(s,a,0)$. That is, with a small enough ϕ_g a procurement shock may make the firms shrink in size. Finally, the results for λ follow from Lemma 6. ■

That is, when firms have no collateral and the only financing comes from revenues, if $\phi_g = \phi_p$ a procurement shock has a similar effect as for unconstrained firms: it leaves private sector activity unchanged and the firm increases in size to serve the public sector demand. If $\phi_g > \phi_p$, a procurement shock increases the supply of credit more than its demand, it allows the firm to grow, it makes it less constrained, and it generates a positive private sector spillover. This is not in contradiction with part (b) in Proposition 6, which states that whenever $\phi_g > \phi_p$ the firm tilts capital towards the public sector. To understand why, imagine that a procurement shock generated no private sector spillover such that $k_p(s,a,1) = \tilde{\phi}_p R_p(s,a,1)$ and $k_g(s,a,1) = \tilde{\phi}_g R_g(s,a,1)$. With $\phi_g > \phi_p$ this would allow a lower marginal (and average) product of capital in the public sector, that is $R_g(s,a,1)/k_g(s,a,1) < R_p(s,a,1)/k_p(s,a,1)$. A positive private sector spillover happens because part of the extra financing in the public sector is used in the private sector, which allows $k_p(s,a,1) > k_p(s,a,0)$ while still having $R_g(s,a,1)/k_g(s,a,1) < R_p(s,a,1)/k_p(s,a,1)$.

Finally, putting the last two propositions together, we can state that,

Proposition 11 *Take the case with $\phi_a > 0$.*

1. *Whenever $\phi_g = \phi_p$, a procurement shock increases firm size, makes the firm more constrained, and generates a negative private sector spillover.*

2. *Whenever $\phi_g < \phi_p$, a procurement shock has the same effects as the case $\phi_g = \phi_p$ unless ϕ_g is small enough, which may lead to a decline in firm size.*

3. *Whenever $\phi_g > \phi_p$, a procurement shock has the same effects as the case $\phi_g = \phi_p$ unless a is small enough, which may relax the firm financial constraint and generate a positive*

private sector spillover.

Proof: Proposition 9 states that whenever $\phi_a > 0$ and $\phi_g = \phi_p = 0$, because of scarce collateral a, a procurement shock does not let the firm borrow more and grow, makes the firm more financially constrained because of the extra demand, and generates a negative private sector spillover to allow production for the public sector. When $\phi_a = 0$ and $\phi_g = \phi_p > 0$ Proposition 10 shows that a procurement shock allows the firm to grow, does not change the severity of the financial constraint, and does not generate any private sector spillover. Hence, putting together both propositions proves (a). For the case (b), $\phi_a > 0$ and $\phi_g < \phi_p$, Proposition 9 and Proposition 10 coincide in stating an increase in the severity of the financial constraint and a negative private sector spillover. Regarding the change in firm size, the collateral channel in Proposition 9 does not allow the firm to grow with a procurement shock, while the revenue channel in Proposition 10 states that the firm will generally grow in size, but it may shrink if ϕ_g is small enough. So this property should be inherited when both channels are active. To show it formally, we can take the difference between the borrowing constraint of any firm under the $d = 1$ and $d = 0$ cases to get,

$$k(s,a,1) - k(s,a,0) = \tilde{\phi}_p\left[R_p(s,a,1) + R_g(s,a,1) - R_p(s,a,0)\right] + \left(\tilde{\phi}_g - \tilde{\phi}_p\right)R_g(s,a,1) \tag{E.33}$$

where we have defined $R_p(s,a,d)$, $R_g(s,a,d)$, $\tilde{\phi}_p$, and $\tilde{\phi}_g$ as in the proof of Proposition 10. Now, if $\phi_g = \phi_p$, the second term in the right-hand-side of equation (E.33) disappears. If the firm keeps capital unchanged after a procurement shock, $k(s,a,1) = k(s,a,0)$, and reallocates some capital to the public sector such that $k_g(s,a,1) > 0$ revenues increase due to the concavity of the revenue functions, that is, $R_p(s,a,1) + R_g(s,a,1) > R_p(s,a,0)$ which increases borrowing capacity and hence $k(s,a,1)$. However, whenever $\phi_p > \phi_g$, the extra borrowing through higher revenues of shifting capital towards the public sector is offset by the lower pledgeability of public sector revenues, as shown in the 2nd term of equation (E.33). In net, it is not clear whether total borrowing capacity and hence $k(s,a,1)$ will go up or down, as it will depend on the size of $\tilde{\phi}_g$. For example, with $\phi_p > \phi_g = 0$ we will have $k(s,a,1) < k(s,a,0)$. To see why, note that with $\phi_g = 0$ equation (E.33) becomes

$$k(s,a,1) - k(s,a,0) = \tilde{\phi}_p\left[R_p(s,a,1) - R_p(s,a,0)\right]$$

Because of the negative private sector spillover whenever $\phi_p > \phi_g$, $R_p(s,a,1) < R_p(s,a,0)$ and thus $k(s,a,1) < k(s,a,0)$. Finally, for the case (c), $\phi_a > 0$ and $\phi_g > \phi_p$, Proposition 9 and Proposition 10 together imply that a procurement shock makes the firm grow because of the extra credit that comes with procurement. Regarding the private sector spillover, let's do as in proof of Proposition 10 and rewrite the difference of the borrowing constraint for

$d = 1$ and $d = 0$ firms as,

$$\begin{aligned}\tilde{\phi}_p \frac{R_p(s,a,0)}{k_p(s,a,0)} &= \tilde{\phi}_p \frac{R_p(s,a,1)}{k_p(s,a,1)} + (1 - u(s,a,1)) \left[\tilde{\phi}_g \frac{R_g(s,a,1)}{k_g(s,a,1)} - \tilde{\phi}_p \frac{R_p(s,a,1)}{k_p(s,a,1)} \right] \\ &+ \phi_a a \left[\frac{1}{k(s,a,1)} - \frac{1}{k(s,a,0)} \right] \end{aligned} \tag{E.34}$$

The second term in equation (E.34) is positive whenever $\phi_g > \phi_p$ (see proof of Proposition 10). Hence, if $a = 0$, equation (E.34) implies $R_p(s,a,1)/k_p(s,a,1) < R_p(s,a,0)/k_p(s,a,0)$, which requires $k_p(s,a,1) > k_p(s,a,0)$, a positive private sector spillover. Now, if $a > 0$, the third term in the right-hand side of equation (E.34) reappears. Because $k(s,a,1) > k(s,a,0)$ whenver $\phi_g > \phi_p$ this term is negative. With low enough a, the sum of the second and third term in the right-hand side of equation (E.34) is positive and hence we still have $R_p(s,a,1)/k_p(s,a,1) < R_p(s,a,0)/k_p(s,a,0)$ and $k_p(s,a,1) > k_p(s,a,0)$. However, with larger a the sum of the second and third term in the right-hand side of equation (E.34) is negative and hence we will have $R_p(s,a,1)/k_p(s,a,1) > R_p(s,a,0)/k_p(s,a,0)$ and $k_p(s,a,1) < k_p(s,a,0)$, that is, a negative private sector spillover. The result for $\lambda(s,a,1) - \lambda(s,a,0)$ follows from Lemma 6. ∎

Hence, whenever $\phi_g > \phi_p$ (our empirically relevant case), a negative private sector spillover happens due to the collateral constraints and despite the earnings-based financial constraint, which pushes in the opposite direction.

Proposition 12 *The value of procurement is strictly positive, that is, $\pi(s,a,1) - \pi(s,a,0) > 0\ \forall s, a$. For $\phi_g \geq \phi_p$, the value of procurement is increasing in net worth a and in productivity s, except for the case of firms with very low net worth whenever $\phi_g > \phi_p$.*

Proof: The first part is trivial. A firm with $d = 1$ can always replicate the profits of a firm with $d = 0$ by choosing $k_p(s,a,1) = k_p(s,a,0)$ and $k_g(s,a,1) = 0$. Because of the concavity of the revenue functions, it is optimal for any firm with $d = 1$ to reallocate capital across sectors, choose $k_g(s,a,1) > 0$, and increase profits. For the effect of net worth a on the value of procurement, we want to show that $\partial[\pi(s,a,1) - \pi(s,a,0)]/\partial a > 0$. Equation (E.30) implies that

$$\frac{\partial[\pi(s,a,1) - \pi(s,a,0)]}{\partial a} = \phi_a \big[\lambda(s,a,1) - \lambda(s,a,0) \big]$$

and the sign of $\lambda(s,a,1) - \lambda(s,a,0)$ is given by Proposition 11. Finally, for the effect of productivity s on the value of procurement we want to show that $\partial[\pi(s,a,1) - \pi(s,a,0)]\partial s > 0$

whenever $\phi_g \geq \phi_p$. Equation (E.31) implies

$$\begin{aligned}\frac{\partial\left[\pi\left(s,a,1\right)-\pi\left(s,a,0\right)\right]}{\partial s} &= \left(1+\phi_p\lambda\left(s,a,1\right)\right)\frac{\partial p_p y_p}{\partial s}+\left(1+\phi_g\lambda\left(s,a,1\right)\right)\frac{\partial p_g y_g}{\partial s} \\ &- \left(1+\phi_p\lambda\left(s,a,0\right)\right)\frac{\partial p_p y_p}{\partial s}\end{aligned}$$

Note that $\frac{\partial p_p y_p}{\partial s} = \frac{1}{\alpha}\frac{k_p}{s}\frac{\partial p_p y_p}{\partial k_p} = \frac{1}{\alpha}\frac{k_p}{s}\frac{r+\delta+\lambda}{1+\lambda\phi_p}$ and an analogous expression holds for $\frac{\partial p_g y_g}{\partial s}$. Substituting these expressions in the above equation gives

$$\frac{\partial\left[\pi\left(s,a,1\right)-\pi\left(s,a,0\right)\right]}{\partial s} = \frac{r+\delta+\lambda\left(s,a,1\right)}{\alpha s}\left[k_p\left(s,a,1\right)+k_g\left(s,a,1\right)\right]-\frac{r+\delta+\lambda\left(s,a,0\right)}{\alpha s}k_p\left(s,a,0\right)$$

With $\phi_g \geq \phi_p$, Proposition 11 states that $k_p\left(s,a,1\right)+k_g\left(s,a,1\right) > k_p\left(s,a,0\right)$. Therefore, whenever $\lambda\left(s,a,1\right) > \lambda\left(s,a,0\right)$ we can guarantee that $\frac{\partial\left[\pi\left(s,a,1\right)-\pi\left(s,a,0\right)\right]}{\partial s} > 0$. Proposition 11 shows this will always happen when $\phi_g = \phi_p$, and also for $\phi_g > \phi_p$ except for very low a. ■

F Equilibrium definition

Let $\mathbf{X} \equiv \mathbf{S}\times\mathbf{A}\times\{0,1\}$ be the state space of the entrepreneur problem, $\mathbf{X}_1 \equiv \mathbf{S}\times\mathbf{A}\times\{1\}$ the subset of the state space for firms with a procurement project, $\mathcal{X}$ a σ-algebra generated by $\mathbf{X}$, and Γ a probability measure over $\mathcal{X}$. Given government policy parameters (Y_g, m_g) and a distribution of entrant entrepreneurs Γ_0 over $\mathcal{X}$, we define the steady state equilibrium of the model as (i) a triplet of prices (w, r, P_g), (ii) a level constant $\bar{p}$ and a tax τ, (iii) firms' policy functions, (iv) macroeconomic aggregates (Y_p, C, A, T), (v) and a probability measure Γ, such that:

1. The household solves its optimization problem, which delivers the modified golden rule condition $\beta\left(1+r\right)=1$ and the budget constraint $C = Ar + w - \tau + T$

2. Entrepreneurs solve their optimization problem, which characterizes the firms' policy functions

3. The final good producers solve their optimization problems and the markets for each intermediate variety clear. This means that,

$$\int_{\mathbf{X}_1} p_g\left(s,a,1\right)y_g\left(s,a,1\right)\mathrm{d}\Gamma = P_g Y_g \quad \text{and} \quad \int_{\mathbf{X}} p_p\left(s,a,d\right)y_p\left(s,a,d\right)\mathrm{d}\Gamma = Y_p$$

4. The factor markets clear

$$\int_{\mathbf{X}} k\left(s,a,d\right)\mathrm{d}\Gamma = \int_{\mathbf{X}} a\,\mathrm{d}\Gamma + A \quad \text{and} \quad \int_{\mathbf{X}} n\left(s,a,d\right)\mathrm{d}\Gamma = 1$$

5. The probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector:

$$\int_{\mathbf{X}} \Pr\left(d' = 1 \mid b\left(s,a,d\right), d, s\right) \mathrm{d}\Gamma = m_g \tag{F.1}$$

6. The budget constraint of the government holds: $P_g Y_g = \tau$

7. The transfers to the household are given by the net accidental bequests,

$$T = (1-\theta)\left[\int_{\mathbf{X}} a'(s,a,d)\mathrm{d}\Gamma - \int_{\mathbf{X}} a\,\mathrm{d}\Gamma_0\right]$$

8. Γ is the stationary distribution of entrepreneurs consistent with the law of motion of the distribution properly obtained from the policy functions of the entrepeneurs, the law of motion of the productivity shocks s, and the probability function of procurement.

9. By Walras law, the market for the private good clears:

$$Y_p = \int_{\mathbf{X}} \left[b\left(s,a,d\right) + c\left(s,a,d\right) + \delta k\left(s,a,d\right)\right] \mathrm{d}\Gamma + C$$

G Aggregate output, productivity, and prices

GDP and aggregate TFP. We can define GDP in the model as

$$Y \;\equiv\; Y_p + P_g Y_g \;=\; \mathrm{TFP}_p K_p^{\alpha} N_p^{1-\alpha} + P_g \mathrm{TFP}_g K_g^{\alpha} N_g^{1-\alpha} \;=\; \mathrm{TFP} K^{\alpha} N^{1-\alpha} \tag{G.1}$$

where $K \equiv K_p + K_g$, $N \equiv N_p + N_g$, and aggregate TFP in units of the private goods is defined by,

$$\mathrm{TFP} \equiv \left(\frac{K_p}{K}\right)^{\alpha}\left(\frac{N_p}{N}\right)^{1-\alpha} \mathrm{TFP}_p + P_g \left(\frac{K_g}{K}\right)^{\alpha}\left(\frac{N_g}{N}\right)^{1-\alpha} \mathrm{TFP}_g \tag{G.2}$$

sectoral TFP. The TFP for the private and public sectors are given by,

$$\mathrm{TFP}_p \equiv \frac{Y_p}{K_p^{\alpha} N_p^{1-\alpha}} = \left\{\int_{[0,1]} \left[s_i \left(\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\alpha}\left(\frac{\overline{\mathrm{MRPN}}_p}{\mathrm{MRPN}_{ip}}\right)^{1-\alpha}\right]^{\sigma_p - 1} \mathrm{d}i\right\}^{\frac{1}{\sigma_p - 1}} \tag{G.3}$$

$$\mathrm{TFP}_g \equiv \frac{Y_g}{K_g^{\alpha} N_g^{1-\alpha}} = \left\{\int_{[0,1]} \left[s_i \left(\frac{\overline{\mathrm{MRPK}}_g}{\mathrm{MRPK}_{ig}}\right)^{\alpha}\left(\frac{\overline{\mathrm{MRPN}}_g}{\mathrm{MRPN}_{ig}}\right)^{1-\alpha}\right]^{\sigma_g - 1} \mathrm{d}i\right\}^{\frac{1}{\sigma_g - 1}} \tag{G.4}$$

$$\text{where} \qquad \frac{1}{\overline{\mathrm{MRPK}}_p} \equiv \int_{[0,1]} \frac{p_{ip}y_{ip}}{P_pY_p}\frac{1}{\mathrm{MRPK}_{ip}}\mathrm{d}i, \quad \frac{1}{\overline{\mathrm{MRPK}}_g} \equiv \int_{I_g} \frac{1}{m_g}\frac{p_{ig}y_{ig}}{P_gY_g}\frac{1}{\mathrm{MRPK}_{ig}}\mathrm{d}i$$

$$\frac{1}{\overline{\mathrm{MRPN}}_p} \equiv \int_{[0,1]} \frac{p_{ip}y_{ip}}{P_pY_p}\frac{1}{\mathrm{MRPN}_{ip}}\mathrm{d}i, \quad \frac{1}{\overline{\mathrm{MRPN}}_g} \equiv \int_{I_g} \frac{1}{m_g}\frac{p_{ig}y_{ig}}{P_gY_g}\frac{1}{\mathrm{MRPN}_{ig}}\mathrm{d}i$$

As shown in Section 4.3.2, there is no heterogeneity in MRPN_{ig} nor MRPN_{ip} in the model. Therefore, sectoral productivities become:

$$\mathrm{TFP}_p \equiv \frac{Y_p}{K_p^\alpha N_p^{1-\alpha}} = \left\{\int_{[0,1]}\left[s_i\left(\frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^\alpha\right]^{\sigma_p-1}\mathrm{d}i\right\}^{\frac{1}{\sigma_p-1}} \tag{G.5}$$

$$\mathrm{TFP}_g \equiv \frac{Y_g}{K_g^\alpha N_g^{1-\alpha}} = \left\{\int_{[0,1]}\frac{1}{m_g}\left[s_i\left(\frac{\overline{\mathrm{MRPK}}_g}{\mathrm{MRPK}_{ig}}\right)^\alpha\right]^{\sigma_g-1}\mathrm{d}i\right\}^{\frac{1}{\sigma_g-1}} \tag{G.6}$$

Absent financial frictions, there would be no heterogeneity in MRPK_{ip} and MRPK_{ig} either and optimal TFP in the private and public sectors (conditional on selection) would be,

$$\mathrm{TFP}_p^* = \left[\int_{[0,1]} s_i^{\sigma_p-1}\mathrm{d}i\right]^{\frac{1}{\sigma_p-1}} \quad \text{and} \quad \mathrm{TFP}_g^* = \left[\int_{I_g}\frac{1}{m_g}s_i^{\sigma_g-1}\mathrm{d}i\right]^{\frac{1}{\sigma_g-1}} \tag{G.7}$$

Relative price of public sector good. Using the definitions of P_g and P_p, the relative price can be written as,

$$\frac{P_g}{P_p} = \frac{\left[\int_{I_g}\frac{1}{m_g}p_{ig}^{1-\sigma_g}\mathrm{d}i\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]} p_{ip}^{1-\sigma_p}\mathrm{d}i\right]^{\frac{1}{1-\sigma_p}}} = \frac{\left[\int_{I_g}\frac{1}{m_g}\left(\frac{1}{s_i}\ \mathrm{MRPK}_{ig}^\alpha\ \mathrm{MRPN}_{ig}^{1-\alpha}\right)^{1-\sigma_g}\mathrm{d}i\right]^{\frac{1}{1-\sigma_g}}}{\left[\int_{[0,1]}\left(\frac{1}{s_i}\ \mathrm{MRPK}_{ip}^\alpha\ \mathrm{MRPN}_{ip}^{1-\alpha}\right)^{1-\sigma_p}\mathrm{d}i\right]^{\frac{1}{1-\sigma_p}}}$$

which follows from the definitions of MRPK_{ip} and MRPN_{ip}, and the production function as,

$$\mathrm{MRPK}_{ip} \equiv \frac{\partial p_{ip}y_{ip}}{\partial k_{ip}} = \alpha\frac{\sigma_p-1}{\sigma_p}\frac{p_{ip}y_{ip}}{k_{ip}}, \quad \mathrm{MRPN}_{ip} \equiv \frac{\partial p_{ip}y_{ip}}{\partial n_{ip}} = (1-\alpha)\frac{\sigma_p-1}{\sigma_p}\frac{p_{ip}y_{ip}}{k_{ip}}$$

$$\Rightarrow \quad p_{ip} = \frac{\sigma_p}{\sigma_p-1}\frac{1}{s_i}\left(\frac{\mathrm{MRPK}_{ip}}{\alpha}\right)^\alpha\left(\frac{\mathrm{MRPN}_{ip}}{1-\alpha}\right)^{1-\alpha}$$

and the same applies for MRPK_{ig} and MRPN_{ig}. Next multiplying and dividing by $\overline{\mathrm{MRPK}}_g^\alpha$ and $\overline{\mathrm{MRPN}}_g^{1-\alpha}$ in the numerator, by $\overline{\mathrm{MRPK}}_p^\alpha$ and $\overline{\mathrm{MRPN}}_p^{1-\alpha}$ in the denominator, and using the fact that our model generates no dispersion in MRPN_{ig}, we obtain,

$$\frac{P_g}{P_p} = \frac{\overline{\mathrm{MRPK}}_g}{\overline{\mathrm{MRPK}}_p} \frac{\left\{\int_{I_g} \frac{1}{m_g} \left[\frac{1}{s_i}\left(\frac{\mathrm{MRPK}_{ig}}{\overline{\mathrm{MRPK}}_g}\right)^{\alpha}\right]^{1-\sigma_g} \mathrm{d}i\right\}^{\frac{1}{1-\sigma_g}}}{\left\{\int_{[0,1]} \left[\frac{1}{s_i}\left(\frac{\mathrm{MRPK}_{ip}}{\overline{\mathrm{MRPK}}_p}\right)^{\alpha}\right]^{1-\sigma_p} \mathrm{d}i\right\}^{\frac{1}{1-\sigma_p}}} = \left(\frac{\overline{\mathrm{MRPK}}_g}{\overline{\mathrm{MRPK}}_p}\right)^{\alpha} \frac{\mathrm{TFP}_p}{\mathrm{TFP}_g}$$

Relative sectoral TFP. Given the definition of TFP_p in equation (G.3), we can write

$$\begin{aligned}
\mathrm{TFP}_p &= \left[m_g \int_{I_g} \frac{1}{m_g}\left(s_i \frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\sigma_p-1} \mathrm{d}i + (1-m_g)\int_{I_g^c} \frac{1}{1-m_g}\left(s_i \frac{\overline{\mathrm{MRPK}}_p}{\mathrm{MRPK}_{ip}}\right)^{\sigma_p-1} \mathrm{d}i\right]^{\frac{1}{\sigma_p-1}} \\
&= \left[m_g \mathrm{TFP}_{p,I_g}^{\sigma_p-1} + (1-m_g)\,\mathrm{TFP}_{p,I_g^c}^{\sigma_p-1}\right]^{\frac{1}{\sigma_p-1}}
\end{aligned}$$

where we have defined TFP_{p,I_g} and TFP_{p,I_g^c} as the average TFP in the private sector within the set of procurement (I_g) and non-procurement (I_g^c) firms respectively. Then, dividing by TFP_g in both sides we get the expression for $\mathrm{TFP}_p/\mathrm{TFP}_g$:

$$\frac{\mathrm{TFP}_p}{\mathrm{TFP}_g} = \left[m_g\left(\frac{\mathrm{TFP}_{p,I_g}}{\mathrm{TFP}_g}\right)^{\sigma_p-1} + (1-m_g)\left(\frac{\mathrm{TFP}_{p,I_g^c}}{\mathrm{TFP}_g}\right)^{\sigma_p-1}\right]^{\frac{1}{\sigma_p-1}} \tag{G.8}$$

The first term in equation (G.8) reflects the within-firm misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0\ \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and the dispersion of MRPK_{ig} declines, which makes $\mathrm{TFP}_{p,I_g}/\mathrm{TFP}_g$ fall. The second term in equation (G.8) reflects both between-firm misallocation and selection into procurement. If firms with higher s self-select into procurement, then $\mathrm{TFP}_{p,I_g^c}/\mathrm{TFP}_g$ declines. If there is more dispersion in MRPK_{ip} between non-procurement firms than in MRPK_{ig} between procurement firms, then $\mathrm{TFP}_{p,I_g^c}/\mathrm{TFP}_g$ is lower. In short, absent financial frictions the only reason for $\mathrm{TFP}_p/\mathrm{TFP}_g \neq 1$ would be the selection of firms into procurement. In the first best (no financial frictions and the government selects the firms with highest s) we would have $\mathrm{TFP}_p/\mathrm{TFP}_g < 1$.

Relative sectoral $\overline{\mathbf{MRPK}}$. Given the definition of $\overline{\mathrm{MRPK}}_p$ in (G.5), we can write

$$\begin{aligned}
\overline{\mathrm{MRPK}}_p &= \left[\frac{R_{p,I_g}}{P_pY_p}\int_{I_g}\frac{p_{ip}y_{ip}}{R_{p,I_g}}\mathrm{MRPK}_{ip}^{-1}\mathrm{d}i + \frac{R_{p,I_g^c}}{P_pY_p}\int_{I_g^c}\frac{p_{ip}y_{ip}}{R_{p,I_g^c}}\mathrm{MRPK}_{ip}^{-1}\mathrm{d}i\right]^{-1} \\
&= \left[\frac{R_{p,I_g}}{P_pY_p}\overline{\mathrm{MRPK}}_{p,I_g}^{-1} + \frac{R_{p,I_g^c}}{P_pY_p}\overline{\mathrm{MRPK}}_{p,I_g^c}^{-1}\right]^{-1}
\end{aligned}$$

where R_{p,I_g} and R_{p,I_g^c} denote total revenues in the private sector by procurement firms and non-procurement firms respectively. Then, dividing by $\overline{\mathrm{MRPK}}_g$ in both sides we obtain the expression for $\overline{\text{MRPK}}_p/\overline{\text{MRPK}}_g$

$$\frac{\overline{\text{MRPK}}_p}{\overline{\text{MRPK}}_g} = \left[\frac{R_{p,I_g}}{P_pY_p}\left(\frac{\overline{\text{MRPK}}_{p,I_g}}{\overline{\text{MRPK}}_g}\right)^{-1} + \frac{R_{p,I_g^c}}{P_pY_p}\left(\frac{\overline{\text{MRPK}}_{p,I_g^c}}{\overline{\text{MRPK}}_g}\right)^{-1}\right]^{-1} \tag{G.9}$$

Whenever $\overline{\text{MRPK}}_p \neq \overline{\text{MRPK}}_g$ there is misallocation of capital across sectors. The first term in equation (G.9) reflects the effects of within-firm misallocation on this between-sector misallocation. With $\sigma_g = \sigma_p$ this term would be equal to 1 if $\phi_g = \phi_p$ or if there were no financial frictions ($\lambda_i = 0 \forall i$). Instead, if $\phi_g > \phi_p$ firms switch their output relatively towards the public sector and hence $\overline{\text{MRPK}}_{p,I_g} > \overline{\text{MRPK}}_g$. The second term in equation (G.9) reflects both between-firm misallocation and selection into procurement.

H Additional moments

In this section, Table H.1 providess information on additional moments, both in the data and in the model. Most of them are untargeted moments in our calibration.

I Identification of model parameters

Our strategy consists of internally calibrating 9 parameters so that the model matches 9 moments. In this appendix we show that our choice of targets is justified by the fact that each of these targets is "particularly informative" of a particular parameter.

First, we draw a large random sequence of parameter vectors from a 10-dimensional hypercube. Second, we solve for the model's steady state and calculate the relevant moments for all parameter combinations. And third, we plot how the 15th percentile, the 50th, and the 85th percentile of a given moment change as we move along the vector of its associated parameter. Intuitively, this figure documents how a particular moment changes with a specific parameter, while the other parameters are allowed to vary randomly. The steeper the slope of the relationship between the parameter values and percentiles of the moment, the stronger the identification. Figure I.1 shows that each moment is especially informative of one parameter.

Table H.1. Additional moments

(A) Selection	Data	Model	**(B) Treatment (LPs)**	Data	Model
Employment*	1.15	1.14	$p_p y_p$ $(h=0)$	-0.127	-0.083
MRPK*	0.15	0.15	$p_p y_p$ $(h=4)$	0.034	0.028
TFPQ	0.56	0.43	credit $(h=0)$	0.053	0.243
Fixed assets	1.44	0.98	credit $(h=4)$	0.051	0.104
Value added	1.56	1.14			
Age	0.26	0.07			
Leverage	0.05	0.05			
$\Pr(d_{t+1}=1 \mid d_t=1)$*	0.60	0.59			
$\Pr(d_{t+2}=1 \mid d_t=1)$	0.71	0.64			
$\Pr(d_{t+3}=1 \mid d_t=1)$	0.76	0.69			

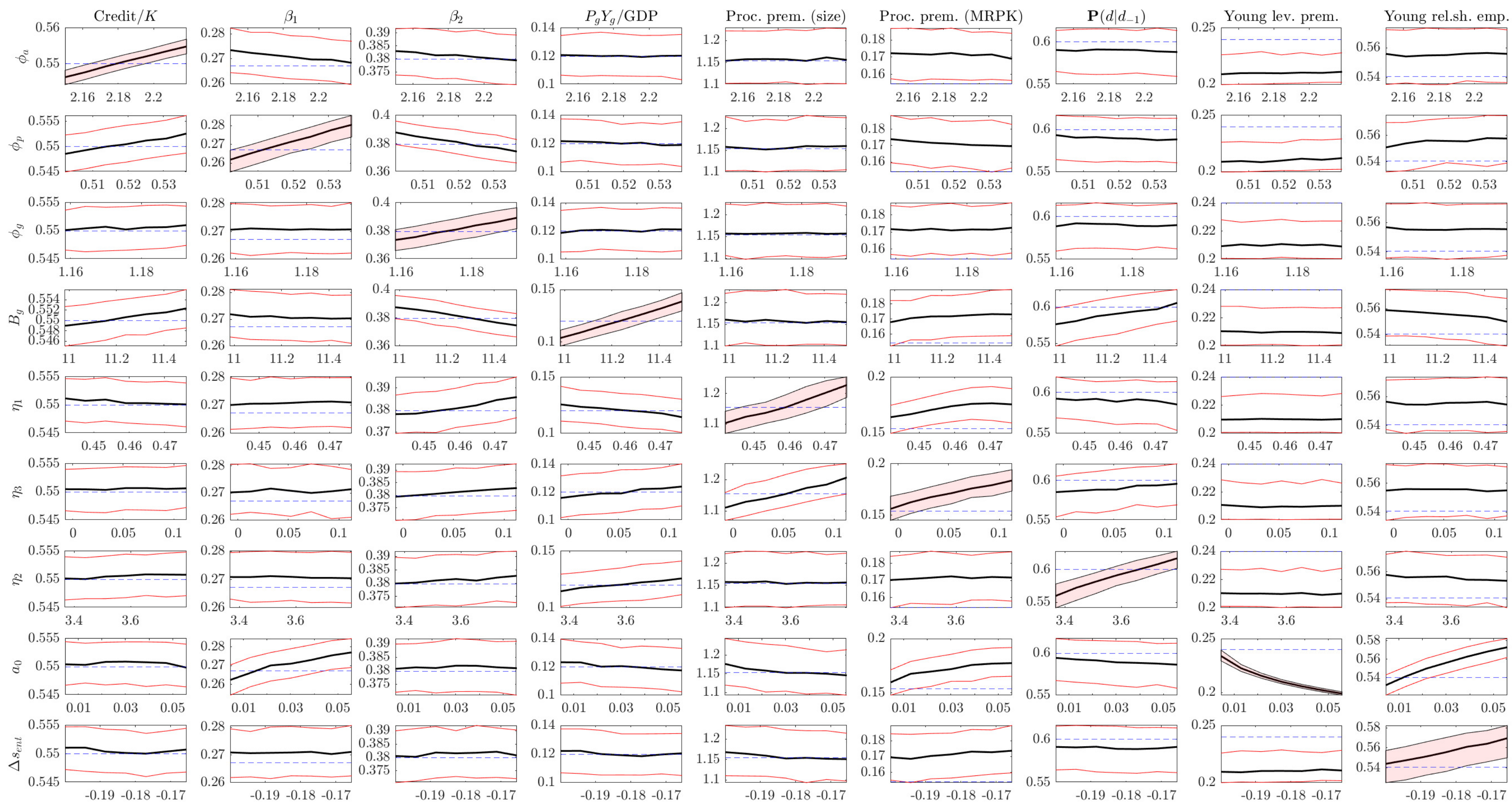
(C) AR1 process	Data	Model	**(D) Procurement sales (share)**	Data	Model
corr(log y_t,log y_{t-1})	0.85	0.89	Mean	0.277	0.299
std(Δ log y_t)	0.67	0.58	pct 25	0.032	0.288
			pct 50	0.105	0.295
			pct 75	0.298	0.304

(E) Firm growth

	Δlog $p_{it}y_{it}$		Δlog n_{it}		Δlog k_{it}	
	Data	Model	Data	Model	Data	Model
Mean	0.15	0.13	0.07	0.13	0.11	0.22
Age 1	0.30	0.27	0.18	0.27	0.18	0.48
Age 2	0.07	0.07	0.02	0.07	0.04	0.13
Age 3	0.05	0.05	0.04	0.01	0.04	0.03

Notes: This table shows a number of moments as measured in our calibrated economy versus the data. The symbol * indicates that the moment has been explicitly targeted in the calibration. The rest are untargeted moments. The top part of **Panel A** reports the *ex-ante* procurement premia for employment, MRPK, fixed assets, age, value added, and TFPQ, as computed as explained in Block #4 of Section 5. The bottom part of **Panel A** reports the (cumulative) probabilities of obtaining procurement contracts in $t+1$, $t+2$, and $t+3$, respectively, conditional on being active in procurement in period t (period t=2006 in the data). **Panel B** shows the point estimates of the local projection regressions shown in Figure 7 for $h=0$ and $h=4$. For the case of leverage (defined as the ratio of credit to fixed assets), the premium is not expressed in log points but in percentage points. **Panel C** shows the one-year autocorrelation of firms' log sales and the standard deviation of firms' sales growth, both in the data (year 2005-2006) and in the model. **Panel D** shows $p_{ig}y_{ig}/p_i y_i$ for firms with $p_{ig}y_{ig} > 0$, both in the data (year 2006) and model. **Panel E** shows the distribution of firm growth in terms of output, employment, and capital, both in the data (year 2005-2006) and model.

Figure I.1. Identification of the model parameters



Notes: This figure plots the relationship between percentiles of a given moment and a specific parameter. The thick black line refers to the median, and the thinner black lines above and below refer to the 85th and 15th percentiles, respectively. The dashed blue line represents the targeted value of the moment. These lines have been generated by solving the steady-state of the model using combinations of parameters drawn from a 9-dimensional hypercube.