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Abstract

I build a tractable random search model with firm dynamics, on-the-job search, and aggregate shocks. Multi-worker firms make recruitment decisions, choose whether to enter or exit the market, and design wage contracts. Tractability is obtained by showing that, under a set of assumptions on the recruitment technology, the decisions of workers and firms depend on the firms' current productivity. I confront the model to salient business cycle moments on the reallocation of workers across the firm productivity distribution derived from firm-level data that the model successfully replicates. I use this framework to quantify the drivers of worker reallocation over the post-war business cycle in Britain.

JEL classification: E3, J63, J64
Keywords: firm dynamics, search, business cycle

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1 Introduction

How do economic fluctuations affect workers? A large literature has documented that recessions coincide with substantial changes in worker flows. Recessions have been shown to markedly increase inflows into unemployment and to decrease the pace at which workers, both employed and unemployed, find new jobs.[1] Given the large degree of firm heterogeneity in the data in terms of productivity, wages, and employment, an important question arising from these regularities is: to what extent do fluctuations in worker flows reallocate workers to better firms?[2] The answer to this question matters for the design of economic policies, such as those that subsidize the search of jobless workers.

In this paper, I develop a rich quantitative framework to measure worker reallocation over the business cycle. The key features of this framework are firm dynamics driven by idiosyncratic productivity shocks, a frictional labor market where workers can search while employed, and aggregate shocks. Conceptually, the model implies that workers have well-defined preferences over firms. In equilibrium, more productive firms deliver better wage contracts, and workers gradually quit to move to these firms over time. I confront the model to salient moments on the reallocation of workers across the firm productivity distribution derived from firm-level data. The model is successful at replicating these moments. I use this framework to quantify the drivers of worker reallocation over the post-war business cycle in Britain

The main features of the model are guided by the stylized facts on the evolution of worker flows over the business cycle and firm heterogeneity. Search frictions in the labor market give rise to transitions in and out of unemployment. With on-the-job search, the model has a counterpart to job-to-job transition flows. These flows are substantial in typical data sets: workers are at least twice as likely to directly transition to another job as to become

[1]See, among others, Blanchard et al. [1990] for unemployment inflows and outflows and Fujita et al. [2020] for US evidence on job transitions. As shown in the paper, these patterns also hold in the British data used to quantify the model.

[2]See, for instance, Syverson [2011] for an analysis of productivity differences. Gibrat [1931] is an early reference on the distribution of firm size.

unemployed.[3] Multi-worker firms are needed to lay a credible foundation for the job ladder at the micro-level, since there is no measure of productivity at the job level in standard data sets. Aggregate shocks, finally, are a pre-requisite to studying business cycles.

A contribution of this paper is to identify a set of model primitives such that the model remains tractable with aggregate shocks. In the model, firms make hiring decisions, choose whether to enter or exit the market, and commit to a state-contingent wage contract. I show that, under specific assumptions on the cost of recruitment, firms' decisions are size-independent and their optimal wage contract is increasing in firm productivity. With these simplifying results, the state-space relevant to firms' decisions reduces to its current productivity, the realization of the aggregate shock, and the employment-weighted distribution of firm productivity. This parsimonious state-space is crucial to the computational tractability of the model. I also show that the optimal wage contract admits a closed-form solution, which implies that wages are well-defined and straightforward to compute in this environment.

These results imply that the preferences of workers over firms map into firm productivity: the job ladder is a productivity ladder.[4] In the model, a natural statistic to summarize the location of workers along the job ladder at any point in time is therefore the employment-weighted distribution of firm productivity. This summary statistic can be further decomposed into a firm component, which summarizes the productivity of active firms at each point in time, and a worker reallocation term, which gives the location of workers on the productivity ladder relative to the set of active firms. This structural decomposition represents a key object of interest, and each of its terms can be quantified within the calibrated model.

Given the close connection between the job ladder and labor productivity at the firm level, I use detailed data on the balance-sheet of British businesses for the period 1997–2018 to discipline the model. I gross up these data into a labor productivity index that

[3]Twice as likely in the US. In the British data I use in the paper, this ratio is around four.

[4]This property is common to a large class of random search model with on-the-job search [Burdett and Mortensen, 1998, Moscarini and Postel-Vinay, 2013, Coles and Mortensen, 2016].

is closely connected to the employment-weighted distribution of firm productivity used to summarize the state of the job ladder in the model. This index can similarly be decomposed into a firm component and an interaction component, an inequality often referred to as the OP decomposition in the literature [Olley and Pakes, 1996], where each term is closely tied to the structural decomposition derived from the model. A novelty of the paper is to use the evolution of this decomposition over the business cycle as a salient moment to assess the worker relocation properties of the model. Specifically, these data show that around 20 percent of the overall fall in the labor productivity index during the Great Recession in Britain is accounted for by the interaction term.

The model is calibrated to match a set of moments related to worker mobility and firm dynamics. Some of these moments come from the cross-section of firms implied by the model, such as the firm productivity distribution and the firm size distribution. These cross-sectional moments provide a foundation for the job ladder implied by the model. Another set of moments is related to the business cycle properties of the model, such as the volatility of worker flows. Importantly, the evolution of each term in the OP decomposition over the recent period is left untargeted. I find that the model successfully matches the OP decomposition of labor productivity derived from the data, which attributes around 20 percent of the variance of labor productivity to worker reallocation. By contrast, the leading job ladder model proposed by Moscarini and Postel-Vinay [2013, 2016] calibrated to the same data attributes nearly all of this variance to worker reallocation. This result is robust across several specifications, and it represents a key validation for the model's implied job ladder.

In the final part of the paper, I use the calibrated model to structurally decompose the reallocation of workers along the job ladder over the recent business cycle in Britain. Through the lens of the baseline model, around 70 percent of the fluctuations in labor productivity are driven by worker reallocation, which can be broken down into 50 percent coming from changes in the productivity of active firms and 20 percent coming from the interaction component. The fact that firm selection positively amplify fluctuations in labor productivity in the model comes from the search component of the model. Low-productivity firms have a lower rate of quits because in searching for new jobs, their employees compete with a larger pool of unemployed workers; therefore, since these firms benefit from unemployment, they are less likely to exit after a negative productivity shock. This channel counteracts the negative direct effect of a productivity shock at the calibrated parameters.

Another contribution of this paper is to propose a numerical solution method suitable to my random search environment with aggregate shocks. This solution method is required for two reasons. First, the model features an infinitely dimensional variable in the state-space (the full employment-weighted distribution of firm productivity). Second, standard linearization techniques [Reiter, 2009] do not apply to my environment because endogenous firm entry-exit makes the firm's problem discontinuous. I therefore rely on a simulation-based approach in which the employment-weighted distribution of firm productivity is approximated by a set of its moments. I also report several tests to check the accuracy of this procedure. Beyond the model considered in this paper, this approach can potentially be useful in other models with aggregate shocks and a similar discontinuity.

Related literature This work is related to the growing literature that combines firm dynamics with frictional labor markets. This literature brings together firm dynamics models in the tradition of Hopenhayn and Rogerson [1993], which maintain the assumption that labor markets clear [Khan and Thomas, 2013, Clementi and Palazzo, 2016, Sedláček and Sterk, 2017], and search and matching models in the tradition of Mortensen and Pissarides [1994], which emphasize a firm-worker match without a meaningful notion of a firm with multiple workers. Multi-worker firms are a pre-requisite to jointly studying firm-level concepts (productivity, employment, job flows) and worker flows (unemployment inflows and outflows). My work complements the recent papers integrating firm dynamics and frictional labor markets by combining three features: (i) firm dynamics, (ii) random search with on-the-job search, and (iii) business cycle fluctuations.

In the existing literature, firm dynamics and search frictions have been integrated using two distinct approaches: directed search and random search. A first series of papers builds on the theoretical results in Menzio and Shi [2011] to introduce firm dynamics in an environment where workers can direct their search to specific jobs [Kaas and Kircher, 2015, Schaal, 2017]. This approach is highly tractable in the presence of aggregate shocks. With the appropriate free-entry condition, all distributions vanish from the state-space and the model can be solved numerically using standard recursive methods. While Kaas and Kircher [2015] abstract from on-the-job search, Schaal [2017] presents an elegant model combining firm dynamics, on-the-job search, and aggregate shocks. But, in a directed search environment, on-the-job search implies a very specific theory of worker reallocation because there is no job ladder: the theory does not specify which firms offer better jobs and systematically poach workers from other firms.[5]

A second strand of this literature introduces firm dynamics in a random search environment. The first contributions abstract from on-the-job search [Elsby and Michaels, 2013, Acemoglu and Hawkins, 2014]. A central feature of the handful of random search models with on-the-search and multi-worker firms is the existence of a job ladder: there is a clear theory of which firms offer better jobs. Implicitly, these models all imply a measure of worker reallocation along the job ladder in response to aggregate shocks. I make this mechanism explicit in the paper and quantify the drivers of worker reallocation in a model disciplined with detailed firm-level data.

The model developed in this paper expands on the random search frameworks with on-the-job search in Moscarini and Postel-Vinay [2013, 2016] and Coles and Mortensen [2016]. Similarly to the wage determination protocol in Moscarini and Postel-Vinay [2013, 2016], I assume that firms can commit to a state-contingent wage contract. I expand on their framework by allowing for firm dynamics while still retaining the tractability of the model. Similarly to Coles and Mortensen [2016], I assume that the cost of recruitment has a specific

[5]The objective in Schaal [2017] is not to quantify worker reallocation but to study the role of uncertainty shocks as drivers of the US business cycle.

functional form and I obtain the same size-independence result. But with state-contingent wage contracts, I can relax their assumption of exogenous firm entry and exit and allow the set of active firms to evolve endogenously over the business cycle. Besides allowing for richer firm dynamics, my work also differs from these important contributions in that it benchmarks the worker reallocation property of the model over the business cycle to summary statistics derived from detailed firm-level data. My simulations suggest that the model by Moscarini and Postel-Vinay [2016] calibrated to the same data tends to overstate the importance of worker reallocation in accounting for aggregate fluctuations in labor productivity.

My environment also maintains the assumption that firms operate a linear production technology. Two contemporaneous papers, Elsby and Gottfries [2022] and Bilal et al. [2022], consider similar environments with decreasing returns to production. Elsby and Gottfries [2022] show under two wage-setting protocols that the job ladder can be characterized in terms of a single variable: the marginal product of labor. A key difference with my framework is that they do not allow for firm entry and exit; all worker reallocation arises through the flows of workers among the same set of firms. Bilal et al. [2022] describe a model related to Elsby and Gottfries [2022] that allows for firm entry and exit. An important difference with my framework is that the theory in Bilal et al. [2022] is agnostic about how the surplus is split within a firm, and wages are therefore not determined. By contrast, wages are well-defined and straightforward to compute in my environment, which I use as an additional check on the reallocation properties of the model.

I see this paper as complementary relative to these two important contributions. My focus is on quantifying worker reallocation along the job ladder over the business cycle. While their analysis is restricted to a comparison of steady states and to perfect-foresight shocks, I propose a solution method to simulate the full model with aggregate shocks. None of these papers confront their theory to firm-level data on the reallocation of workers across the firm productivity distribution over the business cycle.

Outline Section 2 introduces the model. Section 3 defines the equilibrium. Section 4 describes the calibration and numerical solution. Section 5 quantifies the magnitude of worker reallocation within the calibrated model, and Section 6 concludes.

2 A model of firm dynamics with on-the-job search

2.1 Environment

Time $t = 0, 1, 2, \dots$ is discrete and the horizon is infinite. There is a unit measure of infinitely lived and ex-ante identical workers who are either (i) employed earning wage w_t, (ii) unemployed with home production b, or (iii) an entrepreneur attempting to start up a new firm. There is a measure of firms evolving endogenously due to entry and exit. Each firm operates a constant return technology with labor n as the only input and productivity factor $\omega_t p_t$. ω_t is an economy-wide component, which follows a first-order Markov process $\Gamma_\omega(.|\omega_{t-1})$ with realizations in some positive interval $\left[\underline{\omega}, \overline{\omega}\right]$. p_t is a firm-specific component, which follows a first-order Markov process $\Gamma_p(.|p_{t-1})$ with realizations in some positive interval $\left[\underline{p}, \overline{p}\right]$. Both workers and firms are risk-neutral and maximize their expected pay-offs discounted with factor $\beta \in (0, 1)$.

Labor market flows are constrained by search frictions. Unemployed workers sample job offers with some probability $\lambda_t \in (0, 1]$ at time t. Employed workers sample job offers with probability $s\lambda_t \in (0, 1]$, where the exogenous parameter s denotes the search intensity of employed relative to unemployed job seekers. Both unemployed and employed workers sample at most one offer per period t. Each employed worker is separated from her employer with probability δ_t. Workers also transition to unemployment when their current firm decides to exit. Newly unemployed workers do not search in the current period.

Workers, both unemployed and employed, become entrepreneurs in each period t with probability $\mu \in (0, 1]$. Employed workers quit their current job to become entrepreneurs with no opportunity to go back to their previous employer. Potential entrepreneurs draw an initial productivity p_0 from the exogenous distribution Γ_0 and decide whether to enter. They do not search in the current period. They become unemployed if they decide not to enter.

The job destruction rate is assumed to be an exogenous function of aggregate productivity $\delta_t = \delta(\omega_t)$. The probability λ_t that a worker makes contact with a potential employer is instead determined in equilibrium through a matching function. The matching function takes as inputs aggregate ads from firms and aggregate search effort from workers. In each period, a firm with employment n can hire a measure H of workers at a cost $C(H, n)$. The cost $C(H, n)$ corresponds to the production that is lost due to the process of searching for and training new hires. C is assumed to be homogeneous of degree one. The cost of hiring $H = hn$ new workers can then be written $C(H, n) = nC(h, 1) = nc(h)$, where c is positive, continuously differentiable, increasing, convex, and $c(0) = 0$. To hire, the firm has to post $a \geq 0$ job ads before workers have a chance to search.

I introduce some additional notation to formally define the matching technology. Let $M_t(p, n)$ denote the cumulative measure of firms with current productivity at most p and employment size at most n at the onset of period t, before decisions take place. Let $a_t(p, n)$ denote the equilibrium number of ads posted by a firm with current productivity p and employment size n. Let $\chi_t(p, n)$ denote the equilibrium decision of the firm to continue ($\chi_t(p, n) = 1$) or exit ($\chi_t(p, n) = 0$). equilibrium decisions $a_t(p, n)$ and $\chi_t(p, n)$ depend on aggregate variables, such as aggregate productivity ω_t, which are subsumed in the time subscript t for now. The aggregation of all ads posted by continuing firms gives $A_t = \int \chi_t(p, n) a_t(p, n) dM_t(p, n)$, Aggregate search effort is given by $Z_t = (1 - \mu) \left[u_t + (1 - \delta_t) s \int \chi_t(p, n) n dM_t(p, n)\right]$, the measure of $(1 - \mu)$ workers who are not entrepreneurs and are either unemployed at the start of the period $\left(\text{with the unemployment rate } u_t = 1 - \int n dM_t(p, n)\right)$ or employed and not separated adjusting for their relative search intensity s. Finally, the total number of contacts in a period t is given by $\lambda_t Z_t = \eta_t A_t = \min\{Z_t, A_t\}$, where η_t is the probability that a job ad reaches a worker.

Each period t can be divided into the following six phases.

(i) Productivity shocks: Aggregate productivity ω_t and firm-specific productivity p_t are realized.[6]

(ii) Entrepreneurial shock: With probability μ, workers become entrepreneurs. They draw an initial idea with productivity $p_0 \sim \Gamma_0$ and decide whether to enter.

(iii) Firm exit: Firms decide whether to stay on or discontinue their operations based on the realization of the productivity shocks. If they exit, all of their workers become unemployed.

(iv) Exogenous separations: Employees at continuing firms lose their jobs with exogenous probability δ_t.

(v) Search: Firms post vacancies to hire. Both unemployed and employed workers search for jobs. Recruitment at incumbent firms takes place.

(vi) Production and payments: Unemployed workers have home production b. Firms produce with their employees after the search stage. Wages accrue to employed workers. Newly created businesses start producing.

A recursive formulation is used throughout the paper. All value functions in subsequent sections are written from the production and payment stage onward, taking expectation $\mathbf{E}_{t-1}\{.\}$ over the events occurring in period t, conditional on the information available at the end of period $t-1$.

2.2 Wage determination: Contract-posting

Each firm chooses and commits to an employment contract upon entry to maximize the present discounted value of profits, taking the contracts offered by other firms as given. This contract specifies a state-contingent wage payment $w_t(p, n)$. Firms are bound by an equal treatment constraint, which restricts them to offering the same contract to all their

[6] The notation for the cumulative measure of firms $M_t(p, n)$ in the productivity-size space (p, n) is recorded at this point in time in the within-period sequence of events.

employees.[7] With full commitment, the discounted sum of future wage payments can be summarized by an equilibrium contract value $V_t(p,n)$.

I now introduce the notation needed to formally define the firm's problem. The cdf of offered wage contracts is denoted

$$F_t(W) = A_t^{-1} \int \mathbb{1}\left\{V_t(p,n) \leq W\right\} \chi_t(p,n) a_t(p,n) dM_t(p,n), \tag{1}$$

where $\mathbb{1}\{.\}$ is the indicator function. $F_t(W)$ is the fraction of ads posted by firms that offer less than contract value W. Job seekers draw offers from the distribution of values F_t. The distribution of offered wage contracts encapsulates the difficulty in solving the general model. It is a high-dimensional fixed-point object, as it arises from the aggregation of firms' optimal decisions, which themselves require knowledge of the distribution in equilibrium. I subsequently derive several results that drastically simplify Equation (1).

The value of unemployment is given by

$$U_{t-1} = b + \beta \mathbf{E}_{t-1}\Big\{\mu Q_t + (1-\mu)\Big[(1-\lambda_t)U_t + \lambda_t \int \max\left\{\tilde{W}, U_t\right\} dF_t(\tilde{W})\Big]\Big\}. \tag{2}$$

Unemployed workers have home production b in period $t-1$. Conditional on the realization of the shocks in the next period, they become entrepreneurs with chance μ, which gives them continuation value Q_t (defined explicitly below). With chance $(1-\mu)$, they are part of the pool of unemployed job seekers and draw an offer from the distribution of offered contracts F_t with probability λ_t. This offer is accepted if it delivers more than the continuation value of being unemployed U_t.

A firm offering contract value $W_t < U_t$ given the realization of the state variables at time t loses its entire workforce ($n_t = 0$): workers are better off being unemployed if $W_t < U_t$, and they are all offered the same contract due to the equal treatment constraint. The employment

[7] This constraint can be interpreted as a non-discrimination rule among ex-ante identical workers. The contract therefore cannot be contingent on outside offers, which are specific to each worker.

contract therefore specifies firm exit after some realizations of the state. The firm's decision to continue χ_t can be expressed as a function of the employment contract $\chi_t = \mathbb{1}\{W_t \geq U_t\}$. Exit is permanent. Workers are an input in the recruitment technology $nc(h)$, so hiring is not possible with $n = 0$. There is no realization of the state where the firm's present discounted value of profits is negative in equilibrium. The firm can always choose $W_t < U_t$ and get zero profits thereafter, so any contract where the firm's present discounted value of profits is negative cannot be optimal.

A firm offering any contract value $W_t \geq U_t$ given the realization of the state at time t sees a fraction μ of its workforce become entrepreneurs and a fraction $(1-\mu)\delta_t$ separated to unemployment, and the remainder quits to work at other firms at rate $q_t(W_t) = s\lambda_t\big(1 - F_t(W_t)\big)$. The quit rate $q_t(W_t)$ is the probability $s\lambda_t$ that employed workers contact an alternative employer times the probability $\big(1 - F_t(W_t)\big)$ that they draw a better offer. The firm chooses to hire new workers at rate h_t. Employment at a continuing firm with size n_{t-1} at the start of the period evolves according to

$$n_t = (1-\mu)(1-\delta_t)\big[1 - q_t(W_t) + h_t\big]n_{t-1}. \tag{3}$$

The firm posts a measure of ads a_t in accordance with its choice of gross hires in the current period $(1-\mu)(1-\delta_t)h_t n_{t-1}$, given aggregate search frictions. The number of posted ads a_t is implicitly defined by

$$(1-\mu)(1-\delta_t)h_t n_{t-1} = a_t \eta_t Y_t(W_t). \tag{4}$$

Total gross hires at the firm are equal to posted ads a_t times the chance these ads reach a worker η_t times the chance that the worker accepts the firm's employment contract W_t. The acceptance rate $Y_t(W_t)$ is the ratio between the measure of job seekers with current value less than W_t and all job seekers:

$$Y_t(W_t) = \frac{u_t + (1-\delta_t)s \int \mathbb{1}\{V_t(p,n) \leq W_t\}\chi_t(p,n) n dM_t(p,n)}{u_t + (1-\delta_t)s \int \chi_t(p,n) n dM_t(p,n)}. \tag{5}$$

Exiting firms do not hire and therefore do not post vacancies. They do not contribute to the distribution of offered contracts (1), so all offered contracts give at least value U_t. Unemployed workers accept all offers and transition to employment conditional on making a contact.

2.3 The firm's problem

The firm's problem can be written recursively by introducing an additional state variable for the value that the firm has committed to deliver to its workers from period $t-1$ onward [Moscarini and Postel-Vinay, 2013]. Let $\overline{V}$ denote that value. The present discounted value of the firm's profits is given by

$$\begin{aligned} &\Pi_{t-1}\left(p_{t-1}, n_{t-1}, \overline{V}\right) = \\ &\max_{\substack{w,W,\\ h\geq 0}} (\omega_{t-1}p_{t-1} - w)n_{t-1} + \beta \mathbf{E}_{t-1}\Big[\chi \cdot \Big(- c(h)(1-\mu)(1-\delta_t)n_{t-1} + \Pi_t(p_t, n_t, W)\Big)\Big], \end{aligned} \tag{6}$$

subject to the law of motion for employment (3) and the promise-keeping constraint

$$\begin{aligned} \overline{V} = w + \beta \mathbf{E}_{t-1}\Bigg\{ &\mu Q_t + (1-\mu)\Big[\Big((1-\chi) + \delta_t \chi\Big) U_t \\ &+ \chi \cdot (1-\delta_t)\Big(\big(1 - q_t(W)\big)W + s\lambda_t \int \max\big\{\tilde{W}, W\big\} dF_t(\tilde{W})\Big)\Big]\Bigg\}. \end{aligned} \tag{7}$$

In the current period, flow revenues at the firm are $p_{t-1}\omega_{t-1}n_{t-1}$, and its wage bill is wn_{t-1}. In the next period, conditional on the realization of the states, the firm decides whether to remain active $\chi = \mathbb{1}\{W \geq U_t\}$. If it remains active, it chooses its number of gross hires $h(1-\mu)(1-\delta_t)n_{t-1}$ at a cost $c(h)(1-\mu)(1-\delta_t)n_{t-1}$ and posts the corresponding ads in accordance with (4).

The promise-keeping constraint (7) states that the firm's choices must deliver value $\overline{V}$ to its workers, since it is committed to its employment contract.[8] The firm can deliver value $\overline{V}$ by adjusting the wage w and the state-contingent continuation value of its employment contract W. The firm's choice takes into account workers' continuation value. With chance μ, workers become entrepreneurs which gives them value Q_t. Otherwise with probability $(1-\mu)$, they transition to unemployment either because the firm exits $(1-\chi)$ or because they are hit by an exogenous separation shock $(\chi\delta_t)$, which gives them value U_t. Workers who are not separated $\big(\chi \cdot (1-\delta_t)\big)$ draw from the contract offer distribution F_t at rate $s\lambda_t$ and compare their offer with the contract chosen by the firm W. Workers remain with the firm and get value W if they do not get a better offer $\big(1-q_t(W)\big)$.

With constant returns to the production and hiring technology, we can restrict attention to equilibria in which the firm's strategy are independent from its size n:

Result 1 (Size-independence) *The operator defined by firm profits* (6) *admits linear solutions of the form* $\Pi_{t-1}(p_{t-1}, n_{t-1}, \overline{V}) = n_{t-1}J_{t-1}(p_{t-1}, \overline{V})$*, where* $J_{t-1}(p_{t-1}, \overline{V})$ *denotes profits per worker and is independent of firm-size* n_{t-1}*. The corresponding optimal choices for the continuation decision* $\chi_t(p_t)$*, contract* $V_t(p)$*, and hiring rate* $h_t(p)$ *are all independent of firm size* n_{t-1}*.*

The proof is in Appendix A.1.

This firm-size independence result is similar to the result in Coles and Mortensen [2016], but it is obtained under different assumptions on the wage-setting protocol. I assume that firms can commit to a full wage schedule after each realization of the aggregate state, which is reflected in the promise-keeping constraint. Coles and Mortensen [2016] assume that workers form beliefs about the firm's productivity based on the current wage it offers. I further relax some of the restrictions they impose on their environment and allow for endogenous firm

[8]The firm must deliver at least value $\overline{V}$ to satisfy the promise-keeping constraint. This constraint binds at the optimum.

entry and exit.

I stress that there are multi-worker firms in the model because of the equal treatment constraint (it offers the same contract to all employees) and the recruitment technology (the convex recruitment cost). The firm's policies $V_t(p)$ and $h_t(p)$ are independent of firm size, but they determine the firm's quit rate $q_t(V_t(p))$ and gross hiring rate $h_t(p)$. $q_t(V_t(p))$ and $h_t(p)$ pin down the growth rate of employment at the firm $(1-\mu)(1-\delta_t)\big[1-q_t(V_t(p_t))+h_t(p_t)\big]$. Given some initial employment n_0, the firm's law of motion for employment (3) then allows me to keep track of its size. At the estimated parameters, the accumulation of firm-specific shocks generates a realistic firm-size distribution in the cross-section (see Section 4).

2.4 Joint firm-workers' surplus

The firm's problem can be expressed in terms of the discounted production surplus implied by its current employment level n_{t-1}:

$$\Pi_{t-1}\left(p_{t-1}, n_{t-1}, \overline{V}\right) + n_{t-1}\overline{V} = n_{t-1}J_{t-1}\left(p_{t-1}, \overline{V}\right) + n_{t-1}\overline{V} = n_{t-1}S_{t-1}(p_{t-1}). \tag{8}$$

The joint surplus between the firm and its workers is the sum of discounted firm profits $\Pi_{t-1}\left(p_{t-1}, n_{t-1}, \overline{V}\right)$ and the contract value promised to its current workers $n_{t-1}\overline{V}$.

Two results underpin the expression for the joint firm-workers' surplus (8). First, the joint surplus is linear in employment, which follows directly from the linearity of firm profits in Result 1. Second, the joint surplus does not depend on the promised contract value $\overline{V}$. This second result follows from two assumptions: (i) the firm fully commits to the employment contract, and (ii) utility is transferable since workers and firms are risk neutral. Appendix A.2 formally shows that, since $\overline{V}$ moves with the wage one for one, it cancels out in Equation (8) once the wage is substituted out in the firm's discounted profits (6) using the promised-keeping condition (7).

The firm's decision to continue or exit in period t can be expressed as a function of the firm-workers' surplus $S_t(p_t)$. The firm's present discounted value of profits is never negative in equilibrium. The wage contract can always be designed to deliver less than the value of unemployment U_t for this realization of the state, in which case all workers leave the firm (commitment is one-sided) and the current value of its profits is zero. The contract value to which the firm commits from period t onward is therefore greater than U_t if and only if $S_t(p_t) \geq U_t$. The firm's decision to continue can then be written $\chi_t(p_t) = \mathbb{1}\{S_t(p_t) \geq U_t\}$.

Although the promised contract value $\overline{V}$ does not appear in the joint firm-workers' surplus representation $S_{t-1}(p_{t-1})$, wages are still pinned down by the promise-keeping constraint (7). As shown in Appendix A.2, conditional on the decision to continue, firms solve the unconstrained problem

$$\max_{W,h\geq 0} -c(h) + \big(S_t(p_t) - W\big)h + \big(1 - q_t(W)\big)S_t(p_t) + s\lambda_t \int \max\big\{\tilde{W}, W\big\} dF_t(\tilde{W}), \tag{9}$$

from which the optimal state-contingent contract value $V_t(p)$ can be derived. Wages then adjust to satisfy the promise-keeping constraint (7) conditional on the value $\overline{V}$ promised to workers and the optimal state-contingent contract value $V_t(p)$.

2.5 Firm entry

New firms are created by entrepreneurs (workers subject to a μ-shock).[9] Entrepreneurs draw an initial firm productivity from the distribution of business ideas Γ_0 and decide whether to enter given the current aggregate state in period t. Entrepreneurs who decide to enter get the full firm-workers' surplus: their business is purchased by some outside investors and they become the first workers at the new firms.[10] Entrepreneurs' outside option is the value of un-

[9] The arrival rate of business ideas μ is common to unemployed and employed workers. With minor adjustments, this rate could be allowed to differ by employment status. My focus is on the aggregate entry rate, and I therefore keep this parameter generic here.

[10] The promised utility $\overline{V}$ acceptable to entrepreneurs and investors in this transaction is not pinned down because these investors are outside the model. It is assumed investors have deep pockets so that they can deliver the appropriate wage payments following entry. In subsequent period, the state-contingent wage contract is determined by the joint firm-workers maximization problem (9).

employment: a μ-shock forces employed workers out of their current job with no recall option. The value of being an entrepreneur is given by $Q_t = \int \max\big\{S_t(\tilde{p}), U_t\big\} d\Gamma_0(\tilde{p})$. The entry process gives an initial employment $n_0 > 0$ to newborn firms, since entering entrepreneurs become its first workers. There is no meaningful notion of a firm with employment zero in the model because the hiring technology $nc(h)$ requires positive employment.[11] I normalize n_0 to unity: the measure of entering firms is equal to the measure of entrepreneurs deciding to enter.

3 Rank-monotonic equilibria

3.1 Recursive equilibria

There is a notion of an equilibrium independent of calendar time t in the model. I label these equilibria "recursive." The aggregate state is given by $\big(\omega, M\big)$. By assumption, the aggregate shock ω satisfies the Markov property. The measure of firms in the productivity and employment-size space is then sufficient to compute all labor market aggregates. Given the aggregate state $\big(\omega, M\big)$, the firm can compute the acceptance rate (5) conditional on the optimal choice to continue χ, the optimal hiring choice h, and the optimal employment contract value V at all other firms. Given the acceptance rate (5), the optimal posting of ads a follows from the accounting relationship (4). Given firms' optimal posting of ads, the offer distribution follows from the aggregation of ads posted by firms offering different employment contract values (1).

Result 1 shows that we can restrict attention to equilibria in which firms' optimal policies are linear in firm employment n. This result yields the first key simplification of the model's solution since it implies that the employment-weighted measure of firm productivity $L_t(p) = \int_{\tilde{p} \leq p} \int_n n dM_t(\tilde{p}, n)$ is sufficient to compute the acceptance rate and offer distribution.

[11] An alternative entry process would assume a separate hiring technology for new entrants. A strength of my entry protocol is that I can abstract from deriving the optimal contract posted by new entrants, which, in general, would differ from the contract offered by incumbent firms.

Intuitively, to compute the acceptance rate, it is sufficient to know the measure of workers employed at firms with productivity p, since these firms offer the same employment contract by the size-independence result.[12] The aggregate state is therefore equivalently given by $\big(\omega, L\big)$, where the measure L in the state-space is uni-dimensional. Formally:

Definition 1 (Recursive equilibrium) *A recursive equilibrium is a triple of policy functions $\big(\chi, h, V\big)$ and a pair of value functions $\big(S, U\big)$ that depend on the current realization of firm-specific productivity p, the current realization of aggregate productivity ω, and the employment-weighted measure of productivity L. Conditional on all firms following the policies given by $\big(\chi, h, V\big)$, these functions are such that:*

1. *The equations for the acceptance rate* (5)*, ads posting* (4)*, and contract offer distribution* (1) *hold with firms' optimal choices $\chi_t(p) = \chi(p, \omega, L)$, $h_t(p) = h(p, \omega, L)$, and $V_t(p) = V(p, \omega, L)$.*
2. *The contract and hiring functions solve the maximization problem* (9)*. The continuation decision is given by $\chi(p, \omega, L) = \mathbb{1}\big\{S(p, \omega, L) \geq U(\omega, L)\big\}$.*
3. *S and U solve, respectively,* (8) *and* (2)*.*

In Definition 1, a recursive equilibrium is a fixed-point: the firm's policies solve the Bellman equation defined by (8)-(9) taking the policies of other firms as given, and these policies coincide in equilibrium. Characterizing this fixed-point analytically is challenging in general. Solving for the optimal employment contract in the firm's problem (9) requires knowing the distribution of offered contracts, which itself depends on the distribution of employment contracts at all firms.

3.2 Rank-monotonic equilibria

In the remainder of the paper, I focus on the subset of recursive equilibria for which an analytical characterization of the equilibrium is possible. I label this subset of recursive equilibria

[12] This is established formally for the acceptance rate and offer distribution in Appendix A.3.

"rank-monotonic." A rank-monotonic equilibrium (RME) adds the following requirement to the optimal contract in a recursive equilibrium:

Definition 2 (rank-monotonic equilibrium) *A rank-monotonic equilibrium is a recursive equilibrium in which the optimal contract $V(p, \omega, L)$ is weakly increasing in the firm's current realization of productivity p for all ω and L.*

Result 2 gives sufficient conditions on the cost of hiring function that guarantee that any recursive equilibrium is rank-monotonic.

Result 2 (sufficient conditions for RME) *Assume that the Markov process for firm-specific productivity satisfies first-order stochastic dominance and that the conditional distribution Γ_p is everywhere differentiable.*[13] *Assume that the hiring cost function is twice differentiable, increasing, and convex. Assume that the distribution of offered contracts $F(., \omega, L)$ is everywhere differentiable with respect to p. Then, for any recursive equilibrium:*

1. *The firm-workers' surplus defined by Equation (8) is differentiable and weakly increasing in p;*
2. *The equilibrium is rank-monotonic provided $hc''(h)/c'(h) \geq 1$ at all $h \geq 0$.*

Result 2 provides sufficient conditions for a RME in the sense that it is not an existence statement, but a property of the optimal contract conditional on the existence of such a recursive equilibrium. The proof is relegated to Appendix A.4.

The condition on the Markov process for firm-specific productivity in Result 2 requires a form of persistence to guarantee that the firm-workers' surplus (8) is increasing in p. This condition is satisfied by most productivity processes commonly used in the firm dynamics literature.

[13] First-order stochastic requires $\Gamma_p(.|p') \leq \Gamma_p(.|p)$ for $p' > p$ with strict inequality for some productivity level. The differentiability of Γ_p (with respect to both arguments) is required to ensure that the distribution of firm-specific productivity is smooth.

The condition on the cost function $hc''(h)/c'(h) \geq 1$ in Result 2 requires that the cost function have a high degree of convexity. More productive firms offer better contracts to limit quits only to the extent that hiring is sufficiently costly. With identical workers and no learning on the job, the model could potentially generate a large amount of churning at the top of the productivity distribution if employers can easily hire new workers. $hc''(h)/c'(h) \geq 1$ makes hiring costly enough for large h that more productive firms find it optimal to use the retention margin when choosing the rate at which employment changes.

The requirement that $F(., \omega, L)$ is everywhere differentiable rules out recursive equilibria in which the equilibrium contract offer distribution has mass points. Mass points in $F(., \omega, L)$ imply that some or all firms offer the same contract irrespective of their current firm-specific productivity p. The standard argument in random search models with on-the-job search and wage posting to rule out mass points is that firms at a mass point can increase profits by offering jobs paying slightly more, thus poaching workers from other firms [Burdett and Mortensen, 1998]. This argument does not directly translate to this framework because the quit and hiring margins are separately controlled by the firm, respectively through wage contracts and hiring effort. For instance, if all firms offer the value of unemployment at all realizations of the aggregate state, a firm that promises a marginally larger wage contract would not increase its discounted profits. Its hiring costs are the same, and its quit rate is unchanged.

3.3 Additional characterization of rank-monotonic equilibria

Result 2 yields a second series of simplifications of the model's solution. Conditional on the value functions S and U, the model can be fully characterized analytically. These results underpin the numerical solution of the model detailed in Section 4.

The entry decision of entrepreneurs and the exit decision of existing firms can be summarized as a single firm-specific productivity threshold $p_E(\omega, L)$. $p_E(\omega, L)$ is implicitly defined as the firm-specific productivity p solving $S(p, \omega, L) = U(\omega, L)$.

The optimal employment contract can be expressed as a function of the firm-workers' surplus (8) and the value of unemployment (2) only:

Result 3 (employment contract in RME) *In a rank-monotonic equilibrium, the equilibrium contract is given by*

$$V(p,\omega,L) = \frac{uU(\omega,L) + \big(1-\delta(\omega)\big)s\int_{p_E(\omega,L)}^{p} S(\tilde{p},\omega,L)dL(\tilde{p})}{u + \big(1-\delta(\omega)\big)s\Big[L(p) - L\big(p_E(\omega,L)\big)\Big]}. \tag{10}$$

The proof is in Appendix A.5.

Result 3 reveals that the optimal contract is a weighted average between the value of unemployment and the firm-workers' surplus. The optimal contract (10) is reminiscent of the Nash-Bargaining solution used in many standard search and matching models, where the firm-workers' surplus is split between each party conditional on a constant, exogenously given bargaining weight [e.g., Mortensen and Pissarides, 1994]. In this model, the weights are fully endogenous: they evolve with the employment-weighted measure of firm productivity L over the business cycle. The rents that workers can extract from the joint firm-workers' surplus (8) are directly linked to on-the-job search. As the search intensity of employed workers s goes to zero, the optimal contract (10) is given by $V(p,\omega,L) = U(\omega,L)$. Employed workers get the value of unemployment.

The optimal hiring rate $h(p,\omega,L)$ follows directly from solving the firm's joint maximization problem (9). Given optimal contract $V(p,\omega,L)$, the optimal hiring rate follows from inverting the derivative of the cost function in the firm's first-order condition associated with the maximization problem (9): $c'\big(h(p,\omega,L)\big) = S(p,\omega,L) - V(p,\omega,L)$.

In a rank-monotonic equilibrium, the acceptance rate for a firm with current productivity p can be expressed as

$$Y\big(V(p,\omega,L),\omega,L\big) = \frac{u + \big(1-\delta(\omega)\big)s\Big[L(p) - L\big(p_E(\omega,L)\big)\Big]}{u + \big(1-\delta(\omega)\big)s\Big[L(\overline{p}) - L\big(p_E(\omega,L)\big)\Big]}. \tag{11}$$

Because contracts are increasing in p, all searching workers employed at firms with current productivity less than p accept a firm-p employment contract. The distribution of offered contracts can then be shown to simplify to

$$\lambda(\omega, L)F\big(V(p, \omega, L), \omega, L\big) = \int_{p_E(\omega,L)}^{p} \frac{\big(1-\delta(\omega)\big)h(\tilde{p}, \omega, L)}{u + \big(1-\delta(\omega)\big)s\Big[L(\tilde{p}) - L\big(p_E(\omega, L)\big)\Big]} dL_t(\tilde{p}). \quad (12)$$

Equations (11) and (12) are derived in Appendix A.5.

The distribution of offered contracts in an RME (12) fully summarizes the evolution of employment at each level of firm-specific productivity p. Let L^P denote the measure of workers employed at firms with productivity of at most p at the production stage (end of period). Given the aggregate state $\big(\omega, L\big)$, the production stage measure of workers at firms with productivity of at most p is related to the start of period measure L by

$$\begin{aligned} L^P(p) =& \mu\Big[\Gamma_0(p) - \Gamma_0\big(p_E(\omega, L)\big)\Big] \\ &+ (1-\mu)\Big[L(p) - L\big(p_E(\omega, L)\big)\Big]\big(1-\delta(\omega)\big)\Big(1 - q\big(V(p, \omega, L), \omega, L\big)\Big) \\ &+ (1-\mu)u\lambda(\omega, L)F\big(V(p, \omega, L), \omega, L\big). \end{aligned} \quad (13)$$

The first term is the measure of entering entrepreneurs with an initial draw less than p. The second term is the measure of workers still employed after the realization of the shocks who do not find a job at a firm with productivity greater than p: $\Big(1 - q\big(V(p, \omega, L), \omega, L\big)\Big)$. The third term is the measure of unemployed workers who find a job at a firm with productivity of at most p. The end of period and beginning of next period measures are directly linked through $dL_{t+1}(p)/dp = \int_{\underline{p}}^{\overline{p}} dL_t^P(\tilde{p})/d\tilde{p} d\Gamma(p|\tilde{p})$, which corresponds to the "re-shuffling" of workers across productivity levels due to the firm-specific shocks.

4 Quantitative analysis

4.1 Solution method

The size-independence result (Result 1) and the rank-monotonic equilibrium result (Result 2) greatly simplify the firm's problem. Given these results, the relevant state-space is $\big(p, \omega, L\big)$. But since the employment-weighted measure of firm productivity L is an infinitely dimensional object evolving with aggregate shocks, this state-space still represents a challenge for the numerical solution of the model. I then proceed in two steps.

The first step is trivial. I shut down aggregate shocks ($\omega_t = \omega$ for all t) and solve for the corresponding steady state RME. This equilibrium is such that the firm's policies imply a constant measure of workers L at all productivity levels given the law of motion for employment (13).[14]

In the second step, I reintroduce aggregate shocks into the model. An expanding literature building on Reiter [2009] proposes to solve heterogeneous agent models by linearizing around the steady state.[15] But my simulations suggest that taking a derivative around the steady state is highly inaccurate in the context of my model due to the discontinuity implied by the firm's endogenous entry and exit threshold. I therefore rely on a simulation-based approach that adapts ideas from Krusell and Smith [1998] to my setting. Since this solution method is distinct from the original paper, I briefly outline the key ideas here. The details are relegated to Appendix C.2.

The solution method relies on the following two approximations. First, the measure of workers L_t is summarized by a vector $\boldsymbol{m}_t$ of moments. This vector includes the unemployment rate, $m_t^0 = u_t = 1 - L_t(\overline{p})$, and N_m moments $\{m_t^1, \ldots, m_t^{N_m}\}$ from the distribution of workers $L_t/L_t(\overline{p})$. With this approximation of L_t, the aggregate state-space relevant to the

[14] I do not have a proof that this equilibrium exists and is unique. Numerically, I have checked that the algorithm converges to the same solution with alternative initial conditions for a large number of parameters. Details of the corresponding solution algorithm can be found in Appendix C.1.

[15] See, among others, Sedláček and Sterk [2017] and Winberry [2021] for applications to firm dynamics models.

firm is now $\big(\omega_t, \boldsymbol{m}_t\big)$. The second approximation is to parameterize the value of unemployment (2) and the firm-workers' surplus (8) out of the steady state with a flexible polynomial in $\big(\omega_t, \boldsymbol{m}_t\big)$.

Given these approximations, the procedure works as follows. Draw a sequence of aggregate shocks $\{\omega_t\}_{t=1}^{T}$. Conditional on a guess for the coefficients in the polynomial, the aggregate law of motion for employment (13) can be solved forward in time for this sequence of shocks. Conditional on the law of employment along the sequence of shocks, the value of unemployment (2) and the firm-workers' surplus (8) can be solved backward in time. The coefficients in the polynomial can then be updated by running a regression of the simulated value functions on the simulated aggregate states. The solution algorithm proceeds by iterating on these steps until the coefficients in the polynomial converge.

Relative to the solution method proposed in Krusell and Smith [1998], my approach forecasts the agents' value function conditional on the realization of the states. Solving for the agents' value functions in my framework requires knowledge of the full employment-weighted measure of firm productivity, not just the forecast of an aggregate variable, such as the capital stock as in the original paper. This measure is needed to compute the offer distribution aggregating up the recruitment of all firms (12), and it is simulated directly as part of the iterations forward in time. Relative to linearization-based solutions, the main advantage of this approach is that it is robust to the kink in the agents' value functions implied by endogenous entry and exit. Its main disadvantage is that it is computationally more intensive.

I report several robustness checks for the proposed solution algorithm in the Appendix. First, I implement a version of the accuracy test described in den Haan [2010] and show that the procedure performs well according to this metric (Appendix C.3). Second, I experiment with alternative numbers of moments N_m to summarize the measure of workers L_t (Appendix C.4) and justify my choice of $N_m = 2$ used for the results reported in the paper.

4.2 Data

I use three broad types of data from Britain to calibrate the model: worker-level data, firm-level data, and aggregate time series.[16] On the worker side, I compute the unemployment to employment (UE_t), employment to unemployment (EU_t), and job-to-job (EE_t) monthly transition rates in the UK from the British Household Panel Survey (BHPS). This data set is available starting in 1992. These series are derived following the methodology described in Postel-Vinay and Sepahsalari [2019].

On the firm side, I combine several administrative data sets to compute cross-sectional moments on firm employment and labor productivity for a large sample firms every year between 1997 and 2018 [Office for National Statistics, 2019, 2020, 2021]. Importantly for the paper's focus on business cycle fluctuations, these data cover several years before and after the Great Recession (2008q2-2009q3 in the UK). The Annual Respondents Database (ARD) and its successor the Annual Business Survey (ABS) provide yearly firm balance-sheet data for a large repeated cross-section of firms, from which I define the following firm-level measures of labor productivity ($\text{LP}_{i,t}$) and employment cost per worker ($\text{EC}_{i,t}$):

$$\text{LP}_{i,t} = \ln\left(\frac{\text{value added}_{i,t}}{\text{employment}_{i,t}}\right), \qquad \text{EC}_{i,t} = \ln\left(\frac{\text{payroll expenditures}_{i,t}}{\text{employment}_{i,t}}\right).$$

To compute dynamics at the firm-level, I merge these data sets with the Business Structure Database (BSD), which is a yearly panel of the near-universe of firms in Britain but does not have information on their balance-sheets. I refer to this merged sample as "mBSD."

Finally, I use standard aggregate time series to discipline the business cycle properties of the model. These series are obtained from the Office for National Statistics (ONS).

[16]See Appendix B for a full list of data sources and details on the construction of the variables.

4.3 Calibration of steady-state model

A period t is set to a month, and a small subset of parameters are set externally. The discount factor β is set in line with a 5 percent annual discount rate. The employment size of new entrants is normalized to $n_0 = 1$.

Parametrization The steady-state model is solved under the following parametric assumptions. The functional form for the cost of hiring function is guided by the conditions derived in Result 2. I assume that the per-worker cost to the firm of hiring at rate h is given by $c(h) = (c_1+1)^{-1}(c_0 h)^{c_1+1}$, which satisfies the condition in Result 2 provided $c_1 \geq 1$. I enforce this condition directly when searching over the parameter space. I specify the Markov process for the firm-specific productivity shock (p_t) as $\ln p_t = \rho_p \ln p_{t-1} + \sigma_p \varepsilon_t^p,\ \varepsilon_t^p \sim \mathcal{N}(0,1)$. This process satisfies first-order stochastic dominance conditional on its past realizations, which is required for the equilibrium to be rank-monotonic (Result 2).[17] Finally, I assume that draws from Γ_0 (the productivity distribution of new entrants), come from the stationary distribution implied by the process for firm-productivity shocks.

Moment targets These parameters are calibrated from simulations of the model at the steady-state targeting long-run moments. The long-run moments are derived from the pooled data for the period 1997-2018. I heuristically link each parameter to a specific moment in the discussion, but all parameters and moments in Table 1 are related in the actual model.[18]

From the worker-level data, I compute the average UE, EU, and EE transition rates over the period 1997-2018 to calibrate, respectively, the scale of the hiring cost c_0, the exogenous separation rate δ, and the search intensity of employed workers $s \leq 1$ relative to unemployed workers. The calibrated relative search intensity parameter is higher than the values typically obtained for this class of models calibrated to US data, which reflects the large average value

[17] I assume in Section 2 that $p_t \in [\underline{p}, \overline{p}]$, which is the case in practice given the Markov process for firm-specific productivity is discretized.

[18] I confirm this heuristic parameter-moment mapping in Appendix Figure C.2, where I show that small deviations of the parameters around their calibrated values significantly move the corresponding targeted moments in predictable directions.

		Data	Calibration	
			Baseline	$c_1 = 1$
Panel A. Externally set parameters				
β	Discount factor (5 percent annually)	—	0.996	0.996
n_0	Size of entrants (normalization)	—	1.000	1.000
Panel B. Steady-state parameters				
δ	Separation rate	—	4.5E-04	8.7E-04
c_0	Hiring cost:	—	51.562	35.553
c_1	$c(h) = (c_1 + 1)^{-1}(c_0 h)^{c_1+1}$	—	5.908	1.000
s	Relative search effort	—	0.812	0.481
μ	Prob. of start-up shock ($\times$1,000)	—	0.512	0.366
b	Unemployment flow value	—	0.985	0.837
ρ_p	Autocor. AR1 $\ln p_t$	—	0.973	0.947
σ_p	Std. of AR1 $\ln p_t$	—	0.264	0.249
Panel C. Targeted moments				
	Avg. UE_t (BHPS)	0.058	0.058	0.043
	Avg. EU_t (BHPS)	0.003	0.004	0.004
	Avg. EE_t (BHPS)	0.016	0.016	0.012
	Avg. firm size $n_{i,t}$ (ARD)	12.113	12.343	11.861
	Reg. $\Delta \ln n_{i,t+1}$ on $\text{LP}_{i,t}$ (mBSD)	0.136	0.127	0.138
	Frac. job dest. from exits (mBSD)	0.526	0.519	0.496
	Autocor. of $\ln n_{i,t}$ (mBSD)	0.949	0.992	0.991
	IQR of $\text{LP}_{i,t}$ (ARD)	1.129	1.123	0.783
Panel D. Additional moments				
	Pareto tail of empl. size (ARD)	1.066	1.033	1.021
	IQR of $\text{EC}_{i,t}$ (ARD)	1.352	0.914	0.535
	Reg. $\text{EC}_{i,t}$ on $\text{LP}_{i,t}$ (mBSD)	0.704	0.685	0.590
	Reg. $\Delta \ln n_{i,t+1}$ on $\text{EC}_{i,t}$ (mBSD)	0.131	0.162	0.154

Table 1: Calibrated parameters and targeted moments in steady-state model. Panel A reports the parameters calibrated externally. Panel B reports the parameters calibrated to target the selection of data moments reported in Panel C. Panel D reports a selection of additional data moments. The "Baseline" calibration uses all parameters in Panel B. The "$c_1 = 1$" calibration uses all parameters with the exception of c_1. See main text for details.

of the EE transition rate relative to the average UE transition rate in the BHPS (respectively 0.016 and 0.058 on average) compared to the US (respectively 0.02 and 0.21 in the Survey of Income and Program Participation).

The firm-level data are used to discipline the remaining steady-state parameters. All data targets are computed by pooling the data across available years, in line with the steady-state model. The model counterpart to these firm-level data is obtained by simulating a large panel of firms at the steady-state. The probability of starting a firm μ is calibrated to target the average employment of firms $n_{i,t}$ in the data, since μ controls the relative measure of workers to firms conditional on the normalization for n_0. The flow value of unemployment b is disciplined by the share of job destruction resulting from firm exit in the data. This parameter shifts the value of unemployment, so it is related to endogenous exits in the model. The process for firm-specific productivity is calibrated to two moment targets from the firm-level data. ρ_p targets the autocorrelation of firm employment $\ln n_{i,t}$ (12 months apart because the data are yearly).[19] σ_p targets the inter-quartile range (IQR) of labor productivity $\text{LP}_{i,t}$.[20]

A key model parameter to calibrate is the exponent of the cost of hiring function c_1 since it corresponds to the inverse elasticity of the hiring rate to the firm's discounted profits in the model. In the baseline calibration, this parameter is disciplined by the coefficient of a regression of firm-level employment growth $\Delta_{12} \ln n_{i,t+12}$ on $\text{LP}_{i,t}$ controlling for industry-year fixed effects, where the timing reflects the yearly frequency of the data.[21] This strategy yields a value for c_1 close to six. While this value results from the internal calibration of the

[19]I target the autocorrelation of employment because labor productivity can only be constructed in the balance-sheet data, which is a repeated cross-section in the British administrative data. In unreported results, I find that the autocorrelation of labor productivity implied by the model is broadly similar to those derived from firm-level panel data for a selection of European countries.

[20]The inter-quartile range of labor productivity is computed by first residualizing the data on industry-year fixed effects, where industries are defined by SIC07 sections (18 aggregate industries). In unreported results for a selection of European countries, I find that adjusting for workforce quality by expressing labor productivity per unit of employment compensation instead of per employee reduces the inter-quartile range of labor productivity by about 20 percent.

[21]Conditional on keeping all other parameters fixed, I confirm in Appendix Figure C.2 that this moment is informative for identifying c_1 locally. In addition, in unreported results for a selection of European countries, I find that the magnitude of this relationship is similar across a range of regression specifications.

model and is therefore difficult to compare across studies, it is higher than the values used in the previous literature specifying the cost of hiring as a function of the hiring rate. For instance, Merz and Yashiv [2007] estimate an approximately cubic hiring rate cost function ($c_1 = 2$) in a structural model of firm valuation estimated on U.S. data, while Blatter et al. [2012] find that the hiring cost function is approximately quadratic ($c_1 = 1$) in the hiring rate using Swiss survey data.

To assess the sensitivity of the model to the high degree of curvature of the cost of hiring function in the baseline calibration, I also report results for an alternative fully recalibrated version of the model with $c_1 = 1$ (rightmost column in Table 1). This choice corresponds to the lowest value of c_1 that guarantees the equilibrium is rank-monotonic (Result 2). As can be seen in Table 1, it tends to worsen the fit of the model to some of the targeted moments, such as the average UE transition rate and the dispersion of firm-level labor productivity, reinforcing that these moments are jointly informative for this parameter.[22]

In Panel C of Table 1, I also report several non-targeted moments. The model generates a realistic distribution of firm sizes as measured by the Pareto tail of the employment-size distribution.[23] In addition, I also benchmark the firm-level wages implied by the contract-posting assumption in the model by reporting several summary measures of the firm-level employment costs $\text{EC}_{i,t}$. With contract-posting, wages are backed out from the promise-keeping constraint (7) evaluated at the optimal RME contract (10). I find that the model implies a realistic degree of association between EC_{it} and $\text{LP}_{i,t}$, as measured by a univariate regression.[24] Similarly, it also matches the slope of a regression of employment growth

[22]Table 1 shows that the model with $c_1 = 1$ closely fits the regression coefficient of firm-level employment growth $\Delta_{12} \ln n_{i,t+12}$ on $\text{LP}_{i,t}$, which reflects how the calibration balances the full set of targeted moments.

[23]A long tail in the firm-size distribution arises in the model because the process describing the evolution of firm employment is (i) independent of the firm's current employment (Result 1), and (ii) a birth-death process because of firm entry and exit. These two conditions on the process underlying firm employment are precisely those identified by the literature on the emergence of power law distributions in economics [Gabaix, 1999, Reed, 2001]. See Gouin-Bonenfant [2019] for a more extended discussion in the context of a related search model.

[24]Similarly to $\text{LP}_{i,t}$, EC_{it} is first residualized on industry-year fixed effects, where industries are defined by SIC07 sections (18 aggregate industries). Since the data on firms' balance sheets are only available as a repeated cross-section in the ABS/ARD, this coefficient is identified off the cross-section of firms.

$\Delta_{12} \ln n_{i,t+12}$ on EC_{it} controlling for year-industry fixed effects. Finally, the baseline calibration captures around 70 percent of the inter-quartile range of EC_{it} found in the data.

4.4 Calibration of full model

The last set of parameters to be calibrated controls the business cycle properties of the model. These parameters are derived from simulations of the full model with aggregate shocks targeting moments in deviation from trend. Since solving the model with aggregate shocks is computationally expensive, this last set of parameters is calibrated conditional on the steady-state parameters obtained in the previous step.[25]

An extensive literature investigates the cyclical properties of search-and-matching models with aggregate shocks.[26] I complement these studies by documenting the cyclical properties of a random search model with firm dynamics and on-the-job search. My starting point is a standard Markov process for the aggregate component of aggregate productivity (ω_t) specified as $\ln \omega_t = \rho_\omega \ln \omega_{t-1} + \sigma_\omega \varepsilon_t^\omega,\ \varepsilon_t^\omega \sim \mathcal{N}(0,1)$. As I document next, labor market fluctuations are muted with this unique aggregate shock, and I therefore allow a subset of parameters to co-move with ω_t. For instance, I let the exogenous separation rate δ_t depend on ω_t according to $\ln \delta(\omega_t) - \ln \delta = \epsilon_{\delta,\omega} \cdot \ln \omega_t$, where the parameter $\epsilon_{\delta,\omega}$ controls the magnitude of the response to aggregate productivity shocks. I specify the dependence of the subset of parameters that co-move with ω_t similarly.

In line with the existing literature, I assess the cyclical properties of the model through moments of the time series for output and labor market flows. These time series are de-trended in three steps to harmonize data with different frequencies and isolate business cycle fluctuations. I first average all series to yearly frequency. I then de-trend these series using an HP filter with smoothing parameter 100. I finally interpolate the resulting cyclical

[25] A similar two-step strategy (first steady state then aggregate shocks) is also used in the Heterogeneous Agents New Keynesian literature. See for example Bayer et al. [2020].

[26] A small subset of these studies include Shimer [2005], Hagedorn and Manovskii [2008], Elsby and Michaels [2013], Kaas and Kircher [2015], Lise and Robin [2017], and Schaal [2017], among many others.

	Data	Baseline model: Alternative shocks				$c_1 = 1$
		(ω)	(ω, r)	(ω, δ)	(ω, δ, c_0)	(ω, δ, c_0)
Panel A. Aggregate shocks parameters						
ρ_ω	—	0.986	0.753	0.953	0.891	0.941
σ_ω	—	0.003	0.007	0.003	0.002	0.001
$\epsilon_{r,\omega}$	—	—	-147.203	—	—	—
$\epsilon_{\delta,\omega}$	—	—	—	-70.588	-178.807	-183.815
$\epsilon_{c_0,\omega}$	—	—	—	—	-3.703	-3.923
Panel B. Business cycle moments						
$\text{corr}(\ln \text{GDP}_t, \ln \text{GDP}_{t-1})$	0.998	0.984	0.801	0.988	0.988	0.996
$\text{std}(\ln \text{GDP}_t)$	0.019	0.018	0.012	0.021	0.015	0.017
$\text{std}(\text{EU}_t)$	2.3E-03	4.1E-04	1.1E-03	1.6E-03	2.0E-03	1.4E-03
$\text{std}(\text{UE}_t)$	2.7E-04	6.3E-05	2.2E-04	4.2E-04	4.0E-04	4.2E-04
$\text{std}(\text{EE}_t)$	9.3E-04	7.2E-05	2.7E-04	2.2E-04	4.1E-04	3.7E-04

Table 2: Cyclical properties of full model with alternative aggregate shocks. Panel A reports the parameters calibrated to target the moments listed in Panel B. All calibrations are based on the "Baseline" steady-state calibration, except for the rightmost column, which reports results for the $c_1 = 1$ steady-state calibration. All alternative aggregate shock calibrations target all moments listed in Panel B, except for the (ω)-specification, which only targets the autocorrelation and standard deviation of log output. See main text for details.

components to a monthly frequency.[27] This last interpolation step allows me to fit aggregate shocks month-by-month to replicate some key time series within the calibrated model, as I describe in the next section. Another advantage of this approach is to isolate business cycle fluctuations from short-run monthly and quarterly changes, which are likely driven by measurement error.

In Table 2, I describe the parameters (Panel A) and cyclical properties of the full model with aggregate shocks (Panel B). I summarize these properties with five moments of the time series for output and labor market flows in deviation from trend: the autocorrelation of log-output, and the standard deviation of log-output, the EU rate, the UE rate, and the EE rate.

To better understand the role of additional shocks, I consider several alternative aggregate shocks. The (ω)-specification consists of a single aggregate productivity shock calibrated to

[27] I use spline interpolation to obtain smooth series.

replicate the autocorrelation and standard deviation of log output. In line with the well-known results reported for standard search-and-matching models, I find that this single source of aggregate fluctuations generates a limited response of labor market flows [Shimer, 2005]. This response is an order of magnitude lower than what is found in the data. I therefore introduce additional aggregate shocks, which are disciplined by targeting the standard deviation of labor market flows in deviation from trend directly. The (ω, r)-specification adds fluctuations in the discount rate $r = 1/\beta - 1$, an approach explored within the standard framework by Hall [2017].[28] Shocks to the discount factor translate in additional fluctuations in labor flows, but the standard deviation of the job-finding rate is still more than 50 percent lower than in the data. The (ω, δ)-specification instead considers an additional shock to the exogenous separation rate δ, an additional source of aggregate fluctuations considered in several previous studies [see, among others, Moscarini and Postel-Vinay, 2016, Coles and Moghaddasi-Kelishomi, 2018]. This specification also yields a larger response of EU_t and UE_t to aggregate shocks, but the cyclical fluctuations of EE_t remain limited relative to the data. To further increase the size of labor flows, I therefore add a shock to the scale of the hiring cost function c_0 in the (ω, δ, c_0)-specification.[29] The fluctuations in labor market flows entailed by this specification are in line with the standard deviation of the EU rate and the UE rate in the data, though it tends to overstate the former. The model still generates slightly under half the standard deviation of the EE_t rate. Finally, the same combination of aggregate shocks also yields very similar cyclical properties in the steady-state calibration of the model with a lower curvature of the cost of hiring ($c_1 = 1$, rightmost column in Table 2).

[28] I let the discount rate r co-move with ω according to $\ln r(\omega_t) - \ln r = \epsilon_{r,\omega} \cdot \ln \omega_t$ to enforce the restriction $0 < \beta < 1$ at all t.

[29] The parameters in Table 2 can be interpreted as follows within the calibrated model. Looking at the most negative value of $\ln \omega$ occurring at least 1 percent of the time in the simulation, the destruction rate increases by a factor of about 10 (from $6 \cdot 10^{-4}$ at the steady state to $6 \cdot 10^{-3}$ in the (ω, δ) calibration and $7.9 \cdot 10^{-3}$ in the (ω, δ, c_0) calibration), the discount rate r increases by a factor of about 35 in the (ω, r) calibration, and the cost of hiring increases by around 26 percent in the (ω, δ, c_0) calibration. While these shocks are large, these magnitudes are not at odds with prior works. For instance, in their transition dynamics experiment of the Great Recession, Bilal et al. [2022] report requiring a 20-fold increase in the discount rate to replicate the spike in the unemployment rate (see their footnote 42).

In what follows, the (ω, δ, c_0)-specification is used as the baseline calibration of the full model. This specification is chosen because the emphasis is on worker reallocation, and it gives the best fit to aggregate fluctuations in labor market flows, the key moments emphasized in the literature.

5 Worker reallocation over the business cycle

5.1 Data vs model

I construct an index of aggregate labor productivity LP_t directly from the firm-level data as

$$\mathrm{LP}_t = \sum_i \mathrm{ES}_{i,t} \cdot \mathrm{LP}_{i,t}, \quad \mathrm{ES}_{i,t} = \frac{\text{employment}_{i,t}}{\sum_i \text{employment}_{i,t}}, \quad \mathrm{LP}_{i,t} = \ln\left(\frac{\text{value added}_{i,t}}{\text{employment}_{i,t}}\right), \tag{14}$$

where i denotes a firm.[30] To assess the worker reallocation properties of the model, I obtain two sequences of ω-shocks to fit two key time series. I first find a sequence of ω-shocks fitting the index LP_t in deviation from its HP-filter trend. Because this series is aggregated up from the micro data used to calibrate the model at the steady state, it represents my preferred option for finding a sequence of ω-shocks matching the UK business cycle. But a limitation of this time series is that the underlying firm-level data are not available before 1997. I therefore also fit an alternative sequence of shocks going back to 1955 by instead targeting the de-trended cyclical component of log-GDP. In what follows, I refer to the corresponding sequences of shocks as "LP shocks" and "GDP shocks."

I focus on a salient moment of the data closely tied to the job ladder of the model to assess the worker-reallocation properties of the model. Specifically, I decompose the labor productivity index (14) into an unweighted average and an interaction term as

$$\mathrm{LP}_t = \sum_i \mathrm{ES}_{i,t} \cdot \mathrm{LP}_{i,t} = \overline{\mathrm{LP}}_t + \sum_i \left(\mathrm{ES}_{i,t} - \overline{\mathrm{ES}}_t\right)\left(\mathrm{LP}_{i,t} - \overline{\mathrm{LP}}_t\right) = \overline{\mathrm{LP}}_t + \mathrm{OP}_t, \tag{15}$$

[30]This definition is based on Bartelsman et al. [2013].

where $\overline{\text{ES}}_t$ and $\overline{\text{LP}}_t$ denote the corresponding unweighted averages, an equality referred to in the productivity literature as the "OP" decomposition [Olley and Pakes, 1996, Bartelsman et al., 2013].[31] In Equation (15), the second term OP_t is a measure of how well labor is allocated to firms: it increases as more firms with above average productivity have a larger than average employment share. To the best of my knowledge, this paper is the first to confront a search model with aggregate shocks to the evolution of the OP decomposition at business cycle frequency.

Figure 1 shows the fit to the labor productivity index decomposition given in Equation (15) (left column). By construction, the model exactly replicates the evolution of employment-weighted firm productivity $\text{LP}_{i,t}$ and its drop around the Great Recession, since it is targeted to find the shocks (panel a). Panels (c) and (e) show that the drop in LP_t comes from both the average productivity of firms ($\overline{\text{LP}}_t$) and from the interaction term (OP_t). The model is successful at matching both terms in this decomposition, which are not targeted as part of the calibration above.

The right column of Figure 1 also shows the fit of the model to the same decomposition as in Equation (15), but for employment cost per worker $\text{EC}_{i,t}$. The model also matches the evolution of the index of employment cost per worker and its decomposition well. The simulated overall index (panel b) and unweighted component tend to slightly lead the data (panel d) while the interaction term slightly lags the data (panel f), but the magnitude of these changes during the Great Recession is very similar. Again, neither the steady-state calibration nor the full calibration use moments derived from the firm-level employment cost data.

Overall, the fit reported in Figure 1 suggests that the reallocation of workers along the productivity ladder implied by the model during the Great Recession is quantitatively in line with that implied by the firm-level data. This holds both using a measure of labor productivity and a measure of employment costs per worker at the firm level. This also

[31] This equality follows directly from expanding the second term and noting that $\sum_i \text{ES}_{i,t} = 1$ by definition.

holds for the calibration of the model with a lower curvature of the cost of hiring ($c_1 = 1$, red line in Figure 1), suggesting that this property of the model is not driven by the high curvature of the cost of hiring in the baseline calibration.

5.2 Structural decomposition

In a Rank-monotonic equilibrium, the location of workers along the job ladder is fully described by the employment-weighted distribution of workers $L_t^P(p)$. I use the tight link between the labor productivity index defined in Equation (14) and summary statistics of this distribution. This link forms the basis to a structural decomposition of aggregate labor productivity that yields further insights into the drivers of worker reallocation over the business cycle.

Start from the definition of the labor productivity index in Equation (14). In the notation of the model, this definition is given by

$$\mathrm{LP}_t = \int_{p_{E,t}}^{\overline{p}} \ln(\omega_t \tilde{p}) d\bar{L}_t^P(\tilde{p}) = \ln \omega_t + \int_{p_{E,t}}^{\overline{p}} \ln(\tilde{p}) d\bar{L}_t^P(\tilde{p}) = \ln \omega_t + \mathbf{E}_{\bar{L}_t^P}(\ln p). \tag{16}$$

Equation (16) states that LP_t is equal to two terms in the model: the aggregate productivity shock ω_t and the average employment-weighted log-productivity $\mathbf{E}_{\bar{L}_t^P}(\ln p)$.[32] This last term summarizes the position of workers in the firm productivity distribution. Each term in the "OP" decomposition (15) can similarly be expressed in the notation of the model. The unweighted average firm productivity term in this decomposition is given by

$$\overline{\mathrm{LP}}_t = \int_{p_{E,t}}^{\overline{p}} \ln(\omega_t \tilde{p}) d\bar{K}_t^P(\tilde{p}) = \ln \omega_t + \int_{p_{E,t}}^{\overline{p}} \ln(\tilde{p}) d\bar{K}_t^P(\tilde{p}) = \ln \omega_t + \mathbf{E}_{\bar{K}_t^P}(\ln p), \tag{17}$$

where I introduce the notation $\bar{K}_t^P$ for the distribution of firm productivity, the unweighted counterpart to $\bar{L}_t^P$. Equation (17) is the average (log) firm productivity in the economy and summarizes the support of the ladder on which workers move. In contrast to models without

[32] I switch back to subsuming the aggregate states in t for concision.

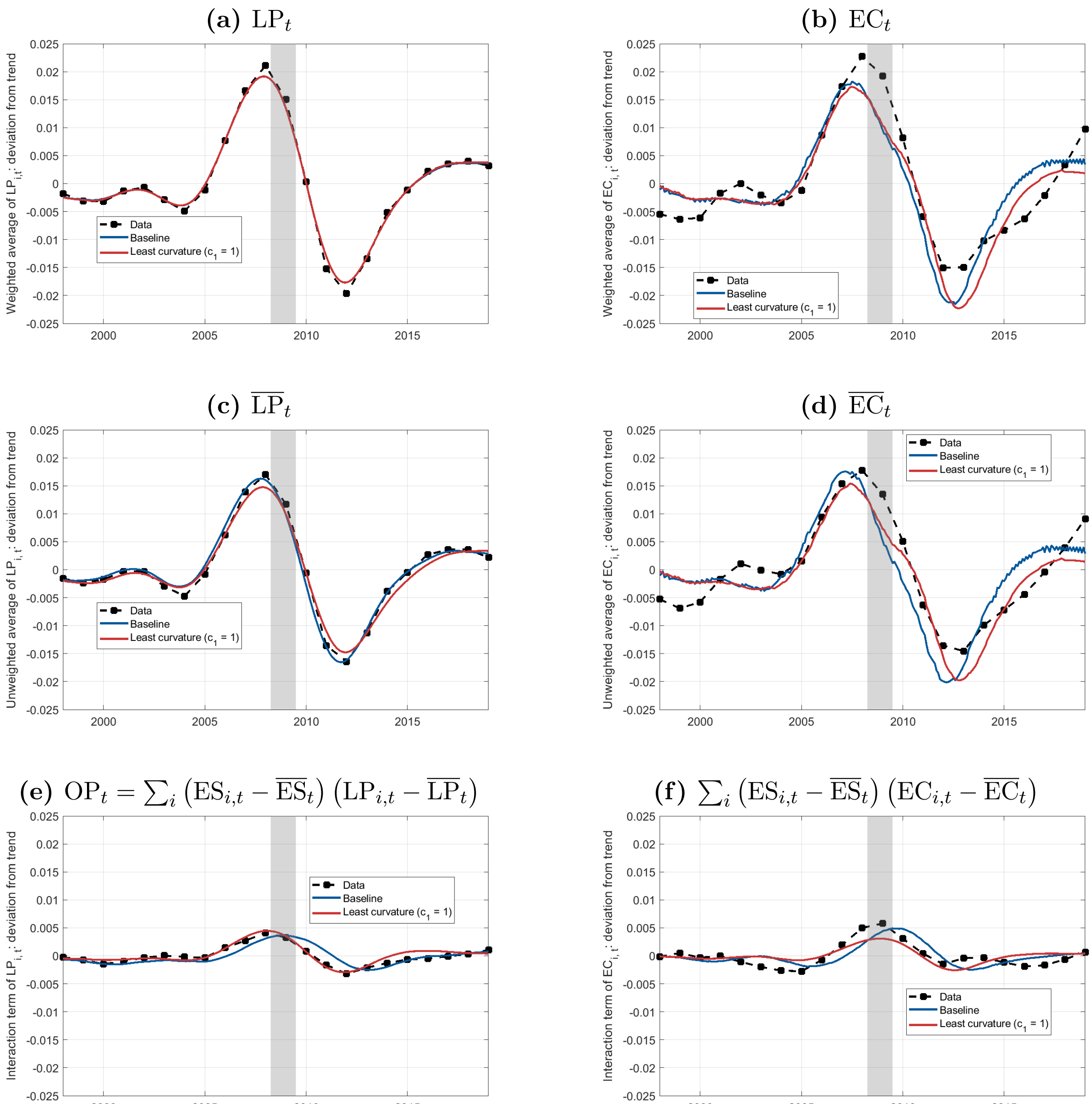

Figure 1: Fit to LP_t and EC_t decompositions. All data series are de-trended using the HP filter with smoothing parameter 100. All model series are shown in deviation from steady-state. Grey band denotes the Great Recession period in Britain. The blue line ("Baseline") is for the baseline model with aggregate shocks (ω, δ, c_0). The red line ("Least curvature") is for the $c_1 = 1$ steady-state calibration with aggregate shocks (ω, δ, c_0).

endogenous entry and exit, here the distribution of firm productivity is endogenous because the entry and exit threshold $p_{E,t}$ results from the decisions of firms. The second term of this decomposition is given by

$$\mathrm{OP}_t = \mathrm{LP}_t - \overline{\mathrm{LP}}_t = \mathbf{E}_{\bar{L}_t^P}(\ln p) - \mathbf{E}_{\bar{K}_t^P}(\ln p). \tag{18}$$

It summarizes the relative location of the weighted and unweighted distributions of firm productivity, and therefore, how high up the ladder workers are, given the current support of the firm productivity ladder. Equation (18) shows that the interaction term in the "OP" productivity decomposition directly maps into summary statistics of key equilibrium distributions implied by the model. In this sense, the fact that the model does well at replicating the evolution of this term over the recent period represents an important validation for the reallocation properties of the model (see Figure 1e).

I use this structural decomposition to study the importance of worker reallocation in the evolution of labor productivity over the post-war business cycle. For this exercise, I use the aggregate shocks obtained from fitting the evolution of log-GDP in deviation from trend, since the shocks fitted to LP_t only start in 1998.

Table 3 reports two variance decompositions of aggregate labor productivity for the baseline version of the model (steady-state parameters and aggregate shocks to ω, δ, and c_0) and three alternative specifications: (i) the version with less curvature in the hiring cost function ($c_1 = 1$), (ii) the version with a single aggregate shock (ω), and (iii) the model described in Moscarini and Postel-Vinay [2016] (hereafter MPV2016). This last model represents a benchmark with the same three aggregate shocks (ω, δ, and c_0) but without firm dynamics, and it is calibrated to the same British data as the baseline model (see Appendix D).

In Panel A of Table 3, I first decompose the variance of LP_t into its observed components ($\overline{\mathrm{LP}}_t$ and OP_t), a summary of the decomposition in Figure 1. The variance decomposition

		Alternative models			
	Data	Baseline	$c_1 = 1$	ω-only	MPV2016
Share of Var(LP_t)					
Panel A. Observed					
$\overline{\mathrm{LP}}_t$	0.830	0.837	0.776	0.928	0.023
OP_t	0.170	0.163	0.224	0.072	0.977
Panel B. Structural					
$\ln \omega_t$	—	0.296	0.290	1.048	0.023
$\mathbf{E}_{\bar{K}_t^P}(\ln p)$	—	0.541	0.486	-0.120	0.000
$\mathrm{OP}_t = \mathbf{E}_{\bar{L}_t^P}(\ln p) - \mathbf{E}_{\bar{K}_t^P}(\ln p)$	—	0.163	0.224	0.072	0.977

Table 3: Variance decomposition of the drivers of worker reallocation over the post-war business cycle. Each line reports the variance components associated with each term as a fraction of Var(LP_t). For example, the OP_t line in Panel A reports [Var(OP_t) + Cov($\overline{\mathrm{LP}}_t, \mathrm{OP}_t$)]/Var($\mathrm{LP}_t$). The model simulations are obtained from the "GDP shocks." All models include the three aggregate shocks (ω, δ, and c_0), except for the "ω-only" column. The columns report the decompositions for alternative versions of the model. See main text for details.

implied by the baseline model and the model with less curvature in the hiring cost function is similar to the data: approximately 80 percent is attributed to average firm productivity ($\overline{\mathrm{LP}}_t$) and 20 percent to worker reallocation (OP_t). By contrast, the model with a single aggregate shock tends to understate the importance of worker reallocation, which points to the importance of additional aggregate shocks to jointly generate the observed cyclical variation in labor market flows (see Table 2) and in worker reallocation. Finally, the MPV2016 model tends to greatly overstate the importance of worker reallocation. This last result, derived from a model calibrated to match similar cross-sectional and cyclical moments, suggests that allowing for firm dynamics is important to match the share of the variance of LP_t coming from worker reallocation.[33]

In Panel B of Table 3, I further decompose the variance of LP_t into its structural components, as outlined in equations (16)–(18). This decomposition shows that the baseline model

[33] Appendix Table D.2 shows that the difference in the variance decomposition between the baseline model and the MPV2016 model is a characteristic of the multi-shock calibration (ω, δ, c_0). In the single-shock calibration (ω), both models attribute a similar share of the variance of LP_t to worker reallocation.

and the model with less curvature in the hiring cost function attribute changes in aggregate labor productivity to similar structural components: 30 percent to the direct effect of the aggregate shock ($\ln \omega_t$), 50 percent to firm selection $\left(\mathbf{E}_{\bar{K}_t^P}(\ln p)\right)$, and 20 percent to worker reallocation $\left(\mathbf{E}_{\bar{L}_t^P}(\ln p) - \mathbf{E}_{\bar{K}_t^P}(\ln p)\right)$. The model with a single aggregate shock yields a very different structural variance decomposition, where the direct impact of $\ln \omega_t$ is the key driver of fluctuations in aggregate labor productivity and firm selection dampens the impact of aggregate productivity shocks. I discuss this point in more detail below. Finally, the structural decomposition of the variance of LP_t in the MPV2016 model is entirely attributed to either the direct impact of the shock or worker reallocation because of the absence of firm entry and exit.[34]

I also use the calibrated model to quantify the evolution of each term in the structural decomposition across post-war recessions in the baseline model. In Figure 2, I use the start date of each recession as a starting point and show the evolution of each term in the structural decomposition of LP_t given by equations (16)–(18) for four years after that date.[35] All series are given in deviation from their value at the start of the recession, and I report the average and a one standard-deviation band across recessions. For comparison, I also show the series specifically for the Great Recession. The takeaway from Figure 2 is that the impact of recessions on the job ladder is persistent relative to the shock. While the aggregate shock is typically close to its pre-recession value after four years (panel a), the overall allocation of workers to firms is still clearly worse relative to the start of the recession at the same horizon (panel b). This is driven both by the selection of firms $\mathbf{E}_{\bar{K}_t^P}(\ln p)$ (panel c) and worker reallocation OP_t (panel d). This exercise also confirms the relative importance of each component documented in Table 3. Three years after the start of a typical recession, the OP_t term accounts for around 20 percent of the overall decrease in worker reallocation.[36] Finally,

[34] This property also holds in models where firm entry and exit is exogenous, such as the model developed by Coles and Mortensen [2016].

[35] The full list of recessions is given in Appendix Table B.1.

[36] In Appendix F, I show the evolution of each of these terms in a counterfactual experiment where I make the flow value of unemployment countercyclical, mimicking the impact of the unemployment insurance extension program in the US. This experiment suggests that the negative effect of worker reallocation on

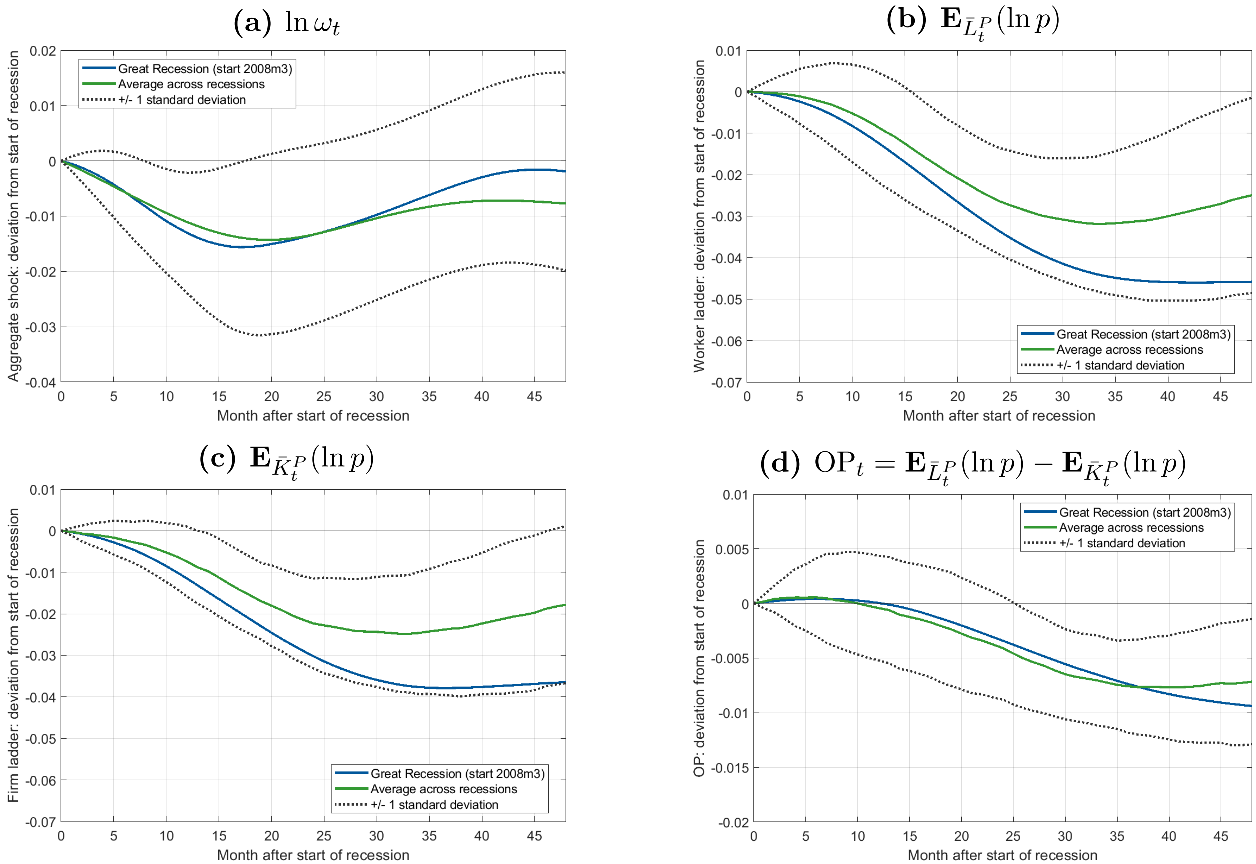

Figure 2: Worker Ladder decomposition across post-war recessions in Britain. Each subpanel reports the evolution of each term in the structural decomposition of LP_t given by equations (16)–(18). The series are given in deviation from their value at the start of each post-war recession in Britain. The blue line is for the Great Recession. The green line averages across all seven recessions in the 1955-2020 sample. Dashed lines give a one standard-deviation error band across recessions.

in terms of worker reallocation along the job ladder, the Great Recession is interpreted as a large recession through the lens of the model, but not unusually large.

5.3 Discussion

Table 3 shows that firm selection differs across model specifications. The model with a single aggregate shock ω points to a negative contribution of firm selection to the variance of LP_t. This comes from a standard "cleansing" mechanism: negative productivity shocks increase the firm entry-exit threshold $p_{E,t}$. Conversely, the baseline specification with additional shocks to δ and c_0 points to a positive contribution of firm selection to the variance of LP_t. This less intuitive property of the model comes from several forces. First, δ-shocks directly affect the share of the surplus going to workers.[37] Second, general equilibrium effects imply that less productive firms are more likely to retain workers since there is a larger pool of unemployed workers from which more productive employers can recruit, while the value of being unemployed decreases. In all, at the calibrated parameters, negative productivity shocks lower the firm entry-exit threshold $p_{E,t}$ in the model with additional shocks. Additional simulations illustrating the impact of alternative aggregate shocks can be found in Appendix E.

I stress two main takeaways from the firm dynamics implied by alternative aggregate shocks. First, while the benchmark model with additional shocks can match the volatility of both output and labor market flows, its firm entry-exit dynamics are at odds with the data given that the total number of firms in the economy is typically pro-cyclical. In this sense, there is still a tension between the model with additional aggregate shocks, which matches the volatility of output and labor market flows, and the model with a single shock, which matches, at least directionally, the cyclicality of firm entry and exit. This property of the model with additional shocks is also potentially shared with other frameworks, especially

aggregate productivity is even larger than documented in the main text with countercyclical unemployment insurance extensions.

[37] It can be shown from the expression for the optimal wage contract (10) that $\partial V(p, \omega, L)/\partial \delta \leq 0$.

since δ-shocks are commonly used in the literature, but it would only become apparent in models with endogenous firm entry and exit, such as the one developed here.

Second, the fact that firms can largely pass on the impact of shocks to workers suggests that wages are too flexible. With contract posting, the relevant variable is the employment wage contract, which conditions workers' decisions to remain employed at the firm, but this model object does not have a direct empirical counterpart. It is, however, possible to compute the pass-through elasticity of wages to productivity shocks in the model. This elasticity is found to be around 0.7, well above the 0.05–0.2 estimates typically found in the literature.[38]

An interesting direction for future research would be to study environments with a similarly rich degree of firm heterogeneity where, in addition, wages are both rigid enough to match the pass-through elasticity the pass-through elasticity found in the data, and where this rigidity is sufficiently binding to affect the firm's job creation and job destruction decisions. This last property is likely to improve the model's ability to match the volatility of labor market flows with a single aggregate shock. Although retaining tractability in such an environment with aggregate shocks is likely to prove challenging, it would further enlarge the set of empirical regularities that can be reproduced within this class of models.

6 Conclusion

I develop a random search model with three key features: (i) on-the-job search, (ii) firm dynamics, and (iii) aggregate shocks. Tractability is retained in this rich environment by identifying a set of conditions on the cost of hiring function such that agents' decisions can be expressed as a function of firm-specific productivity, aggregate productivity, and the employment-weighted distribution of firm productivity. The optimal wage contract offered by firms admits a closed-form solution, so wages are straightforward to compute. Building on

[38] See Table 1 in Card et al. [2017] for a review. I cannot compute this elasticity directly in the British administrative data because the ABS/ARD dataset is a repeated cross-section.

these theoretical results, I propose a numerical solution method suitable to this environment with endogenous firm entry-exit and aggregate shocks.

In the quantitative part of the paper, I fit the model to data on the cross-section of firms and the cyclicality of labor productivity and labor flows. A key novelty of my analysis is to confront the model with firm-level data on worker reallocation across the firm productivity distribution, thus providing a direct quantitative test for its implied job ladder. I find that the model successfully matches the OP decomposition of labor productivity derived from the data, which attributes around 20 percent of the variance of labor productivity to worker reallocation. By contrast, the leading job ladder model proposed by Moscarini and Postel-Vinay [2013, 2016] attributes nearly all of this variance to worker reallocation.

Beyond the framework developed here, using summary statistics on the reallocation of workers across the firm productivity distribution to benchmark this class of models is likely to prove useful in future work.

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Supplementary Materials
"Firm Dynamics and Random Search over the Business Cycle"

Richard Audoly*

A Omitted derivations and proofs

A.1 Size-independent firm profits

The claim is that the functional equation defined by the firm's value function (6) admits solutions of the form $\Pi_{t-1}\left(p_{t-1}, n_{t-1}, \overline{V}\right) = n_{t-1} J_{t-1}\left(p_{t-1}, \overline{V}\right)$ and that the implied firm policies are size-independent.

I start by showing that the functional operator defined by the firm's value (6) preserves linear homogeneity. Assume that $\Pi_t(p_t, n_t, W) = n_t J_t(p_t, W)$. Starting from the equation for firm profits (6), still subject to the law of motion for employment (3) and the promise-keeping constraint (7), the term inside the expectation on the right-hand side rewrites

$$
\begin{aligned}
& -c(h)(1-\mu)(1-\delta_t)n_{t-1} + \Pi_t(p_t, n_t, W) \\
& \quad = -c(h)(1-\mu)(1-\delta_t)n_{t-1} + n_t J_t(p_t, W) \\
& \quad = n_{t-1}(1-\mu)(1-\delta_t)\Big[-c(h) + (1-q_t(W)+h)J_t(p_t, W)\Big]
\end{aligned}
$$

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for any arbitrary decision to continue χ, choice of contract value W, and hiring rate h. The second line substitutes in the guess $n_t J_t(p_t, W)$. The last line uses the law of motion for the firm's workforce. Using this last expression in firm profits (6) gives

$$\Pi_{t-1}\left(p_{t-1}, n_{t-1}, \overline{V}\right) =$$
$$n_{t-1} \max_{w,W,\chi,h\geq 0} \left\{\omega_{t-1}p_{t-1} - w + \beta\mathbf{E}_{t-1}\left[\chi\cdot\Big((1-\mu)(1-\delta_t)\big[-c(h)+(1-q_t(W)+h)J_t(p_t,W)\big]\Big)\right]\right\}.$$

It therefore follows that $\Pi_{t-1}\left(p_{t-1}, n_{t-1}, \overline{V}\right) = n_{t-1}J_{t-1}\left(p_{t-1}, \overline{V}\right)$ with

$$J_{t-1}\left(p_{t-1}, \overline{V}\right) = \max_{w,W,\chi,h\geq 0}\left\{\omega_{t-1}p_{t-1} - w \right.$$
$$\left. + \beta\mathbf{E}_{t-1}\left[\chi\cdot\Big((1-\mu)(1-\delta_t)\big[-c(h)+(1-q_t(W)+h)J_t(p_t,W)\big]\Big)\right]\right\}. \quad \text{(A.1)}$$

This last expression represents a per worker formulation to the firm's problem, still subject to the promise-keeping constraint (7), which shows that linearly homogenous solutions are admissible.

It follows directly from Equation (A.1) that the corresponding optimal choices for the continuation decision $\chi_t(p_t)$, contract $V_t(p)$, and hiring rate $h_t(p)$ are all independent of the firm's employment size n_{t-1}.

A.2 Firm-workers' match surplus

Solving the promise-keeping constraint (7) for w conditional on some state-contingent contract W and the decision to continue χ gives

$$w = \overline{V} - \beta\mathbf{E}_{t-1}\left\{\mu Q_t + (1-\mu)\left[\Big((1-\chi)+\delta_t\chi\Big)U_t \right.\right.$$
$$\left.\left. + \chi\cdot(1-\delta_t)\Big(\big(1-q_t(W)\big)W + s\lambda_t\int \max\big\{\tilde{W}, W\big\}dF_t(\tilde{W})\Big)\right]\right\}.$$

Starting from the definition of $S_{t-1}(p_{t-1})$ and substituting w above in the expression for firm profit per worker (A.1) gives

$$\begin{aligned} S_{t-1}(p_{t-1}) &= J_{t-1}(p_{t-1}, \overline{V}) + \overline{V} \\ &= -\overline{V} + \max_{W,h\geq 0,\chi} \Bigg\{ p_{t-1}\omega_{t-1} \\ &\quad + \beta \mathbf{E}_{t-1}\Bigg[\mu Q_t + (1-\mu)\Bigg(\big(1-\chi\big)U_t + \chi\delta_t U_t \\ &\quad + \chi\cdot(1-\delta_t)\Big[\big(1-q_t(W)\big)W + s\lambda_t \int \max\big\{\tilde{W}, W\big\} dF_t(\tilde{W}) \\ &\quad - c(h) + \big(1-q_t(W)+h\big)\pi_t(p_t, W)\Big]\Bigg)\Bigg]\Bigg\} + \overline{V}. \end{aligned}$$

Using the fact that the continuation decision can be expressed as a function of the joint firm-workers' surplus $\chi_t(p_t) = \mathbb{1}\{S_t(p_t) \geq U_t\}$, taking the max operator inside the expectation, and grouping terms gives

$$\begin{aligned} S_{t-1}(p_{t-1}) = p_{t-1}\omega_{t-1} + \beta \mathbf{E}_{t-1}\Bigg[\mu Q_t + (1-\mu)\Bigg(\big(1-\chi_t(p_t)\big)U_t + \chi_t(p_t)\delta_t U_t \\ + \chi_t(p_t)(1-\delta_t) \max_{W,h\geq 0} \Big\{ -c(h) + \big(1-q_t(W)\big)S_t(p_t) + \big(S_t(p_t) - W\big)h \\ + (1-\delta_t)s\lambda_t \int \max\big\{\tilde{W}, W\big\} dF_t(\tilde{W})\Big\}\Bigg)\Bigg]. \end{aligned} \tag{A.2}$$

A.3 Labor market aggregates with size independence

I report the formal steps to confirm the intuition that the employment-weighted distribution of firm productivity $L_t(p)$ allows firms to compute the acceptance rate (5) and offer distribution (1). With a slight abuse of notation, $L_t(p)$ can be written

$$L_t(p) = \int_{\tilde{p}\leq p} \int_n n dM_t(\tilde{p}, n), \qquad dL_t(p) = \int_n n dM_t(p, n),$$

and the unemployment rate

$$u_t = 1 - \int_p \int_n n dM_t(p,n) = 1 - \int_p dL_t(p).$$

Given Result 1, we can immediately check that the numerator of the acceptance rate (5) can be written

$$\begin{aligned}
&u_t + (1-\delta_t)s \int_p \int_n \mathbb{1}\{V_t(p) \leq W_t\}\chi_t(p) n dM_t(p,n) \\
&\quad = u_t + (1-\delta_t)s \int_p \mathbb{1}\{V_t(p) \leq W_t\}\chi_t(p) \int_n n dM_t(p,n) \\
&\quad = u_t + (1-\delta_t)s \int_p \mathbb{1}\{V_t(p) \leq W_t\}\chi_t(p) dL_t(p).
\end{aligned}$$

A similar derivation can be used for the denominator of the acceptance (5) to show that

$$Y_t(W_t) = \frac{u_t + (1-\delta_t)s \int \mathbb{1}\{V_t(p) \leq W_t\}\chi_t(p) dL_t(p)}{u_t + (1-\delta_t)s \int \chi_t(p) dL_t(p)}. \tag{A.3}$$

Turning to the contract offer distribution, it can be checked from the ads posting condition (4) that posted ads $a_t(p,n)$ are linear in n:

$$a_t(p,n) = \frac{h_t(p)}{\eta_t Y_t(V_t(p))}(1-\mu)(1-\delta_t)n.$$

We can then immediately check that the numerator of the offer distribution (1) can be written

$$\begin{aligned}
&\int_p \int_n \mathbb{1}\left\{V_t(p) \leq W\right\}\chi_t(p) a_t(p,n) dM_t(p,n) \\
&\qquad\qquad = \frac{(1-\mu)(1-\delta_t)}{\eta_t} \int_p \mathbb{1}\left\{V_t(p) \leq W\right\}\chi_t(p) \frac{h_t(p)}{Y_t(V_t(p))} dL_t(p).
\end{aligned}$$

Because the constant terms in front cancel out, a similar derivation for the denominator of the offer distribution (1) gives

$$F_t(W) = \frac{\int \mathbb{1}\left\{V_t(p) \leq W\right\} \chi_t(p) \cdot h_t(p)/Y_t\big(V_t(p)\big) dL_t(p)}{\int \chi_t(p) \cdot h_t(p)/Y_t\big(V_t(p)\big) dL_t(p)}.$$

A.4 Proof of rank-monotonic equilibrium

The proof is similar in spirit to those in Moscarini and Postel-Vinay [2013, 2016]. The goal is to show that the optimal contract is increasing in the firm's current realization of productivity p assuming the existence of a recursive equilibrium (Definition 1). Throughout, I subsume the aggregate state $\big(\omega, L\big)$ in the subscript t for concision.

The key difference with Moscarini and Postel-Vinay [2013, 2016] is that the firm's problem can be considered separately for each worker. Using Result 1, we can restrict attention to equilibria in which the firm's value is linear in its size n, and therefore the firm's problem can be expressed solely in terms of the individual firm-workers' surplus (A.2). As a result, there is no need to account for the effect of contract value on the firm's own size. By contrast, Moscarini and Postel-Vinay [2013, 2016] establish super-modularity so that more productive firms are offering larger employment contracts and are larger in size on the equilibrium path. In this model, the firm's optimal policies are independent of size, so the argument of the proof is simpler.

The proof proceeds in three steps:

- Step 1: Conditional on S being increasing in p, $hc''(h)/c'(h) \geq 1$ for all $h \geq 0$ is sufficient to guarantee that the optimal contract V is weakly increasing in p.
- Step 2: The operator defined by the firm-workers' surplus (A.2) maps differentiable and increasing functions of p into differentiable and increasing functions of p.
- Step 3: Conditional on an equilibrium existing, the value function S is smooth and weakly increasing in p.

Step 1: Sufficient conditions on recruitment cost for an RME

Assume that S_t is increasing in p. Conditional on the firm surviving, the firm's unconstrained problem (9) is given by

$$\Psi_t(p) = \max_{\substack{W\\ h\geq 0}} -c(h) + \big(1 - q_t(W)\big) S_t(p) + h\big(S_t(p) - W\big) + s\lambda_t \int_W^\infty \tilde{W} dF_t(\tilde{W}).$$

At any interior maximum, the firm's optimal choice $\big(h_t, V_t\big)$ must satisfy the following first-order conditions

$$0 = -c'\big(h_t(p)\big) + S_t(p) - V_t(p)$$
$$0 = -q_t'\big(V_t(p)\big)\big(S_t(p) - V_t(p)\big) - h_t(p)$$

where I have implicitly used the assumption that F_t is everywhere differentiable to write $q_t' = -s\lambda_t F_t'$. In addition, the associated Hessian matrix H_t^Ψ must be negative-definite, which requires

$$\det\big(H_t^\Psi\big) = -c''\big(h_t(p)\big)\left[q_t''\big(V_t(p)\big)\Big(S_t(p) - V_t(p)\Big) + q_t'\big(V_t(p)\big)\right] - 1 > 0.$$

The two first-order conditions can be combined to give the following expression in $V_t(p)$

$$-c'\Big(- q_t'\big(V_t(p)\big)\big(S_t(p) - V_t(p)\big)\Big) + S_t(p) - V_t(p) = 0$$

and totally differentiating that last expression with respect to p gives

$$\frac{dV_t(p)}{dp} = \frac{\partial S_t(p)}{\partial p} \cdot \frac{-q_t'\big(V_t(p)\big)c''\big(h_t(p)\big) - 1}{\det(H_t^\Psi)}.$$

In this last expression, $\det\big(H_t^\Psi\big)$ is positive at any maximum. By assumption, the firm-workers' surplus (8) is increasing in p, so $\partial S_t(p)/\partial p \geq 0$. Noting that the two FOCs can be combined to give $-q_t'(V_t(p))c'(h_t(p)) = h_t(p)$, it follows that

$$\frac{dV_t(p)}{dp} \geq 0 \iff -q_t'\big(V_t(p)\big)c''\big(h_t(p)\big) - 1 \geq 0 \iff \frac{h_t(p)c''\big(h_t(p)\big)}{c'\big(h_t(p)\big)} \geq 1.$$

Step 2: Firm-workers' surplus operator preserves smoothness and monotonicity

Assume that S_t is differentiable and increasing in p_t. We want to show that

$$\begin{aligned} S_{t-1}(p_{t-1}) &= p_{t-1}\omega_{t-1} \\ &\quad + \beta \mathbf{E}_{t-1}\left\{\mu Q_t + (1-\mu)\left[\Big(\big(1-\chi_t(p_t)\big) + \delta_t \chi_t(p_t)\Big)U_t + \chi_t(p_t)(1-\delta_t)\Psi_t(p_t)\right]\right\}. \end{aligned}$$

is differentiable and increasing in p_{t-1}, where again Ψ_t denotes the firm's maximization problem

$$\Psi_t(p_t) = \max_{\substack{W \\ h\geq 0}} -c(h) + \big(1-q_t(W)\big)S_t(p_t) + h\big(S_t(p_t) - W\big) + s\lambda_t \int_W^\infty \tilde{W} dF_t(\tilde{W}).$$

Differentiability of S_{t-1} in p_{t-1} follows directly from noting that the expectation in this last expression is differentiable in p_{t-1} as long as the conditional probability density of future productivity is. This is true by assumption.

To show that S_{t-1} is increasing conditional on $\partial S_t(p_t)/\partial p_t \geq 0$, first note that the envelope condition of the firm's optimization problem (9) gives

$$\frac{d\Psi_t(p_t)}{dp_t} = \frac{\partial \Psi_t(p_t)}{\partial p_t} = \Big(1 - q_t\big(V_t(p_t)\big) + h_t(p_t)\Big)\frac{\partial S_t(p_t)}{\partial p_t} \geq 0, \tag{A.4}$$

where $\big(V_t(p_t), h_t(p_t)\big)$ denote the optimal policies in the firm's optimization problem. The term inside the expectation in the firm-workers' surplus is then weakly increasing in p. It is weakly increasing in p by the envelope condition (A.4) on the part of the support of p where the firm continues. It is constant on the part of the support of p where the firm exits. Because the decision to continue can be written in terms of the firm-workers' surplus

$\chi_t(p_t) = \mathbb{1}\{S_t(p_t) \geq U_t\}$, there exists a unique exit productivity threshold in the support of $[\underline{p}, \overline{p}]$.

To complete the proof, the assumption that the Markov process for firm-specific productivity satisfies first-order stochastic dominance is required. With this assumption, conditional on any two distinct previous realizations of p, the conditional densities of future idiosyncratic productivity satisfy a single-crossing property. Let $\hat{p}$ denote this crossing point. Let p_1 and p_2 be two productivity levels such that $p_2 > p_1$. The difference $S_{t-1}(p_2) - S_{t-1}(p_1)$ is then given by

$$S_{t-1}(p_2) - S_{t-1}(p_1) = \omega_{t-1}(p_2 - p_1) + \beta(1-\mu)\Big(\mathbf{E}_{t-1}\big[\kappa_t(p_t)\ |p_2\big] - \mathbf{E}_{t-1}\big[\kappa_t(p_t)\ |p_1\big]\Big),$$

where $\kappa_t(p_t)$ is a notation for the terms inside the expectation

$$\kappa_t(p_t) = \mu Q_t + (1-\mu)\Big[\Big(\big(1-\chi_t(p_t)\big) + \delta_t\chi_t(p_t)\Big)U_t + \chi_t(p_t)(1-\delta_t)\Psi_t(p_t)\Big].$$

I now explicitly condition on the current realization of productivity in the expectation operator. Showing that S_{t-1} is increasing in p amounts to showing that the difference in expectation in the last expression is non-negative. This difference can be rewritten

$$\int_{\underline{p}}^{\overline{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big)dp_t,$$

denoting $\gamma(p_t|p_{t-1})$ the density of p_t conditional on p_{t-1} and the expectation is now taken over the aggregate states. Now, given the crossing-point $\hat{p}$, we can rewrite

$$\begin{aligned}
&\int_{\underline{p}}^{\overline{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big)dp_t \\
&= \int_{\underline{p}}^{\hat{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big)dp_t + \int_{\hat{p}}^{\overline{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big)dp_t
\end{aligned}$$

and, since $\mathbf{E}_{t-1}\big[\kappa_t(p)\big]$ is weakly increasing in p, we can bound the terms in this last expression as

$$\int_{\underline{p}}^{\hat{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big) dp_t \geq \mathbf{E}_{t-1}\big[\kappa_t(\hat{p})\big] \int_{\underline{p}}^{\hat{p}} \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big) dp_t$$

and

$$\int_{\hat{p}}^{\overline{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big) dp_t \geq \mathbf{E}_{t-1}\big[\kappa_t(\hat{p})\big] \int_{\hat{p}}^{\overline{p}} \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big) dp_t,$$

where I use that, by the single-crossing property, $\gamma(p_t|p_2) - \gamma(p_t|p_1) \leq 0$ for $p \in [\underline{p}, \hat{p}]$ and $\gamma(p_t|p_2) - \gamma(p_t|p_1) \geq 0$ for $p \in [\hat{p}, \overline{p}]$. Finally, summing up the last two inequalities, we get

$$\mathbf{E}_{t-1}\big[\kappa_t(p_t)\ |p_2\big] - \mathbf{E}_{t-1}\big[\kappa_t(p_t)\ |p_1\big] = \int_{\underline{p}}^{\overline{p}} \mathbf{E}_{t-1}\big[\kappa_t(p_t)\big] \cdot \big(\gamma(p_t|p_2) - \gamma(p_t|p_1)\big) dp_t \geq 0.$$

This last inequality shows that $S_{t-1}(p_2) \geq S_{t-1}(p_1)$ for $p_2 > p_1$.

Step 3: Smoothness and monotonicity of firm-workers' surplus in equilibrium

The previous step establishes that the operator defining the firm-workers' surplus (A.2) preserves smoothness and monotonicity in p, but it does not prove that any solution to that functional equation, provided it exists, has these properties. This can be shown using a similar argument as in Moscarini and Postel-Vinay [2016, Appendix A], which I briefly outline here.

Fix an arbitrary measure of employment-weighted firm productivity L^* over $\big[\underline{p}, \overline{p}\big]$ and consider the associated firm-workers' surplus operator taking functions of $(p, \omega) \in \big[\underline{p}, \overline{p}\big] \times \big[\underline{\omega}, \overline{\omega}\big]$ as argument. This is a contraction mapping of functions defined on closed intervals of real numbers, so Blackwell's conditions are satisfied and the contraction mapping theorem can be applied. Since this operator also preserves smoothness and monotonicity, the unique solution S_{L^*} is smooth and increasing in p.[1]

[1]More formally, this requires showing that space of smooth and increasing functions over $\big[\underline{p}, \overline{p}\big] \times \big[\underline{\omega}, \overline{\omega}\big]$ is a closed subset of the space of smooth functions over $\big[\underline{p}, \overline{p}\big] \times \big[\underline{\omega}, \overline{\omega}\big]$.

For the general firm-workers' surplus (A.2) operator mapping functions defined over $(p,\omega,L) \in \left[\underline{p},\overline{p}\right] \times \left[\underline{\omega},\overline{\omega}\right] \times \mathcal{F}$, where $\mathcal{F}$ denotes the space of functions over which the measure L is defined, Blackwell's conditions do not apply, since they are restricted to functions defined on closed intervals of real numbers. But, if a solution to the general functional equation S exists, then for every fixed $L^* \in \mathcal{F}$ we have that for all $(p,\omega) \in \left[\underline{p},\overline{p}\right] \times \left[\underline{\omega},\overline{\omega}\right]$, $S(p,\omega,L^*) = S_{L^*}(p,\omega)$. Therefore, provided an equilibrium exists, the value function S is smooth and increasing in p.

A.5 Derivation of employment contract in an RME

This appendix contains the proof of Result 3. The proof proceeds in two steps: (i) it derives the distribution of offered contracts in an RME (12), and (ii) it derives the optimal RME contract (10). Throughout, I subsume the aggregate state (ω, L) in the subscript t for concision.

Offer distribution in an RME

From the equation for the acceptance rate rewritten using size-independence (A.3), we can immediately check that

$$Y_t\big(V_t(p)\big) = \frac{u_t + (1-\delta_t)s\int_{p_{E,t}}^{p} dL_t(\tilde{p})}{u_t + (1-\delta_t)s\int_{p_{E,t}}^{\overline{p}} dL_t(p)} = \frac{u_t + (1-\delta_t)s\Big[L(p) - L\big(p_{E,t}\big)\Big]}{u_t + (1-\delta_t)s\Big[L(\overline{p}) - L\big(p_{E,t}\big)\Big]}.$$

Using the firm's ads posting (4) and the equality $\eta_t A_t = \lambda_t Z_t$, the offer distribution (1) can be rewritten

$$\begin{aligned} F_t(W) &= A_t^{-1}\int_p\int_n \mathbb{1}\left\{V_t(p) \leq W\right\}\chi_t(p)a_t(p,n)dM_t(p,n) \\ &= \int_p \mathbb{1}\left\{V_t(p) \leq W\right\}\frac{\chi_t(p)h_t(p)(1-\mu)(1-\delta_t)}{Z_t\lambda_t Y_t(W)}dL_t(p). \end{aligned}$$

Evaluating this last expression at the optimal contract $V_t(p)$ and using the expression for the acceptance rate $Y_t\big(V_t(p)\big)$ derived above gives

$$\lambda_t F_t\big(V_t(p)\big) = \int_{p_E}^{p} \frac{(1-\delta_t)h_t(\tilde{p})}{u_t + (1-\delta_t)s\big[L_t(\tilde{p}) - L_t(p_E)\big]} dL_t(\tilde{p}).$$

Equilibrium contract in an RME

Start from the first-order condition with respect to the value of contracts in the firm's unconstrained problem (9) for some firm-productivity level p

$$-q_t'(V_t(p))\big[S_t(p) - V_t(p)\big] = h_t(p).$$

The derivative of the quit rate is given by $q_t'(W) = -s\lambda_t F_t'(W)$. In a rank-monotonic equilibrium, the derivative of the offer distribution (12) is given by

$$\lambda_t F_t'\big(V_t(p)\big)\frac{dV_t(p)}{dp} = \frac{(1-\delta_t)h_t(p)l_t(p)}{u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]},$$

with $l_t(p) = dL_t(p)/dp$. Combining the last three expressions yields the following first-order differential equation in V_t

$$\frac{dV_t(p)}{dp} + \frac{s(1-\delta_t)l_t(p)}{u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]}V_t(p) = \frac{s(1-\delta_t)l_t(p)}{u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]}S_t(p)$$

with boundary condition $V_t(p_{E,t}) = U_t$. Noting that

$$\frac{d\ln\Big(u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]\Big)}{dp} = \frac{s(1-\delta_t)l_t(p)}{u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]},$$

the corresponding integrating factor is then

$$\exp\int \frac{s(1-\delta_t)l_t(p)}{u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]}dp = u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big].$$

Along with the boundary condition, $V_t(p_{E,t}) = U_t$, this yields the expression for the optimal contract (10) in the main text

$$V_t(p) = \frac{u_t U_t + (1-\delta_t)s \int_{p_{E,t}}^{p} S_t(\tilde{p}) dL_t(\tilde{p})}{u_t + (1-\delta_t)s\big[L_t(p) - L_t(p_{E,t})\big]}.$$

A.6 Alternative formulation: Firm-workers' net surplus

This Appendix shows that the model solution can be expressed in terms of a single value function. Subtracting the value of unemployment (2) from the firm-workers' surplus (8), the firm's problem can equivalently be expressed in terms of *net* firm-workers' surplus. I omit this notation from the main text so as not to clutter the description of the model. This more compact formulation is used when solving the model.

A.6.1 Firm-workers' net surplus

The net firm-workers' surplus is defined as the firm-workers' surplus net of the value of unemployment:

$$\Sigma_{t-1}(p) = J_{t-1}\big(p, \overline{V}\big) + \overline{V} - U_{t-1} = S_{t-1}(p) - U_{t-1}.$$

Adding and subtracting U_t within the expectation, the firm-workers' surplus (8) can be re-arranged as

$$S_{t-1}(p) \;=\; p_{t-1}\omega_{t-1} + \beta \mathbf{E}_{t-1}\Big[U_t + \mu\big[Q_t - U_t\big] + (1-\mu)\chi_t(p_t)(1-\delta_t)\big[\Psi_t(p_t) - U_t\big]\Big].$$

Using the same strategy, the unemployed worker's value can similarly be rearranged as

$$U_{t-1} = b + \beta \mathbf{E}_{t-1}\Big[U_t + \mu\big[Q_t - U_t\big] + (1-\mu)\lambda_t \int \max\{\tilde{W} - U_t, 0\} dF_t(\tilde{W})\Big].$$

Let $W_\Sigma = W - U_t$ denote the value of the offered contract net of the value of unemployment. Let $F_{\Sigma,t}$ be the corresponding distribution of contracts, so $F_t(W) = F_t(W_\Sigma + U_t) = F_{\Sigma,t}(W_\Sigma)$. The net firm-workers' surplus $\Sigma_{t-1}(p) = S_{t-1}(p) - U_{t-1}$ can then be expressed as

$$\begin{aligned}\Sigma_{t-1}(p) &= p_{t-1}\omega_{t-1} - b \\ &\quad + \beta(1-\mu)\mathbf{E}_{t-1}\left[\chi_t(p_t)\left\{(1-\delta_t)\tilde{\Psi}_t(p) - \lambda_t \int \max\{\tilde{W}_\Sigma, 0\} dF_{\Sigma,t}(\tilde{W}_\Sigma)\right\}\right] \qquad \text{(A.5)}\end{aligned}$$

where $\tilde{\Psi}_t(p)$ is the firm's optimization problem in net surplus form

$$\tilde{\Psi}_t(p) = \max_{W_\Sigma, h \geq 0} \left\{ -c(h) + \big[1 - q_{\Sigma,t}(W_\Sigma)\big]\Sigma_t(p) + h\big[\Sigma_t(p) - W_\Sigma\big] + s\lambda_t \int_{W_\Sigma}^{\infty} \tilde{W}_\Sigma dF_t(\tilde{W}_\Sigma)\right\}.$$

A.6.2 Firm policies as a function of the firm-workers' net surplus in an RME

Since $\Sigma = S - U$ and U does not depend on p, Σ is also increasing in p for every candidate equilibrium. In a RME, the corresponding net contract follows by subtracting $U(\omega, L)$ in the expression for the optimal contract (10), which gives

$$V(p,\omega,L) - U(\omega,L) = V_\Sigma(p,\omega,L) = \frac{\big(1-\delta(\omega)\big)s\int_{p_E(\omega,L)}^{p} \Sigma(\tilde{p},\omega,L)dL(\tilde{p})}{u + \big(1-\delta(\omega)\big)s\Big[L(p) - L\big(p_E(\omega,L)\big)\Big]}. \qquad \text{(A.6)}$$

The optimal hiring rate can also be expressed as

$$c'\big(h(p,\omega,L)\big) = \Sigma(p,\omega,L) - V_\Sigma(p,\omega,L),$$

and the entry/exit decision as

$$\chi(p,\omega,L) = \mathbb{1}\big\{\Sigma(p,\omega,L) \geq 0\big\}.$$

B Data

B.1 Firm-level data

I use two main sources of firm-level administrative data from Britain:

1. The Annual Respondents Database [Office for National Statistics, 2020] and its successor the Annual Business Survey [Office for National Statistics, 2021] give detailed yearly balance-sheet information from the universe of large firms (with more than 250 employees) and a stratified random sample of smaller businesses (with fewer than 250 employees). The Annual Respondents Database (ARD) has data from 1997 to 2008. The Annual Business Survey has data from 2009 onward.

2. The Business Structure Database [Office for National Statistics, 2019] is a snapshot from the registry of all British businesses, but it only has data on some basic variables (employment, estimated turnover, industry). Businesses must satisfy one of two conditions to be included in the Business Structure Database. They must have either a sales turnover above the VAT registration threshold or at least one employee. In practice, these restrictions imply that all but the smallest businesses and the self-employed are included in these data.

Since the Business Structure Database (BSD) does not have information on value added or employment costs, I follow the procedure in Riley et al. [2015] to obtain meaningful aggregates from the Annual Respondents Database (ARD)/Annual Business Survey (ABS). I use the "gross value added at factor costs" and "total employment costs" variables, which are harmonized across survey year by the data provider, as the relevant concepts for a firm's value added and wage bill. I deflate these measures using industry-level deflators provided by the Office for National Statistics. The employment variable is directly taken from the Business Structure Database.

To gross up the data, I construct survey weights directly from the Business Structure

Database, which represents the (near) universe of private sector employment. I define industry×firm-size cells and use the BSD employment counts as weights for the ARD/ABS. In constructing the analysis sample, I drop a few problematic sectors in the ARS/ABS: farming (A), mining & quarrying (B), energy supply (D), water (E), and real estate (L). All sectors dominated by public employment in the UK (education, health care, and social work) are also excluded. Finally, I also trim the top and bottom 2 percent of firms in the distribution of labor productivity, $LP_{i,t}$, in each industry×firm-size cell.

Note that these data sets are not publicly available. Access can be obtained through the UK Data Service (`https://ukdataservice.ac.uk`).

B.2 Worker transition rates

The monthly time series for the worker transition rates (UE_t, EU_t, EE_t) are from Postel-Vinay and Sepahsalari [2019]. These series are derived from the British Household Panel Survey (BHPS) and its successor Understanding Society (UKHLS), a monthly survey of British households. This paper uses data from 1992m1 to 2016m12. Because of the transition from the BHPS to the UKHLS, there is a gap in the series between 2008m8 and 2009m12, which is smoothed over using moving averages. Additional details on the construction of these series can be found in Postel-Vinay and Sepahsalari [2019].[2]

B.3 Additional macro time series

I also use the following aggregate time series, which are publicly available on the Office for National Statistics (ONS) website.

- ABMI: Gross Domestic Product (chained volume, seasonally adjusted), quarterly starting in 1955q1.
- A4YM: Output per Worker (seasonally adjusted), quarterly starting in 1959q3.

[2] I am grateful to the authors for sharing these series and to Pete Spittal for explaining how they are affected by the transition from the BHPS to the UKHLS.

- MGSX: Unemployment rate (aged 16 and over, seasonally adjusted), monthly starting in 1971m1.

B.4 UK recession dates

The UK recession dates are defined as successive quarters of economic growth, as measured by the quarter-on-quarter growth in seasonally adjusted real GDP.[3] Table B.1 lists the recessions used in the paper. This definition is used because there is no British equivalent to the "official" list of recessions defined by the National Bureau of Economic Research for the United States.

Recession start date	Recession end date	Duration (quarters)
31-Mar-57	30-Sep-57	2
30-Jun-61	31-Dec-61	2
30-Jun-73	31-Mar-74	3
31-Mar-75	30-Sep-75	2
31-Dec-79	31-Mar-81	5
30-Jun-90	30-Sep-91	5
31-Mar-08	30-Jun-09	5

Table B.1: List of UK recessions implied by two-quarter rule.

B.5 Robustness of productivity decomposition

I report two robustness checks on the productivity decomposition obtained from firm-level data.

Figure B.1 benchmarks the labor productivity index obtained by grossing up the British micro data to the official labor productivity series (A4 YM) from the ONS. Though the drop and recovery in the ONS series are slightly quicker, overall the two series exhibit a similar pattern in deviation from an HP trend.

[3] The full list is available here: `https://en.wikipedia.org/wiki/List_of_recessions_in_the_United_Kingdom`

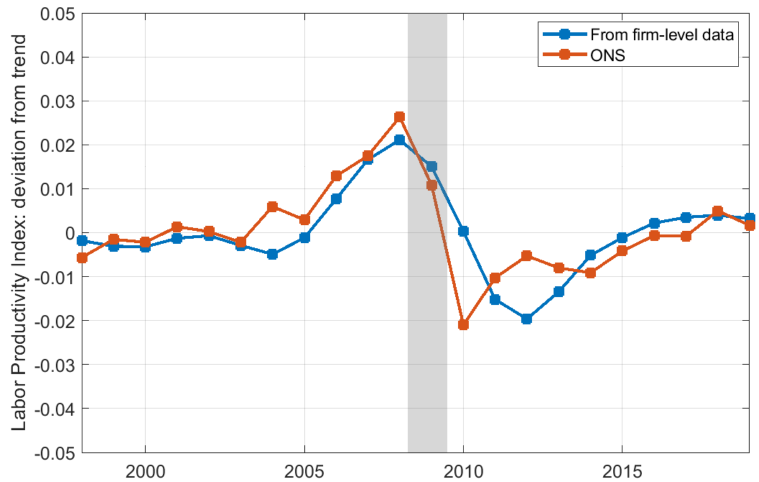

Figure B.1: Benchmark of productivity index from micro data to official series. Both series are shown in deviation from an HP trend with smoothing parameter 100. See main text for details.

Figure B.2 shows the labor productivity decomposition (15) using alternative de-trending methods. These alternative methods (the HP filter and band-pass filter in particular) give very similar results.

C Quantitative analysis

C.1 Steady-state solution

As shown in Appendix A.6, the firm's policies can be expressed in terms of a single value function, the net surplus given in Equation (A.5). The algorithm below is expressed in terms of the net firm-workers' surplus formulation for concision.

Discretization At the steady state, p is the variable in the state-space. I discretize the process for idiosyncratic log-productivity using Tauchen's procedure with $N_p = 401$ points. This gives a grid $\{p_1, \ldots, p_{N_p}\}$ and the associated transition matrix.

This discretization on a thin grid allows me to approximate the relevant policy or value function as being constant on some (small) half-open interval. This gives an intuitive way to integrate against the measure of workers, L, by replacing the integral with the appropriate

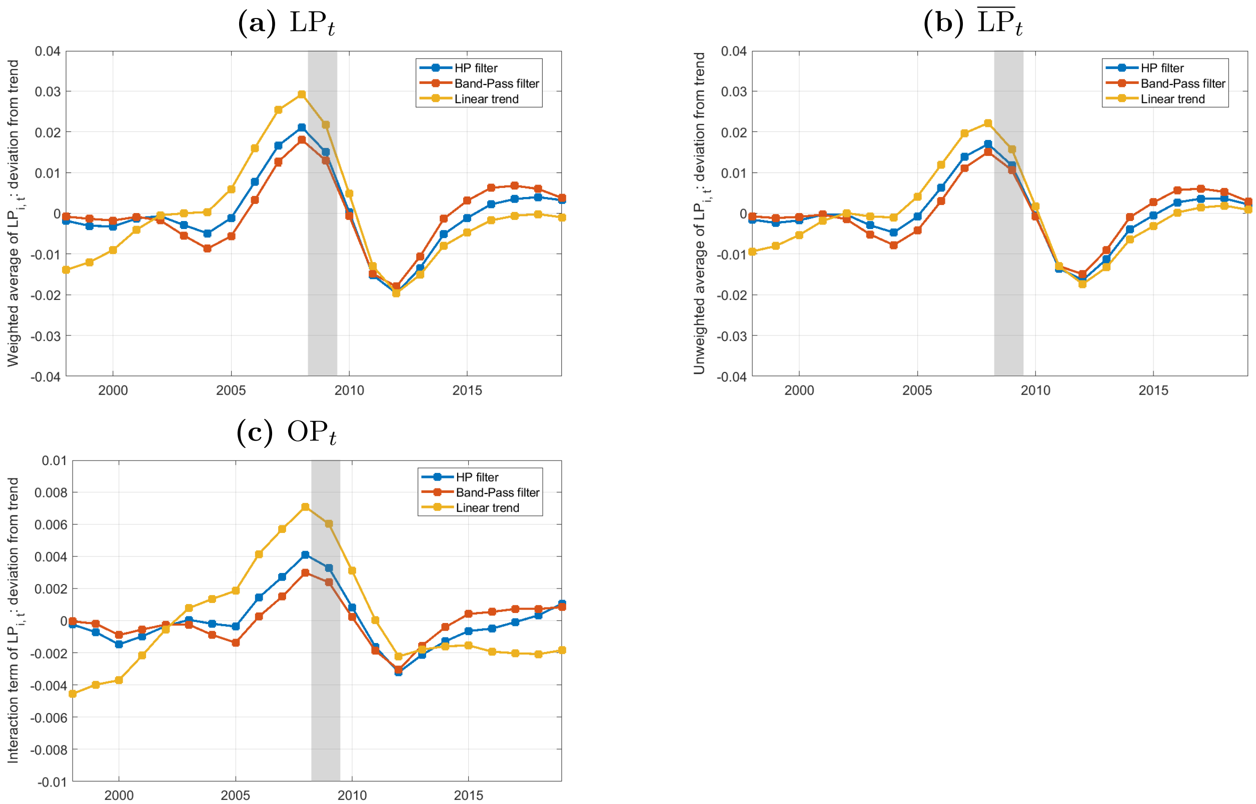

Figure B.2: Labor productivity decomposition (15). Series are shown in deviation from several alternative trends: HP filter (smoothing parameter = 100), band-pass filter (fluctuations restricted to the range of 2 to 14 years), and a linear trend fitted over the sample period. Gray band denotes the Great Recession period in the UK.

employment share weighted sum. As an example, the net optimal contract (A.6) at some productivity node p_k can be approximated as

$$
\begin{aligned}
V_\Sigma(p_k) &= \frac{s(1-\delta)\int_{p_1}^{p_k}\chi(p')\Sigma(p')dL(p')}{u+s(1-\delta)\left(L(p_k)-L(p_E)\right)} = \frac{s(1-\delta)\sum_{i=2}^{k}\int_{p_{i-1}}^{p_i}\chi(p')\Sigma(p')dL(p')}{u+s(1-\delta)\left(L(p_k)-L(p_E)\right)} \\
&\approx \frac{s(1-\delta)\sum_{i=2}^{k}\chi(p_{i-1})\Sigma(p_{i-1})\int_{p_{i-1}}^{p_i}dL(p')}{u+s(1-\delta)\left(L(p_k)-L(p_E)\right)},
\end{aligned}
$$

where the integral in the last expression is simply the fraction of workers employed at firms in the interval between p_{i-1} and p_i.

Algorithm Given the discretization, the algorithm unfolds as follows.

1. Guess initial values for Σ and L on the grid $\{p_1,\dots,p_{N_p}\}$. In line with the RME result, I start with some increasing function of p for the net surplus. I also initialize $L=0$ (all workers initially unemployed).

2. Conditional on values for Σ and L, the agents' optimal policies can be computed. For example, the activity threshold, p_E, is the point at which Σ becomes positive. The optimal contract can be computed from Equation (A.6) and the firm's choice of hiring intensity from the corresponding first-order condition.

3. The net surplus equation and the law of motion for the measure of employed workers imply new values for Σ and L on the grid. The net surplus equation gives an update for Σ in the previous period, while the law of motion for employment yields next period's employment at each productivity level. This does not matter since the algorithm solves for a steady state RME.

4. The final step consists of checking the convergence of L and Σ. If this is the case, the tuple (Σ, L) is the steady state RME. Otherwise, go back to point 2 with the updated values and iterate.

C.2 Aggregate shocks solution

Approximations As explained in the main text, the solution method with aggregate shocks relies on two approximations. First, the measure of employment at firms of different productivity is summarized by a set of $N_m + 1$ moments

$$\begin{aligned}
m_t^0 &= u_t = 1 - \int dL_t(p) \\
m_t^1 &= \int p d\bar{L}_t(p) \\
&\dots \\
m_t^{N_m} &= \int p^{N_m} d\bar{L}_t(p),
\end{aligned} \tag{C.1}$$

where $\bar{L}_t(p) = L_t(p)/L_t(\overline{p})$ denotes the cumulative density associated with the cumulative measure of workers on p.

Second, I parameterize the value functions for the firm-workers' surplus S_t and the unemployed worker U_t with a polynomial. I choose to parameterize these value functions separately since they are positive by definition, so they can be expressed in log-deviation from the steady state. Because preserving the monotonicity of S_t (especially around the entry threshold) is key to the procedure, I use a separate polynomial for each productivity node p_i. The value functions are approximated outside of the steady state as

$$\ln S(p_i, \omega_t, L_t) - \ln \bar{S}(p_i) \approx \hat{S}(p_i, \omega_t, \hat{\boldsymbol{m}}_t; \theta_{p_i}), \quad p_i \in \{p_1, ..., p_{N_p}\},$$

and

$$\ln U(\omega_t, L_t) - \ln \bar{U} \approx \hat{U}(\omega_t, \hat{\boldsymbol{m}}_t; \theta_U),$$

where $\hat{\boldsymbol{m}}_t$ denotes the vector gathering all moments in (C.1) in log-deviation from steady state, while $\bar{S}$ and $\bar{U}$ stand, respectively, for the firm-workers' surplus and the value of unemployment at the steady state.

Algorithm The algorithm for the model solution with aggregate shocks then consists of the four following steps.

1. Draw a sequence of aggregate productivity shocks and guess an initial value for the coefficients of $\hat{S}$ and $\hat{U}$. I initialize them at zero.

2. Simulate the measure of employment forward, starting from the steady-state solution. Conditional on the current values of $\{\theta_U, \theta_{p_1}, \ldots, \theta_{p_{N_p}}\}$, agents make optimal decisions about hiring and contract offers given the current states, which induces a law of motion for employment at each productivity level. The simulated measure of workers is approximated by a set of moments to compute the value functions and policy functions in each period.

3. Update $\hat{S}$ and $\hat{U}$, conditional on the simulation of L_t obtained in the previous step. This requires taking an expectation over future realizations of the aggregate shock. The aggregate shock is discretized using Tauchen's procedure with $N_\omega = 15$ nodes in practice.

4. Run a regression of $\hat{S}$ and $\hat{U}$ on the state variables to update the coefficients. Go back to step 2 and iterate on the coefficients $\{\theta_U, \theta_{p_1}, \ldots, \theta_{p_{N_p}}\}$ until convergence.

I find the coefficients by running separate regressions for the firm-workers' surplus at each p-node on the variables in the state-space. I omit the constant in these regressions, which is equivalent to imposing the constraint that the steady state holds exactly at each node. Since these regressors are at times close to collinear in some iterations, I use a penalized (ridge) regression to regularize the problem. The coefficients for the unemployed worker's value function are found by solving

$$\min_{\theta_U} \sum_t \left[(\ln U_t - \ln \bar{U}) - \hat{U}(\omega_t, \hat{\boldsymbol{m}}_t; \theta_U) \right]^2 + \zeta \theta_U^T \theta_U,$$

where θ_{U_i} denotes individual elements of θ_U, $\zeta > 0$ is the associated regularization parameter, and

$$\hat{U}(\omega_t, \hat{\boldsymbol{m}}_t; \theta_U) = \theta_U^{\omega} \ln \omega_t + \sum_{k=0}^{N_m} \theta_U^{m_k} \left[\ln m_t^k - \ln \bar{m}^k \right].$$

The regularization parameter, $\zeta > 0$, ensures that the matrix of regressors is invertible by adding to it a ζ-diagonal matrix. I proceed similarly to find the coefficients in $\hat{S}(p_i, \omega_t, \hat{\boldsymbol{m}}_t; \theta_{p_i})$ at each productivity node p_i.

I finally allow for less than full updating between each step. With these parametric assumptions, the coefficients $\{\theta_U, \theta_{p_1}, \ldots, \theta_{p_{N_p}}\}$ are elasticities of the value functions with respect to the regressors, which gives some intuition about the appropriate convergence condition.

C.3 Accuracy test

I assess the accuracy of the procedure by adapting ideas from den Haan [2010]. The goal of this test is to check that the error implied by the polynomial approximation does not build up over time. The test proceeds as follows.

1. Draw a new sequence of shocks $\{\omega_t'\}_{t=1}^T$, separate from the sequence used to find the coefficients $\{\theta_U, \theta_{p_1}, \ldots, \theta_{p_{N_p}}\}$.

2. Compute the model solution along $\{\omega_t'\}_{t=1}^T$ in two different ways.

   (a) Simulate the model forward in time using the polynomial approximations. This directly gives the value function $\{\hat{S}_t, \hat{U}_t\}_{t=1}^T$ and moments $\{\hat{\boldsymbol{m}}_t\}_{t=1}^T$.

   (b) Construct the value functions by solving the model back in time. This gives the value functions (in log-deviation from the steady state) $\{\check{S}_t, \check{U}_t\}_{t=1}^T$. Using these value functions to solve for the firm's decisions, the model can be simulated once more forward to obtain the moments $\{\check{\boldsymbol{m}}_t\}_{t=1}^T$.

3. Compute the distance between the two model solutions at each point in time. For each alternative time series $\{X_t\}_{t=1}^{T}$ obtained from step 2, this distance can be expressed as

$$d(\hat{X}_t, \check{X}_t) = 100 \cdot \left| \hat{X}_t - \check{X}_t \right|, \tag{C.2}$$

which is (approximately) in percent given that all time series are in log-deviations from the steady state.

Table C.1 reports summary statistics of the accuracy metric (C.2) for the two model solutions. This measure suggests that the accuracy of the procedure for the two value functions is very good, with a distance of at most 0.065 percent across model solutions. The least accurate part of the simulation procedure comes from differences in the entry-exit threshold, with a distance between the simulated unemployment rates (m_t^0) of at most 1 percent (so 6.0 percent vs 5.9 percent at the calibrated steady state). This suggests that the overall accuracy of the procedure is good.

	Accuracy measure: $100 \cdot \left\| \hat{X}_t - \check{X}_t \right\|$				
Variable	Mean	p75	p90	p95	Max
Value Functions					
S_t	0.002	0.003	0.004	0.005	0.065
U_t	0.002	0.003	0.004	0.005	0.032
Moments summarizing L_t ($\boldsymbol{m}_t$)					
$m_t^0 (:= u_t)$	0.099	0.133	0.242	0.356	0.851
m_t^1	0.005	0.006	0.013	0.021	0.057
m_t^2	0.006	0.008	0.015	0.023	0.063

Table C.1: Accuracy test results.

C.4 Number of moments in approximation

I assess the sensitivity of this solution method to the number of moments used in approximating L_t with the following test. I incrementally introduce up to $N_m = 9$ moments to summarize L_t, and solve the model using the same sequence of aggregate shocks $\{\omega_t'\}_{t=1}^{T}$ using more moments. I can then compute a solution $\hat{S}^{N_m}(p, \omega_t, \hat{\boldsymbol{m}}_t; \theta_{p_i})$ and $\hat{U}^{N_m}(\omega_t, \hat{\boldsymbol{m}}_t; \theta_U)$ along the same sequence of aggregate shocks, where N_m indexes the number of moments included in the approximation. I proceed by defining the following measure of the solution's sensitivity to the inclusion of an additional moment k

$$\Delta_t^{N_m}(\hat{S}_t(p)) = \left|\hat{S}_t^{N_m}(p) - \hat{S}_t^{N_m-1}(p)\right| = \left|\ln S_t^{N_m}(p) - \ln S_t^{N_m-1}(p)\right|.$$

Figure C.1 reports the average and maximum $\Delta_t^{N_m}$ across simulation periods $t = 1, \ldots, T$ and value functions $\{\hat{U}_t, \hat{S}_t(p_1), \ldots, \hat{S}_t(p_{N_p})\}$. The figure shows that after $N_m = 2$, changes in the approximated value functions become smaller than 0.01 percent. All results in the paper are obtained with $N_m = 2$.

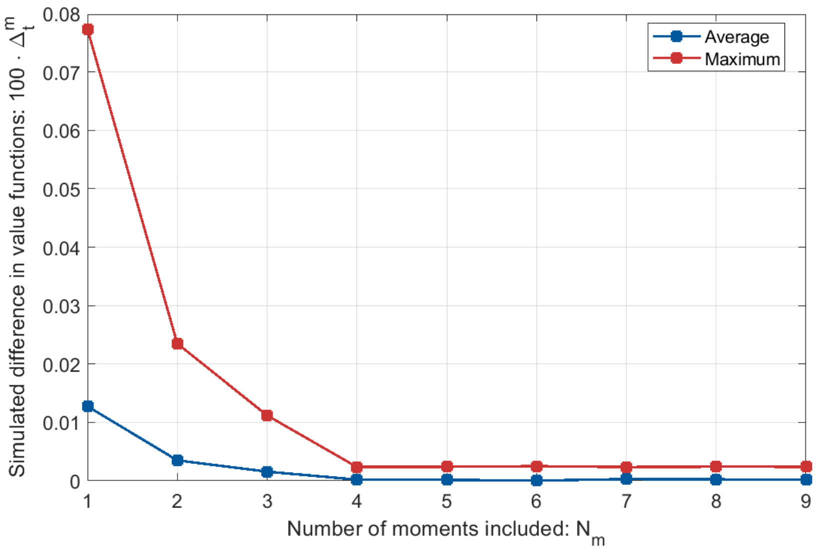

Figure C.1: Robustness of solution method to number of moments N_m included in $\hat{\boldsymbol{m}}_t$

C.5 Local identification of steady-state parameters

Figure C.2 illustrates the local identification of the steady-state parameters in the baseline model. It confirms the parameter-moment pairs introduced heuristically in Section 4.3 of the main text. Small deviations of the parameters around their calibrated baseline values move the corresponding moments in predictable directions. For instance, increasing the hiring cost c_0 decreases the UE transition rate (top-left panel in Figure C.2).

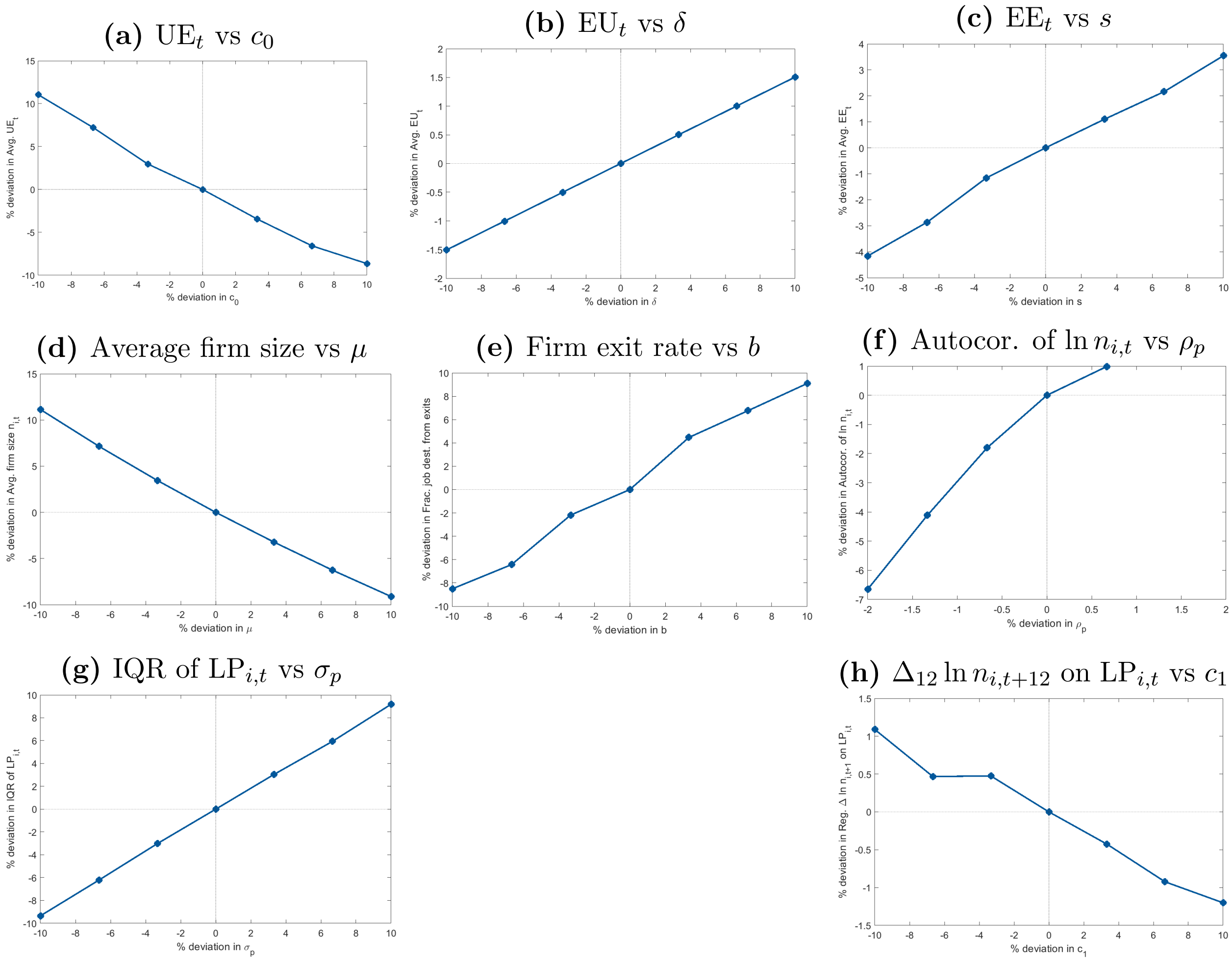

Figure C.2: Numerical local identification of parameters in baseline model. Percentage deviation of targeted moment relative to moment at calibrated parameter, as a function of percentage deviation of parameter from its calibrated value. The moment-parameter pairs correspond to the heuristic mapping discussed in Section 4.3 of the main text. Parameter deviation of $\pm 10\%$ around calibrated baseline value, except for ρ_p, where the deviation is $\pm 2\%$ since $\rho_p < 1$ by assumption.

C.6 Decomposition of aggregate productivity in one-shock model

Figure C.3 reports the fit to the LP$_t$ and EC$_t$ decompositions in the baseline model with a single aggregate shock (on ω only), the counterpart to Figure 1 in the main text. For comparison, I report again the fit from the baseline model with aggregate shocks on ω, δ, and c_0 (blue line), alongside the fit of the one-shock model (red line). As suggested by the results in Table 3, the figure confirms that the one-shock model is less successful in matching the decomposition of aggregate labor productivity than the baseline model with additional aggregate shocks.

D Comparison to MPV2016 model

This appendix gives details on the comparison with the model by Moscarini and Postel-Vinay [2016], hereafter MPV2016.[4] I focus on my calibration of their framework and refer to the original paper for details on the model.

D.1 Steady-state calibration

There are two minor changes relative to the calibration described in MPV2016. First, I assume that the distribution of firm productivity is log-normal, similarly to the ergodic distribution implied by the process for firm productivity in the model developed in this paper. Second, I add some of the firm-level moments I use in the calibration of the baseline model, while MPV2016 focus on labor market flows and aggregate productivity. Specifically, I add the dispersion of firm-level productivity. These changes are intended to create the

Two parameters are taken directly from MPV2016. These are the curvature of the cost of hiring function ($c_1 = 49$ in the notation of my model) and the flow value of unemployment, which is set to $b = 0$ in their baseline specification. Because $c_1 = 49$ can seem like a high degree of curvature, I also experiment with $c_1 = 5$. I cannot use similar moment targets as in the baseline model to discipline these parameters because, in their *steady-state* framework, firm employment is constant (so I cannot meaningfully regress $\Delta \ln n_{i,t+1}$ on $\mathrm{LP}_{i,t}$ since

[4] The authors have several related paper [Moscarini and Postel-Vinay, 2013, 2016]. MPV2016 is closest to the quantitative analysis developed here.

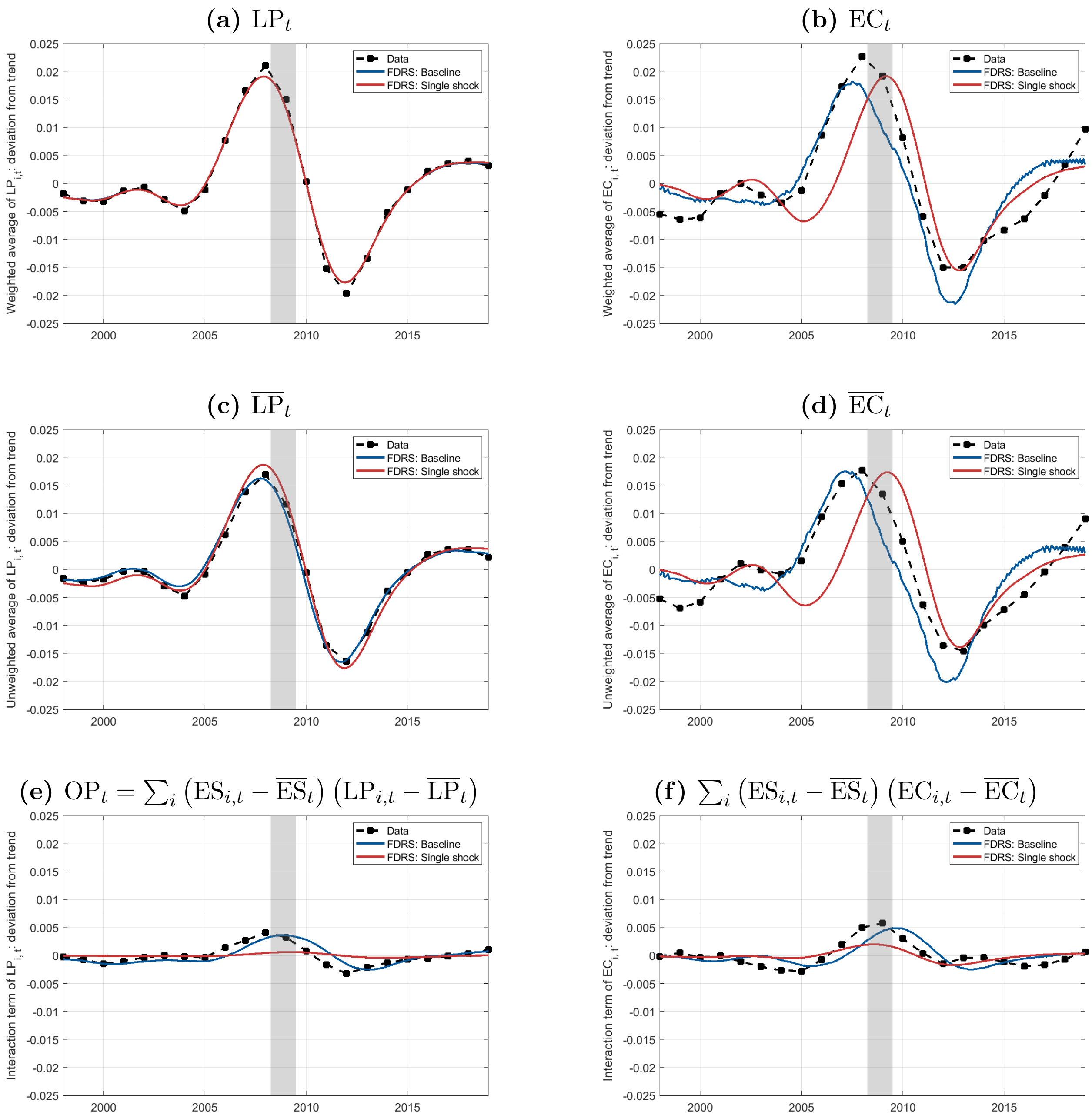

Figure C.3: Fit to LP_t and EC_t decompositions. All data series are de-trended using the HP filter with smoothing parameter 100. All model series are shown in deviation from steady-state. Grey band denotes the Great Recession period in Britain. The blue line ("Baseline") is for the baseline model with aggregate shocks (ω, δ, c_0). The red line ("Single shock") is for the baseline model with a single aggregate shock (ω).

			Data	MPV2016 calibration $c_1 = 49$	$c_1 = 5$
A. Parameters					
	δ	Separation rate	—	0.004	0.004
	c_0	Hiring cost:	—	91.443	89.026
	c_1	$c(h) = (c_1 + 1)^{-1}(c_0 h)^{c_1+1}$	—	49.000	5.000
	s	Relative search effort	—	1.000	1.000
	σ_p	$\ln p \sim \mathcal{N}(0, \sigma_p)$	—	0.879	0.879
	b	Unemployment flow value	—	0.000	0.000
B. Targeted moments (source)					
		Avg. UE_t (BHPS)	0.058	0.063	0.063
		Avg. EU_t (BHPS)	0.003	0.004	0.004
		Avg. EE_t (BHPS)	0.016	0.008	0.008
		IQR of $\mathrm{LP}_{i,t}$ (ARD)	1.129	1.186	1.187
C. Additional moments (source)					
		Avg. firm size $n_{i,t}$ (ARD)	12.113	0.940	0.940
		Pareto tail of empl. size (ARD)	1.066	4.799	3.495
		IQR of $\mathrm{EC}_{i,t}$ (ARD)	1.352	0.803	0.841

Table D.1: Calibration of MPV2016 model at the steady-state. The data moments are described in the main text. Model moments are obtained from simulations of each model at its steady-state.

$\Delta \ln n_{i,t+1} = 0$ at all i by definition) and there is no firm entry and exit.[5]

Table D.1 reports the parameters and simulated moments, along with their data counterparts. With the parameters c_1 and b taken from MPV2016, I calibrate δ, c_0, s, and σ_p to replicate the data moments listed in Panel B of the table. Overall, the model is successful in replicating these moments, although it cannot fully reproduce the job-to-job transition rate observed in the data. Looking at additional data moments, it also does not replicate the average employment of firms and the dispersion of wages.

D.2 Calibration of full model

I follow the same strategy as for the baseline model and introduce aggregate shocks keeping all other parameters at their steady-state value. I use the exact same procedure and data

[5] At the steady-state, their model is similar to Burdett and Mortensen [1998].

	Data	Baseline (ω, δ, c_0)	Baseline (ω)	MPV2016: $c_1 = 5$ (ω, δ, c_0)	MPV2016: $c_1 = 5$ (ω)	MPV2016: $c_1 = 49$ (ω, δ, c_0)	MPV2016: $c_1 = 49$ (ω)
Share of $\mathrm{Var}(\mathrm{LP}_t)$							
Panel A. Observed							
$\overline{\mathrm{LP}}_t$	0.830	0.837	0.928	0.010	0.999	0.023	1.000
OP_t	0.170	0.163	0.072	0.990	0.001	0.977	0.000
Panel B. Structural							
$\ln \omega_t$	—	0.296	1.048	0.010	0.999	0.023	1.000
$\mathbf{E}_{\bar{K}_t^P}(\ln p)$	—	0.541	-0.120	0.000	0.000	0.000	0.000
$\mathrm{OP}_t = \mathbf{E}_{\bar{L}_t^P}(\ln p) - \mathbf{E}_{\bar{K}_t^P}(\ln p)$	—	0.163	0.072	0.990	0.001	0.977	0.000

Table D.2: Variance decompositions in the baseline model and for alternative calibrations of MPV2016 model. (ω, δ, c_0) columns are for calibrations with all three aggregate shocks. (ω) columns are for calibrations with a single aggregate productivity shock. Each line reports the variance components associated with each term as a fraction of $\mathrm{Var}(\mathrm{LP}_t)$. For example, the OP_t line reports $[\mathrm{Var}(\mathrm{OP}_t)+\mathrm{Cov}(\overline{\mathrm{LP}}_t, \mathrm{OP}_t)]/\mathrm{Var}(\mathrm{LP}_t)$. The model simulation is obtained from fitting log-detrended GDP over the post-war period.

targets as for the baseline model. See Section 4 for detail.

D.3 Worker reallocation

I compare the worker reallocation properties of the MPV2016 model to the baseline model in Table D.2. I follow the same approach as in Table 3 in the main text. This exercise confirms that, for both calibrations of the cost function, the MPV2016 model tends to generate excess worker reallocation relative to the data and to the baseline framework in the model with aggregate shocks to ω, δ, and c_0. Conversely, calibrations of the MPV2016 model with only the aggregate productivity shock ω generate variance decompositions close to the one-shock baseline model, with most of the fluctuations in aggregate labor productivity driven by ω.

E Comparative statics

This Appendix illustrates the connection between the structural decomposition of aggregate labor productivity and worker reallocation. I use model simulations to compare the steady states implied by different types of shocks.

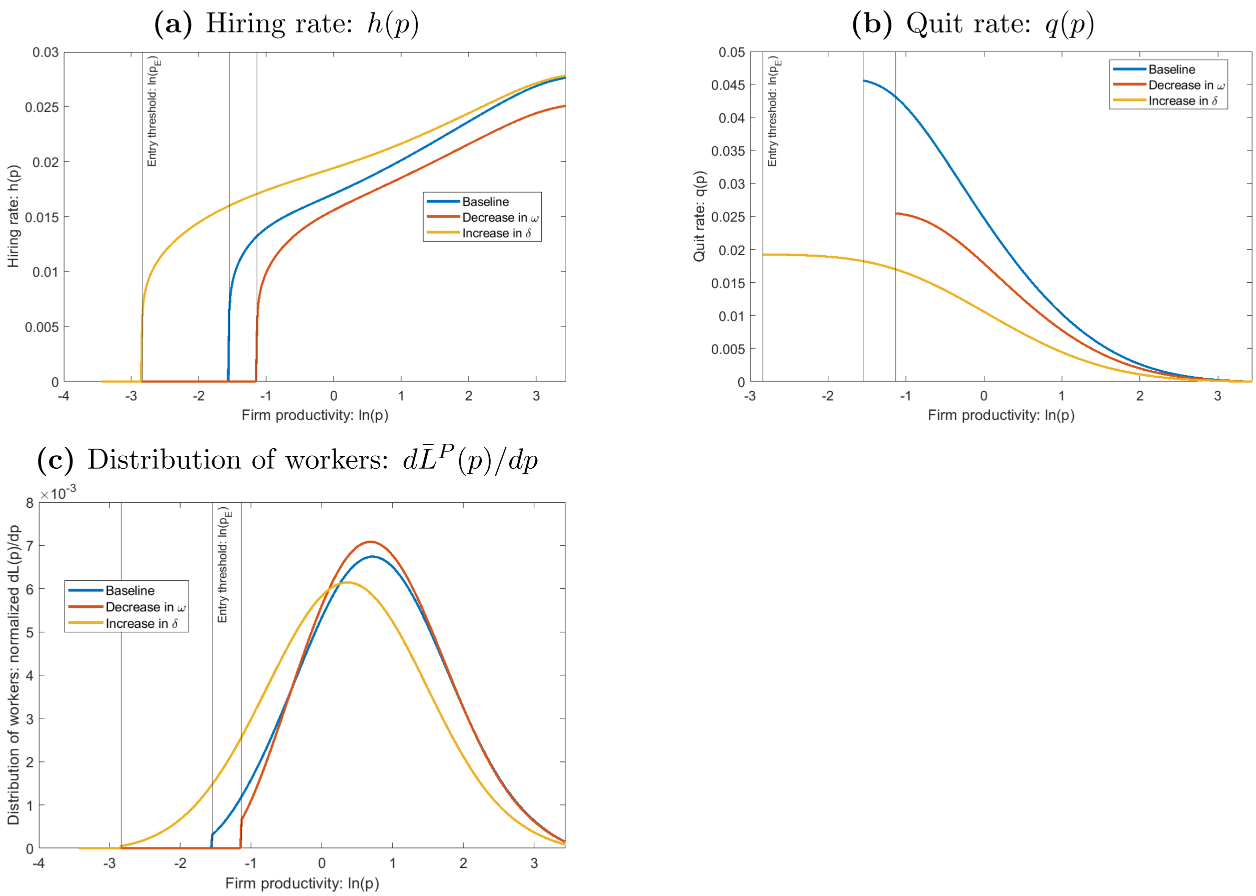

Figure E.1: Equilibrium steady state in response to negative aggregate shocks: a decrease in aggregate productivity ω and an increase in the aggregate separation rate δ. Model simulations at calibrated parameters described in Section C. The vertical line denotes the entry-exit threshold in each scenario.

In a rank-monotonic equilibrium, firms with a higher current realization of productivity p offer wage contracts with greater value, which implies that all job-to-job transitions are toward these firms. This is true for any realization of the aggregate shock provided the conditions in Result 2 hold. But the pace at which workers move to more productive firms varies with aggregate shocks. Similarly, the entry-exit threshold $p_E(\omega, L)$ also changes in response to these shocks. These two forces determine the position of workers along the firm productivity distribution, which is summarized by the employment-weighted measure of firm productivity $L^P(p)$. This reshuffling of workers along the firm productivity distribution represents the key notion of worker reallocation implied by the model.

I illustrate these mechanisms by analyzing the steady state of the model in response to several types of shocks in Figure E.1. I consider two types of negative aggregate shocks commonly used in the macro labor literature: a decrease in aggregate productivity ω and an increase in the aggregate job destruction rate δ [Shimer, 2005, Moscarini and Postel-Vinay, 2016]. All figures are obtained using the parametrization of the model described in the next sections.

A decrease in aggregate productivity ω has the following intuitive impact on the steady state of the model. It first increases the entry-exit productivity threshold ($p_E(\omega, L)$ moves to the right in Figure E.1), since the surplus between a firm and its workers is lower at all p. For the same reason, the hiring rate $h(p)$ is lower at all p (panel a). The rate at which workers are reallocated up the productivity ladder $q(p)$ also falls at all p (panel b). There are fewer chances to make contact with a more productive firm and therefore to move up the ladder in this scenario. Panel (c) summarizes the impact of these changes in the hiring rate $h(p)$ and voluntary quit rate $q(p)$ on the employment-weighted distribution of productivity, where I introduce the notation $\bar{L}^P(p) = L^P(p)/L^P(\overline{p})$ for the normalized measure of workers below p. This distribution moves to the right with the higher entry-exit threshold, but the lower rate of reallocation up the ladder acts as a countervailing force. As a result, the mode of the distribution is roughly identical to the baseline steady state.

An increase in the aggregate rate of job destruction δ has the following less intuitive impact on the steady state of the model. A δ-shock directly affects how the surplus is split between the firm and its workers in the model. To gain some intuition, consider the direct impact of a change in δ on the optimal wage contract (10). It can be shown that $\partial V(p, \omega, L)/\partial \delta \leq 0$. The optimal wage contract specifies that the increased separation risk is, at least to some extent, passed on to workers. The overall impact on the firm-workers' surplus is ambiguous since a lower wage contract also potentially increases the discounted profits of the firm. In Figure E.1, this lower wage contract decreases the entry-exit productivity threshold: less productive firms are viable relative to the baseline steady state equilibrium.

Worker reallocation up the productivity ladder is lower than in the baseline ($q(p)$ is lower than in the baseline at all p in Figure E.1b). The resulting distribution shifts to the left with both less productive firms surviving and less reallocation up the productivity ladder.

This comparison shows that alternative types of negative aggregate shocks do not necessarily have the same impact on worker reallocation. In the quantitative part of the paper (Section C), the calibration of these aggregate shocks is disciplined by the cyclical properties of key time series.

F Contingent unemployment benefits

I consider a counterfactual experiment in which I make the flow value of unemployment countercyclical. I implement this counterfactual by assuming that the flow value of unemployment is given by

$$\ln b(\omega_t) - \ln b = \epsilon_{b,\omega} \cdot \ln \omega_t,$$

where the parameter $\epsilon_{b,\omega} \leq 0$ indexes the dependence of b to the aggregate shocks $\{\omega_t\}$. The baseline calibration with constant b corresponds to the case $\epsilon_{b,\omega} = 0$.

The rationale behind this extension is twofold. First, this counterfactual is in the spirit of the countercyclical unemployment insurance extensions that are implemented in the US when the labor market deteriorates.[6] Second, it introduces an additional degree of downward rigidity in the optimal wage contract (10), since this contract depends on b through the value of unemployment. While a micro-foundation for these rigidities in this environment is beyond the scope of this paper, this exercise gives some preliminary insights into the effect of additional wage rigidities on the reallocation of workers along the job ladder within this

[6]The actual policy indexes the duration of unemployment benefits contingent on the current unemployment rate. I focus on a reduced-form implementation to avoid the need to introduce additional state variables to formally capture the eligibility of workers for unemployment benefits. See Rujiwattanapong [2019] for a model that fully captures the unemployment insurance extension program.

class of models.

I experiment with two alternative values of $\epsilon_{b,\omega} \in \{-100, -50\}$. These values are chosen to yield an increase in the flow value of unemployment commensurate with the increase in the maximum duration of unemployment benefits following a typical US recession.[7] I solve the model again for these values of $\epsilon_{b,\omega}$ and study the structural decomposition in response to the same "GDP shocks."

In Figure F.1, I report the average of each term in the structural decomposition of worker reallocation across post-war recessions, both in the model baseline (the same as in Figure 2) and in each counterfactual scenario. Countercyclical unemployment benefits appear to significantly alter the dynamics of worker reallocation over the business cycle. In contrast with the baseline model, the Firm Ladder term responds positively during a recession in these scenarios. There are fewer low productivity firms as the value of unemployment (and therefore the outside option of workers) increases with larger unemployment benefits. The drop in the ladder interaction term is also larger than in the baseline model. The slowdown in the relocation of workers up the productivity ladder is amplified with countercyclical unemployment benefits. Job-to-job transitions are lower at low-productivity firms in this counterfactual relative to the baseline, since (i) there are more unemployed workers and therefore the job-finding rate λ_t decreases, and (ii) all firms cut recruitment efforts since the optimal wage contract is larger.

Overall, this counterfactual analysis suggests that countercyclical unemployment benefits involve a tradeoff in terms of worker reallocation along the job ladder. They both increase the Firm Ladder term by driving out firms at the lower end of the productivity distribution and decrease the Ladder Interaction term by further slowing down the pace of worker reallocation. The reduction in the Ladder Interaction term following a recession remains a feature of both the baseline model and the model with countercyclical benefits. On net, the positive "Firm Ladder" effect dominates in the two counterfactual scenarios considered, and the allocation

[7] The maximum duration of unemployment benefits typically doubles after a recession in the US over the post-war period [Rujiwattanapong, 2019, Figure 2].

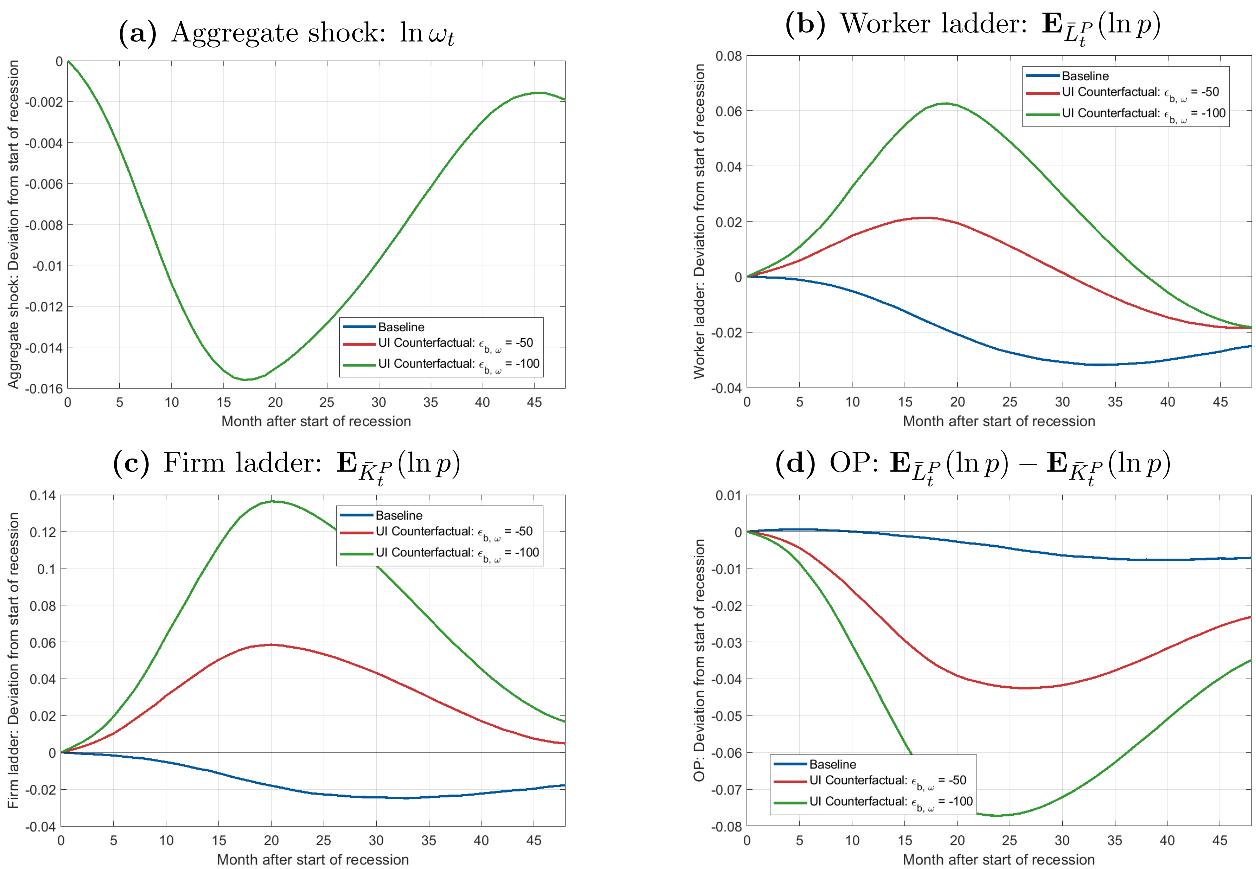

Figure F.1: Decomposition of Worker Ladder in counterfactual scenario across post-war recessions in Britain. Each line gives the average across recessions.

of workers to firms improves following a typical recession relative to the baseline.

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