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Brandyn Bok | Thomas M. Mertens | John C. Williams

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Abstract

The correlation between uncertainty shocks, as measured by changes in the VIX, and changes in breakeven inflation rates declined and turned negative after the Great Recession. This estimated time-varying correlation is shown to be consistent with the predictions of a standard New Keynesian model with a lower bound on interest rates and a trend decline in the natural rate of interest. In one equilibrium of the model, higher uncertainty raises the probability of large shocks that leave the central bank constrained by the lower bound and unable to offset negative shocks. Resulting inflation shortfalls lower average inflation rates.

Williams: Federal Reserve Bank of New York (email: john.c.williams@ny.frb.org). Bok: UCLA (email: bbok@g.ucla.edu). Mertens: Federal Reserve Bank of San Francisco (email: thomas.mertens@sf.frb.org).
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I Introduction

Over the past quarter century, there has been a marked downward trend in the correlation between changes in market-based measures of uncertainty and expected inflation in the United States. At the beginning of this century, this correlation was essentially zero, but it has since turned negative. The emergence of a negative relationship between uncertainty shocks and expected inflation raises the concern that uncertainty shocks now have a more pronounced effect on the economy than two decades ago and thereby pose greater challenges for the conduct of monetary policy.

In this paper, we document this significant repricing in financial markets reflected in changes to financial correlations and analyze them within a macroeconomic model. Our main contributions are two-fold. The first is a theoretical analysis of the effects of uncertainty shocks on expected inflation and interest rates within a standard New Keynesian model. We show how the presence of a lower bound on interest rates affects this correlation. The second is an empirical analysis of the predictions of the theory, which indicate that the decline in the natural real rate of interest to low levels has contributed to the emergence of the negative relationship between uncertainty and inflation expectations evident in the data.

In the theoretical section of the paper, we use a standard New Keynesian model with a lower bound on nominal interest rates for an analytical derivation of the theoretical predictions. For simplicity, we focus on demand shocks as the only source of disturbances. We show the implications of a change in the level of interest rates, as determined by the natural rate of interest, on the joint behavior of uncertainty and inflation expectations. The presence of the lower bound implies the existence of two distinct ergodic distributions for endogenous variables. The distinguishing feature between these equilibria is the probability of a binding lower bound which translates into different levels of inflation expectations. The equilibrium with the lower probability, which we refer to as the target equilibrium, leaves the central bank mostly unconstrained and able to stabilize inflation. In the second equilibrium, which we denote as the liquidity trap equilibrium, the lower bound binds more often which results in more frequent and larger inflation shortfalls. The presence of a lower bound and the type of equilibrium the economy is in are key determinants of the correlation between uncertainty shocks and changes in inflation expectations.

Without a lower bound on interest rates, a mean-preserving increase in uncertainty has no systematic effect on the unconditional means of expected inflation and interest rates. This is because the central bank can use its interest rate policy in response to positive and negative shocks symmetrically to stabilize inflation. In contrast, in the presence of an occasionally binding lower bound on interest rates, a mean-preserving spread to the distribution of shocks affects the unconditional mean rate of inflation, decreasing it in the target equilibrium and increasing it in the liquidity trap equilibrium. In the case of the target equilibrium, an increase in uncertainty implies that policy is constrained in a larger region of the state space. This asymmetric constraint on the ability to respond to negative shocks results in a reduction in the unconditional means of expected inflation and interest rates. In the case of the liquidity trap equilibrium, an increase in uncertainty contracts the region in which the lower bound binds and thereby increases the unconditional means of inflation and interest rates.

In the target equilibrium, a decline in the natural rate of interest leads to the observed effects of changes in uncertainty on inflation expectations when the probability of a binding lower bound is sufficiently small in the case of general distributions. Under additional assumptions such as Gaussian distributions, this result holds for any probability in the target equilibrium.

Second, we empirically assess these theoretical predictions using data from the United States. Therefore, we construct rolling-window correlations between changes in market-based measures of uncertainty, such as the VIX, and changes in expectations of far-forward future inflation and interest rates. The New Keynesian model predicts that, analogously to the correlation for inflation expectations, the correlation between uncertainty shocks and changes in expected interest rates should turn negative as well. Indeed, this correlation fell from close to zero to around -0.5 over the same time period. These changes took place as estimates of the natural rate of interest declined and the incidence of monetary policy being near or at the lower bound increased.

To test this prediction, we regress the correlations for interest rate and inflation expectations on estimates of the natural rate of interest and find that the empirical evidence is qualitatively consistent with the U.S. economy being in the target equilibrium, in line with evidence reported in Mertens and Williams (2021). We furthermore establish the robustness of the results. We show that we obtain consistent results with the target equilibrium of the model, including when removing term premia or using alternative measures of uncertainty shocks and inflation expectations. For the latter, we show that our results are robust to using survey-based measures of expected inflation from the Survey of Professional Forecasters and to using both an aggregate econometric measure of economic financial uncertainty and a news-based economic policy uncertainty index for uncertainty shocks (see Ludvigson et al. (2021) and Baker et al. (2016)).

Finally, we assess the implications of greater uncertainty about aggregate demand for the design of monetary policy. We show that, in the target equilibrium, the benefits of average-inflation targeting relative to standard inflation targeting increase with greater macroeconomic uncertainty. We implement average-inflation targeting through an adjustment in the level of the policy rule for nominal interest rates. Lowering the intercept leads inflation to overshoot the target inflation rate during times when policy is unconstrained. This policy framework thereby makes up for shortfalls in inflation at times when the lower bound on interest rates is strictly binding.

We highlight one caveat to our analysis. To maintain analytical tractability, we do not model time variation in correlations explicitly. Instead, we study changes in the derivative of expectations with respect to uncertainty when there is an exogenous shift in the natural real rate of interest. Therefore, we do not generalize these findings to models of monetary policy with richer dynamics nor address the potential endogeneity of the decline in the natural real rate.

Our analysis relates to several strands of the literature. First, it relates to models that incorporate a lower bound on interest rates into a New Keynesian economy, such as Fuhrer and Madigan (1997), Eggertsson and Woodford (2003), Adam and Billi (2006), Campbell et al. (2012), and Cochrane (2018). Papers in this literature that deal with the multiplicity of equilibria include Benhabib et al. (2001), Mendes (2011), Hills et al. (2016), Nie and Roulleau-Pasdeloup (2021), Cuba-Borda and Singh (2022), and Bilbiie (2022). Our analysis relates to the work by Swanson and Williams (2014) and Mertens and Williams (2021) in that it measures the effects of the lower bound on interest rates from financial market data. Second, it relates to a literature that studies changing correlations in financial markets and links them to macroeconomic outcomes. Baele et al. (2010), Duffee (2022), and Pflueger (2023) document changing correlations between stocks and bonds. Campbell et al. (2020) point out macroeconomic drivers behind this phenomenon. And Bilal (2017) and Gourio and Ngo (2020) argue that the lower bound on interest rates might be behind this finding, while Datta et al. (2021) link the lower bound to the changing equity-oil correlation. Third, this paper contributes to the large and fast-growing literature on uncertainty shocks and their effects on the economy. Seminal contributions on the measurement include Jurado et al. (2015) and Baker et al. (2016). Leduc and Liu (2016) study the effects of uncertainty shocks in a New Keynesian economy while Basu and Bundick (2017), Plante et al. (2017) and Nakata (2017) take the lower bound on interest rates into account.

II Stylized Fact

This section discusses a new stylized fact: Since the mid-2000s, the correlation between uncertainty shocks and changes in long-run inflation expectations has turned negative from a level of close to zero. While we

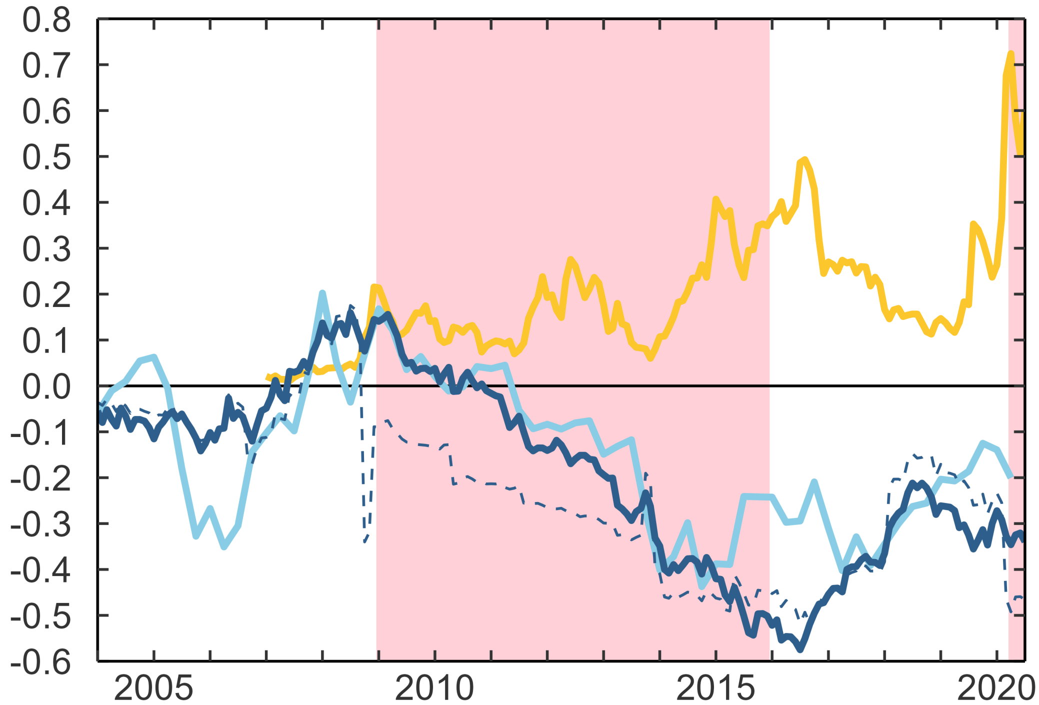

Figure 1: The dark blue line plots the five-year trailing-window correlation (Spearman correlation: solid line, Pearson correlation: dashed line) of monthly changes in the VIX with monthly changes in the five-year, five-year-forward breakeven inflation (BEI) rate; the underlying variables are monthly averages of daily market-close values. The light blue line plots the five-year trailing-window Spearman correlation of quarterly changes in the VIX with quarterly changes in the median SPF forecast of 10-year average CPI inflation. The yellow line plots the caps-implied probability of the zero lower bound binding ten years out. The pink shaded areas represent times when the zero lower bound was binding. The correlation between the probability of a binding lower bound (yellow line) and the baseline correlation (solid dark blue line) is -0.68. The sample period ranges from January 2004 until July 2020.

purposefully keep this section brief, Section V discusses the robustness of these findings.

To demonstrate these patterns, we measure uncertainty shocks as changes in expected volatility in the stock market via the VIX volatility index. For expected inflation, we use market-implied breakeven inflation (BEI) rates, measured as the difference between yields of nominal and inflation-protected Treasury bonds. We use five-year inflation expectations five years ahead to focus on expectations of inflation in the longer term. We retrieve daily time series on each of these variables from Bloomberg.[1] To get monthly observations, we construct changes in monthly averages of the daily market-close values and compute the correlation between these monthly changes in the VIX and changes in inflation expectations for five-year rolling windows. The overall sample for the monthly changes is restricted by the introduction of Treasury inflation-protected securities (TIPS) in 1999, such that the five-year trailing-window correlations begin in January 2004. We

[1] Appendix D.A summarizes the sources and details of all data used in this paper.

end the sample in July of 2020 before the Federal Reserve's announcement of its average-inflation targeting framework (FOMC (2020)).

Figure 1 plots the Spearman and conventional (Pearson) correlations between these measures of uncertainty shocks and changes in inflation expectations at the monthly frequency. Spearman correlations are the conventional (Pearson) correlations of the rank of each observation in the sample. Since they use the rank rather than the observation itself, Spearman correlations are more robust to outliers and thus our preferred measure.[2] The decline in this correlation occurred during the mid-2000s and accelerated as the federal funds rate was lowered to the effective lower bound. It remained highly negative throughout the initial lower bound period and increased after liftoff from the lower bound. After the onset of the COVID-19 pandemic and the ensuing recession, the correlation turned highly negative again.

These changes in the correlation do not seem to be primarily driven by risk premia. Figure 1 contrasts the baseline correlation which uses breakeven inflation expectations as the main measure with the analogous correlation that uses the Survey of Professional Forecasters as a measure of inflation expectations. Specifically, we use the median forecast for 10-year CPI inflation. The two time series comove quite closely, suggesting that there is a structural reason behind their movements.

In this paper, we argue that the lower bound on interest rates can lead to the observed pattern within a basic New Keynesian model. In such an environment, increases in uncertainty raise the possibility that monetary policy is unable to eliminate inflation shortfalls. Concerns about an increased likelihood of a binding zero lower bound and its impact on inflation arose in the late 1990s and early 2000s (see, e.g., Bernanke (2002)). Figure 1 puts the changing correlation and zero lower bound risk ten years out into perspective. Zero lower bound risk is measured as the probability of the 3-month LIBOR rate being at zero ten years out.[3] We focus on long-term zero lower bound risk to ensure that transition dynamics are not driving the results, consistent with the model we present in the next section. As the figure shows, far-out zero lower bound risk tended to be high at times when the break-even inflation-uncertainty correlation was low. Regressing the inflation expectation-uncertainty correlation on the lower bound probability results in a regression coefficient of -1.07 with a Newey-West standard error of 0.54 which is statistically significant at the 5% level.

In the following section, we formalize the link between changing correlation in this section and the lower bound on interest rates.

[2]For the remainder of the paper, we only report Spearman correlations. The analogous charts for conventional Pearson correlations are in Internet Appendix B.A.

[3]The measure is taken from the Center for Monetary Research (2024).

III Model

In this section we study a standard log-linearized three-equation New Keynesian model, as in, e.g., Woodford (2003), to which we add a lower bound on interest rates. For analytical tractability, our model features only demand shocks that are independent and identically distributed (i.i.d.) over time. The model is a simplified version of the one in Mertens and Williams (2021) in that it abstracts from supply-side shocks.[4] Compared to that paper, the model solution in this paper is more concise and facilitates the analysis of uncertainty shocks.

The Phillips curve describes the evolution of inflation π_t

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{1}$$

where $\kappa > 0$ is the sensitivity of inflation to changes in the output gap, $\beta \in (0, 1)$ is the discount factor, and inflation expectations are based on the information set available at time t.

The IS relation describes the behavior of the output gap x_t

$$x_t = \epsilon_t - \alpha(i_t - \mathbb{E}_t \pi_{t+1} - r^*) + \mathbb{E}_t x_{t+1}, \tag{2}$$

where $\alpha > 0$ is the responsiveness of the output gap to deviations of the real interest rate from its long-run natural rate, r^*.

We specify the demand shock as $\epsilon_t = \bar{\epsilon} + \nu_t$ such that we can take comparative statics with respect to its mean $\bar{\epsilon}$.[5] The noise component ν_t has mean zero, is assumed to be i.i.d. over time, and follows a distribution with infinite support, probability density function $g(\cdot)$, and cumulative density function $G(\cdot)$ that satisfies $\lim_{\nu \to -\infty} \nu G(\nu) = 0$.[6]

The central bank sets the nominal rate i_t optimally under discretion but is constrained by a lower bound i^{LB} that lies below the natural real rate of interest; that is, $i^{LB} < r^*$. The central bank aims to minimize a loss function $\mathscr{L}$ that penalizes output gaps as well as deviations of inflation from its target, which is normalized

[4] The addition of supply-side shocks delivers consistent results but makes the exposition more cumbersome.

[5] While changes in this mean can capture changes in the outlook for the economy, all results around the uncertainty go through if we set the mean to zero.

[6] While the results hold more generally, we work with an infinite support of the shock to avoid the distinction of the cutoff for a binding lower bound lying within or outside the support of the distribution.

to zero,[7]

$$\min_{i_t \geq i^{LB}} \mathscr{L} = \min_{i_t \geq i^{LB}} (1-\beta)\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda x_t^2) \right], \tag{3}$$

where $\lambda \geq 0$ is the relative weight the central bank places on the output gap relative to deviations of inflation from target. We focus on optimal monetary policy under discretion, when the central bank lacks the ability to commit to future actions and thus takes inflation expectations as given.[8] The central bank and private sector are assumed to have full knowledge of the model, including the distribution of the shock processes.

The state space of the system consists only of the current realization of the shock ϵ_t and expectations about inflation and output gaps. Due to the shock being purely temporary, its current realization has no predictive content for future realizations of the shock process. As a result, conditional expectations of future variables based on the current information set coincide with unconditional expectations. We thus drop the time subscript t from the expectations operator and solve for the full dynamic equilibrium with unconditional expectations.

It is convenient to define a shadow rate $\mathtt{i}_t$ as

$$\mathtt{i}_t = \theta + r^* + \psi \mathbb{E}\pi_{t+1} + \frac{1}{\alpha}\epsilon_t,$$

where $\psi = 1 + \frac{1}{\alpha\kappa} - \frac{\beta\lambda}{\alpha\kappa(\kappa^2+\lambda)} > 1$. For $\theta = 0$, the shadow rate is the optimal policy rate under discretion without the lower bound constraint. In a later section, we use the parameter θ to introduce different monetary policy frameworks.

In the presence of a lower bound, the optimal discretionary policy is to set the interest rate equal to the shadow rate whenever feasible and to the lower bound otherwise:

$$i_t = \max\{\mathtt{i}_t, i^{LB}\} = \mathtt{i}_t + i_t^{\Delta}. \tag{4}$$

We furthermore define the interest rate wedge i_t^{Δ} as the difference between the policy rate and the shadow rate, $i_t^{\Delta} = i_t - \mathtt{i}_t$, as in the second part of the above equation. Since the policy rate i_t is either equal to the shadow rate $\mathtt{i_t}$ or set above the shadow rate at i^{LB} when the shadow rate falls below the lower bound, the interest rate wedge i_t^{Δ} is always nonnegative.

Equipped with this notation, we rewrite the model equations in terms of the interest rate wedge by

[7]More generally, the same results follow if we assume a nonzero inflation target, with π_t then representing deviations in inflation from target.

[8]The logic of this paper also applies when the central bank follows a Taylor rule.

substituting $\mathtt{i}_t + i_t^{\Delta}$ for the nominal rate i_t. The system of equations (1) and (2) then becomes

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda}\beta\mathbb{E}_t\pi_{t+1} - \alpha\kappa(i_t^{\Delta} + \theta) \tag{5}$$

and

$$x_t = -\frac{\kappa}{\kappa^2 + \lambda}\beta\mathbb{E}_t\pi_{t+1} - \alpha(i_t^{\Delta} + \theta).$$

In these equations for inflation and the output gap, written in terms of the interest rate wedge, the first terms reflect what holds when the lower bound is not binding and when $\theta = 0$, as in the standard New Keynesian model (see Clarida et al. (1999)). Abstracting from the second terms for the moment, the demand shock is fully offset and there is no tradeoff between stabilizing inflation and output, i.e., "divine coincidence" holds. However, the second terms in these equations show that a binding lower bound leads, ceteris paribus, to lower inflation and a negative output gap. Moreover, monetary policy can raise inflation on average by lowering interest rates on average, that is, by setting θ to be negative.

III.A Model Solution

Absent a lower bound and assuming $\theta = 0$, there is a unique equilibrium to equation (5). Taking unconditional expectations of both sides of the equation yields an equilibrium where inflation is at target, $\mathbb{E}\pi_t = 0$, and the nominal interest rate equals the long-run natural rate, $i_t = r^*$.

The presence of a lower bound on interest rates in the model gives rise to the possibility of a second equilibrium.[9] To see this, we split equation (5) into parts depending on whether the lower bound binds in the current period or not

$$\pi_t = \begin{cases} \dfrac{\beta\lambda}{\kappa^2 + \lambda}\mathbb{E}_t\pi_{t+1} - \alpha\kappa\theta & \text{if } \mathtt{i}_t \geq i^{LB} \\ \kappa\epsilon_t - \alpha\kappa(i^{LB} - \theta - r^*) + (1 - \alpha\kappa)\mathbb{E}_t\pi_{t+1} & \text{if } \mathtt{i}_t < i^{LB}. \end{cases} \tag{6}$$

In the deterministic version of our model, each part of equation (6) delivers one equilibrium. When $\theta = 0$, the deterministic steady state stemming from the upper equation is characterized by inflation equal to its target of zero ($\pi = 0$) and the nominal interest rate equal to the long-run natural real rate ($i_t = r^*$). The lower equation gives rise to an equilibrium in which monetary policy is constrained such that the interest rate is permanently at the lower bound, $i_t = i^{LB}$, as in Benhabib et al. (2001). The corresponding

[9]The optimal interest rate rule in equation (4) depends on expectations and is optimal in both equilibria.

steady-state inflation is equal to the difference between the lower bound and the long-run natural real rate, $\pi = i^{LB} - r^* < 0$, and thus below its target of zero when $\theta = 0$.

Characterizing the equilibrium in the stochastic environment with demand shocks requires solving for expectations of future inflation, the output gap, and the interest rate wedge. To this end, we recognize that these expectations are linked in the model via

$$\mathbb{E}\pi_{t+1} = \frac{\kappa}{1-\beta}\mathbb{E}x_{t+1} = -\frac{1}{\psi - 1}(\mathbb{E}i^{\Delta}_{t+1} + \theta). \tag{7}$$

Furthermore, the Fisher equation holds

$$\mathbb{E}\pi_t = \mathbb{E}i_t - r^* - \frac{1}{\alpha}\bar{\epsilon}.$$

Appendix A contains details on the derivation of these equations.

Equation (7) and the Fisher equation imply that it is sufficient to characterize one of the expectations. In the following analysis, we focus on solving for the expected interest rate wedge $\mathbb{E}i^{\Delta}$, which translates into inflation expectations and, by the Fisher equation, into expected interest rates. We start by expressing the expected interest rate wedge as the integral over $i^{LB} - \mathtt{i}_t$, which is the value of i^{Δ}_t when policy is at the lower bound and the interest rate wedge is non-zero. The expected interest rate wedge can then be expressed as

$$\mathbb{E}i^{\Delta} = \int_{-\infty}^{\bar{\nu}^{LB}} (i^{LB} - \mathtt{i}_t) g(\nu_t) d\nu_t = \frac{1}{\alpha}\int_{-\infty}^{\bar{\nu}^{LB}} G(\nu_t) d\nu_t = \frac{1}{\alpha}\mathscr{G}(\bar{\nu}^{LB}), \tag{8}$$

where

$$\bar{\nu}^{LB} = \alpha\left(i^{LB} - \theta - r^* + \frac{\psi}{\psi - 1}(\mathbb{E}i^{\Delta} + \theta)\right) - \bar{\epsilon} \tag{9}$$

defines the cutoff for the innovation to the demand shock below which the lower bound binds.

The reasoning behind equation (8) is as follows. The interest rate wedge is zero whenever policy is unconstrained. When the innovation for the demand shock ν_t falls below the threshold $\bar{\nu}^{LB}$, given by equation (9), policy becomes constrained and the interest rate wedge is $i^{LB} - \mathtt{i}_t = \frac{1}{\alpha}(\bar{\nu}^{LB} - \nu_t)$. Integration by parts leads us from the second term to the third term in equation (8) which is the integral over the cumulative distribution function of the demand shock.

As a result, expected interest rate wedges are the integral over the cumulative distribution function which we label the super-cumulative distribution function $\mathscr{G}$, evaluated at the cutoff value $\bar{\nu}^{LB}$ (fourth term in

(8)). This is to say that $\mathscr{G}'(\nu) = G(\nu)$ where $G(\cdot)$ is the cumulative distribution function associated with the probability density function $g(\cdot)$. As such, the function $\mathscr{G}$ is pinned down by the choice of the underlying distribution and does not depend on other parameter values.

Interest rate wedges arise only when the value of ν_t falls below the threshold $\bar{\nu}^{LB}$. In that case, the interest rate wedge rises linearly as ν_t falls. And the wedge is zero whenever the innovation to demand shocks ν_t is above the threshold. Consequently, the expected interest rate wedge is driven by the probabilities of shock realizations below the threshold $\bar{\nu}^{LB}$ which can be captured by the super-cumulative distribution function.

Translating this into the process for inflation, shocks above $\bar{\nu}^{LB}$ get fully offset by policy and have no effect on inflation, as equation (6) shows. For shock realizations below the threshold, however, demand shocks affect inflation. In particular, the further the demand shock falls below the threshold, the lower inflation will be below target (second part of equation (6)). The super-cumulative distribution function again captures the average effect from these negative shocks.

The super-cumulative distribution function has several properties that facilitate the analysis of the model. First, since one can obtain this function by integrating over the cumulative distribution function, the function converges to zero for $\bar{\nu}^{LB} \to -\infty$ and to infinity for $\bar{\nu}^{LB} \to \infty$. Since its second derivative is the probability density function, it is nonnegative and $\mathscr{G}$ is convex. The comparative statics with respect to uncertainty can be fully characterized as we show below.

Furthermore, the super-cumulative distribution function is available in explicit form for a range of distributions. For example, in the case of a normal distribution for the shock, the super-cumulative function takes the form

$$\mathscr{G}^{\text{norm}}(\bar{\nu}^{LB}) = \bar{\nu}^{LB}\Phi\left(\frac{\bar{\nu}^{LB}}{\sigma}\right) + \sigma^2\phi\left(\frac{\bar{\nu}^{LB}}{\sigma}\right), \tag{10}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function (c.d.f.) of the standard normal distribution, $\phi(\cdot)$ the corresponding probability density function (p.d.f.), and σ is the standard deviation of the demand shock.

Importantly, there is a single fixed-point equation involving the function $\mathscr{G}$ that pins down the solution of the model. To see this, we plug the expected interest rate wedge in (8) into the definition of the cutoff and obtain the equilibrium correspondence

$$\mathscr{G}(\bar{\nu}^{LB}) = \frac{\psi - 1}{\psi}\alpha\left(\frac{1}{\alpha}(\bar{\nu}^{LB} + \bar{\epsilon}) - i^{LB} - \frac{1}{\psi - 1}\theta + r^*\right). \tag{11}$$

Solving this equation for cutoff values delivers the solution to the model as the cutoff determines expected interest rate wedges via equation (8). Appendix A.A contains the derivations and more details.

Figure 2 depicts both sides of equation (11) assuming a normal distribution for the demand shock. The blue line shows the super-cumulative distribution on the left-hand side. The right-hand side is linear in the cutoff $\bar{\nu}^{LB}$ and depicted in the black line.

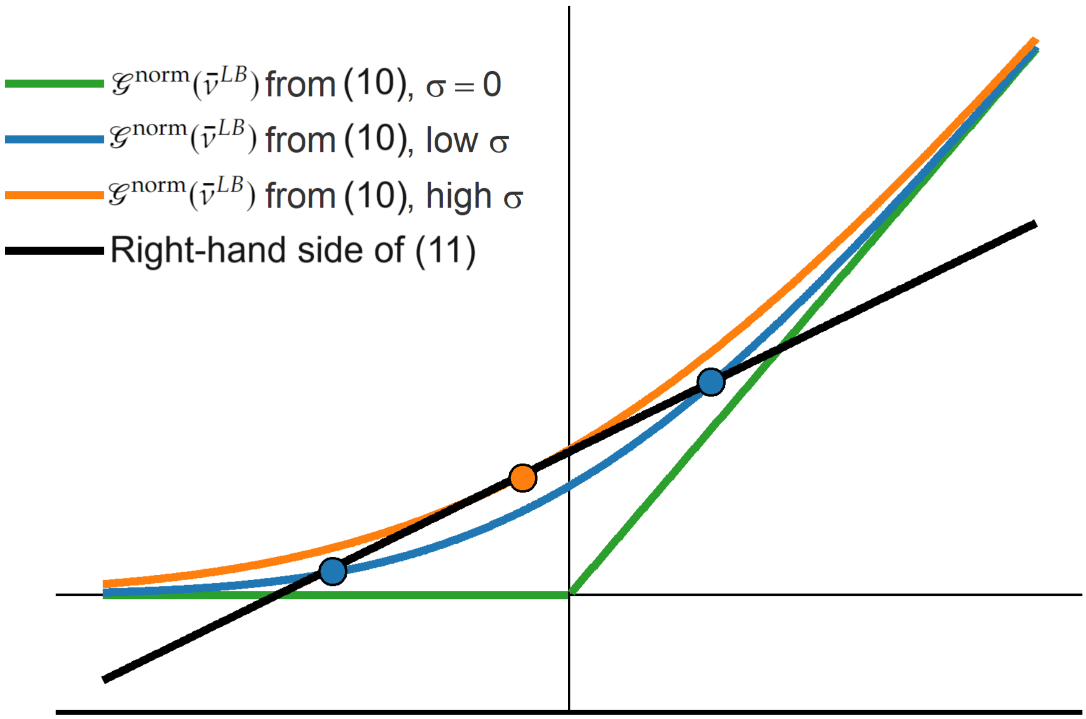

Figure 2: Equilibrium correspondence. The solid straight black line plots $\mathcal{G}(\bar{\nu}^{LB})$ as a linear function of $\bar{\nu}^{LB}$, as given by equation (11). The other lines plot the super-cumulative distribution function $\mathcal{G}^{\text{norm}}(\bar{\nu}^{LB})$ for the case of a normal distribution, as given by equation (10), with mean zero and varying standard deviations σ. The intersection points with the solid black line indicate equilibria for these various values of σ under normality.

Due to the convexity of the super-cumulative distribution function, there can be either zero, one (a knife-edge case), or two equilibria. As we saw above, the deterministic case features two equilibria. We label the equilibrium to the left the *target equilibrium*. In the deterministic case, it turns into the equilibrium in which the lower bound does not bind. With uncertainty, however, the lower bound may become occasionally binding. The equilibrium to the right is a *liquidity trap equilibrium* and is associated with the deterministic equilibrium in which the lower bound binds.

The probability with which the lower bound binds is given by

$$P^{LB} = G(\bar{\nu}^{LB}) = \int_{-\infty}^{\bar{\nu}^{LB}} g(\nu) d\nu. \tag{12}$$

With this definition, we get the following lemma.

Lemma 1 *Depending on the amount of uncertainty, there exist two, one, or zero equilibria in this economy. Furthermore, in the case of two distinct equilibria, there is a threshold* $\frac{\psi-1}{\psi}$ *such that the probability of a binding lower bound is always below the cutoff in the target equilibrium and always above the cutoff in the liquidity trap equilibrium.*

Appendix A.A.2 contains the proof.

In the presence of uncertainty, the lower bound on interest rates may be occasionally binding in both equilibria. When there are two equilibria, the probability of a binding lower bound in the target equilibrium always falls below the cutoff value whereas it exceeds it in the liquidity trap equilibrium. In the knife-edge case of a unique equilibrium, the probability of a binding lower bound is $\frac{\psi-1}{\psi}$. We denote the corresponding cutoff by $\bar{\nu}^*$. Increasing uncertainty beyond this knife-edge case, any further increase in uncertainty leads to non-existence of equilibria.

Figure 3 depicts the distributions for shadow rates in the case with uncertainty. The target equilibrium (blue line) endogenously has a higher mean than in the liquidity trap (red line). In the target equilibrium, there is a cutoff for the demand shock below which the central bank becomes constrained by the lower bound. This is because the central bank aims at offsetting the demand shock and lower realizations cause it to cut interest rates. With the demand shock being sufficiently low, the shadow rate falls below the lower bound.

In the liquidity trap equilibrium, the lower bound always binds in the deterministic steady state. Adding uncertainty results in situations where the demand shock is so high that the shadow rate is above the lower bound and the central bank finds itself unconstrained. The anticipation of a lower probability of a binding lower bound in the future raises inflation expectations which, through the forward-looking Phillips curve, already raise inflation today. As a result, the comparative statics in the liquidity trap go in the opposite direction compared to the ones in the target equilibrium.

The predictions of the liquidity trap equilibrium are not borne out by the data (see Mertens and Williams (2021)). In fact, the evidence presented in this paper can be interpreted as additional evidence against the liquidity trap equilibrium. We therefore present our theoretical findings only for the target equilibrium of the model.

To summarize, without a lower bound on interest rates, the New Keynesian model results in a unique equilibrium where inflation expectations are anchored at the target level. Introducing a lower bound adds a liquidity trap equilibrium that is characterized by a higher probability of a binding lower bound compared to the target equilibrium. We now turn to analyzing the effects of changes in uncertainty on the economy.

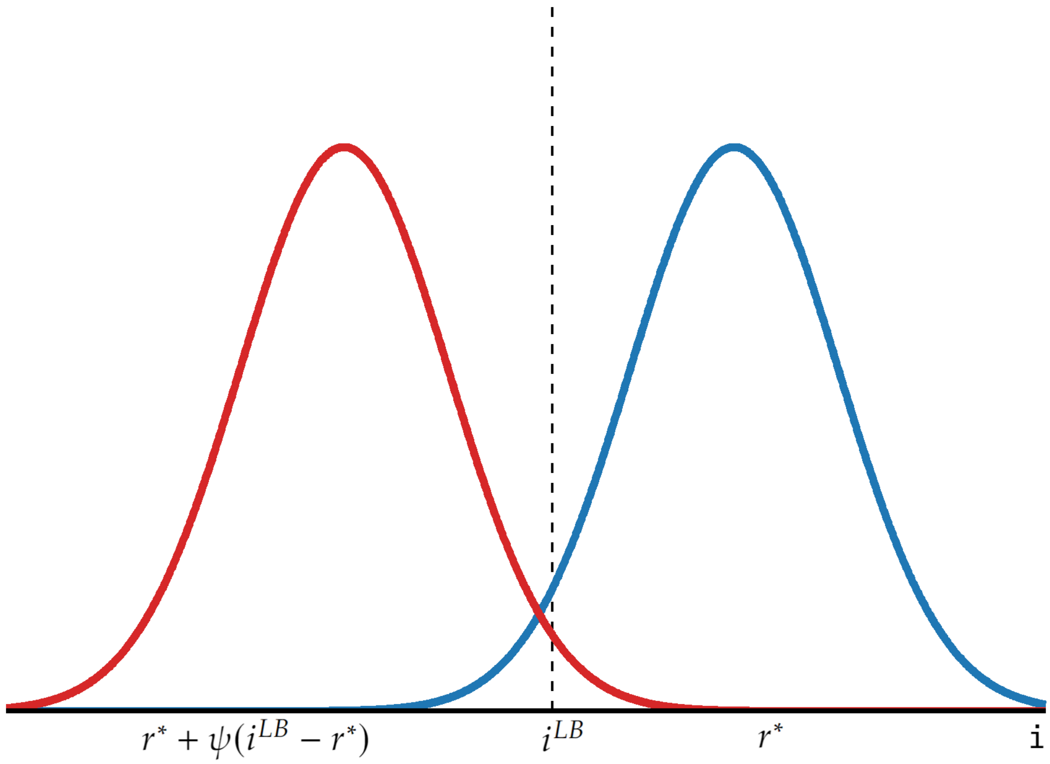

Figure 3: Shadow interest rate distributions in the target (blue) and liquidity trap (red) equilibria. The lower bound on interest rates (dashed line) truncates these shadow rate distributions in the model.

III.B The effects of uncertainty and demand shocks

This section studies the impact of an increase in uncertainty on the economy. It shows that the lower bound plays a crucial role: Without it, an increase in uncertainty has no effect on inflation and output. With a lower bound, higher uncertainty increases the region of the state space in which the lower bound is binding in the target equilibrium. The comparative statics in the liquidity equilibrium have the opposite sign and are at odds with the data. We therefore omit the discussion of this equilibrium here.[10]

In the absence of a lower bound, uncertainty has no effect on the economy since demand shocks can be fully offset. Optimal interest rate policy shields inflation and output from the effect of demand shocks. More volatile demand leads to more volatile interest rates but does not transmit to prices or the real economy.

The lower bound on interest rates changes these predictions. We show that the expected interest rate wedge increases in the target equilibrium. We demonstrate these effects for general distributions with unbounded support for which we consider increases in uncertainty. Let $\varphi_t = \nu_t + s\eta_t$ be a series of mean-preserving spreads of ν_t, indexed by $s \geq 0$, where $\mathbb{E}[\eta_t|\nu_t] = 0$. A larger value of s implies a higher variance

[10]The results for the liquidity trap equilibrium are in Internet Appendix A.C.

of the mean-preserving spread.

A general increase in uncertainty through a mean-preserving spread pushes mass into the tails. In that case, the distribution for the more volatile demand shock is said to second-order stochastically dominate the distribution without the mean-preserving spread. Consequently, the cumulative distribution function rises in the left tail, thereby increasing its integral, and shifting the super-cumulative distribution function up.[11].

Figure 2 depicts these effects for the case of the normal distribution. While the linear relationship between expected interest rate wedges and the cutoff $\bar{\nu}^{LB}$ in the black line is unaffected by an increase in uncertainty, the super-cumulative distribution function increases and the two equilibria move closer together. Further increases in uncertainty eventually lead to a unique equilibrium and non-existence for yet higher levels of uncertainty.

From this figure, it is also evident that the cutoff $\bar{\nu}^{LB}$ rises with uncertainty in the target equilibrium (the point of intersection to the left) and, as a result, the lower bound binds over a larger portion of the support for the demand shock. Proposition 1 summarizes these results.

Proposition 1 (Effects of uncertainty)

Neither a mean-preserving spread nor lower average demand have an effect on inflation expectations absent the lower bound; in the presence of a lower bound, higher uncertainty raises the expected interest rate wedge in a target equilibrium.[12]

We now analyze the derivative of inflation expectations with respect to an increase in the mean-preserving spread $\mathrm{d}\mathbb{E}i^{\Delta}/\mathrm{d}s$. We interpret this expression as the model's analogue of the correlation of changes in expectations due to an uncertainty shock, as measured by a change in the mean-preserving spread. This expression formalizes the discussion of the effects of changes in uncertainty. When there is no lower bound, the function $\mathscr{G}(\nu_t)$ is zero everywhere for all values of s. Thus, the derivative of the wedge vanishes as well, and changes in uncertainty have no effects on the inflation expectations absent the lower bound.

With a lower bound on interest rates, the sign of the effects of changing uncertainty on expectation is determined by the type of equilibrium the economy is in. Proposition 1 demonstrates that increases in uncertainty raise interest rate wedges and thus lower inflation expectations.

We interpret an increase in the VIX as reflecting an increase in uncertainty. There is a literature documenting a negative relationship between increases in the VIX and stock prices. We show that the mechanism

[11]Lemma 3 in Internet Appendix A shows these implications.
[12]Appendix A.B contains the proof.

in our model applies both when the mean of the shock is lower and when uncertainty is higher. Therefore, either potential driver of the VIX triggers the same mechanism in the model. And a negative correlation between uncertainty shocks and average demand would reinforce the effects of the lower bound. With a fall in the mean of the demand shock, inflation expectations fall in the target equilibrium due to a higher probability of a binding lower bound.

$$\frac{dP^{LB}}{d\bar{\epsilon}} = -\frac{\psi - 1}{\psi} \frac{1}{\frac{\psi-1}{\psi} - P^{LB}} g(\bar{\nu}^{LB})$$

The impact of changes in uncertainty on the probability of a binding lower bound is more subtle. While the region of the state space expands with higher uncertainty in the target equilibrium, the probability of a binding lower bound does not always change in the same direction. The reason is that there can be two opposing forces. The cutoff $\bar{\nu}^{LB}$ increases with uncertainty, suggesting that the probability of a binding lower bound should increase. However, a mean-preserving spread has a direct effect on the distribution function that can result in a lower cutoff value.

Lemma 2 establishes that the lower bound probability and the cutoff always move in the same direction in response to increases in uncertainty when the probability of a binding lower bound is sufficiently low.

Lemma 2 *There are values $\underline{\nu}$ and $\bar{\nu}$ such that an increase in uncertainty raises the probability of a binding lower bound whenever $\bar{\nu}^{LB} < \underline{\nu}$ in the target equilibrium. When the demand shock is normally distributed, these results hold globally, i.e.,*

$$\frac{dP^{LB}}{ds} \geq 0 \qquad \textit{for } \bar{\nu}^{LB} < \bar{\nu}^*.$$

For any continuous distribution, lower average demand raises P^{LB} in the target equilibrium.

Appendix A.D contains a proof.

Intuitively, in the target equilibrium, the probability of a binding lower bound increases in the tails with uncertainty since the likelihood of large movements increases. When the demand shock follows a normal distribution, any increase in volatility raises the probability of a binding lower bound, i.e., when the probability of a binding lower bound falls below the cutoff $(\psi - 1)/\psi$.[13]

Together these results indicate that changes in uncertainty should have an effect on inflation expectations

[13]A similar set of results holds under more stringent assumptions. When the distribution is unimodal and symmetric and $\psi = 2$, the results hold globally. When $\psi \in (1, 2)$, we can show that the lower bound probability rises with uncertainty for all target equilibria and, when $\psi > 2$, the lower bound probability declines for higher uncertainty across all liquidity trap equilibria. Internet Appendix A.B contains details.

only in close proximity to the lower bound. Looking at the stylized fact about changing correlations of uncertainty shocks and inflation expectations through this lens, the model is consistent with close to zero correlations in the early 2000s when the lower bound had a low probability of binding. Later in the sample, when the lower bound became a more salient concern, the correlation between uncertainty shocks and inflation expectations turned negative. The following comparative statics show within the model that a decline in the natural real rate of interest is the most likely source of these developments.

III.C Changes in the effects of uncertainty on expectations

This section explores the effects of a change in the natural real rate of interest on the relationship between uncertainty and inflation expectations. We find that, without the lower bound, our model cannot generate the observed change in the effects of increased uncertainty on expected inflation and interest rates. With a lower bound, the equilibrium of the economy again determines the sign of the effects. In the general case, we find that the target equilibrium is consistent with the stylized facts of Section II, provided that the probability of a binding lower bound is sufficiently small. With normally distributed shocks, the results hold for any probability.

We now explore the changing impact of uncertainty on expectations when the natural real rate of interest falls. Therefore, we continue to interpret the correlation between expectations and changes in uncertainty as the derivative of expected interest rate wedges with respect to uncertainty.

Proposition 2 (Changes in the natural real rate and the effects of changes in uncertainty) *Changes in uncertainty do not affect expected interest rate wedges in the absence of a lower bound on interest rates, independent of the level of the natural rate of interest r^*. In the presence of a lower bound, the sensitivity of the expected interest rate wedge to changes in uncertainty declines when the natural real rate of interest falls*

$$\frac{d\frac{\mathbb{E}i^{\Delta}}{ds}}{dr^*} = \frac{d^2\mathbb{E}i^{\Delta}}{dr^* ds} = -\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi} - P^{LB}}\right)^2 \frac{dP^{LB}}{ds}. \tag{13}$$

The analogous expression for average demand $\frac{d^2\mathbb{E}i^{\Delta}}{dr^* d\bar{\epsilon}}$ is unambiguously positive in the target equilibrium.

Appendix A.E contains the proof of these results.

This proposition allows us to sign the direction in which a fall in the natural rate pushes the sensitivity of the expected interest rate wedge to changes in uncertainty. The determining factor is whether uncertainty raises the probability of a binding lower bound. Using Lemma 2, the right-hand side of equation (13) is always negative in the target equilibrium, provided that shocks are normally distributed. The results furthermore hold for any distribution if the likelihood of being constrained is sufficiently low.

As a consequence, the sensitivity of inflation expectations with respect to uncertainty declines as the natural real rate of interest falls. In the liquidity trap equilibrium, this sensitivity rises. Measuring this sensitivity through the correlation of changes in inflation expectations and uncertainty shocks therefore shows that the results in Section II are consistent with the theoretical model presented here. We investigate this connection more formally in Section IV. In other words, a decline in the natural real rate of interest both increases the probability of a binding lower bound and lowers the correlation of inflation expectations with uncertainty shocks in the target equilibrium.

Intuitively, when the natural real rate falls, the policy rate is closer to the lower bound, making it more likely to bind. When shocks display higher uncertainty, the probability of a binding lower bound increases, constraining the ability to offset negative demand shocks. As a consequence, the mean inflation rate declines.

A similar intuition applies to shifts in the distribution of the shock via its mean $\bar{\epsilon}$. Both an increase in uncertainty and a lower mean, i.e., a leftward shift, raise the probability of a binding lower bound. Consequently, both changes in the distribution have the same effect on expectations, as Proposition 2 demonstrates.

We now turn to the question of whether the monetary policy framework has an impact on the transmission of uncertainty to the real sector. Here, we allow the central bank to permanently commit to any policy parameter θ but follows optimal discretionary policy otherwise. Adjusting this parameter amounts to static average-inflation targeting, as laid out in Mertens and Williams (2019, 2020) and Diwan et al. (2020).

Through lowering the interest rate intercept, the central bank provides more stimulus whenever it is not constrained by the lower bound. Consequently, it runs inflation above its target level during times when it is unconstrained to correct for the downward bias in inflation expectations due to the lower bound. That is, the central bank makes up for shortfalls during recessions by overshooting the inflation target during expansions. In that sense, it averages over the business cycle.

We are interested in the effect static average-inflation targeting has on the effects of changes in uncertainty on inflation expectations. We get the following result.

Proposition 3 (Static average-inflation targeting and the effects of changes in uncertainty)
Changes in uncertainty do not affect expected interest rate wedges in the absence of a lower bound on interest rates, independent of the level of the policy intercept θ. In the presence of a lower bound, the following expression determines

whether the sensitivity of the expected interest rate wedge to changes in uncertainty increases or falls with the policy intercept θ

$$\frac{d^2\mathbb{E}i^{\Delta}}{d\theta ds} = \frac{1}{\psi-1}\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi} - P^{LB}}\right)^2 \frac{dP^{LB}}{ds}.$$

The analogous expression for average demand $\frac{d^2\mathbb{E}i^{\Delta}}{d\theta d\bar{\epsilon}}$ is unambiguously negative in the target equilibrium.

Appendix A.F contains the proof of these results.

Proposition 3 demonstrates that a change in the policy intercept can prevent the effects of a fall in the natural rate. A negative intercept θ lowers the expected interest rate wedge and increases inflation on average. This is because the central bank raises inflation above its target when unconstrained to make up, on average, for shortfalls during episodes of a binding lower bound.

As a result, the derivative of the expected interest rate wedge with respect to uncertainty rises when the central bank lowers the intercept of the policy rule. These results hold globally for normally distributed shocks and for low probabilities of a binding bound for any distribution.

III.D Discussion

The model in the previous section features demand shocks as a single source of disturbances. We intentionally leave out markup or other supply-side shocks to highlight a simple set of assumptions that gives rise to the mechanism discussed in this paper. The link between uncertainty shocks and inflation expectations would, however, survive in a model with both demand and markup shocks.

Adding markup shocks would lead to a negative correlation between inflation and the output gap absent a lower bound. Currently, the baseline case has no correlation since interest rate policy can perfectly offset demand shocks away from the lower bound, a feature often referred to as "divine coincidence." The lower bound leads the output gap and inflation to comove together, irrespective of whether supply-side shocks are present or not.

Increases in uncertainty, as measured by the VIX, might be correlated with negative news about the economic outlook. To the extent that this is occuring, the propositions in the previous sections show that the mechanisms for higher uncertainty and a lower mean are mutually reinforcing. Therefore, negative shocks to the first moment and higher second moments can lead to similar changes in correlations.

Although we do not study these implications here, these changing correlation patterns have implications for the correlation of stocks and bonds, a subject of significant attention in the macro-finance literature (see, e.g., Campbell et al. (2020), Duffee (2022), and Pflueger (2023)). The basic finding is that during the 1960s to mid-1990s, the correlation between stock returns and changes in interest rates was negative. This correlation flipped sign in the mid-1990s such that high stock returns have been associated with increasing interest rates since. This latter correlation is consistent with the lower bound being a more salient constraint. Negative shocks close to the lower bound lead to below-target inflation and negative output gaps, consistent with the positive correlation since the mid-1990s (see also Bilal (2017)).

Another issue in comparing the model to the data is that increases in uncertainty measured by the VIX and other similar measures are typically short-lived. The empirical evidence suggests that temporary bursts of uncertainty affect longer-term expectations. For this to be the case in our model with i.i.d. shocks, uncertainty would need to be elevated for a long time. A simple extension of the model with highly persistent shocks could bridge that gap.[14]

In the following section, we document the empirical patterns around inflation expectations and expected interest rates and link them to movements in the natural real rate of interest.

IV Empirical Implications

We approach the empirical tests of the theoretical model in three steps. First, we recognize that the New Keynesian model predicts that the stylized fact should appear in the correlation of uncertainty shocks and changes in expectations of interest rates. After confirming this, we provide evidence that the natural real rate of interest fell during our sample period. Consistent with the theoretical predictions of our model, we show that the correlations of interest rates fell with it. And, lastly, we perform formal tests that link changes in the correlations with the natural real rate of interest.

IV.A Stylized fact for interest rates

The theoretical model of the previous section shows that the stylized fact for inflation expectations should also apply to interest rate expectations. The reason behind this prediction is the Fisher equation: As uncertainty changes inflation expectations, interest rate expectations adjust accordingly.

Figure 4 demonstrates that, as the correlation between uncertainty shocks and changes in inflation expectations turned negative, so did the correlation between uncertainty shocks and changes in expected interest

[14]Internet Appendix C presents a highly stylized extension of the model in which short-lived increases in inflation expectations have lasting effects on inflation expectations.

rates. Here, expected interest rates are measured as the five-year ahead five-year Treasury forward rate (TFR) that we compute using constant maturity Treasury yields.

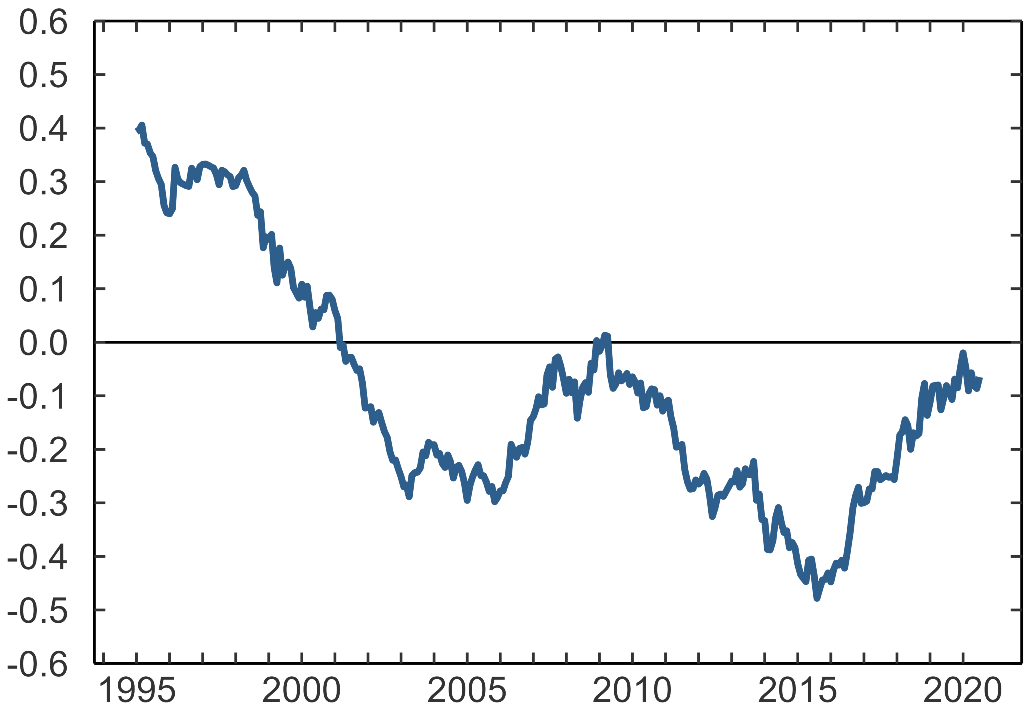

Figure 4: Five-year trailing-window Spearman correlation of monthly changes in the VIX with monthly changes in the five-year ahead five-year forward Treasury constant maturity rate. The sample period ranges from January 1995 until July 2020.

We interpret this declining correlation as the economy initially being shielded from uncertainty shocks, while over time their impact spills over increasingly to the real sector. With monetary policy serving as the primary tool to guard the economy against adverse shocks, the negative correlation between uncertainty shocks and changes in interest rates and inflation expectations, respectively, suggest that the lower bound affected the ability to respond to negative shocks using the policy rate over the sample period.

IV.B The Macroeconomic Driver Behind Changing Correlations

There are two potential drivers behind the declining correlation in the model: The uncertainty of shocks and the natural real rate of interest, r^*. We investigate the plausibility of a change in either parameter.

For the uncertainty to play an increasing role and lead to a larger probability of a binding lower bound, the distribution of shocks would need to widen. There are at least two ways to see that this is an unlikely source of variation in the data. First, the covariance between inflation expectations and uncertainty shocks is consistent with the one for the correlation. While we do not report any further results on this here, there is no obvious sign that there was a trend increase in uncertainty from this analysis.[15] Second, estimates from options data show a decline in uncertainty after the Great Recession, the opposite of what would be necessary to explain the stylized facts in this paper (see Mertens and Williams (2021).

Instead, we explore time variation in r^*. In recent years, a body of research has documented a decline in the natural real rate of interest in the United States and other advanced economies over the past few decades (Williams, 2017).[16] Figure 5 plots the natural interest rate from 1999 to 2020, using daily estimates of the real natural rate from Christensen and Rudebusch (2019a) and daily five-year forward zero-coupon TIPS yields five years ahead from Gürkaynak et al. (2010) as a proxy for the natural rate. Over the sample period, estimates of the natural real rate of interest from Christensen and Rudebusch (2019a) declined by 8 basis points per year on average and five-year five-year forward zero-coupon TIPS yields started out from a higher level and declined by 18 basis points per year on average. The persistent downward trend over time resulted in historically low levels of the natural rate at the end of the sample period.

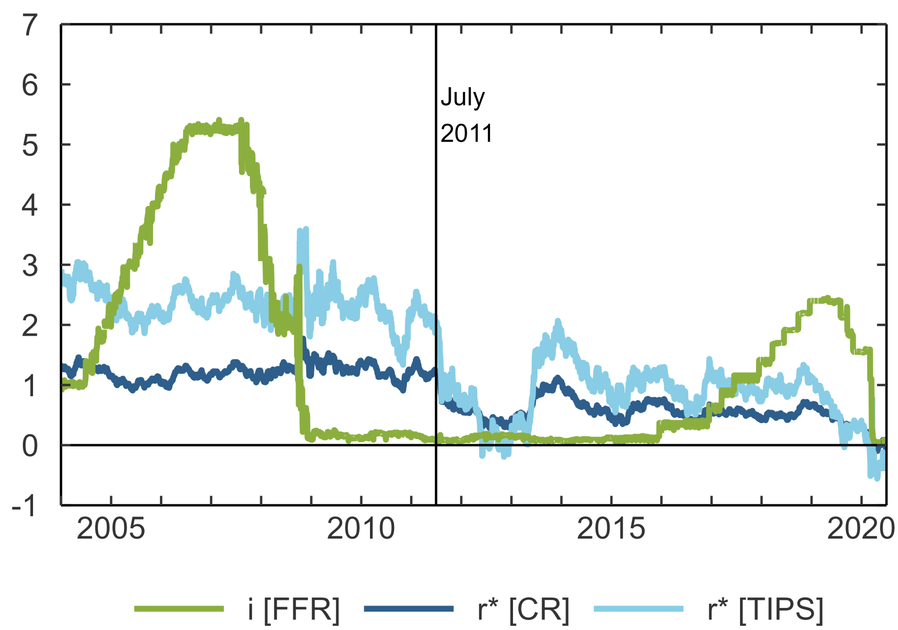

Figure 5: The green line depicts the effective federal funds rate at the daily frequency, while the dark blue line shows daily estimates of the natural real rate of interest from Christensen and Rudebusch (2019a) (CR), and the light blue line shows the five-year ahead five-year forward zero-coupon TIPS rate (TIPS) computed by Gürkaynak et al. (2010) from Treasury real yields. The sample period ranges from January 2004 until July 2020.

[15]See Figure A4 in Section B.B of the Internet Appendix for a chart.

[16]See, for example, Laubach and Williams (2003), Kiley (2015), Lubik and Matthes (2015), Johannsen and Mertens (2016), Holston et al. (2017), Crump et al. (2017), Del Negro et al. (2017) and Christensen and Rudebusch (2019a). Estimates from these models for r^* show a decline consistent with that from the measure used in this paper.

The level of natural rates of interest tends to persist and expectations of future rates should thus, to a significant degree, reflect their current level. There are several reasons for this observation. First, measures of historical levels of r^* display a significant amount of persistence. The estimates aim to capture the low frequency component of real short-term interest rates and, as such, are highly persistent. For example, Holston et al. (2017) show that their estimates of the natural rate are nonstationary. Second, as a case in point, even well into the recovery from the Great Recession of 2007-2009, r^* had not returned to historically normal levels. Therefore, it is likely that long-term influences were holding the natural rate of interest down. Third, many possible explanations for the level of r^*, not only in the United States but internationally, reflect highly persistent forces affecting the global supply and demand for savings. For example, one potential explanation for the decline in r^* during our sample period is a dramatic slowdown in trend real GDP growth in many advanced economies. For a more detailed discussion of the decline in the natural rate, see Williams (2017).

To investigate the possibility of a structural break in the level of r^*, we apply the Andrews (1993) test, which shows the existence of a structural break in all measures of the natural rate.[17] The test identifies July 2011 as the breakpoint for both the CR and TIPS measures of the natural rate. Based on this dating of the structural break in r^*, in the remainder of this subsection we split our overall sample into two subsamples: January 1999 through June 2011, and July 2011 through July 2020. We end the sample in July 2020 before the announcement of the flexible average-inflation targeting framework by the Federal Reserve. Our theoretical analysis suggests that the new framework should lead to a different data-generating process for our variables of interest.

Table 1 reports the average r^* and the average correlation of changes in expected inflation and interest rates with changes in the VIX for each subsample. To test for differences across subsamples, we compute the t-statistic for the null hypothesis that the estimates are the same across subsamples, controlling for serial correlation using the methodology outlined by Wilks (1997).[18] In the earlier subsample, the natural real rate of interest is higher than in the later sample. This feature allows us to assess the theoretical predictions of the model. In the target equilibrium of our theoretical model, a decline in the natural real rate lowers the correlation between uncertainty shocks and changes in expectations about inflation and interest rates further into negative territory, at least in the tails of the distribution. In the liquidity trap equilibrium, the predictions are reversed.

[17] For the CR measure of the natural rate, the $\sup F$ value is 621.55 which occurs in July 2011, and for the TIPS measure, the $\sup F$ value is 662.94 which also occurs in July 2011. In both cases, the null hypothesis of structural stability in the level of r^* is rejected at the 1% level.

[18] Appendix B.A outlines the details of constructing the t-statistic as applied in this context.

Table 1: Differences in moments across subsamples

	mean r^*		mean correlation with ΔVIX			
	r^*_{CR}	r^*_{TIPS}	BEI	BEI	TFR	TFR
frequency	monthly	monthly	daily	monthly	daily	monthly
Jul 2011 - Jul 2020	0.54	0.83	-0.27	-0.33	-0.31	-0.26
Jan 1999 - Jun 2011	1.47	2.88	-0.08	0.00	-0.15	-0.10
difference	-0.93***	-2.05***	-0.19***	-0.33***	-0.16***	-0.16**

Note: ***, **, and * signify that the difference in means across subsamples is statistically significant at the 1% level, 5% level, and 10% level, respectively, accounting for serial correlation. Correlations are the trailing-window Spearman correlation of the change in the variable with the change in the VIX (here referred to as ΔVIX). Spearman correlations are computed over one-year windows at the daily frequency and over five-year windows at the monthly frequency. r^*_{CR} denotes estimates of the natural rate from Christensen and Rudebusch (2019a), and r^*_{TIPS} is the five-year ahead five-year forward zero-coupon TIPS rate computed from Gürkaynak et al. (2010) used as a proxy for the natural rate. BEI is the five-year ahead five-year forward breakeven inflation rate, while TFR is the five-year ahead five-year forward Treasury constant maturity yield.

Consistent with the target equilibrium of our theoretical model, the mean daily correlation, measured over a one-year trailing-window, between changes in five-year ahead five-year forward breakeven inflation expectations (BEI) and changes in the VIX declined from -0.08 in the earlier subsample to -0.27 in the later subsample. This decline is statistically significant at the 1% level.

The analogous mean correlation, again measured at the daily frequency over a one-year trailing-window, between changes in five-year five-year forward Treasury constant maturity yields (TFR) and changes in the VIX declined from -0.15 in the earlier subsample to -0.31 in the later subsample. This difference across subsamples is significant at the 1% level.

These changes in the link between uncertainty shocks and interest rates and inflation, respectively, hold up at different frequencies. As Table 1 shows, the corresponding monthly correlation for breakeven inflation rates, measured over a five-year trailing window, fell from 0.00 to -0.33 across subsamples, a statistically significant decline at the 1% level. Similarly, the correlation for interest rates declined as well with the changes being statistically significant at the 5% level.

IV.C Lower Natural Rates and the Effects of Uncertainty Shocks on Inflation and Interest Rates

This section tests the empirical prediction of our theoretical model that the correlation between uncertainty shocks and movements in inflation expectations and interest rates changes as the natural real rate of interest falls. The target equilibrium of our model predicts a fall in the correlation with a decline in the natural real rate.

To test this prediction, we run a time series regression of the correlation between changes in the VIX and changes in inflation expectations on the real natural rate of interest and an intercept. We repeat this regression for both the nominal and real interest rates.

Table 2: Regression of correlation between changes in VIX and BEI/TFR/TRR on the natural rate of interest

	r^*_{CR}			r^*_{TIPS}		
Dependent Variable: corr(ΔVIX$_t$, Δy_t)	BEI$_t$	TFR$_t$	TRR$_t$	BEI$_t$	TFR$_t$	TRR$_t$
	(1)	(2)	(3)	(4)	(5)	(6)
r^*	0.161***	0.129***	0.075*	0.077***	0.071***	0.041**
	(0.039)	(0.046)	(0.04)	(0.018)	(0.021)	(0.018)
constant	−0.317***	−0.334***	−0.221***	−0.298***	−0.335***	−0.222***
	(0.048)	(0.058)	(0.049)	(0.043)	(0.051)	(0.043)
Frequency	daily	daily	daily	daily	daily	daily
Sample Start	Jan 1999	Jan 1998	Jan 1999	Jan 1999	Jan 1999	Jan 1999
Sample End	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020
Observations	5,374	5,625	5,374	5,371	5,381	5,371
R^2	0.054	0.035	0.013	0.059	0.048	0.017

Note: The dependent variable in the regression is the time series correlation between changes in the VIX and changes in various variables y_t, denoted corr(ΔVIX$_t$, Δy_t), where y_t stands for BEI$_t$, TFR$_t$, and TRR$_t$ across columns as labeled. BEI is the five-year forward breakeven inflation rate five years ahead, TFR is the five-year forward Treasury constant maturity rate five years ahead, and TRR is the analogous real rate. The independent variable in the regression is the natural rate of interest, using the Christensen and Rudebusch (2019b) measure in columns (1)-(3) or the TIPS-based measure in columns (4)-(6). Estimated coefficients are reported with Newey-West (HAC) standard errors in parentheses. ***, **, and * signify that the difference in means across subsamples is statistically significant at the 1% level, 5% level, and 10% level, respectively, accounting for serial correlation.

The correlation between uncertainty shocks and changes in breakeven inflation rates (BEI) serving as the dependent variable is our baseline correlation. We compute it as the correlation from daily data over a 14-day rolling window. Despite obtaining a noisier measure, we choose a short window for computing the correlation to avoid having a highly persistent regressor. We further compute the analogous correlation for changes in the VIX and changes in nominal interest rates as measured by five-year ahead five-year forward Treasury constant maturity yields (TFR). Lastly, we obtain a corresponding correlation for real interest rates. Our measure for the expected real rate (TRR) is the forward real interest rate obtained from Treasury Inflation Protected Securities (TIPS) prices. It is calculated as the difference between the nominal Treasury forward rate (TFR) and break-even inflation (BEI) over the same horizon.

Data for the natural rate of interest, r^*, are the daily estimates of Christensen and Rudebusch (2019a) and daily data on the five-year ahead five-year forward zero-coupon TIPS yield computed from Gürkaynak et al. (2010). The sample period for these regressions is January 1999 to July 2020.

Table 2 shows the regression results for the three dependent variables regressed on the Christensen-Rudebusch measure – in Columns (1), (2), and (3) – as well as the TIPS proxy for r^* – in Columns (4), (5), and (6). It reports the estimated coefficients with Newey-West standard errors for serial correlation.

Together, all empirical specifications show highly significantly positive coefficients on the natural rate of interest. This finding implies that, on average, the correlation between uncertainty shocks and changes in inflation expectations or interest rates falls when the natural rate declines. Column (1) reports the results for the regression of the two-week trailing-window correlation between changes in the VIX and changes in inflation expectations on the CR estimates of r^*. The coefficient on the natural rate is estimated to be 0.161 with a standard error of 0.039. Column (4) runs the analogous specifications using the five-year five-year zero-coupon TIPS rates as a proxy for r^* and yields the same conclusions. The estimated coefficients on the TIPS-measure of r^* in these regressions are slightly lower due to larger fluctuations in this measure compared to the Christensen-Rudebusch r^* over the sample period.

As shown in Column (2) of the table, the regression of the two-week trailing-window correlation between changes in the VIX and changes in long-term expected interest rates on the CR estimates of r^*, we estimate a positive and statistically significant relationship between the responsiveness of interest rate expectations to uncertainty shocks and the natural rate of interest. The estimated coefficient of 0.129 on the CR r^* is statistically significant at the 1% level. The same result holds for the regression on the TIPS proxy for r^* at the daily frequency, shown in Column (5) of the table, with an estimated coefficient on the natural rate lower in magnitude but nonetheless positive and statistically significant.

The analogous regressions for forward real rates show a positive, yet somewhat smaller estimated coefficient on r^*.[19] To investigate whether the theory is informative about this correlation, we compute expected real interest rates from the difference between nominal interest rates and inflation expectations

$$\begin{aligned}\mathbb{E}\left[i_t - \mathbb{E}[\pi_{t+1}]\right] &= -\frac{1}{\psi-1}\theta + r^* + \frac{1}{\alpha}\bar{\epsilon} - \frac{1}{\psi-1}\mathbb{E}i^{\Delta} - \mathbb{E}[\pi_{t+1}] \\ &= r^* + \frac{1}{\alpha}\bar{\epsilon},\end{aligned}$$

where the Fisher equation in the second line follows directly from equation (7).

As the theory section demonstrated, real interest rates are higher at the lower bound compared to a model without a lower bound. However, the presence of a lower bound lowers inflation expectations overall and thus leads to lower real rates away from the lower bound. What the Fisher equation tells us is that, on net, the two effects cancel and the model does not speak directly to a correlation with uncertainty shocks. In other words, the expected real rate is pinned down by the exogenously given natural real rate of interest, r^*.

These results establish a significantly positive link between the natural real rate of interest and the impact of uncertainty shocks on inflation expectations and interest rates, in line with the theoretical model in this paper. The positive sign of the coefficient indicates that this correlation between changes in uncertainty and changes in inflation expectations or interest rates tends to decline when r^* falls, as we observe broadly in the data over our sample period.

IV.D Uncertainty Shocks and the Stock Market

The New Keynesian model is a model for the macroeconomy and, as such, has implications for the real side of the economy. Specifically, one would expect a distribution of output skewed to the downside when the lower bound is likely to bind as the economy moves along the Phillips curve. While there is no liquid financial market that would allow us to observe expectations about real side variables in our model, we can look at close proxies. In this section, we explore whether the link between uncertainty shocks and the stock market changed as the lower bound on interest rates became a more salient concern.

Following the literature on the stock-bond comovement (e.g., Campbell et al. (2017) and Campbell et al. (2020)), we focus on a time series of slope coefficients from rolling-window regressions.[20] We therefore run

[19] The sign of the relationship between the natural rate and the measured correlation between changes in the five-year five-year forward Treasury real rate and uncertainty shocks depends on the frequency at which the correlation is measured (see Internet Appendix B.C). Future research might be able to shed further light on our understanding of real rates.

[20] For comparison, we report the analogous correlations in Internet Appendix Figure A6.

the following regression

$$\Delta \text{VIX}_{t-h} = \alpha_t + \beta_t \Delta \log(\text{S\&P } 500_{t-h}) + \eta_{t-h}$$

for daily data over the 12-month period leading up to time t where Δ represents the daily change in the variable. ΔVIX_{t-h} is the daily change in the VIX at date $t-h$ where h runs through all the days over the prior year. $\Delta \log(\text{S\&P } 500_{t-h})$ is the daily return as measured through the daily change in the logarithm of the S&P 500 valuation. The regression slope β_t captures the time-varying relationship between uncertainty shocks and changes in stock market valuation.

Figure 6 shows the regression slopes over time. Throughout the sample, increases in the VIX are associated with a decline in stock market valuations. This pattern is in line with the standard risk-return trade-off.

Figure 6: One-year trailing-window estimated slope coefficient for the regression of daily changes in the VIX on daily changes in the logarithm of the S&P 500 stock price index. The sample period ranges from January 2004 until July 2020.

This relationship between uncertainty shocks and changes in stock market valuations becomes more negative around the time that the correlation between uncertainty shocks and changes in inflation expectations turns negative. These movements are consistent with the theoretical model in this paper. As the lower bound becomes more likely to bind, higher uncertainty worsens the average outlook for the economy and stock valuations decline.

To show that the decline in the slopes of the regression of the VIX on stock market valuations is consistent with the model, we link the regression slopes to the natural rate of interest. We therefore run the analogous regression to the ones in Table 2 where we regress the regression slopes on the natural rate of interest. As before, we measure r^* both through the Christensen-Rudebusch as well as the TIPS measures. In the case of Christensen-Rudebusch, we obtain a regression coefficient of 0.36 with a Newey-West standard error of 0.16, which is statistically significant at the 5% level. For the TIPS measure, the regression coefficient is 0.17 with a Newey-West standard error of 0.06, which is statistically significant at the 1% level.

In sum, all the empirical results in this paper are consistent with the theoretical prediction of the target equilibrium described in the theoretical section of this paper, and inconsistent with being in a liquidity trap equilibrium. These results supporting the U.S. economy being in a target equilibrium are consistent with the findings of Mertens and Williams (2021) using data from derivatives markets.

V Robustness of Empirical Results

We assess the robustness of our main empirical results along two dimensions, using alternative measures of uncertainty and of inflation expectations. Overall, we find that our qualitative results are robust to these alternative measures, although the level of statistical significance is weaker in some instances.

Our first robustness concerns the length of the sample period that was determined by data availability. To extend the time dimension, we use 5-year forward Treasury yields five years out as in Figure 4. Instead of changes in the VIX, we study changes in realized volatility, a close substitute to the VIX.[21] We compute realized volatility as the backward-looking moving average over 30 days (21 trading days) of squared daily returns in the S&P 500 index starting in 1962.

Figure 7 shows the Spearman correlation between changes in realized volatility and changes in five-year five-year ahead Treasury yield. This correlation broadly hovered around zero between the mid-sixties and the Great Recession. A small correlation is consistent with the natural real rate of interest being sufficiently large for the lower bound not to be a significant constraint on equilibrium quantities. Note that this prediction is only consistent with the target equilibrium. The correlation only turned substantially negative in the aftermath of the Great Recession when the correlation between uncertainty shocks and inflation expectations turned negative as well.

Our second robustness exercise examines alternative measures of inflation expectations. The breakeven

[21]Figure A1 in Internet Appendix B.A shows the close relationship between the VIX and realized volatility during the period when both measures are available.

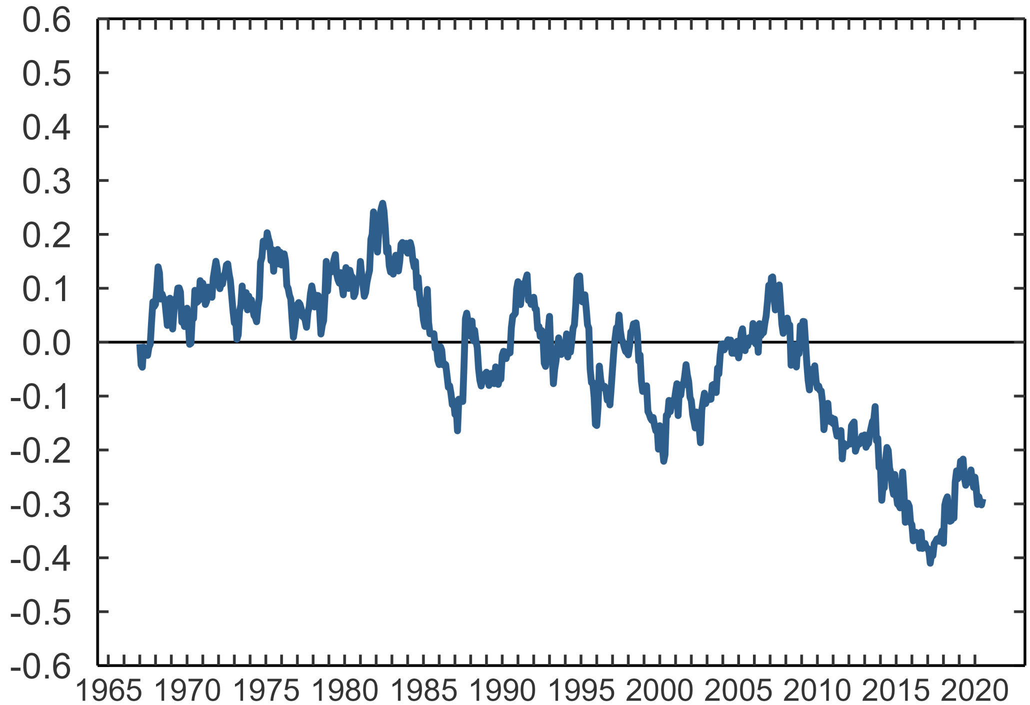

Figure 7: Five-year trailing-window Spearman correlation of monthly changes in realized volatility [computed as the backward-looking moving average over 30 days (21 trading days) of squared daily returns in the S&P 500 index] with monthly changes in the five-year five-year ahead Treasury constant maturity rate. The sample period ranges from January 1967 until July 2020.

inflation measure analyzed earlier is potentially contaminated by time-varying liquidity and risk premiums. To assess the importance of these premiums on our results, we consider two measures of inflation expectations that are arguably less prone to this type of mismeasurement.

The first alternative measure of inflation expectations is the five-year, five-year forward expected inflation rates, which are constructed to remove estimated risk and liquidity premiums by D'Amico et al. (2018) (DKW). The third column of Table 3 reports the subsample mean correlations between changes in the VIX and changes in this measure of inflation expectations at the daily frequency over one-year trailing windows. These results are similar to those using the breakeven inflation rates, and the difference in the mean correlations across subsamples is statistically significant.

The time series for the DKW measure is available at the daily frequency going back in time further than the BEI series. Consistent with our baseline specifications, we turn the DKW measure into monthly averages. This longer time series allows us see how the correlation between changes in long-run inflation expectations and uncertainty shocks behaved in the 1990s. Figure 8 shows that this correlation using the DKW estimates of inflation expectations was positive in the late 1990s, before trending downward over the subsequent two

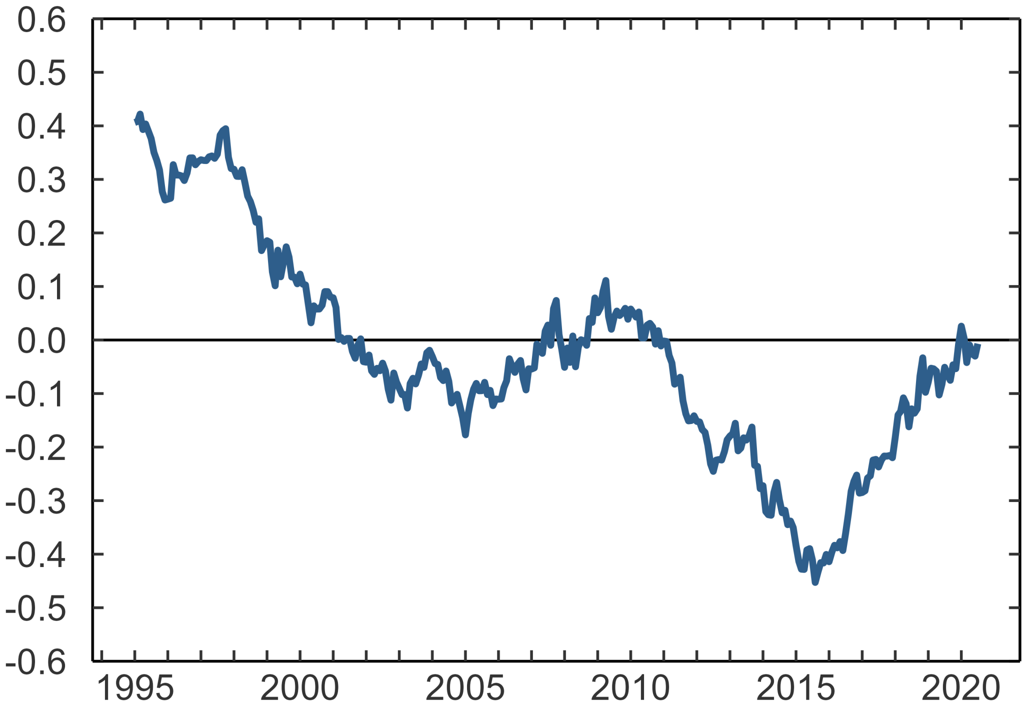

Figure 8: Spearman correlation between changes in the VIX and changes in the five-year, five-year forward expected inflation rate, adjusted for risk premia and the liquidity premium using the estimates of D'Amico et al. (2018): monthly frequency, five-year trailing-window correlation. The sample period ranges from January 1995 until July 2020.

decades.

Our second alternative measure of inflation expectations is the median ten-year CPI inflation expectation from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF).[22] Unlike the financial market data that can be analyzed at higher frequencies, the SPF data are collected quarterly. Therefore, for this measure, we average the VIX over each quarter up through the first month of the quarter to be broadly consistent with the information set available to survey participants. We compute five-year trailing-window correlations with changes in the VIX for this survey-based measure of expected inflation.

The fourth column of Table 3 shows that the results using the SPF measure of inflation expectations are broadly consistent with those using market-based measures. The difference in the mean correlations across subsamples using the SPF is both of similar size to those using market-based measures and is statistically significant. Figure 1 shows the time series of the correlation using the SPF measure of inflation expectations, which declines starting in the mid-2000s and thus follows the same pattern as our baseline correlation.

[22]The SPF also collects five-year ahead forecasts on CPI inflation, but this measure is only available for a shorter sample. We use ten-year inflation expectations for consistency with the sample used throughout this paper.

Third, because VIX may be affected by risk premia and technical factors like dealer positioning, we consider two alternative measures of uncertainty: the one-year-ahead estimate of economic financial uncertainty (EFU) from Ludvigson et al. (2021) and the economic policy uncertainty (EPU) index of Baker et al. (2016). Unlike the VIX, which is priced in financial markets, the EFU index is a weighted average of conditional standard deviations of one-year-ahead forecast errors across a variety of financial indicators, while the EPU index is a news-based measure. The final two columns of Table 3 report subsample comparisons using these measures of uncertainty. These statistics are constructed as in Table 1.

Table 3: Differences in moments across subsamples.

	mean correlation with ΔVIX				mean correlation with ΔBEI	
	BEI	BEI	DKW	SPF	EFU	EPU
frequency	daily	monthly	monthly	quarterly	monthly	monthly
Jul 2011 - Jul 2020	-0.27	-0.33	-0.30	-0.24	-0.23	-0.18
Jan 1999 - Jun 2011	-0.08	0.00	-0.15	-0.02	-0.09	-0.09
difference	-0.19***	-0.33***	-0.15**	-0.22***	-0.14*	-0.09

Note: ***, **, and * signify that the difference in means across subsamples is statistically significant at the 1% level, 5% level, and 10% level, respectively, accounting for serial correlation. Correlation is the trailing-window correlation of the change in the variable with the change in the breakeven inflation rate (here denoted as ΔBEI) or the change in the VIX (here denoted as ΔVIX), as labeled. Correlations are computed over one-year windows at the daily frequency and over five-year windows at the monthly and quarterly frequencies. EFU is the one-year ahead estimate of economic financial uncertainty from Ludvigson et al. (2021), and EPU is the economic policy uncertainty index of Baker et al. (2016). BEI is the five-year ahead five-year forward breakeven inflation rate, and DKW is the five-year ahead five-year forward breakeven inflation rate from D'Amico et al. (2018) which removes risk premia and liquidity premia from the Treasury-implied rate. SPF is the median ten-year ahead CPI inflation rate from the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia.

As seen in the table, the subsample changes in mean correlations using these alternative measures of uncertainty are qualitatively consistent with those using the VIX as the measure of uncertainty (shown in the first two columns of the table). That said, the magnitudes of the declines in the mean correlations are smaller than in the case of the VIX.

These robustness exercises demonstrate that our key findings are not sensitive to the presence of liquidity or risk premiums in breakeven inflation expectations. Although the results using these alternative measures of uncertainty are not as quantitatively or statistically strong as for the market-based measure, qualitatively they are in the same direction and consistent with the model prediction for the target equilibrium.

VI Conclusion

This paper documents a marked downward trend in the correlation between changes in market-based measures of uncertainty and changes in expected inflation and expected interest rates over the past quarter century. These changing correlations have occurred at the same time that estimates of the natural rate of interest declined significantly and the incidence of monetary policy being near or at the lower bound increased. In this paper, we show that this constellation of observations is consistent with the predictions of a standard New Keynesian model with a lower bound on interest rates when the economy is in the target equilibrium. The lower bound on interest rates is crucial because, absent the lower bound, the correlations studied here would be zero. We also show that average-inflation targeting policies can mitigate the effects of uncertainty shocks on the unconditional means of inflation and output.

Throughout this paper, we focus on the theoretical predictions for unconditional moments and empirical counterparts of far-forward expectations. A future extension of this analysis is to examine business-cycle frequency comovements between uncertainty and inflation and interest rate expectations in the presence of persistent shocks to the economy.

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Appendix

A Lemmas and Proofs

A.A Effects of higher uncertainty

A mean-preserving spread is directly linked to the super-cumulative distribution function via the following lemma. The lemma further illustrates that a mean-preserving spread pushes mass into the tails of the distribution.

Lemma 3 (Mean-preserving spread and second-order stochastic dominance)
The following statements are equivalent:

1. *φ_t is a mean-preserving spread of ν_t with p.d.f. $g(s, \varphi_t)$ and c.d.f. $G(s, \varphi_t)$.*
2. *$G(s, \varphi_t)$ second-order stochastically dominates $G(\nu_t)$ for all $s > 0$.*
3. *$\mathscr{G}(s, \varphi) \geq \mathscr{G}(\varphi)$ for all φ and $s > 0$.*
4. *There exist $\underline{\varphi}$ and $\bar{\varphi}$ such that $G(s, \varphi_t) \geq G(\varphi_t)$ for all $\varphi_t \leq \underline{\varphi}$ and $G(s, \varphi_t) \leq G(\varphi_t)$ for all $\varphi_t \geq \underline{\varphi}$.*

Proof: For a proof, see Rothschild and Stiglitz (1970). □

A.B Proof of Proposition 1

Proof: Denote the super-cumulative distribution function of φ_t by $\mathscr{G}(s, \varphi_t)$. We analyze the derivative of inflation expectations with respect to an increase in the mean-preserving spread

$$\frac{\mathrm{d}\mathbb{E}i^{\Delta}}{\mathrm{d}s} = \frac{1}{\alpha}\frac{\psi - 1}{\psi}\frac{\mathscr{G}_1^{\varphi}(s, \bar{\nu}^{LB})}{\frac{\psi-1}{\psi} - P^{LB}}.$$

The numerator is positive, $\mathscr{G}_1(s, \nu_t) > 0$ (see Lemma 3 in Internet Appendix A.A). We furthermore established that the lower bound probability is below the threshold $(\psi - 1)/\psi$ when the economy is in a target equilibrium.

For the comparative statics with respect to the mean-preserving spread, take the derivative with respect to s.

$$\begin{aligned}
\frac{\mathrm{d}\mathbb{E}i^{\Delta}}{\mathrm{d}s} &= \frac{\mathrm{d}}{\mathrm{d}s}\frac{1}{\alpha}\mathscr{G}^{\varphi}(s,\bar{\nu}^{LB}) \\
&= \frac{1}{\alpha}\mathscr{G}_1^{\varphi}(s,\bar{\nu}^{LB}) + \mathscr{G}_2^{\varphi}(s,\bar{\nu}^{LB})\frac{\psi}{\psi-1}\frac{\mathrm{d}\mathbb{E}i^{\Delta}}{\mathrm{d}s} \\
&= \frac{1}{\alpha}\frac{\mathscr{G}_1^{\varphi}(s,\bar{\nu}^{LB})}{1-\frac{\psi}{\psi-1}\mathscr{G}_2^{\varphi}(s,\bar{\nu}^{LB})} \\
&= \frac{1}{\alpha}\frac{\psi-1}{\psi}\frac{\mathscr{G}_1^{\varphi}(s,\bar{\nu}^{LB})}{\frac{\psi-1}{\psi}-\mathscr{G}_2^{\varphi}(s,\bar{\nu}^{LB})}
\end{aligned}$$

□

Turning to changes in average demand, we compute the comparative statics with respect to $\bar{\epsilon}$. From the equilibrium correspondence,

$$\begin{aligned}
\mathscr{G}_2(\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{d\bar{\epsilon}} &= \frac{\psi-1}{\psi}\left(\frac{d\bar{\nu}^{LB}}{d\bar{\epsilon}}+1\right) \\
\iff \frac{d\bar{\nu}^{LB}}{d\bar{\epsilon}} &= -\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi}-P^{LB}}
\end{aligned}$$

From the equation for the cutoff,

$$\frac{d\bar{\nu}^{LB}}{d\bar{\epsilon}} = -1+\alpha\frac{\psi}{\psi-1}\frac{\mathrm{d}\mathbb{E}i^{\Delta}}{\mathrm{d}\bar{\epsilon}}$$

Together,

$$\frac{\mathrm{d}\mathbb{E}i^{\Delta}}{\mathrm{d}\bar{\epsilon}} = \frac{1}{\alpha}\frac{\psi-1}{\psi}\left(1-\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi}-P^{LB}}\right).$$

To see that the sign of this expression switches depending on the equilibrium, notice that the sign is determined by the expression in parentheses. In the target equilibrium, this expression

$$1-\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi}-P^{LB}} = 1-\frac{1}{1-\frac{\psi}{\psi-1}P^{LB}}$$

is negative: for $0 < P^{LB} < \frac{\psi-1}{\psi}$, the denominator in the rightmost expression less than 1 and the whole expression is thus negative.

A.C Derivation of comparative statics to lower bound probability

Differentiating the equilibrium correspondence in equation (11) on both sides with respect to s yields

$$\mathscr{G}_1(s,\bar{\nu}^{LB}) + G(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{ds} = \frac{\psi-1}{\psi}\frac{d\bar{\nu}^{LB}}{ds}.$$

Consequently, we get

$$\frac{d\bar{\nu}^{LB}}{ds} = \frac{\mathscr{G}_1(s,\bar{\nu}^{LB})}{\frac{\psi-1}{\psi} - G(s,\bar{\nu}^{LB})}.$$

To sign the derivative dP^{LB}/ds, we plug the expression for the derivative of the cutoff into the derivative of P^{LB} to get

$$\begin{aligned}\frac{dP^{LB}}{ds} &= G_1(s,\bar{\nu}^{LB}) + g(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{ds} \\ &= G_1(s,\bar{\nu}^{LB}) + g(s,\bar{\nu}^{LB})\frac{\mathscr{G}_1(s,\bar{\nu}^{LB})}{\frac{\psi-1}{\psi} - G(s,\bar{\nu}^{LB})}.\end{aligned} \tag{14}$$

The analogous expression for the derivative with respect to $\bar{\epsilon}$ is

$$\frac{dP^{LB}}{d\bar{\epsilon}} = g(\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{d\bar{\epsilon}} = -\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi} - P^{LB}}g(\bar{\nu}^{LB}). \tag{15}$$

A.D Proof of Lemma 2

We start by differentiating the equilibrium correspondence in (11) to get

$$\frac{\psi-1}{\psi}\frac{d\bar{\nu}^{LB}}{ds} = \mathscr{G}_2(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{ds} + \mathscr{G}_1(s,\bar{\nu}^{LB}).$$

Rewriting this expression, we get

$$\left(\frac{\psi-1}{\psi} - P^{LB}\right)\frac{d\bar{\nu}^{LB}}{ds} = \mathscr{G}_1(s,\bar{\nu}^{LB}) > 0.$$

Consequently, $(\frac{\psi-1}{\psi} - P^{LB})$ and $\frac{d\bar{\nu}^{LB}}{ds}$ are of the same sign. The cutoff is thus increasing in the target equilibrium and decreasing in the liquidity trap equilibrium.

By Lemma 3, the function $\mathscr{G}(\cdot)$ shifts up when uncertainty increases. We furthermore know that $\lim_{\nu\to-\infty}\mathscr{G}(\nu) = 0$ since $\mathscr{G}$ is obtained by integrating over the cumulative density function. Furthermore, for the same reason, the slope of $\mathscr{G}$ converges to 1 for large values

$$\lim_{\nu\to\infty} \mathscr{G}'(\nu) = 1.$$

Lastly, since $\mathscr{G}''(\nu) = g(\nu)$ is the probability density function and thus non-negative, the function $\mathscr{G}(\cdot)$ is convex. At the same time, the right-hand side of the equilibrium correspondence in (11) is independent of uncertainty.

The $\mathscr{G}$-function intersects the right-hand side of the equilibrium correspondence from above to below in the target equilibrium (see Figure 2 for an illustration).

As a result, increasing uncertainty will weakly shift the intersection to the right in the target equilibrium. That is, the equilibrium cutoff $\bar{\nu}^{LB}$ shifts to the right in the target equilibrium, thereby weakly increasing the part of the state space in which the lower bound binds.

Under normality, these results hold globally. The term that determines the sign of the change in the lower bound probability in response to higher uncertainty is

$$\frac{dP^{LB}}{ds} = G_1(s, \bar{\nu}^{LB}) + g(s, \bar{\nu}^{LB}) \frac{\mathscr{G}_1(s, \bar{\nu}^{LB})}{\frac{\psi-1}{\psi} - G(s, \bar{\nu}^{LB})}.$$

Assuming a normal distribution of the demand shock, i.e. $\nu \sim N(0, \sigma^2)$, we write this term as

$$\frac{dP^{LB}}{ds} = \frac{1}{2\pi\sigma} \left(\frac{e^{-\left(\frac{\bar{\nu}^{LB}}{\sigma}\right)^2}}{\frac{\psi-1}{\psi} - \Phi\left(\frac{\bar{\nu}^{LB}}{\sigma}\right)} - \sqrt{2\pi} \frac{\bar{\nu}^{LB} e^{-\frac{1}{2}\left(\frac{\bar{\nu}^{LB}}{\sigma}\right)^2}}{\sigma} \right),$$

where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution. Substituting $\bar{\nu}^{LB} = -\sqrt{2}\sigma\varepsilon$ and recognizing that the term outside the bracket is always positive, we analyze the term in the bracket

$$K(\varepsilon) = \frac{1}{2} \frac{1}{\frac{\psi-1}{\psi} - \Phi(-\sqrt{2}\varepsilon)} + e^{\varepsilon^2} \sqrt{\pi}\varepsilon$$

that is independent of the volatility parameter σ.

We now show that the sign of $K(\varepsilon)$ is negative for $\varepsilon < \varepsilon^*$ where $\varepsilon^* = \Phi^{-1}\left(\frac{\psi-1}{\psi}\right)$ is the point of the singularity and positive for $\varepsilon > \varepsilon^*$. To this end, we proceed in the following steps.

First, when ε approaches negative infinity, $K(\varepsilon) < 0$ approaches negative infinity, i.e. $\lim_{\varepsilon\to-\infty} K(\varepsilon) = -\infty$. Both terms in the expression are negative with one of them being bounded. On the other end, $K(\varepsilon)$ approaches infinity for large values of ε, i.e. $\lim_{\varepsilon \to \infty} K(\varepsilon) = \infty$.

Next, approaching the singularity from the left leads the function value to approach negative infinity, $\lim_{\varepsilon \to_{-} \bar{\varepsilon}^*} K(\varepsilon) = -\infty$. Approaching the singularity from the right leads to positive unbounded function values $\lim_{\varepsilon \to_{+} \bar{\varepsilon}^*} K(\varepsilon) = \infty$.

Third, the function has only one singularity where the denominator of the first term is zero. It is analytic otherwise. Therefore, the function starts from unbounded negative values, reaches a maximum, and then converges to negative infinity when approaching the singularity. On the right-hand side of the singularity, the function starts from a high value, falls to a minimum, and then converges to positive infinity. Next we compute the two extrema and show that the maximum to the left of the singularity sits below zero and the minimum to the right above zero.

To compute the extrema, we take first-order conditions and set them to zero, i.e., $K'(\varepsilon) = 0$, which results in

$$2\pi e^{2\varepsilon^2} \left(1 + 2\varepsilon^2\right) - \frac{1}{\left(\frac{\psi-1}{\psi} - \Phi(-\sqrt{2}\varepsilon)\right)^2} = 0.$$

Rearranging the equation and taking the square root results in two conditions that hold at the extrema

$$\frac{1}{\frac{\psi-1}{\psi} - \Phi(-\sqrt{2}\varepsilon_1)} = -e^{\varepsilon_1^2}\sqrt{2\pi(1+2\varepsilon_1^2)}$$

and

$$\frac{1}{\frac{\psi-1}{\psi} - \Phi(-\sqrt{2}\varepsilon_2)} = +e^{\varepsilon_2^2}\sqrt{2\pi(1+2\varepsilon_2^2)}.$$

Since the left-hand side is negative in the first and positive in the second equation, the two extrema are on opposite sides of the singularity at ε^*.

Evaluating the function value shows that the function value at the maximum to the left of the singularity is below zero. Therefore, replace the first expression in $K(\cdot)$ with the expression from the first-order condition to arrive at

$$K(\varepsilon_1) = \sqrt{\pi} e^{\varepsilon_1^2} \left(\varepsilon_1 - \sqrt{\frac{1}{2} + \varepsilon_1^2}\right) < 0.$$

As a result, $K(\varepsilon_1) < 0$ for all $\varepsilon < \varepsilon^*$. For $\varepsilon > \varepsilon^*$, we get

$$K(\varepsilon_2) = \sqrt{\pi} e^{\varepsilon_2^2} \left(\varepsilon_2 + \sqrt{\frac{1}{2} + \varepsilon_2^2}\right) > 0.$$

Since the maximum to the left of the singularity lies below zero, all function values for $\varepsilon < \varepsilon^*$ are negative. And since the minimum to the right is above zero, all function values for $\varepsilon > \varepsilon^*$ are positive.

The results for lower average demand follow directly from equation (15).

A.E Proof of Proposition 2

Proof: We start from the derivative of the expected wedge with respect to the natural rate

$$\frac{d\mathbb{E}i^{\Delta}}{dr^*} = \frac{1}{\alpha}\mathscr{G}_2(s, \bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{dr^*}. \tag{16}$$

Now, computing the second mixed derivative as

$$\frac{d^2\mathbb{E}i^{\Delta}}{dr^*ds} = \frac{1}{\alpha}\mathscr{G}_{1,2}(s, \bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{dr^*} + \frac{1}{\alpha}\mathscr{G}_{2,2}(s, \bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{dr^*}\frac{d\bar{\nu}^{LB}}{ds} + \frac{1}{\alpha}\mathscr{G}_2(s, \bar{\nu}^{LB})\frac{d^2\bar{\nu}^{LB}}{dr^*ds},$$

where we get the last term by computing

$$\frac{d\bar{\nu}^{LB}}{dr^*} = -\alpha\left(1 - \frac{\psi}{\psi - 1}\frac{d\mathbb{E}i^{\Delta}}{dr^*}\right) \tag{17}$$

and

$$\frac{d^2\bar{\nu}^{LB}}{dr^*ds} = \alpha\frac{\psi}{\psi - 1}\frac{d^2\mathbb{E}i^{\Delta}}{dr^*ds}.$$

Plugging these expressions into the second mixed derivative yields

$$\frac{d^2\mathbb{E}i^{\Delta}}{dr^*ds} = \frac{1}{\alpha}\frac{\psi - 1}{\psi}\frac{1}{\frac{\psi-1}{\psi} - P^{LB}}\left(\mathscr{G}_{1,2}(s, \bar{\nu}^{LB}) + g(s, \bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{ds}\right)\frac{d\bar{\nu}^{LB}}{dr^*}. \tag{18}$$

We combine equations (16) and (17) to get

$$\frac{\psi - 1}{\psi}\left(1 + \frac{1}{\alpha}\frac{d\bar{\nu}^{LB}}{dr^*}\right) = \frac{1}{\alpha}\mathscr{G}_2(s, \bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{dr^*},$$

or, rewritten,

$$\frac{d\bar{\nu}^{LB}}{dr^*} = -\alpha\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi} - P^{LB}}. \tag{19}$$

Taken together and using equation (14) results in

$$\begin{aligned}\frac{d^2\mathbb{E}i^{\Delta}}{dr^*ds} &= -\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^2\left(\mathscr{G}_{1,2}(s,\bar{\nu}^{LB})+g(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{ds}\right)\\ &= -\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^2\frac{dP^{LB}}{ds}.\end{aligned}$$

$\square$

To extend this proof to the mixed derivative with respect to r^* and $\bar{\epsilon}$, start again from equation (16) and differentiate:

$$\frac{d^2\mathbb{E}i^{\Delta}}{dr^*d\bar{\epsilon}} = \frac{1}{\alpha}\frac{dP^{LB}}{d\bar{\epsilon}}\frac{d\bar{\nu}^{LB}}{dr^*}+\frac{1}{\alpha}P^{LB}\frac{d^2\bar{\nu}^{LB}}{dr^*d\bar{\epsilon}}.$$

Using equation (17), we get

$$\frac{d^2\bar{\nu}^{LB}}{dr^*d\bar{\epsilon}} = \alpha\frac{\psi}{\psi-1}\frac{d^2\mathbb{E}i^{\Delta}}{dr^*d\bar{\epsilon}}.$$

Combining these equations with equation (15) to solve for the mixed derivative gives

$$\begin{aligned}\frac{d^2\mathbb{E}i^{\Delta}}{dr^*d\bar{\epsilon}} &= -\frac{1}{\alpha}\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi}-P^{LB}}g(\bar{\nu}^{LB})\left(-\alpha\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)+\frac{1}{\alpha}P^{LB}\left(\alpha\frac{\psi}{\psi-1}\frac{d^2\mathbb{E}i^{\Delta}}{dr^*d\bar{\epsilon}}\right)\\ &= \left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^3 g(\bar{\nu}^{LB})\\ &= -\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^2\frac{dP^{LB}}{d\bar{\epsilon}}.\end{aligned}$$

A.F Proof of Proposition 3

Proof: We follow analogous steps as in Section A.E. We start from the derivative of the expected wedge with respect to the intercept θ in the policy rule

$$\frac{d\mathbb{E}i^{\Delta}}{d\theta} = \frac{1}{\alpha}\mathscr{G}_2(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{d\theta}. \tag{20}$$

Now, computing the second mixed derivative as

$$\frac{d^2\mathbb{E}i^{\Delta}}{d\theta ds} = \frac{1}{\alpha}\mathscr{G}_{1,2}(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{d\theta}+\frac{1}{\alpha}\mathscr{G}_{2,2}(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{d\theta}\frac{d\bar{\nu}^{LB}}{ds}+\frac{1}{\alpha}\mathscr{G}_2(s,\bar{\nu}^{LB})\frac{d^2\bar{\nu}^{LB}}{d\theta ds},$$

where we get the last term by computing

$$\frac{d\bar{\nu}^{LB}}{d\theta} = \alpha\left(\frac{1}{\psi-1} + \frac{\psi}{\psi-1}\frac{d\mathbb{E}i^{\Delta}}{d\theta}\right) \tag{21}$$

and

$$\frac{d^2\bar{\nu}^{LB}}{d\theta ds} = \alpha\frac{\psi}{\psi-1}\frac{d^2\mathbb{E}i^{\Delta}}{d\theta ds}.$$

Plugging these expressions into the second mixed derivative yields

$$\frac{d^2\mathbb{E}i^{\Delta}}{d\theta ds} = \frac{1}{\alpha}\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi} - P^{LB}}\left(\mathscr{G}_{1,2}(s,\bar{\nu}^{LB}) + g(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{ds}\right)\frac{d\bar{\nu}^{LB}}{d\theta}.$$

We combine equations (20) and (21) to get

$$\frac{d\bar{\nu}^{LB}}{d\theta} = \frac{\alpha}{\psi-1} + \frac{\psi}{\psi-1}G(s,\bar{\nu}^{LB})\frac{d\bar{\nu}^{LB}}{d\theta},$$

or, rewritten,

$$\frac{d\bar{\nu}^{LB}}{d\theta} = \frac{\alpha}{\psi}\frac{1}{\frac{\psi-1}{\psi} - P^{LB}}.$$

Taken together and using equation (14) results in

$$\frac{d^2\mathbb{E}i^{\Delta}}{d\theta ds} = \frac{1}{\psi-1}\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi} - P^{LB}}\right)^2\frac{dP^{LB}}{ds}.$$

□

Extending this proof to the mixed derivative of the interest rate wedge with respect to θ and $\bar{\epsilon}$

$$\frac{d^2\mathbb{E}i^{\Delta}}{dr^* d\bar{\epsilon}} = \frac{1}{\alpha}\frac{dP^{LB}}{d\bar{\epsilon}}\frac{d\bar{\nu}^{LB}}{d\theta} + \frac{1}{\alpha}P^{LB}\frac{d^2\bar{\nu}^{LB}}{d\theta d\bar{\epsilon}},$$

where

$$\frac{d^2\bar{\nu}^{LB}}{d\theta d\bar{\epsilon}} = \alpha\frac{\psi}{\psi-1}\frac{d^2\mathbb{E}i^{\Delta}}{d\theta d\bar{\epsilon}}.$$

Therefore,

$$
\begin{aligned}
\frac{d^2\mathbb{E}i^{\Delta}}{d\theta d\bar{\epsilon}} &= \frac{1}{\alpha}\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\frac{dP^{LB}}{d\bar{\epsilon}}\frac{d\bar{\nu}^{LB}}{d\theta} \\
&= \frac{1}{\psi-1}\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^2\frac{dP^{LB}}{d\bar{\epsilon}} \\
&= -\frac{1}{\psi-1}\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^3 g(\bar{\nu}^{LB}).
\end{aligned}
$$

B Technical Details on Time Series Analysis

B.A Controlling for Serial Correlation in Hypothesis Testing on Sample Means

Above we compute sample means of various time series in adjacent subsamples and test the null hypothesis that these estimated means are the same across subsamples. Because the time series for which we compute the mean – both the raw time series for r^* and the trailing-window correlations – are highly persistent, we control for serial correlation to conduct valid inference. To this end, we construct the following t-statistic for this null hypothesis:

$$t = \frac{\bar{y_2}-\bar{y_1}}{\sqrt{\text{Var}(\bar{y_2}-\bar{y_1})}}$$

which is the ratio of the difference in subsample means to the standard error of this difference, where y_1 is the first subsample with n_1 observations and y_2 is the second subsample with n_2 observations, with $\bar{y_i}$ denoting the sample mean of subsample i.

Following Wilks (1997), we use a scaling factor approach to control for serial correlation in our setting. Hence, we compute the variance of the difference in means as follows:

$$\text{Var}(\bar{y_2}-\bar{y_1}) = \hat{\vartheta}_2\frac{s_2^2}{n_2}+\hat{\vartheta}_1\frac{s_1^2}{n_1}$$

with s_i^2 denoting the sample variance of subsample i. For each subsample i, the scaling factor ϑ_i given by $\vartheta_i = 1+2\sum_{k=1}^{n_i-1}[1-k/n_i]\rho_k$ accounts for serial correlation ρ_k at various lags k, and the modified scaling factor $\hat{\vartheta}_i = \vartheta_i\exp(2\vartheta_i/n_i)$ corrects for potential bias in estimating this scaling factor.

Internet Appendix

A Model Derivations: The New Keynesian Model with a Lower Bound

Equations (1) and (2) describe the textbook log-linearized New Keynesian model on which we impose the restriction that demand shocks ϵ_t are i.i.d. With this assumption, we can express these two equations in terms of inflation expectations and without expected output gaps. To this end, we solve equation (1) for the output gap as

$$x_t = \frac{1}{\kappa}(\pi_t - \beta \mathbb{E}_t \pi_{t+1}), \tag{22}$$

iterate forward, and take conditional expectations to get

$$\mathbb{E} x_{t+1} = \frac{1}{\kappa}(\mathbb{E}_t \pi_{t+1} - \beta \mathbb{E}_t \pi_{t+2}).$$

Using the law of iterated expectations, we simplify $\mathbb{E}_t \mathbb{E}_{t+1} \pi_{t+2} = \mathbb{E}_t \pi_{t+2}$. Substituting these expressions for x_t and $\mathbb{E}_t x_{t+1}$ in equation (2) yields

$$\pi_t - \mathbb{E}_t \pi_{t+1} = \kappa[\epsilon_t - \alpha(i_t - \mathbb{E}_t \pi_{t+1} - r^*)] + \beta(\mathbb{E}_t \pi_{t+1} - \mathbb{E}_t \pi_{t+2}).$$

Taking unconditional expectations and using that $\mathbb{E}\pi_t = \mathbb{E}\pi_{t+j} = \mathbb{E}\pi$ yields

$$\pi_t - \mathbb{E}_t \pi_{t+1} = \kappa[\epsilon_t - \alpha(i_t - \mathbb{E}_t \pi_{t+1} - r^*)]. \tag{23}$$

The Fisher relation $\mathbb{E} i = r^* + \mathbb{E}\pi$ follows.

Minimizing the central bank's loss function, taking expectations as given, results in optimal discretionary policy

$$i_t = \max\{\mathtt{i}_t, i^{LB}\}.$$

Policy thus follows the shadow rate whenever feasible, and the policy rate is at the lower bound otherwise.

To write the model in terms of the interest rate wedge, substitute $\mathtt{i}_t + i_t^{\Delta}$ for the nominal interest rate i_t in equation (23) to yield equation (5). Then substitute equation (5) for π_t in equation (22) to yield the corresponding equation for the output gap.

To derive the Fisher equation, take unconditional expectations of equation (23), again using that $\mathbb{E}\pi_t =$

$\mathbb{E}\pi_{t+j} = \mathbb{E}\pi$.

To derive the first and second equalities in equation (7), take unconditional expectations of (1) and (5), respectively, using that $\mathbb{E}\pi_t = \mathbb{E}\pi_{t+1}$.

A.A Derivation for Cutoff of Binding Lower Bound

Start from the following equations:

$$P^{LB} = G(\bar{\nu}^{LB}); \tag{24}$$

$$\bar{\nu}^{LB} = \alpha\left(i^{LB} - \theta - r^* + \frac{\psi}{\psi - 1}(\mathbb{E}i^{\Delta} + \theta)\right) - \bar{\epsilon}; \tag{25}$$

$$\mathbb{E}i^{\Delta} = \frac{1}{\alpha}\int_{-\infty}^{\bar{\nu}^{LB}} (\bar{\nu}^{LB} - \nu)g(\nu)d\nu. \tag{26}$$

A.A.1 Link between expectations, cutoff, and lower bound probability

Equation (25) implies that the link between expectations in equation 7 can be written as

$$-\frac{1}{\psi - 1}(\mathbb{E}i^{\Delta} + \theta) = \frac{1}{\psi}\left(i^{LB} - \theta - r^* - \frac{1}{\alpha}\left[\bar{\epsilon} + \bar{\nu}^{LB}\right]\right) = \frac{1}{\psi}\left(i^{LB} - \theta - r^* - \frac{1}{\alpha}\left[\bar{\epsilon} + G^{-1}(P^{LB})\right]\right).$$

A.A.2 Critical value for P^{LB}

We rewrite equation (25) as

$$-\frac{\psi}{\psi - 1}\mathbb{E}i^{\Delta} = i^{LB} - r^* - \frac{1}{\alpha}\left[\bar{\epsilon} + \bar{\nu}^{LB}\right] + \left(\frac{\psi}{\psi - 1} - 1\right)\theta$$

This equation gives us an affine relationship between the expected wedge $\mathbb{E}i^{\Delta}$ and the cutoff $\bar{\nu}^{LB}$. The slope $\frac{1}{\alpha}\frac{\psi-1}{\psi}$ is positive.

Equation (26) is zero for $\bar{\nu}^{LB} \to -\infty$. It is monotonic since the integrand is nonnegative. Furthermore, it is concave as we shall see below. And it tends to infinity for $\bar{\nu}^{LB} \to \infty$. As a result, it can intersect with the previous line twice, once, or not at all. Now note that

$$\frac{d\mathbb{E}i^{\Delta}}{d\bar{\nu}^{LB}} = \frac{d}{d\bar{\nu}^{LB}}\left[\frac{1}{\alpha}\int_{-\infty}^{\bar{\nu}^{LB}} (\bar{\nu}^{LB} - \nu)g(\nu)d\nu\right] = \frac{1}{\alpha}\int_{-\infty}^{\bar{\nu}^{LB}} g(\nu)d\nu = \frac{1}{\alpha}P^{LB}$$

by Leibniz's rule. This derivative is always positive and increasing in $\bar{\nu}^{LB}$. The two lines thus have the same slope when

$$\bar{P}^{LB} = \frac{\psi - 1}{\psi}.$$

The two intersections between the two lines must thus be to the left and right of the value for ν such that $P^{LB} = \bar{P}^{LB}$. In the case with one intersection, it coincides with the critical lower bound probability.

A.A.3 Expression for expected interest rate wedge through integration by parts

We start from equation (26) and compute the integral over the first term of the integrand:

$$\mathbb{E}i^{\Delta} = \frac{1}{\alpha}\left(\bar{\nu}^{LB}P^{LB} - \int_{-\infty}^{\bar{\nu}^{LB}} \nu g(\nu) d\nu\right). \tag{27}$$

For the second term of the integrand, use the formula for integration by parts to simplify this expression further:

$$\int_{-\infty}^{\bar{\nu}^{LB}} \nu g(\nu) d\nu = \left[\nu G(\nu)\right]_{-\infty}^{\bar{\nu}^{LB}} - \int_{-\infty}^{\bar{\nu}^{LB}} G(\nu) d\nu = \left[\nu G(\nu)\right]_{-\infty}^{\bar{\nu}^{LB}} - \left[\mathscr{G}(\nu)\right]_{-\infty}^{\bar{\nu}^{LB}} = \bar{\nu}^{LB} G(\bar{\nu}^{LB}) - \mathscr{G}(\bar{\nu}^{LB}), \tag{28}$$

where $G(\cdot)$ denotes the cumulative distribution function and $\mathscr{G}(\cdot)$ denotes the super-cumulative distribution function, i.e. the primitive of the c.d.f. Plugging this into the previous expression results in

$$\mathbb{E}i^{\Delta} = \frac{1}{\alpha}\mathscr{G}(\bar{\nu}^{LB}). \tag{29}$$

A.B Details on Symmetric and Unimodal Distributions

For symmetric and unimodal distributions, we get following result.

Lemma 4 (Global result for symmetric unimodal distributions)

In the case of $\psi = 2$ and a unimodal symmetric distribution, a lower natural real rate of interest implies a more negative impact of increased uncertainty on inflation expectations across all target equilibria. Across all liquidity trap equilibria in this special case, a lower natural rate renders the impact of higher uncertainty on inflation expectations less negative. The result also holds for all target equilibria when $\psi \in (1, 2)$ and all liquidity trap equilibria for $\psi > 2$.

Proof: We start by showing that the result of Proposition 2 holds globally for unimodal symmetric distributions when $\psi = 2$. Therefore, start with the observation that, in this case,

$$P^{LB} = \frac{\psi - 1}{\psi} = \frac{1}{2}.$$

Due to the symmetry of the distribution, this case corresponds to a cutoff $\bar{\nu}^{LB} = 0$. An increase in uncertainty for a symmetric unimodal distribution exhibits the single-crossing property for the distributions. As a result, the c.d.f. of the riskier distribution lies above the less risky distribution for all negative values and below for all positive values.

Due to single-crossing of the distributions, the probability of a binding lower bound increases for all negative values and decreases for all positive values with higher uncertainty. Formally, $\mathscr{G}_{1,2}(s,\nu) > 0$ for all $\nu < 0$ and $\mathscr{G}_{1,2}(s,\nu) < 0$ for all $\nu > 0$. Consequently the steps in the proof of Proposition 2 hold globally.

By the same logic, when $\psi \in (1,2)$, the critical value for the lower bound $\frac{\psi-1}{\psi} < \frac{1}{2}$. As a result, all the target equilibria have a lower bound probability of less than 50% and, with a symmetric distribution, the corresponding critical cutoff is in the negative territory. With single crossing, $\mathscr{G}_{1,2}(s,\nu) > 0$ for all target equilibria. When $\psi > 2$, the critical cutoff is in positive territory and the results for the liquidity trap equilibria hold globally. $\square$

A.C Results for liquidity trap equilibria

This section extends the theoretical results of Sections III.B and III.C by including a discussion of liquidity trap equilibria. The comparative statics for liquidity trap equilibria go in the opposite direction as those for the target equilibrium. Since the paper shows that the empirical evidence is consistent with the target equilibrium, one can interpret the data as evidence against the liquidity trap equilibrium.

The cutoff $\bar{\nu}^{LB}$ rises with uncertainty in the target equilibrium as discussed in Proposition 1 and, as a result, the lower bound binds over a larger portion of the support for the demand shock. The opposite prediction holds in a liquidity trap equilibrium. Proposition 4 summarizes these results.

Proposition 4 (Effects of uncertainty)

Neither a mean-preserving spread nor lower average demand have an effect on inflation expectations absent the lower bound; in the presence of a lower bound, higher uncertainty raises (lowers) the expected interest rate wedge in a target *(liquidity trap) equilibrium.*[23]

Proof: From the proof of 1, we get

$$\frac{\mathrm{d}\mathbb{E}i^{\Delta}}{\mathrm{d}\bar{\epsilon}} = \frac{1}{\alpha}\frac{\psi-1}{\psi}\left(1-\frac{\psi-1}{\psi}\frac{1}{\frac{\psi-1}{\psi}-P^{LB}}\right).$$

To see that the sign of this expression switches depending on the equilibrium, notice that the sign is determined by the expression in parentheses. This expression is clearly positive in the liquidity trap equilibrium since there $P^{LB} > \frac{\psi-1}{\psi}$. □

Next, we extend Lemma 2. Lemma 5 establishes that the lower bound probability and the cutoff always move in the same direction in response to increases in uncertainty when the probability of a binding lower bound is sufficiently low (high) in the target (liquidity trap) equilibrium.

Lemma 5 *There are values $\underline{\nu}$ and $\bar{\nu}$ such that an increase in uncertainty raises the probability of a binding lower bound whenever $\bar{\nu}^{LB} < \underline{\nu}$ in the target equilibrium and lowers it for all $\bar{\nu}^{LB} > \bar{\nu}$ in the liquidity trap equilibrium. When the demand shock is normally distributed, these results hold globally, i.e.,*

$$\frac{dP^{LB}}{ds} = \begin{cases} > 0 & \text{for } \bar{\nu}^{LB} < \bar{\nu}^* \quad \text{(target equilibria).} \\ < 0 & \text{for } \bar{\nu}^{LB} > \bar{\nu}^* \quad \text{(liquidity trap equilibria).} \end{cases}$$

For any continuous distribution, lower average demand raises (lowers) P^{LB} in the target (liquidity trap) equilibrium.

Proof:

$$\left(\frac{\psi-1}{\psi} - P^{LB}\right)\frac{d\bar{\nu}^{LB}}{ds} = \mathscr{G}_1(s, \bar{\nu}^{LB}) > 0.$$

Consequently, $(\frac{\psi-1}{\psi} - P^{LB})$ and $\frac{d\bar{\nu}^{LB}}{ds}$ are of the same sign. The cutoff is thus increasing in the target equilibrium and decreasing in the liquidity trap equilibrium. By the same logic as in the proof of Lemma 2, the equilibrium cutoff $\bar{\nu}^{LB}$ shifts to the left in the liquidity trap equilibrium, thereby weakly lowering the probability of a binding lower bound. □

Proposition 5 extends Proposition 2 to include liquidity trap equilibria:

Proposition 5 (Changes in the natural real rate and the effects of changes in uncertainty) *Changes in uncer-*

[23] Appendix A.B contains the proof.

tainty do not affect expected interest rate wedges in the absence of a lower bound on interest rates, independent of the level of the natural rate of interest r^*. In the presence of a lower bound, the following expression determines whether the sensitivity of the expected interest rate wedge to changes in uncertainty increases or falls with the natural real rate of interest

$$\frac{d\frac{\mathbb{E}i^{\Delta}}{ds}}{dr^*} = \frac{d^2\mathbb{E}i^{\Delta}}{dr^*ds} = -\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi} - P^{LB}}\right)^2 \frac{dP^{LB}}{ds}. \tag{30}$$

The analogous expression for average demand $\frac{d^2\mathbb{E}i^{\Delta}}{dr^*d\bar{\epsilon}}$ is unambiguously positive (negative) in the target (liquidity trap) equilibrium.

The proof follows directly from Appendix A.E.

Proposition 5 allows us to sign the direction in which a fall in the natural rate pushes the sensitivity of the expected interest rate wedge to changes in uncertainty. The determining factor is whether uncertainty raises the probability of a binding lower bound. Using Lemma 2, the right-hand side of equation (30) is always negative in the target equilibrium and always positive in the liquidity trap equilibrium, provided that shocks are normally distributed. The results furthermore hold for any distribution if the likelihood of being constrained is sufficiently low (high) in the target (liquidity trap) equilibrium.

As a consequence, the sensitivity of inflation expectations with respect to uncertainty declines in the target equilibrium as the natural real rate of interest falls. In the liquidity trap equilibrium, this sensitivity rises. Measuring this sensitivity through the correlation of changes in inflation expectations and uncertainty shocks therefore shows that the results in Section II are consistent with the target equilibrium and at odds with the liquidity trap equilibrium.

Both an increase in uncertainty and a lower mean of the distribution $\bar{\epsilon}$, i.e., a leftward shift, raise the probability of a binding lower bound in the target equilibrium and lower it in a liquidity trap equilibrium. Consequently, both changes in the distribution have the same effect on expectations, as Proposition 5 demonstrates.

Proposition 6 extends the effects of static average-inflation targeting in Proposition 3 to liquidity equilibria:

Proposition 6 (Static average-inflation targeting and the effects of changes in uncertainty)
Changes in uncertainty do not affect expected interest rate wedges in the absence of a lower bound on interest rates, independent of the level of the policy intercept θ. In the presence of a lower bound, the following expression determines whether the sensitivity of the expected interest rate wedge to changes in uncertainty increases or falls with the policy

intercept θ

$$\frac{d^2\mathbb{E}i^{\Delta}}{d\theta ds} = \frac{1}{\psi-1}\left(\frac{\frac{\psi-1}{\psi}}{\frac{\psi-1}{\psi}-P^{LB}}\right)^2 \frac{dP^{LB}}{ds}.$$

The analogous expression for average demand $\frac{d^2\mathbb{E}i^{\Delta}}{d\theta d\bar{\epsilon}}$ is unambiguously negative (positive) in the target (liquidity trap) equilibrium.

The proof follows immediately from Section A.F.

B Robustness of Empirical Estimates

In this part of the appendix, we show the robustness of our empirical estimates. We first present evidence that the choice of Spearman correlations is not the driving force behind the changing correlation. We further show that the use of regression slopes delivers identical insights relative to using correlations.

B.A Empirical Estimates for Pearson Correlations

This section presents evidence that the conclusions of this paper do not rest on the usage of Spearman correlations. While regular Pearson correlations are more strongly affected by outliers present in the data, they do deliver consistent results. To make this point, we replicate each result using the standard correlation measure. While the results for the correlation between uncertainty and inflation expectations showed the Spearman correlation in the main body of the paper, we show both Pearson and Spearman correlations in Figures A1 and A2 below. Table A1 shows, analogously to Table 1, the estimates for the subsample split based on Pearson correlations. The following two tables show robustness of the main regression specifications. The chart on the DKW measure and the following table show that the robustness exercises are also robust to using Pearson correlations.

B.B Covariance between Changes in Inflation Expectations and Uncertainty Shocks

Figure A4 shows the evolution of the covariance between changes in the five-year ahead five-year-forward breakeven inflation rate with changes in the VIX at the monthly frequency. The solid line shows the minimum covariance determinant, used to control for outliers, and the dashed line is the conventional covariance. The figure for the covariance displays the same pattern as the figure for the correlation, indicating that there is no obvious trend in the level of uncertainty that can explain the results.

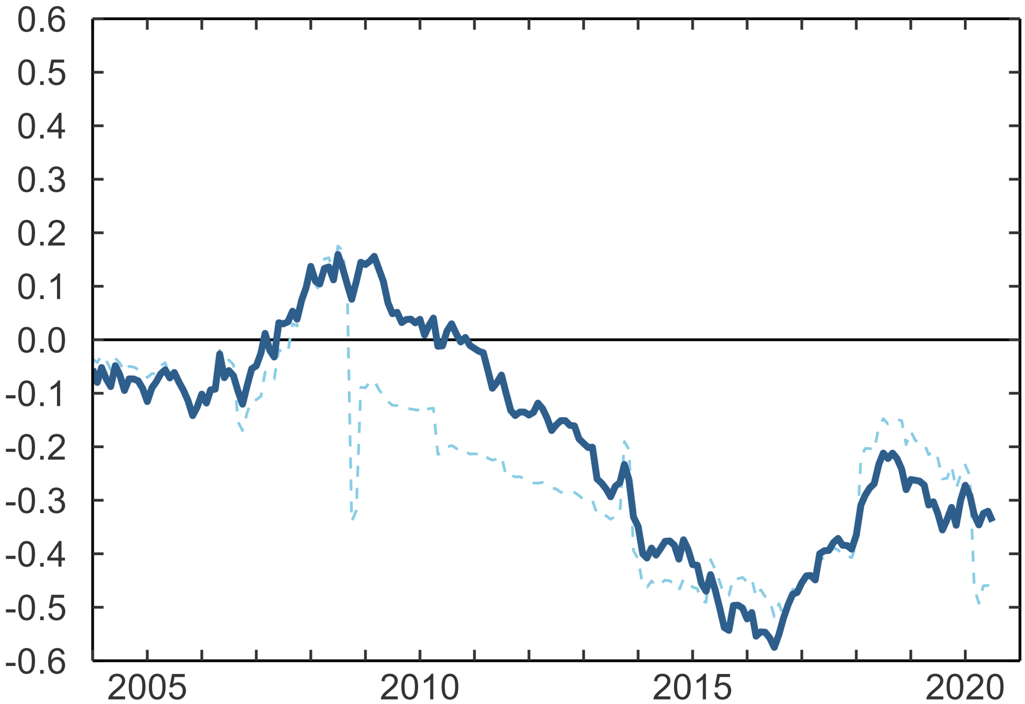

Figure A1: Five-year trailing-window correlation of monthly changes in the VIX with monthly changes in the five-year ahead five-year forward break-even inflation rate rate: Spearman correlation (solid line), Pearson correlation (dashed line). The sample period ranges from January 2004 until July 2020.

Table A1: Differences in moments across subsamples

	mean r^*		mean Pearson correlation with ΔVIX			
	r^*_{CR}	r^*_{TIPS}	BEI	BEI	TFR	TFR
frequency	monthly	monthly	daily	monthly	daily	monthly
Jul 2011 - Jul 2020	0.54	0.83	-0.29	-0.35	-0.33	-0.23
Jan 1999 - Jun 2011	1.47	2.88	-0.09	-0.08	-0.16	-0.08
difference	-0.93***	-2.05***	-0.20***	-0.27***	-0.17***	-0.15**

Note: ***, **, and * signify that the difference in means across subsamples is statistically significant at the 1% level, 5% level, and 10% level, respectively, accounting for serial correlation. Correlations are the trailing-window Pearson correlation of the change in the variable with the change in the VIX (here referred to as ΔVIX). Pearson correlations are computed over one-year windows at the daily frequency and over five-year windows at the monthly frequency. r^*_{CR} denotes estimates of the natural rate from Christensen and Rudebusch (2019a), and r^*_{TIPS} is the five-year ahead five-year forward zero-coupon TIPS rate computed from Gürkaynak et al. (2010) used as a proxy for the natural rate. BEI is the five-year ahead five-year forward breakeven inflation rate, while TFR is the five-year ahead five-year forward Treasury constant maturity yield.

Table A2: Regression of Pearson correlation between changes in VIX and BEI on the natural rate of interest

	Dependent variable:					
	corr(ΔVIX$_t$, ΔBEI$_t$)					
	r^*_{CR}			r^*_{TIPS}		
	(1)	(2)	(3)	(4)	(5)	(6)
r^*	0.192***	0.313***	0.190***	0.092***	0.134***	0.091***
	(0.038)	(0.078)	(0.039)	(0.018)	(0.036)	(0.019)
constant	−0.356***	−0.493***	−0.362***	−0.334***	−0.432***	−0.341***
	(0.045)	(0.106)	(0.042)	(0.040)	(0.098)	(0.036)
Frequency	daily	monthly	within	daily	monthly	within
Sample Start	Jan 1999	Jan 2004	Jan 1999	Jan 1999	Jan 2004	Jan 1999
Sample End	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020
Observations	5,374	199	259	5,371	199	259
R^2	0.072	0.483	0.120	0.078	0.463	0.132

Note: Newey-West (HAC) standard errors. $^*p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table A3: Regression of Pearson correlation between changes in VIX and TFR on the natural rate of interest

	Dependent variable:					
	corr(ΔVIX$_t$, ΔTFR$_t$)					
	r^*_{CR}			r^*_{TIPS}		
	(1)	(2)	(3)	(4)	(5)	(6)
r^*	0.158***	0.129	0.167***	0.086***	0.035	0.090***
	(0.042)	(0.107)	(0.051)	(0.020)	(0.043)	(0.024)
constant	−0.396***	−0.277	−0.417***	−0.395***	−0.218	−0.415***
	(0.051)	(0.194)	(0.058)	(0.045)	(0.175)	(0.050)
Frequency	daily	monthly	within	daily	monthly	within
Sample Start	Jan 1998	Jan 1998	Jan 1998	Jan 1999	Jan 1999	Jan 1999
Sample End	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020
Observations	5,625	271	271	5,381	259	259
R^2	0.049	0.119	0.087	0.066	0.043	0.117

Note: Newey-West (HAC) standard errors. $^*p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.

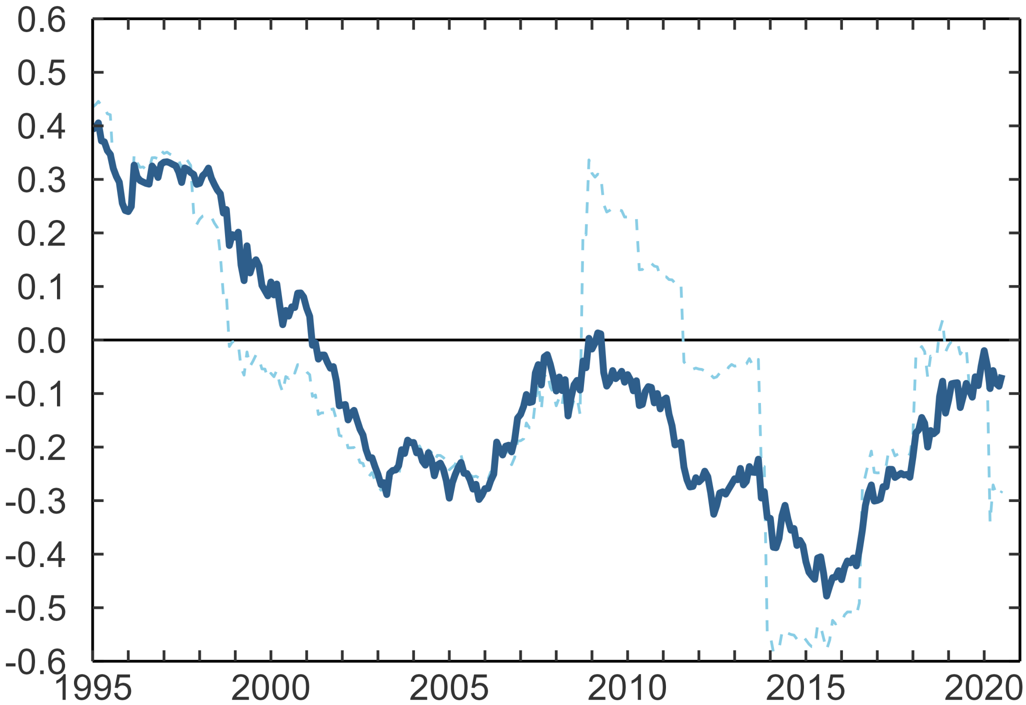

Figure A2: Five-year trailing-window correlation of monthly changes in the VIX with monthly changes in the five-year ahead five-year forward Treasury constant maturity rate: Spearman correlation (solid line), Pearson correlation (dashed line). The sample period ranges from January 1995 until July 2020.

B.C Changes in Natural Rates and Effects of Uncertainty: Additional Evidence

In this section, we provide additional evidence for the link between changes in the effects of uncertainty as natural rates declined. We repeat the daily regressions in Table 2 for different frequencies. First, we report the same specifications using monthly data. And second, we compute the correlations between uncertainty shocks and changes in inflation expectations from daily data within the calendar month and use these as monthly observations.

Table A5 shows the regression of the breakeven inflation BEI-uncertainty shock correlation on the natural rate of interest in various specifications. Columns (1)-(3) show the estimates using the Christensen-Rudebusch measure for the natural rate, columns (4)-(6) show the corresponding estimates using the TIPS measure of the natural rate. Compared to the daily frequency results reported in the main text, the regression labeled "within" using correlations run within calendar months has almost identical magnitude, while the "monthly" regression using correlations computed using monthly average values for both the trailing-window correlations as well as r^* has higher magnitude estimated coefficients across r^* measures. All specifications have a positive sign, and the daily and within specifications are statistically significant at the

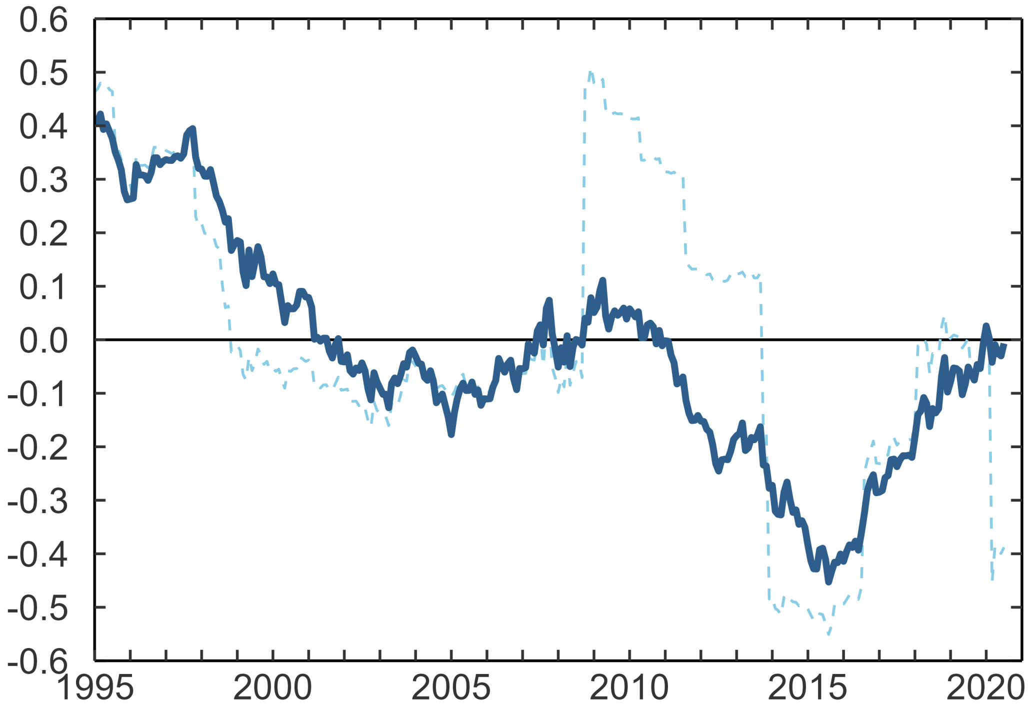

Figure A3: Pearson correlation between changes in the VIX and changes in the five-year, five-year forward expected inflation rate, adjusted for risk premia and the liquidity premium using the estimates of D'Amico et al. (2018): daily frequency, one-year trailing-window correlation. The sample period ranges from January 1995 until July 2020.

1% level.

Table A6 shows analogous results for the regression of the Treasury forward rate TFR-uncertainty shock correlation on the natural rate of interest in various specifications. The estimated coefficients for the "monthly" and "within" specifications are consistent in sign and magnitude with our "daily" specification reported in the main text. The "monthly" specification is statistically significant at the 10% level using the CR measure of r^*, and the "daily" and "within" specifications are statistically significant at the 1% level.

Table A7 shows analogous results for the regression of the real Treasury forward rate TRR-uncertainty shock correlation on the natural rate of interest in various specifications. The estimated coefficients for the "within" specification have the same sign and quantitatively similar magnitude as our baseline "daily" specification reported in the body of the paper and all are significant at the 5% or 10% level. The "monthly" specification here yields an opposite sign but the estimated coefficient is not statistically significantly different from zero.

Table A4: Differences in moments across subsamples for Pearson correlations.

	mean Pearson correlation with ΔVIX				mean Pearson correlation with ΔBEI	
	BEI	BEI	DKW	SPF	EFU	EPU
frequency	daily	monthly	monthly	quarterly	monthly	monthly
Jul 2011 - Jul 2020	-0.29	-0.35	-0.31	-0.12	-0.24	-0.10
Jan 1999 - Jun 2011	-0.09	-0.08	-0.16	0.01	-0.14	-0.06
difference	-0.20***	-0.27***	-0.15***	-0.13*	-0.10	-0.04

Note: ***, **, and * signify that the difference in means across subsamples is statistically significant at the 1% level, 5% level, and 10% level, respectively, accounting for serial correlation. Correlation is the trailing-window correlation of the change in the variable with the change in the breakeven inflation rate (here denoted as ΔBEI) or the change in the VIX (here denoted as ΔVIX), as labeled. Correlations are computed over one-year windows at the daily frequency and over five-year windows at the monthly and quarterly frequencies. EFU is the one-year ahead estimate of economic financial uncertainty from Ludvigson et al. (2021), and EPU is the economic policy uncertainty index of Baker et al. (2016). BEI is the five-year ahead five-year forward breakeven inflation rate, and DKW is the five-year ahead five-year forward breakeven inflation rate from D'Amico et al. (2018) which removes risk premia and liquidity premia from the Treasury-implied rate. SPF is the median ten-year ahead CPI inflation rate from the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia.

Table A5: Regression of correlation between changes in VIX and BEI on the natural rate of interest

	Dependent variable:					
	corr($\Delta\mathrm{VIX}_t$, $\Delta\mathrm{BEI}_t$)					
	r^*_{CR}			r^*_{TIPS}		
	(1)	(2)	(3)	(4)	(5)	(6)
r^*	0.161***	0.395	0.167***	0.077***	0.157	0.081***
	(0.039)	(0.540)	(0.040)	(0.018)	(0.202)	(0.019)
constant	−0.317***	−0.520	−0.330***	−0.298***	−0.423	−0.313***
	(0.048)	(0.608)	(0.045)	(0.043)	(0.444)	(0.038)
Frequency	daily	monthly	within	daily	monthly	within
Sample Start	Jan 1999	Jan 2004	Jan 1999	Jan 1999	Jan 2004	Jan 1999
Sample End	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020
Observations	5,374	199	259	5,371	199	259
R^2	0.054	0.590	0.100	0.059	0.483	0.112

Note: Newey-West (HAC) standard errors. $^*p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.

Table A6: Regression of correlation between changes in VIX and TFR on the natural rate of interest

	Dependent variable:					
	corr(ΔVIX$_t$, ΔTFR$_t$)					
	r^*_{CR}			r^*_{TIPS}		
	(1)	(2)	(3)	(4)	(5)	(6)
r^*	0.129***	0.198*	0.144***	0.071***	0.069	0.079***
	(0.046)	(0.107)	(0.049)	(0.021)	(0.081)	(0.023)
constant	−0.334***	−0.372***	−0.373***	−0.335***	−0.310	−0.373***
	(0.058)	(0.132)	(0.059)	(0.051)	(0.197)	(0.051)
Frequency	daily	monthly	within	daily	monthly	within
Sample Start	Jan 1998	Jan 1998	Jan 1998	Jan 1999	Jan 1999	Jan 1999
Sample End	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020
Observations	5,625	271	271	5,381	259	259
R^2	0.035	0.443	0.071	0.048	0.329	0.097
Note:	Newey-West (HAC) standard errors. $^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.					

Table A7: Regression of correlation between changes in VIX and TRR on the natural rate of interest

	Dependent variable:					
	corr(ΔVIX$_t$, ΔTRR$_t$)					
	r^*_{CR}			r^*_{TIPS}		
	(1)	(2)	(3)	(4)	(5)	(6)
r^*	0.075*	−0.093	0.085**	0.041**	−0.048	0.045**
	(0.040)	(0.366)	(0.041)	(0.018)	(0.107)	(0.018)
constant	−0.221***	−0.073	−0.246***	−0.222***	−0.079	−0.245***
	(0.049)	(0.372)	(0.048)	(0.043)	(0.214)	(0.041)
Frequency	daily	monthly	within	daily	monthly	within
Sample Start	Jan 1999	Jan 2004	Jan 1999	Jan 1999	Jan 2004	Jan 1999
Sample End	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020	Jul 2020
Observations	5,374	199	259	5,371	199	259
R^2	0.013	0.085	0.031	0.017	0.117	0.041
Note:	Newey-West (HAC) standard errors. $^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.					

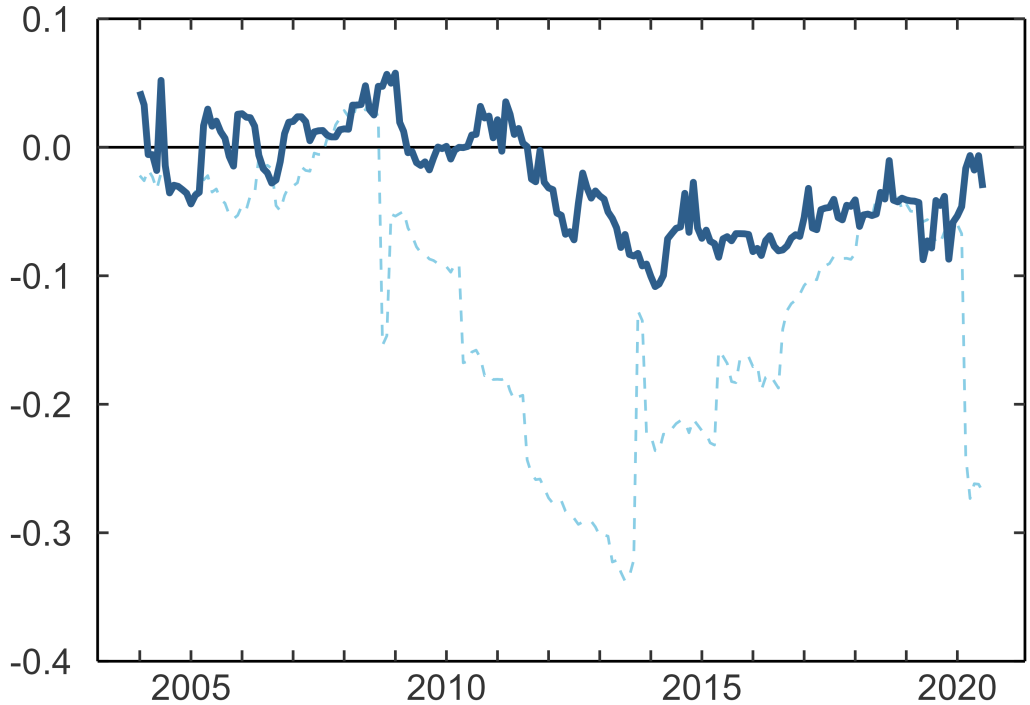

Figure A4: Five-year trailing-window covariance of monthly changes in the VIX with monthly changes in the five-year ahead five-year-forward breakeven inflation (BEI) rate. The solid line is the minimum covariance determinant, and the dashed line is the conventional covariance. The sample period ranges from January 2004 until July 2020.

B.D Regression Slopes as Measures of Comovement

The empirical facts in the paper are based on the correlations of changes in uncertainty and changes in inflation expectations (Figure 1) and interest rates (Figure 4), respectively. Instead of computing correlations that are, by definition, standardized by the standard deviation, we can measure the comovement between these financial market prices using regression slopes. Both measures are in line with the finance literature that focuses on comovements of stock and bond markets. Parts of the literature studies covariances and regression slopes (e.g., Campbell et al. (2017)) while another part has focused on stock-bond correlations (e.g., Duffee (2022)). Therefore, consider the regression

$$\hat{\pi}_{t+1}^{\text{5Y5Y}} - \hat{\pi}_{t}^{\text{5Y5Y}} = \beta_0 + \beta_1 \left(\text{VIX}_{t+1} - \text{VIX}_t\right) + \eta_{t+1},$$

where $\hat{\pi}_{t+1}^{\text{5Y5Y}}$ is the five-year expected inflation rate five years out, as measured by breakeven inflation.

The left panel in Figure A5 shows the time series of the regression coefficients β_1 for monthly regressions over a 5-year rolling window. The time-series pattern for the regression coefficient β_1 follows that of the

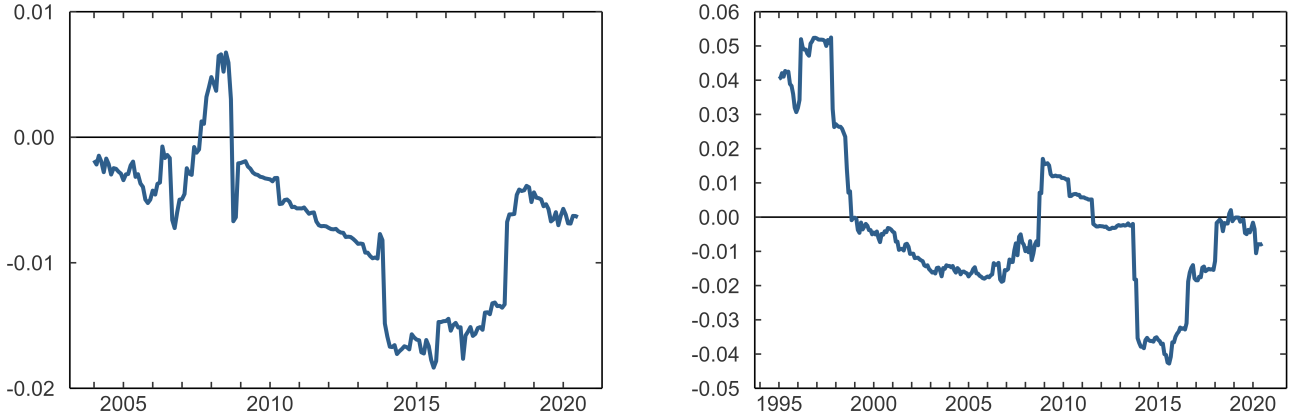

Figure A5: Analogues of Figure 1 and Figure 4 using regression slopes instead of correlations.

correlation. Therefore, the standardization by the level of the standard deviation is not the driving force behind the results.

The right panel in Figure A5 shows the analogous regression coefficients for interest rates. The qualitative pattern for the regression slopes over time is consistent with that for the correlations shown in Figure 4. We therefore conclude that the results of this paper are robust to measuring them via the slopes of a regression.

B.E Correlation between Changes in the S&P 500 and Uncertainty Shocks

This section shows the analogue of Figure 6. Instead of the regression slopes used in the main text, it shows correlations of changes in the log S&P 500 index with changes in the VIX index. Both measures are in line with the finance literature on this topic. For example, Duffee (2022) shows stock-bond correlations while Campbell et al. (2017) show regression slopes. Campbell et al. (2012) show both measures for the stock-bond relation.

Figure A6 shows the time series of the correlation between changes in the stock market and uncertainty shocks. The broad pattern mimics that for other correlations and comovements shown in the paper and is consistent with the theoretical model. While there is a small increase in the correlation at the beginning of the sample, the correlation is generally only mildly negative during the 1990s. It then drops precipitously in the early 2000s and remains strongly negative (below -0.5) until the end of the sample period.

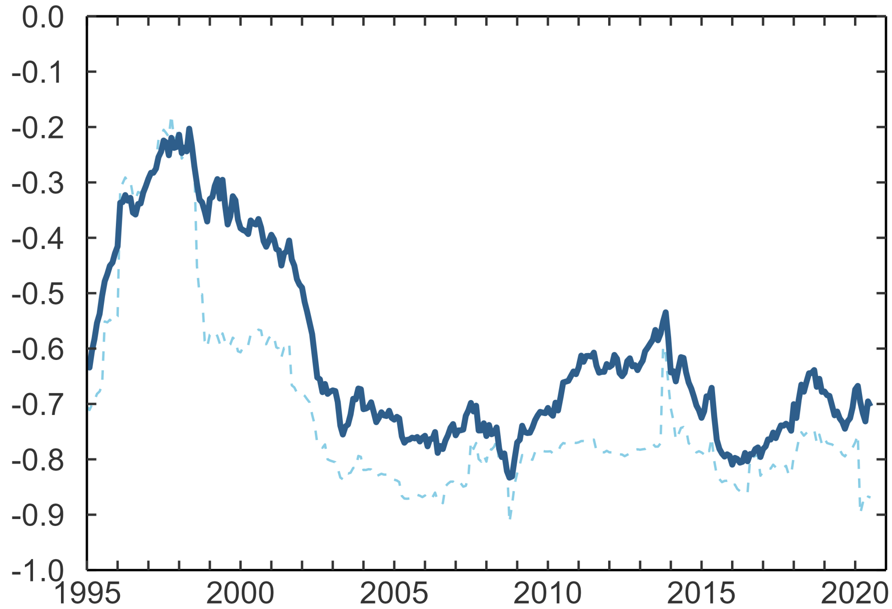

Figure A6: Five-year trailing-window correlation (Spearman correlation: solid dark blue line, Pearson correlation: dashed light blue line) of monthly changes in the VIX with monthly changes in the logarithm of the S&P 500 stock price index. The sample period ranges from January 1995 until July 2020.

C Lasting Effects of Uncertainty Shocks

This section extends the i.i.d. model of the main text with a simple way to capture serial correlation. This version of the model shows that short-lived spikes in uncertainty can have lasting effects on inflation expectations. Since the general problem with serially correlated shocks is notoriously hard to solve, we restrict our analysis to a one-time spike in uncertainty and leave the analysis of a more general model with persistent shocks to future research.

We start from the i.i.d. model with a lower bound on interest rates that plays out in all periods starting in period zero, i.e., $t = 0, \ldots, \infty$. To this model, we add three periods. In the first period, -3, we simply measure uncertainty about future variables. In the second period, -2, the demand shock is realized according to the same distribution as in the i.i.d. model. Howerver, the shock realization now lasts for two periods. That is to say that during -1 and -2, the shocks are perfectly serially correlated. As a result, the uncertainty about the periods -2 and -1 resolves in period -1 and there is no uncertainty about period -1 shocks one period ahead. Figure A7 depicts this shock process graphically.

Let us solve this model backwards. Because the model switches to the i.i.d. model from period 0 on, expectations, the lower bound cutoff, and thus policy functions coincide with those in the i.i.d. model in

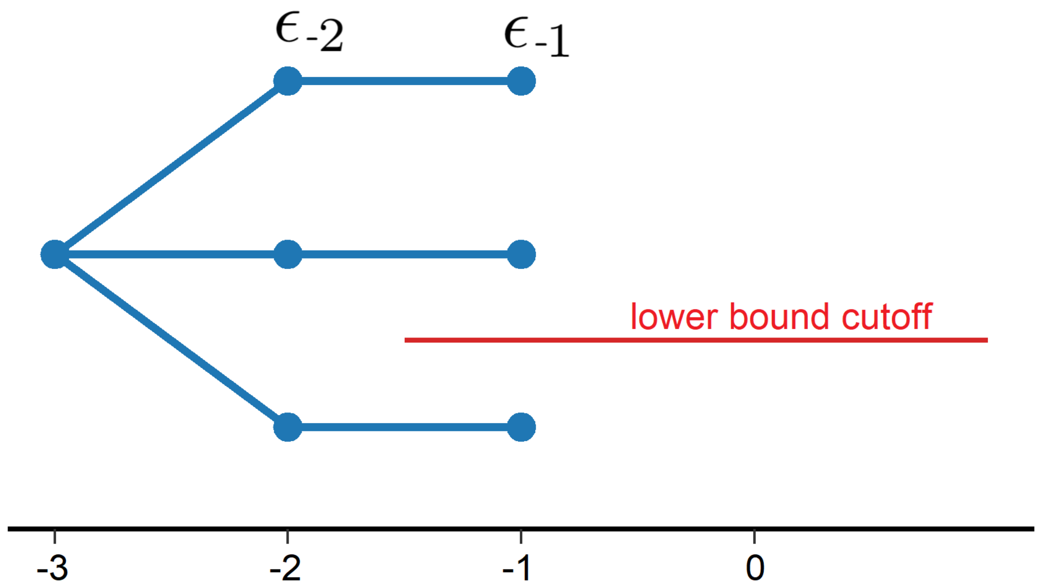

Figure A7: Graphical representation of shocks that persist for two periods without any conditional uncertainty.

period -1. In period -2, however, since next period's shock is know, next period's inflation and output gap are then perfectly predictable:

$$\mathbb{E}_{-2}[\pi_{-1}] = \pi_{-1} \text{ and } \mathbb{E}_{-2}[x_{-1}] = x_{-1}.$$

We get the following results. First, there is a spike in uncertainty in period -3 because shocks in -2 last for two periods. This spike in volatility is short-lived since, conditionally, there is no uncertainty in $t-2$ about $t-1$ variables. Second, as a result of the absence of uncertainty in period -2, the cutoff for the shock ϵ at which the lower bound starts to bind, $\bar{\epsilon}^{LB}_{-2}$, falls in the target equilibrium. One way to see this is to study the shock realization at which the lower bound starts to bind in the i.i.d. model: $\epsilon_{-2} = \epsilon_{-1} = \bar{\epsilon}^{LB}$. In the i.i.d. case, while inflation expectations are below target as usual, inflation can be stabilized. With this logic, we know in period -2 that inflation will be stabilized in period -1. As a result, inflation expectations are more benign and the lower bound cutoff must be lower, leading to overall higher inflation expectations.

Taking these two findings together, a short-lived increase in uncertainty in period -3 can impact *future* inflation expectations — in this case, inflation expectations in period -2 about period -1 inflation.

D Data Sources and Measures

This section presents the data sources and compares our measure of realized volatility to the VIX.

D.A Data Sources

	variable description	frequency	sample	source
Measures of uncertainty				
VIX	CBOE Volatility Index	daily	1990-	Bloomberg: VIX Index
EFU	economic financial uncertainty	monthly	1960-	Jurado et al. (2015)[24]
EPU	news-based policy uncertainty index	daily	1985-	Baker et al. (2016)[25]
Measures of long-term inflation expectations				
BEI	Treasury-implied breakeven inflation rate	daily	1999-	Bloomberg: USGG5Y5Y Index
DKW	dynamic factor model	daily	1983-	D'Amico et al. (2018)[26]
SPF	Survey of Professional Forecasters	quarterly	1991-	Philadelphia Fed - SPF: CPI10 [27]
Measures of long-term interest rate expectations				
TFR	five-year ahead five-year forward Treasury constant maturity rate	daily	1962-	Federal Reserve Board
Measures of the real natural rate of interest (r^*)				
CR	no-arbitrage model-implied	daily	1998-	Christensen and Rudebusch (2019a)[11]
TIPS	five-year ahead five-year forward zero-coupon TIPS rate	daily	1999-	Gürkaynak et al. (2010)[11]

[24]Updated data downloaded from the authors' website: www.sydneyludvigson.com/macro-and-financial-uncertainty-indexes
[25]Updated data downloaded from the authors' website: www.policyuncertainty.com/us_monthly.html - Daily News Index
[26]Data from: www.federalreserve.gov/econres/notes/feds-notes/DKW_updates.csv
[27]Data from: www.philadelphiafed.org/surveys-and-data/data-files

D.B Realized volatility

Figure A1 shows the close connection between VIX and realized volatility during the sample period when both measures are available.

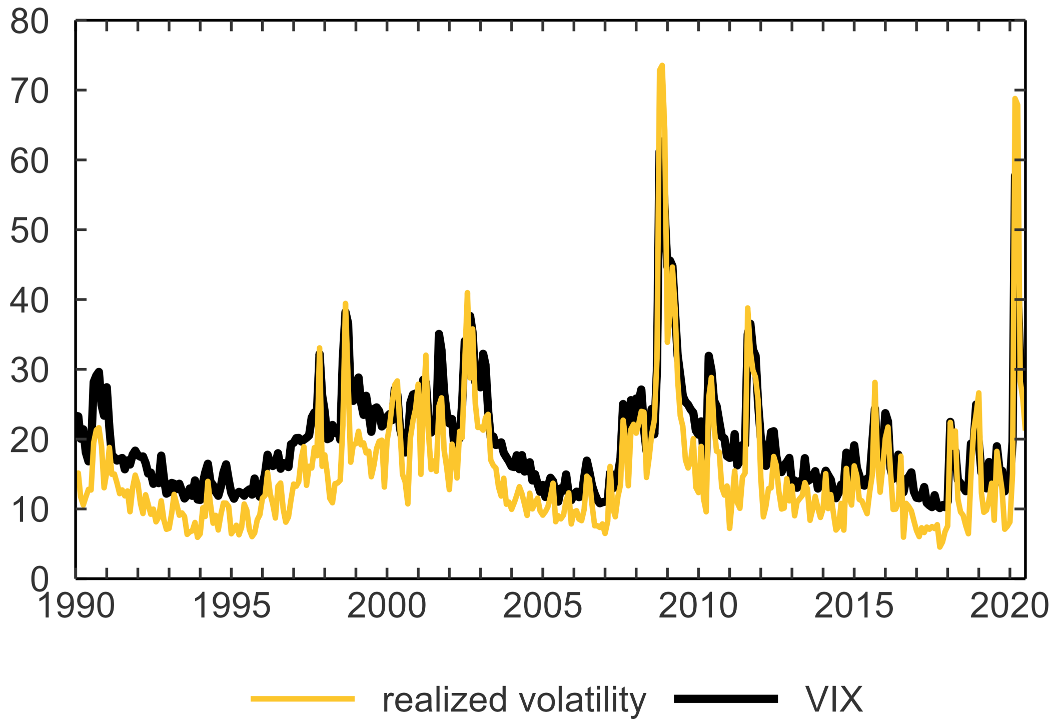

Figure A1: Monthly time series of VIX and realized volatility [computed as the backward-looking moving average over 30 days (21 trading days) of squared daily returns in the S&P 500 index]; both series computed as monthly averages of daily values. The sample period ranges from January 1990 until July 2020.