Excess Volatility of Exchange Rates with Unobservable Fundamentals*

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Abstract

We present tests of excess volatility of exchange rates which impose minimal structure on the data and do not commit to a choice of exchange rate “fundamentals.” Our method builds on existing volatility tests of asset prices, combining them with a procedure that extracts unobservable fundamentals from survey-based exchange rate expectations. We apply our method to data for the three major exchange rates since 1984 and find broad evidence of excess volatility with respect to the predictions of the canonical asset-pricing model of the exchange rate with rational expectations.

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1. Introduction

Whether exchange rates are "too volatile" with respect to the behavior of their underlying determinants is one of the most persistent, and yet unsettled, issues in the international finance debate. The issue is clearly critical for policy purposes: if exchange rates are too volatile with respect to a reasonable benchmark in an efficient market, then there may be grounds for throwing "sand in the gears" of currency markets, so as to slow down changes in exchange rates and keep them in line with those of their "fundamentals" (Eichengreen, Tobin, and Wyplosz, 1995). Conversely, if exchange rate volatility is largely consistent with that predicted by conventional models, then one may simply have to swallow the rates' abrupt swings as the potentially efficient response to underlying shocks.

Applying the volatility tests developed by Shiller (1981) for stock price models, early studies such as Huang (1981), Vander Kraats and Booth (1983), and Wadhwani (1987), first tried to settle this issue. These studies benchmarked exchange rate volatility against the predictions of the monetary model, and popularized the perception that exchange rates over the post-Bretton Woods period had been too volatile for their behavior to be consistent with that of their fundamentals.

Later research, however, cast doubt on these results. Diba (1987), for instance, pointed to the miscalibration of Huang's and Vander Kraats and Booth's tests. When correctly calibrated, these tests failed to provide evidence of excess volatility. More generally, Kleidion (1986), Marsh and Merton (1986) and others pointed to biases in early volatility tests of asset price models--including exchange rate models--which caused the models to be rejected too often in finite samples.
The main econometric problems of early exchange rate volatility tests have later been addressed (see, for instance, Bartolini and Bodnar, 1996; Ghosh, 1992; Gros, 1989; West, 1987). A parallel line of research has also tried to establish if fluctuations in exchange rates could be ascribed to changes in fundamentals, or rather to speculative bubbles or "sunspots" (see Frankel and Rose, 1995, for a survey of this effort, and Jeanne and Masson, 1998, for a more recent contribution.) Like previous work, however, these studies have relied heavily on specific structural models (e.g., the monetary model) as benchmarks for exchange rates’ normal volatility. The problem with this reliance is that the resulting evidence of excess volatility could simply reflect failure of the assumed structural relationships to accurately track exchange rates--not a surprising result, given the poor empirical record of structural exchange rate models in recent decades. Measurement problems also weaken this evidence, which relies on proxying theoretical fundamentals by arbitrary macro-variables, generally ignoring political, psychological, and other unobservable factors. This viewpoint makes the key problem with existing evidence of excess volatility apparent: the less adequate is the model used to define the volatility norm (in particular, the narrower is the choice of fundamentals), the more one is likely to find unexplainable--and hence excessive--changes in exchange rates. While exchange rates are widely viewed to be too volatile (see, among many others, Boertje and Garretsen, 1996; Jeanne and Rose, 1999; Masson, 1998), empirical support for this view remains weak and anchored to very model- and data-specific evidence.

To partially address this gap, this paper presents a method to test for excess exchange rate volatility that requires minimal commitment to an exchange rate model and no commitment at all to a choice of relevant fundamentals. Our approach combines: (i) existing tests of excess
volatility of asset prices (Mankiw, Romer and Shapiro, 1991); (ii) a technique suggested by Flood, Mathieson and Rose (1991) and Flood and Rose (1996) to retrieve unobservable fundamentals from data on exchange and interest rates, assuming uncovered interest parity; and (iii) the insight that one can dispose of the uncomfortable interest parity condition by using survey-based data to measure exchange rate expectations directly.

As a result of this effort, our tests feature several desirable properties. Most importantly, they rely only on the assumptions of rational expectations and of a rather general asset-pricing relationship--namely, the canonical log-linear pricing model. In particular, our tests allow for the widest possible definition of “fundamentals,” requiring no commitment to any measurable or nonmeasurable variables. Our analysis also inherits properties of the methodology of Mankiw, Romer and Shapiro (1991), such as robustness to speculative bubbles and good small-sample performance. Finally, aside from its application to volatility tests, our work suggests an easy way to construct unobservable fundamentals for potentially wide use in empirical work on exchange rates.

2. Volatility Tests of the Canonical Asset-Pricing Exchange Rate Model

Our starting point is the familiar forward-looking, log-linear, exchange rate model:

\[ s_t = f_t + \alpha \left( \mathbb{E}_t[s_{t+1}] - s_t \right), \]

where \( s_t \) denotes the (log) exchange rate, defined as the domestic price of a unit of foreign currency, \( f_t \) denotes variables fundamental to the determination of the exchange rate, and \( \mathbb{E}_t[. . .] \)
denotes the usual rational expectation operator, based on information available at time $t$.

Equation (1) expresses the exchange rate as the sum of current fundamentals and a linear function of its own expected change, and summarizes the "asset market view" of the exchange rate that has become standard in the literature since Mussa (1976). Model (1) can be viewed, inter alia, as the reduced form of a monetary model linking the exchange rate to money supplies and incomes, with money-demand equations possibly derived from money-in-utility (Lucas, 1982) or cash-in-advance assumptions (Stockman, 1980); or it can be interpreted as a more general equilibrium model, possibly involving currency-substitution assumptions (Calvo and Rodriguez, 1977). Also, $f_r$ can be interpreted flexibly, and may include announcements (or signals) of current and future monetary policies, political factors, and a variety of measurable or unobservable variables that affect investors’ demand for currencies. The analogy of model (1) with models of stock valuation is also apparent ($f_r$ plays the role of dividends and $\alpha$ that of the discount factor applied to expected future capital gains), and partly motivates the methodology that follows.

Solving equation (1) forward for $s_r$, up to time $t + h$, yields:

$$ s_r = \sum_{i=0}^{h-1} \left( \frac{\alpha}{(1 + \alpha)^i} E_{i} \left[ f_{t+i} \right] / (1 + \alpha) + \left( \frac{\alpha}{(1 + \alpha)} \right)^h E_{t} \left[ s_{t+h} \right] \right), $$

where $h$ is the holding period for the position opened by purchasing foreign currency at time $t$.

It is common in the literature to assume the absence of speculative bubbles, namely, that

$$ \lim_{i \to \infty} \left( \frac{\alpha}{(1 + \alpha)^i} E_{i} \left[ s_{t+i} \right] = 0, \right. $$

and then solve (2) forward solely as a function of $f_r$. The tests we consider do not require a no-bubble assumption, however, and can be performed directly on (2).

Similarly to Mankiw, Romer and Shapiro’s (1991) work on stock prices, we now define the perfect-foresight (or "fundamental") rate $s_r^*$ as the exchange rate that would prevail if
investors could predict with certainty future fundamentals \( f_{t,i}, i = 1, \ldots, h-1 \), and the future exchange rate \( s_{t+h} \). \( s_t^* \) is obtained by dropping the expectation operator from (2):

\[
 s_t^* = \sum_{i=0}^{h-1} \left( \alpha/(1+\alpha) \right)^i f_{t+i} / (1+\alpha) + \left( \alpha/(1+\alpha) \right)^h s_{t+h},
\]

so that, by definition, \( s_t = \mathbb{E}_t \left[ s_t^* \right] \).

Next, we construct a "benchmark" exchange rate, denoted by \( s_t^o \), from which the volatility of both market and fundamental rates can be measured. The main reason behind the need for a benchmark rate is that exchange rates are typically nonstationary, and therefore volatility must be measured on deviations of exchange rates from a specific (stochastic) trend. Accordingly, the first requirement for the choice of \( s_t^o \) is that the differences \( (s_t - s_t^o) \) and \( (s_t^* - s_t^o) \) should be stationary, to assure their mean square errors to be well defined. The second requirement is that \( s_t^o \) should be known at time \( t \). This condition assures the orthogonality of \( s_t^o \) with rational forecast errors based on information available at \( t \), a property whose usefulness for our tests will soon become clear.

Many different definitions of \( s_t^o \) would satisfy these two requirements, however. We begin by following Mankiw, Romer and Shapiro (1991) who, studying stock price volatility, define \( s_t^o \) as a "naive" price forecast, namely, the price that would prevail in the market if investors expected dividends to follow a random walk. We define \( s_t^o \) as the rate that would prevail if investors expected fundamentals to follow a random walk, i.e., \( f_t = \mathbb{E}_t \left[ f_{t+i} \right], i \geq 0 \). With this assumption, (1) yields \( s_t^o = f_t \). Thus, if fundamentals indeed followed a random walk, so would \( s_t^o \). Note that \( s_t^o \) need be neither a rational nor an empirically accurate forecast. (That said, a random-walk forecast may have claim to realism, as it is usually difficult to distinguish exchange rates from a random walk at short horizons.) The choice of \( s_t^o \) affects the outcome of
the tests, however, by determining the information on which the tests are conditioned, as discussed below and as documented in Section 4 by considering alternative definitions of $s_t^o$.

We can now derive our excess volatility tests as follows. If expectations are formed rationally, the forecast error $\left(s_t^* - s_t\right)$ should be uncorrelated with any variable known at time $t$, including $\left(s_t - s_t^o\right)$. That is:

$$E_t\left[(s_t^* - s_t)(s_t - s_t^o)\right] = 0 .$$  \hspace{1cm} (4)

Therefore, squaring both sides of the identity:

$$\left(s_t^* - s_t^o\right) = \left(s_t^* - s_t\right) + \left(s_t - s_t^o\right) ,$$  \hspace{1cm} (5)

taking expectations, and using (4), yields:

$$E_t\left[\left(s_t^* - s_t^o\right)^2\right] = E_t\left[\left(s_t^* - s_t\right)^2\right] + E_t\left[\left(s_t - s_t^o\right)^2\right] ,$$  \hspace{1cm} (6)

or:

$$q_t = E_t\left[\left(s_t^* - s_t^o\right)^2\right] - E_t\left[\left(s_t^* - s_t\right)^2\right] - E_t\left[\left(s_t - s_t^o\right)^2\right] = 0 .$$  \hspace{1cm} (7)

Thus, model (1) and rational expectations imply $q_t = 0$, $E[q_t] = 0$, and the testable restriction that the sample mean $\bar{q} = \sum_{t=1}^{T} \left[\left(s_t^* - s_t^o\right)^2 - \left(s_t^* - s_t\right)^2 - \left(s_t - s_t^o\right)^2\right]/T$ should be close to zero. After calibrating $\alpha$ and defining suitable fundamentals, $f_t$, we can thus compute $\bar{q}$ and then reject the hypothesis that $E[q_t] = 0$ if $\bar{q}$ is significantly different from zero, using a Generalized Method of Moments distribution for $\bar{q}$ (Bollerslev and Hodrick, 1995). This is a normal distribution:

$$\bar{q} \sim N\left(0 , V/T\right) ,$$  \hspace{1cm} (8)

whose variance, $V/T$, can be estimated using the method proposed by Andrews and Monahan.
(1992), to make the test robust to heteroskedasticity and serial correlation in \( q_t \).  

Finally, we obtain our excess volatility tests by noting that (6) implies the inequalities:

\[
E \left[ (s^*_t - s_{i,t})^2 \right] \leq E \left[ (s^o_t - s^*_t)^2 \right], \quad (9a)
\]

\[
E \left[ (s^*_t - s^o_t)^2 \right] \leq E \left[ (s^o_t - s^*_t)^2 \right]. \quad (9b)
\]

Equation (9a) states that the market exchange rate \( s_t \) should be less volatile around the fundamental rate \( s^*_t \) than the benchmark rate \( s^o_t \), in terms of the usual mean-square error criterion. Similarly, (9b) states that the market exchange rate should be less volatile around the benchmark rate than the fundamental rate. Thus, (9a) and (9b)—failure of which is a sufficient but not necessary condition for \( E[q_t] \neq 0 \)—can be used as excess volatility tests of exchange rates, relative to the predictions of the canonical asset-pricing model of the exchange rate with rational expectations.

A number of steps in the procedure just outlined require discussion. First, we must choose a value of \( \alpha \) to parameterize our tests. Although, in principle, \( \alpha \) could be estimated (Bartolini and Bodnar, 1992; Flood, Rose and Mathieson, 1991), previous efforts have yielded such a broad and imprecise array of estimates that an agnostic approach seems preferable. Hence, as it will be discussed in the next section, we select three values of \( \alpha \) (\( \alpha = 0.1, 1, \) and 5), suggested by previous research as representative of the low, typical, and high ranges of \( \alpha \), respectively, and present results of tests calibrated with these values. Results of tests calibrated with different values of \( \alpha \) are available upon request.
Second, one must define $f_t$. Common practice in the exchange rate literature, particularly in the context of tests of excess volatility, is to specify a structural exchange rate model and let it guide selection and aggregation of fundamentals. For instance, the textbook monetary model defines $f_t$ as a combination of domestic and foreign money supplies and incomes; models with sticky price and money adjustment would include prices and lagged money stocks among fundamentals; and so on.

Instead, here we proceed as follows. We begin by following Flood and Rose (1996) and Flood, Rose and Mathieson (1991), who use model (1) to obtain fundamentals as:

$$f_t = (1 + \alpha)s_t - \alpha E_t[s_{t+1}].$$ (10)

However, while Flood and Rose (1996) and Flood, Rose and Mathieson (1991) estimate expected future exchange rates using uncovered interest parity, i.e., $E_t[s_{t+1}] \equiv s_t + i_t - i_t^*$, we use survey-based exchange rate expectations to measure $E_t[s_{t+1}]$ directly. This procedure allows us to bypass the assumption of uncovered interest parity whose empirical record is, at best, questionable. This completes the parameterization of our test.

To clarify the implications of our method of calculating fundamentals for our tests, first substitute (10) into (3) for all $i$ from $t$ to $t+h$. Next, denote the exchange rate “news” at $t$ (i.e., the forecast error realized at $t$) by $e_t \equiv s_t - E_t[s_t]$, and rearrange terms so as to write $s_t^*$ as:

$$s_t^* = s_t + \sum_{i=1}^{h} \left( \frac{\alpha}{1 + \alpha} \right)^i e_{t-i} \equiv s_t + \eta_t,$$ (11)

where $\eta_t = \sum_{i=1}^{h} \left( \frac{\alpha}{1 + \alpha} \right)^i e_{t-i}$ is the discounted stream of news between $t+1$ and $t+h$. Then
compare (11) with (4). Since, by (11), $s_i^* - s_i = \eta_i$, then a finding that $\bar{q}$ is (significantly) different from zero provides evidence that the composite forecast error $\eta_i$ (which aggregates "news" consistently with model (1)) is correlated with information available at time $t$. Predictability of $\eta_i$, however, is a necessary but not sufficient condition for excess volatility: condition (7) may be violated even if (9a-b) are not. Thus, conditions (9a-b) pose a higher hurdle on the data than condition (7). This suggests that our subsequent ability to identify excess volatility as a cause of the rejection of the textbook rational expectations model of the exchange rate should be viewed as more interesting than the rejection of the model per se. This is especially so, since rejection of the canonical rational expectations model of the exchange rate is fairly widespread, while evidence of excess volatility has so far depended on stringent conditions, including restrictions on money demands, price adjustment, interest parity, and purchasing parity conditions (among others), and narrowly defined fundamentals.

Finally, we note another advantage of our approach, which it inherits directly from our use of the econometric method of Mankiw, Romer and Shapiro (1991). Our tests are independent of the presence (or lack thereof) of speculative exchange rate bubbles. This is because the fundamental rate $s_i^*$ is defined as a discounted stream of fundamentals only up to time $t+h$ (see expression (3)). Based on this definition, a hypothetical bubble term would be incorporated in both $s_i$ and $s_i^*$ through $s_{t+h}$, and hence would not alter the forecast error $(s_i - s_i^*)$.

3. Empirical Results
Data and calibration

We applied our methodology to data for the three major exchange rates: those of the British pound, the Deutsche mark, and the Japanese yen against the U.S. dollar. As noted above, our tests require only data on spot and expected future rates. However, the spot data must be sampled at the same time at which the survey of expectations is conducted, for our procedure to generate fundamentals synchronous with the exchange rates they are supposed to determine. Conveniently, the Financial Times Currency Forecaster archive (previously Currency Forecaster's Digest) provides synchronized monthly spot and survey-based expectations data. From this archive we drew data for all the available forecasting horizons, namely, the rates expected to prevail one, three, six, and twelve months after each survey date. We used the archive’s data from its inception (May 1984), until December 1998, for a total of 176 monthly observations.

Prior to performing our tests, we screened the data using a number of standard tests of unbiasedness and weak efficiency, and found no obvious evidence of “irrationality.” For instance, we found spot rates and survey expectations to be cointegrated with cointegrating vector \([1, -1]\) for all our currencies and forecasting horizons, as required for unbiasedness of expectations (Liu and Maddala, 1992). We also found no evidence of predictability of forecast errors from past exchange rates (e.g., a regression of the one-month forecast errors on the lagged exchange rate yielded \(t\)-statistics of 0.72, 1.58, and 1.42, for the mark/dollar, sterling/dollar, and yen/dollar rates, respectively; higher lags were even less significant). Clearly, however, one of the main points of our analysis is that these tests hardly exhaust the testable implications of rational expectations, when combined with the predictions of the canonical asset-pricing view of the exchange rate. As
shown below, in particular, the volatility restrictions we derived in Section 2 can be confidently rejected using our sample of data, indicating predictability of (suitably weighted sums of) forecast errors, with respect to information sets suggested by model (1). More interestingly, "excess volatility" can be identified as a leading cause of such rejection.

Finally, our tests are parameterized by $\alpha$, a plausible range of which we identified drawing from previous empirical studies. In surveying available evidence, we found studies attempting to estimate $\alpha$ structurally (e.g., as the annualized semi-elasticity of money demand to interest rates) typically yielding estimates between $\alpha=1$ and $\alpha=5$ (Fair, 1987; Goldfeld and Sichel, 1990; Laidler, 1993). By contrast, we found reduced-form estimates of $\alpha$, obtained directly from exchange rate data, to be much smaller, typically ranging between $\alpha=0.1$ and $\alpha=1$ (Bartolini and Bodnar, 1992; Flood, Rose and Mathieson, 1991). Overall, this evidence suggests to view the value $\alpha=1$ as a reasonable baseline for $\alpha$, and the values $\alpha=0.1$ and $\alpha=5$ as representative of the low and high ranges of the same parameter, respectively. (In calibration, these annualized values must be normalized by dividing by the forecasting horizon, defined in units of year.) The next section presents results for tests calibrated with these values.

Baseline results

The top panels of Tables 1-6 report standardized (asymptotically) normal statistics for tests of the null hypothesis $H_0: E[q_i]=0$, along with superscripts “a” and “b,” indicating whether inequalities (9a) and (9b) are violated, respectively. The tables’ bottom panels report Phillips-Perron $Z_t$ tests of unit roots in the series $(s_i - s_i^0)$, $(s_i - s_i^0)$, and $(s_i - s_i^0)$, the stationarity of which is required for the validity of our tests. Our prior is that--given a long enough sample--these series should be
recognized as stationary: we do not expect “market,” “fundamental,” and “forecast” rates to drift infinitely apart as the sample’s length grows to infinity. In particular, having defined the benchmark rate as $s_i^{o} = f_i = (1 + \alpha)s_i - \alpha E_t[s_{i+1}]$ allows for $s_i^{o}$ to be cointegrated with $s_i$ and with $s_i^{*}$ if survey-based expectations are cointegrated with $s_i$, as above documented. However, one must check for potential problems of nonstationarity due to short sample. To this end, we performed unit root tests following the methodology of Andrews and Monahan (1992) to correct for auto-correlation in the residuals of the unit-root regressions. As shown in the tables, our tests strongly confirm theoretical priors, confidently rejecting the null of a unit root in the series $(s_i^{*} - s_i)$, $(s_i - s_i^{o})$, and $(s_i^{*} - s_i^{o})$ at the five-percent significance (the critical value is -2.91) for all three exchange rates and all forecasting horizons, and with even greater confidence in most cases. Alternative stationarity tests we employed yielded essentially the same results.

Tables 1-4 present results for tests computed for different forecasting horizons: one month for Table 1, three months for Table 2, six months for Table 3, and twelve months for Table 4. The holding period $h$ is held constant at three months in these tables; it is allowed to vary from one month to twelve months in Table 5, discussed below.

Two main sets of results emerge from Tables 1-4, the first of which pertains to tests of the hypothesis $E[q_i] = 0$. This hypothesis can be rejected with confidence in all cases (i.e., for all forecasting horizons and all values of $\alpha$) for the mark/dollar rate, and in almost all cases for the sterling/dollar rate (for which the only exceptions occur for relatively high values of $\alpha$ at the forecasting horizon of twelve months) and the yen/dollar rate (for which the same assumptions fail to be confidently rejected only at the forecasting horizon of one month).

The second--and more interesting--set of results documented in Tables 1-4 pertains to the
diagnostics of the model’s rejection, namely, to evidence of violation of inequalities (9a) and (9b).

The evidence, in this respect, is that in almost all cases, excess volatility is a main contributing cause of the null’s rejection: actual spot rates are too volatile around $s^*_t$, around $s^o_t$, or around both, for model (1) to capture our currencies’ behavior in a market where agents form their expectations rationally. In particular, inequality (9a) tends to be rejected for lower values of $\alpha$, while the converse is true for inequality (9b); at least one violation occurs in almost all cases.

We checked whether these results could be attributed to isolated episodes of exchange rate instability (such as the dollar cycle of the mid-1980s or the yen cycle of the mid-1990s), and found evidence suggesting that this was not the case. To this end, we followed a standard procedure and examined the behavior of plots of recursively-generated sample average $\bar{q}$’s. These plots (available upon request) revealed smooth and rapid convergence of the test statistics for the sub-sample $\bar{q}$’s to their full-sample values, indicating robustness to short-lived episodes of instability.

*Alternative specifications of the tests*

To examine the responsiveness of our tests to changes in their configuration, we examined a variety of specifications and of parameter sets. Table 5 reports tests performed for holding periods, $h$, of one, six, and twelve months, while holding $\alpha$ fixed at one, and the forecasting horizon fixed at one month; Table 6 reports tests performed for different definitions of the benchmark rate $s^o_t$ (with $\alpha=1$, and both the forecasting horizon and holding period set at three months).

Tests of the hypothesis $E[q_t]=0$ are broadly insensitive to the choice of holding period for the sterling/dollar and mark/dollar rates; for the yen/dollar rate, however, we cannot reject the null hypothesis $E[q_t]=0$ as the holding period rises to six months or longer. Similarly, evidence
of excess volatility (identified by violation of inequalities (9a) or (9b)) weakens for all rates as the
holding period rises to six months or longer (we find evidence of excess volatility only for the
mark/dollar rate when \( h = 12 \) months; and no evidence of excess volatility when \( h = 6 \) months). To
interpret this evidence, recall that the holding period \( h \) identifies the horizon over which the
present-value relationship (1) is assumed to capture the dynamics of the exchange rate. Thus, the
evidence from Table 5 is that the textbook rational expectations, asset-pricing model of the
exchange rate captures the behavior of our sample currencies better over longer than over shorter
horizons.

Finally, we illustrate in Table 6 the effects on our tests of changes in the definition of the
benchmark rate \( s_t^0 \), which was hitherto set at \( s_t^0 = f_t \). We consider two alternative definitions of
\( s_t^0 \). First, we define \( s_t^0 \) as the model’s prediction at time \( t - 1 \) of the exchange rate expected to
prevail at time \( t \). This rate can be obtained by lagging equation (1) once and solving for \( E_{t-1}[s_t] \),
yielding:

\[
s_t^0 = E_{t-1}[s_t] = \left( \frac{(1 + \alpha)/\alpha}{s_{t-1} - f_{t-1}/\alpha} \right).
\]

(12)

Alternatively, we also define \( s_t^0 \) as \( s_{t-1} \), the last observed value of the market exchange
rate. By this definition of \( s_t^0 \), the volatility (or mean square error) of the difference \( \left( s_t - s_t^0 \right) \)
corresponds to the familiar notion of conditional exchange rate volatility. Both these definitions
meet the requirement that \( s_t^0 \) should be known at time \( t \). As documented in the bottom panel of
Table 6, both definitions also meet the requirement of making the series \( \left( s_t - s_t^* \right), \left( s_t - s_t^0 \right), \) and
\( \left( s_t^* - s_t^0 \right) \) stationary, leaving no doubt on our ability to reject the hypothesis of a unit root.

We report in Table 6 results for our baseline parameterization (\( \alpha = 1, h = 3, \) survey-
forecasting horizon = three months) when the benchmark rate is set at either one of these
definitions. When \( s_t^0 = E_{t-1}[s_t] \), model (1) is strongly rejected for all three exchange rates, although this time the tests reveal no evidence of excess volatility. By contrast, when \( s_t^0 = s_{t-1} \), there is neither evidence against the model as a whole, nor--by necessity--evidence of excess volatility. One should not be surprised by these results, particularly by their apparent contrast with those of Tables 1-4: it is normal for our tests to yield different results when they are conditioned on different time-\( t \) information sets (that is, on different definitions of \( (s_t - s_t^0) \)). This is because, while some of these information sets may be correlated with future forecast errors, other sets may not be so. In particular, lack of evidence of excessive conditional volatility, and our inability to detect a significant correlation between the current innovation \( (s_t - s_{t-1}) \) and future-model-weighted--forecast errors, reflects the well known difficulty of distinguishing exchange rate innovations from pure noise at short horizons.

4. Concluding Remarks

For reasons ranging from calibration errors, small-sample biases, and--most importantly--strong reliance on specific exchange rate models, previous research on exchange rates has failed to provide reliable evidence of the ability of popular exchange rate models to match exchange rate volatilities observed over the post-Bretton Woods period. This paper has tried to partially address this gap by presenting excess volatility tests of exchange rates that require a minimal set of assumptions on the intertemporal behavior of exchange rates and allow for a wide definition of their fundamental determinants, the observability of which is not required for the implementation of the tests. Given the limited structure we have imposed on the data, our evidence of excess volatility with respect to the predictions of the canonical rational expectations, asset-pricing model
of the exchange rate should be viewed as significantly stronger than that available from previous research.

A possible reading of our results (a reading that we favor) is that the problem may rest less with exchange rates being *too volatile*, than with the most popular tools used to explain exchange rates (including rational expectations, and the most stripped-down asset-pricing model of the exchange rate we can think of) simply being unable to generate enough volatility. From this viewpoint, because of the limited assumptions built in our tests, our results may be useful to clarify aspects of the research agenda on exchange rates. Reviewing empirical research on exchange rates conducted in recent decades, for instance, Frankel and Rose (1995) note the sorry record of simple exchange rate models based on observable macro variables (Flood and Taylor, 1996), and note two lines of research promising a better description of exchange rate dynamics: one line of research aimed at incorporating speculative bubbles into exchange rate models; and another one aimed at providing a detailed analysis of currency markets’ micro-structure. Our work points to shortcomings of the simple rational expectations, asset-pricing view of the exchange rate, even when allowing for an open definition of fundamentals and for speculative bubbles. Therefore, our results suggest that if either lifeboat pointed out by Frankel and Rose can rescue structural exchange rate models, it is unlikely to be the first one.
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<th>$\alpha = 5$</th>
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<th>$Z_t$ tests for $s_i^- - s_i$</th>
<th>sterling / dollar</th>
<th>$-7.66$</th>
<th>$-7.49$</th>
<th>$-7.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mark / dollar</td>
<td>$-7.47$</td>
<td>$-7.21$</td>
<td>$-7.23$</td>
<td></td>
</tr>
<tr>
<td>yen / dollar</td>
<td>$-7.81$</td>
<td>$-7.18$</td>
<td>$-7.18$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Z_t$ tests for $s_i - s_i^o$</th>
<th>sterling / dollar</th>
<th>$-8.60$</th>
<th>$-8.60$</th>
<th>$-8.60$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mark / dollar</td>
<td>$-7.44$</td>
<td>$-7.44$</td>
<td>$-7.44$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Z_t$ tests for $s_i^- - s_i^o$</th>
<th>sterling / dollar</th>
<th>$-8.11$</th>
<th>$-8.89$</th>
<th>$-8.86$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mark / dollar</td>
<td>$-7.85$</td>
<td>$-7.91$</td>
<td>$-7.63$</td>
<td></td>
</tr>
<tr>
<td>yen / dollar</td>
<td>$-7.75$</td>
<td>$-8.06$</td>
<td>$-8.24$</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The values in the upper panel are the (asymptotically) standard normal test statistics for the null hypothesis that $E[q_t] = 0$, where $q_t$ is defined by equation (8) in the text. The superscripts “a” and “b” indicate that inequalities (9a) and (9b) were violated, respectively. The 5 percent critical value for the Phillips-Perron $Z_t$ stationarity tests of $\{s_i^- - s_i\}$, $\{s_i - s_i^o\}$, and $\{s_i^- - s_i^o\}$, reported in the lower panel, is -2.91. The sample includes (176-$h$) observations from May 1984 to December 1998.
Table 2
Forecasting horizon = 3 months, holding period = 3 months

<table>
<thead>
<tr>
<th></th>
<th>( \alpha = 0.1 )</th>
<th>( \alpha = 1 )</th>
<th>( \alpha = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility Tests</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-3.19 (^{a,b})</td>
<td>-2.48 (^{b})</td>
<td>-2.47 (^{b})</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-3.84 (^{a,b})</td>
<td>-2.74 (^{b})</td>
<td>-2.64 (^{b})</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-3.33 (^{a})</td>
<td>-2.57 (^{b})</td>
<td>-2.42 (^{b})</td>
</tr>
<tr>
<td>( Z_t ) tests for ( s_{it}^* - s_{it} )</td>
<td>sterling / dollar</td>
<td>-6.58</td>
<td>-5.35</td>
</tr>
<tr>
<td></td>
<td>mark / dollar</td>
<td>-5.91</td>
<td>-4.98</td>
</tr>
<tr>
<td></td>
<td>yen / dollar</td>
<td>-6.93</td>
<td>-5.40</td>
</tr>
<tr>
<td>( Z_t ) tests for ( s_{it} - s_{it}^o )</td>
<td>sterling / dollar</td>
<td>-3.09</td>
<td>-3.09</td>
</tr>
<tr>
<td></td>
<td>mark / dollar</td>
<td>-2.95</td>
<td>-2.95</td>
</tr>
<tr>
<td></td>
<td>yen / dollar</td>
<td>-4.11</td>
<td>-4.11</td>
</tr>
<tr>
<td>( Z_t ) tests for ( s_{it}^* - s_{it}^o )</td>
<td>sterling / dollar</td>
<td>-9.66</td>
<td>-5.87</td>
</tr>
<tr>
<td></td>
<td>mark / dollar</td>
<td>-9.38</td>
<td>-4.80</td>
</tr>
<tr>
<td></td>
<td>yen / dollar</td>
<td>-9.49</td>
<td>-6.94</td>
</tr>
</tbody>
</table>

Notes: The values in the upper panel are the (asymptotically) standard normal test statistics for the null hypothesis that \( \mathbb{E}[q_t] = 0 \), where \( q_t \) is defined by equation (8) in the text. The superscripts “a” and “b” indicate that inequalities (9a) and (9b) were violated, respectively. The 5 percent critical value for the Phillips-Perron \( Z_t \) stationarity tests of \( (s_{it}^* - s_{it}), (s_{it} - s_{it}^o), \) and \( (s_{it}^* - s_{it}^o) \), reported in the lower panel, is -2.91. The sample includes (176-\( h \)) observations from May 1984 to December 1998.
Table 3
Forecasting horizon = 6 months, holding period = 3 months

<table>
<thead>
<tr>
<th></th>
<th>( \alpha = 0.1 )</th>
<th>( \alpha = 1 )</th>
<th>( \alpha = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volatility Tests</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-3.20 a,b</td>
<td>-2.48 a,b</td>
<td>-2.38 b</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-3.45 a,b</td>
<td>-2.87 a,b</td>
<td>-2.60 b</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-4.01 a,b</td>
<td>-3.23 a,b</td>
<td>-2.88 b</td>
</tr>
<tr>
<td>( Z_t ) tests for ( s_t^* - s_t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-5.67</td>
<td>-4.72</td>
<td>-5.20</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-4.79</td>
<td>-4.41</td>
<td>-4.75</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-5.78</td>
<td>-4.64</td>
<td>-4.59</td>
</tr>
<tr>
<td>( Z_t ) tests for ( s_t - s_t^o )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-3.49</td>
<td>-3.49</td>
<td>-3.49</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-3.09</td>
<td>-3.09</td>
<td>-3.09</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-4.39</td>
<td>-4.39</td>
<td>-4.39</td>
</tr>
<tr>
<td>( Z_t ) tests for ( s_t^* - s_t^o )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-10.61</td>
<td>-7.34</td>
<td>-4.14</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-10.42</td>
<td>-5.94</td>
<td>-3.37</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-10.58</td>
<td>-7.92</td>
<td>-5.42</td>
</tr>
</tbody>
</table>

Notes: The values in the upper panel are the (asymptotically) standard normal test statistics for the null hypothesis that \( \mathbb{E}[q_t] = 0 \), where \( q_t \) is defined by equation (8) in the text. The superscripts “a” and “b” indicate that inequalities (9a) and (9b) were violated, respectively. The 5 percent critical value for the Phillips-Perron \( Z_t \) stationarity tests of \( (s_t, s_t^o), (s_t^* - s_t), \) and \( (s_t^* - s_t^o) \), reported in the lower panel, is -2.91. The sample includes (176-h) observations from May 1984 to December 1998.
Table 4
Forecasting horizon = 12 months, holding period = 3 months

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 0.1$</th>
<th>$\alpha = 1$</th>
<th>$\alpha = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility Tests</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling/dollar</td>
<td>-2.63$^{a,b}$</td>
<td>-1.97$^{a,b}$</td>
<td>-1.73$^b$</td>
</tr>
<tr>
<td>mark/dollar</td>
<td>-3.22$^{a,b}$</td>
<td>-2.58$^{a,b}$</td>
<td>-2.35$^b$</td>
</tr>
<tr>
<td>yen/dollar</td>
<td>-4.80$^{a,b}$</td>
<td>-3.85$^{a,b}$</td>
<td>-3.15$^b$</td>
</tr>
<tr>
<td><em><em>$Z_t$ tests for $s_i^</em> - s_i$</em>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling/dollar</td>
<td>-5.75</td>
<td>-4.60</td>
<td>-5.06</td>
</tr>
<tr>
<td>mark/dollar</td>
<td>-4.75</td>
<td>-4.15</td>
<td>-4.69</td>
</tr>
<tr>
<td>yen/dollar</td>
<td>-5.59</td>
<td>-4.29</td>
<td>-4.37</td>
</tr>
<tr>
<td><strong>$Z_t$ tests for $s_i - s_i^o$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling/dollar</td>
<td>-3.32</td>
<td>-3.32</td>
<td>-3.32</td>
</tr>
<tr>
<td>mark/dollar</td>
<td>-3.25</td>
<td>-3.25</td>
<td>-3.25</td>
</tr>
<tr>
<td>yen/dollar</td>
<td>-4.29</td>
<td>-4.29</td>
<td>-4.29</td>
</tr>
<tr>
<td><em><em>$Z_t$ tests for $s_i^</em> - s_i^o$</em>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling/dollar</td>
<td>-11.22</td>
<td>-8.65</td>
<td>-4.78</td>
</tr>
<tr>
<td>mark/dollar</td>
<td>-11.10</td>
<td>-8.26</td>
<td>-4.33</td>
</tr>
<tr>
<td>yen/dollar</td>
<td>-11.30</td>
<td>-8.49</td>
<td>-6.40</td>
</tr>
</tbody>
</table>

*Notes:* The values in the upper panel are the (asymptotically) standard normal test statistics for the null hypothesis that $E[q_t]=0$, where $q_t$ is defined by equation (8) in the text. The superscripts “a” and “b” indicate that inequalities (9a) and (9b) were violated, respectively. The 5 percent critical value for the Phillips-Perron $Z_t$ stationarity tests of $\{s_i^* - s_i\}$, $\{s_i - s_i^o\}$, and $\{s_i^* - s_i^o\}$, reported in the lower panel, is -2.91. The sample includes (176-$h$) observations from May 1984 to December 1998.
Table 5  
α = 1, forecasting horizon = 1 month

<table>
<thead>
<tr>
<th>Volatility Tests</th>
<th>h = 1 month</th>
<th>h = 6 months</th>
<th>h = 12 months</th>
</tr>
</thead>
<tbody>
<tr>
<td>sterling / dollar</td>
<td>-3.57 b</td>
<td>-2.42</td>
<td>-3.50</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-3.50 b</td>
<td>-2.48</td>
<td>-2.87 b</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-3.05 b</td>
<td>-1.06</td>
<td>-1.13</td>
</tr>
</tbody>
</table>

| Zₜ tests for sₜ - sₜ | sterling / dollar | -11.71       | -4.78         | -3.72         |
| mark / dollar        | -11.35       | -4.87        | -3.35         |
| yen / dollar         | -12.43       | -5.02        | -3.88         |

| Zₜ tests for sₜ⁺⁻ sₜ⁹ | sterling / dollar | -9.09        | -9.21         | -9.07         |
| mark / dollar        | -7.53        | -7.68        | -7.45         |
| yen / dollar         | -8.03        | -8.10        | -7.80         |

| Zₜ tests for sₜ⁺⁻ sₜ⁹ | sterling / dollar | -9.75        | -8.36         | -8.69         |
| mark / dollar        | -8.19        | -7.75        | -7.89         |
| yen / dollar         | -8.10        | -6.85        | -6.70         |

**Notes:** The values in the upper panel are the (asymptotically) standard normal test statistics for the null hypothesis that $E[qₜ] = 0$, where $qₜ$ is defined by equation (8) in the text. The superscripts “a” and “b” indicate that inequalities (9a) and (9b) were violated, respectively. The 5 percent critical value for the Phillips-Perron $Zₜ$ stationarity tests of $(sₜ⁺ - sₜ)$, $(sₜ - sₜ⁹)$, and $(sₜ⁺ - sₜ⁹)$, reported in the lower panel, is -2.91. The sample includes (176-h) observations from May 1984 to December 1998.
Table 6
\(\alpha = 1\), forecasting horizon = 3 months, holding period = 3 months

<table>
<thead>
<tr>
<th></th>
<th>(s_t^o = E_{t-1}[s_t])</th>
<th>(s_t = s_{t-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility Tests</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>2.99</td>
<td>-0.70</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>3.56</td>
<td>-0.62</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>2.09</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>(Z_t) tests for (s_t^o - s_t)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-5.30</td>
<td>-5.30</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-4.93</td>
<td>-4.93</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-5.41</td>
<td>-5.41</td>
</tr>
<tr>
<td><strong>(Z_t) tests for (s_t - s_t^o)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-8.63</td>
<td>-11.74</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-7.59</td>
<td>-11.56</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-9.23</td>
<td>-12.20</td>
</tr>
<tr>
<td><strong>(Z_t) tests for (s_t^o - s_t)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sterling / dollar</td>
<td>-4.20</td>
<td>-4.92</td>
</tr>
<tr>
<td>mark / dollar</td>
<td>-3.85</td>
<td>-4.46</td>
</tr>
<tr>
<td>yen / dollar</td>
<td>-4.42</td>
<td>-5.45</td>
</tr>
</tbody>
</table>

**Notes:** The values in the upper panel are the (asymptotically) standard normal test statistics for the null hypothesis that \(E[q_t] = 0\), where \(q_t\) is defined by equation (8) in the text. The superscripts “a” and “b” indicate that inequalities (9a) and (9b) were violated, respectively. The 5 percent critical value for the Phillips-Perron \(Z_t\) stationarity tests of \(\{s_t^o - s_t\}\), \(\{s_t - s_t^o\}\), and \(\{s_t^o - s_t\}\), reported in the lower panel, is -2.91. The sample includes \((176 - h)\) observations from May 1984 to December 1998.
Reference


Mankiw, N. Gregory, David Romer and Matthew D. Shapiro, “Stock Market Forecastability and


Notes

1. This method involves filtering $q_t$ by an AR(1) process and estimating the asymptotic variance $\hat{\Omega}$ of the filtered residuals, $\hat{u}_t$, by $\hat{\Omega} = C(0) + 2 \sum \kappa(j/h) C(j)$, where $C(j) = \sum_{i=1}^{T} \hat{u}_{t-i} \hat{u}_{t+i-j}/T$, $\kappa(j/h)$ is a quadratic spectral kernel, and the bandwidth parameter $h$ is chosen according to an automatic data-based procedure. $\hat{V}$ is then recovered by multiplying $\hat{\Omega}$ by the square of the inverse of the AR filter.

2. The only exception of which we are aware is Gardeazabal, Regúlez, and Vázquez (1997), which tests model (1) by a method of simulated moments without need to identify fundamentals. Difference in econometric methodology and focus (Gardeazabal, Regúlez, and Vázquez do not derive or test the volatility restrictions implied by model (1)) contribute to the difference between our results and theirs. (These authors’ tests, for instance, do not reject model (1)).

3. Though not unchallenged, see Gardeazabal, Regúlez, and Vázquez (1997).

4. Mankiw, Romer, and Shapiro (1991) also show that the volatility test that we apply exhibits better statistical properties than traditional regression-based tests of present value models. In particular, the test’s finite-sample distribution is well approximated by its asymptotic distribution, causing the model to be rejected the correct number of times, on average. Traditional regression tests of asset-price models such as (1), instead, are biased in small samples if prices and fundamentals are non-stationary--as is typically the case with exchange rates--., rejecting the model too often in finite samples.

5. Detailed results are available upon request from the authors. See also Giorgianni (1996) for a detailed analysis.

6. Complete details of our tests, the data, and the needed Gauss program are available upon request.