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Abstract

We propose a macro-finance model in which inflation, growth, and the policy rate are driven by unobservable long-run trends and transitory cycles that investors must infer from aggregate data. Their subjective estimates of these trends, and the uncertainty surrounding them, are priced into the Treasury yield curve in a tractable way through both interest rate expectations and bond risk premia. Empirical estimates reveal an upward smooth trend in the long-run real interest rate (r-star) until the 1980s, and large investor uncertainty with confidence bands on as wide as 3.4 percentage points, contrasting with the volatile rate implied by perfect information models.

JEL classification: C58, E43, E52, G12
Key words: incomplete information, interest rate stars, Bayesian learning, Treasury yields, investor's uncertainty

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1 Introduction

The term structure of Treasury yields constitutes a key lens into bond investors' information sets and their macroeconomic and financial forecasts. Extracting this information is particularly valuable for monetary policymakers, as it helps assess economic conditions and calibrate an appropriate policy stance. Central to this calibration is the long-run neutral real rate of interest — *r-star* — defined as the real interest rate consistent with a stable economy and a policy stance that is neither accommodative nor restrictive (Laubach and Williams 2003). Yet since r-star is inherently unobservable, policymakers must infer it from imperfect models and data, a challenge that Powell (2023) famously described as "navigating by the stars under cloudy skies." The yield curve has emerged as a promising source of information (Bauer and Rudebusch 2020), but how precisely it can be inferred remains contested.[1] Yield-curve-based estimates may convey a false sense of precision: they typically assume that bond market investors perfectly observe r-star.We propose a new model that quantifies how much can be inferred about r-star from the Treasury yield curve when bond investors, like policymakers and economists alike, face fundamental uncertainty about its true level.

Our model is built from three successive layers. In the first layer, we specify standard dynamics of macroeconomic state variables. We consider real GDP growth, inflation, and the nominal monetary policy rate at a quarterly frequency. The long-run trends of these variables are the so-called *macroeconomic stars*: they follow random walks and act as shifting endpoints à la Kozicki and Tinsley (2001), governing long-run macroeconomic forecasts. In the context of the model, r-star is the long-run trend of the real interest rate, the difference between the nominal rate trend and the inflation trend. Cyclical components, by contrast, evolve according to a stationary VAR(1) process whose shocks dissipate in the long run.

The key novelty lies in the second layer, which embeds investor uncertainty about the stars directly into asset pricing. Rather than assuming investors observe trends and cycles separately, we assume they observe only aggregate macroeconomic variables, GDP growth, inflation, the policy rate, and a private information factor whose shocks are correlated with those driving the long-run trends. They perform Bayesian learning each quarter to decompose macroeconomic aggregates into trend and cycle components. Their estimates

1. For instance, the long-run trend estimates of Lubik and Mathes (2015), Johannsen and Mertens (2016), Del Negro, Giannone, Giannoni, and Tambalotti (2017), Holston, Laubach, and Williams (2017), Christensen and Rudebusch (2019), and Bauer and Rudebusch (2020) differ substantially.

of the stars are therefore imperfect and time-varying. The private information factor, being directly observable, acts as a signal about long-run conditions and allows investors to potentially know more than the econometrician.

The main advantage of the framework is that the investors' problem reduces to a direct application of the Kalman (1960) filter, which delivers *subjective* dynamics for all the state variables. Moving from the first to the second layer imposes two meaningful statistical restrictions that sharpen identification. First, the subjective state variables are driven by a reduced set of shocks relative to the perfect-information benchmark, corresponding exactly to investors' forecast errors about aggregate macroeconomic states (Crump et al. 2025). Second, the parameters governing subjective state dynamics are tied to the underlying learning parameters, allowing us to estimate investor uncertainty about the stars directly.

Economically, the learning mechanism delivers two important theoretical predictions. First, investors' interest rate forecast errors are serially correlated (see e.g. Pang 2025). Second, long-horizon interest rate expectations underreact to permanent shocks and overreact to transitory ones, because investors cannot perfectly identify the source of fluctuations and therefore misperceive their persistence (Davis and Segal 2023). We validate both predictions against surveys of professional forecasters, finding regression evidence consistent with this learning mechanism.

The third layer prices the Treasury yield curve. We show that yield movements reflect fluctuations in investors' *subjective* estimates of the stars rather than in the stars themselves. With a standard stochastic discount factor, our model preserves the tractability of Gaussian affine term structure models (ATSMs): pricing formulas are closed-form affine functions of the subjective states (as in Duffie and Kan 1996; Ang and Piazzesi 2003). The key departure from a standard ATSM is that pricing is driven by subjective rather than objective state dynamics — yield curve variation can only reflect information actually observed by investors. Crucially, our framework jointly identifies the parameters governing star uncertainty and the time-varying risk premium that biases r-star extracted from long-term bond yields, all by standard likelihood maximization.

Using Treasury yields from 1961 to 2022 along with a set of survey data about future inflation, growth, and interest rates, our estimation provides four main takeaways. We first find that the investor's uncertainty about the nominal interest rate trend ($i_t^\star$), the inflation trend ($\pi_t^\star$) and the GDP growth trend ($g_t^\star$) are very large, with 95% intervals as wide as about $\pm 225bps$, $\pm 125bps$ and $\pm 80bps$, respectively. This leads to a large uncertainty regarding r-star, with confidence bands of $\pm 170bps$. For example, in the early 2000s, the investor's perceived r-star estimate is approximately at 2%, with a 95% confidence band of [0.3%, 3.7%], reflecting the *fuzzy blur* surrounding r-star. This result confirms an intuitive economic belief, but is in sharp contrast with models assuming perfect information where the term structure of interest rates can nearly perfectly reveal the historical path of the stars. For instance, our model estimated under the assumption of perfect information provides r-star estimates that are both too volatile and too confident.

Second, our model reveals new historical patterns of r-star namely that it is trending upwards during the 1960s-1970s, not downwards, contrasting both with most estimates present in the literature, and that of our model estimated under perfect information. This result emphasizes that investors believed r-star to be low, and the increasing trend reflects the gradual realization –eventually occurring after 1973– that r-star was high during this period, and that monetary policy was being too accommodative. We additionally compute ex-post estimates, that is the historical trajectory of the *subjective* r-star based on the observation of the whole sample from 1960 to 2022, and find that although the r-star values are generally lower than the ex-ante estimates, its qualitative historical trends are virtually unaffected. This finding emphasizes that the historical r-star patterns do not result from limited sample availability from the investor, but rather to the inherent difficulty of inferring r-star values even when she observes with sixty years of data. It confirms the idea of Farmer, Nakamura, and Steinsson (2024) that learning about the long-run is a slow process.

Third, we provide an empirical assessment of the historical values and uncertainty surrounding the perceived $r_t^\star - g_t^\star$, a commonly-used indicator of fiscal space and sovereign state sustainability (see Piketty 2014). Once again, we find a substantial uncertainty surrounding these estimates, with confidence bands approaching $\pm 200 bps$. The time series estimates are consistently negative, but are non-significantly different from zero from the 1980s onwards. This shows overall that investors believe that the probability of the U.S. sovereign being fiscally sound exceeds that of fiscal distress, albeit with significant time variation during the sample.

Fourth, we use our model to produce term premium estimates and show that they are countercyclical, in line with Cochrane and Piazzesi (2005), and the prices of risk are significantly driven by both trends and cycles in distinct ways. We show that r-star is the only long-run trend driving the prices of risk in the stochastic discount factor, and the growth cycle is the only transitory factor, with positive and negative signs, respectively. This contrasts with the findings of Cieslak and Povala (2015) and Bauer and Rudebusch (2020), who find that only the inflation trend or the nominal interest rate trend explain the low-frequency fluctuations of time-varying bond premia. Instead, we find they both intervene significantly with opposite signs, and virtually the same magnitude. This finding also contrasts with the version of model featuring complete information, and is therefore a feature revealed solely by the learning mechanism. We derive the time series of the term premium decomposition, which fluctuates between 0% before the Volcker era and 4% in the early 1980s for the 10y yield. Notably, we show that there is a shift in the term premium estimates in the 1980s, where term premia jumps from low and stable to high and trending downwards, consistently with the shift in the subjective r-star trend observed during that period.

We last use our model as a laboratory to study the effect of transitory monetary policy and persistent inflation target shocks when the investor cannot observe the source of the shocks. We identify structural shocks with sign restrictions and perform an impulse-response analysis. Because the investor cannot distinguish trends from cycles, aggregate shocks are split accordingly with her learning process, similarly to Davis and Segal (2023). We show that the yield curve indeed under-reacts to the long-run inflation target shock and over-reacts to short-run monetary policy shocks, mirroring the fact that the investor underestimates the persistence of permanent shocks, and overestimates that of transitory shocks. This phenomenon can be amplified by the reaction of term premium, especially for monetary policy shocks, because of the reaction in r-star. As the shock propagates, the investor learns about the trend-cycle decomposition and the long-horizon responses converge to those where the shocks are perfectly observed, but we show that this convergence is so slow that it is not achieved even after 10 years. These impulse-response functions show no evidence of a Fed information effect, as contractionary monetary policy shocks do not lead to an increase in growth and inflation forecasts, in line with the results of Bauer and Swanson (2023).

Literature Review. We contribute to three different strands of literature. Our model first builds on the tradition of macrofinance term structure model *à la* Ang and Piazzesi (2003). These models typically blend together financial, potentially unobserved, factors with macroeconomic factors to price the yield curve.[2] In this paper, we set hidden factors to

2. There is a large literature using macroeconomic variables and hidden factors for the valuation of Treasury bonds. Notable examples include, among others Diebold, Piazzesi, and Rudebusch (2005), Ang, Piazzesi, and Wei (2006) and Ang, Bekaert, and Piazzesi (2007), Rudebusch and Wu (2008), Hordahl, Tristani, and Vestin (2006), Bikbov and Chernov (2010), Joslin, Priebsch, and Singleton (2014), Hordahl and Tristani (2012), Wu and Xia (2016), Rousselet (2023).

be macroeconomic trends and cycles, following the tradition of models of shifting endpoints (see e.g. Kozicki and Tinsley 2001; Stock and Watson 2007). We are not the first to introduce trends and cycles in term structure models. Cieslak and Povala (2015) show that the low-frequency trend of inflation is an important driver of the yield curve. Bauer and Rudebusch (2020) find that accounting for the variation of long-run trends is important to efficiently capture the yield curve dynamics, notably that of the nominal interest rate i-star. Feunou and Fontaine (2023) develop a similar model of trends and cycles and focus on the identification of structural shocks. Our model builds on a similar representation of the economy, but adds asymmetric information such that the representative asset pricer does not observe the decomposition of macroeconomic aggregates.

A key novelty in our paper is to leave aside the *full-information* assumption that is typically used in the term structure modelling literature, bringing investor learning into the picture. Our paper thus contributes to a vast strand of literature looking at subjective expectations embedded in asset prices. A substantial strand of the literature has focused on investors learning about the parameters driving the data-generating process of macroeconomic and financial factors (see e.g. Piazzesi and Schneider 2009; Collin-Dufresne et al. 2016; Johannes et al. 2016; Andrei et al. 2019). Giacoletti, Laursen, and Singleton (2020) study a model where the risk factors are known but the parameters move over time and the investor learns about them in Bayesian fashion. Orphanides and Williams (2005) and Kozicki and Tinsley (2005) consider a case where investors perpetually learn and their perceived parameters move every period because they update their beliefs suboptimally according to constant gain learning. We instead follow Bianchi, Lettau, and Ludvigson (2022) and assume that the true long-run trends move. However, our representative investor learns about them optimally according to Bayes rule that is readily given by the Kalman filter, building on the theoretical toy model developed in Feunou, Fontaine, and Roussellet (2023).

The economic mechanism embedded in our learning model considers that investors can receive informative public signals to help them forecast macroeconomic variables in the long-run. The "Fed information effect", that is the ability to learn about the macroeconomy from Fed's actions is one such example (see Nakamura and Steinsson 2018). Hillenbrand (2025) provides empirical evidence that most of the yield trend movements are realized around FOMC announcements, in support of an information channel. In contrast Bauer and Swanson (2023) and Bauer, Pflueger, and Sunderam (2024) argue instead that the learning is primarily done on the monetary policy rule parameters rather than the factors driving the economy. In this paper, we take the view that the optimal long-run macroeconomic forecasts are unknown to the investor, which are technically current states but can also be seen as either parameters or economic factors. Our framework therefore resembles that of Davis and Segal (2023) where investors confuse trends for cycles but our model is the first to embed this type of features in a general dynamic term structure model.

An important source of information about investors subjective beliefs is survey data, and there is a large literature studying the informativeness of such data sources for bond returns. Crump, Eusepi, and Moench (2018, 2024) proposes a trend-cycle model to explain anomalies found in surveys of professional forecaster's data, as documented in Farmer, Nakamura, and Steinsson (2024). Cao, Crump, Eusepi, and Moench (2021) and Crump, Eusepi, Moench, and Preston (2025) provide further support for trend-cycles models to explain forecasters' disagreement about the long-run and the term structure dynamics. They find that forecasters use multivariate models with imperfect information, a version of which is used in our paper. Importantly, close to our paper is that of Pang (2025), who proposes to explain the reaction of the yield curve to inflation news by a trend-cycle model with Bayesian learning similar to our proposed framework. We generalize his learning model to provide empirical estimates of r-star.

Last, our paper relates to the large literature studying how much can be learned about r-star from financial data. Starting with Laubach and Williams (2003) and Holston, Laubach, and Williams (2017), r-star estimates are gathered from an empirical application of a structural economic model. Following the insights of Ang, Piazzesi, and Wei (2006) or Ang, Bekaert, and Piazzesi (2007), the yield curve has been used as a source of information about macroeconomic forecasts. For instance, Christensen and Rudebusch (2016), Abrahams, Adrian, Crump, Moench, and Yu (2016), Breach, D'amico, and Orphanides (2020), Christensen and Rudebusch (2019) and Bauer and Rudebusch (2020) provide stars estimates drawn from the yield curve. In his speech *all the stars we cannot see*, however, Williams (2025) casts doubt on looking at estimates drawn from term structure models and survey measures because they "give a false sense of precision since the reported values do not convey the uncertainty underlying them", and they "are contaminated by [...] risk premiums". Our approach readily takes care of both these issues. Alternatives have been developed in the literature, relying on structural or statistical models, such as Edge, Kiley, and Laforte (2008), Lubik and Mathes (2015), Kiley (2015) Johannsen and Mertens (2016) or Del Negro, Giannone, Giannoni, and Tambalotti (2017) provide estimates of r-star based on macro models. Curdia (2025) provides a new methodology based on a blend of economic theory and statistics. Compared with the previous papers, we rely on the yield curve in addition to our statistical macroeconomic model, but incorporate investors' uncertainty about r-star.

The remainder of the paper is organized as follows. Section 2 provides the key mechanism and some regression evidence of learning. Section 3 details the formulation of our macrofinance model, the filtering problem of the investor, and the resulting bond prices in the economy. In Section 4, we show how to estimate the parameters and factors, notably r-star, as perceived by the investor using the yield curve and observable macroeconomic data. Section 5 provides the parameter and time series estimates, allowing us to unravel the learning process of the investor. Section 6 details the pricing implications of our estimated model. Section 7 presents the effect of permanent and transitory shocks on the yield curve. Section 8 concludes.

2 A Preview on the Learning Mechanism

This section presents a simplified version of the full macrofinance model developed afterwards to provide some intuition about the investor's learning mechanism and its effect on bond yields. We provide empirical evidence that matches the predictions from theory. Formal proofs are gathered in Appendix A.1.

2.1 A Simple Model of Imperfect Information

We first focus on a simple model for the monetary policy rate, denoted by i_t. Interest rate fluctuations are driven by the sum of a *persistent component* (or interest rate trend, $i_t^\star$), and a *transitory component* (or interest rate cycle, $C_{i,t}$). These two components only differ by how quickly their shocks vanish after affecting the nominal rate. At this stage, we assume that $i_t^\star$ is a random walk so its shocks persist forever, while the interest rate cycle is a stationary AR(1) so its shocks gradually die out.

$$i_t^\star = i_{t-1}^\star + \varepsilon_{i,t}^\star \quad \text{and} \quad C_{i,t} = \varphi \cdot C_{i,t-1} + \varepsilon_{i,t}\,, \tag{1}$$

where the shocks $\varepsilon_{i,t}^\star$ and $\varepsilon_{i,t}$ are uncorrelated and independent through time. A representative investor is being faced with the following problem. At each time period, she observes the aggregate monetary policy decision $i_t = i_t^\star + C_{i,t}$ as public information, but not the split into its individual components. The investor then updates her beliefs rationally, and uses bayesian updating through the Kalman filter to compute her best evaluation of the unknown interest rate components. The one-period forecast errors of the investor are then given by:

$$e_{i,t|t-1} := i_t - \mathbb{E}\left(i_t | i_{t-1}, i_{t-2}, \cdots\right) = \underbrace{\varepsilon^{\star}_{i,t} + \varepsilon_{i,t}}_{\text{true shocks}} + (1-\varphi)\underbrace{(i^{\star}_{t-1} - i^{\star}_{t-1|t-1})}_{\text{filtering error}} \tag{2}$$

where $\mathbb{E}(\cdot | i_{t-1}, i_{t-2}, \cdots)$ is the investor's forecast based on her information known at $t-1$, and $i^{\star}_{t-1|t-1} := \mathbb{E}(i^{\star}_{t-1} | i_{t-1}, i_{t-2}, \cdots)$. From the investor's point of view, the true shocks and filtering errors are both undistinguishable and unpredictable based on $t-1$ information. We obtain a first implication: current and future forecast errors can be correlated, because filtering errors are correlated (Kozicki and Tinsley 2001; Pang 2025). If instead the investor directly observed the interest rate components, the covariance between forecast errors would be null since the true shocks $\varepsilon^{\star}_{i,t}$ and $\varepsilon_{i,t}$ are uncorrelated through time.

The second implication of the learning process is that expectations of the short-term interest rate show opposite dynamics following permanent and transitory shocks (Davis and Segal 2023). Consider for instance a permanent shock to the interest rate of $\varepsilon^{\star}_{i,t} = 25$bps. The investor only sees that the interest rate has increased by 25bps, and does not know whether it is due to the permanent or transitory shock. She will attribute part of this increase to the trend, say 5bps, and the remaining 20bps to the cycle as she knows the latter tends to be more volatile than the former. This means that her interest rate expectations will be downward sloping, from about 25bps, to 5bps in the long-run. As time goes by, she gradually realizes that the shock was permanent and corrects her expectations upwards, up to a flat line at 25bps. True permanent shocks therefore imply an initial under-reaction of long-run interest rate expectations, with an upward convergence as time goes by. Consider now a transitory interest rate shock of the same size. Still, the investor will attribute 5bps to the trend and 20bps to the cycle because she only sees the same 25bps aggregate interest rate increase. The initial reaction of interest rate expectations is the same, but as time goes by the investor realizes the shock is transitory and both short and long-run expectations will decrease toward zero, such that long-run interest rate expectations are initially overreacting to transitory shocks.

2.2 Regression Evidence

We empirically test the two previous implications as preliminary evidence of investor's learning about trends and cycles. We use three sets of quarterly data, ranging to a maximum of 1961:Q1 to 2022:Q4. The short-term interest rate is the 3m TBill, investors' expectations are the 1-year ahead interest rate forecasts as provided by the survey of professional forecasters (SPF, available from 1981 onwards), and 3m forward rates constructed from the smoothed yield curve of Liu and Wu (2021). Last, we gather an ex-post estimate for $i_t^\star$ from Bauer and Rudebusch (2020) to construct ex-post transitory and permanent shocks, available from 1971:Q4. An extended data description can be found in Section 4.3.

To test the first implication, we construct interest rate expectation errors as $e_{i,t+4} := i_{t+4} - \mathrm{SPF}_t^{(4)}$. Our choice to look at 4-quarters forecast errors is motivated by data availability of surveys. We then run the following regression:

$$e_{i,t} = \alpha + \beta \cdot e_{i,t+4} + \sum_{j=4}^{h} \gamma_j \cdot e_{i,t-j} + \eta_t \,, \tag{3}$$

Our interest is to know whether β is significant, while the $e_{i,t-j}$ serve as controls for past information. The results are presented in Table 1.

[Insert Table 1 about here.]

Column 1-4 unambiguously show that four-quarter leads are significantly correlated with current forecast errors, no matter how many lags we include in the controls from 4 quarters (column 1), to lags 4 to 7 quarters (column 4). All coefficients are about 0.27, with significance at 1% or 5% depending on the specification. To gain some economic insights, 0.27 would be the conditional correlation between current and future forecast errors if both had the same conditional variance given the past. Later in the paper we also consider the same trend and cycle decomposition for inflation and GDP growth, so we run a second version of the regression (3) controlling by all three past forecast error series and adding their 4-quarter leads as well. These results are presented in columns 5-8, and leave the qualitative result virtually unchanged. Coefficients reduce slightly to about 0.23, and significance tends to be slightly weaker, which is not surprising since the regressions gather three times more explanatory variables than the corresponding baseline, for only about 160 observations.

For our second implication, we first write the model-implied expectations variation of the future short-term interest rate:

$$\begin{aligned} \Delta_h \mathbb{E}_{t+h}\left(i_{t+m+h}\right) &:= \mathbb{E}_{t+h}\left(i_{t+m+h}\right) - \mathbb{E}_t\left(i_{t+m}\right) \\ &= i^\star_{t+h|t+h} - i^\star_{t|t} + \varphi^m \left(C_{i,t+h|t+h} - C_{i,t|t}\right) \,. \end{aligned} \tag{4}$$

If we observed the true permanent and transitory shocks $\varepsilon^{\star}_{i,t+1}$ and $\varepsilon_{i,t+1}$ realized at period $t+1$, as well as the expectation variation, we could run a local projection to test how expectations are corrected over time. The regression would write:

$$\Delta_h \mathbb{E}_{t+h}\left(i_{t+m+h}\right) = \alpha + \beta^{\star}_h \cdot \varepsilon^{\star}_{i,t+1} + \beta_{c,h} \cdot \varepsilon_{i,t+1} + \eta_{t+h}\,, \tag{5}$$

and could be run for different values of h. Based on our discussion in the previous section, we expect $\beta^{\star}_h$ to grow with h, and $\beta_{c,h}$ to decrease with h. We can also look at different expectation horizons m, where higher values of m will make the expectations reflect more trend revisions than cycles.

Since obtaining an entire term structure of interest rate expectations is complicated data-wise, we rely on forward interest rates, which provide expectations of short-term interest rates contaminated by risk premium. The forward rate $f^{(m)}_t$ of the 1-period rate starting in $m-1$ periods therefore represents a biased measure of the investor's expectation $\mathbb{E}_t\left(i_{t+m}\right)$. If we assume that the risk premium is orthogonal to the shocks driving the short-term interest rate, our reasoning following regression (5) using forward variations on the left-hand side still holds. As proxies for permanent and transitory shocks, we rely on the $i^{\star}_t$ series identified by Bauer and Rudebusch (2020). We regress the quarterly difference of the 3m Tbill onto the quarterly difference of $i^{\star}_t$, and take the fitted values as $\varepsilon^{\star}_{i,t}$ and the residuals as $\varepsilon_{i,t}$.

[Insert Table 2 about here.]

Results are presented in Table 2 for the 1y, 3y, 5y and 10y forwards (panels a, b, c, d, respectively), using differences over 0 to 5 quarters. For all forward maturities, we observe exactly the pattern described in the previous section: expectation revisions due to permanent interest rate shocks are significant, show a mild instantaneous response, and grow with the number of quarters used in the variations. For transitory shocks, we observe the opposite evolution, and the effects roughly decrease with the considered horizon. For the 10y forward for instance (panel d), the instantaneous response to permanent shocks is 0.29 and grows to 0.82 after 5 quarters, showing that interest rate expectations are revised upwards gradually. For transitory shocks, the initial effect is about the same at 0.24 but decreases to become insignificantly different from zero afterwards. Last, the effects are on the whole smaller in magnitude for the 10y than for the 5y forward, consistently with the fact that expectation revisions due to the cyclical component have less impact on longer-run forecasts. Remember that the previous reasoning interprets forwards as noisy expectations where the forward premium is independent from the interest rate shocks. In practice this is unlikely to be the case, and the premium is likely to be higher and more volatile for longer-run forwards. The evidence in panels a and b mitigates this concern as short-run forwards are less prone to risk premium contamination and the regressions show a robust pattern for permanent shocks.

Put together, we take this as evidence that investors learn about the trend-cycle decomposition of interest rates, and price financial assets accordingly. We develop a more realistic empirical model hereafter that allows to explore the implications of that learning process on macroeconomic variables and r-star.

3 A Term Structure Model with Unobserved Stars

This section presents a model where macroeconomic variables move around shifting endpoints that are not observed by the representative asset-pricer. In turn, she learns about them through both public and private signals.

3.1 State Variables in the Economy

We consider an economy driven by three macroeconomic variables: the nominal interest rate i_t, inflation π_t, and real growth g_t, gathered together in the vector $M_t = (i_t, \pi_t, g_t)'$. These variables evolve around a set of trends $M_t^\star = \left(i_t^\star, \pi_t^\star, g_t^\star\right)'$ such that they admit the following trend-cycle decomposition:

$$M_t = M_t^\star + C_t, \quad \text{where} \quad \lim_{h \to +\infty} \mathbb{E}_t(M_{t+h}) = M_t^\star\,, \tag{6}$$

and $C_t = M_t - M_t^\star =: \left(C_{i,t}, C_{\pi,t}, C_{g,t}\right)'$. The trends $M_t^\star$ have been widely used in both structural and reduced-form macroeconomic models. They are called alternatively *secular changes* or *shifting endpoints*, and were introduced in the context of yield-curve modelling by Kozicki and Tinsley (2001) and later exploited by Bauer and Rudebusch (2020) and Feunou and Fontaine (2023). Each trend moves according to a random walk, such that:

$$M_t^\star = M_{t-1}^\star + \varepsilon_t^\star, \quad \text{where} \quad \varepsilon_t^\star \sim \mathcal{N}(0, \Sigma^\star)\,. \tag{7}$$

In turn, the cycle variables C_t are stationary and their dynamics are given by a standard VAR(1).

$$C_t = \Phi_c C_{t-1} + \varepsilon_{c,t}, \quad \text{where} \quad \varepsilon_{c,t} \sim \mathcal{N}(0, \Sigma_c)\,, \tag{8}$$

and $\varepsilon_{c,t}$ and $\varepsilon_t^\star$ are uncorrelated. To complete the model, we consider a private signal factor f_t whose role will become clear in the next section. f_t follows an autoregressive process:

$$f_t = \varphi_f \cdot f_{t-1} + \varepsilon_{f,t} + \gamma^{\star\prime} \varepsilon_t^\star \tag{9}$$

where $\varepsilon_{f,t} \sim \mathcal{N}(0, \sigma_f^2)$ and $\gamma^\star$ is a three-dimensional vector. The aggregate shock to the private signal is therefore correlated with structural trend shocks through $\gamma^\star$.

In sum, the economy is driven by seven state variables and seven shocks: the nominal interest rate, inflation, and growth trends and cycles; and the private signal factor. We group these variables into a single state vector denoted by $X_t := \left(M_t^{\star\prime},\, C_t',\, f_t\right)'$. Putting together our previous assumptions, X_t follows a standard Gaussian VAR(1), which we write:

$$X_t = \Phi X_{t-1} + \varepsilon_t, \quad \text{with} \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma)\,. \tag{10}$$

3.2 Imperfect Information and Investor Signal

In standard bond pricing models, the representative investor has perfect information, and observes the entire information set, denoted by $\mathcal{F}_t := \{X_t,\, X_{t-1},\, \ldots\}$, in real time. In our context, perfect information leads us to assume that the investor always knows whether macroeconomic variables move because of trends or cycles. As emphasized in the previous section, this is likely to be counterfactual. We therefore assume that the investor does not observe the individual trend-cycle decomposition. The information set perceived by the investor, $\mathcal{F}_t^i = \{M_t,\, f_t,\, M_{t-1},\, f_{t-1},\, \ldots\}$ contains the aggregate macroeconomic outcomes i_t, π_t and g_t and the private signal factor f_t, present and past.

We can now provide some context on the role of the private signal f_t. First, it is indeed a signal since its aggregate shock is potentially correlated through $\gamma^\star$ with the trend shocks $\varepsilon_t^\star$, which the investor cannot observe. This signal is private since it is is not directly observable by the econometrician but is in the investor's information set and will thus be reflected in asset prices. Second, absent that factor all information is public and asset prices play no role in revealing information to the econometrician. While this could potentially be the case empirically, it is a nested version of our framework where $\gamma^\star$ is zero, and we leave it to the data to determine whether the signal is actually informative to the investor.

Denoting by $Y_t := \left(M_t', \ f_t\right)'$ the variables observed by the investor, we have:

$$Y_t = \mathcal{B}\, X_t\,, \tag{11}$$

Where $\mathcal{B}$ is a (4×7) matrix with one and zeros selecting the right entries of X_t. Equations (10) and (11) form the state-space representation of the economy as perceived by the investor.[3]

3.3 Bayesian Learning About the States

At every time period, the investor updates her beliefs about the state variables rationally according to Bayes rule. Since all variables are normally distributed and the investor's state-space model is linear, the learning reduces to a standard filtering problem. We use the following notations: $X_{t|t} := \mathbb{E}\left(X_t \mid \mathcal{F}_t^i\right)$ is the best estimate of the states given the investor's information set, and $\overline{\mathcal{P}} := \mathbb{V}\left(X_t \mid \mathcal{F}_t^i\right)$. Applying the results of Kalman (1960) and assuming filter stationarity, we have:

$$X_{t|t} = \Phi X_{t-1|t-1} + \overline{\mathcal{K}}\left(Y_t - \mathcal{B}\Phi X_{t-1|t-1}\right)\,, \tag{12}$$

where $\overline{\mathcal{K}}$ is obtained by solving the following system of equations:

$$\overline{\mathcal{K}} = \left(\Phi\overline{\mathcal{P}}\Phi' + \Sigma\right)\mathcal{B}'\overline{\mathcal{V}}^{-1}\,, \quad \overline{\mathcal{P}} = \left(I_7 - \overline{\mathcal{K}}\mathcal{B}\right)\left(\Phi\overline{\mathcal{P}}\Phi' + \Sigma\right)\,, \quad \overline{\mathcal{V}} = \mathcal{B}\left(\Phi\overline{\mathcal{P}}\Phi' + \Sigma\right)\mathcal{B}'\,. \tag{13}$$

The best estimates of the states made by the representative investor are given by her best prediction as of $t-1$, namely $\Phi X_{t-1|t-1}$, corrected by the forecasting error on the observables. The key quantity driving the strength of the update is the Kalman gain $\overline{\mathcal{K}}$, which represents the weights that the investor allocates to each forecasting error to determine her best estimate of the states.[4]

3. It is also possible to have macroeconomic aggregates be imperfect signals about the economy, considering that real-time macroeconomic data releases are provisional and often revised ex-post (Crump et al. 2025). This would lead us to consider vintages of macroeconomic data, and we leave this aside for simplicity.

4. Conditions for stationarity of the filter can be found in Anderson and Moore (1979) and are given by $max\ \left|\text{eig}\left(\Phi - \mathcal{K}_t\mathcal{B}'\right)\right| < 1$.

The *subjective* dynamics of the filtered states is given by a simple VAR(1) with Y_t:

$$\begin{pmatrix} X_{t|t} \\ Y_t \end{pmatrix} = \begin{pmatrix} \Phi & 0 \\ \mathcal{B}\Phi & 0 \end{pmatrix} \begin{pmatrix} X_{t-1|t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} \overline{\mathcal{K}} \\ I_4 \end{pmatrix} \xi_t \,. \tag{14}$$

where $\xi_t := Y_t - \mathcal{B}\Phi X_{t-1|t-1}$ is the forecast error on the investor's observables, and is a normally distributed shock with zero conditional mean and conditional variance-covariance $\overline{\mathcal{V}}$ given $\mathcal{F}^i_{t-1}$. This implies notably that Y_t is also conditionally Gaussian given $\mathcal{F}^i_t$.

3.4 Learning Versus Perfect Information Models: discussion

At this stage, it is useful to compare the implications of the imperfect information assumption with a model where the investor possesses the full information set $\mathcal{F}_t$. If the investor is endowed with full information, she observes all seven factors in Equation (10), and all of the seven shocks that drive them. This is the standard case that the vast majority of the term structure literature is concerned with. In our imperfect information frameowrk however, the perceived shocks ξ_t are identified as the forecast errors to the aggregate macroeconomic variables M_t and the private signal f_t. Therefore, the seven state variables *as perceived by the investor* $X_{t|t}$ only feature four shocks. In the following section, the representative investor will price the term structure of interest rates considering these four shocks only, constituting the sources of risk that she can observe.

An alternative to our approach could be to assume perfect information but have only four structural shocks driving the seven state variables. In that case, we would transform Equation (14) by essentially removing the second set of rows describing Y_t. This approach is the one considered by e.g. Feunou and Fontaine (2023) where the same shocks drive both the trends and cycles. There are however two main differences between perfect information with reduced-rank shocks and imperfect information. First, our dynamics based on learning equate the state variable shocks to the forecasting errors on the investor's observable variables. The alternative would leave the shocks unspecified and provide no such economic interpretation. Second, the learning process constrains $\overline{\mathcal{K}}$ to be a function of Σ, Φ and $\mathcal{B}$ in our model and provides an economically-motivated parametric restriction on the impact that shocks have on the factors. Besides, this greatly reduces the number of parameters to estimate making our specification more parsimonious. Last, our specification with imperfect information is consistent with the empirical evidence presented in Section

2 while a model based on perfect information will in general struggle to reproduce such evidence. In our empirical estimation below, we therefore estimate the imperfect information model and compare it with a perfect information model featuring 7 shocks for comparison purposes.

3.5 Bond Pricing

We assume the presence of no-arbitrage such that the investor values assets with a unique stochastic discount factor (SDF), denoted by $\mathcal{M}_{t+1}$. In our framework, the investor prices shocks to the aggregate macroeconomic variables, that is the ones that are directly observable to her:

$$\log \mathcal{M}_{t+1} = -i_t + \lambda_t' \xi_{t+1} - \frac{1}{2} \lambda_t' \overline{\mathcal{V}} \lambda_t \,, \tag{15}$$

where the last term ensures that $\mathbb{E}\left(\mathcal{M}_{t+1} \,\middle|\, \mathcal{F}_t^i\right) = e^{-i_t}$, and the prices of risk are a function of the subjective state variables (see, Duffee 2002). Consistently with Cochrane and Piazzesi (2005), we impose that only one linear combination of the factors enter the prices of risk, that is $\lambda_t = \alpha\beta' X_{t|t}$ where α is four-dimensional and β has seven entries, such that:

$$\lambda_t = \alpha \cdot \left(\beta_i^\star i_{t|t}^\star + \beta_\pi^\star \pi_{t|t}^\star + \beta_g^\star g_{t|t}^\star + \beta_i C_{i,t|t} + \beta_\pi C_{\pi,t|t} + \beta_g C_{g,t|t} + f_t\right) . \tag{16}$$

Equation (16) specifies that the investor prices shocks to aggregate macroeconomic variables through α, and the time-variation of the prices of risk comes from one factor that comprises both the stationary and integrated parts of the system (the term in brackets in Equation 16). This assumption basically recognizes that the macroeconomic trends can contain information about bond term premiums. Our specification hence encompasses both the framework of Cieslak and Povala (2015) where the bond term premium is a function of inflation only, and Bauer and Rudebusch (2020) where $i_t^\star$ is the only long-run trend that enters the term premium. Our frameworks allows instead for a data-driven determination of the drivers of bond excess returns, trends or cycles.

We show in Appendix A.2 that, similarly to the standard Gaussian ATSM à la Ang and Piazzesi (2003), the SDF defined by the equations above is such that $X_{t|t}$ and Y_t follow a Gaussian VAR(1) under the risk-neutral measure. This allows us to derive the following pricing properties. Consider the price $B_t^{(n)}$ of a nominal zero-coupon bond with residual maturity n. By no-arbitrage, we have:

$$B_t^{(n)} = \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} i_{t+j}\right) \Big| \mathcal{F}_t^i\right] = \exp\left(\mathcal{A}_n + \mathcal{B}_n' X_{t|t}\right) \ , \ . \tag{17}$$

where the loadings are given by standard recursions:

$$\begin{cases} \mathcal{A}_n &= \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}'\overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\overline{\mathcal{K}}'\mathcal{B}_{n-1} \\ \mathcal{B}_n &= -e_\iota + \left(\Phi + \overline{\mathcal{K}}\,\overline{\mathcal{V}}\Lambda\right)'\mathcal{B}_{n-1} \,, \end{cases} \tag{18}$$

and $e_\iota = (1, 0, 0, 1, 0, 0, 0)'$ is a vector containing zeros and ones selecting i_t, such that $X_{t|t}' e_\iota = i_t$ (see Appendix A.3). Since the model is affine, the continuously-compounded yields $R_t^{(n)}$ on nominal bonds are given by affine functions of the state and space variables, such that:

$$R_t^{(n)} = -\frac{\mathcal{A}_n}{n} - \frac{\mathcal{B}_n'}{n} X_{t|t} \, . \tag{19}$$

Our previous computations show two important properties. First, although our model includes Bayesian learning from the representative investor, all the standard features of the Gaussian ATSM are preserved: Treasury yields are affine functions of the risk factors, and are normally distributed (see e.g, Duffie and Kan 1996; Duffie and Singleton 1997). Notice that the pricing formula of Equation (19) involves the risk factors as seen by the representative investor, i.e. $X_{t|t}$ and not X_t. Thus, asset prices only reveal the information that the investor has access to, but not the actual state of the economy. Second, the investor's Kalman gain $\overline{\mathcal{K}}$ is central to the bond pricing formulas as it determines the variance-covariance of the filtered states $X_{t|t}$ and the observables Y_t. This quantity can thus be revealed by asset prices and we can infer the uncertainty that the investor is being faced with.

4 Identifying the Investor's Information

We now consider the estimation problem from the econometrician's point of view. We show that, under adequate identification conditions, we can estimate the parameters of interest with the filtering-based maximum likelihood.

4.1 The Econometrician's State-Space Model

In the economy described above, we do not possess the same information set as the investor, mostly because we do not observe f_t. Instead, we obtain the information set $\mathcal{F}_t^e$ containing the following variables. First, similarly to the investor, we observe the macroeconomic data releases i_t, π_t and g_t. Second, we gather information from the observed yield curve, which contains error. Letting N be the total number of observed zero-coupon yields, and $\mathfrak{R}_t^{(m)}$ be the observed zero-coupon yield of residual maturity m, we have:

$$\mathfrak{R}_t^{(n_j)} = R_t^{(n_j)} + \sigma_r\, e_{r,t}^{(n_j)} \quad \text{where} \quad j \in \{1, \ldots, N\}\,, \quad e_{r,t}^{(n_j)} \sim \mathcal{N}(0,1)\,. \tag{20}$$

Provided the pricing formula (19), the yield curve imperfectly reveals information about the macroeconomic factors as perceived by the investor and the private information f_t. Last, in the spirit of Kim and Orphanides (2012), we have access to survey data that imperfectly reveal the investor's perceived dynamics of the factors. We denote these surveys by $\mathcal{S}_t$, which can provide forecasts of the macroeconomic variables and the yield curve. We come back to how they relate to state variables below. In summary, we have access to $Z_t = \left(M_t',\, \mathfrak{R}_t',\, \mathcal{S}_t'\right)'$. Hence our information set is: $\mathcal{F}_t^e = \{Z_t,\, Z_{t-1},\, \ldots\}$.

All our observable variables relate linearly to the states considered by the investor $X_{t|t}$. Indeed, both M_t and $\mathfrak{R}_t$ are linear combinations of the states, as described above. For the survey data, based on the above computations and according to the Gaussian VAR of Equation (14), the investor's forecast of any linear combination of the states is easily expressed as:

$$\mathbb{E}\left(\omega_x' X_{t+k|t+k} + \omega_y Y_{t+k} \,\middle|\, \underline{Y_t}\right) = \left(\omega_x' + \omega_y' \mathcal{B}\right) \Phi^k X_{t|t}\,. \tag{21}$$

Thus, as long as the variables of interest in survey data are linear combinations of $X_{t+k|t+k}$ or Y_{t+k}, the forecasts will be linear combinations of $X_{t|t}$. This is the case for both macroeconomic and yield forecasts that we are considering. Assuming the survey data are measured with errors, we can stack together all the variables observed by the econometrician and write:[5]

$$Z_t = A + B X_{t|t} + \mathrm{diag}(\sigma_e)\, e_t\,, \tag{22}$$

where the loadings are easily obtained. Taken together, Equations (14) and (22) define a linear state-space model that is readily estimable through the Kalman filter and associated maximum likelihood.

5. We calibrate the survey measurement errors standard deviation to the average forecaster disagreement.

4.2 Estimation Procedure

Having laid down our asset pricing model with Bayesian learning, our interest is twofold. We first want to obtain empirical results for a world where macroeconomic long-run trends are unknown to the investor, and obtain the implications in terms of bond pricing. Second, we want assess the plausibility of our results compared to the same model where investors have access to the full information about trends and cycles in real-time.

We call our model with imperfect information **II**. To estimate the model, we run the following steps. First, we choose a set of candidate parameters, and compute $\overline{\mathcal{P}}$, $\overline{\mathcal{V}}$, and $\overline{\mathcal{K}}$ as the solutions to Equation (13).[6] Second, we use these objects as inputs for our state Equation (14) to compute the loadings of the bond pricing formulas and the forecasts to obtain the parameters of the measurement Equations. Third, we run the Kalman filter algorithm to calculate the log-likelihood and update the candidate parameters accordingly until convergence.

In a second exercise, we estimate our model assuming perfect information, i.e. we consider that the representative investor can perfectly observe the trend-cycle decomposition. We call this model **PI**, and delay the discussion of its results.

4.3 Data

We estimate the model using US quarterly data spanning from 1961:Q3 to 2022:Q4. Real output growth is determined by the year-on-year log-change in the seasonally adjusted real Gross Domestic Product (in billions of chained 2012 dollars). The inflation rate is computed as the year-on-year log-change in the seasonally unadjusted CPI-U index. Both data sets are available on the Federal Reserve Bank of St Louis website. For the treasury yield curve, we use the smoothed zero coupon curve provided by Liu and Wu (2021). We gather annualized yields at 3-month, and 1, 2, 3, 5, 7, and 10-year maturities. Given that this data is provided monthly, we sample the yields at the end of each quarter.

We collect survey data to improve the accuracy of our model estimates and guide the identification of the investor's subjective beliefs. Our sample only contains about 250 dates making it hard for formal econometric tests to distinguish persistent stationary behavior from

6. Since the Kalman gain and filtering variance-covariance matrices are deterministic, we could also consider the pricing exercise in a world where the filter has not yet reached the stationary stage. Pricing formulas would still be available in closed-form, although more complicated. We leave aside this exercise for practical reasons, namely that the obtained results would crucially depend on the initial conditions that we impose on the investor's learning problem.

integrated. Using surveys enables us to better pin down the parameters driving the time series dynamics of the factors, and the factors themselves since the long-run trends will dominate long-run forecasts of all macroeconomic variables and yields (see also Chernov and Mueller (2012), Crump, Eusepi, and Moench (2018), and Breach, D'amico, and Orphanides (2020) for different uses of survey data along with yields).[7] Notice that including surveys only helps us reducing the uncertainty in backing out the factors as perceived by the investor, i.e. around $X_{t|t}$, and that of the parameters but do not play a significant role in reducing the investor's uncertainty regarding her estimation of X_t. In other words, both surveys and yields are potentially informative about the investor's perceived state of the economy.

We gather several survey series. We first consider the mean expected quarterly inflation over the next 10 years from the Philadelphia Fed surveys of professional forecasters. Although this data is available quarterly for long-term inflation expectations, it is only sampled annually for real GDP and 3-month Treasury bills. We can easily add those to our data since the Kalman filter estimation readily takes care of missing data. We also use expected inflation and interest rates data over the next 5 years. Last, we consider short-term survey data from professional forecasters, which provides expected interest rate, inflation, and real growth values for four quarters ahead. Short-horizon SPF series start as early as 1982 for inflation and interest rate, and 1970 for real growth, while longer-horizon forecast series start after 1990.

[Insert Figure 1 and Table 3 about here.]

We present all our available data series used in the estimation in Figure 1 and their summary statistics in Table 3. Over our sample, the average 3m yield is 4.45% and the average inflation is 3.73%, leading to a positive average real rate during the period (see Table 3, panel a). Real growth averages at 2.95%. All macroeconomic variables and yields are highly persistent, except for real growth. Inflation has a first-order autocorrelation of 0.95, and all yields have autocorrelation above 0.97 (Table 3, panel a-b). This is because all variables but real activity show an upward trend during the pre-Volcker period and decline gradually to low levels during the 2010 decade. Inflation mostly stabilizes after the 1980s around 2.5%, and so do the related survey data (see Figure 1, panels a.1 and b.1).

7. Coroneo and Golinski (2023) document that surveys might have issues providing information about the objective dynamics, but are useful to inform the subjective probability measure. In our paper, we treat the Bayesian learning process as a subjective probability measure motivated by statistical theory.

5 What Does the Yield Curve Reveal About the Long-run

5.1 Deciphering Trends and Cycles from Aggregates

Our first output is the parameter estimates, which uncover the role of each factor in the pricing and learning mechanisms. Our estimates are presented in Table 4, and the fitted survey data series presented in Figure 2 show a reasonable fitting performance.

[Insert Table 4 and Figure 2 about here.]

Our estimation provides a first assessment of the importance of trend and cycle factors in macroeconomic shocks. Inflation and growth trend shocks have volatilities of 13bps and 6bps, respectively (variances of 0.017 and 0.003, panel b. of Table 4). In turn, the corresponding cycles have volatilities of 102bps and 200bps, respectively, more than 10 times larger. This means that most of the quarterly variation of macroeconomic variables is accounted for by their cycles, and the trends are slowly-moving, as expected. Remember that in the long-run however, trends dominate macroeconomic movements since they move according to a random walk. For the nominal interest rate, the spread between the trend and cycle volatilities is still important but lower, with 19bps for the former and 68bps for the latter (variances of 0.037 and 0.457, resp.). Our results also indicate that all trends are positively correlated, where correlation between the shocks affecting $i_t^\star$ and $g_t^\star$ is 0.41, 0.91 for shocks to $i_t^\star$ and $\pi_t^\star$, and 0.54 for shocks to $\pi_t^\star$ and $g_t^\star$. Cycles also show positive correlations but lower, especially for the interest rate and inflation cycle at 0.15 (panel b of Table 4).

A complementary analysis comparing the importance of the shocks can be drawn looking at our estimates of the Kalman gain, $\overline{\mathcal{K}}$ using our above notation. These parameters, presented in panel (c) of Table 4, tell us how the investor updates her beliefs about trends and cycles in response to her forecast errors. When there is a positive interest rate surprise, the investor attributes about 15% of the movement to the trend, and 85% to the cycle. The same surprise feeds back to other factors, to a smaller extent. In fact, the effects on other trends and cycles cancel each other out, for both inflation and growth. This results from the cascading effect of the learning mechanism in our model: when the short term interest rate is higher than expected, the investor naturally attributes some of it to an increase in the inflation trend being higher than expected. If observed inflation does not move, this means the inflation cycle must decrease of the same amount.

Real growth surprises are primarily attributed to cycle movements, which represent 97% of the investor's update, such that the growth trend is very smooth compared with observed growth. Positive inflation surprises are split 9% for the trend and 91% for the cycle. Inflation surprises have similar effects on the interest rate trend, but a low effect on growth: for a 100bps positive surprise in inflation, the nominal rate trend goes up by 10bps, roughly equivalent to the inflation trend. As such, subjective r-star estimates increase following surprise interest rate hikes, but stay virtually unchanged following positive inflation surprises.

As a last addition to this decomposition, the yield curve reveals the private piece of information that the investor relies upon, that is f_t. Looking at the last row of Table 4, panel (b), unexpected increases to f_t are mostly indicative of a decrease in long-run growth and inflation (correlations are -0.6 and -0.52, respectively). The covariance with the nominal interest rate trend is insignificantly different from zero. Bond investor private information can be thought of as an indicator of persistent demand shocks to the economy, which goes up when both long-run activity and inflation decrease. This private information will be an important driver of bond yields notably through term premia.

5.2 Subjective Long-Run Trend Paths and Uncertainties

We now focus on the time series of the perceived trends and cycles that are uncovered through asset prices and macroeconomic variables. Our model produces estimates of $i^{\star}_{t|t}$, $\pi^{\star}_{t|t}$, and $g^{\star}_{t|t}$, along with confidence bands contained in $\overline{\mathcal{P}}$ (whose estimates are in Table 4, panel d) that tell us the precision of the investor in evaluating the true stars. The estimated series along with 95% investor's confidence intervals are presented in Figure 3.

[Insert Figure 3 about here.]

Panel (a) of Figure 3 reveals that the interest rate trend tends to increase during the 1960s and 1970s, from 2.5% to 10%, and shows a secular decline from the Volcker era to the more recent period, going back to 2.5%. The same qualitative patterns can be observed for inflation on panel (b). The main difference is that the post-2000 period corresponds to a very stable inflation trend, slightly above 2.5%, in line with the monetary policy stance of inflation stability. The perceived real growth trend is in contrast with the previous two, being very smooth during the whole sample with a gradual decline from the 1970s until the mid-1990s from 3.5% to stabilizing around 2.5%. The growth cycle is thus much more volatile, explaining most of real activity quarterly movements (see panel c of Figure 3).

Our first key finding concerns the size of the investor's uncertainty about the long-run trends, which is represented in orange in Figure 3. For the nominal interest rate, the inflation rate and real activity, the standard deviations around the trends are estimated as 113bps, 62bps and 40bps, respectively (see panel d of Table 4, for variances of 1.267, 0.385 and 0.159, respectively). Converting to 95% confidence bands, the investor's trends estimates are essentially contained in $i^\star_{t|t} \pm 226\text{bps}$, $\pi^\star_{t|t} \pm 124\text{bps}$, and $g^\star_{t|t} \pm 80\text{bps}$. This leads to a wide array of uncertainty around the estimated trends as far as the investor is concerned, consistent with the discrepancies between existing estimates of the secular trends. Our estimates for the investor's perceived $i^\star_{t|t}$ are remarkably similar to the the estimates from Bauer and Rudebusch (2020), presented in red in Figure 3, with comparable size for uncertainty (95% bands of 226bps for our model v. slightly above 200bps for their model). The same observation can be made for our perceived inflation trend and $\pi^\star_{t|t}$ and the series of the perceived target rate from the FRB/US model taken from the Federal Reserve Board website (panel b of Figure 3).

5.3 Comparison with Perfect Information Estimates

At this stage, it is informative to compare the results obtained with the same model assuming perfect information (**PI**), i.e. assuming that the representative investor perfectly knows the trend-cycle decomposition of the macroeconomic variables. The corresponding estimated trends are presented in Figures 4.

[Insert Figure 4 about here.]

First, for $i^\star_t$, $\pi^\star_t$ and $g^\star_t$, the size of the confidence bands largely contrast with the ones obtained previously (green bands in Figure 4). The difference between the two models is striking, particularly for the nominal interest rate on panel (a): the estimated $i^\star_t$ is slightly more volatile that in the **II** model, but its confidence bands are about ±55bps, compared with ±226bps in imperfect information. Assuming perfect information creates an implausibly tight trend estimate since $i^\star_t$ is directly reflected in the term structure of interest rates, and measurement errors are small. Thus a large cross-section of yields can nearly perfectly reveal the trend to the econometrician, and the confidence bands are tiny. This resonates with the idea expressed in Bauer and Rudebusch (2020) that "*a trend hardwired to current interest rates would also result in excessively volatile trend estimates*". This excess volatility is particularly apparent for GDP growth, whose trend is moving fast compared with our imperfect information estimates (see panel a. of Figure 5).

5.4 Perceived R-star

With our subjective $i^{\star}_{t|t}$ and $\pi^{\star}_{t|t}$ estimates, we can compute the model-implied subjective real rate $r^{\star}_{t|t} = i^{\star}_{t|t} - \pi^{\star}_{t|t}$ along with the corresponding investor's confidence bands. We present the resulting series in Figure 5, panel (b).

[Insert Figure 5 about here.]

In terms of investor's uncertainty, the main observation is similar to that of the other trends, and the standard deviation around the perceived r-star estimates is quite wide, at approximately 84bps. The 95% confidence bands are therefore given by $r^{\star}_{t|t} \pm 168bps$, more than a 3 percentage-point window. These estimates are largely in line with policy comments on r-star and its measurement issues. In a 2018 speech, John Williams mentions that "*what appeared to be a bright point of light is really a fuzzy blur, reflecting the inherent uncertainty in measuring r-star*".[8] Therefore, while the yield curve provides useful information about bond investors' beliefs about r-star, it also reveals the large degree of uncertainty regarding their knowledge of the monetary policy stance.

Compared with alternative approaches, our time series estimates are lower than those of Laubach and Williams (2003), represented in blue in Figure 5, especially during the first part of the sample. For the post 1980s period, their series mostly belong to the 95% confidence band of the **II** model estimates, although almost always above. These discrepancies can result from the different definitions and identification assumptions. For most of our sample period however, our estimated series closely match the estimate from Bauer and Rudebusch (2020), who also identify the r-star as a shifting endpoint and not specifically tying it to a structural model. The business cycle fluctuations mostly coincide between all the series. R-star is countercyclical but reacts late and slowly to recessions in our model. For instance, our estimate starts going down right at the onset of the 2008 recession while other estimates turn down some quarters earlier.

A key difference between our model and Laubach and Williams (2003) or the perfect information model is the estimated behavior of the stars during the 1960s and 1970s, resulting in large differences in r-star series. In both LW and our **PI** model, r-star shows a decreasing trend from the mid-1960s to the late 1970s. LW estimates start at 5% and decrease to roughly 2.5%, and the **PI** model goes from 3.75% to 0% during that period (grey solid line, panel b. of Figure 5). In contrast, our baseline model with imperfect information

8. Boocker et al. 2023.

shows an increasing trend during the same period, from 1.5% in 1960 to more than 2.5% in 1980, notwithstanding a period of stability in the second half of the 1970s. This positive trend of the **II** estimates are also in contrast with those obtained by Del Negro, Giannone, Giannoni, and Tambalotti (2017), who show stable estimates around 2% during the two decades. This difference cannot be attributed to ex-ante v. ex-post estimation (we explore this distinction in the next section). Rather, it emphasizes the role of learning and beliefs in our framework. Our model shows that investors believed r-star to be low and rising during these two decades, reflecting the gradual realization that r-star was much higher than anticipated, and that monetary policy was more accommodative than it should have been.

5.5 One- and Two-sided R-star Estimates

Our analysis above comments on the evolution of r-star based on one-sided estimates $r^{\star}_{t|t}$, that is the investor's best estimate of r-star at date t based on her available information up to date t. As time goes by, she can also revise her past evaluations and form two-sided estimates $r^{\star}_{t|T}$, where the series is constructed by looking at the information provided by all observables up to date $T \geqslant t$ (ex-post estimates). These estimates are easily computed using the Kalman smoother. We present the resulting series of one- and two-sided r-star in Figure 6, panel (a), and the corresponding cycles in panel (b), along with their respective confidence bands.

[Insert Figure 6 about here.]

The ex-post series for r-star confirm the qualitative findings of the one-sided series: r-star is showing an increasing trend from the 1960s to the late 1970s, from roughly 1.5% to 2.5%, and starts unambiguously decreasing up to the zero lower bound period where it plateaus at 40bps. The two-sided series show less high-frequency fluctuations, and less sensitivity to the business cycles, consistently with the main features of the Kalman smoother. Uncertainty bands are still wide, albeit smaller at the beginning of the sample for the two-sided estimates. The standard deviation of 84bps across the sample for the one-sided r-star reduces to 63bps in 1960 for the two-sided estimate. This still amounts to a large confidence band of roughly ±1.3%, reflecting the high degree of uncertainty of the investor even if she is endowed with the entire observation sample (see teal bands in panel a, Figure 6). Interestingly, two-sided r-star estimates start increasing slowly from 2016 onwards, from 34bps in 2016:Q1 to 61bps in 2022:Q4, showing that the investor's beliefs of a low r-star era may be changing. This also coincides with an increase of the investor's confidence bands around the two-sided estimates becoming wider by about 12bps on each side, a mechanical effect of the Kalman smoother when reaching the end of the sample.

In panel (b) of Figure 6, we document the corresponding real interest rate cycle that is obtained as the realized real rate minus r-star. Due to the perfect observation of the aggregate real rate, the confidence bands around the one- and two-sided estimates are exactly the same as those of r-star. These series can serve as indicators for the stance of monetary policy, where a negative value represents monetary policy accommodation and a positive value represents monetary policy tightening.

Our estimates show three salient features. First, as indicated above, the 1960s and 1970s cylce estimates become more and more negative, emphasizing the gradual realization of an accommodative monetary policy stance. The overlaying of the two series provide evidence that subsequent observations do not provide much more information about r-star during that period. Second, both series trend down after the Volcker period up to the zero lower bound and Covid period where the monetary policy stance becomes persistently accommodative again, consistently with the unconventional monetary policies put in place, such as forward guidance and quantitative easing. At the end of our sample, the real interest rate cycle starts growing back up again towards zero. Third, one-sided r-star estimates are consistently above two-sided estimates from 1980 onwards, leading to an ex-post evaluation of the monetary policy stance as being more restrictive than perceived at the time. These differences peak during the Greenspan era, with a maximum of 1.20% difference in 2000:Q4. This shows that ex-post, Greenspan policies might be considered more neutral than at the time, and the two-sided cycle estimate is at zero during the burst of the dot-com bubble against -1% for the one-sided estimate.

5.6 Debt Sustainability and Subjective Estimates of $r - g$.

Our model allows us to go one step further and compare the real rate and growth trend estimates, informing the debate on the sign of $r_t^\star - g_t^\star$ and its economic consequences (Piketty 2014; Mankiw 2015). This economic quantity has an enormous importance to assess the fiscal capacity of a sovereign state. In particular, if $r < g$, the government can potentially roll-over its debt forever even if faced with a current deficit, and not worry about debt sustainability (see e.g. Abel and Panageas (2022) and the references therein). While our goal is not to provide a fiscal sustainability assessment, we emphasize that our time series estimates can provide useful information on the investors' view of the long-run $r - g$, the quantity at the heart of the computation of fiscal limits. Our estimates of investors' uncertainty about $r - g$ can be of particular use for policy-makers.

The series are presented in Figure 5, panel c. Since the real interest trend is more volatile than the real growth trend, the difference between the two has the same business cycle fluctuations as the former. $r^\star_{t|t} - g^\star_{t|t}$ is thus countercyclical, and increases during most recessions before the 2000s, but stays mostly unaffected by the dot-com bubble, the great financial crisis, and COVID. Indeed, long-run growth did not fall as much as r-star did in recent recessions, improving sustainability in the long-run. One key feature that can be observed is that the perceived $r^\star_{t|t} - g^\star_{t|t}$ is negative during most of the sample, and is significantly different from zero during the full 1970s decade and after 2020. For the remaining time periods, the time series show estimates that are non-statistically different from zero for the investor. In comparison, the implied estimates from Laubach and Williams (2003) are not as volatile and closer to zero. They reach only slightly negative values at the end of our sample. This shows that yields and surveys help us reveal the information possessed by the investor, above and beyond macroeconomic variables. The uncertainty around the investor's estimates are large, with a standard deviation of $r^\star_{t|t} - g^\star_{t|t}$ of 98bps (straightforward combination of panel d, table 4). Our model implies that investors' uncertainty dampens their ability to evaluate fiscal limits and debt sustainability with precision.

In the end, our model estimates reveal that *(i)* yields and investors' learning can uncover important information about r-star, notably looking at its history in the pre-Volcker era, and *(ii)* assuming the investor has imperfect information produces believable estimates with very wide confidence bands, reflecting the limited information that investors have access to.

6 Term Premium Analysis

6.1 Pricing Performance

Before analyzing the risk premium produced by our model, we confirm that the model provides a good fit to the yield curve. In Table 4, panel (g,) we can see that the **II** model provides a very reasonable term structure fit with only 4 shocks. RMSEs on nominal yields range from 9bps at the 5-year maturity to 25bps at the 10-year maturity, and the other maturities range from 13bps to 19bps RMSEs. In comparison, the perfect information model produces RMSEs ranging from 5bps to 12bps (see panel d of Table A.1 in the

Appendix), significantly lower than our imperfect information model. Provided the number of shocks is almost double in the **PI** model compared to the **II** model, this is only a modest improvement.

Looking at the prices of risk estimates in panel (f) of Table 4, we can directly read which economic factors enter the stochastic discount factor and their contributions. First, estimates of α shows that shocks to the interest rate are priced with a positive sign, while shocks to aggregate inflation and growth are priced with a negative sign (only the latter being significant). Second, estimates of β provide information about the time variation in the prices of risk, and the predictors of bond excess returns. We find that the factor predicting excess returns is equal to:

$$\underset{(0.13)}{0.34\, i^{\star}_{t|t}} - \underset{(0.16)}{0.35\, \pi^{\star}_{t|t}} - \underset{(0.05)}{0.05\, g^{\star}_{t|t}} - \underset{(0.15)}{0.18\, C_{i,t|t}} - \underset{(0.16)}{0.17\, C_{\pi,t|t}} - \underset{(0.09)}{0.21\, C_{g,t|t}} + f_t\,. \tag{23}$$

Only two macroeconomic factors are statistically significant. First, the real interest rate trend $r^{\star}_{t|t}$ is the only long-run factor that enters the price of risk, since $i^{\star}_{t|t}$ and $\pi^{\star}_{t|t}$ have virtually the same coefficient of 0.35 with opposite signs. Second the growth cycle $C_{g,t|t}$ shows a significantly negative coefficient of -0.21. The fact that the inflation trend indirectly enters the price of risk is consistent with Cieslak and Povala (2015), but our model shows that it does mostly due to $r^{\star}_{t|t}$ rather than the inflation trend alone, or to the nominal interest rate trend alone as in Bauer and Rudebusch (2020). The term premium thus contributes to the trend observed in the yields curve, through $r^{\star}_{t|t}$.[9]

These estimates contrast sharply with those of the **PI** model, presented in Table A.1 of the Appendix. Assuming perfect information implies a risk premium factor that is a function of the growth trend (coefficient of -0.925, panel d. of Table A.1), and the real interest rate cycle only (coefficients of -0.276 for $C_{i,t}$ and 0.332 for $C_{\pi,t}$), the other state variables coefficients being insignificantly different from zero. Incorporating incomplete information therefore reveals r-star as a low-frequency driver of bond risk premia, where a higher r-star predicts higher excess returns, a stylized fact that is usually missed by perfect

9. An additional identification assumption that is apparent from Equation (23) is that f_t enters with a coefficient of 1 in the price-of-risk factor. Its presence in the price of risk allows it to appear in the bond premium, and helps us to identify the persistence of the process since f_t is autonomous and does not appear in the expectation of future short-term interest rates (see Equation 9). Therefore we can interpret the idiosyncratic f_t shock, denoted by $\varepsilon_{f,t}$, as a pure premium shock that is orthogonal to structural macroeconomic shocks accordingly with the true DGP. In imperfect information however, the shock can have an impact on the expectation component as investors may confuse it with another structural shock.

information models.

6.2 Term premium decomposition

Remember that yields at any maturity are the combination of an expectation component, i.e. the expected future path of short-term interest rates, and a term premium component resulting from the investor's risk aversion. Since the factor dynamics as perceived by the investor are Gaussian affine, the term premium $TP_t^{(n)}$ is also an affine combination of the factors, such that we have

$$\mathrm{TP}_t^{(n)} = a_n^{(TP)} + b_n^{(TP)'} X_{t|t} \,, \tag{24}$$

and the loadings are easily obtained through closed-form recursions. In standard Gaussian ATSMs, the term premium is homogeneous to the prices of risk times the quantity of risk. The former is specified in the SDF and is linear in $X_{t|t}$, with $\lambda_t = \alpha \cdot \beta' X_{t|t}$, as described in the previous paragraph. Therefore, the perceived states will directly enter the compensation for risk demanded by the investor. Then, the quantity of risk is summarized by the covariance matrix of the factors as perceived by the investor, which is controlled by two key quantities: $\overline{\mathcal{V}}$, the variance covariance matrix of the variables observed by the investor, and $\overline{\mathcal{K}}$, the Kalman Gain summarizing how forecast errors propagate to the state as shocks (see Equation 14). Therefore, all factor estimates and their estimation uncertainties will play a role in the level and dynamics of the term premium produced by our model.

The resulting decomposition produced by our **II** model is presented in Figure 7 for yields of maturities 1y, 3y, 5y and 10y. Consistent with the findings of Bauer and Rudebusch (2020) and Feunou and Fontaine (2023), the expected components closely follow observed yields across all maturities. As such it explains most of the yields variability during the whole sample. This is due to the inclusion of the long-run trend $i_t^\star$, which drives both long-run expectations of the monetary policy rate and its level. The residual term premium grows with maturity, as compensation for duration risk.

[Include Figure 7 about here.]

Our term premium estimates are countercyclical, in line with Cochrane and Piazzesi (2005). They tend to increase during recessions, for all maturities. This is particularly apparent during the two 1980s recessions where the 10y premium jumps to 4% and starts coming slowly down when the recession is over. In other words, the perceived factors driving the prices of risk enter as a countercyclical combination (see Equation 23). This is quite natural provided the estimates entering the price of risk factor: the term premium rises when r-star is elevated, or when growth tends to fall. It also increases when f_t increases, which tends to happen when persistent negative demand shocks are realized. Note that the cyclical properties of term premium are still debated as different term structure models yield different answers (see e.g. Li et al. 2017), but incorporating investor's beliefs favor a countercyclical fluctuation.

The time series of our estimated premia show a shift starting in the 1980s, ending an era of low and stable term premia contained below 2.5%. The term premia estimates are consistent with a delayed realization by bond investors of the slowdown during the 1970s through a slowly growing r-star, and to a low risk-premium factor f_t mimicking the behavior of the real-rate cycle (see Figure A.1 in the Appendix). Only after the beginning of the Volcker disinflation era does the 10-year premium jump to 4%, from a low 0% beforehand. Following that period, TP estimates follow a similar downward trend as the perceived r-star series, thereby casting some caution in common market-based estimates of r-star such as TIPS yields. Adjusting for the trend in term premium is important to obtain reliable r-star estimates with its time trend, a point made notably by Williams and Cho (2025).

More generally, our series are lower and less volatile than the ones produced by standard workhorse term structure models assuming perfect information, such as Adrian, Crump, and Moench (2013) or Kim and Wright (2005). First, consistent with the findings of Li, Meldrum, and Rodriguez (2017), our inclusion of survey forecasts brings the premium down compared to yield only models. Second, these models embed no macroeconomic variables such that they rely heavily on yield factors. These can create wide discrepancies in term premia estimates.

7 Monetary Policy Shocks in Incomplete Information

We last conduct a structural analysis derived from our estimates. We identify structural shocks affecting macroeconomic aggregates and bond prices with sign restrictions in our imperfect information context. We detail the identification assumptions and methodology and derive impulse-response functions of monetary policy shocks. We compare our results to the effects of perfectly observed shocks.

7.1 Structural Identification

Whatever the information set of the investors, the objective data generating process presented in Section 3 contains seven reduced-form residuals: three permanent shocks (nominal $i_t^\star$, inflation $\pi_t^\star$ and growth $g_t^\star$), three transitory shocks (inflation $C_{\pi,t}$, growth $C_{g,t}$ and interest rate $C_{i,t}$), and one risk premium shock (f_t). These reduced-form shocks are a combination of structural economic shocks, and further assumptions have to be made to identify these structural shocks (Sims 1980).

We consider an identification based on sign-restrictions following the example of Cieslak and Pang (2021) (see also Kilian and Lütkepohl (2017) for an overview of structural VAR identification methodologies). Our three structural shocks are in the spirit of standard three-equation New-Keynesian models and are named *supply*, *demand*, and *monetary policy* shocks. We call the permanent monetary shock *inflation target* shock. We assume the following sign-restrictions:

$$\begin{pmatrix} i_t \\ \pi_t \\ g_t \end{pmatrix} \equiv \underbrace{\begin{pmatrix} - & + & + \\ -- & + & + \\ + & + & ? \end{pmatrix}\begin{pmatrix} \mathrm{S}^\star \\ \mathrm{D}^\star \\ \mathrm{IT} \end{pmatrix}}_{\text{permanent shocks}} + \underbrace{\begin{pmatrix} - & + & + \\ - & + & - \\ + & + & - \end{pmatrix}\begin{pmatrix} \mathrm{S} \\ \mathrm{D} \\ \mathrm{M} \end{pmatrix}}_{\text{transitory shocks}}, \tag{25}$$

Equation (25) says that a structural supply shock pushes activity upwards and inflation downwards and lowers the policy rate, while a demand shock pushes both activity and inflation in the same direction thus increases the policy rate. We additionally assume that a permanent positive supply shock raises $r^\star$ by decreasing $i^\star$ less than $\pi^\star$. The inflation target shock raises the long-run inflation, but it also reduces the probability of hitting the zero lower bound, thus increases the interest rate trend. We leave its effect on long-term growth unspecified (the "?" entry in the equation) as we do not take a stance on whether heightened inflation slows down the economy more than the benefits gained from escaping the ZLB. The transitory shocks mimic this structure, where the inflation target shock is replaced by a monetary policy shock that raises the nominal interest rate and decreases inflation and growth, consistently with conventional macroeconomic models.[10] The resulting matrices

10. To obtain the orthogonal matrices, we first compute the lower-triangular Choleski of both $\Sigma^\star$ and Σ_c. We then simulate uniformly 1,000 three-dimensional vectors of angles allowing us to compute a simulated Givens rotation matrix R_s, for simulation $s \in \{1, \ldots, S\}$, and apply these rotations independently to the

are presented in Table 5. We compute impulse-response functions of these shocks onto both macroeconomic variables, trend and cycle decompositions, and the term structure of Treasury yields along with their risk premia decomposition.

[Include Table 5 about here.]

In the following, we focus on the effects of the inflation targeting shock and the monetary policy shock. The effects of supply and demand shocks are presented in Appendix A.5 for completeness.

7.2 Monetary Policy Shocks and R-star

Let us remember the mechanism detailed in Section 2. Because investors confuse trends and cycles, their long-run expectations tend to underreact to permanent shocks, but overreact to transitory shocks. After the shock, investors correct their trend-cycle decomposition as time goes by based on the mean-reversion observed on the aggregate macroeconomic variables. The impulse-response functions of macroeconomic variables to the inflation targeting and transitory policy shocks are presented in Figures 8 and 9, respectively.

[Include Figures 8 and 9 about here.]

We first compare the effects of the two types of shocks on macroeconomic variables, and their perceived trend-cycle decomposition. As imposed in our identification structure, an inflation targeting shock permanently raises both the nominal interest rate and inflation, by about 13bps and 5bps, respectively. The resulting median effect on the real interest rate is positive, at 8bps, although a positive impact is not imposed by our identification scheme. Output growth react mildly negatively, at −2bps, meaning that the cost of raising inflation slightly outweighs the benefit of escaping the ZLB in the long-run. Although all these effects are permanent, the investor cannot see it as it happens and performs her trend-cycle decomposition. I-star increases on impact by 5bps against 8bps for the interest rate cycle, and pi-star increases by 3bps and the inflation cycle by 2bps. As a result, r-star increases mildly by 2bps on impact, the remainder of the real rate effect being soaked up by the cycle (column 3 of Figure 8). The decrease in growth is mostly attributed to the

Choleskys. We compute candidates by rotating the Cholesky and keep the candidate only if it agrees with the signs specified in Equation (25). Last, among the kept candidates, we present the matrices that are the closest to the median matrix in the set of candidates, where the median is taken element-by-element.

cycle, and g-star is virtually unaffected on impact. The dynamics are then conform with the mechanism explained in Section 2, where stars underreacted and start converging to the aggregate effect, and cycles revert back to zero. This convergence is slow: after 10 years, r-star is still below 5bps (compared to 8bps for the true value), mostly because i-star has not converged yet. Because r-star underreacts persistently, monetary policy is believed to be more restrictive than it actually is and transitory growth is perceived as negative.

In contrast, the transitory monetary policy shock raises the nominal interest rate by about 40bps, and decreases inflation by 65bps, thereby raising the real interest rate by 105bps (first row of Figure 9). As imposed by our identification assumptions, output growth decreases by slightly less than 100bps, showing that tightening policy shocks are largely contractionary. Although the true shocks are purely transitory, the investor attributes a small amount to the permanent component: i-star decreases by 4bps on impact, and pi-star by 7bps, leading to an immediate increase in r-star by about 3bps. While most of the negative growth shock is attributed to the cyclical component, the perceived g-star goes down by about 3bps as well. In the aggregate macroeconomic variables, most of the effects die after approximately two to three years. However, the effect on r-star is extremely persistent, and its perceived value is still at 3bps after 10 years, mostly because i-star and pi-star converge at approximately the same speed, but start from different values. Therefore, monetary policy is believed to be slightly more accommodative than it actually is, in contrast with the inflation targeting shock.

Despite their difference in nature, both inflation targeting and transitory monetary policy (one-standard deviation) shocks raise r-star in a very persistent fashion, by about the same amount. Both shocks have a negative impact on growth prospects, thereby casting doubt on a possible Fed information effect.

7.3 Monetary Policy Shocks and the Yield Curve

We now turn to the effects of these shocks on the yield curve. To gain some intuition, remember that long-term yields can be interpreted as expectations of future short-term interest rates under the expectation hypothesis. We therefore expect that the same under/over-reaction noted above for the stars will be a first order effect for long-term yields. Two additional features may blur these effects. First, long-term yields embed risk premia that can react to both trends and cycles, potentially in an opposite direction to the expectation component. Second, structural shocks move all aggregates, trends and cycles, and the effect of inflation and growth may have an impact on the expectation component of long-term yields. The eventual effect is an empirical exercise. The effects of the monetary policy shocks on the yield curve is presented in Figures 10 and 11, for inflation targeting and transitory policy shocks, respectively.

[Include Figures 10 and 11 about here.]

The effect of the two shocks on the yield curve are sharply different, although they both result in a flatter curve. Following the inflation targeting shock, because of the permanent increase of i-star that is gradually understood by investors, the expectation component increases by 12bps at the one-year maturity, 9bps at the five-year maturity and 7bps at the 10-year maturity (see blue dashed lines in Figure 10). The effect is more atoned for longer maturities since i-star, which plays a more important role for longer-run forecasts of interest rates, underreacts. This expectation component then slowly increases at all maturities following the updates in i-star assessments. However, the reaction of the private information factor largely compensates this effect, with the term premium decreasing sharply. As a result, the initial aggregate effect on the yield curve is negative rather than positive (solid black lines in Figure 10). The five-year and 10-year yields go down on impact by 10bps and 12bps, respectively, while the one-year yield goes up by 3bps. After two years, the effect on the risk premium comes back to close-to-zero values, the yields become close to the reaction of their expectation component, and the effect resembles a parallel shift of the yield curve slightly above 10bps. Interestingly, even after 10 years, the reaction of the 10-year maturity yield is still largely below its reaction if the permanent shock was perfectly observed. The effects on the term structure are about 13bps, 16bps and 20bps for the one-, five- and 10-year maturities in perfect information leading to a significant steepening of the term structure of interest rates (black dotted lines in Figure 10), compared with an approximately flat curve at 12bps in imperfect information.

A transitory tightening of monetary policy is better understood by investors in the short-run, contrasting with the inflation targeting shock. The yield curve increases at all maturities, more so at the one-year (+30bps) than for the 10-year (+13bps), reflecting the effects on the expectation component (see black solid lines and blue dashed lines in Figure 11). The immediate effect on the expectation component of the 10-year yield is virtually identical than for the inflation targeting shock, at 7bps. As predicted, the yield curve overreacts at the long-end with respect to a perfect information case because of the persistent effect on the stars, and the long-run effects of the monetary policy shocks are about 5bps for the

10-year yield, against virtually null effects in perfect information (black dotted lines, panel c., Figure 11). In the long-run, the initial flattening of the curve is replaced by a steepening, because of persistent effects on the term premium, albeit low in magnitude. These arise because r-star remains above zero, thereby driving term premium upwards even after 10 years have passed.

In the end, this exercise emphasizes that permanent shocks, such as the inflation targeting shock, may lead to large short-term distortions on the yield curve that may take a long time to correct. Driven by a large and transitory negative premia response and slow learning about the stars by bond investors, the long-end of the curve is lower than it should be in the long-run, by about 10bps. This results partly from the fact that the real interest rate is above r-star, and investors anticipate mean-reversion bringing down interest rates in the future. Conversely, transitory tightening monetary policy shocks flatten the curve less than they should, mostly driven by the fact that investors perceive monetary policy as more accommodative than it actually is. These effects provide some insights in understanding movements in the long-end of the curve, and the strength of the reaction of long-term bond yields to monetary policy shocks when the type of policy is not well understood.

8 Conclusion

This paper provides a new bond pricing framework to obtain term-structure-based estimates of r-star when investors have imperfect information. We formulate a reduced-form macroeconomic model where the monetary policy rate, inflation and growth all follow a trend-cycle decomposition. The representative bond investor cannot observe the decomposition in real-time, but has to infer trends and cycles from observed aggregate outcomes. With a standard stochastic discount factor, we show that our model belongs to the class of standard Gaussian affine term structure models such that its estimation simple. We show that our formulation allows us to obtain estimates of the macroeconomic stars, and of the uncertainty faced by the investor when performing the trend-cycle decomposition. Our estimation on U.S. quarterly data from the 1960s to 2020s reveals that the investor faces a wide uncertainty, with confidence bands around r-star exceeding 1.6% on each side. We provide evidence that r-star was believed to be growing in the 1960s up to the Volcker era, and that it has been unambiguously believed to be decreasing ever since, reflecting gradual learning by investors. In addition, our model provides estimates of $r - g$, frequently used as a fiscal indicator. We find that it is virtually always perceived as non-positive, such that sovereign credit risk is perceived as low in the U.S. We show that the learning mechanism and uncertainty around r-star has a wide array of bond pricing implications. First, r-star is identified the sole long-run trend driving the bond risk premium. Second, rational confusion between r-star and the real interest rate cycle following monetary policy shocks leads the representative investor to perceive monetary policy as more restrictive or accommodative than it actually is, leading the long-end of the yield curve to under and overreact to inflation targeting and transitory monetary policy shocks, respectively.

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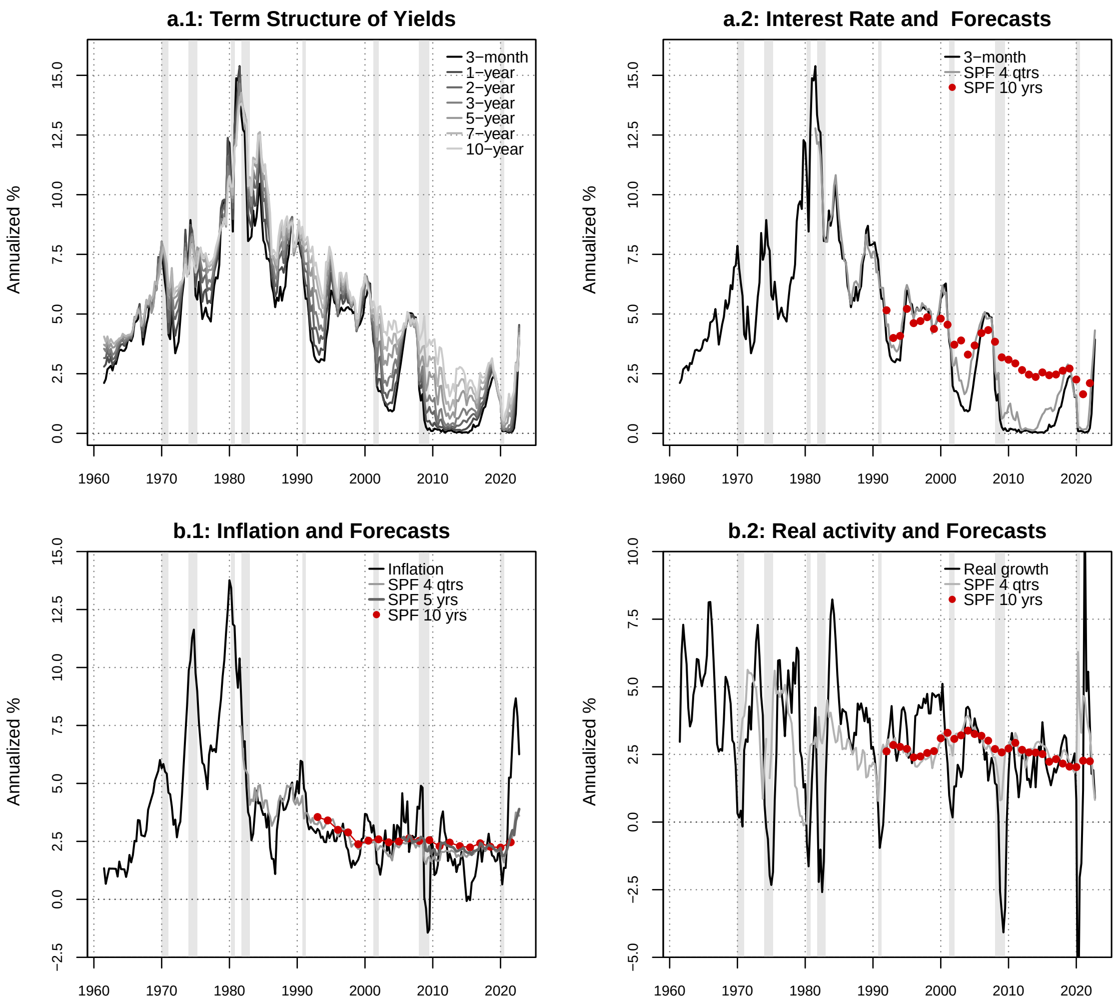
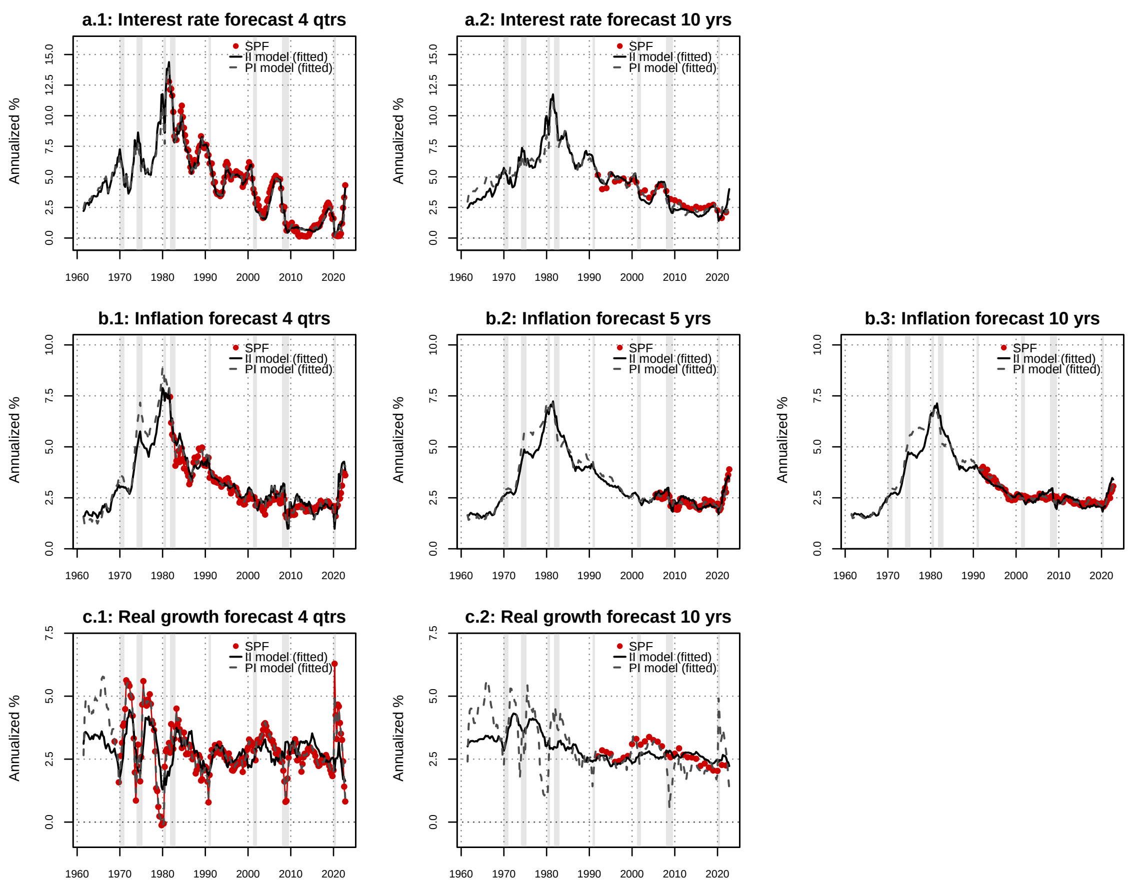
9 Figures

Figure 1: Macroeconomic data



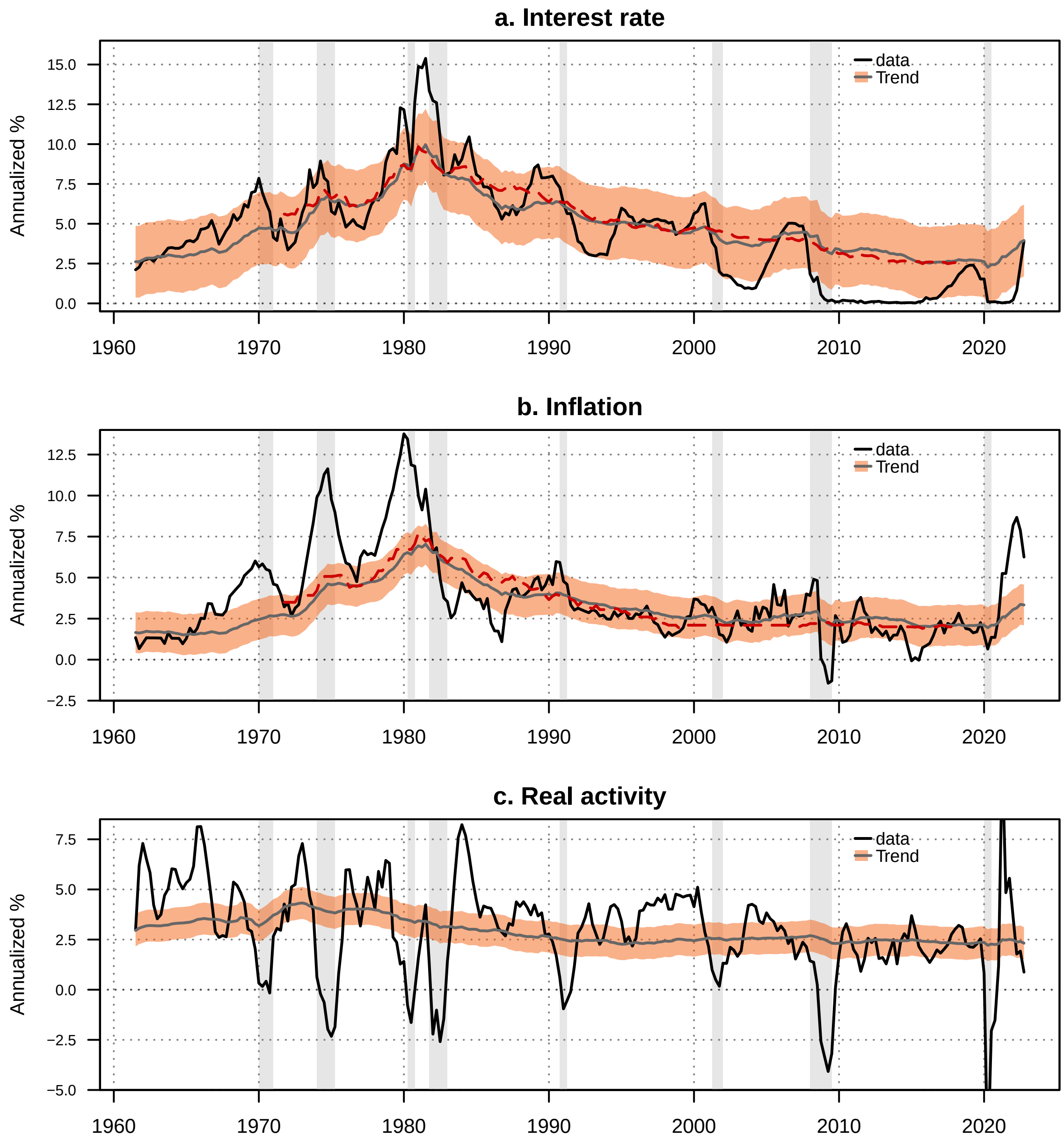
Notes: Panel (a.1) plots the time-series of the term structure of the nominal interest rates with maturities ranging from 3-month to 10-year. The yield data is from Liu and Wu (2021). Panels (a.2), (b.1), and (b.2) present the macroeconomic data alongside the forecasted values from the survey of professional forecasters for the 3-month interest rate, inflation, and real GDP growth, respectively. Inflation is the year-on-year log-change in the seasonally unadjusted CPI-U index and real growth is the year-on-year log-change in seasonally adjusted real GDP. Both data are collected from the Federal Reserve Bank of St Louis website. Forecast values are from the Survey of Professional Forecasters (SPF) conducted by the Federal Reserve Bank of Philadelphia. SPF 4 qtrs represents expected macroeconomic values for the next four quarters. SPF 5 yrs and 10 yrs are the mean expected quarterly macroeconomic values over the next 20 quarters (5 years) and 40 quarters (10 years), respectively. All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure 2: Fitted values of the forecasts across the II and PI models



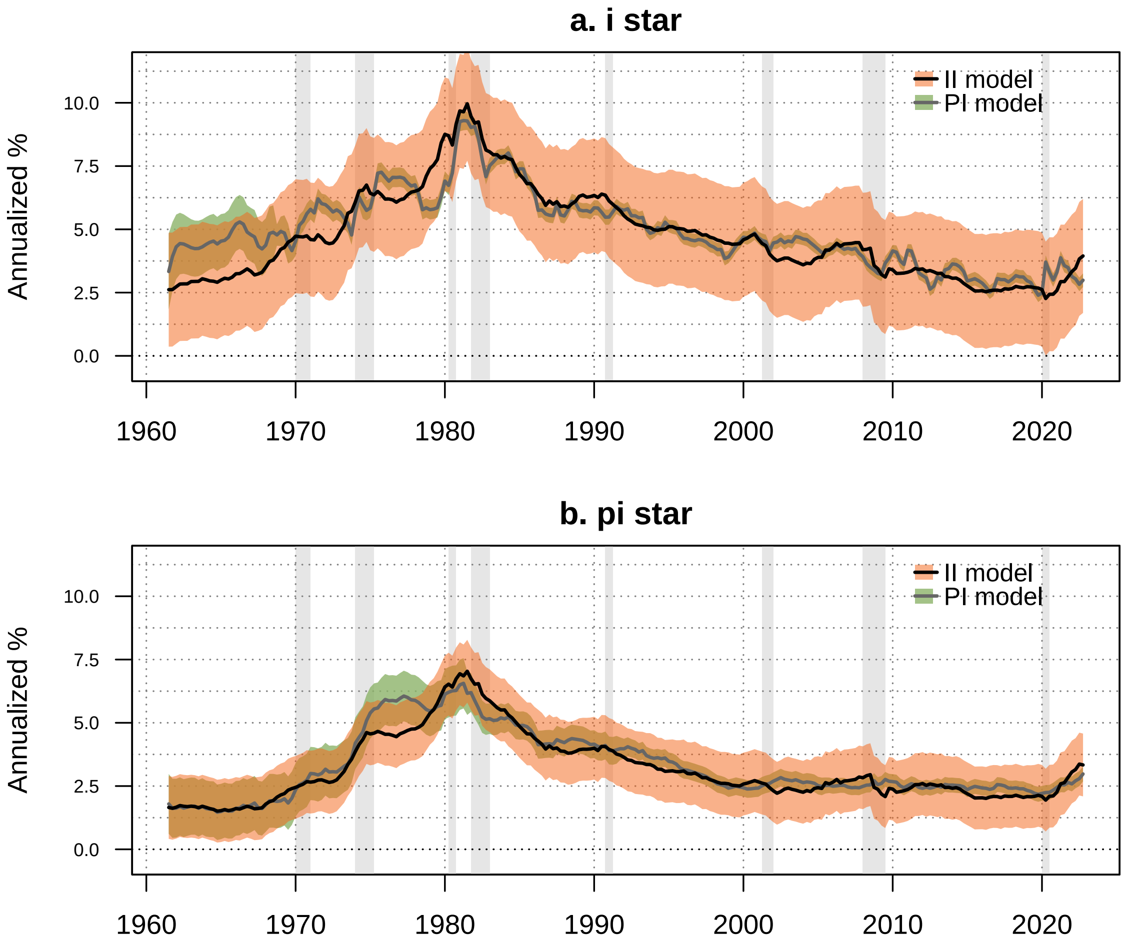
Notes: The figure presents the fitted values for the forecasts across the two models. **II** (**PI**) stands for the model where the representative asset pricer has imperfect information (perfect information). Forecast values are from the Survey of Professional Forecasters (SPF) conducted by the Federal Reserve Bank of Philadelphia. SPF 4 qtrs represents expected macroeconomic values for the next four quarters. SPF 5 yrs and 10 yrs are the mean expected quarterly macroeconomic values over the next 20 quarters (5 years) and 40 quarters (10 years), respectively. The first row (Panels a.1 and a.2), the second row (Panels b.1, b.2, and b.3), and the last row (Panels c.1 and c.2) show the figures for 3-month interest rate, inflation, and real output growth, respectively. All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure 3: Subjective trends in imperfect information



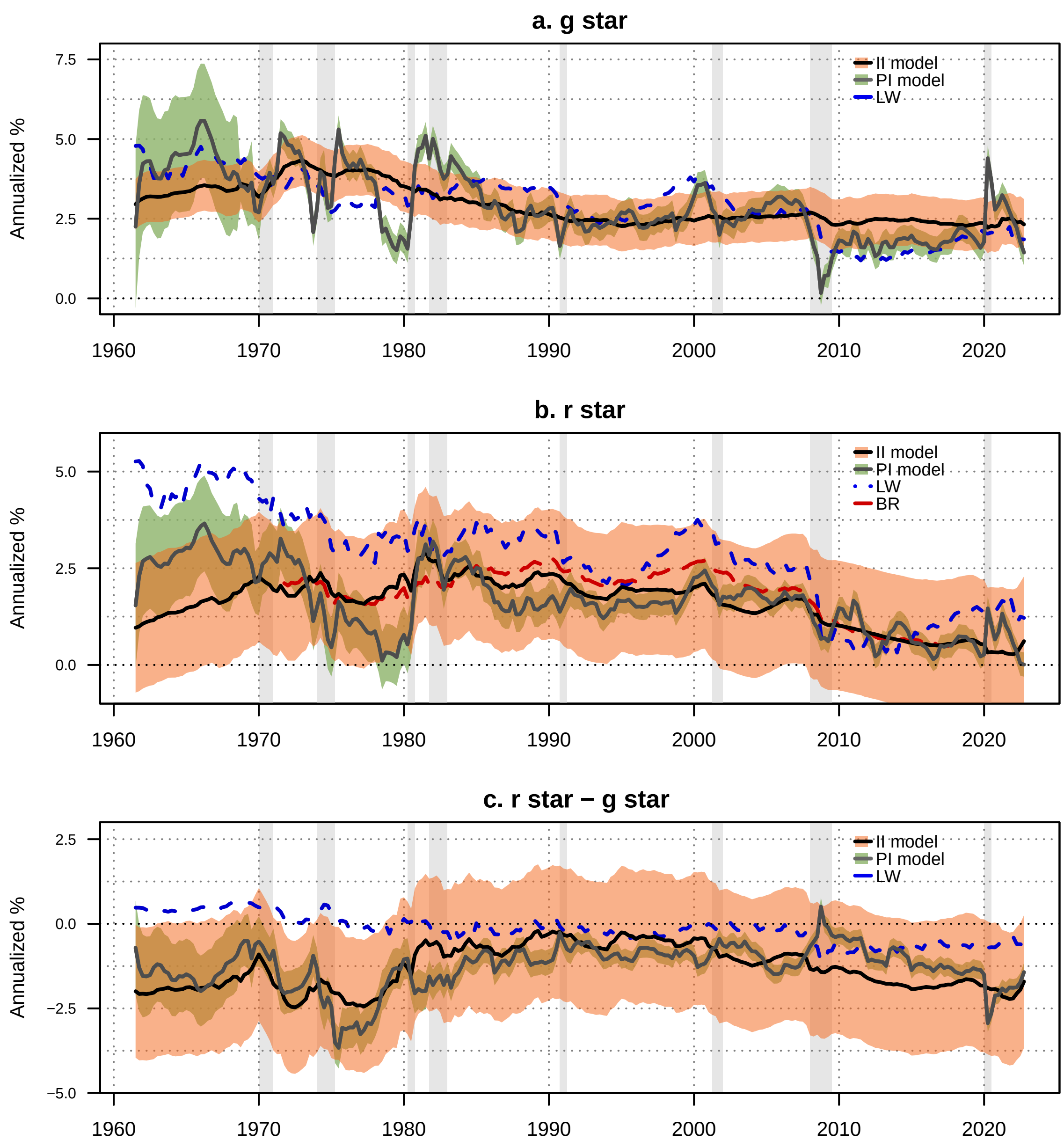
Notes: This figure presents the aggregate interest rate (panel a.), inflation (panel b.) and real activity (panel c.) along with their subjective trends as implied by the model where the representative asset pricer has imperfect information. The shaded areas around the trends and the cycles are the 95% confidence interval as perceived by the representative agent. We plot the estimates of Bauer and Rudebusch (2020) in dashed red. All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure 4: Interest rate and inflation trends across models



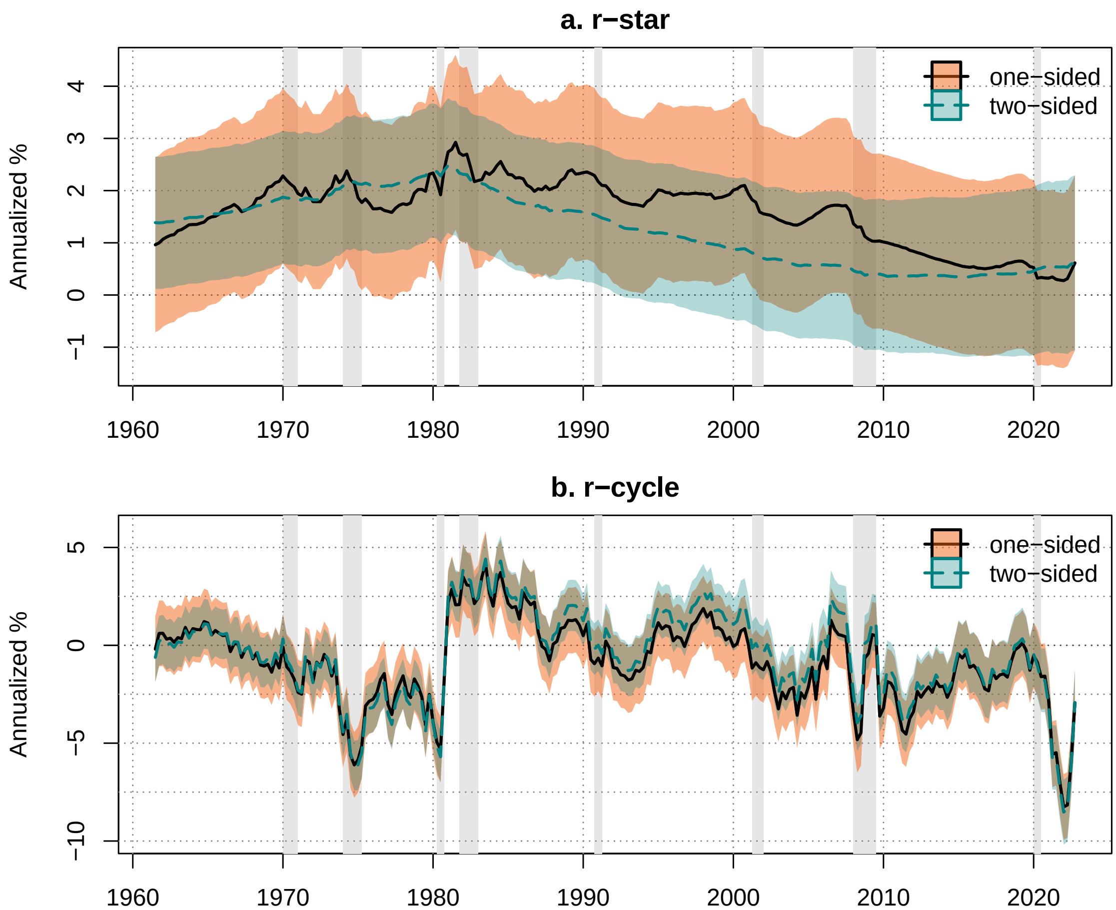
Notes:

Figure 5: Real interest rate and real GDP growth trends across models



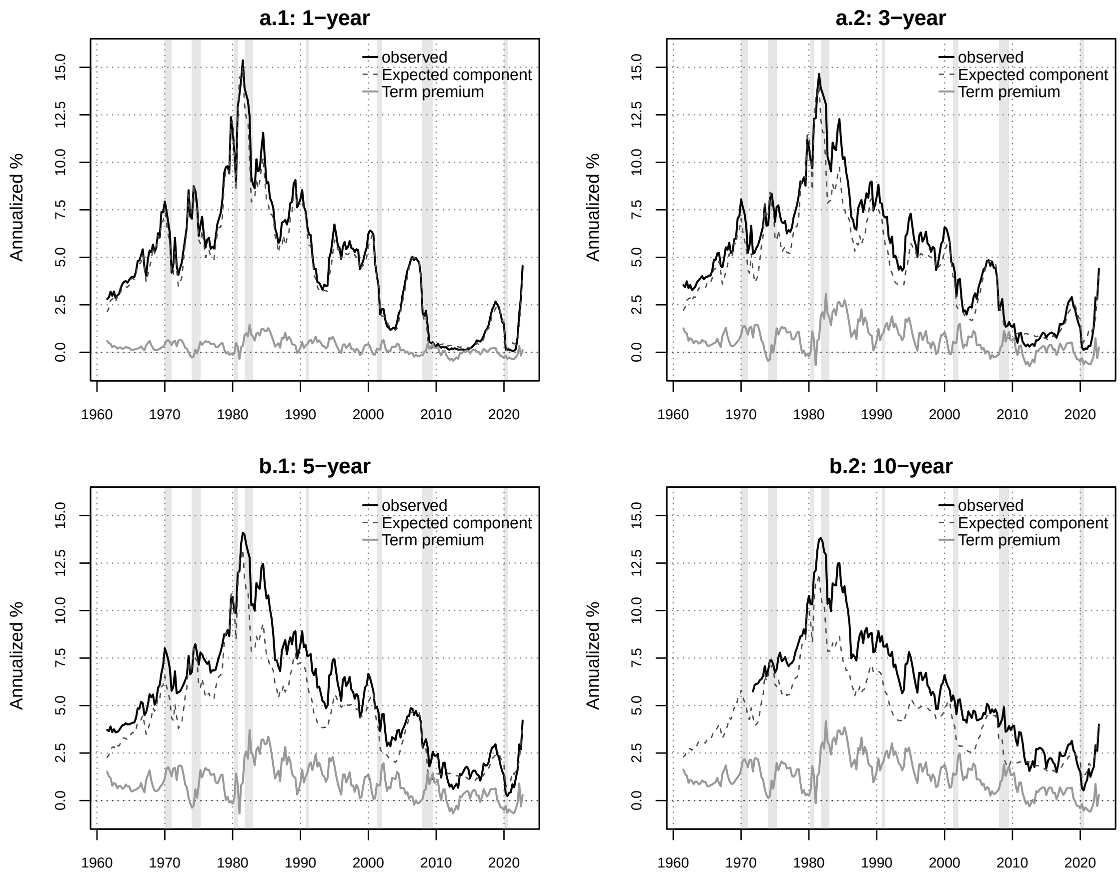
Notes: Panel a., b., and c. plot the real output growth trend $g_t^\star$, the implied real interest rate trend $r_t^\star$, and the difference $r_t^\star - g_t^\star$, respectively. In addition to the **II** (**PI**) model where the representative asset pricer has imperfect information (perfect information), we plot benchmarks from Laubach and Williams (2003), LW; and Bauer and Rudebusch (2020), BR. The shaded areas around those variables are the 95% confidence interval as perceived by the representative agent (for the **II** model) or by the econometrician (for the **PI** model). All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure 6: One-sided and Two-sided r-star Estimates



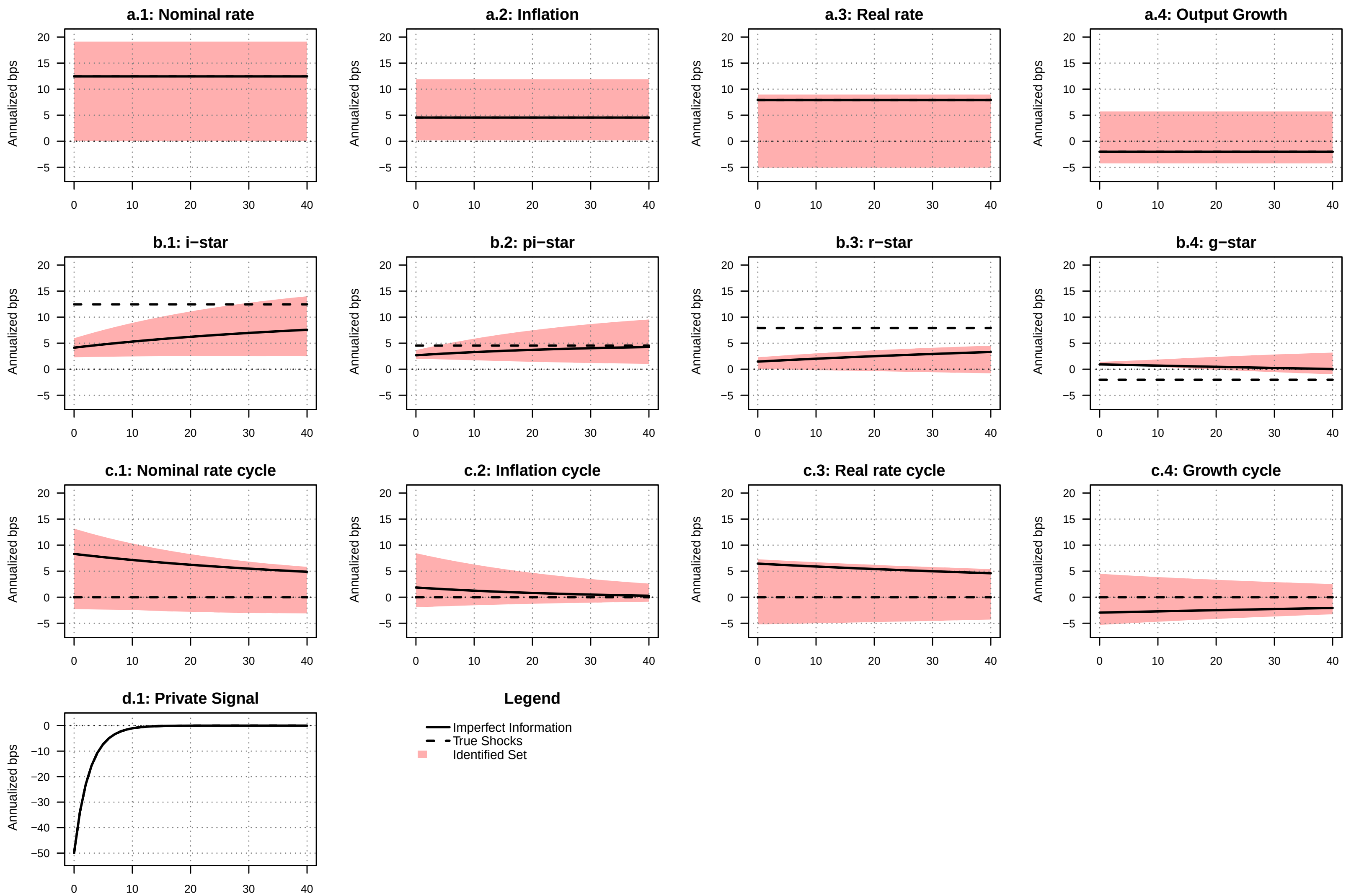
Notes: Panels a. and b. plot the implied real interest rate trend $r_t^\star$, and cycle $C_{r,t}$, respectively. The black solid line represents the one-sided estimates, that is the estimate obtained by investors at each date t with information up to that date (ex-ante). The teal dashed line represents the two-sided estimates that are obtained by investors by using the entire sample (ex-post). Both series are based on the **II** model. Their respective confidence bands are represented in orange and teal. All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure 7: Term premia decomposition in the imperfect information model



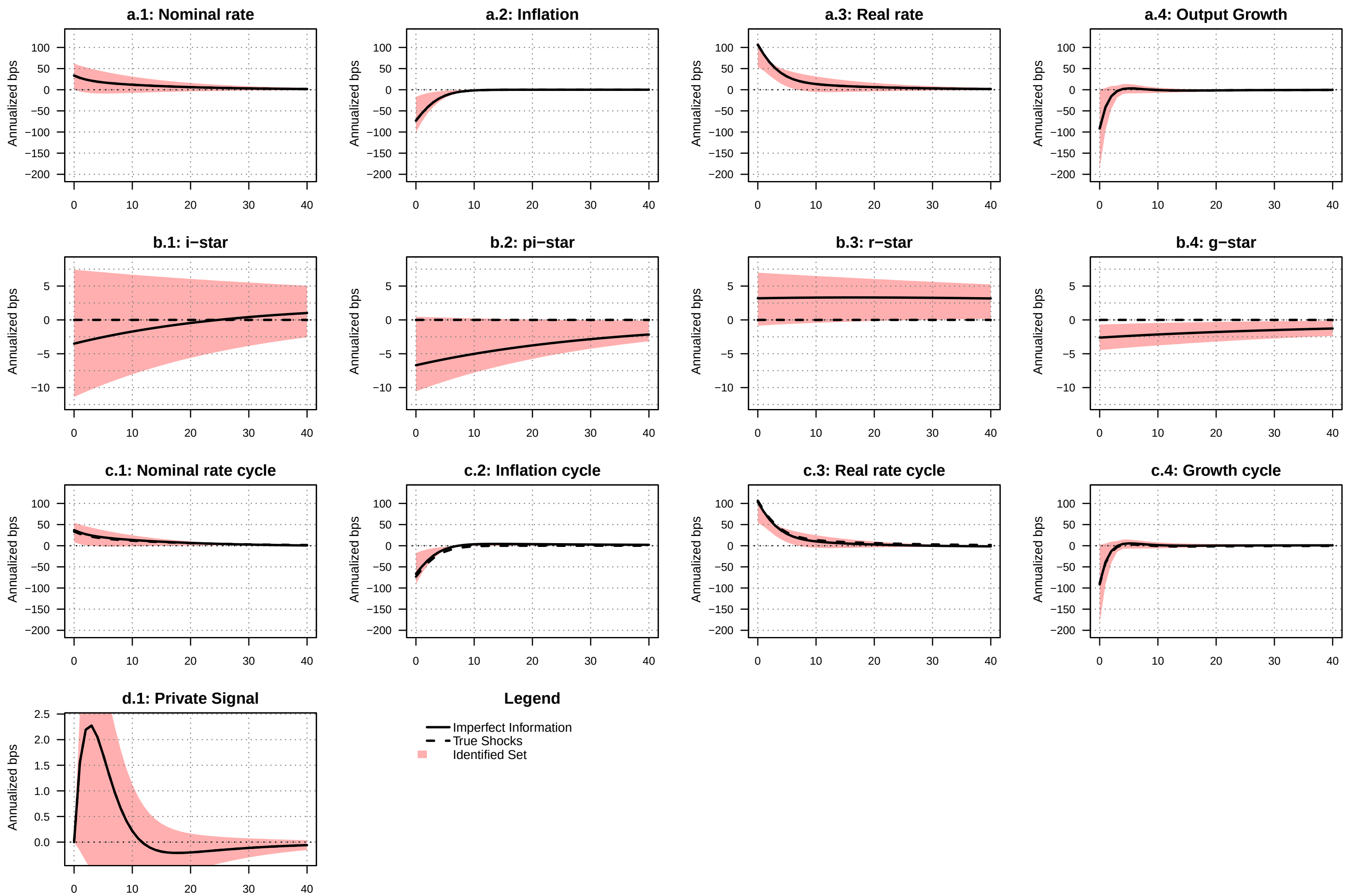
Notes: The figure presents the decomposition of the term structure of yields into the expected component and the term premium for various maturities, as implied by the model where the representative asset pricer has imperfect information. Maturities are 1-year (Panel a.1), 3-year (a.2), 5-year(b.1), and 10-year(b.2). Black solid lines represent the observed yield series, grey dashed lines are the expected component as implied by our model, and solid grey lines are the resulting term premia obtained as the difference between the observed yield and the expected component. All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure 8: IRF of Macroeconomic Variables to an Inflation Targeting Shock in Imperfect Information



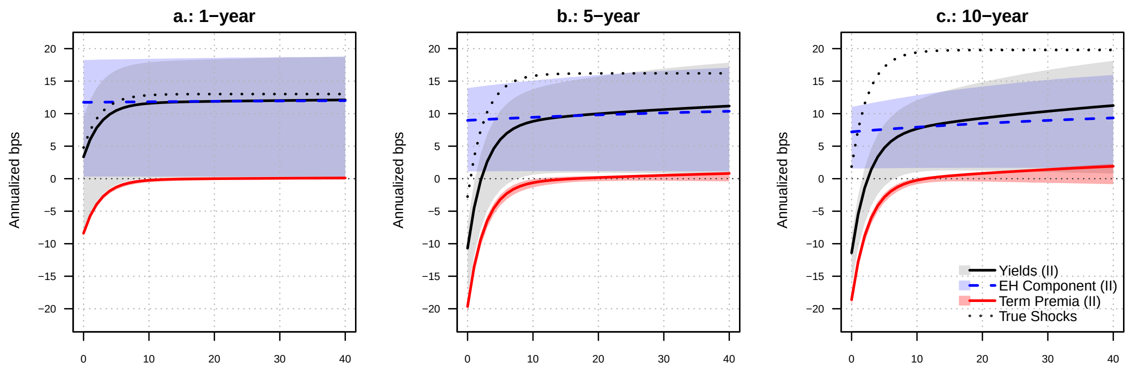
Notes: The figure illustrates the effects of a one standard deviation permanent monetary shock on macroeconomic variables (panels a.1, a.2, and a.3), their trends (panels b.1, b.2, and b.3), their cycles (panels c.1, c.2 and c.3), and the private signal factor (panel d.). The responses are plotted over 40 quarters. **II** stands for the model where the representative asset price has imperfect information. **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed. Red-shaded areas correspond to the range of the effects resulting from our identification based on sign-restriction.

Figure 9: IRF of Macroeconomic Variables to a Transitory Monetary Policy Shock in Imperfect Information



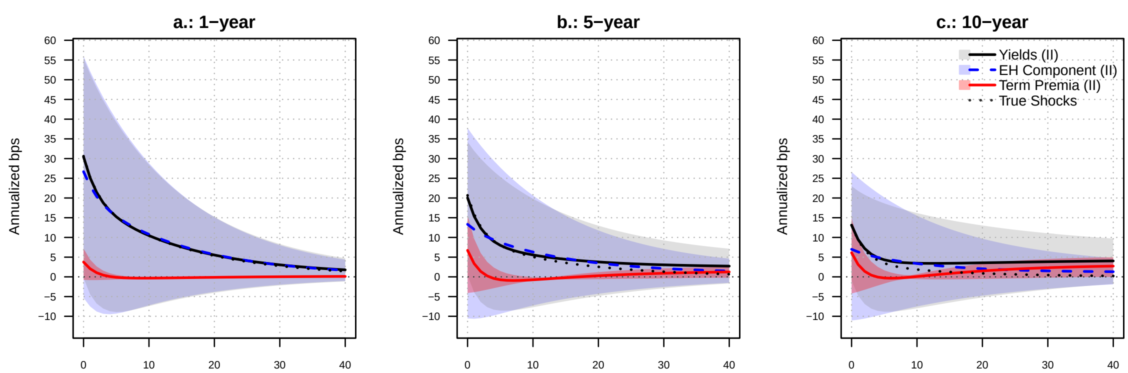
Notes: The figure illustrates the effects of a one standard deviation transitory monetary policy shock on macroeconomic variables (panels a.1, a.2, and a.3), their trends (panels b.1, b.2, and b.3), their cycles (panel c.1, c.2 and c.3), and the private signal factor (panel d.). The responses are plotted over 40 quarters. **II** model is the model where the representative asset price has imperfect information. **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.Red-shaded areas correspond to the range of the effects resulting from our identification based on sign-restriction.

Figure 10: IRF of yields to a permanent monetary shock in imperfect information



Notes: In this figure, we present the responses of 1-, 2-, 3-, 5-, and 7-year yields to a one standard deviation permanent monetary shock in the model where the representative asset pricer has imperfect information (**II**). In addition, we break down these effects into their expected component (EH **II** model) and their term premium (TP **II** model). **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.

Figure 11: IRF of yields to a transitory monetary policy shock in imperfect information



Notes: In this figure, we present the responses of 1-, 2-, 3-, 5-, and 7-year yields to a one standard deviation transitory monetary policy shock in the model where the representative asset pricer has imperfect information. In addition, we break down these effects into their expected component (EH **II** model) and their term premium (TP **II** model). **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.

10 Tables

Table 1: Interest rate forecast errors regressions

	3m TBill forecast errors: $e_{i,t} = \alpha + \beta \cdot e_{i,t+4} + \sum_{j=4}^{7} \gamma_j \cdot e_{i,t-j} + \eta_t$.							
	Realized minus SPF (1981:Q4-2022:Q4)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$e_{i,t+4}$	0.267**	0.281***	0.283***	0.273**	0.237*	0.248**	0.253**	0.217*
	(0.110)	(0.108)	(0.106)	(0.109)	(0.129)	(0.124)	(0.124)	(0.122)
$e_{\pi,t+4}$					0.046	0.036	0.036	0.033
					(0.059)	(0.055)	(0.052)	(0.045)
$e_{g,t+4}$					−0.006	0.009	0.010	0.010
					(0.041)	(0.040)	(0.040)	(0.034)
Controls: lag #	4	4:5	4:6	4:7	4	4:5	4:6	4:7
Observations	157	156	155	154	157	156	155	154
R^2	0.093	0.121	0.120	0.117	0.108	0.131	0.132	0.142

Notes: This table presents the regressions of 3m Tbill rate forecast errors as defined by realized value minus the corresponding SPF data fourth quarter prior, onto its own lags as controls and 4-quarter lead, denoted by $e_{i,t+4}$. Column 1-4 present the results of $e_{i,t} = \alpha + \beta \cdot e_{i,t+4} + \sum_{j=4}^{h} \gamma_j \cdot e_{i,t-j} + \eta_t$, for $h = 4, 5, 6, 7$, respectively. In columns 5-8, we run the same regressions adding the forecast errors on inflation and GDP growth, both as lags and leads. Standard errors are computed with Newey-West whose lag is automatically selected. Significance levels are indicated as * for 10%; ** for 5%; *** for 1%. Data is sampled at the quarterly level and covers 1981:Q1-2022:Q4.

Table 2: Local projections of forward rates onto permanent and transitory shocks

	Panel a: $\Delta_h f^{(1y)}_{t+h} = \alpha + \beta^{\star}_h \cdot \varepsilon^{\star}_{i,t} + \beta_{c,h} \cdot \varepsilon_{i,t} + \eta_{t+h}$						Panel b: $\Delta_h f^{(3y)}_{t+h} = \alpha + \beta^{\star}_h \cdot \varepsilon^{\star}_{i,t} + \beta_{c,h} \cdot \varepsilon_{i,t} + \eta_{t+h}$					
	Number of differenced quarters						Number of differenced quarters					
	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$
Transitory	0.581***	0.428***	0.428**	0.466**	0.548***	0.557**	0.271***	0.186*	0.227**	0.280**	0.275***	0.291*
	(0.040)	(0.155)	(0.177)	(0.195)	(0.201)	(0.279)	(0.042)	(0.096)	(0.104)	(0.119)	(0.096)	(0.158)
Permanent	0.529***	1.196***	1.298***	1.478***	1.692***	1.652***	0.404***	0.674***	0.816***	0.947***	1.115***	1.143***
	(0.141)	(0.185)	(0.208)	(0.254)	(0.249)	(0.353)	(0.106)	(0.162)	(0.127)	(0.170)	(0.189)	(0.209)
Observations	185	185	185	185	185	185	185	185	185	185	185	185
R^2	0.568	0.274	0.244	0.253	0.272	0.229	0.236	0.173	0.181	0.191	0.192	0.172

	Panel c: $\Delta_h f^{(5y)}_{t+h} = \alpha + \beta^{\star}_h \cdot \varepsilon^{\star}_{i,t} + \beta_{c,h} \cdot \varepsilon_{i,t} + \eta_{t+h}$						Panel d: $\Delta_h f^{(10y)}_{t+h} = \alpha + \beta^{\star}_h \cdot \varepsilon^{\star}_{i,t} + \beta_{c,h} \cdot \varepsilon_{i,t} + \eta_{t+h}$					
Transitory	0.145***	0.132***	0.071*	0.114*	0.187***	0.100	0.235*	0.158	0.076	−0.018	−0.142	−0.090
	(0.047)	(0.039)	(0.039)	(0.063)	(0.053)	(0.096)	(0.132)	(0.169)	(0.112)	(0.092)	(0.124)	(0.091)
Permanent	0.380***	0.665***	0.690***	0.817***	0.922***	0.943***	0.293*	0.659**	0.790**	0.718*	0.775*	0.821***
	(0.069)	(0.125)	(0.127)	(0.195)	(0.188)	(0.214)	(0.167)	(0.299)	(0.351)	(0.415)	(0.424)	(0.286)
Observations	185	185	185	185	185	185	185	185	185	185	185	185
R^2	0.137	0.185	0.136	0.146	0.170	0.142	0.093	0.087	0.076	0.049	0.056	0.057

Notes: This table presents local linear projections of forward variations onto proxies of permanent and transitory interest rate shocks. The regression writes:$\Delta_h f^{(m)}_{t+h} = \alpha + \beta^{\star}_h \cdot \varepsilon^{\star}_{i,t} + \beta_{c,h} \cdot \varepsilon_{i,t} + \eta_{t+h}$. Panels a and b consider 3m forwards starting in 9m and 33m, and panels c and d consider 3m forwards starting in 57m and 117m, respectively. Permanent and transitory shocks are computed by regression the one-quarter variation of the 3m TBill rate onto the one-quarter variation of the i-star series from Bauer and Rudebusch (2020). Fitted values are the permanent shocks while the residuals are the transitory shocks. Standard errors are computed from Newey-West with automatically selected lags. Significance levels are indicated as * for 10%; ** for 5%; *** for 1%. Data is sampled at the quarterly level and covers 1971:Q4-2018:Q1.

Table 3: Data summary statistics

	Panel (a): Macroeconomic variables		
	3-month yield (Annualized %)	Inflation (yoy %)	Real growth (yoy %)
Mean	4.447	3.727	2.946
Vol	3.250	2.711	2.377
$\rho(1)$	0.974	0.949	0.789

	Panel (b): Yields (in annualized %)					
Maturity	1-year	2-year	3-year	5-year	7-year	10-year
Mean	4.794	5.016	5.201	5.475	5.692	6.025
Vol	3.305	3.261	3.200	3.063	2.977	3.043
$\rho(1)$	0.975	0.979	0.980	0.982	0.983	0.984

	Panel (c): SPF forecasts						
	Interest rates		Inflation			Real growth	
Horizon	4 qtrs	10 yrs	4 qtrs	5 yrs	10 yrs	4 qtrs	10 yrs
Mean	4.090	3.511	2.919	2.343	2.604	2.846	2.678
Vol	2.975	1.018	1.116	0.373	0.432	1.041	0.378
$\rho(1)$	0.960	0.816	0.918	0.766	0.923	0.824	0.856

Notes: This table presents the descriptive statistics of the variables. The yield data is from Liu and Wu (2021). Inflation is the year-on-year log-change in the seasonally unadjusted CPI-U index and real growth is the year-on-year log-change in seasonally adjusted Real GDP. Both data are collected from the Federal Reserve Bank of St Louis website. Forecast values are from the Survey of Professional Forecasters (SPF) conducted by the Federal Reserve Bank of Philadelphia. SPF 4 qtrs represents expected macroeconomic values for the next four quarters. SPF 5 yrs and 10 yrs are the mean expected quarterly macroeconomic values over the next 20 quarters (5 years) and 40 quarters (10 years), respectively. Mean and Vol present the average and volatility of the series, while $\rho(1)$ is the first order autocorrelation. All values are in annualized percentages. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Table 4: Parameters estimates for the **II** model

a. Autoregressive matrix of cycles and private signal

	$C_{i,t}$	$C_{\pi,t}$	$C_{g,t}$	f_t
$C_{i,t}$	0.940	0.030	0.013	
	(0.005)	(0.010)	(0.007)	
$C_{\pi,t}$	0.015	0.700	0.060	
	(0.011)	(0.014)	(0.017)	
$C_{g,t}$	-0.094	-0.191	0.568	
	(0.007)	(0.011)	(0.010)	
f_t	-0.010	-0.024	-0.001	0.680
	(0.044)	(0.031)	(0.016)	(0.029)

b. Covariance matrix (Σ) and correlation matrix (Q) of factors

	$i_t^\star$	$\pi_t^\star$	$g_t^\star$	$C_{i,t}$	$C_{\pi,t}$	$C_{g,t}$	f_t
$i_t^\star$	0.037	**0.911**	**0.408**				**-0.448**
	(0.006)						
$\pi_t^\star$	0.023	0.017	**0.535**				**-0.516**
	(0.004)	(0.003)					
$g_t^\star$	0.004	0.004	0.003				**-0.600**
	(0.001)	(0.001)	(0.001)				
$C_{i,t}$				0.457	**0.150**	**0.330**	
				(0.031)			
$C_{\pi,t}$				0.104	1.046	**0.457**	
				(0.041)	(0.113)		
$C_{g,t}$				0.445	0.934	3.993	
				(0.068)	(0.176)	(0.285)	
f_t	-0.112	-0.089	-0.045				1.709
	(0.076)	(0.035)	(0.009)				

c. Kalman gain ($\overline{\mathcal{K}}$)

	$i_{t\|t}^\star$	$\pi_{t\|t}^\star$	$g_{t\|t}^\star$	$C_{i,t\|t}$	$C_{\pi,t\|t}$	$C_{g,t\|t}$	f_t
i_t	0.151	0.036	-0.025	0.849	-0.036	0.025	-0.000
π_t	0.101	0.095	-0.008	-0.101	0.905	0.008	-0.000
g_t	0.013	0.010	0.026	-0.013	-0.010	0.974	0.000
f_t	-0.037	-0.036	-0.026	0.037	0.036	0.026	1.000

d. Investor's uncertainty ($\overline{\mathcal{P}}$, correlations in bold)

	$i_t^\star$	$\pi_t^\star$	$g_t^\star$	$C_{i,t}$	$C_{\pi,t}$	$C_{g,t}$	f_t
$i_t^\star$	1.267	**0.680**	**0.045**	**-1.000**	**-0.680**	**-0.045**	**0.000**
$\pi_t^\star$	0.475	0.385	**0.289**	**-0.680**	**-1.000**	**-0.289**	**0.000**

$g_t^\star$	0.020	0.071	0.159	**-0.045**	**-0.289**	**-1.000**	**0.000**
$C_{i,t}$	-1.267	-0.475	-0.020	1.267	**0.680**	**0.045**	**0.000**
$C_{\pi,t}$	-0.475	-0.385	-0.071	0.475	0.385	**0.289**	**0.000**
$C_{g,t}$	-0.020	-0.071	-0.159	0.020	0.071	0.159	**0.000**
f_t	-0.000	-0.000	-0.000	0.000	0.000	0.000	0.000

e. Covariance matrix of investor's forecasting errors ($\overline{\mathcal{V}}$, correlations in bold)

	i_t	π_t	g_t	f_t
i_t	0.497	**0.178**	**0.321**	**-0.121**
π_t	0.131	1.092	**0.462**	**-0.062**
g_t	0.457	0.975	4.081	**-0.015**
f_t	-0.111	-0.085	-0.039	1.709

f. Prices of risk

$\Lambda = \alpha\beta'$

	$i^\star_{t\|t}$	$\pi^\star_{t\|t}$	$g^\star_{t\|t}$	$C_{i,t\|t}$	$C_{\pi,t\|t}$	$C_{g,t\|t}$	f_t	α
i_{t+1}	0.338	-0.346	-0.057	-0.183	-0.170	-0.209	1.000	1.000
	(0.128)	(0.158)	(0.049)	(0.154)	(0.162)	(0.089)	(0.000)	(0.000)
π_{t+1}	-0.044	0.045	0.007	0.024	0.022	0.027	-0.130	-0.130
	(0.272)	(0.280)	(0.045)	(0.147)	(0.138)	(0.175)	(0.813)	(0.813)
g_{t+1}	-0.257	0.263	0.043	0.140	0.129	0.159	-0.760	-0.760
	(0.136)	(0.154)	(0.043)	(0.118)	(0.133)	(0.071)	(0.212)	(0.212)
f_{t+1}	0	0	0	0	0	0	0	0
β	0.338	-0.346	-0.057	-0.183	-0.170	-0.209	1.000	
	(0.128)	(0.158)	(0.049)	(0.154)	(0.162)	(0.089)		

g. RMSE of yields (in annualized bps)

3-mth	1-year	2-year	3-year	5-year	7-year	10-year
0.000	18.760	17.650	14.227	9.017	13.102	24.909

Notes: This table presents the parameter estimates from the **II** model where the representative asset pricer has imperfect information. Panels a. and b. provide the structure of the factors and their covariance matrix. Panels c., d., and e. gives the learning structure within the model. Panels f. and g. show the coefficients of the prices of risks and the pricing errors of the yields, respectively. In the prices of risk specification (panel f.), we impose the normalization that f_t enters with a scale of 1 such that the standard deviation of f_t is identified.

Additionally, we impose that the price of risk enters is exactly that of the nominal interest rate i_{t+1} to identify the scale of α and β. All values in bold are correlations.

Table 5: Identification structure for the II model

a. Structural economic shocks ($\sqrt{\Sigma}$)

	Permanent shocks			Transitory shocks			
	demand	supply	π target	demand	supply	monetary	risk premia
$i_t^\star$	0.143	-0.026	0.125				
$\pi_t^\star$	0.111	-0.054	0.045				
$g_t^\star$	0.052	0.014	-0.020				
$C_{i,t}$				0.531	-0.250	0.335	
$C_{\pi,t}$				0.366	-0.616	-0.730	
$C_{g,t}$				1.682	0.569	-0.916	
f_t							1.000

b. Median matrix (element by element)

	Permanent shocks			Transitory shocks			
	demand	supply	π target	demand	supply	monetary	risk premia
$i_t^\star$	0.152	-0.026	0.109				
$\pi_t^\star$	0.101	-0.052	0.060				
$g_t^\star$	0.046	0.017	0.001				
$C_{i,t}$				0.553	-0.138	0.298	
$C_{\pi,t}$				0.368	-0.577	-0.701	
$C_{g,t}$				1.492	0.615	-0.935	
f_t							1.000

Distance to the median matrix

0.057

Notes: This table presents the identification of structural economic shocks in the **II** model. Panel a. gives the mapping of the seven structural shocks to those of the factors as identified by sign restrictions. This is the matrix that is the closest to the one where the median value is taken element by element across the set-identified matrices. The latter is presented on panel b. The identified set is obtained by drawing 1000 rotation matrices and accepting the ones who fit the sign restrictions.

A Appendix

A.1 Proofs for the simple model

Following the dynamics of Equation (1), we can write the subjective expectation of the interest rate as:

$$\mathbb{E}_{t-1}(i_t) = \mathbb{E}_{t-1}\left(i^\star_{t-1} + \varphi \cdot C_{i,t-1}\right) = i^\star_{t-1|t-1} + \varphi\left(i_{t-1} - i^\star_{t-1|t-1}\right) . \tag{A.1}$$

Thus, the forecast errors are given by:

$$\begin{aligned}
e_{i,t|t-1} &= i_t - \mathbb{E}_{t-1}(i_t) \\
&= i^\star_{t-1} + \varepsilon^\star_{i,t} + \varphi \cdot C_{i,t-1} + \varepsilon_{i,t} - i^\star_{t-1|t-1} - \varphi\left(i_{t-1} - i^\star_{t-1|t-1}\right) \\
&= \varepsilon^\star_{i,t} + \varepsilon_{i,t} + i_{t-1} - (1-\varphi)C_{i,t-1} - i^\star_{t-1|t-1} - \varphi\left(i_{t-1} - i^\star_{t-1|t-1}\right) \\
&= \varepsilon^\star_{i,t} + \varepsilon_{i,t} + i_{t-1} - (1-\varphi)\left(i_{t-1} - i^\star_{t-1}\right) - i^\star_{t-1|t-1} - \varphi\left(i_{t-1} - i^\star_{t-1|t-1}\right) \\
&= \varepsilon^\star_{i,t} + \varepsilon_{i,t} + (1-\varphi)i^\star_{t-1} - i^\star_{t-1|t-1} + \varphi \cdot i^\star_{t-1|t-1} \\
&= \underbrace{\varepsilon^\star_{i,t} + \varepsilon_{i,t}}_{=:e_{i,t}} + (1-\varphi)\left(i^\star_{t-1} - i^\star_{t-1|t-1}\right) .
\end{aligned} \tag{A.2}$$

We now look at the conditional covariance between the forecast errors.

$$\begin{aligned}
& Cov_{t-1}\left(e_{i,t|t-1},\ e_{i,t+1|t}\right) \\
=\ & Cov_{t-1}\left[e_{i,t} + (1-\varphi)\left(i^\star_{t-1} - i^\star_{t-1|t-1}\right),\ e_{i,t+1} + (1-\varphi)\left(i^\star_t - i^\star_{t|t}\right)\right] \\
=\ & (1-\varphi)Cov_{t-1}\left(e_{i,t},\ i^\star_t - i^\star_{t|t}\right) + (1-\varphi)^2 Cov_{t-1}\left(i^\star_{t-1} - i^\star_{t-1|t-1},\ i^\star_t - i^\star_{t|t}\right) \\
=\ & (1-\varphi)\left[Cov_{t-1}\left(e_{i,t},\ i^\star_t - i^\star_{t|t}\right) + (1-\varphi)Cov_{t-1}\left(i^\star_{t-1} - i^\star_{t-1|t-1},\ i^\star_t - i^\star_{t|t}\right)\right]
\end{aligned} \tag{A.3}$$

We have that the filtering errors are recursive:

$$\begin{aligned}
i^\star_t - i^\star_{t|t} &= i^\star_{t-1} + \varepsilon^\star_{i,t} - i^\star_{t-1|t-1} - \mathcal{K}^\star e_{i,t|t-1} \\
&= i^\star_{t-1} + \varepsilon^\star_{i,t} - i^\star_{t-1|t-1} - \mathcal{K}^\star\left[e_{i,t} + (1-\varphi)\left(i^\star_{t-1} - i^\star_{t-1|t-1}\right)\right] \\
&= \left(1 - \mathcal{K}^\star + \mathcal{K}^\star\varphi\right)\left(i^\star_{t-1} - i^\star_{t-1|t-1}\right) + \varepsilon^\star_{i,t} - \mathcal{K}^\star e_{i,t} .
\end{aligned} \tag{A.4}$$

Therefore:

$$
\begin{aligned}
& Cov_{t-1}\left(e_{i,t|t-1},\, e_{i,t+1|t}\right) \\
= & \Big[(1-\varphi)Cov_{t-1}\left(e_{i,t},\, (1-\mathcal{K}^\star+\mathcal{K}^\star\varphi)\left(i^\star_{t-1}-i^\star_{t-1|t-1}\right)+\varepsilon^\star_{i,t}-\mathcal{K}^\star e_{i,t}\right) \\
& +(1-\varphi)^2 Cov_{t-1}\left(i^\star_{t-1}-i^\star_{t-1|t-1},\, (1-\mathcal{K}^\star+\mathcal{K}^\star\varphi)\left(i^\star_{t-1}-i^\star_{t-1|t-1}\right)+\varepsilon^\star_{i,t}-\mathcal{K}^\star e_{i,t}\right)\Big] \\
= & (1-\varphi)Cov_{t-1}\left(e_{i,t},\, \varepsilon^\star_{i,t}-\mathcal{K}^\star e_{i,t}\right) \\
& +(1-\varphi)^2 Cov_{t-1}\left(i^\star_{t-1}-i^\star_{t-1|t-1},\, (1-\mathcal{K}^\star+\mathcal{K}^\star\varphi)\left(i^\star_{t-1}-i^\star_{t-1|t-1}\right)\right) \\
= & (1-\varphi)\left[(1-\mathcal{K}^\star)\,\sigma^{\star^2}-\mathcal{K}^\star\sigma_c^2+(1-\varphi)\,(1-\mathcal{K}^\star+\varphi\mathcal{K}^\star)\,\mathbb{V}_{t-1}\left(i^\star_{t-1}-i^\star_{t-1|t-1}\right)\right] \qquad \text{(A.5)}
\end{aligned}
$$

If the information set is such that $i^\star_{t-1}$ is observed as of $t-1$, then the second term vanishes. If the information set is the same as that of the investor, then the variance is equal to what we denote by $\overline{\mathcal{P}}$ in the model, that is the variance of the filtering error associated with the trend.

It is useful to derive the stationary filter quantities analytically. An application of Equation (13) provides these quantities. It is first useful to note that in the simple model case, the Kalman gain associated with the cycle is equal to $1-\mathcal{K}^\star$ such that the interest rate i_t is perfectly observed. Then, we can show that $\overline{\mathcal{P}}$ has the same four entries in absolute value, and the anti-diagonal has a minus sign. We denote by a the unknown parameter in $\overline{\mathcal{P}}$. Applying the recursions, we have:

$$\overline{\mathcal{V}} = a(1-\varphi)^2+\sigma^{\star^2}+\sigma_c^2\,, \qquad \text{(A.6)}$$

and

$$\mathcal{K}^\star = \frac{1}{\overline{\mathcal{V}}}\left[(1-\varphi)a+\sigma^{\star^2}\right]. \qquad \text{(A.7)}$$

Applying the recursion for $\overline{\mathcal{P}}$, we can show:

$$a = (1-\mathcal{K}^\star)(a+\sigma^{\star^2})+\mathcal{K}^\star\varphi a\,. \qquad \text{(A.8)}$$

Developing the expressions, we find:

$$a^2(1-\varphi)^2+a\sigma^{\star^2}(1-\varphi^2)-\sigma^{\star^2}\sigma_c^2=0\,. \qquad \text{(A.9)}$$

Solving the equation, we find:

$$a = \frac{\sigma^{\star^2}(1+\varphi)}{2(1-\varphi)}\left[\sqrt{1+\frac{4\sigma_c^2}{\sigma^{\star^2}(1+\varphi)^2}}-1\right]. \tag{A.10}$$

Therefore:

$$\mathcal{K}^{\star} = \frac{1+\dfrac{1+\varphi}{2}\left[\sqrt{1+\dfrac{4\sigma_c^2}{\sigma^{\star^2}(1+\varphi)^2}}-1\right]}{\dfrac{\sigma_c^2}{\sigma^{\star^2}}+1+\dfrac{1-\varphi^2}{2}\left[\sqrt{1+\dfrac{4\sigma_c^2}{\sigma^{\star^2}(1+\varphi)^2}}-1\right]} \tag{A.11}$$

A.2 Risk-neutral dynamics

The SDF defines a standard change of measure such that the dynamics of our system are easily expressed under the risk-neutral measure. In particular, the covariance of the system stays the same, and the conditional mean is shifted. The risk-neutral dynamic is the following:

$$\begin{bmatrix} X_{t|t} \\ Y_t \end{bmatrix} \mid \underline{Y_{t-1}} \overset{\mathbb{Q}}{=} \begin{pmatrix} \overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\lambda \\ \overline{\mathcal{V}}\lambda \end{pmatrix} + \begin{bmatrix} \Phi + \overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\Lambda & \mathbf{0} \\ B\Phi + \overline{\mathcal{V}}\,\Lambda & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{t-1|t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} \overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\overline{\mathcal{K}}' & \overline{\mathcal{K}}\,\overline{\mathcal{V}} \\ \overline{\mathcal{V}}'\,\overline{\mathcal{K}}' & \overline{\mathcal{V}} \end{bmatrix}^{1/2} \xi_t^{\mathbb{Q}},$$

$$Z_t \mid \underline{Y_{t-1}} \overset{\mathbb{Q}}{=} F^{\mathbb{Q}} + G^{\mathbb{Q}}\, Z_{t-1} + H^{1/2}\,\xi_t^{\mathbb{Q}}, \quad \text{where} \quad \xi_t^{\mathbb{Q}} \overset{\mathbb{Q}}{\sim} \mathcal{N}(\mathbf{0}, I). \tag{A.12}$$

To derive the proof, we start with the joint distribution in equation 14:

$$\begin{pmatrix} X_{t|t} \\ Y_t \end{pmatrix} \mid Y_{t-1} = \begin{pmatrix} 0 \\ A \end{pmatrix} + \begin{pmatrix} \Phi & 0 \\ B\Phi & 0 \end{pmatrix} \begin{pmatrix} X_{t-1|t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} \overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\overline{\mathcal{K}}' & \overline{\mathcal{K}}\,\overline{\mathcal{V}} \\ \overline{\mathcal{V}}\,\overline{\mathcal{K}}' & \overline{\mathcal{V}} \end{pmatrix}^{1/2} \xi_t$$

$$Z_t = F + GZ_{t-1} + H^{1/2}\xi_t, \quad \text{where} \quad \xi_t \sim \mathcal{N}(\mathbf{0}, I).$$

Since it is Gaussian, the conditional physical Moment Generating function (MGF) is :

$$\begin{aligned}
\varphi^{\mathbb{P}}_{Z,t}(u,v) &\equiv \mathbb{E}_t\left(e^{u'X_{t+1|t+1}\ +\ v'Y_{t+1}}\right) \\
&= \exp\left(u'\Phi X_{t|t} + v'B\Phi X_{t|t} + \frac{1}{2}\left(u'\ \overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\overline{\mathcal{K}}'u + 2u'\overline{\mathcal{K}}\,\overline{\mathcal{V}}v + v'\overline{\mathcal{V}}v\right)\right)
\end{aligned}$$

Using the change in measure, the risk-neutral MGF is :

$$\begin{aligned}
\varphi^{\mathbb{Q}}_{Z,t}(u,v) &= \mathbb{E}^{\mathbb{Q}}_t\left(e^{u'X_{t+1|t+1}\ +\ v'Y_{t+1}}\right) \\
&= \frac{\mathbb{E}^{\mathbb{P}}_t\left\{\exp\left(u'X_{t+1|t+1} + (v+\lambda_t)'Y_{t+1}\right)\right\}}{\varphi^{\mathbb{P}}_{Y,t}(\lambda_t)} \\
&= \exp\Big\{u'\Phi X_{t|t} + (v+\lambda_t)'B\Phi X_{t|t} - \lambda_t'B\Phi X_{t|t} \\
&\quad + \frac{1}{2}\left[u'\overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\overline{\mathcal{K}}'u + 2u'\overline{\mathcal{K}}\,\overline{\mathcal{V}}(v+\lambda_t) + (v+\lambda_t)'\,\overline{\mathcal{V}}\,(v+\lambda_t) - \lambda_t'\overline{\mathcal{V}}\lambda_t\right]\Big\} \\
&= \exp\Big\{u'\left[\overline{\mathcal{K}}\,\overline{\mathcal{V}}\lambda + (\Phi + \overline{\mathcal{K}}\,\overline{\mathcal{V}}\Lambda)X_{t|t}\right] + v'\left[\overline{\mathcal{V}}\lambda + (B\Phi + \overline{\mathcal{V}}\Lambda)X_{t|t}\right] \\
&\quad + \frac{1}{2}\ \left(u'\overline{\mathcal{K}}\,\overline{\mathcal{V}}\,\overline{\mathcal{K}}'u + 2u'\overline{\mathcal{K}}\,\overline{\mathcal{V}}v + v'\overline{\mathcal{V}}v\right)\Big\}
\end{aligned}$$

The last equation gives the risk-neutral dynamic:

A.3 Pricing formulas

$$
\begin{aligned}
B_t^{(h)} &= \mathbb{E}^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{h-1} i_{t+j}\right) \mid \underline{Y_t}\right] = \exp\left(\mathcal{A}_h + \mathcal{B}_h' X_{t|t}\right) \\
&= \mathbb{E}_t^{\mathbb{Q}}\left(e^{-i_t} B_{t+1}^{(h-1)}\right) \\
&= \mathbb{E}_t^{\mathbb{Q}}\left(\exp\left(-i_t + \mathcal{A}_{h-1} + \mathcal{B}_{h-1}' X_{t+1|t+1}\right)\right) \\
&= \exp\left\{-i_t + \mathcal{A}_{h-1} + \mathcal{B}_{h-1}'\left(F_X^{\mathbb{Q}} + G_X^{\mathbb{Q}} X_{t|t}\right) + \frac{1}{2}\mathcal{B}_{h-1}' H_{XX} \mathcal{B}_{h-1}\right\} \\
&= \exp\left\{\mathcal{A}_{h-1} + \mathcal{B}_{h-1}' F_X^{\mathbb{Q}} + \frac{1}{2}\mathcal{B}_{h-1}' H_{XX} \mathcal{B}_{h-1} + \mathcal{B}_{h-1}' G_X^{\mathbb{Q}} X_{t|t} - i_t\right\} \\
&= \exp\left\{\mathcal{A}_{h-1} + \mathcal{B}_{h-1}' F_X^{\mathbb{Q}} + \frac{1}{2}\mathcal{B}_{h-1}' H_{XX} \mathcal{B}_{h-1} + (G_X^{\mathbb{Q}\prime} \mathcal{B}_{h-1} - u_i)' X_{t|t}\right\} \\
&= \exp\left(\mathcal{A}_h + \mathcal{B}_h' X_{t|t}\right) \\
\text{where } \mathcal{A}_h &= \mathcal{A}_{h-1} + \mathcal{B}_{h-1}' F_X^{\mathbb{Q}} + \frac{1}{2}\mathcal{B}_{h-1}' H_{XX} \mathcal{B}_{h-1}, \quad \mathcal{A}_0 = 0 \\
\mathcal{B}_h &= G' \mathcal{B}_{h-1} - u_i, \quad \mathcal{B}_0 = \mathbf{0} \\
u_i &= (1, 0, 0, 1, 0, 0, 0)'
\end{aligned}
$$

where $F_X^{\mathbb{Q}}$ is the top block of $F^{\mathbb{Q}}$, $G_X^{\mathbb{Q}}$ is the top-left block of $G^{\mathbb{Q}}$, and H_{XX} is the top-left block of H.

A.4 Impulse Response Function in the II and PI models

For the **II** model, the impulse response on the factors are obtained through the observed effects on macroeconomic variables:

$$\begin{aligned} Y_t &= BX_t \\ X_t &= \Phi X_{t-1} + \sqrt{\Omega}\,\varepsilon_t \end{aligned}$$

The impact of a structural shock e_i at period t on the macroeconomic variables is :

$$\begin{aligned} Y^s_{t,h} &= \mathbb{E}_t\left[Y_{t+h}|\varepsilon_t = e_i, \underline{Y_{t-1}}\right] = B\Phi^{h-1}\sqrt{\Omega}\,e_i \\ Y^{ns}_{t,h} &= \mathbb{E}_t\left[Y_{t+h}|\varepsilon_t = 0, \underline{Y_{t-1}}\right] = 0 \\ IRF^i_{h,t}(Y_t) &= Y^s_{t,h} - Y^{ns}_{t,h} \end{aligned}$$

The learning structure of the representative asset pricer about the factors is:

$$\begin{aligned} X_{t|t} &= \Phi X_{t-1|t-1} + \overline{\mathcal{K}}\xi_t = \left(I - \overline{\mathcal{K}}B\right)\Phi X_{t-1|t-1} + \overline{\mathcal{K}}Y_t \\ X_{t+h|t+h} &= \left[\left(I - \overline{\mathcal{K}}B\right)\Phi\right]^{h+1} X_{t-1|t-1} + \sum_{i=1}^{h}\left(I - \overline{\mathcal{K}}B\right)^{h-i}\overline{\mathcal{K}}Y_{t+i} \end{aligned}$$

Using the learning, the investor guesses the effect of the shock on the factors:

$$\begin{aligned} X^s_{t+h|t+h} &= \left[\left(I - \overline{\mathcal{K}}B\right)\Phi\right]^{h+1} X_{t-1|t-1} + \sum_{i=1}^{h}\left(I - \overline{\mathcal{K}}B\right)^{h-i}\Phi^{h-i}\overline{\mathcal{K}}Y^s_{t,i} \\ X^{ns}_{t+h|t+h} &= \left[\left(I - \overline{\mathcal{K}}B\right)\Phi\right]^{h+1} X_{t-1|t-1} \end{aligned}$$

$$\begin{aligned} IRF^i_{h,t}(X_{t|t}) &= \mathbb{E}_t\left[X_{t+h}|\underline{Y^s_{t,h}}, \underline{Y_{t-1}}\right] - \mathbb{E}_t\left[X_{t+h}|\underline{Y^{ns}_{t,h}}, \underline{Y_{t-1}}\right] \\ &= X^s_{t+h|t+h} - X^{ns}_{t+h|t+h} \\ &= \sum_{i=1}^{h}\left(I - \overline{\mathcal{K}}B\right)^{h-i}\overline{\mathcal{K}}Y^s_{t,i} \end{aligned}$$

The effects on the yields follows as:

$$
\begin{aligned}
R_t^{(n)} &= -\frac{1}{n}[A_n + B'_{X,n}X_{t|t} + B'_{Y,n}Y_t] \\
IRF^i_{h,t}(R_t^{(n)}) &= -\frac{1}{n}[B'_{X,n}IRF^i_{h,t}(X_{t|t}) + B'_{Y,n}IRF^i_{h,t}(Y_t)]
\end{aligned}
$$

For the **PI** model, since the factors are observed, the effect of a structural shock e_i at period t are derived directly. The response of the factors are:

$$
\begin{aligned}
X_t &= \Phi X_{t-1} + \sqrt{\Omega}\,\varepsilon_t \\
X_{t+h} &= \Phi^{h+1}X_{t-1} + \sum_{i=0}^{h}\Phi^{h-i}\sqrt{\Omega}\,\varepsilon_{t+i} \\
IRF^i_{h,t}(X_t) &= \mathbb{E}[X_{t+h}|\varepsilon_t = e_i, \underline{X_{t-1}}] - \mathbb{E}[X_{t+h}|\varepsilon_t = 0, \underline{X_{t-1}}] \\
IRF^i_{h,t}(X_t) &= \Phi^h\sqrt{\Omega}\,e_i
\end{aligned}
$$

The IRF of macroeconomic variables are:

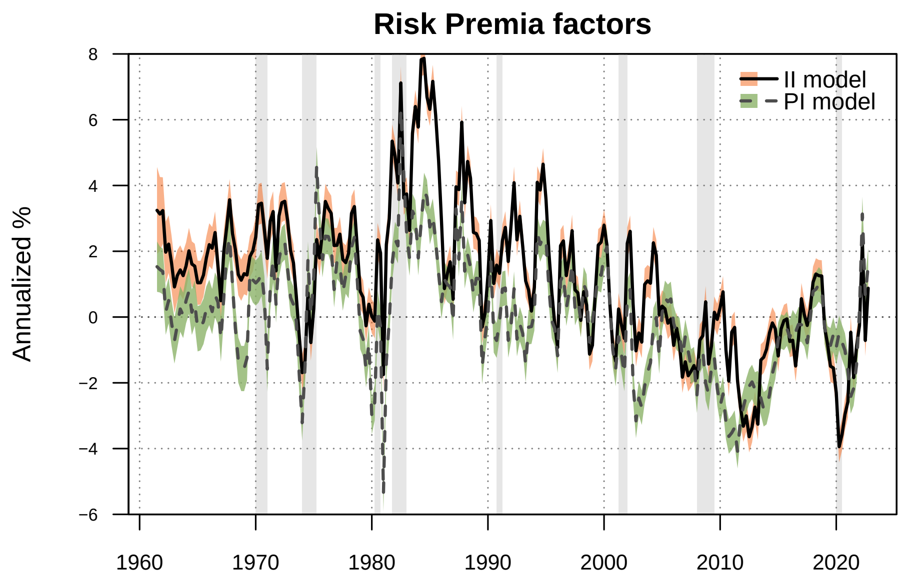
$$
\begin{aligned}
Y_t &= BX_t \\
IRF^i_{h,t}(Y_t) &= B \cdot IRF^i_{h,t}(X_t)
\end{aligned}
$$

For the yields, the effects are:

$$
\begin{aligned}
R_t^{(n)} &= -\frac{1}{n}[A_n + B'_{X,n}X_t + B'_{Y,n}Y_t] \\
IRF^i_{h,t}(R_t^{(n)}) &= -\frac{1}{n}[B'_{X,n}IRF^i_{h,t}(X_t) + B'_{Y,n}IRF^i_{h,t}(Y_t)]
\end{aligned}
$$

A.5 Supplementary Tables and Figures

Figure A.1: Filtered private signal factor across the two models



Notes: The figure presents the filtered private signal factor across the two models. **II** (**PI**) stands for the model where the representative asset pricer has imperfect information (perfect information). The shaded areas around those variables are the 95% confidence interval as perceived by the representative agent (for the **II** model) or by the econometrician (for the **PI** model). Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Table A.1: Parameters estimates for the **PI** model

a. Autoregressive matrix of cycles and private signal

	$C_{i,t}$	$C_{\pi,t}$	$C_{g,t}$	f_t
$C_{i,t}$	0.955	-0.016	0.036	
	(0.007)	(0.011)	(0.019)	
$C_{\pi,t}$	0.043	0.737	0.001	
	(0.028)	(0.026)	(0.026)	
$C_{g,t}$	-0.172	-0.071	0.478	
	(0.021)	(0.036)	(0.041)	
f_t	0.004	-0.028	-0.012	0.836
	(0.024)	(0.016)	(0.027)	(0.028)

b. Covariance matrix (Σ) and correlation matrix (Q) of factors

	$i_t^\star$	$\pi_t^\star$	$g_t^\star$	$C_{i,t}$	$C_{\pi,t}$	$C_{g,t}$	f_t
$i_t^\star$	0.070	**0.382**	**0.719**				
	(0.000)						
$\pi_t^\star$	0.012	0.015	**-0.136**				
	(0.012)	(0.008)					
$g_t^\star$	0.077	-0.007	0.163				
	(0.013)	(0.022)	(0.030)				
$C_{i,t}$				0.446	**0.325**	**0.295**	
				(0.042)			
$C_{\pi,t}$				0.192	0.781	**0.227**	
				(0.058)	(0.076)		
$C_{g,t}$				0.310	0.317	2.489	
				(0.107)	(0.145)	(0.251)	
f_t							1.220

c. Prices of risk

	Λ							
	$i_{t\|t}^\star$	$\pi_{t\|t}^\star$	$g_{t\|t}^\star$	$C_{i,t\|t}$	$C_{\pi,t\|t}$	$C_{g,t\|t}$	f_t	α
i_t	1.143	-0.638	-0.925	-0.276	0.332	-0.194	1.000	1.000
	(0.600)	(0.557)	(0.457)	(0.137)	(0.122)	(0.139)	(0.000)	(0.000)
π_t	-1.923	1.074	1.556	0.464	-0.559	0.327	-1.682	-1.682
	(1.034)	(0.881)	(0.780)	(0.313)	(0.224)	(0.302)	(0.821)	(0.821)
g_t	0.132	-0.074	-0.107	-0.032	0.038	-0.022	0.116	0.116
	(0.459)	(0.255)	(0.368)	(0.118)	(0.134)	(0.087)	(0.424)	(0.424)
β	1.143	-0.638	-0.925	-0.276	0.332	-0.194	1.000	

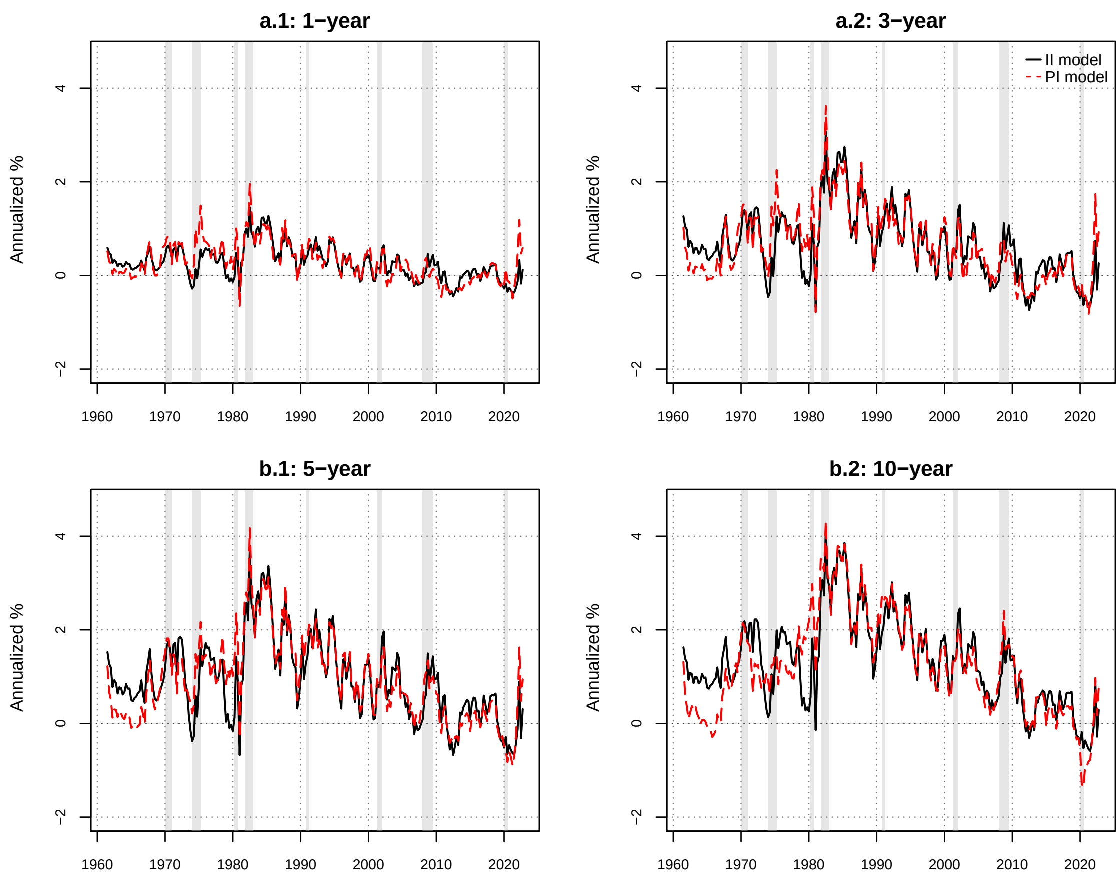
(0.600)	(0.557)	(0.457)	(0.137)	(0.122)	(0.139)	

d. RMSE of yields (in annualized bps)

3-mth	1-year	2-year	3-year	5-year	7-year	10-year
0.000	11.698	4.993	5.162	8.808	6.129	7.306

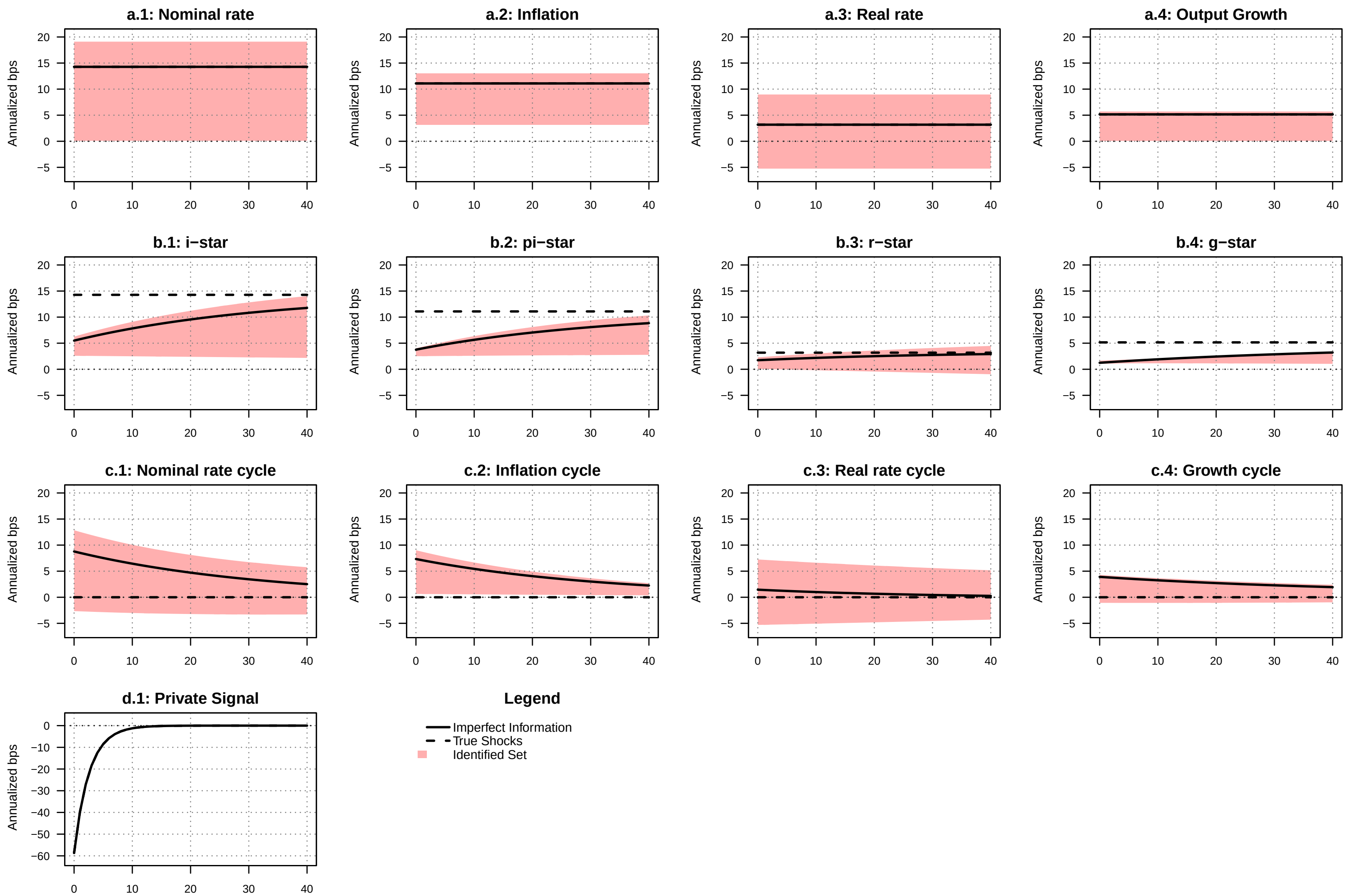
Notes: This table presents the parameter estimates from the **PI** model where the representative asset pricer has perfect information. Panels a. and b. provide the structure of the factors and their covariance matrix. Panel c. and d. shows the structure of the prices of risks and the pricing errors of yields, respectively. All values in bold are correlations.

Figure A.2: Term premia decomposition across models



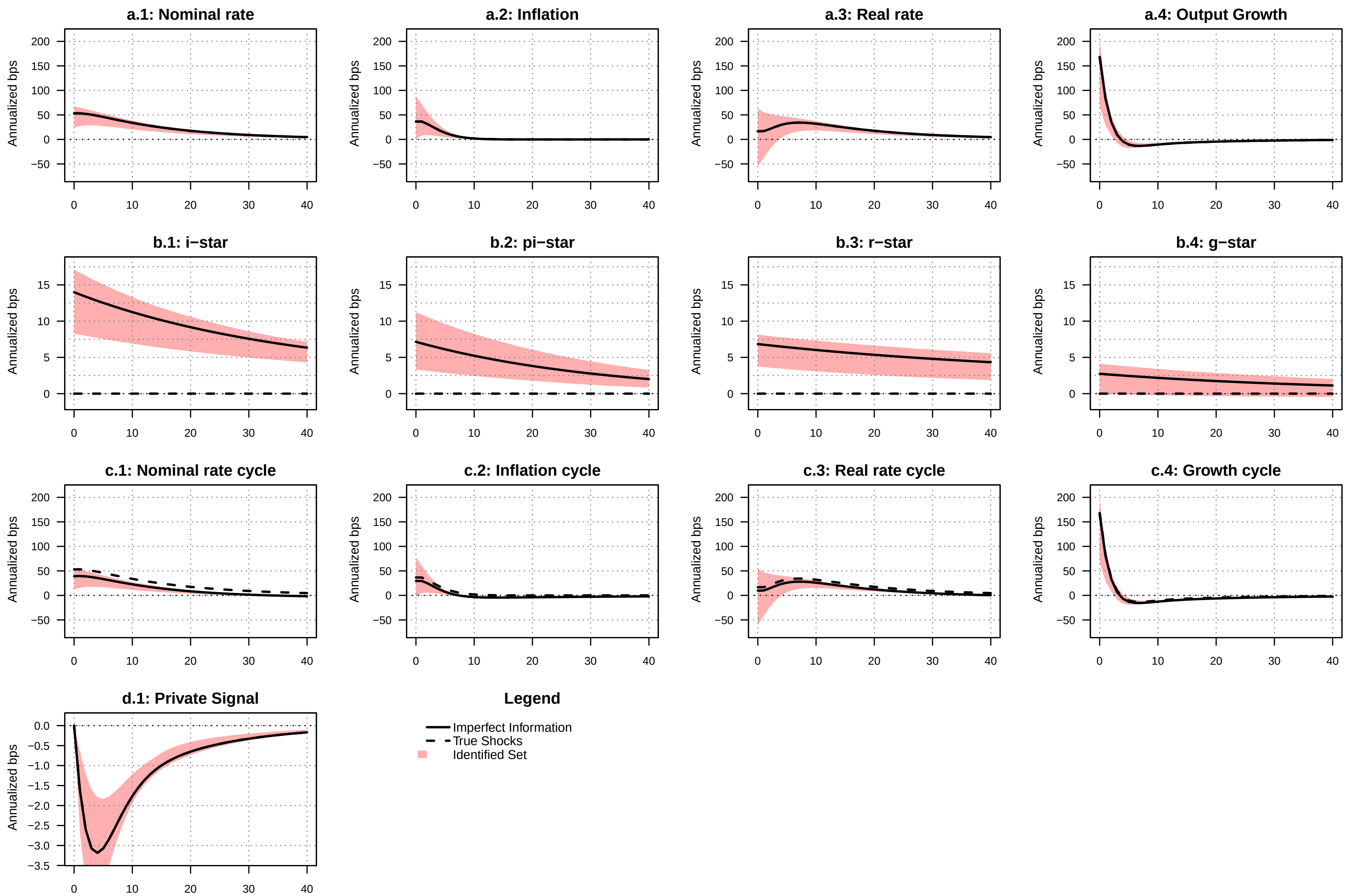
Notes: The figure presents the term structure of the term premium across the two models. **II** stands for the model where the representative asset pricer has imperfect information and is presented in black solid lines (perfect information). The model-implied premium where the investor knows exactly the trend cycle decomposition (**PI**) is presented in red dashed lines. Maturities are 1-year (panel a.1), 3-year (a.2), 5-year (b.1), and 10-year (b.2). All values are in annualized percentages. Grey-shaded areas are NBER recessions. The data are sampled quarterly from 1961:Q3 to 2022:Q4.

Figure A.3: IRF of factors to a permanent demand shock in imperfect information



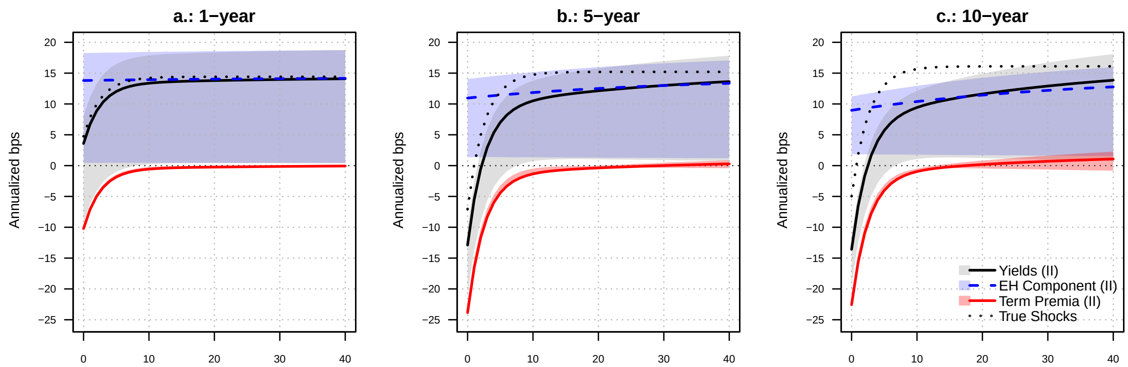
Notes: The figure illustrates the effects of a one standard deviation permanent demand shock on macroeconomic variables (panels a.1, a.2, and a.3), their trends (panels b.1, b.2, and b.3), their cycles (panel c.1, c.2 and c.3), and the private signal factor (panel d.). The responses are plotted over 40 quarters. **II** stands for the model where the representative asset price has imperfect information. **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.Red-shaded areas correspond to the range of the effects resulting from our identification based on sign-restriction.

Figure A.4: IRF of factors to a transitory demand shock in imperfect information



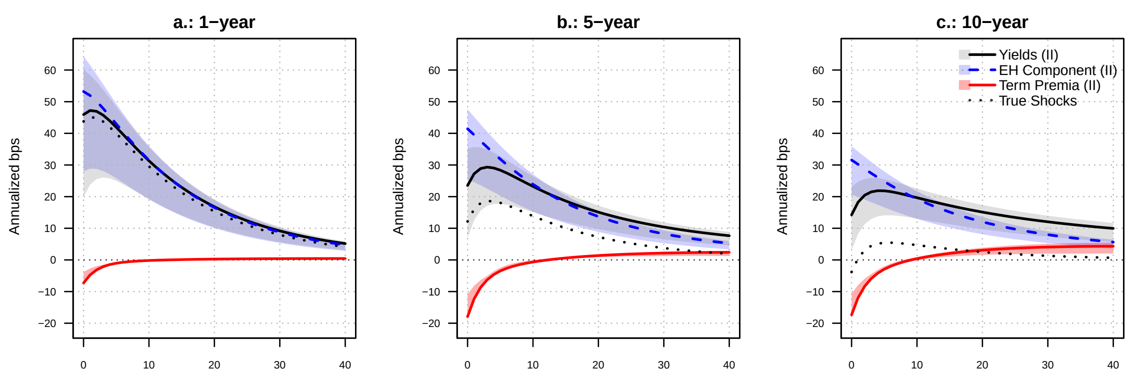
Notes: The figure illustrates the effects of a one standard deviation transitory demand shock on macroeconomic variables (panels a.1, a.2, and a.3), their trends (panels b.1, b.2, and b.3), their cycles (panel c.1, c.2 and c.3), and the private signal factor (panel d.). The responses are plotted over 40 quarters. **II** model is the model where the representative asset price has imperfect information. **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.Red-shaded areas correspond to the range of the effects resulting from our identification based on sign-restriction.

Figure A.5: IRF of yields to a permanent demand shock in imperfect information



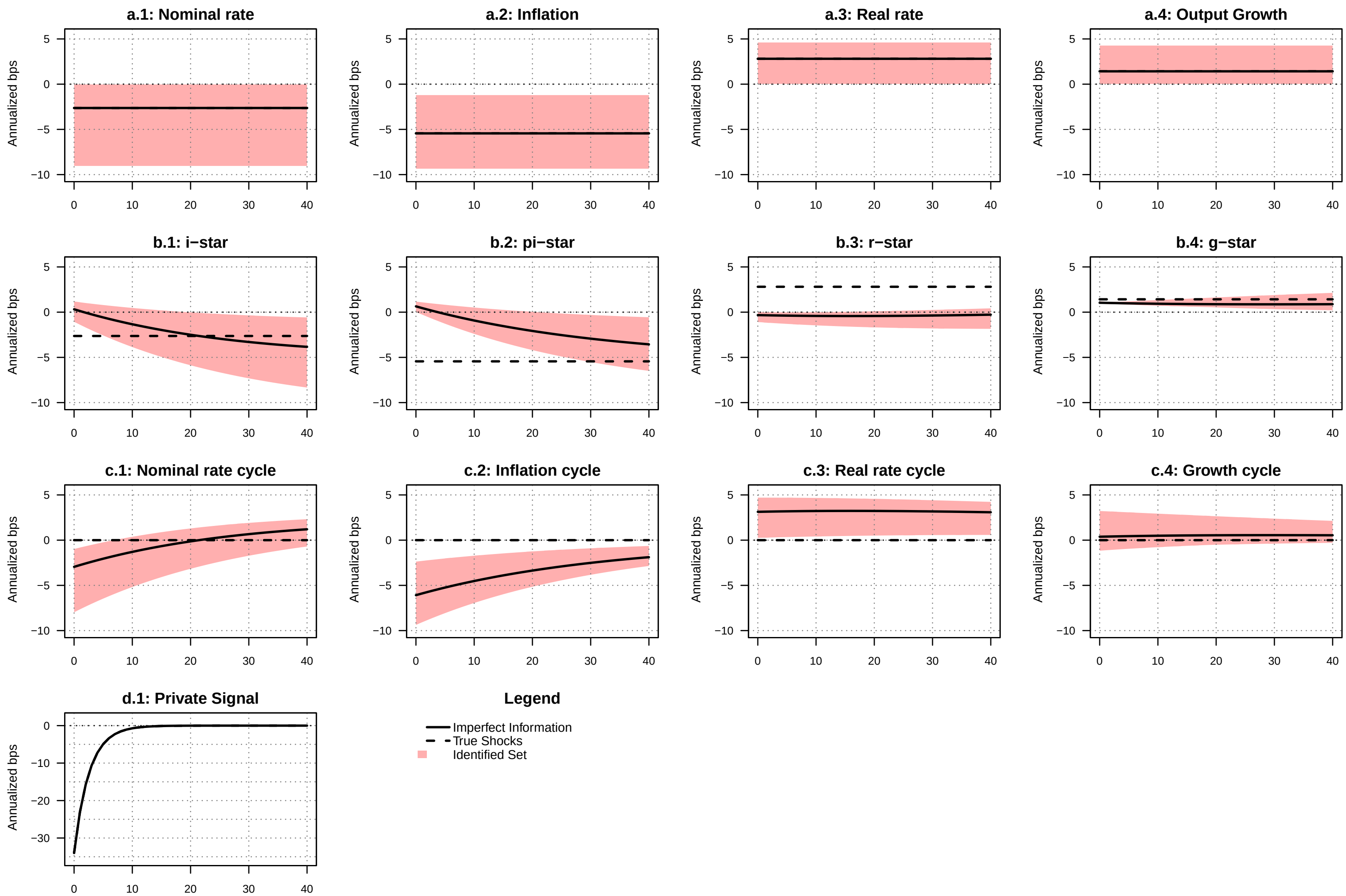
Notes: In this figure, we present the responses of 1-, 2-, 3-, 5-, and 7-year yields to a one standard deviation permanent demand shock in the model where the representative asset pricer has imperfect information (**II**). In addition, we break down these effects into their expected component (EH **II** model) and their term premium (TP **II** model). **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.

Figure A.6: IRF of yields to a transitory demand shock in imperfect information



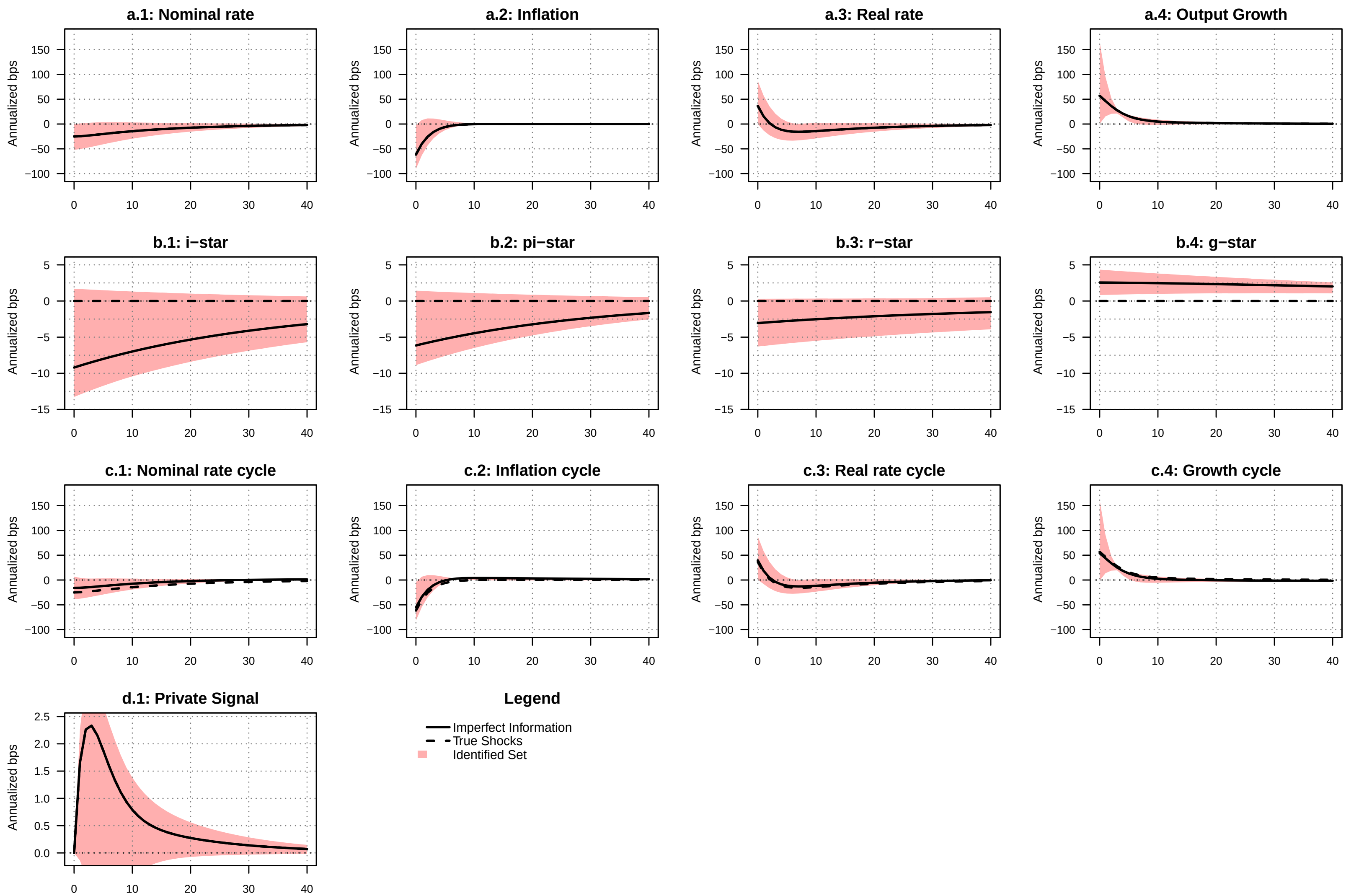
Notes: In this figure, we present the responses of 1-, 2-, 3-, 5-, and 7-year yields to a one standard deviation transitory demand shock in the model where the representative asset pricer has imperfect information (**II** model). In addition, we break down these effects into their expected component (EH **II** model) and their term premium (TP **II** model). **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.

Figure A.7: IRF of factors to a permanent supply shock in imperfect information



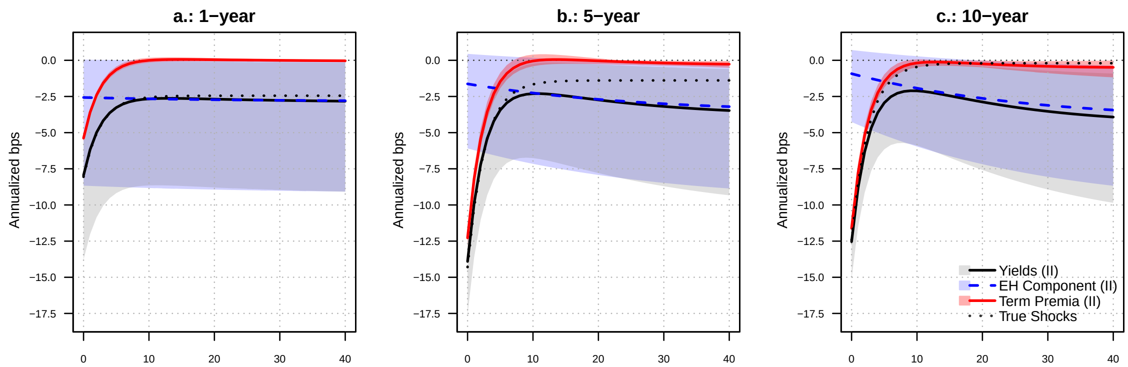
Notes: The figure illustrates the effects of a one standard deviation permanent supply shock on macroeconomic variables (panels a.1, a.2, and a.3), their trends (panels b.1, b.2, and b.3), their cycles (panel c.1, c.2 and c.3), and the private signal factor (panel d.). The responses are plotted over 40 quarters. **II** stands for the model where the representative asset price has imperfect information. **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed. Red-shaded areas correspond to the range of the effects resulting from our identification based on sign-restriction.

Figure A.8: IRF of factors to a transitory supply shock in imperfect information



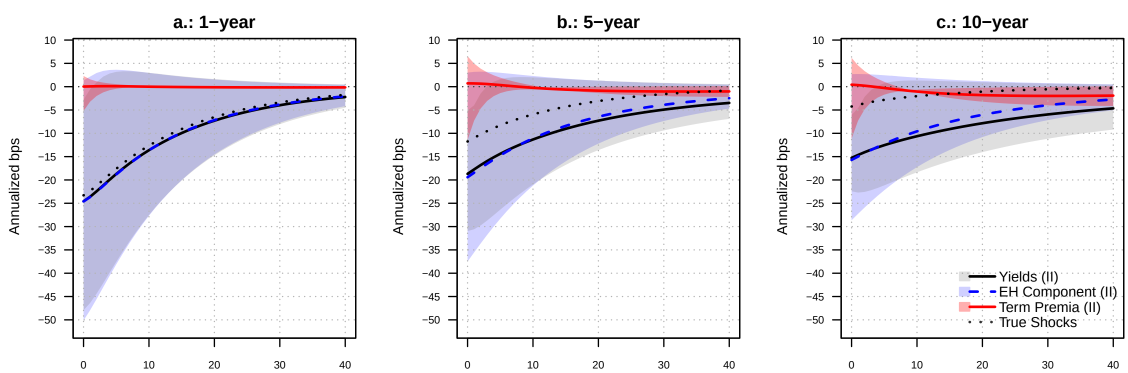
Notes: The figure illustrates the effects of a one standard deviation transitory supply shock on macroeconomic variables (panels a.1, a.2, and a.3), their trends (panels b.1, b.2, and b.3), their cycles (panel c.1, c.2 and c.3), and the private signal factor (panel d.). The responses are plotted over 40 quarters. **II** model is the model where the representative asset price has imperfect information. **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed. Red-shaded areas correspond to the range of the effects resulting from our identification based on sign-restriction.

Figure A.9: IRF of yields to a permanent supply shock in imperfect information



Notes: In this figure, we present the responses of 1-, 2-, 3-, 5-, and 7-year yields to a one standard deviation permanent supply shock in the model where the representative asset pricer has imperfect information (**II**). In addition, we break down these effects into their expected component (EH **II** model) and their term premium (TP **II** model). **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.

Figure A.10: IRF of yields to a transitory supply shock in imperfect information



Notes: In this figure, we present the responses of 1-, 2-, 3-, 5-, and 7-year yields to a one standard deviation transitory supply shock in the model where the representative asset pricer has imperfect information (**II** model). In addition, we break down these effects into their expected component (EH **II** model) and their term premium (TP **II** model). **II** model assuming **PI** represents the counterfactual where the agent uses the parameters of the **II** model, but the shocks are assumed to be perfectly observed.