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Intermediary Balance Sheets and the Treasury Yield Curve

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Abstract

We have documented a regime change in the U.S. Treasury market post-Global Financial Crisis (GFC). We first derived bounds on Treasury yields that account for dealer balance sheet costs, which we call the net short and net long curves. We show that actual Treasury yields moved from the net short curve pre-GFC to the net long curve post-GFC, consistent with the shift in the dealers' net position. We then use a stylized model to demonstrate that increased bond supply and tightening leverage constraints can explain this change in regime. This change, in turn, helps explain negative swap spreads and the co-movement between swap spreads, dealer positions, yield curve slope, and covered-interest-parity violations, and implies changing effects for a wide range of monetary and regulatory policy interventions.

Key words: yield curve, balance sheet constraints, CIP deviations

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To view the authors' disclosure statements, visit
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Introduction

The U.S. Treasury market is one of the most important financial markets in the world. Treasury bonds have long been considered a safe haven for global investors (e.g. Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2012), and Gorton (2017)). In this paper, we document that the dynamics of Treasury yields changed substantially at the time of the global financial crisis (GFC), and propose a framework to explain these changes.

Figure 1 shows certain key ways in which the Treasury market has changed pre- and post-GFC, and illustrates the idea of regimes. Pre-GFC, primary dealers[1] maintained a net short position in the Treasury bonds, and the swap-Treasury spread was positive. Post-GFC, primary dealers switched to holding a net long position in Treasury bonds and the swap spread became negative. The pre-GFC regime featured close to zero deviations from covered interest rate parity (CIP), whereas post-GFC regime features sizable CIP deviations. Furthermore, in the post-GFC period CIP deviations strongly co-move with the swap-Treasury spread, and are negatively correlated with the dealers' Treasury position.

Another striking fact about the Treasury market post-GFC is an extremely tight correlation between dealers' net position and the slope of the Treasury yield curve (measured as the term spread between the 10-year and 3-month yield), as shown Figure 2. The primary dealers' Treasury position is the mirror image of the Treasury slope. When the yield curve is flat (steep), primary dealers load up (scale back) their Treasury position. To the extent that the term spread proxies for the expected return on long-term Treasury bonds, this relationship is at first puzzling as dealers increase (reduce) their net holdings exactly when the expected returns on the Treasury bonds are low (high).

We develop a coherent framework that can explain the facts documented in Figures 1 and 2. Our framework focuses on the interaction of constrained financial intermediaries and return-seeking Treasury investors, and leads naturally to the idea of *regimes* in which intermediaries are net long, net short, or flat with respect to Treasury bonds. We then use our framework to analyze the effects of several unconventional policy and regulatory policy interventions.

We begin by solving for bounds on Treasury bond yields that are consistent with the dealers'

[1] Primary dealers are selected trading counterparties of the Federal Reserve Bank of New York in its implementation of monetary policy. They are also required to bid on a pro-rata basis in all Treasury auctions at reasonably competitive prices.

net positions using a no-arbitrage approach. The key assumption embedded in our approach is that all zero-cost, zero-balance-sheet trading strategies are at least weakly unprofitable under a common stochastic discount factor (SDF). This assumption is implicit in the models of Jermann (2020) and Du, Hébert, and Huber (2022). We construct a "net long" curve and a "net short" curve for Treasury bonds of varying maturities. The net long curve describes a Treasury yield above which the intermediary would always want to be net long Treasury bonds, regardless of its belief of future Treasury yields. Similarly, the net short curve provides a yield below which the intermediary would always be willing to be net short the bond. The net long and net short curves are themselves functions of the risk-neutral expectations of future short-term interest rates implied by the overnight index swap (OIS) rates and future balance sheet costs implied by CIP deviations.[2] We find that actual Treasury yields are quite close to the estimated net short curve pre-GFC and close to the estimated net long curve after-GFC, consistent with the sign of net dealer positions. In other words, our term structure provides a quantitative validation of regime change. We develop the net long and net short curve from the perspective of a securities dealer, but argue that dealers will in effect transmit their balance sheet costs to hedge funds and other levered clients, consistent with Boyarchenko et al. (2018). As a result, the curves we develop are applicable for these levered clients as well.

After establishing the empirical relevance of the net long and net short curves, we embed these bounds in a two-period, two-market supply-demand model for Treasury bonds and synthetic dollar lending (i.e. FX swaps). The model features an intermediary sector and two types of clients for Treasury bonds. The intermediary sector includes dealers, who finance their Treasury bond holdings in the tri-party repo market (or short Treasury bonds in the securities lending market), and levered investors (e.g. relative-value fixed-income hedge funds), who rely on dealer balance sheet to finance Treasury bonds using repos. These intermediaries take net long positions when Treasury yields reach the net long curve, and net short positions when Treasury yields reach the net short curve.

On the client side, we first model real-money investors for Treasury bonds, domestic or foreign, who do not rely on dealers' balance sheets to fund their positions. Real-money investors decide between holding Treasury bills versus long-term Treasury bonds, and their demand for bonds is

[2] Under the assumption that derivatives including swaps are priced by a common SDF, such a risk-neutral measure exists; however, in the presence of arbitrage between cash interest rates and derivatives, this risk-neutral measure will not pin down the price of cash securities such as Treasuries.

an increasing function of the expected excess returns on the Treasury bonds. Second, we model foreign exchange (FX)-hedged foreign investors, who invest in U.S. Treasury bonds and hedge currency risk using short-dated FX swaps. A canonical example of an FX-hedged foreign investor is the Japanese life insurance company, with liabilities all denominated in yen but has large U.S. dollar fixed-income exposure. The FX-hedged foreign investors does not use leverage but nevertheless rely on dealer balance sheets to obtain FX swap hedging. FX-hedged foreign investors' demand increases in the expected return on the Treasury bond net the FX hedging cost.

We close the model by imposing market clearing conditions for Treasury bonds and synthetic dollars, as well as an intermediary balance sheet constraint. The model has a unique equilibrium, which falls into one of the three regimes: intermediaries are either long, short, or flat with respect to Treasury bonds. The model generates different comparative statics across these regimes.

Using these comparative statics, we argue that the increase in Treasury supply and tightening of the balance sheet constraint are sufficient to explain the regime change pre- and post-GFC. As shown in Figure 3, total assets of the broker-dealer sector reached \$6.3 trillion at their pre-GFC peak in March 2008, but contracted sharply during the GFC to \$3.8 trillion in October 2009. Growth of the broker-dealer assets remains stagnant post-GFC. In contrast, the outstanding of total marketable Treasury securities (net holdings of the Federal Reserve) grew five fold since 2008, rising from \$4.7 trillion in January 2008 to about \$22.5 trillion today.

The change in the sign of the swap-Treasury spread post-GFC is consistent in our model with the intermediary sector switching its Treasury position from net short to net long. In particular, when the balance sheet constraint binds post-GFC amid a large Treasury supply, the intermediary maintains a long Treasury position. The negative swap-spread and the CIP deviation co-move, as the spread that the intermediary can earn by using up its balance sheet to go long in the Treasury bond (and hedging the duration risk using interest rate swap) must be comparable to the foregone profits on the CIP trade. The larger the intermediary long position, the higher the shadow cost on the intermediary balance sheet constraint, and the wider the swap-spread and CIP deviations.

Besides matching the key features of the pre- and post-GFC regimes, the equilibrium model also delivers a sharp prediction on the relationship among the intermediary position, intermediary spreads, and the expected returns on the long-term Treasury bonds post-GFC. When the expected excess returns on the Treasury bonds are high (as proxied for by a steep yield curve), client demand for Treasury bonds is high, and the intermediary Treasury position is low in the equilibrium. This prediction is strongly supported in the data as shown in Figure 2. When the curve is flat, these intermediaries face low returns on their Treasury positions, but their hedged returns (hedged with interest rate swaps) are high due to the relationship between the slope of the curve and swap spreads.[3]

In the last section of the paper, we discuss several policy interventions in the context of our two-period model. The model, with minimal modification, is capable of speaking to the effects of quantitative easing and tightening, inter-central-bank swap lines, regulatory exemptions to the supplementary leverage ratio (SLR), and interest rate policy. We discuss the effects of each of these policies, and emphasize how they differ in the long and short regimes. Finally, we discuss the implications of the framework for the ongoing tightening cycle, and draw a parallel with the experience of 2017-2019 tightening cycle.

Our paper is most closely related to Jermann (2020) and Du, Hébert, and Huber (2022), in that we model swap spreads (Jermann (2020)) and CIP violations (Du, Hébert, and Huber (2022)) as arising from leverage constraints on intermediaries. Jermann (2020) emphasizes the importance of leverage constraints on intermediaries as an explanation for negative swap spreads. Relatedly, Favara, Infante, and Rezende (2022) show evidence that SLR shocks have reduced large banks' participation in the U.S. Treasury market. Du, Tepper, and Verdelhan (2018b), Hébert (2020), and Du, Hébert, and Huber (2022) argue that CIP deviations can proxy for the shadow cost of the these constraints. The strong co-movement of CIP violations and swap spreads post-GFC documented in Figure 1 is consistent with these perspectives.

Considering these markets together helps address a puzzle: why would long-maturity (e.g. 30 year) swap spreads be affected by fluctuations in current balance sheet costs? The answer suggested by Du, Hébert, and Huber (2022) is that there is a substantial risk premium associated with the risk that balance sheet costs increase; our quantitative analysis confirms that this risk premium plays a significant role in long maturity swap spreads. Our quantitative term-structure framework is substantially more general than the models employed in these papers, and our two-market equilibrium model is able to address a broader range of policy questions. In contemporaneous work, Hanson, Malkhozov, and Venter (2022) adopt an approach broadly similar to our two-period model to explain the way in which shocks to the demand for interest rate swaps affect swap spreads (see also Klingler and Sundaresan (2019)). Hanson, Malkhozov, and Venter (2022) focus on the swap

[3]The relationship between swap spreads and the yield curve slope in the post-GFC period was documented by Jermann (2020), among others.

market, treating the Treasury market as exogenous, whereas our approach is the reverse. Their interest is in separating supply and demand shocks, whereas our focus is on validating the notion of Treasury market regimes and analyzing policy interventions.

The change from positive to negative swap spreads is related to the observation that long-dated Treasury bonds have lost their "convenience yield," with respect to both domestic and international comparisons. From a domestic perspective, the spread between dollar interest rate swaps and U.S. Treasury bonds turned from positive pre-GFC to negative post-GFC at long maturities (Feldhütter and Lando (2008), Klingler and Sundaresan (2019), Jermann (2020), Augustin, Chernov, Schmid, and Song (2021), and Fleckenstein and Longstaff (2021)). From an international perspective, pre-GFC longer-dated U.S. Treasury yields consistently traded below synthetic dollar yields generated by swapping local currency government bonds issued by other safe havens, such as Germany, into dollars. However, this international "convenience yield" of the U.S. Treasury bonds has also diminished post-GFC (Du, Im, and Schreger (2018a)). More recently, the dislocation of the Treasury bond market during the height of the COVID-19 pandemic in March 2020 has led some authors to question whether U.S. Treasury bonds remain convenient (Duffie (2020) and He, Nagel, and Song (2022)). Relative to this literature, we highlight the importance of the regime change in the dealers' net position and the interaction between dealer balance sheet constraints and client demand for Treasury bonds.

The convenience yield literature has argued that Treasury securities are valued not only by their cash flows, but also by the superior liquidity and safety (Longstaff (2004); Krishnamurthy and Vissing-Jorgensen (2012); Greenwood, Hanson, and Stein (2015); Joslin, Li, and Song (2021)), with macro-finance implications of this convenience discussed in Drechsler, Savov, and Schnabl (2018), Jiang, Krishnamurthy, and Lustig (2021), and Krishnamurthy and Li (2022). This literature has focused on the convenience demand for Treasury bonds from non-intermediary investors. Our results highlight that no convenience demand is needed to explain intermediaries' demand for Treasury bonds. In fact, balance sheet costs make Treasury bonds *inconvenient* for dealers.

1 Institutional Background

In this section, we explain the mechanics about how dealers go long and short Treasury securities, and introduce a balance sheet-neutral Treasury trading strategy.

Let us first consider how dealers go net long Treasury bonds. Suppose that the dealer goes long a zero-coupon Treasury bond of maturity n at time t and unwinds the position at time $t+\tau$ with $\tau \leq n$. At the onset of the trade at time t, the dealer buys the Treasury bond at yield $y_{n,t}$, and finances the position from money market funds (MMF) via the tri-party repo financing rate r_t^{tri}, posting the Treasury bond as collateral (see Panel (a) of Figure 4). There is a small haircut, typically around 2%, for Treasury collateral, so the remaining 2% of the Treasury purchase has to be financed via unsecured borrowing. To simplify the illustration, in Figure 4, we ignore this haircut and assume all financing is via the repo market.

A dealer can hedge the interest rate risk of this trade by entering into a swap contract.[4] Our analysis will focus on overnight index swaps (OIS), in which one party pays a fixed rate of interest in exchange for a series of floating payments indexed to the overnight interbank federal funds rate.[5] The maturity of the swap is typically set the same as the Treasury bond to avoid maturity mismatch. In the context of hedging a long Treasury position, the dealer would pay the fixed rate and receive the floating payments (see Panel (b) of Figure 4). There is a small and relatively stable spread between the federal funds rate the dealer receives in this scenario and the tri-party repo rate the dealer pays to finance the bond purchase. Therefore, by hedging with OIS, the dealers remove the risk of fluctuations in the tri-party repo rate to the first-order.[6] At time $t+\tau$ (Panel (c) of Figure 4), the dealer unwinds the long position by selling the Treasury bond at a new price $e^{-(n-\tau)y_{n-\tau,t+\tau}}$ (where $y_{n-\tau,t+\tau}$ is the log yield at the time of sale) and paying back the repo loan from MMF. The dealer also unwinds the swap contract.

During this trade, the dealer will have a larger balance sheet, equal to the additional Treasury net long position.[7] We will consider trading strategies in which the dealer offsets the balance sheet effects of buying and financing a Treasury bond by reducing another balance-sheet-intensive

[4]Dealers have an incentive to hedge their net interest rate risk, due to risk-based capital requirements, but will typically do so at the trading desk level or the whole book level as opposed to trade-by-trade.

[5]Prior to the GFC, swaps indexed to LIBOR were more commonly used, and recently swaps indexed to SOFR (Secured Overnight Financing Rate) have been introduced. OIS rates are available for our entire sample and are similar to LIBOR swap rates pre-GFC and to SOFR swap rates in the recent period.

[6]An exact interest rate hedge would be selling an interest rate swap indexed to the floating tri-party repo rate. Thus, the difference between tri-party repo rate and OIS rate induces some remaining basis risk.

[7]To a first approximation, the interest rate swap is entirely off-balance-sheet. More precisely, trading interest rate swaps can increase the balance sheet constraint slightly. The total exposure includes initial and variation margins (typically a couple percent of total notional), and an additional 0-1.5% of the swap notional calculated for off-balance sheet interest rate derivative exposure using the Current Exposure Method, depending on the maturity of the interest rate swaps. We ignore the additional balance sheet costs of trading derivatives to simplify our analysis.

activity, covered-interest-rate-parity (CIP) arbitrage, as shown in Figure 6. The mechanism of the CIP trade involves borrowing dollars in the unsecured cash market and lending dollars in the FX swap market for most of the major currency pairs, such as euro-dollar and the dollar-yen (Du, Tepper, and Verdelhan (2018b)). The dealer will be willing to take the Treasury bond position and reduce its CIP activity if this zero-cost, zero-balance-sheet trading strategy earns excess returns under the dealer's SDF.

We have considered CIP arbitrage as an alternative to taking a net Treasury position. In practice, dealers might be engaged in many other activities besides trading Treasurys and CIP. The advantage of considering the CIP trades, however, is that CIP violations have a term structure that allows us to proxy for the shadow cost of dealer balance sheet constraints at multiple maturities (Du, Hébert, and Huber (2022)).

A trading strategy in which the dealer goes net short the Treasury bond works in similar fashion, with a few key differences, and is illustrated in Figure 5. At time t, the dealer posts cash as collateral to borrow a Treasury bond from a security lender, and then short sells it at the current market price $e^{-ny_{n,t}}$. The security lender offers the dealer interest on this cash, at the rate r_t^{sec}, and re-invests the cash at a higher rate, earning a spread. We document in Appendix Section C.2 that the rate security lenders pay to dealers is typically roughly 25 basis points below the tri-party repo rate, even for securities that are not "special", using data from Markit Securities Finance.

The dealer can again choose to hedge the interest rate risk of this strategy with an OIS swap (Panel (b) of Figure 5). In this case, the dealer would pay the federal funds rate and receive the fixed rate. The federal funds rate will in general have a non-trivial but stable ($\approx 25bps$) spread over the security lending rate. Furthermore, the dealer has to pay the security lender bond coupons if there is any. Finally, at date $t+\tau$ (Panel (c) of Figure 5), the dealer buys back the bond and then returns it to the security lender, receiving back the cash collateral. The dealer also unwinds the swap contract.

Similar to the net long Treasury position, the net short position also increases the size of the dealer's balance sheet. The asset in this case is a repo loan to the security lender (the dealer has given the security lender cash in exchange for Treasury collateral), and the liability is a security to be delivered. We can again construct a zero-cost, balance-sheet-neutral trading strategy by assuming that the dealer simultaneously goes net short the Treasury bond and reducing its CIP arbitrage activity (See Figure 7 for an illustration).

2 The Long and Short Treasury Yield Curves

In this section, we construct what we call " net long" and "net short" yield curves. These yield curves represent (approximate) arbitrage bounds at which a dealer would be willing to go net long or net short Treasury bonds. In frictionless models, there is a single yield curve. At yields above this yield curve, dealers would want to go net long bonds, while at lower yields, dealers would want to go net short bonds. In our model, two frictions create a wedge between the yield at which dealers would go net long and the yield at which dealers would go net short.

The first friction is balance sheet costs. As emphasized in the previous section, going either net long or net short a Treasury bond increases the size of dealer balance sheets. This has an opportunity cost, and this opportunity cost acts like a tax on both these trades. It raises the yield dealers require when going net long, while lowering the yield they require when going net short. We proxy for the balance sheet costs using the term structure of CIP deviations.

The second friction is the difference in financing cost in the repo market to finance long position and the cash lending rate in the securities lending market associated with the short position. When a dealer goes net long, they can finance that position at the tri-party repo rate. When a dealer takes a net short position, they receive interest on the cash they lend in exchange for the Treasury bond. If the dealer can find a hedge fund client who owns the bond, they will likely receive a relatively high interest rate. However, if the dealer cannot find such a client, they will instead borrow the security from a security lender, in which case they will receive a rate below the tri-party repo rate. Since our goal is to construct a yield at which the dealer would definitely be willing to go net-short, we will assume the dealer will receive the lower security lending rate as opposed to a higher rate on its cash when shorting.

In what follows, we will proceed in three steps. First, we will introduce a very simple model of dealer behavior, to illustrate the trade-off a dealer faces between Treasury and other arbitrage activities. Second, we generalize the ideas illustrated in this simple model, and construct the net long and net short curves. Third, we discuss the assumptions under which these curves represent arbitrage bounds. Finally, we build a term structure model to estimate the net long and net short curves.

2.1 A Simple Model of Dealers and Arbitrage

We begin by considering the problem of a risk-neutral dealer who can choose between trading a single n-period zero-coupon Treasury bond and CIP arbitrage activities, subject to a fixed balance sheet constraint. Let q^{bond} be the dealer's bond position (in dollars, not notional), with negative values implying short-selling. Let q^{syn} be the dealer's "synthetic dollar" position (in dollars), which we define as the currency hedged investment leg of the CIP arbitrage (the other leg is borrowing dollars). We will assume that the synthetic dollar rate (e.g. the currency-hedged euro rate) is above the dollar borrowing rate, so that the direction of the arbitrage is to buy the synthetic dollars ($q^{syn} > 0$).

The dealer's balance sheet constraint is

$$q^{syn} + |q^{bond}| = \bar{q}, \tag{1}$$

where $\bar{q} > 0$ is the limit on dealer balance sheet size. Note that the absolute value of q^{bond} enters this expression, reflecting the fact that both long and short positions require balance sheet.

Let y be the log-yield on the Treasury bond, and let $p_{\mathbb{Q}}$ be the expected price in the next period (i.e. the expected price at which the trade will be unwound). We assume, following the discussion above, that if the dealer buys the bond, it will finance the position with a mix of tri-party repo financing (r^{tri} is the log repo rate) and unsecured financing (we assume the dealer can borrow at the one-period log OIS rate r^{ois}). The haircut h determines this mix.[8] If the dealer instead short-sells the bond, it receives a log return r^{sec} on its cash.

The dealer's problem is

$$\begin{aligned}\max_{q^{bond},q^{syn}} \max\{q^{bond},0\} \cdot (\underbrace{p_{\mathbb{Q}}}_{\text{sell after one period}} - (1-h) \cdot \underbrace{e^{-ny}e^{r^{tri}}}_{\text{secure financing}} - h \cdot \underbrace{e^{-ny}e^{r^{ois}}}_{\text{unsecured financing}}) \\ + \max\{-q^{bond},0\} \cdot (\underbrace{e^{-ny}e^{r^{sec}}}_{\text{earn security lending rate on short-sold cash}} - \underbrace{p_{\mathbb{Q}}}_{\text{buy back at time 1}}) \\ + q^{syn} \cdot \underbrace{(e^{r^{syn}} - e^{r^{ois}})}_{\text{synthetic lending spread}}\end{aligned} \tag{2}$$

[8]Consistent with the data, we are assuming that the tri-party rate is below the unsecured rate, and that the dealer borrows as much as possible using tri-party.

subject to the balance sheet constraint in (1).

From this problem, it is straightforward to show that dealers are willing to go net long the bond ($q^{bond} > 0$) if the yield $y \geq y^l$, where y^l, which we will call the "net long yield," is defined as

$$e^{-ny^l} \equiv \frac{p_{\mathbb{Q}}}{(1-h)(e^{r^{tri}} - e^{r^{ois}}) + e^{r^{syn}}}, \tag{3}$$

Likewise, the dealer will be willing to go net short if the bond's yield $y \leq y^s$, where y^s, which we will call the "net short yield," is defined as

$$e^{-ny^s} \equiv \frac{p_{\mathbb{Q}}}{e^{r^{sec}} + e^{r^{ois}} - e^{r^{syn}}}. \tag{4}$$

Now suppose that rates and prices are such that the dealer is always willing to make synthetic loans ($q^{syn} > 0$). If this assumption holds, we must have $y \in [y^s, y^l]$. Intuitively, if the dealer is willing to make synthetic loans, the returns on buying or short-selling the Treasury cannot dominate the returns on synthetic lending.

To gain further intuition, consider the case of a one-period bond that matures next period ($p_{\mathbb{Q}} = 1$). The log-linearized version of (3) and (4) is as follows:

$$\begin{aligned} y^l &\approx r^{ois} - (1-h)(r^{ois} - r^{tri}) + r^{cip}, && (5) \\ y^s &\approx r^{ois} - (r^{ois} - r^{sec}) - r^{cip}, && (6) \end{aligned}$$

where $r^{cip} = r^{syn} - r^{ois}$.

The net long yield can differ from the OIS rate for two reasons. First, holding Treasury bonds takes up bank balance sheet, so the yield has to be higher than the OIS rate by a wedge equal to the cost of balance sheet, as measured by r^{cip}. Second, if dealers' repo financing rate is lower than their unsecured funding cost, $r^{tri} < r^{ois}$, then there is a financing benefit to owning the Treasury bond, which makes the dealer willing to accept a lower yield.

The net short yield can differ from the OIS rate for similar reasons. The impact of the balance sheet cost affects the sell yield with a negative sign. The sell yield has to be lower (the price to be higher) to justify dealer's short position, which also takes balance sheet. The sell yield is further lowered if the return on the cash collateral is lower than the dealer's borrowing cost, $r^{ois} > r^{sec}$.

We next extend the logic of these yields to a more general, multi-period setting, constructing what we will call the "net long curve" and "net short curve."

2.2 The Net Long and Net Short Curves

We make three key assumptions: (i) all zero-cost, zero-balance-sheet trading strategies are weakly unattractive under a common SDF (i.e. $0 \geq E[MR]$ in the standard notation), (ii) some synthetic dollar lending occurs in equilibrium, and (iii) interest rate swaps and cross-currency basis swaps are priced by this same SDF. We will interpret this common SDF as a dealer's SDF, under the usual intermediary asset pricing assumption that dealers are active in all of these markets. The first two of these assumptions hold in the simple model above: the strategy of increasing or decreasing q^{bond}, financed with repo or sec. lending, and offsetting the change in balance sheet by changing q^{syn}, is weakly unattractive under the risk-neutral SDF, and by assumption $q^{syn} > 0$. Our third assumption (that the SDF prices swaps) is relevant only in the multi-period context.

Let $\mathbb{Q}$ denote the risk-neutral measure associated with this SDF. Assume that all rates are annual and that each period is one month. Consider the strategy of going long the Treasury bond at yield $y_{n,t}$, financing via repo, and offsetting the balance sheet effect by reducing CIP activity. We must have

$$(1-h) \cdot \underbrace{e^{-ny_{n,t}} e^{r_t^{tri}}}_{\text{secured financing}} + h \cdot \underbrace{e^{-ny_{n,t}} e^{r_t^{ois}}}_{\text{unsecured financing}} + \underbrace{e^{-ny_{n,t}} (e^{\frac{1}{12} r_t^{syn}} - e^{\frac{1}{12} r_t^{ois}})}_{\text{forgone CIP profits}} \geq E_t^{\mathbb{Q}}[e^{-(n-1)y_{n-1,t+1}}]. \quad (7)$$

The left-hand side of this expression represents costs paid at time $t+1$. The repo loan must be repaid (the first term), the unsecured financing must also be repaid (the second term), and the profits of the forgone CIP arbitrage are lost (the third term). These must be weighed against the benefits of selling the bond at time $t+1$ (which was $p_{\mathbb{Q}}$ in our simple model above). Note that because both sides of this equation are defined in terms of $t+1$ payoffs, the discount rate associated with the common SDF is irrelevant.

Define the monthly log interest rate

$$x_{1,t} = \ln((1-h)(e^{\frac{1}{12} r_t^{tri}} - e^{\frac{1}{12} r_t^{ois}}) + e^{\frac{1}{12} r_t^{syn}}), \quad (8)$$

and iterate:

$$e^{-ny_{n,t}} \geq e^{-ny^{l}_{n,t}} = E^{\mathbb{Q}}_{t}[\exp(-\sum_{j=0}^{n-1} x_{1,t+j})]. \tag{9}$$

The yield curve $y^{l}_{n,t}$ defines what we call the "net long curve." This curve represents the point at which dealers would be willing to switch from CIP arbitrage activity to taking a net long position in a Treasury bond. Since zero-cost, zero-balance-sheet strategies are weakly unattractive by assumption, this net long curve is, by induction, an upper bound on Treasury yields.

This net long curve can also be viewed as a lower bound on swap spreads (defined as the difference between OIS swap rates and Treasury yields of matching maturity). Let $r^{cip}_{n,t}$ be the n-period CIP violation (the n-period synthetic dollar rate minus the n-period OIS rate). Linearizing (9) and defining $r^{ois}_{n,t}$ as the n-period OIS swap rate,

$$r^{ois}_{n,t} - y_{n,t} \geq r^{ois}_{n,t} - y^{l}_{n,t} \approx - \underbrace{r^{cip}_{n,t}}_{\text{n-period CIP violation}} + E^{\mathbb{Q}}_{t}[\frac{1}{n}\sum_{j=0}^{n-1} \underbrace{((1-h)(r^{ois}_{t+j} - r^{tri}_{t+j})}_{\text{financing benefit}}]. \tag{10}$$

The difference between short-maturity OIS swap rates and tri-party repo rates is generally small and stable over time. As a result, equation (10) implies that if yields are close to the net long curve, we should expect swap spreads to be negative and close in absolute value to the matched-maturity cross-currency basis.

We should also emphasize the important role of risk premia that is hidden in this expression. The n-period CIP violation $r^{cip}_{n,t}$ is well above the physical ($\mathbb{P}$) measure expectation of future short-maturity CIP violations, as emphasized by Du, Hébert, and Huber (2022). That is, net long curve yields are higher than OIS rates both because of expectations of non-zero future balance sheet costs and because of the risk associated with the possibility that these costs become larger.

We develop the net short curve via similar logic. Consider the strategy of short-selling the Treasury bond at yield $y_{n,t}$, borrowing the bond in the security lending market, and offsetting the balance sheet effect by reducing CIP activity. We must have

$$\underbrace{e^{-ny_{n,t}}}_{\text{sale price}} \underbrace{e^{\frac{1}{12}r^{sec}_{t}}}_{\text{gross return on cash in sec. lending}} \leq \underbrace{E^{\mathbb{Q}}_{t}[e^{-(n-1)y_{n-1,t+1}}]}_{\text{repurchase price}} + e^{-ny_{n,t}} \underbrace{(e^{\frac{1}{12}r^{syn}_{t}} - e^{\frac{1}{12}r^{ois}_{t}})}_{\text{forgone CIP profits}}. \tag{11}$$

The left-hand side of this expression is the cash generated at date $t+1$by selling the bond, posting the cash to the security lending, and receiving the security lending rate. The right-hand side represents the costs of this trade at date $t+1$, including both the cost of repurchasing the bond and the forgone CIP profits. Define the monthly log interest rate

$$x_{2,t} = \ln(e^{\frac{1}{12}r_t^{sec}} + e^{\frac{1}{12}r_t^{ois}} - e^{\frac{1}{12}r_t^{syn}}) \tag{12}$$

and iterate:

$$e^{-ny_{n,t}} \leq e^{-ny_{n,t}^s} = E_t^{\mathbb{Q}}[\exp(-\sum_{j=0}^{n-1} x_{2,t+j})]. \tag{13}$$

The yield curve $y_{n,t}^s$ defines what we call the "net short curve." This curve represents the point at which dealers would be willing to switch from CIP arbitrage activity to taking a net short position in a Treasury bond.

The net short curve defines a lower bound on yields. It can also be viewed as an upper bound on swap spreads. Linearizing (13),

$$r_{n,t}^{ois} - y_{n,t} \leq r_{n,t}^{ois} - y_{n,t}^s \approx \underbrace{r_{n,t}^{cip}}_{\text{n-period CIP violation}} + E_t^{\mathbb{Q}}[\frac{1}{n}\sum_{j=0}^{n-1} \underbrace{(r_{t+j}^{ois} - r_{t+j}^{sec})}_{\text{security lending spread}}]. \tag{14}$$

The difference between the short-maturity OIS swap rate and the security lending rate is positive, and the n-period CIP violation in our definition is also positive and proxies for the balance sheet cost. Taking these two forces together, equation (14) implies that if yields are close to the net short curve, we should expect positive swap spreads.

Our assumptions are sufficient to determine an upper and lower bound on Treasury yields (or, equivalently, on swap spreads), but are not enough to pin down Treasury yields themselves. In a frictionless world with $r_t^{ois} = r_t^{sec} = r_t^{tri}$ and no balance sheet cost, the net long and net short curves converge to one curve and thus exactly pin down Treasury yields. In the presence of frictions, yields can fall anywhere in between the net short and net long curves. We will show, empirically, that yields were close to the net short curve before the GFC and close to the net long curve after the GFC. We will then construct a model in which dealers interact with non-dealers. In this model, the demands of non-dealers will determine where Treasury yields fall within the net short and net long curve bounds.

2.3 Discussion

Before proceeding, we elaborate on the interpretation of these curves and on the role of interest rate swap hedges in this framework.

The Net Short and Net Long Curves as Arbitrage Bounds. The bound $y_{n,t} \in [y^s_{n,t}, y^l_{n,t}]$ is an arbitrage bound if $y_{n-1,t+1} \in [y^s_{n-1,t+1}, y^l_{n-1,t+1}]$ with probability one, which follows from our first two key assumptions. This observation gives rise to the following interpretation: the net long yield is a yield at which a dealer would be willing to go long even if the dealer believed all dealers would be net long the bond in the future with probability one. Likewise, the net short yield is the yield at which a dealer would be willing go short, even if the dealer believed all dealers would be net short in the future with probability one. Yields falling in between these bounds can be loosely interpreted as related to the probability that dealers will be either net long or net short in the future.

In the appendix, section B, we show that even under weaker assumptions, if $y_{n,t} > y^l_{n,t}$, dealers will perceive an arbitrage opportunity. That is, a dealer will be willing to take a long position if $y_{n,t} \geq y^l_{n,t}$, even if it is not guaranteed that $y_{n-1,t+1} \leq y^l_{n-1,t+1}$. The net short curve does not share this property; however, we are able to derive a lower value for yields such that the dealer will always be willing to go short, and show that the net short curve described in the main text is a close approximation of this "partial equilibrium net short curve."

Pre-GFC and Post-GFC. Pre-GFC, synthetic lending rates were close to OIS rates (i.e. CIP violations were roughly zero), and security lending rates were roughly 25 basis points below one-month OIS swap rates. As a result, during this period, $x_{2,t}$ was roughly the one-month OIS rate less 25 basis points. It follows that net short curve yields $y^s_{n,t}$ were lower than matched-maturity OIS rates by about the same amount, which is to say there was a significant positive "swap spread" between OIS rates and the net short curve. In contrast, the net long curve during this period was only about five basis below OIS rates, reflecting the difference between tri-party repo rates and the federal funds rate.

Post-GFC, synthetic lending rates are well above OIS rates, and the spread between OIS and tri-party repo rates is small. In this period, $x_{1,t}$ is approximately equal to the synthetic lending rate. As a result, net long curve yields $y^l_{n,t}$ are approximately the same as synthetic swap rates (i.e. EUR swap rates converted to dollars), which are OIS rates plus the CIP basis of the same maturity. This leads to a significant "negative swap spread" between OIS rates and the net long curve. In contrast, the net short curve now features rates far below OIS rates (large and positive swap spreads), reflecting both the spread between security lending rates and one-month OIS rates and the synthetic-OIS spread (i.e. CIP violations).

Actual swap spreads between OIS rates and Treasury yields went from being large and positive pre-GFC to negative post-GFC. This fact, combined with the discussion above, previews our result that Treasury yields went from being close to the net short curve pre-GFC to close to the net long curve post-GFC.

Hedging with Swaps. It is standard practice to hedge a Treasury position with an interest rate swap. Prior to the GFC, hedging using LIBOR swaps was typical; following the GFC, OIS rose in prominence, and more recently swaps based on repo rates (SOFR swaps) have begun to trade. There are a range of approaches: hedging can be static or dynamic, and based on matching maturity, duration, or cashflows.

Hedging Treasury bonds with interest rates swaps leaves the dealer exposed to mark-to-market risk associated with fluctuations in the Treasury-swap spread. In our log-linearized equations (10) and (14), we can see the swap spreads for the net-long and net-short curve depends on the risk-neutral expectations of future balance sheet constraints and some residual basis rates between money market rates.

Therefore, the net long curve $y^l_{n,t}$ can be hedged with a synthetic dollar swap (by swapping EUR rates to dollars), under the assumption that the tri-party repo vs. short-term OIS swap spread is stable. The net short curve $y^s_{n,t}$ can be hedged via a combination of OIS and synthetic dollar swaps, under the assumption that the spread between security lending rates and short-term OIS swap rates is stable.

If one assumes that a Treasury yield will always be at one end of these boundaries, then the Treasury bond itself can be hedged. But note that the hedge will differ depending on which of the two boundaries is assumed to apply, and in neither case is the hedge a single overnight index swap. The intuition for this result is that balance sheet costs matter, and hedging the Treasury bond with OIS hedges interest rate risk but does not hedge balance sheet cost risk.

Yields and Positions. The net short and net long curves we construct are estimates of yields at which the dealer should be willing to take a net position in a Treasury bond, after accounting for financing and balance sheet costs. This definition leads naturally the prediction that if the yield is at the net long yield, dealers should be net long, and if the yield is at the net short yield, dealers should be net short. These bounds are constructed from possibly unreasonable beliefs about the stochastic process driving bond prices. As a result, we should not be at all surprised if dealers are willing to go long at yields below the net long yield or go short at yields above the net short yield.

Moreover, the yield of a Treasury bond relative to these curves does not directly determine the scale of dealer positions. For example, if the yield is at the net long yield, dealers should be net long, but the quantity by which they will be net long will depend on their balance sheet capacity, risk tolerance, and other considerations that cannot be inferred directly from yield curves. Nevertheless, we should expect net dealer positions to increase when yields move close to the net long yield and to decrease (become net short) when yields move close to the net short yield. We can construct a heuristic mapping from yields to position via this intuition.

Dealers and Levered Clients. In developing the net long and net short curves, we have adopted the perspective of a Treasury dealer. In appendix section A, we argue that these curves are also arbitrage bounds from the perspective of the dealer's levered clients (i.e. hedge funds). Dealers' balance sheet costs are in effect transmitted to these clients via bilateral lending markets, a point emphasized by Boyarchenko et al. (2018). In what follows, we will treat dealers and their levered clients as a consolidated entity.

2.4 The Term Structure Model

To construct the net long and net short curves, we need to construct the risk-neutral expectations of $x_{1,t+j}$ and $x_{2,t+j}$. This is where our third assumption, that interest rate swaps and cross-currency basis swaps are priced by the common SDF, applies. These risk-neutral expectations are determined primarily by the OIS swap curve and the term structure of CIP violations. We will proceed by constructing an SDF (in particular, a term structure model) that fits these swap rates, and then use that SDF to construct the net long and net short curves.

We construct these curves using a term structure model. At the heart of our calculations is a comparison between a Treasury hedged with OIS and a CIP violation of the same maturity. A rough version of this comparison can be done without a model: one simply compares the yield spread on the Treasury vs. OIS with the CIP violation. Our term structure model allows for a more careful version of this comparison. First, it allows us to consider Treasury bonds with maturities that do not exactly line up with the available points of the OIS and CIP curves. Second, it allows us to explicitly account for the residual basis risk between money market rates. Third, it allows us to model the unwinding of the trade when the bond has six months remaining maturity. Fourth, it smooths the OIS and CIP curves, reducing micro-structure-induced noise. Lastly, it accounts for covariances that are omitted from the naive spread calculation.

Our term structure model largely follows the standard approach in the no-arbitrage term structure model literature (Joslin, Singleton, and Zhu, 2011; Joslin, Priebsch, and Singleton, 2014). At first glance, this might seem strange, given that our model necessarily features arbitrage. In particular, our term structure model must match both OIS rates and CIP violations, which is equivalent to matching dollar OIS rates and synthetic dollar OIS rates (CIP violation = synthetic dollar OIS rate − dollar OIS rate). We view the CIP violations as a proxy for dealer balance sheet costs. Due to this balance sheet cost, the term structure model features two different short rates and as a result two different yield curves, as in Augustin et al. (2020). We adopt an essentially identical approach when constructing our model. We use a no-arbitrage approach (as opposed to other methods of yield curve interpolation) on the grounds that derivatives are largely unaffected by leverage and related regulations and that all empirical violations of arbitrage conditions we are aware of involve cash products in addition to derivatives.

We will use lower case symbols to denote scalars or vectors and uppercase symbols to denote matrices, and assume each time period is one month. We follow the convention that all rates and yields are expressed at yearly frequency, so we will scale them by 1/12 to obtain the monthly yield. Let z_t denote a vector of N risk factors (our empirical exercise will have $N = 5$) for our term structure model. We assume that, under the physical (actual) probability measure $\mathbb{P}$, z_t follows a Gaussian AR(1) process,

$$z_{t+1} = k^{\mathbb{P}}_{0,z} + K^{\mathbb{P}}_{1,z} \cdot z_t + (\Sigma_z)^{1/2} \varepsilon^{\mathbb{P}}_{z,t+1}, \varepsilon^{\mathbb{P}}_{z,t+1} \sim N(0, I_N), \tag{15}$$

where I_N is the $N \times N$ identify matrix, $k^{\mathbb{P}}_{0,z}$ is an $N \times 1$ vector of constants, $K^{\mathbb{P}}_{1,z}$ is an $N \times N$ matrix, and Σ_z is a symmetric positive semi-definite $N \times N$ matrix. The intermediary's log stochastic discount factor that prices derivatives is

$$m_{t+1} = -(\delta_0 + \delta_1^T \cdot z_t) - \frac{1}{2}\lambda_t^T \lambda_t + \lambda_t^T \varepsilon_{z,t+1}^{\mathbb{P}}, \tag{16}$$

where $\lambda_t = (\Sigma_z^{-1})(\lambda_0 + \Lambda_1 z_t)$ is the price of risk associated with each shock. We will assume that the profits of derivatives trades are discounted using the OIS curve, consistent with industry practice.[9] That is, $r_t^{ois} = \delta_0 + \delta_1^T \cdot z_t$, where r_t^{ois} is the log of the annualized fixed rate associated with a one-month overnight index swap.

This standard specification leads to a risk-neutral ($\mathbb{Q}$) measure dynamics for the state vector z_t,

$$z_{t+1} = k_{0,z}^{\mathbb{Q}} + K_{1,z}^{\mathbb{Q}} \cdot z_t + (\Sigma_z)^{1/2} \varepsilon_{z,t+1}^{\mathbb{Q}}, \varepsilon_{z,t+1}^{\mathbb{Q}} \sim N(0, I_N), \tag{17}$$

where the parameters $k_{0,z}^{\mathbb{Q}}$ and $K_{1,z}^{\mathbb{Q}}$ are functions of the physical measure parameters and the SDF parameters. It also leads zero coupon-swap rates that are affine in the state vector,

$$r_{n,t}^{ois} = -\frac{12}{n} \ln(E_t^{\mathbb{Q}}[\exp(\sum_{j=0}^{n-1} -\frac{1}{12} r_{t+j}^{ois})]) = a_n^{ois} + (b_n^{ois})^T z_t, \tag{18}$$

where $r_{n,t}^{ois}$ denotes log of the annual fixed rate associated with an n-month OIS swap.

The one-month log CIP violation is likewise assumed to be affine,

$$r_t^{syn} = \hat{\delta}_0 + \hat{\delta}_1 z_t, \tag{19}$$

which generates the recursion

$$r_{n,t}^{syn} = -\frac{12}{n} \ln(E_t^{\mathbb{Q}}[\exp(\sum_{j=0}^{n-1} -\frac{1}{12} r_{t+j}^{syn})]) = a_n^{syn} + (b_n^{syn})^T z_t. \tag{20}$$

We assume that the rates $x_t = (x_{1,t}, x_{2,t}, \frac{1}{12} y_t^{bill})$ are affine functions of our state variable, where $x_{1,t}$ and $x_{2,t}$ are the discount rates associated with our net short and net long curves, and y_t^{bill} is the

[9] The choice to use OIS rather than some other discount rate does not substantially impact our results, as we use the SDF only to price zero-NPV derivatives, whose value is not sensitive to shifts in the level of the discount rate.

log annualized yield on a six-month Treasury bill. We assume that

$$x_t = \gamma_0 + \Gamma_1 z_t + (\Sigma_x)^{\frac{1}{2}} \varepsilon_{x,t}, \varepsilon_{x,t} \sim N(0, I_3). \tag{21}$$

These additional variables can be thought of as akin to the "macro factors" often included in standard term-structure models. The key assumption is that the measurement errors $\varepsilon_{x,t}$ do not enter the SDF (and hence have the same distribution under P and $\mathbb{Q}$). The coefficients γ_0 and Γ_1 can be identified from regressions of these factors on the state variables.[10]

Finally, we assume that the dealer unwinds their position when the remaining maturity reaches six months, at which point the Treasury bond is equivalent to a Treasury bill. We make this assumption to capture the empirical observation that, pre-GFC, bill yields fell below other short-term risk-free rates such as tri-party repo rates (see e.g. Nagel (2016)). Incorporating this effect is important when constructing bounds on a one-year bond (which will become equivalent to a bill relatively soon) but has a minimal effect on long-maturity bonds. The curves we construct are thus

$$e^{-ny^l_{n,t}} = E^{\mathbb{Q}}_t[\exp(-\sum_{j=0}^{n-7} x_{1,t+j}) \exp(-\frac{6}{12} y^{bill}_{t+n-6})], \tag{22}$$

$$e^{-ny^s_{n,t}} = E^{\mathbb{Q}}_t[\exp(-\sum_{j=0}^{n-7} x_{2,t+j}) \exp(-\frac{6}{12} y^{bill}_{t+n-6})]. \tag{23}$$

Under the assumptions of our term structure model, these curves are also affine in the state variables.

2.5 Model Estimations and Predictions

We obtain data from various public sources, as documented in appendix section C. We then estimate the term structure model to fit dollar OIS rates and synthetic dollar OIS rates, using a standard

[10] The tri-party repo and secured lending rates are overnight rates. Given data limitations, we use overnight tri-party repo rates and overnight security lending rates to construct x_t. The 1-month CIP basis data are available, but to avoid the quarter-end effect (Du et al., 2018b), we instead use the 3-month CIP basis to obtain the synthetic rate in x_t. Our estimation reveals that there are unit-root elements in the z_t process. A more sophisticated approach is to restrict that the spreads $x_{1,t} - r^{ois}_t - r^{cip}_t$, $x_{2,t} - r^{ois}_t + r^{cip}_t$, and $x_{2,t} - r^{ois}_t$ are stationary, i.e., zero loadings on the unit-root element. Our main approach is the direct regression of x_t on z_t, but we show in the appendix that results are similar if we impose stationarity on the spreads. See appendix section E for more details.

maximum likelihood approach. The goal of our estimation procedure is to accurately fit and interpolate these curves. Figures 8 and 9 illustrate the fit of our model. For details on the estimation procedure and on related issues such as coupon vs. zero-coupon bonds, and for details related to see appendix section E.6.

In Figure 10, we show the model-implied net long and net short curves, in comparison with the Treasury yield curve. In Figure 11, we subtract matched-maturity OIS rates from all the yield curves in Figure 10.

Several patterns are immediately apparent. Prior to the GFC, yields for one, three, five, and ten-year bonds were often close to the net short curve, consistent with the net position data. In contrast, twenty-year maturity bonds are often close to the net long curve. Our model therefore predicts that dealers would be short intermediate-maturity bonds and long longer-maturity bonds. We validate this prediction in more detailed position data below. We should also note that, because there are substantially more intermediate-maturity than long-maturity bonds outstanding, this pattern naturally generates an overall net short position.[11] After the GFC, all yields for bonds of one-year-maturity or greater are close to the net long yield, suggesting that dealers will be long coupon bonds. This is again consistent with the position data.

Note that the six-month yield is constructed by regressing bill yields on the factors of our term structure model, and by assumption the net long and net short curves are identical at this maturity. We include it in these graphs to illustrate that, for the most part, our model captures the fluctuations in the bill-OIS spread. These fluctuations play an important role in the movements of the net long and net short curves at the one-year maturity point; they play a relatively minimal role for longer maturities. Intuitively, movements in the six-month bill-OIS spread are amortized over longer maturities and hence have only small effects on longer-maturity yields.

We next compare Treasury yields relative to the net short and net long curves to position data for specific bond maturity buckets. As discussed above, a bond's yield relative to the net short and net long curve bounds serves as a heuristic proxy for dealer positions. We define a "relative yield index" by

$$pos_{n,t} = 2 * \frac{y_{n,t} - y^{s}_{n,t}}{y^{l}_{n,t} - y^{s}_{n,t}} - 1. \tag{24}$$

This index takes on a value of one if the yield of the n-month maturity Treasury bond is equal to

[11] Our model has no specific predictions about how dealers should allocate their long/short positions across various arbitrages. In making this argument, we are assuming that, all else equal, larger markets lead to larger positions.

the net long yield, negative one if it is equal to the net short yield, and zero if it is equal to the average of the net long and net short yields.

We then plot this relative yield index against the net dealer position by maturity bucket. We obtain the net primary dealer coupon Treasury bond position in maturity buckets of <3 years, 3-6 years, 6-11 years, and >11 years from the FR2004 primary dealer statistics published by the New York Fed. We then normalize each of these by the total assets of primary dealers, and plot them with the relative yield index at the 2-year, 5-year, 10-year, and 20-year maturities. The bond position and relative yield index are plotted on different axes (because they are not in comparable units), with the zero points on each axis aligned.

For a variety of reasons, we do not expect these series to perfectly align. First, as discussed above, the mapping between how close a yield is to the net long or net short curves and the predicted net dealer position in that maturity is unclear, and may change over time. Second, the arbitrage bounds we construct are motivated by trading strategies that (at least potentially) hold the bond almost to maturity. Dealers also intermediate bonds between clients, and may be willing to buy a bond for the purpose of selling it quickly even if they view the bond as overpriced (close to the net short yield). This kind of intermediation activity acts as a kind of noise in the relationship between net dealer positions and the relative yield index, and is likely accentuated when looking at specific maturity buckets as opposed to overall net dealer positions. Despite these caveats, there is a non-trivial correspondence between the relative yield index and actual positions, as shown in Figure 12. We also construct a weighted average version of the relative yield index, where the weight for each maturity is the fraction of dealer Treasury bond holding at that maturity over total dealer Treasury bond holding. Then we plot this aggregate relative yield index together with total dealer Treasury holding scaled by dealer balance sheet size, as shown in Figure 13.

Summarizing our analysis thus far, Treasury yields have moved from being close to net short arbitrage bounds pre-GFC to being close to net long arbitrage bounds post-GFC, and net primary dealer positions have responded by switching from being net short to net long. Strikingly, the net short and net long curves are constructed by assuming that dealers will remain net short or long going forward. Our results therefore imply that, pre-GFC, dealers were expected by the market to maintain a net short position, and that following the GFC, this expectation flipped and the market now anticipates dealers maintaining a net long position going forward. The relationship between yields and positions we document is consistent with the view that balance-sheet-constrained dealers act as arbitrageurs between the Treasury and swap markets.[12] We next consider the implications of this perspective, with an emphasis on the causes and consequences of the regime shift we have documented.

3 A Model of the Treasury Market

In this section, we will argue that the shift of dealer net positions from short to long has consequences for the price dynamics of the Treasury market. We do so through the lens of a static supply-and-demand model. This model imposes assumptions for the purpose of generating sharp predictions on the role of state-contingency in the Treasury market. In particular, it incorporates a key feedback effect: as dealers and their levered clients expand the size of their short or long positions, the opportunity cost of using their balance sheet increases.

3.1 Model Setup

The model has two dates (zero and one), and a single n-period Treasury bond. Date one exists only for the purpose of determining payoffs; all of our analysis will focus on date zero, and we will omit time subscripts for all date zero rates and yields. We will take as exogenous the date zero log interest rates y^{bill}, r^{tri}, r^{sec}, and r^{ois}, as well as two different expectations concerned future bond prices. We define the dealer's date zero risk-neutral ($\mathbb{Q}$-measure) expectation of date one bond prices as

$$\begin{aligned} p_{\mathbb{Q}} \equiv \exp(-(n-1)y_{\mathbb{Q}}) &\equiv E^{\mathbb{Q}}[\exp(-(n-1)y_{n-1,1})] \\ &= E^{\mathbb{Q}}[\exp(-(n-1)r^{ois}_{n-1,1} + (n-1)(r^{ois}_{n-1,1} - y_{n-1,1}))] \end{aligned} \tag{25}$$

where $r^{ois}_{n-1,1}$ and $y_{n-1,1}$ denote the $(n-1)$-period OIS swap rates and Treasury bond yields at date one, and $y_{\mathbb{Q}}$ denotes the risk-neutral expectation-implied yield at date zero. We define the corresponding physical measure ($\mathbb{P}$) counterpart as $y_{\mathbb{P}}$, so the physical-measure expected future bond price is $\exp(-(n-1)y_{\mathbb{P}})$.

[12] We interpret our results as showing that Treasury yields are often at or near arbitrage bounds given swap prices. However, one could equally say that swap yields are at or near arbitrage bounds given Treasury yields, adopting the perspective of Hanson et al. (2022).

We have written the definition of $y_{\mathbb{Q}}$ in this way to highlight the possible interpretations of comparative statics with respect to $y_{\mathbb{Q}}$. One interpretation, which we emphasize, is that a decrease in $y_{\mathbb{Q}}$ represents a change the OIS curve holding constant the risk-neutral expectation of future swap-Treasury spreads. An equally valid interpretation is as a change in the risk-neutral expectation of future swap-Treasury spreads holding the OIS swap curve constant. It is important to distinguish between comparative statics that hold $y_{\mathbb{P}}$ constant and comparative statics that change both $y_{\mathbb{P}}$ and $y_{\mathbb{Q}}$. The first of these represents a change in risk premia, that latter a change in expected future rates.

These interest rates and expected bond prices will allow us to compute the dealer's net short and net long curves. The key endogenous variables are y, the yield of an n-period zero-coupon bond at date zero, and r^{syn}, the one-period synthetic unsecured risk-free rate at date zero. We focus on the following asset prices that are closely related to the empirical motivations in Figure 1.

- The Treasury term spread, $y - y^{bill}$.
- The synthetic–OIS spread, $r^{syn} - r^{ois}$, which maps to CIP deviations.
- The n-period swap spread $r_n^{ois} - y$, where r_n^{ois} is the date-zero n-period OIS rate,

$$\exp(-nr_n^{ois}) \equiv E^{\mathbb{Q}}[\exp(-r^{ois} - (n-1)r_{n-1,1}^{ois})] \tag{26}$$

- The OIS term spread, $r_n^{ois} - r^{ois}$, which contains both an expectation component and a risk premium component.

We will treat dealers and their levered clients as a single consolidated entity. In the Treasury market, dealers will interact with two kinds of investors. Hedged investors purchase the Treasury bond and swap it to their local currency. Unhedged investors choose between the Treasury bond and Treasury bills. Dealers also have other counterparties in the synthetic lending market; these other counterparties do not participate in the Treasury market. This structure implicitly assumes that the tri-party repo, bill, and interest rate swap markets are infinitely elastic, whereas the Treasury and synthetic borrowing markets are elastic but not infinitely so; we make these assumptions to simplify our exposition.

Our model of dealers is exactly that of section 2.1, with $p_{\mathbb{Q}}$ defined as in (25). Recall that q^{syn} is the quantity (in dollars) of synthetic loans supplied by the dealers at date zero, and q^{bond} be the quantity (in dollars) of bonds owned (positive) or short-sold (negative), and that the dealer balance sheet constraint is $q^{syn} + |q^{bond}| = \bar{q}$.

Let S^{bond} be the supply of bonds (in notional quantities) at date zero, and let D_U^{bond} and D_H^{bond} be the demand (in dollars) for bonds from unhedged and hedged investors, respectively. Market clearing in the bond market requires

$$q^{bond} + D_U^{bond} + D_H^{bond} = \exp(-ny) S^{bond}, \tag{27}$$

the price $\exp(-ny)$ enters this expression to convert the notional supply S^{bond} into dollars.

Unhedged investor demand is a continuously differentiable, strictly positive and increasing function of the expected log excess return of the bond over bills:

$$D_U^{bond} = D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}). \tag{28}$$

Likewise, hedged investor demand is a continuously differentiable, strictly positive and increasing function of the expected log excess return in dollars[13] on the hedged strategy:

$$D_H^{bond} = D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}). \tag{29}$$

When a dealer helps a hedged investor exchange e.g. yen into dollars and hedge using forwards, the dealer will end up with a yen asset (cash) and a dollar liability. As a result, this activity increases the dealer's balance sheet, and is functionally equivalent, from the dealer's perspective, to lending synthetic dollars.

Dealers can also lend synthetic dollars to other counterparties (hedged investors buying corporate bonds, for example). We assume that the demands of these other investors for synthetic dollars are $D^{syn}(r^{syn} - r^{ois})$, where D^{syn} is a continuously differentiable, non-negative and strictly decreasing function of the spread between synthetic dollars and risk-free rates. Market clearing in the synthetic dollar market requires that

$$q^{syn} = D_H^{bond} + D^{syn}(r^{syn} - r^{ois}). \tag{30}$$

[13]For simplicity, we use the hedged return in dollars, as opposed to in local currency; this allows us to ignore second-order terms associated with the covariance between interest rates and exchange rates.

These market clearing conditions, together with the net long and net short inequalities in our simple model (equations (3) and (4)) and the associated implications for dealer positions, define our model.

To guarantee that an equilibrium exists in our model, we need interior solutions for (y, r^{syn}) to satisfy the two market clearing conditions in (27) and (30). Thus, we make the following technical assumptions.

Assumption 1. *We assume that the demand functions are well-behaved:*

- *Excess synthetic loan demand is possible:* $D^{syn}(0) > \bar{q}$*.*
- *Excess synthetic loan supply is possible: For all y,* $\lim_{r^{syn} \longrightarrow \infty} D^{syn}(r^{syn} - r^{ois}) + D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) = 0$*.*
- *Excess bond supply is possible:* $\lim_{y \longrightarrow -\infty} D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) + D_H(ny - r^{ois} - (n-1)y_{\mathbb{P}}) = 0$*.*

Note that excess bond demand is also possible without the need to impose additional assumptions. This is because if we take $y \to \infty$, the right hand side of (27) becomes zero while the left hand side is strictly positive, causing an excess bond demand. We will show that with Assumption 1, the equilibrium solution to the model exists and is unique.

Depending on the sign of q^{bond}, our static model features three possible kinds of equilibria, which we refer to as regimes: a long regime $q^{bond} > 0$, a short regime $q^{bond} < 0$, and an intermediate regime $q^{bond} = 0$. We discuss each of these regimes in turn.

Our focus, when analyzing these regimes, will be on the spread between synthetic dollars and the OIS rate, $r^{syn} - r^{ois}$, which is endogenously determined. In our quantitative analysis, this spread was a key input to the model, and we measured it with CIP violations. In this two-market market, the rate r^{syn} should be understood as the risk-free return the dealer requires for assets held on balance sheet, as opposed to specifically a euro rate swapped to dollars. Under this interpretation, the spread $r^{syn} - r^{ois}$ is the kind of financial intermediation spread that plays a key role in macro-finance models (Brunnermeier and Pedersen, 2009; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2010). For this reason, we emphasize how this financial intermediation spread responds to shocks.

3.2 The Long Regime

We first consider a regime in which dealers are long ($q^{bond} > 0$), in which case $e^{-ny} = e^{-ny^l}$, where the net-long yield y^l is defined in (3) and $p_{\mathbb{Q}}$ can be expressed in terms of $y_{\mathbb{Q}}$ as in (25). Thus,

$$e^{-ny} = \frac{e^{-(n-1)y_{\mathbb{Q}}}}{(1-h)(e^{r^{tri}} - e^{r^{ois}}) + e^{r^{syn}}}. \tag{31}$$

Equation (31) generates a dealer indifference condition between the two endogenous variables r^{syn} and y. This indifference condition suggests that r^{syn} strictly increases with y. Intuitively, the more attractive it is to buy the Treasury bond and hedge with swaps (higher y), the higher the return on synthetic lending (r^{syn}) must be to generate indifference between these two activities.

Combining the market clearing conditions in (27) and (30), the intermediary balance sheet constraint in (1), and using $q^{bond} > 0$, we have

$$\bar{q} - e^{-ny} S^{bond} + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) = D^{syn}(r^{syn} - r^{ois}). \tag{32}$$

The left-hand side of (32) represents the residual balance sheet available for synthetic lending, which in equilibrium must equal the residual demand for synthetic lending. Note that the demand from hedged investors does not appear in this equation, because the dealer balance sheet is unaffected by changes in their demand, holding all else constant.[14] The left-hand side of (32) is strictly increasing in y, while the right-hand side is strictly decreasing in r^{syn}. Therefore, equation (32) generates a kind of market indifference condition, where r^{syn} strictly decreases with y. Intuitively, higher yields lead to more investor demand for bonds, which reduces the balance sheet dealers allocate to bonds, thereby increasing the balance sheet allocated to synthetic dollar lending and reducing the synthetic dollar lending spread.

A long-regime equilibrium (y, r^{syn}) is a point where these two indifference curves intersect and $q^{bond} > 0$, which requires

$$D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) < e^{-ny} S^{bond}. \tag{33}$$

[14] An increase in hedged investor demand reduces the quantity of Treasury bonds dealers must hold, but increases the amount of synthetic borrowing they must finance, and hence has no effect on their balance sheet usage, provided that dealers have a net long bond position.

Because the two indifference curves have opposite slopes, such an equilibrium is unique if it exists.

We next consider various comparative statics associated with a long-regime equilibrium: an increase in bond supply (S^{bond}), dealer balance sheet ($\bar{q}$), an increase in unhedged bond demand (a parallel increase in D_U), an increase in hedged bond demand (a parallel increase in D_H), an increase in the swap market term premium (an increase in $y_{\mathbb{Q}}$ holding $y_{\mathbb{P}}$ constant), and an increase in expected future rates (a parallel increase in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$). The following proposition summarizes our results.

Proposition 1. *In a long regime equilibrium, holding all else constant,*

1. *An increase in S^{bond} leads to an increase in y and an increase in r^{syn},*
2. *A decrease in $\bar{q}$ or a parallel decrease in D_U is equivalent to a expansion of the same size in the dollar supply of bonds.*
3. *A parallel increase in D_H does not change either y or r^{syn},*
4. *An increase in $y_{\mathbb{Q}}$ leads to an increase in y and a decrease in r^{syn},*
5. *An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ leads to an increase in y of less than $\frac{n-1}{n}dy$ and a decrease in r^{syn}.*
6. *A parallel increase in $D^{syn}(\cdot)$ increases both y and r^{syn}.*

Proof. See appendix section F.1. □

An increase in bond supply means that, holding yields constant, less dealer balance sheet is available for synthetic lending. As a result, the market indifference curve increases for each y, which leads to an increase in both synthetic lending spreads and bond yields in equilibrium. A decrease in either dealer balance sheet capacity or in unhedged bond demand is equivalent to an increase in the supply of bonds, and hence generates the same comparative statics. In contrast, an increase in hedged demand has no effect on the equilibrium, because this demand has no net effect on dealer balance sheet constraints; it merely transforms dealer bond holdings into dealer synthetic lending.

An increase in $y_{\mathbb{Q}}$ holding $y_{\mathbb{P}}$ constant is equivalent to an increase in forward swap rates holding expected future swap rates constant, and hence to an increase the swap term premium. Such a change makes owning the Treasury bond and hedging with swaps less attractive to dealers, and as a result the dealer indifference curve decreases for each y. This in turn leads to an increase in y and a decrease in r^{syn}.

An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ represents an increase in expected future rates holding the swap term premium constant. Holding fixed the dollar supply of bonds, such a change would have no effect on r^{syn} and would lead to an increase in y of $\frac{n-1}{n}dy$. However, because it is the notional and not dollar supply of bonds that is held constant, an increase in yields generates a contraction in the dollar supply of bonds. This contraction, in the long regime, has the effect of reducing synthetic lending spreads and dampening the increase in bond yields.

The two indifference curves derived from (31) and (32), and the first and fourth comparative statics in Proposition 1 are illustrated in Figure 14. The market indifference curves are truncated when yields are sufficiently high; this truncation highlights that the long regime equilibrium ceases to exist when client demand exceeds the bond supply.

3.3 The Short Regime

We next consider a regime in which dealers are short ($q^{bond} < 0$). In this regime, the bond yield must exactly equal to the net short yield, $e^{-ny} = e^{-ny^s}$, where the short yield y^s is defined in (4). Expressing $p_{\mathbb{Q}}$ with $y_{\mathbb{Q}}$ as in (25), we obtain

$$e^{-ny} = \frac{e^{-(n-1)y_{\mathbb{Q}}}}{e^{r^{sec}} + e^{r^{ois}} - e^{r^{syn}}} \tag{34}$$

Equation (34) generates a dealer indifference condition, where r^{syn} strictly decreases with y. Intuitively, the less attractive it is to sell the Treasury bond and hedge with swaps (higher y), the lower the return on synthetic lending (r^{syn}) must be to generate indifference between these two activities. This relationship has the opposite sign compared to the long regime.

Combining the market clearing conditions in (27) and (30), balance sheet constraint (1), and using $q^{bond} < 0$, we obtain

$$\bar{q} + e^{-ny}S^{bond} - D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) - 2D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) = D^{syn}(r^{syn} - r^{ois}). \tag{35}$$

The left-hand side of (35) again represents the residual balance sheet available for synthetic lending, which in equilibrium must equal the residual demand for synthetic lending. Note that demand from hedged investors has a double impact on the dealer's balance sheet. All else equal, an increase in this demand will result in dealers taking larger short positions and providing more synthetic financing to hedged investors, both of which use up dealer balance sheet.

The left-hand side of (35) is strictly decreasing in y: unlike the long regime, more demand from investors and less supply require dealers to take larger short positions, using up more balance sheet. Equation (35) therefore generates a kind of market indifference condition, where r^{syn} strictly increases in y.

A short-regime equilibrium (y, r^{syn}) is a point where these two indifference curves intersect and $q^{bond} < 0$, which requires

$$D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) > e^{-ny} S^{bond}. \tag{36}$$

Because the two indifference curves have opposite slopes, such an equilibrium is again unique if it exists.

Our next proposition considers the same set of comparative statics studied previously in the context of the short regime.

Proposition 2. *In a short regime equilibrium, holding all else constant,*

1. *An increase in S^{bond} leads to an increase in y and a decrease in r^{syn};*
2. *An increase in $\bar{q}$ or a parallel decrease in D_U is equivalent to an expansion of the same size in the dollar supply of bonds;*
3. *A parallel increase in D_H leads to a decrease in y and an increase in r^{syn};*
4. *An increase in $y_{\mathbb{Q}}$ leads to an increase in y and an increase in r^{syn};*
5. *An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ leads to an increase in y by less than $\frac{n-1}{n}dy$ and an increase in r^{syn}.*
6. *A parallel increase in $D^{syn}(\cdot)$ increases r^{syn} and decreases y.*

Proof. See appendix section F.2. □

An increase in bond supply in the short regime increases yields (like the long regime) but decreases synthetic lending spreads (unlike the long regime). In the short regime, the larger the supply of bonds the smaller the dealer's required short position, and hence more balance sheet is available for synthetic lending.

Like the long regime, a decrease in unhedged bond demand is equivalent to an increase in bond supply. Unlike the long regime, an increase in dealer balance sheet capacity is equivalent to an increase in supply, because dealers are short bonds instead of being long bonds. Also unlike the long regime, an increase in hedged bond demand leads to a decrease in yields and an increase in synthetic lending spreads; it is equivalent to a contraction in the dollar supply of bonds of twice the magnitude of the demand increase.

An increase in the swap curve term premium makes shorting bonds and hedging with swaps more attractive. The dealer therefore requires a higher synthetic lending spread to be indifferent between synthetic lending and shorting Treasury bonds, which leads in equilibrium to higher yields and higher synthetic lending spreads (the opposite of the long regime).

Figure 15 illustrates the dealer indifference curve (34), the market indifference curve (35), and the first and fourth comparative statics discussed in Proposition 2.

3.4 The Intermediate Regime

The last regime we consider is one in which $q^{bond} = 0$.[15] In this regime, the yield must fall between the net short and net long yields,

$$y^s \leq y \leq y^l, \tag{37}$$

the bond market must clear without dealers taking a position,

$$D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) = e^{-ny}S^{bond}, \tag{38}$$

and the dealer balance sheet constraint is reduced to $q^{syn} = \bar{q}$. Equating q^{syn} with synthetic lending demand, we obtain

$$\bar{q} = D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) + D^{syn}(r^{syn} - r^{ois}). \tag{39}$$

[15] In position data, dealers will never have an exactly zero net Treasury position, for reasons (for example, intermediation activities) that are outside the scope of our model. We view the intermediate regime as describing a situation in which dealers are targeting a roughly net flat position.

The following proposition summarizes the comparative statics in this case. We restrict attention to interior intermediate equilibria, for which $y^s < y < y^l$, and discuss the determinants of regime boundaries below.

Proposition 3. *If an interior intermediate regime exists, it is the only such equilibrium. In an interior intermediate regime equilibrium, holding all else constant,*

1. *An increase in S^{bond} leads to an increase in y and an increase in r^{syn};*
2. *An increase in $\bar{q}$ or a parallel increase in D_U leads to a decrease in both y and r^{syn};*
3. *A parallel increase in D_H leads to a decrease in y and an increase in r^{syn};*
4. *An increase in $y_{\mathbb{Q}}$ leaves both y and r^{syn} unchanged;*
5. *An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ leads to an increase in y by less than $\frac{n-1}{n}dy$ and a decrease in r^{syn}.*
6. *A parallel increase in $D^{syn}(\cdot)$ increases both r^{syn} and y.*

Proof. See appendix section F.3. □

When dealers are not active in the Treasury market, increases in supply lead to higher yields as a consequence of clients demanding higher expected returns in exchange for holding larger positions. Some of these clients are hedged clients, whose increase position size requires additional synthetic dollar financing from dealers, reducing those dealer's ability to lend to other synthetic dollar clients, which results in increasing synthetic lending rates.

As usual, an increase in unhedged demand is equivalent to a decrease in supply. However, the comparative statics with respect to balance sheet and hedged demand are unlike either the long or short regime; both effects are driven by the role of the hedged demand in the Treasury market.

Unlike either the long or the short regime, in the intermediate regime the OIS term premium is disconnected from Treasury yields. Dealers are not actively arbitraging bonds and swaps, and neither type of client trades swaps; as a result, changes in the swap market term premium do not affect the Treasury market. In contrast, increases in expected yields lead to higher yields, exactly as in both the long and the short regime.

3.5 Equilibrium

Let us next consider the factors that determine which of three regimes occur in equilibrium. The following proposition summarizes how some of the comparative statics discussed thus far can change the equilibrium regime.

Proposition 4. *The equilibrium exists and is unique given the exogenous parameters. Holding all else constant,*

1. *There exists an $0 \leq S_S \leq S_B \leq \infty$ such that a short regime equilibrium exists for all $S^{bond} < S_S$, a long regime equilibrium exists for all $S^{bond} > S_B$, and an intermediate regime equilibrium exists for $S^{bond} \in [S_S, S_B]$.*

2. *There exists an $0 \leq y_{Q,S} \leq y_{Q,B} \leq \infty$ such that a short regime equilibrium exists for all $y_{\mathbb{Q}} < y_{Q,S}$, a long regime equilibrium exists for all $y_{\mathbb{Q}} > y_{Q,B}$, and an intermediate regime equilibrium exists for $y_{\mathbb{Q}} \in [y_{Q,S}, y_{Q,B}]$.*

Proof. See appendix section F.4. □

Intuitively, when bonds are scarce, dealers will be short to meet client demands, while when bonds are abundant dealers will be long to fill the shortfall in client demand.

Less intuitively, when the yield curve is steep dealers will be short bonds. Considering the dealer's bond position in isolation, this looks like a money-losing strategy: expected returns are higher when the yield curve is steeper. However, if the swap curve has a higher term premium than the Treasury curve and the dealer is hedging with swaps, then selling bonds and hedging is in fact a profitable strategy. Similarly, buying bonds when the curve is flat looks like a money-losing strategy, but is in fact profitable if the swap curve is even flatter and the dealer hedges. In our model, client demand for Treasury bonds is driven by the expected returns on those bonds; spreads must therefore move in a way that induces dealers to take the opposite position.

Figure 16 illustrates the comparative statics of the model with respect to changes in the OIS curve term premium. The swap-Treasury spread and Treasury term premium are both increasing functions of the swap term premium, but the rate at which they increase depends on the regime. In contrast, the synthetic lending spread is U-shaped: it is high in both the long and short regimes, and low in the intermediate regime.

3.6 Model vs. Data

We now return to the motivating facts described in the introduction and discuss them through the lens of our model. We divide our discussion between the pre-GFC and post-GFC periods. In our view, there are three key differences between these two periods:

- In the pre-GFC period, dealers were able to use large amounts of leverage; in the post-GFC period, leverage constraints were tight. The tightening of leverage constraints was part of regulatory response to the financial crisis, and is discussed by a number of authors, such as Duffie (2017).
- In the pre-GFC period, the supply of Treasury bonds was much smaller than in the post-GFC period. The U.S. government borrowed a large amount during the financial crisis and continued running substantial deficits in the years that followed. The outstanding of marketable Treasury securities grew from $4.7 trillion in 2008 to $22.5 trillion today.
- In the pre-GFC period, short maturity Treasury yields were significantly lower than tri-party repo rates, whereas post-GFC, the two rates are similar. We believe this is a consequence of the Federal Reserve paying interest on excess reserves. In the pre-GFC period, banks were significant holders of Treasury bills at rates well below tri-party repo rates. After the Fed began paying interest on reserves at rates higher than tri-party repo rates, banks substantially reduced their bill ownership, and government-only money market funds substantially increased their bill ownership. Such funds can invest in both bills and tri-party repo; as result, bill yields rose to roughly the level of tri-party repo rates.

There are of course many other differences between the pre- and post-GFC periods. There are also relevant factors in the model (in particular, the shape of the demand curves of clients) that may have changed between these period and about which we have little information. However, we will show using the comparative statics just described that these three facts can explain the stylized facts discussed in the introduction.

To illustrate this point, in Figure 17, we replicate Figure 16, with a much higher balance sheet capacity for dealers, large spreads between bill and tri-party repo rates, and a relatively small supply of Treasury bonds, as a way of capturing the pre-GFC period.

In Figure 17, the equilibrium involves dealers taking a short bond position given a positive OIS term premium. This result is driven primarily by low Treasury supply, which leads (all else equal) to excess client demand. Because balance sheet capacity is large, synthetic dollar lending spreads are small, and there is little relationship between these spreads and swap-Treasury spreads. All of this is consistent with the evidence shown in Figure 1 for the pre-GFC period.

Moreover, the model predicts a large, positive swap-Treasury spread in this environment, for two reasons. First, as a consequence of dealers taking short positions, dealers will borrow at secured lending rates, which are low even relative to repo rates, and therefore require particularly low Treasury yields to justify their short position. Second, because short maturity Treasury yields are substantially below tri-party rates, which are themselves below OIS rates, expectations of date one short-maturity swap spreads are high, which has the effect of increasing date zero swap spreads.

Let us now consider instead the post-GFC period. In this period, Treasury bond supply is high, balance sheet constraints are tight, and short maturity Treasury yields are close to tri-party repo rates. We again replicate Figure 16 in Figure 18, but with a low balance sheet capacity for dealers, no spread between bill and tri-party repo rates, and a relatively large supply of Treasury bonds, as a way of capturing the post-GFC period.

In Figure 18, the equilibrium involves dealers taking a long bond position for realistic levels of the OIS term premium, a result driven primarily by the large Treasury supply. Because balance sheet capacity is constrained, synthetic dollar lending spreads can be large and are negatively correlated with both swap-Treasury spreads and the slope of the OIS term structure.[16] This is consistent with the evidence shown in Figure 1 for the post-GFC period.

Moreover, the model now predicts negative swap spreads during periods in which OIS term structure is flat, again consistent with the post-GFC behavior of swap spreads. This result comes about for several reasons. First, because dealers are long as opposed to short, the relevant financing rate is the tri-party repo rate as opposed to the security lending rate, and hence higher. This results in a higher bond yield, all else equal. However, the tri-party repo rate is still below OIS rates, and as a result the swap spread will be positive when synthetic lending spreads and dealer bond positions are small. But when synthetic lending spreads and dealer bond positions are large, the additional opportunity cost of forgone synthetic lending will be capitalized into the bond price, resulting in bond yields that exceed swap rates.

[16] Here are assuming that the slope of the OIS term structure reflects a mix of term premium and expectations shocks.

3.7 Regimes and Treasury Market Fragility

During the financial crisis of 2008-2009, Treasury yields fell by more than matched maturity OIS swap rates. During the COVID-induced financial turmoil of March 2020, the reverse was true: Treasury yields did not fall by as much as OIS swap rates, and in fact briefly rose (Duffie, 2020; Haddad, Moreira, and Muir, 2021; He, Nagel, and Song, 2022). The different comparative statics across the long and short regimes in our model offer an explanation for this pattern.

In the short regime, an increase in balance sheet costs (as measured by the spread $r^{syn} - r^{ois}$), all else equal, will lead to lower Treasury yields. In contrast, in the long regime, an increase in balance sheet costs will lead, again all else equal, to higher Treasury yields. Both crises were characterized by large increases in arbitrage spreads; the difference was that the market was in the short regime pre-GFC and in the long regime post-GFC.

He, Nagel, and Song (2022) attribute the differences between these two episodes to client demand for Treasury bonds (a dash-for-cash in COVID, a flight-to-safety in the GFC). Our story is compatible with theirs, in the sense that Treasury selling in COVID would increase balance sheet costs (the long regime) and Treasury buying in the GFC would also increase balance sheet costs (the short regime). However, our story does not rely customer demand for Treasury bonds as the causal factor behind the increase in balance sheet costs. In both the GFC and COVID episodes, even if clients had not bought or sold Treasury bonds on net, we expect that balance sheet costs would have risen, and as a result predict that Treasury yields would have moved relative to swap rates upwards in COVID but downwards in the GFC. Quantifying the role of Treasury demand, as opposed to other forces, in explaining the tightening of balance sheet constraints in these episodes is an interesting direction for future research.

4 Implications for Policy

In this section we consider a variety of Federal Reserve policies, and study how the effects of those policies depend on the regimes we have identified in the Treasury market. The specific policies we consider are interest rate policies (including forward guidance), swap lines with other central banks, supplementary leverage ratio (SLR) exemptions, and quantitative easing/tightening (QE/QT). We will use the comparative statics described previously to discuss the impacts of each of these policies. Our analysis will focus on the effects of these policies on Treasury yields and on synthetic dollar rates, which, as discussed above, we view as a proxy for financial intermediation spreads more generally.

Our framework treats the term structure of OIS rates as exogenous. We first discuss of the effects of interest rate policies that changes the level and slope of the OIS curve on Treasury market dynamics. Then in our analysis of the subsequent three policies (swap lines, SLR exemptions, QE/QT), we focus our discussion on the direct quantity and balance sheet effects of these policies, without discussing the additional effects of these policies on the OIS curve and expectations of future rates (i.e. keeping $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ unchanged in our static model).

Before discussing these policies, we should emphasize that all of these policies have effects on inflation, real economic activity, and financial stability that are outside the scope of our model. Policies that increase arbitrage spreads and other financial market distortions can be justified on these grounds. However, combinations of the policies we discuss might achieve these same objectives while avoiding financial market distortions, and it is the goal of our analysis to highlight these possibilities.

4.1 Interest Rate Policy

We first consider the effects of policies that affect OIS rates and rate expectations, including both rate hikes and forward guidance. That is, we define interest rate policy as controlling the current level and future expectations of the federal funds rate (the floating rate for OIS swaps).[17] Our analysis will consider the relationship between shocks to current and future federal funds rates and shocks to Treasury yields.

The comparative statics of our model distinguish between changes in term premium (changes in $y_{\mathbb{Q}}$ holding $y_{\mathbb{P}}$ constant) and changes in expectations (equal changes in both values). Monetary policy likely changes both expectations and risk premia. Hanson and Stein (2015) and Hanson, Lucca, and Wright (2021) argue that rate hikes increase both expected future rates and term premia. There is also evidence that forward guidance affects risk premia in addition to rate expectations (e.g. Rogers, Scotti, and Wright (2018)).

To avoid taking a stand on the exact decomposition between interest rate policy and risk premia, we will describe interest rate policy in terms of the level and slope of the OIS swap curve, under

[17] This sidesteps the issue of how administered rates (such as the interest on reserves rate and ONRRP rates) transmit to the federal funds market, which is beyond the scope of our model.

the premise that a steep slope implies a high risk premium and a flat or inverted slope implies a low risk premium.

A high level of expected rates (a parallel increase in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$) is equivalent, in our model, to a contraction in the dollar supply of bonds, because the supply of bonds is assumed to fixed in notional terms. This effect is potentially offset by forces outside our model– for example, the federal government might issue more debt to cover the additional interest costs associated with high rates. For this reason, we do not emphasize it as the main effect.

Instead, we focus on the slope of the term structure and the term premium. A low term premium in our model in the long regime leads to a contraction in the client demand for Treasury bonds, a further build-up in the dealer's long position, an increase in bond yields, more negative swap spreads, and an increase in synthetic lending spreads (see part four of Proposition 1 or Figure 18). In the short regime, a low term premium instead leads to a decline in synthetic lending spreads (see part four of Proposition 2).

4.2 Central Bank Swap Lines

We next consider the policy of establishing swap lines between the Federal Reserve and other central banks. These swap lines allow foreign central banks to borrow dollars from the Federal Reserve, using their own currency as collateral. The foreign central banks then lend those dollars to their local banks, typically for the purpose of financing a position in dollar-denominated assets.

This procedure allows non-U.S. banks to borrow dollars, and is a substitute for borrowing synthetic dollars via a dealer. We therefore incorporate swap lines into our static model as equivalent to a demand shift in the synthetic lending market (a parallel shift in D^{syn}). The swap lines establish by the Federal Reserve generally have rates determined by policy. If the rate is higher than the prevailing market rate, the facility will go unused, and the equivalent demand shift is zero. If the rate is appealing, it is equivalent to a rate ceiling in the synthetic loan market, and hence to an endogenously sized decrease in the demand for synthetic dollars.[18]

Any demand decrease in the synthetic lending market will lead to reduced synthetic lending

[18] During normal times, because of stigma associated with tapping central bank liquidity facility and moral suasion from central banks discouraging banks to use swap line to fulfill their routine funding needs, the take-up in the swap line is extremely low even when the swap line rate borrowing rate is temporarily below the implied dollar rate from the FX swap market.

spreads in both the long and short regimes (see part six of Propositions 1 and 2). In the long regime, these reduced lending spreads will also lead to reduced Treasury yields (and hence swap spreads), whereas in the short regime reduced lending spreads would lead to increased Treasury yields. Again, both of these effects operate through the relaxation of balance sheet constraints, and the regime determines the relationship between balance sheet tightness and Treasury yields. In both cases, there would be an increase in the demand for Treasury bond financing (either tri-party repo or security lending). Our model assumes these rates are fixed, but in a more complex model these rates might also adjust.

We should note that, in our model, a swap line with a rate equal to the OIS rate and large capacity could drive the synthetic lending spread to zero. This would endogenously result in all of the dealer's balance sheet being allocated to the Treasury trade, and violate Assumption 1. Although we do not formally analyze this case, we should emphasize what does *not* happen: it is not the case that the balance sheet cost faced by dealers goes to zero. Instead, the CIP violation ceases to be a meaningful measure of balance sheet costs. Other financial intermediation spreads would decline (due to the relaxation of dealer balance sheet constraints), but would not go to zero.

Summarizing, in both regimes, swap lines can substitute for dealer balance sheet in the synthetic dollar market, thereby reducing synthetic lending spreads. However, swaps lines have opposite effects on Treasury yields in different regimes: it decreases Treasury yields in the long regime, but increases Treasury yields in the short regime.

4.3 Leverage Ratio Exemptions

We next consider changes to regulatory policy that involve exempting certain kinds of low-risk assets from the SLR calculation. We consider two possible exemptions: exempting Treasury bonds and repo loans against Treasury collateral (exempting Treasurys, for short), and exempting reserves. Similar policies were implemented during the most acute parts of the COVID-induced market disruptions in 2020.

Recall in our static model that we have consolidated dealers and their levered clients into a single entity, based on the analysis of appendix section A. This consolidation is based on the fact that both repo loans that dealer provide to their levered clients to hold Treasury bonds and direct holdings of Treasury bonds increase the size of the dealer's balance sheet. For this reason, it is simpler to consider a policy that exempts both repo loans against Treasury bonds and Treasury bonds directly owned by dealers.[19]

Exempting Treasurys will both free up dealer balance sheet capacity for synthetic lending and remove the need to reduce CIP arbitrage activity when taking a net position in Treasury bonds. This will lead to Treasury yields that are a function only of financing rates (r^{tri} in the long regime, r^{sec} in the short regime), and therefore have the effect of reducing yields in the long regime and increasing yields in the short regime. In both regimes, the SLR exemption will allow dealers to allocate the regulated portion of their balance sheet entirely to synthetic lending ($q^{syn} = \bar{q}$) and lead to a reducing in financial intermediation spreads.

Exempting reserves (or any other assets) frees up the dealer balance sheet space for Treasury holding and synthetic lending, and thus is equivalent to expanding the balance sheet capacity $\bar{q}$ in our static model. In the long regime, this would result in a decline in bond yields and synthetic lending spreads; in the short regime, bond yields would rise while synthetic lending spreads fall (see part two of Propositions 1 and 2).

Both SLR exemption policies will lead to a reduction in financial intermediation spreads, including CIP deviations, whose magnitude depends on the extent to which balance sheet constraints are relaxed. Exempting Treasurys will, in the long regime, reduce Treasury yields by removing balance sheet factors from their pricing entirely, whereas exempting reserves will not have this effect.

4.4 Quantitative Easing and Quantitative Tightening

We define QE (QT) as the Federal Reserve's purchases (or sales/redemptions) of Treasury bonds in the secondary market.[20] The Treasury bonds can ultimately come from (QE) or go to (QT) dealer inventory, bank portfolios (outside the broker-dealer subsidiary), or other non-bank (non-dealer) clients, and are traded in exchange for reserves. Here, we will separately consider the Treasury demand and reserve supply channels of QE/QT. We assume that no SLR exemption is applied to

[19]Exempting one but not the other would shift the net holdings of Treasury bonds from dealers to their levered clients or vice versa, in addition to relaxing balance sheet constraints.

[20]The Federal Reserve and other central banks have at times purchases mortgage, corporate, and other bonds as part of quantitative easing programs. The model described thus far considers only Treasury bonds, and for this reason we restrict attention to Treasury purchases.

reserves or Treasury securities, and set aside the signaling effects of QE (which are covered in Section 4.1).

To isolate the effects of the Treasury demand channel, we consider a hypothetical version of QE in which the Fed purchases Treasury bonds in exchange for Treasury bills. This operation (which is somewhat akin to "Operation Twist") leaves the supply of reserves unchanged. We model this operation, which isolates the Treasury demand channel of QE, as a parallel outward shift in the demand curve for Treasury bonds (D_U) in our static model. Holding fixed money market yields and swap rates, in the long regime quantitative easing will reduce both yields and synthetic lending spreads (see part two of Proposition 1). In contrast, in the short regime, quantitative easing will reduce yields while increasing synthetic lending spreads (see part two of Proposition 2). Both of these effects operate through the balance sheet mechanism; the regime matters because it determines whether balance sheet constraints are tightened or loosened by QE. If the goal of quantitative easing is to lower financial intermediation spreads, then our results imply that QE is effective via the Treasury demand channel in the long regime but not in the short regime.

To isolate the reserve supply channel, we consider a hypothetical purchase of Treasury bills in exchange for reserves, which leaves the supply of longer-maturity Treasury bonds unchanged. Note that the combination of these two operations is QE: the purchase of Treasury bonds in exchange for reserves. Note also that the effects of QT are exactly the opposite of those of QE. The effects of QE through the reserve supply channel are more complex, and depend in particular on whether the Fed's overnight reverse repo (ONRRP) facility is actively used.

Consider first the case without an active ONRRP facility. In this case, the reserves the Fed creates via QE must end up on bank balance sheets.[21] This will be true regardless of whether the ultimate seller of the bonds to the Fed is a bank, dealer, or non-bank. We can incorporate this effect into our model as a reduction in $\bar{q}$.

Suppose the reduction in balance sheet capacity is equal to the increase in bond demand (i.e. that all reserves end up on bank balance sheets). In the long regime, the reserve supply channel ($\bar{q}$) will exactly offset the Treasury demand channel ($D_U(\cdot)$); see (32). In contrast, in the short regime, both the reserve supply channel and the Treasury demand channel will lead to more tightly constrained dealer balance sheets (see (35)).

[21] If the Fed's purchases under QE coincide with Treasury issuance, then some reserves might temporarily go into the Treasury's general account before eventually winding up on bank balance sheets.

With an actively used ONRRP facility, the reserve supply channel will have no effect. The ONRRP facility allow money market funds to make repo loans to the Federal Reserve. If the Fed exchanges reserves for bills, these funds will sell bills and receive deposits at their clearing banks, and then lend those deposits back to the Federal Reserve using the ONRRP facility. Thus, with an active ONRRP facility, the effects of QE operate entirely through the Treasury demand channel.

In summary, in the long regime with an active ONRRP facility and binding balance sheet constraints (the situation as of June 2022), we expect QE/QT to have strong effects on Treasury yields and financial intermediation spreads.

4.5 Implications Monetary Policy Tightening Cycles

In June 2022, the Federal Reserve began to normalize its large balance sheets from the extraordinary response to the COVID pandemic. In addition, the Fed is also expected to increase rates substantially over the next two years while engaging in quantitative tightening. Our framework has important implications for the dynamics of the Treasury market during such a tightening cycle.

We first note that tightening cycles are often associated with with flat or inverted Treasury yield curve, and low expected returns on long-term Treasury bonds. This dampens the real money investors' demand for Treasury bonds, and it is particularly challenging for dealers and levered investors to accommodate a reduction in Fed holdings of Treasury bonds (QT) when client demand is weak.

Consider the experience of the 2017-2019 tightening cycle, in which the Federal normalized its balance sheet for the first time post-GFC and increased the short-term interest rates from the zero-lower-bound to 2.5 percent. During that tightening cycle, dealers' increased their Treasury holdings by about \$100 billion and hedge funds increased their holdings by about \$350 billion, together accounting for the entirety of the \$390 billion Fed balance sheet normalization from October 2017 to September 2019. The swap-Treasury spread and the Treasury cash-futures basis widened considerably over the period. Moreover, the increasingly crowded dealer balance sheet and significant build-up of the levered investor positions may have contributed to the repo market distress in September 2019 and Treasury market dislocation in March 2020.

Consistent with this experience, our framework suggests that the combination of QT with an active ONRRP facility and a flattening curve, in the long regime, can lead to high bond yields, more negative swap spreads, and higher financial intermediation spreads. SLR exemptions and the use of the swap lines established with foreign central banks have the potential to ameliorate these effects.

5 Conclusion

We have documented a regime change in the U.S. Treasury bond market. Prior to the 2008-2009 financial crisis, dealers were net short-sellers of Treasury bonds, swap spreads were positive, and CIP violations were small. Following the GFC, dealers became net buyers of Treasury bonds, swap spreads turned negative, and covered interest parity violations emerged. Our analysis ties these observations together by constructing arbitrage bounds, the net short and net long curves, and providing evidence of dealers-as-arbitrageurs in the Treasury market.

We then discuss the causes and consequences of this regime change. We view the large increase in Treasury supply and the tightening of leverage constraints on dealers as the primary drivers of this regime change. Using a stylized static model, we have argued that this regime shift has amplified the effects of quantitative easing and of the yield curve slope on borrowing spreads. In the post-GFC dealer-long regime our model predicts tighter dealer balance constraints in response to Fed quantitative tightening and a flat or inverted Treasury yield curve, and more elevated financial intermediation spreads. Our analysis suggests that other polices, including the use of swap lines and of exemptions to SLR calculations, can help offset these effects.

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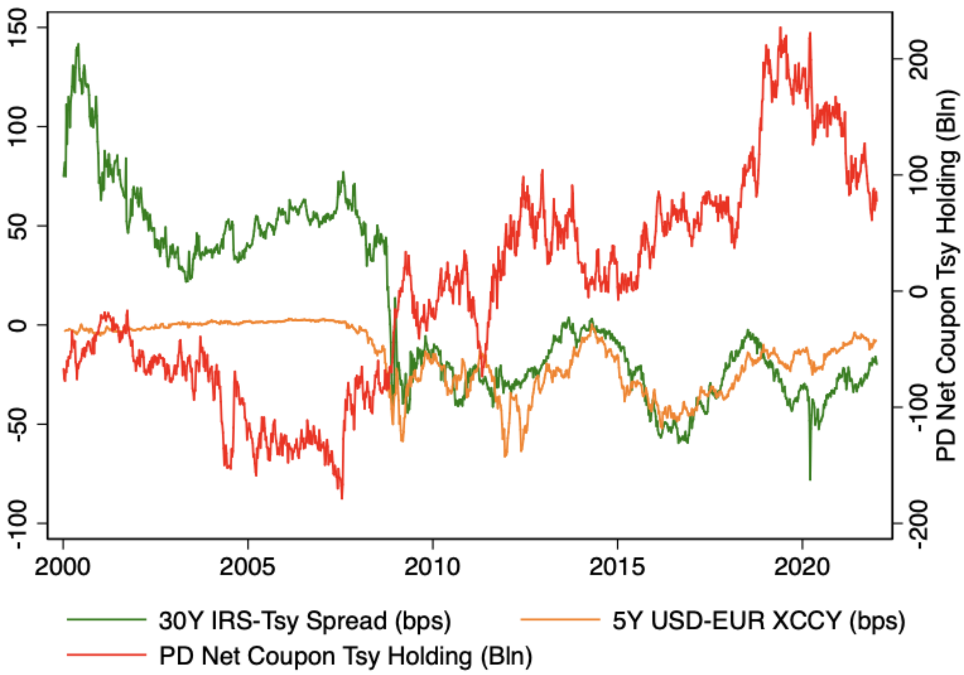
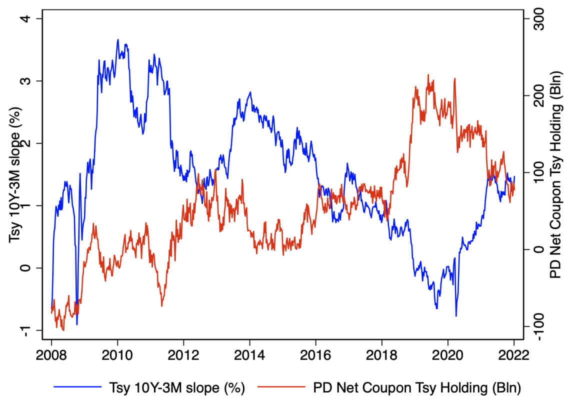
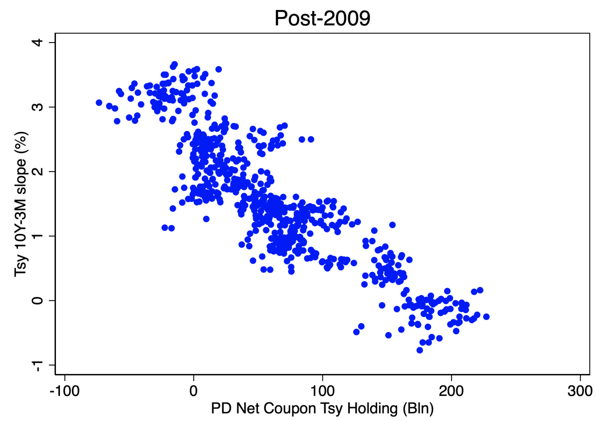
Figure 1: Primary Dealer Treasury Holing, Swap Spreads, and Cross-Currency Basis.



Notes: This figure plots the spread between the 30-year Libor-linked interest rate swap and the U.S. Treasury yield (in green), and the 5-year USD-EUR cross-currency basis (in orange), and net holdings of coupon Treasury bonds. The pricing data are from Bloomberg, and the primary dealer position data are from the publicly available primary dealer statistics published by the Federal Reserve Bank of New York. The quote on the cross-currency basis swap effectively measures the direct dollar interest rate minus the synthetic dollar interest by swapping EUR interest rate into dollars (Du, Tepper, and Verdelhan (2018b)).

Figure 2: Term Spreads and Primary Dealer Treasury Holdings

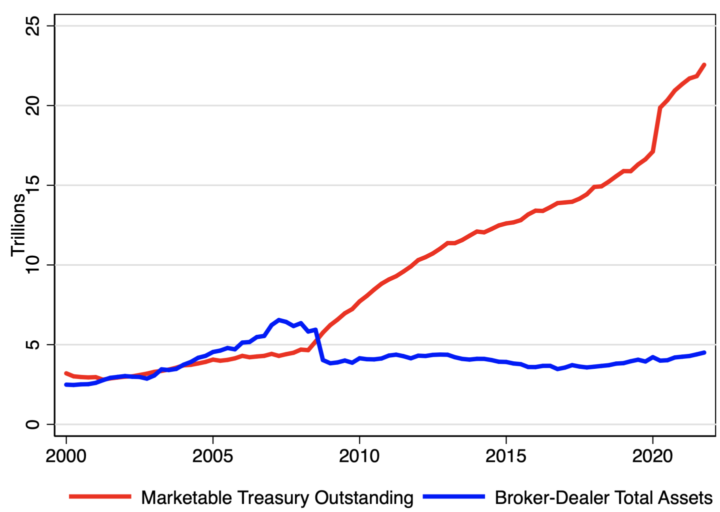




(a) Tresaury Term Spread and Dealer Treasury Holding (b) Treasury Term Spread and Dealer Treasury Holding

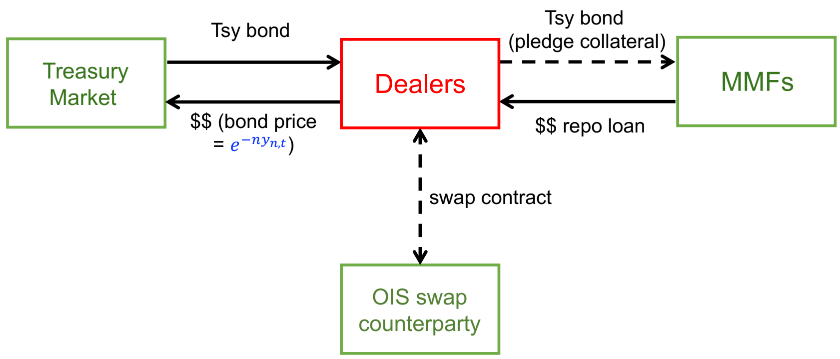
Notes: Panel (a) plots the yield spread between the 10-year Treasury bond and the 3-month Treasury bill (in blue), and the primary dealers' net holdings of Treasury bonds. Panel (b) plots the relationship between the two variables post-2009 in a scatter plot. The pricing data are from Bloomberg, and the primary dealer position data are from the publicly available primary dealer statistics published by the Federal Reserve Bank of New York.

Figure 3: Treasury Supply and Broker-Dealer Total Assets.

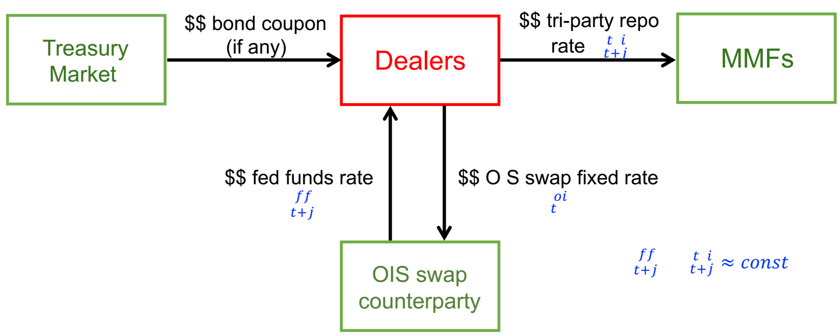


Notes: This figure plots the total marketable Treasury securities outstanding (in red) and the financial assets of the U.S. broker-dealer sector (in blue) in trillions of dollars from Flow of Funds.

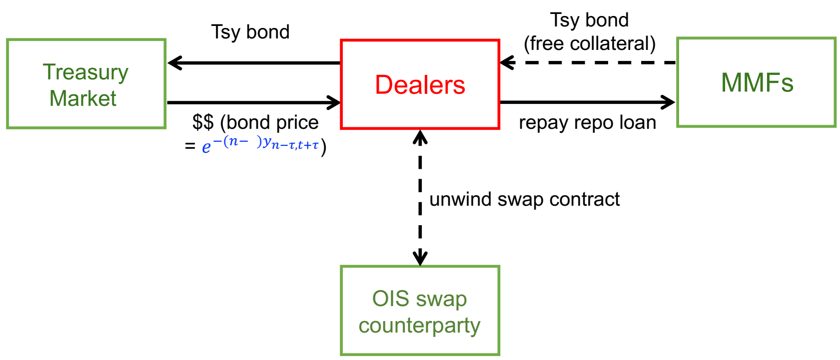
Figure 4: Cash-Flow Illustration for a τ-period Long-Treasury Trade.



(a) Initialization of the Long-Treasury Trade

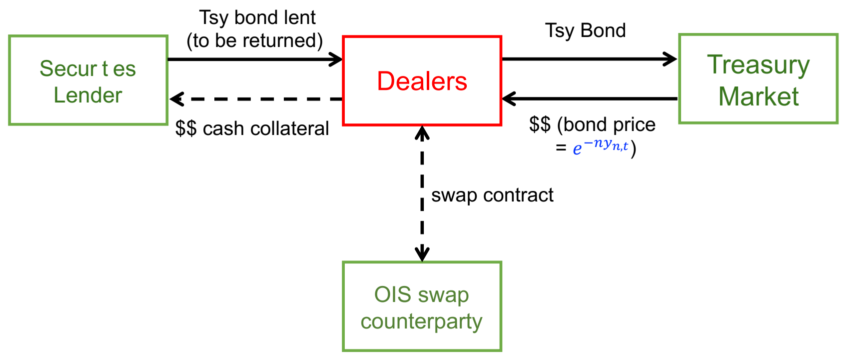


(b) Interim of the Long-Treasury Trade

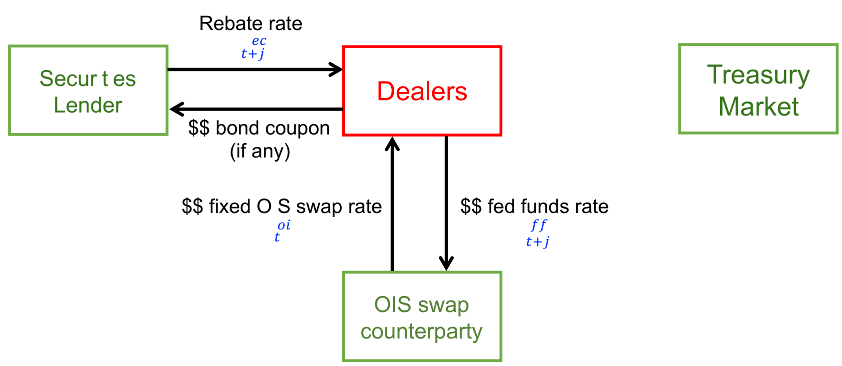


(c) Unwinding of the Long-Treasury Trade

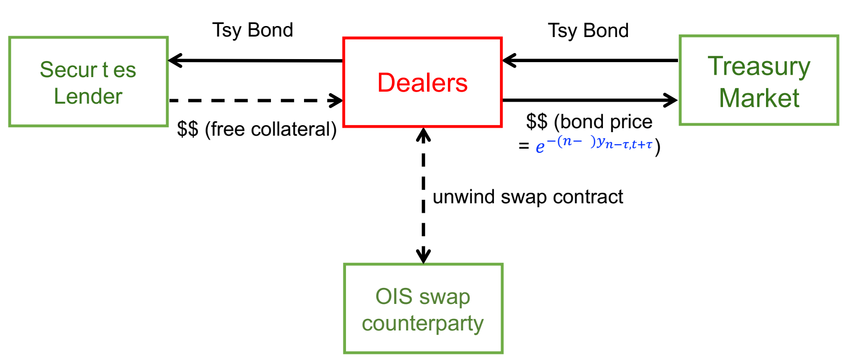
Figure 5: Cash-Flow Illustration for a τ-period Short-Treasury Trade.



(a) Initialization of the Short-Treasury Trade



(b) Interim of the Short-Treasury Trade



(c) Unwinding of the Short-Treasury Trade

Figure 6: Balance Sheet Change for a Long-Treasury Trade in the Long Regime

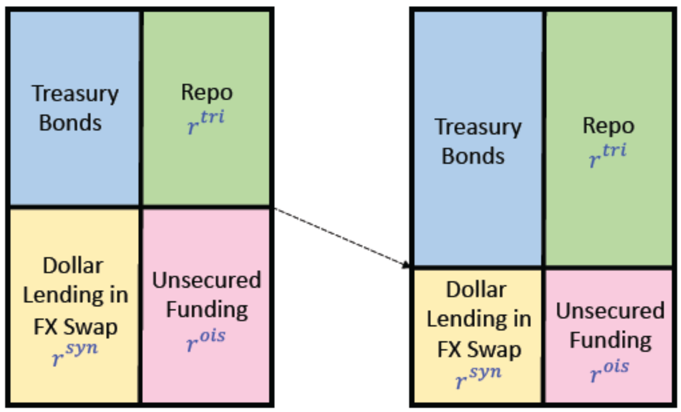


Figure 7: Balance Sheet Change for a Short-Treasury Trade in the Short Regime

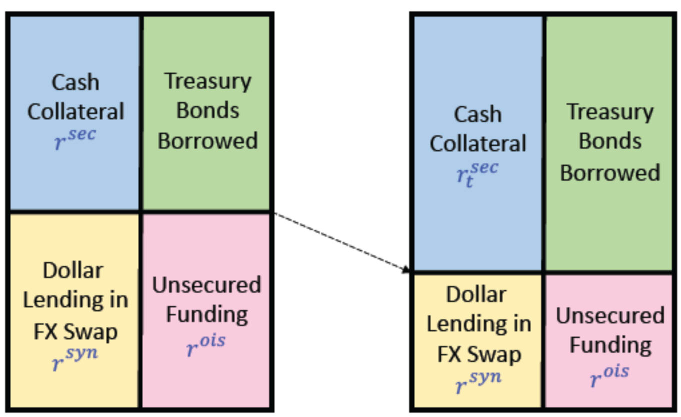
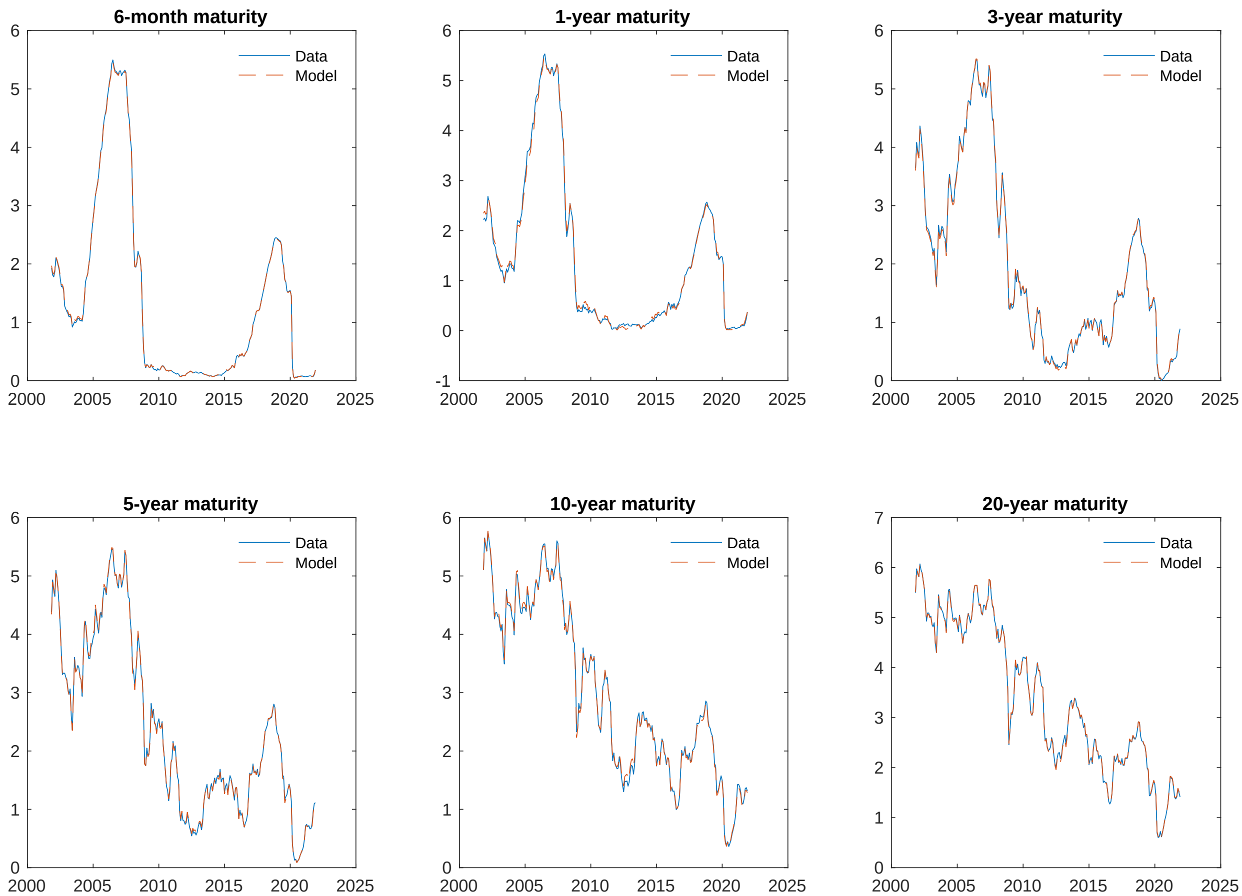
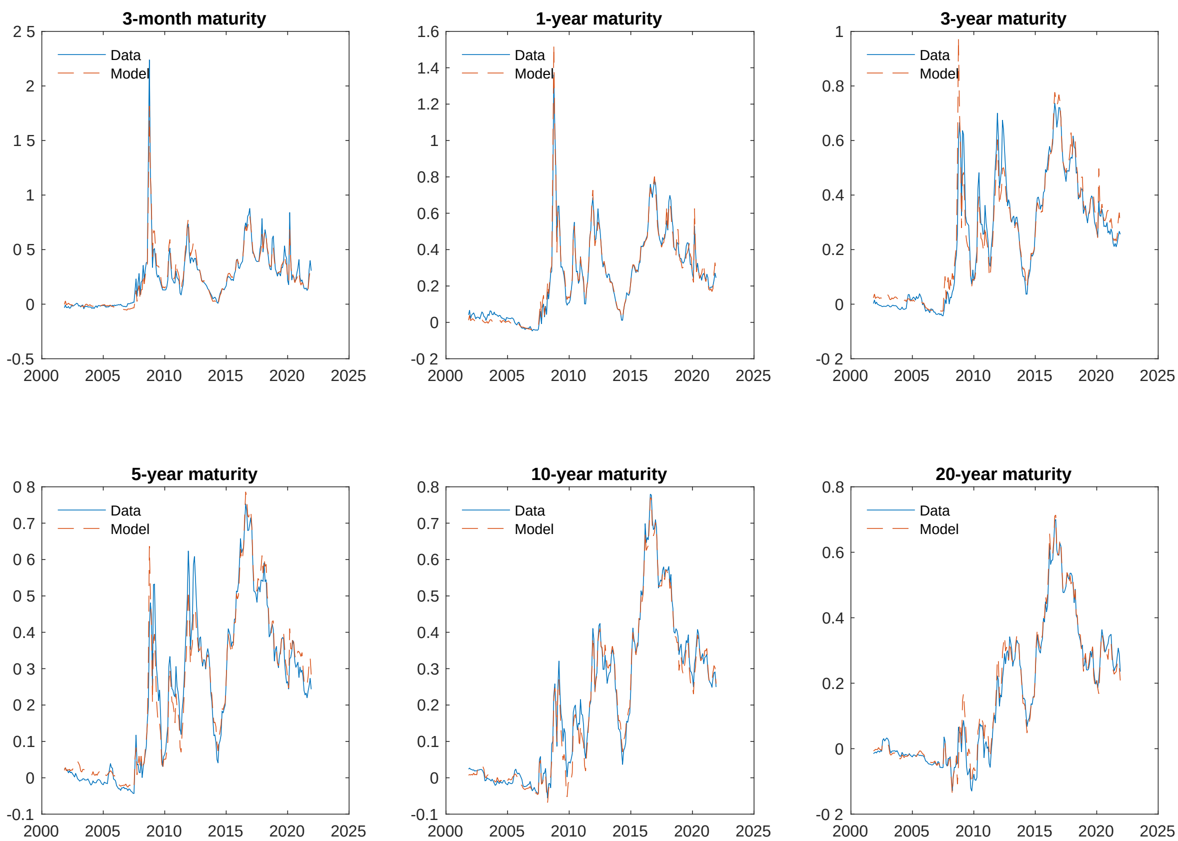


Figure 8: Fit of the TS Model to Dollar OIS Curves.



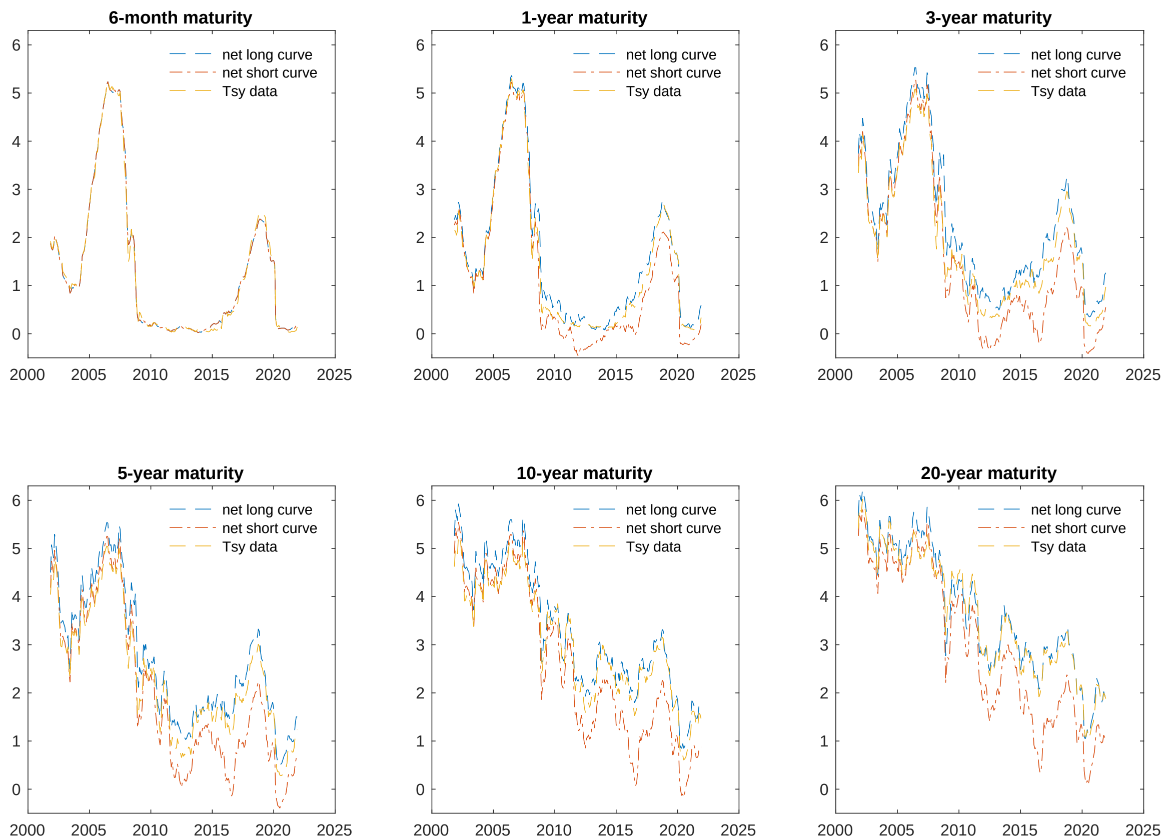
Notes: In this figure, we show the fitting of the dollar OIS curve using our term-structure model. Data are from 2003 to 2021. More details on the term structure model can be found in Section 2.4.

Figure 9: Fit of the TS Model to USD-EUR CIP Deviations.



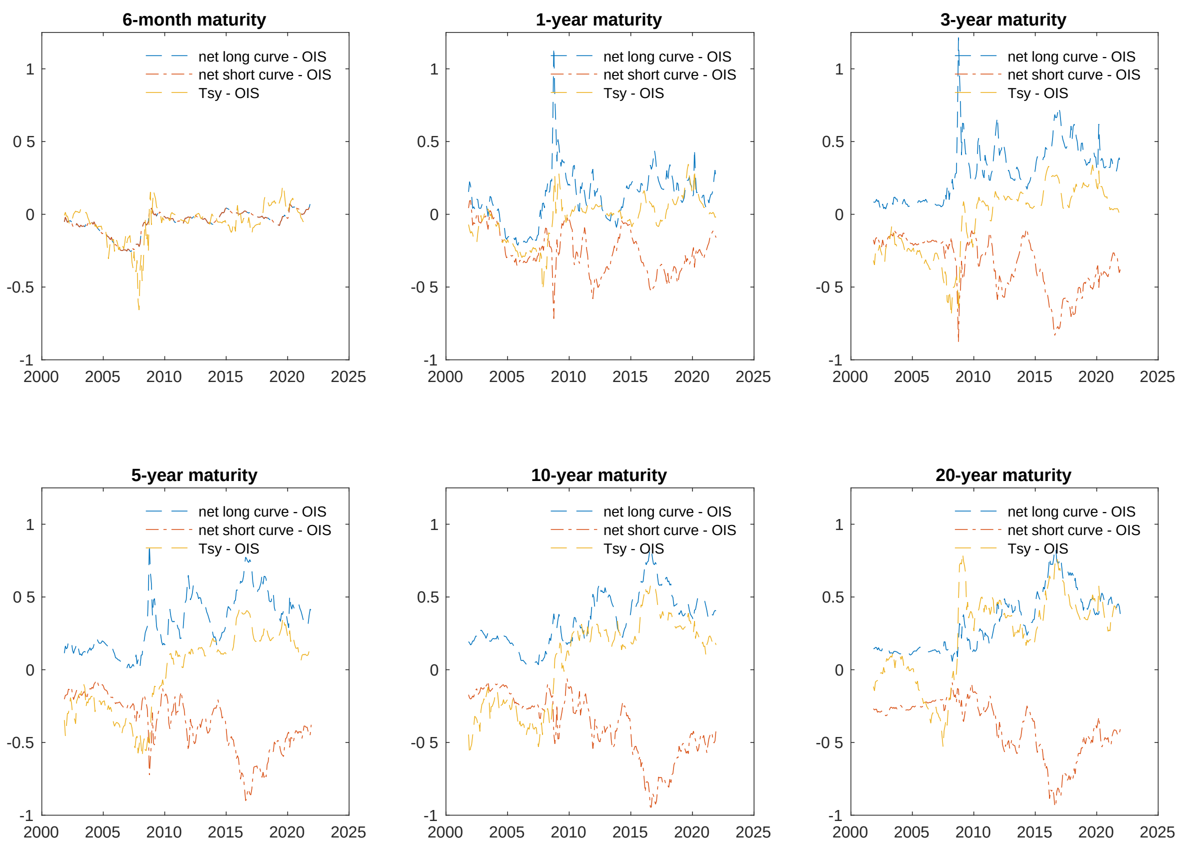
Notes: In this figure, we show the fitting of the OIS-Based EUR-USD CIP Deviations using our term structure model. The CIP deviations are shown as the spread between the synthetic dollar interest rate and the USD OIS rate (the negative of the quoted cross-currency basis). Data are from 2003 to 2021. More details on the term structure model can be found in Section 2.4.

Figure 10: Long and Short Curves.



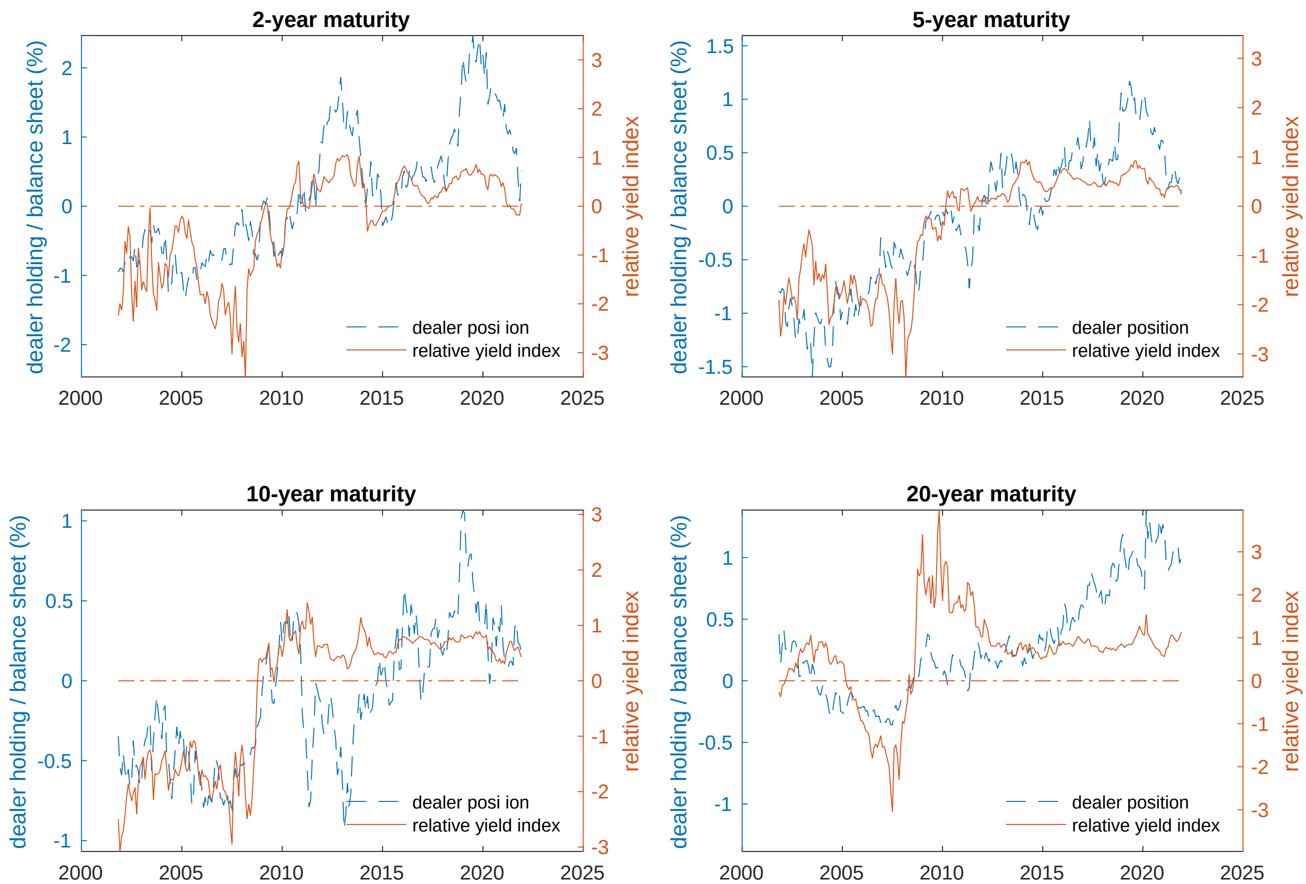
Notes: In this figure, we show the model-implied net-long and net-short curves for Treasury securities, together with the actual Treasury yields. Data are from 2003 to 2021. All yields are par yields. More details on the term structure model can be found in Section 2.4.

Figure 11: Long and Short Curves – OIS Spreads.



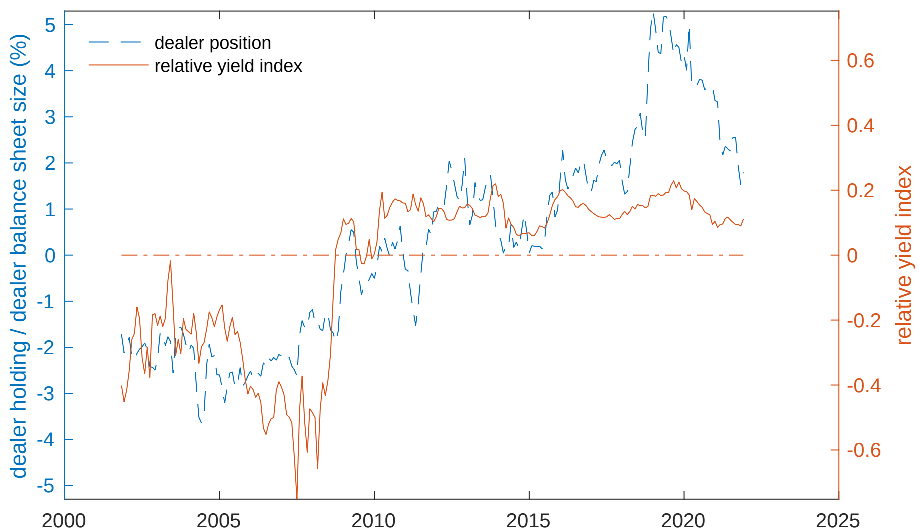
Notes: In this figure, we show the model-implied long and short Treasury curves minus the OIS rates for corresponding maturities, together with the actual Treasury–OIS spreads. Data are from 2003 to 2021. All yields are par yields. More details on the term structure model can be found in Section 2.4.

Figure 12: Relative Yield and Position by Maturity.



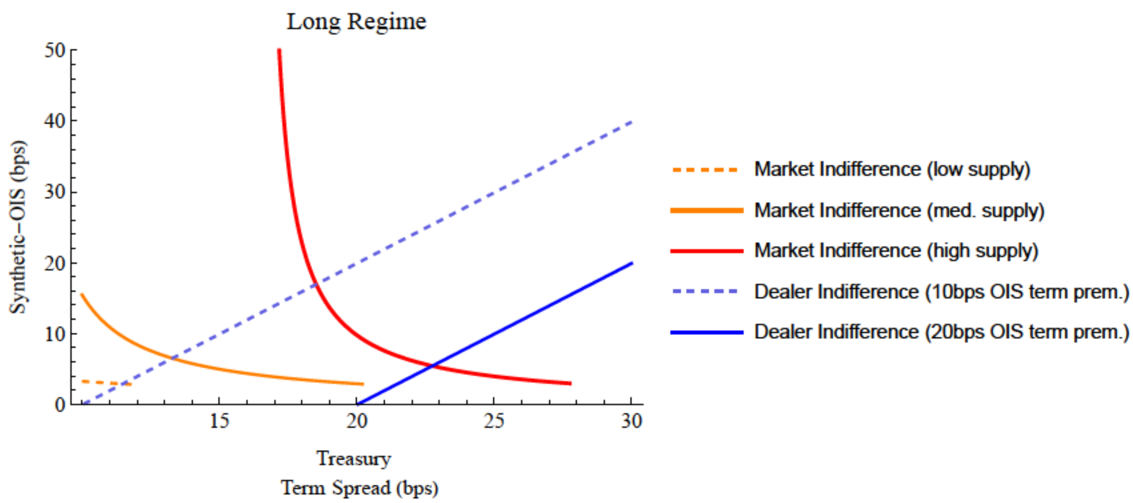
Notes: In this figure, we plot the relative yield index versus actual scaled primary dealer Treasury positions. The relative yield index is defined as 2*(long curve − Treasury curve)/(long curve − short curve) − 1. The scaled primary dealer Treasury position is calculated as the ratio of the primary dealer net position in the Treasury securities for the corresponding maturity bucket (published by Federal Reserve Bank of New York) to total financial assets of the broker-dealer sector (published by Flows of Funds). The position data corresponding to 2-year, 5-year, 10-year, and 20-year yields in the figure are defined based on the following maturity buckets, 1-to-3 years, 3-to-6 years, 6-to-11 years, over 11 years, respectively. Data are from 2003 to 2021.

Figure 13: Relative Yield and Position in Aggregate.



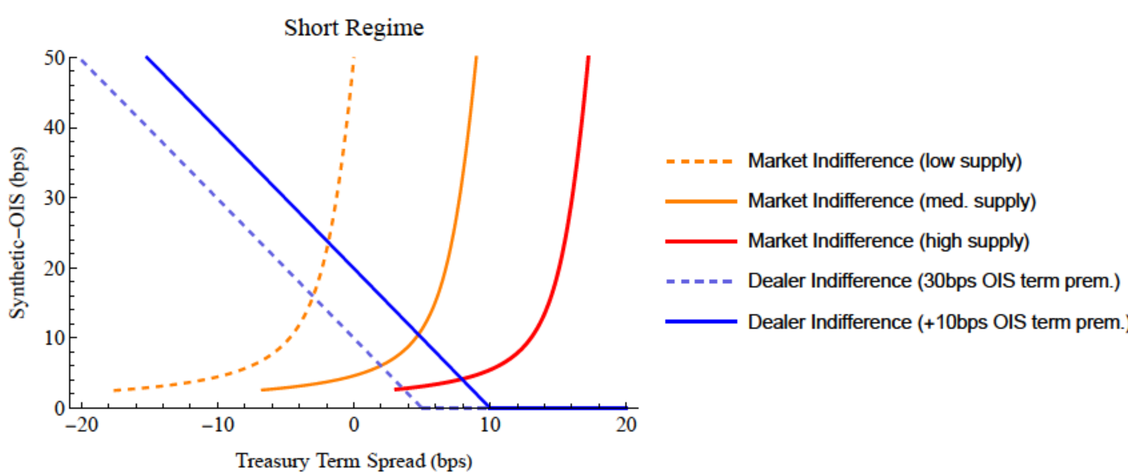
Notes: In this figure, we plot the weighted average of the relative yield index across maturities, versus the scaled primary dealer Treasury positions. For each maturity, the relative yield index is defined as 2*(long curve − Treasury curve)/(long curve − short curve) − 1. Each relative yield is then weighted by coupon Treasury outstanding for the corresponding buckets over total coupon Treasury securities, based on data from the Center for Research in Security Prices. The scaled primary dealer Treasury position is calculated as the ratio of total primary dealer net position in all coupon Treasury securities (published by Federal Reserve Bank of New York) to total financial assets of the broker-dealer sector (published by Flows of Funds). Data are from 2003 to 2021.

Figure 14: Dealer and Market Indifference Curves in the Long Regime.



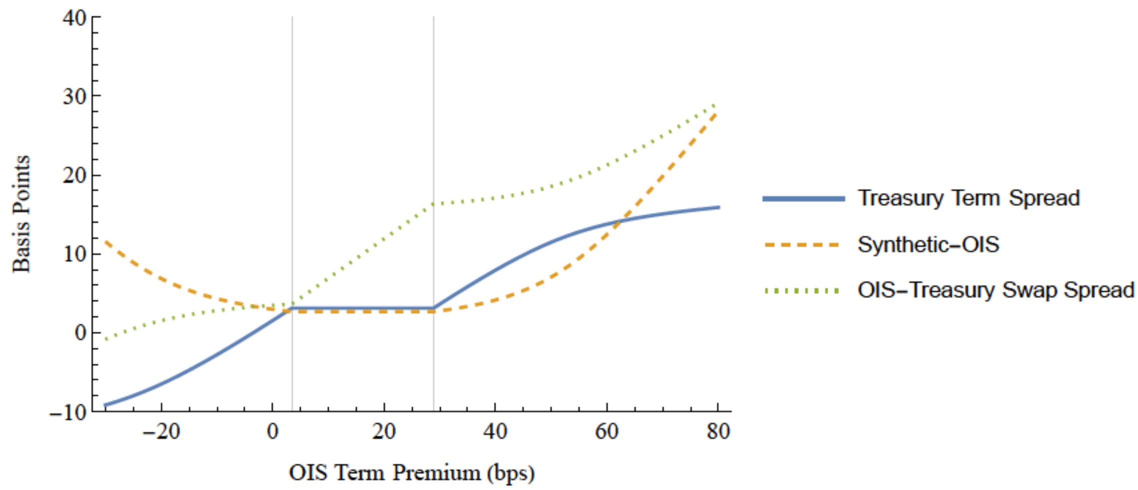
Notes: This figure illustrates the dealer indifference curve defined in (31) and the market indifference curve defined in (32), under three different levels of S^{bond} and two different levels of $y_{\mathbb{Q}}$. The functional forms and parameters used to generate the figure are described in appendix section D.

Figure 15: Dealer and Market Indifference Curves in the Short Regime.



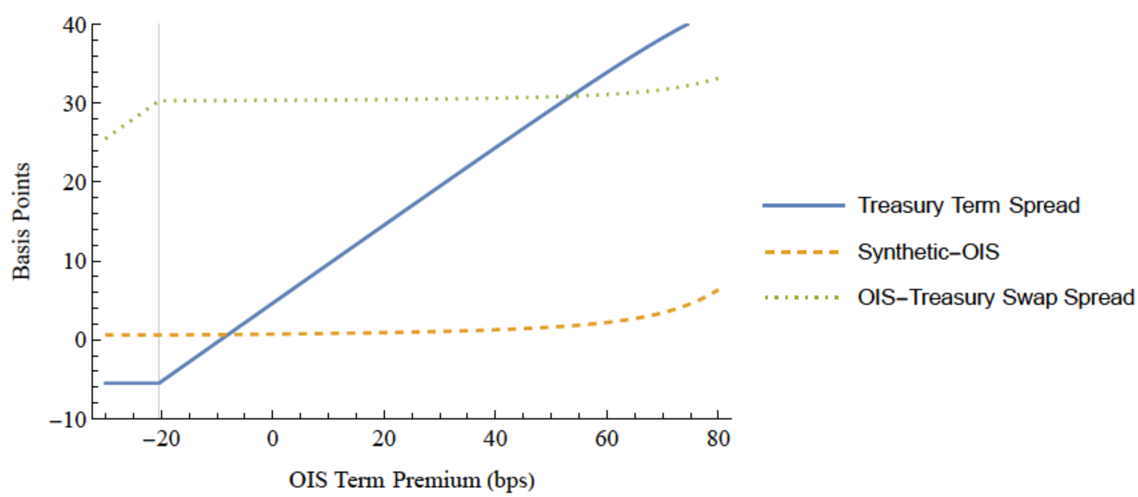
Notes: This figure illustrates the dealer indifference curve defined in (34) and the market indifference curve defined in (35), under three different levels of S^{bond} and two different levels of $y_{\mathbb{Q}}$. The functional forms and parameters used to generate the figure are described in appendix section D.

Figure 16: All-Regimes and the OIS Term Premium.



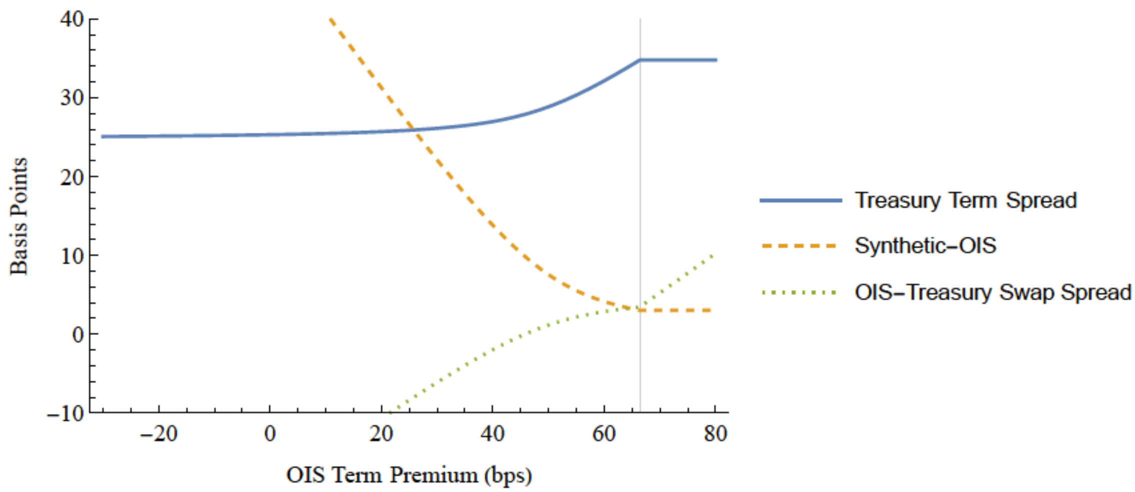
Notes: This figure plots the synthetic lending spread $r^{syn} - r^{ois}$, Treasury term spread $y - y^{bill}$, and swap-Treasury spread $r^{ois}_{n,0} - y$ as a function of the swap curve term premium $r^{ois}_{n,0} - r^{ois}$. The functional forms and parameters used to generate the figure are described in appendix section D.

Figure 17: All-Regimes with Large Dealer Balance Sheet Capacity and Small Bond Supply.



Notes: This figure plots the synthetic lending spread $r^{syn} - r^{ois}$, Treasury term spread $y - y^{bill}$, and swap-Treasury spread $r^{ois}_{n,0} - y$ as a function of the swap curve term premium $r^{ois}_{n,0} - r^{ois}$, holding expected future rates fixed, with a relatively large dealer balance sheet capacity, small bond supply, and large spread between bills and tri-party repo. The functional forms and parameters used to generate the figure are described in appendix section D.

Figure 18: All-Regimes with Small Dealer Balance Sheet Capacity and Large Bond Supply.



Notes: This figure plots the synthetic lending spread $r^{syn} - r^{ois}$, Treasury term spread $y - y^{bill}$, and swap-Treasury spread $r^{ois}_{n,0} - y$ as a function of the swap curve term premium $r^{ois}_{n,0} - r^{ois}$, holding expected future rates fixed, with a relatively small dealer balance sheet capacity, large bond supply, and zero spread between bills and tri-party repo. The functional forms and parameters used to generate the figure are described in appendix section D.

Internet Appendix
"Intermediary Balance Sheets and the Treasury Yield Curve"

Wenxin Du Benjamin Hébert Wenhao Li

A Dealers and Levered Clients

In Section 2, we developed net long and net short curves from the perspective of a securities dealer, yields at which the dealer would be willing to either net long or net short Treasury bonds. In this section, we extend our model to consider the perspective of a levered Treasury investor (e.g. a hedge fund) financed by a security dealer of the kind considered in that section. The main result is that levered clients will have the same net long and net short curves as the dealer that finances them. That is, the net long curve represents a yield at which the levered client would be willing to buy the Treasury bond, irrespective of its beliefs about the stochastic process driving Treasury yields, and a symmetric result holds for the net short curve. This result occurs in spite of the fact that the levered client is not itself directly affected by balance sheet constraints.

This result is important from a general equilibrium perspective. Dealers are never on net long or short a large quantity of Treasury bonds during our sample, relative to the overall Treasury supply. Dealers moved from a net short of roughly 100 hundred billion in 2005 to a net long of 200 hundred billion in 2020. The overall supply of Treasury securities rose from 4 trillion to 22 trillion over the same period.

However, dealers intermediate repo and reverse-repo for their levered clients in much greater quantities– on the order of trillions each day. In this section we will argue that the recipients of much of this financing will act like dealers, and subsequently provide some suggestive evidence on this point.

Consider the following trading strategy for the dealer: finance a client's purchase of a Treasury using bilateral repo, use the resulting collateral to raise financing in the tri-party repo market, and reduce CIP activity so that the trade is balance sheet neutral. In a competitive market, the profits of such a strategy are zero:

$$\underbrace{(1-h)(e^{\frac{1}{12}r_t^{bi}}-e^{\frac{1}{12}r_t^{tri}})}_{\text{Lending/Borrowing Spread}}-\underbrace{(1-h)(e^{\frac{1}{12}r_t^{syn}}-e^{\frac{1}{12}r_t^{ois}})}_{\text{Forgone CIP profits}}=0. \tag{A1}$$

That is, the dealer must be indifferent between matched book repo lending and taking advantage of CIP arbitrage, as both activities use balance sheet. Note that this is expressed per dollar of Treasury collateral, and that we have assumed the same haircut in both markets.

Let's now consider the perspective of a levered client who can purchase a Treasury bond, financed by this intermediary, and can trade derivatives with the securities dealer. Because the levered client can trade derivatives with the dealer, the projection of its stochastic discount factor onto the space of derivative returns must agree with the same projection for the dealer's SDF. Equivalently, the risk-neutral measure Q is shared (within this space) by the levered clients and the dealer.

We will also assume that the levered client can engage in risk-free unsecured borrowing[1] from the unsecured dealer at the synthetic lending rate. The dealer is unwilling to lend at a rate lower than this, as otherwise it would be better off engaging in CIP arbitrage.

Under these assumptions, the levered client considers buying an n-month Treasury and then selling one month later:

$$(1-h)\cdot \underbrace{e^{-ny_{n,t}}e^{r_t^{bi}}}_{\text{secured financing}} + h\cdot \underbrace{e^{-ny_{n,t}}e^{r_t^{syn}}}_{\text{unsecured financing}} \geq E_t^{\mathbb{Q}}[e^{-(n-1)y_{n-1,t+1}}]. \tag{A2}$$

Substituting in (A1), this condition becomes identical to (7). It follows immediately that levered clients must be willing to go net long if the yield reaches the net long curve.

Essentially identical logic applies to the net short curve: the dealers indifference between matched book repo (in the net short case, intermediating between security lenders and short-sellers) and CIP arbitrage converts the levered client's indifference condition to the dealer's indifference condition.

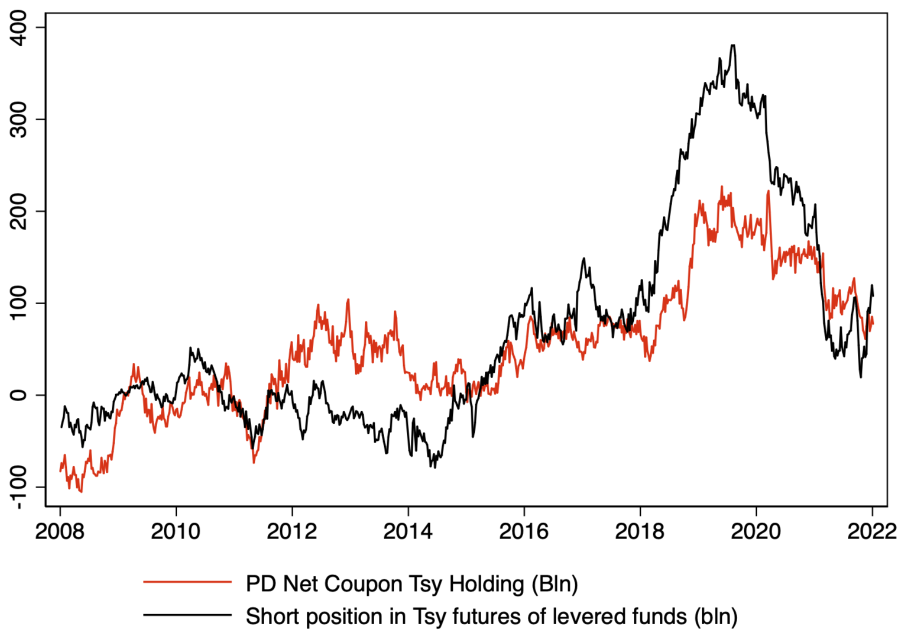
We conclude that levered clients who are dependent on dealers for financing will act as if they face the same balance sheet costs that dealers face, even if they are not themselves directly regulated. As a result, balance sheets costs will influence a substantial segment of the Treasury market, even though dealers are on their own hold a relatively small quantity of Treasury bonds on net.

Below, we provide evidence consistent with this perspective. While the Treasury positions of

[1]It is probably better to think of this as secured borrowing using non-Treasury securities that the dealer cannot itself finance in a repo market.

levered investors are not publicly available, we can infer the holdings of investors that engage in Treasury cash-future trades from Treasury futures positions. Figure A1 plots the primary dealer net coupon holdings and levered funds' short positions in Treasury futures contracts published by the CTFC. For relative value hedge funds that arbitrage Treasury cash-futures basis, a short position in Treasury futures corresponds to a long position in the cash Treasury bonds. We see that primary dealer positions and the levered funds' short Treasury futures position are strongly positively correlated, which is consistent with our result that dealers and levered investors take similar positions and can be considered as a consolidated intermediary.

Figure A1: Primary Dealer Treasury Holdings and Implied Treasury Holding of Levered Investors



Notes: This figure plots the primary dealer's net position in coupon-bearing Treasury securities from Primary Dealer Statistics published by the Federal Reserve Bank of New York, and the short position in the Treasury futures market by levered funds from the Commitments of Traders Report published by the Commodity Futures Trading Commission.

B Partial Equilibrium Arbitrage Bounds

In this appendix section, we construct the net short and net long curves described in the main text as arbitrage bounds under weaker assumptions than those employed in the main text. In particular, in the main text we assumed that zero-cost, zero-balance sheet trades are weakly unattractive under a common SDF (i.e., a version of the no-arbitrage assumption). That assumption leads to $y_{n-1,t+1} \leq y^b_{n-1,t+1}$ with probability one. Here, we instead assume that there could be profitable zero-cost, zero-balance sheet trading strategies under the intermediary's stochastic discount factor. Then we consider the question of whether this intermediary is willing to go net long or net short a Treasury bond, irrespective of the intermediary's preferences or beliefs about the stochastic process driving Treasury yields.

We will assume that this intermediary's SDF prices derivatives, and that the intermediary believes with probability one that $x_{1,t} \geq r^{ois}_t \geq x_{2,t}$, where $x_{1,t}$ and $x_{2,t}$ are defined as in the main text. We discuss the role of this assumption below.

B.1 The Net Long Curve

Consider first the trade in which the intermediary buys a zero-coupon seven-month Treasury bond, and then sells it in one month, at which time the Treasury bond becomes a zero-coupon T-bill. The intermediary can finance this purchase with tri-party repo, up to the standard two percent haircut h, and finance the remainder with unsecured debt. This trade, in combination with a reduction in CIP activity, is a balance-sheet neutral, zero-financing trade. The intermediary is therefore willing to get net long if this strategy is weakly appealing under the SDF that prices derivatives. Let $\mathbb{Q}$ denote the risk-neutral measure associated with this SDF. We assume that r^{ois}_t is the log risk-free rate associated with this SDF.[2]

Let $y^b_{7,t}$ denote a yield at which this trade is attractive to the dealer, and define $y^b_{6,t} = y^{bill}_t$. The

[2]That is, we assume the one-month OIS swap rate is the intermediary's unsecured borrowing rate. This assumption is consistent with the empirical observation that the one-month OIS rate closely tracks other unsecured rates, for example the one-month highly rated financial commercial paper rate. It is also consistent with the industry practice of using the OIS curve to discount derivative cashflows. Lastly, it is consistent with the observation that the unsecured borrowing rate is the appropriate discount rate for off-balance-sheet cashflows, under our generalized no-arbitrage assumption.

dealer will be indifferent between employing and not employing this trading strategy if

$$\underbrace{e^{-\frac{7}{12}y^b_{7,t}}}_{\text{Purchase price}}\Big(\underbrace{he^{\frac{1}{12}r^{ois}_t}}_{\text{Unsecured Financing}}+\underbrace{(1-h)e^{\frac{1}{12}r^{tri}_t}}_{\text{Repo Financing}}+\underbrace{(e^{\frac{1}{12}r^{syn}_t}-e^{\frac{1}{12}r^{ois}_t})}_{\text{Forgone CIP profits}}\Big)=\underbrace{E^{\mathbb{Q}}_t[e^{-\frac{6}{12}y^{bill}_{t+1}}]}_{\text{Sale price}}.$$

Cheap financing ($r^{tri}_t < r^{ois}_t$) makes the trade attractive and hence decreases the required yield, while the opportunity cost of using balance sheet ($r^{syn}_t > r^{ois}_t$) has the opposite effect. We assume that $x_{1,t} \geq r^{ois}_t$, which is consistent with the post-GFC data and implies that

$$e^{\frac{1}{12}r^{syn}_{t+j}}-e^{\frac{1}{12}r^{ois}_{t+j}} \geq (1-h)(e^{\frac{1}{12}r^{ois}_{t+j}}-e^{\frac{1}{12}r^{tri}_{t+j}}).$$

This assumption states that the balance sheet cost exceeds the financing advantage. It can be justified on the grounds that, if it did not hold, it would be efficient for dealers to purchase Treasury bills from money market funds, financed by repo loans from those same money market funds. This would lead to large dealer balance sheets, causing the leverage constraint to tighten, and hence cannot be part of an equilibrium.

Let us now define a yield curve , $y^l_{n,t}$, such that the dealer will be certainly be willing to purchase an n-month Treasury bond, regardless of her preferences or beliefs, if its yield exceeds this value. This will be the net long curve. We will conjecture and verify that the curve defined recursively by

$$e^{-\frac{n}{12}y^l_{n,t}}((1-h)(e^{\frac{1}{12}r^{tri}_t}-e^{\frac{1}{12}r^{ois}_t})+e^{\frac{1}{12}r^{syn}_t}))=E^{\mathbb{Q}}_t[e^{-\frac{n-1}{12}y^l_{n-1,t+1}}]$$

has this property. That is, the net long curve is defined by the discount rate $e^{x_{1,t}}=(1-h)(e^{\frac{1}{12}r^{tri}_t}-e^{\frac{1}{12}r^{ois}_t})+e^{\frac{1}{12}r^{syn}_t}$, as in the main text.

Fix some $n>7$ and suppose $y^l_{m,t}$ is defined by this recursion for all $m \in \{6,\dots,n-1\}$. Consider a trading strategy that purchases the bond, finances the trade with repo and unsecured borrowing, offsets the balance sheet cost by reducing CIP activity, and unwinds at the first moment at which the bond yield becomes weakly lower than $y^l_{m,t}$. Let τ denote the months elapsed and let $y_{n-\tau,t+\tau} \leq y^l_{n-\tau,t+\tau}$ be the bond price at which the trade is unwound. According to the strategy, we have $y_{m,t} \geq y^l_{m,t}$ for all $m \in \{6,\dots,n-1\}$. Further, $\tau \leq n-6$ is guaranteed because by assumption, the intermediary always unwinds the trade once the bond has six-month remaining maturity.

The intermediary will be willing to engage in this strategy provided that

$$\underbrace{e^{-\frac{n}{12}y_{n,t}}}_{\text{purchase price}} + E_t^{\mathbb{Q}}[\sum_{j=0}^{\tau-1} \underbrace{e^{-\sum_{k=0}^{j} r^{ois}_{t+k}}}_{\text{discount rate}} \underbrace{e^{-\frac{n-j}{12}y_{n-j,t+j}}}_{\text{interim bond price}} (\underbrace{(1-h)(e^{\frac{1}{12}r^{tri}_{t+j}} - e^{\frac{1}{12}r^{ois}_{t+j}})}_{\text{repo financing benefits}} + \underbrace{(e^{\frac{1}{12}r^{syn}_{t+j}} - e^{\frac{1}{12}r^{ois}_{t+j}})}_{\text{forgone CIP profits}})]$$
$$\leq E_t^{\mathbb{Q}}[\underbrace{e^{-\sum_{k=0}^{\tau-1} r^{ois}_{t+k}}}_{\text{discount rate}} \underbrace{e^{-\frac{n-\tau}{12}y_{n-\tau,t+\tau}}}_{\text{sale price}}]$$

Since derivatives are priced by the intermediary, hedging does not affect the profit in the above trade. We could add a hedging component to this equation, so that certain future fluctuations in the financing rate are fixed at the beginning of the trade, as illustrated by Figure 4. We omit this extra zero-cost component for simplicity.

However, this strategy cannot be fully hedged by interest rates swaps. First, the time τ at which the bond yield falls below $y^l_{m,t}$ is uncertain, as is the ultimate sale price. Second, the interim price of the bond before τ affects the size of the trade that needs to be financed, and consequently both the benefit of cheap financing via tri-party repo and the opportunity cost of the balance sheet. The effects of intermediate bond prices occur because the intermediary uses short term, as opposed to term, financing, and because the assets are marked to market. Thus, even if it were possible to perfectly hedge all of the relevant interest rates, the attractiveness of this trade would depend in part on the intermediary's beliefs about the stochastic process driving bond yields.

However, the worse case scenario for the sale price is that it is exactly equal to the unwinding threshold, $y_{n-\tau,t+\tau} = y^l_{n-\tau,t+\tau}$. Under the assumption that $x_{1,t} \geq r^{ois}_t$, the worse case scenario for the intermediate bond yields is that they are as low as possible (i.e. $y_{n-j,t+j} = y^l_{n-j,t+j}$), which is to say that the trading strategy uses up the maximum possible balance sheet capacity. Consequently, the intermediary will definitely be willing to buy the bond if, for all possible stopping times τ ,

$$e^{-\frac{n}{12}y_{n,t}} \leq -E_t^{\mathbb{Q}}[\sum_{j=0}^{\tau-1} e^{-\frac{n-j}{12}y^l_{n-j,t+j}} e^{-\sum_{k=0}^{j} r^{ois}_{t+k}}((1-h)(e^{\frac{1}{12}r^{tri}_{t+j}} - e^{\frac{1}{12}r^{ois}_{t+j}}) + (e^{\frac{1}{12}r^{syn}_{t+j}} - e^{\frac{1}{12}r^{ois}_{t+j}}))]$$
$$+ E_t^{\mathbb{Q}}[e^{-\sum_{k=0}^{\tau-1} r^{ois}_{t+k}} e^{-\frac{n-\tau}{12}y^l_{n-\tau,t+\tau}}]$$

and this is in fact the tightest possible bound. Rewriting the definition of net long curve, we obtain

$$e^{-\Sigma_{k=0}^{j-1} r^{ois}_{t+k}} e^{-\frac{n-j}{12} y^{l}_{n-j,t+j}} = -e^{-\Sigma_{k=0}^{j} r^{ois}_{t+k}} e^{-\frac{n}{12} y^{l}_{n-j,t+j}} \big((1-h)(e^{\frac{1}{12} r^{tri}_{t+j}} - e^{\frac{1}{12} r^{ois}_{t+j}}\big) \\ + e^{-\Sigma_{k=0}^{j} r^{ois}_{t+k}} E^{\mathbb{Q}}_{t+j} [e^{-\frac{n-j-1}{12} y^{l}_{n-j-1,t+j+1}}]$$

for any j, and thus it also holds for any bounded stopping time τ. By the definition of the net long curve, this inequality is equivalent to

$$e^{-\frac{n}{12} y_{n,t}} \leq e^{-\frac{n}{12} y^{l}_{n,t}}$$

Thus, the intermediary will be willing to buy the bond, regardless of the nature of the intermediary's preferences and beliefs about the bond price process, if $y_{n,t} \geq y^{l}_{n,t}$.

We conclude that the intermediary's demand for a zero-coupon bond should be high if its yield exceeds the net long curve yield. This demand is limited only by the intermediary's leverage constraint: at some point, the intermediary will have switched entirely to doing the Treasury arbitrage as opposed to other arbitrages, at which point $r^{syn}_t - r^{ois}_t$ is no longer a valid measure of the opportunity cost of balance sheet. We therefore predict that if a bond's yield exceeds the buy yield, the intermediary's demand should be substantial.

B.2 The Net Short Curve

We next develop parallel logic for the case of short-selling. In this case, we assume that the intermediary borrows the security from a securities lender in exchange for cash equal to the market value of the security, and receives a log interest rate $r^{sec}_t < r^{tri}_t$ on the cash lent.

The intermediary will be willing to short a seven-month bond at yield $y_{7,t}$ if

$$\underbrace{e^{-\frac{7}{12} y_{7,t}}}_{\text{Sale price}} \underbrace{e^{\frac{1}{12} r^{sec}_t}}_{\text{Gross return on cash in sec. lending}} \geq \underbrace{E^{\mathbb{Q}}_t [e^{-\frac{6}{12} y^{bill}_{t+1}}]}_{\text{Repurchase price}} + e^{-\frac{7}{12} y_{7,t}} \underbrace{(e^{\frac{1}{12} r^{syn}_t} - e^{\frac{1}{12} r^{ois}_t})}_{\text{Forgone CIP profits}}. \tag{B-1}$$

Note that the sign of the forgone CIP profits has changed, relative to the analogous equation for the net long curve, reflecting the fact that both buying and short-selling increase the size of the balance sheet. In equation (B-1), moving the right-hand-side OIS term to the left and dividing both sides by $\exp(\frac{1}{12}r_t^{ois})$, we obtain

$$e^{-\frac{7}{12}y_{7,t}} \geq e^{-\frac{7}{12}y_{7,t}}e^{-\frac{1}{12}r_t^{ois}}\left(e^{\frac{1}{12}r_t^{syn}} - e^{\frac{1}{12}r_t^{sec}}\right) + e^{-\frac{1}{12}r_t^{ois}}E_t^{\mathbb{Q}}[e^{-\frac{6}{12}y_{t+1}^{bill}}] \tag{B-2}$$

Under the assumption that yields are weakly positive, $y_{7,t} \geq 0$, the intermediately is definitely willing to short if

$$e^{-\frac{7}{12}y_{7,t}} \geq e^{-\frac{7}{12}y_{7,t}^s} \equiv e^{-\frac{1}{12}r_t^{ois}}\left(e^{\frac{1}{12}r_t^{syn}} - e^{\frac{1}{12}r_t^{sec}}\right) + e^{-\frac{1}{12}r_t^{ois}}E_t^{\mathbb{Q}}[e^{-\frac{6}{12}y_{t+1}^{bill}}]. \tag{B-3}$$

Following the same spirit, let us define $y_{n,t}^s$ recursively for $n \geq 8$ as

$$e^{-\frac{n}{12}y_{n,t}^s} = e^{-\frac{1}{12}r_t^{ois}}\left(e^{\frac{1}{12}r_t^{syn}} - e^{\frac{1}{12}r_t^{sec}} + E_t^{\mathbb{Q}}[e^{-\frac{n-1}{12}y_{n-1,t+1}^s}]\right), \tag{B-4}$$

which can be interpreted as the pricing equation for a bond with a monthly coupon of $e^{\frac{1}{12}r_t^{syn}} - e^{\frac{1}{12}r_t^{sec}}$, discounted using the OIS curve.

As above, fix some $n > 7$ and suppose $y_{m,t}^s$ is defined as above. Consider a trading strategy that short-sells the bond, borrows the bond from a securities lender, offsets the balance sheet cost by reducing CIP activity, and unwinds at the first moment at which the bond yield becomes weakly higher than $y_{m,t}^s$. Let τ denote this time and let $y_{n-\tau,t+\tau} \geq y_{n-\tau,t+\tau}^s$ be the bond price at which the trade is unwound. According to the strategy, we have $y_{m,t} \leq y_{m,t}^s$ for all $m \in \{6, 7, \cdots, n-1\}$. Further, $\tau \leq n-6$ is guaranteed by the assumption that dealers always unwinds the trade once the bond has six-month remaining maturity. The intermediary will be willing to engage in this strategy provided it is profitable,

$$e^{-\frac{n}{12}y_{n,t}} \geq E_t^{\mathbb{Q}}[\sum_{j=0}^{\tau-1} e^{-\frac{n-j}{12}y_{n-j,t+j}}e^{-\sum_{k=0}^{j} r_{t+k}^{ois}}(e^{\frac{1}{12}r_{t+j}^{syn}} - e^{\frac{1}{12}r_{t+j}^{sec}})] + E_t^{\mathbb{Q}}[e^{-\sum_{k=0}^{\tau-1} r_{t+k}^{ois}}e^{-\frac{n-\tau}{12}y_{n-\tau,t+\tau}}]. \tag{B-5}$$

Note that, because $r_t^{sec} < r_t^{syn}$, the worst-case scenario is the one that makes intermediate bond prices as high as possible. Unlike the net long curve, the fact that $y_{n-j,t+j} < y_{n-j,t+j}^s$ is of no help is generating a bound. In this case, we instead assume a lower bound on yields, $y_{m,t} \geq 0$, motivated the possibility of substitution to cash. In the worst-case scenario, the pricing condition becomes

$$e^{-\frac{n}{12}y_{n,t}} \geq E_t^{\mathbb{Q}}[\sum_{j=0}^{\tau-1} e^{-\Sigma_{k=0}^{j} r_{t+k}^{ois}}(e^{\frac{1}{12}r_{t+j}^{syn}} - e^{\frac{1}{12}r_{t+j}^{sec}})] + E_t^{\mathbb{Q}}[e^{-\Sigma_{k=0}^{\tau-1} r_{t+k}^{ois}} e^{-\frac{n-\tau}{12}y_{n-\tau,t+\tau}^{s}}]. \tag{B-6}$$

For all stopping times τ (bounded above by $n-6$), this is equivalent to

$$e^{-\frac{n}{12}y_{n,t}} \geq e^{-\frac{n}{12}y_{n,t}^{s}}, \tag{B-7}$$

which is to say that the intermediary will be willing to short-sell if yields are below $y_{n,t}^{s}$, irrespective of intermediary's preferences or beliefs about future bond prices.[3]

Finally, we will illustrate that to a first-order approximation, the net-short curve in this appendix is the same as the net-short curve (23) in the main text. Ignoring the covariance terms, the net-short curve in this appendix is

$$1 + \frac{1}{12}r_t^{ois} - \frac{n}{12}y_{n,t}^{s} \approx \frac{1}{12}r_t^{syn} - \frac{1}{12}r_t^{sec} + E_t^{\mathbb{Q}}[1 - \frac{n-1}{12}y_{n-1,t+1}^{s}]$$

$$ny_{n,t}^{s} \approx r_t^{sec} - (r_t^{syn} - r_t^{ois}) - (n-1)E_t^{\mathbb{Q}}[y_{n-1,t+1}^{s}]$$

$$ny_{n,t}^{s} \approx E_t^{\mathbb{Q}}\left[\sum_{j=0}^{n-7}(r_t^{sec} - (r_t^{syn} - r_t^{ois})) + \frac{6}{12}y_{t+n-6}^{bill}\right]$$

It is straightforward to show that equation (23) in the main text also leads to the same linear approximation.

C Data

C.1 Data Sources

We obtain the Treasury term structure from Bloomberg, for maturities 0.25, 0.5, 1, 3, 5, 10, 15, 20, and 30 years, all at daily frequency. The T-bills are from the ticker "GB", representing actively traded T-bill yields, and the non-bills are from the ticker "C082", representing the widely-used

[3] Subject to the caveat that the intermediary must believe in the zero lower bound. Our formulas can be readily generated to other (non-zero) lower bounds, at the expense of additional notation.

Bloomberg fair value Treasury yield curve.

We obtain OIS term structure denominated in USD from Bloomberg for maturities 0.25, 0.5, 1, 3, 5, 10, 15, 20, and 30 years, all at daily frequency. The ticker is "USSO" and data are from Nov 1996 to Dec 2021.

We construct the synthetic dollar lending rate from Euro (EUR). For this purpose, we obtain OIS term structure for EUR for maturities 1, 3, 5, 10, 15, 20, and 30 years. The EUR OIS data are from Aug 2009 to Dec 2021.

Then we obtain the above-one-year maturity LIBOR basis at a daily frequency for EUR-USD from Bloomberg. The EUR-USD LIBOR basis covers Nov 1999 to Dec 2021, and includes the following maturities (in years): 1, 3, 5, 10, 15, 20, 30. EUR 3-month LIBOR basis is from Bloomberg, and they are at daily frequency from Jan 2000 to Dec 2021.

To construct the OIS basis, we also collect EUR inter-bank interest-rate swap (IRS) term structure from Bloomberg at daily frequency. EUR IRS data are from Sep 1999 to Dec 2021. Then we construct the OIS basis for each maturity as

$$\text{EUR-USD OIS basis} = \text{EUR-USD LIBOR basis} + (\text{USD OIS} - \text{USD IRS}) - (\text{EUR OIS} - \text{EUR IRS})$$

where each term has the same maturity.

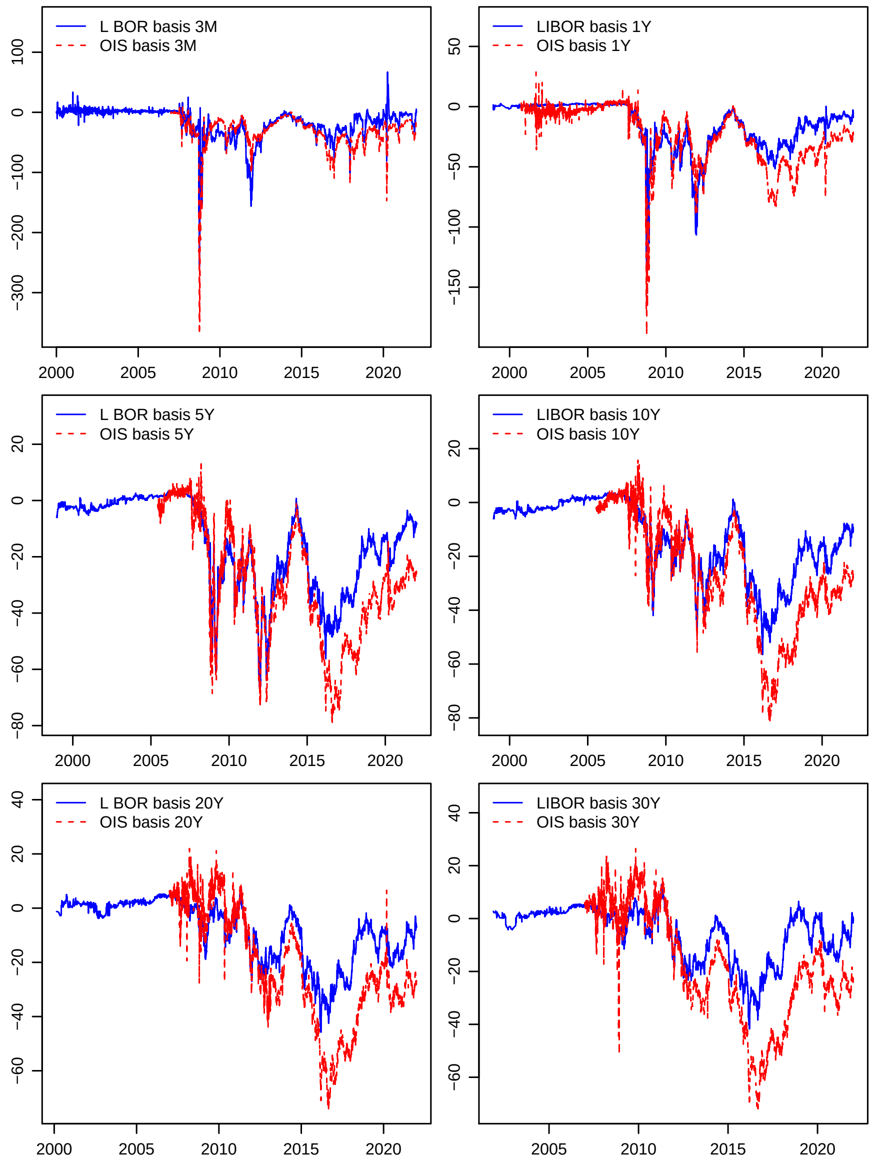
Due to data limitation, we use the “hybrid OIS basis”, defined as follows:

- Whenever OIS data are available, we construct the OIS basis from the LIBOR basis and LIBOR-OIS basis swap.
- When OIS data are not available (only happens before 2008), we use the LIBOR basis instead

This approach is essentially OIS basis throughout the whole sample period because OIS basis and LIBOR basis are almost the same before the global financial crisis. A comparison between the OIS basis and the LIBOR basis is shown in Figure A2.

On the quantity side, dealer net holdings are Treasury securities are based on the primary dealer statistics published by the Federal Reserve Bank of New York.

Figure A2: Comparison of LIBOR EUR Basis and OIS EUR Basis

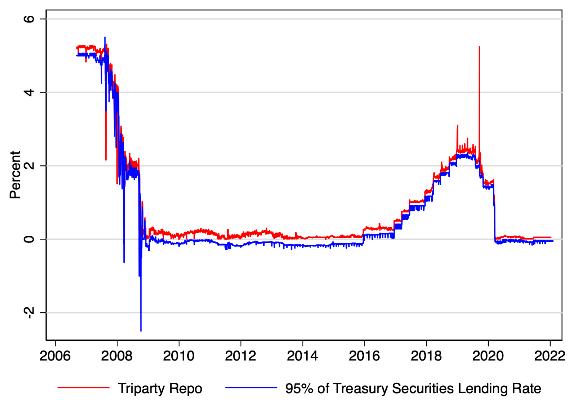


Notes: This figure illustrates the LIBOR EUR-USD basis and the OIS EUR-USD basis. The cross-currency basis is defined as the dollar rate minus the synthetic rate, which is exactly the opposite to the CIP violations we used in the model.

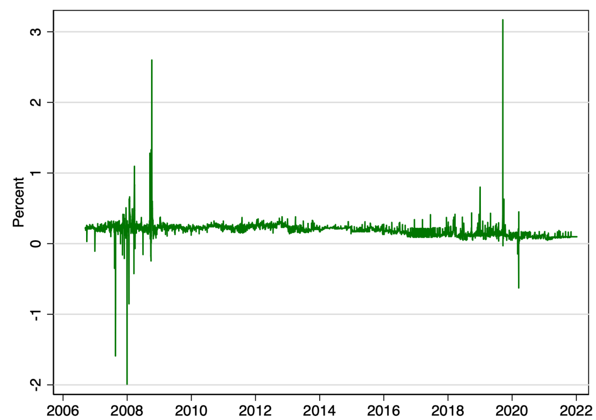
C.2 Treasury Securities Lending Rebate Rates

We use data from Market Securities Finance to calculate the rebate rate on the cash collateral when the dealer is borrowing Treasury bonds from a security lender. Figure A3 shows that the 95 percentile of all Treasury securities lending rebate rate is consistently below the triparty repo rate. The spread between triparty and rebate rate is about 20 basis points on average and quite stable throughout our sample.

Figure A3: Comparison Between Securities Lending Rebate and Triparty Repo Rates



(a) 95 pct of Rebate vs. Triparty



(b) Spread between 95 pct of Rebate and Triparty

Notes: Panel (a) plots the yield spread between the 10-year Treasury bond and the 3-month Treasury bill (in blue), and the primary dealers' net holdings of Treasury bonds. Panel (b) plots the relationship between the two variables post-2009 in a scatter plot.

D Functional Forms and Parameters for Figures

This appendix section describes the functional form and parameter assumptions used to generate Figures 14, 15, 16, 17, and 18. These functional form and parametric assumptions are for illustrative purposes only and do not represent a calibration of the model.

We assume a constant elasticity functional form for the Treasury demand curves. Note that both of these demand curves are functions of the hedged bond log risk premium $\pi_{n,H}$ (the expected excess log return using r^{syn} as the risk-free rate). We assume that

$$D_H(\pi_{n,H}) = D_{H,0}\exp(\eta_H \pi_{n,H}) \tag{D-1}$$

where $D_{H,0} > 0$ represents the demand at zero risk premium. The parameter $\eta_H > 0$ is the semi-elasticity of bond demand to the log risk premium.

We similarly assume that

$$D_U(\pi_{n,U}) = D_{U,0}\exp(\eta_U \pi_{n,U}), \tag{D-2}$$

where $\pi_{n,U}$ is the log risk premium with respect to Treasury bills, with $D_{U,0} > 0$ and $\eta_U > 0$. Note that $\pi_{n,H}$ and $\pi_{n,U}$ are log risk premia; an η_U or η_H of 50 implies a roughly $1.35x$ change in demand given a 1% excess return.

For the synthetic demand curve, we assume that

$$D^{syn}(x) = D_0^{syn} x^{-\xi}, \tag{D-3}$$

where $x = r^{syn} - r^{ois}$ is the spread in basis points and $\xi > 0$ is the elasticity of demand to the spread. This functional form imposes an Inada-type condition that ensures that demand is large as the spread becomes close to zero.

Note that these functional forms satisfy Assumption 1, irrespective of the parameters employed.

We use three sets of parameters to generate the figures used in the main text. The illustrative parameters are chosen to generate clear graphs, and in particular have the property that the regime can change given modest changes in term premium or bond supply. The pre-GFC parameter set perturbs this parameter set using a smaller Treasury supply, larger dealer balance sheet capacity, and larger repo-bill spread. The post-GFC parameter set uses instead a large bond supply, comparatively tight dealer balance sheet, and zero repo-bill spread.

The parameters are chosen under the assumption of an annual holding period and that the bond is a two-year bond ($n = 2$). The table below lists the sets of parameters we employ. Note that Figures 14 and 15 plot dealer indifference curves for different levels of $y_{\mathbb{Q}}$, holding all else constant. Likewise, Figures 16, 17, and 18 have the OIS term premium on the x-axis, which is equivalent to $y_{\mathbb{Q}}$ (holding $y_{\mathbb{P}}$ constant). For this reason, we do not list $y_{\mathbb{Q}}$ in the set of parameters below.

Table A1: Parameters for Figures

Parameter	Illustrative Value	Pre-GFC	Post-GFC
S^{bond}	10.5	9.5	14.5
$\bar{q}$	2	7	2
y^{bill} (bps)	95	65	95
$y_{\mathbb{P}}$ (bps)	95	65	95
r^{ois} (bps)	100		
r^{tri} (bps)	95		
h	2%		
r^{sec} (bps)	75		
D_0^{syn}	4		
ξ	1		
$D_{U,0}$	9.5		
$D_{H,0}$	0.5		
$\eta_U = \eta_H$	50		

E Details of the Term Structure Model

The term structure model consists $\mathbb{P}$ and $\mathbb{Q}$ dynamics

$$z_{t+1} = k_{0,z}^{\mathbb{P}} + K_{1,z}^{\mathbb{P}} \cdot z_t + (\Sigma_z)^{1/2} \varepsilon_{z,t+1}^{\mathbb{P}}, \varepsilon_{z,t+1}^{\mathbb{P}} \sim N(0, I_N),$$

$$z_{t+1} = k_{0,z}^{\mathbb{Q}} + K_{1,z}^{\mathbb{Q}} \cdot z_t + (\Sigma_z)^{1/2} \varepsilon_{z,t+1}^{\mathbb{Q}}, \varepsilon_{z,t+1}^{\mathbb{Q}} \sim N(0, I_N)$$

The state variable vector z_t is 5-by-1, include the first three PCs of OIS term structure (i_t^{PC1}, i_t^{PC2}, and i_t^{PC3}) and the first two PCs of the cross-currency basis term structure ($r_t^{cip,PC1}$ and

$r_t^{cip,PC2}$),

$$z_t = \begin{bmatrix} i_t^{PC1} \\ i_t^{PC2} \\ i_t^{PC3} \\ r_t^{cip,PC1} \\ r_t^{cip,PC2} \end{bmatrix}.$$

The monthly OIS rate and the monthly synthetic rate are both affine functions of the state vector,

$$\frac{1}{12} r_t^{ois} = \delta_0 + (\delta_1)^T z_t,$$
$$\frac{1}{12} r_t^{syn} = \hat{\delta}_0 + (\hat{\delta}_1)^T z_t,$$

For pricing Treasury securities, we also need the state vector $x_t = (x_{1,t}, x_{2,t}, x_{3,t})$, constructed from the data as

$$x_{1,t} = \ln((1-h)(e^{\frac{1}{12}r_t^{tri}} - e^{\frac{1}{12}r_t^{ois}}) + e^{\frac{1}{12}r_t^{syn}})$$
$$x_{2,t} = \ln(e^{\frac{1}{12}r_t^{sec}} + e^{\frac{1}{12}r_t^{ois}} - e^{\frac{1}{12}r_t^{syn}})$$
$$x_{3,t} = \frac{1}{12} y_t^{bill}$$

To operationalize the term structure model and reduce dimensionality, we assume that the vector x_t is affine in the state vector z_t,

$$x_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{bmatrix} = \gamma_0 + \Gamma_1 z_t + (\Sigma_x)^{\frac{1}{2}} \varepsilon_{x,t}, \varepsilon_{x,t} \sim N(0, I_3).$$

All state variables $x_{k,t}, k \in \{1,2,3\}$ represent yields at the monthly frequency. However, due to the lack of data, we use overnight tri-party rate and overnight security lending rate as proxies for the monthly counterparts. Furthermore, the one-month CIP basis is subject to a quarter-end effect, where the one-month CIP basis spikes at the end of each quarter due to capital regulation, as documented by Du et al. (2018b). To avoid such effect, we instead use the three-month CIP basis to construct the synthetic rate. The underlying assumption is that the rate difference due to maturity difference between one month and three months is negligible.

From our estimations, variance matrix Σ_x is close to zero (the maximum eigen value of Σ_x is about 7×10^{-5}, and much smaller than the maximum eigenvalue of Σ_z which is 4×10^{-3}). To simplify expositions, we set $\Sigma_z = 0$ and limit the actual state space to be five-dimensional. Thus, we will proceed with

$$x_t = \gamma_0 + \Gamma_1 z_t$$

In what follows, we first show the derivations of the OIS term structure and the basis term structure. Then we provide details on how the model generates dealer net long and net short curves. Next, we discuss the conversions between zero-coupon yields and par yields. Finally, we discuss how to estimate the model.

E.1 OIS Term Structure

The zero-coupon OIS term structure is the "risk-free rate" term structure in our model. Denote the swap rate as $i_{n,t}$. The swap exchanges floating payment pegged to the short-term OIS rate $i_t (\equiv r_t^{ois})$ to the fixed swap rate $i_{n,t}$. By construction, the floating leg and the fixed leg should have the same present value. Thus,

$$\exp(ni_{n,t})E_t^{\mathbb{Q}}[\exp(\sum_{k=1}^{n} -i_{t+k-1})] = 1$$

Conjecture

$$ni_{n,t} = A_n + B_n z_t$$

Then we have

$$\begin{aligned}
\exp(-n \cdot i_{n,t}) = \exp(-A_n - B_n z_t) &= E_t^{\mathbb{Q}}[\exp(-\sum_{k=1}^{n} i_{t+k-1})] \\
&= E_t^{\mathbb{Q}}[E_{t+1}^{\mathbb{Q}}[\exp(-\sum_{k=1}^{n-1} i_{(t+1)+k-1})]\exp(-i_t)] \\
&= E_t^{\mathbb{Q}}[\exp(-A_{n-1} - B_{n-1}z_{t+1} - \delta_0 - \delta_1 z_t)] \\
&= E_t^{\mathbb{Q}}[\exp(-A_{n-1} - B_{n-1}k_{0,z}^{\mathbb{Q}} - B_{n-1}K_{1,z}^{\mathbb{Q}} z_{t+1} + \frac{1}{2}B_{n-1}\Sigma_z(B_{n-1})^T - \delta_0 - \delta_1 z_t)]
\end{aligned}$$

which implies

$$
\begin{aligned}
A_n &= \delta_0 + A_{n-1} + B_{n-1} k^{\mathbb{Q}}_{0,z} - \frac{1}{2} B_{n-1} \Sigma_z (B_{n-1})^T \\
B_n &= \delta_1 + B_{n-1} K^{\mathbb{Q}}_{1,z}
\end{aligned}
$$

for all $n \geq 1$. The starting values are $A_0 = B_0 = 0$.

E.2 Synthetic-Rate Term Structure

We denote the synthetic rate as $i_{n,t} + r^{cip}_{n,t} \equiv r^{syn}_{n,t}$, i.e., composed of both OIS rate and the cross-currency basis. Conjecture that the cumulative synthetic rate is affine in the state vector,

$$
n(r^{cip}_{n,t} + i_{n,t}) = A^{syn}_n + B^{syn}_n z_t
$$

Then we have

$$
\begin{aligned}
&\exp(-n(r^{cip}_{n,t} + i_{n,t})) = \exp(-A^{syn}_n - B^{syn}_n z_t) \\
&= E^{\mathbb{Q}}_t[\exp(\sum_{k=1}^{n}(-r^{cip}_{t+k-1} - i_{t+k-1}))] \\
&= E^{\mathbb{Q}}_t[E^{\mathbb{Q}}_{t+1}[\exp(\sum_{k=1}^{n-1}(-r^{cip}_{(t+1)+k-1} - i_{(t+1)+k-1}))]\exp(-r^{cip}_t - i_t)] \\
&= E^{\mathbb{Q}}_t[\exp(-A^{syn}_{n-1} - B^{syn}_{n-1} z_{t+1} - (\delta_0 + \hat{\delta}_0) - (\delta_1 + \hat{\delta}_1) z_t)] \\
&= E^{\mathbb{Q}}_t[\exp(-A^{syn}_{n-1} - B^{syn}_{n-1} k^{\mathbb{Q}}_{0,z} - B^{syn}_{n-1} K^{\mathbb{Q}}_{1,z} z_{t+1} + \frac{1}{2} B^{syn}_{n-1} \Sigma_z (B^{syn}_{n-1})^T - (\delta_0 + \hat{\delta}_0) - (\delta_1 + \hat{\delta}_1) z_t)]
\end{aligned}
$$

The above equation is the present value of a CIP strategy that earns the CIP deviations, and the values is the same as the long-term CIP discounted at the long-term discount rate. Then we obtain the following iteration:

$$
\begin{aligned}
A^{syn}_n &= \delta_0 + \hat{\delta}_0 + A^{syn}_{n-1} + B^{syn}_{n-1} k^{\mathbb{Q}}_{0,z} - \frac{1}{2} B^{syn}_{n-1} \Sigma_z (B^{syn}_{n-1})^T \\
B^{syn}_n &= \delta_1 + \hat{\delta}_1 + B^{syn}_{n-1} K^{\mathbb{Q}}_{1,z}
\end{aligned}
$$

with the starting values $A^{syn}_0 = B^{syn}_0 = 0$.

E.3 Treasury Net Long Curve

Next, we derive the iteration steps for the Treasury net long curve.

$$e^{-\frac{n}{12}y^l_{n,t}}e^{x_{1,t}} = E_t^Q[e^{-\frac{n-1}{12}y^l_{n-1,t+1}}]$$

We use $\iota_1 = (1,0,0)$ to denote the indicator vector of the first element, so $x_{1,t} = \iota_1 x_t = \iota_1(\gamma_0 + \Gamma_1 z_t)$. Conjecture that the cumulative yield is affine in the state vector,

$$\frac{n}{12}y^l_{n,t} = A^l_n + B^l_n z_t$$

For all $n \geq 7$, the iteration is

$$\begin{aligned}
\exp(-\left(A^l_n + B^l_n z_t\right)) &= E_t^Q[e^{-\frac{n-1}{12}y^l_{n-1,t+1} - \iota_1(\gamma_0+\Gamma_1 z_t)}] \\
&= E_t^Q[\exp(-\left(A^l_{n-1} + B^l_{n-1}z_{t+1} + \iota_1(\gamma_0 + \Gamma_1 z_t)\right))] \\
&= E_t^Q[\exp(-\left(A^l_{n-1} + B^l_{n-1}(k^{\mathbb{Q}}_{0,z} + K^{\mathbb{Q}}_{1,z} \cdot z_t + (\Sigma_z)^{1/2}\varepsilon^{\mathbb{Q}}_{z,t+1}) + \iota_1(\gamma_0 + \Gamma_1 z_t)\right))] \\
&= E_t^Q[\exp(-\left(A^l_{n-1} + B^l_{n-1}k^{\mathbb{Q}}_{0,z} + \iota_1\gamma_0 - \frac{1}{2}B^l_{n-1}\Sigma_z(B^l_{n-1})' + (B^l_{n-1}K^{\mathbb{Q}}_{1,z} + \iota_1\Gamma_1)\cdot z_t\right))]
\end{aligned}$$

which implies the iteration equation

$$\begin{aligned}
A^l_n &= \iota_1\gamma_0 + A^l_{n-1} + B^l_{n-1}k^{\mathbb{Q}}_{0,z} - \frac{1}{2}B^l_{n-1}\Sigma_z(B^l_{n-1})^T \\
B^l_n &= \iota_1\Gamma_1 + B^l_{n-1}K^{\mathbb{Q}}_{1,z}
\end{aligned}$$

At $n = 6$, we have

$$\frac{6}{12}y^l_{6,t} = \frac{6}{12}y^{bill}_t = 6x_{3,t} = 6\iota_3(\gamma_0 + \Gamma_1 z_t)$$

with initial values

$$A^l_6 = 6\iota_3\gamma_0, \quad B^l_6 = 6\iota_3\Gamma_1$$

E.4 Treasury Net Short Curve

Next, we derive the iteration steps for the Treasury net short curve.

$$e^{-\frac{n}{12}y^s_{n,t}}e^{x_{2,t}} = E^Q_t\left[e^{-\frac{n-1}{12}y^s_{n-1,t+1}}\right]$$

Similar arguments as in the last section will lead to cumulative yield

$$\frac{n}{12}y^s_{n,t} = A^s_n + B^s_n z_t$$

where

$$A^s_n = \iota_2\gamma_0 + A^s_{n-1} + B^s_{n-1}k^{\mathbb{Q}}_{0,z} - \frac{1}{2}B^s_{n-1}\Sigma_z(B^s_{n-1})^T$$
$$B^s_n = \iota_2\Gamma_1 + B^s_{n-1}K^{\mathbb{Q}}_{1,z}$$

At $n = 6$, we have

$$\frac{6}{12}y^s_{6,t} = \frac{6}{12}y^{bill}_t = 6x_{3,t} = 6\iota_3(\gamma_0 + \Gamma_1 z_t)$$

with initial values

$$A^s_6 = 6\iota_3\gamma_0, \quad B^s_6 = 6\iota_3\Gamma_1$$

E.5 Par Curve and Zero Curve Conversion

In our term structure model, all the yields are zero-coupon yields. In the data, on the other hand, yields are par yields. The ideal way to resolve the mismatch is asking the model to convert all zero-coupon yields into par yields. However, the model is solved thousands of times when we estimate it, and the extra conversion significantly slows the estimation process. Thus, we do the following:

- We convert the OIS term structure and the CIP basis term structure into zero-coupon yields for model estimation purpose.
- Once we finish estimating the model, then we generate the net long and net short zero-coupon curves, and convert them into par yields.

For the par-to-zero conversion, we follow the standard Svensson (1994) method that fits the whole yield curve with a parsimonious functional form and infer the zero yields.

For the zero-to-par conversion, we directly use the definition. We want to transform the annualized zero-coupon yields $r_{n,t}$ into annualized par yields $r^{par}_{n,t}$ with coupon payment every 6 months. Then for a coupon-bond of maturity n, the pricing relationship is

$$q^{par}_{n,t} = \frac{r^{par}_{n,t}}{2}\left(e^{-\frac{6}{12}r_{t,6}} + e^{-\frac{12}{12}r_{t,12}} + \cdots + e^{-\frac{n}{12}r_{n,t}}\right) + e^{-\frac{n}{12}r_{n,t}} \tag{E-1}$$

For a bond at the par, the price is $q^{par}_{n,t} = 1$, indicating the par yield as

$$r^{par}_{n,t} = 2 \times \frac{1 - e^{-\frac{n}{12}r_{n,t}}}{e^{-\frac{6}{12}r_{t,6}} + e^{-\frac{12}{12}r_{t,12}} + \cdots + e^{-\frac{n}{12}r_{n,t}}} \tag{E-2}$$

E.6 Model Estimation

We estimate the model to fit the OIS and basis term structure. Then we use regression-implied coefficients γ_0 and Γ_1 to obtain the model-implied net long and net short curves. Denote the observed OIS yield of maturity n at time t as

$$\hat{i}_{n,t} = i_{n,t} + e^{ois}_{n,t}, \quad e^{ois}_t \sim \mathcal{N}(0, \Sigma_{ois})$$

and the observed basis as

$$\hat{r}^{cip}_{n,t} = r^{cip}_{n,t} + e^{basis}_{n,t}, \quad e^{basis}_t \sim \mathcal{N}(0, \Sigma_{basis})$$

We denote the stacked OIS yields (across different maturities) as $\hat{i}_t$, and the stacked basis rates as $\hat{r}^{cip}_t$. For the estimation step, the set of parameters is $\Theta = \{k^{\mathbb{Q}}_{0,z}, K^{\mathbb{Q}}_{1,z}, k^{\mathbb{P}}_{0,z}, K^{\mathbb{P}}_{1,z}, \Sigma_z, \Sigma_{ois}, \Sigma_{basis}, \delta_0, \delta_1, \hat{\delta}_0, \hat{\delta}_1\}$. The objective of the estimation is to maximize the log likelihood that the observed yields are generated by the model,

$$\mathcal{L}(\{\hat{i}_t, \hat{r}^{cip}_t, z_t\}_{t\in \text{data}}; \Theta)$$

Denote the log likelihood an N-variable normal variable Z with mean μ and variance matrix Σ as $\mathscr{G}(Z,\mu,\Sigma)$. Then the objective function is

$$\mathscr{L}(\{\hat{i}_t,\hat{r}_t^{cip},z_t\}_{t\in\text{data}};\Theta)=$$
$$\sum_{t\in\text{data}}\left(\underbrace{\mathscr{G}(z_t-k_{0,z}^{\mathbb{P}}-K_{1,z}^{\mathbb{P}}\cdot z_{t-1},0,\Sigma_z)}_{\text{state variable physical dynamics}}+\underbrace{\mathscr{G}(\hat{i}_t-i_t,0,\Sigma_{ois})}_{\text{OIS fitting}}+\underbrace{\mathscr{G}(\hat{r}_t^{cip}-r_t^{cip},0,\Sigma_{basis})}_{\text{basis fitting}}\right)$$

The whole estimation problem is thus

$$\max_{k_{0,z}^{\mathbb{Q}},K_{1,z}^{\mathbb{Q}},k_{0,z}^{\mathbb{P}},K_{1,z}^{\mathbb{P}},\Sigma_z,\Sigma_{ois},\Sigma_{basis},\delta_0,\delta_1,\hat{\delta}_0,\hat{\delta}_1}\mathscr{L}(\{\hat{i}_t,\hat{r}_t^{cip},z_t\}_{t\in\text{data}};\Theta) \tag{E-3}$$

To reduce dimensionality, we assume that the covariance matrices for observation errors are in the form of $\Sigma_{ois}=\sigma_{ois}I$ and $\Sigma_{basis}=\sigma_{basis}I$.

Compared to the classical term structure estimation problem, the key challenge of this problem is that we need to estimate two inter-linked term structures simultaneously. However, the canonical form transformation in Joslin et al. (2011) only applies to one term structure. To resolve the challenge and at the same time taking advantage of the canonical form, we design the following two-step procedure that applies the canonical form to each individual term struccture as initialization (the initial values for this high-dimensional optimization problem are quite important):

1. Divide the state-space into two blocks, an OIS block, $z_t^{ois}=(z_{1,t},z_{2,t},z_{3,t})$, and a basis block $z_t^{basis}=(z_{4,t},z_{5,t})$. Similarly, we denote the associated sub-group risk-neutral dynamic parameters as $k_{0,z^{ois}}^{\mathbb{Q}},K_{1,z^{ois}}^{\mathbb{Q}}$ and $k_{0,z^{basis}}^{\mathbb{Q}},K_{1,z^{basis}}^{\mathbb{Q}}$. Denote the sub-group physical dynamic parameters as $k_{0,z^{ois}}^{\mathbb{P}},K_{1,z^{ois}}^{\mathbb{P}}$ and $k_{0,z^{basis}}^{\mathbb{P}},K_{1,z^{basis}}^{\mathbb{P}}$. Also divide the observations into the OIS group and basis group. Then apply the standard canonical form estimation procedure to two models separately,

$$\mathscr{L}(\{\hat{i}_t,z_t\}_{t\in\text{data}}^{ois};k_{0,z^{ois}}^{\mathbb{Q}},K_{1,z^{ois}}^{\mathbb{Q}},k_{0,z^{ois}}^{\mathbb{P}},K_{1,z^{ois}}^{\mathbb{P}},\Sigma_z^{ois},\Sigma_{ois},\delta_0^{ois},\delta_1^{ois})$$

$$\mathscr{L}(\{\hat{i}_t,z_t\}_{t\in\text{data}}^{basis};k_{0,z^{basis}}^{\mathbb{Q}},K_{1,z^{basis}}^{\mathbb{Q}},k_{0,z^{basis}}^{\mathbb{P}},K_{1,z^{basis}}^{\mathbb{P}},\Sigma_z^{basis},\Sigma_{basis},\delta_0^{basis},\delta_1^{basis})$$

where the short rate in the first estimation is $\delta_0^{ois}+\delta_1^{ois}*z_t^{ois}$, and the short rate in the second estimation is $\delta_0^{basis}+\delta_1^{basis}*z_t^{basis}$ is a two-dimensional vector that loads on z_t^{basis}. The covariance matrix Σ_z^{ois} is 3×3 and Σ_z^{basis} is 2×2. After estimating the above dynamics, we construct an initialization of the original problem as

$$k_{0,z}^{\mathbb{Q}} = \begin{pmatrix} k_{0,z^{ois}}^{\mathbb{Q}} \\ k_{0,z^{basis}}^{\mathbb{Q}} \end{pmatrix}, \quad K_{1,z}^{\mathbb{Q}} = \begin{pmatrix} K_{1,z^{ois}}^{\mathbb{Q}} & \\ & K_{1,z^{basis}}^{\mathbb{Q}} \end{pmatrix}, \quad \Sigma_z = \begin{pmatrix} \Sigma_z^{ois} & \\ & \Sigma_z^{basis} \end{pmatrix}$$

$$\delta_0 = \delta_0^{ois}, \quad \delta_1 = \begin{pmatrix} \delta_1^{ois} \\ 0 \\ 0 \end{pmatrix}, \quad \hat{\delta}_0 = \delta_0^{ois} + \delta_0^{basis}, \quad \hat{\delta}_1 = \begin{pmatrix} \delta_1^{ois} \\ \delta_1^{basis} \end{pmatrix}$$

We initialize the physical dynamic parameters $(k_{0,z}^{\mathbb{P}}, K_{1,z}^{\mathbb{P}})$ simply from linear regressions,

$$z_t \sim k_{0,z}^{\mathbb{P}} + K_{1,z}^{\mathbb{P}} \cdot z_{t-1}$$

2. Then we feed these initial values to the whole estimation problem (E-3), and apply the optimization package in Matlab to optimize over the whole high-dimensional parameter space. We use the equivalent implementation of the CIP short rate (instead of the synthetic lending short rate), $r_t^{syn} - r_t^{ois}$, and the corresponding loading $\hat{\delta}_0 - \delta_0 + (\hat{\delta}_1 - \delta_1) z_t$.

After we finish estimating the key parameter set Θ, we proceed to obtain γ_0 and Γ_1 via a simple linear regressions,

$$x_t \sim \gamma_0 + \Gamma_1 z_t$$

We find that the residual standard errors for this linear regression are one order of magnitude smaller than Σ_z. In other words, we are able to obtain very accurate approximation of x_t through the state vector z_t, so adding the extra estimation error to the above approximation in the model will not cause much difference, but it requires augmenting the state space. For this reason, we make the assumption that x_t is spanned by z_t in the main model.

Finally with estimated Θ and (γ_0, Γ_1), we are able to obtain the Treasury net long and net short curves. We convert these curves into par curves to be comparable with the Treasury yield data.

E.7 Stationarity Restrictions

Treasury yields can be non-stationary, but the spread between an OIS rate and the matched-maturity Treasury yield cannot diverge due to arbitrage incentives in financial markets. Our main approach does not impose such a restriction for simplicity. In this subsection, we discuss how to impose stationarity on the process x_t and show that results are broadly similar.

First, our estimation reveals that z_t contains unit-root processes. In particular, the $\mathbb{Q}$-dynamics of z_t contains unit-root elements. Denote the eigenvalue decomposition of $K_{1,z}^{\mathbb{Q}}$ as

$$K_{1,z}^{\mathbb{Q}} = VDV^{-1}$$

where D is an diagonal matrix that contains all the eigenvalues of $K_{1,z}^{\mathbb{Q}}$, and V is the matrix of all the column eigenvectors for $K_{1,z}^{\mathbb{Q}}$. We find that two among the five eigenvalues have absolute values above 0.999, which is a strong sign of unit root.

To operationalize the stationarity restriction, we rotate the state vector z_t to $\tilde{z}_t = V^{-1} z_t$, and rewrite the $\mathbb{Q}$-dynamics in (17) as

$$\tilde{z}_{t+1} = V^{-1} k_{0,z}^{\mathbb{Q}} + D\tilde{z}_t + V^{-1}(\Sigma_z)^{1/2} \varepsilon_{z,t+1}^{\mathbb{Q}}, \varepsilon_{z,t+1}^{\mathbb{Q}} \sim N(0, I_N)$$

We denote the spread vector as

$$\hat{x}_t = x_t - \begin{pmatrix} r_t^{ois} + r_t^{cip} \\ r_t^{ois} - r_t^{cip} \\ r_t^{ois} \end{pmatrix}$$

Then we project $\hat{x}_t$ on the stationary components of $\tilde{z}_t$, i.e., three of five with (absolute values of) eigenvalues below 0.999. The loadings on the non-stationary components are set as zeros. Then we denote the whole projection as

$$\hat{x}_t = \tilde{\gamma}_0 + \tilde{\Gamma}_1 \tilde{z}_t$$

Next, we rotate back to z_t,

$$\hat{x}_t = \tilde{\gamma}_0 + \tilde{\Gamma}_1 V^{-1} z_t$$

Thus, we obtain

$$x_t = \tilde{\gamma}_0 + \tilde{\Gamma}_1 V^{-1} z_t + \begin{pmatrix} r_t^{ois} + r_t^{cip} \\ r_t^{ois} - r_t^{cip} \\ r_t^{ois} \end{pmatrix}$$

In the implementation, we find that there are complex-number eigenvalues, so the resulting $\tilde{\gamma}_0$ and $\tilde{\Gamma}_1 V^{-1}$ are also complex numbers. Nevertheless, the imaginary parts are quite small so we only keep the real parts.

With the projection of x_t on z_t, we are able to derive the Treasury net long and net short curves. We illustrate the results in Figure A4. We find that results are very close to the baseline results in Figure 11. Furthermore, all other results, such as the relative yield index matching the movements in dealer position, are quite similar. For conciseness, we omit other results in this appendix.

F Proofs

F.1 Proof of Proposition 1 (Long Regime)

Define the function

$$f_1^{long}(y, r^{syn}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}) = e^{-(ny-(n-1)\varepsilon)} - \frac{\exp(-(n-1)y_{\mathbb{Q}})}{(1-h)(e^{r^{tri}} - e^{r^{ois}}) + e^{r^{syn}}}$$

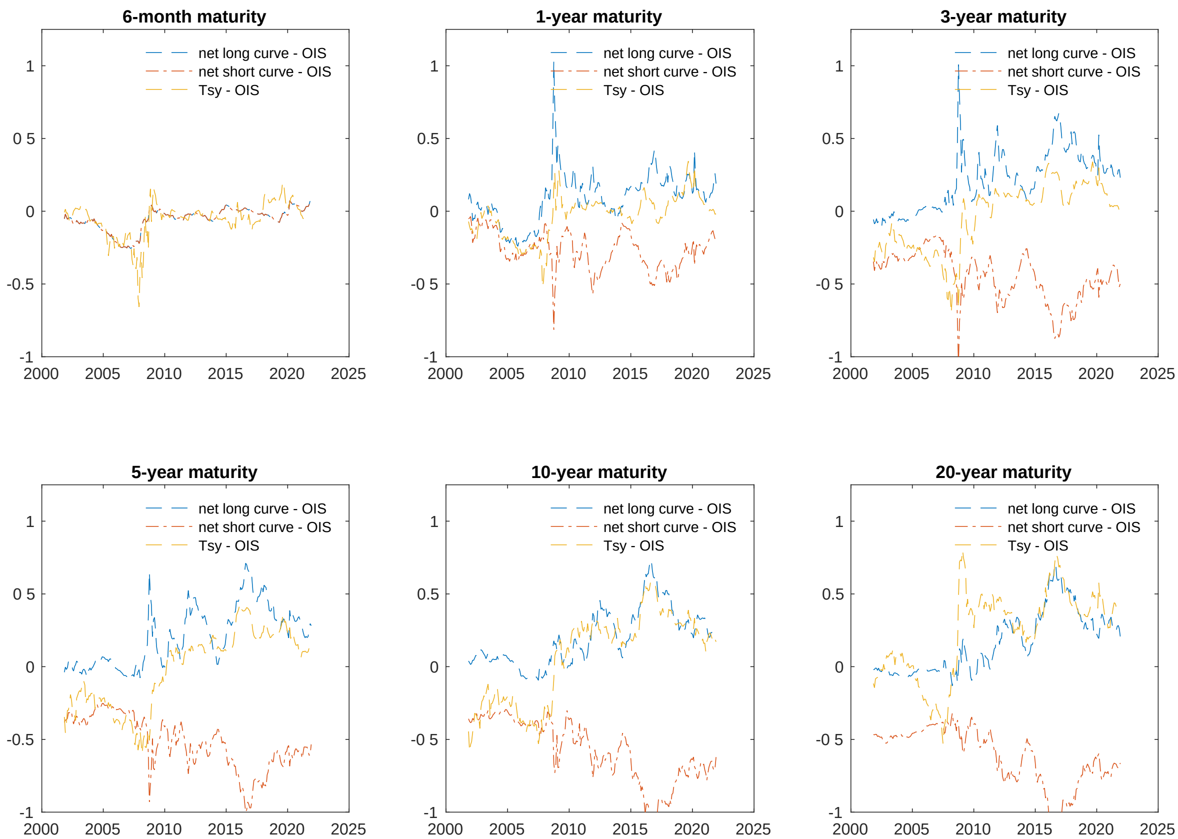
and the function

$$\begin{aligned} &f_2^{long}(y, r^{syn}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}) = \\ &\bar{q} - e^{-(ny-(n-1)(\varepsilon-\omega))} S^{bond} + D_U(ny - y^{bill} - (n-1)(y_{\mathbb{P}} + \varepsilon)) + \delta_U - \left(D^{syn}(r^{syn} - r^{ois}) + \delta_{syn}\right). \end{aligned}$$

By assumption, D_U and D^{syn} are continuously differentiable, and hence f_1 and f_2 are continuously differentiable.

Suppose there exists, given the exogenous values $y_{\mathbb{P}}, r^{ois}, r^{tri}, y^{bill}$ and some initial point ($S^{bond} > 0, \bar{q} > 0, y_{\mathbb{Q}}, \varepsilon = 0, \omega = 0, \delta_U = 0, \delta_H = 0, \delta_{syn} = 0$), a solution

$$\begin{bmatrix} f_1^{long}(y^*, r^{syn*}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, 0,0,0,0,0) \\ f_2^{long}(y^*, r^{syn*}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, 0,0,0,0,0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$



Notes: In this figure, we show the model-implied long and short Treasury curves minus the OIS rates for corresponding maturities, together with the actual Treasury–OIS spreads. We use the alternative projection method as in Appendix Section E.7. Data are from 2003 to 2021. All yields are par yields.

such that

$$D_H(ny^* - r^{syn*} - (n-1)y_{\mathbb{P}}) + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) < e^{-ny^*} S^{bond}.$$

Such a point constitutes an equilibrium.

Observe that

$$\frac{\partial f_1^{long}(\cdot)}{\partial y} < 0, \frac{\partial f_1^{long}(\cdot)}{\partial r^{syn}} > 0, \frac{\partial f_2^{long}(\cdot)}{\partial y} > 0, \frac{\partial f_2^{long}(\cdot)}{\partial r^{syn}} > 0,$$

and consequently

$$\begin{bmatrix} \frac{\partial f_1^{long}(\cdot)}{\partial y} & \frac{\partial f_1^{long}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{long}(\cdot)}{\partial y} & \frac{\partial f_2^{long}(\cdot)}{\partial r^{syn}} \end{bmatrix}$$

is invertible (its determinant is strictly negative).

It follows that the equilibrium (y^*, r^{syn*}), if it exists, is unique. Suppose not and there exists another equilibrium (y, r^{syn}) in the long regime. If $r^{syn} > r^{syn*}$, then $\tilde{y} > y^*$ according to $f_1^{long}(\tilde{y}, r^{syn}) = 0$. By monotonicity of f_2^{long}, we have $f_2^{long}(y, r^{syn}) > f_2^{long}(y^*, r^{syn*}) = 0$, which contradicts to (y, r^{syn}) being an equilibrium. A symmetric argument rules out all $r^{syn} < r^{syn*}$. Thus, the equilibrium solution to y^* is unique. Strict monotonicity ensures the uniqueness of y^*.

By the implicit function theorem,

$$\begin{bmatrix} \frac{\partial y^*(\cdot)}{\partial x} \\ \frac{\partial r^{syn*}(\cdot)}{\partial x} \end{bmatrix} = - \begin{bmatrix} \frac{\partial f_1^{long}(\cdot)}{\partial y} & \frac{\partial f_1(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2(\cdot)}{\partial y} & \frac{\partial f_2(\cdot)}{\partial r^{syn}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f_1^{long}(\cdot)}{\partial x} \\ \frac{\partial f_2^{long}(\cdot)}{\partial x} \end{bmatrix}$$

for any $x \in \{S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}\}$. Observe that the signs of the negative inverse matrix are

$$sgn\left(- \begin{bmatrix} \frac{\partial f_1^{long}(\cdot)}{\partial y} & \frac{\partial f_1^{long}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{long}(\cdot)}{\partial y} & \frac{\partial f_2^{long}(\cdot)}{\partial r^{syn}} \end{bmatrix}^{-1} \right) = sign\left(\begin{bmatrix} \frac{\partial f_2^{long}(\cdot)}{\partial r^{syn}} & -\frac{\partial f_1^{long}(\cdot)}{\partial r^{syn}} \\ -\frac{\partial f_2^{long}(\cdot)}{\partial y} & \frac{\partial f_1^{long}(\cdot)}{\partial y} \end{bmatrix} \right) = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}.$$

We solve for the comparative statics as follows:

1. An increase in S^{bond}: $\frac{\partial f_1^{long}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} < 0$, and therefore $\frac{\partial y^*(\cdot)}{\partial x} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.

2. A decrease in $\bar{q}$ or a decrease in δ_U: $\frac{\partial f_1^{long}(\cdot)}{\partial x} = 0$ and

$$\frac{\partial f_2^{long}(\cdot)}{\partial x} = -\frac{\partial f_2^{long}(\cdot)}{e^{-ny} \cdot \partial(S^{bond})}.$$

   Thus, the decrease in $\bar{q}$ or δ_U is equivalent to the same same size expansion in the dollar supply of bonds.

3. An increase in δ_H has $\frac{\partial f_1^{long}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} = 0$, and therefore $\frac{\partial y^*(\cdot)}{\partial x} = 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} = 0$.

4. An increase in $y_{\mathbb{Q}}$ has $\frac{\partial f_1^{long}(\cdot)}{\partial x} > 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} = 0$ and thus $\frac{\partial y^*(\cdot)}{\partial x} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} < 0$.

5. An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ is equivalent to an increase ε by $\Delta\varepsilon = dy$ in both f_1 and f_2 and an increase in ω by $\Delta\omega = dy$. The increase in ε causes an $\frac{n-1}{n}\Delta\varepsilon$ increase in y and no change in r^{syn}. The increase in ω has $\frac{\partial f_1^{long}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} > 0$, and thus thus $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} < 0$. Taking the two effects together, clearly r^{syn*} will decrease. To determine the sign on y^*, we can evaluate the change of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ directly and obtain $\frac{\partial f_1^{long}(\cdot)}{\partial x} > 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} < 0$, which implies y^* will increase. In summary, we find that the increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ increases y^* by less than $\frac{n-1}{n}\Delta\varepsilon$ and decreases r^{syn*}. Furthermore, the absolute value of the effect of ω is smaller than that of ε, indicating that the total effect is still to increase bond yield.

   Taking the two effects together, we find that the increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ increase y^* by less than $\frac{n-1}{n}\Delta\varepsilon$ and decreases r^{syn*}.

6. An increase in δ_{syn} has $\frac{\partial f_1^{long}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} < 0$, and thus $\frac{\partial y^*(\cdot)}{\partial x} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.

F.2 Proof of Proposition 2 (Short Regime)

Define the function

$$f_1^{short}(y, r^{syn}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}) = e^{-(ny-(n-1)\varepsilon)} - \frac{\exp(-(n-1)y_{\mathbb{Q}})}{e^{r^{sec}} + e^{r^{ois}} - e^{r^{syn}}}$$

and the function

$$
\begin{aligned}
f_2^{short}(y, r^{syn}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}) =& \bar{q} + e^{-(ny-(n-1)(\varepsilon-\omega))} S^{bond} \\
& - D_U(ny - y^{bill} - (n-1)(y_{\mathbb{P}} + \varepsilon)) - \delta_U \\
& - \left(D^{syn}(r^{syn} - r^{ois}) + \delta_{syn}\right) \\
& - 2(D_H(ny - y^{syn} - (n-1)(y_{\mathbb{P}} + \varepsilon)) + \delta_H)
\end{aligned}
$$

By assumption, D_U, D_H, and D^{syn} are continuously differentiable, and hence f_1^{short} and f_2^{short} are continuously differentiable.

Suppose there exists, given the exogenous values $y_{\mathbb{P}}, r^{ois}, r^{tri}, y^{bill}$ and some initial point $(S^{bond} > 0, \bar{q} > 0, y_{\mathbb{Q}}, \varepsilon = 0, \omega = 0, \delta_U = 0, \delta_H = 0, \delta_{syn} = 0)$, a solution

$$
\begin{bmatrix} f_1^{short}(y^*, r^{syn*}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, 0, 0, 0, 0, 0) \\ f_2^{short}(y^*, r^{syn*}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, 0, 0, 0, 0, 0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},
$$

such that

$$
D_H(ny^* - r^{syn*} - (n-1)y_{\mathbb{P}}) + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) > e^{-ny^*} S^{bond}.
$$

Such a point constitutes an equilibrium.

Observe that

$$
\frac{\partial f_1^{short}(\cdot)}{\partial y} < 0, \frac{\partial f_1^{short}(\cdot)}{\partial r^{syn}} < 0, \frac{\partial f_2^{short}(\cdot)}{\partial y} < 0, \frac{\partial f_2^{short}(\cdot)}{\partial r^{syn}} > 0,
$$

and consequently

$$
\begin{bmatrix} \frac{\partial f_1^{short}(\cdot)}{\partial y} & \frac{\partial f_1^{short}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{short}(\cdot)}{\partial y} & \frac{\partial f_2^{short}(\cdot)}{\partial r^{syn}} \end{bmatrix}
$$

is invertible (its determinant is strictly negative).

It follows that the equilibrium (y^*, r^{syn*}), if it exists, is unique. Suppose not and there exists another pair (r^{syn}, y) that satisfies the eqilibrium in the short regime. If $r^{syn} > r^{syn*}$, we must have $y < y^*$ due to $f_1^{short}(y, r^{syn}) = f_1^{short}(y^*, r^{syn*})$. (if no such $\tilde{y}$ exists, r^{syn} cannot be part of an equilibrium). It follows that $f_2^{short}(y, r^{syn}) > f_2^{short}(y^*, r^{syn*}) = 0$, and hence r^{syn} cannot be part of an equilibrium. A symmetric argument rules out all $r^{syn} < r^{syn*}$, and strict monotonicity ensures the uniqueness of y^*.

By the implicit function theorem,

$$\begin{bmatrix} \frac{\partial y^*(\cdot)}{\partial x} \\ \frac{\partial r^{syn*}(\cdot)}{\partial x} \end{bmatrix} = - \begin{bmatrix} \frac{\partial f_1^{short}(\cdot)}{\partial y} & \frac{\partial f_1^{short}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{short}(\cdot)}{\partial y} & \frac{\partial f_2^{short}(\cdot)}{\partial r^{syn}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f_1^{short}(\cdot)}{\partial x} \\ \frac{\partial f_2^{short}(\cdot)}{\partial x} \end{bmatrix}$$

for any $x \in \{S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H\}$. Observe that the signs of the negative inverse matrix are

$$sgn\left(- \begin{bmatrix} \frac{\partial f_1^{short}(\cdot)}{\partial y} & \frac{\partial f_1^{short}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{short}(\cdot)}{\partial y} & \frac{\partial f_2^{short}(\cdot)}{\partial r^{syn}} \end{bmatrix}^{-1} \right) = sign\left(\begin{bmatrix} \frac{\partial f_2^{short}(\cdot)}{\partial r^{syn}} & -\frac{\partial f_1^{short}(\cdot)}{\partial r^{syn}} \\ -\frac{\partial f_2^{short}(\cdot)}{\partial y} & \frac{\partial f_1^{short}(\cdot)}{\partial y} \end{bmatrix} \right) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

We solve for the comparative statics as follows:

1. An increase in S^{bond}: $\frac{\partial f_1^{short}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{short}(\cdot)}{\partial x} > 0$, and therefore $\frac{\partial y^*(\cdot)}{\partial x} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} < 0$. Thus, bond yield y^* increases, but the synthetic rate r^{syn*} decreases.
2. An increase in $\bar{q}$ or a decrease in δ_U (i.e., a parallel decrease in D_U): $\frac{\partial f_1^{short}(\cdot)}{\partial x} = 0$ and

$$\frac{\partial f_2^{short}(\cdot)}{\partial \bar{q}} = -\frac{\partial f_2^{short}(\cdot)}{\partial \delta_U} = \frac{\partial f_2^{short}(\cdot)}{e^{-ny} \cdot \partial (S^{bond})}.$$

   Thus, the increase in $\bar{q}$ or the same decrease in δ_U are equivalent to the same same size expansion in the dollar supply of bonds.
3. An increase in δ_H has $\frac{\partial f_1^{short}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{short}(\cdot)}{\partial x} < 0$, and therefore $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.
4. An increase in $y_{\mathbb{Q}}$ has $\frac{\partial f_1^{short}(\cdot)}{\partial x} > 0, \frac{\partial f_2^{short}(\cdot)}{\partial x} = 0$ and thus $\frac{\partial y^*(\cdot)}{\partial x} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.
5. An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$: this change is equivalent to an increase ε by dy in both f_1^{short} and f_2^{short} and an increase in ω by dy. The increase in ε causes an $\frac{n-1}{n}\Delta\varepsilon$ decrease in y and no change in r^{syn*}. The increase in ω has $\frac{\partial f_1^{short}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{short}(\cdot)}{\partial x} < 0$, and thus thus $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$. Taking the two effects together, clearly r^{syn*} will increase. To determine the sign on y^*, we can evaluate the change of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ directly and

obtain $\frac{\partial f_1^{short}(\cdot)}{\partial x} > 0, \frac{\partial f_2^{short}(\cdot)}{\partial x} > 0$, which implies y^* will increase. In summary, we find that the increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ increases y^* by less than $\frac{n-1}{n}\Delta\varepsilon$ and increases r^{syn*}.

6. An increase in δ_{syn} has $\frac{\partial f_1^{long}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{long}(\cdot)}{\partial x} < 0$, and thus $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.

F.3 Proof of Proposition 3 (Intermediate Regime)

Define the functions

$$f_1^{int}(y, r^{syn}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}) = \bar{q} - e^{-(ny-(n-1)(\varepsilon-\omega))} S^{bond} - \left(D^{syn}(r^{syn} - r^{ois}) + \delta_{syn}\right) + D_U(ny - y^{bill} - (n-1)(y_{\mathbb{P}} + \varepsilon)) + \delta_U$$

and

$$f_2^{int}(y, r^{syn}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H, \delta_{syn}) = \bar{q} - (D_H(ny - r^{syn} - (n-1)(y_{\mathbb{P}} + \varepsilon)) + \delta_H) - \left(D^{syn}(r^{syn} - r^{ois}) + \delta_{syn}\right).$$

By assumption, D_U, D_H, and D^{syn} are continuously differentiable, and hence f_1^{int} and f_2^{int} are continuously differentiable.

Suppose there exists, given the exogenous values $y_{\mathbb{P}}, r^{ois}, r^{tri}, y^{bill}$ and some initial point ($S^{bond} > 0, \bar{q} > 0, y_{\mathbb{Q}}, \varepsilon = 0, \omega = 0, \delta_U = 0, \delta_H = 0, \delta_{syn} = 0$), a solution

$$\begin{bmatrix} f_1^{int}(y^*, r^{syn*}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, 0, 0, 0, 0, 0) \\ f_2^{int}(y^*, r^{syn*}; S^{bond}, \bar{q}, y_{\mathbb{Q}}, 0, 0, 0, 0, 0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

such that

$$y^s < y^* < y^l$$

Such a point constitutes an interior equilibrium.

Observe that

$$\frac{\partial f_1^{int}(\cdot)}{\partial y} = n e^{-ny} S^{bond} + n D_U' > 0$$

$$\frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} = -(D^{syn})' > 0$$

$$\frac{\partial f_2^{int}(\cdot)}{\partial y} = -nD_H' < 0$$

$$\frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} = D_H' - (D^{syn})' > 0$$

Then the determinant of the derivative matrix

$$\begin{bmatrix} \frac{\partial f_1^{int}(\cdot)}{\partial y} & \frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{int}(\cdot)}{\partial y} & \frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} \end{bmatrix}$$

is positive, which implies that the derivative matrix is invertible.

It follows that the equilibrium (y^*, r^{syn*}), if it exists, is unique. Suppose not and there exists another pair (r^{syn}, y) that satisfies the equilibrium in the intermediate regime. If $r^{syn} > r^{syn*}$, we must have $y > y^*$ due to $f_2^{int}(y, r^{syn}) = f_2^{int}(y^*, r^{syn*})$. It follows that $f_1^{int}(y, r^{syn}) > f_1^{int}(y^*, r^{syn*}) = 0$, and hence r^{syn} cannot be part of an equilibrium. A symmetric argument rules out all $r^{syn} < r^{syn*}$. Strict monotonicity also guarantees the uniqueness of y^*.

By the implicit function theorem,

$$\begin{bmatrix} \frac{\partial y^*(\cdot)}{\partial x} \\ \frac{\partial r^{syn*}(\cdot)}{\partial x} \end{bmatrix} = - \begin{bmatrix} \frac{\partial f_1^{int}(\cdot)}{\partial y} & \frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{int}(\cdot)}{\partial y} & \frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f_1^{int}(\cdot)}{\partial x} \\ \frac{\partial f_2^{int}(\cdot)}{\partial x} \end{bmatrix}$$

for any $x \in \{S^{bond}, \bar{q}, y_{\mathbb{Q}}, \varepsilon, \omega, \delta_U, \delta_H\}$. Observe that the signs of the negative inverse matrix are

$$sgn\left(- \begin{bmatrix} \frac{\partial f_1^{int}(\cdot)}{\partial y} & \frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} \\ \frac{\partial f_2^{int}(\cdot)}{\partial y} & \frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} \end{bmatrix}^{-1} \right) = sign\left(- \begin{bmatrix} \frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} & -\frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} \\ -\frac{\partial f_2^{int}(\cdot)}{\partial y} & \frac{\partial f_1^{int}(\cdot)}{\partial y} \end{bmatrix} \right) = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}.$$

We solve for the comparative statics as follows:

1. An increase in S^{bond}: $\frac{\partial f_1^{int}(\cdot)}{\partial x} < 0, \frac{\partial f_2^{int}(\cdot)}{\partial x} = 0$, and therefore $\frac{\partial y^*(\cdot)}{\partial x} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$. Thus, both the bond yield y^* and the synthetic rate r^{syn*} increase.

2. An increase in $\bar{q}$: $\frac{\partial f_1^{int}(\cdot)}{\partial x} > 0$ and $\frac{\partial f_2^{int}(\cdot)}{\partial \bar{q}} > 0$. Thus, we have $\frac{\partial r^{syn*}(\cdot)}{\partial x} < 0$. To determine the

sign of $\frac{\partial y^*(\cdot)}{\partial x}$, we note that

$$\frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} = D_H' - (D^{syn})' > \frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} = -(D^{syn})' > 0$$

$$\frac{\partial f_1^{int}(\cdot)}{\partial \bar{q}} = \frac{\partial f_2^{int}(\cdot)}{\partial \bar{q}} = 1$$

Thus,

$$\frac{\partial y^*(\cdot)}{\partial x} \propto \frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} - \frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} < 0$$

3. An increase in δ_U: $\frac{\partial f_1^{int}(\cdot)}{\partial x} > 0$ and $\frac{\partial f_2^{int}(\cdot)}{\partial \bar{q}} = 0$. Thus, we have $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} < 0$.

4. An increase in δ_H has $\frac{\partial f_1^{int}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{int}(\cdot)}{\partial x} < 0$, and therefore $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.

5. An increase in $y_{\mathbb{Q}}$ has $\frac{\partial f_1^{int}(\cdot)}{\partial x} = 0, \frac{\partial f_2^{int}(\cdot)}{\partial x} = 0$ and thus $\frac{\partial y^*(\cdot)}{\partial x} = 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} = 0$.

6. An increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$: this change is equivalent to an increase ε by dy in both f_1^{int} and f_2^{int} and an increase in ω by dy. The increase in ε causes an $\frac{n-1}{n}dy$ increase in y and no change in r^{syn*}. The increase in ω has $\frac{\partial f_1^{short}(\cdot)}{\partial x} > 0, \frac{\partial f_2^{short}(\cdot)}{\partial x} = 0$, and thus $\frac{\partial y^*(\cdot)}{\partial x} < 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$. To determine the total effect on y^*, we can evaluate the change of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ directly and obtain $\frac{\partial f_1^{int}(\cdot)}{\partial x} < 0, \frac{\partial f_2^{int}(\cdot)}{\partial x} > 0$, which implies that the total effect on y^* is positive. In summary, we find that the increase of dy in both $y_{\mathbb{Q}}$ and $y_{\mathbb{P}}$ increases y^* (by less than $\frac{n-1}{n}dy$) and increases r^{syn*}.

7. An increase in δ_{syn} has $\frac{\partial f_1^{long}(\cdot)}{\partial x} = \frac{\partial f_2^{long}(\cdot)}{\partial x} = -1$, and thus $\frac{\partial y^*(\cdot)}{\partial x} \propto \frac{\partial f_2^{int}(\cdot)}{\partial r^{syn}} - \frac{\partial f_1^{int}(\cdot)}{\partial r^{syn}} > 0$ and $\frac{\partial r^{syn*}(\cdot)}{\partial x} > 0$.

F.4 Proof of Proposition 4

Propositions 1, 2, and 3 establish that there is at most one equilibrium in each regime. To proceed, we first prove that across all possible regimes, the equilibrium is unique. Then we show the existence of an equilibrium. Finally, we will show how bond supply S^{bond} and the risk premium $y_{\mathbb{Q}}$ affects the equilibrium regime.

Define

$$f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = e^{-ny} - \frac{\exp(-(n-1)y_{\mathbb{Q}})}{(1-h)(e^{r^{tri}} - e^{r^{ois}}) + e^{r^{syn}}}$$

$$f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = \bar{q} - e^{-ny} S^{bond} - D^{syn}(r^{syn} - r^{ois}) + D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}).$$

$$f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = e^{-ny} S^{bond} - D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) - D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}).$$

$$f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = e^{-ny} - \frac{\exp(-(n-1)y_{\mathbb{Q}})}{e^{r^{sec}} + e^{r^{ois}} - e^{r^{syn}}}$$

$$\begin{aligned} f_5(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = {} & \bar{q} + e^{-ny} S^{bond} - D^{syn}(r^{syn} - r^{ois}) - D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) \\ & - 2D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}). \end{aligned}$$

where f_1 is the residual of long-regime dealer indifference equation (31), f_2 is the residual of the long-regime market indifference curve (32), f_3 is the residual of the bond-market clearing condition in (27), f_4 is the residual of short-regime dealer indifference equation (34), and f_5 is the residual of the short-regime market indifference curve (35).

In equilibrium, bond market clearing (27) and synthetic lending market clearing (30) implies

$$f_3 = q^{bond}$$

$$D_H + D^{syn} = q^{syn}$$

By assumption, $r^{ois} > r^{tri} > r^{sec}$, and in any equilibrium, $r^{syn} \geq r^{ois}$. It follows that

$$2e^{r^{syn}} \geq 2e^{r^{ois}} > e^{r^{sec}} + 2e^{r^{ois}} - e^{r^{tri}} > e^{r^{sec}} + e^{r^{ois}} + (1-h)(e^{r^{ois}} - e^{r^{tri}}),$$

and hence that

$$e^{r^{syn}} + (1-h)(e^{r^{tri}} - e^{r^{ois}}) > e^{r^{sec}} + e^{r^{ois}} - e^{r^{syn}}.$$

It follows that

$$f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}).$$

In a long-regime equilibrium, $q^{bond} > 0$, so

$$\bar{q} = q^{bond} + q^{syn}$$

Therefore,

$$\begin{aligned} f_5(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) &= \bar{q} + f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) - D^{syn}(r^{syn} - r^{ois}) - D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) \\ &= \bar{q} + q^{bond} - q^{syn} \\ &= 2q^{bond} \\ &> 0 \end{aligned}$$

Furthermore, the equilibrium conditions in the long equilibrium indicates

$$f_1 = f_2 = 0, \quad f_3 = q^{bond} > 0$$

In a short-regime equilibrium, $q^{bond} < 0$, so

$$\bar{q} = -q^{bond} + q^{syn}$$

Therefore,

$$\begin{aligned} f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) &= \bar{q} - f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) - D^{syn}(r^{syn} - r^{ois}) - D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) \\ &= \bar{q} - q^{bond} - D^{syn}(r^{syn} - r^{ois}) - D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}) \\ &= \bar{q} - q^{bond} - q^{syn} \\ &= -2q^{bond} \\ &> 0 \end{aligned}$$

Furthermore, the equilibrium conditions in the short equilibrium indicates

$$f_4 = f_5 = 0, \quad f_3 = q^{bond} < 0$$

In an intermediate-regime equilibrium, $f_3 = q^{bond} = 0$, so

$$\bar{q} = q^{syn}$$

and

$$f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = \bar{q} - q^{bond} - q^{syn} = 0$$
$$f_5(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = \bar{q} + q^{bond} - q^{syn} = 0$$

Furthermore, the intermediate-regime equilibrium requires that the yield is between the long and short thresholds, so

$$f_1 \geq 0 \geq f_4$$

Note that f_1, f_3, and f_5 are decreasing in y and increasing in r^{syn}, whereas f_2 is increasing in both y and r^{syn} and f_4 is decreasing in both in both y and r^{syn}.

We next show that the existence of either a long or a short equilibrium rules out the existence of another kind of equilibrium. Since all equilibria involve $q^{bond} > 0, q^{bond} < 0$, or $q^{bond} = 0$, it follows that an intermediate-regime equilibrium cannot coexist with other equilibria as well (i.e., the uniqueness of the intermediate-regime equilibrium holds once we prove the other two). Thus, the equilibrium if exists must be unique.

F.4.1 Uniqueness of a Long Regime Equilibrium

Suppose there is a $(y_{long}, r^{syn}_{long})$ that is a long equilibrium. Equilibrium conditions imply

$$f_4(y_{long}, r^{syn}_{long}; \cdot) < 0 = f_1(y_{long}, r^{syn}_{long}; \cdot) = f_2(y_{long}, r^{syn}_{long}; \cdot)$$

$$f_3(y_{long}, r^{syn}_{long}; \cdot) > 0, \quad f_5(y_{long}, r^{syn}_{long}; \cdot) > 0$$

The goal is to show that there cannot be another equilibrium in the short or the intermediate regime.

1. Now suppose there is another equilibrium (y, r^{syn}) that is in the intermediate regime, which implies

$$f_2(y, r^{syn}; \cdot) = f_3(y, r^{syn}; \cdot) = f_5(y, r^{syn}; \cdot) = 0$$

$$f_1(y, r^{syn}; \cdot) \geq 0 \geq f_4(y, r^{syn}; \cdot)$$

If $r^{syn} > r^{syn}_{long}$, we must have $y > y_{long}$ by $f_3(y, r^{syn}; \cdot) < f_3(y_{long}, r^{syn}_{long}; \cdot)$, but in this case,

$$f_2(y, r^{syn}; \cdot) > f_2(y_{long}, r^{syn}_{long}; \cdot) = 0,$$

which results in a contradiction.

If $r^{syn} < r^{syn}_{long}$, we have have $y < y_{long}$ by $f_1(y, r^{syn}; \cdot) > f_1(y_{long}, r^{syn}_{long}; \cdot)$, but in this case

$$f_2(y, r^{syn}; \cdot) < f_2(y_{long}, r^{syn}_{long}; \cdot) = 0,$$

whcih reuslts in a contradiction.

If $r^{syn} = r^{syn}_{long}$, it is not possible to simultaneously increase f_1 and decrease f_3 by changing y, and therefore no intermediate equilibrium exists.

Consequently, there is no alternative equilibrium in the intermediate regime.

2. Now suppose there is another equilibrium (y, r^{syn}) that is in the short regime,which implies

$$f_1(y, r^{syn}; \cdot) > 0 = f_4(y, r^{syn}; \cdot) = f_5(y, r^{syn}; \cdot)$$

$$f_2(y, r^{syn}; \cdot) > 0, \quad f_3(y, r^{syn}; \cdot) < 0$$

If $r^{syn} > r^{syn}_{long}$, we must have $y > y_{long}$ by $f_3(y, r^{syn}) < f_3(y_{long}, r^{syn}_{long}; \cdot)$, but in this case $f_4(y, r^{syn}; \cdot) < f_4(y_{long}, r^{syn}_{long}; \cdot) < 0$, which leads to a contradiction.

If $r^{syn} < r^{syn}_{long}$, we have have $y < y_{long}$ by $f_1(y, r^{syn}; \cdot) > f_1(y_{long}, r^{syn}_{long}; \cdot)$, but in this case $f_2(y, r^{syn}; \cdot) < f_2(y_{long}, r^{syn}_{long}; \cdot) = 0$, which again leads to a contradiction.

If $r^{syn} = r^{syn}_{long}$, it is not possible to simultaneously increase f_1 and decrease f_3 by changing y, and therefore no short equilibrium exists.

Consequently, there is no alternative equilibrium in the long regime.

F.4.2 Uniqueness of a Short Regime Equilibrium

Suppose there is a $(y_{short}, r^{syn}_{short})$ that is a short equilibrium. Equilibrium conditions imply

$$f_1(y_{short}, r^{syn}_{short}; \cdot) > 0 = f_4(y_{short}, r^{syn}_{short}; \cdot) = f_5(y_{short}, r^{syn}_{short}; \cdot)$$

$$f_2(y_{short}, r^{syn}_{short}; \cdot) > 0, \quad f_3(y_{short}, r^{syn}_{short}; \cdot) < 0$$

1. Now suppose there is another equilibrium (y, r^{syn}) in the intermediate regime, which implies

$$f_2(y, r^{syn}; \cdot) = f_3(y, r^{syn}; \cdot) = f_5(y, r^{syn}; \cdot) = 0$$

$$f_1(y, r^{syn}; \cdot) \geq 0 \geq f_4(y, r^{syn}; \cdot)$$

If $r^{syn} > r^{syn}_{short}$, we must have $y > y_{short}$ by $f_5(y, r^{syn}; \cdot) = f_5(y_{short}, r^{syn}_{short}; \cdot)$, but in this case $f_2(y, r^{syn}; \cdot) > f_2(y_{short}, r^{syn}_{short}; \cdot) > 0$, which leads to a contradiction.

If $r^{syn} < r^{syn}_{short}$, we have have $y < y_{short}$ by $f_5(y, r^{syn}; \cdot) = f_5(y_{short}, r^{syn}_{short}; \cdot)$, but in this case $f_4(y, r^{syn}; \cdot) > f_4(y_{short}, r^{syn}_{short}; \cdot) = 0$, which leads to a contradiction.

If $r^{syn} = r^{syn}_{short}$, then by $f_5(y, r^{syn}; \cdot) = 0$ we must have $y = y_{short}$. However, then this leads to $f_3(y, r^{syn}; \cdot) = f_3(y_{short}, r^{syn}_{short}; \cdot) < 0$, which is a contradiction.

Consequently, there is no alternative equilibrium in the intermediate regime.

2. Now suppose there is another equilibrium (y, r^{syn}) in the long regime, which implies

$$f_4(y, r^{syn}; \cdot) < 0 = f_1(y, r^{syn}; \cdot) = f_2(y, r^{syn}; \cdot)$$

$$f_3(y, r^{syn}; \cdot) > 0, \quad f_5(y, r^{syn}; \cdot) > 0$$

If $r^{syn} > r^{syn}_{short}$, we have have $y > y_{short}$ by $f_1(y, r^{syn}; \cdot) < f_1(y_{short}, r^{syn}_{short}; \cdot)$, but in this case $f_2(y, r^{syn}; \cdot) > f_2(y_{short}, r^{syn}_{short}; \cdot) > 0$, which is a contradiction.

If $r^{syn} < r^{syn}_{short}$, we must have $y < y_{short}$ by $f_3(y, r^{syn}) > f_3(y_{short}, r^{syn}_{short}; \cdot)$, but in this case $f_4(y, r^{syn}; \cdot) > f_4(y_{short}, r^{syn}_{short}; \cdot) = 0$, which is a contradiction.

If $r^{syn} = r^{syn}_{short}$, it is not possible to simultaneously increase f_3 and decrease f_1 by changing y, and therefore no long equilibrium exists.

Consequently, there is no alternative equilibrium in the short regime.

F.4.3 Equilibrium Existence

Next, we prove the existence of the equilibrium. The high-level idea is to construct the equilibrium as a convex mapping from a compact and convex set to itself, and then apply the Kakutani fixed-point theorem.

First, we show the compactness of the state space (y, r^{syn}).

Compactness of the y dimension

In any equilibrium, we must have

$$f_3(y, r^{ois}; S^{bond}, y_{\mathbb{Q}}) \le f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) \le \bar{q}.$$

Because f_3 is decreasing in y, there is a y_{min} such that

$$f_3(y_{min}, r^{ois}; S^{bond}, y_{\mathbb{Q}}) > \bar{q},$$

and any equilibrium must have $y \ge y_{min}$. We must also have, in any equilibrium,

$$f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) \ge -\bar{q},$$

which yields, by $D_H \ge 0$,

$$e^{-ny} S^{bond} - D_U(ny - y^{bill} - (n-1)y_{\mathbb{P}}) \ge -\bar{q}.$$

Defining y_{max} by

$$e^{-ny_{max}} S^{bond} - D_U(ny_{max} - y^{bill} - (n-1)y_{\mathbb{P}}) = -\bar{q},$$

it follows that $y \le y_{max}$.

Compactness of the r^{syn} dimension

Define r^{min} as

$$D^{syn}(r^{min} - r^{ois}) = \bar{q},$$

By assumption $D^{syn}(0) > \bar{q}$ and D^{syn} is a strictly decreasing function, we have $r^{min} - r^{ois} > 0$. For any $r^{syn} < r^{min}$,

$$D^{syn}(r^{syn} - r^{ois}) > \bar{q},$$

which violates the synthetic market clearing condition in (30). Consequently, in any equilibrium, $r^{syn} \ge r^{min}$.

Next, we will find an upper bound r^{max} such that for any $r^{syn} > r^{max}$, one of the market clearing conditions are violated. First, we note that there exists a r_1^{max} such that for all $r^{syn} > r_1^{max}$, for any feasible y we consider, i.e. $y \in [y_{min}, y_{max}]$,

$$e^{-ny}(1-h)(e^{r^{tri}} - e^{r^{ois}}) + e^{r^{syn}} > \exp(-(n-1)y_{\mathbb{Q}}) > e^{-ny}(e^{r^{sec}} + e^{r^{ois}} - e^{r^{syn}}).$$

which says that $y \in (y^s, y^l)$ and thus the equilibrium is in the intermediate regime and dealer chooses $q^{bond} = 0$, and supply $\bar{q}$ to the synthetic lending market. We will show that if r^{syn} is too large, the synthetic lending market demand will fall below this supply.

Define synthetic lending demand as

$$m(y, r^{syn}) = D^{syn}(r^{syn} - r^{ois}) + D_H(ny - r^{syn} - (n-1)y_{\mathbb{P}}).$$

which decreases in r^{syn}. There exists a $r^{max} \geq r_1^{max}$ such that, for all $r^{syn} > r^{max}$ and $y \in [y_{min}, y_{max}]$,

$$m(y, r^{syn}) < \bar{q},$$

which breaks the synthetic lending market clearing condition.

Consequently, if $r^{syn} \geq r^{max}$, no equilibrium can exist.

Convex and Closed Correspondence

So far we have found a compact and convex space $\mathscr{C} = [y_{min}, y_{max}] \times [r^{min}, r^{max}]$ where the equilibrium (y, r^{syn}) must belong. Next, we define the correspondence for the equilibrium and prove that it is convex and closed.

The mapping we construct will constitute four dimensions, including (y, r^{syn}), the dealer bond position q^{bond}, and dealer synthetic lending q^{syn}.

From the dealer optimization problem, the demand correspondence only depends on (y, r^{syn}) and is defined as follows

$$Q(y, r^{syn}) = \begin{cases} \{(\bar{q}, 0)\} & \text{if } f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < 0 \\ \{(q^{bond}, q^{syn}) \in \mathbb{R}_+^2 : q^{bond} + q^{syn} = \bar{q}\} & \text{if } f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0, \\ \{(q^{bond}, q^{syn}) \in \mathbb{R}_- \times \mathbb{R}_+ : -q^{bond} + q^{syn} = \bar{q}\} & \text{if } f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0, \\ \{(-\bar{q}, 0)\} & \text{if } f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) > 0, \\ \{(0, \bar{q})\} & otherwise. \end{cases}$$

The first case $f_1 < 0$ is the only-long region where $y > y^l$. The second case $f_1 = 0$ is the long region where $y = y^l$. The third case $f_4 = 0$ is the sell region where $y = y^s$. The fourth case $f_4 > 0$ is the sell-only region where $y < y^s$. The fifth case is the intermediate region where $y^s < y < y^l$.

Define the aggregate excess demand correspondence as

$$Z(y, r^{syn}) = \{(z_1, z_2\} \in \mathbb{R}^2 : (z_1 + f_3(y, r^{syn}; \cdot), m(y, r^{syn}) - z_2) \in Q(y, r^{syn})\}.$$

Here, z_1 represents the excess demand for bonds, and z_2 is the excess demand for synthetic loans. By definition, $f_3(\cdot)$ is the bond supply less non-intermediary demand, and hence $f_3(\cdot) + z_1$ must equal the intermediary demand q^{bond}. Likewise, $m(\cdot)$ is synthetic loan demand, and $m(\cdot) - z_2$ must equal the synthetic loan supply q^{syn}.

Note that this correspondence is non-empty, u.h.c. (by the u.h.c. property of q, which ultimately arises from the continuity of f_1, f_4, and the continuity of f_3 and m). Note that it is also convex-valued, a property it inherits from Q. Define the maximum and minimum possible excess demands by

$$\begin{aligned}
z_{max}^{bond} &= \max_{(y, r^{syn}) \in [y_{min}, y_{max}] \times [r^{min}, r^{max}]} \bar{q} - f_3(y, r^{syn}; \cdot), \\
z_{min}^{bond} &= \min_{(y, r^{syn}) \in [y_{min}, y_{max}] \times [r^{min}, r^{max}]} -\bar{q} - f_3(y, r^{syn}; \cdot), \\
z_{max}^{syn} &= \max_{(y, r^{syn}) \in [y_{min}, y_{max}] \times [r^{min}, r^{max}]} m(y, r^{syn}), \\
z_{\min}^{syn} &= \min_{(y, r^{syn}) \in [y_{min}, y_{max}] \times [r^{min}, r^{max}]} m(y, r^{syn}) - \bar{q},
\end{aligned}$$

Now define a price player, who solves, given any vector $(z_1, z_2) \in [z_{min}^{bond}, z_{max}^{bond}]$,

$$\max_{(y, r^{syn}) \in [y_{min}, y_{max}] \times [r^{min}, r^{max}]} (y, r^{syn}) \cdot \begin{bmatrix} -z_1 \\ z_2 \end{bmatrix}.$$

Let $p^*(z)$ be the optimal policy correspondence, and note that it is non-empty, u.h.c., and convex-valued (which follows from the concavity of the objective).

Now define the correspondence

$$g(y, r^{syn}, z) = \begin{bmatrix} p^*(z) \\ Z(y, r^{syn}) \end{bmatrix}$$

which maps $[y_{min}, y_{max}] \times [r^{min}, r^{max}] \times [z^{bond}_{min}, z^{bond}_{max}] \times [z^{syn}_{min}, z^{syn}_{max}]$ to itself. Note that this set is compact, and by the u.h.c. properties of p^* and Z and the compactness of this set, g has a closed graph. Consequently, by Kakutani's fixed point theorem, a fixed point (y^*, r^{syn*}, z^*) exists.

By construction, at y_{min},

$$f_3(y_{min}, r^{ois}; S^{bond}, y_{\mathbb{Q}}) > \bar{q},$$

and consequently all values $Z_1(y, r^{syn})$ are negative. The best response of the price player at this point would be y_{max}, and hence there cannot be a fixed point with $y^* = y_{min}$. Essentially the same logic rules out $y^* = y_{max}$. Similarly, if $r^{syn*} = r^{min}$, then all values of $Z_2(y^*, r^{syn*})$ are positive, and the price player's best response is r_{max}, and hence this cannot be a fixed point. Likewise, if $r^{syn*} = r^{max}$, then all values of $Z_2(y^*, r^{syn*})$ are negative, and the price player's best response is $r^{syn} = r^{min}$. It follows that the fixed point is interior, and hence that $z^* = Z(y^*, r^{syn*}) = (0, 0)$. Note that a fixed point with $z^* = (0, 0)$ cannot exist in which there is no supply of synthetic lending; consequently, the equilibrium is either a long regime equilibrium,

$$f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0,$$

a short regime equilibrium,

$$0 = f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}),$$

or an intermediate equilibrium,

$$f_4(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < 0 < f_1(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}).$$

F.4.4 Bond Supply and Equilibrium Regime

To prove that the existence of cutoffs S_S and S_B with $0 \leq S_S \leq S_B \leq \infty$, such that the short-regime, the intermediate regime, and the long-regime fall into the three regions, we simply prove that there is a ranking of the equilibrium along the supply of bonds S.

Consider $S^{bond} = S$. According to the previous proofs, an equilibrium (y, r^{syn}) exists and must be unique.

Long Equilibrium

First, we show that if S corresponds to a long-regime equilibrium, then for any $\bar{S} > S$, the equilibrium $(\bar{y}, \bar{r}^{syn})$ must also be a long equilibrium.

Suppose instead the equilibrium for $S^{bond} = \bar{S}$ is a short-regime equilibrium with $(\bar{y}, \bar{r}^{syn})$. Then we must have $y = y^l > y^s = \bar{y}$. Furthermore, $f_2(y, r^{syn}; S, y_{\mathbb{Q}}) = 0$ and $f_2(\bar{y}, \bar{r}^{syn}; S, y_{\mathbb{Q}}) > f_2(\bar{y}, \bar{r}^{syn}; \bar{S}, y_{\mathbb{Q}}) > 0$. By monotonicity of f_2, we must have $\bar{r}^{syn} > r^{syn}$. Therefore, by monotonicty of f_5, we have

$$f_5(\bar{y}, \bar{r}^{syn}; \bar{S}, y_{\mathbb{Q}}) > f_5(y, r^{syn}; \bar{S}, y_{\mathbb{Q}})$$

However, in the long regime, $f_5(\bar{y}, \bar{r}^{syn}; \bar{S}, y_{\mathbb{Q}}) = 0$, and in the short regime, $f_5(y, r^{syn}; \bar{S}, y_{\mathbb{Q}}) > f_5(y, r^{syn}; S, y_{\mathbb{Q}}) > 0$, which leads to a contradiction. Thus, $\bar{S}$ cannot correspond to a short equilibrium.

Next, suppose that $\bar{S}$ corresponds to an intermediate-regime equilibrium. Then we have $y = y^l \geq \bar{y}$, $f_3(\bar{y}, \bar{r}^{syn}; \bar{S}, y_{\mathbb{Q}}) = 0$. For the equilibrium of S, we have $f_3(y, r^{syn}; \bar{S}, y_{\mathbb{Q}}) > f_3(y, r^{syn}; S, y_{\mathbb{Q}}) > 0$. By the monotonicity of f_3, $r^{syn} > \bar{r}^{syn}$. Thus, $f_2(\bar{y}, \bar{r}^{syn}; S, y_{\mathbb{Q}}) < f_2(y, r^{syn}; S, y_{\mathbb{Q}})$.

However, by the properties of long and intermediate regimes, we also have $f_2(y, r^{syn}; S, y_{\mathbb{Q}}) > 0$ and $f_2(\bar{y}, \bar{r}^{syn}; S, y_{\mathbb{Q}}) > f_2(\bar{y}, \bar{r}^{syn}; \bar{S}, y_{\mathbb{Q}}) = 0$, which leads to a contradiction.

In summary, if S is a long-regime equilibrium, for any $\bar{S} > S$, the equilibrium $(\bar{y}, \bar{r}^{syn})$ must also be a long-regime equilibrium.

Short Equilibrium

Second, we show that if S corresponds to a short-regime equilibrium, then for any $\underline{S} < S$, the equilibrium solution $(\underline{y}, \underline{r}^{syn})$ must also be a short-regime equilibrium.

Suppose that instead the equilibrium for $S^{bond} = \underline{S}$ is a long-regime equilibrium. Then we must have $y = y^s < y^l = \underline{y}$. Furthermore, $f_5(\underline{y}, \underline{r}^{syn}; S, y_{\mathbb{Q}}) > f_5(\underline{y}, \underline{r}^{syn}; \underline{S}, y_{\mathbb{Q}}) > 0$, and $f_5(y, r^{syn}; S, y_{\mathbb{Q}}) = 0$. By monotonicity of f_5, we get $\underline{r}^{syn} > r^{syn}$. Thus, by monotonicity of f_2, we obtain

$$f_2(y, r^{syn}; S, y_{\mathbb{Q}}) < f_2(\underline{y}, \underline{r}^{syn}; S, y_{\mathbb{Q}})$$

However, by the properties of long and short regimes, we must have $f_2(\underline{y}, \underline{r}^{syn}; S, y_{\mathbb{Q}}) < f_2(\underline{y}, \underline{r}^{syn}; \underline{S}, y_{\mathbb{Q}}) = 0$, and $f_2(y, r^{syn}; S, y_{\mathbb{Q}}) > 0$, which leads to a contradiction.

Suppose that the equilibrium for $S^{bond} = \underline{S}$ is an intermediate-regime equilibrium. Then we must have $y = y^s \leq \underline{y}$. Furthermore, $f_5(\underline{y}, \underline{r}^{syn}; S, y_{\mathbb{Q}}) > f_5(\underline{y}, \underline{r}^{syn}; \underline{S}, y_{\mathbb{Q}}) = 0$, and $f_5(y, r^{syn}; S, y_{\mathbb{Q}}) = 0$. By monotonicity of f_5, we get $\underline{r}^{syn} > r^{syn}$. Thus, by monotonicity of f_2, we obtain

$$f_2(y, r^{syn}; S, y_{\mathbb{Q}}) < f_2(\underline{y}, \underline{r}^{syn}; S, y_{\mathbb{Q}})$$

However, by the properties of short and intermediate regimes, we must have $f_2(y, r^{syn}; S, y_{\mathbb{Q}}) > 0$ and $f_2(\underline{y}, \underline{r}^{syn}; S, y_{\mathbb{Q}}) < f_2(\underline{y}, \underline{r}^{syn}; \underline{S}, y_{\mathbb{Q}}) = 0$, which leads to a contradiction.

In summary, if S is a short-regime equilibrium, for any $\underline{S} < S$, the equilibrium $(\underline{y}, \underline{r}^{syn})$ must also be a short-regime equilibrium.

Regime Ranking

From the above discussions, we know that there must be cutoffs S_S and S_B with $0 \leq S_S \leq S_B \leq \infty$, such that a short-regime equilibrium exists in the left region, an intermediate-regime equilibrium exists in the middle region, and a long-regime equilibrium exists in the right region. However, we still have to prove whether the intervals are open or closed.

Intervals for Regimes

We now show that the interval of long-regime equilibrium should be (S_B, ∞) instead of $[S_B, \infty)$. Suppose that $S^{bond} = S$ is a long-regime equilibrium, with solutions (y, r^{syn}), and q^{bond}. By definition, $q^{bond} > 0$.

In the long regime, $f_3(y, r^{syn}; S, y_{\mathbb{Q}}) = q^{bond}$. We know that $q^{bond} > 0$ increases in the total supply of bond and the mapping is continuous. Therefore, there exists a smaller bond supply $S^{bond} = S - \varepsilon$ for $\varepsilon > 0$, such that the new equilibrium still has $q^{bond} > 0$. Consequently, the interval of S^{bond} for the long-regime equilibrium must be an open set.

Similarly, the interval for the short-regime equilibrium must also be an open set, $(-\infty, S_B)$.

F.4.5 Term Premium and Equilibrium Regime

Next, we study how the term premium $y_{\mathbb{Q}}$ affects the equilibrium. Consider $y_{\mathbb{Q}}$. According to previous proofs, an equilibrium solution (y, r^{syn}) exists and is unique.

Long Equilibrium

First, we show that if $y_{\mathbb{Q}}$ corresponds to a long-regime equilibrium, then for any $\bar{y}_{\mathbb{Q}} > y_{\mathbb{Q}}$, the equilibrium $(\bar{y}, \bar{r}^{syn})$ must also be a long equilibrium.

Suppose instead the equilibrium for $\bar{y}_{\mathbb{Q}}$ is a short-regime equilibrium with $(\bar{y}, \bar{r}^{syn})$. Then we must have $y = y^l > y^s = \bar{y}$. Furthermore, $f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0$ and $f_2(\bar{y}, \bar{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_2(\bar{y}, \bar{r}^{syn}; S^{bond}, \bar{y}_{\mathbb{Q}}) > 0$. By monotonicity of f_2, we must have $\bar{r}^{syn} > r^{syn}$. Therefore, by monotonicty of f_5, we have

$$f_5(\bar{y}, \bar{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) > f_5(y, r^{syn}; S^{bond}, y_{\mathbb{Q}})$$

However, in the short regime, $f_5(\bar{y}, \bar{r}^{syn}; S^{bond}, \bar{y}_{\mathbb{Q}}) = f_5(\bar{y}, \bar{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0$, and in the long regime, $f_5(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) > 0$, which leads to a contradiction. Thus, $\bar{y}_{\mathbb{Q}}$ cannot correspond to a short equilibrium.

Next, suppose that $\bar{y}_{\mathbb{Q}}$ corresponds to an intermediate-regime equilibrium. Then we have $y = y^l \geq \bar{y}$, $f_3(\bar{y}, \bar{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_3(\bar{y}, \bar{r}^{syn}; S^{bond}, \bar{y}_{\mathbb{Q}}) = 0$. For the long-regime equilibrium of $y_{\mathbb{Q}}$, we have $f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) > 0$. By the monotonicity of f_3, $r^{syn} > \bar{r}^{syn}$. Thus, $f_2(\bar{y}, \bar{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) < f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}})$.

However, by the properties of long and intermediate regimes, we also have $f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0$ and $f_2(\bar{y}, \bar{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_2(\bar{y}, \bar{r}^{syn}; S^{bond}, \bar{y}_{\mathbb{Q}}) = 0$, which leads to a contradiction.

In summary, if $y_{\mathbb{Q}}$ is a long-regime equilibrium, for any $\bar{y}_{\mathbb{Q}} > y_{\mathbb{Q}}$, the equilibrium $(\bar{y}, \bar{r}^{syn})$ must also be a long equilibrium.

Short Equilibrium

Second, we show that if $y_{\mathbb{Q}}$ corresponds to a short-regime equilibrium, then for any $\underline{y}_{\mathbb{Q}} < y_{\mathbb{Q}}$, the equilibrium $(\underline{y}, \underline{r}^{syn})$ must also be a short-regime equilibrium.

Suppose that instead the equilibrium for $\underline{y}_{\mathbb{Q}}$ is a long-regime equilibrium. Then we must have $y = y^s < y^l = \underline{y}$. Furthermore, $f_5(\underline{y}, \underline{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_5(\underline{y}, \underline{r}^{syn}; S^{bond}, \underline{y}_{\mathbb{Q}}) > 0$, and $f_5(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) = 0$. By monotonicity of f_5, we get $\underline{r}^{syn} > r^{syn}$. Thus, by monotonicity of f_2, we obtain

$$f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < f_2(\underline{y}, \underline{r}^{syn}; S^{bond}, y_{\mathbb{Q}})$$

However, by the properties of long and short regimes, we must have

$$f_2(\underline{y}, \underline{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_2(\underline{y}, \underline{r}^{syn}; S^{bond}, \underline{y}_{\mathbb{Q}}) = 0,$$

and $f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) > 0$, which leads to a contradiction.

Suppose that the equilibrium for $\underline{y}_{\mathbb{Q}}$ is an intermediate-regime equilibrium. Then we must have $y = y^s \leq \underline{y}$. Furthermore, $f_3(\underline{y}, \underline{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_3(\underline{y}, \underline{r}^{syn}; S^{bond}, \underline{y}_{\mathbb{Q}}) = 0$, and $f_3(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < 0$. By monotonicity of f_3, we get $\underline{r}^{syn} > r^{syn}$. Thus, by monotonicity of f_2, we obtain

$$f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) < f_2(\underline{y}, \underline{r}^{syn}; S^{bond}, y_{\mathbb{Q}})$$

However, by the properties of short and intermediate regimes, we must have $f_2(y, r^{syn}; S^{bond}, y_{\mathbb{Q}}) > 0$ and $f_2(\underline{y}, \underline{r}^{syn}; S^{bond}, y_{\mathbb{Q}}) = f_2(\underline{y}, \underline{r}^{syn}; S^{bond}, \underline{y}_{\mathbb{Q}}) = 0$, which leads to a contradiction.

In summary, if $y_{\mathbb{Q}}$ is a short-regime equilibrium, for any $\underline{y}_{\mathbb{Q}} < y_{\mathbb{Q}}$, the equilibrium $(\underline{y}, \underline{r}^{syn})$ must also be a short-regime equilibrium.

Regime Ranking

From the above discussions, we know that there must be cutoffs y_S and y_B with $0 \leq y_S \leq y_B \leq \infty$, such that a short-regime equilibrium, an intermediate-regime equilibrium, and a long-regime equilibrium exits in the left, middle and right regions. However, we still need to determine whether those intervals are open or closed sets.

Intervals for Regimes

We now show that the interval of long-regime equilibrium should be (y_B, ∞) instead of $[y_B, \infty)$. Suppose that $y_{\mathbb{Q}}$ is a long-regime equilibrium, with solutions (y, r^{syn}), and q^{bond}. By definition, $q^{bond} > 0$.

In the long regime, we know that $q^{bond} > 0$ is a continuous function of $y_{\mathbb{Q}}$. Therefore, there exists a smaller risk-neutral expectation $y_{\mathbb{Q}} - \varepsilon$, where the new equilibrium is still in the long regime with $q^{bond} > 0$. Consequently, the interval of $y_{\mathbb{Q}}$ for the long-regime equilibrium must be an open set.

Similarly, the interval of $y_{\mathbb{Q}}$ for the short-regime equilibrium must also be an open set.