STAFF
REPORTS

NO. 1126
OCTOBER 2024

REVISED
SEPTEMBER 2026

Firm Wage Setting, On-the-Job Search, and the Inflationary Effects of Supply Shocks

Justin Bloesch | Seung Joo Lee | Jacob P. Weber

FEDERAL RESERVE BANK *of* NEW YORK

Firm Wage Setting, On-the-Job Search, and the Inflationary Effects of Supply Shocks

Justin Bloesch, Seung Joo Lee, and Jacob P. Weber

Federal Reserve Bank of New York Staff Reports, no. 1126
October 2024; revised September 2026
https://doi.org/10.59576/sr.1126

Abstract

We argue that if firms set wages and workers search on-the-job, then pass-through from prices to wages is weak, limiting the cumulative inflationary effects of supply shocks. We derive a tractable general equilibrium model with firm wage setting and on-the-job search, yielding an empirically-realistic wage Phillips curve tying worker quits to wage growth. In our model, price level increases reduce real wages at both workers' current job and their outside options, muting firms' incentive to raise wages. Quantitative exercises reveal that counterfactually assuming union wage-setting can overstate wage and price inflation following oil shocks.

JEL classification: E31, E52
Key words: monopsony, inflation, cost-of-living shocks, on-the-job search

---

Weber: Federal Reserve Bank of New York (email: jake.weber@ny.frb.org). Bloesch: Cornell University (email: jb2722@cornell.edu). Lee: University of Oxford (email: seung.lee@sbs.ox.ac.uk). This paper was previously circulated under the titles "Do Cost-of-Living Shocks Pass Through to Wages?" and "Firm Wage Setting and On-the-Job Search Limit Wage-Price Spirals." The authors thank the editor, Ayşegül Şahin, and two anonymous referees for their feedback and suggestions. They also thank Daron Acemoglu, Keshav Dogra, Ryan Chahrour, Renato Faccini, Andrea Ferrero, John Grigsby, Marianna Kudlyak, Arvind Krishnamurthy, Pierre-Olivier Gourinchas, Walker Ray, Robert Shimer, Paolo Surico, Iván Werning, and seminar participants at the Reserve Bank of Australia, Bank of England, Federal Reserve Bank of New York, West Coast Search and Matching Conference, Oxford, UCL, Cornell, CUHK, HKUST, Korea University, Sogang University, Ewha Womans University, 2024 SED, CEBRA, Midwest Macro Annual Meetings, 2025 Econometric Society World Congress, ECB Inflation: Drivers and Dynamics Conference, and 2026 ASSA Meeting for helpful comments and discussion.

This paper presents preliminary findings and is being distributed to economists and other interested readers solely to stimulate discussion and elicit comments. The views expressed in this paper are those of the author(s) and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System. Any errors or omissions are the responsibility of the author(s).

To view the authors' disclosure statements, visit
https://www.newyorkfed.org/research/staff_reports/sr1126.html.

1 Introduction

Through the COVID pandemic and economic recovery, economies throughout the world experienced large supply shocks and rapid price inflation and nominal wage growth, raising concerns of a supply shock-induced wage-price spiral. While there is substantial evidence that wage costs are passed through into prices,[1] there is little consensus regarding how much, and by what mechanisms, wages respond to higher prices. Mainstream macroeconomic models used to understand the determinants of nominal wage growth assume that unions representing workers set wages and imply that higher prices will pass through to wages. However, union membership has declined dramatically in many advanced economies, and evidence suggests that wage posting is the most common, if not dominant, method of wage determination in the United States.[2] In light of this evidence and the recent experience of inflation, we ask: *when firms set both prices and wages, how much and by what mechanisms do workers' wages and overall inflation respond to supply shocks?*

To answer this, we derive a two-sector macroeconomic model where workers search on the job, and firms set both prices and wages. Workers consume two goods: an endowment good (e.g., unprepared food or energy) and a services bundle that uses both labor and endowment goods as intermediates. Firms set a common wage for all workers subject to nominal rigidities in the form of standard, convex adjustment costs, deriving their power to set wages from search frictions and from workers' idiosyncratic preferences over employers. When choosing wages, a firm considers that paying a higher wage decreases costly turnover: a firm fills costly vacancies more quickly and discourages its workers from quitting by paying a higher wage. With on-the-job search, wages are determined in equilibrium primarily by firms competing for already-employed workers, as the threat of workers quitting into unemployment is low.

We derive the wage Phillips curve in this environment, and show how it can be written as a simple relationship between nominal wage growth and log deviations in the quits rate and the unemployment rate alone. Calibrated to match U.S. data on worker flows, our model's wage Phillips curve states that the quits rate has the most explanatory power for wage growth, while the unemployment rate—the forcing variable in standard sticky-wage models such as Galí (2011)—has almost no weight. This is consistent with the finding that the worker quits rate predicts nominal wage growth very well in U.S. data (Faberman and Justiniano, 2015)[3] and dominates unemploy-

[1] Heise et al. (2022) show that pass-through of wage costs to services prices is high, while pass-through in goods-producing industries is lower. Harasztosi and Lindner (2019) similarly find nearly complete pass-through of wages to prices in non-tradable industries.

[2] See Hall and Krueger (2012); Lachowska et al. (2022); Di Addario et al. (2023).

[3] Karahan et al. (2017) and Moscarini and Postel-Vinay (2017) find that job-to-job transitions dominate the unemployment-to-employment job finding rate in predicting wage levels. Quits and job-to-job transitions are highly correlated but are defined differently. Quits include all voluntary worker separations, including to non-employment. Job-to-job transitions may include involuntary separations where workers are quickly re-employed.

ment in predicting wage growth (e.g., Heise et al., 2026b), providing a validation of the model.

We then consider the effects of a negative supply shock, modeled as a negative shock to the quantity of the endowment good. We show that in the model's wage Phillips curve, the direct pass-through from this shock to wages via higher consumer prices is zero. While a higher price level lowers a worker's real wage, the higher price level also worsens the worker's outside options: both the worker's real wage in other jobs and real consumption while unemployed also fall. With log utility, a higher price level thus has no effect on the worker's propensity to quit their job, either to another job or to unemployment. Similarly, the difficulty of recruiting workers is unchanged by the shock. With no changes in the firm's ability to recruit or retain workers, firms optimally do not change their nominal wage.

In general equilibrium, whether the negative supply shock affects nominal wage growth depends on the shock's effects on labor demand, as changes in relative prices can affect labor demand through expenditure switching (between services and endowment goods) or substitution in production (between labor and endowment good intermediates). We show formally that if monetary policy perfectly stabilizes employment, the cumulative inflation generated from a negative supply shock is similarly zero: nominal wages remain unchanged, price inflation temporarily rises, real wages temporarily fall, but the price level returns to its pre-shock level as the supply shock recedes.

Next, we consider a version of the model where increases in the price level erode the real wage but do not similarly erode the value of unemployment, motivated by the idea that the value of unemployment may be derived from leisure or home production. This extension allows for positive direct pass-through of prices to wages: if the value of unemployment is insulated from higher consumer prices while real wages are not, higher prices lowers the value of taking a job, making unemployed workers harder to recruit, thus incentivizing firms to set higher wages. However, we show that on-the-job search quantitatively limits this pass-through. With realistic rates of on-the-job search, firm wage setting is dominated by competition for already-employed workers, whose propensity to accept outside job offers is unaffected by a higher price level. As a consequence, a negative supply shock that raises consumer prices has trivial effects on firms' optimal wage.

We then calibrate the model and report the effects of supply shocks in general equilibrium under a realistic monetary policy rule where the central bank puts a large but finite weight on stabilizing employment. We treat the endowment good as oil, which means setting a low substitutability of oil in both production (as an intermediate with labor) and in final consumption. We show that the effects of the shock on cumulative price and wage inflation is small in both our baseline case and our extension with a real value of unemployment. We additionally show that shutting down on-the-job search in the case with real unemployment benefits dramatically raises the effect of the shock on cumulative price and wage inflation. Lastly, we simulate the same shock in an equivalently

calibrated counterfactual model where unions set wages based on Erceg et al. (2000). We show that the union model generates dramatically more price and wage inflation, in line with our extension where we shut down on-the-job search.

Overall, our results suggest that in economies like the United States where few workers operate under collective bargaining agreements with cost-of-living adjustments, and where firms' wage setting decisions reflect competition for already-employed rather than for unemployed workers, the ability for workers to reclaim real wages in response to a supply shock that raises their cost of living is limited. The model's prediction of weak pass-through accords with empirical results in Bernanke and Blanchard (2024) that "catch-up" of wages to unexpected price increases was close to zero for most advanced economies during the COVID period, and recent survey evidence finding that U.S. workers believe that the pass-through from aggregate inflation to wages is low (Hajdini et al., 2023; Yaremko and Baek, 2026). We conclude that there is little scope for supply-shock induced wage-price spirals fueled by workers' ability to command higher nominal wages in response to higher nominal prices, even when monetary policy aggressively stabilizes employment.

Recent studies have also explored supply shocks and the response of wages in equilibrium: Lorenzoni and Werning (2023a,b) assume workers set wages via unions and Gagliardone and Gertler (2025) assume workers bargain and wages are rigid in real terms. We depart from these studies by analyzing an economy where firms have monopsony power and workers search on the job. Since evidence suggests that few workers' wages are indexed to inflation in the United States at present, we avoid assuming that wages are rigid in real terms, which generates automatic adjustment along with inflation.[4] Similarly, while there is a long tradition of tractably incorporating nominal wage rigidity in New Keynesian models by assuming workers are unionized following Erceg et al. (2000), we assume that workers are not unionized, as only 11.3% of U.S. workers were unionized as of 2022 (Shierholz et al., 2023); we also show that doing so is important for our results, as assuming unions set wages does imply a sizable pass-through from prices to wages in response to an oil supply shock.

A recent literature explores how workers' beliefs or search behavior respond to inflationary episodes in the presence of nominal wage rigidity (Pilossoph and Ryngaert, 2026; Afrouzi et al., 2026). An important property of the models in this literature is that the distribution of workers' outside options improves in nominal terms as inflation occurs, either because the distribution of offered real wages is exogenously assumed to be fixed, or because the marginal revenue product of labor exogenously rises with inflation. In our general equilibrium setting, we allow worker's outside option to evolve endogenously in response to shocks, and show how the effect of negative

[4]Evidence on the use of cost-of-living adjustments (COLAs) comes from studies of large union contracts, which now cover only a small share of U.S. employment. Even within unionized workers, the share covered by contracts with COLAs has shrunk dramatically since the 1970s. See Christiano et al. (2016), footnote 4.

supply shocks on wages is much smaller than the effect of monetary policy shocks which raise labor demand and the marginal revenue product of labor.

While stylized, our model is consistent with a range of recent microeconomic evidence on how wages are determined. Our model captures the result in Jäger et al. (2020) that wages are insensitive to the flow value of unemployment benefits, even for workers who were hired directly from unemployment. This feature arises for two reasons. First, in our model calibrated to U.S. data, the value of unemployment is significantly below the value of employment, so even sizable changes in the flow value of unemployment benefits does not make unemployment a credible outside option. Second, because firms post wages rather than bargain, all workers are paid the same regardless of their previous employment status. This common wage policy is supported by the finding in Di Addario et al. (2023) that workers' prior employer has small effects on workers' current wages in a large majority of occupations, as well as results in Hall and Krueger (2012) and Lachowska et al. (2022) that most workers in the United States face wage posting rather than bargaining. Lastly, our model features finite elasticities of hiring and separation rates with respect to firms' wage policies (i.e., when a firm raises its wages, workers join the firm more quickly and leave the firm more slowly), which has been extensively documented in the monopsony literature (e.g., Datta, 2023; Bassier et al., 2022).

There is a large macro-labor literature that embeds search frameworks and labor market frictions in macroeconomic models to study implications for the business cycle. de la Barrera i Bardalet (2024) develops a similar model of labor market monopsony with on-the-job search and finds that increased monopsony power flattens the wage Phillips curve. Moscarini and Postel-Vinay (2023) study a model where workers search on the job and bargain when they receive an outside offer, resulting in a wage Phillips curve in which the distribution of wages and misallocation of workers matters for wage growth. Instead, we assume wage-posting and study a simpler setting without misallocation, where workers and firms are homogeneous in their productivity, and study the implications for pass-through from supply shocks to prices.

While prior research has modeled the relationship between job-to-job mobility and wage growth,[5] our setting provides a tractability advantage: if firms are ex-ante identical and adjust prices and wages subject to pricing frictions à la Rotemberg (1982), then our model features a symmetric equilibrium with a single wage alongside endogenous worker flows between firms (and unemployment). This single-wage outcome is compatible with workers' on-the-job search due to the presence of idiosyncratic, worker-specific preference shocks over workplaces, so that workers will sometimes choose to switch jobs even when firms offer identical wages. The result is a tractable

[5]For example, Moscarini and Postel-Vinay (2016) extend the wage posting model of Burdett and Mortensen (1998) into a dynamic setting, and Birinci et al. (2026) assume a three-party bargaining protocol to determine wages. Faccini and Melosi (2023) study the relationship of job-to-job mobility and price inflation.

labor block with job-to-job mobility that can be easily integrated into the standard New Keynesian modeling framework widely used both in the macroeconomics literature (Christiano et al., 2005; Smets and Wouters, 2007) and for policy analysis at central banks. Additionally, our model has proven useful for policymakers' reduced-form empirical analysis of the U.S. wage Phillips curve (Heise et al., 2026a, 2024; Williams, 2024) by providing a *tractable* structural explanation for the strong correlation between wage growth and certain measures of labor market tightness (namely, quits and vacancies per effective searcher).

Layout Section 2 presents stylized facts from U.S. data, demonstrating the tight correlation between quits and wage inflation that motivates our model's assumptions of wage posting and on-the-job search, as well as a weak relationship between price and wage inflation. Section 3 presents our benchmark, tractable New Keynesian model with on-the-job search and firms' wage posting. Section 4 derives the wage Phillips curve in our model and presents analytical results. Section 5 calibrates the model to U.S. data and shows that an oil shock causes less wage and price inflation in our model than would occur in a standard union wage-setting model or in our model when on-the-job search is shut down. This section also discusses the effect of monetary policy shocks and several extensions to the tractable model presented in the main text, along with their implications for the propagation of inflationary supply shocks. Section 6 concludes.

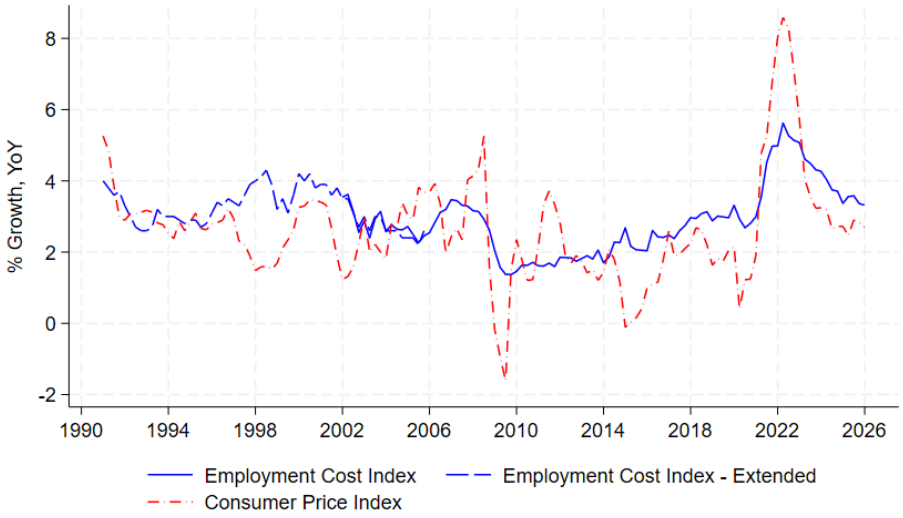
2 Stylized Facts on Aggregate Nominal Wage Growth

Before proceeding to our formal framework, we present two stylized facts about the relationship between aggregate nominal wage growth, labor market variables, and price inflation. First, nominal wage growth, measured by the employment cost index, and price inflation are weakly related at high frequencies. Second, nominal wage growth is tightly correlated with the quits rate.

Figure 1 plots the time series of four quarter growth in the employment cost index of wages and salaries for private industries and four quarter growth in the consumer price index, beginning in the fourth quarter of 1990 when quits data become available. Outside of the post-COVID period, nominal wage growth and price inflation are weakly correlated. Divergence between price inflation and wage growth is particularly visible in 2011 and 2015, when inflation rose and subsequently fell, while nominal wage growth was changing only gradually.

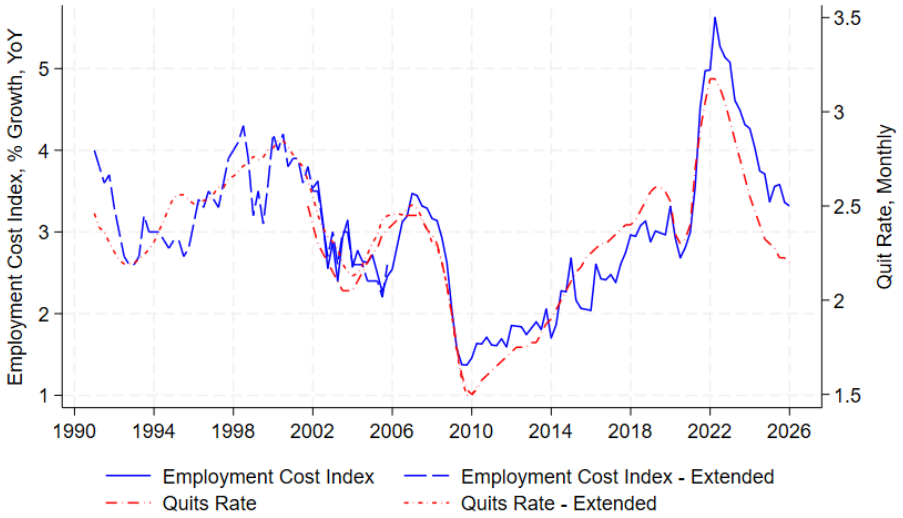
Figure 2 plots the relationship between the four quarter moving average of the quits rate, which is measured as quits per hundred employees from the Job Openings and Labor Turnover Survey (JOLTS), and the four quarter growth in the employment cost index (ECI). This figure shows an extension of the result documented by Faberman and Justiniano (2015) that quits are highly related to growth in the employment cost index, and is related to results documented by Moscarini and

Figure 1: Wage Growth and Headline Price Inflation



Notes: Between 1990-2019, nominal wage growth and price inflation were weakly correlated. Nominal wage growth and price inflation both surged during the COVID pandemic and recovery.

Figure 2: Wage Growth and Quits



Notes: There is a strong correlation between quits, measured here as the four quarter moving average of quits per hundred employees from the Job Openings and Labor Turnover Survey (JOLTS), and four quarter growth in the employment cost index, wages and salaries for private industries.

Postel-Vinay (2017) and Karahan, Michaels, Pugsley, Şahin and Schuh (2017) that nominal wage growth is well-predicted by job-to-job transitions.[6] The fact that a large share of quits are in fact

[6]The empirical measure of quits encompasses several types of labor market transitions, including direct job-to-job

job-to-job transitions motivates the inclusion of on-the-job search in our model; we will show later that including a realistic quantity of on-the-job search (i.e., by calibrating our model to U.S. data) has important implications for the pass-through of supply shocks to nominal wages.

We next revisit the findings in this prior literature and present OLS regressions of wage inflation on quits and price inflation. We also include unemployment and a measure of the unemployment gap (the difference between the unemployment rate and the natural rate of unemployment), which are the traditional labor market indicators for estimating wage Phillips curves (Galí, 2011).[7] Formally, in quarter t, letting Q_t denote quits, U_t denote the unemployment rate, and π_t^w and π_t denote quarterly nominal wage and price inflation, respectively, we estimate the following regression:[8]

$$\pi_t^w = \beta_0 + \beta_Q \ln Q_t + \beta_U \ln U_t + \beta_\pi \pi_{t-1} + \varepsilon_t. \tag{1}$$

Table 1 reports the results: the empirical estimate $\hat{\beta}_Q$ is much larger than $\hat{\beta}_U$.[9] Indeed, $\hat{\beta}_U$ is not generally significant at conventional levels and is not of the expected sign once we include quits. These results are robust to the inclusion of the COVID pandemic and recent recovery. In the following sections, we will develop a model that is capable of matching the empirical correlations in equation (1): specifically, we will write down a structural wage Phillips curve of the similar form of (1), where we will have both a relatively large positive value of $\hat{\beta}_Q$ but a relatively small (and potentially positive) value of $\hat{\beta}_U$.

Table 1 also reports the coefficient of lagged price inflation on wage growth $\hat{\beta}_\pi$. The coefficient on inflation is fairly small, with elasticities all below 0.15. Comparing Columns (1) and (2), we see that the coefficient on price inflation falls when adding quits, suggesting that the unemployment rate is failing to pick up fluctuations in both wage and price inflation that the quits rate absorbs.

moves without a spell of non-employment, job-to-job transitions with a period of non-employment, and voluntary quits into non-employment. Qiu (2022) finds that approximately 3% of workers transition from employment to non-participation each month, with most of these transitions being voluntary. In addition, Elsby et al. (2010) report that only 16% of workers who quit their jobs subsequently enter unemployment.

[7] We take our estimate of the natural rate of unemployment rate from the CBO, acknowledging that there is a wide literature and range of estimates of the natural rate of unemployment (e.g., Crump et al., 2019, 2024). For the quits data, from 2010Q3-2023, we use the private sector quits rate JTS1000QUR from the St. Louis Federal Reserve FRED database, aggregated by averaging at the quarterly level. Prior to 2000, we use the quarterly private sector quits rate from Davis et al. (2012). Between 2001Q1 and 2010Q2, we use the average of these two series. Similarly for the employment cost index, we use the employment cost index wages and salaries series for private industry workers ECIWAG from FRED beginning in 2005. Prior to 2001, we use the SIC industry basis of the employment cost index for private industry wages and salaries, series ECS20002I, from the Bureau of Labor Statistics. From 2001-2005, we take the average of these two wage series.

[8] We compute the inflation terms as log differences: letting W_t be the nominal wage and P_t be the price level, $\Pi_t^w \equiv 100 \times (\ln W_t - \ln W_{t-1})$ and $\Pi_{t-1} \equiv 100 \times (\ln P_{t-1} - \ln P_{t-2})$.

[9] Column 1 should be interpreted as "for a 100% increase in the unemployment rate (i.e., double the unemployment), there is an .485% point decrease in quarterly gross wage growth". Likewise, Column 2 implies that 100% increase in the quits rate (i.e., double the quits) would result in 0.9737% point increase in quarterly wage growth. The standard deviation of log quits and log unemployment are 0.17 and 0.28, respectively, from 1990Q2-2026Q1.

Table 1: Time Series Regression of Wage Growth on Labor Market Variables, 1990Q2-2026Q1

VARIABLES	(1) ECI	(2) ECI	(3) ECI	(4) ECI	(5) ECI	(6) ECI
$\ln U_t$	-0.4754***	-0.0894	0.1375			
	(0.0681)	(0.0892)	(0.1054)			
$\ln Q_t$		0.8992***	1.1175***	0.9118***	0.9487***	1.0695***
		(0.1485)	(0.1875)	(0.1642)	(0.1283)	(0.1597)
$\ln U_t - \ln U_t^*$				-0.0840		
				(0.0971)		
$\ln U_{t-1}$					-0.0599	-0.0494
					(0.0743)	(0.0812)
Π_{t-1}	0.1412***	0.0886***	0.0365	0.0877***	0.0877***	
	(0.0480)	(0.0331)	(0.0321)	(0.0330)	(0.0326)	
Observations	144	144	119	144	144	144

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Notes: Results from a quarterly regression of wage growth measured using the Employment Cost Index (ECI) on unemployment, the quits rate, and lagged price inflation, as specified in equation (1). While Column (1) shows that a regression of wage growth on unemployment alone yields a familiar negative sign, including quits reduces the significance to below conventional levels as seen in Column (2). Columns (3) and (4) show that this result is robust to dropping the COVID pandemic and recovery, and also to measuring unemployment in log deviations from the natural rate as estimated by the CBO (U.S. Congressional Budget Office, 2026), $\ln(U_t) - \ln(U_t^*)$. Columns (5) and (6) demonstrate that using lagged U_t does not alter the results. Standard errors are Newey-West with 4 lags.

Column (3) shows that prior to COVID, there was no significant relationship between price inflation and wage growth after conditioning on quits. Including the unemployment gap in Columns (4) or lagging unemployment in columns (5) and (6), respectively, does not meaningfully change the results.

In Section 5.1, we will use this evidence on the relationship between quits, unemployment, and wage growth to calibrate our model's wage Phillips curve.

3 Model

This section builds a model where firms post wages and workers search on the job, and calibrates that model to U.S. data. In laying out the model, we first describe the problem of a firm posting wages in the presence of recruiting costs and where workers search on-the-job. When deciding

whether to raise wages, the firm trades off between a higher wage bill and lower turnover costs. Higher wages lower turnover costs by increasing the probability that the firm recruits a particular searcher, regardless of whether that searcher is already employed or unemployed (the recruiting rate), while also lowering the probability that incumbent workers leave (the separation rate). Because the firm's problem does not depend directly on the price level in partial equilibrium, an increase in workers' cost of living can only affect wages through its effects on these recruiting or separation rates.

We then describe the solution to the worker's problem, which determines firms' recruiting and separation rates. Because firms are trading off wage and turnover costs, changes in the price level affect optimal wages only to the extent that the price level affects the firm's ability to recruit and retain workers. Since a change in the price level affects the real wages offered by all firms proportionally, and if workers have log utility, then a higher price level has no effect on the relative desirability of outside jobs. Therefore, changes in the price level can relatively improve workers' outside option only if they make unemployment relatively more attractive. If this is the case, this can lead to pass-through from prices to wages, as firms must now offer a higher wage to retain the same number of workers as before. However, these considerations are quantitatively small when (i) most workers already vastly prefer a job to unemployment and/or when (ii) most searching workers already have a job, rendering the value of unemployment irrelevant when considering whether to accept a new job offer. Moreover, this channel need not exist at all if changes in the price level do not affect the desirability of unemployment, as in our baseline case described below.

Consumption There are two goods in the economy: an endowment good X_t and services Y_t. They are combined into an aggregate consumption good, C_t, according to the CES function

$$C_t = \left(\alpha_Y^{\frac{1}{\eta}} Y_t^{\frac{\eta-1}{\eta}} + \alpha_X^{\frac{1}{\eta}} X_t^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{2}$$

with corresponding aggregate price index[10]

$$P_t = \left(\alpha_Y P_{y,t}^{1-\eta} + \alpha_X P_{x,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}. \tag{3}$$

Workers are hired by firms to produce services Y_t, so that their real wage is determined by the nominal wage offered in that sector divided by the aggregate price level P_t.

We assume that the endowment good (X) can be used as an input by firms in the service sector, and denote the demand of firms for this good by $\bar{X}_t$. Given the total endowment supply, X_t^s, market

[10] We assume $\alpha_X + \alpha_Y = 1$ with $\alpha_X > 0$ and $\alpha_Y > 0$ as usual.

clearing implies

$$X_t + \bar{X}_t = X_t^s.$$

Perfectly competitive retailers sell the endowment supply X_t^s to both firms and workers and rebate their profits to households, following Gagliardone and Gertler (2025).

Service Sector We assume that perfectly-competitive retailers bundle service types j according to a standard Dixit-Stiglitz production function with an associated ideal price index:

$$\begin{aligned} Y_t &= \left(\int \left(Y_t^j \right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \\ P_{y,t} &= \left(\int \left(P_{y,t}^j \right)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}, \end{aligned}$$

yielding product demand for variety j:

$$\frac{Y_t^j}{Y_t} = \left(\frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon}. \tag{4}$$

The production function for output Y_t^j of firm j is given by

$$Y_t^j = A_t^j \left(\alpha_N^{\frac{1}{\bar{\eta}}} \left(N_{jt} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\bar{X}_t^j \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \right)^{\frac{\bar{\eta}}{\bar{\eta}-1}}, \tag{5}$$

where N_{jt} and $\bar{X}_t^j$ represent the demand for labor and intermediate inputs, respectively. In equilibrium, $\int_j \bar{X}_t^j dj = \bar{X}_t$. Note that from (5),

$$\frac{\partial Y_t^j}{\partial N_{jt}} = \left(A_t^j \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \alpha_N^{\frac{1}{\bar{\eta}}} \left(\frac{N_{jt}}{Y_t^j} \right)^{-\frac{1}{\bar{\eta}}} = \alpha_N^{\frac{1}{\bar{\eta}}} \left[\frac{\left(N_{jt} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}}}{\left(A_t^j \right)^{\bar{\eta}-1} \left(\alpha_N^{\frac{1}{\bar{\eta}}} \left(N_{jt} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\bar{X}_t^j \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \right)} \right]^{\frac{-1}{\bar{\eta}-1}} \tag{6}$$

which is increasing in $\bar{X}_t^j$ for any level of $\bar{\eta}$. Consequently, a reduction in $\bar{X}_t^j$ leads to a decline in the marginal product of labor when $\alpha_{\bar{X}} > 0$.

Firm's Wage-Posting Problem We now turn to the determination of nominal wages. The firm j produces with labor and the endowment good according to (5). Firm j sets nominal wages W_{jt} each period, which is assumed to be the same for all workers in the firm, including new hires. Workers separate from firm j with probability $S_t(W_{jt}|\{W_{kt}\}_{k\neq j})$ each period, with $S_t'(W_{jt}|\{W_{kt}\}_{k\neq j}) < 0$: firms retain a higher share of workers each period by paying a higher wage, given other firms'

wages. The firm can recruit workers by posting vacancies V_{jt}, and the probability that a vacancy successfully results in a hire is $R_t(W_{jt}|\{W_{kt}\}_{k\neq j})$, with $R_t'(W_{jt}|\{W_{kt}\}_{k\neq j}) > 0$. How retention and separation functions $R_t(W_{jt}|\{W_{kt}\}_{k\neq j})$ and $S_t(W_{jt}|\{W_{kt}\}_{k\neq j})$ depend on wages set by other service firms will be derived after we describe households' and workers' problems in Section 3.1.

The firm pays a convex, per-vacancy hiring cost, $c\left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi} W_t$, to post V_t vacancies, where W_t is the aggregate wage, $c > 0$ and $\chi \geqslant 0$. Finally, the firm is also subject to price and wage adjustment frictions à la Rotemberg (1982). Given this, each firm j maximizes the present discounted value of profits, solving

$$\max_{\substack{\{P^j_{y,t}\},\{Y^j_t\},\{N_{jt}\}\\ \{W_{jt}\},\{V_{jt}\},\{\bar{X}^j_t\}}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P^j_{y,t} Y^j_t - W_{jt} N_{jt} - c\left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi} V_{jt} W_t - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt} N_{jt} - P_{x,t} \bar{X}^j_t \right) \tag{7}$$

subject to the law of motion for employment

$$N_{jt} = (1 - S_t(W_{jt}))N_{j,t-1} + V_{jt} R_t(W_{jt}), \tag{8}$$

the product demand equation (4), and the service sector production function (5).

From inspecting (7) and (8), we can see that the service sector firm chooses the wage (and other choice variables) taking as given the choices of other service sector firms (embodied in the price index and aggregate output of the service sector), parameters, and the separation and recruiting rates $S_t(\cdot)$ and $R_t(\cdot)$. Since the vacancy-posting cost is denominated in labor (i.e., priced by the aggregate wage W_t), the aggregate price level P_t does not appear directly in (7). Thus, in partial equilibrium, the only way that changes in the price level can impact the firm's wage setting decision is through changes in $S_t(\cdot)$ and $R_t(\cdot)$, which will be determined by the solution to workers' optimization problem described in Section 3.1.

The next section presents the optimization problems faced by households and workers, which jointly determine the recruiting and separation functions faced by firms. With these components in place, we then will define a symmetric equilibrium where all firms choose the same allocations.

3.1 Households and Workers

This section derives the household and worker block of the model. We deviate from the standard assumption in the New Keynesian literature of perfect consumption insurance within the household by assuming that households only imperfectly insure the consumption of workers who are unem-

ployed, consistent with evidence that unemployed workers consume less than employed workers (e.g., Chodorow-Reich and Karabarbounis, 2016). We assume that workers themselves choose whether to take a particular job offer, making employment decisions based on relative wages and consumption levels, in addition to idiosyncratic firm-specific preference shocks. Workers' mobility decisions aggregate up into the firms' recruiting and separation functions. Households smooth aggregate consumption within the household over time, yielding a standard Euler equation, making the labor block easy to integrate into a standard New Keynesian setting.

Frictional Markets Workers and firms match according to random search in a frictional market. As mentioned above, each firm j posts V_{jt} vacancies, and aggregate vacancies are $V_t = \int V_{jt} dj$. At the beginning of each period, a share of workers $s \in [0, 1]$ exogenously separate and enter unemployment without an opportunity to search that period. Also each period, employed workers who do not exogenously separate can search on the job with some constant, exogenous probability $\lambda_{EE} \in [0, 1]$, and unemployed workers always search.[11]

Let U_t denote the unemployment rate. Then, the total mass of searchers, $\mathcal{S}_t$, is given by $\mathcal{S}_t = (1 - s)\lambda_{EE}(1 - U_{t-1}) + U_{t-1}$, which consists of on-the-job searchers in the current period and unemployed workers carried over from the previous period.

Matching is random and follows a constant returns to scale matching function $M_t(V_t, \mathcal{S}_t)$, given by

$$M_t(V_t, \mathcal{S}_t) = m\mathcal{S}_t^{1-\nu} V_t^{\nu}, \tag{9}$$

with $\nu \in (0, 1)$ and the matching efficiency parameter $m > 0$. Labor market tightness is $\theta_t = \frac{V_t}{\mathcal{S}_t}$. The job finding rate for workers, $f(\theta_t) = \frac{M_t}{\mathcal{S}_t}$, is increasing in tightness θ_t, and the probability that a vacancy is matched with a worker $g(\theta_t) = \frac{M_t}{V_t}$ is a decreasing function of tightness θ. The share of searchers who are employed is $\phi_{E,t} = \frac{(1-s)\lambda_{EE}(1-U_{t-1})}{\mathcal{S}_t}$, and the share of searchers who are unemployed is $\phi_{U,t} = 1 - \phi_{E,t} = \frac{U_{t-1}}{\mathcal{S}_t}$. Under (9), the job finding rate for workers $f(\theta_t)$ and vacancy-filling rate $g(\theta_t)$ are given by

$$f(\theta_t) = m\theta_t^{\nu},$$
$$g(\theta_t) = m\theta_t^{\nu-1}.$$

Households A representative household has a unit mass $i \in [0, 1]$ of members who can work. Households seek to maximize the discounted present value of their members' utility, which is log in consumption. Without loss of generality, we assume that unemployed household members must

[11] This simplifying assumption abstracts from the possibility that on-the-job search intensity rises with the price level (Hajdini et al., 2023; Pilossoph and Ryngaert, 2026; Raposo, 2025). In Online Appendix E, we relax this assumption and show that our main results continue to hold when on-the-job search effort is determined endogenously.

each have the same consumption level, C_t^u.[12] Then letting $C_t(i, j(i))$ denote the consumption of worker i in state $j(i)$, where $j(i)$ indicates the firm i is employed at, the household's objective function becomes

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[U_t \ln(C_t^u) + \int_0^{1-U_t} \ln\left(C_t(i, j(i))\right) di\right].$$

The household is allowed to choose C_t^u (effectively, an unemployment benefit) and also a linear tax/subsidy $1 - \tau_t$ on its members. Employed workers consume their income each period:

$$C_t\left(i, j(i)\right) = \tau_t \frac{W_{j(i)t}}{P_t} \tag{10}$$

subject to the following budget constraint: Let D_t denote nominal dividend payments from service-sector firms—which earn profits with monopoly and monopsony power—and from perfectly competitive goods firms, which receive the endowment X_t^s, sell it, and rebate the proceeds to households. Let B_t denote nominal bond holdings in zero net supply that pay the nominal interest rate i_t. Finally, define

$$\bar{W}_t \equiv \frac{1}{1-U_t} \int_0^{1-U_t} W_{j(i)t} di$$

as the average wage earned by employed workers. The household budget constraint is then given by

$$U_t C_t^u = \frac{D_t}{P_t} - \frac{B_t}{P_t} + \frac{(1+i_{t-1})B_{t-1}}{P_t} + (1-\tau_t)(1-U_t)\frac{\bar{W}_t}{P_t}. \tag{11}$$

Unemployment Insurance Policy To make further progress in delivering a tractable model with households' standard consumption Euler equation, we impose an *ad hoc* consumption sharing rule within the household. In specific, we will consider the following rule: letting $\varpi \in \{0, 1\}$,

$$b_t = \varpi b + (1-\varpi)\xi^{-1}\frac{\bar{W}_t}{P_t} \tag{12}$$

where b and ξ are constants, and $\bar{W}_t$ is the average wage of employed workers, satisfying $\bar{W}_t = W_t$ in the symmetric equilibrium. Unemployed workers consume $C_t^u = \tau_t b_t$ each period.

We use this specification to nest two kinds of unemployment insurance policies: under $\varpi = 0$, unemployed workers' consumption becomes a *constant* fraction $1/\xi$ of employed workers' average consumption:

$$\frac{\bar{C}_t^e}{C_t^u} = \xi, \tag{13}$$

[12] This is not restrictive, as given our other assumptions the household will always choose to equalize consumption across unemployed agents due to diminishing marginal utility of consumption.

where $\xi \geqslant 1$ and $\bar{C}_t^e \equiv \frac{1}{1-U_t}\int_0^{1-U_t} C(i,j(i))di$ is the average consumption of employed. This rule allows us to capture the fact that the ratio of unemployed and employed consumption is relatively constant over the business cycle (Chodorow-Reich and Karabarbounis, 2016). Moreover, this can be interpreted as the outcome of a household facing an incentive–insurance trade-off. By providing less insurance to unemployed workers and making unemployment relatively less attractive (that is, increasing ξ), the household strengthens members' incentives to accept employment, albeit at the cost of reducing consumption for unemployed workers, whose marginal utility of consumption is higher.

In contrast, when $\varpi = 1$, the model implies that unemployed workers are guaranteed a constant level of consumption b that is indexed to inflation. This formulation provides a convenient way to capture a range of model extensions, such as home production of final consumption goods or utility derived from leisure when consumption and leisure are substitutes.[13]

We retain the general formulation in (12) when deriving the equilibrium conditions and then analyze the two benchmark cases, $\varpi = 0$ and $\varpi = 1$, separately. In practice, we will take as our preferred specification the case where $\varpi = 0$: in Section 5 we will show that this version of the model with $\varpi = 0$ captures the fact that labor income *falls* after an inflationary oil shock in U.S. data (Del Canto et al., 2025)[14]—unlike in a standard union model or versions of our model where $\varpi = 1$.

Euler Equation Under the general consumption sharing rule (12), we show in Online Appendix B.2 that the standard consumption Euler equation with log utility holds:

$$C_t^{-1} = \frac{1}{1+\rho}\frac{1+i_{t,t+1}}{\Pi_{t+1}}C_{t+1}^{-1}, \tag{14}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the aggregate inflation.

3.1.1 Workers

Workers get utility from the log of consumption plus an idiosyncratic preference draw ι.[15] ι represents how much workers like their current job at firm j, which is redrawn every period and is i.i.d. Workers draw a similar preference shock each period during unemployment. Workers are myopic[16] and consider their utility only one period at a time, which for worker i in state j is given

[13] See Appendix B.1 for a more detailed discussion of the parallels between this approach and models in which the value of unemployment arises from additional leisure time.

[14] Del Canto et al. (2025) find that nominal household weekly earnings falls in response to an oil shock.

[15] We abstracted from idiosyncratic shocks ι in the households problem, because the household takes these shocks as given.

[16] In Online Appendix H, we relax this assumption and analyze how a negative endowment shock passes through to wages when workers are forward-looking. We find that the degree of wage pass-through with forward-looking workers

by

$$\mathcal{V}_t(i,j) = \ln\left(C_t(i,j)\right) + \iota_{ijt}.$$

Workers are allowed to search on the job with probability λ^{EE}, and conditional on searching, are matched with a vacancy with probability $f(\theta)$. We assume that workers are allowed to transition into unemployment with probability λ^{EU}. Consider a worker i currently employed at firm j who successfully matches with firm k's vacancy. She will move to firm k only if $\mathcal{V}_t(i,k) \geqslant \mathcal{V}_t(i,j)$. We define $s_{jk}(W_{jt}, W_{kt})$ as the probability that the worker is poached from firm j to firm k, conditional on matching with firm k's vacancy.

For tractability, we assume that ι follows a Type-1 extreme value distribution with scale parameter γ^{-1}. Following the consumption rule (10), the separation probability $s_{jk}(W_{jt}, W_{kt})$ is given by

$$s_{jk}\left(W_{jt}, W_{kt}\right) = \frac{\left(\tau_t \frac{W_{kt}}{P_t}\right)^{\gamma}}{\left(\tau_t \frac{W_{kt}}{P_t}\right)^{\gamma} + \left(\tau_t \frac{W_{jt}}{P_t}\right)^{\gamma}} = \frac{W_{kt}^{\gamma}}{W_{kt}^{\gamma} + W_{jt}^{\gamma}}, \tag{15}$$

which is decreasing in W_{jt}: if firm j pays a higher wage, workers are less likely to be poached. Notice also that due to log utility, the probability a worker switches jobs is only a function of the relative *nominal* wage. The worker takes as given the top-up rate τ_t set by the household and the price level P_t, both of which are unchanged regardless of which job the worker chooses.

Now consider a worker who is deciding whether to quit into unemployment. Since a worker i who is currently employed at firm j quits into unemployment only if $\mathcal{V}(i,j) \leqslant \mathcal{V}(i, \text{unemployed})$, thus the probability that a worker voluntarily quits into unemployment $s_{ju}(W_{jt})$ is given by

$$s_{ju}(W_{jt}) = \frac{(\tau_t b_t)^{\gamma}}{(\tau_t b_t)^{\gamma} + \left(\tau_t \frac{W_{jt}}{P_t}\right)^{\gamma}} = \frac{b_t^{\gamma}}{b_t^{\gamma} + \left(\frac{W_{jt}}{P_t}\right)^{\gamma}}, \tag{16}$$

where $C_t^u = \tau_t b_t$ from the consumption sharing rule (12). Under log utility, whether or not the price level cancels out of the separation rate depends on how b_t is determined in (12). If $\varpi = 0$—that is, when unemployed workers consume a constant fraction $1/\xi$ of the average consumption of employed workers—we have $b_t = \xi^{-1}\frac{\bar{W}_t}{P_t}$ and

$$s_{ju}(W_{jt}) = \frac{\left(\frac{\bar{W}_t}{\xi}\right)^{\gamma}}{\left(\frac{\bar{W}_t}{\xi}\right)^{\gamma} + {W_{jt}}^{\gamma}}$$

with no P_t in it.[17] In practice, we consider alternative specifications depending on the value of ϖ

is quantitatively very similar to that in our baseline model with myopic workers, suggesting that our main results are not sensitive to this assumption.

[17] In general, if it is some fixed nominal income, $b_t = \bar{b}/P_t$, the price level cancels out and the separation probability

and discuss their implications for the wage Phillips curve below.

These individual transition probabilities, $S_{jk}(\cdot)$ and $S_{ju}(\cdot)$, aggregate up into the firm's separation rate $S(W_{jt})$: each period, a share of workers $s \in (0,1)$ exogenously separate while the remaining share, $(1-s)$, separates endogenously when they receive an opportunity they prefer to their current job—either another employment opportunity or the option to quit into unemployment. Recall that $f(\theta_t)\lambda_{EE}$ denotes the probability that an employed worker is allowed to search on the job and matches to another firm, and that λ_{EU} denote the probabilities that an employed worker is allowed to consider quitting into unemployment. The separation rate is given by

$$\begin{aligned} S_t(W_{jt}) &\equiv S_t(W_{jt}|\{W_{kt}\}_{k\neq j}) \\ &= s + (1-s)\left[\lambda_{EE} f(\theta_t)\int s_{jk}(W_{jt},W_{kt})z(W_{kt})dk + \lambda_{EU} s_{ju}(W_{jt})\right], \end{aligned} \tag{17}$$

where $z(W_{kt})$ is an endogenous density function of outside posted wages. Note in (17) that $S_t(\cdot)$ is a decreasing function of W_{jt}, i.e. $S_t'(W_{jt}) < 0$, since all of it's components are decreasing in W_{jt}; in other words, the firm's separation rate falls as the wage rises.

Analogous to the individual separation probabilities, there are also probabilities that a matched worker is recruited by the firm, conditional on whether the worker is currently employed or unemployed. Consider a worker employed at firm k that encounters firm j's vacancy. The probability that firm j successfully poaches the worker $r_{kj}(W_{kt},W_{jt})$ is:

$$r_{kj}(W_{kt},W_{jt}) = \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma}{\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{W_{jt}^\gamma}{W_{kt}^\gamma + W_{jt}^\gamma}, \tag{18}$$

which is increasing in W_{jt} and is a function of relative wages.

Now consider an unemployed worker who is matched with firm j's vacancy. The probability that the worker takes the job with firm j is defined as $r_{uj}(W_{jt})$ and is equal to

$$r_{uj}(W_{jt}) = \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma}{(\tau_t b_t)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma} = \frac{\left(\frac{W_{jt}}{P_t}\right)^\gamma}{b_t^\gamma + \left(\frac{W_{jt}}{P_t}\right)^\gamma}, \tag{19}$$

which is increasing in W_{jt}.[18]

is a function of relative nominal payments only.

[18] Again, when $\varpi = 0$, we have $b_t = \xi^{-1}\frac{\bar{W}_t}{P_t}$ and

$$r_{uj}(W_{jt}) = \frac{W_{jt}^\gamma}{\left(\frac{\bar{W}_t}{\xi}\right)^\gamma + W_{jt}{}^\gamma},$$

We can use (18) and (19) to write firm j's recruiting rate, defined as the share of vacancies that successfully result in hiring a worker. Recalling that $g(\theta_t)$ denotes the probability that a vacancy is matched with a worker, and that $\phi_{E,t}$ and $\phi_{U,t}$ denote the share of searchers who are employed and unemployed, respectively, we can write the recruiting rate as:

$$R_t(W_{jt}) \equiv R_t(W_{jt}|\{W_{kt}\}_{k\neq j}) = g(\theta_t)\left[\phi_{E,t}\int_k r_{kj}(W_{kt}, W_{jt})\omega(W_{kt})dk + \phi_{U,t}r_{uj}(W_{jt})\right]. \quad (20)$$

where $\omega(W_{kt})$ is the distribution of wages that workers are currently employed at. The recruiting rate $R_t(W_{jt})$ is an increasing function because all of its components r_{kj} and r_{uj} are also increasing in W_{jt}. In other words, a higher wage improves the firm's odds of recruiting workers through its vacancies.

Recruiting and Separation Rate Elasticities The recruiting and separation rate elasticities are defined as follows:

$$\varepsilon_{R,W_{jt}} \equiv \frac{\partial R_t(W_{jt})}{\partial W_{jt}}\frac{W_{jt}}{R_t(W_{jt})}, \quad (21)$$

$$\varepsilon_{S,W_{jt}} \equiv \frac{\partial S_t(W_{jt})}{\partial W_{jt}}\frac{W_{jt}}{S_t(W_{jt})}. \quad (22)$$

The recruiting elasticity $\varepsilon_{R,W_{jt}}$ captures the extent to which firm j can increase its recruiting rate by raising its wage W_{jt}, while the separation rate elasticity $\varepsilon_{S,W_{jt}}$ captures the extent to which firm j can reduce its separation rate by raising its wage.

3.2 Symmetric Equilibrium

We close the model using a simple Taylor rule that allows for a monetary policy shock, $\varepsilon_{i,t}$. Letting N denote steady-state employment, the Taylor rule is defined as[19]

$$1 + i_t = (1 + i)\left(\frac{\Pi_{Y,t}}{\Pi_Y}\right)^{\phi_\pi}\left(\frac{N_t}{N}\right)^{\phi_N}\exp(\varepsilon_{i,t}), \quad (23)$$

where $\Pi_{Y,t} \equiv \frac{P_{Y,t}}{P_{Y,t-1}}$ denotes inflation in service-sector prices, and we set $\phi_\pi = 1.5$. We target services inflation to be consistent with modern central banks' focus on core inflation, looking ahead to our quantitative exercises where we will calibrate X to be oil. We will vary the parameter ϕ_N, which governs the policy response to labor market stabilization in addition to inflation, with

where P_t cancels out of the formula.

[19]Note in (23) that $\Pi_Y = 1$ at the steady state.

the benchmark value set to $\phi_N = 1$. Note that $\phi_N \to \infty$ corresponds to the case in which monetary policy perfectly stabilizes the labor market, achieving $N_t = N$.

With the policy rule in (23), we solve for a symmetric equilibrium. The equilibrium consists of sequences of endogenous prices and quantities satisfying the following conditions: (i) firms choose identical allocations, such that $A_t^j = A_t$, $W_{jt} = W_t$, $N_{jt} = N_t$, $V_{jt} = V_t$, $P_{y,t}^j = P_{y,t}$, $\bar{X}_t^j = \bar{X}_t$; (ii) workers and households maximize utility; (iii) firms maximize profits; (iv) product markets clear; and (v) labor market flows are internally consistent.

We linearize these necessary conditions around a non-stochastic steady state within a symmetric equilibrium and solve for the unique equilibrium responses to unanticipated date-0 shocks, followed by a perfect-foresight transition (i.e., "MIT shocks"). Online Appendix D demonstrates that, under our calibration, the proposed symmetric equilibrium is indeed sustainable; specifically, no firm j has an incentive to deviate from it.[20] Although equilibrium features a common wage across firms, idiosyncratic shocks still generate worker flows between unemployment and different firms. This structure delivers a highly tractable dynamic model with on-the-job search.

3.2.1 Wage Phillips Curve

To clarify the relationship between separation rates, recruiting rates, and wage-setting decisions of firms, we derive a wage Phillips curve from the firm's first-order conditions under a symmetric equilibrium in which all firms offer the same aggregate wage W_t.

Under the symmetric equilibrium, the wage Phillips curve expresses nominal wage growth as exclusively a function of aggregate, endogenous labor market variables: vacancies, employment, recruiting and separation rates, and recruiting and separation elasticities, again with no direct role for aggregate price index P_t. With $\Pi_t^w \equiv \frac{W_t}{W_{t-1}}$, the wage Phillips curve is given by:

$$\begin{aligned}\psi^w \left(\Pi_t^w - 1\right) \Pi_t^w + 1 =& c(1+\chi) \left(\frac{V_t}{N_{t-1}}\right)^\chi \left[\frac{V_t}{N_t} \varepsilon_{R,W_t} + (-\varepsilon_{S,W_t}) \frac{N_{t-1}}{N_t} \frac{S_t(W_t)}{R_t(W_t)}\right] \\ &+ \frac{1}{1+\rho} \psi^w \left(\Pi_{t+1}^w - 1\right) \left(\Pi_{t+1}^w\right)^2 \frac{N_{t+1}}{N_t}.\end{aligned} \tag{24}$$

Therefore, in any symmetric equilibrium where firms solve an optimization problem of the form of (7), the wage Phillips curve will be a function of the current and expected future paths of the job vacancy rate V_t, employment N_t, the recruiting and separation rates $R_t(W_t)$ and $S_t(W_t)$, and their elasticities, denoted $\varepsilon_{R,W_t} > 0$ and $\varepsilon_{S,W_t} < 0$ following conventions in the monopsony literature; see e.g. Bloesch et al. (2026).[21]

[20] Specifically, Online Appendix D demonstrates numerically that a sufficiently large variance in idiosyncratic preference shocks and a sufficiently high probability of quitting into unemployment ensure equilibrium stability.

[21] Appendix B.2 derives the firm's first-order conditions in (7), including the price Phillips curve and wage Phillips

With $\Pi_{y,t} \equiv \frac{P_{y,t}}{P_{y,t-1}}$, the price Phillips curve in a symmetric equilibrium is given by

$$(\Pi_{y,t} - 1)\,\Pi_{y,t} = \frac{\epsilon}{\psi}\left(\frac{\mu_t}{P_{y,t}} - \frac{\epsilon - 1}{\epsilon}\right) + \frac{1}{1+\rho}\,(\Pi_{y,t+1} - 1)\,(\Pi_{y,t+1})^2\,, \tag{25}$$

where μ_t is the firm marginal cost, so that this yields the standard markup expression $P_{y,t} = \frac{\epsilon}{\epsilon-1}\mu_t$ in a zero inflation steady state or in the absence of adjustment costs in price setting ($\psi = 0$). Headline inflation is given by

$$\Pi_t = \Pi_{y,t}\left(\frac{\alpha_Y + \alpha_X\left(\frac{P_{X,t}}{P_{Y,t}}\right)^{1-\eta}}{\alpha_Y + \alpha_X\left(\frac{P_{X,t-1}}{P_{Y,t-1}}\right)^{1-\eta}}\right)^{\frac{1}{1-\eta}}. \tag{26}$$

Interpretation This wage Phillips curve (24) captures how competition for workers affects firms' optimal wage growth. The first term $(\frac{V_t}{N_{t-1}})^{\chi}$ captures the convex cost of posting vacancies: since firms must post vacancies to attract workers, higher marginal vacancy posting costs raises the value of both recruiting a worker the firm has matched with as well as retaining existing workers. If recruiting a worker is more valuable, then firms will want to pay higher wages. The next term, within brackets, includes the recruiting elasticity term ε_{R,W_t}, which captures how sensitive the probability of hiring a matched worker is to the wage. If this recruiting elasticity is elevated, then workers' acceptance probability will be more sensitive to the wage, increasing the incentive at the margin for a firm to raise its wage. This ε_{R,W_t} is multiplied by the number of vacancies V_t. Next is the separation elasticity term ε_{S,W_t}. This elasticity is negative, so the negative of the separation elasticity is positive. A more steeply negative separation elasticity means that workers' likelihood of quitting is more sensitive to wages, so the more negative this value is, the greater the incentive to raise wages at the margin. Lastly, we have the separation rate $S_t(W_t)$ and recruiting rate $R_t(W_t)$. A higher separation rate indicates that workers have more opportunities to quit, increasing pressure for firms to raise wages. Analogously, when the recruiting rate $R_t(W_t)$ is higher, workers are easier to hire, lowering the pressure for firms to raise wages.

4 Analytical Results

In this section, we simplify the nonlinear wage Phillips curve in equation (24) by log-linearizing our model around its non-stochastic steady state. The log-linear wage Phillips curve provides first order accurate descriptions of the model's dynamic, nonlinear relationships, and allows us to show that wage pressures can be summarized by a small number of labor market tightness variables,

curve (24).

similar to the regressions presented in Table 1 (up to a forward-looking wage inflation term). These equations are useful for building intuition about what drives wage inflation in the model, and for answering the question of whether prices pass through directly to wages, holding labor market variables fixed. The section concludes by using these log-linear wage Phillips curves to prove that when the central bank perfectly stabilizes employment, wage inflation is also perfectly stabilized in our baseline model with $\varpi = 0$.

First, for a generic real unemployment benefit b_t defined in equation (12), we begin by defining the ex ante probability that an unemployed workers accepts a job at the wage $\tilde{w}_t \equiv \frac{W_t}{P_t}$ as

$$\mathcal{C}_t \equiv \frac{\tilde{w}_t^{\gamma}}{\tilde{w}_t^{\gamma} + b_t^{\gamma}}.$$

This job acceptance term $\mathcal{C}_t$ will be important for understanding the "pass-through" of prices to wages, as the price level will enter the wage Phillips curve only through this term.

Next, we log-linearize the wage Phillips curve in (24).[22] Let $\check{x}_t$ denote the log deviation of x_t from its steady-state value. The following proposition presents the general log-linearized form of the wage Phillips curve implied by our model.

Proposition 1 (Wage Phillips Curve in $(\theta, U, \mathcal{C})$) *For generic $\varpi \in [0, 1]$, the first-order approximation to wage inflation can be written as*

$$\check{\Pi}_t^w = \Phi_V^{(\varpi)} \check{\theta}_t + \widetilde{\Phi}_U^{(\varpi)} \check{U}_{t-1} + \Phi_{\mathcal{C}}^{(\varpi)} \check{\mathcal{C}}_t + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w, \tag{27}$$

Further, under the limit $s \to 0$ and $\mathcal{C} \to 1$,[23]

$$\begin{aligned} \Phi_V^{(\varpi)} &\to \frac{\kappa}{\psi^w}(1+\chi)\mathfrak{m} && > 0 \\ \Phi_{\mathcal{C}}^{(\varpi)} &\to -\frac{\kappa}{\psi^w}\mathcal{J}\left(\mathfrak{m}(1+S)\right) && < 0 \end{aligned}$$

where $K_\phi \equiv \frac{1-\phi_E}{1-U} > 0$ and $\mathcal{J} \equiv \frac{\lambda_{EU}}{0.5\lambda_{EE}f} > 0$, $\kappa \equiv c(1+\chi)\left(\frac{V}{N}\right)^{1+\chi} > 0$, and inverse monopsony power as $\mathfrak{m} \equiv \varepsilon_R - \varepsilon_S$. Moreover,

$$\widetilde{\Phi}_U^{(\varpi)} \to \frac{\kappa}{\psi^w} K_\phi \mathfrak{m}\left[\chi - S - a_\phi(1+S)\right],$$

[22] The log-linearized price Phillips curve can be similarly derived from (25) as follows:

$$\check{\Pi}_{y,t} = \frac{\epsilon - 1}{\psi} \underbrace{\ln\left(\frac{\mu_t / P_{y,t}}{(\epsilon-1)/\epsilon}\right)}_{\equiv (\mu_t / \check{P}_{y,t})} + \frac{1}{1+\rho}\check{\Pi}_{y,t+1}.$$

[23] Under the calibration to U.S. data reported in Table 2, we have $s \simeq 0$ and $\mathcal{C} \simeq 1$ in the steady state.

with

$$a_\phi \equiv \frac{\phi_E}{2-\phi_E} \in (0,1),$$

so the unemployment coefficient is generally ambiguous for generic $\chi > 0$, and is strictly negative when $\chi = 0$.

Proof. We first show that the model's linearized wage Phillips curve can be written in terms of the present discounted value of three objects: vacancies $\check{V}_t$ (with coefficient $\Phi_V^{(\varpi)}$), unemployment $\check{U}_{t-1}$ (with coefficient $\Phi_U^{(\varpi)}$), and $\check{\mathcal{C}}_t$ (with coefficient $\Phi_{\mathcal{C}}^{(\varpi)}$). We then substitute out $\check{V}_t$ for $\check{\theta}_t$ and $\check{U}_{t-1}$, using the fact that $\check{\theta}_t = \check{V}_t - \eta_{\theta,U}\check{U}_{t-1}$ where $\eta_{\theta,U}$ is a function of parameters and steady-state objects. This implies that the coefficient $\Phi_V^{(\varpi)}$ on $\check{\theta}_t$ is the same as the one on $\check{V}_t$, and the coefficient on $\check{U}_{t-1}$ is $\widetilde{\Phi}_U^{(\varpi)} = \Phi_U^{(\varpi)} + \eta_{\theta,U}\Phi_V^{(\varpi)}$. See the full proof and derivation in Online Appendix B.3 ■

The coefficient on θ is generally positive, and reflects the logic of the firm's problem: firms view wage increases and vacancies as two alternative tools to recruit and retain workers. In equilibrium, they use both, so we expect tightness and wage growth to commove strongly.[24] The coefficient on U is small and negative when $\chi = 0$, reflecting composition effects: when unemployment is high, firms are more likely to meet unemployed searchers, who are less wage-sensitive in their job taking decisions than employed searchers. This puts downward pressure on the wage as firms would prefer to recruit through vacancies instead; indeed, without on-the-job search, this composition effect vanishes and our wage Phillips curve simplifies to state that V/U is the correct measure of wage pressures in the wage Phillips curve.[25] When instead $\chi > 0$, so that vacancy costs are convex in V/N, there is an offsetting effect: a high unemployment rate coincides with low N, raising the marginal cost of a vacancy and tilting firms towards recruiting through wages instead. This can yield a positive coefficient on unemployment, $\widetilde{\Phi}_U^{(\varpi)}$, as we will see in Table 4 below. This explains why $\chi = 0$ is necessary to sign the coefficient on U, even given assumptions on s and $\mathcal{C}$.

Finally, the last term $\check{\mathcal{C}}_t$ captures the ease of recruiting workers from unemployment at current real wage levels, with a coefficient which is generally negative: if $\check{\mathcal{C}}_t$ is high, workers are easier to recruit from unemployment. That puts downward pressure on wages, all else equal. If instead $\check{\mathcal{C}}_t$ is low, perhaps because real unemployment benefits are high, that makes workers harder to recruit from unemployment, and puts upward pressure on the wage.

[24] Proposition 1 also clarifies the relationship between recruiting and separation elasticities and the slope of the wage Phillips curve: lower $\mathfrak{m}$, which implies higher monopsony power, reduces the slope, all else equal.

[25] Online Appendix B.4 demonstrates that in our baseline model with $\varpi = 0$, in the absence of on-the-job search ($\lambda_{EE} = 0$) and with linear vacancy costs ($\chi = 0$), the wage Phillips curve can be expressed solely as a function of labor market tightness, $\theta \equiv V/U$, consistent with the existing literature (Gagliardone and Gertler, 2025).

To see this, consider the wage Phillips curve written in terms of $\check{\mathcal{C}}_t = (1-\mathcal{C})\gamma\left(\check{\tilde{w}}_t - \check{b}_t\right)$:

$$\check{\Pi}^w_t = \Phi^{(\varpi)}_V \check{\theta}_t + \widetilde{\Phi}^{(\varpi)}_U \check{U}_{t-1} + \underbrace{\Phi^{(\varpi)}_{\mathcal{C}}(1-\mathcal{C})\gamma}_{\equiv \Phi^{(\varpi)}_{\frac{w}{b}} < 0}\left(\check{\tilde{w}}_t - \check{b}_t\right) + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}. \tag{28}$$

The formulation in (28) makes it clear that if it is easy to recruit workers from unemployment—real wages are much higher than real unemployment benefits—then that puts downward pressure on wages. Therefore, even if the central bank perfectly stabilizes the labor market, pass-through of prices to wages may still arise. This occurs because changes in the price level can alter workers' likelihood of choosing employment over unemployment at a given nominal wage W_t and labor market tightness θ_t, effectively generating shifts in aggregate labor supply.

Given that the quits rate in the symmetric equilibrium is

$$Q_t = (1-s)\left[\frac{\lambda_{EE}}{2} f(\theta_t) + \lambda_{EU}(1-\mathcal{C}_t)\right],$$

we can rewrite the log-linearized wage Phillips curve in (27) in terms of quits, unemployment, and acceptance probabilities, $(Q, U, \mathcal{C})$, as follows:

Proposition 2 (Wage Phillips Curve in $(Q, U, \mathcal{C})$) *For generic $\varpi \in [0,1]$, the first-order approximation to wage inflation admits the representation*

$$\check{\Pi}^w_t = \beta^{(\varpi)}_Q \check{Q}_t + \beta^{(\varpi)}_U \check{U}_{t-1} + \beta^{(\varpi)}_{\mathcal{C}} \check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}.$$

Under the limit $s \to 0$ and $\mathcal{C} \to 1$, $\beta^{(\varpi)}_Q > 0$, $\beta^{(\varpi)}_U$ is generally ambiguous (and strictly negative when $\chi = 0$), and $\beta^{(\varpi)}_{\mathcal{C}}$ is generally ambiguous.

Proof. Using the definition $Q_t = S_t - s$, we show that $\check{Q}_t$ depends only on $\check{\theta}_t$ and $\check{\mathcal{C}}_t$. We then use this linear relationship to replace $\check{\theta}_t$ in Proposition 1's wage Phillips curve with $\check{Q}_t$ and $\check{\mathcal{C}}_t$. This implies that the coefficient on unemployment remains unchanged: $\beta^{(\varpi)}_U \equiv \widetilde{\Phi}^{(\varpi)}_U$; see Online Appendix B.3 for a full derivation.[26] ■

This quits formulation is useful for comparing against results in Table 1. As we will show, in our calibration to U.S. data on labor market flows, the model predicts that the coefficient on quits is much larger in magnitude than the coefficient on unemployment, as observed in U.S. time series data and emphasized by Heise et al. (2026b).

To build intuition, we now consider two alternative specifications for the unemployment benefit in (12) separately: the cases $\varpi = 0$ and $\varpi = 1$.

[26] We show in Online Appendix I that adding hiring costs has minimal effect on our model-implied log-linearized wage Phillips curve.

Case of $\varpi = 0$ In our baseline case ($\varpi = 0$), $\mathcal{C}_t = \frac{\xi^\gamma}{1+\xi^\gamma}$ is constant reflecting results in Chodorow-Reich and Karabarbounis (2016) that the ratio of unemployed and employed consumption is relatively constant over the business cycle. In this case, the wage Phillips curve (27) simplifies to:

$$\check{\Pi}_t^w = \Phi_V \check{\theta}_t + \widetilde{\Phi}_U \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w. \tag{29}$$

Here, we can show that stabilizing the labor market, i.e., $\phi_N \to \infty$ so that $\check{N}_t = 0$, which stabilizes θ and U, also stabilizes wages. There is no pass-through from prices to wages. We summarize this result as follows.

Proposition 3 (Employment Stabilization and Wage Inflation) *Consider the log-linear wage Phillips curve (29) under $\varpi = 0$. If monetary policy stabilizes employment at its steady-state level following a shock ($\phi_N \to \infty$) so that $\check{N}_t = 0$ for all t, then it follows that $\check{U}_t = \check{\theta}_t = 0$ for all t, and that wage inflation is stabilized:*

$$\check{\Pi}_t^w = 0 \quad \textit{for all } t.$$

Proof. Provided in Online Appendix B.3.3.[27] ■

This conclusion does not rely on the limit $s \to 0$, $\mathcal{C} \to 1$ (which was only necessary to sign the coefficients in earlier propositions) or any other parameter restrictions beyond $\varpi = 0$. Note that this result does not hold in a union wage setting model, which generally does exhibit positive pass-through from an inflationary supply shock to wages under the same assumptions for monetary policy; see Proposition 5 in Appendix C.1.

We next consider the alternative case where $\varpi = 1$, where stabilizing the labor market does not perfectly stabilize wage inflation, and discuss how our assumptions and calibration for the structure of the labor market mute the pass-through from prices to wages in the wage Phillips curve even in this case.

Case of $\varpi = 1$ In an alternative specification with inflation-indexed unemployment insurance, $b_t = b$ is constant, implying that

$$\mathcal{C}_t \equiv \frac{\tilde{w}_t^\gamma}{\tilde{w}_t^\gamma + b^\gamma}.$$

Recall that $\tilde{w}_t \equiv W_t/P_t$ is the level of the real wage. As the price level rises, $\mathcal{C}_t$ declines because the real wage falls—given sticky nominal wages—while the real value of unemployment benefits remains fixed at b. This reduction in workers' willingness to accept employment places upward pressure on wages (with $\Phi_{\mathcal{C}}^{(\varpi)} < 0$), as firms must offer more attractive compensation to recruit workers from unemployment.

[27] Online Appendix B.6 formally demonstrates that this result depends in part on the assumption that vacancy costs are denominated in labor. If vacancy costs are instead denominated in final goods, headline inflation is passed through to wages even when monetary policy fully stabilizes the labor market.

The magnitude of this effect, however, depends on the structure of the labor market. To see this, note that under $\varpi = 1$ the wage Phillips curve in equation (28) simplifies to:

$$\check{\Pi}^w_t = \Phi^{(\varpi)}_V \check{\theta}_t + \widetilde{\Phi}^{(\varpi)}_U \check{U}_{t-1} + \Phi^{(\varpi)}_{\frac{w}{b}} \check{\tilde{w}}_t + \frac{1}{1+\rho} \check{\Pi}^w_{t+1}. \tag{30}$$

Note that the price level enters the wage Phillips curve only through the real wage term $\check{\tilde{w}}_t$. We will therefore refer to $\Phi^{(\varpi)}_{\frac{w}{b}} \equiv \Phi^{(\varpi)}_{\mathcal{C}}(1-\mathcal{C})\gamma < 0$ as the coefficient that governs the *direct* pass-through of prices to wages. In our model calibrated to U.S. data, this coefficient is small: most job seekers are already employed, and workers rarely quit into unemployment, implying $\mathcal{C} \simeq 1$. Consequently, the outside option of unemployment plays only a limited role in firms' wage-setting decisions. By contrast, in alternative calibrations of the model without on-the-job search and with more generous unemployment benefits, the coefficient $\Phi^{(\varpi)}_{\frac{w}{b}}$ is substantially larger. In such environments, changes in the value of unemployment have a much stronger effect on wage setting, generating more pronounced propagation from inflationary supply shocks to both prices and wages. Next, we quantify these differences in Section 5.

5 Quantitative Results

In this section, we describe how we calibrate the model and proceed to solve for the effects of an unanticipated shock to the supply of oil, followed by a perfect-foresight transition (i.e., an "MIT shock"). We report results for both the case of $\varpi = 0$ (our baseline case, where the relative consumption of the employed and unemployed is held fixed), and the case of $\varpi = 1$ (and the value of unemployment is fixed in real terms, motivated by home production or the assumption that the value of unemployment is largely leisure). We show that in both cases an oil price shock causes less wage and price inflation than in a standard union wage-setting model, and that this is partly due to the assumption of on-the-job search: in the case where $\varpi = 1$, we show that eliminating on-the-job search results in wage and price inflation that is closer to what we observe in the union model.

5.1 Calibration

We calibrate the baseline search model to match labor-market flows and the sensitivity of recruiting and separation rates to firm wage policies. For labor market flows, the calibration targets U.S. data during 2015–2019, capturing the approximately full-employment conditions before the COVID shock. Data on the unemployment rate and separation rate come from the Bureau of Labor Statistics (BLS) and the Job Openings and Labor Turnover Survey (JOLTS). For the sensitivity

of recruiting and separation rates to wages, we choose the inverse scale parameter for workers' idiosyncratic preferences, $\gamma = 6$, to match the recruiting and separation elasticities estimated in Bassier et al. (2022). We use conventional values where possible: for example, $\epsilon = 11$ yields a 10% steady-state markup, and the matching elasticity is $\nu = 1/2$, following the labor literature summarized by Petrongolo and Pissarides (2001); see also Faberman et al. (2022). We choose $\rho = .004$ which is a standard value for a monthly model (implying a steady-state annual interest rate of 4.9%). We choose $\psi = 1000$ to imply a monthly price Phillips curve slope in terms of real marginal cost of .01, broadly consistent with existing estimates of price stickiness.[28] We set the wage stickiness parameter $\psi^w = 9000$ in our baseline case to match wage Phillips curve estimates in terms of quits in quarterly U.S. data given in Table 1.[29]

For the baseline case with $\varpi = 0$ we set the consumption ratio $\xi = 2$, which is higher than in Chodorow-Reich and Karabarbounis (2016) but closer to what maximizes steady-state utility for the household in this setting. When $\varpi = 1$, we choose $b = .4$ to target the same consumption ratio. For the matching efficiency $m = 0.6$, we choose it to target a realistic value for the unemployment rate U in line with U.S. data from 2015-2019.

For comparison, we also consider the union wage-setting model of Erceg, Henderson and Levin (2000) in the quantitative simulations presented in Section 5.[30] The union wage-setting model uses the same baseline macroeconomic calibration but replaces firm wage posting and search with union wage setting over differentiated labor types. The additional union parameters are chosen to ensure that steady-state labor supply N is the same as in our baseline case with $\varpi = 0$, and that the slope of the wage Phillips curve is the same as in Erceg, Henderson and Levin (2000).[31]

Finally, in all models, the supply of oil X^s is chosen to normalize the steady-state price of oil to 1, following Gagliardone and Gertler (2025). The other oil share and elasticity parameters are taken from Gagliardone and Gertler (2025). Table 2 lists the model's calibrated parameters, some of which are chosen to target the steady-state moments in Table 3.

Wage Phillips Curve Coefficients Table 4 reports the model-implied coefficients of the wage Phillips curve under the calibration in Table 2. Specifically, it presents $\Phi_V^{(\varpi)}$, $\widetilde{\Phi}_U^{(\varpi)}$, and $\Phi_{\frac{w}{b}}^{(\varpi)}$ from Proposition 1, and $\beta_Q^{(\varpi)}$, $\beta_U^{(\varpi)}$, and $\beta_{\frac{w}{b}}^{(\varpi)} \equiv \beta_{\mathcal{C}}^{(\varpi)}\gamma(1-\mathcal{C})$ from Proposition 2. Consistent

[28] Given our other parameter choices, this is consistent with a monthly price change frequency of 10%, which is broadly in line with U.S. data presented in Nakamura and Steinsson (2013), Table 1; and a *quarterly* slope in terms of real marginal cost of about .09, which is on the high end but well within the confidence interval for recent estimates in Gagliardone et al. (2025) based on Belgian microdata: see Table 2, Panel C, Columns (2)-(4).

[29] We choose ψ^w so that a regression on quarterly data aggregated from monthly time-series data generated by the model's response to an oil supply shock in Figure 5 matches the coefficient on quits reported in Table 1, Column 6.

[30] Online Appendix C provides a formal analysis of the union wage-setting model.

[31] While it is difficult to directly compare the slopes of the wage Phillips curves in the two models, since they have different forms, we note that our calibration implies that the union wage model and our baseline model predict about the same response for wage inflation in response to a monetary policy shock; see Appendix Figure A.1

Table 2: Parameters in the Monthly Benchmark New Keynesian Model

Parameter	Value	Meaning	Reason
ϖ	0	Choice of unemployment benefit policy	Fix $C_t^e/C_t^u = \xi$
b	.4	Value of real unemployment benefit	Fix steady-state $C^e/C^u = \xi$ under $\varpi = 1$
ξ	2	Consumption ratio: C_t^e/C_t^u	See notes below
λ_{EE}	.14	OTJ search probability	Match EE rates in Fujita et al. (2024)
λ_{EU}	.15	Opportunity-to-quit probability	Match voluntary EU rate in Qiu (2022)
s	.012	Exogenous separation rate	Match JOLTS monthly separation rate
γ	6	Scale parameter^{-1} of i.i.d. preferences	Match $\varepsilon_{R,W} - \varepsilon_{S,W}$
ϵ	11	Elasticity of substitution of services	Target 10% steady-state markup
ψ	1000	Services price adjustment cost	Target monthly price PC slope $\frac{\epsilon-1}{\psi} = .01$
ψ^w	9000	Wage adjustment cost	Target Table 1 Col. 6 (see text)
η	.02	Services/endowment-good EOS in Consumption	Gagliardone and Gertler (2025)
$\bar{\eta}$	.37	Services/endowment-good EOS in Production	Gagliardone and Gertler (2025)
α_X	.02	Endowment-good share in Consumption	Gagliardone and Gertler (2025)
$\alpha_{\bar{X}}$	.03	Endowment-good share in Production	Gagliardone and Gertler (2025)
ρ_X	.964	Endowment-good shock persistence	Gagliardone and Gertler (2025)
ν	.5	Elasticity of Cobb-Douglas matching function	Petrongolo and Pissarides (2001)
m	.6	Matching efficiency	Target unemployment rate U
χ	1	Convexity in vacancy posting costs	Quadratic costs
c	60	Hiring cost shifter	Target vacancy rate V
ρ	.004	Discount rate	Monthly model
ϕ_π	1.5	Monetary policy rule weight on services inflation	Standard value (Taylor, 1993)
ϕ_N	1	Monetary policy rule weight on employment	

Notes: This table reproduces the benchmark monthly calibration from the main calibration section. The consumption ratio is set to $\xi = 2$, higher than in Chodorow-Reich and Karabarbounis (2016) but close to the value that maximizes steady-state household utility in this setting.

Table 3: Selected Moments in Model Steady State and Data

Targeted Moment	Meaning	Model	Data	Source
$\varepsilon_{R,W} - \varepsilon_{S,W}$	Recruiting less separation elasticities	4.0	4.2	Bassier et al. (2022)
U	Unemployment rate	.047	.044	BLS
V	Vacancy rate	.044	.042	JOLTS
S	Monthly separation rate	.035	.036	JOLTS
Q	Monthly quits rate	.023	.022	JOLTS
$(1-s)\lambda_{EE}f(\theta)/2$	Monthly EE Rate	.021	.022	Fujita et al. (2024)
$(1-s)\lambda_{EU}/(1+(w/b)^\gamma)$	Monthly EU Quits Rate	.002	.002	Qiu (2022)

Notes: The labor-market targets in the fourth column use 2015–2019 U.S. data to capture the pre-COVID full-employment labor market.

with empirical evidence of Section 2, our structural wage Phillips curve places substantial weight on quits, while the coefficient on unemployment is close to zero. Because our calibration implies

$s \approx 0$ and $\mathcal{C} \approx 1$, the signs on the coefficients align with the limiting predictions in the propositions, where $\widetilde{\Phi}_U^{(\varpi)} \equiv \beta_U^{(\varpi)} > 0$ due to $\chi > 0$.[32] Similarly, $\beta_{\frac{w}{b}}^{(\varpi)}$ inherits an ambiguous sign from ambiguity of the sign of $\beta_{\mathcal{C}}^{(\varpi)}$, described in Proposition 2, and turns out to be positive in our calibration.

Table 4: Structural Wage Phillips Curve Coefficients

Panel A: Tightness θ_t, Unemployment U_{t-1}, and the Wage–Benefit Gap

$$\check{\Pi}_t^w = \Phi_V^{(\varpi)}\check{\theta}_t + \widetilde{\Phi}_U^{(\varpi)}\check{U}_{t-1} + \Phi_{\frac{w}{b}}^{(\varpi)}\left(\check{\tilde{w}}_t - \check{b}_t\right) + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$$

Source	$\Phi_V^{(\varpi)}$	$\widetilde{\Phi}_U^{(\varpi)}$	$\Phi_{\frac{w}{b}}^{(\varpi)}$
Baseline Model ($\varpi = 0$)	2.2e-04	1.2e-05	
Baseline Model: Fixed Real UI Benefit ($\varpi = 1$)	2.2e-04	1.3e-05	-8.3e-05

Panel B: Quits Q_t, Unemployment U_{t-1}, and the Wage–Benefit Gap

$$\check{\Pi}_t^w = \beta_Q^{(\varpi)}\check{Q}_t + \beta_U^{(\varpi)}\check{U}_{t-1} + \beta_{\frac{w}{b}}^{(\varpi)}\left(\check{\tilde{w}}_t - \check{b}_t\right) + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$$

Source	$\beta_Q^{(\varpi)}$	$\beta_U^{(\varpi)}$	$\beta_{\frac{w}{b}}^{(\varpi)}$
Baseline Model ($\varpi = 0$)	4.8e-04	1.2e-05	
Baseline Model: Fixed Real UI Benefit ($\varpi = 1$)	4.8e-04	1.3e-05	2.3e-04

Notes: These variations on the baseline model's monthly wage Phillips curve broadly align with Section 2 in putting much more weight on vacancies or quits than unemployment. We also see that the coefficient on changes in the relative value of employment $\check{\tilde{w}}_t - \check{b}_t$ is relatively small, suggesting minimal impact on wage inflation even when it is allowed to vary in the case with $\varpi = 1$. Note that in our baseline case with $\varpi = 0$, both $\Phi_{\frac{w}{b}}^{(\varpi)}$ and $\beta_{\frac{w}{b}}^{(\varpi)}$ multiply a zero, and play no role. Finally, the notation $\Phi_V^{(\varpi)}$ reflects that the coefficient on θ in the θ-U representation in the wage Phillips curve is the same as the coefficient on V in the V-U representation of the wage Phillips curve; similarly, $\widetilde{\Phi}_U^{(\varpi)} \equiv \beta_U^{(\varpi)}$ as the numerical values suggest. See Appendix B.3.

Note the inclusion of the quits rate reverses the sign of $\beta_{\frac{w}{b}}^{(\varpi)}$. When unemployment benefits are indexed to inflation ($\varpi = 1$), an increase in the price level reduces the relative value of employment $\check{\tilde{w}}_t - \check{b}_t$, making it more difficult for firms to recruit and retain workers and generating upward pressure on nominal wages. However, as real wages decline and unemployment becomes relatively more attractive, quits increase too. Consequently, the quits rate contains information about the level of $\frac{w}{b}$. Once this information is accounted for, the coefficient on the real wage term may become positive—though of course, it multiplies a zero in our preferred baseline case where $\varpi = 0$, which is why the table leaves this entry blank for that model.

We conclude that the model is broadly consistent with the empirical evidence on aggregate nominal wage growth in the U.S. labor market, in predicting that quits or labor market tightness θ matters much more for wage growth than the unemployment rate (Heise et al., 2026b).

[32] Setting $\chi = 0$ would imply a negative coefficient, as previously discussed in Section 4.

5.2 Pass-Through from Prices to Wages

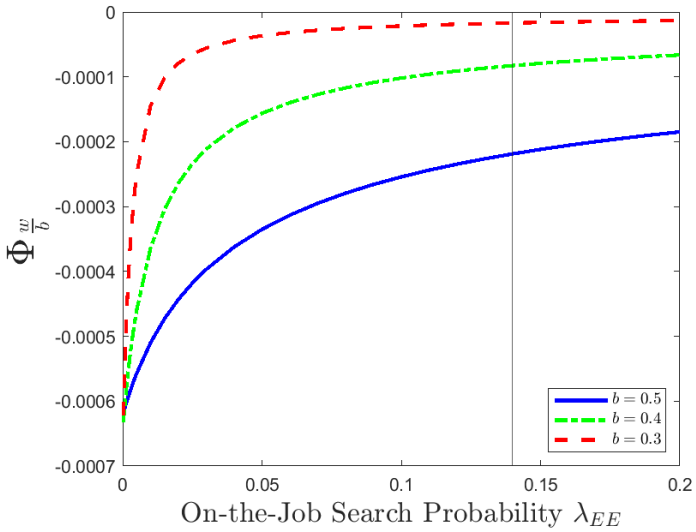
In Section 4, we showed that if the central bank perfectly stabilized the labor market, there could still be room for pass-through from prices to wages if higher prices made it harder to recruit unemployed workers—which occurs when $\varpi = 1$ in (12). This effect is moderated by the coefficient $\Phi_{\frac{w}{b}}$ on b and the real wage $\tilde{w}_t$ in equation (28). We can see that this coefficient is quite small: recall

$$\Phi_{\frac{w}{b}}^{(\varpi)} \equiv \Phi_{\mathcal{C}}^{(\varpi)} \underbrace{(1-\mathcal{C})}_{\approx 0} \gamma$$

where $1-\mathcal{C}$ is small because $\mathcal{C}$ is close to 1, and in Table 4 we can see that $\Phi_{\mathcal{C}}^{(\varpi)}$ is small as well.

Because the definition of the coefficient is a complicated collection of steady-state terms (even in the limit cases used in Section 4), we turn to a numerical exercise to show what happens as we vary λ_{EE}, holding all other parameters fixed and $\varpi = 1$. Figure 3 illustrates the mechanism at work. Under fixed real unemployment benefits, eliminating on-the-job search by letting $\lambda_{EE} \to 0$ raises the magnitude of the pass-through coefficient $\Phi_{\frac{w}{b}}^{(\varpi)}$ in the wage Phillips curve equation (28). As a result, the sensitivity of wage inflation to changes in the real wage increases, thereby raising the extent of price-to-wage pass-through.

Figure 3: On-the-Job Search Reduces Scope for Pass-Through in the Wage Phillips Curve

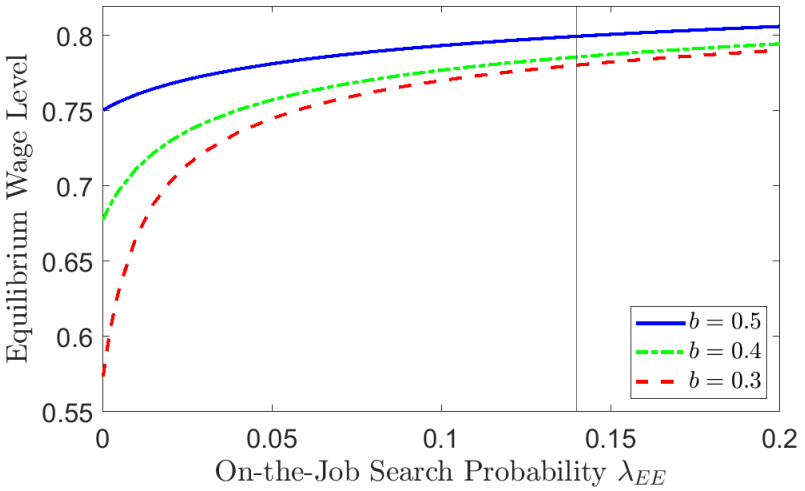


Notes: In the case with fixed real unemployment benefits ($\varpi = 1$), eliminating on-the-job search by sending $\lambda_{EE} \to 0$ means that the magnitude of the coefficient $\Phi_{\frac{w}{b}}^{(\varpi)}$ in the wage Phillips curve equation (28) rises. This raises scope for pass-through from price changes to wages.

The reason is that, in the presence of on-the-job search, the value of unemployment becomes

less relevant for wage-setting decisions. Firms correctly perceive themselves as competing primarily for already employed workers rather than for unemployed workers. As a result, increases in prices—which reduce the relative desirability of employment compared with unemployment—have only a limited effect on wage inflation. Nevertheless, when unemployment benefits b are higher, unemployment becomes a more credible outside option. Therefore with higher unemployment benefits, a marginal increase in the price level and thus improved relative desirability of non-employment makes it more difficult for firms to recruit workers from unemployment. As a result, the magnitude of $\Phi^{(\varpi)}_{\frac{w}{b}}$ increases with b.

Figure 4: On-the-Job Search Mutes the Effect of Changing Unemployment Benefits on Equilibrium Real Wages



Notes: In the case with fixed real unemployment benefits ($\varpi = 1$), eliminating on-the-job search by sending $\lambda_{EE} \to 0$ implies that changes in unemployment benefit b have larger effects on the equilibrium real wage (denominated in the price of aggregate consumption), as seen by examining the gaps between the blue solid line and dashed red line. This occurs because without on-the-job search, firms set wages mostly considering the problem of recruiting unemployed workers, which makes the level of b important in their wage-setting problem. At our value of $\lambda = .14$ calibrated to U.S. data, where firms mostly recruit from other firms, we see that the same changes in b have almost no change in the equilibrium real wage offered, as in the data: the three lines nearly lie on top of each other at this point.

To see this intuition another way, Figure 4 shows that the sensitivity of the steady-state real wage to shifts in unemployment benefits is dampened in the presence of on-the-job search. Specifically, we calculate the steady-state level of wages for different values of real unemployment benefits b and on-the-job search probabilities λ_{EE}. With even a modest amount of on-the-job search, we generate the empirical finding in Jäger et al. (2020) that the value of unemployment benefits

has little effect on the equilibrium wage level. As $\lambda_{EE} \to 0$, changing the real unemployment benefit has larger effects on the equilibrium real wage, seen by examining the gaps between the blue solid line and dashed red line. At our value of $\lambda_{EE} = .14$ calibrated to U.S. data, we observe that the same changes in b yield almost no change in the equilibrium real wage offered by firms, as in the data. Thus, beyond the fact that incorporating on-the-job search is important to capture the fact that quits are mostly job-to-job quits rather than quits into nonemployment, on-the-job search helps capture the near irrelevance of unemployment benefits for the wage.

5.3 Response of Price and Wage Inflation to Oil Shocks

This section analyzes the response of multiple versions of our model to a temporary decrease in the supply of oil. Specifically, Figure 5 presents the effects of a decreased supply of the endowment good X_t^s of 10%, which is calibrated to be oil as discussed above.[33]

We compare the results in three variants of our model with firm wage-setting and on-the-job search (OTJS) against a standard union wage-setting model (Erceg, Henderson and Levin, 2000). The three variants of our model include the baseline case with $\varpi = 0$, the case with fixed, real unemployment benefits ($\varpi = 1$), and a variant of the real unemployment benefits model ($\varpi = 1$) with $\lambda_{EE} \to 0$, to illustrate the role of on-the-job search in dampening the response of price and wage inflation in response to the oil shock. This shows the general equilibrium analogue to our analysis in Figure 3: shutting down OTJS both increases the magnitude of $\Phi_{\frac{w}{b}}^{(\varpi)}$ and increases the cumulative effect of an oil shock on wages and prices.[34]

Overall, while all models predict a similar response for the real wage, our baseline case with a consumption sharing rule ($\varpi = 0$) predicts the least wage and price inflation (the solid blue line). Assuming inflation-indexed (real) unemployment benefits ($\varpi = 1$) yields more inflation (the blue dash-dotted line), but still much less than the union wage-setting model—in part due to our assumption of a quantitatively realistic amount of OTJS. Eliminating OTJS in the fixed real UI case (the blue dotted line) results in wage and price inflation much closer to the union wage-setting model (the dashed orange line).

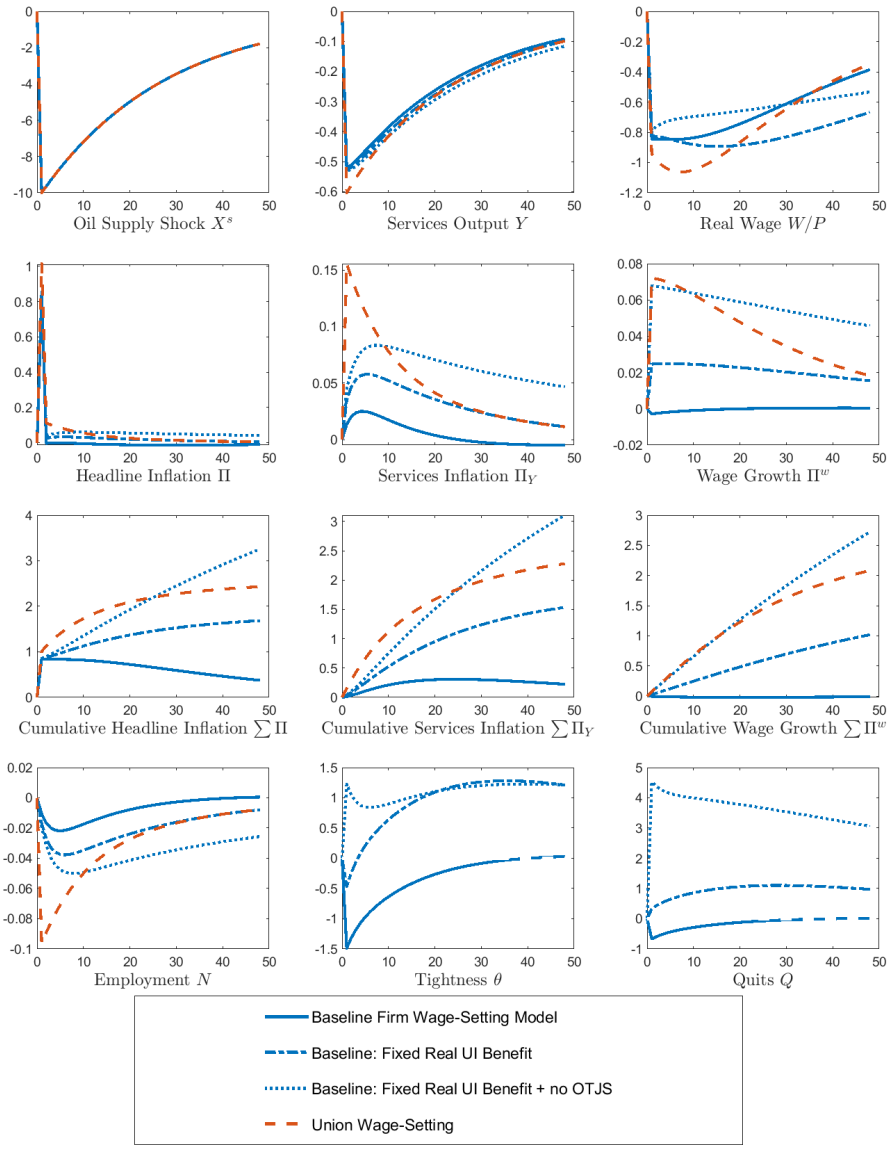
Note that the response of wage inflation to the negative shock in X_t^s is negative in our baseline case of $\varpi = 0$, leading to a decline in overall nominal labor income. This is consistent with Del Canto et al. (2025). In the model, this occurs because households switch their spending away from services, which lowers labor demand. Note also that the negative cumulative response of

[33] Following Gagliardone and Gertler (2025), we assume that X_t^s follows an $AR(1)$ process with a monthly persistence of 0.964 after an initial MIT shock.

[34] When changing λ_{EE}, we recalibrate the matching efficiency parameter m to maintain a realistic value of the unemployment rate U. Without this adjustment, lowering λ_{EE} dramatically lowers the unemployment rate, because fewer on-the-job searchers un-congests search for unemployed workers at any given level of vacancies, leading to lower equilibrium unemployment.

wage inflation in response to the oil shock differs from our result in Proposition 3, which states that there is no nominal wage growth if the central bank stabilizes employment. This is because in our quantitative exercises, the central bank's Taylor Rule puts finite weight on stabilizing employment: $\phi_N = 1$, which is less aggressive than perfectly stabilizing employment as in Proposition 3.

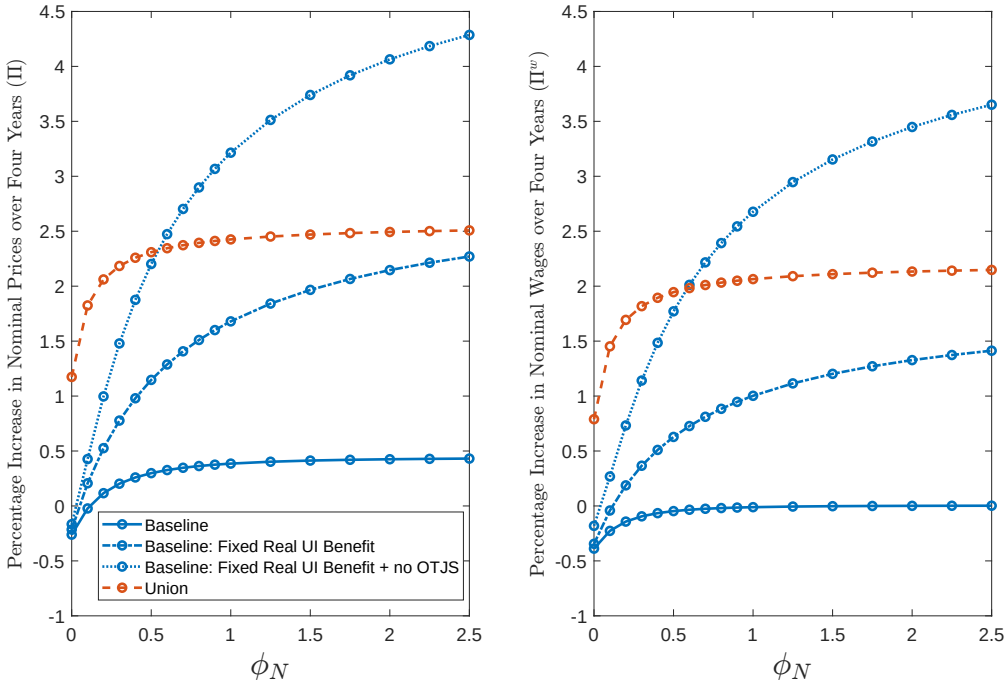
Figure 5: Impulse Response to a 10% Negative Shock to Supply of Endowment Good



Notes: This figure presents the effects of a decreased supply of the endowment good X_t^s, calibrated to be oil. We compare the results in three variants of our model with firm wage-setting and on-the-job search (OTJS) against a standard union wage-setting model. The union wage model predicts the most price and wage inflation in response to the shock (the dashed orange line). Our baseline case with a consumption sharing rule ($\varpi = 0$) predicts the least wage and price inflation (the solid blue line). Assuming inflation-indexed (real) unemployment benefits ($\varpi = 1$) yields more inflation (the blue dash-dotted line), but still much less than the union wage-setting model—in part due to our assumption of a quantitatively realistic amount of OTJS. Eliminating OTJS in the fixed real UI benefit model (the blue dotted line) results in wage and price inflation much closer to the union wage-setting model.

This difference in results highlights that in general equilibrium, the cumulative response of inflation to any shock depends on the conduct of monetary policy. Therefore we show in Figure 6 the cumulative 4-year growth in the aggregate price level Π and aggregate nominal wage growth Π^w in the same four models as in the previous figure for different values of ϕ_N. For all models, cumulative inflation in response to the oil shock increases as the central bank tries to more aggressively stabilize employment. As the second panel shows, for $\varpi = 0$, the cumulative nominal wage growth for $\phi_N = 1$ in the baseline case is negative. As ϕ_N increases, the negative response of nominal wage growth gets smaller, and the cumulative response of wage growth will converge to 0 as the central bank perfectly stabilizes employment $\phi_N \to \infty$.

Figure 6: Impulse Response to a 10% Negative Shock to Supply of Endowment Good



Notes: This figure presents the cumulative inflation after four years in four models: our baseline variant with a consumption sharing rule ($\varpi = 0$), fixed real unemployment benefits ($\varpi = 1$), fixed real unemployment benefits and no on-the-job search ($\varpi = 1$, $\lambda^{EE} \to 0$) , and the union model, with different values of the weight on stabilizing employment ϕ_N in the central bank's Taylor Rule.

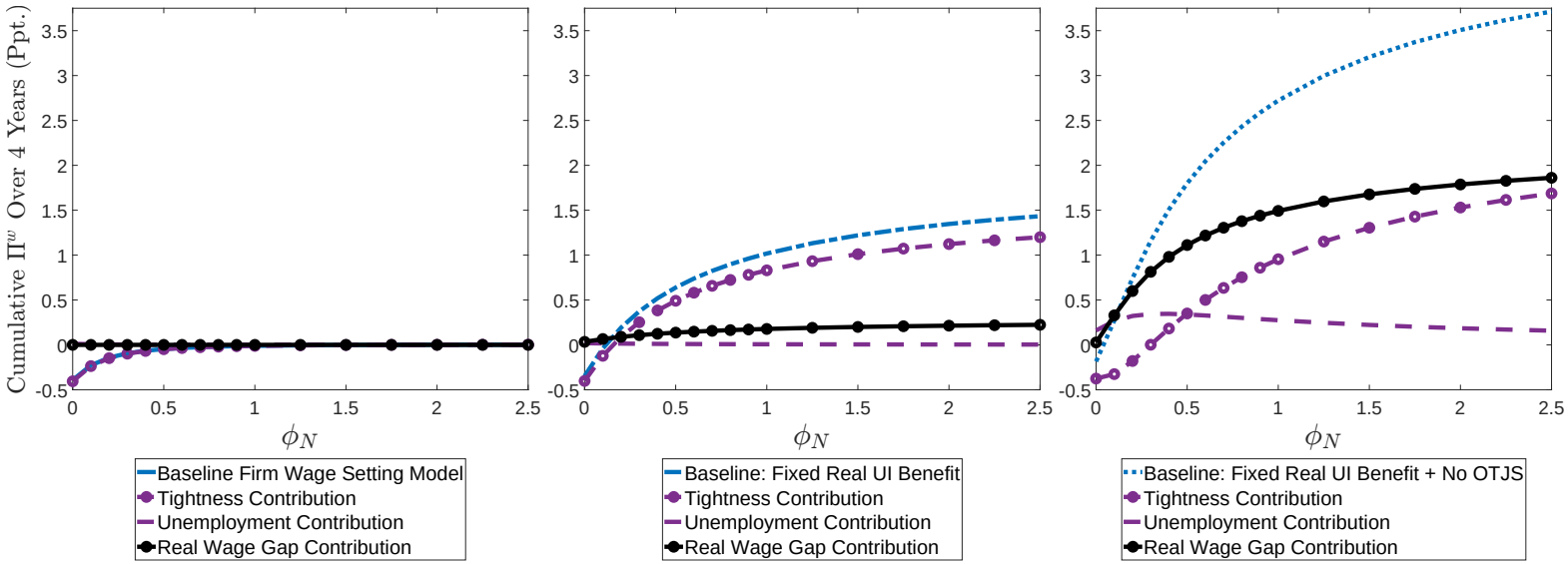
Further, note that the wage Phillips curve states that we can decompose the wage inflation response in any of our firm wage setting models into three parts: a component driven by deviations of labor market tightness θ, a component driven by deviations of unemployment, and a residual part that reflects direct pass-through from prices to wages in the wage Phillips curve due to a decline in the real wage. Figure 7 does so, using the wage Phillips curve in Panel A of Table 4. Each panel decomposes cumulative wage inflation at a 48 month horizon according to the wage Phillips curve in a different model as we vary ϕ_N: the three contributions mechanically add up to the blue

lines for total cumulative 48 month wage inflation in each model, which are the same blue lines as printed on the right-hand side panel of Figure 6. In the baseline case with $\varpi = 0$, the contribution of the real wage gap term is zero, since the price level has no effect on nominal wage growth. When monetary policy does not respond to employment $\phi_N = 0$, the contribution of tightness to wage inflation is negative, as the central bank targets only inflation at the expense of stabilizing the labor market. As the weight on employment ϕ_N increases, the central bank increasingly stabilizes the labor market, lessening the negative contribution of tightness on wage inflation.

In the model variant with fixed real unemployment benefits ($\varpi = 1$), the contribution of the real wage gap to cumulative wage inflation is non-zero, but small, accounting for only 0.2 percentage points of wage inflation over 4 years. The response of wage inflation to the oil shock largely comes from the tightness term, which arises from an interaction of workers' labor supply and monetary policy. As prices increase and real wages fall relative to the value of unemployment benefits, workers are less likely to accept jobs from unemployment and are more likely to quit into unemployment, which is equivalent to a decrease in their labor supply. This means that the central bank can no longer jointly stabilize both tightness θ and unemployment: a higher vacancy rate and therefore tightness would be needed to maintain the same unemployment rate. Since the monetary policy rule focuses on stabilizing employment (and therefore unemployment), a higher value of ϕ_N does a better job of stabilizing unemployment U_t but requires increasingly greater vacancy rates and tightness to achieve that, which generates more cumulative wage inflation as ϕ_N increases.

Finally, in the variant with fixed real unemployment benefits ($\varpi = 1$) and no on-the-job search ($\lambda_{EE} \to 0$), the contribution of tightness to cumulative wage inflation is very similar to the case in the second panel with $\varpi = 1$ and $\lambda_{EE} = 0.14$. What is different is that the direct pass-through due to lower real wages is much larger. For example, when $\phi_N = 1$, the contribution of the real wage gap term in the case with $\lambda_{EE} \to 0$ is 1.5 percentage points, almost an order of magnitude larger than the contribution of the real wage gap in the second panel when $\lambda_{EE} = 0.14$. This demonstrates the result in Figure 3 that the magnitude of direct pass-through of higher prices (via lower real wages) to wage inflation $\Phi_{\frac{w}{b}}$ increases as λ_{EE} falls.

Figure 7: Cumulative Wage Inflation Decomposition: Oil Shock



Notes: Each panel decomposes wage inflation according to the wage Phillips curve written in θ: the three contribution lines mechanically add up to the blue lines in each model. In both the baseline case $(\varpi = 0)$ and the fixed real unemployment benefit case $(\varpi = 1)$, Panels one and two, labor market tightness θ explains the vast majority of wage inflation (nearly 100% in the baseline model). In the fixed real UI benefits case $(\varpi = 1)$, eliminating on-the-job search (sending $\lambda_{EE} \to 0$) means that the magnitude of the coefficient $\Phi_{\frac{w}{b}}^{(\varpi)}$ in the wage Phillips curve (28) rises. This raises the contribution of the pass-through from prices to wages, so that labor market tightness only explains about half of observed wage inflation.

5.3.1 Monetary Policy Shocks

In Online Appendix A, we conduct the same exercise as in Figure 5 but for monetary policy shocks, and discuss how the response is affected by the presence of on-the-job search by varying λ_{EE}. We also explore variations of the model with firm wage setting and real unemployment benefits ($\varpi = 1$) that generate more pass-through from prices to wages: namely, raising unemployment benefits b and eliminating on-the-job search λ_{EE}.

The results show that in general, wage growth remains higher in the model with fixed real unemployment benefits ($\varpi = 1$) than in our preferred calibration with a consumption sharing rule ($\varpi = 0$), with the elimination of on-the-job search generating even more inflation in response to an expansionary monetary policy shock. This is consistent with the results in Figure 5 for oil price shocks. However, the union model predicts less wage inflation in response to the same monetary policy shock than our firm wage-setting models with $\varpi = 1$ (and predicts similar wage inflation for our baseline calibration of $\varpi = 0$).

Note that the reason our firm-wage setting models predict much more wage inflation in response to a monetary policy shock is because labor market tightness is predicted to move more than for an oil price shock—not because of greater direct pass-through from prices to wages in the wage Phillips curve. To show this, we replicate the exercise in Figure 7, plotting the contributions of labor market tightness θ, unemployment, and the gap between the real wage and unemployment benefits to wage inflation. The conclusions are similar to those drawn from the oil shock: in both the baseline model ($\varpi = 0$) and the fixed real unemployment benefit model ($\varpi = 1$), labor market tightness explains the vast majority of inflation in response to a monetary policy shock, as in the case for an oil shock. In the model with fixed real unemployment benefits ($\varpi = 1$), eliminating on-the-job search by sending $\lambda_{EE} \to 0$ raises the contribution of the pass-through from prices to wages, so that labor market tightness only explains about half of observed wage inflation.

This finding underpins our central claim: it is not the case that always and everywhere, in response to any shock, our firm-wage setting model would predict little wage inflation (or less than in a union wage setting model). Rather, the point is that the channels are different: in our model with firm-wage setting and on-the-job search, there is little scope for direct pass-through from prices to workers' wages in the wage Phillips curve. The path of labor market tightness θ (and closely related quits) is nearly a sufficient statistic for wage inflation, in calibrations of our model that incorporate a realistic amount of on-the-job search.

5.4 Robustness

We now briefly explain various extensions in the Online Appendix that enrich our baseline model. We find that none of the extensions changes our main conclusion that there is little response of

wages to an inflationary supply shock.

5.4.1 Wage Dispersion, Endogenous Search Effort, and CRRA Utility

Our simple model abstracts from a potentially key mechanism in how workers would respond to inflationary supply shocks: increasing search for better paying jobs. In Online Appendix E, we extend the model so that firms have heterogeneous costs of posting vacancies, workers have constant relative risk aversion (CRRA) utility, and workers can choose their search effort subject to a convex cost. As a result, the equilibrium exhibits wage dispersion, and workers can endogenously change their search effort in response to shocks.

We then show the response of cumulative inflation and wage growth are similarly muted as in our baseline model, despite the addition of a wage ladder and endogenous search effort. The extension can match a rich array of moments such as the elasticity of search with respect to real wages of -0.5 in the cross-section as in Faberman et al. (2022) but has little effect on our headline conclusions.

5.4.2 Dual Labor Markets

In Online Appendix G, we extend the baseline model by allowing firms to differentiate between employed and unemployed workers, offering distinct wages to each group and posting separate numbers of vacancies. In effect, this introduces segmentation between labor markets for workers hired from unemployment (U-to-E) and for those already employed (E-to-E). The extended model follows Cheremukhin and Restrepo-Echavarria (2024), who show that a dual vacancy model—one that separates the search processes of these two groups—fits the data better than traditional models assuming a homogeneous labor market.

Within this setting, we find that workers hired from unemployment receive lower wages than those poached from other firms, generating an endogenous job ladder among workers. However, when workers have log utility, wage pass-through from a cost-of-living shock remains absent when monetary policy stabilizes the labor market. With finite ϕ_N, cumulative inflation and nominal wage growth remain quantitatively small for both E-to-E workers and U-to-E workers.

5.4.3 Hours Margin

In Online Appendix F, we extend the baseline model to allow workers to derive utility from both consumption and leisure. In this model, firms post not only wages but also the number of working hours, and workers take both dimensions into account when deciding whether to stay in their current job or switch to a new one.

We find that our benchmark result, zero pass-through to wages, continues to hold in this environment when monetary policy stabilizes the labor market. The reason is that both job-switching probabilities and hours remain independent of the aggregate price level. Due to log utility, income and substitution effects cancel out when the price level increases, meaning that workers' preference for one job over another, as well as their preference for a marginal hour of work, is unchanged. This result is consistent with Hajdini et al. (2023), who use evidence from a randomized controlled trial to show that higher inflation does not increase workers' perceived likelihood of raising hours with their current employer.

5.4.4 Forward-Looking Workers

In Online Appendix H, we analyze how a negative endowment shock passes through to wages when workers are forward-looking. In this environment, workers base their decisions not only on current wages but also on the expected future wage path promised by the firm, in addition to idiosyncratic preference shocks. Despite this richer framework, we find that the degree of pass-through to wages is quantitatively very similar to that in the baseline model with myopic workers.

We also discuss several technical issues in detail, including how we address the firm's commitment problem arising from time inconsistency.[35]

6 Conclusion

This paper develops a tractable New Keynesian model where firms set both prices and wages, subject to nominal rigidities in price and wage setting, and workers search on the job. Calibrated to match U.S. data on worker flows, the model implies that the quits rate is the dominant predictor of nominal wage growth in the wage Phillips curve, consistent with recent U.S. data. We analyze the propagation of supply shocks (e.g., food or energy) in the model economy. Because firms set wages to avoid costly turnover, such shocks pass through to wages only to the extent that higher cost of living improves workers' outside options, such as competing jobs or unemployment, relative to their current job. As a higher price level lowers real wages at all jobs evenly, and unemployment is rarely a credible outside option in the modern U.S. economy, we find that inflationary supply shocks have little to no effect on relative outside options and therefore wages, consistent with recent evidence that oil shocks decrease labor earnings from Del Canto et al. (2025).

[35] In particular, firms have an incentive to offer relatively low wages initially while promising higher wages in the future to attract workers. Because this incentive arises in every period, the model features a classic time-inconsistency problem. We provide a novel and tractable solution by adapting the timeless perspective approach from the literature on optimal monetary policy under commitment. Relatedly, Rudanko (2023) studies time inconsistency in a model of firm wage setting and directed search.

While stylized, our model is consistent with a range of recent microeconomic evidence on how wages are determined, including the result in Jäger et al. (2020) that wages are insensitive to the flow value of unemployment benefits, and direct evidence on the preponderance of wage posting (Hall and Krueger, 2012; Lachowska et al., 2022; Di Addario et al., 2023). Admittedly, our simple model does abstract from the fact that there are a minority of unionized workers in the United States, and workers with automatic COLAs, for whom prices would pass through into wages more directly. However, our results suggest that in a setting such as the United States where few workers operate under collective bargaining agreements with cost-of-living adjustments, and where firms' wage setting decision reflects competition for already-employed rather than for unemployed workers, the ability for most workers to reclaim real wages in response to a supply shock that raises their cost of living is limited. We conclude that there is limited scope for supply-shock-induced wage–price spirals driven specifically by workers' ability to obtain higher nominal wages in response to rising prices.

References

Afrouzi, Hassan, Andres Blanco, Andres Drenik, and Erik Hurst, "A Theory of How Workers Keep up with Inflation," *The Quarterly Journal of Economics*, 2026, p. qjag007.

Auclert, Adrien, Matthew Rognlie, and Ludwig Straub, "The Intertemporal Keynesian Cross," *Journal of Political Economy*, 2024, *132* (12), 4068–4121.

Bassier, Ihsaan, Arindrajit Dube, and Suresh Naidu, "Monopsony in Movers: The Elasticity of Labor Supply to Firm Wage Policies," *Journal of Human Resources*, 2022, *57* (S), S50–s86.

Bernanke, Ben and Olivier Blanchard, "An Analysis of Pandemic-Era Inflation in 11 Economies," 2024. Hutchins Center Working Paper 91.

Birinci, Serdar, Fatih Karahan, Yusuf Mercan, and Kurt See, "Labor Market Shocks and Monetary Policy," *Journal of Monetary Economics*, 2026, p. 103928.

Bloesch, Justin, Birthe Larsen, and Anders Yding, "Monopsony with Recruiting," 2026. Working Paper.

Burdett, Kenneth and Dale T Mortensen, "Wage differentials, Employer Size, and Unemployment," *International Economic Review*, 1998, pp. 257–273.

Canto, Felipe Del, John Grigsby, Eric Qian, and Conor Walsh, "Are Inflationary Shocks Regressive? A Feasible Set Approach," *The Quarterly Journal of Economics*, 2025, *140* (4), 2685–2747.

Cheremukhin, Anton and Paulina Restrepo-Echavarria, "The Dual Beveridge Curve," *Working Paper*, 2024.

Chodorow-Reich, Gabriel and Loukas Karabarbounis, "The Cyclicality of the Opportunity Cost of Employment," *Journal of Political Economy*, 2016, *124* (6), 1563–1618.

Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of political Economy*, 2005, *113* (1), 1–45.

_ , Martin S Eichenbaum, and Mathias Trabandt, "Unemployment and business cycles," *Econometrica*, 2016, *84* (4), 1523–1569.

Crump, Richard K., Stefano Eusepi, Marc Giannoni, and Ayşegül Şahin, "A Unified Approach to Measuring u*," *Brookings Papers on Economic Activity*, 2019, *50* (1), 143–238.

_ , _ , _ , and _ , "The Unemployment-Inflation Trade-off Revisited: The Phillips Curve in COVID Times," *Journal of Monetary Economics*, 2024, *145*, 103580.

Datta, Nikhil, "The Measure of Monopsony: the Labour Supply Elasticity to the Firm and its Constituents," 2023. Working Paper.

Davis, Steven J, R Jason Faberman, and John Haltiwanger, "Labor market flows in the cross section and over time," *Journal of monetary economics*, 2012, *59* (1), 1–18.

de la Barrera i Bardalet, Marc, "Monopsony and Nominal Rigidities," 2024. Working Paper.

Di Addario, Sabrina, Patrick Kline, Raffaele Saggio, and Mikkel Sølvsten, "It Ain't Where You're From, It's Where You're At: Hiring Origins, Firm Heterogeneity, and Wages," *Journal of Econometrics*, 2023, *233* (2), 340–374.

Elsby, Michael W, Bart Hobijn, and Ayşegül Şahin, "The Labor Market in the Great Recession," 2010. National Bureau of Economic Research Working Paper No. 15979.

Erceg, Christopher J, Dale W Henderson, and Andrew T Levin, "Optimal monetary policy with staggered wage and price contracts," *Journal of Monetary Economics*, 2000, *46* (2), 281–313.

Faberman, R Jason and Alejandro Justiniano, "Job Switching and Wage Growth," *Chicago Fed Letter*, 2015.

Faberman, R. Jason, Andreas I. Mueller, Ayşegül Şahin, and Giorgio Topa, "Job Search Behavior Among the Employed and Non-Employed," *Econometrica*, 2022, *90* (4), 1743–1779.

Faccini, Renato and Leonardo Melosi, "Job-to-job Mobility and Inflation," *The Review of Economics and Statistics*, 2023, pp. 1–45.

Fujita, Shigeru, Giuseppe Moscarini, and Fabien Postel-Vinay, "Measuring Employer-to-Employer Reallocation," *American Economic Journal: Macroeconomics*, 2024, *16* (3), 1–51.

Gagliardone, Luca and Mark Gertler, "Oil Prices, Monetary Policy and Inflation Surges," *American Economic Journal: Macroeconomics*, 2025. Forthcoming.

_ , _ , Simone Lenzu, and Joris Tielens, "Anatomy of the Phillips curve: micro evidence and macro implications," *American Economic Review*, 2025, *115* (11), 3941–3974.

Galí, Jordi, "The Return of the Wage Phillips Curve," *Journal of the European Economic Association*, 2011, *9* (3), 436–461.

Hajdini, Ina, Edward S Knotek, John Leer, Mathieu Pedemonte, Robert W Rich, and Raphael Schoenle, "Low Passthrough from Inflation Expectations to Income Growth Expectations: Why People Dislike Inflation," 2023. Federal Reserve Bank of Cleveland Working Paper.

Hall, Robert E and Alan B Krueger, "Evidence on the Incidence of Wage Posting, Wage Bargaining, and On-the-Job Search," *American Economic Journal: Macroeconomics*, 2012, *4* (4), 56–67.

Harasztosi, Péter and Attila Lindner, "Who Pays for the Minimum Wage?," *American Economic Review*, 2019, *109* (8), 2693–2727.

Heise, Sebastian, Fatih Karahan, and Ayşegül Şahin, "The Missing Inflation Puzzle: The Role of the Wage-Price Pass-Through," *Journal of Money, Credit and Banking*, 2022, *54* (S1), 7–51.

_ , Jeremy Pearce, and Jacob P. Weber, "Measuring Labor Market Tightness: Data Update and New Web Feature," *Liberty Street Economics*, January 2026. Federal Reserve Bank of New York blog.

_ , _ , and Jacob P Weber, "Wage Growth and Labor Market Tightness," *Journal of Monetary Economics*, 2026, p. 103903.

_ , _ , and Jacob Weber, "A New Indicator of Labor Market Tightness for Predicting Wage Inflation," *Liberty Street Economics*, 2024.

Jäger, Simon, Benjamin Schoefer, Samuel Young, and Josef Zweimüller, "Wages and the Value of Nonemployment," *The Quarterly Journal of Economics*, 2020, *135* (4), 1905–1963.

Karahan, Fatih, Ryan Michaels, Benjamin Pugsley, Ayşegül Şahin, and Rachel Schuh, "Do Job-to-Job Transitions Drive Wage Fluctuations Over the Business Cycle?," *American Economic Review*, 2017, *107* (5), 353–358.

Lachowska, Marta, Alexandre Mas, Raffaele Saggio, and Stephen A Woodbury, "Wage Posting or Wage Bargaining? A Test using Dual Jobholders," *Journal of Labor Economics*, 2022, *40* (S1), S469–S493.

Lorenzoni, Guido and Iván Werning, "Inflation is conflict," Technical Report, National Bureau of Economic Research 2023.

_ **and** _ , "Wage-Price Spirals," *Brookings Papers on Economic Activity*, 2023, *2023* (2), 317–367.

Michaillat, Pascal and Emmanuel Saez, "Resolving New Keynesian Anomalies with Wealth in the Utility Function," *Review of Economics and Statistics*, 2021, *103* (2), 197–215.

Moscarini, Giuseppe and Fabien Postel-Vinay, "Wage Posting and Business Cycles," *American Economic Review, Papers and Proceedings*, 2016, *106* (5), 208–213.

_ **and** _ , "The Relative Power of Employment-to-Employment Reallocation and Unemployment Exits in Predicting Wage Growth," *American Economic Review*, 2017, *107* (5), 364–368.

_ **and** _ , "The Job Ladder: Inflation vs. Reallocation," 2023. National Bureau of Economic Research Working Paper No. 31466.

Nakamura, Emi and Jón Steinsson, "Price rigidity: Microeconomic evidence and macroeconomic implications," *Annu. Rev. Econ.*, 2013, *5* (1), 133–163.

Petrongolo, Barbara and Christopher A Pissarides, "Looking into the black box: A survey of the matching function," *Journal of Economic literature*, 2001, *39* (2), 390–431.

Pilossoph, Laura and Jane M. Ryngaert, "Job Search, Wages, and Inflation," NBER Working Paper 33042, National Bureau of Economic Research 2026. Originally issued October 2024; revised July 2026.

Qiu, Xincheng, "Vacant Jobs," 2022. Working Paper.

Raposo, Ines Goncalves, "Inflation expectations, wages and on-the-job search," 2025. Working Paper.

Rotemberg, Julio J., "Sticky Prices in the United States," *Journal of Political Economy*, 1982, *90* (6), 1187–1211.

Rudanko, Leena, "Firm Wages in a Frictional Labor Market," *American Economic Journal: Macroeconomics*, 2023, *15* (1), 517–550.

Shierholz, Heidi, Margaret Poydock, and Celine McNicholas, "Unionization increased by 200,000 in 2022: Tens of millions more wanted to join a union, but couldn't," 2023.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97* (3), 586–606.

Taylor, John B., "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, December 1993, *39* (1), 195–214.

U.S. Congressional Budget Office, "Noncyclical Rate of Unemployment [NROU]," https://fred.stlouisfed.org/series/NROU 2026. Accessed June 18, 2026.

Williams, John C., "All About Data," Speech 98995, Federal Reserve Bank of New York October 2024.

Yaremko, Vitaliia and ChaeWon Baek, "The Puzzle of a Missing Wage-Price Spiral: Experimental Evidence on Inflation Expectations and Labor Supply," *Available at SSRN 6446379*, 2026.

Online Appendix for
Firm Wage Setting and On-the-Job Search Limit Wage-Price Spirals

JUSTIN BLOESCH SEUNG JOO LEE JACOB P. WEBER

Appendix A Additional Quantitative Results

In this section, we present results on the effect of monetary policy shocks in the model, and discuss how they are affected by the presence of on-the-job search by varying λ_{EE}. We also explore variations of the model with firm wage setting and inflation-indexed unemployment benefits that yield more of a wage price spiral: namely, raising unemployment benefits and eliminating on-the-job search.

A.1 Effects of Monetary Policy Shocks

Figure A.1 plots the response of numerous variables for several variants of the model subject to an expansionary Taylor rule shock with moderate persistence ($\varepsilon_{i,t} = \rho\varepsilon_{i,t-1}$ with $\rho = 0.8$). Specifically, we compare three variants of our model with firm wage-setting and on-the-job search (OTJS) against a union wage-setting model, as we did for oil price shocks in Figure 5.

In this case the union model predicts a similar wage inflation response (the dashed orange line) to the baseline model,[1] in contrast to our results for oil shocks. Our baseline model with a consumption sharing rule ($\varpi = 0$) still predicts less wage and price inflation (the solid blue line) than the model with inflation indexed unemployment benefits ($\varpi = 1$; the blue dash-dotted line). Eliminating OTJS in the fixed-b model (the blue dotted line) again raises the response of price and wage inflation, as in the case of the oil price shock. We conclude that the absence of OTJS amplifies the response of inflationary monetary policy shocks, as for the case of inflationary oil price shocks, though the effects are moderate.

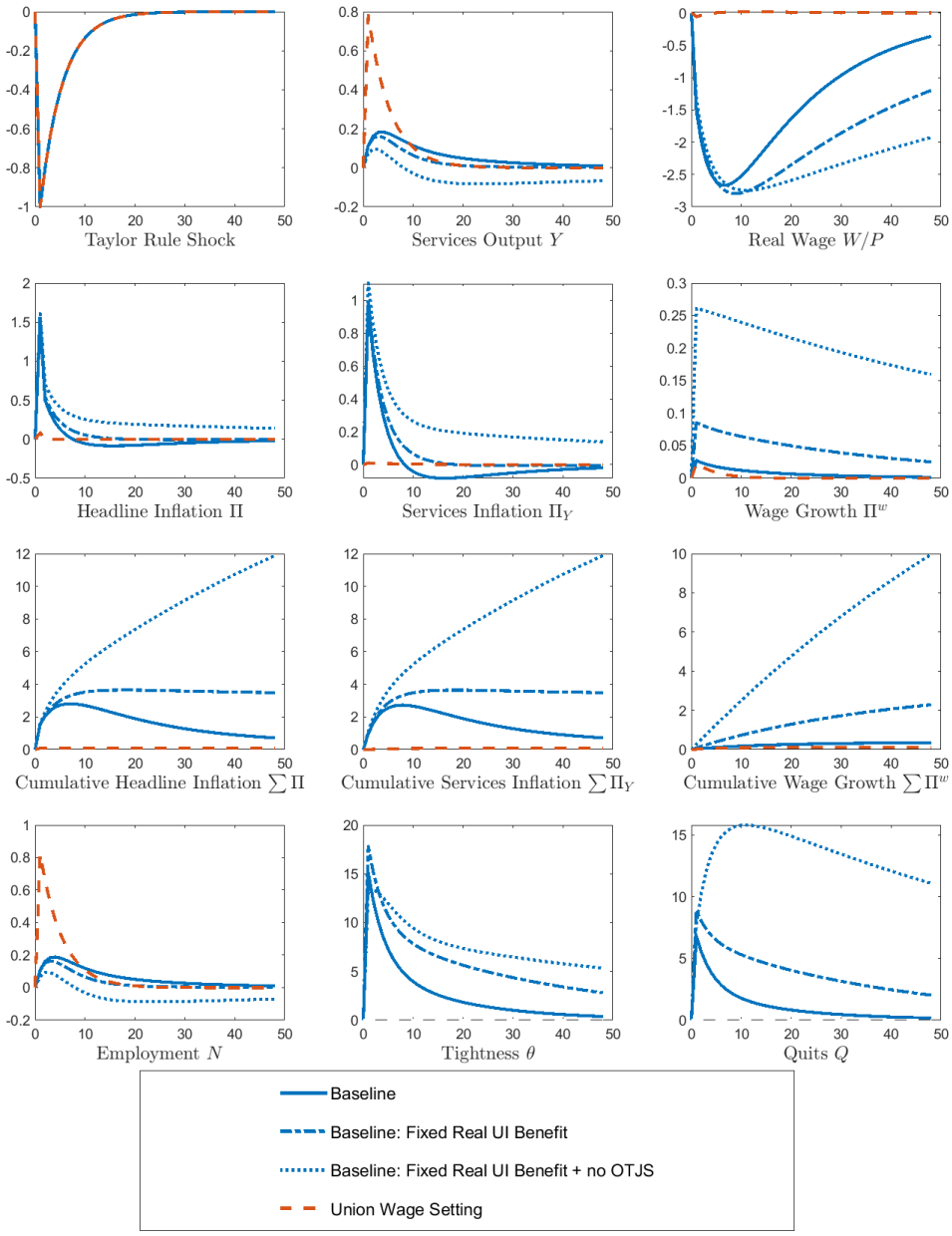
Note that the reason our firm-wage setting models predict more wage inflation in response to a monetary policy shock is because labor market tightness is predicted to move

[1] This implies that, despite comparing two frameworks with fundamentally different microfoundations (our model versus union-based models), they imply comparable slopes of the wage Phillips curves. Note that this is not a calibration target.

more than for an oil price shock—not because of greater direct pass-through from prices to wages in the wage Phillips curve. To show this, Figure A.2 plots the contributions of labor market tightness θ, unemployment, and the gap between the real wage and unemployment benefits in the wage Phillips curve in Panel A of Table 4. The conclusions are similar to those drawn from the oil shock in Figure 7: in both the baseline model ($\varpi = 0$) and the fixed real unemployment benefit model ($\varpi = 1$), labor market tightness explains the vast majority of inflation in response to a monetary policy shock, as in the case for an oil shock. In the model with fixed real unemployment benefits ($\varpi = 1$), eliminating on-the-job search by sending $\lambda_{EE} \to 0$ raises the contribution of the pass-through from prices to wages, so that labor market tightness only explains about half of observed wage inflation.

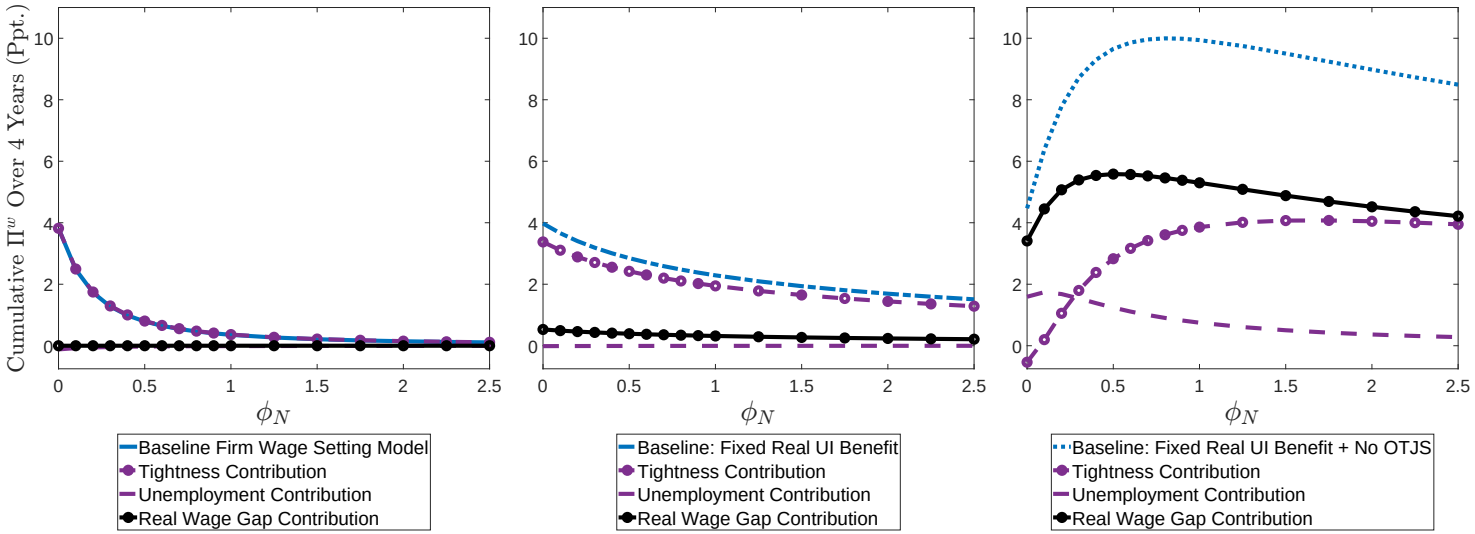
In sum, while changing ϕ_N matters for the overall level of wage inflation (the blue lines), we conclude that this is due to how labor market tightness responds to monetary policy in our models with on-the-job search. Only in the absence of on-the-job search ($\lambda_{EE} \to 0$) do we find a significant role for factors beyond labor market tightness in determining wage growth: specifically, inflation induces changes in the real wage, which makes workers more likely to prefer unemployment (under $\varpi = 1$) and puts pressure on firms to raise wages back to their steady-state level.

Figure A.1: Impulse Response to an Expansionary Monetary Policy Shock



Notes: Effects of an expansionary Taylor rule shock, comparing three variants of our model with firm wage-setting and on-the-job search (OTJS) against a union wage-setting model. The union model predicts the least price and wage inflation (the dashed orange line), in contrast to our results for oil shocks. Our baseline model with a consumption sharing rule ($\varpi = 0$) still predicts less wage and price inflation (the solid blue line) than the model with inflation indexed unemployment benefits ($\varpi = 1$); the blue dash-dotted line). Eliminating OTJS in the fixed-b model (the blue dotted line) again raises the response of price and wage inflation, as in the case of the oil price shock.

Figure A.2: Cumulative Wage Inflation Decomposition: Monetary Policy Shock



Notes: Each panel decomposes wage inflation according to the wage Phillips curve written in θ: the three contribution lines mechanically add up to the blue lines in each model. In both the baseline case $(\varpi = 0)$ and the fixed real unemployment benefit case $(\varpi = 1)$, Panels one and two, labor market tightness θ explains the vast majority of wage inflation (nearly 100% in the baseline model). In the fixed real UI benefits case $(\varpi = 1)$, eliminating on-the-job search (sending $\lambda_{EE} \to 0$) means that the magnitude of the coefficient $\Phi_{\frac{w}{b}}^{(\varpi)}$ in the wage Phillips curve (28) rises. This raises the contribution of the pass-through from prices to wages, so that labor market tightness only explains about half of observed wage inflation.

A.2 Conditions for Larger Wage and Price Responses to Oil Shocks

In this section, we build on the results in Figure 5 to show how modifying the model with $\varpi = 1$ (i.e., inflation-indexed unemployment benefits) can result in significantly more price and wage inflation. In addition to noting that eliminating OTJS can increase the propagation of the oil shock into wage and price inflation, we now also show that making unemployment more attractive can also increase this propagation.

Specifically, we increase real unemployment benefits b by 20% relative to the benchmark calibration, reducing steady-state $\mathcal{C}$ and increasing the magnitude of the coefficient $\Phi_{\frac{w}{b}}$ in the wage Phillips curve equation (28). As Figure A.3 shows, a higher real value of unemployment benefits amplifies the response of wage and price inflation to oil price shocks.

Appendix B Derivations and Proofs

B.1 Worker's Problem with Utility From Leisure

This section reviews the worker's optimization problem and derives the probability that a worker chooses a particular job j over an alternative offer k or unemployment. We show that introducing utility from leisure, in addition to consumption, does not generally overturn the result that the price level has no effect on workers' optimal choices, unless the elasticity of substitution between consumption and leisure differs from one.

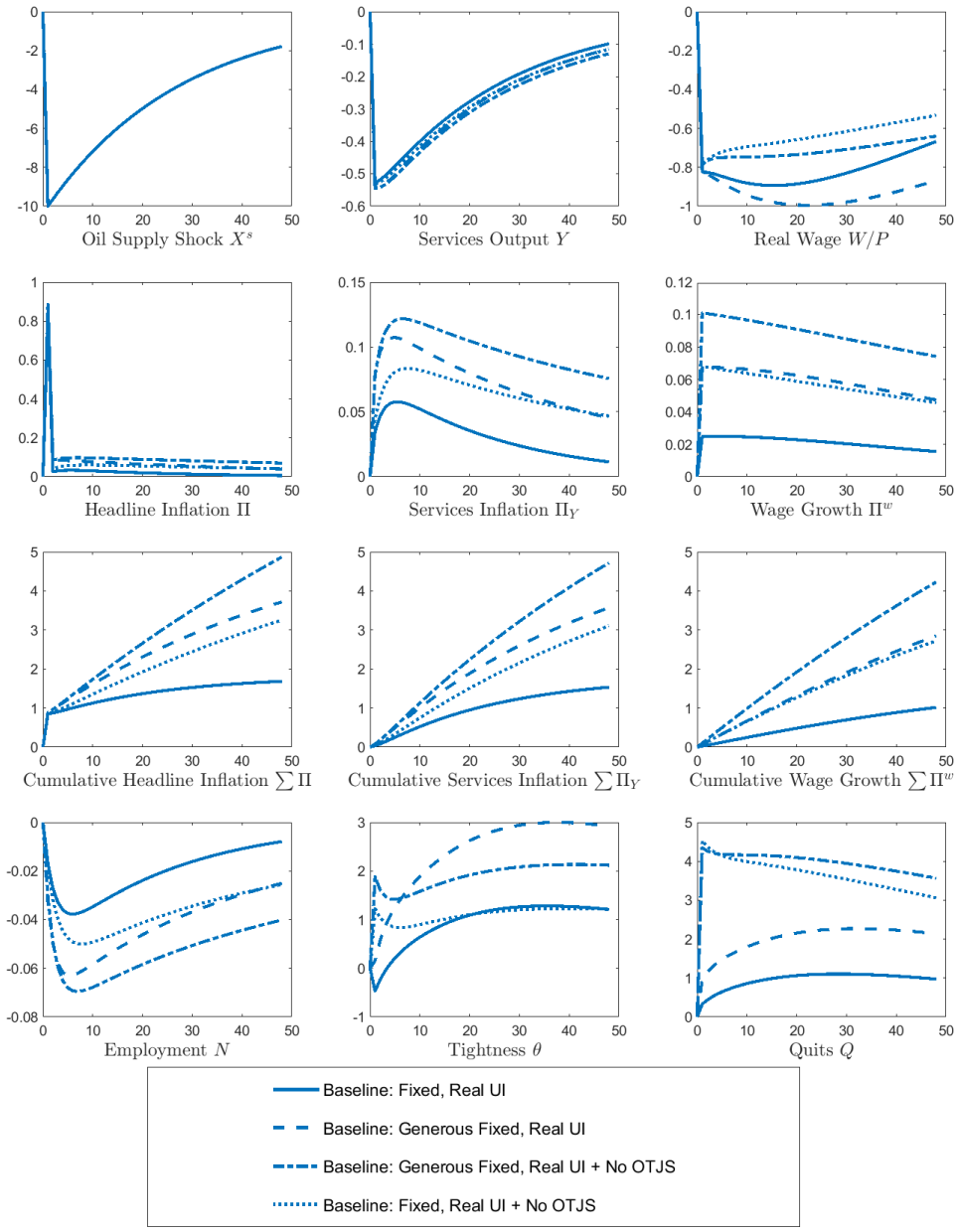
Discrete choice with Type-1 extreme value preference draws Suppose worker i in state j (which could be working at firm j, for example), gets utility $\mathcal{U}(ijt)$ plus a draw ι_{ijt} that is distributed type-1 extreme value:

$$\mathcal{V}_t(i,j) = \mathcal{U}(ijt) + \iota_{ijt}$$

Let ι_{ijt} have scale parameter $\frac{1}{\gamma}$. Then given options two states j and k, the probability that the worker chooses j is

$$\frac{\exp(\gamma \mathcal{U}(ijt))}{\exp\left(\gamma \mathcal{U}(ijt)\right) + \exp\left(\gamma \mathcal{U}(ikt)\right)}.$$

Figure A.3: Impulse Response to an Oil Shock: $\varpi = 1$



Notes: Compares the baseline model in the main paper with $\varpi = 1$ against a model without OTJS, as in Figure 5, and then repeats this exercise for an economy with higher unemployment benefits $b = .48$, or 20% higher than in the baseline case with $\varpi = 1$. This increases the sensitivity of unemployed workers' job-taking decision with respect to the wage and results in more price and wage inflation in response to the shock.

Suppose now that utility $\mathcal{U}$ is a function of log consumption: $\mathcal{U}(ijt) = \log(C_t(i,j))$. This is the case in the main text. Then the probability of choosing j is

$$\frac{C_t(i,j)^\gamma}{C_t(i,j)^\gamma + C_t(i,k)^\gamma}.$$

Case with a more general utility function Consider now the more general form:

$$\mathcal{V}(ijt) = \log\left(U\left(C_t(i,j), \ell_t(i,j)\right)\right) + \iota_{ijt}$$

where $\ell_t(i,j)$ is the leisure worker i gets in state j at time t, which nests the above case. For simplicity, we denote utility while unemployed by $U\left(C_t(i,u), \ell_t(i,u)\right)$, and while employed by $U\left(C_t(i,e), \ell_t(i,e)\right)$; the probability of an unemployed worker taking a job when matched is now:

$$\frac{1}{1 + \left(\frac{U(C_t(i,u),\ell_t(i,u))}{U(C_t(i,e),\ell_t(i,e))}\right)^{\gamma}}. \tag{B.1}$$

Claim 1 *In partial equilibrium (i.e. holding all other equilibrium prices and quantities fixed) the probability that an unemployed worker takes a job in our general setting, (B.1), is invariant to changes in the price level P_t if and only if $\frac{\partial}{\partial P_t}\frac{U(C_t(i,u),\ell_t(i,u))}{U(C_t(i,e),\ell_t(i,e))} = 0$. This is the case with CES preferences if the elasticity of substitution is equal to unity.*

Proof.

consider the case with CES preferences:

$$U = \left(aC^{\frac{\rho-1}{\rho}} + (1-a)\ell^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}$$

where ρ is the elasticity of substitution. We write the utility function as $U(C,\ell) = U\left(\frac{I}{P},\ell\right)$, imposing the budget constraint $C = \frac{I}{P}$ for both employed and unemployed workers. The two groups differ only in their nominal expenditure I—denoted by I_e for employed workers and I_u for unemployed workers.[2] Noting constant returns to scale (CRS) yields

$$\frac{U\left(\frac{I_t(i,u)}{P_t}, \ell_t(i,u)\right)}{U\left(\frac{I_t(i,e)}{P_t}, \ell_t(i,e)\right)} = \frac{U\left(I_t(i,u), P_t\ell_t(i,u)\right)}{U\left(I_t(i,e), P_t\ell_t(i,e)\right)},$$

[2]Here, the result does not depend on the case where we impose a tax-and-transfer scheme to keep $I_e/I_u = C_e/C_u$ constant over the business cycle as in the case of $\varpi = 0$ in (12).

and using the property of CES functions: $\frac{\partial}{\partial P}U(I,P\ell) = (1-a)U(\cdot)^{\frac{1}{\rho}}(P\ell)^{-\frac{1}{\rho}}l$, we can show:

$$\begin{aligned}\frac{\partial}{\partial P_t}\frac{U\left(\frac{I_t(i,u)}{P_t},\ell_t(i,u)\right)}{U\left(\frac{I_t(i,e)}{P_t},\ell_t(i,e)\right)} &= \frac{(1-a)P_t^{-\frac{1}{\rho}}}{U(I_t(i,e),P_t\ell_t(i,e))} \\ &\cdot\Bigg[U(I_t(i,u),P_t\ell_t(i,u))^{\frac{1}{\rho}}\ell_t(i,u)^{1-\frac{1}{\rho}} \\ &- U(I_t(i,e),P_t\ell_t(i,e))^{\frac{1}{\rho}}\ell_t(i,e)^{1-\frac{1}{\rho}}\frac{U(I_t(i,u),P_t\ell_t(i,u))}{U(I_t(i,e),P_t\ell_t(i,e))}\Bigg]\end{aligned}$$

which converges to 0 as $\rho \to 1$, corresponding to the Cobb–Douglas case. ■

B.2 Model Derivations and Proofs

Euler Equation with Variable Unemployment Benefit Regimes We begin by deriving the Euler equation for the general case of ϖ specified in (12).

Suppose that when unemployed, household members are guaranteed some quantity b_t of real consumption goods and receive no other income. The household takes b_t, market wages W_t, and the price level P_t as given, but can smooth all members' consumption by choosing a proportional tax/subsidy rate $1-\tau_t$ each period, multiplying each type of worker's income by τ_t. This yields the following consumption levels for unemployed and employed workers, respectively: $C_t^u = \tau_t b_t$ and

$$C_t^e = \tau_t \frac{W_t}{P_t},$$

and total consumption

$$C_t = \left(U_t b_t + (1-U_t)\frac{W_t}{P_t}\right)\tau_t. \tag{B.2}$$

Continuing on to derive the Euler equation, the household maximizes:

$$\begin{aligned}&\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(U_t\ln(C_t^u)+(1-U_t)\ln(C_t^e)\right) \\ =&\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(U_t\ln(b_t)+(1-U_t)\ln\left(\frac{W_t}{P_t}\right)+\ln\tau_t\right).\end{aligned}$$

The household's budget constraint can be written as:

$$\tau_t \left(U_t b_t + (1 - U_t) \frac{W_t}{P_t} \right) + \frac{B_t}{P_t} = \frac{D_t}{P_t} + (1 - U_t) \frac{W_t}{P_t} + \frac{(1 + i_{t-1,t}) B_{t-1}}{P_t}.$$

where dividends consist of profits from the competitive sale of the endowment good X_t^s as well as profits earned by service firms. The household's Lagrangian function is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \left(U_t \ln(b_t) + (1 - U_t) \ln \left(\frac{W_t}{P_t} \right) + \ln \tau_t \right.$$
$$\left. + \lambda_t \left[-\tau_t \left(U_t b_t + (1 - U_t) \frac{W_t}{P_t} \right) - \frac{B_t}{P_t} + \frac{D_t}{P_t} + \frac{(1 + i_{t-1,t}) B_{t-1}}{P_t} \right] \right).$$

The household's only choice variables are τ_t and choice of bonds B_t, which are in zero net supply. The first order conditions are

$$\mathcal{L}_{\tau_t} = 0 : \frac{1}{\tau_t} = \lambda_t \left(U_t b_t + (1 - U_t) \frac{W_t}{P_t} \right),$$

$$\mathcal{L}_{B_t} = 0 : \frac{\lambda_t}{P_t} = \left(\frac{1}{1+\rho} \right) \lambda_{t+1} \frac{(1 + i_{t,t+1})}{P_{t+1}}.$$

Plugging the aggregate consumption equation (B.2) into the first order condition on τ_t yields the standard Euler equation (14) in the main text:

$$C_t^{-1} = \frac{1}{1+\rho} \frac{1 + i_{t,t+1}}{\pi_{t,t+1}} C_{t+1}^{-1}. \tag{B.3}$$

Recruiting and Separation Rate Elasticities Now we calculate the recruiting and separation rate elasticities under the conjectured symmetric equilibrium. From equations (21) and (22), the key derivatives are:

$$\frac{\partial R_t(W_{jt})}{\partial W_{jt}} = g(\theta_t) \left[\phi_{E,t} \int \frac{\partial r_{kj}(W_{kt}, W_{jt})}{\partial W_{jt}} \omega(W_{kt}) \, dk + \phi_{U,t} \frac{\partial r_{uj}(W_{jt})}{\partial W_{jt}} \right],$$
$$\frac{\partial S_t(W_{jt})}{\partial W_{jt}} = (1 - s) \left[\lambda_{EE} f(\theta_t) \int \frac{\partial s_{jk}(W_{jt}, W_{kt})}{\partial W_{jt}} z(W_{kt}) \, dk + \lambda_{EU} \frac{\partial s_{ju}(W_{jt})}{\partial W_{jt}} \right].$$

Given the logit formulas (15), (16), (18), and (19) in the main text, we obtain

$$\frac{\partial r_{kj}}{\partial W_{jt}} = \frac{\gamma}{W_{jt}} \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma \left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma}{\left[\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma\right]^2} = \frac{\gamma}{W_{jt}} r_{kj}(1 - r_{kj}),$$

$$\frac{\partial r_{uj}}{\partial W_{jt}} = \frac{\gamma}{W_{jt}} \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma (\tau_t b_t)^\gamma}{\left[(\tau_t b_t)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma\right]^2} = \frac{\gamma}{W_{jt}} r_{uj}(1 - r_{uj}),$$

$$\frac{\partial s_{jk}}{\partial W_{jt}} = -\frac{\gamma}{W_{jt}} \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma \left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma}{\left[\left(\tau_t \frac{W_{kt}}{P_t}\right)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma\right]^2} = -\frac{\gamma}{W_{jt}} s_{jk}(1 - s_{jk}),$$

$$\frac{\partial s_{ju}}{\partial W_{jt}} = -\frac{\gamma}{W_{jt}} \frac{\left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma (\tau_t b_t)^\gamma}{\left[(\tau_t b_t)^\gamma + \left(\tau_t \frac{W_{jt}}{P_t}\right)^\gamma\right]^2} = -\frac{\gamma}{W_{jt}} s_{ju}(1 - s_{ju}).$$

Hence,

$$\frac{\partial r_{kj}}{\partial W_{jt}} > 0, \qquad \frac{\partial r_{uj}}{\partial W_{jt}} > 0, \qquad \frac{\partial s_{jk}}{\partial W_{jt}} < 0, \qquad \frac{\partial s_{ju}}{\partial W_{jt}} < 0,$$

so $\varepsilon_{R,W_{jt}} > 0$ and $\varepsilon_{S,W_{jt}} < 0$ generically.

These expressions simplify enormously in a symmetric equilibrium with $W_{jt} = W_t$ for all j. Define $\tilde{w}_t \equiv W_t/P_t$. Then in a symmetric equilibrium, we have

$$r_{kj} = \frac{1}{2}, \qquad s_{jk} = \frac{1}{2},$$
$$r_{uj} = \frac{\tilde{w}_t^\gamma}{b_t^\gamma + \tilde{w}_t^\gamma} \equiv \mathcal{C}_t, \qquad s_{ju} = \frac{b_t^\gamma}{b_t^\gamma + \tilde{w}_t^\gamma} = 1 - \mathcal{C}_t,$$

and

$$R_t = g(\theta_t)\left(\frac{\phi_{E,t}}{2} + \phi_{U,t}\mathcal{C}_t\right), \tag{B.4}$$

$$S_t = s + (1-s)\left(\frac{\lambda_{EE} f(\theta_t)}{2} + \lambda_{EU}(1 - \mathcal{C}_t)\right). \tag{B.5}$$

Using $\partial r_{kj}/\partial W_{jt} = \gamma/(4W_t)$, $\partial s_{jk}/\partial W_{jt} = -\gamma/(4W_t)$, and

$$\frac{\partial \mathcal{C}_t}{\partial W_{jt}} = \frac{\gamma}{W_t}\mathcal{C}_t(1 - \mathcal{C}_t),$$

we obtain

$$\frac{\partial R_t}{\partial W_{jt}} = g(\theta_t)\frac{\gamma}{W_t}\left(\frac{\phi_{E,t}}{4} + \phi_{U,t}\mathcal{C}_t(1-\mathcal{C}_t)\right),$$
$$\frac{\partial S_t}{\partial W_{jt}} = -(1-s)\frac{\gamma}{W_t}\left(\frac{\lambda_{EE}f(\theta_t)}{4} + \lambda_{EU}\mathcal{C}_t(1-\mathcal{C}_t)\right).$$

Therefore the recruiting and separation elasticities are given by

$$\varepsilon_{R,t} = \frac{\gamma\left(\frac{\phi_{E,t}}{4} + \phi_{U,t}\mathcal{C}_t(1-\mathcal{C}_t)\right)}{\frac{\phi_{E,t}}{2} + \phi_{U,t}\mathcal{C}_t} > 0,$$
$$\varepsilon_{S,t} = -\frac{(1-s)\gamma\left(\frac{\lambda_{EE}f(\theta_t)}{4} + \lambda_{EU}\mathcal{C}_t(1-\mathcal{C}_t)\right)}{s + (1-s)\left(\frac{\lambda_{EE}f(\theta_t)}{2} + \lambda_{EU}(1-\mathcal{C}_t)\right)} < 0.$$

Hence both elasticities depend on unemployment insurance b_t (through $\mathcal{C}_t = \tilde{w}_t^\gamma/(b_t^\gamma + \tilde{w}_t^\gamma)$), on the search composition $(\phi_{E,t}, \phi_{U,t} = 1 - \phi_{E,t})$, on tightness through $f(\theta_t)$, and on model parameters $(\gamma, s, \lambda_{EE}, \lambda_{EU})$.

B.2.1 Firm Optimization

We now derive the wage Phillips curve (24), along with the remaining first-order conditions. Recall that firm j solves

$$\max_{\substack{\{P^j_{y,t}\},\{Y^j_t\},\{N_{jt}\}\\ \{W_{jt}\},\{V_{jt}\},\{\bar{X}^j_t\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(P^j_{y,t}Y^j_t - W_{jt}N_{jt} - c\left(\frac{V_{jt}}{N_{j,t-1}}\right)^\chi V_{jt}W_t - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt}N_{jt} - P_{x,t}\bar{X}^j_t\right) \tag{B.6}$$

subject to

$$N_{jt} = (1 - S(W_{jt}))N_{j,t-1} + R(W_{jt})V_{jt}, \tag{B.7}$$
$$Y^j_t = A^j_t\left(\alpha_N^{\frac{1}{\bar{\eta}}}(N_{jt})^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}}\left(\bar{X}^j_t\right)^{\frac{\bar{\eta}-1}{\bar{\eta}}}\right)^{\frac{\bar{\eta}}{\bar{\eta}-1}}, \tag{B.8}$$
$$\frac{Y^j_t}{Y_t} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}. \tag{B.9}$$

Setting up the Lagrangian yields:

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^{t}\Bigg(& P_{y,t}^{j}Y_{t}^{j} - W_{jt}N_{jt} - c\left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi}V_{jt}W_{t} - \frac{\psi}{2}\left(\frac{P_{y,t}^{j}}{P_{y,t-1}^{j}} - 1\right)^{2}Y_{t}^{j}P_{y,t}^{j} \\ & - \frac{\psi^{w}}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^{2}W_{jt}N_{jt} - P_{x,t}\bar{X}_{t}^{j} \\ & + \lambda_{t}\left[(1 - S(W_{jt}))N_{j,t-1} + R(W_{jt})V_{jt} - N_{jt}\right] \\ & + \mu_{t}\left[-Y_{t}^{j} + A_{t}^{j}\left(\alpha_{N}^{\frac{1}{\bar{\eta}}}\left(N_{jt}\right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}}\left(\bar{X}_{t}^{j}\right)^{\frac{\bar{\eta}-1}{\bar{\eta}}}\right)^{\frac{\bar{\eta}}{\bar{\eta}-1}}\right] \\ & + \Omega_{t}\left[-\frac{Y_{t}^{j}}{Y_{t}} + \left(\frac{P_{y,t}^{j}}{P_{y,t}}\right)^{-\epsilon}\right]\Bigg).\end{aligned}$$

Taking the first-order conditions with respect to $P_{y,t}^{j}$, we obtain:

$$\begin{aligned}\Omega_{t} = \frac{P_{y,t}^{j}Y_{t}}{\epsilon}\Bigg[1 - \psi\left(\frac{P_{y,t}^{j}}{P_{y,t-1}^{j}} - 1\right)\left(\frac{P_{y,t}^{j}}{P_{y,t-1}^{j}}\right) - \underbrace{\frac{\psi}{2}\left(\frac{P_{y,t}^{j}}{P_{y,t-1}^{j}} - 1\right)^{2}}_{\simeq 0} \\ + \frac{1}{1+\rho}\psi\left(\frac{P_{y,t+1}^{j}}{P_{y,t}^{j}} - 1\right)\left(\frac{P_{y,t+1}^{j}}{P_{y,t}^{j}}\right)^{2}\frac{Y_{t+1}^{j}}{Y_{t}^{j}}\Bigg].\end{aligned} \tag{B.10}$$

Taking the first-order conditions with respect to Y_{t}^{j}, we obtain:

$$\mu_{t} + \frac{\Omega_{t}}{Y_{t}} = P_{y,t}^{j}\left(1 - \underbrace{\frac{\psi}{2}\left(\frac{P_{y,t}^{j}}{P_{y,t-1}^{j}} - 1\right)^{2}}_{\simeq 0}\right). \tag{B.11}$$

Taking the first-order conditions with respect to N_{jt}, we obtain:

$$\lambda_t = \frac{1}{1+\rho}\lambda_{t+1}(1 - S(W_{j,t+1})) + \mu_t (A_t^j)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \alpha_N^{\frac{1}{\bar{\eta}}} \left(\frac{N_{jt}}{Y_t^j}\right)^{-\frac{1}{\bar{\eta}}} - W_{jt}$$
$$+ \frac{1}{1+\rho} c\chi \left(\frac{V_{j,t+1}}{N_{jt}}\right)^{\chi+1} W_{t+1} - \underbrace{\frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2}_{\simeq 0} W_{jt}. \tag{B.12}$$

Taking the first-order conditions with respect to W_{jt}, we obtain:

$$\lambda_t = \frac{1}{\left(\varepsilon_{S,W_{jt}} \frac{S_t^j}{W_{jt}} N_{t-1}^j - \varepsilon_{R,W_{jt}} \frac{R_t^j}{W_{jt}} V_t^j\right)} \Bigg[- N_t^j - \psi^w \left(\frac{W_{jt}}{W_{j,t-1}} - 1\right) \frac{W_{jt}}{W_{j,t-1}} N_t^j - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 N_{jt}$$
$$+ \frac{1}{1+\rho}\psi^w \left(\frac{W_{j,t+1}}{W_{jt}} - 1\right)\left(\frac{W_{j,t+1}}{W_{jt}}\right)^2 N_{j,t+1} \Bigg]. \tag{B.13}$$

Taking the first-order conditions with respect to V_{jt}, we obtain:

$$\lambda_t = \frac{c(\chi+1)W_t}{R(W_{jt})}\left(\frac{V_t^j}{N_{t-1}^j}\right)^{\chi}. \tag{B.14}$$

Finally, taking the first-order conditions with respect to $\bar{X}_t^j$, we obtain:

$$P_{x,t} = \mu_t \cdot (A_t^j)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\frac{\bar{X}_t^j}{Y_t^j}\right)^{-\frac{1}{\bar{\eta}}}, \tag{B.15}$$

where it follows that when N_{jt} is held constant, a decrease in $\bar{X}_t^j$ results in an increase in $P_{x,t}$.

Price Phillips Curve Note that combining the first order conditions for prices and output yields the following Price Phillips curve in a symmetric equilibrium, after dropping terms

that are approximately zero: defining $\Pi^j_{y,t} = P^j_{y,t}/P^j_{y,t-1}$,

$$\frac{\mu^j_t}{P^j_{y,t}} = \frac{\epsilon-1}{\epsilon} + \frac{\psi}{\epsilon}\left(\Pi^j_{y,t}-1\right)\Pi^j_{y,t} - \frac{\psi}{\epsilon}\frac{1}{1+\rho}\left(\Pi^j_{y,t+1}-1\right)\left(\Pi^j_{y,t+1}\right)^2,$$

where μ^j_t can be interpreted as marginal cost, so that this yields the standard markup expression $P^j_y = \frac{\epsilon}{\epsilon-1}\mu^j$ in a zero inflation steady state or in the absence of adjustment costs in price setting ($\psi = 0$). In a symmetric equilibrium this becomes

$$\left(\Pi_{y,t}-1\right)\Pi_{y,t} = \frac{\epsilon}{\psi}\left(\frac{\mu_t}{P_{y,t}} - \frac{\epsilon-1}{\epsilon}\right) + \frac{1}{1+\rho}\left(\Pi_{y,t+1}-1\right)\left(\Pi_{y,t+1}\right)^2,$$

so that inflation will be determined by the present discounted value of firm's real marginal costs. To a first order approximation, this becomes:

$$\check{\Pi}_{y,t} = \frac{\epsilon-1}{\psi}\underbrace{\ln\left(\frac{\mu_t/P_{y,t}}{(\epsilon-1)/\epsilon}\right)}_{\equiv\,(\mu_t/\check{P}_{y,t})} + \frac{1}{1+\rho}\check{\Pi}_{y,t+1}. \tag{B.16}$$

Wage Phillips Curve and Pass-Through The wage Phillips curve is derived from (B.13) and (B.14):

$$\begin{aligned}
&\frac{c(\chi+1)W_t}{R(W_{jt})}\left(\frac{V^j_t}{N^j_{t-1}}\right)^{\chi} \\
&= \frac{1}{\left(\varepsilon_{S,W_{jt}}\frac{S^j_t}{W_{jt}}N^j_{t-1} - \varepsilon_{R,W_{jt}}\frac{R^j_t}{W_{jt}}V^j_t\right)}\Bigg[-N^j_t - \psi^w\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)\frac{W_{jt}}{W_{j,t-1}}N^j_t - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)^2 N_{jt} \\
&\qquad\qquad + \frac{1}{1+\rho}\psi^w\left(\frac{W_{j,t+1}}{W_{jt}}-1\right)\left(\frac{W_{j,t+1}}{W_{jt}}\right)^2 N_{j,t+1}\Bigg],
\end{aligned}$$

which in our symmetric equilibrium becomes:

$$\begin{aligned}
&\frac{c(\chi+1)W_t}{R_t}\left(\frac{V_t}{N_{t-1}}\right)^{\chi} \\
&= \frac{1}{\left(\varepsilon_{S,t}\frac{S_t}{W_t}N_{t-1} - \varepsilon_{R,t}\frac{R_t}{W_t}V_t\right)}\Bigg[- N_t - \psi^w\left(\frac{W_t}{W_{t-1}}-1\right)\frac{W_t}{W_{t-1}}N_t - \frac{\psi^w}{2}\left(\frac{W_t}{W_{t-1}}-1\right)^2 N_t \\
&\qquad\qquad + \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}}{W_t}-1\right)\left(\frac{W_{t+1}}{W_t}\right)^2 N_{t+1}\Bigg].
\end{aligned} \tag{B.17}$$

B.3 Log-Linearizing the Wage Phillips Curve for Generic ϖ

We now prove Propositions 1, 2, and 3 in the main paper. First, recall that Proposition 1 is given as follows:

Proposition 1 (Wage Phillips Curve in $(\theta, U, \mathcal{C})$) *For generic $\varpi \in [0,1]$, the log-linear approximation to wage inflation can be written as*

$$\check{\Pi}^w_t = \Phi^{(\varpi)}_V \check{\theta}_t + \widetilde{\Phi}^{(\varpi)}_U \check{U}_{t-1} + \Phi^{(\varpi)}_{\mathcal{C}} \check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}, \tag{B.18}$$

Further, under the limit $s \to 0$ and $\mathcal{C} \to 1$,

$$\begin{aligned}
\Phi^{(\varpi)}_V &\to \frac{\kappa}{\psi^w}(1+\chi)\mathfrak{m} && > 0 \\
\Phi^{(\varpi)}_{\mathcal{C}} &\to -\frac{\kappa}{\psi^w}\mathcal{J}\left(\mathfrak{m}(1+S)\right) && < 0
\end{aligned}$$

where $K_\phi \equiv \frac{1-\phi_E}{1-U} > 0$ and $\mathcal{J} \equiv \frac{\lambda_{EU}}{0.5\lambda_{EE}f} > 0$, $\kappa \equiv c(1+\chi)\left(\frac{V}{N}\right)^{1+\chi} > 0$, and $\mathfrak{m} \equiv \varepsilon_R - \varepsilon_S$. Moreover,

$$\widetilde{\Phi}^{(\varpi)}_U \to \frac{\kappa}{\psi^w}K_\phi \mathfrak{m}\left[\chi - S - a_\phi(1+S)\right],$$

with

$$a_\phi \equiv \frac{\phi_E}{2-\phi_E} \in (0,1),$$

so $\widetilde{\Phi}^{(\varpi)}_U$ is generally ambiguous for generic $\chi > 0$, and is strictly negative when $\chi = 0$.

In proving this proposition, we first log-linearize the wage Phillips curve (B.17) around the steady state of a symmetric equilibrium for a generic ϖ. We will go as far as we can and

then specialize to $\varpi = 0$ and $\varpi = 1$ below. First, we use the following definitions:

$$T_t \equiv \frac{V_t}{N_{t-1}}, \qquad g_t \equiv \frac{N_t}{N_{t-1}}, \qquad N_t = 1 - U_t,$$

$$\mathcal{C}_t \equiv \frac{\tilde{w}_t^{\gamma}}{\tilde{w}_t^{\gamma} + b_t^{\gamma}}, \qquad \tilde{w}_t \equiv \frac{W_t}{P_t}, \qquad b_t = \varpi b + (1-\varpi)\xi^{-1}\tilde{w}_t.$$

Since $\varepsilon_{S,t} < 0$ as proved above, define

$$\bar{\varepsilon}_{S,t} \equiv -\varepsilon_{S,t} > 0$$

to avoid taking logs of negative objects.

Step 1: Rewrite the Nonlinear Wage Phillips Curve Using $\Pi_t^w \equiv W_t/W_{t-1}$, (B.17) is equivalent to

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w - 1\right)^2 + \psi^w\left(\Pi_t^w - 1\right)\Pi_t^w + 1 = c(1+\chi)T_t^{\chi} g_t^{-1}\left(T_t\varepsilon_{R,t} + \frac{S_t}{R_t}\bar{\varepsilon}_{S,t}\right) \\ + \frac{\psi^w}{1+\rho}\left(\Pi_{t+1}^w - 1\right)\left(\Pi_{t+1}^w\right)^2 g_{t+1}.\end{aligned} \tag{B.19}$$

Step 2: First-Order Approximation Let $\check{X}_t \equiv \ln X_t - \ln X$ for $X_t > 0$. Define[3]

$$\kappa \equiv c(1+\chi)\frac{T^{\chi+1}}{g} = c(1+\chi)\left(\frac{V}{N}\right)^{1+\chi}$$

We rewrite (B.19) as:

$$0 = F\left(\ln(\Pi_t^w), \ln(\Pi_{t+1}^w), \ln(S_t), \ln(R_t), \ln(\varepsilon_{R,t}), \ln(\bar{\varepsilon}_{S,t}), \ln(T_t), \ln(g_t), \ln(g_{t+1})\right),$$

and take a linear approximation around a zero inflation (both wage and price) steady state with variables $\ln(\Pi_t^w)$, $\ln(\Pi_{t+1}^w)$, $\ln(S_t)$, $\ln(R_t)$, $\ln(\varepsilon_{R,t})$, $\ln(\bar{\varepsilon}_{S,t})$, $\ln(T_t)$, $\ln(g_t)$, and $\ln(g_{t+1})$. We first calculate derivatives of $F(\cdot)$ with respect to each variable as follows:

$$\begin{aligned}F_{\ln(\Pi_t^w)} &= \psi^w \Pi_t^w\left(2(\Pi_t^w - 1) + \Pi_t^w\right) \\ F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w g_{t+1}}{1+\rho}\left(\Pi_{t+1}^w(\Pi_{t+1}^w)^2 + (\Pi_{t+1}^w - 1)2(\Pi_{t+1}^w)^2\right)\end{aligned}$$

[3]Note that $g = 1$ at the steady state.

$$
\begin{aligned}
F_{\ln(S_t)} &= -c(1+\chi)T_t^{\chi} g_t^{-1} \frac{S_t}{R_t}\bar{\varepsilon}_{S,t}, \quad F_{\ln(R_t)} = c(1+\chi)T_t^{\chi} g_t^{-1} \frac{S_t}{R_t}\bar{\varepsilon}_{S,t} \\
F_{\ln(\varepsilon_{R,t})} &= -c(1+\chi)T_t^{\chi+1} g_t^{-1}\varepsilon_{R,t}, \quad F_{\ln(\bar{\varepsilon}_{S,t})} = -c(1+\chi)T_t^{\chi} g_t^{-1} \frac{S_t}{R_t}\bar{\varepsilon}_{S,t} \\
F_{ln(T_t)} &= -c(1+\chi)g_t^{-1}\left((1+\chi)T_t^{\chi+1}\varepsilon_{R,t} + \chi T_t^{\chi}\frac{S_t}{R_t}\bar{\varepsilon}_{S,t}\right) \\
F_{\ln(g_t)} &= c(1+\chi)T_t^{\chi} g_t^{-1}\left(T_t\varepsilon_{R,t} + \frac{S_t}{R_t}\bar{\varepsilon}_{S,t}\right) \\
F_{\ln(g_{t+1})} &= -\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w - 1\right)(\Pi_{t+1}^w)^2 g_{t+1},
\end{aligned}
$$

which at the steady state with zero wage inflation (and $g = 1$) are given by:

$$
\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w, \quad F_{\ln(\Pi_{t+1}^w)} = -\frac{\psi^w}{1+\rho} \\
F_{\ln(S_t)} &= -c(1+\chi)T^{\chi} g^{-1}\frac{S}{R}\bar{\varepsilon}_S = -c(1+\chi)T^{\chi+1}\bar{\varepsilon}_S \\
F_{\ln(R_t)} &= c(1+\chi)T^{\chi} g^{-1}\frac{S_t}{R_t}\bar{\varepsilon}_{S,t} = c(1+\chi)T^{\chi+1}\bar{\varepsilon}_S \\
F_{\ln(\varepsilon_{R,t})} &= -c(1+\chi)T^{\chi+1}\varepsilon_R \\
F_{\ln(\bar{\varepsilon}_{S,t})} &= -c(1+\chi)T^{\chi+1}\bar{\varepsilon}_S \\
F_{ln(T_t)} &= -c(1+\chi)T^{\chi+1}\left((1+\chi)\varepsilon_R + \chi\bar{\varepsilon}_S\right) \\
F_{\ln(g_t)} &= c(1+\chi)T^{\chi+1}\left(\varepsilon_R + \bar{\varepsilon}_S\right) > 0 \\
F_{\ln(g_{t+1})} &= 0
\end{aligned}
$$

We can see that the assumption above that $\frac{\psi^w}{2}\left(\Pi_t^w - 1\right)^2 \approx 0$ is correct in the sense that it drops out in our first-order approximation. We also find that in a zero inflation steady-state, there is no role for expectations of future employment growth in our first-order approximation.

Then the first order approximation of $F(\cdot)$ around its steady state is given by

$$
\begin{aligned}
0 =& \psi^w\check{\Pi}_t^w - \frac{\psi^w}{1+\rho}\check{\Pi}_{t+1}^w - \kappa\bar{\varepsilon}_S\left(\check{S}_t - \check{R}_t\right) - \kappa\left(\bar{\varepsilon}_S\check{\bar{\varepsilon}}_{S,t} + \varepsilon_R\check{\varepsilon}_{R,t}\right) \\
& - \kappa(\chi\bar{\varepsilon}_S + (1+\chi)\varepsilon_R)\check{T}_t + \kappa(\varepsilon_R + \bar{\varepsilon}_S)\check{g}_t,
\end{aligned}
$$

which we can be rearranged to

$$\begin{aligned}\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \frac{\kappa}{\psi^w}\Big[\bar{\varepsilon}_S\left(\check{S}_t - \check{R}_t\right) + \varepsilon_R\check{\varepsilon}_{R,t} + \bar{\varepsilon}_S\check{\bar{\varepsilon}}_{S,t} \\ + \left((1+\chi)\varepsilon_R + \chi\bar{\varepsilon}_S\right)\check{T}_t - \left(\varepsilon_R + \bar{\varepsilon}_S\right)\check{g}_t\Big].\end{aligned} \tag{B.20}$$

From the employment law of motion $g_t = (1 - S_t) + R_t T_t$, we obtain

$$\check{g}_t = S\left(\check{R}_t + \check{T}_t - \check{S}_t\right). \tag{B.21}$$

Substituting (B.21) into (B.20) gives

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \frac{\kappa}{\psi^w}\Big[\Delta_\varpi\left(\check{S}_t - \check{R}_t\right) + \Lambda_\varpi\check{T}_t + \varepsilon_R\check{\varepsilon}_{R,t} + \bar{\varepsilon}_S\check{\bar{\varepsilon}}_{S,t}\Big], \tag{B.22}$$

where

$$\Delta_\varpi \equiv \bar{\varepsilon}_S + S\left(\varepsilon_R + \bar{\varepsilon}_S\right), \qquad \Lambda_\varpi \equiv \varepsilon_R + (\chi - S)\left(\varepsilon_R + \bar{\varepsilon}_S\right). \tag{B.23}$$

Monopsony and Slope of the Wage Phillips Curve Define net recruitment power (inverse monopsony) as

$$\mathfrak{m} \equiv \varepsilon_R + \bar{\varepsilon}_S = \varepsilon_R - \varepsilon_S.$$

Then (B.23) becomes

$$\Delta_\varpi = \bar{\varepsilon}_S + S\mathfrak{m}, \qquad \Lambda_\varpi = \varepsilon_R + (\chi - S)\mathfrak{m}.$$

Hence

$$\frac{\partial\Delta_\varpi}{\partial\mathfrak{m}} = S > 0, \qquad \frac{\partial\Lambda_\varpi}{\partial\mathfrak{m}} = \chi - S.$$

Without taking the limit of any parameters, note that in the model's benchmark calibration ($\chi = 1 > S$), lower $\mathfrak{m}$ (stronger monopsony power) lowers both slope loadings. Therefore, the wage Phillips curve is flatter when monopsony power is stronger.

We then reduce (B.22) to $(\check{V}_t, \check{U}_{t-1}, \check{\mathcal{C}}_t)$ plus the one-step-ahead wage inflation term.

Step 1: Tightness and Search Composition From

$$\theta_t = \frac{V_t}{\mathcal{S}_t}, \qquad \mathcal{S}_t = (1-s)\lambda_{EE}(1 - U_{t-1}) + U_{t-1},$$

it follows that

$$\check{\theta}_t = \check{V}_t - \eta_{\theta,U}\check{U}_{t-1}, \qquad \eta_{\theta,U} \equiv \frac{U\left(1-(1-s)\lambda_{EE}\right)}{(1-s)\lambda_{EE}(1-U)+U} = \frac{1-U-\phi_E}{1-U}. \tag{B.24}$$

Also

$$\check{T}_t = \check{V}_t + \frac{U}{1-U}\check{U}_{t-1}. \tag{B.25}$$

Let

$$\eta_f \equiv \nu, \qquad \eta_g \equiv 1-\nu.$$

so $\check{f}_t = \eta_f \check{\theta}_t$ and $\check{g}_t = -\eta_g \check{\theta}_t$.[4]

Step 2: Log-Decomposition of $R_t, S_t, \varepsilon_{R,t}, \bar{\varepsilon}_{S,t}$ Define, for $X_t \in \{R_t, S_t, \varepsilon_{R,t}, \bar{\varepsilon}_{S,t}\}$,

$$g^{(\varpi)}_{X,V} \equiv \left.\frac{\partial \ln X_t}{\partial \ln V_t}\right|_{ss}, \quad g^{(\varpi)}_{X,U} \equiv \left.\frac{\partial \ln X_t}{\partial \ln U_{t-1}}\right|_{ss}, \quad g^{(\varpi)}_{X,\mathcal{C}} \equiv \left.\frac{\partial \ln X_t}{\partial \ln \mathcal{C}_t}\right|_{ss}.$$

Then

$$\check{X}_t = g^{(\varpi)}_{X,V}\check{V}_t + g^{(\varpi)}_{X,U}\check{U}_{t-1} + g^{(\varpi)}_{X,\mathcal{C}}\check{\mathcal{C}}_t. \tag{B.26}$$

We derive these g-loadings next.

Step 3: Reduced-Form Generic-ϖ Wage Phillips Curve Substituting (B.25) and (B.26) into (B.22) gives

$$\check{\Pi}^w_t = \Phi^{(\varpi)}_V \check{V}_t + \Phi^{(\varpi)}_U \check{U}_{t-1} + \Phi^{(\varpi)}_{\mathcal{C}}\check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}, \tag{B.27}$$

with

$$\Phi^{(\varpi)}_V = \frac{\kappa}{\psi^w}\left[\Lambda_\varpi + \Delta_\varpi\left(g^{(\varpi)}_{S,V} - g^{(\varpi)}_{R,V}\right) + \varepsilon_R g^{(\varpi)}_{\varepsilon_R,V} + \bar{\varepsilon}_S g^{(\varpi)}_{\bar{\varepsilon}_S,V}\right], \tag{B.28}$$

$$\Phi^{(\varpi)}_U = \frac{\kappa}{\psi^w}\left[\frac{U}{1-U}\Lambda_\varpi + \Delta_\varpi\left(g^{(\varpi)}_{S,U} - g^{(\varpi)}_{R,U}\right) + \varepsilon_R g^{(\varpi)}_{\varepsilon_R,U} + \bar{\varepsilon}_S g^{(\varpi)}_{\bar{\varepsilon}_S,U}\right], \tag{B.29}$$

$$\Phi^{(\varpi)}_{\mathcal{C}} = \frac{\kappa}{\psi^w}\left[\Delta_\varpi\left(g^{(\varpi)}_{S,\mathcal{C}} - g^{(\varpi)}_{R,\mathcal{C}}\right) + \varepsilon_R g^{(\varpi)}_{\varepsilon_R,\mathcal{C}} + \bar{\varepsilon}_S g^{(\varpi)}_{\bar{\varepsilon}_S,\mathcal{C}}\right]. \tag{B.30}$$

[4]Note that expressing the model in this way makes clear that the specific choice of matching function does not affect the analysis that follows.

Connection to $\varpi = 0$ and $\varpi = 1$ If $\varpi = 0$, then $b_t = \xi^{-1}\tilde{w}_t$ and $\mathcal{C}_t = \frac{\xi^\gamma}{1+\xi^\gamma}$ is constant, so $\check{\mathcal{C}}_t = 0$ and (B.27) collapses to a form with only $(\check{V}_t, \check{U}_{t-1})$. If $\varpi = 1$, then $b_t = b$ and $\check{\mathcal{C}}_t = \gamma(1-\mathcal{C})\check{\tilde{w}}_t$.

Wage Phillips Curve in $(\theta, U, \mathcal{C})$ By using the fact that $\check{\theta}_t = \check{V}_t - \eta_{\theta,U}\check{U}_{t-1}$, we can rewrite (B.27) as:

$$\check{\Pi}^w_t = \Phi^{(\varpi)}_V \check{\theta}_t + \underbrace{\left(\Phi^{(\varpi)}_U + \eta_{\theta,U}\Phi^{(\varpi)}_V\right)}_{\equiv \tilde{\Phi}^{(\varpi)}_U} \check{U}_{t-1} + \Phi^{(\varpi)}_{\mathcal{C}} \check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}. \tag{B.31}$$

which is consistent with the log-linearized wage Phillips curve in Proposition 1.

B.3.1 Expressions for Wage Phillips Curve Coefficients

Given the log-linerized wage Phillips curve (B.31), this section derives the loadings in (B.26) by directly log-linearizing each object and collecting coefficients on $\check{V}_t$ (or $\check{\theta}_t$), $\check{U}_{t-1}$, and $\check{\mathcal{C}}_t$. For notational simplicity, let

$$f_t \equiv f(\theta_t), \quad g_t \equiv g(\theta_t), \quad A_t \equiv \frac{\phi_{E,t}}{2} + (1-\phi_{E,t})\mathcal{C}_t, \quad B_t \equiv \frac{\lambda_{EE}}{2} f_t + \lambda_{EU}(1-\mathcal{C}_t),$$

$$D_t \equiv \frac{\phi_{E,t}}{4} + (1-\phi_{E,t})\mathcal{C}_t(1-\mathcal{C}_t), \quad E_t \equiv \frac{\lambda_{EE}}{4} f_t + \lambda_{EU}\mathcal{C}_t(1-\mathcal{C}_t),$$

with steady-state counterparts (A, B, D, E). Also

$$\check{\theta}_t = \check{V}_t - \eta_{\theta,U}\check{U}_{t-1}, \qquad \check{f}_t = \eta_f \check{\theta}_t, \qquad \check{g}_t = -\eta_g \check{\theta}_t$$

from (B.24), and

$$\check{\phi}_{E,t} = -\frac{1-\phi_E}{1-U}\check{U}_{t-1}.$$

which follows from the definition.[5]

1. The Recruiting Rate R_t Starting from

$$R_t = g_t\left(\frac{\phi_{E,t}}{2} + (1-\phi_{E,t})\mathcal{C}_t\right) = g_t A_t.$$

[5]Define $a_t \equiv (1-s)\lambda_{EE}(1-U_{t-1})$, so $\phi_{E,t} = \frac{a_t}{a_t + U_{t-1}}$. Then $\check{\phi}_{E,t} = \check{a}_t - \left(\frac{a}{a+U}\check{a}_t + \frac{U}{a+U}\check{U}_{t-1}\right) = (1-\phi_E)\check{a}_t - (1-\phi_E)\check{U}_{t-1}$. Since $\check{a}_t = -\frac{U}{1-U}\check{U}_{t-1}$, this gives $\check{\phi}_{E,t} = -\frac{1-\phi_E}{1-U}\check{U}_{t-1}$.

Log-linearizing:

$$\check{R}_t = \check{g}_t + \check{A}_t.$$

For $\check{A}_t$,

$$\check{A}_t = \underbrace{\frac{\phi_E\left(\frac{1}{2}-\mathcal{C}\right)}{A}}_{\equiv\omega_{A,\phi}}\check{\phi}_{E,t} + \underbrace{\frac{(1-\phi_E)\mathcal{C}}{A}}_{\equiv\omega_{A,\mathcal{C}}}\check{\mathcal{C}}_t.$$

Hence

$$\check{R}_t = -\eta_g(\check{V}_t - \eta_{\theta,U}\check{U}_{t-1}) - \omega_{A,\phi}\frac{1-\phi_E}{1-U}\check{U}_{t-1} + \omega_{A,\mathcal{C}}\check{\mathcal{C}}_t.$$

Collecting terms:

$$\begin{aligned}
g^{(\varpi)}_{R,V} &= -\eta_g,\\
g^{(\varpi)}_{R,U} &= \eta_g\eta_{\theta,U} - \omega_{A,\phi}\frac{1-\phi_E}{1-U},\\
g^{(\varpi)}_{R,\mathcal{C}} &= \omega_{A,\mathcal{C}} = \frac{(1-\phi_E)\mathcal{C}}{\frac{\phi_E}{2}+(1-\phi_E)\mathcal{C}}.
\end{aligned}$$

2. The Separation Rate S_t Write

$$S_t = s + (1-s)B_t, \qquad B_t = \frac{\lambda_{EE}}{2}f_t + \lambda_{EU}(1-\mathcal{C}_t).$$

Then

$$\check{S}_t = \underbrace{\frac{(1-s)B}{S}}_{\equiv\omega_S}\check{B}_t, \qquad \check{B}_t = \underbrace{\frac{0.5\lambda_{EE}f}{B}}_{\equiv\omega_{B,f}}\check{f}_t \underbrace{-\frac{\lambda_{EU}\mathcal{C}}{B}}_{\equiv\omega_{B,\mathcal{C}}}\check{\mathcal{C}}_t.$$

Using $\check{f}_t = \eta_f(\check{V}_t - \eta_{\theta,U}\check{U}_{t-1})$,

$$\check{S}_t = \omega_S\omega_{B,f}\eta_f(\check{V}_t - \eta_{\theta,U}\check{U}_{t-1}) + \omega_S\omega_{B,\mathcal{C}}\check{\mathcal{C}}_t.$$

So

$$\begin{aligned}
g^{(\varpi)}_{S,V} &= \omega_S\omega_{B,f}\eta_f,\\
g^{(\varpi)}_{S,U} &= -g^{(\varpi)}_{S,V}\eta_{\theta,U},\\
g^{(\varpi)}_{S,\mathcal{C}} &= \omega_S\omega_{B,\mathcal{C}} = -\frac{(1-s)\lambda_{EU}\mathcal{C}}{s+(1-s)\left(0.5\lambda_{EE}f+\lambda_{EU}(1-\mathcal{C})\right)}.
\end{aligned}$$

3. The Recruiting Elasticity $\varepsilon_{R,t}$ With

$$\varepsilon_{R,t} = \gamma \frac{D_t}{A_t}, \qquad D_t = \frac{\phi_{E,t}}{4} + (1-\phi_{E,t})\mathcal{C}_t(1-\mathcal{C}_t),$$

we get

$$\check{\varepsilon}_{R,t} = \check{D}_t - \check{A}_t.$$

Also

$$\check{D}_t = \underbrace{\frac{\phi_E\left(\frac{1}{4} - \mathcal{C}(1-\mathcal{C})\right)}{D}}_{\equiv \omega_{D,\phi}} \check{\phi}_{E,t} + \underbrace{\frac{(1-\phi_E)\mathcal{C}(1-2\mathcal{C})}{D}}_{\equiv \omega_{D,\mathcal{C}}} \check{\mathcal{C}}_t.$$

Therefore

$$\check{\varepsilon}_{R,t} = -(\omega_{D,\phi} - \omega_{A,\phi})\frac{1-\phi_E}{1-U}\check{U}_{t-1} + (\omega_{D,\mathcal{C}} - \omega_{A,\mathcal{C}})\check{\mathcal{C}}_t,$$

so

$$\begin{aligned}
g^{(\varpi)}_{\varepsilon_R,V} &= 0, \\
g^{(\varpi)}_{\varepsilon_R,U} &= -(\omega_{D,\phi} - \omega_{A,\phi})\frac{1-\phi_E}{1-U}, \\
g^{(\varpi)}_{\varepsilon_R,\mathcal{C}} &= \omega_{D,\mathcal{C}} - \omega_{A,\mathcal{C}} = \mathcal{C}\left[\frac{(1-\phi_E)(1-2\mathcal{C})}{\frac{\phi_E}{4} + (1-\phi_E)\mathcal{C}(1-\mathcal{C})} - \frac{1-\phi_E}{\frac{\phi_E}{2} + (1-\phi_E)\mathcal{C}}\right].
\end{aligned}$$

4. The Separation Elasticity $\bar{\varepsilon}_{S,t}$ Using $\bar{\varepsilon}_{S,t} \equiv -\varepsilon_{S,t} > 0$:

$$\bar{\varepsilon}_{S,t} = (1-s)\gamma\frac{E_t}{S_t}, \qquad E_t = \frac{\lambda_{EE}}{4}f_t + \lambda_{EU}\mathcal{C}_t(1-\mathcal{C}_t).$$

Hence

$$\check{\bar{\varepsilon}}_{S,t} = \check{E}_t - \check{S}_t,$$

with

$$\check{E}_t = \underbrace{\frac{0.25\lambda_{EE}f}{E}}_{\equiv \omega_{E,f}} \check{f}_t + \underbrace{\frac{\lambda_{EU}\mathcal{C}(1-2\mathcal{C})}{E}}_{\equiv \omega_{E,\mathcal{C}}} \check{\mathcal{C}}_t.$$

Substituting $\check{f}_t = \eta_f(\check{V}_t - \eta_{\theta,U}\check{U}_{t-1})$ and $\check{S}_t$ from above:

$$\check{\bar{\varepsilon}}_{S,t} = (\omega_{E,f} - \omega_S\omega_{B,f})\eta_f(\check{V}_t - \eta_{\theta,U}\check{U}_{t-1}) + (\omega_{E,\mathcal{C}} - \omega_S\omega_{B,\mathcal{C}})\check{\mathcal{C}}_t.$$

So

$$
\begin{aligned}
g^{(\varpi)}_{\bar{\varepsilon}_S,V} &= (\omega_{E,f} - \omega_S \omega_{B,f})\eta_f, \\
g^{(\varpi)}_{\bar{\varepsilon}_S,U} &= -g^{(\varpi)}_{\bar{\varepsilon}_S,V}\eta_{\theta,U}, \\
g^{(\varpi)}_{\bar{\varepsilon}_S,\mathcal{C}} &= \omega_{E,\mathcal{C}} - \omega_S \omega_{B,\mathcal{C}} \\
&= \mathcal{C}\left[\frac{\lambda_{EU}(1-2\mathcal{C})}{0.25\lambda_{EE}f + \lambda_{EU}\mathcal{C}(1-\mathcal{C})} + \frac{(1-s)\lambda_{EU}}{s + (1-s)\left(0.5\lambda_{EE}f + \lambda_{EU}(1-\mathcal{C})\right)}\right].
\end{aligned}
$$

B.3.2 Sign Conditions for $\Phi^{(\varpi)}_V, \Phi^{(\varpi)}_U$, and $\Phi^{(\varpi)}_{\mathcal{C}}$ under $s \to 0$ and $\mathcal{C} \to 1$

In this section, we determine the signs of these coefficients under the assumptions that $s \to 0$ and $\mathcal{C} \to 1$, conditions that closely match our baseline calibration. Intuitively, layoffs are rare, and unemployed workers almost always prefer employment when given the opportunity to choose among jobs. This analysis completes the proof of Proposition 1.

Limit of the Relevant Loadings As $s \to 0$ and $\mathcal{C} \to 1$:

$$
\omega_S \to 1, \qquad \omega_{B,f} \to 1, \qquad \omega_{E,f} \to 1, \qquad \omega_{A,\phi} \to -\frac{\phi_E}{2-\phi_E}, \qquad \omega_{D,\phi} \to 1.
$$

Hence

$$
g^{(\varpi)}_{S,V} \to \eta_f, \qquad g^{(\varpi)}_{R,V} = -\eta_g, \qquad g^{(\varpi)}_{\bar{\varepsilon}_S,V} \to 0, \qquad g^{(\varpi)}_{\varepsilon_R,V} = 0, \tag{B.32}
$$

$$
g^{(\varpi)}_{S,U} \to -\eta_{\theta,U}\eta_f, \qquad g^{(\varpi)}_{R,U} \to \eta_g\eta_{\theta,U} - \omega_{A,\phi}\frac{1-\phi_E}{1-U}, \qquad g^{(\varpi)}_{\bar{\varepsilon}_S,U} \to 0, \tag{B.33}
$$

$$
g^{(\varpi)}_{\varepsilon_R,U} \to -(\omega_{D,\phi} - \omega_{A,\phi})\frac{1-\phi_E}{1-U} < 0, \tag{B.34}
$$

and for $\mathcal{C}$-loadings,

$$
g^{(\varpi)}_{S,\mathcal{C}} \to -\frac{2\lambda_{EU}}{\lambda_{EE}f} < 0, \tag{B.35}
$$

$$
g^{(\varpi)}_{R,\mathcal{C}} \to \frac{1-\phi_E}{1-\phi_E/2} > 0, \tag{B.36}
$$

$$
g^{(\varpi)}_{\bar{\varepsilon}_S,\mathcal{C}} \to -\frac{2\lambda_{EU}}{\lambda_{EE}f} < 0, \tag{B.37}
$$

$$
g^{(\varpi)}_{\varepsilon_R,\mathcal{C}} \to -\frac{4(1-\phi_E)}{\phi_E} - \frac{1-\phi_E}{1-\phi_E/2} < 0. \tag{B.38}
$$

Also, $\eta_f + \eta_g = 1$.

Simplified Limit Expression for $\Phi_V^{(\varpi)}$ Using (B.28) with (B.32):

$$\Phi_V^{(\varpi)} \to \frac{\kappa}{\psi^w}\left[\Lambda_\varpi + \Delta_\varpi(\eta_f + \eta_g)\right] = \frac{\kappa}{\psi^w}\left(\Lambda_\varpi + \Delta_\varpi\right).$$

Since

$$\Lambda_\varpi + \Delta_\varpi = (\varepsilon_R + (\chi - S)(\varepsilon_R + \bar{\varepsilon}_S)) + (\bar{\varepsilon}_S + S(\varepsilon_R + \bar{\varepsilon}_S)) = (1+\chi)(\varepsilon_R + \bar{\varepsilon}_S),$$

we get

$$\Phi_V^{(\varpi)} \to \frac{\kappa}{\psi^w}(1+\chi)(\varepsilon_R + \bar{\varepsilon}_S) = \frac{\kappa}{\psi^w}(1+\chi)\mathfrak{m} > 0 \qquad (\chi \geqslant 0). \tag{B.39}$$

So in this limit, the vacancy coefficient is unambiguously positive.

Simplified Limit Expression for $\Phi_U^{(\varpi)}$ Define

$$K_\phi \equiv \frac{1-\phi_E}{1-U}, \qquad A_U \equiv \eta_{\theta,U} - \omega_{A,\phi}K_\phi > 0, \qquad B_U \equiv (\omega_{D,\phi} - \omega_{A,\phi})K_\phi > 0.$$

Using (B.29) with (B.33)–(B.34):

$$\Phi_U^{(\varpi)} \to \frac{\kappa}{\psi^w}\left[\frac{U}{1-U}\Lambda_\varpi - \Delta_\varpi A_U - \varepsilon_R B_U\right]. \tag{B.40}$$

Hence $\Phi_U^{(\varpi)}$ is generally sign-ambiguous in this limit. A sufficient-and-necessary condition for $\Phi_U^{(\varpi)} < 0$ is

$$\frac{U}{1-U}\Lambda_\varpi < \Delta_\varpi A_U + \varepsilon_R B_U. \tag{B.41}$$

Since $\Lambda_\varpi = \varepsilon_R + (\chi - S)(\varepsilon_R + \bar{\varepsilon}_S)$ is increasing in χ, larger χ tends to push $\Phi_U^{(\varpi)}$ upward.

Coefficient on U_{t-1} in the $(\theta, U, \mathcal{C})$ Representation From (B.31), the unemployment loading is

$$\widetilde{\Phi}_U^{(\varpi)} \equiv \Phi_U^{(\varpi)} + \eta_{\theta,U}\Phi_V^{(\varpi)}.$$

Using (B.39) and (B.40):

$$\widetilde{\Phi}_U^{(\varpi)} \to \frac{\kappa}{\psi^w}\left[\left(\eta_{\theta,U} + \frac{U}{1-U}\right)\Lambda_\varpi + \Delta_\varpi\omega_{A,\phi}K_\phi - \varepsilon_R B_U\right]. \tag{B.42}$$

Because $\varepsilon_R B_U = \varepsilon_R \left(1 + \frac{\phi_E}{2-\phi_E}\right) K_\phi$ and $\eta_{\theta,U} + \frac{U}{1-U} = \frac{1-\phi_E}{1-U} = K_\phi$, we can pull out K_ϕ to obtain:

$$\widetilde{\Phi}_U^{(\varpi)} \to \frac{\kappa}{\psi^w} K_\phi \left[\Lambda_\varpi + \Delta_\varpi \omega_{A,\phi} - \varepsilon_R \left(1 + \frac{\phi_E}{2-\phi_E}\right)\right]. \tag{B.43}$$

Start from (B.43) in the limit and set

$$a_\phi \equiv \frac{\phi_E}{2-\phi_E} \in (0,1).$$

Since $\omega_{A,\phi} \to -a_\phi$, we have

$$\widetilde{\Phi}_U^{(\varpi)} \to \frac{\kappa}{\psi^w} K_\phi \left[\Lambda_\varpi - a_\phi \Delta_\varpi - \varepsilon_R(1 + a_\phi)\right]. \tag{B.44}$$

Therefore, we can show that:[6]

$$\widetilde{\Phi}_U^{(\varpi)} \to \frac{\kappa}{\psi^w} K_\phi \mathfrak{m} \left[\chi - S - a_\phi(1+S)\right]. \tag{B.45}$$

Simplified Limit Expression for $\Phi_{\mathcal{C}}^{(\varpi)}$ Using (B.30) with (B.35)–(B.38):

$$\Phi_{\mathcal{C}}^{(\varpi)} \to \frac{\kappa}{\psi^w} \left[\Delta_\varpi \left(g_{S,\mathcal{C}}^{(\varpi)} - g_{R,\mathcal{C}}^{(\varpi)}\right) + \varepsilon_R g_{\varepsilon_R,\mathcal{C}}^{(\varpi)} + \bar{\varepsilon}_S g_{\bar{\varepsilon}_S,\mathcal{C}}^{(\varpi)}\right].$$

Under $s \to 0, \mathcal{C} \to 1$, $\Phi_{\mathcal{C}}^{(\varpi)}$ simplifies as follows: note that this limit implies that $U \to 0$, so $\phi_E \to 1$. Then defining $\mathcal{J} \equiv \frac{\lambda_{EU}}{0.5\lambda_{EEf}} > 0$, the loadings satisfy

$$g_{R,\mathcal{C}}^{(\varpi)} \to 0, \qquad g_{\varepsilon_R,\mathcal{C}}^{(\varpi)} \to 0, \qquad g_{S,\mathcal{C}}^{(\varpi)} \to -\mathcal{J}, \qquad g_{\bar{\varepsilon}_S,\mathcal{C}}^{(\varpi)} \to -\mathcal{J},$$

Hence

$$\Phi_{\mathcal{C}}^{(\varpi)} \to -\frac{\kappa}{\psi^w} \mathcal{J} \left(\Delta_\varpi + \bar{\varepsilon}_S\right).$$

In this limit, $\varepsilon_R \to \bar{\varepsilon}_S \to \mathfrak{m}/2$, so

$$\Delta_\varpi = \bar{\varepsilon}_S + S\mathfrak{m} \to \mathfrak{m}\left(\frac{1}{2} + S\right), \qquad \Delta_\varpi + \bar{\varepsilon}_S \to \mathfrak{m}(1+S).$$

[6] Using $\mathfrak{m} \equiv \varepsilon_R + \bar{\varepsilon}_S$, $\Lambda_\varpi = \varepsilon_R + (\chi - S)\mathfrak{m}$, and $\Delta_\varpi = \bar{\varepsilon}_S + S\mathfrak{m}$, the bracket in (B.44) simplifies as

$$\begin{aligned}\Lambda_\varpi - a_\phi \Delta_\varpi - \varepsilon_R(1+a_\phi) &= [\varepsilon_R + (\chi - S)\mathfrak{m}] - a_\phi[\bar{\varepsilon}_S + S\mathfrak{m}] - \varepsilon_R(1+a_\phi) \\ &= (\chi - S - a_\phi S)\mathfrak{m} - a_\phi(\bar{\varepsilon}_S + \varepsilon_R) = [\chi - S - a_\phi(1+S)]\mathfrak{m}.\end{aligned}$$

Therefore

$$\Phi_{\mathcal{C}}^{(\varpi)} = -\frac{\kappa}{\psi^w}\mathcal{J}\left(\mathfrak{m}(1+S)\right) \tag{B.46}$$

This completes the proof of Proposition 1. We now turn to Proposition 2, which describes the wage Phillips curve in terms of quits:

Proposition 2 (Wage Phillips Curve in $(Q, U, \mathcal{C})$) *For generic $\varpi \in [0,1]$, the log-linearized wage Phillips curve can be written as*

$$\check{\Pi}_t^w = \beta_Q^{(\varpi)}\check{Q}_t + \beta_U^{(\varpi)}\check{U}_{t-1} + \beta_{\mathcal{C}}^{(\varpi)}\check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w,$$

Under the same limit $s \to 0$ and $\mathcal{C} \to 1$, $\beta_Q^{(\varpi)} > 0$, $\beta_U^{(\varpi)}$ is generally ambiguous (and strictly negative when $\chi = 0$), and $\beta_{\mathcal{C}}^{(\varpi)}$ is generally ambiguous.

Proof of Proposition 2. From $Q_t = S_t - s = (1-s)B_t$, we have $\check{Q}_t = \check{B}_t$ and therefore

$$\check{Q}_t = \omega_{B,f}\eta_f\check{\theta}_t + \omega_{B,\mathcal{C}}\check{\mathcal{C}}_t,$$

so that

$$\check{\theta}_t = \frac{1}{\omega_{B,f}\eta_f}\check{Q}_t - \frac{\omega_{B,\mathcal{C}}}{\omega_{B,f}\eta_f}\check{\mathcal{C}}_t.$$

Substituting this into (B.31) gives

$$\check{\Pi}_t^w = \underbrace{\frac{\Phi_V^{(\varpi)}}{\omega_{B,f}\eta_f}}_{\equiv\beta_Q^{(\varpi)}}\check{Q}_t + \underbrace{\left(\Phi_U^{(\varpi)} + \eta_{\theta,U}\Phi_V^{(\varpi)}\right)}_{\equiv\beta_U^{(\varpi)}}\check{U}_{t-1} + \underbrace{\left(\Phi_{\mathcal{C}}^{(\varpi)} - \Phi_V^{(\varpi)}\frac{\omega_{B,\mathcal{C}}}{\omega_{B,f}\eta_f}\right)}_{\equiv\beta_{\mathcal{C}}^{(\varpi)}}\check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w. \tag{B.47}$$

As in the previous cases, we can generally only sign these expressions in the limit case of $s \to 0$ and $\mathcal{C} \to 1$. In that limit, we have $\Phi_V^{(\varpi)} > 0$ and $\Phi_{\mathcal{C}}^{(\varpi)} < 0$. So since $\omega_{B,f}\eta_f \in (0,1)$, the quits coefficient $\beta_Q^{(\varpi)} > \Phi_V^{(\varpi)} > 0$ is positive. However, since $\omega_{B,\mathcal{C}} < 0$, the coefficient on $\check{\mathcal{C}}_t$ is of ambiguous sign.[7]

[7] To see this, use $\omega_{B,\mathcal{C}}/\omega_{B,f} = -\lambda_{EU}\mathcal{C}/(0.5\lambda_{EE}f)$,

$$-\frac{\omega_{B,\mathcal{C}}}{\omega_{B,f}\eta_f} \xrightarrow[s\to 0,\ \mathcal{C}\to 1]{} \frac{\lambda_{EU}}{0.5\lambda_{EE}f\,\eta_f} \equiv \frac{\mathcal{J}}{\eta_f}, \qquad \text{given } \mathcal{J} \equiv \frac{\lambda_{EU}}{0.5\lambda_{EE}f} > 0,$$

and recall that we already showed under this limit that

$$\Phi_{\mathcal{C}}^{(\varpi)} \to -\frac{\kappa}{\psi^w}\mathcal{J}\left(\mathfrak{m}(1+S)\right), \qquad \Phi_V^{(\varpi)} \to \frac{\kappa}{\psi^w}(1+\chi)\mathfrak{m}.$$

Note also that the sign of the unemployment coefficient $\beta_U^{(\varpi)} = \widetilde{\Phi}_U^{(\varpi)}$ is ambiguous without further restrictions on parameters. When $\chi = 0$, we have $\widetilde{\Phi}_U^{(\varpi)} < 0$ in the limit as shown above, so $\beta_U^{(\varpi)}$ is negative in that case.

■

B.3.3 Proof of No Pass-Through Result

Proposition 3 (Employment Stabilization and Wage Inflation) *Consider the log-linear wage Phillips curve (29) under $\varpi = 0$. If monetary policy stabilizes employment at its steady-state level following a shock ($\phi_N \to \infty$) so that $\check{N}_t = 0$ for all t, then it follows that $\check{U}_t = \check{\theta}_t = 0$ for all t, and that wage inflation is stabilized:*

$$\check{\Pi}_t^w = 0 \quad \textit{for all } t.$$

Proof of Proposition 3. From (29), it is sufficient to show that $\check{N}_t = 0$ is consistent with $\check{U}_t = 0$ and $\check{\theta}_t = 0$. Note that holding $N_t = N$ constant implies that the number of searchers $\mathcal{S} = (1-s)\lambda_{EE}N + \lambda_{EU}(1-N)$ is constant; the shares $\phi_{E,t}$ and $\phi_{U,t}$ are thus also constant. This means the tightness term θ_t is constant if and only if V_t is constant.

To prove V_t is constant, i.e., $\check{V}_t = 0$, we first write the law of motion for employment when $N_t = N_{t-1} = N$, plugging in for the separation and recruiting rates (B.5) and (B.4), to yield $V_t \cdot R_t = N \cdot S_t$ that can be written as:

$$V_t g\left(\frac{V_t}{\mathcal{S}}\right)\left(\phi_E \frac{1}{2} + \phi_U\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right) - Nf\left(\frac{V_t}{\mathcal{S}}\right)(1-s)\frac{\lambda_{EE}}{2} = N\left[s + (1-s)\left(\lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right)\right].$$

Now using our definition for g, rewrite the left hand side in terms of f:

$$\mathcal{S} f\left(\frac{V_t}{\mathcal{S}}\right)\left(\phi_E \frac{1}{2} + \phi_U\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right) - Nf\left(\frac{V_t}{\mathcal{S}}\right)(1-s)\frac{\lambda_{EE}}{2} = N\left[s + (1-s)\left(\lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right)\right]$$

Therefore plugging in for $\beta_{\mathcal{C}}^{\varpi}$ yields:

$$\beta_{\mathcal{C}}^{(\varpi)} \to \frac{\kappa}{\psi^w}\mathcal{J}\,\mathfrak{m}\left[\frac{1+\chi}{\eta_f} - (1+S)\right].$$

Since $\eta_f < 1$, we have if $\chi > S$, then $\beta_{\mathcal{C}}^{(\varpi)} > 0$ in this limit. In our calibration with small S and $\eta_f = 1/2$, it is easy to see the coefficient will generally be positive.

leading to

$$f\left(\frac{V_t}{\mathcal{S}}\right) = \frac{N\left[s + (1-s)\left(\lambda_{EU}\left(\frac{1}{1+\xi^\gamma}\right)\right)\right]}{\mathcal{S}\left(\phi_E\frac{1}{2} + \phi_U\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right) - N(1-s)\frac{\lambda_{EE}}{2}}$$

Thus, there is a unique solution for $V_t = V$ for a given N. So we conclude that when monetary policy stabilizes N_t, V_t and θ_t are also jointly stabilized under $\varpi = 0$. ■

B.4 No On-the-Job Search ($\lambda_{EE} = 0$) and Linear Vacancy Costs ($\chi = 0$)

Moreover, under the assumptions of no on-the-job search ($\lambda_{EE} = 0$) and linear vacancy costs ($\chi = 0$), the log-linearized wage Phillips curve admits a further simplification, as stated in the following proposition. We focus on the case of $\varpi = 0$ in (12).

Proposition 4 (No On-the-Job Search and Linear Vacancy Costs) *Suppose $\lambda_{EE} = 0$ and $\chi = 0$. If $\mathcal{C}_t$ is constant (as in the $\varpi = 0$ case), the wage Phillips curve can be written as*

$$\check{\Pi}^w_t = \frac{\kappa}{\psi^w}\left[\Delta_\varpi \eta_g \check{\theta}_t + \Lambda_\varpi \check{T}_t\right] + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}.$$

With the additional limiting assumptions $s \to 0$ and $\mathcal{C} \to 1$, this reduces to

$$\check{\Pi}^w_t = \frac{\kappa}{\psi^w}\gamma\eta_g\,\check{\theta}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}.$$

Hence, in this special case, labor-market tightness is a sufficient statistic for wage inflation, as in Gagliardone and Gertler (2025).

Proof of Proposition 4. Imposing $\lambda_{EE} = 0$ and $\chi = 0$, the search pool is just unemployment:

$$\mathcal{S}_t = U_{t-1}, \qquad \theta_t = \frac{V_t}{U_{t-1}}, \qquad \check{\theta}_t = \check{V}_t - \check{U}_{t-1}, \qquad \eta_{\theta,U} = 1.$$

Also

$$\check{T}_t = \check{V}_t + \frac{U}{1-U}\check{U}_{t-1}.$$

Under $\lambda_{EE} = 0$, the following objects simplify to

$$R_t = g_t\mathcal{C}_t, \qquad S_t = s + (1-s)\lambda_{EU}(1-\mathcal{C}_t), \qquad \varepsilon_{R,t} = \gamma(1-\mathcal{C}_t),$$

$$\bar{\varepsilon}_{S,t} = (1-s)\gamma \frac{\lambda_{EU}\mathcal{C}_t(1-\mathcal{C}_t)}{S_t}.$$

Hence, from the derived loadings in Appendix B.3,

$$g^{(\varpi)}_{R,V} = -\eta_g, \quad g^{(\varpi)}_{R,U} = \eta_g, \quad g^{(\varpi)}_{S,V} = g^{(\varpi)}_{S,U} = 0,$$

$$g^{(\varpi)}_{\varepsilon_R,V} = g^{(\varpi)}_{\varepsilon_R,U} = 0, \quad g^{(\varpi)}_{\bar{\varepsilon}_S,V} = g^{(\varpi)}_{\bar{\varepsilon}_S,U} = 0.$$

Plugging these into (B.28)–(B.29) gives

$$\Phi^{(\varpi)}_V = \frac{\kappa}{\psi^w}\left(\Lambda_\varpi + \Delta_\varpi \eta_g\right), \qquad \Phi^{(\varpi)}_U = \frac{\kappa}{\psi^w}\left(\frac{U}{1-U}\Lambda_\varpi - \Delta_\varpi \eta_g\right). \tag{B.48}$$

Therefore the (V,U) block can be written as

$$\begin{aligned}\Phi^{(\varpi)}_V \check{V}_t + \Phi^{(\varpi)}_U \check{U}_{t-1} &= \frac{\kappa}{\psi^w}\left[\Delta_\varpi \eta_g(\check{V}_t - \check{U}_{t-1}) + \Lambda_\varpi\left(\check{V}_t + \frac{U}{1-U}\check{U}_{t-1}\right)\right] \\ &= \frac{\kappa}{\psi^w}\left[\Delta_\varpi \eta_g \check{\theta}_t + \Lambda_\varpi \check{T}_t\right]. \end{aligned} \tag{B.49}$$

The $\check{\mathcal{C}}_t$ term remains in general:

$$\Phi^{(\varpi)}_{\mathcal{C}} = \frac{\kappa}{\psi^w}\left[\Delta_\varpi\left(g^{(\varpi)}_{S,\mathcal{C}} - g^{(\varpi)}_{R,\mathcal{C}}\right) + \varepsilon_R g^{(\varpi)}_{\varepsilon_R,\mathcal{C}} + \bar{\varepsilon}_S g^{(\varpi)}_{\bar{\varepsilon}_S,\mathcal{C}}\right].$$

Thus, for generic ϖ, no on-the-job search and linear vacancy costs reduce the wage Phillips curve to

$$\check{\Pi}^w_t = \frac{\kappa}{\psi^w}\left[\Delta_\varpi \eta_g \check{\theta}_t + \Lambda_\varpi \check{T}_t\right] + \Phi^{(\varpi)}_{\mathcal{C}}\check{\mathcal{C}}_t + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}. \tag{B.50}$$

If in addition $\check{\mathcal{C}}_t = 0$ in the $\varpi = 0$ case, then

$$\check{\Pi}^w_t = \frac{\kappa}{\psi^w}\left[\Delta_\varpi \eta_g \check{\theta}_t + \Lambda_\varpi \check{T}_t\right] + \frac{1}{1+\rho}\check{\Pi}^w_{t+1}. \tag{B.51}$$

Finally, with the additional limiting assumptions $s \to 0$ and $\mathcal{C} \to 1$ we obtain

$$S \to 0, \qquad \varepsilon_R \to 0, \qquad \bar{\varepsilon}_S \to \gamma,$$

so those results, along with $\chi = 0$, imply:

$$\Delta_\varpi = \bar{\varepsilon}_S + S(\varepsilon_R + \bar{\varepsilon}_S) \to \gamma, \qquad \Lambda_\varpi = \varepsilon_R - S(\varepsilon_R + \bar{\varepsilon}_S) \to 0,$$

and therefore

$$\check{\Pi}_t^w = \frac{\kappa}{\psi^w}\gamma\eta_g\,\check{\theta}_t + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w. \tag{B.52}$$

Thus, in this special case, θ_t is a sufficient statistic for wage inflation, with $\theta_t = \frac{V_t}{U_{t-1}}$ when $\lambda_{EE} = 0$. ■

B.5 Model File and Equilibrium Conditions

We define a symmetric equilibrium as one in which (a) all firms optimize and choose identical values for choice variables, such as $W_{jt} = W_t$, $V_{jt} = V_t$, $P_{y,t}^j = P_{y,t}$, $\bar{X}_t^j = \bar{X}_t$, (b) all workers optimize, (c) households optimize, and (d) all markets clear. This subsection maps objects in `sim_one_sector_supply_shock.mod` to Paper notation and documents the equilibrium conditions used in the code.

In terms of reading the mod file, note two conventions:

- `wbar` and `realLambda` are in units of $P_{y,t}$ (service-price units), not P_t.
- Parameter "c" in the paper is not the same as "`c`" in the code. In the code, paper c is implemented as `c*tau^(-chi)`.

Mapping from Mod-File Names to Objects for Core Endogenous Variables

$$\texttt{wbar} \leftrightarrow \frac{W_t}{P_{y,t}}, \quad \texttt{realLambda} \leftrightarrow \frac{\lambda_t}{P_{y,t}}, \quad \texttt{realwage} \leftrightarrow \frac{W_t}{P_t}, \quad \texttt{realmc} \leftrightarrow \frac{\mu_t}{P_t}.$$

$$\texttt{U} \leftrightarrow U_t,\ \texttt{N} \leftrightarrow N_t,\ \texttt{V} \leftrightarrow V_t,\ \texttt{theta} \leftrightarrow \theta_t,\ \texttt{ftheta} \leftrightarrow f(\theta_t),\ \texttt{gtheta} \leftrightarrow g(\theta_t),$$

$$\texttt{phiE} \leftrightarrow \phi_{E,t},\ \texttt{phiU} \leftrightarrow \phi_{U,t},\ \texttt{R} \leftrightarrow R_t,\ \texttt{S} \leftrightarrow S_t,\ \texttt{eR} \leftrightarrow \varepsilon_{R,t},\ \texttt{eS} \leftrightarrow \varepsilon_{S,t}.$$

$$\texttt{piY} \leftrightarrow \Pi_{y,t},\ \texttt{pi} \leftrightarrow \Pi_t,\ \texttt{piW} \leftrightarrow \Pi_t^w,\ \texttt{irate} \leftrightarrow 1+i_t,\ \texttt{realrate} \leftrightarrow 1+r_t.$$

$$\texttt{Y} \leftrightarrow Y_t,\ \texttt{C} \leftrightarrow C_t,\ \texttt{X} \leftrightarrow X_t,\ \texttt{Xbar} \leftrightarrow \bar{X}_t,\ \texttt{Xs} \leftrightarrow X_t^s,\ \texttt{PXoverPY} \leftrightarrow \frac{P_{x,t}}{P_{y,t}},\ \texttt{PoverPY} \leftrightarrow \frac{P_t}{P_{y,t}}.$$

The code variable `b` corresponds to the unemployment flow payoff in the worker discrete-choice block, expressed in terms of the real wage. `dRw` and `dSw` are the wage-slope terms used in the wage first-order condition implementation (they map to the derivative terms underlying $\varepsilon_{R,t}$ and $\varepsilon_{S,t}$).

For Parameters

`gamma` $\leftrightarrow \gamma$, `rho` $\leftrightarrow \rho$, `varpi` $\leftrightarrow \varpi$, `fixb` $\leftrightarrow b$, `s` $\leftrightarrow s$, `m` $\leftrightarrow m$, `lambdaEE` $\leftrightarrow \lambda_{EE}$,

`lambdaEU` $\leftrightarrow \lambda_{EU}$, `eps` $\leftrightarrow \epsilon$, `chi` $\leftrightarrow \chi$, `psi` $\leftrightarrow \psi$, `psiW` $\leftrightarrow \psi^w$, `wtob` $\leftrightarrow \xi$, `sig` $\leftrightarrow \sigma$, `eta` $\leftrightarrow \eta$,

`etaBar` $\leftrightarrow \bar{\eta}$, `nu` $\leftrightarrow \nu$, `alphaX` $\leftrightarrow \alpha_X$, `alphaY` $\leftrightarrow \alpha_Y$, `alphaXbar` $\leftrightarrow \alpha_{\bar{X}}$, `A` $\leftrightarrow A$, `tau` $\leftrightarrow \tau$.

Unemployment Benefit Regime `mod.1` implements the policy $b_t = \varpi b + (1-\varpi)\xi^{-1}(W_t/P_t)$. In the code, this is written using `realwage`.

Household Optimization `mod.2` is the Euler equation under general CRRA preferences in aggregate form, corresponding to the generalized household block (the log case in (B.2) and surrounding Euler derivation is the $\sigma = 1$ special case).

Firm Optimization `mod.3` is the symmetric-equilibrium wage first-order condition from combining the B.2 wage condition (B.13) with the vacancy condition (B.14), i.e. the same object as (B.17) written in level form with code conventions. For convenience, since we log linearize the model, we drop terms that are zero to first order.

`mod.4` is the vacancy first-order condition in symmetric equilibrium, directly from (B.14) with the code scaling convention for vacancy costs.

`mod.5` is the Rotemberg price Phillips condition based on (B.10)–(B.11) in symmetric equilibrium.

`mod.6` is the employment-envelope condition from (B.12), rewritten with `realmc` and the product-price unit conventions.

`mod.7` is the intermediate-input first-order condition from (B.15) in ratio form, i.e. a condition for $P_{x,t}/P_{y,t}$.

Labor Market Equilibrium `mod.8` is accounting identity $N_t + U_t = 1$.

`mod.9` is the tightness definition $\theta_t = V_t/\mathcal{S}_t$ with $\mathcal{S}_t = (1-s)\lambda_{EE}(1-U_{t-1}) + U_{t-1}$.

`mod.10` and `mod.11` are the Cobb-Douglas matching-rate formulas $f(\theta_t) = m\theta_t^{\nu}$ and $g(\theta_t) = m\theta_t^{\nu-1}$.

`mod.12` and `mod.13` are the search-composition shares $\phi_{E,t}$ and $\phi_{U,t} = 1 - \phi_{E,t}$.

`mod.14` is the recruiting rate R_t from (B.4) evaluated under symmetry, using the logit acceptance probabilities in (18)–(19).

`mod.15` is the separation rate S_t from (B.5) under symmetry, using (15) and (16).

`mod.16` is the employment law of motion $N_t = (1 - S_t)N_{t-1} + R_t V_t$ from (B.7).

`mod.17` and `mod.18` are the wage-slope objects entering the wage first-order condition, consistent with the derivative structure in the derivation for recruiting and separation elasticities.

`mod.19` and `mod.20` define elasticities as $eR = dRw/R$ and $eS = dSw/S$, matching the elasticity definitions.

Goods Market Equilibrium `mod.21` is the symmetric production relation corresponding to (5); with the CES powers on both N_t and $\bar{X}_t$, it now matches the generalized production block.

`mod.22` is the CES final-goods aggregator for C_t from (2).

`mod.23` is the endowment resource constraint $X_t + \bar{X}_t = X_t^s$.

Monetary Policy and Inflation Block `mod.24` is the Fisher equation $(1 + r_t) = (1 + i_t)/\Pi_{t+1}$.

`mod.25` Specifies monetary policy, assuming the nominal rate i_t follows a Taylor rule targeting service inflation and employment.

`mod.26` is aggregate inflation implied by the price aggregator (3) (a gross-inflation ratio).

`mod.27` is wage inflation: $\Pi_t^w = W_t/W_{t-1}$, implemented using `wbar` and $\Pi_{y,t}$.

`mod.28` is the relative-price relation $P_{x,t}/P_{y,t}$ implied by CES intratemporal allocation from (2)–(3).

`mod.29` is the level relation $P_t/P_{y,t}$ from (3).

B.6 Denomination of Various Costs and No Pass-Through

This section analyzes the baseline model in Section 3 to elaborate the conditions under which there is no pass-through from prices to wages. In our baseline model with $\varpi = 0$, there is no pass-through from prices to wages *regardless of the level of* η as shown in Proposition 3: in response to an X_t^s shock, when monetary policy perfectly stabilizes N_t, it also perfectly stabilizes wage inflation.

We show that this result depends in part on the assumption that vacancy costs are denominated in labor. If vacancy costs are instead denominated in final goods, headline inflation is transmitted to wages even when monetary policy fully stabilizes the labor market. The title

of this section emphasizes that the analysis does not hinge on whether prices or wages are sticky; rather, what matters is that nominal rigidities enable monetary authorities to stabilize employment, N_t.

To demonstrate the role of the denomination of adjustment costs, we generalize the firms problem slightly: let P_t^V denote the nominal price in which vacancy costs are denominated, and let P_t^ψ be the nominal price in which wage adjustment costs are denominated (which we will show will not matter). Then firm j maximizes present-discounted revenues, less costs (abstracting from price adjustment costs and the use of X as an intermediate in firms' production, which do not affect the wage Phillips curve), given by

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(P_{y,t}^j Y_t^j - W_{jt}N_t^j - c(V_t^j)^{1+\chi}(N_{t-1}^j)^{-\chi}P_t^V - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)^2 N_t P_t^\psi\right),$$

subject to the law of motion for employment,

$$N_t^j = (1-S_t(W_{jt}))N_{t-1}^j + V_t^j R_t(W_{jt})$$

and production and demand functions for Y_t^j. Combining the firm's first-order conditions for V_t^j and W_{jt}, and assuming a symmetric equilibrium, yields a nonlinear wage Phillips curve given by

$$\begin{aligned}\psi^w\left(\Pi_t^w-1\right)\Pi_t^w P_t^\psi + W_t =& P_t^V c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^\chi\left(\frac{V_t}{N_t}\varepsilon_{R,W_t} - \frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\ &+\frac{\psi^w}{1+\rho}\left(\Pi_{t+1}^w-1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}P_{t+1}^\psi.\end{aligned}$$

Gather the labor market tightness terms in $Z_t \equiv c(1+\chi)\left(\frac{V_t}{N_{t-1}}\right)^\chi\left(\frac{V_t}{N_t}\varepsilon_{R,W_t} - \frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)$ and log-linearize, defining $\pi_t^V \equiv \frac{P_t^V}{P_{t-1}^V}$, let $\check{\omega}_t \equiv \sum_{s=0}^t(\pi_t^w - \pi_t^V)$, we can obtain

$$\check{\pi}_t^w = \frac{ZP^V}{\psi^w P^\psi}\sum_{s=0}^{\infty}\left(\frac{1}{1+\rho}\right)^s\left(\check{Z}_{t+s} - \check{\omega}_{t+s}\right).$$

As shown in the proof of Proposition 3 under $\varpi = 0$, when monetary policy stabilizes N, i.e., $\check{N}_t = 0$, it stabilizes other labor market variables such as θ (or V) and elasticities. It then

follow that $\check{Z}_t = 0$, so the wage Phillips curve reduces to:

$$\check{\pi}_t^w = \frac{Z P^V}{\psi^w P^\psi} \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^s (-\check{\omega}_{t+s}) .$$

If the cost of posting vacancies is denominated in labor, so $P_t^V = W_t$, then $-\check{\omega}_t = 0$ and monetary policy stabilizes wage growth as well as employment.

B.7 Comparative Statics for Weights on (V, U)

This section illustrates that, under $\varpi = 0$ (which implies $\check{\mathcal{C}}_t = 0$) in (12), the log-linearized wage Phillips curve in (B.27), expressed in terms of vacancies and unemployment (V, U), places greater weight on vacancies than on unemployment under our calibration.

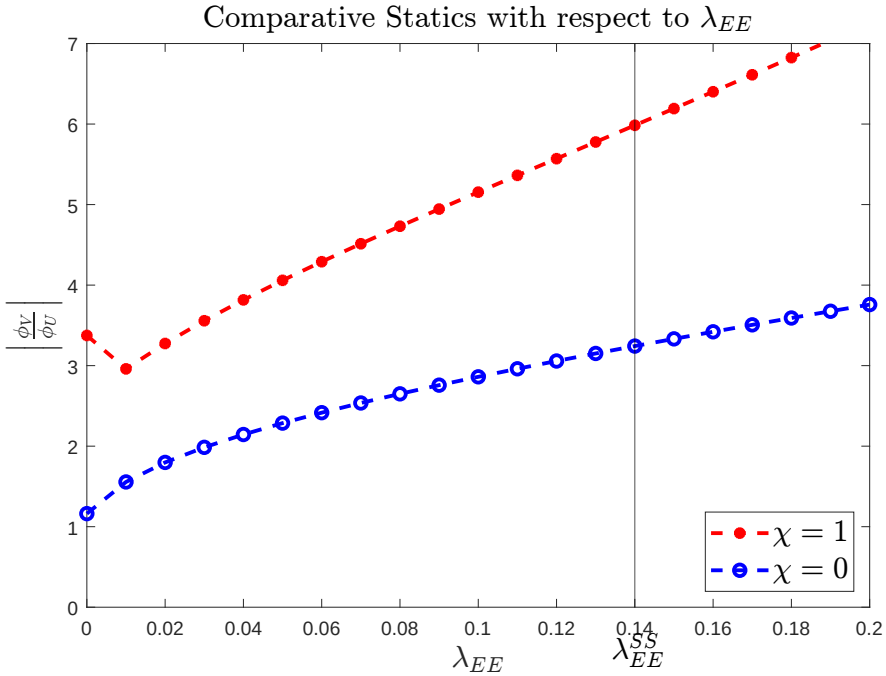
$$\check{\Pi}_t^w = \Phi_V \check{V}_t + \Phi_U \check{U}_{t-1} + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w \tag{B.53}$$

for coefficients $\Phi_V > 0$ and $\Phi_U < 0$ provided in Appendix B.3. Based on our calibration in Table 2, we now illustrate how varying the probability of being allowed to search on the job, λ_{EE}, affects the predictions of the model about the relative importance of vacancies, as opposed to unemployment, in the wage Phillips curve (B.53).

Since both $\Phi_V > 0$ and $\Phi_U < 0$ are complex collections of model parameters and steady-state values, to consider their relative magnitudes, we proceed numerically, and specialize to particular parameter choices. Figure B.4 shows how the relative importance of vacancies in explaining wage growth, represented by the ratio of coefficients in (B.53), $\left| \frac{\Phi_V}{\phi_U} \right|$, increases monotonically in on-the-job search intensity λ_{EE} under the benchmark calibration $\chi = 1$ and also when $\chi = 0$, or a linear cost of posting a vacancy which is commonly assumed in the search literature. The limit case where $\chi = 0$ and $\lambda_{EE} \to 0$ is of particular interest as a benchmark: as Proposition 4 shows, at the limit where $\lambda_{EE} \to 0$ and $\chi \to 0$, $\left| \frac{\Phi_V}{\Phi_U} \right|$ converges to one and wage growth becomes solely a function of market tightness $\theta_t = \frac{V_t}{U_{t-1}}$ in line with the literature (Gagliardone and Gertler, 2025).

We acknowledge that simply pointing out the coefficient on V is larger than on U does not technically imply that variations in U are less important in explaining wage growth: if U has a much higher variance than V, it can have a small coefficient while still playing a large role. To show more formally how rising $\left| \frac{\Phi_V}{\Phi_U} \right|$ diminishes the importance of unemployment in explaining wage growth, consider the variance decomposition of wage growth in the model

Figure B.4: In Economies with More On-the-job Search, Vacancies Matter More in the Wage Phillips Curve



Notes: The red, starred line plots the effects of a change in on-the-job search intensity λ_{EE}, holding all other model parameters constant at their values in Table 2, on the ratio of the coefficients in equation (B.53): $\check{\Pi}_t^w = \Phi_V \check{V}_t + \Phi_U \check{U}_{t-1} + \frac{1}{1+\rho}\check{\Pi}_{t+1}^w$. The blue dotted line repeats the exercise but with $\chi = 0$, a linear cost of vacancy posting. The vertical line marks the value for λ_{EE} used in our benchmark calibration. The relative importance of vacancies in explaining wage inflation, compared with unemployment, increases with both λ_{EE} and χ.

under the assumption that we can ignore the inflation expectations term:[8]

$$\text{Var}\left(\check{\Pi}_t^w\right) \propto \left(\frac{\Phi_V}{\Phi_U}\right)^2 \text{Var}\left(\check{V}_t\right) + \text{Var}\left(\check{U}_{t-1}\right) + \underbrace{2\frac{\Phi_V}{\Phi_U}}_{<0} \text{Cov}\left(\check{V}_t, \check{U}_{t-1}\right) \tag{B.54}$$

Now, consider Figure B.4, which increases λ_{EE} holding other parameters constant, raising $\left|\frac{\Phi_V}{\Phi_U}\right|$. Given that the covariance term $\text{Cov}\left(\check{V}_t, \check{U}_{t-1}\right)$ in (B.54) is strongly negative (both empirically, and also in any reasonably calibrated model), the importance of unemployment in

[8]For example, assuming firms have constant inflation expectations, $\mathbf{E}_t\Pi_{t+1}^w = \Pi^w$, or "adaptive" expectations $\mathbf{E}_t\Pi_{t+1}^w = \Pi_t^w$. Alternatively, we might view (B.54) as an approximation when ρ is high, permitting us to ignore the many covariance cross-terms complicating the expression when solved forward.

explaining wage growth falls monotonically as we increase the amount of on-the-job search, and convexity of the vacancy costs, in the model.

Appendix C The Union Wage Setting Model

This appendix lays out the standard union model following Erceg et al. (2000) used in Section 5's quantitative exercises. We define the new model parameters listed in Table C.1; all other model parameters are the same as those in our baseline model and calibrated to the same values as in Table 2.

In this appendix, we also demonstrate analytically how our framework with wage posting and on-the-job search differs from this model. In both the union model and our wage-posting framework, the pass-through of endowment shocks X ultimately depends on labor supply responses, reflecting how such shocks alter the relative desirability of work and non-work

However, a key distinction lies in the role of non-labor income. In union wage-setting models, the degree of pass-through depends on the amount of non-labor income generated by the endowment good shock X, which in turn depends on the value of η, the elasticity of substitution between X and Y in equation (2). If the shock generates substantial non-labor income, the resulting *wealth effect* reduces households' labor supply, placing upward pressure on nominal wage growth. In contrast, in our model with firm-wage setting and on-the-job search, unemployment is rarely an attractive outside option. As a result, wages are determined primarily by firms' competition for already employed job seekers, and changes in the relative desirability of employment versus unemployment have only a limited effect on wage determination.

For simplicity, we formally prove the relationship between η and pass-through for a special case of the union model with flexible prices and zero profits, and where $\bar{X}_t \equiv 0$, or equivalently $\alpha_{\bar{X}} = 0$ in (5), so that X is not used as an input in service-sector production. The results are gathered in Proposition 5 below.

C.1 Pass-Through with Differentiated Labor and Union Wage Setting

The union model replaces the frictional labor market with differentiated labor types and union wage setting following Erceg, Henderson and Levin (2000), but assuming Rotemberg (1982)-style convex adjustment costs following Auclert et al. (2024).[9] The representative household/union chooses consumption, bonds, and wages for labor types to maximize

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[\ln C_t - \vartheta_t \int_0^1 \frac{N_t(i)^{1+\phi}}{1+\phi}\, di - \frac{\psi^w}{2} \int_0^1 \left(\frac{W_t(i)}{W_{t-1}(i)} - 1\right)^2 di\right], \tag{C.1}$$

[9] To a first order approximation, the wage Phillips curve is the same as that in Erceg et al. (2000).

subject to the budget constraint

$$C_t + \frac{B_t}{P_t} = \frac{D_t}{P_t} + \int_0^1 \frac{W_t(i)}{P_t} N_t(i)\, di + \frac{(1 + i_{t-1,t}) B_{t-1}}{P_t},$$

and labor demand for each type, where consumption C_t remains a CES aggregate of X and services output Y given in (2), with an elasticity of substitution η. A labor packer aggregates labor types according to

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \qquad W_t = \left(\int_0^1 W_t(i)^{1-\epsilon_w} di \right)^{\frac{1}{1-\epsilon_w}},$$

implying labor demand $N_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_w} N_t$. Each union chooses the nominal wage for its labor type subject to wage adjustment costs à la Rotemberg (1982); firms hire labor in a spot market and choose prices, labor, and intermediate inputs facing CES demand (4) and production function (5) to maximize profits.[10]

The symmetric-equilibrium marginal rate of substitution is $MRS_t = \vartheta_t C_t N_t^{\phi}$, and in steady state the union sets

$$\frac{W}{P} = \frac{\epsilon_w}{\epsilon_w - 1} MRS.$$

This setup with convex adjustment costs instead of staggered wage setting yields the standard log-linearized wage Phillips curve of Erceg et al. (2000), in which the slope is governed by the curvature of the labor disutility function, ϕ, and the wage adjustment cost parameter, ψ^w:

$$\check{\Pi}_t^w = \frac{\epsilon_w}{\psi^w} \frac{N\, MRS}{C} \left(\check{MRS}_t - \check{\tilde{w}}_t \right) + \frac{1}{1+\rho} \check{\Pi}_{t+1}^w, \tag{C.2}$$

[10] Formally, firm j chooses $\{P_{y,t}^j\}, \{Y_t^j\}, \{N_{jt}\}, \{\bar{X}_t^j\}$ to maximize:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \left(P_{y,t}^j Y_t^j - W_{jt} N_{jt} - \frac{\psi}{2} \left(\frac{P_{y,t}^j}{P_{y,t-1}^j} - 1 \right)^2 Y_t^j P_{y,t}^j - P_{x,t} \bar{X}_t^j \right)$$

subject to

$$Y_t^j = A_t^j \left(\alpha_N^{\frac{1}{\bar{\eta}}} \left(N_{jt} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\bar{X}_t^j \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \right)^{\frac{\bar{\eta}}{\bar{\eta}-1}},$$

$$\frac{Y_t^j}{Y_t} = \left(\frac{P_{y,t}^j}{P_{y,t}} \right)^{-\epsilon}.$$

where $\check{MRS}_t = \check{\chi}_t + \check{C}_t + \phi\check{N}_t$ and $\check{\tilde{w}}_t = \check{W}_t - \check{P}_t$. Here, we can observe that, similar to the firm wage-setting model with inflation-indexed unemployment benefits—that is, the case of $\varpi = 1$ in (12)—there exists a potential mechanism for wage–price spiral dynamics. When prices rise, the real wage declines, creating upward pressure on nominal wages as unions attempt to restore wages toward their desired target level.

Indeed, we can show that, when the central bank fully stabilizes employment ($\phi_N \to \infty$), so that $N_t = N$, following a negative shock to the endowment good X_0 at time $t = 0$, wages increase only when $\eta < 1$, that is, when the elasticity of substitution between X_t and Y_t is sufficiently low. This result is summarized in the following proposition: for simplicity, we assume perfect competition, flexible price-setting, and $\bar{\alpha}_X = 0$ in this proposition (departing from the calibration used in the main text's quantitative exercises).

Proposition 5 (Employment Stabilization and Wage Inflation (Union Wage-Setting Models)) *Consider the log-linear wage Phillips curve (C.2) in the union wage-setting frameworks. Assume prices are flexible and equal to marginal cost ($\psi = 0; \epsilon \to \infty$) and that X is not used as an intermediate ($\bar{\alpha}_x = 0$). If monetary policy stabilizes employment at its steady-state level following a shock ($\phi_N \to \infty$)—satisfying $\check{N}_t = 0$—then wages increase in response to a negative X_0 shock:*

$$\check{\Pi}^w_0 > 0,$$

if $\eta < 1$. Thus, even when employment is perfectly stabilized, a negative endowment shock generates positive wage inflation if the elasticity of substitution, η, is sufficiently low.

This result contrasts with Proposition 3, where wage inflation remains stabilized regardless of the value of η. Note that Proposition 3's no pass-through result also did not rest on any particular assumptions for $\bar{\alpha}_x, \epsilon$, or ψ. We make these simplifying assumptions here because our goal is to illustrate the presence of wealth effects in the union model.[11]

The proof is as follows:

Proof. The log-linearized wage Phillips curve in (C.2) can be expressed as follows:

$$\check{\Pi}^w_t = \beta\mathbb{E}_t\{\check{\Pi}^w_{t+1}\} + \lambda\left(-\widetilde{W}_t + \widetilde{P_tC_t} + \phi\check{N}_t\right).$$

for $\lambda = \frac{\epsilon_w}{\psi^w}\frac{N\,MRS}{C} > 0$. To make progress, we rewrite the equation in terms of relative prices:

[11] The assumption of flexible prices is not substantive: in an earlier draft, we showed that a model with neoclassical labor supply and sticky prices also demonstrates pass-through to wages in the case where $\eta < 1$.

$$\check{\Pi}_t^w = \beta \mathbb{E}_t\{\check{\Pi}_{t+1}^w\} + \lambda \left(-\widetilde{\frac{W_t}{P_{y,t}}} + \widetilde{\frac{P_t C_t}{P_{y,t}}} + \phi \check{N}_t \right). \tag{C.3}$$

Consider the household's budget constraint in equilibrium. Using the fact that bonds are in zero net supply and expanding the dividend term D_t by letting d_t denote dividends paid by service-sector firms—which will be zero given our assumption that service prices are flexible and firms are perfectly competitive—the budget constraint can be written as follows:

$$P_t C_t = W_t N_t + P_{x,t} X_t + d_t.$$

How will each term in this equation respond to an X_t shock? Rewriting, we have:

$$\frac{P_t C_t}{P_{y,t}} = \frac{W_t}{P_{y,t}} N_t + \frac{P_{x,t}}{P_{y,t}} X_t + \frac{d_t}{P_{y,t}}. \tag{C.4}$$

First note that CES demand implies

$$\frac{P_t}{P_{y,t}} = \left(\frac{Y_t}{C_t} \right)^{\frac{1}{\eta}} \alpha_y^{-\frac{1}{\eta}}.$$

Under the case where monetary policy stabilizes N_t, and hence Y_t, we have up to first order that $\check{C}_t = \alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C}\right)^{\frac{\eta-1}{\eta}} \check{X}_t$ and

$$\widetilde{\frac{P_t}{P_{y,t}}} = -\frac{1}{\eta} \left(\alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C} \right)^{\frac{\eta-1}{\eta}} \check{X}_t \right), \tag{C.5}$$

so that when X_t declines, the price of aggregate consumption relative to the labor-intensive service good, $\frac{P_t}{P_{Y,t}}$, increases. In other words, purchasing one unit of the consumption bundle requires more units of the Y-good. We also obtain the following expression for aggregate spending:

$$\widetilde{\frac{P_t C_t}{P_{y,t}}} = \frac{\eta - 1}{\eta} \left(\alpha_x^{\frac{1}{\eta}} \left(\frac{X}{C} \right)^{\frac{\eta-1}{\eta}} \check{X}_t \right).$$

Note that aggregate nominal spending may either increase or decrease following a decline in X_t, depending on η. Under Cobb–Douglas preferences ($\eta = 1$), aggregate nominal spending remains unchanged. If $\eta < 1$, then $\frac{P_0 \check{C}_0}{P_{y,0}}$ rises in response to a negative X_0 shock and $\frac{P_t \check{C}_t}{P_{y,t}}$ is zero in other periods ($t > 0$) when there is no shock. From (C.4), we see the middle term, $\frac{P_{x,t}}{P_{y,t}} X_t$, in its deviation from steady state, is zero when there is no shock. Thus, we obtain for

all $t > 0$:

$$0 = \frac{WN}{PC}\frac{\widetilde{W_t}}{P_{y,t}} + \frac{d}{PC}\check{d}_t.$$

Under the assumption of flexible prices and zero profits ($\psi = 0, \epsilon \to \infty$) then we have that $\check{d}_t = 0$ and thus $\frac{\widetilde{W_t}}{P_{y,t}} = 0$. As a result, the forward looking wage Phillips curve (C.3) implies $\check{\Pi}^w_t = 0$ for all $t > 0$, and the wage Phillips curve for the initial period greatly simplifies to

$$\check{\Pi}^w_0 = \lambda\left(-\widetilde{W_0} + \widetilde{P_0 C_0}\right)$$

Given that wage inflation is defined as $\check{\Pi}^w_t = \widetilde{W_t} - \widetilde{W}_{t-1}$ with $\widetilde{W}_{-1} = 0$, we can write

$$\widetilde{W_0} = \lambda(-\widetilde{W_0} + \widetilde{P_0 C_0})$$

Divide by $P_{y,0}$ to apply our above results for $\frac{P_0 \check{C}_0}{P_{y,0}}$ and find that when $\eta < 1$, the right hand side is positive for a negative X shock, and we thus have positive pass-through to wages, i.e., $\check{\Pi}^w_0 > 0$. This completes the proof of Proposition 4.

■

Discussion: The sign and magnitude of the response in wages depends on the strength of *income* and *substitution* effects, governed by the elasticity of intertemporal substitution of consumption utility, and *wealth* effects when workers are endowed both with leisure and the good X_t. Because utility of consumption is log, income and substitution effects on labor supply from a lower real wage cancel out. Therefore wealth effects will determine the labor supply response of households.

The direction of those wealth effects are governed by η, the degree of substitution between X_t and Y_t. When $\eta < 1$, a negative endowment shock in X^s_t generates positive wealth effects, as prices for the endowment good rise more than one-for-one with the decline in quantity. This makes the household want to supply less labor and increase leisure, so firms must raise wages if N_t is to be stabilized at its pre-shock level.

In contrast, Proposition 3 effectively shuts down the wealth-effect channel by construction. By fixing the consumption ratio between employed and unemployed household members ($\varpi = 0$), workers' labor supply incentives remain unchanged following the shock. As a result, there is no pass-through from the cost-of-living shock to wages, regardless of the value of η.

C.2 Union Model Calibration

As Section 5 shows, the union wage-setting model generates substantially greater amplification of both wage and price inflation than our model under the baseline calibration and in the more general specification where X serves as an input into service-sector production and we allow for nominal rigidities in price-setting as well as wage-setting. We use the following parameters for our union wage-setting model, given in Table C.1. While we assume convex adjustment costs, rather than Calvo-type price rigidities, we calibrate the model so that the slope of the wage Phillips curve is identical to that used in Erceg et al. (2000).

Table C.1: Additional Parameters in the Union Wage-Setting Model

Parameter	Value	Meaning	Reason
ϕ	1	Curvature of labor disutility	Unit Frisch elasticity, following Christiano et al. (2005)
ϵ_w	21	Elasticity of substitution across labor types	5% steady-state wage markup, following Christiano et al. (2005)
ϑ_t	.9036	Scales disutility of labor	Chosen to target the same steady-state employment N as the baseline model
ψ^w	5750	Adjustment cost	Targets the implied monthly wage Phillips curve slope from Erceg et al. (2000) of 0.003

Notes: All other parameters are set equal to the baseline model.

Appendix D Equilibrium Stability

In this section, we describe parameters in the model that ensure equilibrium stability. There are three parameters that matter the most: the inverse scale parameter of the workers' idiosyncratic preferences over jobs γ, the frequency that workers consider quitting into unemployment λ_{EU}, and wage rigidity ψ^w. In general, high variance of workers' preferences (low γ) gives firms more wage setting power and encourages stability and prevents firms from deviating from the equilibrium wage to separately target unemployed workers. High values of λ_{EU} discourages firms from paying a wage of 0 to employed workers who have no outside options in a given period. High wage rigidity ψ^w prevents firms from oscillating between wage strategies.

The equilibrium in our baseline calibration ($\gamma = 6$, $\lambda_{EU} = 0.15$) is stable, even without price or wage rigidity. We show how the equilibrium breaks down for high values of γ ($\gamma = 12$) or low values of λ_{EU} ($\lambda_{EU} = 0.01$) in the absence of wage rigidity. We show that our baseline value of wage rigidity $\psi^w = 9000$ restores equilibrium stability, even for these alternative parameter values.

D.1 Baseline Calibration

In this section, we demonstrate that under our baseline calibration, the proposed symmetric equilibrium is an equilibrium. In particular, we show that both a sufficiently large variance of idiosyncratic preference shocks and a sufficiently high probability of quitting into unemployment are crucial for maintaining the stability of this equilibrium.

We begin by solving for the candidate steady-state symmetric equilibrium, in which all firms adopt identical wage policies and all first-order conditions and market-clearing constraints are satisfied. To do so, we solve the firm's optimization problem using value function iteration, allowing us to evaluate the desirability of different wage and vacancy strategies. We initially assume that there are no price and wage rigidities at the firm level, as these do not influence firm decisions in a non-stochastic steady state. We set $\gamma = 6$ and $\lambda_{EU} = 0.15$ as in our standard calibration (Table D.2).

Parameter	Value
γ	6
λ_{EU}	.15

Table D.2: Baseline Calibration

Figure D.5 illustrates a firm's value function in period t for different wage choices in the same period, with the number of vacancies optimally determined conditional on the firm's chosen wage, while all other firms maintain their steady-state wage and vacancy levels. Under the calibration in Table D.2, note that it is indeed *globally* optimal for an individual firm to choose the steady-state wage level, denoted in the figures below as $\bar{w}$.

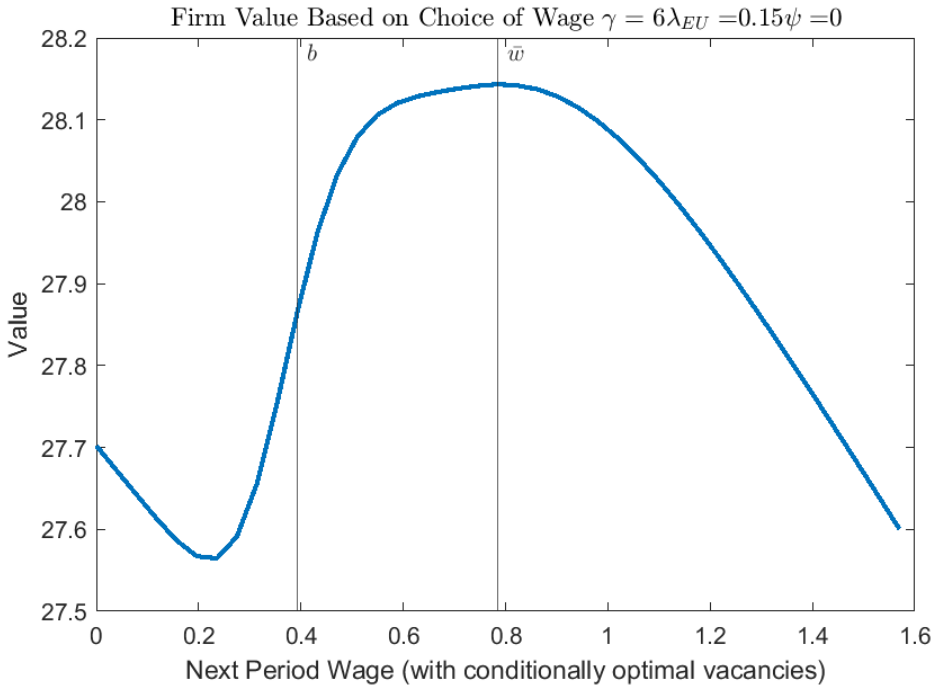


Figure D.5: Individual Firm Value as a Function of Wage Choices ($\gamma = 6$)

Figure D.6 presents the recruiting and separation functions as functions of the current wage, assuming that all other firms maintain their steady-state wage and vacancy levels. As discussed in the main text, both functions are smooth and monotonic.

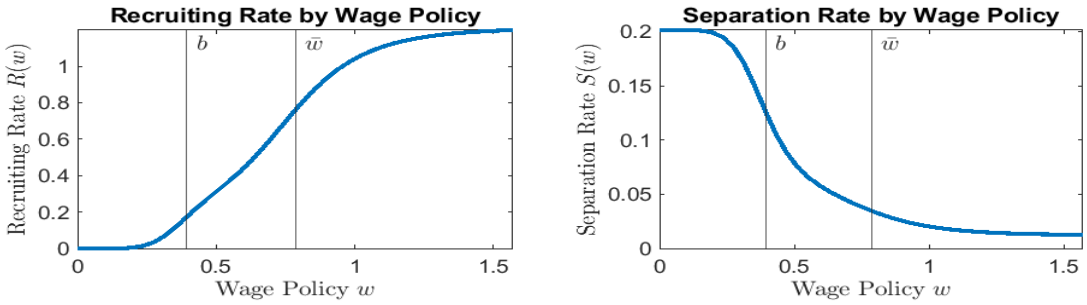


Figure D.6: Recruiting and Separation Rates as a Function of Wage Choices ($\gamma = 6$)

Equilibrium Stability Figure D.7 presents the optimal level of employment N_t for the next period, given the previous period's employment N_{t-1}, which may deviate from its steady-state level.[12] When N_{t-1} is below (above) $\bar{N}$, the steady-state level of employment, employment increases (decreases) over time until it converges to $\bar{N}$. This pattern indicates that the candidate steady-state equilibrium is indeed stable.

Figure D.7: Employment Dynamics for an Individual Firm under $\gamma = 6$

Figure D.8 depicts the optimal policy functions as a function of last period's employment, N_{t-1}. It supports the stability interpretation: a larger firm (with higher N_{t-1}) sets a lower wage W_t and vacancy rate $\frac{V_t}{N_{t-1}}$, indicating that larger (smaller) firms reduce (increase) their size until converging to the steady-state level.

[12] Note that all other firms maintain their steady-state wage, vacancy, and employment levels.

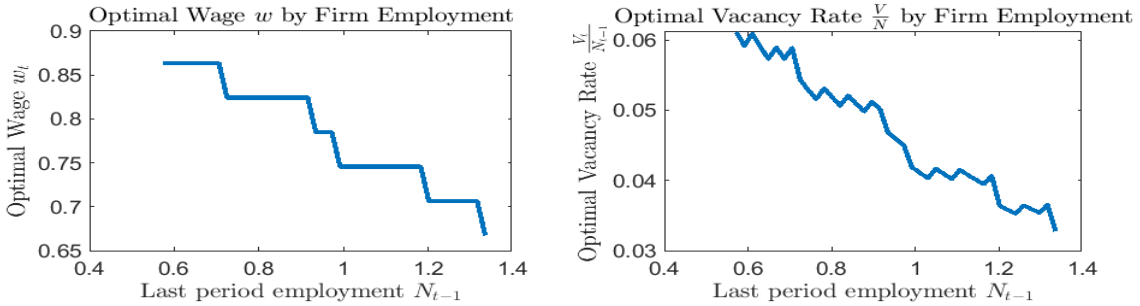


Figure D.8: Wage and Vacancy Rate Policies as Functions of Employment N_{t-1} ($\gamma = 6$)

D.2 Low Variance of Idiosyncratic Preference Draws

We now consider a lower variance of idiosyncratic preference draws γ^{-1}, by setting a higher value of $\gamma = 12$. This increases workers' wage sensitivity, meaning that smaller differences in wages have a larger impact on their mobility choices.

Figure D.9 shows that the recruiting and separation rates become more "stair-stepped." For example, a wage slightly above the unemployment benefit b attracts nearly all unemployed workers, while almost no employed workers will accept such a wage. These shapes of the recruiting and separation functions influence an individual firm's incentives when setting its wage and vacancy policies. For example, a firm may choose to hire only matched unemployed workers—rather than those engaged in on-the-job search—by posting a wage slightly above b, knowing that unemployed workers will accept. By doing so, the firm can reduce its wage bill and optimally downsize.

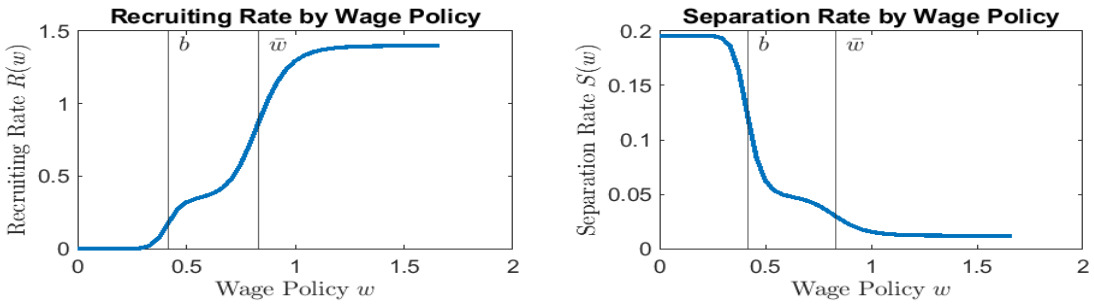


Figure D.9: Recruiting and Separation Rates as a Function of Wage Choices ($\gamma = 12$)

Oscillatory Behavior In this case, the firm's optimal strategy exhibits oscillatory behavior: in one period it offers high wages and posts many vacancies to attract workers, and in the next period it sets a wage above b but below the market average. Then, the firm's size (i.e.,

employment) also oscillates accordingly. Indeed, Figure D.10 shows that the firm's value function, as a function of the current wage, has two global maxima: one slightly above b and another slightly above $\bar{W}$. Given that all other firms maintain their steady-state wage, vacancy, and employment levels, the firm is indifferent between these two wage options and thus optimally oscillates between them, generating wage dispersion. This finding is related to the wage dispersion result in Burdett and Mortensen (1998): as workers care less about idiosyncratic shocks and care only about wages, small differences in wages can create large changes in job acceptance of workers. Combined with our simplifying assumption of worker myopia, this creates a perverse outcome where firms can take advantage of myopic workers who do not have an outside employment offer every period, while periodically attracting unemployed workers at low wages.

Figure D.10: Individual Firm Value as a Function of Wage Choices ($\gamma = 12$)

Figure D.11 illustrates the firm's optimal employment N_t, given the previous period's employment N_{t-1}, which may differ from its steady-state level.[13] In contrast to the standard calibration case in Figure D.7, with a lower variance of idiosyncratic preference draws ($\gamma = 12$), the firm's size oscillates continuously, alternating between high and low employment levels around the steady state.

[13]Note that all other firms maintain their steady-state wage, vacancy, and employment levels.

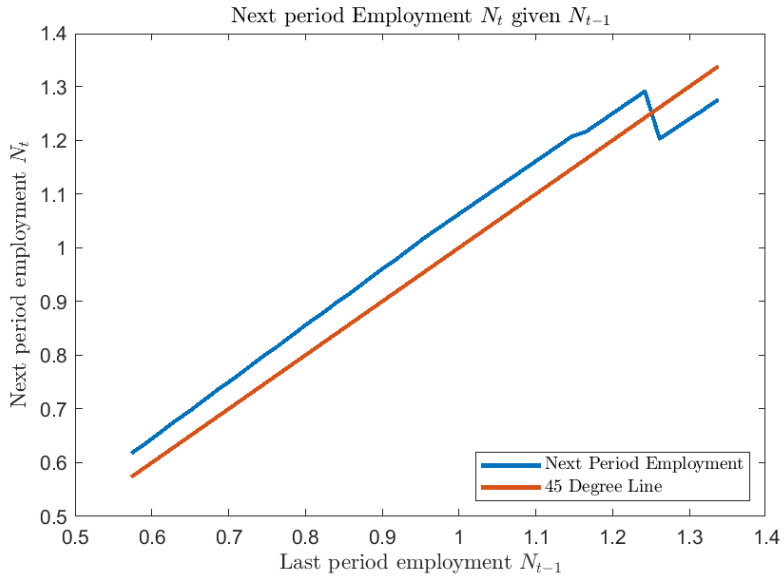


Figure D.11: Employment Dynamics for an Individual Firm under $\gamma = 12$

Figure D.12 depicts the optimal policy functions as a function of last period's employment, N_{t-1}. Notice the significant discontinuity in the optimal wage and vacancy rate policies around the steady state, which drives the oscillatory behavior: a large firm (with higher N_{t-1}) in period t sets a low wage just above b and a low vacancy rate, reducing its size in the following period. In period $t+1$, the downsized firm then sets high wages and vacancy rates, increasing its size for period $t+2$.

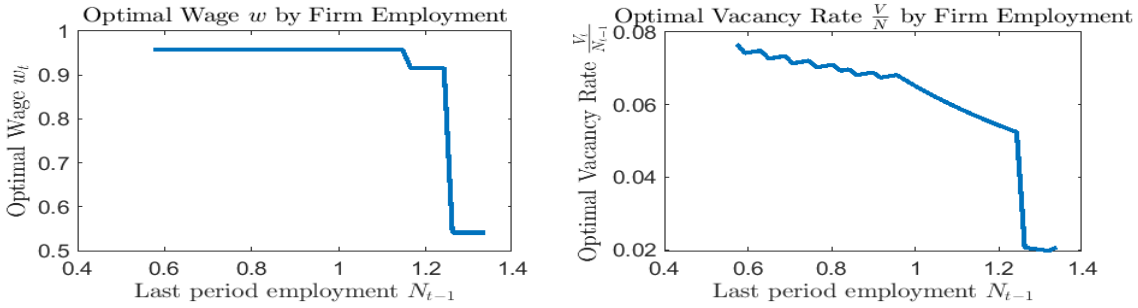


Figure D.12: Wage and Vacancy Rate Policies as Functions of Employment N_{t-1} ($\gamma = 12$)

Interpretation Therefore, we conclude that our single-wage symmetric equilibrium remains compatible with on-the-job search, provided that idiosyncratic, worker-specific preference shocks over workplaces have sufficiently high variance, which corresponds to a low enough γ.

Note that even though we abstract from nominal wage and price rigidities in this section, incorporating these frictions would only reduce the firm's profitability of deviating from the single-wage steady state and adopting the oscillatory policy.

D.3 Low λ_{EU}, i.e., Few Opportunities to Quit into Unemployment

This section considers a setting with more limited opportunities of workers to quit into unemployment by setting $\lambda_{EU} = 0.01$ instead of 0.15, while restoring the baseline value of $\gamma = 6$ for the inverse scale parameter of preference shocks.

Figure D.13 shows that the recruiting and separation functions remain smooth, but the level of the separation function is significantly lower than the baseline case in Figure D.6, particularly in the region where wages are low. As a result, even when firms offer very low wages, most workers remain with their employers—effectively *trapped*, since only a small fraction receive outside job offers each period. This creates an incentive for firms to occasionally set very low wages.

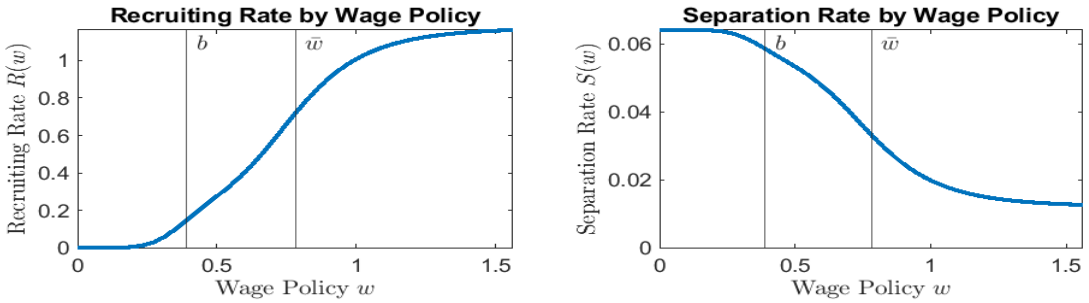


Figure D.13: Recruiting and Separation Rates as a Function of Wage Choices ($\lambda_{EU} = 0.01$)

Oscillatory Behavior In this case, the firm's optimal strategy exhibits oscillatory behavior, similar to the high-γ scenario: in one period, it offers high wages and posts many vacancies to attract workers, while in the next, it sets wages to "zero". Firms can do so without significant concern about worker attrition, as few employees receive outside offers or quit into unemployment. As a result, the firm's size (employment) also fluctuates over time. Indeed, Figure D.14 shows that the firm's value function, as a function of the current wage, has two global maxima: one at "zero" and another slightly above $\bar{W}$. Given that all other firms maintain their steady-state wage, vacancy, and employment levels, the firm is indifferent between these two wage options and thus optimally oscillates between them, resulting in wage dispersion.

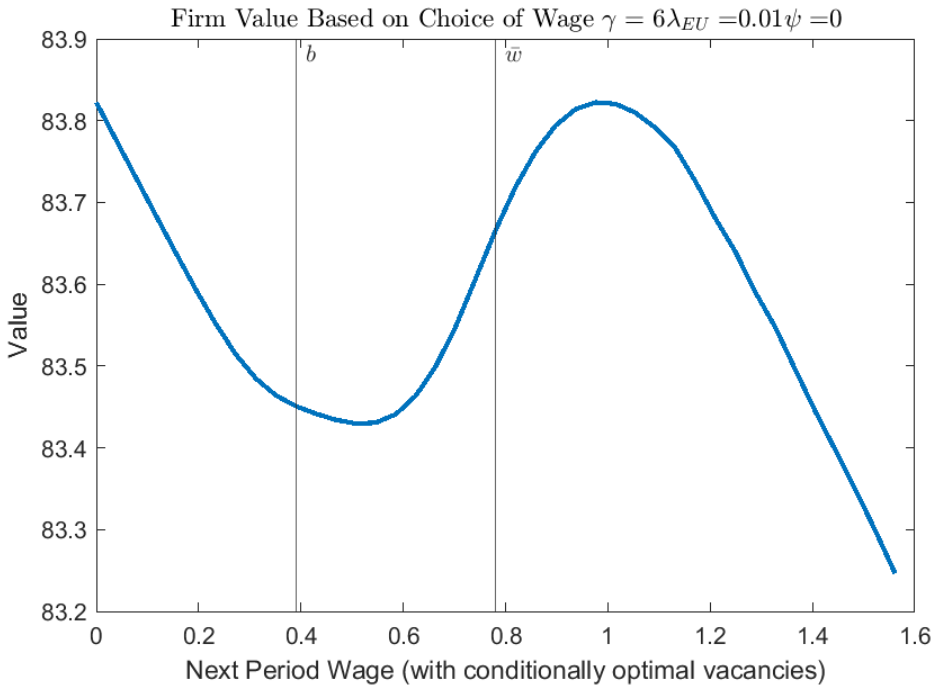


Figure D.14: Individual Firm Value as a Function of Wage Choices ($\lambda_{EU} = 0.01$)

Figure D.15 illustrates the firm's optimal employment N_t, given the previous period's employment N_{t-1}, which may differ from its steady-state level.[14] In contrast to the standard calibration case in Figure D.7, with a lower probability of quitting into unemployment ($\lambda_{EU} = 0.01$), the firm's size oscillates continuously, alternating between high and low employment levels around the steady state.

Figure D.16 illustrates the optimal policy functions as a function of last period's employment, N_{t-1}. There is a pronounced discontinuity in the optimal wage and vacancy policies around the steady state, which drives the oscillatory behavior: a large firm (with higher N_{t-1}) in period t sets wages to *zero* and posts a low vacancy rate, thereby reducing its size in the next period. In period $t+1$, the downsized firm responds by setting high wages and vacancy rates, increasing its size in period $t+2$.

Interpretation We conclude that our single-wage symmetric equilibrium remains compatible with on-the-job search, provided there is a sufficiently high probability that workers transition into unemployment, corresponding to a high enough λ_{EU}. Note that our calibration of λ_{EU} is drawn from Qiu (2022) and is independent of the model.

[14] Note that all other firms maintain their steady-state wage, vacancy, and employment levels.

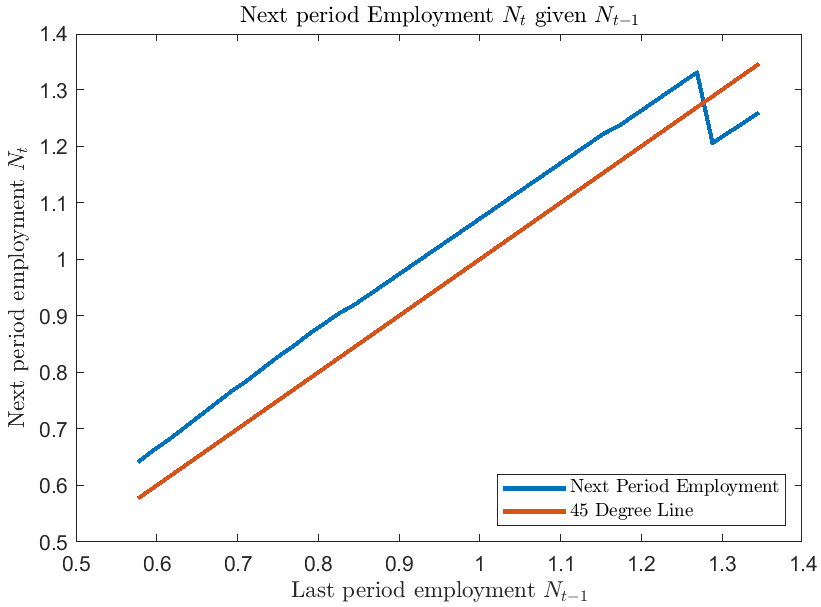


Figure D.15: Employment Dynamics for an Individual Firm under $\lambda_{EU} = 0.01$

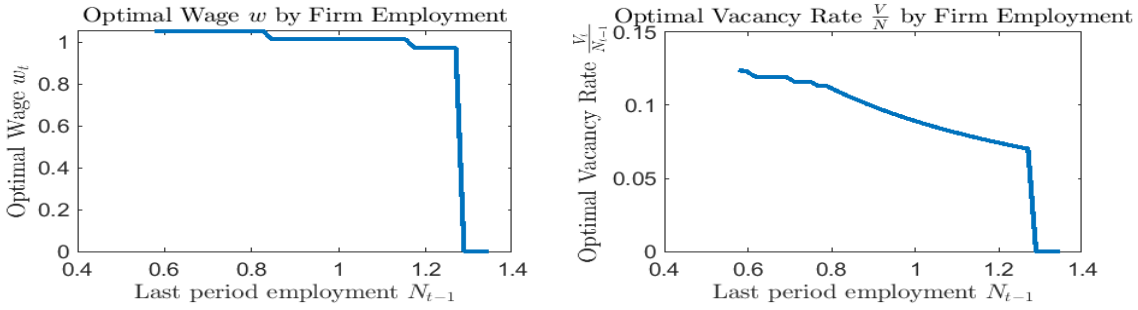


Figure D.16: Wage and Vacancy Rate Policies as Functions of Employment N_{t-1} ($\lambda_{EU} = 0.01$)

Price and Wage Nominal Rigidities So far, we have focused on the firm's problem using value function iteration assuming that firms face no adjustment costs to prices or wages. Introducing price and wage adjustment costs will further promote equilibrium stability, as the adjustment costs make oscillatory behavior more costly.

Under our benchmark calibration for price and wage rigidities ($\psi = 1000$, $\psi^W = 9000$), Figure D.17 shows that even when γ increases to 12, the individual firm's value function remains *globally* maximized at the steady-state wage. This implies that each firm optimally chooses the steady-state wage when other firms do the same. Therefore, nominal rigidities further enhance equilibrium stability, even at higher levels of γ.

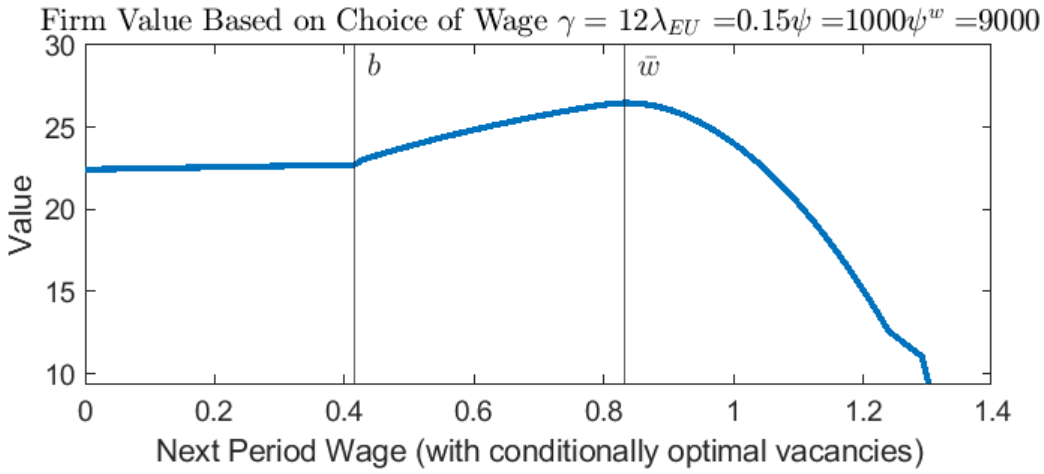


Figure D.17: Individual Firm Value as a Function of Wage Choices ($\gamma = 12$): under Benchmark Price and Wage Rigidity

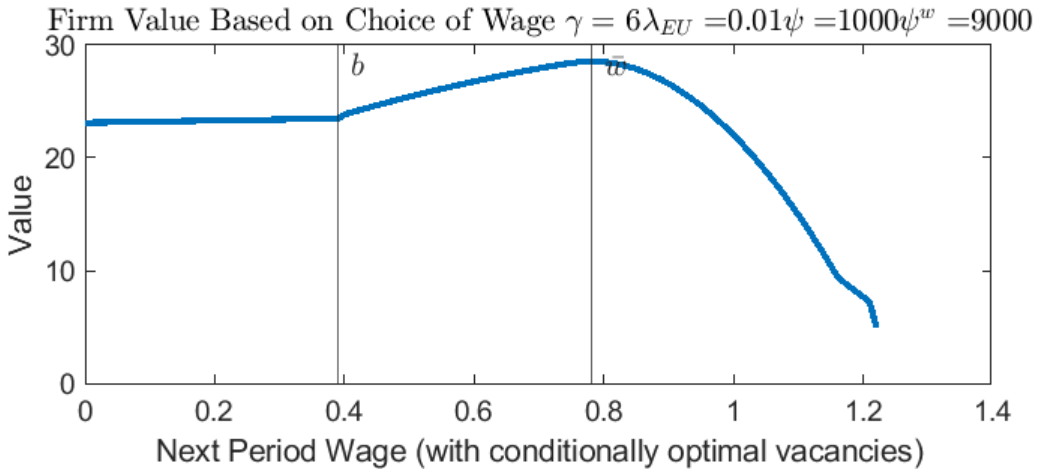


Figure D.18: Individual Firm Value as a Function of Wage Choices ($\lambda_{EU} = 0.01$): under Benchmark Price and Wage Rigidity

Appendix E Wage Dispersion, Endogenous Search Effort, and CRRA Utility

In the main text, we focused on a model that generated no wage dispersion in equilibrium and assumed no ability for workers to search harder for higher wage jobs in response to inflationary shocks. While tractable and simple, the stylized setup abstracts from various important questions about how workers may respond to inflationary supply shocks. For example, how does workers' search effort respond to inflationary supply shocks in general equilibrium? Can workers' search effort responses contribute to amplification of inflationary shocks?

In this section, we generalize the model to allow for a job ladder, endogenous on-the-job search effort, and CRRA utility where workers have a coefficient of relative risk aversion $\sigma > 1$, moving away from the log case assumed in the main text. We introduce a job ladder by letting firms have heterogeneous costs of posting a vacancy: firms with more expensive hiring offer higher wages in equilibrium. Workers can choose their intensity of job search subject to a convex cost. Also, allowing for CRRA utility means that inflation may be more costly in proportional terms for workers in low-wage jobs than for workers in high-wage jobs, encouraging workers in low-wage jobs to increase on-the-job search more in response to inflationary shocks.

Summary Our findings can be summarized as follows. With a job ladder and endogenous search effort, workers in low-wage jobs search harder than workers in high-wage jobs in steady state. We calibrate the model to match the elasticity of search effort with respect to real wages (within a period) of -0.5 as in Faberman et al. (2022): workers in jobs that have 10% lower wages search 5% more frequently. We then use the model to study the response of search effort and wage growth in response negative endowment shocks. Broadly speaking, the results are similar to the main text: an endowment shock lowers real wages but in general equilibrium has limited pass-through to nominal wages, with effects on worker search effort that depend on how workers' preferences are specified.

The choice of worker preferences has nuanced implications for the response of worker search effort in response to negative supply shocks that increase the cost of living. If workers have log utility as in the baseline model (i.e., $\sigma \to 1$), then the direct effects of a higher price level from the negative endowment shock has *no direct effect* on search effort. This non-response is the result of two offsetting forces. When the shock raises prices, it lowers the

gain in real wages that is attained by switching to a higher wage job, lessening the incentive to search. At the same time, a higher cost of living makes workers poorer, inducing them to search more. Under log utility, these forces exactly offset, resulting in no change in search effort, no effect on wages, and thus no propagation of the initial price shock to wages.

These results change when workers become more risk averse (i.e., $\sigma > 1$). In this case, the lower real income from a negative endowment shock has a larger proportionate negative effect on utility for workers with lower incomes, incentivizing them to search harder. This affects unemployed workers the most, who increase their search effort by the most. The extra congestion from unemployed workers, combined with lower demand for labor due to the endowment shock, depresses the incentive for employed workers to search. This means that in response to an oil shock, workers in low-wage jobs leave their search effort effectively unchanged, while workers in high-wage jobs *decrease* their search effort, as the appeal of taking a job with lower wages but good idiosyncratic amenities becomes less desirable.

Our findings can be consistent with the survey evidence from Pilossoph and Ryngaert (2026), who find an elasticity of on-the-job search likelihood with respect to inflation expectations of around $0.6 - 0.9$. That is, if workers in our model expect their wage to be eroded by inflation but do not internalize that inflation erodes the real value of wages in other jobs, then workers would increase their search effort in response to higher inflation expectations by a similar magnitude as found in Pilossoph and Ryngaert (2026).

E.1 Model Extension: Job Ladder Framework

In this section, we extend the baseline model—where firms post symmetric wages and vacancies while facing price and nominal rigidities—by introducing wage heterogeneity. Specifically, we assume a unit mass of firms with high vacancy costs (hereafter, H-firms) and an equal mass of firms with lower vacancy costs (L-firms). In equilibrium, H-firms always offer higher wages than L-firms, creating a job ladder for workers.[15]

We also introduce endogenous search effort into the benchmark symmetric equilibrium model. Specifically, rather than assuming constant values for λ_{EE} and $\lambda_{UE} = 1$,[16] we derive these as choice variables that depend on relative wages and match the elasticity of search effort with respect to real wages. We show that the benchmark result of zero pass-through to wages continues to hold in this setting with log utility. This outcome arises because, under

[15] Wage heterogeneity in the model arises from differences in vacancy creation costs across firms, rather than from heterogeneous productivity levels across worker-firm pairs. This is for tractability—even if we introduce firm-level productivity differences, the qualitative results would remain the same.

[16] In the baseline model, note that all unemployed workers fully engage in job search, i.e., $\lambda_{UE} = 1$.

log preferences, the desirability of changing jobs via job-switching is independent of the aggregate price level. CRRA utility leads to some adjustments in search effort, particularly among unemployed and lower-income workers, while maintaining a relatively modest pass-through from cost-of-living shocks to wages.

Final Aggregator The final services firm bundles services Y_t^H and Y_t^L from the H and L service firms, respectively as follows.

$$Y_t = \left(\frac{Y_t^H}{\alpha_h}\right)^{\alpha_h} \left(\frac{Y_t^L}{1-\alpha_h}\right)^{1-\alpha_h} \tag{E.1}$$

where $\alpha_h = 0.5$ in our calibration. The price index for services is then a geometric weighted average of H- and L-prices, given by

$$P_t^Y = \left(P_t^H\right)^{\alpha_h} \left(P_t^L\right)^{1-\alpha_h}. \tag{E.2}$$

From equations (E.1) and (E.2), the relative price of H and L services is given by

$$\frac{P_t^H}{P_t^L} = \frac{\alpha_h}{1-\alpha_h}\frac{Y_t^L}{Y_t^H}.$$

The final consumption good is a composite of Y_t, as defined in equation (E.1), and X_t:

$$C_t = \left(\alpha_Y^{\frac{1}{\eta}} Y_t^{\frac{\eta-1}{\eta}} + \alpha_X^{\frac{1}{\eta}} X_t^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}},$$

as in the main body. Total endowment resources add up

$$\bar{X}_t^H + \bar{X}_t^L + X_t = X_t^s.$$

We will focus on negative shocks to X_t^s as in the main text.

E.1.1 Services Aggregators

Service outputs from the H and L industries, Y_t^H and Y_t^L, are in turn aggregated using a constant elasticity of substitution (CES) function:

$$Y_t^H = \left(\int_j \left(Y_t^{H,j} \right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$
$$Y_t^L = \left(\int_j \left(Y_t^{L,j} \right)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

where each individual firm j in each industry faces the following standard demand function:

$$Y_t^{H,j} = \left(\frac{P_t^{H,j}}{P_t^H} \right)^{-\epsilon} Y_t^H, \tag{E.3}$$

$$Y_t^{L,j} = \left(\frac{P_t^{L,j}}{P_t^L} \right)^{-\epsilon} Y_t^L. \tag{E.4}$$

The production function for output Y_t^j of firm j is given by

$$Y_t^{H,j} = A_t^{H,j} \left(\alpha_N^{\frac{1}{\bar{\eta}}} \left(N_t^{H,j} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\bar{X}_t^{H,j} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \right)^{\frac{\bar{\eta}}{\bar{\eta}-1}} \tag{E.5}$$

$$Y_t^{L,j} = A_t^{L,j} \left(\alpha_N^{\frac{1}{\bar{\eta}}} \left(N_t^{L,j} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\bar{X}_t^{L,j} \right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \right)^{\frac{\bar{\eta}}{\bar{\eta}-1}}, \tag{E.6}$$

The cost of creating vacancies for H-firms and L-firms is respectively given by

$$c_H \left(\frac{V_{jt}}{N_{j,t-1}} \right)^{\chi} V_{jt} W_t \tag{E.7}$$

$$c_L \left(\frac{V_{jt}}{N_{j,t-1}} \right)^{\chi} V_{jt} W_t \tag{E.8}$$

where W_t denotes the average wage level. We later impose $A_t^{H,j} = A_t^{L,j} = A_t$ for all firms j, whether H-type or L-type. As stated above, we will assume $c_H > c_L$, leading to $W_t^H > W_t^L$ in equilibrium where $W_t^{H(L)}$ denotes the wage level in the H- and L-industries, respectively.

E.1.2 Workers and Endogenous Search Effort

We define $\lambda_{i,t}$ as the search effort of worker i in period t. With endogenous search effort, total matches are a constant returns to scale function of total vacancies and total search activity, which is $\int_i \lambda_{i,t} di$:

$$M\left(V_t, \int_i \lambda_{i,t} di\right),$$

where the labor market tightness θ_t is defined as

$$\theta_t = \frac{V_t}{\int_i \lambda_{i,t} di}. \tag{E.9}$$

The total number of vacancies V_t is the sum of H- and L-firm vacancies, that is, $V_t = V_t^H + V_t^L$. We also define the share of vacancies created by H- and L-firms relative to the total as $\sigma_t^{H(L)}$:

$$\sigma_t^H = \frac{V_t^H}{V_t}, \quad \text{and} \quad \sigma_t^L = \frac{V_t^L}{V_t}.$$

Consumption Sharing Considering how households allocate consumption between employed and unemployed workers, we continue to assume that an employed worker i at firm j faces an effective tax rates $1 - \tau_t$. Consequently, their consumption is:

$$C_{ijt} = \tau_t \frac{W_{jt}}{P_t}, \tag{E.10}$$

where W_{jt} is the wage at firm j. In contrast, unemployed workers receive a nominal unemployment benefit b_t, giving them consumption of b_t. We maintain the baseline consumption rule from the main text with $\varpi = 0$:

$$\frac{\bar{W}_t}{P_t b_t} = \xi, \tag{E.11}$$

where $\bar{W}_t \equiv \frac{\int_j W_{jt} dj}{N_t}$ is the average wage of all workers in the household. As in the main text, we calibrate $\xi = 2$.

Search Effort Based on the above consumption formulas for employed and unemployed workers, we can write the static value function for an unemployed worker as follows.

The static value function for an employed worker at firm j is given by

$$V_{i,j} = \max_{\lambda_{i,t}^{EE}} \left(1 - f(\theta_t)\lambda_{i,t}^{EE}\right) \frac{\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} + f(\theta)\lambda_{i,t}^{EE} \left[\sigma_t^H \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) + \sigma_t^L \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) \right] - e^E\left(\lambda_{i,t}^{EE}\right).$$

where $e^E\left(\lambda_{i,t}^{EE}\right)$ denotes the search cost of an employed worker. The static value function for an unemployed worker is similarly given by

$$V_{i,U} = \max_{\lambda_{i,t}^{UE}} \left(1 - f(\theta_t)\lambda_{i,t}^{UE}\right) \frac{(b_t)^{1-\sigma} - 1}{1-\sigma} + f(\theta)\lambda_{i,t}^{UE} \left[\sigma_t^H \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{(b_t)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) + \sigma_t^L \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{(b_t)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) \right] - e^U\left(\lambda_{i,t}^{UE}\right).$$

where $e^U\left(\lambda_{i,t}^{UE}\right)$ denotes the search cost of an unemployed worker.

Parameterization We parameterize the search effort cost functions as follows:

$$e^E\left(\lambda_{i,t}^{EE}\right) = \frac{1}{1+\nu}\left(\frac{1}{\lambda_0^{EE}}\right)^{\nu}\left(\lambda_{i,t}^{EE}\right)^{1+\nu}$$

and

$$e^U\left(\lambda_{i,t}^{UE}\right) = \frac{1}{1+\nu}\left(\frac{1}{\lambda_0^{UE}}\right)^{\nu}\left(\lambda_{i,t}^{UE}\right)^{1+\nu}$$

for some constants ν, λ_0^{EE} and λ_0^{UE}, which will be calibrated later.

With these functional forms, the first-order condition on $\lambda_{i,t}^{EE}$ yields

$$\lambda_{i,t}^{EE}(W_{jt}) = \left(f(\theta) \left[\sigma_t^H \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) \right.$$
$$+ \sigma_t^L \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right)$$
$$\left. - \frac{\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right] \Bigg)^{\frac{1}{v}} \lambda_0^{EE}. \tag{E.12}$$

The first-order condition on $\lambda_{i,t}^{UE}$ yields

$$\lambda_{i,t}^{UE} = \left(f(\theta) \left[\sigma_t^H \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{(b_t)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) \right.$$
$$\left. + \sigma_t^L \frac{1}{\gamma} \log \left(\exp\left(\gamma \frac{(b_t)^{1-\sigma} - 1}{1-\sigma} \right) + \exp\left(\gamma \frac{\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - 1}{1-\sigma} \right) \right) - \frac{(b_t)^{1-\sigma} - 1}{1-\sigma} \right] \Bigg)^{\frac{1}{v}} \lambda_0^{UE}. \tag{E.13}$$

E.1.3 Worker Flows

With CRRA utility for workers, the separation function for firm j depends directly on the price level P_t and can be expressed as

$$S(W_{jt}|P_t) = s + (1-s) \Bigg[f(\theta_t) \lambda_{i,t}^{EE}(W_{jt}) \Bigg(\sigma_t^H \frac{1}{1 + \exp\left(\frac{\gamma}{1-\sigma} \left(\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} \right) \right)}$$
$$+ \sigma_t^L \frac{1}{1 + \exp\left(\frac{\gamma}{1-\sigma} \left(\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} \right) \right)} \Bigg)$$
$$+ \lambda_{EU} \frac{1}{1 + \exp\left(\frac{\gamma}{1-\sigma} \left(\left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma} - (b_t)^{1-\sigma} \right) \right)} \Bigg]. \tag{E.14}$$

Evaluating (E.14) at H-firms ($W_{jt} = W_t^H$), we obtain

$$S(W_t^H|P_t) = s + (1-s)\Bigg[f(\theta_t)\lambda_{i,t}^{EE}(W_t^H)\left(\frac{\sigma_t^H}{2} + \sigma_t^L \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma}\right)\right)}\right) + \lambda_{EU}\frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - (b_t)^{1-\sigma}\right)\right)}\Bigg].$$

Analogously, for L-firms:

$$S(W_t^L|P_t) = s + (1-s)\Bigg[f(\theta_t)\lambda_{i,t}^{EE}(W_t^L)\left(\frac{\sigma_t^L}{2} + \sigma_t^H \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma}\right)\right)}\right) + \lambda_{EU}\frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - (b_t)^{1-\sigma}\right)\right)}\Bigg],$$

where $\lambda_{i,t}^{EE}(W_t^H)$ and $\lambda_{i,t}^{EE}(W_t^L)$ are provided in equations (E.18) and (E.20).

Likewise, with CRRA utility for workers, the recruiting function for firm j depends directly on the price level P_t and is given by

$$R(W_{jt}|P_t) = g(\theta_t)\Bigg[\phi_t^H \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}\right)\right)} + \phi_t^L \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}\right)\right)} + \phi_t^U \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left((b_t)^{1-\sigma} - \left(\frac{\tau_t W_{jt}}{P_t}\right)^{1-\sigma}\right)\right)}\Bigg]. \tag{E.15}$$

where $\phi_t^{H(L)}$ is the share of on-the-job searchers currently employed at H(L)-firms, and ϕ_t^U is the share of searchers who are unemployed, where $\phi_t^H + \phi_t^L + \phi_t^U = 1$. Evaluating equation

(E.15) at $W_{jt} = W_t^H$ (i.e., for H-firms) gives

$$R(W_t^H|P_t) = g(\theta_t)\left[\frac{\phi_t^H}{2} + \phi_t^L \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma}\right)\right)} + \phi_t^U \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left((b_t)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma}\right)\right)}\right].$$

Analogously, for L-firms:

$$R(W_t^L|P_t) = g(\theta)\left[\frac{\phi_t^L}{2} + \phi_t^H \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left(\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma}\right)\right)} + \phi_t^U \frac{1}{1+\exp\left(\frac{\gamma}{1-\sigma}\left((b_t)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma}\right)\right)}\right].$$

E.1.4 Firm's Problem

A firm j operating in the H services market solves the problem

$$\max_{\substack{\{P_{y,t}^{H,j}\},\{Y_t^{H,j}\},\{\bar{X}_t^j\} \\ \{N_{jt}\},\{W_{jt}\},\{V_{jt}\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \left(P_{y,t}^{H,j}Y_t^{H,j} - W_{jt}N_{jt} - c^H\left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi}V_{jt}W_t - \frac{\psi}{2}\left(\frac{P_{y,t}^{H,j}}{P_{y,t-1}^{H,j}} - 1\right)^2 Y_t^{H,j}P_{y,t}^{H,j} - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt}N_{jt} - P_{x,t}\bar{X}_t^j\right) \tag{E.16}$$

subject to the law of motion for employment

$$N_{jt} = (1 - S_t(W_{jt}))N_{j,t-1} + V_{jt}R_t(W_{jt}) \tag{E.17}$$

where $S_t(W_{jt})$ and $R_t(W_{jt})$ are given by (E.14) and (E.15), respectively. Firms also face the constraint on product demand from equation (E.3), with its production function given by (E.5).

The firm's problem in the L services market is formulated similarly.

E.2 Search Elasticities and Calibration

In constructing a symmetric equilibrium, we assume that all the firms selling in the H market face the same vacancy cost parameter c^H, and similarly all the firms selling in the L market face the same vacancy cost c^L.

Evaluating (E.12) at $W_{jt} = W_t^H$, using $\sigma_t^H + \sigma_t^L = 1$, the search effort at H-firms is given by

$$\lambda_{i,t}^{EE}(W_t^H) = \left(f(\theta) \left[\sigma_t^H \frac{1}{\gamma} \log(2) + \sigma_t^L \frac{1}{\gamma} \log \left(1 + \exp \left(\gamma \frac{\left(\frac{\tau_t W_t^L}{P_t} \right)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t} \right)^{1-\sigma}}{1-\sigma} \right) \right) \right] \right)^{\frac{1}{v}} \lambda_0^{EE}, \tag{E.18}$$

with the search elasticity given by

$$\varepsilon_{\lambda_{it}^{EE}, W_{jt}}(W_t^H) = -\frac{\gamma}{v} \left(\frac{\tau_t W_t^H}{P_t} \right)^{1-\sigma} \frac{\left(\frac{\sigma_t^H}{2} + \frac{\sigma_t^L}{1+\exp\left(\gamma \frac{\left(\frac{\tau_t W_t^H}{P_t} \right)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t} \right)^{1-\sigma}}{1-\sigma} \right)} \right)}{\sigma_t^H \log(2) + \sigma_t^L \log \left(1 + \exp \left(\gamma \frac{\left(\frac{\tau_t W_t^L}{P_t} \right)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t} \right)^{1-\sigma}}{1-\sigma} \right) \right)}. \tag{E.19}$$

Evaluating (E.12) at $W_{jt} = W_t^L$, the search effort at L-firms is given by

$$\lambda^{EE}(W_t^L) = \left(f(\theta) \left[\sigma_t^H \frac{1}{\gamma} \log \left(1 + \exp \left(\gamma \frac{\left(\frac{\tau_t W_t^H}{P_t} \right)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t} \right)^{1-\sigma}}{1-\sigma} \right) \right) + \sigma_t^L \frac{1}{\gamma} \log(2) \right] \right)^{\frac{1}{v}} \lambda_0^{EE}, \tag{E.20}$$

with the search elasticity given by

$$\varepsilon_{\lambda_{it}^{EE},W_{jt}}(W_t^L) = -\frac{\gamma}{v}\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} \frac{\left(\frac{\sigma_t^L}{2} + \frac{\sigma_t^H}{1+\exp\left(\gamma\frac{\left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma}}{1-\sigma}\right)}\right)}{\sigma_t^L \log(2) + \sigma_t^H \log\left(1+\exp\left(\gamma\frac{\left(\frac{\tau_t W_t^H}{P_t}\right)^{1-\sigma} - \left(\frac{\tau_t W_t^L}{P_t}\right)^{1-\sigma}}{1-\sigma}\right)\right)}. \tag{E.21}$$

(E.18) and (E.20) imply that optimal search effort is lower at H-firms than at L-firms. This is intuitive, as workers at L-firms have stronger incentives to engage in on-the-job search in order to move to H-firms. Equations (E.19) and (E.21) indicate that on-the-job search effort becomes more responsive to a firm's wage as real wages decreases, with this effect being more pronounced for unemployed workers and workers at L-firms.[17] This is important because, when a negative supply shock reduces real wages, unemployed workers and workers at L-firms will intensify their on-the-job search.

E.2.1 Log Utility

We calibrate the model under log utility ($\sigma \to 1$). All parameters in common with our baseline model in the main body are the same as in Section 5.1. To generate wage dispersion, firms in the two sectors have different hiring costs: $c^H = 120$ and $c^L = 30$. We calibrate ν, λ_0^{EE}, and λ_0^{UE} to match $\lambda^{EE} = 0.14$ and $\lambda^{UE} = 1$ at the steady state (as in our benchmark calibration) and to match the elasticity of search effort with respect to real wages (within a period) of -0.5 provided in Faberman et al. (2022). This means setting the baseline search intensities $\lambda_0^{EE} = 0.21$, $\lambda_0^{UE} = 1.2$, and $\nu = 8.5$, as can be seen in Table E.3. We obtain the steady state values of real wage and search effort in Table E.4, $W^L/P = .727$ and $W^H/P = .851$. Workers in low wage jobs search harder: $\lambda^{EE}(W_L) = 0.146$ and $\lambda^{EE}(W_H) = 0.134$. Calculating the elasticity of search with respect to real wages at these two points yields and elasticity $\varepsilon_{\lambda,W_j|\{P,(W/P)_{-j}\}} = -.508$.

[17] This occurs because the function $x^{1-\sigma}$ is concave in x when $\sigma > 1$.

Table E.3: Calibration: Log Utility

Parameter	Log Utility ($\sigma = 1$)
ν	8.5
λ_0^{EE}	.21
λ_0^{UE}	1.2

Table E.4: Steady State Outcomes

Variable	Unemployed	Low-wage	High-wage
Real Wage	.383	.727	.851
Search Effort λ^{EE}	.996	0.146	0.134
Search elasticity w.r.t W_j: $\varepsilon_{\lambda,W_j\|\{P,(W/P)_{-j}\}}$	-	-.460	-.557
Search elasticity w.r.t P : $\varepsilon_{\lambda,P\|\{W_j,W_{-j}\}}$	0	0	0

Notes: This table reports the steady-state real wage level, on-the-job search effort, and elasticity of search effort with respect to real wages for workers at high-wage (H) and low-wage (L) firms. The elasticity of search effort with respect to real wages calculated across the two points $\log(\lambda^{EE}(W^H/P)/\lambda^{EE}(W^L/P))/\log((W^H/P)/(W^L/P)) = -.508$. The last row reports the elasticity of search effort with respect to the price level, holding other firms' nominal wage fixed.

Search Effort and Changes in the Price Level In the previous section, we showed that in the cross-section, workers in lower-wage jobs search harder while on-the-job: being lower in the wage distribution, these workers have more to gain from on-the-job search. However, the elasticity we calculated is not necessarily informative for how on-the-job search effort responds to a jump in the price level. To understand this, we want to calculate the following object

$$\varepsilon_{\lambda,P|\{W_j,W_{-j}\}},$$

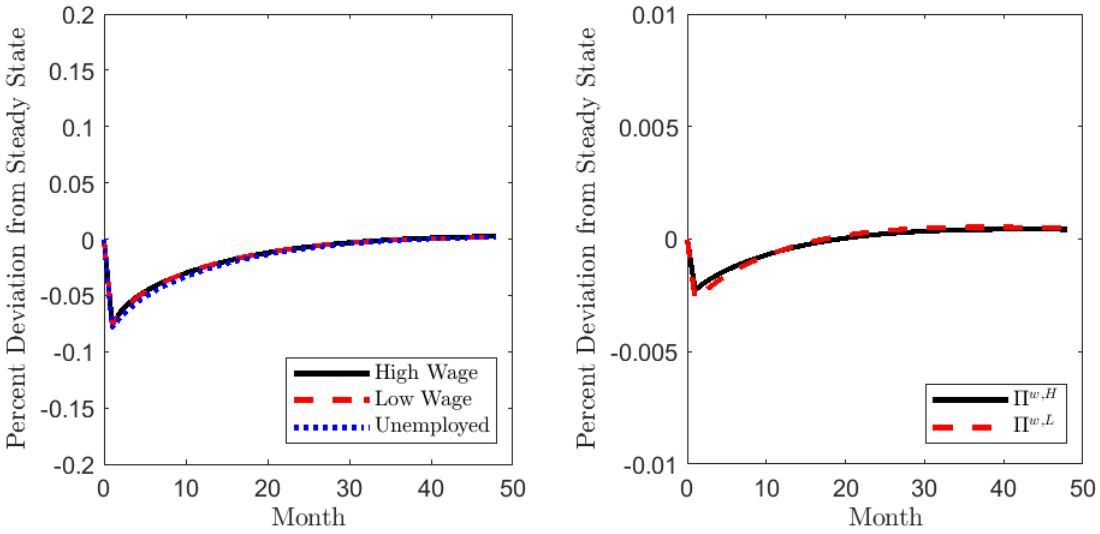
which is the partial-equilibrium response of search effort with respect to the price level, holding both firm j's wage and other firms' $-j$ nominal wages fixed. As can be seen in equations (E.18) and (E.20), under log utility $\sigma \to 1$ the price level P_t drops out of the equation. That means for both workers in high-wage and low-wage jobs, changes in the price level have *no effect* on optimal search effort. That is $\varepsilon_{\lambda,P|\{W_j,W_{-j}\}} = 0$ for both workers in high-wage and low-wage jobs, as well as for unemployed workers. These results are reported in the bottom row of Table E.4

Inflationary Supply Shocks Even though search effort does not respond directly to the price level (under log utility), search effort may endogenously move for other reasons in response to inflationary supply shocks in general equilibrium. We therefore simulate our extended model with two wages and endogenous search effort in response to an inflationary supply shock.

Figure E.19 plots the response of search effort and wage inflation to a 10% negative shock to the quantity of X^s. Consistent with the results from Figure 5 of the main text, demand for labor falls and this is not entirely offset by the central bank. Vacancies fall on impact, and since the returns to search fall with labor market tightness declines (see how search effort increases with workers' job finding rate $f(\theta)$ in equations (E.18) and (E.20)), search effort by all workers modestly declines. Notably there is *no* difference in the response of search effort among workers in high-wage and low-wage jobs and unemployed workers: the price level has no direct impact on search effort, and lower labor market tightness equally and negatively affects workers' incentive to search in high and low wage jobs.

Figure E.19: Response of Search Effort and Wage Growth to a 10% Negative Shock to X^s

(a) Search Effort (b) Wage Inflation



Responses in Search Effort and Wages to a 10% Negative X^s Shock (Log Utility)

As for the effect on nominal wage growth, the second panel of Figure (E.19) shows that nominal wage growth is negative in response to the shock, and so the cumulative response of nominal wage growth is negative, similar to our baseline case in Figure 5 of the main text.

Thus with log utility, the addition of a job ladder and endogenous on-the-job search does not change our results from the baseline model that wages do not respond to inflationary supply shocks.

E.3 Results: CRRA Utility $\sigma > 1$

In the previous section, log utility over consumption eliminated the effect of the price level on search effort. We now explore the implications of workers with CRRA utility, and specifically if workers are risk averse ($\sigma > 1$). This means that as workers become poorer from a higher cost of living, they will value the same percent increase in income by more, opening up more scope for changes in search effort in response to shocks that affect the price level.

To illustrate the dynamics in the model, we choose value $\sigma = 5$, which is a high degree of risk aversion. We re-calibrate ν, λ_0^{EE}, and λ_0^{UE} to hit an average on-the-job search effort of 0.14, search effort by unemployed workers equal to 1, and the elasticity of search with respect to real wages in the cross section of -0.5 from Faberman et al. (2022). Table E.5 reports the parameters, and Table E.6 reports the real wage, search effort, and cross-section search effort elasticity with respect to real wages.

Table E.5: Calibration: CRRA Utility ($\sigma = 5$)

Parameter	Value
ν	19
λ_0^{EE}	.17
λ_0^{UE}	.96

As the bottom row of Table E.6 shows, now the price level has a direct effect on search effort. The effect is strongest among the unemployed. Because unemployed workers have the lowest consumption, an increase in the price level and a decline in their real consumption is the most painful, motivating unemployed workers to search harder, increasing search effort by 2.1% for every 10% increase in prices.

Next, workers in low-wage jobs increase their search effort modestly, by 0.54% from a 10% increase in prices. These workers are relatively low-income and care more about higher wages when prices rise, and subsequently increase their search effort.

For workers in high-wage jobs, part of the value of search comes from finding lower-wage jobs with high idiosyncratic values of utility. When prices rise and real consumption falls, these voluntary wage cuts become less desirable, lowering these workers' incentive to search on the job.

Table E.6: Steady State Outcomes

Variable	Unemployed	Low-wage	High-wage
Real Wage	.408	.809	.866
Search Effort λ^{EE}	1	.143	.138
Search elasticity w.r.t W_j: $\varepsilon_{\lambda,W_j\|\{P,(W_{-j}/P)\}}$	-	-.524	-.483
Search elasticity w.r.t P : $\varepsilon_{\lambda,P\|\{W_j,W_{-j}\}}$	.211	.054	- .077

Notes: This table reports the steady-state real wage level, on-the-job search effort, and elasticity of search effort with respect to real wages for workers at high-wage (H) and low-wage (L) firms. The elasticity of search effort with respect to real wages calculated across the two points $\log(\lambda^{EE}(W^H/P)/\lambda^{EE}(W^L/P))/\log((W^H/P)/(W^L/P)) = -.504$. The last row reports the elasticity of search effort with respect to the price level, holding other firms' nominal wage fixed.

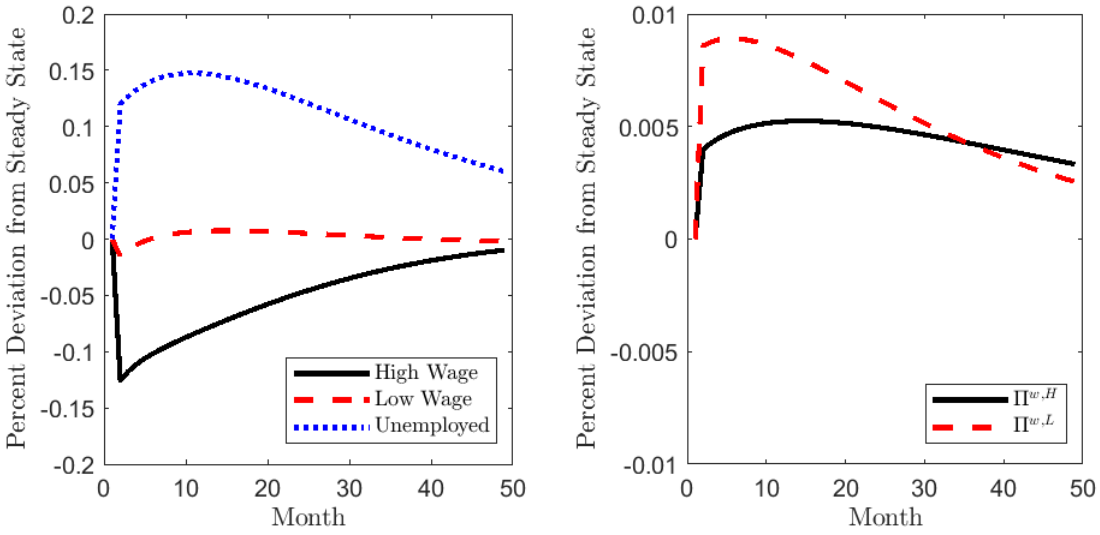
Figure E.20 presents the impulse responses of key variables to a 10% shock to X_t^s. Consistent with the partial equilibrium values of $\varepsilon_{\lambda,P|\{W_j,W_{-j}\}}$, the left panel illustrates that the response of search effort to a supply shock is the most positive among the unemployed and most negative among workers at high-wage firms. Notably, the search effort of workers in low-wage jobs hardly moves. This is because, even though higher prices have a direct positive effect on search effort among workers in low-wage jobs, the weakening of the labor market from lower labor demand lowers the return to search for all workers. Furthermore, the greater search effort by the unemployed congests the labor market for employed workers, lowering labor market tightness and further lessening the incentive for employed workers to search on the job.

The right panel shows the impulse responses of wage growth. In contrast to the log-utility specification, nominal wage growth becomes positive under CRRA utility with $\sigma = 5$. This is because workers become more wage sensitive, which increases the magnitude of $\varepsilon_{R,w}$ and $\varepsilon_{S,w}$, incentivizing firms to raise wages. However, the response of wages remains quantitatively very small: note that the units mean that the wage growth of low wage workers peak at below 0.01% per month. The cumulative response of nominal wage growth is less that 0.3% for both high- and low-wage jobs after 48 quarters. Thus, introducing greater risk aversion does not alter our main conclusion: the cumulative responses of both wage and price inflation to a supply shock remain modest.

Figure E.20: Response of Search Effort and Wage Growth to 10% Negative Shock to X^s

(a) Search Effort

(b) Wage Inflation



Responses in Search Effort and Wages to a 10% Negative X^s Shock (CRRA Utility)

E.4 Comparison with Pilossoph and Ryngaert (2026)

Pilossoph and Ryngaert (2026) use the Real-Time Population Survey (RTS) and Survey of Consumer Expectations (SCE) to estimate the response of on-the-job search activity to changes inflation expectations, finding elasticities of search for new work in the range of 0.6 to 0.9. The authors interpret these results through the lens of a model where workers have CRRA utility, and that the wages of incumbents are potentially less indexed to inflation than the wages of new hires. In their notation, with inflation expectations $\tilde{\pi}$, incumbent wages adjust at rate $\gamma_1\tilde{\pi}$ and new hire wage adjust at $\gamma_2\tilde{\pi}$, with $0 \leqslant \gamma_1 \leqslant \gamma_2 \leqslant 1$.

In our exercise with CRRA utility and $\sigma = 5$, we found an elasticity of on-the-job search with respect to the price level P_t : $\varepsilon_{\lambda,P|\{W_j,W_{-j}\}}$ to be quantitatively small: around 0.05 for workers in low-wage jobs and negative for workers in high-wage jobs. What accounts for the difference between our model's estimates and the findings in Pilossoph and Ryngaert (2026)? First, our elasticity is calculated in a rational expectations equilibrium where there is no indexing of either incumbent or new hire wages to inflation (in the notation of Pilossoph and Ryngaert (2026), this would be $\gamma_1 = \gamma_2 = 0$).

Now consider our model in partial equilibrium where workers believe that their current nominal wage will not change with inflation, but the outside distribution of wages is com-

pletely inflation-indexed.[18] (In Pilossoph and Ryngaert (2026), this would be $\gamma_1 = 0$ and $\gamma_2 = 1$.) We can express this elasticity as $\varepsilon_{\lambda,P|\{W_j,(W/P)_{-j}\}}$.[19] In our baseline calibration, this elasticity is equal to 0.524 for workers in low-wage jobs and 0.483 for workers in high-wage jobs, not far from Pilossoph and Ryngaert (2026) estimates of 0.6-0.9.

Why is the elasticity of search effort with respect to the price level holding outside real wages fixed $\varepsilon_{\lambda,P|\{W_j,(W/P)_{-j}\}}$ so much higher than the equivalent elasticity when outside nominal wages fixed $\varepsilon_{\lambda,P|\{W_j,W_{-j}\}}$? It is fairly straightforward: if outside offers are growing while a workers' own current wage is not, this dramatically increases the returns to search effort.

Given our calibration, it also should not be surprising that our calculated value of $\varepsilon_{\lambda,P|\{W_j,(W/P)_{-j}\}}$ is around 0.5. After all, higher prices raising the outside distribution of wages with inflation one-for-one while holding the wage at firm j fixed is equivalent to lowering firm j's relative wage. This is conceptually the same exercise as in Faberman et al. (2022), which calculates the elasticity of search with respect to real wages, holding all else fixed, in the cross-section: $\varepsilon_{\lambda,P|\{W_j,(W/P)_{-j}\}} = -\varepsilon_{\lambda,W_j|\{P,(W/P)_{-j}\}}$.

Our exercise highlights that the response of search effort and the distribution of real new hire wages to inflationary shocks depends on the type of shock. In our calibrated baseline model, an inflationary supply shock *lowers* demand for labor and incentivizes firms to post lower wages for new hires. If incumbent worker wages were sticky but new hires wages were flexible, new hire wages would fall below incumbent wages in response to the shock we study, discouraging workers from searching on the job. Thus, we conclude that inflationary supply shocks generate a limited response of worker on-the-job search effort, and the inclusion of endogenous search effort into the model does not overturn our headline result that inflationary supply shocks generate little wage growth and cumulative inflation.

[18] Afrouzi et al. (2026) similarly assume full indexation of new hire wages.

[19] This notation indicates that when calculating the elasticity $\varepsilon_{\lambda,P|\{W_j,(W/P)_{-j}\}}$, we hold W_j and $\frac{W_{-j}}{P}$ fixed.

Appendix F Hours Margin

In this section, we assume workers derive utility not only from consumption but also from leisure. They also get utility from preference shocks. Workers are still myopic and consider their utility only one period at a time, which for worker i at firm j in period t is given by

$$V_{ijt} = \log(C_{ijt}\ell_{ijt}) + \iota_{ijt}, \tag{F.1}$$

where ι_{ijt} is distributed type-1 extreme value with scale parameter $\frac{1}{\gamma}$ as in the main text and $\ell_{ijt} \in [0, 1]$ is leisure.[20]

As demonstrated by Hajdini et al. (2023) through a randomized controlled trial (RCT), higher inflation does not raise workers' perceived likelihood of increasing their hours with their current employer. Accordingly, we model hours explicitly, by allowing firms to post both wages and the number of working hours. Workers consider both factors when deciding whether to remain in their current job or move to a new one.

Specifically, we assume that each firm j sets not only its wage W_t^j but also the number of hours H_t^j employees are required to work for. When the cost of living rises, workers may be willing to supply more hours, which gives firms an incentive to offer positions with longer working hours to attract them. However, we will show that our benchmark result—namely, zero pass-through to wages—continues to hold even in this alternative framework when $\varpi = 0$. This is because, under the preference specification in (F.1), both job-switching probabilities and the tradeoff between consumption and leisure on the margin are independent of the aggregate price level.

As in the main text with $\varpi = 0$, we will consider the following rule for unemployment benefit b_t:

$$b_t = \xi^{-1}\frac{\overline{W_t H_t}}{P_t}, \tag{F.2}$$

where ξ are constants, and $\overline{W_t H_t}$ is the average value of $W_t^j H_t^j$ across employed workers, satisfying $\overline{W_t H_{tt}} = W_t H_t$ under the symmetric equilibrium. Unemployed workers consume

[20] We can view this utility as a special case of Balanced Growth, or King-Plosser-Rebelo (KPR), preferences:

$$V_{ijt} = \underbrace{\left(\frac{C_{ijt}}{1-\sigma}^{1-\sigma}\nu(\ell_{ijt})\right)}_{\text{KPR Preferences}} + \iota_{ijt}.$$

With $\sigma = 1$ $\nu(x) \equiv \log(x)$, KPR preference becomes (F.1).

$C_t^u = \tau_t b_t$ each period given a tax rate of $1 - \tau_t$, and the consumption of an employed worker i at firm j is

$$C_{ijt} = \tau_t \frac{W_t^j}{P_t} H_t^j,$$

while leisure is given by

$$\ell_{ijt} = 1 - H_t^j.$$

Unemployed workers are assumed to enjoy full leisure, corresponding to $\ell_{iut} = 1$.

Recruiting and separation probabilities In this setting, the probability that a worker currently employed at firm k chooses firm j upon matching depends on both the wages and the hours offered by each firm, and is given by:

$$\begin{aligned}\text{Prob}[k \to j] &= \frac{(C_{ijt}\ell_{ijt})^\gamma}{(C_{ijt}\ell_{ijt})^\gamma + (C_{ijt}\ell_{ikt})^\gamma} \\ &= \frac{\left(\frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma}{\left(\frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma + \left(\frac{W_t^k}{P_t} H_t^k \left(1 - H_t^k\right)\right)^\gamma}.\end{aligned}$$

Similarly, the probability that a worker prefers employment at firm j over unemployment is given by[21]

$$\text{Prob}[\text{U} \to j] = \frac{\left(\tau_t \frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma}{\left(\tau_t \frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma + (\tau_t b_t)^\gamma},$$

where b_t denotes the nominal unemployment benefit provided by the household.

Assuming the symmetric equilibrium, where $H_t^j = H_t$ and $W_t^j = W_t \; \forall k \neq j$, we have

$$\begin{aligned}R(W_t^j, H_t^j) &= g(\theta_t) \left(\phi_t^E \frac{\left(\frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma}{\left(\frac{W_t^j}{P_t} H_t^j (1 - H_t^j)\right)^\gamma + \left(\frac{W_t}{P_t} H_t (1 - H_t)\right)^\gamma} + \phi_t^U \frac{\left(\tau_t \frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma}{\left(\tau_t \frac{W_t^j}{P_t} H_t^j \left(1 - H_t^j\right)\right)^\gamma + (\tau_t b_t)^\gamma} \right) \\ &= g(\theta_t) \left(\phi_t^E \frac{1}{\left(1 + \left(\frac{W_t H_t (1 - H_t)}{W_t^j H_t^j (1 - H_t^j)}\right)^\gamma\right)} + \phi_t^U \frac{1}{\left(1 + \left(\frac{P_t b_t}{W_t^j H_t^j (1 - H_t^j)}\right)^\gamma\right)} \right),\end{aligned}$$

[21] Note that unemployment corresponds to $H_t = 0$.

from which we obtain

$$\frac{\partial R(W_t^j, H_t^j)}{\partial H_t^j} = -g(\theta_t)\frac{\gamma(1-2H_t^j)}{H_t^j(1-H_t^j)}\left(\phi_t^E \frac{\left(\frac{W_t H_t(1-H_t)}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma}{\left(1+\left(\frac{W_t H_t(1-H_t)}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma\right)^2} + \phi_t^U \frac{\left(\frac{P_t b_t}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma}{\left(1+\left(\frac{P_t b_t}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma\right)^2}\right),$$

and

$$\frac{\partial R(W_t^j, H_t^j)}{\partial W_t^j} = g(\theta_t)\frac{\gamma}{W_t^j}\left(\phi_t^E \frac{\left(\frac{W_t H_t(1-H_t)}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma}{\left(1+\left(\frac{W_t H_t(1-H_t)}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma\right)^2} + \phi_t^U \frac{\left(\frac{P_t b_t}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma}{\left(1+\left(\frac{P_t b_t}{W_t^j H_t^j(1-H_t^j)}\right)^\gamma\right)^2}\right).$$

Analogously, the separation function is given by

$$\begin{aligned} S(W_t^j, H_t^j) &= s + (1-s)\Bigg(f(\theta_t)\lambda^{EE}\frac{\left(\frac{W_t}{P_t}H_t(1-H_t)\right)^\gamma}{\left(\frac{W_t^j}{P_t}H_t^j(1-H_t^j)\right)^\gamma + \left(\frac{W_t}{P_t}H_t(1-H_t)\right)^\gamma} \\ &\qquad + \lambda^{EU}\frac{(\tau_t b_t)^\gamma}{\left(\tau_t\frac{W_t^j}{P_t}H_t^j(1-H_t^j)\right)^\gamma + (\tau_t b_t)^\gamma}\Bigg) \\ &= s + (1-s)\left(f(\theta_t)\lambda^{EE}\frac{1}{\left(1+\left(\frac{W_t^j H_t^j(1-H_t^j)}{W_t H_t(1-H_t)}\right)^\gamma\right)} + \lambda^{EU}\frac{1}{\left(1+\left(\frac{W_t^j H_t^j(1-H_t^j)}{P_t b_t}\right)^\gamma\right)}\right), \end{aligned}$$

from which we obtain

$$\frac{\partial S(W_t^j, H_t^j)}{\partial H_t^j} = -(1-s)\frac{\gamma(1-2H_t^j)}{H_t^j(1-H_t^j)}\left(f(\theta)\lambda^{EE}\frac{\left(\frac{W_t^j H_t^j(1-H_t^j)}{W_t H_t(1-H_t)}\right)^\gamma}{\left(1+\left(\frac{W_t^j H_t^j(1-H_t^j)}{W_t H_t(1-H_t)}\right)^\gamma\right)^2} + \lambda^{EU}\frac{\left(\frac{W_t^j H_t^j(1-H_t^j)}{P_t b_t}\right)^\gamma}{\left(1+\left(\frac{W_t^j H_t^j(1-H_t^j)}{P_t b_t}\right)^\gamma\right)^2}\right),$$

and

$$\frac{\partial S(W_t^j, H_t^j)}{\partial W_t^j} = -(1-s)\frac{\gamma}{W_t^j}\left(f(\theta)\lambda^{EE}\frac{\left(\frac{W_t^j H_t^j(1-H_t^j)}{W_t H_t(1-H_t)}\right)^\gamma}{\left(1+\left(\frac{W_t^j H_t^j(1-H_t^j)}{W_t H_t(1-H_t)}\right)^\gamma\right)^2} + \lambda^{EU}\frac{\left(\frac{W_t^j H_t^j(1-H_t^j)}{P_t b_t}\right)^\gamma}{\left(1+\left(\frac{W_t^j H_t^j(1-H_t^j)}{P_t b_t}\right)^\gamma\right)^2}\right).$$

We define ε_{S,W_t^j} as

$$\varepsilon_{S,W_t^j} \equiv \frac{W_t^j}{S(W_t^j, H_t^j)}\frac{\partial S(W_t^j, H_t^j)}{\partial W_t^j}.$$

Likewise, we define ε_{S,H_t^j} as

$$\varepsilon_{S,H_t^j} \equiv \frac{H_t^j}{S(W_t^j, H_t^j)} \frac{\partial S(W_t^j, H_t^j)}{\partial H_t^j}.$$

Note that under the consumption-sharing rule in (F.2)—that is, when the unemployment benefit b_t is set so that unemployed workers consume a constant fraction, ξ, of the average consumption of employed workers—both $R(W_t^j, H_t^j)$ and $S(W_t^j, H_t^j)$ are independent of the aggregate price index P_t in a symmetric equilibrium. Consequently, even in this extended model, where firms choose hours in addition to wages, the zero pass-through result continues to hold.

This can be demonstrated analytically through the following firm optimization problem.

Firm's Problem For analytical tractability, this section assumes $\bar{X}_t \equiv 0$, or equivalently $\alpha_{\bar{X}} = 0$, so that X is not used as an input in service-sector production. Therefore, Firm j solves:

$$\max_{\substack{\{P_{y,t}^j\},\{Y_t^j\},\{N_t^j\} \\ \{W_t^j\},\{V_t^j\},\{H_j^t\}}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P_{y,t}^j Y_t^j - W_t^j N_t^j H_t^j - c\left(\frac{V_t^j}{N_{t-1}^j}\right)^{\chi} V_t^j W_t - \frac{\psi}{2}\left(\frac{P_{y,t}^j}{P_{y,t-1}^j} - 1\right)^2 Y_t^j P_{y,t}^j \right.$$
$$\left. - \frac{\psi^w}{2}\left(\frac{W_t^j}{W_{t-1}^j} - 1\right)^2 W_t^j N_t^j \right), \tag{F.3}$$

subject to

$$N_t^j = (1 - S(W_t^j, H_t^j))N_{t-1}^j + R(W_t^j, H_t^j)V_t^j, \tag{F.4}$$
$$Y_t^j = A_t^j N_t^j H_t^j, \tag{F.5}$$
$$\frac{Y_t^j}{Y_t} = \left(\frac{P_{y,t}^j}{P_{y,t}}\right)^{-\epsilon}. \tag{F.6}$$

Note that (F.4) shows that both the separation and recruiting functions depend on hours H_t^j. In addition, equation (F.5) indicates that firm output is determined not only by employment N_t^j but also by the number of hours worked H_t^j.

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P_{y,t}^j Y_t^j - W_t^j N_t^j H_t^j - c\left(\frac{V_t^j}{N_{t-1}^j}\right)^{\chi} V_t^j W_t - \frac{\psi}{2}\left(\frac{P_{y,t}^j}{P_{y,t-1}^j} - 1\right)^2 Y_t^j P_{y,t}^j \right.$$

$$- \frac{\psi^w}{2}\left(\frac{W_t^j}{W_{t-1}^j} - 1\right)^2 W_t^j N_t^j + \lambda_t[(1 - S(W_t^j, H_t^j))N_{t-1}^j + R(W_t^j, H_t^j)V_t^j - N_t^j]$$
$$+ \mu_t[-Y_t^j + A_t^j N_t^j H_t^j] + \Omega_t\left[-\frac{Y_t^j}{Y_t} + \left(\frac{P_{y,t}^j}{P_{y,t}}\right)^{-\epsilon}\right]\Bigg).$$

Taking the first-order condition with respect to $\{P_{y,t}^j\}$, we obtain

$$\Omega_t = \frac{P_{y,t}^j Y_t}{\epsilon}\left[1 - \psi\left(\frac{P_{y,t}^j}{P_{y,t-1}^j} - 1\right)\frac{P_{y,t}^j}{P_{y,t-1}^j} - \frac{\psi}{2}\left(\frac{P_{y,t}^j}{P_{y,t-1}^j} - 1\right)^2 + \frac{\psi}{1+\rho}\left(\frac{P_{y,t+1}^j}{P_{y,t}^j} - 1\right)\left(\frac{P_{y,t+1}^j}{P_{y,t}^j}\right)^2 \frac{Y_{t+1}^j}{Y_t^j}\right]. \tag{F.7}$$

Taking the first-order condition with respect to $\{Y_t^j\}$, we obtain

$$\mu_t + \frac{\Omega_t}{Y_t} = P_{y,t}^j\left(1 - \frac{\psi}{2}\left(\frac{P_{y,t}^j}{P_{y,t-1}^j} - 1\right)^2\right). \tag{F.8}$$

Taking the first-order condition with respect to $\{N_t^j\}$, we obtain

$$\lambda_t = \frac{\lambda_{t+1}}{1+\rho}(1 - S(W_{t+1}^j, H_{t+1}^j)) + \mu_t A_t^j H_t^j - W_t^j H_t^j + \frac{1}{1+\rho} c\chi\left(\frac{V_{t+1}^j}{N_t^j}\right)^{\chi+1} W_{t+1} - \frac{\psi^w}{2}\left(\frac{W_t^j}{W_{t-1}^j} - 1\right)^2 W_t^j$$

(F.9)

Taking the first-order condition with respect to $\{W_t^j\}$, we obtain

$$\lambda_t = \frac{1}{\left(\varepsilon_{S,W_t^j}\frac{S}{W_t^j}N_{t-1}^j - \varepsilon_{R,W_t^j}\frac{R}{W_t^j}V_t^j\right)}\Bigg[- N_t^j H_t^j - \psi^w\left(\frac{W_t^j}{W_{t-1}^j} - 1\right)\frac{W_t^j}{W_{t-1}^j}N_t^j - \frac{\psi^w}{2}\left(\frac{W_t^j}{W_{t-1}^j} - 1\right)^2 N_t^j$$
$$+ \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}^j}{W_t^j} - 1\right)\left(\frac{W_{t+1}^j}{W_t^j}\right)^2 N_{t+1}^j\Bigg]. \tag{F.10}$$

Taking the first-order condition with respect to $\{V_t^j\}$, we obtain

$$\lambda_t = \frac{c(\chi+1)W_t}{R(W_t^j, H_t^j)}\left(\frac{V_t^j}{N_{t-1}^j}\right)^{\chi}. \tag{F.11}$$

Finally, taking the first-order condition with respect to $\{H_t^j\}$, we obtain

$$\lambda_t = \frac{1}{\left(\varepsilon_{S,H_t^j}\frac{S_t^j}{H_t^j}N_{t-1}^j - \varepsilon_{R,H_t^j}\frac{R_t^j}{H_t^j}V_t^j\right)}\left(\mu_t A_t^j N_t^j - W_t^j N_t^j\right). \tag{F.12}$$

Merging equations (F.11) and (F.12), we can obtain

$$\frac{c(\chi+1)W_t}{R(W_t^j,H_t^j)}\left(\frac{V_t^j}{N_{t-1}^j}\right)^{\chi}\left(\varepsilon_{S,H_t^j}\frac{S_t^j N_{t-1}^j}{H_t^j} - \varepsilon_{R,H_t^j}\frac{R_t^j V_t^j}{H_t^j}\right) = \mu_t A_t^j N_t^j - W_t^j N_t^j,$$

which in our symmetric equilibrium becomes

$$\frac{\mu_t A_t}{\bar{W}_j} = \frac{1}{N_t}\frac{c(\chi+1)}{R(W_t,H_t)}\left(\frac{V_t}{N_{t-1}}\right)^{\chi}\left(\varepsilon_{S,H_t^j}\frac{S_t N_{t-1}}{H_t} - \varepsilon_{R,H_t^j}\frac{R_t V_t}{H_t}\right) + 1.$$

Wage Phillips curve and pass-through In this case, the wage Phillips curve is derived from (F.10) and (F.11):

$$\begin{aligned}
&\frac{c(\chi+1)W_t}{R(W_t^j,H_t^j)}\left(\frac{V_t^j}{N_{t-1}^j}\right)^{\chi} \\
&= \frac{1}{\left(\varepsilon_{S,W_t^j}\frac{S}{W_t^j}N_{t-1}^j - \varepsilon_{R,W_t^j}\frac{R}{W_t^j}V_t^j\right)}\Bigg[-N_t^j H_t^j - \psi^w\left(\frac{W_t^j}{W_{t-1}^j}-1\right)\frac{W_t^j}{W_{t-1}^j}N_t^j - \frac{\psi^w}{2}\left(\frac{W_t^j}{W_{t-1}^j}-1\right)^2 N_t^j \\
&\qquad + \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}^j}{W_t^j}-1\right)\left(\frac{W_{t+1}^j}{W_t^j}\right)^2 N_{t+1}^j\Bigg],
\end{aligned}$$

which in our symmetric equilibrium becomes:

$$\begin{aligned}
&\frac{c(\chi+1)W_t}{R(W_t,H_t)}\left(\frac{V_t}{N_{t-1}}\right)^{\chi} \\
&= \frac{1}{\left(\varepsilon_{S,W_t^j}\frac{S}{W_t}N_{t-1} - \varepsilon_{R,W_t^j}\frac{R}{W_t}V_t\right)}\Bigg[-N_t H_t - \psi^w\left(\frac{W_t}{W_{t-1}}-1\right)\frac{W_t}{W_{t-1}}N_t - \frac{\psi^w}{2}\left(\frac{W_t}{W_{t-1}}-1\right)^2 N_t \\
&\qquad + \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}}{W_t}-1\right)\left(\frac{W_{t+1}}{W_t}\right)^2 N_{t+1}\Bigg].
\end{aligned} \tag{F.13}$$

F.1 Analytical Results

Recall in this case that $R(W_t^j, H_t^j)$ and $S(W_t^j, H_t^j)$ are independent of the aggregate price index P_t in a symmetric equilibrium, implying that

$$\frac{1}{\left(\varepsilon_{S,W_t^j}\frac{S}{W_t}N_{t-1} - \varepsilon_{R,W_t^j}\frac{R}{W_t}V_t\right)}$$

in (F.13) is independent of P_t as well.

Imagine that the central bank stabilizes the labor market in response to a shock to X_t^s; that is, labor market variables such as N_t and V_t remain stabilized.[22] Note from (F.13) that there is no price aggregator, implying that a higher price level does not pass through to equilibrium wages W_t when the central bank stabilizes the labor market. This result can be summarized as follows:

Proposition 6 (Employment Stabilization and Wage Inflation) *Consider the wage Phillips curve (F.13) under $\varpi = 0$. If monetary policy stabilizes employment at its steady-state level following a negative endowment shock ($\phi_N \to \infty$)—satisfying $N_t = N$—then wage inflation is stabilized:*

$$\check{\Pi}_t^w = 0 \quad \textit{for all } t.$$

In this equilibrium, hours are also stabilized, so that $H_t = H$ for all t.

Thus, we conclude that the introduction of an hours margin does not change the zero pass-through result of the baseline model in the main text.

F.2 Quantitative Results

We now simulate the extended model under the baseline calibration (Table 2). The one exception is that in the baseline consumption rule ($\varpi = 0$), we set $\xi = 4$, so employed workers consume 4 times what unemployed workers consume. We end up with hours in equilibrium $H = .469$, so $\frac{C^E H}{C^U} \approx 2$. This keeps the acceptance probabilities across employment and unemployment roughly equal to what their values in the baseline model in the main text.

For monetary policy, we consider two alternative policy rules. Under the first rule, the

[22] Note that this monetary policy rule is equivalent to a Taylor rule with $\phi_N \to \infty$.

central bank follows

$$1 + i_t = (1 + i) \left(\frac{\Pi_{Y,t}}{\Pi_Y} \right)^{\phi_\pi} \left(\frac{N_t}{N} \right)^{\phi_N} \exp(\varepsilon_{i,t}), \tag{F.14}$$

where the central bank responds to service-sector inflation and employment, consistent with the baseline setup in the main text. Alternatively, the central bank may follow

$$1 + i_t = (1 + i) \left(\frac{\Pi_{Y,t}}{\Pi_Y} \right)^{\phi_\pi} \left(\frac{N_t H_t}{N H} \right)^{\phi_N} \exp(\varepsilon_{i,t}), \tag{F.15}$$

where policy responds to total hours worked, $N_t H_t$, rather than employment alone, N_t.

We consider the policy rule (F.15) largely for illustrative purposes. In neoclassical models and union wage-setting models, labor supply is typically measured in terms of total hours worked rather than employment counts. Thus, total labor input $N_t H_t$ in our extended model provides a more comparable measure to labor supply in those environments.

The blue solid line in Figure F.21 illustrates the model's response under the consumption-sharing rule (F.2) and the policy rule (F.14). Consistent with the results in the main text, a negative endowment shock—that is, a decline in X_t^s—reduces demand for services and leads to declines in wages, hours worked, and vacancies. Targeting $N_t H_t$, rather than N_t alone, leads to higher wage inflation. However, the overall wage response remains smaller than in the benchmark specification presented in the main text.

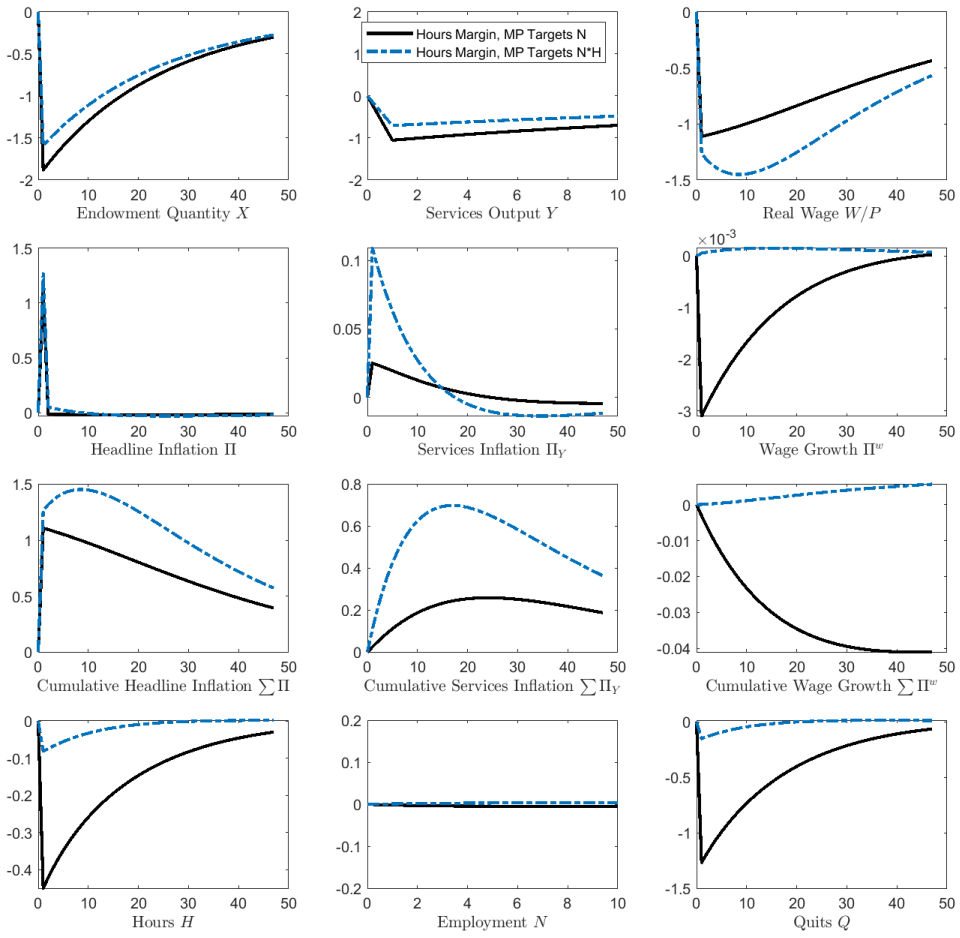


Figure F.21: Model Behavior with an Hours Margin Following a Negative Endowment Shock Under the Baseline Calibration

Appendix G Dual Labor Markets: Separately Targeting Unemployed Workers

In this section, we extend the benchmark model by allowing firms to target employed and unemployed workers differently, offering distinct wages to each group and posting different numbers of vacancies.[23] We show that workers who are hired out of unemployment receive lower wages relative to workers poached from other firms. In addition, wages for poached workers are more procyclical than those for workers who are hired out of unemployment.

Considering how households allocate consumption between employed and unemployed workers, we continue to assume that an employed worker i at firm j faces an effective tax rates $1-\tau_t$. Consequently, their consumption is:

$$C_{ijt} = \tau_t \frac{W_{ijt}}{P_t},$$

where W_{ijt} is the wage of worker i at firm j. Unemployed workers receive a real unemployment benefit b_t, which is

$$b_t = \varpi b + (1-\varpi)\xi^{-1}\frac{\bar{W}_t}{P_t} \tag{G.1}$$

where b and ξ are constants, and $\bar{W}_t$ is the average wage of employed workers.[24] Finally, unemployed workers consume $C_t^u = \tau_t b_t$ each period.

Tightness Note that labor market tightness for workers hired from unemployment is given by

$$\theta_t^U = \frac{V_t^U}{U_{t-1}}, \tag{G.2}$$

where V_t^U represents the total number of vacancies targeting unemployed workers.

[23] In effect, we assume that labor markets for currently employed and unemployed workers are segmented. This setting is consistent with Cheremukhin and Restrepo-Echavarria (2024), who show that a dual vacancy model—one that separates the search processes of unemployed and employed workers—fits the data better than traditional models that assume a homogeneous labor market.

[24] In the symmetric equilibrium described below, it is given by

$$\bar{W}_t = W_t^E \frac{N_t^E}{N_t^E + N_t^U} + W_t^U \frac{N_t^U}{N_t^E + N_t^U}.$$

Likewise, labor market tightness for workers hired from employment at other firms is:

$$\theta_t^E = \frac{V_t^E}{(1-s)\ \lambda_{EE}\ (1-U_{t-1})}, \tag{G.3}$$

where V_t^E is the total number of vacancies targeting employed workers who search on the job. For tractability, we assume that workers hired from unemployment and workers hired from other firms both exhibit the same on-the-job search intensity, λ_{EE}.

Workers hired from unemployment Workers who are hired from unemployment by firm j are paid W_t^{Uj}. Firm j faces the following recruiting function when hiring unemployed workers:

$$R_t^{Uj} = g(\theta_t^U)\frac{(\tau_t W_t^{Uj})^\gamma}{(\tau_t W_t^{Uj})^\gamma + (\tau_t P_t b_t)^\gamma}. \tag{G.4}$$

Since workers hired from unemployment engage in on-the-job search with intensity λ_{EE}, the separation function for this segment of the labor market is given by:

$$S_t^{Uj} = s + (1-s)\left(\lambda^{EE} f(\theta_t^E)\frac{(W_t^E)^\gamma}{(W_t^{Uj})^\gamma + (W_t^E)^\gamma} + \lambda_{EU}\frac{(P_t b_t)^\gamma}{(W_t^{Uj})^\gamma + (P_t b_t)^\gamma}\right), \tag{G.5}$$

where W_t^E denotes the (average) wage offered to workers hired from employment at other firms. Note that in the symmetric equilibrium (i.e., $W_t^{Uj} = W_t^U$), the dual structure of the labor market implies—unlike in the baseline model—that $\frac{(W_t^E)^\gamma}{(W_t^{Uj})^\gamma + (W_t^E)^\gamma} \neq \frac{1}{2}$.

The law of motion for workers in firm j who were hired out of unemployment is given by:

$$N_t^{Uj} = (1 - S_t^{Uj}(W_t^{Uj}))N_{t-1}^{Uj} + R_t^{Uj}(W_t^{Uj})V_t^{Uj}.$$

Workers hired from employment Workers poached from other firms by firm j are paid W_t^{Ej}. Firms recruiting among the employed may encounter two types of workers: those who previously moved from one firm to another (E-to-E) and earn W_t^{Ek}, and those whose only employment since leaving unemployment has been a single job (U-to-E), earning W_t^{Uk}.

Let ϕ_t^E denote the share of searchers with more than one job in their most recent employment spell, and ϕ_t^U be the share of searchers who have held only their current job during their current employment spell. The recruiting function for workers already employed can then be written as

$$R_t^{Ej} = g(\theta_t^E)\left(\phi_t^E \frac{(W_t^{Ej})^\gamma}{(W_t^{Ej})^\gamma + (W_t^E)^\gamma} + \phi_t^U \frac{(W_t^{Ej})^\gamma}{(W_t^{Ej})^\gamma + (W_t^U)^\gamma}\right). \tag{G.6}$$

When these workers search on the job, they are, by definition, searching within the pool of workers making an E-to-E transition. Accordingly, the separation function for poached workers is given by:

$$S_t^{Ej} = s + (1-s)\left(\lambda^{EE} f(\theta_t^E) \frac{(W_t^E)^\gamma}{(W_t^{Ej})^\gamma + (W_t^E)^\gamma} + \lambda_{EU} \frac{(P_t b_t)^\gamma}{(W_t^{Ej})^\gamma + (P_t b_t)^\gamma}\right). \tag{G.7}$$

Note that in the symmetric equilibrium (i.e., $W_t^{Ej} = W_t^E$), the dual structure of the labor market implies—as in the baseline model, but unlike in (G.5)—that $\frac{(W_t^E)^\gamma}{(W_t^{Ej})^\gamma + (W_t^E)^\gamma} = \frac{1}{2}$.

The law of motion for poached workers in firm j is:

$$N_t^{Ej} = (1 - S_t^{Ej}(W_t^{Ej}))N_{t-1}^{Ej} + R_t^{Ej}(W_t^{Ej})V_t^{Ej}.$$

Finally, we separately define recruiting and separation elasticities for the two markets: ε^E_{R,W_t}, ε^U_{R,W_t}, ε^E_{S,W_t}, and ε^U_{S,W_t}, as in the main text.

G.1 Firm's Problem

Firm j combines its E-to-E and U-to-E workers, N_t^{Ej} and N_t^{Uj}, into total employment,

$$N_t^j = N_t^{Ej} + N_t^{Uj},$$

and uses this labor input together with intermediate inputs $\bar{X}_t^j$ in production. It solves the following problem, accounting for the dual structure of the labor market:

$$\max_{\substack{\{P_{y,t}^j\},\{Y_t^j\},\{\bar{X}_t^j\},\{N_t^{Ej}\},\{N_t^{Uj}\} \\ \{W_t^{Ej}\},\{W_t^{Uj}\},\{V_t^{Ej}\},\{V_t^{Uj}\}}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \pi_t \tag{G.8}$$

where

$$\pi_t \equiv P^j_{y,t} Y^j_t - W^{Ej}_t N^{Ej}_t - W^{Uj}_t N^{Uj}_t - c\left(\frac{V^{Ej}_t}{N^{Ej}_{t-1}}\right)^{\chi} V^{Ej}_t W^E_t - c\left(\frac{V^{Uj}_t}{N^{Uj}_{t-1}}\right)^{\chi} V^{Uj}_t W^U_t$$
$$- \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W^{Ej}_t}{W^{Ej}_{t-1}} - 1\right)^2 W^{Ej}_t N^{Ej}_t - \frac{\psi^w}{2}\left(\frac{W^{Uj}_t}{W^{Uj}_{t-1}} - 1\right)^2 W^{Uj}_t N^{Uj}_t, \tag{G.9}$$

subject to

$$N^{Uj}_t = (1 - S^{Uj}_t(W^{Uj}_t)) N^{Uj}_{t-1} + R^{Uj}_t(W^{Uj}_t) V^{Uj}_t, \tag{G.10}$$
$$N^{Ej}_t = (1 - S^{Ej}_t(W^{Ej}_t)) N^{Ej}_{t-1} + R^{Ej}_t(W^{Ej}_t) V^{Ej}_t, \tag{G.11}$$
$$Y^j_t = A^j_t \left(\alpha_N^{\frac{1}{\bar\eta}} \left(N^{Ej}_t + N^{Uj}_t\right)^{\frac{\bar\eta-1}{\bar\eta}} + \alpha_{\bar X}^{\frac{1}{\bar\eta}} \left(\bar X^j_t\right)^{\frac{\bar\eta-1}{\bar\eta}}\right)^{\frac{\bar\eta}{\bar\eta-1}}, \tag{G.12}$$
$$\frac{Y^j_t}{Y_t} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}. \tag{G.13}$$

We assume the same wage adjustment cost function à la Rotemberg (1982) for workers who are hired from unemployment and poached workers. Taking the Lagrangian, we obtain:

$$\begin{aligned}
\mathcal{L} := \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \Bigg[& \left(P^j_{y,t} Y^j_t - W^{Ej}_t N^{Ej}_t - W^{Uj}_t N^{Uj}_t - c\left(\frac{V^{Ej}_t}{N^{Ej}_{t-1}}\right)^{\chi} V^{Ej}_t W^E_t - c\left(\frac{V^{Uj}_t}{N^{Uj}_{t-1}}\right)^{\chi} V^{Uj}_t W^U_t \right. \\
& - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W^{Ej}_t}{W^{Ej}_{t-1}} - 1\right)^2 W^{Ej}_t N^{Ej}_t \\
& \left. - \frac{\psi^w}{2}\left(\frac{W^{Uj}_t}{W^{Uj}_{t-1}} - 1\right)^2 W^{Uj}_t N^{Uj}_t \right) \\
& + \lambda_{U,t}(1 - S^{Uj}_t(W^{Uj}_t)) N^{Uj}_{t-1} + R^{Uj}_t(W^{Uj}_t) V^{Uj}_t - N^{Uj}_t) \\
& + \lambda_{E,t}(1 - S^{Ej}_t(W^{Ej}_t))) N^{Ej}_{t-1} + R^{Ej}_t(W^{Ej}_t) V^{Ej}_t - N^{Ej}_t) \\
& + \mu_t \left(A^j_t \left(\alpha_N^{\frac{1}{\bar\eta}} \left(N^{Ej}_t + N^{Uj}_t\right)^{\frac{\bar\eta-1}{\bar\eta}} + \alpha_{\bar X}^{\frac{1}{\bar\eta}} \left(\bar X^j_t\right)^{\frac{\bar\eta-1}{\bar\eta}}\right)^{\frac{\bar\eta}{\bar\eta-1}} - Y^j_t \right) \\
& + \Omega_t \left(\left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon} - \frac{Y^j_t}{Y_t} \right) \Bigg].
\end{aligned}$$

Optimality conditions In a symmetric equilibrium where $P_{y,t}^j = P_{y,t}$, $V_t^{Uj} = V_t^U$, $V_t^{Ej} = V_t^E$, $W_t^{Uj} = W_t^U$, $W_t^{Ej} = W_t^E$, $\bar{X}_t^j = \bar{X}_t$, $A_t^j = A_t$ for all j, we can characterize the first-order conditions and derive the price and wage Phillips curves for each labor market—one for poached workers (E-to-E) and one for workers hired from unemployment (U-to-E). Here we only characterize the wage Phillips curve for W_t^E and W_t^U.

For poached workers (E-to-E), the wage Phillips curve for W_t^E is given by

$$\begin{aligned}\psi^w\left(\Pi_t^{E,w}-1\right)\Pi_t^{E,w}+1 =& c(1+\chi)\left(\frac{V_t^E}{N_{t-1}^E}\right)^{\chi}\left(\frac{V_t^E}{N_t^E}\varepsilon_{R,W_t}^E-\frac{N_{t-1}^E}{N_t^E}\frac{S^E(W_t^E)}{R^E(W_t^E)}\varepsilon_{S,W_t}^E\right)\\ &+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^{E,w}-1\right)(\Pi_{t+1}^{E,w})^2\frac{N_{t+1}^E}{N_t^E}.\end{aligned} \tag{G.14}$$

where $\Pi_t^{E,w} \equiv \frac{W_t^E}{W_{t-1}^E}$. Similarly, for workers who are hired right out of unemployment (U-to-E), the wage Phillips Curve for W_t^U is given by

$$\begin{aligned}\psi^w\left(\Pi_t^{U,w}-1\right)\Pi_t^{U,w}+1 =& c(1+\chi)\left(\frac{V_t^U}{N_{t-1}^U}\right)^{\chi}\left(\frac{V_t^U}{N_t^U}\varepsilon_{R,W_t}^U-\frac{N_{t-1}^U}{N_t^U}\frac{S^U(W_t^U)}{R^U(W_t^U)}\varepsilon_{S,W_t}^U\right)\\ &+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^{U,w}-1\right)(\Pi_{t+1}^{U,w})^2\frac{N_{t+1}^U}{N_t^U},\end{aligned} \tag{G.15}$$

where $\Pi_t^{U,w} \equiv \frac{W_t^U}{W_{t-1}^U}$.

G.2 Analytical Results

We assume $\varpi = 0$ in (G.1) for this subsection. Recall in this case that $R_t^{Ej}(W_t^{Ej})$, $R_t^{Uj}(W_t^{Uj})$, $S_t^{Ej}(W_t^{Ej})$ and $S_t^{Uj}(W_t^{Uj})$ are independent of the aggregate price index P_t in a symmetric equilibrium, implying that

$$\frac{V_t^E}{N_t^E}\varepsilon_{R,W_t}^E-\frac{N_{t-1}^E}{N_t^E}\frac{S^E(W_t^E)}{R^E(W_t^E)}\varepsilon_{S,W_t}^E$$

in (G.14) and

$$\frac{V_t^U}{N_t^U}\varepsilon_{R,W_t}^U-\frac{N_{t-1}^U}{N_t^U}\frac{S^U(W_t^U)}{R^U(W_t^U)}\varepsilon_{S,W_t}^U$$

in (G.15) are independent of P_t as well.

We first assume that the central bank stabilizes the labor market in response to a shock to X_t^s; that is, the monetary authority adjusts the policy rate so that labor market variables

such as N_t^E, N_t^U, V_t^E and V_t^U remain stabilized.[25] Note from (G.14) and (G.15) that there is no price aggregator, implying that cost-of-living shocks do not pass through to wages. This result can be summarized as follows:

Proposition 7 (Employment Stabilization and Wage Inflation) *Consider the wage Phillips curves for W_t^E and W_t^U, given by (G.14) and (G.15), respectively, under $\varpi = 0$. If monetary policy stabilizes employment at its steady-state level following a negative endowment shock ($\phi_N \to \infty$)—satisfying $N_t = N$—then wage inflation is stabilized:*[26]

$$\check{\Pi}_t^{E,w} = \check{\Pi}_t^{U,w} = 0 \quad \textit{for all } t.$$

In this equilibrium, all other labor market variables are also stabilized.

The black solid line in Figure G.22 corresponds to this case, showing that, when monetary policy stabilizes the labor market with $\phi_N \to \infty$, both W_t^E and W_t^U, remain unchanged over time.

G.3 Quantitative Results

We now simulate the model under the baseline calibration in the main text. For monetary policy, we consider the benchmark Taylor rule:

$$1 + i_t = (1 + i) \left(\frac{\Pi_{Y,t}}{\Pi_Y} \right)^{\phi_\pi} \left(\frac{N_t}{N} \right)^{\phi_N} \exp(\varepsilon_{i,t}), \tag{G.16}$$

where the central bank responds to service-sector inflation and employment. We focus on the baseline consumption sharing rule $\varpi = 0$. In this calibration, workers who have made a job-to-job transition earn higher wages: $W^E/P = .794$ and $W^U/P = .685$. The majority of workers having reached the higher wage: $N^E = .639$ and $N^U = .334$.

The blue dot-dashed line in Figure G.22 shows the model's response under the baseline consumption-sharing rule (G.1) and the policy rule (G.16). As in the results in the main text, a negative endowment shock—that is, a decline in X_t^s—reduces demand for services and leads to declines in wages, employment, and vacancies. Note that E-to-E wage inflation declines by slightly more more than U-to-E wage inflation.[27] Also note that cumulative wage growth is negative and closely resembles that generated by our baseline one-wage model.

[25] Note that this monetary policy rule is equivalent to a Taylor rule with $\phi_N \to \infty$.

[26] Here, $N_t = N_t^E + N_t^U$.

[27] On impact under $\phi_N = 1$: $\Pi^{E,w}$ falls by about .007%, while $\Pi^{U,w}$ falls by about .005%.

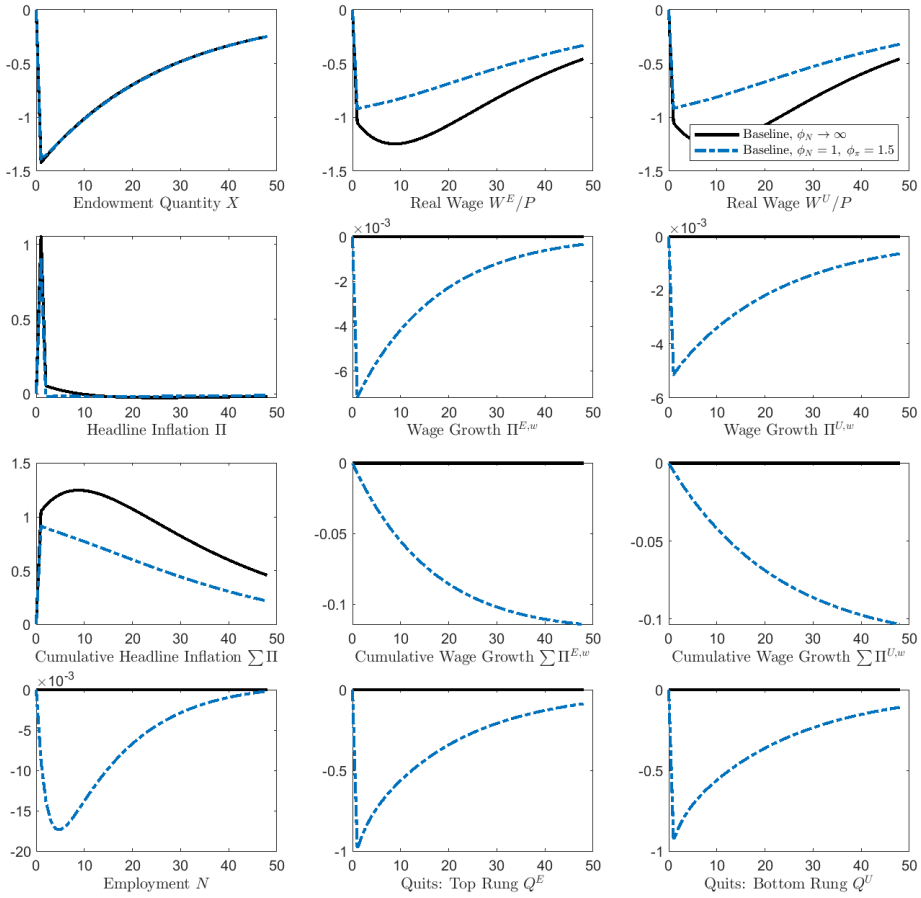


Figure G.22: Model Behavior with Dual Labor Markets Separately Targeting Unemployed Workers, Following a Negative Endowment Shock Under the Baseline Calibration

Appendix H Forward-Looking Workers

In this section, we relax the assumption that workers are completely myopic. While relaxing this assumption increases the complexity of the model, we show here that it barely matters for the dynamics: the response of wage inflation and other key labor market variables to a negative shock in X_t^s is nearly identical, assuming the monetary authority stabilizes output as well as inflation. For analytical tractability, this section assumes $\bar{X}_t \equiv 0$, or equivalently $\alpha_{\bar{X}} = 0$ in (5), so that X is not used as an input in service-sector production.

To understand why, note that the only part of the model that changes is the firm's problem, and more specifically, the first-order condition for wages. Even relaxing myopia, it continues to be the case that stabilizing output Y_t (and hence N_t) stabilizes the firm's first-order condition for wages and hence wage inflation. However, this is in spite of the fact that this first order condition becomes much more complicated because separation and recruiting rates at time t now depend not just on the wage at time t, but also on the wage promised for all $T > t$. The reason for this is that now when a worker considers joining a firm, they know they may remain there for more than one period. They thus consider not just the wage offered today, but also the wage promised in the future.

From technical standpoint, this makes the firm's problem *time-inconsistent*: when making a plan for wages, it is optimal to promise to offer a high wage in the future in order to raise the recruiting rate today "for free." But when the time arrives to pay the higher wage, the firm will be tempted to renege. This issue shows up in the firm's problem as a first-order condition for wages which is non-stationary, as the first period is special, which introduces additional difficulty both in solving and discussing the model.

To make progress, and keep the model with forward looking workers as similar as possible to that in the main text to ease comparison, we again study the effect of an unanticipated "MIT shock" to the endowment X^s that becomes known at some time $t = 0$. At that date, we assume that firms are paying wages that they committed to at some point in the distant past, $t = -\infty$ so that at $t = -1$ the economy has converged to a long run steady state which we will describe below.

In letting firms respond to this MIT shock, we solve for the responses that the firms would have found optimal had they known about the shock back in the infinite past. More formally, we assume firms knew at $t = -\infty$ that there was some chance p of the shock occurring at time t, and solve for the economy's response to the shock under that optimal plan as $p \to 0$, so that the shock is entirely unanticipated.[28]

[28] In the context of time-inconsistent optimal monetary policy, this is equivalent to the "timeless approach"

As in the main text, we will solve for a symmetric, perfect-foresight equilibrium where all firms set the same wage. Unlike in the main text, we will solve for the solution to the nonlinear models (with and without myopia) to better compare their behavior and make the point that the assumption of myopic workers is largely-innocuous simplifying assumption.

H.1 Firm's Problem With Dynamic Separation and Recruiting Rates

Recall that the only choice facing workers occurs when they are offered a chance to take a job. With probability $\lambda_{EE} f(\theta_t)$, workers meet alternative jobs, and with probability λ_{EU} they are allowed to consider quitting into unemployment. When workers meet alternate jobs, or consider the unemployment state, they draw idiosyncratic utilities ι from a type-1 extreme value distribution with scale parameter γ. They then choose the state which yields the greatest utility: for example, let $\mathcal{V}_{jt}$ be the value of being employed at firm j and $\bar{\mathcal{V}}_t$ be the value of being employed at the other matched firm. Then a worker at firm j who matches with another firm of value $\bar{\mathcal{V}}_t$ maximizes:

$$\max\{\mathcal{V}_{jt} + \iota_{jt}, \bar{\mathcal{V}}_t + \iota_{kt}\}.$$

Given this, the expected value of having a choice between two value functions next period is $\mathcal{V}^E(\mathcal{V}^a, \mathcal{V}^b) = \frac{1}{\gamma} \ln\left(\exp\left(\gamma \mathcal{V}^a\right) + \exp\left(\gamma \mathcal{V}^b\right)\right)$. So the worker's Bellman equation at firm j is thus the following: letting $\mathcal{V}^u_t$ be the value of the unemployed state at time t, and assuming all firms other than j have identical value $\bar{V}_t$ (which is without loss of generality, given that we will search for a symmetric equilibrium later),

$$\begin{aligned}\mathcal{V}_{jt} =& \ln\left(\frac{\tau_t W_{jt}}{P_t}\right) + \beta s \mathcal{V}^u_{t+1} \\ &+ \beta(1-s)\left[\lambda_{EE} f(\theta_{t+1}) \mathcal{V}^E(\mathcal{V}_{j,t+1}, \bar{\mathcal{V}}_{t+1}) + \lambda_{EU} \mathcal{V}^E(\mathcal{V}_{j,t+1}, \mathcal{V}^u_{t+1}) + (1 - \lambda_{EE} f(\theta_{t+1}) - \lambda_{EU}) \mathcal{V}_{j,t+1}\right] \\ \equiv& \mathcal{F}_t(\mathcal{V}_{j,t+1})\end{aligned}$$

where we write this function $\mathcal{F}_t$ as time-varying because of its dependence on the real wage w_t, tax rate $1 - \tau_t$, tightness θ_{t+1}, and $\bar{\mathcal{V}}_{t+1}$. We also assume workers discount the future at rate β, which need not equal $\frac{1}{1+\rho}$. This allows us to easily nest the myopic worker case where $\frac{1}{1+\rho} > \beta = 0$. In practice, we solve a version of model where workers discount the future with $\beta = .96$ which corresponds to an annual discount rate of $\beta^{12} \approx .60$, consistent with experimental evidence; see Michaillat and Saez (2021) for a discussion and summary

to computing optimal policy under commitment.

of this evidence. So we can plug in the following:

$$\mathcal{V}^E(\mathcal{V}_{j,t+1}, \bar{\mathcal{V}}_{t+1}) = \frac{1}{\gamma} \ln\left(\exp\left(\gamma \mathcal{V}_{j,t+1}\right) + \exp\left(\gamma \bar{\mathcal{V}}_{t+1}\right)\right)$$
$$\mathcal{V}^E(\mathcal{V}_{j,t+1}, \mathcal{V}^u_{t+1}) = \frac{1}{\gamma} \ln\left(\exp\left(\gamma \mathcal{V}_{j,t+1}\right) + \exp\left(\gamma \mathcal{V}^u_{t+1}\right)\right)$$

To see that $\mathcal{V}_{jt}$ depends on the whole path of nominal wages, $\{W_{jt}\}_{t=0}^{\infty}$, we can evaluate the following derivative: for $\mathcal{S} \geqslant 1$,

$$\begin{aligned}\frac{d\mathcal{V}_{jt}}{dW_{j,t+\mathcal{S}}} &= \frac{d}{dW_{j,t+\mathcal{S}}} \mathcal{F}_t\left(\mathcal{F}_{t+1}\left(\ldots \mathcal{F}_{t+\mathcal{S}-1}(\mathcal{V}_{j,t+\mathcal{S}}) \ldots\right)\right) \\ &= \mathcal{F}'_t(\mathcal{V}_{j,t+1}) \times \mathcal{F}'_{t+1}(\mathcal{V}_{j,t+2}) \times \ldots \times \mathcal{F}'_{t+\mathcal{S}-1}(\mathcal{V}_{j,t+\mathcal{S}}) \frac{d\mathcal{V}_{j,t+\mathcal{S}}}{dW_{j,t+\mathcal{S}}}\end{aligned}$$

where

$$\mathcal{F}'_t(\mathcal{V}_{j,t+1}) = \beta(1-s)\left[\frac{\lambda_{EE} f(\theta_{t+1})}{1+\exp\left(\gamma(\bar{\mathcal{V}}_{t+1} - \mathcal{V}_{j,t+1})\right)} + \frac{\lambda_{EU}}{1+\exp\left(\gamma(\mathcal{V}^u_{t+1} - \mathcal{V}_{j,t+1})\right)} + 1 - \lambda_{EE} f(\theta_{t+1}) - \lambda_{EU}\right]$$

and since the final term $\frac{d\mathcal{V}_{j,t+\mathcal{S}}}{dW_{j,t+\mathcal{S}}} = \frac{1}{W_{j,t+\mathcal{S}}} > 0$, $\frac{d\mathcal{V}_{jt}}{dW_{j,t+\mathcal{S}}} > 0$. Promising a higher wage in the future makes a job offer at firm j more attractive, and helps recruit workers today.

In a symmetric equilibrium with $\bar{\mathcal{V}}_t = \mathcal{V}_{jt}$ always, and where the household fixes $\bar{\mathcal{V}}_t - \mathcal{V}^u_t$ at some value $\ln \xi$ (by varying the consumption of the unemployed),[29] this simplifies greatly to

$$\mathcal{F}'_t(\mathcal{V}_{j,t+1} = \bar{\mathcal{V}}_{t+1}) = \beta(1-s) \underbrace{\left[\frac{\lambda_{EE} f(\theta_{t+1})}{2} + \frac{\lambda_{EU}}{1+\xi^{-\gamma}} + 1 - \lambda_{EE} f(\theta_{t+1}) - \lambda_{EU}\right]}_{\equiv P(\theta_{t+1}) < 1}.$$

Thus in a symmetric equilibrium, and for $\mathcal{S} \geqslant 1$ we can write down these derivatives as:

$$\frac{d\mathcal{V}_{jt}}{dW_{j,t+\mathcal{S}}} = \frac{(\beta(1-s))^{\mathcal{S}} \prod_{k=1}^{\mathcal{S}} P(\theta_{t+k})}{W_{j,t+\mathcal{S}}},$$

[29] With log utility and myopia ($\beta = 0$) with $\varpi = 0$ in the consumption-sharing rule (12), we have $\bar{\mathcal{V}}_t - \mathcal{V}^u_t = \ln C^e_t - \ln C^u_t = \ln \xi$ implying $\frac{C^e_t}{C^u_t} = \xi$ as before.

and for $\mathcal{S} = 0$:

$$\frac{d\mathcal{V}_{jt}}{dW_{j,t}} = \frac{1}{W_{j,t}}.$$

So we obtain

$$P(\theta_{t+1}) = -\frac{\lambda_{EE} f(\theta_{t+1})}{2} + \frac{\lambda_{EU}}{1+\xi^{-\gamma}} + 1 - \lambda_{EU}$$

which is decreasing in θ_{t+1}. It can be understood as follows: as future market tightness θ_{t+1} increases, it is easier for workers to switch firms in which they work in the future, so a future wage increase leads to less increase in the current value $\mathcal{V}_{jt}$ at period t.

Now that we know the value function $\mathcal{V}_{jt}$ depends on the whole path of wages, we rewrite our recruiting and separation rates as follows:

$$R_t\left(\mathcal{V}_{jt}\right) \equiv g(\theta_t)\left[\phi_{E,t}\left(\frac{\exp(\gamma(\mathcal{V}_{jt}-\bar{\mathcal{V}}_t))}{1+\exp(\gamma(\mathcal{V}_{jt}-\bar{\mathcal{V}}_t)}\right) + \phi_{U,t}\left(\frac{\exp(\gamma(\mathcal{V}_{jt}-\mathcal{V}^u_t))}{1+\exp(\gamma(\mathcal{V}_{jt}-\mathcal{V}^u_t)}\right)\right]$$

$$S_t\left(\mathcal{V}_{jt}\right) \equiv s + (1-s)\left[\lambda_{EE} f(\theta_t)\left(\frac{\exp(-\gamma(\mathcal{V}_{jt}-\bar{\mathcal{V}}_t))}{1+\exp(-\gamma(\mathcal{V}_{jt}-\bar{\mathcal{V}}_t)}\right) + \lambda_{EU}\left(\frac{\exp(-\gamma(\mathcal{V}_{jt}-\mathcal{V}^u_t))}{1+\exp(-\gamma(\mathcal{V}_{jt}-\mathcal{V}^u_t)}\right)\right]$$

Where we write that R_t and S_t are time varying because of changes labor market conditions (tightness θ_t and employed/unemployed searcher shares $\phi_{E,t}$ and $\phi_{U,t}$) and competition from both other firms and unemployment ($\bar{\mathcal{V}}_t$ and $\mathcal{V}^u_t$).

Note that this impacts only the first-order conditions for vacancies and wages, and the law of motion for employment. We can no longer derive a simple wage Phillips Curve (though our price Phillips curve is, happily, unchanged). Putting changes in red, the firm's problem is now

$$\max_{\substack{\{P^j_{y,t}\},\{N_{jt}\}\\ \{W_{jt}\},\{V_{j,t}\}}} \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t\left(P^j_{y,t}Y^j_t - W_{jt}N_{jt} - c\left(\frac{V_{j,t}}{N_{j,t-1}}\right)^{\chi}V_{j,t}W_t - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}}-1\right)^2 Y^j_t P^j_{y,t}\right.$$
$$\left. - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}}-1\right)^2 W_{jt}N_{jt}\right)$$

subject to

$$N_{jt} = (1 - S_t(\mathcal{V}_{jt}))N_{j,t-1} + R_t(\mathcal{V}_{jt})V_{j,t}.$$

Output is produced with labor (with $\bar{X}^j_t = 0$) with the linear production: $Y^j_t = A^j_t N_{jt}$, and Dixit-Stiglitz demand, so $\frac{Y^j_t}{Y_t} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}$, hence $N_{jt} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}\frac{Y_t}{A^j_t}$ with $\epsilon > 1$.

Here we can observe why only the first-order condition with respect to wages will change:

the only new component is that the derivatives of the recruiting and separation rates R_t and S_t will be different. Note that their levels will be unchanged: in a symmetric equilibrium, R_t and S_t will have the same functional forms as before, in the main text: formally in a symmetric equilibrium with $\mathcal{V}_{jt} = \bar{\mathcal{V}}_t$, and assuming households set $\bar{\mathcal{V}}_t - \mathcal{V}_t^u = \ln \xi$ by adjusting the consumption of unemployed households, i.e., adjusting τ_t, we have:

$$R_t \equiv g(\theta_t)\left[\phi_{E,t}\left(\frac{1}{2}\right) + \phi_{U,t}\left(\frac{\xi^\gamma}{1+\xi^\gamma}\right)\right]$$
$$S_t \equiv s + (1-s)\left[\lambda_{EE} f(\theta_t)\left(\frac{1}{2}\right) + \lambda_{EU}\left(\frac{\xi^{-\gamma}}{1+\xi^{-\gamma}}\right)\right].$$

Note that we continue to assume that the representative household imposes taxes and transfers to satisfy a standard Euler equation: while this may no longer necessarily be optimal in this case, we make this assumption to facilitate direct comparison with the model in the main text.

Equilibrium and Steady State We focus on one particular equilibrium where $P_{y,t}^j = P_{y,t}$, $V_{j,t} = V_t$, $W_{jt} = W_t$, $A_t^j = A_t \ \forall j$. All equations in our model in the main body, and the firms problem, are unchanged except for the first order condition for the wage, W_t: defining for convenience

$$\Delta_t \equiv N_t + \psi^w(\Pi_t^w - 1)\Pi_t^w N_t - \frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w - 1\right)\left(\Pi_{t+1}^w\right)^2 N_{t+1},$$
$$\Lambda_t \equiv \frac{\lambda_t}{P_{y,t}}\left(V_t R_t' - S_t' N_{t-1}\right)$$

where λ_t is the co-state variable associated with the law of motion for N_t, we can write the first-order condition for wage W_t as

$$\text{for } t = 0: \quad \frac{W_0}{P_{y,0}}\Delta_0 = \Lambda_0$$
$$\text{for } t \geqslant 1: \quad \frac{W_t}{P_{y,t}}\Delta_t = \Lambda_t + \beta(1-s)\frac{P(\theta_t)}{\Pi_{y,t}}\left(\frac{W_{t-1}}{P_{y,t-1}^y}\Delta_{t-1}\right).$$

In the long run, assuming that firms committed to a wage plan arbitrarily far in the past ($t = -\infty$), we should converge to a steady state satisfying the first-order condition for W_t with $t \geqslant 1$, i.e. not the equation in the initial period. Thus, we look for a solution which solves the following: in a zero-inflation steady state, and noting that $P(\theta_t) < 1$, the wage

satisfies:

$$\frac{W}{P^y}\Delta = \frac{\Lambda}{1-\beta(1-s)P(\theta)}.$$

Note that if $\beta = 0$, and workers are myopic, we recover the expression for the real wage in steady state for the myopic worker model in the main body. Given this equation, and all the other equations in the model, we can again solve numerically for a zero-inflation steady state of the model (assuming the monetary authority targets $\Pi_Y = 1$).

This makes it easy to see the time inconsistency in the firm's optimal wage plan: if we plug in the long run steady state here for the $t = 0$ constraint, and consider what firms choose for the real wage given Λ, Δ, θ, we see that the long run steady state does not satisfy the first-order condition at $t = 0$: the wage is too high by a factor of $1 - \beta(1 - s)P(\theta)$. In short, given the chance to re-optimize, firms choose a lower wage than the one committed to in the infinite past, because that future commitment once helped with contemporary recruitment.

Convergence to the Long Run Steady State We assume an economy as described above without aggregate risk and where firms made their wage plans an infinitely long time ago, so that the economy is now in the long run steady state described above. When firms first set their wage plan, they initially choose to promise a wage below the long run value, and later would want to renege, but this is not permitted here. Figure H.23 plots the transition to this long run steady state if we allow firms to choose new wage plans. All impulse responses are shown as percent deviations from the long-run steady state, so that we can confirm the intuition described above for the wage. Re-optimization acts like an expansionary shock: firms immediately lower nominal wages, but increase in size by recruiting through promising higher nominal wages in the future and by posting more vacancies. This causes market tightness to rise in the aggregate.

Response to a Negative MIT Shock to X_t^s When firms commit to their wage plan at some time $t = -\infty$, we assume that firms know that an X_t^s MIT shock might hit the economy at date $t = 0$, but that the probability of that MIT shock is effectively zero (in the limit). However, we can characterize what their wage plan looks like in the case that the shock hits: this is plotted in Figure H.24. That wage plan respects the first-order condition for wages for $t \geqslant 1$, not $t = 0$, written above.

Replicating the theoretical experiment in the main text, where the central bank perfectly stabilizes $N_t = N$ in response to the shock, yields identical impulse response functions for

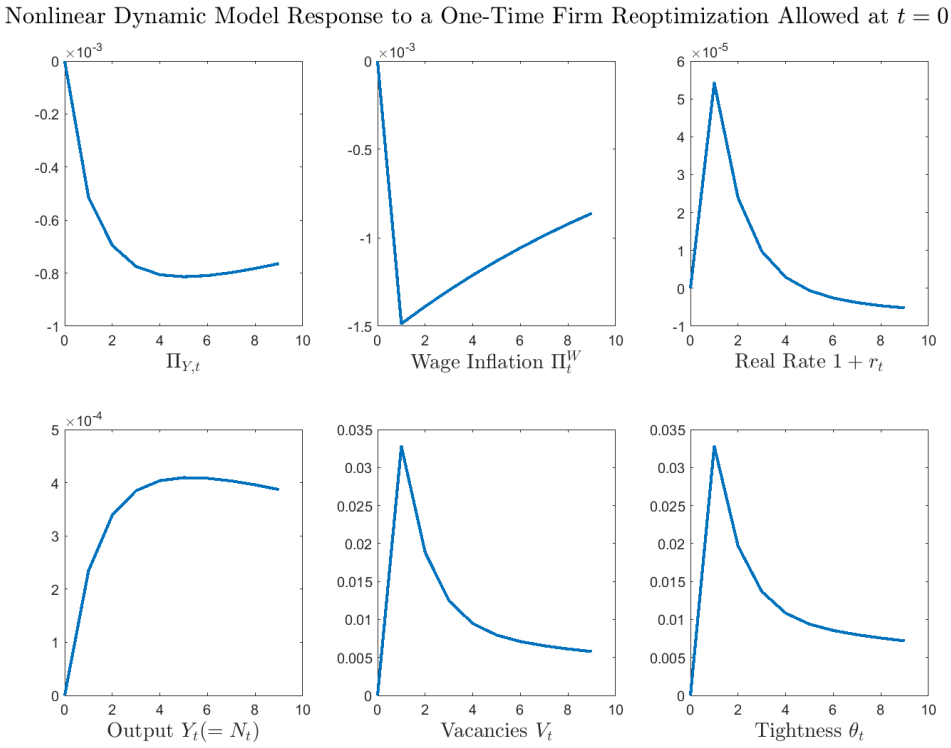


Figure H.23: The effects of allowing firms to reoptimize and choose new paths for wages and all other choice variables, which they then commit to following forever. All impulse responses are shown as percent deviations from the long run steady state.

both models.

We then plot the case in which the central bank follows the Taylor rule (23) that targets service inflation and employment. The responses in both models remain very similar, though they are no longer identical: As in the baseline model result (Figure 5), wage inflation falls below zero—and is slightly higher on impact than the "myopic" model.

We conclude by noting that the assumption of myopic workers is a largely innocuous simplifying assumption which is not critical to obtaining the results in the main text.

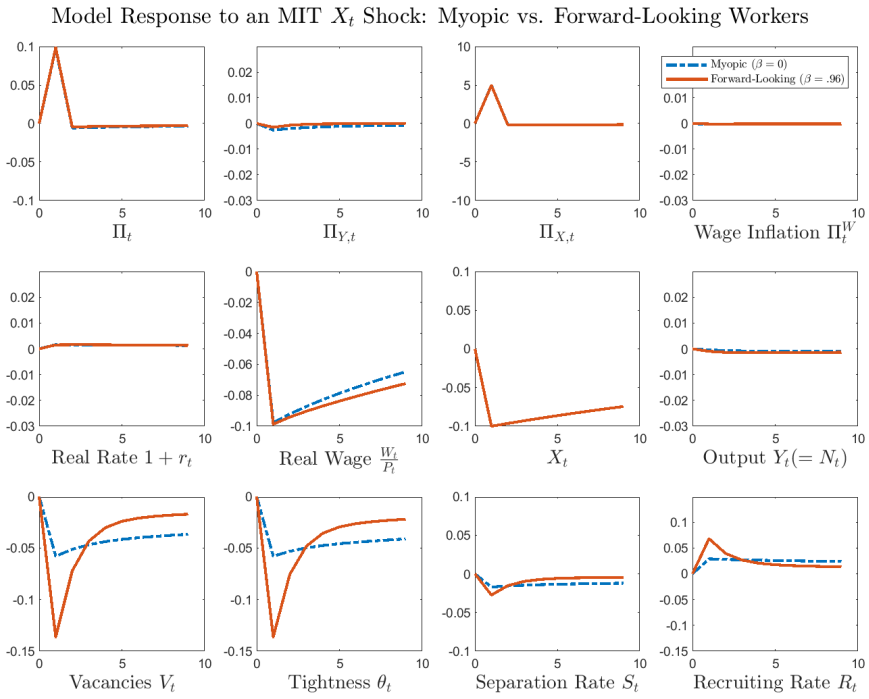


Figure H.24: The effects of a negative shock to X_t assuming the central bank follows an active Taylor rule in (23) under the baseline calibration. The responses in both models are very similar: the monetary authority responds to the inflationary shock by raising real interest rates. The price of domestic output falls and domestic consumption Y_t remains basically flat. Wage inflation remains extremely modest. All impulse responses are shown as percent deviations from the long run steady state.

Appendix I With Hiring Costs

Now, in addition to the direct vacancy-creating costs in the firm optimization (E.16), we assume that an intermediate firm pays a hiring cost which is convex in the number of new employees hired in each period. In this environment, the firm j maximizes

$$\max_{\substack{\{P^j_{y,t}\},\{N_{jt}\}\\ \{W_{jt}\},\{V_{jt}\}}} \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P^j_{y,t} Y^j_t - W_{jt} N_{jt} - c_v \left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi_v} V_{jt} W_t - \underbrace{c_h \left(\frac{H^j_t}{N_{j,t-1}}\right)^{\chi_h} H^j_t W_t}_{\text{Hiring cost}} \right.$$
$$\left. - \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt} N_{jt} \right)$$

subject to

$$N_{jt} = (1 - S_t(W_{jt}))N_{j,t-1} + \underbrace{R_t(W_{jt})V_{jt}}_{\equiv H^j_t}.$$

As in the main text, we will have that physical output is produced with labor and the endowment good:

$$Y^j_t = A^j_t \left(\alpha_N^{\frac{1}{\bar{\eta}}} \left(N_{jt}\right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} + \alpha_{\bar{X}}^{\frac{1}{\bar{\eta}}} \left(\bar{X}^j_t\right)^{\frac{\bar{\eta}-1}{\bar{\eta}}} \right)^{\frac{\bar{\eta}}{\bar{\eta}-1}}, \tag{I.1}$$

$$\frac{Y^j_t}{Y_t} = \left(\frac{P^j_{y,t}}{P_{y,t}}\right)^{-\epsilon}. \tag{I.2}$$

The Lagrangian then can be written as:

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left(P^j_{y,t} Y^j_t - W_{jt} N_{jt} - c_v \left(\frac{V_{jt}}{N_{j,t-1}}\right)^{\chi_v} V_{jt} W_t - \underbrace{c_h \left(\frac{V_{jt} R(W_{jt})}{N_{j,t-1}}\right)^{\chi_h} V_{jt} R_t(W_{jt}) W_t}_{\text{Hiring cost}} \right.$$
$$- \frac{\psi}{2}\left(\frac{P^j_{y,t}}{P^j_{y,t-1}} - 1\right)^2 Y^j_t P^j_{y,t} - \frac{\psi^w}{2}\left(\frac{W_{jt}}{W_{j,t-1}} - 1\right)^2 W_{jt} N_{jt}$$
$$\left. + \lambda^j_t \left[-N_{jt} + \underbrace{V_{jt} R_t(W_{jt})}_{=H^j_t} + (1 - S_t(W_{jt}))N_{j,t-1} \right] \right).$$

where the labor demand N_{jt} is given by (I.1) and (I.2).

First-Order Conditions We similarly write the first-order conditions under the symmetric equilibrium, where $N_{jt} = N_t$, $W_{jt} = W_t$, $\bar{X}_t^j = \bar{X}_t$, $V_{jt} = V_t$, and $\lambda_t^j = \lambda_t$. The first-order condition with V_{jt} in the symmetric equilibrium is given by

$$-c_v(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V} W_t - c_h(1+\chi_h)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_h} (R_t(W_t))^{1+\chi_h} W_t + \lambda_t R_t(W_t) = 0,$$

which leads to

$$\lambda_t = c_v \frac{(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}}{R_t(W_t)} W_t + c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h} W_t, \tag{I.3}$$

where λ_t can be interpreted as the shadow value of a worker. It is increasing in c_h, a shifter in the hiring cost function. The first-order condition with W_{jt} in the symmetric equilibrium is given by

$$\begin{aligned}
&\frac{\psi^w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 + \psi^w\left(\frac{W_t}{W_{t-1}} - 1\right)\frac{W_t}{W_{t-1}} + 1 \\
&= \lambda_t\left(R_t'(W_t)\frac{V_t}{N_t} - \frac{N_{t-1}}{N_t}S_t'(W_t)\right) - c_h(1+\chi_h)R_t'(W_t)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}\frac{V_t}{N_t}W_t \\
&\quad + \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}}{W_t} - 1\right)\frac{W_{t+1}}{W_t^2}W_{t+1}\frac{N_{t+1}}{N_t}.
\end{aligned} \tag{I.4}$$

Combining equations (I.3) and (I.4), we obtain

$$\begin{aligned}
&\frac{\psi^w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 + \psi^w\left(\frac{W_t}{W_{t-1}} - 1\right)\frac{W_t}{W_{t-1}} + 1 \\
&= \left(\underbrace{W_t c_v \frac{(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}}{R_t(W_t)} + W_t c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}}_{=\lambda_t}\right)\left(R_t'(W_t)\frac{V_t}{N_t} - \frac{N_{t-1}}{N_t}S_t'(W_t)\right) \\
&\quad - c_h(1+\chi_h)R_t'(W_t)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}\frac{V_t}{N_t}W_t \\
&\quad + \frac{1}{1+\rho}\psi^w\left(\frac{W_{t+1}}{W_t} - 1\right)\frac{W_{t+1}}{W_t^2}W_{t+1}\frac{N_{t+1}}{N_t}.
\end{aligned} \tag{I.5}$$

Equation (I.5) can be rewritten as

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&W_t c_v\frac{(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}}{R_t(W_t)}\left(R_t'(W_t)\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}S_t'(W_t)\right)\\&+W_t c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}\left(-\frac{N_{t-1}}{N_t}S_t'(W_t)\right)\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2\frac{N_{t+1}}{N_t},\end{aligned}$$

which lead to the following wage Phillips curve with hiring costs:

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&c_v(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}\left(\varepsilon_{R,W_t}\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\&+\underbrace{c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}S_t(W_t)\frac{N_{t-1}}{N_t}(-\varepsilon_{S,W_t})}_{\text{New term (hiring costs)}}\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2\frac{N_{t+1}}{N_t}.\end{aligned}\tag{I.6}$$

Special Cases In the case of convex vacancy costs ($\chi_v>0$) and linear hiring costs ($\chi_h=0$), the wage Phillips curve (I.6) can be written as:

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&c_v(1+\chi_V)\left(\frac{V_t}{N_{t-1}}\right)^{\chi_V}\left(\varepsilon_{R,W_t}\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\&+c_h S_t(W_t)\frac{N_{t-1}}{N_t}(-\varepsilon_{S,W_t})\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2\frac{N_{t+1}}{N_t}.\end{aligned}$$

If instead we have linear vacancy costs, i.e., $\chi_v=0$ and convex hiring costs, i.e., $\chi_h>0$, the wage Phillips curve (I.6) is given by

$$\begin{aligned}\frac{\psi^w}{2}\left(\Pi_t^w-1\right)^2+\psi^w\left(\Pi_t^w-1\right)\Pi_t^w+1=&c_v\left(\varepsilon_{R,W_t}\frac{V_t}{N_t}-\frac{N_{t-1}}{N_t}\frac{S_t(W_t)}{R_t(W_t)}\varepsilon_{S,W_t}\right)\\&+c_h(1+\chi_h)\left(\frac{H_t}{N_{t-1}}\right)^{\chi_h}S_t(W_t)g_t^{-1}(-\varepsilon_{S,W_t})\\&+\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w-1\right)\left(\Pi_{t+1}^w\right)^2 g_{t+1},\end{aligned}$$

where we define $g_t \equiv \frac{N_t}{N_{t-1}}$ as employment growth.

No Vacancy Cost To illustrate that the log-linearized wage Phillips curve retains the same structure when hiring costs are imposed in place of vacancy costs, consider the case in which $c_v \to 0$ and $c_h > 0$. With $B_t \equiv \frac{H_t}{N_{t-1}} = R_t T_t$ where $T_t \equiv \frac{V_t}{N_{t-1}}$ as defined in Appendix B.3, we can express equation (I.6) as

$$0 = F\left(\ln(\Pi_t^w), \ln(\Pi_{t+1}^w), \ln g_t, \ln g_{t+1}, \ln B_t, \ln \varepsilon_{S,W_t}, \ln S_t\right),$$

where

$$\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w \Pi_t^w \left(2(\Pi_t^w - 1) + \Pi_t^w\right), \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w g_{t+1}}{1+\rho}\left(\Pi_{t+1}^w(\Pi_{t+1}^w)^2 + (\Pi_{t+1}^w - 1)2(\Pi_{t+1}^w)^2\right), \\
F_{\ln g_t} &= -c_h(1+\chi_h)B_t^{\chi_h} \cdot S_t \cdot (-g_t^{-1})(-\varepsilon_{S,W_t}), \\
F_{\ln(g_{t+1})} &= -\frac{1}{1+\rho}\psi^w\left(\Pi_{t+1}^w - 1\right)(\Pi_{t+1}^w)^2 g_{t+1}, \\
F_{ln(B_t)} &= -c_h(1+\chi_h)\chi_h \cdot B_t^{\chi_h} \cdot S_t \cdot g_t^{-1}(-\varepsilon_{S,W_t}), \\
F_{\ln(\epsilon_{S,W_t})} &= c_h(1+\chi_h)B_t^{\chi_h} g_t^{-1}\epsilon_{S,W_t}, \\
F_{\ln(S_t)} &= c_h(1+\chi_h)B_t^{\chi_h} g_t^{-1}\epsilon_{S,W_t}.
\end{aligned}$$

which at the steady state with zero wage inflation (and $g = 1$) are given by:

$$\begin{aligned}
F_{\ln(\Pi_t^w)} &= \psi^w, \\
F_{\ln(\Pi_{t+1}^w)} &= -\frac{\psi^w}{1+\rho}, \\
F_{\ln g_t} &= c_h(1+\chi_h)B^{\chi_h} \cdot S \cdot (-\varepsilon_S) \equiv \kappa_h > 0, \\
F_{\ln(g_{t+1})} &= 0, \\
F_{ln(B_t)} &= -\chi_h \kappa_h, \\
F_{\ln(\epsilon_{S,W_t})} &= -\kappa_h, \\
F_{\ln(S_t)} &= -\kappa_h,
\end{aligned}$$

with which (I.6) becomes:

$$0 = \psi^w \check{\Pi}_t^w - \frac{\psi^w}{1+\rho}\check{\Pi}_{t+1}^w + \kappa_h\left(\check{g}_t - \chi_h \check{B}_t - \check{\varepsilon}_{S,W_t} - \check{S}_t\right),$$

leading to the following linearized wage Phillips curve:

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \underbrace{\frac{\kappa_h}{\psi^w}}_{>0}\left(\check{S}_t + \check{\varepsilon}_{S,W_t} + \chi_h \check{B}_t - \check{g}_t\right). \tag{I.7}$$

Equation (I.7) has a straightforward interpretation. Higher separation, $\check{S}_t$, and a more negative separation elasticity,[30] that is, $\check{\varepsilon}_{S,W_t} > 0$, both increase wage growth. When $\chi_h > 0$ (i.e., convex hiring costs), a higher $\check{B}_t$ raises the marginal cost of hiring additional workers, inducing firms to increase wages in order to retain existing employees. Finally, higher employment growth, $\check{g}_t$, reduces firms' incentives to raise wages, as worker retention become less important.

More Simplification To rearrange (I.7) so that it is represented in vacancy and unemployment, we use the projection approach of Appendix B.3 as follows:

$$\begin{aligned}\check{S}_t &= g_{S,V}^{(\varpi)}\check{V}_t + g_{S,U}^{(\varpi)}\check{U}_{t-1}\\ \check{\varepsilon}_{S,W_t} &= g_{\varepsilon_S,V}^{(\varpi)}\check{V}_t + g_{\varepsilon_S,U}^{(\varpi)}\check{U}_{t-1},\end{aligned} \tag{I.8}$$

and

$$\check{B}_t = \check{R}_t + \check{T}_t = \left(g_{R,V}^{(\varpi)} + 1\right)\check{V}_t + \left(g_{R,U}^{(\varpi)} + \frac{U}{1-U}\right)\check{U}_{t-1}, \tag{I.9}$$

with

$$\begin{aligned}\check{g}_t &= S(\check{R}_t + \check{T}_t - \check{S}_t)\\ &= S\left(g_{R,V}^{(\varpi)} + 1 - g_{S,V}^{(\varpi)}\right)\check{V}_t + S\left(g_{R,U}^{(\varpi)} + \frac{U}{1-U} - g_{S,U}^{(\varpi)}\right)\check{U}_{t-1}.\end{aligned} \tag{I.10}$$

Based on equations (I.8), (I.9), and (I.10), equation (I.7) can be written as

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \Phi_V^{H,(\varpi)}\check{V}_t + \Phi_U^{H,(\varpi)}\check{U}_{t-1}, \tag{I.11}$$

where

$$\Phi_V^{H,(\varpi)} = \frac{\kappa_h}{\psi^w}\left[\left(g_{S,V}^{(\varpi)} + g_{\varepsilon_S,V}^{(\varpi)} + \chi_h\left(g_{R,V}^{(\varpi)} + 1\right) - S\left(g_{R,V}^{(\varpi)} + 1 - g_{S,V}^{(\varpi)}\right)\right]$$

[30] As in Online Appendix B.2, in log-linearizing the wage Phillips curve (I.6), we define the deviation in the (negative) separation elasticity as

$$\check{\varepsilon}_{S,W_t} = \frac{\varepsilon_{S,W_t} - \varepsilon_S}{\varepsilon_S},$$

which implies that $\check{\varepsilon}_{S,W_t} > 0$ when ε_{S,W_t} is more negative than its steady-state value ε_S.

and

$$\Phi_U^{H,(\varpi)} = \frac{\kappa_h}{\psi^w}\left[g_{S,U}^{(\varpi)} + g_{\varepsilon_S,U}^{(\varpi)} + \chi_h\left(g_{R,U}^{(\varpi)} + \frac{U}{1-U}\right) - S\left(g_{R,U}^{(\varpi)} + \frac{U}{1-U} - g_{S,U}^{(\varpi)}\right)\right].$$

Based on $\check{Q}_t = g_{Q,V}^{(\varpi)}\check{V}_t + g_{Q,U}^{(\varpi)}\check{U}_{t-1}$, equation (I.11) can be written as

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \underbrace{\frac{\Phi_V^{H,(\varpi)}}{g_{Q,V}^{(\varpi)}}}_{\equiv\beta_Q^{H,(\varpi)}}\check{Q}_t + \underbrace{\left(\Phi_U^{H,(\varpi)} - \Phi_V^{H,(\varpi)}\frac{g_{Q,U}^{(\varpi)}}{g_{Q,V}^{(\varpi)}}\right)}_{\equiv\beta_U^{H,(\varpi)}}\check{U}_{t-1} \tag{I.12}$$

$$= \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \beta_Q^{H,(\varpi)}\check{Q}_t + \beta_U^{H,(\varpi)}\check{U}_{t-1}, \tag{I.13}$$

which is a function of quits $\check{Q}_t$ and unemployment $\check{U}_{t-1}$.

No On-the-Job Search As in Appendix B.4, we consider the case where $\lambda_{EE} \to 0$. In this case, we show in Appendix B.4 that $\check{S}_t \to 0$, $\check{\varepsilon}_{S,W_t} \to 0$, and

$$\check{R}_t = -\frac{\theta^2}{1+\theta^2}\check{\theta}_t = -\frac{\theta^2}{1+\theta^2}(\check{V}_t - \check{U}_{t-1}). \tag{I.14}$$

We also know that

$$\check{B}_t = \check{R}_t + \check{T}_t = \underbrace{\check{R}_t + \check{V}_t}_{=\check{H}_t} + \frac{U}{1-U}\check{U}_{t-1} \tag{I.15}$$

where

$$\check{H}_t = \check{R}_t + \check{V}_t = \frac{1}{1+\theta^2}\check{V}_t + \frac{\theta^2}{1+\theta^2}\check{U}_{t-1}. \tag{I.16}$$

Finally

$$\check{g}_t = S\left(\check{R}_t + \check{T}_t - \check{S}_t\right) = S\left(\check{H}_t + \frac{U}{1-U}\check{U}_{t-1}\right). \tag{I.17}$$

Plugging (I.14), (I.15), (I.16), and (I.17), our log-linearized wage Phillips curve (I.7) under hiring costs can be written as

$$\check{\Pi}_t^w = \frac{1}{1+\rho}\check{\Pi}_{t+1}^w + \frac{\kappa_h}{\psi^w}(\chi_h - S)\check{B}_t, \tag{I.18}$$

where the right-hand side is written in $\check{B}_t$ only. Under $\chi_h = 1$, with $\chi_h > S$, higher $\check{B}_t$ raises

wage growth since it increases the marginal cost of hiring a new worker due to the assumed convexity.

By contrast, when $\chi_h = 0$, an increase in $\check{B}_t$ dampens wage growth. With linear hiring costs, the marginal cost of adding workers remains constant, so a higher $\check{B}_t$ leads to excessive hiring and prompts firms to lower wages.