Recent Changes in the US Business Cycle*

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Abstract

The US business cycle expansion that started in March 1991 is the longest on record. This paper uses statistical techniques to examine whether this expansion is a onetime unique event or whether its length is a result of a change in the stability of the US economy. Bayesian methods are used to estimate a common factor model that allows for structural breaks in the dynamics of a wide range of macroeconomic variables. We find strong evidence that a reduction in volatility is common to the series examined. Further, the reduction in volatility implies that future expansions will be considerably longer than the historical average.

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1 Introduction

The US economy is experiencing an unprecedented business cycle phase as the current expansion continues somewhat shakily into its 11th year (the expansion started in March 1991).[1] The National Bureau of Economic Research has been analyzing the US business cycle for over 146 years (1854 through 2000). Until the current expansion, the longest was the 1960s expansion lasting 9 years, while the second longest persisted for 7.5 years in the 1980s. These previous long expansions produced speculation about the death of the business cycle, with discussion generally peaking about the time that the expansions actually ended. This expansion, however, occurs in a period in which there has been only one recession in eighteen-year interval (the previous two long expansions were not preceded by long expansions) . This paper uses statistical techniques to examine whether this current episode is somehow different from the previous ones, given this distinguishing feature.

This recent experience of longer expansions is associated with an increased stability of US GDP growth. This has been documented by a number of authors, who find a structural break in its volatility at the beginning of 1984 (McConnell and Perez 2000, Kim and Nelson, 1999, Koop and Potter, 2000). As discussed in Potter (2000), the decline in volatility implies that recessions, defined as two consecutive negative quarters of GDP growth, are likely to be very infrequent in the future. This paper expands on this previous research by examining changes in the behavior of a wide range of macroeconomic time series (see Van Dijk and Sensier 2001 also examine volatility changes in a number of US time series). In particular, we consider two measures each of industrial output, consumption, personal income, and employment. These series, in addition to GDP, are similar to the ones studied by the NBER to date business cycles.

In order to allow comparison to earlier results on GDP, we develop a common factor model that allows for structural breaks. This model attempts to extract the common dynamics in the series examined. Previously, common factor models have been estimated with recurrent breaks generated by switches in an unobserved Markov process (see Chauvet 1998). In this paper, Bayesian methods are used to estimate the common factor model and to produce posterior means of three statistical measures of changes in the

[1] Hall (2001) suggests that the end of the current expansion has not occured as of March 5th 2001.

common cycle.

First, we examine changes in the inverse of the coefficient of variation[2] and the frequency of negative growth in the common factor. Thus, we capture both the drop in volatility and any implications for the frequency of contractions. Next, we examine properties of the common factor in the frequency domain. The idea here is to examine whether the decrease in volatility is uniform at all frequencies or whether some of the changes are more pronounced at business cycle frequencies. Finally, we consider the expected time to the next recession (see Potter 2000) implied by the common factor model before and after the break.

We find strong statistical evidence that the volatility of the business cycle common factor is lower than in the past. The estimated breakpoint is around 1984 and agrees with the previous studies of US GDP alone. In contrast to Blanchard and Simon (2001) we also find evidence that the dynamics of the business cycle have changed . In particular since the breakpoint the common factor displays reduced volatilities at business cycle frequencies, but higher volatility at lower frequencies. Finally, we match the estimates of expected time to the next recession of 10 to 20 years as in Potter (2000).

The outline of the paper is as follows: Section 2 describes the data using the first quarter of 1984 as the breakpoint in business cycle fluctuations. Section 3 develops the statistical model and the techniques used to estimate it. Section 4 describes some methods for measuring the change in the business cycle. Section 5 contains the results. Section 6 offers some conclusions.

2 Data Description

We use data for the period from 1959Q2 to 2000Q4.[3] We model the common behavior of the following time series in addition to GDP:

1. Output: Total Industrial and Manufacturing
2. Consumption: Total and Retail Sales

[2] That is, the mean divided by the standard deviation.

[3] Starting the sample this late ensures reasonably uniform data collection for the time series. Earlier discussions of changes in the business cycle have faced problems regarding possible inconsistencies in the time series over time (see Romer 1994, Diebold and Rudebusch 1992, and Watson 1994).

3. Income: Total Personal and Wage and Salary

4. Employment: Total Payroll and Hours

We deflate the consumption and income measures by the personal consumption deflator. The data is transformed by taking logarithmic differences and converted to (approximate) annualized growth rates by multiplying by 400.

In order to get some intuition regarding the changes in the business cycle, we split the sample in two with a break date in 1984Q1. This is approximately the date estimated by McConnell and Perez (2000) and others in their investigation of a structural break in the volatility of GDP by itself. For each series we calculate the average growth rate , the standard deviation of growth, the coefficient of variation (standard deviation divided by the mean), and the frequency of negative quarters.

Table 1 contains the results for the first subsample and Table 2 for the second part.[4]

Table 1: 1959Q2 to 1983Q4

Series	Ave. Growth	St. Dev.	CV	% Neg Q
GDP	3.41	4.33	1.27	18
Industrial Production	3.49	8.62	2.47	26
Manufacturing Output	3.59	9.45	2.63	27
Income	3.77	3.15	0.83	13
Wages and Salary	3.19	3.68	1.15	22
Consumption	3.62	3.31	0.91	11
Retail Sales	2.46	6.87	2.79	32
Payroll Employment	2.25	2.52	1.12	16
Aggregate Hours	1.90	3.28	1.73	22

As can be seen from comparing the tables, there is little difference in average growth rates, except for retail sales and personal income.[5] However, there has been a marked reduction in volatility of all series. In fact, only retail sales and wages and salaries show an average growth rate greater than

[4]The calculations in Table 2 contain the effects of expected tax code changes in 1993 and 1994. The possibility of these changes in the tax code produced substantial income shifting between quarters at the end and start of the year and artificially inflate the volatility measure.

[5]Both effects are likely due to statistical issues with the series.

their standard deviation. The reduction in the volatility of manufacturing output is particularly striking and was the focus of the analysis of McConnell and Perez (2000), who attributed it to improved inventory control in durable manufacturing.

Table 2: 1984Q1 to 2000Q4

Series	Ave. Growth	St. Dev	CV	% Neg Q
GDP	3.39	2.13	0.63	6
Industrial Production	3.39	3.37	0.99	9
Manufacturing Output	3.84	3.67	0.95	10
Income	3.16	2.73	0.86	7
Wages and Salaries	3.23	3.86	1.20	9
Consumption	3.46	2.06	0.60	4
Retail Sales	2.96	4.51	1.52	26
Payroll Employment	2.13	1.34	0.63	9
Aggregate Hours	2.07	1.84	0.89	12

Figure 1 plots the 4 quarter moving average of growth rates of the dataset and GDP.[6] In addition, two horizontal lines at 1% and 5% growth are plotted to help recognizing changes in the volatility of the cycle. The business cycle pattern is clear in the time series, and there is direct visual evidence of a more muted business cycle recently.

3 Statistical Model and Methods

3.1 Common Factor Model

We define Y_t to be the $K \times 1$ vector of growth rates used to estimate the common factor, C_t. The statistical model is

$$\mathsf{Y}_t = \boldsymbol{\lambda} C_t + \mathsf{V}_t,$$

where $\boldsymbol{\lambda}$ is the $K \times 1$ vector of factor loadings and the common factor is

$$C_t = \begin{cases} \alpha_1 + \phi_{1p}(L)C_{t-1} + \sigma_1\varepsilon_t & \text{if } t \leq \tau \\ \\ \alpha_2 + \phi_{2p}(L)C_{t-1} + \sigma_2\varepsilon_t & \text{if } t > \tau. \end{cases}$$

[6] We do not label the individual series, since our intent is to show the common cyclical pattern.

The measurement error vector V_t has an autoregressive structure:

$$\mathsf{V}_t = \Theta_1 \mathsf{V}_{t-1} + \cdots + \Theta_q \mathsf{V}_{t-q} + \mathsf{U}_t,$$

where the innovations to the common factor, $\varepsilon_t \sim IIDN(0,1)$,and the measurement error, $\mathsf{U}_t \sim IIDN(\mathsf{0}, \Sigma_K), \Sigma_K$ diagonal[7], are independent of each other at all leads and lags. Finally, the autoregressive matrices are diagonal.

$$\Theta_i = \begin{bmatrix} \theta_{1i} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \theta_{1K} \end{bmatrix}.$$

The model can be written in state space form, where we assume, without loss of generality but considerable saving in notation, that $p = q + 1$. First, we define the following:

1. Define $\mathsf{Y}_t^* = (\mathsf{I}_k - \Theta(L))\mathsf{Y}_t$.
2. Define $\mathsf{C}_t^* = [C_t, \ldots, C_{t-p+1}]^{\prime}$.
3. Define the $K \times (q+1)$ matrix H by:

$$\begin{bmatrix} \lambda_1 & -\lambda_1\theta_{11} & \cdots & -\lambda_1\theta_{q1} \\ \lambda_2 & -\lambda_2\theta_{12} & \cdots & -\lambda_2\theta_{q2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_K & -\lambda_K\theta_{1K} & \cdots & -\lambda_K\theta_{qK} \end{bmatrix}.$$

4. Define the $p \times p$ matrix A by:

$$\begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_p \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ 0 & \ddots & 1 & 0 \end{bmatrix}.$$

The state space form has measurement equation:

$$\mathsf{Y}_t^* = \mathsf{H}\mathsf{C}_t^* + \mathsf{U}_t$$

[7] Given the similarity of some of the series, one could also assume that Σ_{k} had a particular block structure.

and transition equation:

$$\mathsf{C}_t^* = \begin{cases} \mathsf{a}_1 + \mathsf{A}_1\mathsf{C}_{t-1}^* + \mathsf{W}_1\varepsilon_t & \text{if } t \leq \tau \\ \\ \mathsf{a}_2 + \mathsf{A}_2\mathsf{C}_{t-1}^* + \mathsf{W}_2\varepsilon_t & \text{if } t > \tau, \end{cases}$$

where $\mathsf{W}_i = [\sigma_i, 0, \cdots, 0]'$ and $\mathsf{a}_i = [\alpha_i, 0, \ldots, 0]'$, $i = 1, 2$, are $(p \times 1)$ vectors. Below we will also sometimes summarize the conditional mean coefficients in each regime by the $p \times 1$ vector $\boldsymbol{\phi}_i = (\phi_{1i}, \ldots, \phi_{pi})$ or the $(p+1) \times 1$ vector $\boldsymbol{\beta}_i = (\alpha_i, \phi_{1i}, \ldots, \phi_{pi})$. Let φ represent all the parameters of the common factor model with a structural break and χ represent the (smaller by β_2, σ_2, τ) set of parameters of the common factor model without a break.

As is true of all single factor models there is an identification issue between the factor loadings and the scaling of the innovation to the common factor(see Chauvet 1998). We normalize the first element of the factor loading vector setting it equal to 1. Also, unlike Stock and Watson (1989), we do not demean and standardize the observable variables before the analysis. Thus, not only will $\boldsymbol{\lambda}$ be estimated from the joint dynamics of the observed time series, it will also depend on information on their relative means and variances. One could allow for individual means to be estimated for each time series, but this will center the dynamic factor model at zero and not provide an unambiguous definition of negative values for the time series. If the sample size got large, our methods could cause problems as $\boldsymbol{\lambda}$ would be over-identified. However, given that we are allowing for a structural break in the time series model for the unobserved factor and autocorrelated measurement errors, there is no over-identification in the finite sample.

The modeling of the structural break here is quite simple: all series have to experience the break at the same time. There are more sophisticated choices one could use involving changes in the factor loadings with the model for the common factor remaining constant across the breakpoint. However, this suffers from the problem that some of the changes in the business cycle might not be summarized by a drop in volatility alone. In particular, there are two sources of volatility in the time series model for the factor: the first is the innovation variance; the second is the persistence of shocks produced by the autoregressive mechanism. The introduction of a structural break in the common factor model makes it possible to distinguish between these two sources. However, it also makes the requirements for finding a break more stringent, since it has to occur at the same time for all series.

3.2 Bayesian Methods

If the breakpoint τ were known, estimation by classical or Bayesian methods would be standard. Both methods use the Kalman filter to construct the likelihood function. The Kalman filter iterations are given by:

1. Prediction Step: the conditional mean of the factor is,

$$\mathsf{C}^*_{t+1|t} = \begin{cases} \mathsf{a}_1 + \mathsf{A}_1 \mathsf{C}^*_{t|t} & \text{if } t < \tau \\ \mathsf{a}_2 + \mathsf{A}_2 \mathsf{C}^*_{t|t} & \text{if } t \geq \tau \end{cases}.$$

   The conditional variance of the factor is,

$$\mathsf{P}_{t+1|t} = \begin{cases} \mathsf{A}_1 \mathsf{P}_{t|t} \mathsf{A}_1^{\prime} + \mathsf{W}_1 \mathsf{W}_1^{\prime} & \text{if } t < \tau \\ \mathsf{A}_2 \mathsf{P}_{t|t} \mathsf{A}_2^{\prime} + \mathsf{W}_2 \mathsf{W}_2^{\prime} & \text{if } t \geq \tau \end{cases}.$$

   Plugging the conditional mean into the measurement equation we have the forecast error:

$$\mathsf{Y}^*_{t+1} - \widehat{\mathsf{Y}}^*_{t+1|t} = \mathsf{H}(\mathsf{C}^*_t - \mathsf{C}^*_{t+1|t}) + \mathsf{U}_t,$$

   and variance:

$$E\left[(\mathsf{Y}^*_{t+1} - \widehat{\mathsf{Y}}^*_{t+1|t})(\mathsf{Y}^*_{t+1} - \widehat{\mathsf{Y}}^*_{t+1|t})^{\prime}\right] = \mathsf{H}\mathsf{P}_{t+1|t}\mathsf{H}^{\prime} + \Sigma_K.$$

2. Updating Step: first, the Kalman Gain matrix is constructed:

$$\mathsf{G}_{t+1} = \mathsf{P}_{t+1|t}\mathsf{H}^{\prime} \left\{ E\left[(\mathsf{Y}^*_{t+1} - \widehat{\mathsf{Y}}^*_{t+1|t})(\mathsf{Y}^*_{t+1} - \widehat{\mathsf{Y}}^*_{t+1|t})^{\prime}\right]\right\}^{-1}.$$

   Then, this is used to include the new information in the conditional mean of the factor

$$\mathsf{C}^*_{t+1|t+1} = \mathsf{C}^*_{t+1|t} + \mathsf{G}_{t+1}\left(\mathsf{Y}^*_{t+1} - \widehat{\mathsf{Y}}^*_{t+1|t}\right),$$

   and to update the conditional variance:

$$\mathsf{P}_{t+1|t+1} = (\mathsf{I}_p - \mathsf{G}_{t+1}\mathsf{H})\,\mathsf{P}_{t+1|t}.$$

The problem here is that the breakpoint is not known. One approach would be to estimate the model for each possible breakpoint and then choose the value that maximizes the likelihood function. Such an approach is computationally feasible, but suffers from two major drawbacks. First, the sampling distribution of the likelihood ratio statistic for the breakpoint is not known and would require additional simulations to generate it. Second, and perhaps most importantly, such methods do not allow one to include uncertainty about the breakpoint in assessing the changes in the business cycle. Potter (2000) shows that inferences conditional on a breakpoint are very fragile for classical estimation of recession frequencies and, implicitly, for other measures of the reduction in amplitude of the business cycle as well.

For our purposes, we treat the breakpoint as unknown and use Bayesian methods to extract the sample evidence about its likelihood and date. The Bayesian methods involve using a Gibbs sampler to generate random draws from the posterior distribution by utilizing a sequence of conditioning distributions. In particular, the Gibbs sampler generates random draws of τ that allows one to act as if the breakpoint was known. Furthermore, a random draw of the common factor is generated as part of the iterations of the Gibbs sampler. The recursion used to generate the random draw of the common factor is as follows (see Carter and Kohn, 1994). Here the recursion is conditional on the draw of the parameters of the factor model:

1. The last iteration of the Kalman filter yields:

   $$\mathsf{C}^*_T \sim N(\mathsf{C}^*_{T|T}, \mathsf{P}_{T|T}).$$

   Thus, using standard methods one can draw a realization $\widetilde{\mathsf{C}}^*_T$, from this multivariate normal. Then the draw of the most recent value of the common factor is given by:

   $$\widetilde{C}_T = \mathsf{s}\widetilde{\mathsf{C}}^*_T,$$

   where $\mathsf{s} =[1, 0, \ldots, 0]$ is a $p \times 1$ selection vector. In practice, one only needs to draw from the univariate normal with mean given by the first element of $\mathsf{C}^*_{T|T}$ and variance by the first diagonal element of $\mathsf{P}_{T|T}$.

2. Given a draw at $t + 1$ based on draws from $t + 2$ to T, the information from the Kalman filter iterations is incorporated as if the filter were

running backwards, combining prior information from the initial forward run of the filter with the 'sample' information generated by the random draw:

$$f_t = \widetilde{C}_{t+1} - \begin{cases} \alpha_1 + \phi_{1p}(L)C_{t|t} & \text{if } t < \tau \\ \\ \alpha_2 + \phi_{2p}(L)C_{t|t} & \text{if } t \geq \tau \end{cases},$$

$$p_t = \begin{cases} \boldsymbol{\phi}_1^{\prime} \mathsf{P}_{t|t} \boldsymbol{\phi}_1 + \sigma_1^2 & \text{if } t < \tau \\ \\ \boldsymbol{\phi}_2^{\prime} \mathsf{P}_{t|t} \boldsymbol{\phi}_2 + \sigma_2^2 & \text{if } t \geq \tau \end{cases},$$

$$\mathsf{g}_t = \begin{cases} \mathsf{P}_{t|t} \boldsymbol{\phi}_1 / p_t & \text{if } t < \tau \\ \\ \mathsf{P}_{t|t} \boldsymbol{\phi}_2 / p_t & \text{if } t \geq \tau \end{cases},$$

$$\mathsf{C}_{t|T}^* = \mathsf{C}_{t|t}^* + \mathsf{g}_t f_t,$$

$$\mathsf{P}_{t|T} = \begin{cases} \left(\mathsf{I}_p - \mathsf{g}_t \boldsymbol{\phi}_1^{\prime}\right) \mathsf{P}_{t|t} & \text{if } t < \tau \\ \\ \left(\mathsf{I}_p - \mathsf{g}_t \boldsymbol{\phi}_2^{\prime}\right) \mathsf{P}_{t|t} & \text{if } t \geq \tau \end{cases}.$$

Thus, after observing the whole sample $\mathsf{C}_t^* \sim N(\mathsf{C}_{t|T}^*, \mathsf{P}_{t|T})$, Standard methods can be used to obtain a random draw of the common factor at time t.

3. This iteration stops with $\mathsf{C}_p^* \sim N(\mathsf{C}_{p|T}^*, \mathsf{P}_{p|T})$, which is used to simultaneously draw the first p observations of the common factor.

Given this realized sequence of the common factor, one can apply the methods described in detail in Potter (2000) to investigate the breakpoint. In brief one treats the draw of the common factor as an observed time series. For each possible breakpoint one calculates the marginal likelihood of the data. The collection of marginal likelihoods suitably normalized is the conditional posterior distribution of the breakpoint.

3.2.1 Estimation by Gibbs Sampler

We initialize the Gibbs sampler by running the Kalman filter on the observed data assuming: a break date at 1984Q1, factor loadings equal to unity, measurement error equal to 1/4 of the observed variance, and point estimates obtained for GDP with a sample split in 1984Q1 as initial guesses for the parameters of the common factor. The results of the Kalman filter are then used to draw a sequence of realizations for the common factor.

The ordering of the Gibbs sampler is:

1. Conditional on $\{\widetilde{C}_t\}$, $\Theta(L), \Sigma_K$ we draw the $K \times 1$ vector of factor loadings $\boldsymbol{\lambda}$ from (independent) normal distributions. For the generic loading λ_k we have the sample information:

$$\left[\sum_{t=p+1}^{T} C_{kt}^{*2}\right]^{-1}, \sum_{t=p+1}^{T} C_{kt}^{*} Y_{kt}^{*},$$

   where $C_{kt}^{*} = C_t - \theta_{1k} C_{t-1} - \cdots - \theta_{qk} C_{t-q}$. Let V_{λ_k} be the variance of the Gaussian prior on λ_k and M_{λ_k} be its prior mean. Then the posterior draw is from normal distribution with mean

$$\frac{V_{\lambda_k}^{-1} M_{\lambda_k} + \sum_{t=p+1}^{T} C_{kt}^{*} Y_{kt}^{*}}{V_{\lambda_k}^{-1} + \sum_{t=p+1}^{T} C_{kt}^{*2}}$$

   and variance

$$\left[V_{\lambda_k}^{-1} + \sum_{t=p+1}^{T} C_{kt}^{*2}\right]^{-1}.$$

   For the first element of $\boldsymbol{\lambda}$ we impose the prior belief that it is equal to 1.

2. Conditional on $\{\widetilde{C}_t\}$, $\Theta(L), \boldsymbol{\lambda}$ we draw the measurement error variances from independent gamma distributions. For the generic measurement error Σ_{kk} we have the sample information

$$\sum_{t=p+1}^{T} (Y_{kt}^{*} - \lambda_k C_{kt}^{*})^2, T - p,$$

   which is combined with the prior degrees of freedom of ξ and sum of squares ξs^2 to obtain the posterior degrees of freedom $\xi + T - p$ and sum of squares $\xi s^2 + \sum_{t=p+1}^{T} (Y_{kt}^{*} - \lambda_k C_{kt}^{*})^2$.

3. Conditional on $\{\widetilde{C}_t\}$,$\boldsymbol{\lambda}, \Sigma_K$ we draw the measurement autoregressive coefficients from independent multivariate Gaussian distributions. For the generic measurement error autoregression k the sample information is:

$$\left[\mathsf{Z}_k^{\prime}\mathsf{Z}_k\right]^{-1}, \mathsf{Z}_k^{\prime}\mathsf{W}_k,$$

where $\mathsf{W}_k = [Y_{kq+1}, \cdots, Y_{kT}]'$ and

$$\mathsf{Z}_k = \begin{bmatrix} Y_{kq} & \cdots & Y_{k1} \\ Y_{kq+1} & \cdots & Y_{k2} \\ \vdots & \vdots & \vdots \\ Y_{kT-1} & \cdots & Y_{kT-q} \end{bmatrix}.$$

This is combined with the prior Gaussian distribution,$N(0, V_{\theta_k})$ on the autoregressive coefficients in the standard way to obtain a posterior variance of:

$$\left[V_{\theta_k}^{-1} + \mathsf{Z}_k^{\prime}\mathsf{Z}_k\right]^{-1}$$

and posterior mean of

$$\left[V_{\theta_k}^{-1} + \mathsf{Z}_k^{\prime}\mathsf{Z}_k\right]^{-1}\left[\mathsf{Z}_k^{\prime}\mathsf{W}_k\right].$$

4. Conditional on $\{\widetilde{C}_t\}$ we calculate the posterior distribution of τ and the marginal likelihood of $\{\widetilde{C}_t\}$ under both the structural break model and the no break model. This requires that a normal-inverted gamma prior be used for both before and after the break values of the parameters of the common factor model (see Potter, 2000). We use the conditional posterior distribution of τ to draw a particular breakpoint.

5. Conditional on $\{\widetilde{C}_t\}, \tau$ we draw the autoregessive model parameters for before and after the break from the inverted-gamma normal distribution. These draws of the autogressive parameters are used to calculate various measures of changes in the common factor before and after the break.

6. Conditional on $\Theta(L), \boldsymbol{\lambda}, \Sigma_K, \tau, \beta_1, \beta_2, \sigma_1, \sigma_2$ the Kalman filter is run on the observed data. The filter is initialized at the stationary distribution for $\{C_t\}$ implied by β_1, σ_1. Then, using the recursions described above, a draw of $\{\widetilde{C}_t\}$ is obtained and we return to step 1. The posterior mean for the smoothed factor is produced directly from a similar set of recursions, with the draw $\widetilde{C}_t$ replaced by $C_{t|T}$.

3.2.2 Evidence for a structural break

We can assess the sample evidence in favor of a structural break in the common factor model by comparing the average likelihood of the observed time series with and without a break. This calculation would directly involve multiple integration but can be simplified using the following tricks. The Bayes factor is the marginal likelihood of the no break model divided by the marginal likelihood of the break model:

$$B_{\text{no break vs.break}} = \frac{\int l(\mathsf{Y}|\chi)b(\chi)d\chi}{\int l(\mathsf{Y}|\varphi)b(\varphi)d\varphi}$$

Using the basic likelihood identity (Chib 1995) we have:

$$\int l(\mathsf{Y}|\varphi)b(\varphi)d\varphi = \frac{l(\mathsf{Y}|\varphi)b(\varphi)}{p(\varphi|\mathsf{Y})}$$

for all points in the parameter space. In particular, consider the transformation of the parameter space for the common factor model from $(\beta_1, \sigma_1, \beta_2, \sigma_2, \tau)$ to $(\beta_1, \sigma_1, \beta_2 - \beta_1, \sigma_2/\sigma_1, \tau)$. If we evaluate the transformation at $\beta_2 - \beta_1 = 0, \sigma_2/\sigma_1 = 1$, then there is no information in the likelihood function about τ. As discussed in Koop and Potter (1999), one can use this lack of identification to simplify marginal likelihood calculations using the Savage-Dickey Density ratio. In order to simplify the calculations we assume that the prior over the shared common factor parameters is the same for the break and no break models. In this case we have:

$$\begin{aligned} & \frac{\int l(\{\widetilde{C}_t\}|\beta_1, \sigma_1)b(\beta_1, \sigma_1)d\beta_1 d\sigma_1}{\int l(\{\widetilde{C}_t\}|\beta_1, \sigma_1, \beta_2, \sigma_2, \tau)b(\beta_1, \sigma_1, \beta_2, \sigma_2, \tau)d\beta_1 d\sigma_1 d\beta_2 d\sigma_2 d\tau} \\ = & \frac{\int p(\beta_2 - \beta_1 = 0, \sigma_2/\sigma_1 = 1, \tau|\{\widetilde{C}_t\}, \mathsf{Y}, \boldsymbol{\varphi}^-)d\tau}{b(\beta_2 - \beta_1 = 0, \sigma_2/\sigma_1 = 1|\boldsymbol{\varphi}^-)}, \end{aligned}$$

where $\boldsymbol{\varphi}^-$ signifies the parameter space excluding the parameters of the common factor model. Using the methods of Potter (2000) one can directly calculate the marginal likelihoods at each iteration of the Gibbs sampler. If the conditional posterior density is then averaged across draws of $\boldsymbol{\varphi}^-$ and $\{\widetilde{C}_t\}$ from the Gibbs sampler we will have

$$\frac{\int p(\beta_2 - \beta_1 = 0, \sigma_2/\sigma_1 = 1, \tau|\mathsf{Y})d\tau}{b(\beta_2 - \beta_1 = 0, \sigma_2/\sigma_1 = 1)},$$

which is the Savage Dickey ratio for the Bayes factor of a no break common factor model vs a structural break common factor model.

4 Measuring Changes in the Business Cycle

This section develops some simple methods for describing changes in the business cycle. The first measure uses the (inverse) coefficient of variation of the common factor model before and after the break, as well as the frequency of negative growth in the common factor. The second measure is the spectrum of the time series model for the common factor before and after the break. In particular, we focus on whether the change in the business cycle is best summarized as a reduction in volatility of the shocks hitting the economy as argued in Blanchard and Simon (2001). Finally, we discuss a measurement of the frequency of recessions, that is, the expected time to a recession starting from the stationary distribution of the common factor.

All the measures considered are nonlinear functions of the estimated parameters. As shown in Potter (2000), traditional classical techniques for estimating these nonlinear functions can produce substantial bias. Further, it is very difficult to obtain sampling distributions that allow for uncertainty over the break point. The approach here will be to use the realizations of the Gibbs sampler to calculate the various nonlinear functions, and then average across realizations to obtain the posterior means.

4.1 Coefficient of Variation

The coefficient of variation is the (inverse) ratio of the mean of a random variable to its standard deviation. There are two general ways in which recessions could have become less frequent (or expansions lengthier): first, the volatility of fluctuations could have been reduced; second, the growth rate could be higher for the same level of volatility. The description of the data above suggests that the former is the main source of the recent changes in the U.S. business cycle. The common factor model is constructed in such a way that one of the main set of moments it must match, when scaled by the relevant factor loading, is the mean growth rate of the various time series. Thus, rather than examining its volatility directly, it is necessary to study it relative to the mean. Further, the sources of the volatility of the observed time series are both the factor and the individual measurement errors processes. Thus, the coefficient of variation allows us to check whether any of the reduction in volatility in the observed series is accounted for indirectly by the measurement error process.

In terms of the parameters of the time series model for the common factor, we have in the AR(2) case:

$$\frac{\alpha}{1-\phi_1-\phi_2} \times \frac{\sqrt{\frac{1+\phi_2}{1-\phi_1}\left\{(1-\phi_2)^2-\phi_1^2\right\}}}{\sigma},$$

which we can calculate at each iteration of the Gibbs sampler for the parameter draws before and after the break, and then average to form the posterior mean of this quantity.

If the time series model was known, then the inverse of the coefficient of variation would allow direct calculation of the probability of a negative quarter using the cumulative distribution function of a normal. Since the time series model is not known, we also calculate the implied probability of a negative quarter at each iteration of the Gibbs sampler and average this quantity. This gives us some indication of how informative is the posterior mean of the inverse coefficient of variation about the rest of its posterior distribution.

4.2 Spectral Methods

The spectrum of the common factor time series model has the following representation:

$$f(\omega) = \frac{\sigma^2}{2\pi}\left|\frac{1}{\phi[\exp(-i\omega)]}\right|^2.$$

Recall that the spectrum decomposes a time series in terms of cycles of different frequencies, where the average height of the spectrun represents the overall variance of the time series. Clearly, one source of change in the business cycle is the reduction in the volatility of shocks as measured by σ^2. If this were the only source of change in the common factor model before and after the break, then the two spectra should have exactly the same shape, but with a different scaling. In other words, cycles at all frequencies would have been reduced by an equal amount. We can check for changes in other parts of the spectrum by using Kolgomorov's formula:

$$\sigma^2 = 2\pi \exp\left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln\left(f(\omega)\, d\omega\right) \right\}.$$

Thus, one can decompose the difference in the innovation variances before and after the break as:

$$\begin{aligned} \ln(\sigma_1^2) - \ln(\sigma_2^2) &= \frac{1}{\pi} \int_0^{\pi} \ln\left(\frac{f_1(\omega)}{f_2(\omega)}\right) d\omega \\ &= \ln\left(\frac{\sigma_1^2}{\sigma_2^2}\right) \frac{1}{\pi} \int_0^{\pi} \ln\left(\frac{|\phi_2[\exp(-i\omega)]|^2}{|\phi_1[\exp(-i\omega)]|^2}\right), \end{aligned}$$

where we see that

$$\int_0^{\pi} \ln\left(\frac{|\phi_2[\exp(-i\omega)]|^2}{|\phi_1[\exp(-i\omega)]|^2}\right) = \pi$$

for all stationary autoregressive operators. Hence, if the only change arised from the volatility of the shocks, then the log ratio of the two spectra would be constant. In particular,

$$\ln\left(\frac{f_1(\omega)}{f_2(\omega)}\right) = \ln\left(\frac{\sigma_1^2}{\sigma_2^2}\right).$$

Frequencies in which the log spectral density ratio is greater than the log ratio of the innovation variances have cycles that have been muted more than average, whereas frequencies for which the ratio is less have cycles that have been muted less than average. In particular, it is possible that even though the overall variance has dropped, some frequencies are now absolutely more variable than they were in the past:

$$\ln\left(\frac{f_1(\omega)}{f_2(\omega)}\right) < 1$$

4.3 Calculating Time to Next Recession

Consider the statement: "recessions occur once every 4 years on average". One interpretation of this statement is that the average duration of an expansion is 3 years. This interpretation would be exact if the average duration of a recession was 1 year. However, if recessions lasted less than one year on average, then the duration of expansions would be longer than 3 years. Alternatively, consider the statement that the average duration of an expansion is 10 years. Does this imply that recessions occur every 10th year? The answer is, not necessarily. Suppose that recessions, if they occur, last for 10 years also on average. In addition, assume that, unconditionally, recessions are as likely as expansions. Then, the **frequency of years in which the economy is in recession** is about one out of two years. However, if we had just reached the trough of the business cycle it is true that we would expect the next recession to happen in 10 years. We focus on a hybrid of these two concepts of the duration of expansions and the frequency of recessions.

We mainly focus on probabilities generated assuming that the initial conditions of the series are drawn from their stationary distribution. We also examine a current recession prediction that conditions on the most recent economic data. In order to start, we need to convert the subjective criteria of the NBER into a quantitative statement about common factor. We follow a long literature and assume that a recession is equivalent to two **consecutive** quarters of negative growth. In our case it is negative growth in the common factor rather than GDP. Based on this criteria, we need to distinguish between forecasting a recession at a certain date in the future, and the expected time to the next recession. The former is captured, for example, by statements such as:

$$P[\text{Recession in 2002Q3/4}] = P[C_{\mathsf{2002}Q\mathsf{3}} < 0, C_{\mathsf{2002}Q\mathsf{4}} < 0] = 0.10,$$

that is, the probability of a recession at the end of the year 2002 is 10%. Notice that this leaves unanswered the question of whether there will be a recession in the year 2001. We are interested in the different exercise of finding the probability of the next recession. Consider first the probability of a recession hitting at time t given that there are no recessions before this date:

$$\begin{aligned} & P[\text{First Recession at time t}|\text{no recession from } t = 2, \ldots t-1] \\ = \; & P[C_{t-\mathsf{1}} < 0, C_t < 0|\text{rec}_{t-\mathsf{1}} = 0], t = 2, 3, \ldots, \end{aligned}$$

where $\mathrm{rec}_t = 1$ if $\mathrm{rec}_{t-1} = 1$ or $C_t < 0$ and $C_{t-1} < 0$ and $\mathrm{rec}_{t-1} = 0$.

In order to convert this sequence of conditional probabilities into probabilistic statements about the timing of the next recession, we need to multiply this term by the probability that no recession has hit before time t :

$$\begin{aligned} P[\mathrm{rec}_t &= 1|\mathrm{rec}_1 = 0, C^1] \\ &= P[C_{t-1} < 0, C_t < 0|\mathrm{rec}_{t-1} = 0, C^1]P[\mathrm{rec}_{t-1} = 0|\mathrm{rec}_1 = 0, C^1], \end{aligned}$$

where $C^1 = \{C_1, C_0, C_{-1}, \dots\}$.

This probability is itself conditional on the probability of the initial state, $P[\mathrm{rec}_1 = 0, C^1]$. Again, in a pure forecasting exercise the initial state would be determined by current conditions. In contrast, the initial state here is taken to be a draw from the stationary distribution of the common factor. However, in both cases we have $P[\mathrm{rec}_1 = 0] = 1$ by assumption.

This assumption defines a unique sequence of the probabilities indexed by the forecast horizon t and C^1. Here we assume that C^1 is a draw from the stationary distribution. This will place some weight on starting from a recession position, $C_1 < 0$ and $C_0 < 0$. If this probability were large, then the calculations below would underestimate the length of expansions as described above. The alternative would be to consider the initial condition to be the turning point from a recession to an expansion: $C_1 > 0$ and $C_{-1} < 0$ and $C_{-2} < 0$. This would provide a measure of the average length of an expansion. However, this measure would tend to underestimate the frequency of recessions, if the duration of recessions were similar to that of expansions, as discussed above.

As recessions become less likely, there will be little disagreement between the measures. Computationally, in the case that recessions are infrequent, it is much easier to start from the stationary distribution. Further, since we are making random draws from the posterior it is important to reduce the ambiguity of any measures calculated. We also examine the case where C^1 is given by the contemporaneous (2000Q4) estimate of the common factor.

Assuming that $t = 2, \dots$, then the probability of no recession before a certain date is given by:

$$\begin{aligned} &P[\text{No Recession thru periods } 2, 3, \dots s] \\ &= 1 - \sum_{t=2}^{s} P[C_{t-1} < 0, C_t < 0|\mathrm{rec}_{t-1} = 0]P[\mathrm{rec}_{t-1} = 0]. \end{aligned}$$

and $rec_1 = 0$, with initial conditions for C^1 from the stationary distribution.

The expected waiting time to the next recession, starting from the stationary distribution of the time series is then given by:

$$\sum_{s=1}^{\infty} P[\text{No Recession before } s].$$

Note that the first term in the sum is 1, since, by assumption, the earliest a recession can happen is after two periods.

This gives the expected waiting time to the recession event in quarters. One can invert this expected weighting time to obtain an approximate long run probability of recession at the quarterly frequency. This quantity multiplied by 4 to annualize would then be the approximate frequency of recessions assuming the majority of recessions were less than one year in length.

4.3.1 Simulation Techniques

Simulation methods can be used to calculate the expected time to recession from a known time series model. Consider, for example, a Gaussian autoregression of the form:

$$C_t = \alpha_0 + \phi_1 C_{t-1} + \cdots + \phi_p C_{t-p} + \sigma v_t,$$

One starts by constructing initial conditions from stationary distribution of the time series. Next, draws of the random shock v_t are propagated by the time series model for N periods. Suppose that J collections of length N are simulated. One forms the time series for each draw from the Gibbs sampler, φ:

$$R_n^j(\varphi) = 1[C_{n-1}^j < 0, C_n^j < 0 \text{ and } R_{n-1}^j = 0] + R_{n-1}^j(\varphi),$$

with $R_1^j(\varphi) = 0$ and then the sequence of averages:

$$\overline{R}_n^J(\varphi) = \frac{1}{J} \sum_{j=1}^{J} R_n^j(\varphi),$$

which converges to $P_\varphi[\text{rec}_n\text{=}1|\text{rec}_1 = 0]$ as $J \to \infty$ for each draw φ.

Now, as we increase the number of iterations I on the Gibbs sampler we have:

$$\frac{1}{I} \sum_{i=1}^{I} \overline{R}_n^J(\varphi_i) \to E[\overline{R}_n^J(\varphi)],$$

where the expectation is over the posterior draws of φ. Alternatively, for each draw φ we can form an expected recession time from:

$$\sum_{n=1}^{N}\left(1-\overline{R}_n^J(\varphi)\right),$$

and then examine the posterior distribution across the draws of φ. The posterior mean of this variable is clearly the same as

$$\sum_{n=1}^{N}\left(1-\frac{1}{I}\sum_{i=1}^{I}\overline{R}_n^J(\varphi_i)\right),$$

but it provides information on the variation in recession probabilities across draws. These posterior features are once again calculated for the period before and after the break.

In the case of current prediction, we also have to condition on the p most recent values of the common factor at each draw, and use the time series model for the period after the break. Thus, at each iteration of the Gibbs sampler we use the random draws of the parameters of the time series model and the current draws of the common factor to construct a time series model and, then, simulate J series of length N.

5 Priors and Results

5.1 Properties of the Prior Distribution

We focus on the choice of the hyperparameters of the Normal-inverted gamma priors for the common factor parameters. These priors are the most important for the interpretation of the sample evidence. We begin by eliciting a prior, which is relatively noninformative, but accords with our subjective prior beliefs. To simplify matters, it is assumed that the prior means and covariances are zero for all the conditional mean parameters in the model (i.e. the $\boldsymbol{\beta}_i$'s are centered over zero). The prior variance for the intercept is taken to be 4. We use a shrinkage prior on the autogressive coefficients with the first autogressive lag having a variance of 1 and subsequent variances reduced by 0.5^{p-1}.

In particular, we assume that the marginal prior variance for $\boldsymbol{\beta}_i$ is $E(\sigma_i^2)cB_{p+1}$, where $B_{p+1}$is a $(p+1)\times(p+1)$ diagonal matrix with $(1,1)'th$ element 4 and all other diagonal elements given by the shrinkage prior. Degrees of freedom ($\underline{\nu}$) for the inverted gamma priors are 3 for both before and after break to ensure the first two marginal prior moments exist for all parameters. The other hyperparameter of the inverted gamma prior is $\underline{s}$. This hyperparameter is defined so that $E(\sigma_i^2) = \frac{\upsilon s^2}{\underline{\nu}-2}$.

We set $\underline{s}^2 = 20/3$, $c = 1/20$. This implies a very flat prior for σ_i^2, but one that has mean 20. Since the data are measured as (annual) percentage changes (e.g. 1.0 implies a 1.0 percent change in the growth rate) and annualized, this choice of $\underline{s}^2$ is sensible in light of the typical U.S. fluctuations. In addition, it is centered well above the size of fluctuations observed recently.

By the symmetry of the prior distribution of the intercept around zero, the prior immediately places some of its weight on recessions happening in two quarters. If we fix the autoregressive lag at two, we can simulate from the prior imposing stationarity. This gives a prior mean for expected time to recession of 8 years. The median of the prior distribution is considerably lower at 2 years. In the posterior analysis, stationarity is not directly imposed in calculating the marginal likelihood, but it is imposed to generate the draws of the parameters of the common factor model. The Cumulative Distribution Function of the prior expected waiting time is shown in Figure 2. Note how relatively flat it is after 10 years.[8]

5.2 Results

The model is estimated with $q = 1, p = 2$. The growth rate of GDP is chosen as the variable with factor loading equal to unity. The Gibbs sampler was run with a burn-in phase of 1000 iterations and a further $10,000$ iterations. Figure 3 shows a recursive estimate of the Bayes factor for no break vs. a break. As it can be seen in the plot, after some initial imprecision the Bayes factor settles down and appears to have converged.

Figure 4 plots the posterior mean of the smoothed common factor (i.e., we averaged the estimates of the smoothed factor across posteriors draws). We can see from Figure 4 that the factor is negative during the NBER-dated recessions and positive elsewhere. The factor also appears to pick up the "growth recessions" of 1967 and the mid 1980s, as well as the recent abrupt slowdown in the United States.

[8] In practice, only 14 out of 10,000 draws violated the stationarity condition in the posterior sample.

Table 3 contains various correlations between the estimated common factor and the time series used to estimate it. The whole sample correlation is from the posterior mean of the common factor. The Before Break and After Break calculations are based on the average correlations across draws of the Gibbs Sampler. Over the whole sample, the factor has a correlation of 89% with GDP, 93% with industrial production and manufacturing output, and 84% with payroll employment. The correlation is lowest with consumption at 68%, and retail sales at 60%. Notice how these whole sample correlations are not good guides to the pattern of correlations after the break. In particular, measures of the "old economy," such as industrial production have become less correlated with the factor.

Table 3: Correlations with Common Factor

Series	Whole Sample	Before Break	After Break
GDP	0.89	0.89	0.88
Industrial Production.	0.93	0.95	0.82
Manufacturing	0.93	0.94	0.83
Income	0.76	0.81	0.72
Wage/Salary	0.79	0.91	0.66
Consumption	0.68	0.70	0.63
Retail Sales	0.60	0.62	0.50
Payroll Employment	0.84	0.85	0.81
Aggregate Hours	0.87	0.88	0.80

The estimated Bayes factor in favor of no break is $.00025$, (i.e., about $4000:1$ in favor of a break) as can be seen in Figure 3 considering the last value in the plot. This is a less overwhelming evidence than in Potter (2001), where the odds in favor of a break were found to be much higher. On one hand, one might expect that using more time series would strengthen the evidence in favor of a break if it was actually present. On the other hand, as discussed above, this is a very crude model for the break, since it imposes a simultaneous break in all the time series. Overall, there is strong evidence for a break, but it is quite possible that it did not occur simultaneous in all the series examined.

Given that a break did occur, the most likely location of the break is 1983/4, as can be observed in Figure 5. There is virtually no probability attached to breaks elsewhere. In particular, the previous breakpoint for growth rates in 1973, used in many studies, gets virtually zero probability.

The posterior mean of the ratio of the mean to standard deviation before the break is 0.783, which from the normal CDF implies 22% of quarters having negative growth. The posterior mean after the break is 1.305, which from the normal CDF implies only 10% of quarters having negative growth. However, while the posterior mean of the frequency of negative quarters before the break is in agreement at 22%, the posterior mean after the break is 12%, suggesting some draws of the common factor after the break had less stability. Since the observed average frequency of negative quarters in the dataset is 21% before the break located in the first quarter of 1984, and 10% after the break, we have a reasonably close match to sample averages.

The posterior mean of the spectrum before and after the break, shown in Figure 6, is hard to interpret because the after break spectrum has a considerable amount of power at very low frequencies. The relative changes are easier to see in Figure 7, which shows the log spectral ratio and log innovation variance for the common factor model. From this figure, we can see that the innovation is only about 50% as volatile after the break than before (this is very similar to the estimates of McConnell and Perez 2000). Figure 7 further indicates that this reduction in volatility is not uniform across frequencies. One can observe that there has been a more than proportional drop in the volatility of cycles at frequencies traditionally associated with the business cycle. However, Figures 6 and 7 indicate a proportionate **increase** in the volatility at both lower and higher frequencies, which is **absolute** in the case of low frequencies.

These results are in contrast to those for GDP alone, in which nearly all the reduction in volatility is due to a drop in the innovation variance (see Blanchard and Simon 2001). This result is also consistent with the changes in the volatility of consumption relative to GDP and wage income, shown in Tables 1 and 2. After the break, it can be seen that consumption has a very similar volatility to GDP and wage income, whereas its volatility is considerable lower before the break. This suggests that more of the fluctuations in GDP and wage income are permanent since 1984.[9]

Finally, we consider measures of the expected time to the next recession or frequency of recession. Figure 8 contains the cumulative probability of the first recession by year for the common factor model, before and after the break and for current conditions in 2000Q4. The difference between the CDFs

[9]Total personal income does not show quite the same pattern, but this is more difficult to interpret because of definitional issues regarding what is included in total income.

before and after break is large and similar to that found in Potter (2000). Note that neither the post-break CDF nor current CDF attain the value of 1. Instead, after 100 years they are only 0.96. In the simulations, the number of quarters forecasted (N) was 400. This suggests that the posterior mean of the expected time to recession after the break will be underestimated. Figure 9 contains the posterior distribution of the expected waiting time before the break.

The overall mean is estimated to be 4.5 years, compared to 5 years in the sub-sample before 1984. There is very little probability attached to expected waiting time greater than 10 years. Figure 10 contains the posterior distribution for the model after the break and for current conditions. The posterior mean after the break or currently is estimated to be 19.5 years (compared to the 1 recession experienced since 1984). In practice it turned out that the differences between the unconditional estimates and the current estimates were very small. This is consistent with the view of Hall (2001) that up to the end of 2000 there is little indication of the end of the expansion.

The underlying distribution of the probability of the next recession is skewed towards longer time periods. Thus, other measures of time to the next recession contain useful information. For example, the posterior median of the expected time to recession is 4 years before the break and is around 15 years after the break. The posterior probability that the expected time to recession is less than 9 years (the average of the last two expansions) is 0.23 after the break compared to 0.97 before the break. Finally, the probability that the next recession occurs before 10 years is 0.86 before the break and 0.50 after the break.

Overall, the results on the frequency of recessions are very similar to those found in Potter (2000) based on GDP alone, but the dynamics of the common factor are very different from that of GDP. This is mostly clearly observed in the large relative changes in the frequency domain. If we drop GDP out of the variables used to identify the common factor, the expected time to the next recession after the break falls to 14 years (the median to 10 years) and posterior odds in favor of a break drop to 25 : 1.

6 Conclusions

This paper has presented strong statistical evidence that the business cycle as traditionally described in terms of recession and expansions is dampening.

Although the statistical evidence presented here is more robust than previous claims regarding the possible end of the business cycle, it is still dependent on history providing an accurate view of the future.

However, history can be interpreted in more than one way. Clearly, there is overwhelming evidence that economic fluctuations have become less volatile in the United States. On the other hand, if the economy has experienced a structural break in favor of stability, one might reasonably argue that a break in the other direction towards increased instability could occur in the future.

For example, consider the conjecture that there is a probability p that we will return to the greater instability of the long historical record. Further, suppose that the movement to the increased instability will be caused by a recession. Then, this gives a probability of recession in the year 2001 of $p + (1 - p)0.05$, since the calculations above imply a current probability of recession within one year of approximately 5%. If one has a subjective belief in a recession in 2001of q, this would be supported by $p = (q - 0.05)/0.95$. For example, if one thought the recession probability was 2/9 (the historical record), then the probability of returning to greater instability would be about 16%. Alternatively, if one thought the probability of recession was 50%, then the probability of returning to greater instability would be a relatively high 48%.

The United States experience is somewhat different from other G-7 countries. In the 1950s and, to some extent, in the 1960s these countries did not suffer the same intensity of the business cycle as they were catching up to the United States. However, from the mid-1970s to the mid-1980s, the intensity of the business cycle was reasonably similar across these G-7 countries. This pattern has now reversed itself with the United States experiencing a more muted business cycle than other G-7 countries, particularly Japan. One could be more confident that the changes in the US business cycle were permanent if other G-7 countries also started to experience similar reductions in the volatility of their fluctuations. Blanchard and Simon (2001) find some limited evidence that this is true with the exception of Japan, but the evidence appears to be weaker than in the US.

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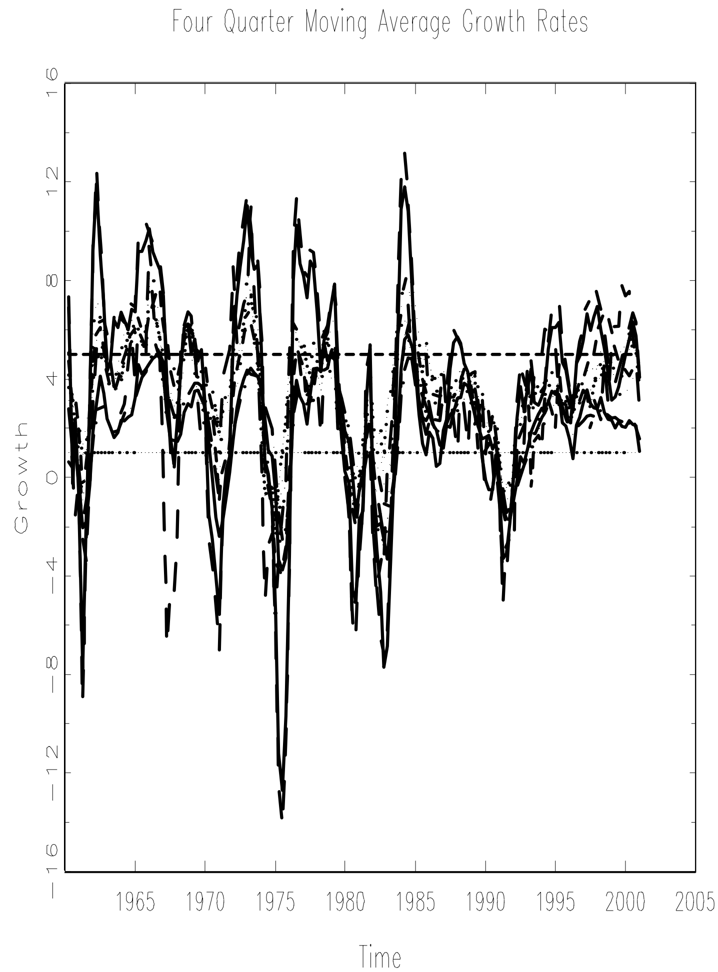

Figure 1: 4 quarter growth rates of the data set

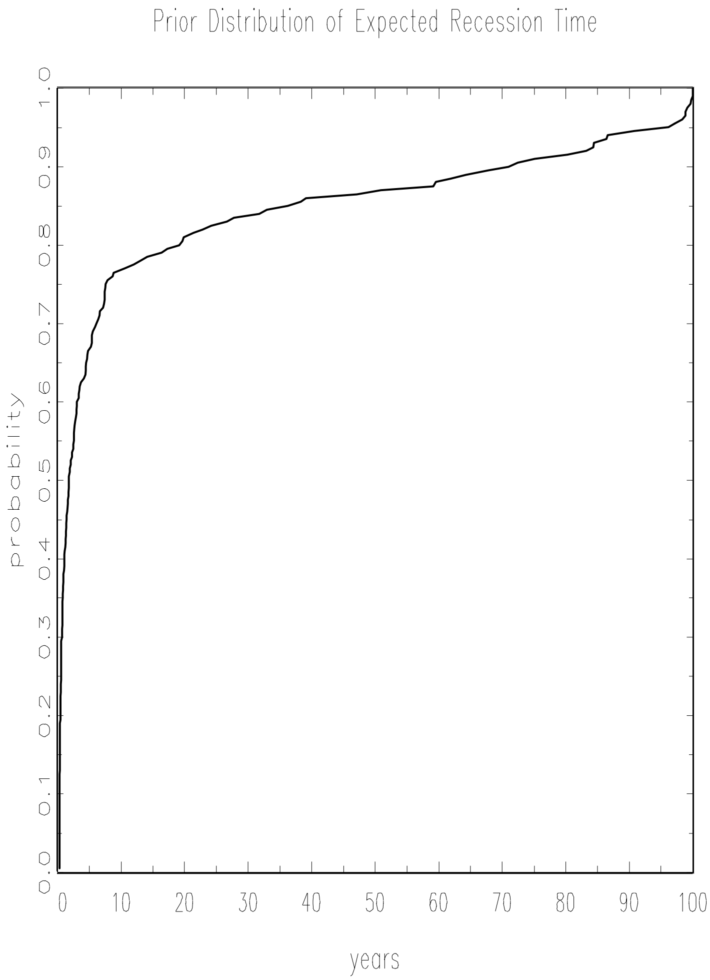

Figure 2: CDF of Common Factor Prior in terms of expected time to recession

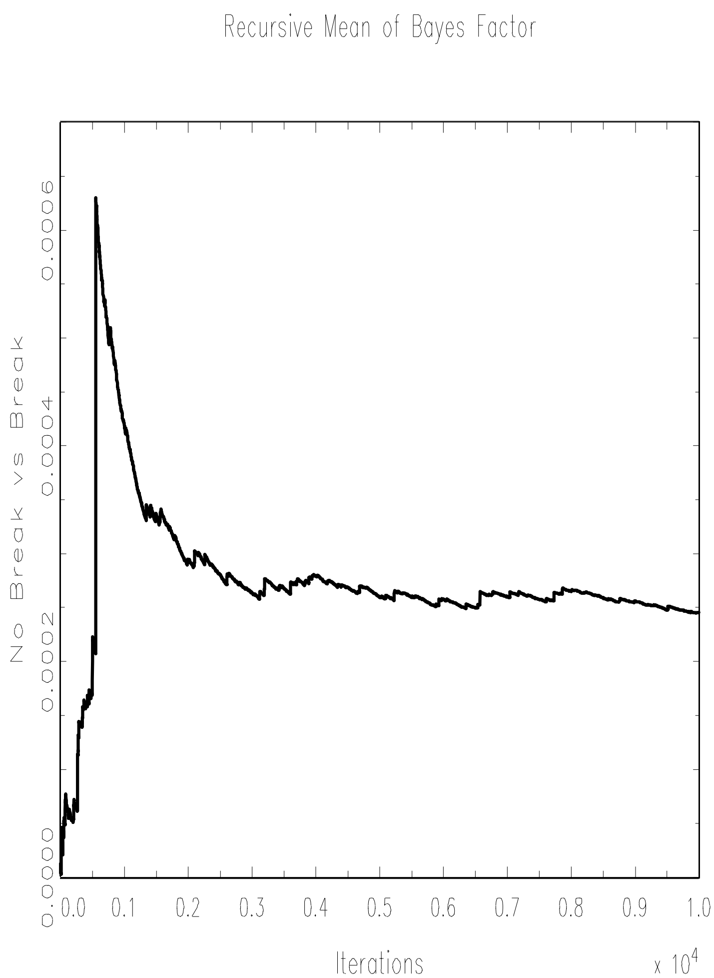

Figure 3: Evidence for Convergence of Gibbs Sampler

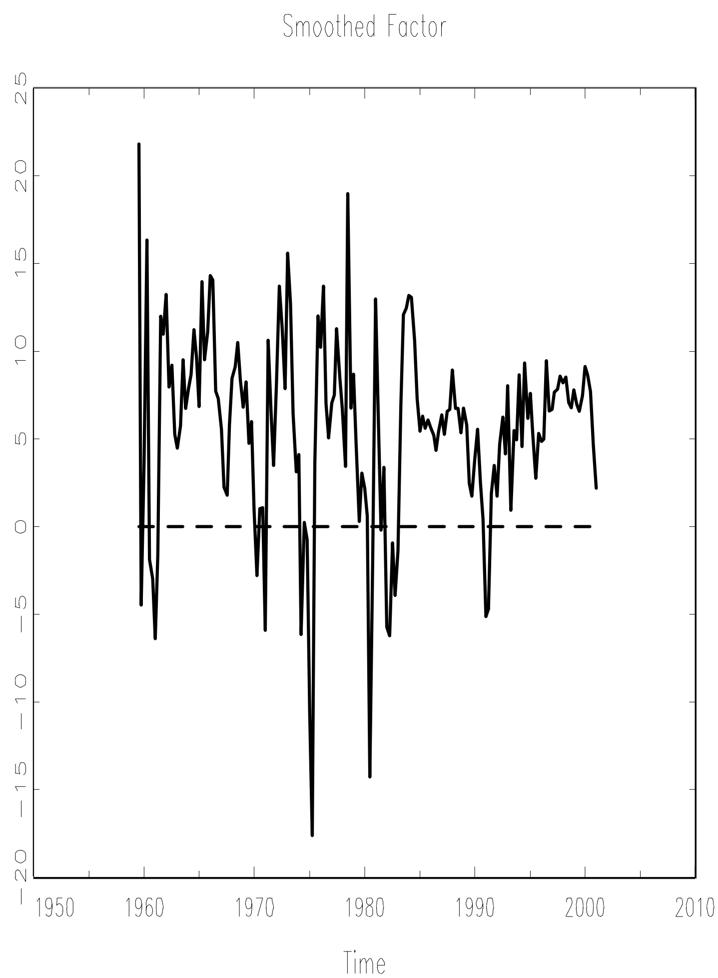

Figure 4: Posterior Mean of Common Factor

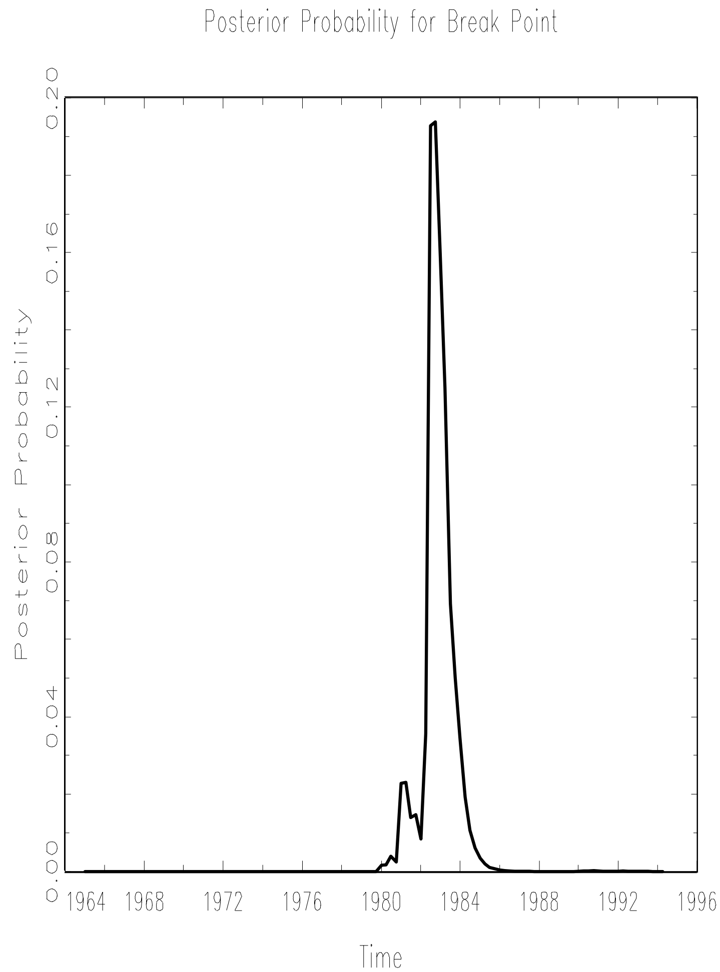

Figure 5: Posterior Distribution for Breakpoint

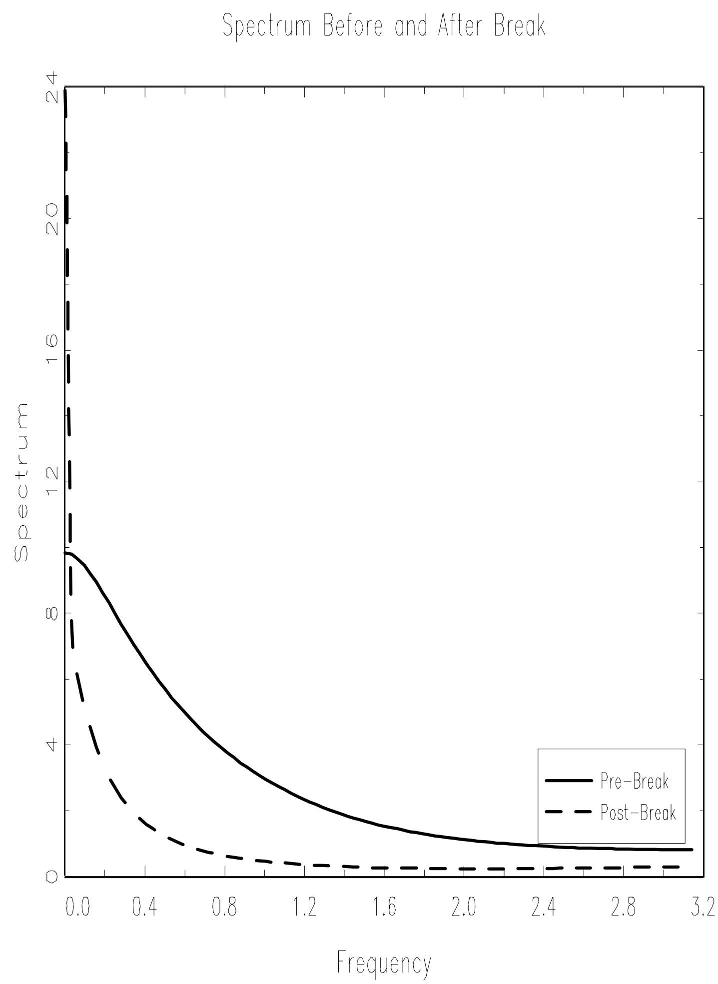

Figure 6: Spectra Before and After the Break

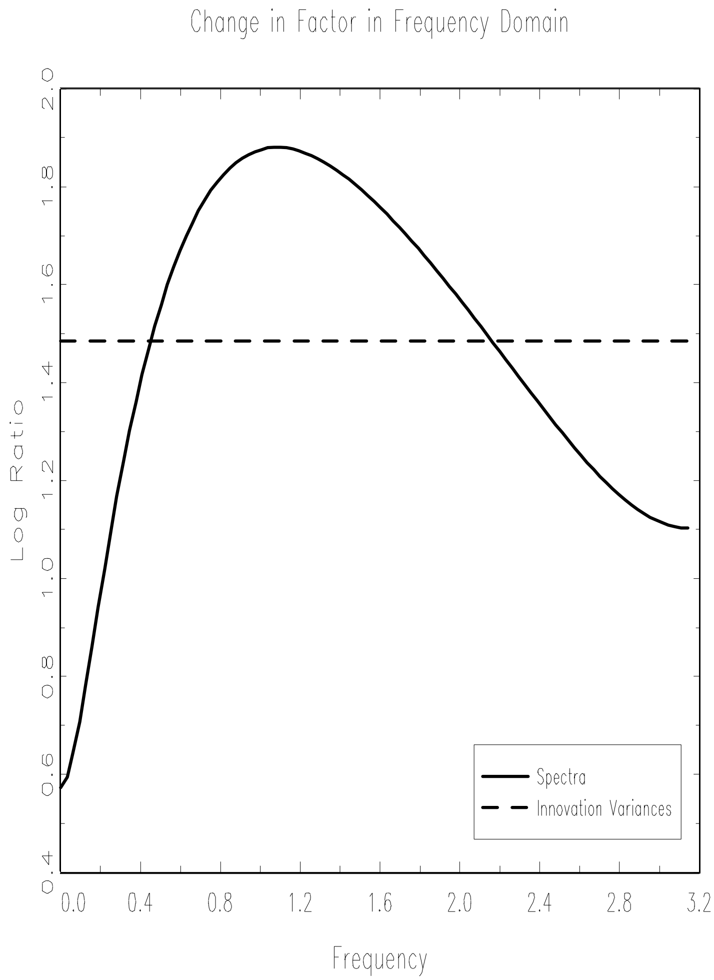

Figure 7: Log Ratio of Normalized Spectra

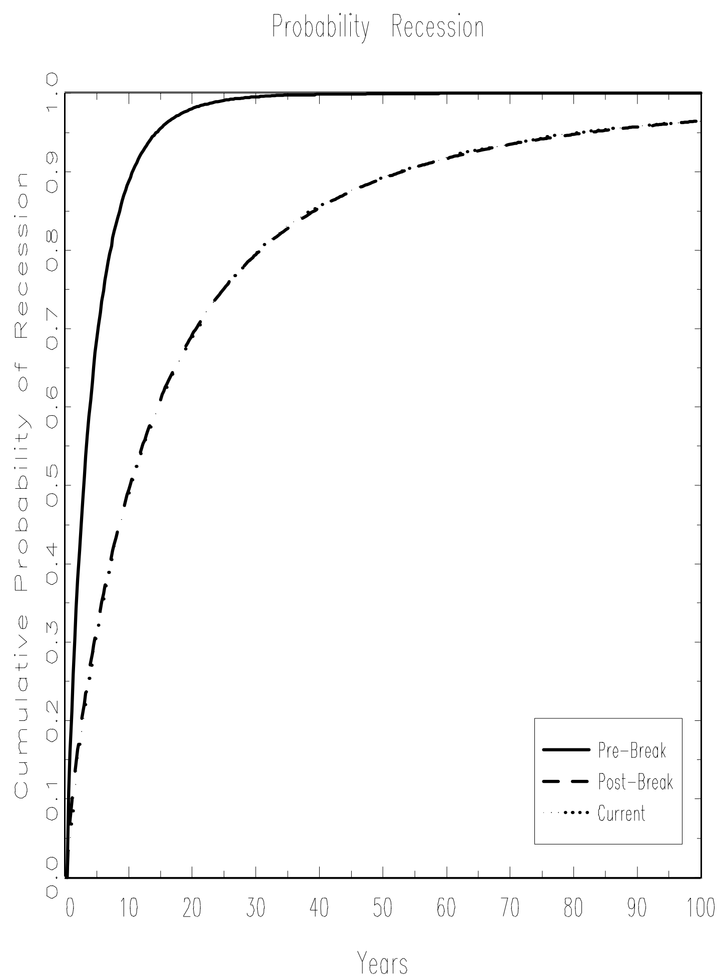

Figure 8: CDFs for Probability of Recession after x years

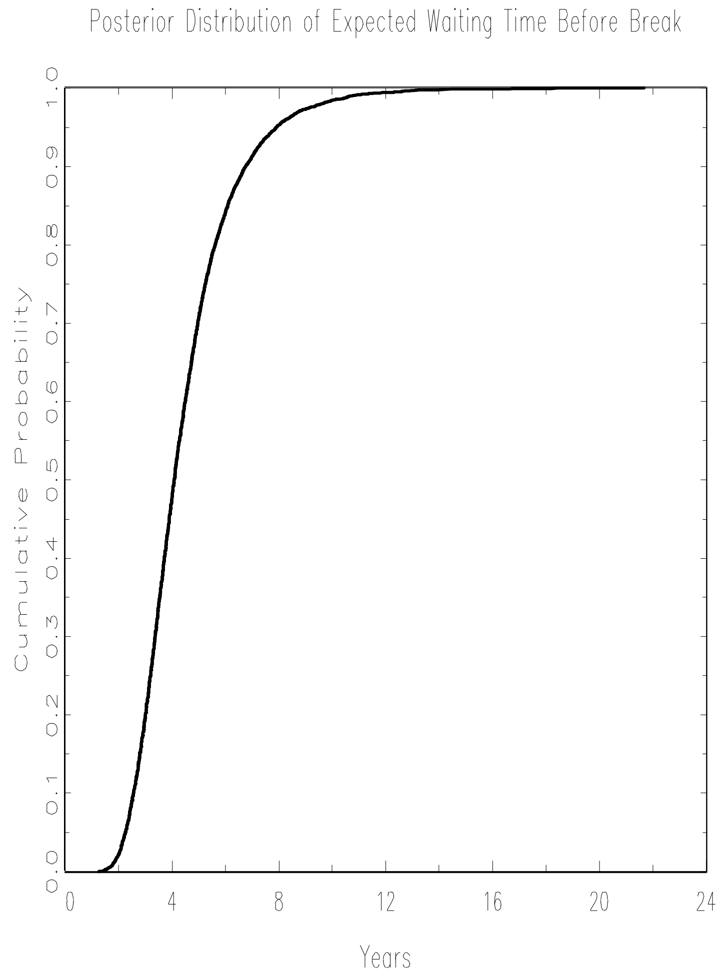

Figure 9: Posterior Dist. of Expected Waiting Time Before Break

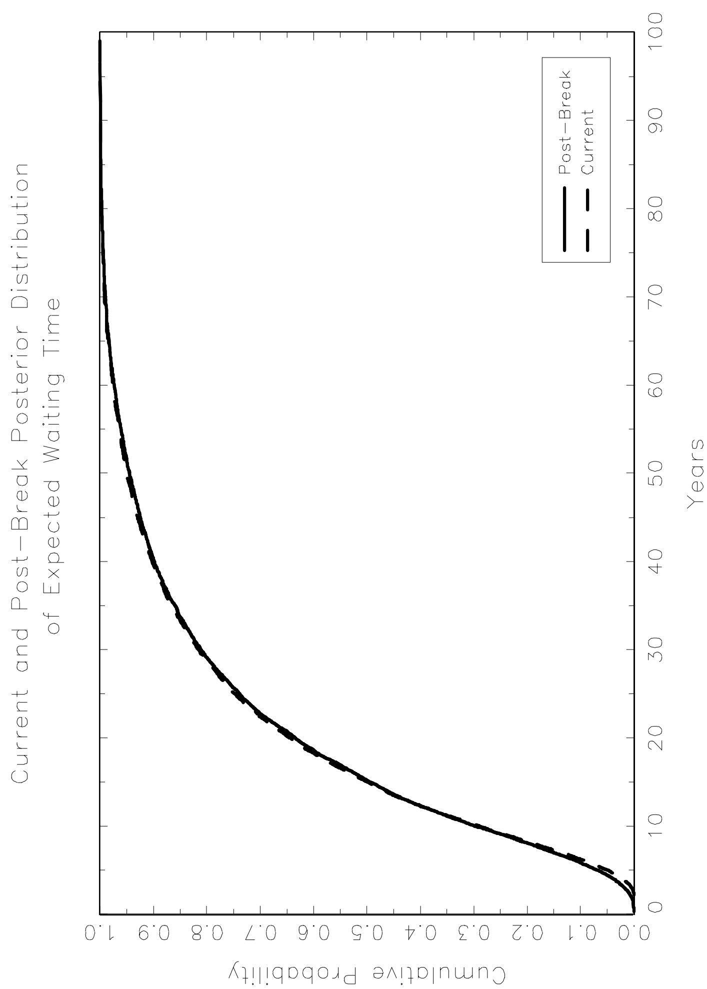

Figure 10: Post. Dist of Expected Waiting Time After Break and Current