Government Procurement and Access to Credit: Firm Dynamics and Aggregate Implications

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Abstract

We provide a framework to study how different allocation systems of public procurement affect firm dynamics and long-run macroeconomic outcomes. We build a newly created panel data set of administrative data for Spain that merges credit register loan data, quasi-census firm-level data, and public procurement projects. We show evidence consistent with the hypothesis that procurement contracts provide valuable collateral for firms, and more so than sales to the private sector. We then build a model of firm dynamics with both asset-based and earnings-based borrowing constraints and a government that buys goods and services from private sector firms, and use it to quantify the long-run macroeconomic consequences of alternative procurement allocation systems. We find that policies that promote the participation of small firms have sizeable macroeconomic effects, but their net impact on aggregate output is ambiguous. These policies help small firms grow and overcome financial constraints, which increases output in the long run. However, they also reduce saving incentives for large firms, decreasing output. The relative extent of these two forces, and hence which of them dominates, crucially depends on the type of financial frictions and the specific way the policy is implemented.

Key words: government procurement; financial frictions; capital accumulation; aggregate productivity

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1 Introduction

Governments play a key role in economic activity. They set taxes and transfers, they are large employers, and they purchase goods and services from the private sector. The purchases of goods and services are done by awarding public procurement contracts to private firms. The size of public procurement varies over time and across countries, but it consistently represents a large fraction of GDP —12.8% in the OECD countries, 14% in EU countries, and 9.3% in the United States.\(^1\) Because of its large size and high level of discretion, governments use the public procurement process to allocate resources to specific sectors or firms. In the U.S., for example, the Small Business Act aims to “ensure that a fair proportion of federal contracts is awarded to small business”.\(^2\) Similarly, in the EU, promoting the participation of small firms is at the core of the European Commission’s agenda for public procurement regulation.\(^3\) Given these different possible designs of procurement programs, it is surprising that we have little understanding of how the procedure of awarding public procurement contracts to private firms may affect the macroeconomy.

In this paper we study the effects of public procurement on firm outcomes and the macroeconomy. We argue that the long-run macroeconomic impact of public procurement depends crucially on the severity as well as on the type of firm-level financial frictions. As a consequence, procedural differences in the awarding of contracts to firms can have first-order effects on macroeconomic outcomes. In particular, we show that granting procurement contracts to small firms —either by directly targeting smaller firms or by slicing large contracts into smaller ones— helps these firms grow and overcome financial constraints in the long run, but the aggregate effects can reduce GDP.

Framework. We carry out an analysis that integrates a novel firm-level dataset with a macroeconomic model of firm dynamics. Our dataset merges administrative data on public procurement, credit allocation at the bank-firm level, and firm outcomes for the Spanish economy over the 2000-2013 period. Our model builds on the

\(^1\)See EU Commission’s web and OECD for details.
\(^2\)See Report from Congressional Research Service for details.
\(^3\)See the Public Sector Directive 2014/24/EU for details. There has been strong support from the European Parliament for explicit regulation that discriminates in favor of small firms: “From this House, we must insist that...administrative bodies incorporate terms into their tender specifications that facilitate positive discrimination in favor of SMEs and remove contractual provisions that hinder their participation.” See EU Parliament Debate.
canonical framework of firm dynamics with financial frictions (e.g., Midrigan and Xu 2014) and incorporates two novel elements. First, there is a government that purchases goods and services from private sector firms. Firms that are willing to sell to the government must make a risky investment in advance, which reflects the costs of preparing a good proposal and increasing the chances to win the contract. Second, we allow for both asset- and earnings-based borrowing constraints. That is, firms not only borrow against their assets but also against their earnings.4

**Motivating facts.** We start by documenting four empirical facts that motivate our paper. First, we show a positive relationship between obtaining a procurement contract and firms' credit growth at impact. Second, we show that this credit growth is mostly explained by an increase in loans for which no tangible collateral is posted. Given that we observe firms’ credit at the quarterly level, we can run these sets of regressions including firm×year fixed effects, which allows us to control for time-varying firms’ unobservables at the annual level. Third, we exploit loan applications data to show that this association arises, at least partly, due to an easing of financial constraints as opposed to an increase in firms’ demand for credit: obtaining a procurement contract is associated with an increase in the probability of getting a loan conditional on applying for it. These facts together point towards procurement contracts serving as collateral that relaxes borrowing constraints. Finally, we provide evidence consistent with the fact that winning a procurement contract eases firms’ financial constraints more than selling to the private sector. In particular, we find that the composition of sales, i.e., private vs. procurement, matters in explaining the evolution of firms’ credit: we estimate the effect of procurement to be positive and significant even when we control for the change in firms’ total sales.

**What we do.** The core of our analysis consists in using our model to study the interplay between procurement and the macroeconomy. We calibrate the model to reproduce several micro-moments related to firm selection into procurement and to firm dynamics after winning a procurement contract, as well as to macro-moments.

**Selection.** In terms of selection, we show a strong pattern based on firm size. In particular, we document that firms that end up participating in procurement are 72% bigger in terms of value-added before they do so. We refer to this difference as the “procurement size premium.” Our model generates a procurement size premium...
through two state variables of firms: productivity (TFPQ) and net worth. As is standard in models with firm heterogeneity, the value of participating in a given market—the procurement market in this case—depends on firms’ ability to deliver large projects (e.g., Melitz (2003) in the context of international trade). In our model, this ability uniquely depends on firms’ TFPQ in the case of financially unconstrained firms. However, for constrained firms, that ability also depends on their financing capacity, which itself depends on firms’ net worth (e.g., Chaney (2016) also in the context of international trade). In our baseline calibration, where we match the 72% value-added procurement size premium, our model implies a procurement premium of 36% in terms of TFPQ and 53% in terms of net worth.

**Treatment.** Regarding the treatment effect of procurement on credit growth, the model is calibrated to reproduce a structural regression in which the change in constrained firms’ leverage, i.e., total credit divided by fixed assets, depends on two variables: the change in total earnings divided by fixed assets and the change in total earnings from procurement divided by fixed assets. The model structure implies that the coefficient associated with the former pins down the parameter that governs the pledgeability of firms’ earnings from selling to the private sector, whereas the coefficient associated with the latter pins down the difference in the pledgeability of earnings from procurement relative to the pledgeability of private earnings. We run this regression for firms that are likely to be financially constrained in our data, i.e., young firms, and find that firms can pledge 42% of their annual earnings from selling to the private sector and 110% of their annual earnings from procurement.

There are several possible explanations for why government contracts are more pledgeable than sales to the private sector. The government may be less likely to default than private firms, winning a government contract may reduce the uncertainty about a firm’s total demand in the future, or getting a contract might be a signal of a firm’s higher productivity. We introduce this feature in our framework, and hence we do not take a stand on its causes.

In terms of the dynamics of real variables, the model is able to reproduce the (un-targeted) fact that procurement generates a positive long-run effect on firms’ performance. For example, both in the model and in the data, we find that procurement is associated with a cumulative increase of firms’ sales to the private sector of

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5 This structural identification of the earnings-based constraints is similar to the one used by Li (2022) for the case of private sector earnings only.
around 5% in the fourth year after obtaining the contract. The high pledgeability of procurement contracts together with the extra profits generated by them reinforces the self-financing channel previously emphasized in the literature (Moll, 2014). In this respect, public procurement is a powerful policy tool to help small firms overcome financial frictions and achieve closer to optimal size in the long run. Importantly, we find that this positive long-run effect takes place despite the fact that procurement temporarily crowds out constrained firms’ sales to the private sector. In our model, this within-firm negative spillover occurs because financially constrained firms have to split their scarce collateral to serve both procurement and private sector operations. The fact that government sales can be collateralized partly alleviates but does not eliminate this problem.

**Policy counterfactuals.** To assess the interplay between procurement and the macroeconomy, we use our calibrated model to perform some expenditure-neutral counterfactual experiments that consist of reallocating procurement contracts across firms while keeping government expenditure unchanged. In particular, we compare our benchmark economy with counterfactual economies in which a higher share of procurement contracts is allocated to small firms. Our preferred counterfactual, which consists of promoting small firms’ participation by directly targeting them in the procurement allocation system, aims to reproduce the “set-aside” policies for small businesses implemented by the U.S Small Business Administration. In practice, we reproduce this policy by targeting a procurement size premium of 50% —as opposed to the 72% in the baseline calibration— while keeping the fraction of firms from which the government buys constant.

**Macroeconomic implications.** As a result of the policy, we find that aggregate GDP would go up by 2.07%, of which around 1/6 is explained by an increase in TFP and the rest is explained by an increase in aggregate capital. The increase in TFP is the result of an increase in TFP in the private sector —which is explained by a reduction in misallocation across firms due to the reinforcement of the self-financing channel— and a reduction of TFP in the procurement sector — which is explained by the fact that the selection pattern based on firms’ TFPQ weakens. That is, reaching out to small firms implies buying goods from less efficient firms.

**Three different channels.** To understand the mechanisms behind the evolution of capital accumulation and GDP, we conduct a decomposition of the policy experiment’s effects that allows us to isolate three different channels. The first channel
is a negative short-run partial equilibrium effect on directly affected firms, which is the result of aggregating the crowding-out effects at impact mentioned above. We find that this channel would reduce GDP by 0.10%. The second channel is a positive long-run partial equilibrium effect also on directly affected firms, which is the aggregate consequence of the strengthening of the self-financing channel. We find that this channel would increase GDP by 2.19%. The third channel is a negative long-run general equilibrium effect coming from the change in capital accumulation incentives of all firms (not only those that ex-post obtain procurement projects) and their responses to general equilibrium price changes. One of the main reasons why firms accumulate financial wealth in our model is the fact that they expect to obtain a public procurement contract at some point. That is, obtaining a procurement contract acts as a large demand shock in response to which firms want to expand their capital stock, causing even relatively big firms to accumulate precautionary savings. Intuitively, productive firms want to have enough net worth so that they minimize the probability of being constrained in case an opportunity for a big procurement contract is realized. A procurement policy that targets smaller firms very aggressively will remove savings incentives for middle-size and large firms. We find that this channel shrinks the output gains associated to the reform.

**An alternative counterfactual.** We also conduct a policy experiment that consists in promoting small firms’ participation by reducing the average size of contracts. This reform aims to mimic the European Commission’s strategy to increase the presence of small firms in public procurement in Europe.\(^6\) In practice, we implement this reform by solving for an economy in which the fraction of firms to which the government allocates contracts increases from 3.8% (which is our target in the baseline economy) to 13.8%, and the average contract size decreases accordingly so that government expenditure remains unchanged. Our main finding is that this alternative policy counterfactual would reach out to a higher number of small firms than in our preferred counterfactual but would actually generate a fall in aggregate GDP of 2.68%. The reason is that the reduction in incentives of relatively big firms to accumulate capital – the long-run general equilibrium effect – is particularly high as a result of the fall in the average size of procurement contracts. Our results hence imply that the particular way in which the promotion of small firms is implemented is crucial to understand its aggregate effects.

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\(^6\)See Trybus (2014) for details.
Importance of high pledgeability of government contracts. Finally, we find that the aggregate effects of these counterfactuals would be significantly less expansionary in a world in which earnings from government contracts exhibit the same pledgeability as earnings from selling to the private sector. For example, for the case of our preferred counterfactual, we find that GDP gains would be around 80% lower than when running the same counterfactual under our baseline calibration (0.38% vs. 2.07%). By reducing the extent to which borrowing capacity increases when participating in procurement, the above-mentioned positive long-run partial equilibrium effect weakens. This result points towards the importance of the extra collateral provided by government contracts when evaluating the aggregate effects of changes in the procurement allocation system.

1.1 Related literature

There is practically no literature that analyzes how the microeconomic aspects of public procurement can affect the macroeconomy. One recent exception is Cox et al. (2021), who document several new facts using micro-level data on public procurement contracts awarded by the U.S Federal Government, and investigate how accounting for these facts—in particular that government spending is concentrated in sectors where prices are more sticky—can affect the short run fiscal transmission mechanism in a New Keynesian model. Our interest instead is in quantifying the long-run macroeconomic effects of different procurement allocation systems.

Governments have been pointed to as directly responsible for the long-run economic performance of countries through the implementation of policies that distort the allocation of resources across firms. Some examples are credit subsidies to state-owned-enterprises (Song et al., 2011), the reservation of goods for small firms (García-Santana and Pijoan-Mas, 2014), labor market regulations (Garicano et al., 2016), tariffs (Berthou et al., 2019), or capital markets regulation (Bau and Matray, 2021). However, one of the most important roles that governments play in modern economies, i.e., their role as buyers of goods and services from private sector firms, has been overlooked. We focus on this by analyzing specific size-dependent procurement policies aimed at helping small firms. In this respect, our work is related to Guner et al. (2008), Restuccia and Rogerson (2008), and Hsieh and Klenow (2009), who show the importance of firms’ idiosyncratic distortions in affecting misallocation across firms and aggregate productivity.
Our focus on firm-level financial frictions as a channel through which public procurement can affect the macroeconomy builds on the literature that quantifies the effects of financial constraints on aggregate output and productivity (Buera et al., 2011; Midrigan and Xu, 2014; David and Venkateswaran, 2019; Gopinath et al., 2017; Catherine et al., Forthcoming). A few papers in this literature have studied the interplay of financial frictions with different forms of taxation (Erosa and González, 2019; Itskhoki and Moll, 2019; Guvenen et al., 2019; Blanco and Baley, 2022) but none has focused on the expenditure side of government policies. Our finding that the type of financial frictions matters in understanding the effects of procurement on the macroeconomy is also related to recent papers that show that the type of financial frictions, i.e., earnings- vs. asset-based, and not only their severity, plays a crucial role in explaining important economic outcomes: the gains from trade liberalization (Brooks and Dovis, 2020), aggregate productivity (Li, 2022), macroeconomic fluctuations (Drechsel, 2021), and the transmission of monetary policy (Caglio et al., 2021).

Our results on the treatment effects of winning procurement contracts on firms are related to the recent literature analyzing the relationship between public procurement and firm dynamics. Ferraz et al. (2016) and Lee (2021) use quasi-experimental designs for Brazil and South Korea, respectively to show that firms winning procurement contracts have a positive and permanent effect on firms’ performance. Hebous and Zimmermann (2021) document for the U.S. a positive relationship between winning a procurement contract and firm investment, and show that the effect disappears when looking at firms that are less likely to be financially constrained. Our results are consistent with all this body of research. We provide novel evidence on loan acceptances and on the fact that only non-collateralized credit increases, which along with the other empirical facts that we document, can be taken as direct evidence of earnings-based financial constraints that are alleviated with procurement projects. Additionally, our results on the short-run crowding out of sales to the private sector by procurement sales are related to recent papers that investigate within-firm spillover effects across markets, like Almunia et al. (2021) with domestic versus foreign markets and Alfaro-Ureña et al. (Forthcoming) with multinational corporations versus other buyers. Finally, Cappelletti and Giuffrida (2021) use data for Italy to show that firms that receive public procurement contracts survive longer, a dimension of the data that

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7See Buera et al. (2015) for a survey of this literature.
we do not explore.

The rest of the paper is structured as follows. Section 2 describes the construction of the dataset and provides summary statistics. Section 3 provides our empirical evidence organized in five stylized facts. Section 4 presents the model of firm dynamics with procurement. Section 5 discusses how we parameterize the model. Section 6 describes our benchmark economy. Section 7 provides the main quantitative results. Section 8 concludes.

2 Data and Summary Statistics

Our empirical work is based on merging three large datasets at the firm level. First, we construct a novel dataset on Spanish public procurement contracts published by the official bulletin of the Spanish Central Government (Boletín Oficial del Estado, BOE) over the 2000-2013 period. We have information on the type of good or service provided, the institution awarding the contract, the initial bidding and final price of the contract, the type of procedure used to allocate the contract, and the firm(s) that won the contract. Second, we use more standard firm-level data on balance sheets and income statements of the quasi-universe of Spanish companies between 2000 and 2016, a dataset that is maintained by the Banco de España and taken from the Spanish Commercial Registry. And third, we use credit registry for Spain, which contains detailed information (e.g., whether or not a non-personal collateral was posted on a particular loan) on all outstanding loans over 6,000 euros to non-financial firms granted by all banks operating in Spain. Additionally, it contains rich information on loan applications. Online Appendix A provides details about the different data sources and samples that we use.

Types and size of procurement contracts in BOE. For many people, procurement is associated with large infrastructure projects. However, only 20% of the contracts in our BOE data are in the construction sector and the median size of procurement projects in construction (0.74 million euros) is of the same magnitude as the median size in the other categories reported by BOE: services (0.42), consulting (0.37), supplies (0.37), and other sectors (0.35) respectively. The major differences in project size across sectors appear in the right tail of the distribution, with the top 1% of projects in construction being much larger than in other sectors. We also note that there is a large number of relatively small projects in all sectors: 25% of projects
have a value of less or equal to 230,000 euro in construction, 200,000 euro in services, and 170,000 in consulting and in supplies. See Table A.I in the Online Appendix for details. Although we do not have direct information about the duration of the contracts in our sample, we were able to collect information about the duration of the contracts awarded in Spain in the year 2015. Around 71% of the contracts have a duration which is one year or less, and 91% have a duration which is two years or less.\(^8\)

**Presence of procurement firms.** Looking at the firm-level data, we find that procurement firms are present in most industries of the economy: firms with at least one procurement contract in a given year operate in 71 out of the 91 industries based on NACE 2-digit classification. The share of procurement firms in our data is 0.5% percent, but it varies a lot across industries, with the highest fraction—around 15%— in industries like “Manufacture of coke and refined petroleum products” and “Manufacturing of Pharmaceutical Products.” Because procurement firms tend to be larger, the share of employment, sales, assets, or credit of procurement firms tend to be larger than the share of firms, see Table A.II in the Online Appendix for details.

**Procurement vs. non-procurement firms.** We find that firms participating in procurement are significantly larger and older on average, but there is considerable overlap in the support of the size and age distribution for procurement and non-procurement firms (see Table A.III in the Online Appendix). For example, the average number of employees of a procurement firm is around 6 times larger than for the rest of the firms (73.56 vs. 12.75), total sales are 7 times larger (8.9 millions of euro vs. around 1.2 million), and procurement firms are 9 years older (20 vs 11 years). Yet, around 25% of procurement firms have less than 16 employees, have revenues that are lower than 1.14 million euro, and are 12 or fewer years old. We also find that conditional on having at least one procurement project, there is a lot of variation on the importance of these projects as a fraction of firms’ total revenue. The average ratio of all the procurement value to total revenue is 0.20, with 25th, 50th, and 75th percentiles of 0.01, 0.03 and 0.10 respectively. Finally, we observe large differences between procurement and non-procurement firms in terms of their composition of credit. In particular, procurement firms seem to rely more on non-collateralized credit (86% vs 71% on average) despite holding higher levels of assets.

\(^8\)As a reference for a different country, Cox et al. (2021) find that the median contract in the U.S. has a duration of 31 days and 90% of contracts last less than one year.
3 Motivating Empirical Evidence

We begin by documenting several facts related to the effects of procurement on firms’ outcomes. First, we show a positive relationship between obtaining a procurement contract and firms’ credit growth. Second, we show that this credit growth is mostly explained by an increase in credit for which no tangible collateral is posted. Crucially, given our empirical specification and our data at the quarterly frequency, these reduced-form regressions allow for fixed effects that control for time-varying firms’ unobservables at the annual level, such as demand or productivity. Third, we exploit loan-level data to show that this association arises, at least partly, due to an easing of financial constraints, in contrast to an increase in firms’ demand for credit. We interpret these facts together as evidence for procurement contracts serving as collateral in earnings-based borrowing constraints. Finally, we provide evidence consistent with the fact that winning a procurement contract eases firms’ financial constraints more than selling to the private sector. For facts one and two we use two different samples: a sample of firms that obtain at least one procurement project between 2000 and 2013 (the main sample) and the sample of firms for which information and ranking of the other bidders is available (bidders sample). For facts 3 and 4, we can only use the main sample. For fact 4, we work with annual frequency because firms’ sales to the private sector come from firm-level balance sheet information, which is only available at the annual frequency.

3.1 Procurement and credit growth

We start by regressing firms’ credit growth on a dummy variable for procurement as follows:

\[
\Delta \log l_{it} = \alpha_{iy} + \alpha_{st} + \beta_1 \text{PROC}_{it} + \beta_2 \log l_{it-1} + \varepsilon_{it}
\]

where the dependent variable \(\Delta \log l_{it}\) is the annualized quarterly growth of credit (loans) of firm \(i\) between quarter \(t-1\) and quarter \(t\) defined as \(\Delta \log l_{it} \equiv \log l_{it} - \log l_{it-1}\), winsorized between \(-1\) (-100%) and \(+2\) (+200%). The regressor \text{PROC}_{it}\) is a dummy variable that takes value one if the firm obtained a procurement contract in quarter \(t\).

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9For the main sample, one could alternatively use a sample with all firms, but results would be very similar. This is because all our specifications use firm fixed effects, and hence the identification of the effects of procurement comes from the panel variation and not from comparing firms that participate in procurement on a regular basis with firms that never compete for procurement. Results are available upon request.
We include the firm’s lagged credit at \( t - 1 \) to control for the fact that firms with large outstanding loan volumes may mechanically have less room for credit growth than firms with smaller outstanding loan levels.\(^{10}\) We further include a stringent set of fixed effects. In particular, we use firm×year fixed effects, \( \alpha_{iy} \), in order to capture firm-level characteristics that vary over time at the yearly (\( y \)) level. Importantly, these fixed effects help control for several factors that may otherwise bias the estimation. First, as they vary at the year level, they pick up the overall firm-level trend of credit growth and thus helps assuage the concern of any potential bias arising from differences in trends pre/post “treatment” by procurement events. Second, these fixed effects control for firm-level unobserved variables that may change at annual level such as productivity or demand. We further include 4-digit sector×quarter effects, \( \alpha_{st} \), which control for both sector and macroeconomic conditions that vary over time. Therefore, identification of the key parameter of interest, \( \beta_1 \), comes from the variation of a firm’s credit growth across quarters within a year conditional on obtaining a procurement contract.

Table 1, column (1), presents the results of this regression for the main sample. The estimate of \( \beta_1 \) is positive and significant at the one-percent level.\(^{11}\) The estimated coefficient implies that winning a procurement contract in a quarter translates into an increase of credit growth of 5.5 percentage points annually.

We next use the sample of procurement projects where we have information on all bidders as well as the final ranking. Doing so allows us to run regressions analogous to (1), except that we can identify the association between a firm’s ranking in a given auction and its ensuing credit growth. To be more precise, we run two regressions similar to specification (1) at the auction level. In the first regression, we include all bidders and the PROC variable indicates which firm wins the auction (‘First’ place). Table 1, column (2), shows the results. We find that the winner of a procurement contract has higher credit growth relative to the firms it competes against in a given auction. Note that identification of the coefficient is exploiting the full time series of bidders, so the comparison is based on the within-auction group of firms but also with respect to each firm’s annual credit growth given the inclusion of firm×year effects. The coefficient on the winner is 0.073, which indicates that winning the auction is associated to a 7.3 percentage points higher credit growth annually.

\(^{10}\)The estimation results without lagged credit are similar and are available upon request.
\(^{11}\)We cluster standard errors at the firm-level in all regressions unless otherwise noted.
Table 1. Credit Growth and Procurement

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
<th>Bidders only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>First (2)</td>
</tr>
<tr>
<td>PROC_{it}</td>
<td>0.055(^a)</td>
<td>0.073(^a)</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.028)</td>
</tr>
<tr>
<td>log(Credit_{it-1})</td>
<td>-0.410(^a)</td>
<td>-0.175(^a)</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>Observations</td>
<td>700,780</td>
<td>8,310</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.786</td>
<td>0.360</td>
</tr>
<tr>
<td>Sector×quarter FE</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Firm×year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Auction FE</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: Results from estimating the relationship between total credit growth and procurement participation (PROC) by regression (1): with firms obtaining at least one procurement project over 2000-13 in column (1), and with firms who participated in procurement contests over 2013–15 in columns (2) and (3), where the PROC dummy indicates the winning firm (‘First’) in column (2) and the runner-up firm (‘Second’) in column (3). All regressions use quarterly data. Standard errors clustered at the firm level; \(^a\) indicates significance at the 1% level, \(^b\) at the 5% level, and \(^c\) at the 10% level.

While the bidder firms’ sample is more restrictive than the full sample, we are reassured that we are picking up an unbiased “procurement effect” for a few reasons. First, the point estimates of PROC in columns (1) and (2) are remarkably similar, even though the sample and variation exploited are slightly different. Second, we are able to control for firm×year effects in both regressions, thus helping dilute productivity or demand firm-specific effects at the annual level. Third, an identification threat would be that productivity or demand shocks at the quarterly level are correlated with the concession of procurement projects such that winning the contract may be a proxy for these shocks. Therefore, our estimate may capture the effect of being ranked above other firms as opposed to the effect of obtaining the procurement contract. In Column (3) of Table 1 we drop the winner of the procurement contest and the PROC dummy now indicates which firm was runner-up (‘Second’ place). We run this second regression to make sure that winning the contract, as opposed to the relative ranking, is what is really associated with differences in credit growth across auction participants. The estimated coefficient on PROC implies that there is no statistical difference in quarterly credit growth for the firm that placed second relative to other losers of the auction. Fourth, as in any diff-in-diffs type of environment, it
could be the case that winner firms are in different credit trajectories than non-winner firms. In principle, this should be captured by the firm×year effects. Still, in Online Appendix A.8, we provide evidence which shows that this is not the case. In particular, we show that the evolution of credit growth for winners and non-winners was similar before the auction and that it diverged afterwards. Finally, we show in the Online Appendix (Table A.IV), the procurement dummy is positive, significant, and relatively stable when running regression (1) across sub-samples based on quartiles of firms’ (i) total assets, (ii) employment, (iii) net worth, or (iv) age.

3.2 Procurement and the composition of credit

We next decompose the increase in credit associated with winning a procurement contract into that coming from collateralized vs. non-collateralized credit, which will help us motivate the type of financial constraint we will use in Section 4. To this end, we use the information on the composition of firms’ loans, which indicates whether these loans require collateral or not to be posted by a firm to receive financing from a bank. We therefore run a similar regression as (1), constructing the dependent variable at the firm×credit-type×quarter level, and split the estimation between collateralized and non-collateralized credit growth.

Table 2 presents the main results, where \(c\) denotes the additional collateral/non-collateral dimension that we exploit in the data. Looking at the main sample, we see that a procurement contract is not significantly correlated with the growth rate of collateralized credit in column (1). However, when turning to column (2) we see a positive and significant association with a firm obtaining a procurement contract and non-collateralized credit growth. The results with the bidders sample, in columns (3) and (4), mimic the findings for the main sample. That is, a firm winning a contract experiences significantly larger growth in non-collateralized loans relative to losing firms, but there is no differential for collateralized loan growth. Regressions for the second vs. the rest samples in columns (5) and (6) do not yield any significant estimates. Overall, these findings point to the growth rate in overall credit associated with obtaining a procurement contract observed in Table 1 being driven by the growth in loans that do not require tangible-assets backing.
Table 2. Composition of Credit Growth and Procurement

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
<th>Bidders only</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>PROC&lt;sub&gt;it&lt;/sub&gt;</td>
<td>0.001</td>
<td>0.070&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.011</td>
<td>0.080&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.005)</td>
<td>(0.029)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>log(Credit&lt;sub&gt;ict&lt;/sub&gt;-1)</td>
<td>-0.474&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.421&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.449&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.192&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.073)</td>
<td>(0.040)</td>
</tr>
</tbody>
</table>

| Observations        | 224,011   | 557,873      | 2,690               | 8,110               |
| R-squared           | 0.791     | 0.764        | 0.357               | 0.368               |
| Sector×quarter FE   | Yes       | Yes          | No                  | No                  |
| Firm×year FE        | Yes       | Yes          | Yes                 | Yes                 |
| Quarter FE          | No        | No           | Yes                 | Yes                 |
| Auction FE          | No        | No           | Yes                 | Yes                 |
|                     |           |              |                     |                     |

Notes: Results from estimating the relationship between collateralized (Collat.) and non-collateralized (NoCollat.) credit growth and procurement participation (PROC) by regression (1) with firms obtaining at least one procurement project over 2000-13 in columns (1) and (2), and with firms who participated in procurement contests over 2013-15 in columns (3)-(6) respectively, where the PROC dummy indicates the winning firm (‘First’) in columns (3)-(4) and the runner-up firm (‘Second’) in columns (5)-(6). All regressions use quarterly data. Standard errors clustered at the firm level; <sup>a</sup> indicates significance at the 1% level, <sup>b</sup> at the 5% level, and <sup>c</sup> at the 10% level.

3.3 Procurement and firms’ borrowing capacity

The previous evidence shows a positive relationship between procurement participation and credit growth. However, it is silent about whether the observed increase in credit actually comes from a relaxation of firms’ financial constraints or simply from the fact that firms demand more credit when they expand their operations. We next ask whether firms are able to use their procurement contracts to access credit more easily at the extensive margin. A unique piece of information contained in the Banco de España’s credit registry allows us answer this question: the information on the loan application process for firms and banks. In particular, we can see whether a firm has applied to a given bank and whether the loan application has been accepted or rejected throughout our sample period. We use this information to help identify an increase in firms’ borrowing capacity. To do so, we run regressions at the firm-bank level and relate the probability of firms obtaining a loan to whether they have received a procurement contract using the following linear probability specification:

\[
\text{Loan granted}_{ibt} = \alpha_{ib} + \alpha_{bt} + \alpha_{st} + \beta \text{PROC}_{it} + \varepsilon_{ibt}
\]  

(2)
where the variable ‘Loan granted’ is a 0/1 dummy variable that is turned on when the firm \( i \) receives a loan from bank \( b \) in quarter \( t \) conditional on the firm applying for it during that same quarter. We include firm×bank fixed effects, \( \alpha_{ib} \), which implies that we are identifying the coefficient \( \beta \) on the procurement variable via the variation within a firm-bank relationship over time. We further control for overall bank credit supply in a given period with bank×quarter fixed effect \( \alpha_{bt} \), and for macroeconomic events with sector×quarter fixed effects \( \alpha_{st} \).

**Table 3. Probability of a New Loan and Procurement**

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>PROC(_{it})</td>
<td>0.024(^a)</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
</tr>
<tr>
<td>Observations</td>
<td>36,857</td>
</tr>
<tr>
<td>R-squares</td>
<td>0.395</td>
</tr>
<tr>
<td>Firm×bank FE</td>
<td>Yes</td>
</tr>
<tr>
<td>Bank×quarter FE</td>
<td>No</td>
</tr>
<tr>
<td>Sector×quarter FE</td>
<td>No</td>
</tr>
</tbody>
</table>

*Notes:* Results from estimating the relationship between loan participation and procurement participation (PROC) by regression (2) with firms obtaining at least one procurement project over 2000-13 using quarterly data. Standard errors clustered at the firm level; \(^a\) indicates significance at the 1% level, \(^b\) at the 5% level, and \(^c\) at the 10% level.

Table 3 shows the results from running this regression. We include only firm×bank fixed effects in column (1), and add the time-varying bank and sector fixed effects in column (2). Overall, regardless of the specification, the probability of receiving a bank loan conditional on having applied for it increases by approximately 2 percent in the quarter that a firm wins a procurement project.

### 3.4 Differential impact of earnings from procurement on firms’ credit

Our fourth piece of evidence is that public revenues can be pledged to a larger extent than revenues from the private sector. To show this, we use the main sample at the annual frequency to run regression (1) with total revenue growth as an extra control on the right hand side. Notice that we introduce the growth rate of revenue between \( t \) and \( t+1 \). The reason is that the sales carried out by the firm in \( t \) will manifest in
Table 4. Differential impact of earnings from procurement on firms’ credit

<table>
<thead>
<tr>
<th></th>
<th>All firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
</tbody>
</table>
| PROC
t                  | 0.053\textsuperscript{a} 0.043\textsuperscript{a} 0.041\textsuperscript{a} |
| (0.006) (0.006) (0.006) |
| Sales growth
t+1     | 0.107\textsuperscript{a} 0.027\textsuperscript{b} 0.024\textsuperscript{c} |
| (0.020) (0.011) (0.011) |
| log(Credit
t−1)     | -0.047\textsuperscript{a} -0.267\textsuperscript{a} -0.274\textsuperscript{a} |
| (0.003) (0.023) (0.023) |
| Observations          | 86,537   | 86,096   | 83,652   |
| R-squared             | 0.051    | 0.282    | 0.330    |
| Year FE               | Yes      | Yes      | No       |
| Firm FE               | No       | Yes      | Yes      |
| Sector×Year FE        | No       | No       | Yes      |

Notes: Results from estimating an equation similar to (1) but using the annual sample and controlling for the change in firms’ total sales. Standard errors double clustered at the firm and year levels; \textsuperscript{a} indicates significance at the 1% level, \textsuperscript{b} at the 5% level, and \textsuperscript{c} at the 10% level.

the data in \( t+1 \).\textsuperscript{12} Revenue growth captures revenues from both private and public sector.

We show the results of running this type of regressions in Table 4, whose columns reflect specifications using different sets of fixed effects. In all cases, we find that the coefficient associated to the change in total sales is positive and significant, which is consistent with the idea that firms can borrow against their earnings. Importantly, we also find that the procurement dummy remains positive and significant. If earnings-based constraints were similar for both types of revenues, the extra effect of a dummy for procurement should be null. If the effect of the procurement dummy is positive (negative), this reflects public revenues being more (less) pledgeable than private revenues. In Section 5, we will exploit a similar type of variation to identify some of the model parameters.

4 The Model

We set up a model of privately held heterogeneous firms. We build on standard models of firm dynamics with collateral constraints —as Midrigan and Xu (2014),

\textsuperscript{12}This timing is also consistent with the one we will use in our model.
Moll (2014), or Buera and Moll (2015)—and extend this setting to allow for (a) earnings-based borrowing constraints, (b) a public sector demanding goods from private firms, (c) downward-sloping demands in both the private and public sectors, and (d) a choice to compete for procurement projects.

4.1 Technology

Time is discrete and we omit the subscript $t$ unless it is strictly needed. The economy is populated by a continuum of size 1 of heterogeneous infinitely-lived households indexed by $i$. Each household is also an entrepreneur running a firm that produces a differentiated intermediate good $y_i$. There are two final goods in the economy: the “private sector” good, $Y_p$, used by households to consume, invest in productive capital, or prepare applications for procurement projects, and the “public sector” good $Y_g$, purchased by the government to produce (useless) public consumption.

**Final goods.** The two final goods are assembled by two final good producers combining the differentiated intermediate goods $y_i$ through the following CES aggregators:

$$Y_p = \left( \int_{[0,1]} y_i^{\sigma_p} \sigma_p^{-1} \, dy_i \right)^{\sigma_p^{-1}}$$

and

$$Y_g = m_g \left( \int_{I_g} y_i^{\sigma_g} \sigma_g^{-1} \, dy_i \right)^{\sigma_g^{-1}}$$

with $\sigma_p, \sigma_g > 1$ (3) where $I_g$ is the subset of goods purchased by the public sector and $m_g$ is the measure of this set. Note that $Y_g$ is corrected by $m_g$ to prevent love for variety. We also note that $I_g$ (and the implied $m_g$) is a policy variable and the identity of firms in this set is discussed below. The final goods producers are perfectly competitive and choose the optimal demand of intermediate goods $y_{ip}$ and $y_{ig}$, respectively, to maximize profits taking intermediate good prices $p_{ip}$ and $p_{ig}$, final good prices $P_p$ and $P_g$, and the set $I_g$ as given. We assume that firms compete independently in each sector and face the following downward-sloping demands,

$$p_{ip} = B_p y_{ip}^{-1/\sigma_p}$$

and

$$p_{ig} = B_g y_{ig}^{-1/\sigma_g}$$

where for convenience we define $B_p = P_p^{1/\sigma_p} Y_p^{1-1/\sigma_p}$ and $B_g = m_g^{-1/\sigma_g} P_g Y_g^{1/\sigma_g}$.

Governments purchase only a fraction of goods and services provided by the private economy mainly because their needs are different than the needs of private households and firms. By removing ‘love-for-variety’ we want to eliminate this trivial effect from the analysis of the effects of the number of contracts offered.
intermediate good $i$ may differ because intermediate good $i$ producer has monopoly power over its variety and may be selling different quantities to each market. $Y_g$ is the demand of the public good from the government and is a policy variable in the model, while $Y_p$ is the demand of the private good from the households and it is determined in equilibrium. The aggregate prices $P_p$ and $P_g$ of the private and public goods are given by the usual aggregators:

$$P_p = \left( \int_{[0,1]} p_{ip}^{1-\sigma_p} \, di \right)^{1-\sigma_p} \quad \text{and} \quad P_g = \left( \int_{I_g} \frac{1}{m_{ig}} p_{ig}^{1-\sigma_g} \, di \right)^{1-\sigma_g} \quad (5)$$

We will use the final private good as the numeraire, so we set $P_p = 1$ in what is to follow.

**Intermediate inputs.** The intermediate inputs are produced by heterogeneous firms. At any period in time, these firms are characterized by their idiosyncratic stochastic productivity $s_i$, their capital stock $k_i$ (which depreciates at rate $\delta$), their debt level $l_i$ (when $l_i > 0$ the firm is a net borrower), and whether they currently hold a procurement project $d_i = 1$ or not $d_i = 0$. Output $y_i$ is given by a simple CRS production function, $y_i = f(s_i, k_i) = s_i k_i$, that depends on capital $k_i$ and managerial productivity $s_i$. The firm-specific $s_i$ follows a first order Markov process, specified in more detail below. If a procurement project is active ($d_i = 1$) a fraction of output $u_i$, chosen by the firm, is sold to the private sector and a fraction $1 - u_i$ is sold to the public sector, otherwise all output is sold to the private sector. Our simple production function implies that $u_i$ is also the fraction of capital used for the production of the private sector variety, that is, $k_{ip} = u_i k_i$.

### 4.2 Participation in public procurement

The government has control over the subset $I_g$ of goods purchased by the public sector, and a choice of the subset $I_g$ naturally implies its measure $m_g$. In order to introduce structure in this choice, we consider that the government follows a simple stochastic rule for the allocation of procurement contracts based on the quality of the proposals. In particular, we assume that firms who wish to sell to the government next period ($d_{it+1} = 1$) must invest an amount of private sector good $b_{it} > 0$ today. This quantity may reflect the costs of learning how the process works, the actual costs of preparing a proposal, or the costs of establishing connections with government officials. There is always uncertainty in the outcome of the application, which reflects the fact that
“equally capable” firms usually compete in the same auction with only one winner.\textsuperscript{14} The probability of being able to sell to the government next period depends on the amount invested, \( Pr\left(d_{it+1} = 1 \mid b_{it}\right) = g\left(b_{it}\right) = 1 - e^{-\eta_0 b_{it}} \) with \( \eta_0 > 0 \) and \( 1 > \eta_1 > 0 \) to ensure positive and diminishing returns. Also notice that \( \lim_{b \to 0} \frac{\partial g(b)}{\partial b} \to \infty \), so there will always be an interior solution in the optimal choice of \( b_{it} \). This probability function captures in reduced form the competition for procurement projects. As such, we think of \( \eta_0 \) as an equilibrium object that ensures that the fraction of firms obtaining a procurement project equals the measure \( m_g \) of goods purchased by the public sector. Hence, the probability of procurement depends on firms’ own actions through \( b_{it} \) as well as on the actions of all other firms through the equilibrium object \( \eta_0 \).\textsuperscript{15} The winners of the competition for procurement form the set \( I_g \) in that period.

### 4.3 Entry and exit

A fraction \( 1 - \theta \) of households die every period and are replaced by the same number of new households running new firms. To avoid changing the composition of the goods produced in the economy, the entrant households produce the varieties left vacant by the exiting households. Dying households leave accidental bequests that for simplicity are taken by the government. Entrant households start with a joint distribution of financial wealth and productivity \( \Gamma_0 \) and with no procurement project. The wealth of the entrants is provided by the government. Alternatively, we could have assumed that all accidental bequests go the newborns, but we want to break this link in order to have the flexibility to choose the amount of financial wealth for entrants.

### 4.4 Preferences and constraints

Firms are owned by entrepreneurs. Entrepreneurs have CRRA preferences over consumption with curvature \( \mu_i \), and their objective is to maximize the discounted sum

\textsuperscript{14}In practice, the final ranking of firms is decided based on a number of attributes as the price, quality, and technical requirements. Therefore, firms always face uncertainty about how the public entity awarding the contract will perceive them and their competitors fulfilling these attributes.

\textsuperscript{15}Alternatively, we could have followed a more structural approach in modelling the competition for public contracts. For instance, in a different setting, Michelacci and Pijoan-Mas (2012) model competition for jobs with a job finding probability depending on individual human capital relative to the average human capital of the economy. Yet, our formulation is flexible and does not require taking a stand on the complex procurement competition process.
of utilities.\textsuperscript{16} They obtain income only from their firm so their budget constraint is given by:

\[ c_{it} + b_{it} + k_{it+1} - l_{it+1} \leq p_{ipt}y_{ipt} + p_{igt}y_{igt} + (1 - \delta)k_{it} - (1 + r_t)l_{it} - \text{tax}_{it} \]  

(6)

where \( \text{tax}_{it} = \tau [p_{ipt}y_{ipt} + p_{igt}y_{igt} - (r_t + \delta)k_{it}] \) denotes the proportional taxes on profits paid by entrepreneur \( i \) at time \( t \). The tax function is purposely simple because we focus on revenue neutral counterfactuals.\textsuperscript{17} As it is standard in the literature, we only allow for one-period debt contracts \( l_t \) that pay a risk-free interest rate \( r_t \). The amount of debt is limited by the repayment capacity of the firm through a combination of earnings-based and asset-based collateral constraint. In particular, the amount of debt of a firm coming into \( t + 1 \) is limited by,

\[ l_{it+1} \leq \phi_a k_{it+1} + \phi_p p_{ipt+1}y_{ipt+1} + \phi_g p_{igt+1}y_{igt+1} \]  

(7)

If \( \phi_a = 0, \phi_p = 0, \text{ and } \phi_g = 0 \) no external finance is available and all production needs to be self-financed. With \( \phi_a > 0 \) the firm can lever up. With \( \phi_p > 0 \) and \( \phi_g > 0 \) firms can borrow against the revenues generated in the private and the public sector respectively.\textsuperscript{18}

### 4.5 Timing and state space

Regarding the non-procurement part of the model, we follow the timing convention commonly used in the firm dynamics literature. First, we assume that resources devoted to consumption are spent at the beginning of each period \( t \). Second, we

\textsuperscript{16}Modeling firms as being run by entrepreneurs with curvature in preferences over consumption could also be justified by firms’ dividend-smoothing motives, as empirically documented, e.g. Leary and Michaely (2011).

\textsuperscript{17}Note that this simple tax function is allowing for tax deductibility of depreciation \( \delta k_{it} \) and of the (interest) opportunity cost of capital \( r_t k_{it} \). Since \( l_{it} \leq k_{it} \) whenever the firm’s net worth is non-negative, we are implicitly allowing the firm to deduct more than just the interest payments on debt \( r_t l_{it} \).

\textsuperscript{18}An alternative and more structural borrowing constraint would limit repayment \( (1 + r_{t+1})l_{it+1} \) explicitly by a fraction of undepreciated capital \( (1 - \delta)k_{it+1} \) plus revenues,

\[ (1 + r_{t+1})l_{it+1} \leq \ddot{\phi}_a (1 - \delta)k_{it+1} + \ddot{\phi}_p p_{ipt+1}y_{ipt+1} + \ddot{\phi}_g p_{igt+1}y_{igt+1} \]

In steady state with constant \( r \) this specification would be equal to (7) with the redefinitions: \( \ddot{\phi}_a \equiv \frac{(1-\delta)\ddot{\phi}_a}{1+r}, \ddot{\phi}_p \equiv \frac{\ddot{\phi}_p}{1+r}, \text{ and } \ddot{\phi}_g \equiv \frac{\ddot{\phi}_g}{1+r} \). In counterfactual exercises, increases (decreases) in the equilibrium \( r \) would tighten (loosen) the borrowing constraints. Our formulation ignores this effect, but this is quantitatively second order, as seen by the results from our counterfactuals below.
assume that production in $t+1$ is carried out using capital installed at the end of period $t$. Third, we assume that the household survival shock and the firms’ productivity in $t+1$ are revealed (in this order) before firms decide how much capital to install for next period, $k_{it+1}$, and how much debt to issue for next period, $l_{it+1}$. Regarding the variables related to procurement, we follow a similar logic. The amount of resources devoted to increase the probability of being active in procurement in $t+1$, i.e., $b_{it}$, is spent at the beginning of each period $t$. Whether or not the firm is successful and becomes active in procurement in $t+1$, i.e, $d_{it+1} = 1$, is revealed at the same time as productivity in $t+1$ and right after the survival shock. This means that procurement applications of dying households are ignored by the government and hence dying households are not awarded a procurement project that cannot be delivered.

**Alternative specification.** These assumptions on timing simplify the state-space dimensionality of the problem. In particular, let $a_{it+1} \equiv k_{it+1} - l_{it+1}$ be the firm’s net worth to be carried to next period in units of private good today. Then we can redefine the budget constraint as

$$c_{it} + b_{it} + a_{it+1} \leq (1 - \tau) [p_{ipt}y_{ipt} + p_{igt}y_{igt} - (r_t + \delta)k_{it}] + (1 + r_t)a_{it} \quad (8)$$

The collateral constraint becomes

$$k_{it} \leq \phi_a a_{it} + \phi_p p_{ipt}y_{ipt} + \phi_g p_{igt}y_{igt} \quad (9)$$

where the parameters in the borrowing constraint are re-defined as:

$$\phi_a \equiv \frac{1}{1 - \varphi_a} \in [1, \infty), \quad \phi_p \equiv \frac{\varphi_p}{1 - \varphi_a} \in [0, \infty), \quad \phi_g \equiv \frac{\varphi_g}{1 - \varphi_a} \in [0, \infty) \quad (10)$$

Hence, the production decisions (capital and sales composition) are intratemporal, while the accumulation of net worth and the investment in procurement are intertemporal. This allows to split the firm’s problem in two: a static production problem and a dynamic consumption-saving problem. Next, we describe them in turn.

### 4.6 The static production problem

The intratemporal production problem is characterized by firm productivity $s$, firm net worth $a$, and the availability of a procurement project $d$. For simplicity we drop the firm subindex $i$. Firms with $d = 0$ only have to choose their optimal size $k$ subject to the borrowing constraint, while firms with $d = 1$ also decide on the fraction of
output \( u \in [0, 1] \) sold to the private sector. We can write the formal maximization problem for the firm of type \((s, a, d = 1)\) as,

\[
\pi(s, a, 1) = \max_{k, u} \{ p_p y_p + p_g y_g - (r + \delta) k \}
\]

subject to:

\[
\begin{align*}
    p_p y_p &= B_p \left[ u \right]^\sigma_p \sigma_p - 1; \\
    p_g y_g &= B_g \left[ (1 - u) \right]^\sigma_g \sigma_g - 1
\end{align*}
\]

\( k \in [0, \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g]; \ u \in [0, 1] \)

while for the firm of type \((s, a, d = 0)\) all the terms \( p_g y_g \) trivially disappear and \( u \) becomes equal to 1. Let \( \lambda \) be the multiplier of the intratemporal borrowing constraint and let’s consider the general case with \( d = 1 \). The optimal choices are described by the following FOC:

\[
\begin{align*}
    (1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial u} + (1 + \lambda \phi_f) \frac{\partial p_g y_g}{\partial u} &= 0 \quad (11) \\
    (1 + \lambda \phi_p) \frac{\partial p_p y_p}{\partial k} + (1 + \lambda \phi_f) \frac{\partial p_g y_g}{\partial k} &= r + \delta + \lambda \quad (12) \\
    \lambda \geq 0, \ \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k &\geq 0, \ \lambda \left[ \phi_a a + \phi_p p_p y_p + \phi_g p_g y_g - k \right] = 0 \quad (13)
\end{align*}
\]

These optimality conditions show how financial frictions distort the two decisions faced by firms: production composition and firm size. Equation (11) characterizes the composition of sales. With \( \lambda = 0 \), the optimal choice requires the equalization of the marginal revenues obtained from each sector. Because of the concave revenue functions in both sectors, there is always an interior solution to this problem. With binding financial constraints \((\lambda > 0)\), production is shifted towards the sector whose output can be better collateralized. For instance, if procurement contracts offer better collateral value than sales to the private sector \((\phi_g > \phi_p)\) the optimal choice requires lower marginal revenues from public procurement relative to the marginal revenues from the private sector, which happens when production is shifted towards the public sector and away from the private sector.

Equation (12) determines optimal firm size. With \( \lambda = 0 \) the optimal choice requires to equalize the marginal revenue product of capital to its cost, which is just \( r + \delta \). With binding financial constraints \((\lambda > 0)\), the effective cost of capital is \( \frac{r + \delta + \lambda}{1 + \lambda \sigma_p} \) for sales to the private sector and \( \frac{r + \delta + \lambda}{1 + \lambda \sigma_g} \) for sales to the public sector. The multiplier of the financial constraint \( \lambda \) has two opposite effects on the cost of capital: on the one hand it increases the cost of capital as in standard asset-based financial constraints, but on the other hand it decreases the cost of capital because a fraction of the generated output
Figure 1. Solution of the static profit maximization problem

Notes: This figure shows the solution to the firm's problem. Panel (a) shows the size of the firm represented by the amount of capital \( k(s, a, d) \); Panel (b) shows the multiplier of the financial constraint \( \lambda(s, a, d) \); Panel (c) shows the profits \( \pi(s, a, d) \). All of them are plotted against firm’s productivity \( s \), for two different levels of net worth, and for the cases \( d = 0 \) and \( d = 1 \).

can also be collateralized. We will restrict \( \phi_p \) and \( \phi_g \) as indicated in Assumption 1 below to ensure that the earnings-based constraints cannot self-finance the optimal capital of the unconstrained problem, that is, to ensure that the financial constraints are binding for at least the entrepreneurs with zero net worth. Otherwise all firms would be unconstrained, see Lemma 2 and Proposition 2 in Online Appendix B. An implication of Assumption 1 is also that the values of \( \phi_p \) and \( \phi_g \) are below \( (r + \delta)^{-1} \). This implies that the effective costs of capital for the private and public sector, \( \frac{r + \delta + \lambda}{1 + \lambda \phi_p} \) and \( \frac{r + \delta + \lambda}{1 + \lambda \phi_g} \), are monotonically increasing in \( \lambda \), which in turn means that financially constrained firms operate with less capital, see Lemma 1 and Proposition 1 in Online Appendix B.

**Assumption 1** The model parameters satisfy the following boundary constraints:
\[ \phi_p < \frac{\sigma_p - 1}{\sigma_p} (r + \delta)^{-1} \text{ and } \phi_g < \frac{\sigma_g - 1}{\sigma_g} (r + \delta)^{-1}. \]

where \( \frac{\sigma_p - 1}{\sigma_p} (r + \delta)^{-1} \) and \( \frac{\sigma_g - 1}{\sigma_g} (r + \delta)^{-1} \) are the capital to revenues ratios for the unconstrained problem in the private and public sector respectively.

**Static policy functions.** The solution of this problem yields optimal choices \( k(s, a, d) \) and \( u(s, a, d) \), an associated shadow value of the financial constraint \( \lambda(s, a, d) \), and a profit function \( \pi(s, a, d) \). In Online Appendix B we characterize analytically
these objects for both non-procurement \( (d = 0) \) and procurement firms \( (d = 1) \) whenever \( \sigma_g = \sigma_p \). In Figure 1, we illustrate the numerical solution for both cases with the parameterization discussed in Section 5. First, as it is common in standard models of firm dynamics with collateral constraints, constrained firms with no procurement see their capital and profits increase with net worth (while the shadow value of the borrowing constraints declines) until the point in which the financial constraints stop binding and net worth plays no role. Second, different from models with only asset-based collateral constraints, financially constrained firms without procurement increase capital and profits when productivity increases. This happens through the earnings-based constraint, which allows more productive firms to generate more revenues at the same level of net worth and hence expand production. Note also that more productive firms are more financially constrained at any level of net worth (their shadow value of the borrowing constraint is larger) because the expansion of borrowing possibilities with \( s \) is lower than the increase in the optimal size. Third, looking at firms with procurement, the fraction of output sold by constrained firms to the private sector is decreasing in productivity \( s \) and increasing in net worth \( a \), which simply says that more financially constrained firms, conditional on participating, have a higher fraction of their capital allocated to the production of goods sold to the government. This last result is true under \( \phi_g > \phi_p \) and it would be the opposite if \( \phi_g < \phi_p \). Finally, note also that capital, profits, and the shadow value of the borrowing constraint for firms with procurement evolve with \( s \) and \( a \) as in the case without procurement.

A procurement shock. We can also analyze the static effect of a procurement shock by comparing the solutions of the \( d = 1 \) and \( d = 0 \) cases at any value of the state variables \( s \) and \( a \). For unconstrained firms, a procurement shock leaves operations in the private sector unchanged and increases firm size (and profits) to serve the public demand. This is due to the constant returns to scale production assumption and the absence of adjustment costs. For constrained firms, a procurement shock tightens the financial constraint whenever \( \phi_g \leq \phi_p \). With \( \phi_g = \phi_p \) this is because the firm with \( d = 1 \) has two demands to serve, which are equally pledgeable, and has the same net worth to finance capital in the two different markets. As a result the firm scales down the operations in the private sector to free up collateral for the production in the public sector, which generates a negative within-firm private sector spillover of the procurement contract, that is, \( k_p (s, a, 1) \equiv u (s, a, 1) k (s, a, 1) < k (s, a, 0) \). When \( \phi_g < \phi_p \) the financial situation is aggravated because the public sector demand can be
self-financed to a lesser extent than the private sector one and the negative private sector spillover is larger. When $\phi_g > \phi_p$, instead, public procurement may alleviate the firm financial situation because the public sector demand can be self-financed to a larger extent. This will only be relevant for firms with little or no wealth, which will be less constrained when obtaining a procurement project and will use the extra financing capacity coming from the public sector to scale up operations in the private sector. This is precisely stated in Proposition 13 in Online Appendix B. In our numerical exercises with $\phi_g > \phi_p$, with a realistic calibration, and endogenously accumulated net worth distributions, however, a procurement shock always increases firm size, makes the firm more constrained, and almost always generates a negative spillover on the private sector sales for constrained firms that obtain procurement. Finally, a procurement shock always increases profits. Among unconstrained firms, this is more so for the more productive ones because more productive firms can deliver larger projects. Among constrained firms, and for the empirically relevant case $\phi_g > \phi_p$, this is more so for the more productive and the richer firms, because these two variables determine the capacity to deliver large projects. The only exception is for the firms with little or no wealth discussed above, in which case the increase of profits with procurement actually falls with net worth, see Proposition 15 in Online Appendix B.

4.7 The dynamic problem

The *dynamic consumption-saving* problem can be written in recursive form,

$$V(s, a, d) = \max_{c, b, a'} \left\{ u(c) + \beta \theta E_{s', a'|s, b}[V(s', a', d')] \right\}$$

subject to:

$$E_{s', a'|s, b}[V(s', a', d')] = g(b) E_{s'|s}V(s', a', 1) + (1 - g(b)) E_{s'|s}V(s', a', 0)$$

$$c + b + a' = (1 + r) a + (1 - \tau) \pi(s, a, d) \quad \text{and} \quad a' \geq 0$$

The first constraint says that the expected firm’s value for tomorrow is an average of the firm’s value under procurement, i.e., $d' = 1$, and no procurement, i.e., $d' = 0$, weighted by the endogenous probability of procurement $g(b)$. This is why the expectations operator $E$ depends on $b$ in addition to $s$. The FOC for the choices of
Figure 2. Decision rules

Notes: The first and second panels show the net saving rules \( g_s(s, a, d) \) for firms of different levels of productivity and net worth, both for non-procurement and procurement firms respectively. The third and fourth panels show the endogenous probability of obtaining procurement contracts evaluated at the optimal rules \( g_b(s, a, d) \) for firms with different levels of productivity and net worth, both for non-procurement and procurement firms respectively.

\( a' \) and \( b \) are:

\[
\begin{align*}
    u_c(c) &\geq \beta \theta \mathbb{E}_{s', d' \mid s, b} \left[ \left( 1 + r + (1 - r) \frac{\partial \pi(s', a', d')}{\partial a'} \right) u_c(c') \right] \\
    u_c(c) &= \beta \theta \frac{\partial g(b)}{\partial b} \mathbb{E}_{s' \mid s} [V(s', a', 1) - V(s', a', 0)]
\end{align*}
\]  

(14)  

(15)

The first equation is the standard Euler equation that emerges in models of heterogeneous firms with financial constraints. If a firm is expected to be financially constrained next period in the static profit maximization problem, that is \( \partial \pi(s, a, d) / \partial a = \phi_s \lambda(s, a, d) > 0 \), then there is an extra return above \( r \) to accumulating net worth that is given by the increase in (after tax) profits due to relaxing the firm’s collateral constraint, see Online Appendix B. The second equation determines the optimal spending in \( b \): the entrepreneur will equalize its marginal utility of consumption to the marginal return of \( b \), which is given by the expected increase of the firm’s value coming from the possibility of selling to the government. Because of the properties of \( g(b) \) and because \( \mathbb{E}_{s' \mid s} [V(s', a', 1) - V(s', a', 0)] > 0 \) the right hand side declines with \( b \).

Decision rules. Figure 2 illustrates the net saving decision \( a' - a \) of firms without and with procurement (first and second panel respectively). At low levels of net worth there is a hump-shaped relationship between net savings and net worth that is driven by the tradeoff between smoothing consumption vs. relaxing future borrowing constraints, a feature present in similar models like Midrigan and Xu (2014). At

\[19\text{Proposition 15 in Online Appendix B shows that } \pi(s, a, 1) - \pi(s, a, 0) > 0, \text{ and } V(s, a, 1) - V(s, a, 0) \text{ inherits this property as } d \text{ plays no other role than increasing profits } \pi \text{ in the value function.}\]
larger levels of wealth, the saving behavior follows the logic in Aiyagari (1994): net
savings decrease monotonically with net worth and there is a target level of wealth
that is larger for larger productivity \( s \). This figure also shows big differences between
procurement and non-procurement firms in terms of saving decisions. In particular,
procurement firms save more conditional on their current net worth \( a \) and productivity
shock \( s \). This difference is driven by the fact that profits are higher for firms that are
active in procurement, which relaxes their budget constraint and hence allow them
to save more without sacrificing too much consumption.

Figure 2 also shows the function \( g(b) \) evaluated at the actual choice of \( b \) for firms
with different levels of net worth and productivity, both for non-procurement (third
panel) and procurement firms (fourth panel). The first thing to notice is that high-net
worth firms invest more resources in increasing their probability of being able to sell to
the government. This emerges as a result of an interesting trade off. In the dynamic
problem there are two competing mechanisms to lessen borrowing constraints. On
the one hand, households can accumulate wealth to relax the asset-based constraint
and increase profits next period (right hand side of equation (14)). On the other
hand, they can alternatively invest in applications for procurement projects that will
relax the earnings-based constraint if \( \phi_g > \phi_b \) and allow to increase revenues and
accumulate net worth in any case (right hand side of (15)). Online Appendix B
shows that \( \frac{\partial^2 \pi(s,a,d)}{\partial a^2} < 0 \), which means that the return of accumulating net worth is
lower for firms with more net worth. The profit premium of a procurement project
\( \pi(s,a,1) - \pi(s,a,0) \) increases with net worth for constrained firms, see Proposition 15
in Online Appendix B, and so does \( V(s,a,1) - V(s,a,0) \), which means that the
return of investment in procurement is larger for firms with more net worth. This
happens because selling to the government does not relax the borrowing constraints
completely, which means that firms still rely on their own assets for determining
the size of their procurement contracts. This reflects a “size effect”: the bigger the
procurement projects the firm expects to be able to deliver, the higher the expected
profits that participating in procurement generates. Therefore, we obtain the result
that the investment in procurement projects increases with firm net worth.

The second thing to notice is that there are almost no differences between procure-
ment and non-procurement firms. The reason is that, conditional on \( b \), the probability
of obtaining contracts tomorrow is independent from whether the firm is active in pro-
curement today. Procurement firms spend a bit more on \( b \) though for low levels of \( a \),
which simply reflects the fact that these firms have more available resources at hand.

4.8 Steady state equilibrium

A steady state equilibrium requires that (a) entrepreneurs solve their optimization problem; (b) the probability measure over the state space of households is stationary; (c) the market for the private good clears; (d) the market for the public good clears; (e) the probability of obtaining procurement projects is consistent with the measure of goods bought by the public sector; (f) the budget constraint of the government holds; (g) by Walras law, the credit market clears. A more detailed definition of the equilibrium is provided in Online Appendix C.1. Several comments are in order. First, the parameter $\eta_0$ driving the average probability of a procurement project is an equilibrium object that ensures meeting equilibrium condition (e). It summarizes in reduced form the competition for projects. Second, the government can accumulate financial wealth $D$, which serves as an aggregate counterpart for the loans of entrepreneurs such that loans do not need to be in zero net supply in condition (g). Indeed, $D$ will be calibrated to match the total amount of debt relative to capital held by firms in the data at a targeted interest rate $r$. Third, condition (f) establishes that the government budget constraint in steady state is such that procurement is financed by taxes, plus interest revenues from the stationary amount of government wealth $D$, plus accidental bequests left by dying entrepreneurs, minus the initial net worth provided by the government to newly born entrepreneurs (which is dictated by the exogenously fixed distribution of entrants $\Gamma_0$). Finally, we note that the aggregate objects determined in general equilibrium that are relevant for the optimization problem of households are $Y_p$, $r$, $\tau$, and $P_g$.

4.9 Two types of misallocation

Our model generates two types of misallocation. First, the presence of financial frictions generates misallocation of capital across firms. This is a type of misallocation that is well understood by the literature that studies the effects of financial frictions on aggregate productivity, see for instance Midrigan and Xu (2014) or Moll (2014). After some manipulations of the firm’s FOCs (equations (11) and (12)), defining $k_p = uk$ and $k_g = (1 - u)k$, we obtain the following expressions:

$$\text{MRPK}_{ip} \equiv \frac{\partial p_y y_p}{\partial k_p} = \frac{r + \delta + \lambda}{1 + \lambda \phi_p}$$

and

$$\text{MRPK}_{ig} \equiv \frac{\partial p_g y_g}{\partial k_g} = \frac{r + \delta + \lambda}{1 + \lambda \phi_g}$$
Unconstrained firms ($\lambda = 0$) equalize their marginal products of capital to $r + \delta$ and hence operate at their optimal size. In contrast, constrained firms ($\lambda > 0$) face an effectively higher cost of capital in the private and public sectors $(r + \delta + \lambda)/(1 + \lambda \phi_p)$ and $(r + \delta + \lambda)/(1 + \lambda \phi_g)$ respectively and hence operate at a suboptimal scale, that is, they operate at inefficiently high $\text{MRPK}_{ip}$ and $\text{MRPK}_{ig}$ compared to unconstrained firms.

**Misallocation across firms.** In an economy with binding financial frictions misallocation has two consequences: first, the average marginal revenue products in the private and public sectors, $\overline{\text{MRPK}}_p$ and $\overline{\text{MRPK}}_g$ defined in Online Appendix C, will be inefficiently high and the average capital in each sector, $K_p$ and $K_g$, inefficiently low. Second, because $\lambda$ depends on the firm state variables, $s$ and $a$, there will be heterogeneity in the $\text{MRPK}_{ip}$ and $\text{MRPK}_{ig}$ across constrained firms, which lowers $\text{TFP}_p$ and $\text{TFP}_g$ (defined in Online Appendix C). This type of misallocation across firms within a sector is similar to the one emphasized by Hsieh and Klenow (2009), with the key difference that in our model the same firm may produce in two sectors at the same time, and hence the marginal products of capital are firm-sector specific.

**Misallocation within firms.** Second, the model also generates misallocation of capital *within firms.* As it is apparent from the two equations above, unconstrained firms ($\lambda = 0$) equalize their marginal products across the two sectors. Constrained firms ($\lambda > 0$), instead, shift their production towards the sector that gives higher collateral value. In particular, whenever $\phi_g > \phi_p$ the marginal product of capital from selling to the private sector, $\text{MRPK}_{ip}$, will be inefficiently large relative to the one from selling to the government, $\text{MRPK}_{ig}$. This has again two consequences. First, the average marginal revenue product in the private sector, $\overline{\text{MRPK}}_p$, will be inefficiently higher than the one in the public sector, $\overline{\text{MRPK}}_g$. Second, the dispersion in $\lambda$ across firms generates a larger dispersion in $\text{MRPK}_{ig}$ than in $\text{MRPK}_{ip}$, which lowers $\text{TFP}_g$ more than $\text{TFP}_p$. Therefore, and for both reasons, there would be efficiency gains from reallocating capital from the public to the private sector within the firm.

## 5 Calibration

The model period is one year. We classify the model parameters into four different blocks. The first block contains parameters related to preferences, technology, and productivity that we set to predetermined values. We calibrate the parameters in
the other three blocks such that in equilibrium the model matches several moments measured in the data for the year 2006.

**Block #1: preferences and technology.** The first subset of parameters in this block are the relative risk aversion coefficient $\mu$, which we set equal to 2; the CES elasticities $\sigma_p$ and $\sigma_g$, which we set both to 3; the discount factor $\beta$, which we set to 0.94; and the annual depreciation rate $\delta$, which we set to 0.10. These are within the range of standard values in the literature. In this block, we also include the parameters governing firms’ idiosyncratic productivity. We assume that the log of a firm’s productivity process $s$ evolves over time according to an AR(1) process with Gaussian shocks and unconditional mean $\bar{s} \equiv \mathbb{E}[\log(s)]$. Because we already calibrate a large number of parameters internally, we exogenously set the autocorrelation coefficient $\rho_s$ to 0.80 and the standard deviation of the innovations $\sigma_s$ to 0.30, as estimated by Ruiz-García (2020) using the same dataset of firms. Since $\rho_s$ and $\sigma_s$ have been found to be critical for assessing the aggregate effects of financial frictions in this class of models (e.g., Midrigan and Xu (2014); Moll (2014)), we will check ex-post how well our model matches moments that are usually used to pin down these two parameters. We discretize the process following the Rouwenhorst method, allowing for $N_s = 5$ different states.

**Block #2: financial constraints.** Our model contains three parameters governing firms’ financial constraints: $\phi_a$, $\phi_p$, and $\phi_g$. We choose a value of $\phi_a$ so that the model matches the credit-to-capital ratio observed in our micro-level data, 0.55. Regarding $\phi_p$ and $\phi_g$, we proceed as follows. Given the credit constraint in equation (7), and after dividing by $k$ and taking first differences, changes in firms’ leverage for constrained firms are given by:

$$
\Delta \left( \frac{l_{it}}{k_{it}} \right) = \varphi_p \Delta \left( \frac{p_{it}y_{it}}{k_{it}} \right) + (\varphi_g - \varphi_p) \Delta \left( \frac{p_{igt}y_{igt}}{k_{it}} \right)
$$

(16)

where $l_{it}/k_{it}$ is the firms’ leverage, i.e., total credit divided by fixed assets; $p_{it}y_{it}/k_{it}$ is the firms’ total average product of capital, measured as total value added (minus wages) divided by total fixed assets; $p_{igt}y_{igt}/k_{it}$ is the firms’ value added (minus wages) coming from selling to the government divided by the firm’s total stock of capital. Therefore, for constrained firms, the coefficients from an OLS regression directly pin down $\varphi_p$ and $(\varphi_g - \varphi_p)$, which together with $\phi_a$ allow to recover $\phi_p$ and $\phi_g$ (see equation (10)).

The construction of “output”, i.e., $p_{it}y_{it}$ in the data requires some explanation.
Following the recent literature on earnings-based constraints (e.g., Drechsel (2021)), we assume that the flow variable that firms can collateralize is EBITDA: sales net of overhead and labor costs, without subtracting investment, interest payments or taxes. Because we do not have labor in our model, that variable is equal to the firm’s value added added $p_{ipt}y_{ipt} + p_{igt}y_{igt}$. However, in the data, we compute the counterpart of that variable as:

$$p_{ipt}y_{ipt} + p_{igt}y_{igt} = VA_i - \text{wage bill}_i$$  \hspace{1cm} (17)

Because we do not observe the use of factors separately for what is used to deliver sales to the private vs. the government sector. To compute value added generated by selling to the government, we assume that the intermediate goods and the labor share in total expenditure is constant within the firm, i.e., it does not change depending on whether the firm sells to the private sector or the government.

**Block #3: participation and size of procurement.** There are four parameters driving the size and participation in procurement. The parameters $Y_g$ and $m_g$ are policy parameters governing the relative size of procurement in the economy and the fraction of goods bought by the government. We set $Y_g$ to match the share of procurement in GDP equal to 12.1%, which is the value we measure in the Spanish national accounts in the year 2006. We set $m_g$ equal to the share of firms that participate in procurement, which we calculate to be 3.8%.\(^{20}\) Regarding the probability function of winning a contract we proceed as follows. We calibrate the level parameter $\eta_0$ to ensure that the the fraction of firms doing procurement equals the fraction of goods bought by the government, $m_g$, which is the equilibrium condition (e). Regarding the curvature parameter $\eta_1$, we identify it by matching the selection pattern of firms into procurement observed in the data. We proceed as follows. In the data, we select firms with no procurement contracts between 1999 and 2005. Then, we classify as procurement firms those firms that obtain at least one contract in 2006. We de-

\(^{20}\)In our sample only 0.5% percent of firms participate in procurement in a given year. However, our procurement data only captures 13% of the total procurement value measured in national accounts (see Online Appendix A). Assuming that the number of procurement firms relative to the value of procurement is the same in our sample as in the whole population, we can scale up the share of procurement firms to 3.8% ($0.005 / 0.13 = 0.038$). This is probably a lower bound for the number of firms active in procurement because our procurement dataset is biased towards contracts awarded by the central government, which are bigger than the contracts awarded by other governments layers (e.g., local governments), and hence are probably more concentrated in a few firms. To the best of our knowledge, there are no official statistics on the number of firms selling to governments in Spain. As a reference from a different country, Lee (2021) calculates that 5.3% participate in the procurement market in South Korea.
fine the “procurement premium” as the relative difference in size (measured by value added) between procurement and non-procurement firms in 2005 (only exploiting variation across firms within the same 4-digit industry). That is, we want the model to match the ex-ante difference in size between procurement and non-procurement firms. We measure this procurement premium to be 72%. The intuition why the parameter $\eta_1$ affects the selection of firms into procurement is as follows. When $\eta_1$ approaches zero, the probability function $g(b)$ exhibits strong diminishing marginal returns in $b$: the marginal increase in probability falls quickly as firms invest more. This makes differences in $b$ across firms inconsequential for their probability of selling to the government, and hence generates very little selection, with complete randomness in allocation when $\eta_1 = 0$. Conversely, when $\eta_1$ approaches 1, the diminishing marginal returns are small: the marginal increase in probability falls slowly with $b$ as firms invest more. This implies that differences in $b$ translate into big differences in the probability of participating in procurement, which generates a strong selection pattern.

**Block #4: rest of the parameters.** We use firms’ average productivity level $\bar{s}$ to match the capital-to-output ratio observed in our firm-level data. The reason why this moment is informative of the average productivity in the economy has to do with our $AK$ assumption on firms’ technology. Using our measure of output and firms’ fixed-capital stock, we compute an aggregate capital-output ratio of 3.88. To discipline government’s wealth $D$, we target an equilibrium interest rate equal to 5%. Finally, we calibrate the survival probability $\theta = 0.95$ to the firms’ exit rate in Spain of 0.05.

### 5.1 Calibration results

Our model matches all the targeted moments. Panel A in Table 5 shows the definition of the parameters as well as their inferred values. Panel B shows the description of moments and their value in the data and in the model.

**Block # 1.** Although we exogenously set the parameters governing the AR(1) productivity process to $\rho_s = 0.80$ and $\sigma_s = 0.30$, we check that the model matches well two non-targeted moments that are informative about these two parameters: the one-year autocorrelation of firms’ output and the standard deviation of firms’ output growth. In our Spanish data, we find the one-year autocorrelation of firms’ log sales to be 0.89 (which compares to 0.82 implied by our model) and the standard deviation
of firms’ sales growth to be 0.57 (which compares to 0.49 implied by our model).

**Block # 2.** With respect to financial frictions, we find $\phi_a = 2.17$, which implies $a = 1 - 1/\phi_a = 0.54$. Therefore, our calibration implies that firms can collateralize 54% of their capital stock. Regarding the earnings-based constraints, Table 6 presents the results from running the empirical counterpart of equation (16) for firms that receive procurement contracts in at least two consecutive years.\(^{21}\) Because that equation should hold with equality only for firms whose financial constraint is binding, we further restrict the sample to firms that are likely to be financially constrained according to our model, i.e., young firms. In particular, column (1) restricts the

\(^{21}\)This is what we need in order to exploit intensive margin variation in $\Delta p_{igt} y_{igt}$. 

---

**Table 5. Calibration**

<table>
<thead>
<tr>
<th>Panel A: parameters</th>
<th>Panel B: Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block 1</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Base. $\phi_p = \varphi_g$</td>
</tr>
<tr>
<td>$\sigma_{p}$</td>
<td>2.00</td>
</tr>
<tr>
<td>$\sigma_{g}$</td>
<td>3.00</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.94</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.10</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.80</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.30</td>
</tr>
</tbody>
</table>

| Block 2             |                  |
| $\phi_a$            |       2.17       |       |
| $\phi_p$            |       0.92       | 0.55  |
| $\phi_g$            |       2.40       | 0.38  |

| Block 3             |                  |
| $\eta_0$            |       0.21       |       |
| $\eta_1$            |       0.53       |       |
| $Y_g$               |       0.83       |       |
| $m_g$               |       0.038      |       |

| Block 4             |                  |
| D                   |       0.86       |       |
| $\bar{s}$           |       -6.51      | 3.88  |
| $\theta$            |       0.95       |       |

Notes: This table summarizes our baseline calibration. All moments, with the exception of the regression coefficients, have been computed for the year 2006. Government lending $D$ is expressed as a fraction of total credit in the model economy. In column (1), we show the parameter values in our baseline calibration. In column (2), we show the parameter values in our alternative calibration where we set $\phi_p = \varphi_g$ (see Section 7.4) for details. Notice that we do not report data and model’s moments separately because the model matches the data moments perfectly in the two calibrations.
<table>
<thead>
<tr>
<th>Table 6. Change in Leverage and Procurement</th>
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<tbody>
<tr>
<td><strong>∆p_ity/kt</strong></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td><strong>∆p_lgtylgt/kt</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R-squared</td>
</tr>
<tr>
<td>Sector×year FE</td>
</tr>
<tr>
<td>Sample by age</td>
</tr>
</tbody>
</table>

**Notes:** This table presents results from estimating the relationship between the change in firm’s leverage and the change in its average product of capital and change in its earnings coming from selling to the government divided by the firm’s total stock of capital. Regression (16) is estimated with firms obtaining at least one procurement project over 2000-13 using annual data. Standard errors are clustered at the firm level; <sup>a</sup> indicates significance at the 1% level, <sup>b</sup> at the 5% level, and <sup>c</sup> at the 10% level.

sample to firms that are ten years or younger (the median of the age distribution), while columns (2) and (3) further cut the sample to nine or eight years and less, respectively. Both the coefficients on total and government earnings are positive and statistically significant, indicating that ϕ<sub>p</sub> > 0 and that ϕ<sub>g</sub> > ϕ<sub>p</sub>. For example, in our preferred specification, i.e., column (1), our estimates imply values for ϕ<sub>p</sub> and ϕ<sub>g</sub> of 0.42 and 1.10 respectively. Hence, we find that firms can pledge 42% of the annual earnings from selling to the private sector and 110% of their annual earnings from selling to the government. Together with ϕ<sub>a</sub> = 0.54, these numbers translate into ϕ<sub>p</sub> = 0.92 and ϕ<sub>g</sub> = 2.40.<sup>22</sup> These numbers mean that firms can increase their capital by 92% of their annual earnings in the private sector and by 240% of their annual earnings from selling to the government. These two last numbers are the result of a multiplier effect: firms can borrow against their revenues, allowing them to buy more capital, which can be partly collateralized to obtain further credit. This is an important interaction: how earnings-based constraints affect a firm’s ability to grow also depends on the value of ϕ<sub>a</sub>.

**Blocks # 3 and 4.** Regarding the probability function of winning procurement contracts, we find the level η₀ to be equal to 0.21 and the slope η₁ to be equal to 0.53.

<sup>22</sup>We note that both ϕ<sub>p</sub> and ϕ<sub>g</sub> satisfy Assumption 1, which means that capital cannot be self-financed through the earnings-based constraints, see Lemma 2 in Online Appendix B. This is true despite ϕ<sub>g</sub> > 1 because the optimal unconstrained capital to output ratio in procurement is \( \frac{\sigma_g^{-1}}{\sigma_g} (r + \delta)^{-1} = 4.44 \), which means that ϕ<sub>g</sub> should equal 4.44 and ϕ<sub>g</sub> should equal 2.04 for procurement to be self-financed.
To match the aggregate capital-output ratio of 3.88 the model needs an average log productivity $\bar{s} = -6.51$. Finally, the model needs a high level of government lending to match an interest rate $r$ of 5%. In particular, the amount of government lending represents around 86% of the total amount of credit in the economy.

6 The Benchmark Economy

In this Section we describe three dimensions of our benchmark economy: the selection pattern of firms into procurement, the treatment effect of a procurement shock on firm dynamics, and the macroeconomic consequences of procurement.

6.1 Selection

We have calibrated our model economy to match a “procurement premium” in value added of 72% (by measuring the relative size of procurement vs. non-procurement firms before they obtain a procurement project). As discussed in Section 4.6, the value of procurement, $V(s, a, d = 1) - V(s, a, d = 0)$, is increasing in firms’ ability to deliver large projects, which is determined by productivity $s$ for unconstrained firms and also by net worth $a$ for constrained firms. Hence, in the model firms self-select into procurement based on their productivity $s$ and their net worth $a$. When we compute the “procurement premium” for $s$ and $a$ we find that procurement firms are ex-ante 36% more productive and hold ex ante 53% more net wealth.

6.2 Treatment

We next describe the treatment effects of procurement on firm dynamics implied by our calibrated model. First, we use model-simulated data to estimate a local projection regression of a procurement shock on private sales, which we show in the red line in Figure 3 (see Online Appendix D for details). We find that procurement generates a crowding out of sales to the private sector on impact (a 35% decline in sales to the private sector), and a crowding in during the subsequent years. As discussed in Section 4.6, a constrained firm has to split resources between the two sectors, despite the extra credit generated by selling to the government, which explains the fall in sales to the private sector on impact. In fact, the firm obtaining procurement becomes more constrained at impact, i.e. higher $\lambda$, as a result of the increase in demand
(see Figure A.III in the Online Appendix). However, the new profits generated from procurement allow the firm to accumulate more net worth over time. This higher level of net worth will ease the firm’s financial constraint (lowering \( \lambda \)) and hence allow it to increase output in the private sector in the subsequent periods. We observe a similar pattern in the data. Using the main sample of Section 3, we estimate the same local projection regression and find that a procurement contract is associated with a fall in sales to the private sector of around 4% at impact and a subsequent increase over time, see blue line in Figure 3. Hence, both in the data and in the model, the crowding out effect disappears and actually gets reversed over time: four years after obtaining a procurement contract, firms increase their sales to the private sector by around 5% in the model and 6% in the data.

Second, we find that the procurement contract has a permanent positive effect on \( b \), which is the result of an increase in the firm’s cash on hand. In particular, we find an increase of around 10% estimated four years after the firm obtains a contract (see Figure A.III). This is a channel through which the model endogenously generates some persistence in firms’ participation in procurement. To have a sense of how strong this channel is as compared to the data, we compute the probability of obtaining a contract over the next three years, i.e., \( t + 1, t + 2, \) or \( t + 3, \) conditional on a firm having a contract in \( t \). In the data, this number is quite high, 75%, which compares to its model counterpart of 16%. That is, our model generates around 1/5 of the persistence observed in the data.
6.3 The macroeconomy

We report the most relevant aggregate numbers of the benchmark economy in column (1) of Table 7. We find significant differences in aggregate TFP across the two sectors. In particular, our model implies that TFP in the procurement sector is 21% higher than its counterpart in the private sector (0.308 vs. 0.255), see Online Appendix C for derivations. This difference is mainly due to selection on \( s \). To see why, we note that absent financial frictions but keeping the same selection on \( s \) into procurement, the first-best level of TFP in the procurement sector would be 19% higher than its equivalent in the public sector. The calibrated economy displays modest levels of misallocation in both sectors, but significantly more in the private one: the aggregate TFP gains of equalizing MRPK across firms are 4.7% in the private sector and 3.3% in procurement. There are two reasons for the difference of misallocation between the two sectors. The first one is \( \phi_g > \phi_p \), which increases procurement firms’ borrowing capacity relative to firms producing only in the private sector. The second reason is that firms with higher net worth self select into procurement, which reduces the dispersion in net worth across procurement firms and hence the dispersion in \( \lambda \). These two reasons together imply a variance in the log MRPK across firms in the procurement sector which is around 18% lower than that in the private sector: 0.023 vs. 0.026.

Relative prices. Finally, we look at the price of public goods relative to private goods, which will be important to understand some of the results from the policy counterfactuals. The relative price can be written as:

\[
\frac{P_g}{P_p} = \frac{\overline{MRPK}_g TFP_p}{\overline{MRPK}_p TFP_g}
\]

where \( \overline{MRPK}_g, \overline{MRPK}_p, TFP_g, \) and \( TFP_p \) are the weighted average marginal revenue products and sectorial TFP’s (see Online Appendix C for derivations). As in standard multi-sector models, the ratio of relative prices is inversely related to sectorial TFPs. But equation (18) also implies that the relative price is positively related to the ratio of average marginal revenue products in each sector. That is, a relatively high sectorial “wedge” is associated with a higher relative price. Because firms active in procurement are on average more financially constrained, i.e., have a higher \( \lambda \), the average wedge in the procurement sector is higher. In particular, in our benchmark economy, \( \overline{MRPK}_g \) is around 8% higher than \( \overline{MRPK}_p \). However, as mentioned above,
TFP$_g$ is 21% higher than TFP$_p$. All together the relative price of public goods is $P_g/P_p = 0.899$.

7 Policy Experiments

Our empirical evidence in Section 3 and model results in Section 6 show that procurement contracts help firms grow out of their financial constraints. At the same time, in Section 5 we have seen that smaller firms, typically the most constrained, participate less in procurement. This suggest that making procurement contracts available to smaller firms may lead to aggregate output gains. For this reason, in this Section we quantify the aggregate effects of reforming the public procurement allocation system through expenditure-neutral changes that favor small firms.

7.1 Counterfactual 1: Targeting the selection pattern

We first run an experiment that consists of encouraging the participation of smaller firms and hence change the selection pattern of firms into procurement. This counterfactual aims to reproduce the type of “set-aside” policies for small businesses implemented by the U.S. Small Business Administration. Policies of this type could be those that facilitate access to the competition for procurement contracts —like better publicity or direct assistance to prepare the process— or measures to provide more transparency of the whole process —which should diminish the importance of political connections. To model this type of policies, we reduce the parameter $\eta_1$ so that the model generates procurement size premium of 50%, as opposed to the 72% in the baseline calibration. That is, we solve for a new economy in which the procurement system gives relatively lower weight to firms’ investment in $b$, making it easier for small firms to participate. We also change $\eta_0$ and $Y_g$ so that the fraction of procurement firms $m_g$ and total government expenditure $P_g Y_g$ remain unchanged.

We present the main results from this exercise in column (1) of Table 7, which shows the relative change of some relevant variables compared to their counterparts in the benchmark economy.$^{23}$ We use aggregate GDP in private good units as the main measure to assess the macroeconomic impact of the policy. We report two different

$^{23}$We report the difference not the relative change for variables that are already in shares, i.e., the percentage of procurement firms and the share of procurement in GDP, as well as for the interest rate $r$, the tax $\tau$, and the parameters $\eta_0$ and $\eta_1$.  

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measures of GDP: “nominal” GDP, which uses the relative price of procurement $P_g$ in the reformed economy, and “real” GDP, which keeps the price $P_g$ of the benchmark economy. We also report changes in the levels of capital in the two sectors as well as in the aggregate, together with changes in variables related to misallocation and aggregate TFP.

**Aggregate output.** We find that the reform increases nominal GDP by around 2.07%. Since we are keeping $P_g Y_g$ constant in our experiment, this increase comes entirely from a 2.36% increase in $Y_p$. We find an increase in “real” GDP of 1.18%, which is lower than in “nominal” GDP. The reason is that the relative price of procurement $P_g$ increases as a result of the policy experiment, making the provision of public goods by the government more costly than in the benchmark economy. We will come back to the increase in $P_g$ below.

**TFP vs. $K$.** We can decompose the increase in “nominal” GDP into that coming from capital accumulation vs. TFP. We find that most of the increase in GDP (around 72%) is accounted for by an increase in the aggregate stock of capital $K$, which increases by 1.88%. In Section 7.3 below, we will provide more details that will help understand the evolution of the stock of capital. The rest is explained by an increase of 0.29% in aggregate “nominal” TFP, which is the result of a slight increase in TFP$_p$ (0.02%), a large reduction in TFP$_g$ (5.72%), and the above-mentioned increase in $P_g$. When keeping constant $P_g$ to its value in the benchmark economy, our model predicts a decrease in “real” TFP (0.69%) due to the fall in TFP$_g$.

**Why do TFP$_p$ and TFP$_g$ change?** The increase in TFP$_p$ is the result of the beneficial effects of procurement on wealth accumulation of small firms. In the new steady state, firms that had a relatively high MRPK$_{ip}$ in the benchmark economy are more likely to be active in procurement, which allows them to accumulate more assets and hence operate with a higher level of capital in the private sector (see Section 6). This reallocation of procurement contracts towards relatively high MRPK$_{ip}$ firms implies a reduction in the dispersion of MRPK$_{ip}$ and hence in misallocation. The decrease in TFP$_g$ is explained by a change in the selection of firms into procurement and hence by the change in the composition of procurement firms. In particular, procurement firms in this counterfactual economy have lower productivity $s$ (6.47% less) and lower net worth $a$ (13.90% less). As a result, procurement firms are less

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24This compares to a 12% increase in GDP of eliminating financial frictions in our economy (setting $\phi_a \to \infty$).
productive and more constrained (as reflected by a larger $\lambda$). This leads to the decline in $\text{TFP}_g$ due to both lower $s$ (lower first-best productivity) and lower $a$ (more misallocation).

**Why does $P_g$ increase?** As explained above, the relative price of public goods depends on the ratio of sectorial wedges times the inverse of relative sectorial TFPs, see equation (18). We have just seen that the ratio $\text{TFP}_p/\text{TFP}_g$ increases substantially, which raises $P_g$. In addition, because procurement firms are now more constrained on average, their average wedge $\overline{\text{MRPK}}_g$ also increases, which pushes $P_g$ further up.

### 7.2 Counterfactual 2: Decreasing contract size

We perform a second experiment that consists in reducing the size of contracts to reach out to more firms, while keeping the same level of expenditure $P_gY_g$. This experiment is motivated by the fact that decreasing the size of procurement contracts as a tool to promote the participation of small firms is at the core of the European Commission’s agenda for public procurement regulation. In practice, we solve for a counterfactual economy in which the fraction of firms from which the government buys, $m_g$, increases by 10 percentage points, i.e., from 3.8% to 13.8%.

We do so by increasing $\eta_0$ and adjusting $Y_g$ so that $P_gY_g$ remains unchanged. We also keep $\eta_1$ unchanged.

We show the results from running this policy experiment in column 3 of Table 7. In contrast to the previous counterfactual, we find that the reform reduces nominal GDP by around 2.68%. Out of this decline, around 85% is explained by a fall in aggregate capital and the rest by a decline in aggregate TFP. As in the previous counterfactual, we will explain the behavior of capital accumulation below (see Section 7.3).

Also different from the first counterfactual, the model predicts a decline in $P_g$, which means that the provision of public goods by the government becomes cheaper. As in the other counterfactual, $\text{TFP}_p/\text{TFP}_g$ goes up, which should raise $P_g$. However, $P_g$ decreases in this counterfactual because the average wedge in the procurement sector $\overline{\text{MRPK}}_g$ decreases by 6.09% instead of increasing. As in the previous counter-

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25 This represents a large change in the average size of contracts. In the counterfactual economy, the average size of the contract is 27% of that in the benchmark economy. The European Commission is not explicit about how much governments should decrease the size of the contracts: “[…] Such division could be done on a quantitative basis, making the size of the individual contracts better correspond to the capacity of SMEs […]” (see the Public Sector Directive 2014/24/EU for details.)
Table 7. Counterfactuals

<table>
<thead>
<tr>
<th></th>
<th>Panel A: $\phi_g &gt; \phi_p$</th>
<th>Panel B: $\phi_g = \phi_p$</th>
</tr>
</thead>
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<tr>
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<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>Benchmark</td>
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</tr>
<tr>
<td>Output</td>
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<td>$Y_p$</td>
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<tr>
<td>$Y_g$</td>
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<td>GDP</td>
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<tr>
<td>Capital</td>
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<tr>
<td>$K_p$</td>
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</tr>
<tr>
<td>$K_g$</td>
<td>2.710</td>
<td>-1.81%</td>
</tr>
<tr>
<td>$K_p + K_g$</td>
<td>24.094</td>
<td>1.88%</td>
</tr>
<tr>
<td>Productivity</td>
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<td>TFP</td>
<td>0.255</td>
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</tr>
<tr>
<td>TFP $p$</td>
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<td>0.29%</td>
</tr>
<tr>
<td>real TFP</td>
<td>0.258</td>
<td>-0.69%</td>
</tr>
<tr>
<td>MRPK $p$</td>
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<tr>
<td>MRPK $g$</td>
<td>0.277</td>
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</tr>
<tr>
<td>TFP $p$ gain</td>
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<td>-0.37%</td>
</tr>
<tr>
<td>TFP $g$ gain</td>
<td>0.033</td>
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<td>Prices/tax</td>
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</tr>
<tr>
<td>$r$</td>
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<td>0.000</td>
</tr>
<tr>
<td>$\tau$</td>
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<tr>
<td>Procurement</td>
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<tr>
<td>% firms</td>
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<tr>
<td>Share GDP</td>
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<td>$\eta_0$</td>
<td>0.209</td>
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<tr>
<td>$\eta_1$</td>
<td>0.527</td>
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<tr>
<td>ratio mean $s$</td>
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<tr>
<td>ratio mean $a$</td>
<td>1.727</td>
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<tr>
<td>ratio mean $\lambda$</td>
<td>2.973</td>
<td>8.74%</td>
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Notes: Panel A of shows the results from running the two policy experiments under our baseline calibration, i.e., $\phi_g > \phi_p$. Panel B shows the results from running the experiments for the alternative calibration in which we impose that $\phi_g = \phi_p$. Columns (1) and (4) show the variables from the respective benchmark economies. Columns (2) and (5) show the results from running counterfactual 1, which consists in changing $\eta_0$ and $\eta_1$ so that the procurement premium decreases from 72% to 50% (while keeping the % of procurement firms equal to 3.8%) and changing the average size of contracts accordingly so that $P_g Y_g$ remains constant. Columns (3) and (6) show the results from running counterfactual 2, which consists in increasing $\eta_0$ so that the model generates a % of procurement firms of 13.8% and decreasing the average size of contracts accordingly so that $P_g Y_g$ remains constant, while keeping $\eta_1$ constant.
### Table 8. Channels

<table>
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<tr>
<th></th>
<th>Panel A: Count. 1</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Benchmark</td>
<td>Step 1</td>
<td>Step 2</td>
<td>Full</td>
</tr>
<tr>
<td>$Y_p$</td>
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<td>-0.12%</td>
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<td>2.36%</td>
</tr>
<tr>
<td>$K$</td>
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<td>2.09%</td>
<td>1.88%</td>
</tr>
<tr>
<td>TFP</td>
<td>0.258</td>
<td>0.05%</td>
<td>0.10%</td>
<td>0.29%</td>
</tr>
<tr>
<td>GDP</td>
<td>6.214</td>
<td>-0.10%</td>
<td>2.19%</td>
<td>2.07%</td>
</tr>
<tr>
<td>$r$</td>
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<td>0.000</td>
<td>0.000</td>
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<table>
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<th>Panel B: Count. 2</th>
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<td>Step 1</td>
<td>Step 2</td>
<td>Full</td>
</tr>
<tr>
<td>$Y_p$</td>
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<td>-3.05%</td>
</tr>
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<td>5.22%</td>
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<tr>
<td>TFP</td>
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<td>-1.01%</td>
<td>-0.73%</td>
<td>-0.43%</td>
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<tr>
<td>GDP</td>
<td>6.214</td>
<td>-0.81%</td>
<td>4.45%</td>
<td>-2.68%</td>
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<td>$r$</td>
<td>0.050</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
</tbody>
</table>

**Notes:** This table shows the results from running different versions of our model. Columns (0) and (4) show the value of the variables both in our benchmark economy and in the new steady state. Column (1) refers to the “Short-run partial equilibrium effect.” Column (2) refers to the “Long-run partial equilibrium effect.” Importantly, in columns (1) and (2), we solve the model by using the $\eta_0$ and $\eta_1$ that we use to compute the new steady states, and adjust $Y_g$ so that $P_g Y_g$ remains unchanged.

Factual, the policy allocates procurement contracts to smaller firms, which leads to a pool of procurement firms that are both less productive and have less net worth. However, different from the previous counterfactual this policy also reduces the average contract size. This makes procurement firms less constrained (instead of more constrained as in the previous counterfactual) because they have a smaller demand to serve (the average $\lambda$ of procurement firms declines by 36%) and hence the wedge in the procurement sector is smaller.

### 7.3 Channels

The main factor driving changes in aggregate GDP between the two steady states is capital accumulation. In the first counterfactual, a higher $K$ explains around 72% of the rise in GDP. In the second counterfactual, a lower $K$ explains around 85% of the fall in GDP. In this section, we provide more details about the specific mechanisms driving changes in $K$, as well as some other interesting effects that our model generates.

To do so, in Table 8 we solve for two different intermediate versions of the two
counterfactual economies that aim to isolate the different channels at play. In column (0) we report again statistics of our benchmark economy, in columns (1) and (2) statistics of these intermediate exercises, and in column (3) we report again statistics of the complete counterfactual economy. We refer to this last column as “Full,” capturing the idea that it contains all the different mechanisms we want to isolate in the previous columns.\textsuperscript{26}

Short-run partial equilibrium effect. As we discussed in previous sections, a procurement shock ($d = 0 \rightarrow d = 1$) makes constrained firms decrease their output sold in the private sector at impact. This within-firm spillover manifests with the opposite sign when a firm becomes inactive in procurement ($d = 1 \rightarrow d = 0$). Our policy reforms reallocate procurement contracts across firms, and hence generate crowding out effects for some firms (constrained firms that start selling to the government), crowding in effects for others (constrained firms that stop selling or sell smaller contracts to the government), and no change for the rest (constrained firms unaffected by the policy or unconstrained firms).

We want to measure the aggregate effects of this short run crowding out/in effects. To do so, we solve the static firm’s problem using the parameters that characterize the new procurement allocation system, i.e., the new $\eta_0$ and $\eta_1$, without taking any general equilibrium or dynamic effects into account. This means that (a) we keep

\textsuperscript{26}Importantly, in columns (1) and (2), we solve the model by using the $\eta_0$ and $\eta_1$ that we use to compute the complete counterfactual, and adjust $Y_g$ so that $P^*_g Y_g$ remains unchanged.
$Y_p$, $r$, and $\tau$ as in the benchmark economy (and hence markets do not clear and the government’s budget constraint is not satisfied), and (b) we keep the distribution of $a$ and $b$ unchanged.

In Figure 4, we provide some evidence on how these crowding out/in effects operate for different types of firms. In particular, we plot the relative change in total $p_p y_p$ across groups of firms with different levels of net worth $a$. We do so by ordering firms based on the benchmark distribution of $a$, splitting them in four groups so that each group accounts for $1/4$ of the production of aggregate $Y_p$, and calculating the change in total $p_p y_p$ produced by each group, as caused by the procurement reform at impact. We find that the crowding out effect dominates within the first, second, and third bins of the distribution of $a$, and that the opposite is true for the fourth bin. This result reflects the fact that our policy reforms consists of reallocating procurement contracts from relatively big to relatively small firms. That is, firms with relatively lower $a$ are more likely to be “new procurement firms” as a result of the policy change, whereas firms with relatively higher $a$ are less likely to do procurement.

Looking at column (1) of Table 8, we find that the policy reform generates a short-run partial equilibrium fall in the private sector output $Y_p$ and hence in GDP of the two counterfactuals. In particular if prices and the distributions of $a$ and $b$ were fixed, the policy reform would generate a fall in GDP of 0.10% in the first counterfactual of 0.81% in the second one.

**Long-run partial equilibrium effect.** Financially constrained firms that get procurement projects increase their revenues and can accumulate net worth at a faster pace and hence increase their private sector activity in the long run. In step 2, we quantify the aggregate effects of this strengthening of the self-financing mechanism. To do that, we solve for a new steady state distribution of $a$ and $s$ of our model under the new $\eta_0$ and $\eta_1$, but imposing that the policy functions of the dynamic problem $c(s, a, d)$, $a'(s, a, d)$ and $b(s, a, d)$, as a ratio of the entrepreneurs’ cash on hand $(1+r)a+(1-\tau)\pi(s, a, d)$, remain unchanged. Our goal is to isolate the mechanical accumulation effect that income from procurement generates on directly affected firms, without taking into account adjustments in dynamic decisions. We also abstract from general equilibrium effects by using the same $r$ and $\tau$ as in our benchmark economy. Column 2 of Table 8 shows a significant positive effect of this channel on the macroeconomy. In counterfactual 1, the implied capital stock and GDP are $2.09\%$ and $2.19\%$ higher than in the benchmark. In counterfactual 2, their counterparts are
5.22% and 4.45% higher. Hence, if we keep the policy functions of the dynamic problem and the interest rate unchanged, reforming the procurement allocation system in a way that favors small firms would generate a positive aggregate effect because it allows more constrained firms to accumulate more net worth, grow out of their financial constraints, and hence produce more in the long run. The enhancement of the self-financing channel is more effective in counterfactual 2, that is, in the policy experiment that reduces contract sizes to reach out to a higher number of firms.

**Full effect.** Finally, we study the aggregate effect of the changes in the policy functions of the dynamic problem and prices in general equilibrium. We find that the reforms reduce the incentives for big firms to accumulate assets over time, which shrinks (in counterfactual 1) or even turns negative (in counterfactual 2) the output gains associated to the reforms.

One of the main reasons why firms accumulate assets in our model is the fact that they expect to obtain a public procurement contract at some point. That is, obtaining a procurement contract is a big demand shock in response to which firms want to expand their invested capital stock, causing even relatively big firms to accumulate precautionary savings. Intuitively, productive firms want to have enough net worth so that they minimize the probability of being constrained in case the procurement shock is realized. In a context in which the average contract size is lower or in which obtaining a contract is less likely, this precautionary savings motive becomes weaker.

In *Figure 5*, we show how this reduction in firms’ incentives manifests in our model. This figure simulates the average life cycle profile of a cohort of firms in the benchmark economy (solid blue line), in the counterfactual 1 (dashed green line), and in the counterfactual 2 (dashed red line). In particular, we take a large number of newborn firms that draw the highest productivity state in every period and stochastically obtain procurement projects according to the probability \( g(b) \) and their choices of \( b \). We focus on firms that draw the highest productivity state in every period, to capture the fact that changes in saving incentives across the different economies will be specially apparent in firms that expect to operate at large scales.

We find that the three economies exhibit common patterns. As they age, firms become larger —accumulate more net worth, operate with more capital, and sell more both in the private and procurement sector— and less financially constrained. The panels for \( p_y y_g \) and \( d \) show the differences in the procurement allocation systems across the three economies. The probability of participating in procurement, given
Figure 5. Firms’ life cycle profiles

Notes: This figure shows the “average” life cycle profile of a large number of firms, all drawing the highest productivity level $s$ in every period, simulated in our model under three different scenarios: the benchmark economy (blue line), the counterfactual 1 (green line), and the counterfactual 2 (red line). This particular figure uses firms with a productivity shock which is the highest among the five productivity shocks that we use to solve our model.

by $d$, is the highest in counterfactual 2 (there is a higher number of contracts) and the lowest in counterfactual 1 (the number of contracts is the same but high $s$ firms are less likely to get them). In terms of the revenues from procurement, $p_y y_t$, the highest ones are under the benchmark economy, despite the fact that the probability of getting a contract is significantly higher in the counterfactual 1. This is driven by the fact that contracts are considerably bigger in the benchmark.

The most important finding from this figure has to do with the evolution of $a$ and $k$ over the firms’ life cycle. We find that net worth accumulation by high $s$ firms is the highest in the benchmark economy. In counterfactual 1, these firms prefer to accumulate slightly less because, although the size of contracts is still big, it is less likely for high $s$ firms to obtain them. The big difference becomes visible in counterfactual 2, where firms’ net worth accumulation is significantly lower. This also becomes apparent when looking at the evolution of $k$. In the counterfactual 2

\[\text{Note: Instead, in counterfactual 1, all firms with } s \text{ lower than the highest state, accumulate more net worth compared to the baseline (this is not shown in the paper, but it is available upon request).}\]
economy, firms reach their optimal size at an age that is considerably earlier than in
the other two economies. To provide some intuition on this result, let’s go back to
the Euler equation given by equation (14). The strength of the precautionary savings
motive (or self financing channel) is given by the term $\frac{\partial \pi(s',a',d')}{\partial a'}$, which is equal to
$\phi_a \lambda'$ (see Online Appendix B.2 and B.3). That is, the expected value of the financial
constraint multiplier represents an extra return to asset accumulation. In other words,
firms that expect to be financially constrained next period will accumulate more assets
today.

The panel $\lambda - \lambda_{Base}$ in Figure 5 compares the $\lambda$’s in the two counterfactual
economies with the one in the benchmark economy. We find that the $\lambda$’s tend to
be smaller in the two counterfactuals for high $s$ firms, and particularly so in coun-
terfactual 2. Importantly, these differences become bigger as firms approach their
optimal size in the counterfactual from below, which points towards the fall in incep-
tives to accumulate assets being particularly high for relatively bigger firms. This is
driven by the fact that, in counterfactual 2 the procurement contracts are smaller so
firms do not need much financial capacity to expand in order to service them, and in
counterfactual 1 the old high productivity firms become less likely to win procurement
contracts as these are being reallocated to firms with lower $a$ and $s$.

7.4 The importance of $\phi_g > \phi_p$

In our final exercise we want to show the quantitative importance of the fact that
revenues from public procurement help obtain credit to a larger extent than revenues
from the private sector ($\phi_g > \phi_p$). To do so, in this section we compare the macroe-
conomic effects of the policy reforms in a world where $\phi_g = \phi_p$. We apply the same
 calibration strategy presented in Section 5, but imposing that $\varphi_p = \varphi_g$ both equal to
the value of $\varphi_g = 0.42$ in the baseline, and ignore the targets associated with $\varphi_p$ and $\varphi_g$
in the calibration. In column 2 of Table 5, we show that most of the parameters are
similar to those found in our baseline calibration. Because the model has to generate
the same credit-to-capital ratio as before and $\phi_g$ is lower by construction, $\phi_a$ must be
higher, mechanically increasing $\phi_p$.

In columns 4, 5, and 6 of Table 7, we show the benchmark economy and its as-
sociated counterfactual exercises for this new calibration. Comparing the benchmark
economy in both cases we can understand the role of $\phi_g$ in the aggregate economy.
There are three main results. First, as discussed in Section 4.9, there is no within-
firm misallocation when \( \phi_g = \phi_p \), that is, the fraction of output sold to the private and public sectors does not depend on the financial situation of the firm. Second, whenever \( \phi_g = \phi_p \) the pattern of selection into procurement in terms of net worth \( a \) is more acute. This is because it is harder to finance procurement with lower \( \phi_g \), and hence procurement becomes relatively more attractive for firm with more financing capacity. And third, with \( \phi_g = \phi_p \) the public good become more expensive relatively to private good. This is the result of an increase in the two components of \( P_g/P_p \). The ratio \( \text{TFP}_p/\text{TFP}_g \) increase slightly because there is more misallocation and hence lower TFP in the government sector due to firms operating in procurement being more financially constrained. The ratio \( \overline{\text{MRPK}}_g/\overline{\text{MRPK}}_p \) increases mainly due to the loss of within-firm misallocation, which decreases capital in the procurement sector for all firms.

Regarding the procurement reforms, our main finding is that if government contracts were equally pledegable as revenues from selling to the private sector, changes in the procurement system that facilitate the presence of small firms would be associated with worse macroeconomic outcomes. In the case of keeping the average size of contracts but increasing the strength of diminishing returns to \( b \), i.e., counterfactual 1, the increase in nominal GDP would be around 1.69 percentage points smaller. In the case of reducing the average size of contracts, i.e., counterfactual 2, we find that the fall in nominal GDP would be more than twice as big as in the baseline calibration. In fact, we find that GDP would also fall as a result of counterfactual 1 when we measure the change in GDP in real terms. The reason for these results is as follows. When \( \phi_g = \phi_p \), the private sector negative spillover of procurement in the short run is larger because there is no extra financing through public revenues to alleviate the problem of scarce collateral; see Proposition 13 in Online Appendix B. In addition, by reducing the extent to which borrowing capacity increases when participating in procurement, the long-run positive effects also weaken. Overall, procurement is less effective in helping constrained firms increase their production.

8 Conclusion

In this paper, we quantify the macroeconomic impact of changes in the public procurement allocation system. To do so, we use a comprehensive framework that builds on three steps: selection, treatment, and the interplay between procurement and the
We use our framework to evaluate some of the policy reforms that are currently implemented in the US or are in the industrial policy agenda of the European Commission. In particular, we quantify the long-run macroeconomic effects of a size-dependent expenditure-neutral policy reform that consists of facilitating small firms’ participation by either targeting them in the allocation process or by breaking down big projects into smaller ones.

Our results point towards the presence of long-run positive effects for directly affected firms, but also suggest the existence of important changes in big firms’ dynamic behaviors that could shrink the expansionary effects or even make them negative. Our findings show that both the sign and size of these effects and hence the overall macroeconomic impact of this type of policies crucially depends on the severity and type of financial frictions in the economy. But they also depend on the type of reform, which determines how the change in procurement harms larger firms. These findings suggest that the optimal procurement allocation system in a country would depend on the specific institutional characteristics of the economy.

We view our contribution as part of a broader research agenda on the macroeconomic effects of government procurement, a policy that is surprisingly understudied. In our work, we only investigate the long-run consequences of expenditure-neutral changes in the procurement allocation system. Issues like the short-term consequences of reforms, or the potential implications for the effectiveness of fiscal policy are still unexplored. We emphasize that pursuing this research agenda will deliver important policy implications.

References


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