

NO. 1053 FEBRUARY 2<u>023</u>

REVISED
DECEMBER 2023

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Marco Del Negro, Julian di Giovanni, and Keshav Dogra *Federal Reserve Bank of New York Staff Reports*, no. 1053 February 2023; revised December 2023

JEL classification: E12, E31, E52, Q54

Abstract

We develop a multi-sector New Keynesian model to analyze the inflationary effects of climate policies. Climate policies need not be inflationary, but can generate an inflation-output tradeoff whose size depends on how flexible prices are in the "dirty" and "green" sectors relative to the rest of the economy, and on whether climate policies consist of taxes or subsidies. A quantitative version of the model calibrated to U.S. data on input-output linkages and sectoral heterogeneity in emissions and price stickiness suggests that an increase in carbon taxes would generate a sizable tradeoff: containing the impact on headline or core inflation would lead to a deep recession. But while sizeable, the tradeoff is relatively short-lived as it wanes after one year.

Key words: green transition, inflation, central bank's tradeoffs, input-output linkages

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This paper presents preliminary findings and is being distributed to economists and other interested readers solely to stimulate discussion and elicit comments. The views expressed in this paper are those of the author(s) and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System Any errors or omissions are the responsibility of the author(s).

1 Introduction

Climate change will have widespread effects on the economy. One prescient concern is climate change's impact on price stability. Indeed, some policymakers have argued that we face a "new age of energy inflation" (Schnabel, 2022), whereby central banks may be forced to live with a persistently higher level of inflation as a result of both the physical effects of climate change and the transition to a low-carbon economy. While this idea may seem intuitive, as a general statement it is arguably incorrect: the adjustment in *relative* prices induced by climate change or policies can in principle occur under any level of inflation. Moreover, monetary policymakers may still have the necessary levers to meet their inflation targets, though doing so may involve a tradeoff with other targets, such as the output gap.

The goal of this paper is to study these tradeoffs using both analytics and a rich quantitative input-output model. Specifically, we ask how the *green transition* – policies such as carbon taxes that reduce greenhouse gas emissions in order to limit global warming – affects monetary policymakers' ability to pursue price stability. We focus on the inflationary effects of climate *policy* because the green transition is an immediate concern for central bankers as policies aimed at discouraging high emission activities and/or promoting clean energy have been already put in place in many advanced economies, and more are likely to come. We find that the green transition does not force monetary policymakers to tolerate higher inflation, but can potentially generate a tradeoff for policymakers. Two key factors drive this tradeoff. First, the relative *stickiness* of prices in the "dirty", "green" (clean energy), and the "other" sector (the rest of the economy) is a key determinant as to whether monetary policy is able to keep inflation at its target while also stabilizing output at its natural level. Second, the two types of climate policies analyzed in this paper – either a tax on the dirty sector or a subsidy on the green sector – have dramatically different implications for inflation and the tradeoff faced by the monetary policymaker.

We begin by documenting empirically the relationship between the "dirtiness" of a sector, as measured by emissions per value added, and price stickiness. We find that prices in "dirty" sectors tend to be more flexible than in the rest of the economy (section 2).

Next, we employ a simple framework to develop some analytical intuition about the forces at play (section 3). We begin by studying the effects of a tax on the dirty sector, and use a two-sector New Keynesian model where the dirty sector represents high-emission activities and the other sector stands in for the rest of the economy, initially abstracting from the green sector. Each sector is monopolistically competitive and features nominal rigidities; importantly, the degree of price stickiness can vary across sectors.² In this simple model, there are no input-output linkages

¹We do not study the impact of the physical effects of climate change itself on inflation, partly because the implications of climate change for the economy, even if potentially large, are also very uncertain and hence more difficult to discuss.

²In this model production uses only labor, and there is no capital nor investment. We suspect that a more complex

between the two sectors. Thus, "dirty goods" should be thought of as a stand-in for goods and services with relatively high greenhouse gas emissions, both direct and indirect, while "other goods" represent all other consumption.

The key lessons from the simple model are as follows. First, with fully flexible prices climate policies would not pose any problem for an inflation targeting central bank—hence any tradeoff is necessarily related to the presence of nominal rigidities. This is because the adjustment in relative prices can take place under any level of overall inflation. Second, in the empirically realistic case where prices are more flexible in carbon-intensive sectors, the transition creates a tradeoff between keeping inflation low and closing the output gap. Intuitively, the tradeoff arises because the central bank needs to nudge inflation in the sticky sector down so that the needed adjustment in relative prices occurs with an overall inflation level that is in line with its target. But this nudge involves cooling down the economy. If the central bank is not willing to do that, it may have to accept temporarily high inflation. Finally, if instead climate policy primarily takes the form of subsidizing a green sector with relatively flexible prices, rather than taxing a dirty sector, our conclusions are reversed: the green transition is deflationary unless monetary policy engineers a positive output gap.

To investigate the quantitative importance of our results, we calibrate a 396-sector version of the model using input-output tables from the Bureau of Economic Analysis (BEA), Cotton and Garga (2022)'s data set on sectoral price stickiness based on Producer Price Index (PPI) microdata, and sector-level emissions data from the EIA and EPA derived using the methodology of Shapiro et al. (2018) (section 4). This multi-sector version of the model is key for our quantitative analysis for several reasons. First, the notion that dirty sector prices are flexible while clean sectors' prices are sticky, which is embedded in some two sector models in the literature, is an oversimplification (as shown in section 2). There are several sectors that are quite dirty in terms of emissions—in that they use a lot of fossil fuels as inputs—whose prices are quite sticky. For that matter, even the prices of some energy sectors, like coal, are not all that flexible. Our granular multisector IO model lets us assign to each sector the correct level of stickiness and emissions, via the input-output matrix, without having to make oversimplifying assumptions.

Second, the literature studying monetary policy in network economies (e.g., Ghassibe, 2021; La'O and Tahbaz-Salehi, 2022; Rubbo, 2023; Afrouzi and Bhattarai, 2023) has emphasized the importance of networks in the transmission of both monetary policy and relative price shocks, such as those we study here. In particular, a quantitative analysis needs to take into account the fact that dirty sector output (e.g., energy) is an input to other sectors, and that this may slow down the adjustment in relative prices, since taxes to the dirty sector increase marginal costs for the rest of

economy where the green transition amounts to subsidizing/penalizing capital accumulation in the clean/dirty sector will yield very similar conclusions, although we do not explicitly consider such an economy in our analysis.

the economy. Third, the multi-sector model allows us to investigate how the degree of substitution in production between (dirty) energy and other factors of production impacts the inflation-output gap tradeoff under different monetary policy regimes. Finally, we are able to consider how imposing a carbon tax at different points along the supply chain (e.g., upstream vs. downstream) impacts our main results.

We find that in this input-output model an increase in carbon taxes creates a sizable tradeoff between stabilizing inflation and the output gap. Under strict headline or even core inflation
targeting, a gradual increase in the carbon tax from 0 to 20 (2012) dollars per metric ton of CO2
leads to a dramatic recession. Even trying to contain rather than completely suppress the inflationary impact of the increase in taxes—for example bringing 12-month headline or core inflation
below 1 and .5 percent one year after the shock, respectively—leads to a very large recession, with
an average output gap of 30 and 8 percent over the year following the announcement of the tax.
We find that nominal wage stickiness is important in generating this tradeoff. If wages were fully
flexible, they would tank following the tax increase, counteracting the increase in marginal costs
due to rising energy prices and moderating the effect on core inflation. As suggested by the simple
two-sector model described earlier, heterogeneous price-stickiness is also crucial in generating this
result: In a counterfactual economy where all sectors have the same degree of price flexibility, the
same increase in carbon taxes would be much less inflationary.

At the same time, while the impact of the carbon tax on headline and core inflation is substantial, it is fairly short lived. A central bank committed to closing the output gap would be forced to tolerate a rise in headline and core inflation of almost 2 and 1 percent, respectively, after one year, but little inflation afterwards. In this sense, we find that the tradeoff due to the green transition is quantitatively not very different from those typically associated with large fluctuations in energy prices: the rise in inflation is sizable, but mostly temporary, in that it does not last past one year. All in all, we conclude that the green transition has limited implications for price stability. At least for the size of the carbon tax we consider, a central bank that 'looks through' the effects of the green transition in order to prevent a recession is unlikely to find its credibility damaged by the ensuing inflation, which is considerably smaller than that experienced during compared with the post-COVID inflationary outbreak.

Related Literature. While there are already a number of important empirical and model-based works discussing the impact of climate change and climate policies on the macroeconomy, none of them to our knowledge formally studies the tradeoffs faced by central banks, and the drivers of this tradeoff.

³Within our model, this reduces long-run emissions by 40 percent, broadly in line with the Biden administration's commitment to reduce emissions to 50-52 percent of 2005 levels by 2030 under the Paris climate accord (https://www.whitehouse.gov/climate/, https://www.epa.gov/system/files/documents/2023-04/Data-Highlights-1990-2021.pdf).

Schnabel (2022) argues that physical and transition risks arising from climate change may be inflationary. Schnabel classifies three sources of climate-driven inflation. First, "climateflation," where climate change increases the probability of natural disasters and severe weather events, which lead to droughts, supply chain problems and other production disruptions that may put upward pressure on prices. This climateflation captures possible physical risks of climate change, which we do not study here for the reasons discussed above. Second, "fossilflation," where the use of policies such as carbon taxes to discourage the use of fossil fuels and reduce emissions may place upward pressure on prices. This fossilflation is at the heart of the climate policy in our proposed analytical framework. Third, "greenflation," which arises from price increases in scarce commodities (e.g., lithium for batteries) as a result of the increased demand from the green energy sector. This mechanism is unlikely to change our qualitative result that subsidies to the green sector are, on the whole, deflationary, although it is in principle quantitatively important in attenuating this effect.

Most recent studies on the impact of transition policies have focused on their effects on output (eg, Metcalf and Stock, Forthcoming), but a growing number also studied the implications for inflation. Using VAR-based evidence, Känzig (2022) finds that a carbon policy shock in Europe leads to a persistent rise in energy prices (1 percent on impact, by construction) and a decline in emissions, as one would expect.⁵ The responses of headline prices are about one-fifth of the response in energy prices, while prices in the rest of the economy (core) barely respond. Industrial production declines for about two years after the shock. Importantly, the policy rate essentially does not change. All in all, these responses are consistent with the simple model outlined below, where energy prices are more flexible than core prices, and policy lets nominal energy prices do all the adjustment in relative prices. Using local projections, Konradt and Weder di Mauro (2021) find that while carbon taxes implemented in Europe and Canada impact relative prices, they have no significant impact on overall inflation.⁶ However, they also find that for a subset of European countries where monetary policies are constrained, the effect on inflation is positive and significant, in line with Känzig (2022) and with related work by McKibbin et al. (2021).

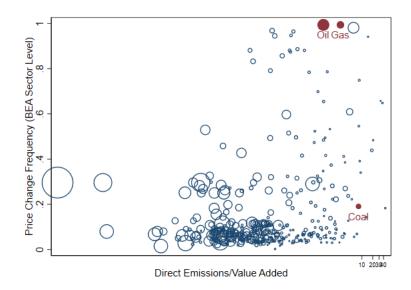
Some authors, like us, have used New Keynesian frameworks to study the inflationary impact of transition policies. Bartocci et al. (2022) use a two-country model with an energy sector, calibrated to the euro area and the rest of the world, and find that an increase in carbon taxes generates recessionary effects, which are ameliorated by accommodative monetary policy. Ferrari and Nispi Landi (2022) focus instead on the role of expectations in determining whether emission taxes are infla-

⁴Faccia et al. (2021) examine how rising temperatures may impact inflation via higher food prices using a panel of cross-country data, and find that while hot summers may drive up prices in the short run, the effects are either negative or insignificant in the medium term. Ciccarelli and Marotta (2021) find evidence that physical risks work as negative demand shocks while transition policies resemble downward supply movements.

⁵Känzig (2022) uses a high-frequency identification approach based on changes in carbon future prices from the European Union Emissions Trading System immediately following regulatory events.

⁶This result is supported in recent work by Moessner (2022), who estimates the impact of emissions trading systems and carbon taxes on a broad set of price indexes using a dynamic panel model for 35 countries.

Figure 1. Mean price change frequency of a good in a given sector vs CO2 emissions/value added across 396 sectors in the United States



Notes: This figure plots a bin scatter of the sector-level mean price change frequency against the sector-level CO2 emissions to value added. The emissions ratio is expressed in terms of kilotons of CO2 emitted per millions of US\$ value added produced and is based on the direct usage of fossil fuels (oil, gas, or coal) in production, and is plotted on a log scale. Circle sizes are based on the toal sector-level value added within a bin. Regressions of level on level or log-level yield a positive and significant coefficient.

tionary or deflationary. Ferrari and Pagliari (2021) and Airaudo et al. (2023) consider optimal policy under the the green transition in the world economy and in a small open economy, respectively. None of these studies emphasizes the importance of relative price stickiness in determining the tradeoffs faced by monetary policy, or consider a realistically calibrated network economy, as we do. In contemporaneous work issued after the publication of our working paper, Olovsson and Vestin (2023) and Nakov and Thomas (2023) use simple New Keynesian models with an energy sector to study the tradeoffs faced by monetary policymakers during the green transition, along the lines of our analysis in section 3. Finally, our work relates to the literature on heterogeneity in price stickiness across sectors (Carvalho, 2006; Nakamura and Steinsson, 2010) and to the aforementioned recent literature on networks and monetary policy (Ghassibe, 2021; La'O and Tahbaz-Salehi, 2022; Rubbo, 2023; Afrouzi and Bhattarai, 2023).

2 Price Rigidities and Emissions

In this section we document empirically the relationship between the "dirtiness" of a sector, as measured by emissions per value added, and price stickiness. Specifically, we collect data on price

Table 1. Mean price change frequency and CO2 emissions value added for other vs. dirty sectors in the United States

| Sector | CO2/VA | Price Δ Freq. |
|--------|--------|----------------------|
| Other | 0.049 | 0.148 |
| Dirty | 1.326 | 0.205 |

Notes: Mean price change frequency and CO2 emissions per value added for other vs. dirty sectors in the United States. Calculations are based on averaging of underlying sectoral information for sub-sectors in Other and Dirty, respectively, where we split the 396 sub-sectors into two based on their emissions levels. See Figure 1 and Appendix A for underlying sectoral information and for data sources.

rigidity and emissions intensity at the sector level. We source information on price rigidity from Cotton and Garga (2022) that calculates the frequency of price changes at the goods level as the ratio of the number of price changes to the number of sample months. We compute the sector-level average of these measures as our baseline measure of price rigidity. We follow the methodology of Shapiro et al. (2018) to construct sector-level emission measures based each sectors use of fossil fuels (oil, gas, and coal) in production along with fossil fuel emissions data from the EPA and EIA. As in Shapiro et al. (2018), a sector's emissions can be calculated based on its direct use of the fossil fuel sectors in production as well as a total measure that incorporates the indirect usage via the full input-output network. A sector-level emissions intensity is defined as the ratio of CO2 emissions to value added, using 2012 BEA IO data, to obtain a measure is expressed in terms of kilotons of CO2 emitted per millions of US\$ value added. Appendix A describes all details of our data construction.

Figure 1 plots a bin scatter of the sector-level price rigidity measure against the CO2/VA ratio for 396 U.S. sectors based on *direct* emissions' creation in production. The size of each bin's circle corresponds to the sum of the value added of all sectors in a given bin, so large circles represent large value added. We further include individual solid points for the three fossil fuel industries, though they are also included in their respective bin. Figure 1 shows that sectors with higher CO2 emissions (relative to value added) tend to have a higher average frequency of price change. Table 1 presents the average frequency for two equally sized groups of sectors with emissions above and below the median, respectively, and shows that the frequency of price changes is about 0.15 for dirty sectors and 0.20 for the rest of the economy. Figure 1 also shows that the simple notion that dirty sector prices are flexible while clean sectors prices are sticky is not correct, however. There are several sectors that are quite dirty in terms of emissions—in that they use a lot of fossil fuels as inputs—whose prices are quite sticky. Even the prices of some energy sectors, like coal, are not

⁷Cotton and Garga (2022) construct their data set by using CPI and PPI price change data from Nakamura and Steinsson (2008) and then creating a crosswalk for the goods and services with reported price frequency change data to 2017 NAICS in order to create sector-level measures of price changes.

⁸Formally, total emissions are calculated using the input-output table's Leontief inverse.

all that flexible. The granular multisector IO model we present in section 4 lets us assign to each sector the correct level of stickiness and emissions, via the input-output matrix, without having to make oversimplifying assumptions.

3 Analytical Results from a Two-Sector Model

The goal of this section is to deliver qualitative insights using the simplest possible model. We therefore start with a two-sector model, where the sectors are a dirty high-emissions sector that the government wants to tax, and the rest of the economy. This two-sector model initially ignores a green sector that produces low-emissions goods (such as clean energy) and that the government may want to subsidize, but we will consider subsidies later in section 3.3. Thus, our two-sector model is a relatively standard New Keynesian economy except that household consumption is an aggregate of dirty goods, which may be taxed, and other goods. The economy consists of a representative household, monopolistically competitive firms, a fiscal authority, and a central bank.

Households. The representative household solves

$$\max_{\{C_t, C_t^o, C_t^d, C_t^o(\cdot), C_t^d(\cdot), L_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ \ln C_t - bL_t \}
\text{s.t. } \int_0^1 P_t^d(j) C_t^d(j) dj + \int_0^1 P_t^o(j) C_t^o(j) dj + \frac{1}{1+i_t} B_{t+1} = W_t L_t + T_t + B_t
C_t = (C_t^o/\gamma)^{\gamma} (C_t^d/(1-\gamma))^{1-\gamma}
C_t^i = \left(\int_0^1 C_t^i(j)^{\frac{\varepsilon_t^i - 1}{\varepsilon_t^i}} dj \right)^{\frac{\varepsilon_t^i}{\varepsilon_t^i - 1}}, i = o, d,$$

where W_t denotes the nominal wage rate and T_t denotes net transfers from the government and monopolistically competitive firms. Consumption C_t is a Cobb-Douglas aggregate of consumption of other goods C_t^o and dirty goods C_t^d , each of which is in turn a CES aggregate of the varieties $C_t^o(j), C_t^d(j)$ produced by monopolistically competitive producers, with elasticities of substitution ε_t^o and ε_t^d respectively. Since the baseline model does not feature an input-output structure, dirty goods should be thought of as a stand-in for goods and services with relatively high greenhouse gas emissions, both direct and indirect, while other goods represent all other consumption.

The household's optimality conditions imply the standard relationships

$$C_{t}^{o} = \gamma C_{t}(P_{t}/P_{t}^{o}) = \gamma C_{t}S_{t}^{1-\gamma},$$

$$C_{t}^{d} = (1-\gamma)C_{t}(P_{t}/P_{t}^{d}) = (1-\gamma)C_{t}S_{t}^{-\gamma},$$

$$C_{t}^{i}(j) = \left(\frac{P_{t}^{i}(j)}{P_{t}^{i}}\right)^{-\varepsilon_{t}^{i}}C_{t}^{i}, i = o, d, j \in [0, 1],$$

$$\frac{W_{t}}{P_{t}} = bC_{t},$$

$$1 = \beta \mathbb{E}_{t} \left[(1+i_{t})\frac{C_{t}}{C_{t+1}}\frac{P_{t}}{P_{t+1}} \right],$$

where $P_t = (P_t^o)^{\gamma} (P_t^d)^{1-\gamma}$ is the aggregate price level, and $S_t = \frac{P_t^d}{P_t^o}$ denotes the price of dirty goods relative to other goods.

Firms. The monopolistically competitive producer of variety $j \in [0,1]$ in sector i=o,d faces a tax \mathcal{T}_t^i (which may be negative, i.e. a subsidy) per unit of output produced. We will assume $\mathcal{T}_t^o \leq 0$ and $\mathcal{T}_t^d \geq 0$, i.e. other goods may be subsidized, while dirty goods may be taxed (a proxy for carbon taxes and regulations). Firms produce using a linear technology $Y_t^i(j) = A_t^i L_t^i(j)$ with labor as the only input and face quadratic costs of adjusting prices. We assume these adjustment costs as "psychic" (or, equivalently, they are transfers to households) i.e. they will not appear in aggregate resource constraints. Thus, the nominal marginal cost for a firm in sector i equals $M_t^i = \frac{W_t}{A_t^i} + \mathcal{T}_t^i$.

A natural interpretation of the tax is that greenhouse gas emissions are proportional to production of dirty goods, and the government taxes these emissions. However, we do not explicitly model the emissions generated by the dirty sector, the effect of emissions on climate, or the effect of climate on welfare and economic outcomes (eg, see Golosov et al., 2014; Känzig, 2022). Our focus is on the effect of climate policy on inflation over the medium term; while a change in climate policy will affect emissions and hence climate change, the effect of policy on inflation via this channel is likely to be small over the horizon we are interested in.

The firm solves

$$\max E_0 \sum_{t=0}^{\infty} Q_{t|0} \left\{ (P_t^i(j) - M_t^i) Y_t^i \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\varepsilon_t^i} - \frac{\Psi^i}{2} \left(\frac{P_t^i(j)}{P_{t-1}^i(j)} - 1 \right)^2 P_t^i Y_t^i \right\},\,$$

where $Q_{s|t} = \beta^{s-t} \frac{P_t C_t}{P_s C_s}$ denotes the representative household's nominal stochastic discount factor (SDF). Taking the first-order conditions, assuming a symmetric equilibrium and using market

⁹Section 3.2.3 discusses the effect of incorporating input-output linkages, but we relegate the derivation of these results to Appendix B.2.

clearing to simplify the SDF terms yields the sectoral Phillips curves

$$\Pi_{t}^{i}(\Pi_{t}^{i}-1) = \frac{\varepsilon_{t}^{i}}{\Psi^{i}} \left(\frac{M_{t}^{i}}{P_{t}^{i}} - \frac{1}{\mu_{t}^{i}} \right) + \mathbb{E}_{t} \left\{ \beta \Pi_{t+1}^{i}(\Pi_{t+1}^{i}-1) \right\}, \ i = o, d,$$

where $\Pi_t^i = \frac{P_t^i}{P_{t-1}^i}$ denotes inflation in sector i, and we define $\mu_t^i = \frac{\varepsilon_t^i}{\varepsilon_t^i - 1}$ to be the desired (gross) markup in sector i = o, d. The cost of price adjustment Ψ^i may differ between sectors. In particular, prices in the dirty sector may be more flexible ($\Psi^d < \Psi^o$), or even fully flexible ($\Psi^d = 0$).

The relative price S_t evolves according to

$$S_t = \frac{\Pi_t^d}{\Pi_t^o} S_{t-1}. \tag{1}$$

CPI inflation is defined as $\Pi_t = (\Pi_t^o)^{\gamma} (\Pi_t^d)^{1-\gamma}$.

Monetary and fiscal policy. The monetary authority sets the nominal interest rate i_t ; the fiscal authority sets taxes \mathcal{T}_t^o , \mathcal{T}_t^d and adjusts the lump sum transfer to households as necessary to maintain a balanced budget. We assume government debt is in zero net supply $(B_t = 0, \forall t)$ which is without loss of generality since the economy features Ricardian equivalence. Rather than specify a particular monetary policy rule, we will study outcomes under various different rules.

Market clearing. In equilibrium markets clear for goods in each sector and for labor:

$$C_t^i = Y_t^i = A_t^i L_t^i, \ i = o, d,$$

 $L_t^o + L_t^d = L_t,$

Model solution. To solve the model, note that real marginal costs (deflated by prices in each sector) can be written as

$$\frac{M_t^i}{P_t^i} = \frac{W_t}{P_t^i A_t^i} + \frac{\mathcal{T}_t^i}{P_t^i} = \frac{W_t}{P_t A_t^i} \frac{P_t}{P_t^i} + \frac{\mathcal{T}_t^i}{P_t^i} = \frac{bY_t}{A_t^i} \frac{P_t}{P_t^i} + \frac{\mathcal{T}_t^i}{P_t^i},$$

where $\frac{P_t}{P_t^o} = S_t^{1-\gamma}$ and $\frac{P_t}{P_t^d} = S_t^{-\gamma}$. Thus, we can substitute out for marginal costs to obtain

$$\Pi_t^o(\Pi_t^o - 1) = \frac{\varepsilon_t^o}{\Psi^o} \left(b Y_t \frac{S_t^{1-\gamma}}{A_t^o} + \frac{\mathcal{T}_t^o}{P_t^o} - \frac{1}{\mu_t^o} \right) + \mathbb{E}_t \left\{ \beta \Pi_{t+1}^o(\Pi_{t+1}^o - 1) \right\}, \tag{2}$$

$$\Pi_t^d(\Pi_t^d - 1) = \frac{\varepsilon_t^d}{\Psi^d} \left(b Y_t \frac{S_t^{-\gamma}}{A_t^d} + \frac{\mathcal{T}_t^d}{P_t^d} - \frac{1}{\mu_t^d} \right) + \mathbb{E}_t \left\{ \beta \Pi_{t+1}^d(\Pi_{t+1}^d - 1) \right\}. \tag{3}$$

Taxes $\frac{\mathcal{T}_t^d}{P_t^d} > 0$ are isomorphic to a positive cost-push shock or increase in dirty firms' desired markups: they tend to increase inflation. Taking $\frac{\mathcal{T}_t^o}{P_t^o}$ and $\frac{\mathcal{T}_t^d}{P_t^d}$ as given, (2) and (3) together with (1) gives us three equations in four unknowns, Π_t^o , Π_t^d , S_t , Y_t . Given a specification of the monetary policy rule and the path of taxes, these equations fully characterize equilibrium.

Taxes. We wish to study the macroeconomic effects of climate policy, modeled as the effect of an increase in taxes on the dirty sector \mathcal{T}_t^d . Rather than working with taxes directly, however, it is convenient to define the 'virtual markup' $\tilde{\mu}_t^i$ such that

$$\frac{1}{\widetilde{\mu}_t^i} = \frac{1}{\mu_t^i} - \frac{\mathcal{T}_t^i}{P_t^i}, \ i = o, d.$$

An increase in real taxes on dirty goods $\frac{\mathcal{T}_t^d}{P_t^d}$ is isomorphic to an increase in dirty goods producers' desired markup μ_t^d : both imply an increase in that sector's virtual markup $\widetilde{\mu}_t^d$, inducing producers to prefer lower output and higher prices.

In the experiments described below, we assume productivity is constant in each sector, and model climate policy as follows. The economy is initially in a steady state with zero CPI inflation $(\Pi_t = 1)$ and real taxes (or subsidies) consistent with $\widetilde{\mu}_{-1}^o = 1$, $\widetilde{\mu}_{-1}^d = 1$. At date 0, it becomes common knowledge that the tax on dirty goods will increase such that μ_t^d converges to a higher long-run level $\widetilde{\mu}_{\infty}^d > \widetilde{\mu}_0^d$:

$$\ln \widetilde{\mu}_t^d - \ln \widetilde{\mu}_{\infty}^d = \rho^{t+1} (\ln \widetilde{\mu}_{-1}^d - \ln \widetilde{\mu}_{\infty}^d),$$

where ρ governs the speed with which convergence to the long-run level occurs.

3.1 The Long Run and the Flexible-Price Benchmark

In this section we briefly describe the steady state after the taxes on the dirty sector have been implemented and the effect of nominal rigidities has vanished. We also discuss the effect that the whole dynamic path of taxes would have in a counterfactual economy where prices were *always* perfectly flexible.

Flexible-price equilibrium. While we are ultimately interested in the effect of the green transition on inflation, which is only a meaningful topic in an economy with nominal rigidities, the flexible-price equilibrium provides a useful benchmark. In the flexible price limit ($\Psi^i = 0, i = o, d$), firms are free to set prices in each sector equal to their desired markup over marginal cost, and the Phillips curves (2) and (3) become

$$bY_t \frac{S_t^{1-\gamma}}{A_t^o} = \frac{1}{\widetilde{\mu}_t^o},\tag{4}$$

$$bY_t \frac{S_t^{-\gamma}}{A_t^d} = \frac{1}{\widetilde{\mu}_t^d}.$$
 (5)

Relative prices, output, and hours worked in the flexible price equilibrium are given by

$$\begin{split} S_t &= \frac{\widetilde{\mu}_t^d}{\widetilde{\mu}_t^o} \frac{A_t^o}{A_t^d}, \\ Y_t &= \frac{1}{b} \left(\frac{A_t^o}{\widetilde{\mu}_t^o} \right)^{\gamma} \left(\frac{A_t^d}{\widetilde{\mu}_t^d} \right)^{1-\gamma} := Y_t^*, \\ Y_t^i &= \frac{1}{b} \frac{A_t^i}{\widetilde{\mu}_t^i}, \ i = o, d, \\ L_t &= \frac{1}{b} \left[\frac{\gamma}{\widetilde{\mu}_t^o} + \frac{1-\gamma}{\widetilde{\mu}_t^d} \right], \\ &= \left[\gamma \left(\frac{\widetilde{\mu}_t^d}{\widetilde{\mu}_t^o} \right)^{1-\gamma} + (1-\gamma) \left(\frac{\widetilde{\mu}_t^d}{\widetilde{\mu}_t^o} \right)^{-\gamma} \right] \frac{Y_t}{(A_t^o)^{\gamma} (A_t^d)^{1-\gamma}}. \end{split}$$

We refer to the flexible price level of output Y_t^* as potential output.

Here we note that throughout, whenever we discuss efficiency, we ignore externalities associated with higher output of dirty goods, which are not modeled here. Implicitly, these externalities are the reason that the government would want to reduce the output of the dirty sector. What we call the 'efficient' level of output features higher dirty-sector output than would be socially desirable.

Even when prices are fully flexible, the equilibrium may not be efficient owing to the distortions arising from taxes and/or monopolistic competition. In the efficient flexible price equilibrium (which maximizes the utility of the representative household), these distortions are absent and $\widetilde{\mu}_t^o = \widetilde{\mu}_t^d = 1$. As is standard in New Keynesian models, this requires subsidizing output to offset monopolistic distortions, $\frac{T_t^i}{P_t^i} = -\frac{1}{\widetilde{\mu}_t^i}$. Relative prices are then purely driven by relative costs of production, $S_t = A_t^o/A_t^d$, aggregate output is $Y_t = \frac{1}{b}(A_t^o)^{\gamma}(A_t^d)^{1-\gamma}$, sectoral output is $Y_t^i = \frac{1}{b}A_t^i$, i = o, d, and labor supply is $L_t = \frac{1}{b}$. As mentioned above, we assume the economy starts out in the efficient steady state, $\widetilde{\mu}_{-1}^o = \widetilde{\mu}_{-1}^d = 1$.

New steady state under a higher carbon tax. We will study the effect of an increase in taxes on the dirty sector, which raises $\widetilde{\mu}_t^d > 1$. Under flexible prices, this increases the relative price of dirty goods to $S_t = \mu_t^d A_t^o / A_t^d > A_t^o / A_t^d$, and reduces dirty sector output to $\frac{1}{b} \frac{A_t^d}{\widetilde{\mu}_t^d} < \frac{1}{b} A_t^d$. Given our assumptions on household preferences, this tax neither increases nor decreases output in the other sector, and therefore it reduces aggregate potential output. Note that the proportional reduction in the output of the dirty sector, relative to the efficient level of production, equals $\frac{1}{\widetilde{\mu}_t^d}$. Thus, the policy we study can also be interpreted as a quantity target which reduces dirty sector output by some percentage amount relative to its efficient level.

When productivity and $\widetilde{\mu}_t^i$ are both constant, the flexible-price equilibrium is also a zero-inflation steady state of the sticky-price economy, featuring $\Pi_t^o = \Pi_t^d = \Pi_t = 1$. Given our assumptions on

taxes, the economy transitions from the efficient steady state to a new steady state with higher relative prices $S_t = \widetilde{\mu}_{\infty}^d S_{-1}$ and lower aggregate output $Y_{\infty} = (\widetilde{\mu}_{\infty}^d)^{-(1-\gamma)} Y_{-1}$.

In this new steady state the relative price of the dirty good – relative to the price for the rest of the economy's output – is going to be higher, because taxes increase the marginal cost of producing dirty output. For this same reason, dirty output is going to be scarcer, which is the point of taxes in the first place. In the main experiment we consider, taxes on the dirty sector will not affect the flexible-price level of output in the other sector, and so the overall level of output will also be lower than before. Since we consider a gradual increase in taxes, this decline in the flexible-price level of output, Y_t^* . takes place gradually over time. More generally, whether taxes on the dirty sector affect production in the other sector would depend on whether these taxes are used to subsidize the rest of the economy or not, as well as on the degree of substitutability in consumption between dirty and non-dirty output (our baseline model assumes a unit elasticity of substitution, i.e. Cobb-Douglas preferences). Regardless, the central feature of the green transition is that it features a decline in both the absolute size and the share of the dirty sector.

To achieve such an outcome, dirty output needs to eventually become more expensive in relative terms. Is the green transition then inflationary? Not necessarily. A change in relative prices can be achieved in many ways – by increasing the nominal price of dirty goods or lowering the price of rest-of-the-economy output. Either combination works, and the ultimate result in terms of inflation depends entirely on monetary policy. If prices are flexible, monetary policy only determines nominal variables and not real allocations. Since the choice of the central bank has no consequence for real activity, there is no reason why it would choose an inflation rate different from its objective. In sum, when prices are flexible, the green transition per se is neither inflationary nor deflationary. Any inflationary effects of the green transition therefore must have to do with nominal rigidities.

3.2 The Role of Nominal Rigidities

If nominal rigidities are present, taxes on the dirty sector may present the central bank with a tradeoff between efficiently facilitating the green transition and maintaining low inflation. The nature of this tradeoff, however, depends crucially on the relative degree of nominal rigidities in the dirty sector and the rest of the economy, as discussed below.¹¹

 $^{^{10}}$ As shown in Woodford (2003) in a flexible price economy the central bank pins down expected (and hence average) inflation by its choice of the nominal interest rate, via the Fisher equation, given that the real interest rate in such economy always equals r^* , that is, it is independent from monetary policy.

¹¹Indeed, the literature has often argued that shocks to the relative price of energy are inflationary precisely because prices in this sector are relatively flexible (Gordon, 1975; Aoki, 2001; Rubbo, 2023). Since the energy sector also accounts for the majority of greenhouse gas emissions, one might suspect that prices are more flexible in dirty sectors of the economy.

3.2.1 Relative prices and Sectoral Phillips Curves

We now study the behavior of inflation during the transition to a new steady state with higher taxes on dirty goods. We loglinearize the system around the 'new' zero-inflation steady state consistent with $\tilde{\mu}_{\infty}^d$. Without yet specifying a monetary policy rule, this yields four equations which can be used to understand the macroeconomic tradeoffs introduced by the green transition:

$$\pi_t^o = \kappa^o(y_t - y_t^* + (1 - \gamma)(s_t - s_t^*)) + \beta \mathbb{E}_t \pi_{t+1}^o, \tag{6}$$

$$\pi_t^d = \kappa^d (y_t - y_t^* - \gamma(s_t - s_t^*)) + \beta \mathbb{E}_t \pi_{t+1}^d, \tag{7}$$

$$s_t = s_{t-1} + \pi_t^d - \pi_t^o, (8)$$

$$\pi_t = \gamma \pi_t^o + (1 - \gamma) \pi_t^d. \tag{9}$$

Here lower case variables denote log-deviations from the new, high-tax steady state: y_t denotes (the log-deviation of) aggregate output and $s_t := p_t^d - p_t^o$ denotes the price of dirty goods relative to other goods. $y_t^* := -(1-\gamma)\mu_t^d$ and $s_t^* := \mu_t^d$ denote the log-deviations of the flexible price values of y_t and s_t , and their evolution is given by

$$\mu_t^d = \rho^{t+1} \mu_{-1}^d, \quad \mu_{-1}^d \le 0, \quad s_{-1} = \mu_{-1}^d < 0.$$

(where with some abuse of notation μ_t^d denotes the log-deviation of $\widetilde{\mu}_t^d$ from steady state). Note that $\mu_0^d < 0$ means that μ_t^d is initially below its new steady state value.

Equations (6) and (7) are Phillips curves for the other and dirty sectors. These equations relate inflation in the two sectors (π_t^o and π_t^d respectively) to the deviation of aggregate output y_t and relative prices s_t from their flexible price levels, y_t^* and s_t^* . These starred variables do not depend on monetary policy, but do depend on climate policy: as described above, the gradual introduction of a tax on dirty goods will gradually increase s_t^* , and reduce potential output y_t^* , towards their new steady state levels. The sectoral Phillips curve slopes κ^o and κ^d measure the degree of price flexibility in the other and dirty sectors respectively; again, Table 1 suggests that the empirically relevant case is $\kappa^d > \kappa^o$. Equation (8) is an accounting identity stating that the change in relative prices equals the difference in sectoral inflation rates. Finally, equation (9) defines overall CPI inflation (where γ and $1-\gamma$ denote the expenditure shares of the other and dirty sectors respectively).

Why do relative prices enter the sectoral Phillips curves (6) and (7)? As in a 1-sector New Keynesian model, inflation in each sector depends on the marginal cost in that sector. This in turn depends on that sector's product wage, i.e. nominal wages deflated by the price of that sector's output. For any given real wage (i.e. nominal wages deflated by the CPI), an increase in the relative price of dirty goods s_t increases the product wage in the other sector, adding to inflationary pressure there, and reduces the product wage in the dirty sector. Mathematically (abstracting from changes

in productivity and taxes):

$$\underbrace{mc_t^i}_{\text{marginal cost}} = \underbrace{w_t - p_t^i}_{\text{product wage}} = \underbrace{w_t - p_t}_{\text{real wage}} - \underbrace{(p_t^i - p_t)}_{\text{relative price}} = \begin{cases} y_t + (1 - \gamma)s_t & \text{for } i = o, \\ y_t - \gamma s_t & \text{for } i = d. \end{cases}$$

3.2.2 The Role of Relative Price Stickiness

To understand what happens in our model economy when nominal prices are slow to adjust, it is instructive to first consider a few special cases.

Case 1: Other prices fixed, dirty prices flexible. Start with the extreme case where dirty prices are fully flexible ($\kappa^d = \infty$), while they are completely sticky in nominal terms – that is, fixed – for the remainder of the economy ($\kappa^o = 0$). In this case, our system reduces to

$$s_t = s_t^* + \frac{1}{\gamma} (y_t - y_t^*), \tag{10}$$

$$\pi_t = (1 - \gamma)\pi_t^d = (1 - \gamma)\Delta s_t. \tag{11}$$

Equation (10) states that the relative price of dirty goods can rise above its flexible price level when the output gap is positive (which raises wages and costs in the dirty sector); equation (11) states that inflation is driven by dirty sector prices since other prices are fixed. In such a situation it is obvious that the green transition would be inflationary: the only way to reduce the share of the dirty sector, and increase the relative price of dirty goods, is for the dirty prices to move up (from (8), if $\pi_t^o = 0$, implementing $\Delta s_t > 0$ requires $\pi_t^d > 0$). Since all other prices are fixed, overall inflation needs to move up as well (from the definition of CPI inflation (7), if $\pi_t^d > 0$ and $\pi_t^o = 0$, $\pi_t > 0$). Changes in relative prices are necessarily associated with aggregate inflation.¹²

Case 2: Other prices sticky, dirty prices flexible. Now maintain the assumption that dirty prices are fully flexible ($\kappa^d = \infty$), but suppose that prices for the rest of the economy are sticky, but not completely rigid ($0 < \kappa^o < \infty$). In this case, our system becomes

$$s_t = s_t^* + \frac{1}{\gamma} (y_t - y_t^*), \tag{12}$$

$$\pi_t^o = \frac{\kappa^o}{\gamma} (y_t - y_t^*) + \beta \mathbb{E}_t \pi_{t+1}^o, \tag{13}$$

$$\pi_t = \pi_t^{o} + (1 - \gamma)\Delta s_t. \tag{14}$$

Here the central bank has a choice: as (14) shows, the central bank can engineer whatever level of overall inflation π_t it wants, while still allowing relative prices s_t to increase, by picking inflation in the non-dirty sector π_t^o . However, since π_t^o is determined by the Phillips curve (13), the only way

¹²Olovsson and Vestin (2023) show that if monetary authorities only care about inflation in the other sector π_t^o , which is fixed, then the flexible price equilibrium is implemented. This is apparent from equations (10) and (11): if π_t^d fully adjusts so that relative prices are the same as in the flexible price equilibrium, then $y_t = y_t^*$.

to achieve this objective amounts to picking the level of output gap $y_t - y_t^*$ for the economy. In turn, this gives rise to the tradeoff mentioned at the beginning of this section.

For concreteness, suppose the central bank has a zero inflation target. In order to implement such a target, and at the same time to achieve the required adjustment in relative prices, if dirty output prices are rising there needs to be deflation in the rest of the economy. Such deflation can only be accomplished by having a negative output gap, that is, a recession. Hence it is still true that the green transition is *per se* neither inflationary nor deflationary. But in order to achieve the desired level of inflation the central bank needs to exert some influence on aggregate economic activity, so as to affect marginal costs in the sticky sector. Intuitively, in the presence of stickiness the required nominal adjustment in the sticky sector needs a push from the central bank. This push is not costless, as it hinges on the output gap and therefore generates a tradeoff.

If prices are sticky also in the dirty sector, $\kappa^d < \infty$, as will be the case in the numerical examples discussed in the next section, the conclusions do not change. As long as prices are stick *ier* in the rest of the economy, the central bank can only achieve zero overall inflation by generating a contraction in economic activity. Conversely, if prices were stickier in the dirty sector, implementing zero inflation would require a boom in economic activity.

Case 3: Prices equally sticky in both sectors. However, in the knife-edge case where stickiness is the same in both sectors ($\kappa^o = \kappa^d \equiv \kappa$), no output gap is needed to achieve the required adjustment in relative prices. Nominal prices in both sectors are just as sluggish, and will gradually adjust in opposite directions without affecting overall inflation. Mathematically, our system becomes

$$\pi_t = \kappa(y_t - y_t^*) + \beta \mathbb{E}_t \pi_{t+1}, \tag{15}$$

$$\Delta s_t = -\kappa (s_t - s_t^*) + \beta \mathbb{E} \Delta s_{t+1}. \tag{16}$$

That is, we can write a standard aggregate Phillips curve for CPI inflation in terms of an output gap which does not depend directly on relative prices. Similarly, relative prices are governed by a second order difference equation which depends on their flexible price level s_t^* , but not directly on output.¹³ In this special case (but *only* in this case!), aggregate inflation is fully determined by the aggregate output gap, while the evolution of relative prices depends on fundamental factors and is unaffected by monetary policy. Despite the green transition, the monetary authority can close the output gap while implementing zero inflation.

To understand this result, recall that changes in relative prices have opposite-signed effects on

¹³Solving this equation yields $s_t = \lambda s_{t-1} + \psi s_t^*$, where $\lambda = \frac{1 + \beta + \kappa - \sqrt{(1 + \beta + \kappa)^2 - 4\beta}}{2\beta} \in (0,1)$, $\psi = \frac{\kappa}{1 + \kappa + \beta(1 - \rho - \lambda)} > 0$. In the limit as prices in both sectors become fully rigid $(\kappa \to 0)$, $\lambda \to 1$, and $\psi \to 0$, i.e. relative prices are fixed $(s_t = s_{t-1})$ and do not move towards their flexible price level; in the limit as both sectors become fully flexible $(\kappa \to \infty)$, $\lambda \to 0$, $\psi \to 1$, i.e. relative prices jump instantly to their flexible price level $(s_t = s_t^*)$.

marginal costs in the two sectors: an increase in the relative price of dirty goods s_t raises marginal cost for the clean sector, and reduces it for the dirty sector. These effects must cancel out for (expenditure-weighted) average marginal cost for the economy as a whole:

$$\overline{mc}_t := \gamma mc_t^o + (1 - \gamma)mc_t^d = w_t - p_t - \underbrace{\left[\gamma(p_t^o - p_t) + (1 - \gamma)(p_t^d - p_t)\right]}_{=0 \text{ by definition of } p_t}.$$

In general, average marginal costs are not what determines aggregate inflation. Instead, marginal costs in the more flexible price sector have an outsized effect on aggregate inflation. But in the special case where $\kappa^o = \kappa^d = \kappa$, it is average marginal costs that matter: we can simply aggregate the sectoral Phillips curves to get the same aggregate Phillips curve as in a one-sector model, equation (15).

There are two things to note about this special case. First, a zero output gap $(y_t - y_t^*)$ still implies that the *level* of output is declining in line with potential y_t^* . But in itself, this does not necessarily indicate an adverse tradeoff. Presumably when setting the tax on the dirty sector, the fiscal authority traded off the cost of lower output against the (unmodeled) benefit from lower carbon emissions. The monetary authority would not want to completely offset the effect of the tax and prevent dirty output from declining, even if it was feasible to do so.

Second, while in this case the central bank can keep aggregate output equal to its flexible price level while maintaining zero inflation, it does not follow that relative prices and sectoral output are equal to their flexible-price levels. Nominal rigidities slow down the adjustment of relative prices (this is easiest to see in the limiting case where $\kappa \to 0$; clearly if prices are fixed in both sectors, (8) implies that relative prices and sectoral output shares can never adjust). In fact, in this case monetary policy cannot do anything to speed up the transition. Not only do relative prices not affect aggregate inflation; by the same token, the aggregate level of economic activity does not affect relative prices.

The general case: Some numerical examples. When moving beyond the special cases just described, one way to illustrate the monetary policy tradeoffs associated with the green transition is to compare outcomes under two extreme policies: strict inflation targeting, which sets $\pi_t = 0$, and strict output gap targeting, which sets $y_t - y_t^* = 0$. The figures below present a numerical example (this is not intended to be quantitative, and the parameterization and results are only illustrative). The calibration is described in detail in Appendix B.1. The red lines show a calibration with $\kappa^d = \kappa^o = 0.01$; blue-dashed lines illustrate the case with flexible prices in the dirty sector $(\kappa^d = \infty)$; magenta-dotted lines illustrate an intermediate case where the slope of the Phillips curve is 5 times larger in the dirty sector, $\kappa^d = 0.05$. Black dotted lines show the flexible-price levels of s_t , y_t , and dirty sector output y_t^d . Dirty output (shown in the bottom-right panel) is given by $y_t^d = y_t - \gamma s_t$; this variable can also be interpreted as the level of emissions, and so the difference between the colored lines and black dotted lines in the bottom right panel illustrates how nominal

rigidities slow down the green transition, relative to the flexible price benchmark. Figure 2 plots dynamics under strict inflation targeting $\pi_t = 0$. Figure 3 plots dynamics under strict output gap targeting, $y_t = y_t^*$. All variables are plotted as log-deviations relative to the new steady state featuring lower output and a higher relative price s_t .

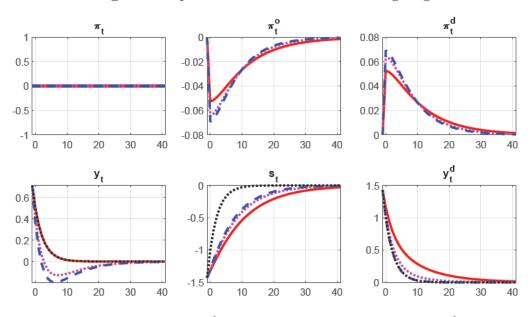


Figure 2. Dynamics under strict inflation targeting

Notes: Red lines denote calibration with $\kappa^o = \kappa^d$, dashed blue lines denote calibration with $\kappa^d = \infty$, magenta dotted lines denote calibration with $\kappa^d = 5\kappa^o$, and dotted black lines denote flexible price allocations.

As described above, when $\kappa^o = \kappa^d$, inflation targeting is equivalent to output gap targeting and so the red lines are identical across the two figures. Output remains equal to potential and declines towards its new lower steady state level. The relative price of dirty goods increases, but more slowly than in a flexible price economy since prices take time to adjust. Inflation in the dirty goods sector is balanced by deflation in the clean goods sector.

When prices are more flexible in the dirty goods sector, the equivalence between inflation targeting and output gap targeting breaks down. Maintaining an unchanged inflation target ($\pi_t = 0$) requires implementing a larger decline in output, i.e. a negative output gap: output undershoots its longer-run level. Conversely, keeping output equal to potential requires tolerating an initial increase in overall inflation. A higher degree of price flexibility in the dirty sector makes this tradeoff between output gap and overall inflation stabilization more pronounced. Under output gap targeting, marginal costs increase in the dirty sector and fall in the clean sector (owing to lower economic activity), but the increase in costs in the dirty sector has a larger impact on sectoral inflation since prices in this sector are more flexible (compare the dotted-magenta and dashed-blue lines in the top-middle and top-right panels of Figure 3). Thus, overall inflation increases (top-left

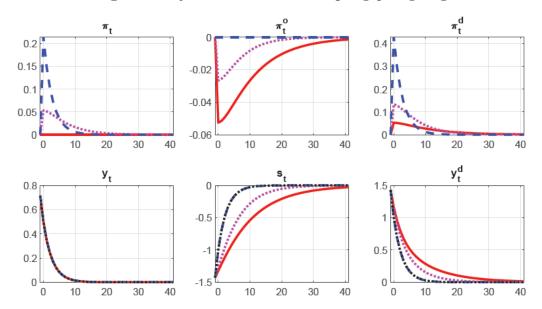


Figure 3. Dynamics under strict output gap targeting

Notes: Red lines denote calibration with $\kappa^o = \kappa^d$, dashed blue lines denote calibration with $\kappa^d = \infty$, magenta dotted lines denote calibration with $\kappa^d = 5\kappa^o$, and dotted black lines denote flexible price allocations.

panel). Offsetting this and stabilizing overall inflation would require reducing economic activity even more to bring down marginal costs and prevent π_t^d from spiking.

3.2.3 Input-Output Linkages

We now briefly discuss how our conclusions would change if we allow the rest of the economy to use dirty output as an input in production (this extension is described in detail in Appendix B.2). Some of the intuition obtained from this very stylized IO model will be useful in interpreting the results from the full-fledged multi-sector model discussed in section 4.

Consider first the flexible-price economy, and suppose the policymaker introduces a tax on dirty output in order to engineer the same proportional reduction in the gross output of the dirty sector as in our baseline model. Holding the consumption share of dirty goods $(1 - \gamma)$ fixed, the same reduction in dirty output now implies a larger reduction in aggregate potential output y^* , since it also curtails production in the rest of the economy which uses dirty goods as an input. However, a given reduction in dirty sector output can now be achieved with a smaller change in relative prices s^* , because dirty goods are used as an input to produce other goods, and so a tax on dirty goods raises costs for the other sector.

With nominal rigidities, input-output linkages also quantitatively affect the tradeoff monetary authorities face between stabilizing inflation and closing the output gap, although our qualitative results remain largely unchanged. The Phillips curve for the dirty sector (7) remains the same,

since dirty goods producers still only use labor as an input. The Phillips curve for the other sector becomes

$$\pi_t^o = \kappa^o \left[(1 - \omega_{od})(y_t - y_t^*) + (1 - \gamma + \gamma \omega_{od})(s_t - s_t^*) \right] + \beta \mathbb{E}_t \pi_{t+1}^o$$
(17)

where $\omega_{od} > 0$ denotes the cost share of dirty goods in the production of other goods (and $1 - \omega_{od}$ the labor share). Higher usage of dirty output by the other sector $\omega_{od} > 0$ makes the other sector's Phillips curve less sensitive to aggregate economic activity, but more sensitive to the relative price of dirty goods. Intuitively, an increase in dirty sector prices now increases marginal costs directly via the price of inputs, as well as indirectly by increasing the product wage for a given real wage.

Suppose prices are perfectly flexible in the dirty sector $(\kappa^d = \infty)$ but somewhat sticky in the other sector $(0 < \kappa^o < \infty)$. As in our baseline model, since the relative price of dirty goods s_t is increasing, a central bank committed to stabilizing CPI inflation π_t must engineer deflation in the other sector, which requires a negative output gap; however, this tradeoff is less severe than in our baseline model. The relationship between inflation in the other sector π_t^o and the output gap is the same as in our baseline. This is because the lower slope of the dirty sector Phillips curve with respect to the output gap $(\kappa^o(1-\omega_{od}))$ is exactly compensated by the higher sentitivity to $s_t - s_t^*$. Since the output gap also has an effect on the relative price s_t (dirty goods producers set prices equal to marginal costs, which depend on wages and hence on the output gap) the overall effect is identical. Thus as in our baseline, stabilizing the output gap implies zero inflation in the other sector, and so positive CPI inflation. However, since the required increase in relative prices is less dramatic than in our baseline, the gap between dirty and other sector inflation is smaller, and so overall CPI inflation is lower, though still positive.¹⁴

Again, the presence of a tradeoff depends on the assumption that prices are more flexible in the dirty sector. Recall that when prices are equally sticky in both sectors ($\kappa^o = \kappa^d$), there is no tradeoff between stabilizing CPI inflation and closing the output gap in our baseline economy. With input-output linkages, since firms produce not only for consumers but for other firms, CPI inflation is not the relevant benchmark: instead, there is no tradeoff between stabilizing PPI inflation and closing the output gap.¹⁵ PPI is higher than CPI inflation in the scenarios we consider, since it puts a higher weight on dirty goods prices which are increasing during the transition. Thus, while IO linkages may not change the tradeoff faced by a PPI-targeting central bank, they do make the tradeoff less severe for a CPI-targeting central bank. In fact, when $\kappa^d = \kappa^o$, the sign of the tradeoff reverses: stabilizing CPI inflation requires running a positive output gap, i.e. preventing output from falling as much as potential.

In sum, in the empirically realistic case where dirty sector prices are significantly more flexible,

¹⁴Mathematically, $\pi_t = \pi_t^o + (1 - \gamma)\Delta s_t$; given π_t^o , a smaller Δs_t implies smaller π_t .

¹⁵Since CPI equals PPI in our baseline economy without intermediate inputs, this implies that IO linkages do not change the tradeoff between stabilizing PPI and closing the output gap.

the standard tradeoff remains, but IO linkages generally make it less severe. This is a somewhat surprising result, as one might have thought that IO linkages would put the central bank in a more difficult spot. Again though, while the tradeoff between stabilizing CPI inflation and the output $gap \ y_t - y_t^*$ is less severe, IO linkages also increase the decline in y_t^* , so closing the output gap implies a steeper decline in the *level* of output y_t .

3.3 Subsidies versus Taxes

In the experiment described above, the green transition is implemented through taxes on a dirty sector. In reality, climate policy often (perhaps increasingly) instead consists of subsidizing "clean" sectors – such as renewable energy or electric vehicles – which are substitutes for polluting activities. Fully extending our analysis to allow for such subsidies would require (at least) a three-sector model which explicitly models substitution between dirty and clean consumption. And arguably in order to understand the motivation for (and potential advantages of) a subsidy-centered approach, it would also be necessary to model the effect of subsidies on investment and endogenous technical change, as well as the distributional effects of various policies. But as a very first pass, we can analyse the inflationary effect of subsidies by ignoring the dirty sector altogether, and reinterpreting the sector of our economy with more flexible prices as a clean sector whose production the government wishes to subsidize.

In this case, the logic of our analysis above goes through exactly but with a minus sign, and the conclusions are reversed. In the long run, subsidies reduce the relative price of clean goods, and increase their production. If all prices were fully flexible, this change in relative prices need not be deflationary. Even with nominal rigidities, the monetary authority could in principle offset the effect of subsidies on aggregate inflation by appropriately choosing the level of the output gap. But if the clean sector's prices adjust more rapidly than in the rest of the economy, strict inflation targeting requires engineering a *positive* output gap. An output gap-targeting central bank would be forced to tolerate lower inflation.

In this sense, the model suggests that *in principle* policies such as the Inflation Reduction Act passed by Congress – the climate component of which primarily consists of subsidies to the clean energy sector rather than taxes on polluting activities – could actually be deflationary as advertised. Needless to add, our model is not designed to quantitatively assess whether this is actually the case.

4 Multisector model

We now extend our analysis to consider a quantitative multisector model with production networks.

4.1 Model Environment

There are n sectors, i = 1, ..., n.

Households The representative household solves

$$\max_{\{C_{t}, \{C_{t}^{i}, C_{t}^{i}(\cdot)\}_{i=1}^{n}\}_{t=0}^{\infty}} \mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \ln C_{t} - \widetilde{b} \int_{0}^{1} L_{t}(\iota) d\iota \right\}
\text{s.t. } \sum_{i=1}^{n} \int_{0}^{1} P_{t}^{i}(j) C_{t}^{i}(j) dj + \frac{1}{1+i_{t}} B_{t+1} = \int_{0}^{1} W_{t}(\iota) L_{t}(\iota) d\iota + T_{t} + B_{t}$$

$$C_{t} = \left[\sum_{i} (\gamma_{i})^{\frac{1}{\zeta}} (C_{t}^{i})^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}$$

$$C_{t}^{i} = \left(\int_{0}^{1} C_{t}^{i}(j)^{\frac{\varepsilon^{i}-1}{\varepsilon^{i}}} dj \right)^{\frac{\varepsilon^{i}}{\varepsilon^{i}-1}}, i = 1, ..., n$$
(18)

where W_t denotes the nominal wage and T_t denotes net transfers from the government and monopolistically competitive firms. Consumption C_t is a CES aggregate of the products C_t^i produced by each of the n sectors, each of which is in turn a CES aggregate of the varieties $C_t^i(j)$ produced by a continuum of monopolistically competitive producers in that sector. The elasticity of substitution between varieties in sector i is ε^i . As is standard, this implies that the consumer price index P_t equals $\left[\sum_{i=1}^n \gamma_i(P_t^i)^{-(\zeta-1)}\right]^{-\frac{1}{\zeta-1}}$, where $P_t^i = \left[\int_0^1 (P_t^i(j))^{-(\varepsilon^i-1)} dj\right]^{-\frac{1}{\varepsilon^i-1}}$ for all i and the demand for variety j of good i is $C_t^i(j) = C_t^i \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon^i}$.

Unions There are a continuum of differentiated labor varieties $L_t(\iota)$, $\iota \in [0, 1]$. Competitive labor packers combine these into 'labor services' L_t using the CES technology

$$L_t = \left(\int_0^1 L_t(\iota)^{\frac{\varepsilon^w - 1}{\varepsilon^w}} d\iota\right)^{\frac{\varepsilon^w}{\varepsilon^w - 1}}$$

and sell labor services to the firms at nominal price W_t . This yields the standard demand curve $L_t(\iota) = L_t \left(\frac{W_t(\iota)}{W_t}\right)^{-\varepsilon^w}$ where $W_t = \left[\int_0^1 W_t(\iota)^{1-\varepsilon^w} d\iota\right]^{\frac{1}{1-\varepsilon^w}}$. The wage of each variety of labor ι is set by a monopolistically competitive union with Calvo frictions, which can adjust the wage with probability $1-\theta_w$ each period, and solves the problem

$$\max_{W_t^*} \sum_{k=0}^{\infty} (\theta_w \beta)^k \left[\frac{W_t^*}{P_{t+k} C_{t+k}} - \widetilde{b} \right] L_{t+k} \left(\frac{W_t^*}{W_{t+k}} \right)^{-\varepsilon^w}$$
(19)

Firms Firms in sector i have the constant returns to scale production function

$$X_t^i = A_t^i \left[\alpha_i^{\frac{1}{\eta}} (L_t^i)^{\frac{\eta - 1}{\eta}} + (1 - \alpha_i)^{\frac{1}{\eta}} (I_t^i)^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}$$

where A_t^i is a Hicks-neutral productivity shifter, η is the elasticity of substitution between labor and intermediate inputs, and I_t^i is a CES aggregate of 'energy' E_t^i and 'non-energy inputs' N_t^i :

$$I_t^i = \left[\varsigma_i^{\frac{1}{\nu}} (E_t^i)^{\frac{\nu-1}{\nu}} + (1 - \varsigma_i)^{\frac{1}{\nu}} (N_t^i)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$$

where ν is the elasticity of substitution between energy and non-energy inputs, and ς_i is the energy share of inputs for sector i. E_t^i and N_t^i are, in turn, aggregates of energy and non-energy intermediate goods, respectively:

$$E_t^i = \left[\sum_j (\omega_{ij}^E)^{\frac{1}{\xi}} (X_t^{ij})^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

$$N_t^i = \left[\sum_j (\omega_{ij}^N)^{\frac{1}{\xi}} (X_t^{ij})^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

where $\sum_{j=1}^{n} \omega_{ij}^{E} = \sum_{j=1}^{n} \omega_{ij}^{N} = 1$, $\omega_{ij}^{E} = 0$ if j is a non-energy good, and $\omega_{ij}^{N} = 0$ if j is an energy good.

Here X_t^{ij} denotes the quantity of good j used by firms in sector i. Note that the share of different energy goods in the energy aggregate that is relevant for firms in sector i, E_t^i , may differ from sector to sector. The index of intermediate inputs used by firms in sector k and produced by a firm in sector i, is given by the same CES aggregate as household's consumption of sector i goods:

$$X_t^{ki} = \left(\int_0^1 X_t^{ki}(j)^{\frac{\varepsilon^i - 1}{\varepsilon^i}} dj\right)^{\frac{\varepsilon^i}{\varepsilon^i - 1}}.$$

which yields the standard CES demand curve $X_t^{ki}(j) = X_t^{ki} \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon^i}$.

The cost minimization problem of a sector i firm is

$$M^{i}X^{i} = \min_{L_{t}^{i}, I_{t}^{i}, E_{t}^{i}, N_{t}^{i} \{X_{t}^{ij}\}_{j=1}^{n}} W_{t}L_{t}^{i} + \sum_{j} P_{t}^{j}X_{t}^{ij} + \mathcal{T}_{t}e_{i}X_{t}^{i}$$

$$\text{s.t. } A_{t}^{i} \left[\alpha_{i}^{\frac{1}{\eta}} (L_{t}^{i})^{\frac{\eta-1}{\eta}} + (1 - \alpha_{i})^{\frac{1}{\eta}} (I_{t}^{i})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \geq X_{t}^{i}$$

$$\left[\zeta_{i}^{\frac{1}{\nu}} (E_{t}^{i})^{\frac{\nu-1}{\nu}} + (1 - \zeta_{i})^{\frac{1}{\nu}} (N_{t}^{i})^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \geq I_{t}^{i}$$

$$\left[\sum_{j} (\omega_{ij}^{E})^{\frac{1}{\xi}} (X_{t}^{ij})^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \geq E_{t}^{i}$$

$$\left[\sum_{j} (\omega_{ij}^{N})^{\frac{1}{\xi}} (X_{t}^{ij})^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \geq N_{t}^{i}$$

Here \mathcal{T}_t is the nominal carbon tax per unit of emissions, and e_i is the emissions intensity of sector i, i.e. the volume of emissions produced by producing one unit of gross output of good i. e_i is assumed to be fixed. We also define M_t^i to be the firm's marginal cost inclusive of the carbon tax.

We will calibrate e_i to raw emissions (which are nonzero only for oil and gas extraction and coal mining). Thus, we assume that the carbon tax is imposed upstream, at the point of fuel production: crude oil is taxed at the point it leaves the refinery, gas at the point it enters a pipeline, coal as it leaves the mine. While in principle a carbon tax can be levied at various different points in the supply chain, it is often argued that an upstream system is best as the number of fuel distributors is much smaller than the number of end users, making an upstream tax much easier to administer (Metcalf and Weisbach, 2009). The tax is based on the imputed carbon emissions per unit of fuel, whether or not the fuel is actually used to produce emissions (for example, in our model if a plastics manufacturer uses crude oil without producing emissions, it still pays the same price as an oil refinery).

Firms face Calvo-type price rigidities. Each period, a firm in sector i can change their price with probability $1 - \theta_i$; otherwise it remains unchanged. The optimal price for a resetting firm in sector i at date t, P_i^{i*} , solves

$$\max \sum_{k=0}^{\infty} Q_{t+k|t} \theta_i^k [P_t^{i*} - M_t^i] X_{t+k}^i \left(\frac{P_t^{i*}}{P_{t+k}^i} \right)^{-\varepsilon^i}$$
 (21)

where $Q_{t+k|t} = \beta^k \frac{P_t C_t}{P_{t+k} C_{t+k}}$ denotes households' nominal stochastic discount factor between dates t and t + k.¹⁶

Monetary policy We will consider outcomes under the same two monetary policy rules as in our simple two-sector model: (i) strict inflation targeting:

$$\Pi_t := \frac{P_t}{P_{t-1}} = 1$$

and (ii) strict output gap targeting, in which the central bank keeps aggregate consumption (and hence value added) equal to its level C_t^* in a counterfactual flexible-price economy where $\theta_i = 0$ for all i:

$$C_t = C_t^*$$

As in the two-sector model, we consider experiments in which a carbon tax is announced at date 0 and gradually transitions to some long-run level:

$$\tau_{t+1} - \tau^* = \rho(\tau_t - \tau^*), \tau_0 = 0, \text{ where } \tau_t := \frac{\mathcal{T}_t}{P_t}$$

 $^{^{16}}$ Strictly speaking, since we have no aggregate shocks, this is known as of date t and is just the price of a k-period bond.

Market clearing The market clearing conditions for each sector and for the labor market are

$$C_t^i + \sum_{j=1}^n X_t^{ji} = X_t^i, i = 1, ..., n$$
(22)

$$\sum_{i=1}^{n} L_t^i = L_t \tag{23}$$

4.2 Calibration

We calibrate the model at a monthly frequency and set $\beta=0.96^{(1/12)}$. We set the elasticity of substitution between consumption goods $\eta=2$, in line with Carvalho et al. (2021) and within the range of estimates for upper-level elasticities of substitution in Hobijn and Nechio (2019). We set the elasticity of substitution between labor and intermediate inputs $\eta=0.6$, in line with the elasticities of substitution between materials and non-materials for manufacturing plants in 2007 estimated by Oberfield and Raval (2021). The elasticity of substitution between energy and non-energy inputs ν can also be interpreted as the elasticity of businesses' demand for energy. As summarized by Bachmann et al. (2022), estimates of the short-run price elasticity of gas and energy demand mostly lie between 0.15 and 0.25; we therefore set $\nu=0.2$. Finally, we set the elasticity of substitution between intermediate inputs $\xi=0.1$, towards the upper end of the range of estimates reported by Atalay (2017).

We calibrate the monthly frequency of price change $1 - \theta_i$ based on price adjustment data from Cotton and Garga (2022) as described in Appendix A. This data is not available for BEA industries operating in the public sector; importantly, given our focus on energy, this includes electric utilities.¹⁷ The U.S. Energy Information Administration reports that in 2018, nearly half of all major US electric utilities tried to change electricity rates by filing a rate case.¹⁸ We therefore set the monthly frequency of price change $\theta = 0.5^{1/12}$ for both the Federal electric utilities and state and local government electric utilities sectors. We drop the other 8 government sectors for which price adjustment data are unavailable, together with the 'customs' and 'households' BEA sectors. After dropping these 10 sectors, and splitting oil and gas extraction into two separate sectors, this leaves us with 396 sectors rather than the 405 in the BEA tables. We manually classify some of these sectors as 'food' or 'energy'. As described below, as a baseline we set the wage stickiness parameter $\theta_w = 0.9^{1/3}$ following Del Negro et al., 2015, but consider robustness to alternative values, including flexible wages ($\theta_w = 0$).

¹⁷The complete set of industries without price adjustment data are: Federal general government (defense), Federal general government (nondefense), Federal electric utilities, Other federal government enterprises, State and local government ducational services, State and local government hospitals and health services, State and local government other services, State and local government passenger transit, State and local government electric utilities, Other state and local government enterprises.

¹⁸See https://www.eia.gov/todayinenergy/detail.php?id=40133.

Our model features a closed economy in which consumption is the only source of final demand. In the data, there are other sources of final demand, including net exports, and there are also imports and exports of intermediate goods. We deal with this by interpreting the data as a 'fictitious closed economy' in which imports are produced by domestic producers, and model consumption equals the sum of consumption, investment, government purchases and exports (without subtracting imports) in the data. In addition, some of the output produced by our remaining 396 sectors is used as intermediate inputs by the omitted sectors. To obtain a consistent input-output matrix, for each remaining sector, we subtract from its gross output the usage of its output by omitted sectors. That is, we set

$$P^{i}X^{i} = TotalOutput_{i} - \sum_{j \in Omitted} IntermediateUsage_{ij} + Imports_{i}$$

$$P^{i}C^{i} = FinalDemand_{i} + Imports_{i}$$

where $IntermediateUsage_{ij}$ denotes sector j's usage of i's product.

In our model exercises, we start from a steady state with zero carbon tax, and we abstract away from any other tax. Thus, we calibrate each sector's total costs M^iX^i in the initial steady state as the sum of that sector's compensation and intermediate inputs expenditure. We also calibrate the labor share of total costs $\tilde{\alpha}_i$, the energy share of intermediates expenditure $\tilde{\zeta}_i$, and the share of each sector's energy and nonenergy intermediates spending allocated to each sector j, $\tilde{\omega}_{ij}^E$ and $\tilde{\omega}_{ij}^E$ respectively. Consumption expenditure shares $\tilde{\gamma}_i$ are simply calculcated as $\frac{P^iC^i}{\sum_j P^jC^j}$. Appendix C shows that the log-linearized dynamics of the model, and the percentage change in all steady state variables following a change in taxes in the nonlinear model, depend only on these share variables, and not on the structural parameters α_i, γ_i , etc.

Finally, we calibrate e_i based on each sector's total raw CO2 emissions, which are calculated using EIA and EPA data as described in Appendix A. Again, raw emissions are nonzero for only three sectors: oil and gas extraction and coal mining.

4.3 Results

We consider a gradual increase in the carbon tax, announced at date 0, from 0 to 20 dollars per metric ton of CO2, where the size of the tax is chosen so as to achieve a 40 percent reduction in emissions when the tax is fully implemented. This is broadly in line with the Biden administration's commitment to reduce emissions to 50-52 percent of 2005 levels by 2030.¹⁹ As in section 3 we assume that the tax is implemented gradually following an autoregressive process with $\rho = 0.4^{1/12}$ (the path of the tax is shown in figure A1 in the appendix).

¹⁹Seehttps://www.whitehouse.gov/climate/. Net emissions in 2021 were already 17 percent below 2005 levels https://www.epa.gov/system/files/documents/2023-04/Data-Highlights-1990-2021.pdf).

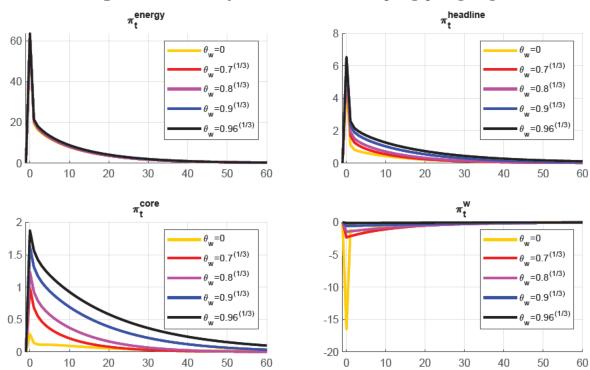


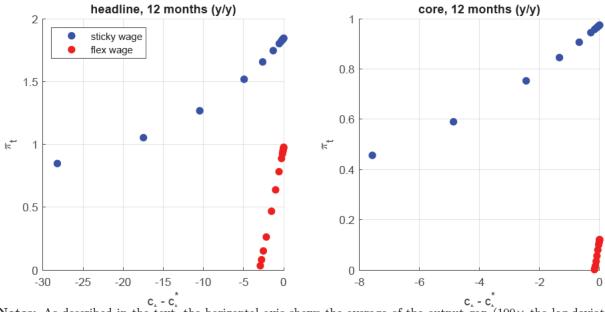
Figure 4. Inflation dynamics under strict output gap targeting

Notes: All lines show annualized log-deviations of each variable relative to the new steady state, i.e. we plot $1200 \times$ the log deviation.

Figure 4 shows the inflationary implications of the carbon tax under strict output gap targeting. The four panels show energy, headline, core, and wage inflation (monthly annualized) for different values of the degree of wage stickiness. Energy prices, which are very flexible, respond strongly to the rise in the carbon tax especially on impact. Since the tax increases gradually, energy prices keep rising throughout the horizon. Since labor input plays a limited role in the production of energy, energy price inflation is essentially the same regardless of wage stickiness. Wage stickiness instead plays a key role for the effects of the tax on both headline and core inflation. When wages are fully flexible ($\theta_w = 0$, yellow lines) the impact of the tax on core inflation is very small—less than a quarter of a percentage point. Headline inflation rises on impact together with energy inflation, but then quickly falls below 50 basis points for the reminder of the horizon. This is not because the increase in marginal costs associated with the rising cost of energy is small. Rather, the limited response of core inflation is due to the fact that wages, if flexible, adjust rapidly to the new lower steady state level and fall on impact by more than 15 percentage points, thereby reducing marginal costs and almost fully compensating for the rising cost of energy.

Nominal wage rigidity prevents this sudden adjustment from taking place and results in a more sizable and long lasting increase in both core and headline inflation. We show the impulse responses

Figure 5. Inflation/output tradeoff at different horizons



Notes: As described in the text, the horizontal axis shows the average of the output gap (100× the log-deviation of $c_t - c_t^*$ from its steady state value of zero) over the first 12 months after the announcement of the carbon tax. The vertical axis shows year-over-year inflation over the same period (i.e. the sum of 100× the log-deviation of π_t from its steady state value of zero, over the first 12 months). The left panel shows headline inflation and the output gap, under monetary policy rules of the form $\alpha \pi_t + (1 - \alpha)(c_t - c_t^*) = 0$, for various values of α (strict output gap targeting corresponds to $\alpha = 0$). Blue dots show outcomes under sticky wages ($\theta_w = 0.9^{1/3}$); red dots show flexible wages ($\theta_w = 0$). The right panel shows core inflation and the output gap, and considers rules which put weight α on core inflation, $1 - \alpha$ on the output gap and no weight on headline inflation.

for values of θ_w ranging from 0.7 (quarterly, as estimated in Smets and Wouters, 2007) to the higher values obtained in papers using post-Great recession data and/or models with financial frictions (0.8 in Christiano et al., 2014; 0.9 in Del Negro et al., 2015; 0.96 in Del Negro et al., 2017). With $\theta_w = 0.7$ (quarterly) nominal wages still drop by more than 2 percent on impact (see figure A2), arguably a large amount, but nonetheless the impact on core inflation is non negligible. With higher degrees of wage rigidity that are more consistent with the evidence on downward nominal wage stickiness (Grigsby et al., 2021) the response of inflation is even more notable, with 12-month headline and core inflation at 2 and 1 percent above target, respectively, one year after the announcement. The reason behind the rise in core inflation is that energy sectors, in spite of having relatively small value added, provide important inputs for core sectors whose prices are sticky, as shown in figure 1, and whose output is an input for other sticky sectors (Afrouzi and Bhattarai, 2023). The rise in marginal costs due to the carbon tax therefore propagates through the network, leading to core inflation.

While the experiment is different, it is instructive to compare the relative magnitude of the VAR responses of the various inflation measures to a carbon policy shock in Känzig (2022). Känzig finds that a shock leading to a 1 percent increase in energy prices increases headline and core inflation

by about 0.15 and 0.06 percent, respectively. In the sticky wages version of our model, the increase energy price inflation is one order of magnitude larger than the increase in headline inflation, while core inflation is about half of it. Nominal wages in Känzig (2022) do not react for a few quarters and fall only eventually, indicating some degree of nominal wage stickiness. Finally, the policy rate barely moves, consistent with the assumption behind figure 4 that policy is not trying to slow growth in order to combat inflation (industrial production falls some, but so does output in our model).

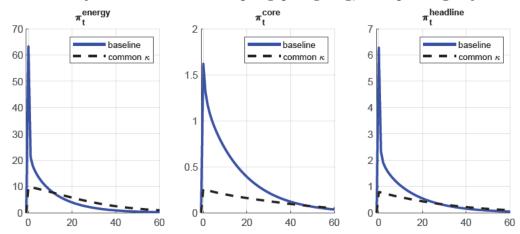
Figure 4 showed how much (above target) headline and core inflation the economy would have to endure if the policymakers pursued strict output gap targeting. Figure 5 shows the menu of outcomes under different preferences of the policymaker. That is, the dots in figure 5 depict outcome for inflation relative to target on the y-axis and the consumption gap in the x-axis (recall that consumption and output are the same in the model) under rules of the form $\alpha \pi_t + (1-\alpha)(c_t - c_t^*) = 0$ for different values of α (strict output gap targeting corresponding to $\alpha = 0$). The π_t entering the rule and depicted in the panels is headline inflation for the left panel and core inflation for the right panel. The tradeoff is shown in terms of average annualized inflation and average gap between actual and flexible price consumption after one year.²⁰

The red dots show the tradeoff under flexible wages. Even under flexible wages the tradeoff is not negligible for headline inflation. In order to bring 12-month headline inflation below .5 percent a small recession is needed: c is about 1.5 percent below c^* on average in the twelve months after the shock (recall that this output contraction is on top of the contraction in the fully flexible star economy, which is depicted in figure A1 of the appendix). The tradeoff in terms of core inflation under flexible wages is negligible however. Hence these results indicate that the green transition is fairly costless for a policymaker willing to "see through" the rise in energy prices and and to focus on core inflation, in agreement with Olovsson and Vestin (2023). This is not so with sticky wages however (blue dots). When $\theta_w = 0.9$ bringing 12-month headline inflation below 1 percent or 12-month core inflation below .5 percent after one year entails an average gap over the previous 12 months of 30 and 8 percent, respectively. The green transition has significant consequences for core inflation in our model, and any attempt to fight this inflation comes at the cost of a very large recession.

As in our simple two-sector economy, the reason that carbon taxes create an adverse tradeoff for a policymaker seeking to stabilize consumer price inflation and the output gap is that prices are relatively flexible in sectors with high direct and indirect emissions, and relatively sticky in other sectors. This suggests that the adverse tradeoff would largely disappear if, counterfactually, prices

 $^{^{20}}$ 12-month averages for inflation are usually the object of interest for policymakers. Likewise, the average gap summarizes the depth of the recession on average over the previous year. Figure A3 in the appendix provides instantaneous values for horizons 0, 6, and 12 months, so one can appreciate the evolution of the output gap over time for different values of α .

Figure 6. Inflation dynamics under strict output gap targeting; same price rigidity across sectors



Notes: Solid blue lines denote inflation under our baseline calibration with heterogenous price stickiness, dashed black lines show inflation in our counterfactual where all sectors have the same degree of price stickiness ($\theta_i = 0.92$ at a monthly frequency). All panels show annualized log-deviations, i.e. $1200 \times$ the log-deviation of inflation, under strict output-gap targeting.

were equally sticky (i.e. the sectoral Phillips curve slope κ were the same) in all sectors. Indeed, this is what Figure 6 shows: under strict output gap targeting both headline and core inflation rise a fraction of their increase in our baseline simulation, even with sticky wages.²¹ Quite simply, if energy prices were also sticky the rise in the carbon tax would not trigger as much energy inflation. Therefore the increase in headline and core inflation, and hence the tradeoffs faced by policymakers, are also smaller in this case.

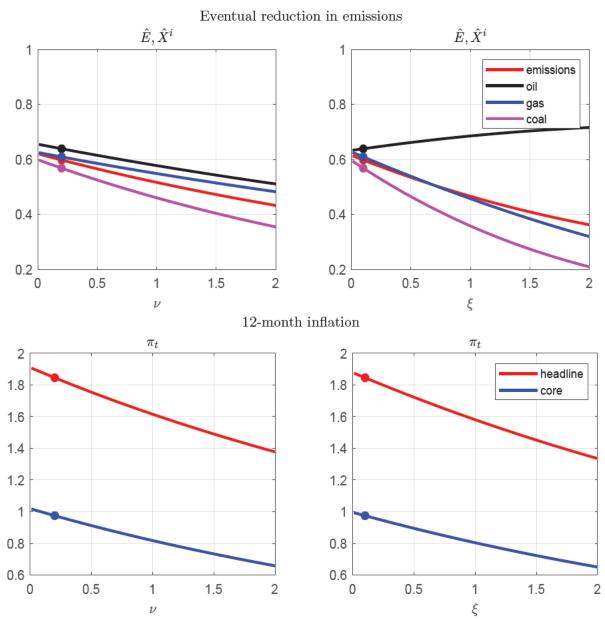
In sum, we find that the green transition does generate a tradeoff for monetary policymakers, in that they can stabilize inflation only at the cost of a very large recession. If they choose to stabilize the output gap instead, they would have to face a burst of inflation, with 12-month core reaching 1 percent above target after one year. To put this number in context, the last time before COVID that core PCE inflation in the US was 1 percent above target was in July 1992. Yet, this burst of inflation is arguably temporary, in spite of the fact that the carbon tax is implemented gradually over a five year period and is ultimately permanent, as it is almost over after one year. As such, the tradeoff faced by policymakers appears not to be very different from that associated with a large burst in energy prices. As we know from the literature on the optimal response to cost push shocks (Aoki, 2001), the optimal response to such shocks mostly consists in accommodating the temporary

²¹As we now show, our quantitative multisector model yields qualitatively the same prediction as the simple model with IO linkages. To construct the counterfactual economy with homogeneous price stickiness, we set the nominal-gross-output-weighted average duration of a price change equal to its value in our benchmark economy. Carvalho (2006) argues that it makes sense to calibrate the *duration* of price change in a homogeneous-firm economy to the average duration in a heterogeneous price economy, rather than calibrating in terms of the frequency of price adjustment. Because of Jensen's inequality these approaches are not equivalent. (The average duration of a price set by a firm with Calvo parameter θ_i is $\frac{1}{1-\theta_i}$.) The average duration is 12.7 months, which gives $\theta=0.92$ at a monthly frequency and $\kappa=0.007$. In this counterfactual economy, $\kappa_i=\kappa=0.007$ for all sectors.

burst in inflation, although the answer of course depends on the policymakers preferences.

4.4 Robustness to the elasticity of substitution

Figure 7. Robustness to the elasticity of substitution



Notes: Top row: lines show emissions and gross output in each of the polluting sectors in the new steady state as a fraction of their values in the old steady state, as a function of the elasticity of substitution between energy and non-energy inputs ν (left panel) and the elasticity of substitution between intermediate input varieties ξ (right panel), fixing all other parameters at their baseline calibration. Bottom row: lines show year-over-year headline (red line) and core (blue line) inflation over the 12 months following the announcement of the tax shock (i.e. the sum of $100 \times$ the log-deviation of π_t from zero over the first 12 months), as a function of the same elasticities.

Of course, one may rightly wonder to what extent these conclusions are robust to our modeling

choices and to our calibration. We can at least address the latter question. The impact of a carbon tax on emissions depends on how easy it it for producers and consumers to substitute away from fossil-fuel intensive goods and inputs. In our model, this is primarily governed by the elasticity of substitution between different inputs of a given type ξ , and the elasticity between energy and non-energy inputs ν . The top two panels of figure 7 are somewhat reassuring, as they show that our baseline choice of elasticities is quite low (circles indicate our baseline calibration), and that lowering them further would increase the reduction in eventual emissions induced by the 20 dollar carbon tax, but not by much.²²

The bottom two panels of figure 7 show the extent to which the inflationary dynamics shown in Figure 2 are robust to changing the elasticity of substitution parameters. Specifically, the panels plot average annualized headline and core inflation after 12 months as a function of ν and ξ . The larger the elasticities ν and ξ , the lower the inflationary impact of the carbon tax as upstream sectors are able to substitute energy with other inputs. The reason why this is the case for ν , the elasticity of substitution between energy and non-energy inputs, is readily understood. However, ξ , the elasticity of substitution between different non-energy inputs, and between different types of energy input, also matters, for two reasons. First, some non-energy sectors are more affected by the increase in energy prices than other, as they use more energy, and being able to substitute away from these sectors lowers the inflationary impact. Second, a higher ξ makes it easier to substitute from dirty to clean energy, mitigating the effect of the carbon tax on the overall price of energy.²³ The figure shows however that the lines are rather flat: one would have to increase the elasticities by an order of magnitude relative to our baseline calibration for the inflation response to be significantly different.

5 Conclusion

It has been argued that the green transition will be inflationary. In this paper we investigated whether this is the case using both a simple two-sector model, in order to gain intuition, and a quantitative input-output model with almost 400 sectors calibrated to data on input-output linkages and sectoral heterogeneity in emissions and price stickiness. We show that whether the green transition is inflationary crucially depends on (1) price stickiness, (2) central bank policy, (3) whether the green transition consists of taxes or subsidies. If prices were flexible there would be no reason for the green transition to be inflationary or deflationary, regardless of (3). If prices of non-dirty goods and services in the economy are stickier than prices in the dirty sectors – a

²²Figure 7 plots the long-run level level of emissions after the tax is fully implemented, as a fraction of emissions prior to the introduction of the tax. The entire path of emissions is shown in figure A1 in the appendix.

²³In our model, clean energy is included in the "Electric power generation, transmission, and distribution" sector. The 2012 BEA input-output tables, on which our calibration is based, do not distinguish between renewable and dirty fuel sources within electric power generation.

realistic situation – then policies aimed at reducing production in the dirty sector impose a tradeoff on the central bank between stabilizing inflation and closing the output gap. These conclusions are reversed if the green transition consists of subsidies to a clean energy sector, as for example in the recent Inflation Reduction Act, as long as prices in this sector are more flexible than in the rest of the economy. Our quantitative model suggests that an increase in carbon taxes from 0 to 20 (2012) dollars would generate a sizable tradeoff: containing the impact on headline or core inflation would lead to a deep recession. But while sizeable, the tradeoff is relatively short-lived as it wanes after one year.

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Appendix A Data Construction

This appendix describes our procedure to construct and merge price frequency change and emissions data with BEA input-output tables.

Price change frequencies. Sectoral price change frequency data are sourced from Cotton and Garga (2022), who use price change frequencies from Nakamura and Steinsson (2008). Specifically, Cotton and Garga (2022) use the price change frequencies of CPI and PPI products from Nakamura and Steinsson (2008), and transform these product price change frequencies into sectoral price change frequencies, with sectors defined at the 2017 six-digit NAICS level.

Emissions. Emissions data are constructed from a combination of data from the Bureau of Economic Analysis (BEA), Energy Information Administration (EIA), and Environmental Protection Agency (EPA). We follow the methods used by Shapiro et al. (2018) to compute total emissions by sector and direct emissions by sector.

First, we construct augmented versions of the 2012 BEA input-output tables, in which we expand the number of sectors in order to disentangle oil and gas usage across the US production network. We use the BEA's industry-by-commodity "make" table M and commodity-by-industry "use" table U: square 405×405 matrices originally, respectively representing the production and use of each commodity by each industry in dollar terms. Specifically, each element M_{ij} of M represents the production of commodity j by industry i, while each element U_{ij} of U represents the use of commodity i by industry j.

We split industry 211000 (Oil and gas extraction) into two. We update the "make" table by assigning output in the new sectors such that the oil industry produces all of the old industry's output of commodity 324110 (Petroleum refineries), the gas industry produces all of the old industry's output of commodity 325120 (Industrial gas manufacturing), and the residual output is assigned to make the new oil and gas sectors' relative level of production line up with consumption of oil and gas per the EIA.²⁴ We then update the "use" table by splitting the commodity usage of the old industry again in line with the relative output of the sector – a choice that assumes the new oil and gas sectors individually have the same commodity mix in inputs as the old combined sector. The new make and use tables \overline{M} and \overline{U} thus have dimensions of 406×405 and 405×406 respectively.

We then follow the standard method of constructing an industry-level input-output coefficients matrix by normalizing each column of \overline{M} by total commodity usage (i.e., the sum of the elements

²⁴Since the EIA's data are in terms of energy produced, rather than dollars spent, we transform the EIA energy use data into dollar amounts using the 2012 average prices of Brent crude and natural gas per energy unit (pulled from Haver).

in that column) to create a "market shares" matrix m, and similarly normalize each column of \overline{U} by total industry output (once again, the sum of elements in that column) to create a "direct requirements" matrix u. Multiplying m by u then gives us the industry-by-industry technical coefficients matrix A, in which each element A_{ij} represents the dollars of industry i's commodity production that industry j must use in order to produce a dollar of output. This matrix helps define the following (rather famous) equilibrium:

$$X = AX + Y \implies X = (I - A)^{-1}Y,$$

where for a number of industries N, X denotes the N-dimensional vector of industry gross output, Y denotes a given N-dimensional vector of final demand, and $(I - A)^{-1}$ denotes the $N \times N$ Leontief inverse or "total requirements matrix", which shows the amount of output required from each industry in total (not merely directly, but throughout the supply chain) to meet the given vector of final demand Y.

We compute total and direct emissions by sector using the Leontief inverse $(I-A)^{-1}$ and IO coefficients matrix A respectively. To do so, we construct a N-dimensional row vector c of 'raw' CO_2 emissions by energy type, which is 0 everywhere except for the entries associated with the oil, gas, and coal industries from the BEA tables. In those three entries, we take the EIA energy usage data for each source, and multiply it by the corresponding emissions intensity factor from the EPA, where these intensities show the amount of CO_2 emitted per unit of energy produced for a given energy source. (This vector c corresponds to $\{e_iX_i\}$ in the quantitative model presented in section 4.) We then premultiply the Leontief inverse and the IO coefficients matrix by c, producing in each case an N-dimensional row vector showing carbon emissions associated with each industry. The row vector cA gives emissions associated with energy usage in the production process for each industry, while the row vector $c(I-A)^{-1}$ shows total emissions associated with an industry's production, all the way upstream in its supply chain.

Crosswalk. In order to combine the price change frequencies with the emissions data described above, we create a crosswalk between the BEA's 2012 input-output sector definitions and the 2017 six-digit NAICS sectors in which terms Cotton and Garga (2022) present their price change frequencies. The BEA's sectors are closely related to the 2012 NAICS sectors, and a concordance is provided in the IO tables between those two types of codes. We thus follow two steps. First, we link the 2017 and 2012 six-digit NAICS codes using an existing concordance between them. Then, we use the 2012 BEA-2012 NAICS concordance to complete the link.

The first caveat is that the link between the 2012 NAICS and the BEA input-output codes is not perfect, in the sense that the NAICS codes vary in the level of aggregation at which they map into the BEA codes. While some NAICS codes map into the BEA codes at the six-digit level, others

only map in at the five- or four-digit level, with some going as low as at the two-digit level. This is an issue in the sense that each six-digit NAICS code has a price change frequency associated with it, which means that BEA codes have multiple price change frequencies associated with them if they concord with multiple six-digit NAICS codes. To resolve this issue, we take an average across the "candidate" price change frequencies for each BEA code, which then leaves us with a unique code for each input-output sector.

The other caveat is that even after conducting the merge described above, some BEA codes did not have any candidate price change frequencies associated with them due to the Cotton and Garga (2022) data not covering the entire set of NAICS six-digit codes. To deal with this issue, we apply the average price change frequency across the closest set of comparable BEA codes to any BEA code which lacks a match. Specifically, we cut the BEA codes from six digits down to five, search for sectors that match with them at this five-digit level, take the average price change frequency across these "comparable" sectors, and apply it to the unmatched sector. If no five-digit match with price change frequency data exists, we try four digits, three digits, and so on until a price change frequency is assigned to the sector. Using this method, we are able to assign to each BEA sector (with the exception of a few BEA industries operating in the public sector) a price change frequency.

Appendix B Model Details

B.1 Calibration for Figures in Two-sector Model

When plotting the figures for our two-sector model, we set $\beta=0.99, \ \gamma=0.5$, and $\kappa^o=0.01$. In terms of the path of climate policy, we set $\mu_0^d=-1$, implying a long-run reduction in the size of the dirty sector of $\frac{e^{\mu_0^d}-e^0}{e^{\mu_0^d}}\approx 63\%$. We set $\rho=0.7$, implying that taxes on the dirty sector increase relatively rapidly towards their new higher steady state level.

B.2 Input-Output Linkages

We now allow for the two sectors to use each other's products as intermediate inputs. Firms in sector i = o, d have the constant returns to scale production function

$$X_t^i = A_t^i (X_t^{io})^{\omega_{io}} (X_t^{id})^{\omega_{id}} (L_t^i)^{\omega_{il}},$$

where $\omega_{io} + \omega_{id} + \omega_{il} = 1$ (in our baseline model, $\omega_{io} = \omega_{id} = 0, \omega_{il} = 1$). Here X_t^{od} (for example) denotes the quantity of dirty goods used by firms in the "other" sector. In particular, if $\omega_{od} > 0$, production of other goods requires dirty goods as input. The index of intermediate inputs used by firms in sector k and produced by a firm in sector i is given by the same CES aggregate as household's consumption of sector i goods. Thus, the demand for the variety produced by firm j

in sector i still has the form

$$X_t^i(j) = X_t^i \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon_t^i}.$$

Now, however, X_t^i denotes *gross* output of sector i, which in general will differ from value added or net output (which we still refer to as Y_t^i or Y_t for sectoral and aggregate net output respectively). The market clearing conditions for each sector are

$$C_t^i + X_t^{oi} + X_t^{di} = X_t^i = A_t^i (X_t^{io})^{\omega_{io}} (X_t^{id})^{\omega_{id}} (L_t^i)^{\omega_{il}}, i = o, d.$$

Nominal marginal cost for a firm in sector i now equals

$$M_t^i = \frac{1}{A_t^i} \left(\frac{P_t^o}{\omega_{io}}\right)^{\omega_{io}} \left(\frac{P_t^d}{\omega_{id}}\right)^{\omega_{id}} \left(\frac{W_t}{\omega_{il}}\right)^{\omega_{il}} + \mathcal{T}_t^i.$$

Deflating by product prices in each sector, real marginal costs are given by

$$\begin{split} \frac{M_t^o}{P_t^o} &= \frac{1}{\omega_o A_t^o} (bY_t)^{\omega_{ol}} S_t^{\omega_{od} + \omega_{ol} (1 - \gamma)} + \frac{\mathcal{T}_t^o}{P_t^o}, \\ \frac{M_t^d}{P_t^d} &= \frac{1}{\omega_d A_t^d} (bY_t)^{\omega_{dl}} S_t^{-\omega_{do} - \omega_{dl} \gamma} + \frac{\mathcal{T}_t^d}{P_t^d}, \end{split}$$

where we define $\omega_i = (\omega_{io})^{\omega_{io}} (\omega_{id})^{\omega_{id}} (\omega_{ld})^{\omega_{ld}}$, i = o, d. Cost minimization implies that the quantities of intermediate inputs and labor used by sector i are given by

$$\begin{split} X_t^{io} &= \frac{X_t^i}{A_t^i} \left(\frac{P_t^o}{\omega_{io}} \right)^{\omega_{io}-1} \left(\frac{P_t^d}{\omega_{id}} \right)^{\omega_{id}} \left(\frac{W_t}{\omega_{il}} \right)^{\omega_{il}}, \\ X_t^{id} &= \frac{X_t^i}{A_t^i} \left(\frac{P_t^o}{\omega_{io}} \right)^{\omega_{io}} \left(\frac{P_t^d}{\omega_{id}} \right)^{\omega_{id}-1} \left(\frac{W_t}{\omega_{il}} \right)^{\omega_{il}}, \\ L_t^i &= \frac{X_t^i}{A_t^i} \left(\frac{P_t^o}{\omega_{io}} \right)^{\omega_{io}} \left(\frac{P_t^d}{\omega_{id}} \right)^{\omega_{id}} \left(\frac{W_t}{\omega_{il}} \right)^{\omega_{il}-1}. \end{split}$$

Flexible-price benchmark. In the flexible price equilibrium, we have

$$\begin{split} \frac{1}{\widetilde{\mu}_t^o} &= \frac{1}{\omega_o A_t^o} (bY_t)^{\omega_{ol}} S_t^{\omega_{od} + \omega_{ol} (1 - \gamma)}, \\ \frac{1}{\widetilde{\mu}_t^d} &= \frac{1}{\omega_d A_t^d} (bY_t)^{\omega_{dl}} S_t^{-\omega_{do} - \omega_{dl} \gamma}, \end{split}$$

which implicitly define equilibrium net output Y_t and relative prices S_t . The quantities of inputs used by sector i satisfy

$$X_t^{io} = \frac{\omega_{io}}{P_t^o} P_t^i X_t^i \frac{1}{\widetilde{\mu}_t^i}, \qquad X_t^{id} = \frac{\omega_{id}}{P_t^d} P_t^i X_t^i \frac{1}{\widetilde{\mu}_t^i}, \qquad L_t^i = \frac{\omega_{il}}{W_t} P_t^i X_t^i \frac{1}{\widetilde{\mu}_t^i}$$

Substituting these into the resource constraints, we obtain a relation between the value of net and gross output:

$$\begin{split} P_t^o X_t^o &= \gamma P_t Y_t + \frac{\omega_{oo}}{\widetilde{\mu}_t^o} P_t^o X_t^o + \frac{\omega_{do}}{\widetilde{\mu}_t^d} P_t^d X_t^d, \\ P_t^d X_t^d &= (1-\gamma) P_t Y_t + \frac{\omega_{od}}{\widetilde{\mu}_t^o} P_t^o X_t^o + \frac{\omega_{dd}}{\widetilde{\mu}_t^d} P_t^d X_t^d. \end{split}$$

Dividing through by P_t , we can represent this in matrix form as

$$oldsymbol{s} \circ oldsymbol{x} = oldsymbol{\gamma} Y_t + oldsymbol{\Omega}' \widetilde{oldsymbol{\mu}}^{-1} (oldsymbol{s} \circ oldsymbol{x}),$$

where \circ denotes the element-wise product, $\mathbf{s} = (P_t^o/P_t, P_t^d/P_t)'$, $\mathbf{x} = (X_t^o, X_t^d)'$, $\mathbf{\gamma} = (\gamma, 1 - \gamma)'$, $\widetilde{\boldsymbol{\mu}}$ denotes the diagonal matrix with $\widetilde{\mu}_t^i$ on the diagonal, and $\mathbf{\Omega}$ denotes the 2×2 matrix of intermediate input shares ω_{ij} , i = o, d, j = o, d. Rearranging, we have

$$s \circ x = (I - \Omega' \widetilde{\mu}^{-1})^{-1} \gamma Y_t,$$

$$x = Y_t s^{-1} \circ \left[(I - \Omega' \widetilde{\mu}^{-1})^{-1} \gamma \right].$$

In our baseline model without input-output linkages, Ω is a matrix of zeros, and the vector of sectoral gross output is simply $\mathbf{x} = Y_t \mathbf{s}^{-1} \circ \gamma$, which is the vector of sectoral net output or consumption.

To simplify the analysis, we will focus on the case where $\omega_{od} > 0$, $\omega_{oo} = \omega_{dd} = \omega_{do} = 0$, $\omega_{ol} = 1 - \omega_{od}$, $\omega_{dl} = 1$. That is, the only linkage is that dirty goods are used by the other sector as inputs. In this case, flexible-price net output and relative prices are given by

$$S_{t} = \left(\frac{\omega_{o} A_{t}^{o}}{\widetilde{\mu}_{t}^{o}}\right) \left(\frac{\omega_{d} A_{t}^{d}}{\widetilde{\mu}_{t}^{d}}\right)^{-(1-\omega_{od})},$$

$$Y_{t} = \frac{1}{b} \left(\frac{\omega_{o} A_{t}^{o}}{\widetilde{\mu}_{t}^{o}}\right)^{\gamma} \left(\frac{\omega_{d} A_{t}^{d}}{\widetilde{\mu}_{t}^{d}}\right)^{1-\gamma+\gamma\omega_{od}},$$

$$Y_{t}^{d} = (1-\gamma)Y_{t}S_{t}^{-\gamma} = \frac{1-\gamma}{b} \left(\frac{\omega_{d} A_{t}^{d}}{\widetilde{\mu}_{t}^{d}}\right).$$

Turning from net to gross output, in this special case we have

$$(I - \mathbf{\Omega}' \widetilde{\boldsymbol{\mu}}^{-1})^{-1} = \begin{pmatrix} 1 & 0 \\ -\omega_{od}(\widetilde{\boldsymbol{\mu}}_t^o)^{-1} & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ \omega_{od}(\widetilde{\boldsymbol{\mu}}_t^o)^{-1} & 1 \end{pmatrix},$$

and the formula above implies

$$X_t^o = \gamma Y_t S_t^{1-\gamma}, \qquad X_t^d = Y_t S_t^{-\gamma} [\gamma \omega_{od}(\widetilde{\mu}_t^o)^{-1} + 1 - \gamma] = \frac{\gamma \omega_{od}(\widetilde{\mu}_t^o)^{-1} + 1 - \gamma}{b} \left(\frac{\omega_d A_t^d}{\widetilde{\mu}_t^d}\right).$$

The use of dirty output as an intermediate input $(\omega_{od} > 0)$ increases this sector's gross output, all else equal. In particular, the share of expenditures on dirty goods as a fraction of gross output is $\frac{\gamma \omega_{od}(\widetilde{\mu}_t^o)^{-1} + 1 - \gamma}{\gamma \omega_{od}(\widetilde{\mu}_t^o)^{-1} + 1} > 1 - \gamma$. Nonetheless, as in our baseline, $\widetilde{\mu}_t^d$ can still be interpreted as the proportional reduction in dirty sector output under flexible prices.

The firm's problem has the same structure as before, except that gross output X_t^i replaces net output Y_t^i (we assume price adjustment costs are also scaled by gross output):

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{t|0} \left\{ (P_t^i(j) - M_t^i) X_t^i \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\varepsilon_t^i} - \frac{\Psi^i}{2} \left(\frac{P_t^i(j)}{P_{t-1}^i(j)} - 1 \right)^2 P_t^i X_t^i \right\}.$$

Taking FOCs and assuming a symmetric equilibrium, we have:²⁵

$$\Pi_{t}^{i}\left(\Pi_{t}^{i}-1\right) = \frac{\varepsilon_{t}^{i}}{\Psi^{i}}\left(\frac{M_{t}^{i}}{P_{t}^{i}} - \frac{1}{\mu_{t}^{i}}\right) + \beta \frac{Y_{t}}{Y_{t+1}} \frac{\Pi_{t+1}^{i}}{\Pi_{t+1}} \frac{X_{t+1}^{i}}{X_{t}^{i}} \Pi_{t+1}^{i} \left(\Pi_{t+1}^{i}-1\right).$$

Defining the 'virtual markup' $\tilde{\mu}_t^i$ as before, and log-linearizing around a zero inflation steady state, we have the sectoral Phillips curves:

$$\pi_t^o = \kappa^o((1 - \omega_{od})y_t + [1 - \gamma + \gamma\omega_{od}]s_t) + \beta \mathbb{E}_t \pi_{t+1}^o,$$

$$\pi_t^d = \kappa^d(y_t - \gamma s_t + \mu_t^d) + \beta \mathbb{E}_t \pi_{t+1}^d.$$

While the dirty sector Phillips curve is unchanged from our baseline, $\omega_{od} > 0$ makes the other sector's Phillips curve less sensitive to aggregate economic activity, but more sensitive to the relative price of dirty goods. Log-linearizing the expressions above describing the flexible-price levels of Y_t and S_t , we have $y_t^* = -(1 - \gamma + \gamma \omega_{od})\mu_t^d$ and $s_t = (1 - \omega_{od})\mu_t^d$, which can be used to obtain the Phillips curves in the main text. Note that the same proportional reduction in dirty output μ_t^d results in a larger reduction in aggregate output when $\omega_{od} > 0$.

With $\omega_{od} > 0$, if prices are equally flexible in both sectors ($\kappa^o = \kappa^d = \kappa$), stabilizing CPI inflation will not close the output gap. Multiplying the two Phillips curves by their consumption expenditure weights γ and $1 - \gamma$, summing, and using the expressions for y_t^* and s_t^* , we obtain the CPI Phillips curve

$$\pi_t = \kappa \left[(1 - \gamma \omega_{od})(y_t - y_t^*) + \gamma \omega_{od}(s_t - s_t^*) \right] + \beta \mathbb{E}_t \pi_{t+1}.$$

Since $s_t < s_t^*$ during the transition, stabilizing CPI inflation allows output to run somewhat above potential (though recall that potential output is itself declining more sharply than in our baseline model without IO linkages). If instead we weight the two Phillips curves by their *gross* expenditure shares $\frac{\gamma}{\gamma\omega_{od}+1}$ and $\frac{\gamma\omega_{od}+1-\gamma}{\gamma\omega_{od}+1}$, we obtain the PPI Phillips curve²⁶

$$\pi_t^{PPI} := \frac{\gamma}{\gamma \omega_{od} + 1} \pi_t^o + \frac{\gamma \omega_{od} + 1 - \gamma}{\gamma \omega_{od} + 1} \pi_t^d = \frac{\kappa}{\gamma \omega_{od} + 1} (y_t - y_t^*) + \beta \mathbb{E}_t \pi_{t+1}^{PPI}.$$

In this special case, it is stabilizing PPI inflation that is equivalent to closing the output gap. PPI puts a higher weight on dirty sector prices, which are increasing during the transition. Consequently, stabilizing PPI inflation would require a more aggressive monetary policy response than stabilizing CPI.

²⁵Since net output need not equal gross output, the term $\frac{Y_t}{Y_{t+1}} \frac{\Pi_{t+1}^i}{\Pi_{t+1}} \frac{X_{t+1}^i}{X_t^i}$ is not necessarily equal to 1 as in our baseline model. This will not affect the linearized Phillips curve given that we log-linearize around a zero-inflation steady state.

²⁶Here, as in the experiments in our baseline, we assume $\tilde{\mu}_t^o = 1$, i.e. there is a constant subsidy to correct distortions from monopolistic competition in the other sector.

If prices are sticky (or even perfectly fixed) in the other sector, but perfectly flexible in the dirty sector, then as in our baseline, the dirty sector Phillips curve reduces to $s_t = \frac{1}{\gamma}(y_t + \mu_t^d)$. Substituting into the Phillips curve for the other sector, we have

$$\pi_t^o = \frac{\kappa^o}{\gamma} (y_t - y_t^*) + \beta \mathbb{E}_t \pi_{t+1}^o,$$

as in our baseline economy without IO linkages. When prices in the dirty sector are completely flexible, stabilizing inflation in the rest of the economy implements flexible price allocations; stabilizing overall CPI inflation instead requires a negative output gap.

To recap: if prices are equally flexible in both sectors ($\kappa^d/\kappa^o = 1$), stabilizing CPI inflation implies running output above potential, but if prices are infinitely more flexible in the dirty sector ($\kappa^d/\kappa^o = \infty$), stabilizing CPI implies running output below potential as in our baseline. Is there some degree of relative price flexibility at which stabilizing inflation closes the output gap, and there is no tradeoff? Yes: this will be the case when

$$\frac{\kappa^d}{\kappa^o} = 1 + \frac{\gamma \omega_{od}}{1 - \gamma}.$$

Intuitively, in this knife-edge case, the difference in price flexibility exactly offsets the difference between PPI and CPI weights. To prove this, assume that $\kappa^d = \left(1 + \frac{\gamma \omega_{od}}{1 - \gamma}\right) \kappa^o$ and add the expenditure-weighted Phillips curves to obtain the CPI Phillips curve:

$$\pi_{t} = \gamma \kappa^{o} \left[(1 - \omega_{od}) y_{t} + (1 - \gamma + \gamma \omega_{od}) s_{t} \right] + (1 - \gamma + \gamma \omega_{od}) \kappa^{o} \left[(y_{t} - y_{t}^{*}) - \gamma (s_{t} - s_{t}^{*}) \right] + \beta \mathbb{E}_{t} \pi_{t+1}$$

$$= \kappa^{o} (y_{t} - y_{t}^{*}) + \beta \mathbb{E}_{t} \pi_{t+1}.$$

Appendix C Multisector model

C.1 Optimality conditions

The household's optimization problem (18) implies the following. First, the demand for each sector is given by

$$C_t^i = \gamma_i C_t \left(\frac{P_t^i}{P_t}\right)^{-\zeta} \tag{C.1}$$

where the consumer price index P_t is given by

$$P_{t} = \left[\sum_{i=1}^{n} \gamma_{i} (P_{t}^{i})^{-(\zeta-1)}\right]^{-\frac{1}{\zeta-1}}$$
 (C.2)

Second, the demand for variety j in sector i is given by

$$C_t^i(j) = C_t^i \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\zeta} \tag{C.3}$$

where each sectoral price index P_t^i is an aggregate of prices set by producers in that sector:

$$P_t^i = \left[\int_0^1 (P_t^i(j))^{-(\varepsilon^i - 1)} dj \right]^{-\frac{1}{\varepsilon^i - 1}}$$
 (C.4)

The solution to the union's optimization problem (19) yields the optimality condition

$$W_t^* = b \frac{\sum_{k=0}^{\infty} (\theta_w \beta)^k N_{t+k} \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon^w}}{\sum_{k=0}^{\infty} (\theta_w \beta)^k \frac{1}{P_{t+k} C_{t+k}} N_{t+k} \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon^w}}$$
(C.5)

where we define $b = \frac{\varepsilon^w}{\varepsilon^w - 1}\tilde{b}$. Note that in zero-inflation steady state, or in the flexible-wage limit with $\theta_w = 0$, we have $\frac{W_t}{P_t} = bC_t$. The wage W_t evolves according to

$$W_t = \left[\theta_w(W_{t-1}^i)^{1-\varepsilon^w} + (1-\theta_w)(W_t^*)^{1-\varepsilon^w}\right]^{\frac{1}{1-\varepsilon^w}} \tag{C.6}$$

Attaching multipliers $\widetilde{M}_t^i, P_t^{I,i}, P_t^{E,i}, P_t^{N,i}$ to the constraints, the firm's cost minimization problem (20) yields the optimality conditions

$$\begin{split} L_t^i &= (A_t^i)^{\eta-1} X_t^i \alpha_i \left(\frac{W_t}{\widetilde{M}_t^i}\right)^{-\eta} \\ I_t^i &= (A_t^i)^{\eta-1} X_t^i (1-\alpha_i) \left(\frac{P_t^{I,i}}{\widetilde{M}_t^i}\right)^{-\eta} \\ E_t^i &= \varsigma_i I_t^i \left(\frac{P_t^{E,i}}{P_t^{I,i}}\right)^{-\nu} \\ N_t^i &= (1-\varsigma_i) I_t^i \left(\frac{P_t^{N,i}}{P_t^{I,i}}\right)^{-\nu} \\ X_t^{ij} &= \omega_{ij}^E E_t^i \left(\frac{P_t^j}{P_t^{E,i}}\right)^{-\xi} + \omega_{ij}^N N_t^i \left(\frac{P_t^j}{P_t^{N,i}}\right)^{-\xi} \end{split}$$

which implies that

$$\widetilde{M}_{t}^{i} = \frac{1}{A_{t}^{i}} \left[\alpha_{i} \left(W_{t} \right)^{-(\eta - 1)} + \left(1 - \alpha_{i} \right) \left(P_{t}^{I,i} \right)^{-(\eta - 1)} \right]^{-\frac{1}{\eta - 1}}$$

$$P_{t}^{I,i} = \left[\varsigma_{i} \left(P_{t}^{E,i} \right)^{-(\nu - 1)} + \left(1 - \varsigma_{i} \right) \left(P_{t}^{N,i} \right)^{-(\nu - 1)} \right]^{-\frac{1}{\nu - 1}}$$

$$P_{t}^{E,i} = \left[\sum_{j} \omega_{ij}^{E} \left(P_{t}^{j} \right)^{-(\xi - 1)} \right]^{-\frac{1}{\xi - 1}}$$

$$P_{t}^{N,i} = \left[\sum_{j} \omega_{ij}^{N} \left(P_{t}^{j} \right)^{-(\xi - 1)} \right]^{-\frac{1}{\xi - 1}}$$

 \widetilde{M}_t^i can be interpreted as the nominal marginal cost excluding the carbon tax. The full nominal marginal cost is

$$M_{t}^{i} = \frac{1}{A_{t}^{i}} \left[\alpha_{i} (W_{t})^{-(\eta-1)} + (1 - \alpha_{i}) \left(P_{t}^{I,i} \right)^{-(\eta-1)} \right]^{-\frac{1}{\eta-1}} + \mathcal{T}_{t} e_{i}$$

$$= \frac{1}{A_{t}^{i}} \left[\alpha_{i} (W_{t})^{-(\eta-1)} + (1 - \alpha_{i}) \left[\sum_{j} \omega_{ij} \left(P_{t}^{j} \right)^{-(\xi-1)} \right]^{\frac{\eta-1}{\xi-1}} \right]^{-\frac{1}{\eta-1}} + \mathcal{T}_{t} e_{i}$$

The solution to the firm's optimal pricing problem (21) implies that the price set by a resetting firm in sector i at date t, P_t^{i*} satisfies

$$\sum_{k=0}^{\infty} Q_{t+k|t} \theta_i^k \left[P_t^{i*} - \frac{\varepsilon^i}{\varepsilon^i - 1} M_t^i \right] X_{t+k}^i \left(\frac{P_t^{i*}}{P_{t+k}^i} \right)^{-\varepsilon^i} = 0$$
 (C.7)

The sectoral price index P_t^i evolves according to

$$P_t^i = \left[\theta_i(P_{t-1}^i)^{1-\varepsilon^i} + (1-\theta_i)(P_t^{i*})^{1-\varepsilon^i}\right]^{\frac{1}{1-\varepsilon^i}}$$
 (C.8)

C.2 Solving the log-linearized model

We log-linearize the model around an arbitrary zero-inflation steady state, assuming no shocks and no changes in exogenous variables except for the carbon tax. We log-linearize all variables except the carbon tax, which we linearize; this allows for the case in which the steady state carbon tax $\mathcal{T}=0$. Importantly, we do not need to fully solve for steady state in order to linearize the model. We only need to know price flexibility, input shares and emissions intensity by sector in steady state.

Solving for key variables Log-linearizing (C.5) and (C.6) yields

$$w_{t}^{*} = (1 - \beta \theta_{w}) \sum_{k=0}^{\infty} (\beta \theta_{w})^{k} (p_{t+k} + c_{t+k})$$
$$w_{t} = \theta_{w} w_{t-1} + (1 - \theta_{w}) w_{t}^{*}$$

Combining and defining $\pi_t^w = w_t - w_{t-1}$, $\omega_t = w_t - p_t$, we have the nominal wage Phillips curve

$$\pi_t^w = \kappa_w(c_t - \omega_t) + \beta \pi_{t+1}^w \tag{C.9}$$

where $\kappa_w := \frac{(1 - \beta \theta_w)(1 - \theta_w)}{\theta_w}$. (Again, note that with flexible wages we have $\kappa_w = \infty$ and $\omega_t = c_t$.) Real wages evolve according to

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t \tag{C.10}$$

Similarly, log-linearizing (C.7) around a steady state with $\Pi^i = 1$, we have

$$p_t^{i*} = (1 - \beta \theta_i) \sum_{k=0}^{\infty} (\beta \theta_i)^k m_{t+k}^i$$

where lower case variables denote log-deviations. Log-linearizing (C.8), we have

$$p_t^i = \theta_i p_{t-1}^i + (1 - \theta_i) p_t^{i*}$$

Combining and defining $\pi_t^i = p_t^i - p_{t-1}^i$, we have the standard sectoral Phillips curve

$$\pi_t^i = \kappa_i(m_t^i - p_t^i) + \beta \pi_{t+1}^i \tag{C.11}$$

where $\kappa_i := \frac{(1 - \beta \theta_i)(1 - \theta_i)}{\theta_i}$.

Constant returns to scale imply that

$$M_t^i X_t^i = W_t L_t^i + \sum_{i=1}^n P_t^j X_t^{ij} + \mathcal{T}_t e_i X_t^i$$

Log-linearizing around steady state, since inputs are chosen to minimize cost, Shephard's lemma implies that

$$M^{i}X^{i}m_{t}^{i} = WL^{i}w_{t} + \sum_{j} P^{j}X^{ij}p_{t}^{j} + e_{i}X^{i}\widehat{\mathcal{T}}_{t}$$

or in real terms,

$$m_t^i - p_t = m_t^i - p_t^i + s_t^i = \frac{WL^i}{M^i X^i} \omega_t + \sum_j \frac{P^j X^{ij}}{M^i X^i} s_t^j + \frac{e_i X^i}{M^i X^i} \widehat{\tau}_t$$
 (C.12)

where we define (the log-deviation of) real sectoral prices, deflated by CPI, to be $s_t^i := p_t^i - p_t$.

Combining (C.11) and (C.12), we have sectoral Phillips curves in terms of value added, relative prices and taxes:

$$\pi_t^i = \kappa_i \left[\frac{WL^i}{M^i X^i} \omega_t + \sum_j \frac{P^j X^{ij}}{M^i X^i} s_t^j + \frac{e_i X^i}{M^i X^i} \widehat{\tau}_t - s_t^i \right] + \beta \pi_{t+1}^i, i = 1, ..., n$$
 (C.13)

To calibrate these equations, we need

- 1. sectoral price adjustment frequencies $1 \theta_i$ (to get κ_i)
- 2. labor shares $\frac{WL^i}{M^iX^i}$. (These are all shares of total costs M^iX^i , rather than revenues P^iX^i .)
- 3. intermediate input shares $\frac{P^j X^{ij}}{M^i X^i}$
- 4. emissions shares $\frac{e_i}{M^i X^i}$

The dynamics of relative prices must also satisfy

$$\Delta s_t^i = \pi_t^i - \pi_t, \ i = 1, ..., n \tag{C.14}$$

CPI inflation is defined by

$$\pi_t = \sum_{i=1}^n \widetilde{\gamma}_i \pi_t^i \tag{C.15}$$

where $\tilde{\gamma}_i := \frac{P^i C^i}{PC}$ denotes steady state consumption expenditure shares, which may differ from γ_i . Given a path for $\hat{\tau}_t$, (C.13), (C.14), (C.15), (C.9) and (C.10) constitute a system of 2n+3 equations in 2n+4 endogenous variables $(\{s_t^i, \pi_t^i\}_{i=1}^n, c_t, \pi_t, \omega_t, \pi_t^w)$. To close the system, we need to specify a monetary policy rule. The linearized system can then be used to study the impulse response to a shock to carbon taxes.

Solving for other variables Having solved for $\{s_t^i, \pi_t^i\}_{i=1}^n, c_t, \pi_t, \omega_t, \pi_t^w$, we can solve for other variables of interest – sectoral quantities and aggregate emissions. Log-linearizing sectoral final consumption demand (C.1), we have

$$c_t^i = c_t - \zeta s_t^i \tag{C.16}$$

Log-linearizing goods market clearing (22) (and multiplying and dividing by steady state prices in order to relate the coefficients to observable values), we have

$$x_t^j = \sum_{j=1}^n \frac{P^i X^{ji}}{P^i X^i} x_t^{ji} + \frac{P^i C^i}{P^i X^i} c_t^i$$
 (C.17)

It is convenient to denote the set of energy and non-energy goods by \mathcal{E} and \mathcal{N} respectively. Log-linearizing the equations characterizing demand for intermediate goods, we have

$$\begin{split} x_t^{ij} &= e_t^i - \xi(p_t^j - p_t^{E,i}) \text{ if } i \in \mathcal{E} \\ x_t^{ij} &= n_t^i - \xi(p_t^j - p_t^{N,i}) \text{ if } i \in \mathcal{N} \\ e_t^i &= i_t^i - \nu(p_t^{E,i} - p_t^{I,i}) \\ n_t^i &= i_t^i - \nu(p_t^{N,i} - p_t^{I,i}) \\ i_t^i &= x_t^i - \eta(p_t^{I,i} - \widetilde{m}_t^i) \\ p_t^{I,i} &= \widetilde{\varsigma}_i p_t^{E,i} + (1 - \widetilde{\varsigma}_i) p_t^{N,i} \\ p_t^{E,i} &= \sum_j \widetilde{\omega}_{ij}^E p_t^j \\ p_t^{N,i} &= \sum_j \widetilde{\omega}_{ij}^N p_t^j \end{split}$$

where \widetilde{m}_t^i is the log-deviation of \widetilde{M}_t^i , nominal marginal cost excluding the carbon tax; $\widetilde{\varsigma}_i := \frac{P^E, i E^i}{P^I, i I^i} = \varsigma_i \left(\frac{P^{E,i}}{P^I, i}\right)^{1-\nu}$ is the share of energy inputs in sector i's overall intermediates expenditure; $\widetilde{\omega}_{ij}^E := \mathbf{1}\{j \in \mathcal{E}\}\frac{P_j X_{ij}}{P^{E,i} E^i} = \omega_{ij}^E \left(\frac{P^j}{P^{E,i}}\right)^{1-\xi}$ is the share of input j in sector i's expenditure on energy (and equals zero if j is not an energy input); and $\widetilde{\omega}_{ij}^N := \mathbf{1}\{j \in \mathcal{N}\}\frac{P_j X_{ij}}{P^{N,i} N^i} = \omega_{ij}^N \left(\frac{P^j}{P^{N,i}}\right)^{1-\xi}$ is the share of input j in sector i's expenditure on non-energy inputs (and equals zero if j is an energy input). In general, reduced-form parameters with tildes denote revenue shares, which generally differ from the corresponding structural parameters without shares except in the Cobb-Douglas case. For example, the share of energy inputs in sector i's overall intermediates expenditure $\widetilde{\varsigma}_i$ depends on relative prices and thus may differ from the structural parameter ς_i .

Since emissions per unit of gross output are fixed, the same argument based on Shephard's lemma as made above establishes that

$$\widetilde{m}_t^i = \frac{WL^i}{\widetilde{M}^i X^i} w_t + \frac{P^{I,i} I^i}{\widetilde{M}^i X^i} p_t^{I,i}$$

So, if $j \in \mathcal{E}$ we have

$$\begin{split} x_t^{ij} &= x_t^i - \eta \left(p_t^{I,i} - \frac{WL^i}{\widetilde{M}^i X^i} w_t - \frac{P^{I,i} I^i}{\widetilde{M}^i X^i} p_t^{I,i} \right) - \nu (p_t^{E,i} - p_t^{I,i}) - \xi (p_t^j - p_t^{E,i}) \\ &= x_t^i - \eta \left(s_t^{I,i} - \frac{WL^i}{\widetilde{M}^i X^i} (w_t - p_t) - \frac{P^{I,i} I^i}{\widetilde{M}^i X^i} s_t^{I,i} \right) - \nu (s_t^{E,i} - s_t^{I,i}) - \xi (s_t^j - s_t^{E,i}) \\ &= x_t^i - \left[\eta \left(1 - \frac{P^{I,i} I^i}{\widetilde{M}^i X^i} \right) - \nu \right] s_t^{I,i} + \eta \frac{WL^i}{\widetilde{M}^i X^i} \omega_t - (\nu - \xi) s_t^{E,i} - \xi s_t^j \\ &= x_t^i - \Theta^{ijt} \end{split}$$

where
$$\Theta_t^{ij} := \left[\eta \left(1 - \frac{P^{I,i}I^i}{\widetilde{M}^iX^i} \right) - \nu \right] s_t^{I,i} - \eta \frac{WL^i}{\widetilde{M}^iX^i} \omega_t + (\nu - \xi) s_t^{E,i} + \xi s_t^j$$
, and we define
$$s_t^{I,i} = \widetilde{\varsigma_i} s_t^{E,i} + (1 - \widetilde{\varsigma_i}) s_t^{N,i}$$

$$s_t^{E,i} = \sum_j \widetilde{\omega}_{ij}^E s_t^j$$

$$s_t^{N,i} = \sum_j \widetilde{\omega}_{ij}^N s_t^j$$

For non-energy inputs $(j \in \mathcal{N})$ we instead have

$$\Theta_t^{ij} := \left[\eta \left(1 - \frac{P^{I,i}I^i}{\widetilde{M}^iX^i} \right) - \nu \right] s_t^{I,i} - \eta \frac{WL^i}{\widetilde{M}^iX^i} \omega_t + (\nu - \xi) s_t^{N,i} + \xi s_t^j$$

Since Θ_t^{ij} is a known function of relative prices and real wages, we can substitute $x_t^i - \Theta^{ij_t}$ back into the market clearing condition (C.17):

$$x_{t}^{i} = \sum_{j=1}^{n} \frac{P^{i} X^{ji}}{P^{i} X^{i}} (x_{t}^{j} - \Theta_{t}^{ji}) + \frac{P^{i} C^{i}}{P^{i} X^{i}} c_{t}^{i}$$

and invert this equation to solve for sectoral gross output x_t^i . Finally, aggregate emissions can be defined as $E_t = \sum_{i=1}^n e_i X_t^i$. Log-linearizing,

$$e_t = \sum_{i=1}^n \frac{e_i X^i}{E} x_t^i$$

where $e^i X^i$ is steady state direct emissions of sector i (which we calibrate). Note that in principle, the log-deviation of emissions can be written solely as a function of aggregate consumption c_t and relative prices $\{s^i\}_{i=1}^n$.

C.3 Effects of a permanent tax increase in the nonlinear model

Next, we describe how to compute the new steady state following a permanent increase in carbon taxes. Recall that we have

$$\widetilde{M}_{t}^{i} = \frac{1}{A_{t}^{i}} \left[\alpha_{i} \left(W_{t} \right)^{-(\eta - 1)} + (1 - \alpha_{i}) \left(P_{t}^{I,i} \right)^{-(\eta - 1)} \right]^{-\frac{1}{\eta - 1}}$$

$$P_{t}^{I,i} = \left[\varsigma_{i} \left(P_{t}^{E,i} \right)^{-(\nu - 1)} + (1 - \varsigma_{i}) \left(P_{t}^{N,i} \right)^{-(\nu - 1)} \right]^{-\frac{1}{\nu - 1}}$$

$$P_{t}^{E,i} = \left[\sum_{j} \omega_{ij}^{E} \left(P_{t}^{j} \right)^{-(\xi - 1)} \right]^{-\frac{1}{\xi - 1}}$$

$$P_{t}^{N,i} = \left[\sum_{j} \omega_{ij}^{N} \left(P_{t}^{j} \right)^{-(\xi - 1)} \right]^{-\frac{1}{\xi - 1}}$$

In zero-inflation steady state, we have $P^i = \mu^i M^i = \mu_i (\widetilde{M}^i + \mathcal{T}e_i)$ for every sector, i.e. (dividing by the CPI P to get relative prices and using W/P = bC):

$$\frac{S^{i}}{\mu^{i}} = \frac{1}{A^{i}} \left[\alpha_{i} (bC)^{-(\eta - 1)} + (1 - \alpha_{i}) \left(S^{I,i} \right)^{-(\eta - 1)} \right]^{-\frac{1}{\eta - 1}} + \tau e_{i}$$

$$= \frac{1}{A^{i}} \left[\alpha_{i} (bC)^{-(\eta - 1)} + (1 - \alpha_{i}) \left\{ \varsigma_{i} \left(\sum_{j} \omega_{ij}^{E} \left(S^{j} \right)^{-(\xi - 1)} \right)^{\frac{\nu - 1}{\xi - 1}} + (1 - \varsigma_{i}) \left(\sum_{j} \omega_{ij}^{N} \left(S^{j} \right)^{-(\xi - 1)} \right)^{\frac{\nu - 1}{\xi - 1}} \right\}^{\frac{\eta - 1}{\nu - 1}} \right]^{-\frac{1}{\eta - 1}}$$

$$+ \tau e_{i}$$

The definition of the CPI implies that

$$1 = \left[\sum_{i=1}^{n} \gamma_i(S^i)^{-(\zeta-1)} \right]^{-\frac{1}{\zeta-1}}$$

Given parameters, this is a system of n+1 equations in n+1 unknowns. In what follows, we use hats to denote gross percentage changes relative to some initial steady state: $S^i = \overline{S}^i \hat{S}^i$, etc.

We can now rewrite the system of equations in terms of the n+1 variables \hat{C} , $\{\hat{S}_i\}$: we can solve for these percentage changes without having to calibrate variables such as productivity, relative prices, etc. in the initial steady state. To simplify notation, we will also work with the variables $\{\hat{S}^{E,i}, \hat{S}^{N,i}, \hat{S}^{I,i}\}$, which are known functions of \hat{S}_i . First, we have

$$\begin{split} & \frac{\overline{S}^{i}\widehat{S}^{i}}{\mu^{i}} = \frac{1}{A^{i}} \left[\alpha_{i} \left(b\overline{C}\widehat{C} \right)^{-(\eta-1)} + (1 - \alpha_{i}) \left(\overline{S}^{I,i} \widehat{S}^{I,i} \right)^{-(\eta-1)} \right]^{-\frac{1}{\eta-1}} + \tau e_{i} \\ & \widehat{S}^{i} = \left[\alpha_{i} \left(\frac{b\overline{C}}{A^{i}(\overline{M}^{i}/\overline{P})} \right)^{-(\eta-1)} \left(\widehat{C} \right)^{-(\eta-1)} + (1 - \alpha_{i}) \left(\frac{\overline{S}^{I,i}}{A^{i}(\overline{M}^{i}/\overline{P})} \right)^{-(\eta-1)} \left(\widehat{S}^{I,i} \right)^{-(\eta-1)} \right]^{-\frac{1}{\eta-1}} + \tau \frac{e_{i}\mu^{i}}{\overline{S}^{i}} \\ & = \frac{\overline{\widetilde{M}}^{i}}{\overline{M}^{i}} \left[\alpha_{i} \left(\frac{b\overline{C}}{A^{i}(\overline{\widetilde{M}}^{i}/\overline{P})} \right)^{-(\eta-1)} \left(\widehat{C} \right)^{-(\eta-1)} + (1 - \alpha_{i}) \left(\frac{\overline{S}^{I,i}}{A^{i}(\overline{\widetilde{M}}^{i}/\overline{P})} \right)^{-(\eta-1)} \left(\widehat{S}^{I,i} \right)^{-(\eta-1)} \right]^{-\frac{1}{\eta-1}} + \tau \frac{e_{i}\mu^{i}}{\overline{S}^{i}} \\ & = (1 - \widetilde{e}^{i}\overline{\tau}) \left[\widetilde{\alpha}_{i} \left(\widehat{C} \right)^{-(\eta-1)} + (1 - \widetilde{\alpha}_{i}) \left(\widehat{S}^{I,i} \right)^{-(\eta-1)} \right]^{-\frac{1}{\eta-1}} + \tau \widetilde{e}_{i} \end{split}$$

Next,

$$\widehat{S}^{I,i} = \left[\varsigma_i \left(\frac{\overline{S}^{E,i}}{\overline{S}^{I,i}} \right)^{-(\nu-1)} \left(\widehat{S}^{E,i} \right)^{-(\nu-1)} + (1 - \varsigma_i) \left(\frac{\overline{S}^{N,i}}{\overline{S}^{I,i}} \right)^{-(\nu-1)} \left(\widehat{S}^{N,i} \right)^{-(\nu-1)} \right]^{-\frac{1}{\nu-1}}$$

$$= \left[\widetilde{\varsigma}_i \left(\widehat{S}^{E,i} \right)^{-(\nu-1)} + (1 - \widetilde{\varsigma}_i) \left(\widehat{S}^{N,i} \right)^{-(\nu-1)} \right]^{-\frac{1}{\nu-1}}$$

Finally, we have

$$\widehat{S}^{E,i} = \left[\sum_{j} \widetilde{\omega}_{ij}^{E} \left(\widehat{S}^{j} \right)^{-(\xi-1)} \right]^{-\frac{1}{\xi-1}}$$

$$\widehat{S}^{N,i} = \left[\sum_{j} \widetilde{\omega}_{ij}^{N} \left(\widehat{S}^{j} \right)^{-(\xi-1)} \right]^{-\frac{1}{\xi-1}}$$

Thus, we can write the first n equations as

$$\frac{\widehat{S}^{i}}{1 - \widetilde{e}^{i}\overline{\tau}} = \left[\widetilde{\alpha}_{i} \left(\widehat{C} \right)^{-(\eta - 1)} + (1 - \widetilde{\alpha}_{i}) \left\{ \widetilde{\varsigma}_{i} \left(\sum_{j} \widetilde{\omega}_{ij}^{E} \left(\widehat{S}^{j} \right)^{1 - \xi} \right)^{\frac{\nu - 1}{\xi - 1}} + (1 - \widetilde{\varsigma}_{i}) \left(\sum_{j} \widetilde{\omega}_{ij}^{N} \left(\widehat{S}^{j} \right)^{1 - \xi} \right)^{\frac{\nu - 1}{\xi - 1}} \right\}^{\frac{\eta - 1}{1 - \eta}} + \frac{\tau \widetilde{e}_{i}}{1 - \widetilde{e}^{i}\overline{\tau}}$$

where $\widetilde{\alpha}_i := \frac{\overline{WL}^i}{\overline{\widetilde{M}}^i \overline{X}^i} = \alpha_i \left(\frac{\overline{w}}{\overline{A^i \widetilde{M}}^i / \overline{P}} \right)^{-(\eta - 1)}$ is the labor share of pretax marginal cost for sector i in

the initial steady state;
$$1 - \widetilde{\alpha}_i := \frac{\overline{P^{I,i}}\overline{I}^i}{\overline{M}^i\overline{X}^i} = \left(\frac{\overline{S}^{I,i}}{A^i\overline{\widetilde{M}}^i/\overline{P}}\right)^{-(\eta-1)} (1 - \alpha_i)$$
 is the share of intermediate

inputs in pretax marginal costs (note that the two shares sum to 1); $\tilde{e}_i = \frac{e_i}{\overline{M^i/P}}$ is sector *i*'s direct emissions, relative to total marginal costs; and the other reduced-form parameters with tildes are defined as above. Similarly, the the (n+1)th equation becomes:

$$1 = \left[\sum_{i=1}^{n} \widetilde{\gamma}^{i} (\hat{S}^{i})^{-(\zeta-1)}\right]^{-\frac{1}{\zeta-1}}$$

where $\widetilde{\gamma}_i = \gamma_i(\overline{S}^i)^{-(\zeta-1)}$ is the consumption share of sector i in the initial steady state. So, in order to solve this system of n+1 equations in n+1 unknowns (the percentage change in each relative price and aggregate consumption), we need the same information that we already used to solve the linearized model. Given τ and the initial shares, the percentage changes $\{\widehat{S}^i\}$, \widehat{C} can be solved for without needing to solve for all variables or impose normalizations.

Having solved for the new steady state, we will linearize around the new steady state to compute the transition. In order to linearize around the new steady state, we need to recompute all the share parameters (reduced form parameters with tildes). Note first that

$$\begin{split} \frac{\widetilde{M}^{i}}{P} &= \frac{M^{i}}{P} - \tau e_{i} \\ &= \frac{\overline{M}^{i}}{\overline{P}} \widehat{S}^{i} - \tau e_{i} \\ \frac{\widetilde{M}^{i}/P}{\overline{\overline{M}^{i}/\overline{P}}} &= \frac{\overline{M}^{i}/\overline{P}}{\overline{\overline{M}^{i}/\overline{P}}} \widehat{S}^{i} - \tau \frac{e_{i}}{\overline{\overline{M}^{i}/\overline{P}}} \\ &= \frac{\overline{M}^{i}/\overline{P}}{\overline{M}^{i}/\overline{P} - \overline{\tau} e_{i}} \widehat{S}^{i} - \frac{\overline{M}^{i}/\overline{P}}{\overline{M}^{i}/\overline{P} - \overline{\tau} e_{i}} \tau \frac{e_{i}}{\overline{M}^{i}/\overline{P}} \\ &= \frac{1}{1 - \widetilde{e}_{i}\overline{\tau}} \widehat{S}^{i} - \tau \frac{\widetilde{e}_{i}}{1 - \widetilde{e}_{i}\overline{\tau}} \end{split}$$

In the special case where the initial steady state features no carbon tax ($\bar{\tau} = 0$), this becomes

$$\frac{\widetilde{M}^i/P}{\overline{\widetilde{M}}^i/\overline{P}} = \widehat{S}^i - \tau \widetilde{e}_i$$

Thus in the general case, we have

$$\begin{split} \widetilde{\alpha}_{i}^{n} &= \widetilde{\alpha}_{i} \left(\hat{C} \right)^{-(\eta - 1)} \left(\frac{1}{1 - \widetilde{e}_{i} \overline{\tau}} \widehat{S}^{i} - \tau \frac{\widetilde{e}_{i}}{1 - \widetilde{e}_{i} \overline{\tau}} \right)^{\eta - 1} \\ 1 - \widetilde{\alpha}_{i}^{n} &= (1 - \widetilde{\alpha}_{i}) \left(\sum_{j} \widetilde{\omega}_{ij} (\hat{S}^{j})^{-(\xi - 1)} \right)^{\frac{\eta - 1}{\xi - 1}} \left(\frac{1}{1 - \widetilde{e}_{i} \overline{\tau}} \widehat{S}^{i} - \tau \frac{\widetilde{e}_{i}}{1 - \widetilde{e}_{i} \overline{\tau}} \right)^{\eta - 1} \\ \widetilde{\zeta}_{i}^{n} &= \widetilde{\zeta}_{i} \left(\frac{\widehat{S}^{E, i}}{\widehat{S}^{I, i}} \right)^{1 - \nu} \\ 1 - \widetilde{\zeta}_{i}^{n} &= (1 - \widetilde{\zeta}_{i}) \left(\frac{\widehat{S}^{N, i}}{\widehat{S}^{I, i}} \right)^{1 - \nu} \\ \widetilde{\omega}_{ij}^{E, n} &= \widetilde{\omega}_{ij}^{E, n} \left(\frac{\widehat{S}^{j}}{\widehat{S}^{E, i}} \right)^{1 - \xi} \\ \widetilde{\omega}_{ij}^{N, n} &= \widetilde{\omega}_{ij}^{N, n} \left(\frac{\widehat{S}^{j}}{\widehat{S}^{N, i}} \right)^{1 - \xi} \\ \widetilde{e}_{i}^{n} &= \frac{e_{i}}{M^{i}/P} = \frac{\widetilde{e}_{i}}{\widehat{S}^{i}} \end{split}$$

It is straightforward to verify that the objects we have called $\widetilde{\alpha}_i^n$ and $1-\widetilde{\alpha}_i^n$ sum to 1. Their sum is

$$\begin{split} \widetilde{\alpha}_{i} \left(\hat{C} \right)^{-(\eta - 1)} \left(\frac{1}{1 - \widetilde{e}_{i} \overline{\tau}} \widehat{S}^{i} - \tau \frac{\widetilde{e}_{i}}{1 - \widetilde{e}_{i} \overline{\tau}} \right)^{\eta - 1} + (1 - \widetilde{\alpha}_{i}) \left(\sum_{j} \widetilde{\omega}_{ij} (\hat{S}^{j})^{-(\xi - 1)} \right)^{\frac{\eta - 1}{\xi - 1}} \left(\frac{1}{1 - \widetilde{e}_{i} \overline{\tau}} \widehat{S}^{i} - \tau \frac{\widetilde{e}_{i}}{1 - \widetilde{e}_{i} \overline{\tau}} \right)^{\eta - 1} \\ &= \left(\frac{\widehat{S}^{i} - \tau \widetilde{e}_{i}}{1 - \widetilde{e}_{i} \overline{\tau}} \right)^{-(\eta - 1)} \left(\frac{1}{1 - \widetilde{e}_{i} \overline{\tau}} \widehat{S}^{i} - \tau \frac{\widetilde{e}_{i}}{1 - \widetilde{e}_{i} \overline{\tau}} \right)^{\eta - 1} = 1 \\ \widetilde{\gamma}_{i}^{n} &= \frac{\widetilde{\gamma}_{i} (\widehat{S}^{i})^{1 - \zeta}}{\sum_{j} \widetilde{\gamma}_{j} (\widehat{S}^{j})^{1 - \zeta}} \end{split}$$

It is also straightforward to verify that the objects we have called $\widetilde{\varsigma}_i^n$ and $1 - \widetilde{\varsigma}_i^n$ sum to 1 and that $\sum_i \widetilde{\omega}_{ij}^{E,n} = \sum_i \widetilde{\omega}_{ij}^{N,n} = 1.$

When log-linearizing around the new steady state and computing dynamics, we need to specify initial conditions for endogenous state variables (relative prices and real wages), expressed as log-deviations relative to the new steady state. These is simply

$$s_{-1}^i = -\ln \hat{S}^i, \ \omega_{-1}^i = -\ln \hat{C}$$

Finally, we solve for the new steady state values of gross sectoral output and average emissions. It is convenient to treat energy and non-energy sectors separately. Sector *i*'s intermediate demand for an energy good $j \in \mathcal{E}$ satisfies

$$\frac{S_{j}X^{ij}}{(\widetilde{M}^{i}/P)X^{i}} = (1 - \widetilde{\alpha}_{i}^{n})\widetilde{\varsigma}_{i}^{n}\widetilde{\omega}_{ij}^{E,n}$$

$$\frac{X^{ij}}{X^{i}} = (1 - \widetilde{\alpha}_{i}^{n})\widetilde{\varsigma}_{i}^{n}\widetilde{\omega}_{ij}^{E,n}\frac{(M^{i}/P - \tau e_{i})}{S_{j}} = (1 - \widetilde{\alpha}_{i}^{n})\widetilde{\varsigma}_{i}^{n}\widetilde{\omega}_{ij}^{E,n}(1 - \tau \widetilde{e}_{i}^{n})\frac{\overline{M^{i}/P}}{\overline{S}^{j}}\frac{\widehat{S}^{i}}{\widehat{S}^{j}}$$

Thus, for an energy sector $i \in \mathcal{E}$, using market clearing we have

$$C^{i} + \sum_{j} X^{ji} = X^{i}$$

$$\overline{C}^{i} \widehat{C}(\widehat{S}^{i})^{-\zeta} + \sum_{j=1}^{n} (1 - \widetilde{\alpha}_{j}^{n}) \widetilde{\varsigma}_{j}^{n} \widetilde{\omega}_{ji}^{E,n} (1 - \tau \widetilde{e}_{j}^{n}) \frac{\overline{M^{j}/P}}{\overline{S}^{i}} \frac{\widehat{S}^{j}}{\widehat{S}^{i}} \overline{X}^{j} \widehat{X}^{j} = \overline{X}^{i} \widehat{X}^{i}$$

$$\frac{\overline{C}^{i}}{\overline{X}^{i}} \widehat{C}(\widehat{S}^{i})^{-\zeta} + \sum_{j=1}^{n} (1 - \widetilde{\alpha}_{j}^{n}) \widetilde{\varsigma}_{j}^{n} \widetilde{\omega}_{ji}^{E,n} (1 - \tau \widetilde{e}_{j}^{n}) \frac{\overline{M^{j}/P}}{\overline{S}^{j}} \frac{\overline{S}^{j}}{\overline{S}^{i} \overline{X}^{j}} \frac{\widehat{S}^{j}}{\widehat{S}^{i}} \widehat{X}^{j} = \widehat{X}^{i}$$

Similarly, for a non-energy sector $i \in \mathcal{N}$ we have

$$\frac{\overline{C}^i}{\overline{X}^i}\widehat{C}(\widehat{S}^i)^{-\zeta} + \sum_{j=1}^n (1 - \widetilde{\alpha}_j^n)(1 - \widetilde{\varsigma}_j^n)\widetilde{\omega}_{ji}^{N,n}(1 - \tau \widetilde{e}_j^n) \frac{\overline{M^j/P}}{\overline{S}^j} \frac{\overline{S}^j}{\overline{S}^i \overline{X}^i} \frac{\widehat{S}^j}{\widehat{S}^i} \widehat{X}^j = \widehat{X}^i$$

This a linear system in $\{\hat{X}^i\}$, which we can invert to solve for $\{\hat{X}^i\}$. The change in emissions is then

$$\hat{E} = \sum_{i} \frac{e_{i} \overline{X}^{i}}{\overline{E}} \hat{X}^{i}$$

i.e. we just need to weight the proportional change in each sector's gross output by that sector's initial share of total emissions.

Appendix D Appendix Tables and Figures

 τ_{\star} (\$/metric ton CO2)

Figure A1. Dynamics of the carbon tax, flexible price consumption/output, and emissions.

Notes: Left panel shows τ_t , the level of the carbon tax in dollars per metric ton of CO2 emissions; middle and right panels show flexible-price consumption and emissions, respectively, as $100 \times \log$ -deviations from the new steady state.

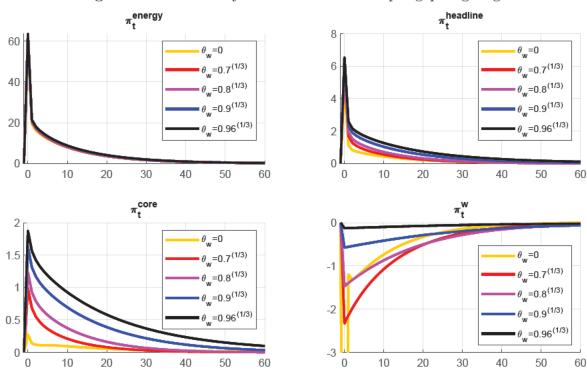


Figure A2. Inflation dynamics under strict output gap targeting

Notes: All lines show annualized log-deviations of each variable relative to the new steady state, i.e. we plot $1200 \times$ the log deviation.

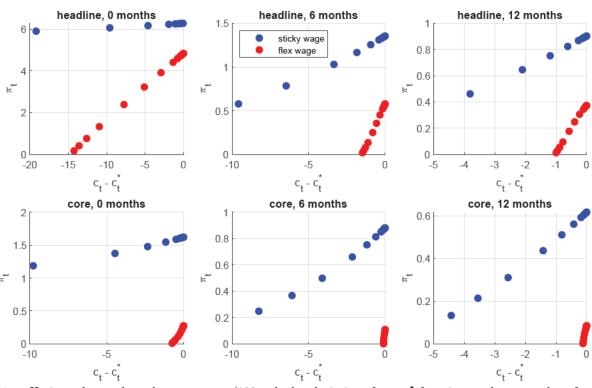


Figure A3. Inflation/output tradeoff at different horizons

Notes: Horizontal axes show the output gap (100× the log-deviation of $c_t - c_t^*$ from its steady state value of zero) in the period the tax is announced (left panels), 6 months after (middle panels) and 12 months after (right panels). Vertical axes show annualized month-over-month inflation at the same horizons (i.e.1200× the log-deviation of π_t from its steady state value of zero). The top row panel shows headline inflation and the output gap, under monetary policy rules of the form $\alpha \pi_t + (1 - \alpha)(c_t - c_t^*) = 0$, for various values of α (strict output gap targeting corresponds to $\alpha = 0$). Blue dots show outcomes under sticky wages ($\theta_w = 0.9^{1/3}$); red dots show flexible wages ($\theta_w = 0$). The bottom row shows core inflation and the output gap, and considers rules which put weight α on core inflation, $1 - \alpha$ on the output gap and no weight on headline inflation.

Table A1. Mean price change frequency of a good in a given sector and CO2 emissions/value added across 396 sectors in USA in 2012

| Sector | Code | CO2/VA | Price Δ Frequency |
|---|--------|--------|--------------------------|
| Cement manufacturing | 327310 | 44.180 | 0.184 |
| Lime and gypsum product manufacturing | 327400 | 36.810 | 0.649 |
| Secondary smelting and alloying of aluminum | 331314 | 32.718 | 0.658 |
| Pulp mills | 322110 | 20.248 | 0.484 |
| Other petroleum and coal products manufacturing | 324190 | 18.580 | 0.439 |
| Alumina refining and primary aluminum production | 331313 | 14.633 | 0.942 |
| Ground or treated mineral and earth manufacturing | 327992 | 13.230 | 0.143 |
| Mineral wool manufacturing | 327993 | 9.107 | 0.341 |
| Wet corn milling | 311221 | 8.566 | 0.392 |
| Poultry and egg production | 112300 | 8.292 | 0.749 |
| Asphalt paving mixture and block manufacturing | 324121 | 8.079 | 0.547 |
| Coal mining | 212100 | 7.901 | 0.191 |
| Clay product and refractory manufacturing | 327100 | 7.137 | 0.070 |
| Miscellaneous nonmetallic mineral products | 327999 | 5.951 | 0.057 |
| Petroleum refineries | 324110 | 5.739 | 0.982 |
| Industrial gas manufacturing | 325120 | 5.135 | 0.223 |
| Natural gas distribution | 221200 | 4.472 | 0.610 |
| Textile and fabric finishing and fabric coating mills | 313300 | 4.106 | 0.128 |
| Paperboard mills | 322130 | 4.063 | 0.238 |
| Iron and steel mills and ferroalloy manufacturing | 331110 | 3.373 | 0.271 |
| Seafood product preparation and packaging | 311700 | 3.319 | 0.386 |
| Concrete pipe, brick, and block manufacturing | 327330 | 3.256 | 0.070 |
| All other converted paper product manufacturing | 322299 | 3.125 | 0.073 |
| Asphalt shingle and coating materials manufacturing | 324122 | 3.044 | 0.413 |
| Glass and glass product manufacturing | 327200 | 2.830 | 0.057 |
| Gas extraction | 211000 | 2.477 | 0.994 |
| Cut stone and stone product manufacturing | 327991 | 2.317 | 0.032 |
| Printing ink manufacturing | 325910 | 1.942 | 0.088 |
| Nonferrous metal foundries | 331520 | 1.859 | 0.092 |
| Petrochemical manufacturing | 325110 | 1.854 | 0.223 |
| Other concrete product manufacturing | 327390 | 1.801 | 0.072 |
| Electric lamp bulb and part manufacturing | 335110 | 1.742 | 0.126 |
| Ready-mix concrete manufacturing | 327320 | 1.735 | 0.113 |
| Other nonmetallic mineral mining and quarrying | 2123A0 | 1.681 | 0.049 |
| Other household nonupholstered furniture | 33712N | 1.508 | 0.049 |
| Synthetic dye and pigment manufacturing | 325130 | 1.482 | 0.223 |
| Paper mills | 322120 | 1.464 | 0.282 |
| Abrasive product manufacturing | 327910 | 1.446 | 0.031 |

Table A1 – continued from previous page

| Sector | Code | CO2/VA | Price Δ Frequency |
|--|--------|--------|--------------------------|
| Aluminum product manufacturing from purchased aluminum | 33131B | 1.365 | 0.374 |
| Polystyrene foam product manufacturing | 326140 | 1.328 | 0.051 |
| Other Basic Inorganic Chemical Manufacturing | 325180 | 1.299 | 0.139 |
| Fats and oils refining and blending | 311225 | 1.256 | 0.533 |
| Fabric mills | 313200 | 1.228 | 0.076 |
| Soybean and other oilseed processing | 311224 | 1.058 | 0.787 |
| Urethane and other foam product (except polystyrene) manufacturing | 326150 | 1.032 | 0.032 |
| Copper, nickel, lead, and zinc mining | 212230 | 1.011 | 0.096 |
| Laminated plastics plate, sheet (except packaging), and shape manufacturing | 326130 | 1.000 | 0.053 |
| Fiber, yarn, and thread mills | 313100 | 0.980 | 0.065 |
| Dry, condensed, and evaporated dairy product manufacturing | 311514 | 0.958 | 0.417 |
| Breakfast cereal manufacturing | 311230 | 0.923 | 0.495 |
| Sugar and confectionery product manufacturing | 311300 | 0.888 | 0.177 |
| Ferrous metal foundries | 331510 | 0.884 | 0.057 |
| Flour milling and malt manufacturing | 311210 | 0.877 | 0.269 |
| Ice cream and frozen dessert manufacturing | 311520 | 0.859 | 0.149 |
| Nonferrous Metal (except Aluminum) Smelting and Refining | 331410 | 0.836 | 0.655 |
| Nonferrous metal (except copper and aluminum) rolling, drawing, extruding and alloying | 331490 | 0.831 | 0.485 |
| Oil extraction | 211000 | 0.826 | 0.994 |
| Copper rolling, drawing, extruding and alloying | 331420 | 0.813 | 0.348 |
| Coffee and tea manufacturing | 311920 | 0.755 | 0.087 |
| Rubber and plastics hoses and belting manufacturing | 326220 | 0.716 | 0.032 |
| Iron, gold, silver, and other metal ore mining | 2122A0 | 0.702 | 0.096 |
| Stone mining and quarrying | 212310 | 0.690 | 0.049 |
| Other basic organic chemical manufacturing | 325190 | 0.640 | 0.307 |
| Synthetic rubber and artificial and synthetic fibers and filaments manufacturing | 3252A0 | 0.594 | 0.126 |
| Carbon and graphite product manufacturing | 335991 | 0.583 | 0.055 |
| Fruit and vegetable canning, pickling, and drying | 311420 | 0.583 | 0.064 |
| Dog and cat food manufacturing | 311111 | 0.581 | 0.699 |
| Paper Bag and Coated and Treated Paper Manufacturing | 322220 | 0.566 | 0.066 |
| All other food manufacturing | 311990 | 0.561 | 0.326 |
| Frozen food manufacturing | 311410 | 0.501 | 0.194 |
| Adhesive manufacturing | 325520 | 0.486 | 0.098 |
| Other animal food manufacturing | 311119 | 0.477 | 0.524 |
| Cheese manufacturing | 311513 | 0.457 | 0.474 |
| Steel product manufacturing from purchased steel | 331200 | 0.443 | 0.075 |
| Water transportation | 483000 | 0.432 | 0.298 |
| Electric power generation, transmission, and distribution | 221100 | 0.410 | 0.291 |
| Snack food manufacturing | 311910 | 0.408 | 0.127 |

Table A1 – continued from previous page

| Table A1 – continued from previous page | | | |
|---|-----------------------|--------|--------------------------|
| Sector | Code | CO2/VA | Price Δ Frequency |
| Nonupholstered wood household furniture manufacturing | 337122 | 0.402 | 0.065 |
| Sanitary paper product manufacturing | 322291 | 0.399 | 0.073 |
| Flavoring syrup and concentrate manufacturing | 311930 | 0.381 | 0.057 |
| Carpet and rug mills | 314110 | 0.377 | 0.064 |
| Cookie, cracker, pasta, and tortilla manufacturing | 3118A0 | 0.371 | 0.209 |
| Stationery product manufacturing | 322230 | 0.346 | 0.073 |
| Breweries | 312120 | 0.338 | 0.066 |
| Tire manufacturing | 326210 | 0.337 | 0.069 |
| Fluid milk and butter manufacturing | 31151A | 0.330 | 0.785 |
| Other commercial and service industry machinery manufacturing | 333318 | 0.312 | 0.039 |
| Seasoning and dressing manufacturing | 311940 | 0.276 | 0.039 |
| Other rubber product manufacturing | 326290 | 0.276 | 0.032 |
| All other chemical product and preparation manufacturing | 3259A0 | 0.265 | 0.088 |
| Power, distribution, and specialty transformer manufacturing | 335311 | 0.251 | 0.047 |
| Fishing, hunting and trapping | 114000 | 0.244 | 0.882 |
| Paperboard container manufacturing | 322210 | 0.235 | 0.073 |
| Wineries | 312130 | 0.232 | 0.066 |
| Distilleries | 312140 | 0.223 | 0.066 |
| Optical instrument and lens manufacturing | 333314 | 0.209 | 0.037 |
| Fertilizer manufacturing | 325310 | 0.208 | 0.416 |
| Pesticide and other agricultural chemical manufacturing | 325320 | 0.195 | 0.042 |
| Soft drink and ice manufacturing | 312110 | 0.195 | 0.263 |
| Industrial and commercial fan and blower and air purification equipment manufacturing | 333413 | 0.190 | 0.041 |
| Plastics pipe, pipe fitting, and unlaminated profile shape manufacturing | 326120 | 0.190 | 0.161 |
| Showcase, partition, shelving, and locker manufacturing | 337215 | 0.175 | 0.038 |
| Other textile product mills | 314900 | 0.161 | 0.062 |
| Propulsion units and parts for space vehicles and guided missiles | 33641A | 0.158 | 0.060 |
| Greenhouse, nursery, and floriculture production | 111400 | 0.156 | 0.887 |
| Veneer, plywood, and engineered wood product manufacturing | 321200 | 0.152 | 0.251 |
| Plastics material and resin manufacturing | 325211 | 0.150 | 0.317 |
| Bread and bakery product manufacturing | 311810 | 0.149 | 0.041 |
| Storage battery manufacturing | 335911 | 0.144 | 0.055 |
| Poultry processing | 311615 | 0.142 | 0.497 |
| All other miscellaneous electrical equipment and component manufacturing | 335999 | 0.122 | 0.031 |
| Other crop farming | 111900 | 0.120 | 0.965 |
| Medicinal and botanical manufacturing | 325411 | 0.119 | 0.037 |
| Plastics bottle manufacturing | 326160 | 0.118 | 0.087 |
| Plastics packaging materials and unlaminated film and sheet manufacturing | 326110 | 0.114 | 0.184 |
| Machine tool manufacturing | 333517 | 0.111 | 0.029 |
| - | · · | | tinued on part name |

Table A1 – continued from previous page

| Sector | Code | CO2/VA | Price Δ Frequency |
|---|--------|--------|--------------------------|
| Dairy cattle and milk production | 112120 | 0.103 | 0.948 |
| Travel trailer and camper manufacturing | 336214 | 0.099 | 0.086 |
| Vegetable and melon farming | 111200 | 0.099 | 0.875 |
| Rail transportation | 482000 | 0.094 | 0.241 |
| Forestry and logging | 113000 | 0.094 | 0.882 |
| Photographic and photocopying equipment manufacturing | 333316 | 0.092 | 0.038 |
| Fabricated pipe and pipe fitting manufacturing | 332996 | 0.090 | 0.066 |
| Couriers and messengers | 492000 | 0.090 | 0.208 |
| Animal (except poultry) slaughtering, rendering, and processing | 31161A | 0.088 | 0.516 |
| Sawmills and wood preservation | 321100 | 0.082 | 0.324 |
| All other wood product manufacturing | 3219A0 | 0.081 | 0.109 |
| Water, sewage and other systems | 221300 | 0.079 | 0.107 |
| Motor vehicle body manufacturing | 336211 | 0.076 | 0.057 |
| Air transportation | 481000 | 0.076 | 0.598 |
| Pipeline transportation | 486000 | 0.067 | 0.262 |
| Air and gas compressor manufacturing | 333912 | 0.066 | 0.050 |
| Institutional furniture manufacturing | 337127 | 0.066 | 0.046 |
| Paint and coating manufacturing | 325510 | 0.063 | 0.098 |
| Wood kitchen cabinet and countertop manufacturing | 337110 | 0.062 | 0.052 |
| Grain farming | 1111B0 | 0.058 | 0.858 |
| Fluid power process machinery | 33399B | 0.057 | 0.043 |
| All other forging, stamping, and sintering | 33211A | 0.057 | 0.074 |
| In-vitro diagnostic substance manufacturing | 325413 | 0.056 | 0.055 |
| Lighting fixture manufacturing | 335120 | 0.056 | 0.025 |
| Dry-cleaning and laundry services | 812300 | 0.053 | 0.033 |
| Office supplies (except paper) manufacturing | 339940 | 0.050 | 0.017 |
| Motor vehicle steering, suspension component (except spring), and brake systems manufacturing | 3363A0 | 0.050 | 0.109 |
| Switchgear and switchboard apparatus manufacturing | 335313 | 0.050 | 0.038 |
| Spring and wire product manufacturing | 332600 | 0.047 | 0.038 |
| Truck transportation | 484000 | 0.045 | 0.107 |
| Railroad rolling stock manufacturing | 336500 | 0.044 | 0.074 |
| Transit and ground passenger transportation | 485000 | 0.042 | 0.066 |
| Soap and cleaning compound manufacturing | 325610 | 0.041 | 0.066 |
| Coating, engraving, heat treating and allied activities | 332800 | 0.040 | 0.054 |
| Support activities for printing | 323120 | 0.037 | 0.003 |
| Office furniture and custom architectural woodwork and millwork manufacturing | 33721A | 0.036 | 0.038 |
| Oilseed farming | 1111A0 | 0.036 | 0.946 |
| Religious organizations | 813100 | 0.035 | 0.079 |
| Museums, historical sites, zoos, and parks | 712000 | 0.034 | 0.079 |

Table A1 – continued from previous page

| Sector | Code | CO2/VA | Price Δ Frequency |
|--|--------|--------|--------------------------|
| Other industrial machinery manufacturing | 33329A | 0.033 | 0.039 |
| Millwork | 321910 | 0.031 | 0.043 |
| Beef cattle ranching and farming, including feedlots and dual-purpose ranching and farming | 1121A0 | 0.031 | 0.969 |
| Motor home manufacturing | 336213 | 0.030 | 0.068 |
| Wiring device manufacturing | 335930 | 0.030 | 0.050 |
| Motor vehicle electrical and electronic equipment manufacturing | 336320 | 0.029 | 0.109 |
| Heating equipment (except warm air furnaces) manufacturing | 333414 | 0.029 | 0.055 |
| Motor vehicle gasoline engine and engine parts manufacturing | 336310 | 0.029 | 0.109 |
| Custom roll forming | 332114 | 0.028 | 0.056 |
| Drilling oil and gas wells | 213111 | 0.028 | 0.321 |
| Postal service | 491000 | 0.028 | 0.165 |
| Other plastics product manufacturing | 326190 | 0.028 | 0.043 |
| Tobacco product manufacturing | 312200 | 0.028 | 0.100 |
| Fruit and tree nut farming | 111300 | 0.027 | 0.792 |
| Automotive equipment rental and leasing | 532100 | 0.027 | 0.494 |
| Speed changer, industrial high-speed drive, and gear manufacturing | 333612 | 0.026 | 0.055 |
| Ship building and repairing | 336611 | 0.026 | 0.075 |
| Leather and allied product manufacturing | 316000 | 0.026 | 0.117 |
| Doll, toy, and game manufacturing | 339930 | 0.025 | 0.027 |
| Other nonresidential structures | 2332D0 | 0.025 | 0.115 |
| Motor vehicle metal stamping | 336370 | 0.025 | 0.109 |
| Amusement parks and arcades | 713100 | 0.025 | 0.079 |
| Scenic and sightseeing transportation and support activities for transportation | 48A000 | 0.025 | 0.262 |
| Mining and oil and gas field machinery manufacturing | 333130 | 0.024 | 0.038 |
| Turbine and turbine generator set units manufacturing | 333611 | 0.024 | 0.141 |
| Curtain and linen mills | 314120 | 0.023 | 0.021 |
| Industrial process furnace and oven manufacturing | 333994 | 0.022 | 0.034 |
| Residential maintenance and repair | 230302 | 0.022 | 0.115 |
| Motor vehicle transmission and power train parts manufacturing | 336350 | 0.021 | 0.109 |
| Hardware manufacturing | 332500 | 0.021 | 0.067 |
| Elementary and secondary schools | 611100 | 0.021 | 0.054 |
| Other amusement and recreation industries | 713900 | 0.020 | 0.079 |
| Mechanical power transmission equipment manufacturing | 333613 | 0.020 | 0.084 |
| Printing | 323110 | 0.020 | 0.054 |
| Sporting and athletic goods manufacturing | 339920 | 0.019 | 0.047 |
| Transportation structures and highways and streets | 2332C0 | 0.019 | 0.115 |
| Facilities support services | 561200 | 0.019 | 0.068 |
| Manufacturing and reproducing magnetic and optical media | 334610 | 0.019 | 0.039 |
| Manufacturing structures | 233230 | 0.018 | 0.115 |

Table A1 – continued from previous page

| Sector | Code | CO2/VA | Price Δ Frequence |
|--|--------|--------|--------------------------|
| Toilet preparation manufacturing | 325620 | 0.018 | 0.028 |
| Health care structures | 233210 | 0.017 | 0.115 |
| Nonresidential maintenance and repair | 230301 | 0.017 | 0.115 |
| Other general purpose machinery manufacturing | 33399A | 0.017 | 0.036 |
| Waste management and remediation services | 562000 | 0.017 | 0.094 |
| Small electrical appliance manufacturing | 335210 | 0.017 | 0.042 |
| Plumbing fixture fitting and trim manufacturing | 332913 | 0.017 | 0.052 |
| Funds, trusts, and other financial vehicles | 525000 | 0.016 | 0.058 |
| Packaging machinery manufacturing | 333993 | 0.016 | 0.040 |
| Household cooking appliance manufacturing | 335221 | 0.016 | 0.094 |
| Metal tank (heavy gauge) manufacturing | 332420 | 0.016 | 0.034 |
| Boat building | 336612 | 0.016 | 0.110 |
| Industrial mold manufacturing | 333511 | 0.016 | 0.032 |
| Power-driven handtool manufacturing | 333991 | 0.016 | 0.074 |
| Other fabricated metal manufacturing | 332999 | 0.015 | 0.095 |
| Dental laboratories | 339116 | 0.015 | 0.064 |
| Dental equipment and supplies manufacturing | 339114 | 0.015 | 0.084 |
| News syndicates, libraries, archives and all other information services | 5191A0 | 0.014 | 0.022 |
| Other aircraft parts and auxiliary equipment manufacturing | 336413 | 0.014 | 0.046 |
| Cutlery and handtool manufacturing | 332200 | 0.013 | 0.036 |
| Civic, social, professional, and similar organizations | 813B00 | 0.013 | 0.079 |
| Computer terminals and other computer peripheral equipment manufacturing | 334118 | 0.013 | 0.034 |
| Other Motor Vehicle Parts Manufacturing | 336390 | 0.013 | 0.109 |
| Sign manufacturing | 339950 | 0.013 | 0.030 |
| Cutting and machine tool accessory, rolling mill, and other metalworking machinery manufacturing | 33351B | 0.012 | 0.035 |
| Office and commercial structures | 2332A0 | 0.012 | 0.115 |
| Truck trailer manufacturing | 336212 | 0.012 | 0.061 |
| Biological product (except diagnostic) manufacturing | 325414 | 0.012 | 0.072 |
| Ball and roller bearing manufacturing | 332991 | 0.012 | 0.038 |
| Other support activities for mining | 21311A | 0.012 | 0.321 |
| Other ambulatory health care services | 621900 | 0.012 | 0.080 |
| Power boiler and heat exchanger manufacturing | 332410 | 0.011 | 0.039 |
| Turned product and screw, nut, and bolt manufacturing | 332720 | 0.011 | 0.020 |
| Metal crown, closure, and other metal stamping (except automotive) | 332119 | 0.011 | 0.038 |
| Other educational services | 611B00 | 0.011 | 0.092 |
| Metal can, box, and other metal container (light gauge) manufacturing | 332430 | 0.011 | 0.081 |
| Other furniture related product manufacturing | 337900 | 0.011 | 0.054 |
| Material handling equipment manufacturing | 333920 | 0.010 | 0.039 |
| Community food, housing, and other relief services, including rehabilitation services | 624A00 | 0.010 | 0.048 |
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|---|--------|--------|--------------------------|
| Sector | Code | CO2/VA | Price Δ Frequency |
| Multifamily residential structures | 233412 | 0.010 | 0.115 |
| Power and communication structures | 233240 | 0.010 | 0.115 |
| Other residential structures | 2334A0 | 0.010 | 0.115 |
| General and consumer goods rental | 532A00 | 0.010 | 0.100 |
| Valve and fittings other than plumbing | 33291A | 0.010 | 0.038 |
| Lawn and garden equipment manufacturing | 333112 | 0.010 | 0.068 |
| Construction machinery manufacturing | 333120 | 0.010 | 0.084 |
| Ornamental and architectural metal products manufacturing | 332320 | 0.009 | 0.076 |
| Commercial and industrial machinery and equipment rental and leasing | 532400 | 0.009 | 0.297 |
| Full-service restaurants | 722110 | 0.009 | 0.051 |
| Animal production, except cattle and poultry and eggs | 112A00 | 0.009 | 0.833 |
| Ammunition, arms, ordnance, and accessories manufacturing | 33299A | 0.009 | 0.064 |
| Other major household appliance manufacturing | 335228 | 0.009 | 0.094 |
| Educational and vocational structures | 233262 | 0.009 | 0.115 |
| Limited-service restaurants | 722211 | 0.008 | 0.119 |
| Plate work and fabricated structural product manufacturing | 332310 | 0.008 | 0.077 |
| Special tool, die, jig, and fixture manufacturing | 333514 | 0.008 | 0.033 |
| Automobile manufacturing | 336111 | 0.008 | 0.273 |
| All other food and drinking places | 722A00 | 0.008 | 0.045 |
| Machine shops | 332710 | 0.008 | 0.051 |
| Military armored vehicle, tank, and tank component manufacturing | 336992 | 0.007 | 0.060 |
| Support activities for agriculture and forestry | 115000 | 0.007 | 0.882 |
| Upholstered household furniture manufacturing | 337121 | 0.007 | 0.046 |
| Residential mental health, substance abuse, and other residential care facilities | 623B00 | 0.007 | 0.057 |
| All other transportation equipment manufacturing | 336999 | 0.007 | 0.060 |
| Ophthalmic goods manufacturing | 339115 | 0.006 | 0.036 |
| Primary battery manufacturing | 335912 | 0.006 | 0.055 |
| Services to buildings and dwellings | 561700 | 0.006 | 0.061 |
| Gambling industries (except casino hotels) | 713200 | 0.006 | 0.079 |
| Communication and energy wire and cable manufacturing | 335920 | 0.005 | 0.048 |
| Other computer related services, including facilities management | 54151A | 0.005 | 0.063 |
| Motor and generator manufacturing | 335312 | 0.005 | 0.056 |
| Semiconductor machinery manufacturing | 333242 | 0.005 | 0.039 |
| Single-family residential structures | 233411 | 0.005 | 0.115 |
| All other miscellaneous manufacturing | 339990 | 0.005 | 0.040 |
| Other electronic component manufacturing | 33441A | 0.004 | 0.047 |
| Motor vehicle seating and interior trim manufacturing | 336360 | 0.004 | 0.109 |
| Photographic services | 541920 | 0.004 | 0.095 |
| Directory, mailing list, and other publishers | 5111A0 | 0.004 | 0.079 |
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Table A1 – continued from previous page

| Sector | Code | CO2/VA | Price Δ Frequency |
|---|--------|--------|--------------------------|
| Satellite, telecommunications resellers, and all other telecommunications | 517A00 | 0.004 | 0.260 |
| Motor vehicle and motor vehicle parts and supplies | 423100 | 0.004 | 0.038 |
| Farm machinery and equipment manufacturing | 333111 | 0.004 | 0.068 |
| Pharmaceutical preparation manufacturing | 325412 | 0.004 | 0.055 |
| Accommodation | 721000 | 0.004 | 0.428 |
| Child day care services | 624400 | 0.004 | 0.069 |
| Individual and family services | 624100 | 0.004 | 0.028 |
| Relay and industrial control manufacturing | 335314 | 0.004 | 0.047 |
| Spectator sports | 711200 | 0.004 | 0.066 |
| Other communications equipment manufacturing | 334290 | 0.004 | 0.047 |
| Grocery and related product wholesalers | 424400 | 0.004 | 0.251 |
| Pump and pumping equipment manufacturing | 33391A | 0.004 | 0.050 |
| Other support services | 561900 | 0.004 | 0.068 |
| Warehousing and storage | 493000 | 0.004 | 0.186 |
| Jewelry and silverware manufacturing | 339910 | 0.003 | 0.020 |
| Motorcycle, bicycle, and parts manufacturing | 336991 | 0.003 | 0.060 |
| Performing arts companies | 711100 | 0.003 | 0.079 |
| Periodical Publishers | 511120 | 0.003 | 0.079 |
| Commercial and industrial machinery and equipment repair and maintenance | 811300 | 0.003 | 0.107 |
| Aircraft engine and engine parts manufacturing | 336412 | 0.003 | 0.066 |
| Other engine equipment manufacturing | 333618 | 0.003 | 0.057 |
| Grantmaking, giving, and social advocacy organizations | 813A00 | 0.003 | 0.079 |
| Industrial process variable instruments manufacturing | 334513 | 0.003 | 0.035 |
| Surgical appliance and supplies manufacturing | 339113 | 0.003 | 0.051 |
| Newspaper publishers | 511110 | 0.002 | 0.079 |
| Air conditioning, refrigeration, and warm air heating equipment manufacturing | 333415 | 0.002 | 0.089 |
| Guided missile and space vehicle manufacturing | 336414 | 0.002 | 0.060 |
| Data processing, hosting, and related services | 518200 | 0.002 | 0.122 |
| Other financial investment activities | 523900 | 0.002 | 0.058 |
| Semiconductor and related device manufacturing | 334413 | 0.002 | 0.047 |
| Medical and diagnostic laboratories | 621500 | 0.002 | 0.080 |
| Business support services | 561400 | 0.002 | 0.068 |
| Other durable goods merchant wholesalers | 423A00 | 0.002 | 0.038 |
| Heavy duty truck manufacturing | 336120 | 0.002 | 0.273 |
| Offices of other health practitioners | 621300 | 0.002 | 0.112 |
| Investigation and security services | 561600 | 0.002 | 0.068 |
| Automatic environmental control manufacturing | 334512 | 0.002 | 0.040 |
| Book publishers | 511130 | 0.002 | 0.079 |
| Nursing and community care facilities | 623A00 | 0.002 | 0.057 |

Table A1 – continued from previous page

| Sector | Code | CO2/VA | Price Δ Frequency |
|--|--------|--------|--------------------------|
| Promoters of performing arts and sports and agents for public figures | 711A00 | 0.002 | 0.079 |
| Building material and garden equipment and supplies dealers | 444000 | 0.002 | 0.199 |
| Apparel manufacturing | 315000 | 0.002 | 0.024 |
| Light truck and utility vehicle manufacturing | 336112 | 0.002 | 0.167 |
| Motor vehicle and parts dealers | 441000 | 0.001 | 0.263 |
| Computer storage device manufacturing | 334112 | 0.001 | 0.106 |
| Specialized design services | 541400 | 0.001 | 0.063 |
| Personal care services | 812100 | 0.001 | 0.031 |
| Machinery, equipment, and supplies | 423800 | 0.001 | 0.038 |
| Other nondurable goods merchant wholesalers | 424A00 | 0.001 | 0.251 |
| Nondepository credit intermediation and related activities | 522A00 | 0.001 | 0.058 |
| Sound recording industries | 512200 | 0.001 | 0.091 |
| Securities and commodity contracts intermediation and brokerage | 523A00 | 0.001 | 0.058 |
| Gasoline stations | 447000 | 0.001 | 0.459 |
| Veterinary services | 541940 | 0.001 | 0.087 |
| Wholesale electronic markets and agents and brokers | 425000 | 0.001 | 0.145 |
| Hospitals | 622000 | 0.001 | 0.063 |
| Household laundry equipment manufacturing | 335224 | 0.001 | 0.094 |
| Architectural, engineering, and related services | 541300 | 0.001 | 0.063 |
| Management of companies and enterprises | 550000 | 0.001 | 0.115 |
| Office administrative services | 561100 | 0.001 | 0.068 |
| Food and beverage stores | 445000 | 0.001 | 0.286 |
| Watch, clock, and other measuring and controlling device manufacturing | 33451A | 0.001 | 0.043 |
| Monetary authorities and depository credit intermediation | 52A000 | 0.001 | 0.035 |
| Offices of dentists | 621200 | 0.001 | 0.100 |
| Surgical and medical instrument manufacturing | 339112 | 0.001 | 0.085 |
| Scientific research and development services | 541700 | 0.001 | 0.063 |
| Junior colleges, colleges, universities, and professional schools | 611A00 | 0.001 | 0.075 |
| Home health care services | 621600 | 0.001 | 0.080 |
| Internet publishing and broadcasting and Web search portals | 519130 | 0.001 | 0.022 |
| Aircraft manufacturing | 336411 | 0.001 | 0.069 |
| Automotive repair and maintenance | 811100 | 0.001 | 0.156 |
| Analytical laboratory instrument manufacturing | 334516 | 0.001 | 0.040 |
| Household refrigerator and home freezer manufacturing | 335222 | 0.001 | 0.094 |
| Printed circuit assembly (electronic assembly) manufacturing | 334418 | 0.001 | 0.047 |
| Irradiation apparatus manufacturing | 334517 | 0.001 | 0.040 |
| Telephone apparatus manufacturing | 334210 | 0.001 | 0.047 |
| All other miscellaneous professional, scientific, and technical services | 5419A0 | 0.001 | 0.091 |
| Personal and household goods repair and maintenance | 811400 | 0.001 | 0.054 |

Table A1 – continued from previous page

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|--|--------|------------|--------------------------|
| Sector | Code | CO2/VA | Price Δ Frequency |
| Outpatient care centers | 621400 | 0.001 | 0.080 |
| Petroleum and petroleum products | 424700 | 0.001 | 0.251 |
| All other retail | 4B0000 | 0.001 | 0.183 |
| Other personal services | 812900 | 0.001 | 0.075 |
| Household appliances and electrical and electronic goods | 423600 | 0.001 | 0.038 |
| Travel arrangement and reservation services | 561500 | 0.001 | 0.076 |
| Computer systems design services | 541512 | 0.001 | 0.063 |
| Environmental and other technical consulting services | 5416A0 | 0.001 | 0.063 |
| Electronic and precision equipment repair and maintenance | 811200 | 0.001 | 0.111 |
| Electronic computer manufacturing | 334111 | 0.001 | 0.299 |
| Software publishers | 511200 | 0.001 | 0.108 |
| Clothing and clothing accessories stores | 448000 | 0.001 | 0.327 |
| Professional and commercial equipment and supplies | 423400 | 0.001 | 0.038 |
| Nonstore retailers | 454000 | 0.000 | 0.530 |
| Wireless telecommunications carriers (except satellite) | 517210 | 0.000 | 0.269 |
| Drugs and druggists' sundries | 424200 | 0.000 | 0.251 |
| Audio and video equipment manufacturing | 334300 | 0.000 | 0.084 |
| Other real estate | 531ORE | 0.000 | 0.297 |
| Totalizing fluid meter and counting device manufacturing | 334514 | 0.000 | 0.033 |
| Motion picture and video industries | 512100 | 0.000 | 0.091 |
| General merchandise stores | 452000 | 0.000 | 0.284 |
| Electromedical and electrotherapeutic apparatus manufacturing | 334510 | 0.000 | 0.055 |
| Health and personal care stores | 446000 | 0.000 | 0.142 |
| Advertising, public relations, and related services | 541800 | 0.000 | 0.063 |
| Accounting, tax preparation, bookkeeping, and payroll services | 541200 | 0.000 | 0.055 |
| Radio and television broadcasting | 515100 | 0.000 | 0.128 |
| Broadcast and wireless communications equipment | 334220 | 0.000 | 0.047 |
| Lessors of nonfinancial intangible assets | 533000 | 0.000 | 0.297 |
| Electricity and signal testing instruments manufacturing | 334515 | 0.000 | 0.024 |
| Management consulting services | 541610 | 0.000 | 0.063 |
| Death care services | 812200 | 0.000 | 0.089 |
| Independent artists, writers, and performers | 711500 | 0.000 | 0.091 |
| Offices of physicians | 621100 | 0.000 | 0.029 |
| Wired telecommunications carriers | 517110 | 0.000 | 0.252 |
| Custom computer programming services | 541511 | 0.000 | 0.063 |
| Search, detection, and navigation instruments manufacturing | 334511 | 0.000 | 0.051 |
| Cable and other subscription programming | 515200 | 0.000 | 0.128 |
| Direct life insurance carriers | 524113 | 0.000 | 0.080 |
| Legal services | 541100 | 0.000 | 0.016 |

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| Sector | Code | CO2/VA | Price Δ Frequency |
|--|--------|--------|--------------------------|
| Insurance agencies, brokerages, and related activities | 524200 | 0.000 | 0.080 |
| Employment services | 561300 | 0.000 | 0.068 |
| Insurance carriers, except direct life | 5241XX | 0.000 | 0.080 |
| Tenant-occupied housing | 531HST | 0.000 | 0.297 |
| Owner-occupied housing | 531HSO | 0.000 | 0.297 |
| Private households | 814000 | 0.000 | 0.079 |
| Customs duties | 4200ID | 0.000 | 0.145 |