Is the integration of world asset markets necessarily beneficial in the presence of monetary shocks?

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Abstract

This paper evaluates the consequences of the integration of international asset markets when goods markets are characterized by price rigidities. Using an open economy general equilibrium model with volatility in the money markets, we show that such an integration is not universally beneficial. The country with the more volatile shocks will benefit whereas the country where the volatility of shocks is moderate will suffer. The welfare effects reflect changes in the terms of trade that occur because forward-looking price setters adjust to the changes in exchange rate volatility brought about by the integration of international asset markets.

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1 Introduction

A prominent feature of the world economy in recent years is the increased integration of financial markets worldwide. The number of countries that accept Article VIII of the IMF nearly tripled from 55 in 1980 to 145 in 1998 (IMF (1999)). The implications of the globalization of asset markets are many and varied. On the benefit side, it allows countries to finance investment projects by giving them access to foreign funds. It also provides agents with a broader menu of assets, allowing them to better diversify idiosyncratic risk. Several studies have assessed the gains from international risk sharing. Some found them to be limited (Cole and Obstfeld (1991), Obstfeld and Rogoff (1996 ch. 5), Tesar (1995)), whereas others found larger gains (Davis, Nalewaik and Willen (2000), van Wincoop (1999)). Lewis (2000) contrasts the different approaches. These benefits however come along with significant costs. Several contributions have stressed the risks associated with panics and mania in financial markets (Bhagwati (1998), Calvo (2000), Calvo and Mendoza (2000)). Whether such problems are large enough to offset the benefits from integration remains debated (Obstfeld (1998), Rogoff (1999)).

This paper analyzes how the integration of international asset markets interacts with imperfections in good markets. Such imperfections play a central role in the analysis of optimal policies in open economies. In particular, several recent studies discuss optimal monetary policy using micro-founded models encompassing these imperfections. A non-exhaustive list includes Benigno and Benigno (2000), Devereux and Engel (2000, 1998), Engel (2000), Gali and Monacelli (1999), Ghironi (2000), Obstfeld and Rogoff (2000).[1]

Most of the studies of international monetary interactions however do not consider how different international asset markets structures affect the results.[2] This paper undertakes such an analysis using a standard micro-founded general equilibrium open economy model

[1] For a more complete list see the webpage on monetary policy rules in open economies, http://socrates.berkeley.edu/~gbenigno/mpoe.htm.

[2] An exception is Engel (2000).

based on Obstfeld and Rogoff (1998). Despite the extensive use of variants of this model, it has not yet be used to address the question of international asset markets integration. It is well suited for the exercise as it provides us with a well grounded welfare criterion, namely the utility of the representative agent, and includes market imperfections in the form of monopolistic competition and price rigidities. The general equilibrium nature of the model also allows us to take the various channels through which the integration of asset markets affect the economy into account. This is especially important as indirect effects through the forward looking setting of prices can drastically affect the results of the analysis, as stressed by Devereux and Engel (1998).

We use a version of the model allowing for real exchange rate fluctuations by considering deviations from the law of one price, a feature supported by empirical studies (Engel and Rogers (2000), Engel (1999)). Unlike earlier contributions (Chari, Kehoe and McGrattan (2000), Devereux and Engel (2000, 1998), Devereux, Engel and Tille (1999)) we do not restrict the analysis to a setup with complete international asset markets. Instead, we also derive a variant where international asset markets are simply non-existent. The integration of international asset markets can then be analyzed as a move from the second to the first setup.

We consider a model with two countries that are both affected by monetary shocks. The volatility of these shocks is however different across countries. We distinguish between a low volatility country, which can be interpreted as an industrialized country, and a high volatility country, which can represent an emerging market. The major finding is that the integration of international asset markets does not benefit all countries. Instead, the high volatility country benefits whereas the low volatility country looses. The main impact comes through the terms of trade. By increasing the volatility of the exchange rate, the integration of international asset markets induces exporting firms to lower their prices because their revenue is convex in the exchange rate. This is the dominant effects for firms located in the low volatility country. In the high volatility country however, this effect is dominated by the fact that exchange rate fluctuations reduces the exporters' revenue precisely when their marginal utility of income is high, which leads them to increase their prices. These price changes boost the competitiveness of goods produced in the low volatility country. However this comes at the cost of a worsening of the terms of trade which reduces the purchasing power of households in the low volatility country, ultimately making them worse off.

The possibility of adverse effects from asset markets integration has been recognized in earlier studies, such as Bhagwati (1998), Gertler and Rogoff (1990), Osler (1991). These contributions point that integration can be detrimental to the emerging market. By contrast this paper stresses that the industrialized country can be adversely affected. It also shows that under reasonable parameters the magnitudes of the welfare effect is sizeable, as it amounts to between 0.6 and 3.5 percent of consumption.

The paper is organized as follows. Section 2 presents the structure of the model. Section 3 analyzes the positive impact of volatility. The discussion focuses on the main results and their intuitive interpretation, with the detailed steps being presented in the Appendix. The normative implications are discussed in section 4. The results are illustrated by a numerical example in section 5. Section 6 analyzes two extensions and shows that the results remain valid. Section 7 concludes.

2 The model

2.1 Households optimization

The world is made of two countries, denoted by 1 and 2. A mass n of households are residents of country 1 and a mass $1-n$ are residents of country 2. Households consume a continuum of goods of different brands. Brands are indexed by v, with n brands produced in country 1 ($v \in [0,n)$) and $1-n$ brands produced in country 2 ($v \in [n,1]$). The objective of household x in country 1 is to maximize:

$$U_{1t}(x) = E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{\left[C_{1t+s}(x)\right]^{1-\rho}}{1-\rho} + \chi_1 \ln\left(\frac{M_{1t+s}(x)}{P_{1t+s}}\right) - \eta L_{1t+s}(x) \right] \tag{1}$$

The first term captures the utility from consuming a basket of goods, $C_1(x)$. ρ is the coefficient of risk aversion and we adopt the usual assumption that $\rho > 1$. The second term captures the liquidity services from the real currency holdings, where $M_1(x)$ is the amount of country 1 currency held by household x, and P_1 is the aggregate price level in country 1. The last term captures the disutility of hours worked, $L_1(x)$.

The consumption basket $C_1(x)$ is composed of goods produced in country 1 and 2:

$$C_{1t}(x) = \frac{[C_{11t}(x)]^n [C_{12t}(x)]^{1-n}}{n^n (1-n)^{1-n}}$$

where $C_{1j}(x)$ is the consumption by household x living in country 1 of goods produced in country j. $C_{11}(x)$ and $C_{12}(x)$ are in turn baskets of the various brands produced in each country:

$$\begin{aligned} C_{11t}(x) &= \left[n^{-\frac{1}{\lambda}} \int_0^n [C_{11t}(x,v)]^{\frac{\lambda-1}{\lambda}} dv \right]^{\frac{\lambda}{\lambda-1}} \\ C_{12t}(x) &= \left[(1-n)^{-\frac{1}{\lambda}} \int_n^1 [C_{12t}(x,v)]^{\frac{\lambda-1}{\lambda}} dv \right]^{\frac{\lambda}{\lambda-1}} \end{aligned}$$

where $C_{1j}(x,v)$ is the consumption by household x living in country 1 of brand v produced in country j. The intra-temporal allocation of consumption across the available brands reflects the brands prices. The demands from household x living in country 1 for a country 1 brand and a country 2 brand are given by:

$$C_{11t}(x,v) = \left[\frac{P_{11t}(v)}{P_{11t}}\right]^{-\lambda} \frac{P_{1t}C_{1t}(x)}{P_{11t}} \tag{2}$$

$$C_{12t}(x,v) = \left[\frac{P_{12t}(v)}{P_{12t}}\right]^{-\lambda} \frac{P_{1t}C_{1t}(x)}{P_{12t}} \tag{3}$$

where $P_{1j}(v)$ is the price, in country 1 currency, of a brand v produced in country j and sold in country 1. P_{1j} is the price index, in country 1 currency, of goods produced in country j and sold in country 1. P_1 is the price index, in country 1 currency, of all goods sold in country 1. The price indexes represent the minimum expenditure required to purchase one unit of the corresponding basket. The consumption allocation for an household living in country 2 takes a similar form.

In addition of domestic money households can hold a complete range of contingent securities. We separate the set of available securities in two subsets distinguished by the currency denomination of the payoff. Securities in the first (second) subset pay 1 unit of country 1 (2) currency, conditional on the realization of the associated state of the world. Households can always hold securities that pay off in their domestic currency, but may be prohibited from holding securities paying off in the other country currency. This possible prohibition is the distinguishing feature between the two variants we consider:

- If households can hold securities paying off in the other country currency, they can trade assets with foreign households. Assets markets are therefore complete across and within countries. We refer to this case as the **comp** model,

- If households can *not* hold securities paying off in the other country currency, they cannot trade assets with foreign households. There is therefore no possibility of international risk sharing: assets markets are complete within countries but not across countries. The impossibility of holding claims across countries implies that trade flows in goods are balanced in each state of the world, and we refer to this model as the **bal** model.

In both models, the budget constraint of household x living in country 1 is given by:

$$\begin{aligned} & P_{1t}C_{1t}(x) + M_{1t}(x) + \mathsf{BP}_{1t+1}(x) \\ = \; & \mathsf{BR}_{1t}(x) + M_{1t-1}(x) + \Pi_{1t}(x) + W_{1t}L_{1t}(x) - T_{1t}(x) \end{aligned} \tag{4}$$

where $\mathsf{BP}_{1t+1}(x)$ denotes the purchases by household x living in country 1 of contingent securities that pay off in period $t+1$. $\mathsf{BR}_{1t}(x)$ is the payoff in period t on similar securities purchased in period $t-1$. $\Pi_{1t}(x)$ denotes the lump sum dividend revenue from the firms owned by household x and $T_{1t}(x)$ is a lump sum tax. The budget constraint for a household living in country 2 is similar.

We now discuss the specification of the BP and BR terms. In the **comp** model, all households can trade in all securities, so the BP_i and BR_i terms cover securities denominated in country 1 currency, as well as securities denominated in country 2 currency, for $i = 1, 2$. In the **bal** model by contrast, households can trade only in the securities paying off in the currency of their country of residence. The BP_i and BR_i terms then cover only securities denominated in country i currency, for $i = 1, 2$. The key feature of the **bal** model is that households cannot trade securities with their counterparts abroad, not that households are restricted in terms of the currency denomination of the securities they can trade.

Household x living in country 1 maximizes (1) subject to (4), with her counterpart in country 2 solving a similar problem. The Appendix shows that in both the **comp** and **bal** models we obtain:

$$\frac{W_{1t}}{P_{1t}} = \eta C_{1t}^{\rho} \tag{5}$$

$$\frac{M_{1t}}{P_{1t}} = C_{1t}^{\rho} \frac{\chi_1}{1 - d_1} \tag{6}$$

where we dropped the x index as all households within a country are identical. (5) is the labor supply relation equating the marginal utility of consumption adjusted by the real wage to the marginal cost of effort. (6) is the money market equilibrium where the marginal utility of monetary balances adjusted for the discount factor is equal to the marginal utility of consumption.[3] Similar relations hold for country 2:

$$\frac{W_{2t}}{P_{2t}} = \eta C_{2t}^{\rho} \tag{7}$$

$$\frac{M_{2t}}{P_{2t}} = C_{2t}^{\rho} \frac{\chi_2}{1 - d_2} \tag{8}$$

(5)-(8) are independent of the structure of asset markets and hold in both the **comp** and the **bal** models. We consider that in both models governments simply repay the seignorage revenue in the form of lump sum transfers: $M_{it} - M_{it-1} = -T_{it}$, for $i = 1, 2$.

2.2 Firms optimization

Firms produce goods using a constant returns to scale technology with labor as the only input. Our model is characterized by price rigidities as firms chose their prices before shocks are realized. Following a shock, output is demand determined and firms accommodate changes in demand at the preset prices. Following Devereux and Engel (2000) we assume that firms choose two prices: one in their own currency for domestic sales and one in foreign

[3] The Appendix shows that the discount factor $d_{1t,t+1} = \beta \left[P_{1t} C_{1t}^{\rho}\right] \left[P_{1t+1} C_{1t+1}^{\rho}\right]^{-1}$ is constant at d_1. This result reflects our assumption of a logarithmic utility for real balances.

currency for export sales. As prices are preset in the consumers' currency the law of one price does not necessarily hold. The price charged by a firm for sales in country 1 can differ from the price it charges in country 2 adjusted for the exchange rate. We do not assume that the law of one price holds for two reasons. First, the law of one price is not supported by empirical evidence (Engel (1999), Engel and Rogers (2000)). Second, deviations from purchasing power parity, stemming from the failure of the law of one price, are a crucial element of our analysis. As shown by Obstfeld and Rogoff (2000, 1998) households never choose to hold any claims on the other country when purchasing power parity holds, and the results are the same whether international asset markets are non-existent, incomplete, or complete.

The optimization problem of each firm is to maximize its expected profits one period ahead. Firms located in a country being owned by the households residing in that country, the profits are discounted by the marginal utility of income. A firm in country 1 chooses a price P_{11t} in country 1 currency charged to domestic customers, and a price P_{21t} in country 2 currency charged to customers abroad. The optimal prices are given by:

$$P_{11t} = \frac{\lambda}{\lambda - 1} \frac{E_{t-1} W_{1t} C_{1t}^{1-\rho}}{E_{t-1} C_{1t}^{1-\rho}} \tag{9}$$

$$P_{21t} = \frac{\lambda}{\lambda - 1} \frac{E_{t-1} W_{1t} C_{1t}^{-\rho} C_{2t}}{E_{t-1} S_t C_{1t}^{-\rho} C_{2t}} \tag{10}$$

where S is the nominal exchange rate is defined in terms of units of country 1 currency per unit of country 2 currency, so that an increase in S corresponds to a depreciation of the country 1 currency. To highlight the intuition, we focus on the second equation and rewrite it as:

$$P_{21t} E_{t-1} S_t C_{1t}^{-\rho} C_{2t} = \frac{\lambda}{\lambda - 1} E_{t-1} W_{1t} C_{1t}^{-\rho} C_{2t}$$

The left hand side represents the expected marginal revenue in country 1 currency, $P_{21t} E_{t-1} S_t$, and the right hand-side represents the expected marginal cost, $E_{t-1} W_{1t}$. The terms on both sides are weighted by $C_{1t}^{-\rho} C_{2t}$, which captures the marginal utility of revenue for the firm owner, $C_{1t}^{-\rho}$, and the strength of demand in the market, C_{2t}.

Intuitively the firm owner has to choose a price knowing that ex-post shocks will occur which will make her wish to set a different price if she could. If she equally cares about all possible future states, she will be equally likely to wish to increase or decrease her price ex-post. She then chooses the price so that the expected marginal revenue exceeds the expected marginal cost by a markup reflecting her monopoly power: $P_{21t}E_{t-1}S_t = \lambda(\lambda-1)^{-1} E_{t-1}W_{1t}$. The firm owner however does not weight all states equally. Instead she cares more about states where her marginal utility of income is high ($C_{1t}^{-\rho}$ is large) and / or demand is strong (C_{2t} is large). If in such states she is equally likely to want to lower her price or increase it, she will still set the expected revenue at a markup over the expected marginal cost.

The firm owner however chooses a price such that the expected marginal revenue exceeds the expected marginal cost if her margin is lower in the states that she cares more about. This occurs if the exchange rate S_t is negatively correlated with $C_{1t}^{-\rho}C_{2t}$. An appreciation of country 1 currency (a low value of S_t) reduces the revenue for each unit sold, expressed in country 1 currency. This occurs precisely in the states about which the firm owner cares more, so she chooses a higher price ex ante. She also chooses a higher price if the marginal cost W_{1t} is positively correlated with $C_{1t}^{-\rho}C_{2t}$, that is if the cost is higher, hence the profit margin lower, in the states she cares more about.

Similarly, a firm in country 2 chooses a price P_{12t} in country 1 currency charged to customers abroad, and a price P_{22t} in country 2 currency charged to domestic customers:

$$P_{12t} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}W_{2t}C_{1t}C_{2t}^{-\rho}}{E_{t-1}S_t^{-1}C_{1t}C_{2t}^{-\rho}} \tag{11}$$

$$P_{22t} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}W_{2t}C_{2t}^{1-\rho}}{E_{t-1}C_{2t}^{1-\rho}} \tag{12}$$

Note that the optimal price setting equations are independent of the structure of asset markets. The output of the representative country 1 and country 2 firms are given by:

$$Y_{1t} = n\frac{P_{1t}C_{1t}}{P_{11t}} + (1-n)\frac{P_{2t}C_{2t}}{P_{21t}} \quad , \quad Y_{2t} = n\frac{P_{1t}C_{1t}}{P_{12t}} + (1-n)\frac{P_{2t}C_{2t}}{P_{22t}} \tag{13}$$

2.3 The relation between consumption and the real exchange rate

The results so far hold whether international asset markets are complete or non existent. We now turn to the dimension of the model that is affected by the structure of international asset markets, namely the relation between the real exchange rate and relative consumption.

In the **comp** model, optimal risk sharing implies that the real exchange rate is always equal to the ratio of marginal utilities of consumption, as shown in Chari, Kehoe and McGrattan (2000) and Devereux and Engel (2000):

$$\frac{S_t P_{2t}}{P_{1t}} = \frac{C_{1t}^{\rho}}{C_{2t}^{\rho}} \tag{14}$$

Intuitively (14) shows that the benefit from an additional unit of income in either currency is the same regardless whether it is paid to households in country 1 or country 2.

(14) does not hold in the **bal** model as households cannot trade securities across borders. As there are no cross-country asset holdings, trade in goods must be balances each period. Using the import demand in country 1 (3) and its equivalent in country 2, we write:

$$\begin{aligned} S_t P_{21t} n \left(1-n\right) \frac{P_{2t} C_{2t}}{P_{21t}} &= P_{12t} n \left(1-n\right) \frac{P_{1t} C_{1t}}{P_{12t}} \\ \Rightarrow \frac{S_t P_{2t}}{P_{1t}} &= \frac{C_{1t}}{C_{2t}} \end{aligned} \tag{15}$$

(15) provides a relation between the real exchange rate and relative consumptions in the **bal** model,[4] which is the counterpart of the optimal risk sharing relation (14) in the **comp** model.

(14) and (15) show that the integration of international asset markets, i.e. a move from the **bal** model to the **comp** model, impacts households by changing the sensitivity of relative consumption to the real exchange rate. This clearly shows the central role played by the real exchange rate: in any setup where the purchasing power parity holds, that is $S_t P_{2t} = P_{1t}$, the integration of international asset markets is irrelevant as consumption is the same in both countries: $C_{1t} = C_{2t}$. An example is a setup with nominal rigidities and complete exchange rate pass-through, as shown by Obstfeld and Rogoff (2000, 1998). Another example is a flexible price setup. Indeed, in the absence of price rigidities the structure of the model implies that the integration of international asset markets is neutral. This holds even if we consider non-monetary sources of volatility such as government spending or productivity shocks.

[4] The proportionnality between the real exchange rate and relative consumption reflects our assumption of a unit elasticity of substitution between goods produced in different countries.

As we consider that $\rho > 1$, (14) and (15) show that the integration of international asset markets reduces the sensitivity of relative consumption to the real exchange rate. Conversely, the real exchange rate is more volatile for a given volatility of relative consumption when international asset markets are complete.

3 The impact of monetary volatility

Having presented the mechanisms of the model, we now turn to the analysis of volatility. Following earlier contributions (Devereux and Engel (2000), Devereux, Engel and Tille (1999)) we focus on shocks in the money market equilibrium (6) and (8). More specifically we assume that nominal balances follow a log normal distribution:

$$m_{1t} = E_{t-1} m_{1t} + \varepsilon_{1t} \quad , \quad m_{2t} = E_{t-1} m_{2t} + \varepsilon_{2t}$$

$$\begin{vmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{vmatrix} \rightsquigarrow N \left(\begin{vmatrix} 0 \\ 0 \end{vmatrix}, \begin{vmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{vmatrix} \right) \tag{16}$$

where lower cases denote logarithms. Without loss of generality we assume that monetary shocks are uncorrelated across countries. The shocks to the money market equilibrium can be interpreted as shocks to either the money supply or the money demand. They bring uncertainty in the amount of real balances required to finance a given degree of consumption. A high volatility can then be interpreted as a situation where financial intermediaries are not stable and it is possible that a large amount of real balances be required to finance a given consumption.

Without loss of generality, we assume that the money market is less volatile in country 2 than in country 1: $\sigma_1^2 > \sigma_2^2$. Country 1 can be interpreted as an emerging market with volatile financial institutions, whereas country 2 represents an industrialized country where financial markets are more stable.

Both the **comp** and the **bal** models are characterized by the absence of any linkage between periods through cross-country wealth effects. In the **bal** model cross-country assets are non-existent and households cannot save any fraction of their income in the form of claims on households in other countries. The situation is similar in the **comp** model where any unexpected increase in income is immediately transferred to the other country through the insurance market. As the entire impact of a shock occurs in one period in both models, we can focus our analysis entirely on one period t. We consider how shocks in period t affect the ex-post realization of the variables in period t, along with the impact on the expectations from the point of view of period $t-1$ when firms choose their prices.

3.1 Volatility of consumption and the exchange rate

The volatility of consumption can be computed from the money market equilibrium relations (6) and (8) which can be expressed in logarithms as:

$$m_{1t} - p_{1t} = \ln\left(\frac{\chi_1}{1-d_1}\right) + \rho c_{1t} \quad , \quad m_{2t} - p_{2t} = \ln\left(\frac{\chi_2}{1-d_2}\right) + \rho c_{2t}$$

As all prices are preset in the consumer's currency, p_{1t} and p_{2t} are not affected by the shocks, and the variance of the logarithms of consumption follows from (16):

$$\sigma^2(c_{1t}) = \rho^{-2}\sigma_1^2 > \sigma^2(c_{2t}) = \rho^{-2}\sigma_2^2$$

where $\sigma^2(c_{it})$ is the variance of the logarithm of consumption in country i. The volatility of consumption directly reflects the volatility of the domestic money market and is higher in the emerging market (country 1). The degree of integration of international asset markets does not affect the money market equilibrium (6) and (8). The variance of consumption is therefore the same in the **comp** and the **bal** models.

The volatility of the exchange rate can easily be computed from (14) and (15) as follows:

$$\begin{aligned} \sigma^2(s_t)_{\mathbf{bal}} &= Var(c_{1t} - c_{2t}) = \rho^{-2}\left(\sigma_1^2 + \sigma_2^2\right) \\ \sigma^2(s_t)_{\mathbf{comp}} &= Var(\rho(c_{1t} - c_{2t})) = \left(\sigma_1^2 + \sigma_2^2\right) \end{aligned}$$

$$\Rightarrow \sigma^2(s_t)_{\mathbf{comp}} > \sigma^2(s_t)_{\mathbf{bal}} \tag{17}$$

where $\sigma^2(s_t)$ is the variance of the logarithm of the exchange rate. As $\rho > 1$ the exchange rate is more volatile in the **comp** model. This occurs because the volatility of consumption is the same whether international asset markets are complete or non existent. The exchange rate is then more volatile in the **comp** because it is more sensitive to relative consumption, as shown by (14)-(15). The amplitude of the co-movements between the exchange rate and consumption is also larger:

$$\begin{aligned}\sigma\left(s_t, c_{1t}\right)_{\mathbf{comp}} &= \rho\left(\sigma^2\left(c_{1t}\right) - \sigma\left(c_{1t}, c_{2t}\right)\right) = \rho\sigma\left(s_t, c_{1t}\right)_{\mathbf{bal}} \\ \sigma\left(s_t, c_{2t}\right)_{\mathbf{comp}} &= \rho\left(\sigma\left(c_{1t}, c_{2t}\right) - \sigma^2\left(c_{2t}\right)\right) = \rho\sigma\left(s_t, c_{2t}\right)_{\mathbf{bal}}\end{aligned} \tag{18}$$

3.2 Ex-ante prices and expected levels

As stressed in Devereux and Engel (1998), the impact of volatility is not limited to the second moments of the variables. It also affects the ex-ante price setting, which in turns has an impact on the expected levels of consumption and effort. The detailed analysis of the impact through prices and expected levels is presented in the Appendix, and we focus on the main results and the underlying intuition for simplicity.

3.2.1 Optimal prices

In the **comp** model, all prices charged in a given country are identical:

$$\begin{aligned}P_{11t\mathbf{comp}} &= P_{12t\mathbf{comp}} = P_{1t\mathbf{comp}}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t\mathbf{comp}}}{E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}} \\ P_{21t\mathbf{comp}} &= P_{22t\mathbf{comp}} = P_{2t\mathbf{comp}}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t\mathbf{comp}}}{E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}}\end{aligned}$$

In the **bal** model, the price of domestically produced goods and imports is however different, and we write:

$$\begin{aligned}P_{11t\mathbf{bal}} &= P_{1t\mathbf{bal}}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t\mathbf{bal}}}{E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}} \quad , \quad P_{21t\mathbf{bal}} = P_{2t\mathbf{bal}}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t\mathbf{bal}}}{E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}} \\ P_{12t\mathbf{bal}} &= P_{1t\mathbf{bal}}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t\mathbf{bal}}}{E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}} \quad , \quad P_{22t\mathbf{bal}} = P_{2t\mathbf{bal}}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t\mathbf{bal}}}{E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}}\end{aligned}$$

The form of the prices for domestic goods, P_{11t} and P_{22t}, is the same in both models. The expressions for the prices of imported goods, P_{12t} and P_{21t}, are however different. This shows that the direct impact of the degree of asset markets integration is on the price of imported goods. Intuitively, this reflects the changes in the co-movements of the exchange rate and consumptions.

To illustrate the intuition behind this mechanism in a simple way, we focus on the case where there is no volatility in country 2: $\sigma_2^2 = 0$. Prices are set to equate the marginal revenue (unit revenue x demand x discount) and a markup over the marginal cost (unit cost x demand x discount). From (11), the price of imports from country 2 into country 1 is written as:

$$P_{12t}E_{t-1}\left[\frac{1}{S_t}\cdot C_{1t}\cdot C_{2t}^{-\rho}\right] = \frac{\lambda}{\lambda-1}E_{t-1}\left[W_{2t}\cdot C_{1t}\cdot C_{2t}^{-\rho}\right]$$

The price is set so that the expected marginal revenue, captured by $P_{12t}E_{t-1}S_t^{-1}$, is a markup over the expected marginal cost, $E_{t-1}W_{2t}$. Both the expected revenue and the expected cost are adjusted by the strength of demand in the market, captured by C_{1t}, and the marginal utility of income for the firm owner, captured by $C_{2t}^{-\rho}$.

The optimal price expression can be rewritten in logarithms, so the analysis is not sensitive to terms reflecting Jensen's inequality.[5] We use the property of the log-normal distribution,[6] and recall that there is no volatility in country 2, to write:

$$p_{12t} \propto -\frac{1}{2}\sigma^2(s_t) + \sigma(s_t, c_{1t})$$

where $\propto$ denotes a relation of proportionality.

The integration of international asset markets, that is a move from the **bal** model to the **comp** model, has two opposite direct effects on p_{12t}. First, the increase in the exchange rate volatility (17) tends to reduce the price. Intuitively this reflects the convexity of the marginal expected revenue, $P_{12t}E_{t-1}S_t^{-1}$, with respect to the logarithm exchange rate. Because of this convexity the higher volatility of the exchange rate increases the expected revenue, leading the firm to lower its price to keep the marginal revenue in line with the marginal cost.

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[5] For instance the analysis is not affected by the definition of the exchange rate. Defining $Z_t = S_t^{-1}$, we can see that:

$$\begin{aligned} z_t &= -s_t \Rightarrow E_{t-1}z_t = -E_{t-1}s_t \\ E_{t-1}Z_t &= E_{t-1}\left[\frac{1}{S_t}\right] \neq \frac{1}{E_{t-1}S_t} \end{aligned}$$

An analysis in logarithms is the same whether we consider z_t or s_t, but an analysis in levels would be affected.

[6] Namely: if $x = \ln X \rightsquigarrow N(\mu, \sigma^2)$ then $EX^a = \exp\left[a\mu + \frac{1}{2}a^2\sigma^2\right]$.

Second, the higher covariance between the exchange rate and the strength of demand $\sigma(s_t, c_{1t})$ tends to reduce the price. As shown by (18) this covariance is positive. An appreciation of country 1 currency (a low value of S_t^{-1}) reduces the revenue on each unit sold in country 1, expressed in terms of country 2 currency. As $\sigma(s_t, c_{1t}) > 0$ this occurs precisely when demand is strong (C_{1t} is high). As states of strong demand are precisely the states that the firm owner cares more about, she sets a higher ex ante price to partially offset the reduced revenue in such states. By increasing the magnitudes of the covariance between the exchange rate and the strength of demand, the integration of international asset markets reinforces this channel.

We can show that the impact through the exchange rate volatility is stronger than the effect on the covariance between the exchange rate and demand, so that the direct impact of the integration of international asset markets is to lower the price:

$$
\begin{aligned}
& -\frac{1}{2}\left[\sigma^2(s_t)_{\mathbf{comp}} - \sigma^2(s_t)_{\mathbf{bal}}\right] + \left[\sigma(s_t, c_{1t})_{\mathbf{comp}} - \sigma(s_t, c_{1t})_{\mathbf{bal}}\right] \\
= & -\frac{(\rho-1)^2}{2}\sigma^2(c_{1t}) < 0
\end{aligned}
$$

We can undertake a similar analysis for the price of imports from country 1 to country 2. From (10) we write:

$$
P_{21t}E_{t-1}\left[S_t \cdot C_{2t} \cdot C_{1t}^{-\rho}\right] = \frac{\lambda}{\lambda-1}E_{t-1}\left[W_{1t} \cdot C_{2t} \cdot C_{1t}^{-\rho}\right]
$$

$P_{21t}E_{t-1}S_t$ reflects the marginal revenue, and $E_{t-1}W_{1t}$ is the marginal cost. Both are adjusted by the strength of demand in the market, captured by C_{2t}, and the marginal utility of income for the firm's owner, captured by $C_{1t}^{-\rho}$. We can express this relation in logarithms as:

$$
p_{21t} \propto -\frac{1}{2}\sigma^2(s_t) + \rho\sigma(s_t, c_{1t})
$$

The integration of asset markets again has two opposite direct impacts on p_{21t}. First, the increase in the exchange rate volatility tends to reduce the price. As before this reflects the convexity of the marginal expected revenue, $P_{21t}E_{t-1}S_t$, with respect to the logarithm of the exchange rate. The higher volatility of the exchange rate increases the expected revenue, leading the firm to lower its price.

Second, the higher covariance between the exchange rate and consumption in country 1, $\sigma(s_t, c_{1t})$ tends to reduce the price. This impact now reflects the co-movements of the exchange rate and the marginal utility of income for the firm owner, $C_{1t}^{-\rho}$. A depreciation of country 1 currency (a low value of S_t) reduces the revenue from each unit sold in country 2, expressed in country 1 currency. As $\sigma(s_t, c_{1t}) > 0$, this happens precisely when the marginal utility of income for the firm owner is high ($C_{1t}^{-\rho}$ is high). As the firm owner cares more about states where her marginal utility is high, she sets a higher price ex ante to offset the reduction of revenue in these states. By increasing the magnitude of $\sigma(s_t, c_{1t})$, the integration of international asset markets magnifies this channel.

We can show that the impact through the exchange rate volatility is the weakest of the two effects and the direct impact of the integration of asset markets is to increase the price:

$$
\begin{aligned}
& -\frac{1}{2}\left[\sigma^2(s_t)_{\mathbf{comp}} - \sigma^2(s_t)_{\mathbf{bal}}\right] + \rho\left[\sigma(s_t, c_{1t})_{\mathbf{comp}} - \sigma(s_t, c_{1t})_{\mathbf{bal}}\right] \\
= & \ \frac{(\rho-1)^2}{2}\sigma^2(c_{1t}) > 0
\end{aligned}
$$

The direct impact of the integration of international asset markets on prices is therefore a reduction of import prices in the country with high volatility (country 1), and an increase in import prices in the country with low volatility (country 2). Goods produced in country 2 are more competitive in both markets, leading all consumers to switch their consumption towards these goods. This increases the expected output of firms in country 2. The change in import prices however also leads to a higher consumer price index in country 2, thereby lowering the purchasing power of country 2 households.

Our analysis so far has only focused on the direct impact through the exchange rate volatility and the co-movements between the exchange rate and consumptions. We now build on this first step by deriving the solution of the model for the expected values of consumption and output.

3.2.2 Expected consumptions

From the expressions for the optimal prices, we can derive the solution for expected consumption. The Appendix presents the detailed steps, and we focus on the final results.

Table 1 presents the expected consumption in both countries and both models. Expected consumption is higher in the high volatility country (country 1) than in the low volatility country in the **comp** model. In the **bal** model, expected consumption is the same in both countries:

$$E_{t-1}C_{1t\mathbf{comp}} > E_{t-1}C_{1t\mathbf{bal}} = E_{t-1}C_{2t\mathbf{bal}} > E_{t-1}C_{2t\mathbf{comp}} \tag{19}$$

The integration of asset markets rises increases consumption in the high volatility country and lowers it in the other country. As pointed in Devereux-Engel-Tille (1999) consumption is an index of consumption of goods from both countries. An increase in the expected consumption index can then reflect an increase in the expected consumption of each good, or a decrease in the variance of relative consumption between goods produced in different countries (reflecting changes in relative prices). As all consumer prices are preset, exchange rate fluctuations do no affect the relative price of goods and consumption changes are evenly shared across goods produced in different countries. The changes in the expected consumption index hence do not reflect changes in the volatility of consumption of goods from country 1 relative to the consumption of goods from country 2, but changes in expected consumptions. In other words, the changes in the expected consumption index has the intuitive interpretation of a change in expected consumption, instead of reflecting the particular features of the index.

We can also derive the expected consumption of domestic and imported goods in each model. As shown in the Appendix, we first derive the expected power consumption, defined as $E_{t-1}C_{it}^{1-\rho}$, leading to the results in table 2 and the following ranking:

$$E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} > E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} > E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} > E_{t-1}C_{2t\mathbf{bal}}^{1-\rho} \tag{20}$$

We then derive the expected consumption of domestic and imported goods in country 1, presented in table 3. From the ranking (20) we see that in country 1 the expected consumption of domestic goods is reduced by the integration of assets markets, whereas the expected consumption of imports increases. We can derive the corresponding results for country 2, as shown in table 4. Households in country 2 increase their expected consumption of domestic goods and reduce their consumption of imports. Note that the increase in the expected consumption of goods produced in country 2 is stronger for the households living in country 1:

$$\frac{E_{t-1}C_{12t\mathbf{comp}}}{E_{t-1}C_{12t\mathbf{bal}}} = \frac{E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}}{E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}} > \frac{E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}}{E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}} = \frac{E_{t-1}C_{22t\mathbf{comp}}}{E_{t-1}C_{22t\mathbf{bal}}} > 1$$

Similarly, the reduction in the expected consumption of goods produced in country 1 is stronger for the households of country 2:

$$\frac{E_{t-1}C_{21t\mathbf{comp}}}{E_{t-1}C_{21t\mathbf{bal}}} = \frac{E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}}{E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}} < \frac{E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}}{E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}} = \frac{E_{t-1}C_{11t\mathbf{comp}}}{E_{t-1}C_{11t\mathbf{bal}}} < 1$$

3.2.3 Expected effort

The expected output, and effort, for each household is computed from (13) and the optimal prices, as shown in the Appendix. The results are given in table 5. Using the ranking (20) we show that the integration of international asset markets generates a reduction in the output of country 1. This occurs because all households shift their consumption towards the goods produced in country 2 as they are now cheaper. Similarly, the output of country 2 increases to meet the additional demand from all households.

$$E_{t-1}Y_{1t\mathbf{bal}} > E_{t-1}Y_{1t\mathbf{comp}} = E_{t-1}Y_{2t\mathbf{comp}} > E_{t-1}Y_{2t\mathbf{bal}} \tag{21}$$

The integration of international asset markets generates an output boom in the low volatility country and a contraction in the high volatility country. In both countries the change in output is led by exports.

3.2.4 Expected profits

Having computed the expected output, it is interesting to compute the expected profits for each firm. As firms use a linear technology we separate each firm between a domestic firm, producing for domestic sales, and an export firm for clarity. We measure the expected value of profits for the firm owner, that is the expectation of sales net of wages, adjusted for the marginal utility of revenue. For comparability profits are scaled by the size of the market in which the firm sells its goods. $E_{t-1}\Pi_{jit}$ denotes the expected profit for a firm producing in country i for sales in country j. For instance, the expected profits of a domestic firm in country 1 are:

$$E_{t-1}\Pi_{11t} = \frac{1}{n} n E_{t-1} C_{1t}^{-\rho} \frac{1}{P_{1t}} \left(P_{11t} - W_{1t}\right) \frac{P_{1t}C_{1t}}{P_{11t}}$$

Similarly, the expected profits of an export firm in country 1 are:

$$E_{t-1}\Pi_{21t} = \frac{1}{1-n} \left(1-n\right) E_{t-1} C_{1t}^{-\rho} \frac{1}{P_{1t}} \left(S_t P_{21t} - W_{1t}\right) \frac{P_{2t}C_{2t}}{P_{21t}}$$

The results for firms located in country 1 are given in table 6, and we can show that:

$$E_{t-1}\Pi_{11t\mathbf{bal}} = E_{t-1}\Pi_{21t\mathbf{bal}} > E_{t-1}\Pi_{11t\mathbf{comp}} > E_{t-1}\Pi_{21t\mathbf{comp}} \tag{22}$$

The integration of international markets reduces expected profits in the high volatility country, especially for export firms. We can undertake similar steps for firms producing in country 2. The results are given in table 7 and are ranked as follows:

$$E_{t-1}\Pi_{12t\mathbf{comp}} > E_{t-1}\Pi_{22t\mathbf{comp}} > E_{t-1}\Pi_{22t\mathbf{bal}} = E_{t-1}\Pi_{12t\mathbf{bal}} \tag{23}$$

The integration of international asset markets boosts the profits of firms operating in country 2, especially for export firms.

3.3 Summary of the positive findings

Before proceeding to the welfare analysis, we review the positive implications of the model. The integration of international asset markets, i.e. a switch from the **bal** to the **comp** model, increases the volatility of the exchange rate as well as the magnitude of the co-movements between the exchange rate and consumptions. This impact on the second moments is taken into account by exporting firms in their price setting decisions. The higher volatility of the exchange rate leads firms to lower their prices as their revenue is convex with respect to the exchange rate. This is the dominant force for firms in country 2 which reduce the price they charge for sales in country 1. For country 1 firms however, the higher exchange rate volatility is more than offset by the fact that revenues are low precisely in the states firms owners care more about. This leads them to increase the prices they charge in country 2.

The integration of international asset markets therefore increases the price of exports from country 1, while reducing the price of exports from country 2. Goods produced in country 2 are then more competitive and all consumers worldwide switch towards them. This consumption switching boosts output and expected profits in country 2, especially for the export firms. The opposite occurs in country 1.

The export led growth in country 2 is however offset by a deterioration of its terms of trade, as shown in the Appendix. The price of imported goods is higher in country 2, while the price of domestic goods remains unchanged. The purchasing power of consumers in country 2 is reduced, leading to a lowering of expected consumption. The opposite occurs in country 1 where expected consumption is higher thanks to the improvement of the terms of trade.

4 Welfare results

How does the integration of world markets affect the welfare in each country? We are interested in the welfare effect ex-ante, instead of the consequences of a particular shock. The welfare metric we choose is the expectation of the representative household's utility (1). As the impact of shocks is limited to one period, we evaluate the following measure of welfare for each country i:

$$E_{t-1}U_{it} = \frac{E_{t-1}C_{it}^{1-\rho}}{1-\rho} - \eta E_{t-1}Y_{it}$$

where we used $L_{it} = Y_{it}$. Following earlier contributions in the literature (Devereux and Engel (2000), Obstfeld and Rogoff (2000)), we focus on the impact through consumption and effort and abstract from the impact through real balances by considering that χ_i is a small number.[7]

The welfare impact of integrating international asset markets can easily be established

[7] Our results are reinforced if we take the welfare impact of real balances into account. The only impact of asset markets integration on $\chi_i E_{t-1} \ln\left(M_{it}P_{it}^{-1}\right)$ occurs through consumer prices. As prices are reduced in the high volatility country, the direct impact of real balances is beneficial. The opposite occurs in the low volatility country.

from our rankings of consumption (20) and effort (21). Recalling that $\rho > 1$, we derive:

$$E_{t-1}U_{1t\mathbf{bal}} < E_{t-1}U_{1t\mathbf{comp}} < E_{t-1}U_{2t\mathbf{comp}} < E_{t-1}U_{2t\mathbf{bal}} \tag{24}$$

The integration of international asset markets unambiguously benefits the high volatility country (country 1), at the expense of the low volatility country (country 2). It increases the purchasing power of households in the high volatility country through a reduction of import prices. By contrast households in the low volatility country experience a reduction of their purchasing power. The impact through consumption is reinforced by changes in expected efforts. As worldwide demands shifts towards goods produced in the low volatility country, households there have to work more. By contrast, households in the high volatility country enjoy more leisure. (24) also shows that the low volatility country is always better off than the high volatility country. The integration of asset markets reduces this gap without bringing it to zero.

5 A numerical example

Our results can be illustrated by a simple numerical example. We set the elasticity of substitution across brands, λ, to 6, implying a steady state markup of 20% over marginal cost. To abstract from country size effect we set n to 0.5. The weight of effort, η, is set to 1. We consider three possible values for ρ: 2, 3 and 4. We assume that there is no volatility in country 2, and set the standard deviation of consumption volatility in country 1 to 10%:[8]

$$\sigma^2(c_{1t}) = (0.1)^2 \quad , \quad \sigma^2(c_{2t}) = 0$$

We consider the impact from a complete integration of international asset markets, i.e. a move from the **bal** model to the **comp** model. The first step is to evaluate the impact on the volatility of the exchange rate, and its co-movements with consumption. Table 8 indicates the standard error of the exchange rate ($\sqrt{Var(s_t)}$), and the square root of its covariance with

[8] For clarity, we choose to hold the volatility of consumptions ($\sigma^2(c_{1t})$, $\sigma^2(c_{2t})$) fixed, instead of holding the volatility of monetary shocks (σ_1^2, σ_2^2) fixed.

consumption in the high volatility country ($\sqrt{Covar\left(s_t, c_{1t}\right)}$), both expressed as percentages. Both measures are significantly increased by the integration of international asset markets.

The changes in volatility brought in turn affect the price setting by firms. The percentage changes of consumer prices are shown in table 9. The change in volatility has no impact for the price of domestically produced goods, but leads to a significant reduction in the price of import in country 1, and an increase in import prices in country 2. Goods produced in country 2 are more competitive in both markets leading to a consumption switching away from goods produced in country 1. Households in country 1 benefit from lower prices, whereas their counterparts in country 2 experience a reduction in their purchasing power.

The change in consumer prices affect expected consumption in both countries as shown by table 10. The high volatility country experiences an overall consumption boom, as the increase in imports consumption is stronger than the contraction in consumption of domestic goods. The opposite is true in the low volatility country where the higher consumption of domestic goods is not strong enough to offset the contraction in imports, leading to an overall reduction in consumption.

Table 11 presents the impact on expected outputs and profits. The increase of the relative price of goods produced in country 1 leads to a contraction of output in country 1 and an expansion in country 2. The pattern of profits is similar: they are reduced for firms in the high volatility country and increased for their counterparts in the low volatility country. The changes in profits are stronger for export firms.

We now turn to the welfare effects. To give a straightforward intuitive interpretation to our results we express them in terms of equivalent consumption changes. These changes are defined as the percentage change in expected consumption from the **bal** model, holding consumption volatility and expected effort at their values in the **bal** model, that leads to the welfare level reached in the **comp** model. Specifically, the equivalent change in country i is the change between $E_{t-1}C_{it\mathbf{bal}}$ and $E_{t-1}C_{it\mathbf{equiv}}$ where $E_{t-1}C_{it\mathbf{equiv}}$ is the solution of:

$$E_{t-1}U_{it\mathbf{comp}} = \frac{1}{1-\rho}\left[E_{t-1}C_{it\mathbf{equiv}}\right]^{1-\rho}\exp\left[\rho\frac{\rho-1}{2}Var\left(c_{it\mathbf{bal}}\right)\right] - \eta E_{t-1}Y_{it\mathbf{bal}}$$

Table 12 presents the equivalent percentage changes in expected consumption. The integration of international asset has sharply different impact on the two countries. The high volatility country experiences a gain equivalent to a $0.6-3.5$ percent increase in expected consumption, with the low volatility country suffering from a loss of a slightly lower magnitude. These magnitudes are sizable, especially compared to the small gains from international asset markets integration found in some studies (Cole and Obstfeld (1991), Tesar (1995)) As the gain for country 1 slightly exceeds the loss for country 2, the integration of international asset markets is moderately beneficial on average.

Interestingly monopolistic competition is not the driving force behind the results. All the positive effects illustrated in tables 1 to 11 do not depend on the exact value of λ. Only the welfare effects in table 12 are affected. The impact is however small. In addition, a lower degree of monopolistic competition (a higher λ) actually magnifies the welfare results.

6 Extensions

6.1 Reduced volatility through integration

Our analysis so far has focused on the impact of international asset markets integration, holding the volatility of shocks constant. We could however argue that the volatility is not invariant with respect to the structure of asset markets. For instance, the integration of international asset markets can allow for the diffusion of more efficient banking and financial practices, thereby strengthening the financial sector in the countries where it was initially weak.

To assess the impact of reduced volatility, we consider that the integration of international asset markets does not affect the volatility in country 2 but brings the volatility in country 1 all the way down to the level prevailing in country 2:

$$\sigma^2(c_{1t\mathbf{compRV}}) = \sigma^2(c_{2t}) < \sigma^2\left(c_{1t\mathbf{bal}}\right)$$

where **compRV** denotes the model with complete international asset markets and reduced volatility in country 1, whereas **comp** denotes the model with complete international asset markets and unchanged volatility.

The Appendix shows that the impact on expected consumption in country 2 is the same whether volatility is reduced or not. By contrast, the integration of international asset markets reduces expected consumption in country 1 when it is accompanied by reduced volatility:

$$E_{t-1}C_{1t\mathbf{compRV}} < E_{t-1}C_{1t\mathbf{bal}} \quad , \quad E_{t-1}C_{2t\mathbf{compRV}} = E_{t-1}C_{2t\mathbf{comp}}$$

The reduction of expected effort in country 1 brought by the integration of international asset markets is especially strong when volatility is reduced. By contrast expected effort increases in country 2, although by less than when volatility remains constant:

$$\begin{aligned} E_{t-1}Y_{1t\mathbf{compRV}} &< E_{t-1}Y_{1t\mathbf{comp}} < E_{t-1}Y_{1t\mathbf{bal}} \\ E_{t-1}Y_{2t\mathbf{bal}} &< E_{t-1}Y_{2t\mathbf{compRV}} < E_{t-1}Y_{2t\mathbf{comp}} \end{aligned}$$

Turning the welfare effect, the consumption component is affected by two opposite influences in country 1. The reduced volatility of consumption is beneficial, whereas the reduction in expected consumption is harmful. We can show that the first channel is stronger and the consumption component of welfare in country 1 is higher:

$$\frac{1}{1-\rho}E_{t-1}C_{1t\mathbf{compRV}}^{1-\rho} > \frac{1}{1-\rho}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} > \frac{1}{1-\rho}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}$$

Country 1 therefore benefits from the integration of international asset markets by even more when it is accompanied by a reduction of volatility:

$$E_{t-1}U_{1t\mathbf{compRV}} > E_{t-1}U_{1t\mathbf{comp}} > E_{t-1}U_{1t\mathbf{bal}}$$

Country 2 is still adversely affected as expected consumption decreases and expected effort increases. The magnitude of the effect is however smaller than in the case with constant volatility:

$$E_{t-1}U_{2t\mathbf{comp}} < E_{t-1}U_{2t\mathbf{compRV}} < E_{t-1}U_{2t\mathbf{bal}}$$

Intuitively, the reduction of volatility dampens the worsening of country 2's terms of trade, but does not removes it entirely. Our conclusions remain valid even if $\sigma_2^2 = 0$, in which case there is no uncertainty in the **compRV** model. If country 2 has a perfectly stable financial sector it is still adversely affected by the integration of international asset markets, even when this integration entirely removes the volatility in foreign financial markets.

The magnitude of the welfare effect under reduced volatility is illustrated in table 13. It presents the increase in expected consumption that would be required in the **bal** model, holding effort and consumption volatility constant, to reach the welfare level attained in the model with complete international asset markets and reduced volatility. Compared with table 12, the benefit from integration in country 1 is made significantly larger by the reduction of volatility. By contrast, the adverse impact in country 2 is only moderately reduced.

6.2 Asymmetric exchange rate pass-through

Our analysis assumes that all export prices are set in the consumer currency. This assumption can seem too extreme if we think of country 1 as an emerging market. Instead, a setup with asymmetric pass-through may be more appropriate. In such a setup firms in country 1 still set their export prices in country 2 currency. By contrast, firms in country 2 set their export prices in their own currency and pass exchange rate fluctuations through to consumer prices in country 1. Engel (2000) argues that such a setup can be more relevant than a symmetric framework with zero pass-through. We can undertake our analysis under an asymmetric pass-through setup. The steps are more complex and are given in the Appendix. The main finding is that our results remain valid.

Setting $\sigma_2^2 = 0$ for simplicity we find that the integration of international asset markets still increases the exchange rate volatility. However it now reduces the volatility of consumption in country 1:

$$\sigma^2 (s_t)_{\mathbf{comp}} > \sigma^2 (s_t)_{\mathbf{bal}} \quad , \quad \sigma^2 (c_{1t})_{\mathbf{comp}} < \sigma^2 (c_{1t})_{\mathbf{bal}}$$

Intuitively the exchange rate dampens the impact of monetary shocks on consumption. An unexpected increase in M_{1t} leads to a depreciation of the country 1 currency. This depreciation in turn increases the price of imports and partially offsets the impact of the monetary shock on real balances. As consumption is proportional to real balances, the fluctuations in import prices dampen the impact of monetary shocks. By increasing the volatility of the exchange rate, the integration of international asset markets reinforces this dampening effect.

We can show that expected consumption in country 2 is still lower in the **comp** model than in the **bal** model. The impact on expected consumption in country 1 is ambiguous in general. However, we can show that if both countries are equally large ($n = 0.5$) expected consumption is reduced in the **comp** model.

To illustrate the effect of integrating of international asset markets under asymmetric pass-through, we compute a numerical example using the same parameter values as for the baseline model. The only difference is the volatility of monetary shocks in country 1. In our baseline model, we considered a 1 percent variance of consumption in country 1 ($\sigma^2(c_{1t}) = 0.01$). This translate into the following variance of monetary shocks ($\sigma_1^2 = \rho^2\sigma^2(c_{1t})$): 0.04 for $\rho = 2$, 0.09 for $\rho = 3$ and 0.16 for $\rho = 4$. Using these values for the underlying shocks we find that our baseline results remain valid. The integration of asset markets increases the volatility of the exchange rate. It also reduces expected consumption in both countries. This is however offset by a reduction of consumption volatility in country 1. Expected effort is reduced in country 1 and increased in country 2. In terms of welfare, table 14 indicates the equivalent changes in expected consumption. Country 1 benefits whereas country 2 looses. The magnitudes are smaller than under symmetric pass-through (table 12) but remain substantial. Interestingly the gain for country 1 is now smaller that the loss for country 2, implying that the average effect of asset market integration is actually detrimental.

7 Conclusion

This paper shows that the impact of the integration of international asset markets in the presence of monetary shocks is sensitive to imperfections in the goods markets. Whereas such an integration is neutral if segmented asset markets are the only distortion, it can be harmful in the presence of other distortions such as nominal rigidities. We assess the effect of international asset markets integration in a general equilibrium 2-country model where the two countries differ by the degree of volatility in their money market.

We show that in such a setup the integration of international asset markets affects the terms of trade between the two countries as firms set their prices in a forward looking manner. The welfare impacts significantly differ between the two countries. The benefits accrue entirely to the high volatility country whereas the low volatility country is made worse off, despite a gain in competitiveness leading to an export led boom and an increase in profits. Intuitively the worsening of the terms of trade of the low volatility country reduces the purchasing power of households.

We extend the setup in two directions. We allow for the integration of international asset markets to reduce the volatility of underlying shocks. We also consider a more realistic, albeit more complex, structure of exchange rate pass-through. Our results remain valid under these extensions.

The analysis shows that the impact of international asset markets integration cannot be evaluated abstracting from the structure of the rest of the economy. The model presented remains simple and extensions are a avenue for future research. Of particular interest are a richer modelization of the financial sector, as well as the inclusion of different sources of volatility.

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Table 1: Expected consumption		
Model	Country 1	Country 2
bal	$E_{t-1}C_{1t\mathbf{bal}} = \Omega^{-\frac{1}{\rho}} \exp\left[\frac{\rho-1}{2}\Sigma\right]$	$E_{t-1}C_{2t\mathbf{bal}} = E_{t-1}C_{1t\mathbf{bal}}$
comp	$E_{t-1}C_{1t\mathbf{comp}} = \Omega^{-\frac{1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})\right]$	$E_{t-1}C_{2t\mathbf{comp}} = \Omega^{-\frac{1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right]$
where: $\Sigma = n\sigma^2(c_{1t}) + (1-n)\sigma^2(c_{2t})$ $\Omega = \lambda\eta(\lambda-1)^{-1}$		

Table 2: Expected power consumption		
Model	Country 1	Country 2
bal	$E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} = \Omega^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})\right] \times \exp\left[(1-n)\frac{(\rho-1)^2}{2}\Delta\right]$	$E_{t-1}C_{2t\mathbf{bal}}^{1-\rho} = \Omega^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right] \times \exp\left[-n\frac{(\rho-1)^2}{2}\Delta\right]$
comp	$E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} = \Omega^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})\right]$	$E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} = \Omega^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right]$
where: $\Delta = \sigma^2(c_{1t}) - \sigma^2(c_{2t})$ $\Omega = \lambda\eta(\lambda-1)^{-1}$		

Table 3: Composition of consumption, country 1		
Model	Goods produced in country 1	Goods produced in country 2
bal	$E_{t-1}C_{11t\mathbf{bal}} = n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}$	$E_{t-1}C_{12t\mathbf{bal}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}$
comp	$E_{t-1}C_{11t\mathbf{comp}} = n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}$	$E_{t-1}C_{12t\mathbf{comp}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}$

Table 4: Composition of consumption, country 2		
Model	Goods produced in country 1	Goods produced in country 2
bal	$E_{t-1}C_{21t\mathbf{bal}} = n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}$	$E_{t-1}C_{22t\mathbf{bal}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}$
comp	$E_{t-1}C_{21t\mathbf{comp}} = n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}$	$E_{t-1}C_{22t\mathbf{comp}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}$

Table 5: Expected effort		
Model	Country 1	Country 2
bal	$E_{t-1}Y_{1t\mathbf{bal}} = \Omega^{-1}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}$	$E_{t-1}Y_{2t\mathbf{bal}} = \Omega^{-1}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}$
comp	$E_{t-1}Y_{1t\mathbf{comp}} = \Omega^{-1}\left[\begin{array}{c} nE_{t-1}C_{1t\mathbf{comp}}^{1-\rho} \\ +(1-n)\,E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}\end{array}\right]$	$E_{t-1}Y_{2t\mathbf{comp}} = E_{t-1}Y_{1t\mathbf{comp}}$
where: $\Omega = \lambda\eta\,(\lambda-1)^{-1}$		

Table 6: Expected profits, firms located in country 1		
Model	Goods sold in country 1	Goods sold in country 2
bal	$E_{t-1}\Pi_{11t\mathbf{bal}} = \lambda^{-1}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}$	$E_{t-1}\Pi_{21t\mathbf{bal}} = \lambda^{-1}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}$
comp	$E_{t-1}\Pi_{11t\mathbf{comp}} = \lambda^{-1}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}$	$E_{t-1}\Pi_{21t\mathbf{comp}} = \lambda^{-1}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}$

Table 7: Expected profits, firms located in country 2		
Model	Goods sold in country 1	Goods sold in country 2
bal	$E_{t-1}\Pi_{12t\mathbf{bal}} = \lambda^{-1}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}$	$E_{t-1}\Pi_{22t\mathbf{bal}} = \lambda^{-1}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}$
comp	$E_{t-1}\Pi_{12t\mathbf{comp}} = \lambda^{-1}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}$	$E_{t-1}\Pi_{22t\mathbf{comp}} = \lambda^{-1}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}$

Table 8: Impact of asset markets integration: Standard deviation of the exchange rate (%)			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
$\sqrt{Var\,(s_t)}$			
Bal model	10	10	10
Comp model	20	30	40
$\sqrt{Covar\,(s_t, c_{1t})}$			
Bal model	10	10	10
Comp model	14	17	20

Table 9: Impact of asset markets integration: Percentage change in consumer prices			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
Country 1: P_{1t}	-0.50	-1.49	-2.96
P_{11t}	0	0	0
P_{12t}	-1.00	-2.96	-5.82
Country 2: P_{2t}	0.50	1.51	3.05
P_{21t}	1.01	3.05	6.18
P_{22t}	0	0	0

Table 10: Impact of asset markets integration: Percentage change in expected consumption			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
Country 1: $E_{t-1}C_{1t}$	0.25	0.50	0.75
$E_{t-1}C_{11t}$	-0.25	-1.00	-2.23
$E_{t-1}C_{12t}$	0.75	2.02	3.82
Country 2: $E_{t-1}C_{2t}$	-0.25	-0.50	-0.75
$E_{t-1}C_{21t}$	-0.75	-1.98	-3.68
$E_{t-1}C_{22t}$	0.25	1.01	2.28

Table 11: Impact of asset markets integration: Percentage change in expected output and profits			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
Country 1: $E_{t-1}Y_{1t}$	-0.50	-1.49	-2.95
$E_{t-1}\Pi_{11t}$	-0.25	-1.00	-2.23
$E_{t-1}\Pi_{21t}$	-0.75	-1.98	-3.68
Country 2: $E_{t-1}Y_{2t}$	0.50	1.51	3.05
$E_{t-1}\Pi_{12t}$	0.75	2.02	3.82
$E_{t-1}\Pi_{22t}$	0.25	1.01	2.28

Table 12: Impact of asset markets integration: Welfare effect Equivalent percentage change in expected consumption			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
Country 1: $E_{t-1}C_{1t\mathbf{equiv}}$	0.67	1.78	3.42
Country 2: $E_{t-1}C_{2t\mathbf{equiv}}$	-0.66	-1.72	-3.10

Table 13: Impact of asset markets integration and reduced volatility: Welfare effect Equivalent percentage change in expected consumption			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
Country 1: $E_{t-1}C_{1t\mathbf{equiv}}$	1.39	2.75	4.70
Country 2: $E_{t-1}C_{2t\mathbf{equiv}}$	-0.46	-1.31	-2.52

Table 14: Impact of asset markets integration under assymetric pass-through: Welfare effect Equivalent percentage change in expected consumption			
	$\rho = 2$	$\rho = 3$	$\rho = 4$
Country 1: $E_{t-1}C_{1t\mathbf{equiv}}$	0.39	1.10	2.16
Country 2: $E_{t-1}C_{2t\mathbf{equiv}}$	-0.46	-1.40	-2.76

8 Appendix

8.1 Appendix 1: Household optimization

We start by focusing on the dimensions of the solution that are the same in the **comp** and the **bal** model. Household x living in country 1 chooses a sequence of state contingent consumption $C_1(x,h)$, money holding $M_1(x,h)$, contingent assets holdings and hours worked $L_1(x,h)$ to maximize:

$$\begin{aligned}
\mathcal{L} \;=\; & \sum_{s=0}^{\infty}\beta^s\sum_{h_{t+s}}\Pr(h_{t+s})\left\{\frac{[C_{1t+s}(x,h_{t+s})]^{1-\rho}}{1-\rho}\right. \\
& \left.+\chi_1\ln\left(\frac{M_{1t+s}(x,h_{t+s})}{P_{1t+s}(h_{t+s})}\right)-\eta L_{1t+s}(x,h_{t+s})\right\} \\
& -\sum_{s=0}^{\infty}\beta^s\sum_{h_{t+s}}\lambda_{1t+s}(x,h_{t+s})\{P_{1t+s}(h_{t+s})C_{1t+s}(x,h_{t+s}) \\
& +M_{1t+s}(x,h_{t+s})-M_{1t+s-1}(x,h_{t+s-1}) \\
& +\mathsf{BP}_{1t+s+1}(x,h_{t+s})-\mathsf{BR}_{1+st}(x,h_{t+s}) \\
& -\Pi_{1t+s}(x,h_{t+s})-W_{1t+s}(h_{t+s})L_{1t+s}(x,h_{t+s})-T_{1t+s}(x,h_{t+s})\}
\end{aligned} \tag{25}$$

where $\Pr(h_{t+s})$ is the probability that state h_{t+s} occurs and $\lambda_{1t+s}(x,h_{t+s})$ is the Lagrange multiplier on the associated budget constraint. We focus on period t and $t+1$ for simplicity and write the first order conditions with respect to consumption, money holdings and hours worked as:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial C_{1t}(x)} &= 0\Rightarrow\frac{[C_{1t}(x)]^{-\rho}}{P_{1t}}=\lambda_{1t}(x) \\
\frac{\partial\mathcal{L}}{\partial C_{1t+1}(x,h_{t+1})} &= 0\Rightarrow\frac{[C_{1t+1}(x,h_{t+1})]^{-\rho}}{P_{1t+1}(h_{t+1})}=\frac{\lambda_{1t+1}(x,h_{t+1})}{\Pr(h_{t+1})} \\
\frac{\partial\mathcal{L}}{\partial M_{1t}(x)} &= 0\Rightarrow\chi_1\frac{1}{M_{1t}(x)}=\lambda_{1t}-\beta\sum_{h_{t+1}}\lambda_{1t+1}(x,h_{t+1}) \\
\frac{\partial\mathcal{L}}{\partial L_{1t}(x)} &= 0\Rightarrow\frac{\eta}{W_{1t}}=\lambda_{1t}(x)
\end{aligned} \tag{26}$$

As all households within a country are identical, we drop the index x and rewrite the conditions as:

$$\frac{W_{1t}}{P_{1t}}=\eta C_{1t}^{\rho} \qquad , \qquad \frac{M_{1t}}{P_{1t}}=C_{1t}^{\rho}\frac{\chi_1}{1-E_t d_{1t,t+1}}$$

where $d_{1t,t+1}$ is the discount factor between the two periods:

$$d_{1t,t+1} = \beta \frac{P_{1t} C_{1t}^{\rho}}{P_{1t+1} C_{1t+1}^{\rho}} \tag{27}$$

following Obstfeld and Rogoff (1998), we can show that the discount factor is a constant if M_{1t} follows a random walk with drift:

$$\frac{M_{1t+1}}{M_{1t}} = (1+\mu)\,\epsilon_{t+1}$$

with ϵ is and i.i.d. shock with mean 1. From (26) we can write the first order condition with respect to money balances as:

$$1 = \frac{\chi_1 P_{1t} C_{1t}^{\rho}}{M_{1t}} + \beta E_t \frac{P_{1t} C_{1t}^{\rho}}{P_{1t+1} C_{1t+1}^{\rho}}$$

Conjecturing that the ratio between M_{1t} and $P_{1t} C_{1t}^{\rho}$ is constant across states, we show that $d_{1t,t+1}$ is a constant and write:

$$\frac{M_{1t}}{P_{1t}} = C_{1t}^{\rho} \frac{\chi_1}{1-d_1}$$

Similar steps can be undertaken for an household living in country 2, and we obtain:

$$\frac{W_{2t}}{P_{2t}} = \eta C_{2t}^{\rho} \qquad , \qquad \frac{M_{2t}}{P_{2t}} = C_{2t}^{\rho} \frac{\chi_2}{1-d_2}$$

We now turn our focus on the role of contingent securities. $B_{ijt+1}(x, h_{t+1})$ denotes the holding by household x living in country i of securities paying 1 unit of country j currency in period $t+1$ if the state of the world is h_{t+1}. $q_{jt}(h_{t+1})$ is the price in country j currency and period t of a security paying 1 unit of country j currency in period $t+1$ if the state of the world is h_{t+1}. For simplicity we focus on the optimal holdings of securities purchased in period t and paying off in period $t+1$. In the **comp** model, all households can trade in all securities:

$$\begin{aligned}
\mathsf{BP}_{1t+1}(x) &= \sum_{h_{t+1}} q_{1t}(h_{t+1}) B_{11t+1}(x, h_{t+1}) + \sum_{h_{t+1}} S_t q_{2t}(h_{t+1}) B_{12t+1}(x, h_{t+1}) \\
\mathsf{BR}_{1t}(x) &= B_{11t}(x, h_t) + S_t B_{12t}(x, h_t) \\
\mathsf{BP}_{2t+1}(x) &= \sum_{h_{t+1}} S_t^{-1} q_{1t}(h_{t+1}) B_{21t+1}(x, h_{t+1}) + \sum_{h_{t+1}} q_{2t}(h_{t+1}) B_{22t+1}(x, h_{t+1}) \\
\mathsf{BR}_{2t}(x) &= S_t^{-1} B_{21t}(x, h_t) + B_{22t}(x, h_t)
\end{aligned}$$

where S is the nominal exchange rate (units of country 1 currency per unit of country 2 currency). The Lagrangian (25) for a household living in country 1 is then written as:

$$\begin{aligned}\mathcal{L} &= \Theta - \lambda_{1t}(x)\left[\sum_{h_{t+1}} q_{1t}(h_{t+1}) B_{11t+1}(x, h_{t+1}) + \sum_{h_{t+1}} S_t q_{2t}(h_{t+1}) B_{12t+1}(x, h_{t+1})\right] \\ &\quad -\beta \sum_{h_{t+s}} \lambda_{1t+1}(x, h_{t+1}) \left[-B_{11t+1}(x, h_{t+1}) - S_{t+1}(h_{t+1}) B_{12t+1}(x, h_{t+1})\right]\end{aligned}$$

where Θ is a term unaffected by the holdings of securities. The optimal conditions are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial B_{11t+1}(x, h_{t+1})} &= 0 \Rightarrow q_{1t}(h_{t+1}) = \beta \frac{\lambda_{1t+1}(x, h_{t+1})}{\lambda_{1t}(x)} \\ \frac{\partial \mathcal{L}}{\partial B_{12t+1}(x, h_{t+1})} &= 0 \Rightarrow q_{2t}(h_{t+1}) = \beta \frac{\lambda_{1t+1}(x, h_{t+1})}{\lambda_{1t}(x)} \frac{S_{t+1}(h_{t+1})}{S_t}\end{aligned}$$

Undertaking similar steps for a household living in country 2 leads to:

$$\begin{aligned}q_{1t}(h_{t+1}) &= \beta \frac{\lambda_{2t+1}(x, h_{t+1})}{\lambda_{2t}(x)} \frac{S_t}{S_{t+1}(h_{t+1})} \\ q_{2t}(h_{t+1}) &= \beta \frac{\lambda_{2t+1}(x, h_{t+1})}{\lambda_{2t}(x)}\end{aligned}$$

Recall that:

$$\frac{C_{2t}^{-\rho}}{P_{2t}} = \lambda_{2t} \qquad , \qquad \frac{C_{2t+1}^{-\rho}(h_{t+1})}{P_{2t+1}(h_{t+1})} = \frac{\lambda_{2t+1}(h_{t+1})}{\Pr(h_{t+1})}$$

Combining the conditions for households living in different countries, we show that:

$$\frac{\lambda_{2t+1}(h_{t+1})}{\lambda_{2t}} S_t = \frac{\lambda_{1t+1}(h_{t+1})}{\lambda_{1t}} S_{t+1}(h_{t+1})$$

$$\Rightarrow \frac{C_{1t}^{-\rho}}{C_{2t}^{-\rho}} \frac{S_t P_{2t}}{P_{1t}} = \frac{S_{t+1}(h_{t+1}) P_{2t+1}(h_{t+1})}{P_{1t+1}(h_{t+1})} \frac{C_{1t+1}^{-\rho}(h_{t+1})}{C_{2t+1}^{-\rho}(h_{t+1})}$$

The real exchange rate is proportional to the ratio of consumption in any state of the world:

$$\frac{S_t P_{2t}}{P_{1t}} = \Gamma \frac{C_{1t}^{\rho}}{C_{2t}^{\rho}} \qquad , \qquad \frac{S_{t+1} P_{2t+1}}{P_{1t+1}} = \Gamma \frac{C_{1t+1}^{\rho}}{C_{2t+1}^{\rho}} \qquad \forall h_{t+1}$$

where Γ is a constant that reflects the initial conditions as shown by Chari, Kehoe and McGrattan (2000). For simplicity, we consider that the no country holds claims on the other initially, so that $\Gamma = 1$.

In the **bal** model, households can trade only in the securities paying off in the currency of their country of residence:

$$
\begin{aligned}
\mathsf{BP}_{1t+1}(x) &= \sum_{h_{t+1}} q_{1t}(h_{t+1}) B_{11t+1}(x, h_{t+1}) \\
\mathsf{BR}_{1t}(x) &= B_{11t}(x, h_t) \\
\mathsf{BP}_{2t+1}(x) &= \sum_{h_{t+1}} q_{2t}(h_{t+1}) B_{22t+1}(x, h_{t+1}) \\
\mathsf{BR}_{2t}(x) &= B_{22t}(x, h_t)
\end{aligned}
$$

The Lagrangian (25) for a household living in country 1 is now written as:

$$
\mathcal{L} = \Theta - \lambda_{1t}(x) \sum_{h_{t+1}} q_{1t}(h_{t+1}) B_{11t+1}(x, h_{t+1}) - \beta \sum_{h_{t+s}} \lambda_{1t+1}(x, h_{t+1}) \left[-B_{11t+1}(x, h_{t+1}) \right]
$$

Using (27), the optimality condition is:

$$
q_{1t}(h_{t+1}) = \beta \frac{\lambda_{1t+1}(x, h_{t+1})}{\lambda_{1t}(x)} \Rightarrow q_{1t}(h_{t+1}) = \Pr(h_{t+1}) d_1
$$

Following similar steps for a household living in country 2, we write:

$$
q_{2t}(h_{t+1}) = \beta \frac{\lambda_{2t+1}(x, h_{t+1})}{\lambda_{2t}(x)} \Rightarrow q_{2t}(h_{t+1}) = \Pr(h_{t+1}) d_2
$$

The optimality conditions for securities holdings do not generate a relation between relative consumption and the real exchange rate. Instead, the prices of securities simply adjust so that the holdings are exactly zero, as households can only trade with other households that are identical to them.

8.2 Appendix 2: Firms optimization

We denote the quantity sold by a firm producing brand v in country j by $Y_{jt}(v)$. Aggregating the demand from a country 1 household (2)-(3) and their counterparts for a country 2 household, we write:

$$
\begin{aligned}
Y_{1t}(v) &= n \left[\frac{P_{11t}(v)}{P_{11t}} \right]^{-\lambda} \frac{P_{1t} C_{1t}}{P_{11t}} + (1-n) \left[\frac{P_{21t}(v)}{P_{21t}} \right]^{-\lambda} \frac{P_{2t} C_{2t}}{P_{21t}} \\
Y_{2t}(v) &= n \left[\frac{P_{12t}(v)}{P_{12t}} \right]^{-\lambda} \frac{P_{1t} C_{1t}}{P_{12t}} + (1-n) \left[\frac{P_{22t}(v)}{P_{22t}} \right]^{-\lambda} \frac{P_{2t} C_{2t}}{P_{22t}}
\end{aligned}
$$

The optimization problem of the firm is to maximize its expected profits one period ahead. Firms located in a country being owned by the households residing in that country, the profits are discounted by the marginal utility of income. A firm in country 1 sets its prices to maximize:

$$E_{t-1}\left\{\frac{C_{1t}^{-\rho}}{P_{1t}}\left[P_{11t}(v)-W_{1t}\right]n\left[\frac{P_{11t}(v)}{P_{11t}}\right]^{-\lambda}\frac{P_{1t}C_{1t}}{P_{11t}}\right\}$$
$$+E_{t-1}\left\{\frac{C_{1t}^{-\rho}}{P_{1t}}\left[S_tP_{21t}(v)-W_{1t}\right](1-n)\left[\frac{P_{21t}(v)}{P_{21t}}\right]^{-\lambda}\frac{P_{2t}C_{2t}}{P_{21t}}\right\}$$

All prices being preset, the only stochastic terms are the consumptions, the wage and the exchange rate. Taking the fist order conditions with respect to $P_{11t}(v)$ and $P_{21t}(v)$, and recalling that all firms are identical in equilibrium, leads to (9)-(10).

Similarly, a firm in country 2 maximizes:

$$E_{t-1}\left\{\frac{C_{2t}^{-\rho}}{P_{2t}}\left[S_t^{-1}P_{12t}(v)-W_{2t}\right]n\left[\frac{P_{12t}(v)}{P_{12t}}\right]^{-\lambda}\frac{P_{1t}C_{1t}}{P_{12t}}\right\}$$
$$+E_{t-1}\left\{\frac{C_{2t}^{-\rho}}{P_{2t}}\left[P_{22t}(v)-W_{2t}\right](1-n)\left[\frac{P_{22t}(v)}{P_{22t}}\right]^{-\lambda}\frac{P_{2t}C_{2t}}{P_{22t}}\right\}$$

The first order conditions with respect to $P_{12t}(v)$ and $P_{22t}(v)$ lead to (11)-(12).

8.3 Appendix 3: Optimal prices and expected consumption

8.3.1 The comp model

In the **comp** model, we derive the optimal prices by combining the labor supplies (5) and (7), the optimal risk sharing relation (14) and the optimal pricing equations (9)-(12). After some algebra, we write:

$$\begin{aligned}P_{11t} &= P_{12t}=P_{1t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t}}{E_{t-1}C_{1t}^{1-\rho}}\\ P_{21t} &= P_{22t}=P_{2t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t}}{E_{t-1}C_{2t}^{1-\rho}}\end{aligned}$$

All consumer prices charged in a given country are identical. Using the definition of the consumer price index we establish that:

$$P_{1t} = P_{11t}^{n}P_{12t}^{1-n}=P_{1t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t}}{E_{t-1}C_{1t}^{1-\rho}}\Rightarrow E_{t-1}C_{1t}^{1-\rho}=\frac{\lambda\eta}{\lambda-1}E_{t-1}C_{1t}$$

$$P_{2t} = P_{21t}^{n}P_{22t}^{1-n} = P_{2t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t}}{E_{t-1}C_{2t}^{1-\rho}} \Rightarrow E_{t-1}C_{2t}^{1-\rho} = \frac{\lambda\eta}{\lambda-1}E_{t-1}C_{2t}$$

As variables are log normal, we use the property that if $x = \ln X \rightsquigarrow N(\mu, \sigma^2)$ then $EX^a = \exp\left[a\mu + \frac{1}{2}a^2\sigma^2\right]$ to write:

$$E_{t-1}C_{1t}^{1-\rho} = \left[E_{t-1}C_{1t}\right]^{1-\rho}\exp\left[\frac{\rho(\rho-1)}{2}\sigma^2(c_{1t})\right]$$

Using our results so far, we compute the expected consumption levels as:

$$\begin{aligned} E_{t-1}C_{1t\mathbf{comp}} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}}\exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})\right] \\ E_{t-1}C_{2t\mathbf{comp}} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}}\exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right] \end{aligned}$$

Furthermore, we can establish that:

$$\begin{aligned} E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})\right] \\ E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right] \end{aligned}$$

Turning to the allocation of consumption between domestic of foreign goods, we use the consumption demands (2)-(3) and their counterparts in country 2 to write:

$$\begin{aligned} C_{11t} &= n\frac{P_{1t}}{P_{11t}}C_{1t} \quad , \quad C_{12t} = (1-n)\frac{P_{1t}}{P_{12t}}C_{1t} \\ C_{21t} &= n\frac{P_{2t}}{P_{21t}}C_{2t} \quad , \quad C_{22t} = (1-n)\frac{P_{2t}}{P_{22t}}C_{2t} \end{aligned}$$

Using our results for the optimal prices, we obtain:

$$\begin{aligned} E_{t-1}C_{11t\mathbf{comp}} &= n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} \quad , \quad E_{t-1}C_{12t\mathbf{comp}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} \\ E_{t-1}C_{21t\mathbf{comp}} &= n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} \quad , \quad E_{t-1}C_{22t\mathbf{comp}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} \end{aligned}$$

8.3.2 The bal model

When international asset markets are not integrated, we proceed as for the **comp** model, using (15) instead of (14). After some algebra we write:

$$\begin{aligned} P_{11t} &= P_{1t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t}}{E_{t-1}C_{1t}^{1-\rho}} \quad , \quad P_{21t} = P_{2t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t}}{E_{t-1}C_{1t}^{1-\rho}} \\ P_{12t} &= P_{1t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t}}{E_{t-1}C_{2t}^{1-\rho}} \quad , \quad P_{22t} = P_{2t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t}}{E_{t-1}C_{2t}^{1-\rho}} \end{aligned}$$

From the price index we write:

$$\begin{aligned}
P_{1t} &= P_{11t}^{n} P_{12t}^{1-n} = P_{1t} \frac{\lambda\eta}{\lambda-1} \left[\frac{E_{t-1}C_{1t}}{E_{t-1}C_{1t}^{1-\rho}}\right]^{n} \left[\frac{E_{t-1}C_{1t}}{E_{t-1}C_{2t}^{1-\rho}}\right]^{1-n} \\
P_{2t} &= P_{21t}^{n} P_{22t}^{1-n} = P_{2t} \frac{\lambda\eta}{\lambda-1} \left[\frac{E_{t-1}C_{2t}}{E_{t-1}C_{1t}^{1-\rho}}\right]^{n} \left[\frac{E_{t-1}C_{2t}}{E_{t-1}C_{2t}^{1-\rho}}\right]^{1-n}
\end{aligned}$$

Combining these two equations leads to:

$$\begin{aligned}
\left[E_{t-1}C_{1t}^{1-\rho}\right]^{n} \left[E_{t-1}C_{2t}^{1-\rho}\right]^{1-n} &= \frac{\lambda\eta}{\lambda-1} E_{t-1}C_{1t} \\
\left[E_{t-1}C_{1t}^{1-\rho}\right]^{n} \left[E_{t-1}C_{2t}^{1-\rho}\right]^{1-n} &= \frac{\lambda\eta}{\lambda-1} E_{t-1}C_{2t}
\end{aligned}$$

As the left-hand side is the same in both relations, the expected consumption is equal across countries:

$$E_{t-1}C_{1t\mathbf{bal}} = E_{t-1}C_{2t\mathbf{bal}}$$

We can now re-write any of the two relations as:

$$\left[E_{t-1}C_{1t\mathbf{bal}}\right]^{-\rho} \exp\left[n\frac{\rho(\rho-1)}{2}\sigma^{2}(c_{1t})\right] \exp\left[(1-n)\frac{\rho(\rho-1)}{2}\sigma^{2}(c_{2t})\right] = \frac{\lambda\eta}{\lambda-1}$$

$$\Rightarrow E_{t-1}C_{1t\mathbf{bal}} = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}} \exp\left[\frac{\rho-1}{2}\left(n\sigma^{2}(c_{1t}) + (1-n)\sigma^{2}(c_{2t})\right)\right]$$

The expected consumption in the **bal** model is between the values for the two countries in the **comp** model. We can also write:

$$\begin{aligned}
E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^{2}(c_{1t})\right] \\
&\quad \exp\left[(1-n)\frac{(\rho-1)^{2}}{2}\left(\sigma^{2}(c_{1t}) - \sigma^{2}(c_{2t})\right)\right] \\
E_{t-1}C_{2t\mathbf{bal}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^{2}(c_{2t})\right] \\
&\quad \exp\left[-n\frac{(\rho-1)^{2}}{2}\left(\sigma^{2}(c_{1t}) - \sigma^{2}(c_{2t})\right)\right]
\end{aligned}$$

As in the **comp** model, we can derive the allocation of expected consumption across domestic and foreign goods by using the optimal prices:

$$\begin{aligned}
E_{t-1}C_{11t\mathbf{bal}} &= E_{t-1}C_{21t\mathbf{bal}} = n\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} \\
E_{t-1}C_{12t\mathbf{bal}} &= E_{t-1}C_{22t\mathbf{bal}} = (1-n)\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}
\end{aligned}$$

8.4 Appendix 4: Expected output

We compute the expected effort from (13):

$$\begin{aligned} E_{t-1}Y_{1t} &= n\frac{P_{1t}E_{t-1}C_{1t}}{P_{11t}} + (1-n)\frac{P_{2t}E_{t-1}C_{2t}}{P_{21t}} \\ E_{t-1}Y_{2t} &= n\frac{P_{1t}E_{t-1}C_{1t}}{P_{12t}} + (1-n)\frac{P_{2t}E_{t-1}C_{2t}}{P_{22t}} \end{aligned}$$

We combine these relations with the optimal prices (9)-(12) to derive the solution for expected output. In the **comp** model, we write:

$$E_{t-1}Y_{1t\mathbf{comp}} = \frac{\lambda-1}{\lambda\eta}\left[nE_{t-1}C_{1t\mathbf{comp}}^{1-\rho} + (1-n)E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}\right] = E_{t-1}Y_{2t\mathbf{comp}}$$

Similarly, we show that in the **bal** model:

$$\begin{aligned} E_{t-1}Y_{1t\mathbf{bal}} &= \frac{\lambda-1}{\lambda\eta}\left[nE_{t-1}C_{1t\mathbf{bal}}^{1-\rho} + (1-n)E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}\right] = \frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} \\ E_{t-1}Y_{2t\mathbf{bal}} &= \frac{\lambda-1}{\lambda\eta}\left[nE_{t-1}C_{2t\mathbf{bal}}^{1-\rho} + (1-n)E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}\right] = \frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho} \end{aligned}$$

8.5 Appendix 5: Expected profits

We now turn to the expected profits. Starting with country 1, we use the demand (2) to write the expected profits for a domestic firm as:

$$E_{t-1}\Pi_{11t} = \frac{1}{n}nE_{t-1}C_{1t}^{-\rho}\frac{1}{P_{1t}}(P_{11t} - W_{1t})\frac{P_{1t}C_{1t}}{P_{11t}}$$

where n^{-1} is the scaling parameters as the firm sells goods to n customers, $C_{1t}^{-\rho}P_{1t}^{-1}$ is the marginal utility of revenue, $P_{11t} - W_{1t}$ is the unit margin and $P_{1t}P_{11t}^{-1}C_{1t}$ is the demand from each consumer. Using the labor supply (5) we write:

$$E_{t-1}\Pi_{11t} = E_{t-1}C_{1t}^{1-\rho} - \eta\frac{P_{1t}E_{t-1}C_{1t}}{P_{11t}}$$

We can similarly write the expected profits of an export firm in country 1 as follows:

$$\begin{aligned} E_{t-1}\Pi_{21t} &= \frac{1}{1-n}(1-n)E_{t-1}C_{1t}^{-\rho}\frac{1}{P_{1t}}(S_tP_{21t} - W_{1t})\frac{P_{2t}C_{2t}}{P_{21t}} \\ &= E_{t-1}\frac{S_tP_{2t}}{P_{1t}}C_{1t}^{-\rho}C_{2t} - \eta\frac{P_{2t}E_{t-1}C_{2t}}{P_{21t}} \end{aligned}$$

Using our results for the optimal prices and the relations between the real exchange rate and relative consumptions, (14) and (15), we write:

$$\begin{aligned} E_{t-1}\Pi_{11t\mathbf{comp}} &= \frac{1}{\lambda}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} \quad , \quad E_{t-1}\Pi_{21t\mathbf{comp}} = \frac{1}{\lambda}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} \\ E_{t-1}\Pi_{11t\mathbf{bal}} &= \frac{1}{\lambda}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} \quad , \quad E_{t-1}\Pi_{21t\mathbf{bal}} = \frac{1}{\lambda}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} \end{aligned}$$

We can derive similar expressions for the expected profits of export and domestic firms in country 2:

$$\begin{aligned} E_{t-1}\Pi_{12t} &= \frac{1}{n}nE_{t-1}C_{2t}^{-\rho}\frac{1}{P_{2t}}\left(\frac{P_{21t}}{S_t} - W_{2t}\right)\frac{P_{1t}C_{1t}}{P_{12t}} \\ &= E_{t-1}\frac{P_{1t}}{S_tP_{2t}}C_{2t}^{-\rho}C_{1t} - \eta\frac{P_{1t}E_{t-1}C_{1t}}{P_{12t}} \\ E_{t-1}\Pi_{22t} &= \frac{1}{1-n}\left(1-n\right)E_{t-1}C_{2t}^{-\rho}\frac{1}{P_{2t}}\left(P_{22t} - W_{2t}\right)\frac{P_{2t}C_{2t}}{P_{22t}} \\ &= E_{t-1}C_{2t}^{1-\rho} - \eta\frac{P_{2t}E_{t-1}C_{2t}}{P_{22t}} \end{aligned}$$

Using the optimal prices, we can write:

$$\begin{aligned} E_{t-1}\Pi_{12t\mathbf{comp}} &= \frac{1}{\lambda}E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} \quad , \quad E_{t-1}\Pi_{22t\mathbf{comp}} = \frac{1}{\lambda}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} \\ E_{t-1}\Pi_{12t\mathbf{bal}} &= \frac{1}{\lambda}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho} \quad , \quad E_{t-1}\Pi_{22t\mathbf{bal}} = \frac{1}{\lambda}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho} \end{aligned}$$

8.6 Appendix 6: Expected terms of trade

The terms of trade are defined as the unit revenue from export divided by the unit cost of imports:

$$TOT_{1t} = \frac{S_tP_{21t}}{P_{12t}} = \frac{1}{TOT_{2t}}$$

To avoid any spurious conclusion reflecting Jensen inequality,[9] we start by analyzing the logs of the terms of trade:

$$tot_{1t} = s_t + p_{21t} - p_{12t} = -tot_{2t}$$

[9] As $E_{t-1}TOT_{1t} \neq \left(E_{t-1}TOT_{2t}\right)^{-1}$.

We start with the **comp** model. From the optimal prices and the optimal risk sharing condition (14), we write the expected terms of trade as:

$$\begin{aligned} E_{t-1}tot_{1t} &= \rho E_{t-1}c_{1t} - \rho E_{t-1}c_{2t} + \ln E_{t-1}C_{2t} - \ln E_{t-1}C_{2t}^{1-\rho} \\ &\quad + \ln E_{t-1}C_{1t}^{1-\rho} - \ln E_{t-1}C_{1t} \end{aligned}$$

Using the properties of the log normal distribution and some algebra we obtain:

$$E_{t-1}tot_{1t\mathbf{comp}} = \frac{\rho(\rho-2)}{2}\left[\sigma^2(c_{1t}) - \sigma^2(c_{2t})\right]$$

Turning to the **bal** model, we use the optimal prices and the balanced trade condition to write:

$$\begin{aligned} E_{t-1}tot_{1t} &= E_{t-1}c_{1t} - E_{t-1}c_{2t} + \ln E_{t-1}C_{2t} - \ln E_{t-1}C_{1t}^{1-\rho} \\ &\quad + \ln E_{t-1}C_{2t}^{1-\rho} - \ln E_{t-1}C_{1t} \end{aligned}$$

Recall that $E_{t-1}C_{1t\mathbf{bal}} = E_{t-1}C_{2t\mathbf{bal}} \Rightarrow E_{t-1}c_{1t} + \frac{1}{2}\sigma^2(c_{1t}) = E_{t-1}c_{2t} + \frac{1}{2}\sigma^2(c_{2t})$. After some algebra, we establish:

$$E_{t-1}tot_{1t\mathbf{bal}} = -\frac{\rho(\rho-1)+1}{2}\left[\sigma^2(c_{1t}) - \sigma^2(c_{2t})\right]$$

We can then establish that country 1 terms of trade are more favorable in the **comp** model:

$$E_{t-1}tot_{1t\mathbf{comp}} - E_{t-1}tot_{1t\mathbf{bal}} = \frac{(2\rho-1)(\rho-1)}{2}\left[\sigma^2(c_{1t}) - \sigma^2(c_{2t})\right] > 0$$

8.7 Appendix 7: Domestic prices

Our analysis shows that the integration of asset markets reduces the price of import from country 2 in country 1, and increase the price for import from country 1 in country 2. This Appendix computes the equilibrium impact on the prices of domestic goods, P_{11t} and P_{22t}.

From the optimal price (9) we know that the form of P_{11t} is the same in both the **comp** and the **bal** models:

$$P_{11t} = P_{1t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{1t}}{E_{t-1}C_{1t}^{1-\rho}} = P_{1t}\frac{\lambda\eta}{\lambda-1}\left[E_{t-1}C_{1t}\right]^{\rho}\exp\left[-\frac{\rho(\rho-1)}{2}\sigma^2(c_{1t})\right]$$

where we used $E_{t-1}C_{1t}^{1-\rho} = [E_{t-1}C_{1t}]^{1-\rho} \exp\left[\frac{\rho(\rho-1)}{2}\sigma^2(c_{1t})\right]$. From the money demand (6) we can write:

$$\begin{aligned}
P_{1t}C_{1t}^{\rho} &= \frac{1}{\chi_1}M_{1t}(1-d_1) \\
&\Rightarrow P_{1t}E_{t-1}C_{1t}^{\rho} = \frac{1-d_1}{\chi_1}E_{t-1}M_{1t} \\
&\Rightarrow P_{1t} = \frac{1-d_1}{\chi_1}[E_{t-1}C_{1t}]^{-\rho}\exp\left[-\rho\frac{\rho-1}{2}\sigma^2(c_{1t})\right]E_{t-1}M_{1t}
\end{aligned}$$

Therefore:

$$P_{11t} = \frac{1-d_1}{\chi_1}\frac{\lambda\eta}{\lambda-1}E_{t-1}M_{1t}\exp\left[-\rho(\rho-1)\sigma^2(c_{1t})\right]$$

All the terms on the right-hand side are the same in both the **comp** and the **bal** model.

We can undertake similar steps for the price of domestic goods in country 2. We know that in both the **comp** and the **bal** model:

$$P_{22t} = P_{2t}\frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}C_{2t}}{E_{t-1}C_{2t}^{1-\rho}} = P_{2t}\frac{\lambda\eta}{\lambda-1}[E_{t-1}C_{2t}]^{\rho}\exp\left[-\frac{\rho(\rho-1)}{2}\sigma^2(c_{2t})\right]$$

From the money demand (8) we write:

$$P_{2t} = \frac{1-d_2}{\chi_2}[E_{t-1}C_{2t}]^{-\rho}\exp\left[-\rho\frac{\rho-1}{2}\sigma^2(c_{2t})\right]E_{t-1}M_{2t}$$

Therefore:

$$P_{22t} = \frac{1-d_2}{\chi_2}\frac{\lambda\eta}{\lambda-1}E_{t-1}M_{2t}\exp\left[-\rho(\rho-1)\sigma^2(c_{2t})\right]$$

The integration of asset markets has no impact on the price of domestic goods, P_{11t} and P_{22t}, in both countries.

Our results also lead to simple expressions for the impact of asset market integration on consumer prices. We can show that:

$$\begin{array}{ccccc}
\dfrac{P_{1t\mathbf{comp}}}{P_{1t\mathbf{bal}}} < 1 & , & \dfrac{P_{11t\mathbf{comp}}}{P_{11t\mathbf{bal}}} = 1 & , & \dfrac{P_{12t\mathbf{comp}}}{P_{12t\mathbf{bal}}} < 1 \\
\dfrac{P_{2t\mathbf{comp}}}{P_{2t\mathbf{bal}}} > 1 & , & \dfrac{P_{21t\mathbf{comp}}}{P_{21t\mathbf{bal}}} > 1 & , & \dfrac{P_{22t\mathbf{comp}}}{P_{22t\mathbf{bal}}} = 1
\end{array}$$

8.8 Appendix 8: Reduced volatility through integration

We solve a variant of the model where the integration of international asset markets affects the volatility of shocks. We assume that the integration does not affect the volatility in country 2 (the low volatility country) but brings the volatility in country 1 down to the level of country 2. The solution for the **bal** model is as before, whereas the solution for the **comp** model becomes:

$$
\begin{aligned}
\sigma^2(c_{1t\mathbf{compRV}}) &= \sigma^2(c_{2t}) \\
E_{t-1}C_{1t\mathbf{compRV}} &= E_{t-1}C_{2t\mathbf{comp}} = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right] \\
E_{t-1}C_{1t\mathbf{compRV}}^{1-\rho} &= E_{t-1}C_{2t\mathbf{comp}}^{1-\rho} = \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}} \exp\left[\frac{\rho-1}{2}\sigma^2(c_{2t})\right]
\end{aligned}
$$

Where **compRV** denotes the model with complete markets and reduced volatility, and **comp** denotes the model with complete markets and unchanged volatility. Expected consumption in country 2 is not affected by the reduced volatility in country 1, whereas it is reduced in country 1. Turning to expected output we write:

$$E_{t-1}Y_{1t\mathbf{compRV}} = E_{t-1}Y_{2t\mathbf{compRV}} = \frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}$$

Using our results for the case where volatility remains constant, we infer:

$$
\begin{aligned}
E_{t-1}Y_{1t\mathbf{compRV}} &< E_{t-1}Y_{1t\mathbf{comp}} < E_{t-1}Y_{1t\mathbf{bal}} \\
E_{t-1}Y_{2t\mathbf{bal}} &< E_{t-1}Y_{2t\mathbf{compRV}} < E_{t-1}Y_{2t\mathbf{comp}}
\end{aligned}
$$

The welfare of both countries in the comp model is unambiguously higher when the volatility in country 1 is reduced, compared to the case with constant volatility. Recall that welfare is given by:

$$
\begin{aligned}
E_{t-1}U_{1t\mathbf{comp}} &= \frac{E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}}{1-\rho} - \frac{\lambda-1}{\lambda}\left[nE_{t-1}C_{1t\mathbf{comp}}^{1-\rho} + (1-n)\,E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}\right] \\
E_{t-1}U_{2t\mathbf{comp}} &= \frac{E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}}{1-\rho} - \frac{\lambda-1}{\lambda}\left[nE_{t-1}C_{1t\mathbf{comp}}^{1-\rho} + (1-n)\,E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}\right]
\end{aligned}
$$

Comparing the outcomes with and without the reduction in volatility, we know that $E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}$ is not affected and $E_{t-1}C_{1t\mathbf{compRV}}^{1-\rho} < E_{t-1}C_{1t\mathbf{comp}}^{1-\rho}$. Therefore, the reduction of volatility benefits both countries through a reduction in expected effort compared to the case with constant volatility, and also country 1 through the consumption term.

The integration of international asset markets is therefore especially beneficial to country 1 when it is accompanied by a reduction of its volatility. Country 2 is still adversely affected, although the reduction in volatility reduces the magnitude of the loss.

8.9 Appendix 9: Asymmetric pass-through

We consider a case where exchange rate pass-through is asymmetric. Firms in country 1 set their export price for sales to country 2 in country 2 currency. By contrast firms in country 2 set their price for sales to country 1 in country 2 currency. For simplicity, we focus on the case where there is no volatility in country 2 ($\sigma_2^2 = 0$)

8.9.1 Optimal price setting

A firm in country 1 chooses a price in country 1 currency for domestic sales, $P_{11t}(v)$, and a price in country 2 currency for export sales, $P_{21t}(v)$, in order to maximize:

$$E_{t-1}\left\{\frac{C_{1t}^{-\rho}}{P_{1t}}\left[P_{11t}(v) - W_{1t}\right] n \left[\frac{P_{11t}(v)}{P_{11t}}\right]^{-\lambda} \frac{P_{1t}C_{1t}}{P_{11t}}\right\}$$
$$+E_{t-1}\left\{\frac{C_{1t}^{-\rho}}{P_{1t}}\left[S_t P_{21t}(v) - W_{1t}\right](1-n)\left[\frac{P_{21t}(v)}{P_{21t}}\right]^{-\lambda} \frac{P_{2t}C_{2t}}{P_{21t}}\right\}$$

Using the labor supply (5) and the fact that firms are identical in equilibrium, the optimal prices are:

$$P_{11t} = \frac{\lambda\eta}{\lambda - 1}\frac{E_{t-1}P_{1t}C_{1t}}{E_{t-1}C_{1t}^{1-\rho}} \quad , \quad P_{21t} = \frac{\lambda\eta}{\lambda - 1}\frac{E_{t-1}P_{2t}C_{2t}}{E_{t-1}\frac{S_t P_{2t}}{P_{1t}}C_{2t}C_{1t}^{-\rho}}$$

A firm in country 2 chooses a price in country 2 currency for domestic sales, $P_{22t}(v)$, and a price in country 2 currency for export sales ($\tilde{P}_{12t}(v)$). Both prices are in the same currency, but are not necessarily equal. The firm maximizes:

$$E_{t-1}\left\{\frac{C_{2t}^{-\rho}}{P_{2t}}\left[\tilde{P}_{12t}(v) - W_{2t}\right] n \left[\frac{\tilde{P}_{12t}(v)}{\tilde{P}_{12t}}\right]^{-\lambda} \frac{P_{1t}C_{1t}}{S_t\tilde{P}_{12t}}\right\}$$
$$+E_{t-1}\left\{\frac{C_{2t}^{-\rho}}{P_{2t}}\left[P_{22t}(v) - W_{2t}\right](1-n)\left[\frac{P_{22t}(v)}{P_{22t}}\right]^{-\lambda} \frac{P_{2t}C_{2t}}{P_{22t}}\right\}$$

The optimal prices are given by:

$$\tilde{P}_{12t} = \frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}S_t^{-1}P_{1t}C_{1t}}{E_{t-1}\frac{P_{1t}}{S_tP_{2t}}C_{1t}C_{2t}^{-\rho}} \quad , \quad P_{22t} = \frac{\lambda\eta}{\lambda-1}\frac{E_{t-1}P_{2t}C_{2t}}{E_{t-1}C_{2t}^{1-\rho}}$$

8.9.2 Volatility

The volatility of consumptions and the exchange rate is driven by the money market equilibria (6) and (8). Omitting constant terms we write them in logarithms as:

$$m_{1t} - p_{1t} \propto \rho c_{1t} \quad , \quad m_{2t} - p_{2t} \propto \rho c_{2t}$$

p_{2t} is not stochastic as all consumer prices are preset in country 2. p_{1t} fluctuates as exchange rate fluctuations are passed through to import prices: $p_{1t} \propto (1-n)\, s_t$. We therefore write:

$$c_{1t} \propto \frac{1}{\rho}m_{1t} - \frac{1}{\rho}(1-n)\, s_t \quad , \quad c_{2t} \propto \frac{1}{\rho}m_{2t}$$

As we focus on the case where $\sigma_2^2 = 0$ consumption is non-stochastic in country 2.

Starting with the **bal** model, we write the balanced trade condition (15) in logarithms as:

$$s_t + p_{2t} - p_{1t} = c_{1t} - c_{2t} \Rightarrow s_t \propto \frac{1}{n}c_{1t}$$

Substituting in the money market equilibria we obtain

$$c_{1t} \propto \frac{n}{1+n(\rho-1)}m_{1t} \quad , \quad s_t \propto \frac{1}{1+n(\rho-1)}m_{1t}$$

These relations imply the following variance / covariance structure:

$$\begin{aligned} \sigma^2(c_{1t})_{\mathbf{bal}} &= (1+n(\rho-1))^{-2}\, n^2\sigma_1^2 \\ \sigma^2(s_t)_{\mathbf{bal}} &= (1+n(\rho-1))^{-2}\, \sigma_1^2 \\ \sigma(s_t, c_{1t})_{\mathbf{bal}} &= (1+n(\rho-1))^{-2}\, n\sigma_1^2 \end{aligned}$$

Turning to the **comp** model, we write the optimal risk sharing condition (14) in logarithms as:

$$s_t + p_{2t} - p_{1t} = \rho c_{1t} - \rho c_{2t} \Rightarrow s_t \propto \frac{\rho}{n}c_{1t}$$

Substituting in the money market equilibria we obtain:

$$c_{1t} \propto \frac{n}{\rho} m_{1t} \quad , \quad s_t \propto m_{1t}$$

Which implies the following variance / covariance structure:

$$\begin{aligned} \sigma^2 (c_{1t})_{\mathbf{comp}} &= \rho^{-2} n^2 \sigma_1^2 \\ \sigma^2 (s_t)_{\mathbf{comp}} &= \sigma_1^2 \\ \sigma (s_t, c_{1t})_{\mathbf{comp}} &= \rho^{-1} n \sigma_1^2 \end{aligned}$$

Comparing the **bal** and **comp** model, we see that:

$$\begin{aligned} \sigma^2 (c_{1t})_{\mathbf{comp}} &< \sigma^2 (c_{1t})_{\mathbf{bal}} \\ \sigma^2 (s_t)_{\mathbf{comp}} &> \sigma^2 (s_t)_{\mathbf{bal}} \\ \sigma (s_t, c_{1t})_{\mathbf{comp}} &> \sigma (s_t, c_{1t})_{\mathbf{bal}} \Leftrightarrow \rho > \left(\frac{1-n}{n}\right)^2 \end{aligned}$$

8.9.3 Prices and expected consumption

The bal model We start with the prices in country 2 where no prices react to the exchange rate. Recalling the definition of the consumer price index, $P_{2t} = P_{21t}^n P_{22t}^{1-n}$, we combine the optimal prices to write:

$$1 = \frac{\lambda\eta}{\lambda - 1} C_{2t} \left(\frac{1}{E_{t-1} C_{1t}^{1-\rho}}\right)^n \left(\frac{1}{C_{2t}^{1-\rho}}\right)^{1-n}$$

Using the properties of the log normal distribution, some algebra leads to:

$$\begin{aligned} 0 &= \ln\left(\frac{\lambda\eta}{\lambda-1}\right) + n(\rho-1) E_{t-1} c_{1t} - n\frac{(\rho-1)^2}{2}\sigma^2(c_{1t}) \\ &\quad + [1 + (1-n)(\rho-1)] c_{2t} \end{aligned}$$

Turning to the prices in country 1, we combine the definition of the consumer price index, $P_{1t} = P_{11t}^n \left(S_t \tilde{P}_{12t}\right)^{1-n}$, with the optimal prices to write:

$$1 = \frac{\lambda\eta}{\lambda - 1} \left(\frac{E_{t-1} S_t^{1-n} C_{1t}}{E_{t-1} C_{1t}^{1-\rho}}\right)^n \left(\frac{E_{t-1} S_t^{-n} C_{1t}}{C_{2t}^{1-\rho}}\right)^{1-n}$$

After some algebra, we can rewrite this expression as:

$$\begin{aligned} 0 &= \ln\left(\frac{\lambda\eta}{\lambda-1}\right)+\frac{n(1-n)}{2}\sigma^2(s_t) \\ &\quad +\left[1+n(\rho-1)\right]E_{t-1}c_{1t}+\frac{1-n(\rho-1)^2}{2}\sigma^2(c_{1t}) \\ &\quad +(1-n)(\rho-1)c_{2t} \end{aligned}$$

Combining our results, we derive the expected logarithm consumptions as:

$$\begin{aligned} E_{t-1}c_{1t} &= -\frac{1}{\rho}\ln\left(\frac{\lambda\eta}{\lambda-1}\right)-n(1-n)\frac{1+(1-n)(\rho-1)}{2\rho}\sigma^2(s_t) \\ &\quad -\frac{1}{2}\sigma^2(c_{1t})+\frac{\rho-1}{2}n\sigma^2(c_{1t}) \\ c_{2t} &= -\frac{1}{\rho}\ln\left(\frac{\lambda\eta}{\lambda-1}\right)+n(1-n)\frac{n(\rho-1)}{2\rho}\sigma^2(s_t)+\frac{\rho-1}{2}n\sigma^2(c_{1t}) \end{aligned}$$

This allows us to derive the expected consumption levels as follows:

$$\begin{aligned} E_{t-1}C_{1t\mathbf{bal}} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}}\exp\left[-n(1-n)\frac{1+(1-n)(\rho-1)}{2\rho}\sigma^2(s_t)_{\mathbf{bal}}+\frac{\rho-1}{2}n\sigma^2(c_{1t})_{\mathbf{bal}}\right] \\ C_{2t\mathbf{bal}} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}}\exp\left[n(1-n)\frac{n(\rho-1)}{2\rho}\sigma^2(s_t)_{\mathbf{bal}}+\frac{\rho-1}{2}n\sigma^2(c_{1t})_{\mathbf{bal}}\right] \end{aligned}$$

In addition, we derive the expected power consumptions as:

$$\begin{aligned} E_{t-1}C_{1t\mathbf{bal}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\left[\begin{array}{c}(\rho-1)n(1-n)\frac{1+(1-n)(\rho-1)}{2\rho}\sigma^2(s_t)_{\mathbf{bal}} \\ +\rho\frac{\rho-1}{2}\sigma^2(c_{1t})_{\mathbf{bal}}-\frac{(\rho-1)^2}{2}n\sigma^2(c_{1t})_{\mathbf{bal}}\end{array}\right] \\ C_{2t\mathbf{bal}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\left[\begin{array}{c}-(\rho-1)n(1-n)\frac{n(\rho-1)}{2\rho}\sigma^2(s_t)_{\mathbf{bal}} \\ -\frac{(\rho-1)^2}{2}n\sigma^2(c_{1t})_{\mathbf{bal}}\end{array}\right] \end{aligned}$$

The comp model We now turn to the **comp** model. Combining the consumer price index in country 2, $P_{2t}=P_{21t}^{n}P_{22t}^{1-n}$, with the optimal prices, we obtain the expected consumption:

$$C_{2t\mathbf{comp}}=\left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}}$$

Turning to country 1, we combine the optimal prices with the expression for the consumer price index, $P_{1t}=P_{11t}^{n}\left(S_t\tilde{P}_{12t}\right)^{1-n}$, to write:

$$1=\frac{\lambda\eta}{\lambda-1}\frac{\left(E_{t-1}S_t^{1-n}C_{1t}\right)^{n}\left(E_{t-1}S_t^{-n}C_{1t}\right)^{1-n}}{E_{t-1}C_{1t}^{1-\rho}}$$

After some algebra, we obtain:

$$\begin{aligned} 0 &= \ln\left(\frac{\lambda\eta}{\lambda-1}\right)+\rho E_{t-1}c_{1t}-\frac{(1-\rho)^2}{2}\sigma^2(c_{1t}) \\ &+\frac{n}{2}\left((1-n)^2\sigma^2(s_t)+\sigma^2(c_{1t})\right)+\frac{(1-n)}{2}\left(n^2\sigma^2(s_t)+\sigma^2(c_{1t})\right) \end{aligned}$$

Which leads us to the expected consumption:

$$E_{t-1}C_{1t\mathbf{comp}}=\left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}}\exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})_{\mathbf{comp}}-\frac{n(1-n)}{2\rho}\sigma^2(s_t)_{\mathbf{comp}}\right]$$

We can also compute the expected power consumptions as:

$$\begin{aligned} E_{t-1}C_{1t\mathbf{comp}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}}\exp\left[\frac{\rho-1}{2}\sigma^2(c_{1t})_{\mathbf{comp}}+\frac{n(1-n)(\rho-1)}{2\rho}\sigma^2(s_t)_{\mathbf{comp}}\right] \\ C_{2t\mathbf{comp}}^{1-\rho} &= \left(\frac{\lambda\eta}{\lambda-1}\right)^{\frac{\rho-1}{\rho}} \end{aligned}$$

Comparing the **bal** and **comp** model, we can show that:

$$\begin{aligned} \frac{C_{2t\mathbf{comp}}}{C_{2t\mathbf{bal}}} &= \exp\left[-\frac{\rho-1}{2\rho}\frac{n^2}{1+n(\rho-1)}\sigma_1^2\right]<1 \\ \frac{C_{2t\mathbf{comp}}^{1-\rho}}{C_{2t\mathbf{bal}}^{1-\rho}} &= \exp\left[\frac{n^2(\rho-1)^2}{2(1+n(\rho-1))}\sigma_1^2\right]>1 \end{aligned}$$

The impact on expected consumption in country 1 is ambiguous in general. Note however that if we consider two countries of identical size ($n=0.5$) we can write:

$$\frac{C_{1t\mathbf{comp}}}{C_{1t\mathbf{bal}}}=\exp\left[-\frac{(\rho-1)^2n^3(\rho+n)}{2\rho^2(1+n(\rho-1))^2}\sigma_1^2\right]<1$$

8.9.4 Expected effort

In both models, the expected efforts are given by:

$$\begin{aligned} E_{t-1}Y_{1t} &= \frac{\lambda-1}{\lambda\eta}\left[nE_{t-1}C_{1t}^{1-\rho}+(1-n)E_{t-1}\frac{S_tP_{2t}}{P_{1t}}C_{2t}C_{1t}^{-\rho}\right] \\ E_{t-1}Y_{2t} &= \frac{\lambda-1}{\lambda\eta}\left[nE_{t-1}\frac{P_{1t}}{S_tP_{2t}}C_{1t}C_{2t}^{-\rho}+(1-n)E_{t-1}C_{2t}^{1-\rho}\right] \end{aligned}$$

In the **bal** model, we use our previous results to write:

$$E_{t-1}Y_{1t\mathbf{bal}}=\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{1t\mathbf{bal}}^{1-\rho}\quad,\quad E_{t-1}Y_{2t\mathbf{bal}}=\frac{\lambda-1}{\lambda\eta}E_{t-1}C_{2t\mathbf{bal}}^{1-\rho}$$

Whereas in the **comp** model, we obtain:

$$E_{t-1}Y_{1t\mathbf{comp}} = E_{t-1}Y_{2t\mathbf{comp}} = \frac{\lambda - 1}{\lambda\eta}\left[nE_{t-1}C_{1t\mathbf{comp}}^{1-\rho} + (1-n)\,E_{t-1}C_{2t\mathbf{comp}}^{1-\rho}\right]$$