On Both Sides of the Quality Bias in Price Indexes

Bart Hobijn
Federal Reserve Bank of New York*

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Abstract

It is often argued that price indexes do not fully capture the quality improvements of new goods in the market. Because of this shortcoming, price indexes are perceived to overestimate the actual price increases that occur. In this paper, I argue that the quality bias in price indexes is just as likely to be upward as it is to be downward. I show how both the sign and the magnitude of the quality bias in the most commonly applied price index methods are determined by the cross-sectional variation of prices per quality unit across the product models sold in the market.

I do so by simulating a model of a market that includes monopolistically competing suppliers of the various product models and a representative consumer with CES (constant elasticity of substitution) preferences. I illustrate the bias in the commonly applied price index methods by comparing their estimates of inflation with the theoretical inflation rate implied by the data-generating process.

Keywords: Price index theory, hedonic price methods, quality bias, monopolistic competition.
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* Correspondence: Bart Hobijn, Domestic Research Function, Federal Reserve Bank of New York, 33 Liberty Street, 3rd floor, New York, NY 10045. E-mail: bart.hobijn@ny.frb.org. The views expressed in this paper are those of the author and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System as a whole. I would like to thank Alexis Antoniades for his excellent research assistance.

1. Introduction

There is a widespread consensus among economists that price index methods tend to overestimate actual inflation in markets where there is a rapid turnover of goods due to technological progress. The Boskin (1996) commission made this point with respect to the U.S. Consumer Price Index, while Gordon (1990) used hedonic price indexes to correct for this bias in equipment price indexes.

There is, however, also a small number of studies that challenge this conventional wisdom. Studies by by Triplett (1972,2002), Feenstra (1995), as well as Hobijn (2001) have each made the point that quality adjustments in price index methods might actually lead to an understatement of inflation.

This paper follows up on the above papers by introducing a parsimonious theoretical model that can generate both a positive as well as a negative quality bias in the most commonly applied price indexes. The value added of this approach is that it allows for the study of the factors that determine the sign and magnitude of the quality bias in a stylized framework. This contrasts strongly with the methodology that is traditionally applied in the price index literature.

A large part of the literature on price indexes compares various price indexes calculated for the same dataset. This is for example the approach of Aizcorbe and Jackman (1993), Manser and McDonald (1988), and Braithwait (1980) when assessing the magnitude of substitution bias as well as of Aizcorbe, Corrado, and Doms (2000) and Silver and Heravi (2002) in the comparison of hedonic and matched model price indexes.

Such an approach allows us to consider the sensitivity of price indexes to the choice of method applied. It does not, however, enable us to make any normative statements about which index method is 'better' than another. Such normative statements on price indexes are all based on the extensive theoretical price index literature, which focuses on properties like idealness, exactness, and superlativity of price index formula.

It turns out that the theoretical results derived in this paper contradict some of the properties of price indexes that are presumed in this applied strand of the literature. Three results stand out in particular.

The most important is that the theoretical model in this paper confirms the claims by Triplett (1972,2002), Feenstra (1995), and Hobijn (2001) that the quality bias in price indexes is not by definition upward. Moreover, the sign and magnitude of the bias turn out *not to depend* on the overall level of inflation. Instead they depend on the cross-sectional behavior of prices per quality unit across models sold in the market during the same period.

Secondly, the existence and sign of this bias does not depend on the specific price index formula applied. I show how the application of the most popular price index formulas, like Laspeyres, Paasche, Geometric Mean, Fisher Ideal, and Tornqvist, all lead to similar magnitudes of the quality bias.

Finally, hedonic price indexes suffer from the same quality bias as matched model indexes. Hence, the theoretical results here seem to disagree with the presumption that hedonic price indexes do a better job at correcting prices for quality improvements, as made in, among many, Pakes (2002) and Hulten (2002).

The particular theoretical model that I use for my analysis in this paper is that of a market with a representative consumer with CES preferences over a set of models sold. This setup is very similar to Dixit and Stiglitz (1977) and Hornstein (1993). The main difference is that the market that I consider has a countably finite number of models and suppliers. The advantage of this choice of model is that price index theory for CES preferences is extremely well developed. Sato (1976) derived the ideal exact price index for CES preferences when the same models are sold in both the base- and measurement periods. Feenstra (1994) extended Sato's index to an exact matched model index that can be used when the universes of models sold in both periods do not coincide.

The resulting methodology in this paper is closely related to the Monte Carlo methodology in econometrics. In this sense, I follow Lloyd (1975) who also used numerical simulation methods to derive and quantify certain properties of price indexes. In Lloyd's (1975) study the focus was on the substitution bias in price indexes while here the focus is on their quality bias.

The structure of the paper is as follows. In the next section I introduce the form of the CES preferences that I consider in the rest of the paper and derive the theoretical price level that price indexes are meant to measure. In Section 3 I then illustrate graphically how conventional price index methods might yield a downward quality bias for these preferences. This graphical description is essentially an informal version of the results that are derived formally in the context of the theoretical model. I introduce this theoretical model in Section 4. I consider its demand and supply side and show how its Pure Strategy Nash equilibrium exists and is unique. In Section 5 I then proceed by deriving some general results for the sign of the quality bias in matched model and hedonic price indexes calculated for a specific parameterization of the model. Finally, in Section 6 I present the results of a set of numerical simulations of the model and illustrate how these simulations confirm the results shown in Sections 3 through 5. Section 7 concludes.

2. CES-preferences and the theoretical price level

The aim of this paper is to be able to make normative statements about price index methods and to say which ones perform better, in certain situations, than others. In order to make these normative statements we need to define what it is we would like our price index methods to measure. Since Konüs (1939) the main focus of price index theory has been on constructing a cost-of-living index (COLI). The aim of a COLI is to track the (percentage) changes in the minimum expenditures required to reach a certain base-level of utility over time[1].

The minimum amount of expenditures that is necessary to reach a certain utility level crucially depends on the underlying preferences of the consumer. Hence, the theoretical price level that price index methods are after depends on the preferences of the consumer. In reality, a market consists of a spectrum of consumers with different preferences. It turns out that it is not always possible, in such cases, to specify the theoretical price level because aggregate demand does not always behave as if it is generated by a well-behaved aggregate utility representation.

The focus of this paper is not on the conditions for the existence of an aggregate utility representation for aggregate demand. What I will do is simply use one of the best developed aggregate utility representations for which it has been proven that it can be interpreted as the aggregate utility function of a market with a continuum of heterogeneous agents. This aggregate utility representation is Constant Elasticity of

[1] I will focus on consumer price indexes throughout this paper. The theory presented in this paper is also applicable to producer price indexes, which are aimed at tracking the minimum cost required to obtain a base quantity of output over time.

Substitution (CES) preferences. Anderson, de Palma, and Thisse (1993) introduced the microfoundations of CES preferences and showed how they can be interpreted as the aggregate utility representation of a market consisting of a continuum of heterogeneous agents.

Let $X_{i,t}$ be the quantity consumed of good i at time t, where I will assume that good $i=0$ is the numeraire good. C_t is the universe of goods sold at time t. I will assume that aggregate demand in the market in the theoretical model behaves as if resulting from the utility maximizing decision of a representative consumer with the utility function

$$U_t = \left(\sum_{i \in C_t \setminus \{0\}} \left(a_i X_{i,t} \right)^{\frac{1}{1+\lambda}} \right)^{1+\lambda} X_{0,t}^{\alpha} \text{ where } \lambda>0 \text{ and } 0<\alpha<1 \tag{1}$$

This is a relatively standard CES utility function, where $\sigma \equiv (1+\lambda)/\lambda$ is the constant elasticity of substitution. The only non-standard features of (1) are that the quantities for goods $i \in C_t \setminus \{0\}$ are multiplied by a quality parameter a_i and that the numeraire good, $i=0$, is included.

Let $p_{i,t}$ be the price of a unit of good i. Since $i=0$ is the numeraire good, I will assume that $p_{0,t}=1$ for all t. In the rest of this paper, I will focus on the construction of a price index for the set of goods, which I will call models in the future, that are contained in the CES part of (1). That is, my focus is on the measurement of the price level of the set of models $i \in C^*_t$ where $C^*_t = C_t \setminus \{0\}$.

We are thus confronted with two sets of goods, i.e. the numeraire good and the models for which we would like to measure an aggregate price level. Diewert (2001) shows that, because the preferences in (1) are separable between $X_{0,t}$ and the other goods, the aggregate price level for the models $i \in C^*_t$ is well defined. In particular, aggregate demand for the models $i \in C^*_t$ will be as if it was generated by the representative agent maximizing the amount of utility obtained from these models for the expenditures solely on these models. This implies that the theoretical price aggregate for the set of models $i \in C^*_t$ is the CES price aggregate as applied in, among many, Dixit and Stiglitz (1977), Hornstein (1993), and Feenstra (1994). This aggregate, the value at time t of which I will denote by P_t^T, reads

$$P_t^T = \left[\sum_{i \in C_t^*} \left(p_{it} / a_i \right)^{-1/\lambda} \right]^{-\lambda} \tag{2}$$

It is a CES aggregate of the prices per quality unit for all models that are traded in the market. This price aggregate represents the money cost of a unit of utility obtained from the consumption of the competing varieties in the set of models C_t^*. This money cost does not depend on the base-level of utility because the preferences are homothetic.

The aim of price index methods is to construct an index that approximates, up to a constant, the path of P_t^T. In particular, the index methods are meant to estimate the period by period percentage change in P_t^T. Throughout this paper, I will focus on the percentage change in P_t^T between periods $t=0$ and $t=1$. I will refer to the percentage change in P_t^T between those two periods as the theoretical inflation rate and will denote it as

$$\pi^T = \frac{P_1^T - P_0^T}{P_0^T} \tag{3}$$

It represents the percentage change of the money cost of a unit of utility between periods 0 and 1.

If one would know all the preference parameters in (1) then it would not be difficult to calculate the theoretical inflation rate in (3). In practice, however, the preference parameters are not observed. That is, we do not exactly know the elasticity of substition, i.e. $\sigma \equiv (1+\lambda)/\lambda$. Neither do we know the quality embodied in each unit sold for each model, i.e. a_i. In fact, when we apply price index methods we do not even know by what preference representation aggregate demand is generated. There are basically two lines of thought here, which I will both pursue in this paper.

The first line assumes that aggregate preferences belong to a certain class and then uses this restriction to obtain an estimate of (3). For the CES preferences the index that exactly measures the theoretical inflation rate is the one derived by Sato (1976). The details of this index are described in Table 1. Sato's index is valid under the assumption that the universes of models sold in both periods are the same, such that $C_0^* = C_1^*$. It is a proper price index in the sense that it only depends on observables, namely expenditure shares and prices.

The requirement of coinciding sets of models being sold in both periods renders the Sato (1976) index inapplicable at many lower levels of aggregation. Many markets have a high rate of product turnover, as illustrated in Aizcorbe, Corrado, and Doms (2000) for the market for Intel CPU units and in Silver and Heravi (2002) for the market for laundry machines. Hence, it is thus essential to develop price index methods that allow for dynamic universes of models that change over time, i.e. $C_0^* \neq C_1^*$. Feenstra (1994) extends Sato's result to a quasi-index that is exact for CES preferences with non-overlapping universes of models. Feenstra's is a quasi-index because it depends on the unobserved elasticity of substitution, which has to be estimated to implement the index. It is described in Table 1 and I will discuss its intuition in more detail later on.

Price index theory is thus very well developed for CES preferences. We know the form of the exact indexes both when the universe of models is static as well as when it is dynamic. The problem is that in many practical cases it is a big leap to assume that demand is generated by aggregate CES preferences. This brings us to the second line of thought. This line is to construct price indexes that do not exactly measure (3) but instead reasonably approximate it for a very broad class of preferences.

This is the approach most commonly chosen for the calculation of aggregate statistics. Classical price index theory, among others Konüs (1939), Frisch (1936) and Fisher (1922), yielded many important results for the case in which the universe of models is static. Konüs (1939) introduced the concepts of a cost-of-living index and substitution bias in price indexes that price a fixed basket of goods. Frisch (1936) showed how Konüs's substitution bias result implied that for homothetic preferences the change in the true cost of living is bounded from above by the Laspeyres index and from below by the Paasche index. Fisher (1922) showed how the geometric mean of the Laspeyres and Paasche indexes constitutes an ideal index in the sense that both the price and quantity indexes have the same functional form.

A large part of the literature has focused on the question which price index formula approximates (3) in the 'most reasonable' way. Like Fisher (1922), Diewert (1976), as well as Lloyd (1975), Braithwait (1980), Manser and McDonald (1988), and Aizcorbe and Jackman (1993).

A much smaller part of the literature has focused on the construction of 'reasonable' approximations to (3) in case of dynamic universes of models. The problem when the universes of models are dynamic is that the prices of new goods are not observed in the first period, while the prices of obsolete goods are not observed in the second period. It is thus not possible to measure the percentage change in the prices between both periods for new and obsolete goods.

Two approaches are generally considered when dealing with this problem. The first, known as matched model indexes, makes specific assumptions about the relative price per quality unit of the new models versus the old models. These assumptions are such that they imply that the change in the overall price level can be estimated solely as a function of the price changes of the models that are sold in both periods, i.e. that are matched. Triplett (2002) contains an overview of the different matched model methods and the possible biases that they induce.

The second, known as hedonic price indexes, uses a regression model that relates the price of a model in a certain period to its characteristics to impute the unobserved prices for the new and obsolete models. This imputation completes the set of prices needed to apply conventional price index methods developed for overlapping universes of models. After the price imputation of the missing price observations, indexes are then constructed using conventional price index methods.

3. A graphical illustration of the main argument

The conventional wisdom is that the introduction and obsolescence of goods in a market would cause standard price index methods to overstate the actual inflation rate. The Boskin (1996) commission report as well as its recent reassessment by Lebow and Rudd (2001) both contain extensive descriptions of this conventional wisdom. There are three main reasons why this is argued to be the case. The first reason, designated *quality bias* by the Boskin (1996) commission, is that current price indexes do not properly capture the quality improvements embodied in new (or improved) models. By underestimating these quality improvements, price indexes will attribute too much of changes in expenditures to changes in prices rather than to changes in quantities. The second reason, designated *product bias* by the Boskin (1996) commission, argues that prices of new goods tend to drop faster than those of established models. Because new goods and models are only included in the sample of goods used to calculate the price index with a certain delay, the initial price drops early in the product cycle are not captured by current price indexes. Finally, there is the *substitution bias*. This bias is due to some price indexes, including the CPI and most price indexes calculated in Europe, being fixed weighted price indexes which do not capture the increases in welfare from consumers being able to substitute new goods for goods that they were previously consuming.

In the rest of this paper I will mainly focus on the *quality bias* and ignore issues related to the latter two sources of bias. In general it is hard to argue against statistical agencies including new models and goods more timely in their samples and reducing the potential sources of product bias. Furthermore, the issue of substitution bias is currently being addressed, at least for the U.S. CPI, by the joint publication of a fixed weighted as well as a chain weighted price index. The latter is meant to account for the substitution bias. See Bureau of Labor Statistics (2002) for a detailed description.

The main point of this paper is that the *quality bias* in price indexes is not solely a source of upward bias. Instead, the quality bias induced by most commonly applied price index methods can be both upward as well as downward. Before I illustrate this in a formal mathematical economic framework, I first describe the main intuition of the argument graphically in this section. The graphical description in this section is based on Figure 1 through Figure 3.

The top panel of Figure 1 depicts two hypothetical price schedules, for $t=0$ and $t=1$, of a set of models that differ according to their quality levels, a_i. I will assume that a_i is not directly observed. Therefore, the researcher observes the price of each model, i.e. p_{it}, but does not know its relative position on the x-axis. As explained in the previous section, what is important for the price level associated with the CES preferences that I consider is not the actual price levels, p_{it}, but the price per unit of quality, p_{it}/a_i, for each model. Panel (b) of Figure 1 depicts the associated schedule of prices per quality unit. Panel (c) contains the same price per quality unit schedule and adds some of the notation that I will use in the rest of this paper.

Just like in the previous section C^*_t denotes the set of models sold in period t, while P_t^T denotes the theoretical price level at time t. Note that I have chosen to draw the example such that $P^T_0<P^T_1$. That is, in the graphical example the actual price level increases between periods $t=0$ and $t=1$, such that there is positive inflation. In each period the set of models sold, i.e. C^*_t, consists of a group of models that are not sold in the other period, i.e. the set A_t-B_t, as well as a group of models that are 'matched' in the sense that they are sold in both periods, i.e. the set B_t-D_t.

What I will now illustrate is that, even though the theoretical price inflation is positive for these hypothetical price schedules, most commonly applied price index methods will tend to measure negative inflation instead. That is, in this graphical example standard price index methods will tend to *underestimate* actual inflation rather than overestimate it, as the consensus view suggests. I will illustrate this for both matched model as well as hedonic price indexes.

There are several ways in which matched model indexes are calculated. They each make different identifying assumptions about the relative price per quality unit of the obsolete and new models in the market.

The first method, often referred to as 'direct comparison', assumes that the obsolete and new models can be directly compared in the sense that they embody the same levels of quality. Because this method assumes that there are no quality improvements between the old and new vintages of models, this method is never applied in markets with rapid product turnover due to technological progress, like those for computers and other electronic products for example. Because I will focus on markets with quality improvements in the products sold, I will disregard this method in the rest of this paper.

The second method, known as 'link-to-show-no-price-change', assumes that the price per quality unit is the same for the obsolete and new models. In this case, the relative price of the obsolete and new models is assumed to be fully attributable to quality improvements. Aizcorbe (2001) uses this assumption for example to identify the parts in semiconductor price changes attributable to quality changes and price changes respectively. Note that, as Triplett (2002) describes in more detail, this method *overestimates* inflation only when the price per quality unit of the new models is lower than that of the obsolete models. In that case the method overestimates the price per quality unit for the new models and thus will overestimate inflation. The reverse is true in our graphical example here. In the example the price per quality unit is higher for the new models than for the old models. Consequently, the method will overestimate quality changes and *underestimate* the actual level of inflation.

The final matched model method that is frequently applied is the "Implicit Price – Implicit Quantity"[2] method (IP-IQ). This method is based on the identifying assumption that the overall price change equals the price change in the set of matched models. When one makes this assumption, the price levels of the unmatched models are not needed to measure inflation. Hence, in this case the unmatched models are ignored, i.e. "deleted", and standard price index methods are applied to the set of models that is sold in both periods, i.e. the matched models.

Figure 2 illustrates the application of the IP-IQ method in our graphical example. The set of matched models in the example is the intersection of C^*_0 and C^*_1. Consequently, the IP-IQ method will compare the B_0-D_0 part of the period *0* price schedule with the B_1-D_1 part of that of period *1*. For all models in this range the prices are falling. The IP-IQ method will thus, incorrectly, find a drop in the overall price level. The simplest way to see why the IP-IQ method underestimates inflation in this case is to compare the relative prices of the deleted sections A_0-B_0 and A_1-B_1 with the matched parts of the price schedules.

For the deleted part A_0-B_0 in period *0* we obtain that the prices per quality unit are lower than the prices per quality unit on the matched part of the schedule, B_0-D_0. Consequently, the deletion of the below average prices on the A_0-B_0 part of the price schedule will lead to an inferred price level in period *0* that is higher than the actual level. Similarly, when the above average prices of part A_1-B_1 are deleted in period *1*, the prices of the matched models, i.e. B_1-D_1, reflect a price level that is lower than the actual price level. That is, because the price per quality unit is increasing in quality and the worst models become obsolete while the new models are of the highest quality, the price level in period *0* is overestimated while the price level in period *1* is underestimated. The combination of these two measurement errors leads to an unambiguous downward bias in the measured inflation rate, independent of which price index formula is applied.

One final thing is worth noting about this argument. That is that the bias incurred due to the application of the IP-IQ method does not depend on the overall inflation rate. Instead, it completely depends on the cross-sectional behavior of the prices per quality unit as a function of the quality units embodied in the models sold in the market. I will show this in a numerical example later on. This result contrasts sharply with the argument in Triplett (2002) who argues that "The errors produced by the IP-IQ method are symmetric, in the sense that when prices are falling the IP-IQ method tends also to miss price declines. … Prices have generally been falling for electronic products, including IT products. When the IP-IQ method is used to construct price indexes for electronic products, the price indexes are biased upward because they do not adequately measure price declines that accompany new introductions". The example here suggests that what matters for the IP-IQ bias in IT product inflation is not whether prices are *declining over time* but rather whether prices per quality unit are *declining in the amount of quality embodied* in the models.

[2] This is also often referred to as the "deletion" method.

Hedonic price indexes are, in some sense, the opposite of IP-IQ matched model indexes. That is, where the IP-IQ method 'deletes' the observed prices of the unmatched models, hedonic methods 'insert' the unobserved prices of the unmatched models. This insertion, or more correctly 'imputation', is done by estimating a hedonic price equation that relates the price of a model in a particular period to a set of its quality characteristics and then using this equation to predict what the unobserved prices of the unmatched models would have been.

Over the past five years, hedonic price indexes have been implemented for an increasing number of goods for U.S. aggregate statistics. See Landefeld and Grimm (2000) as well as Moulton (2001), for example, for a discussion of the application of hedonic price indexes in the U.S. national accounts. The main reason why hedonic price indexes are adopted for an increasing number of goods is the practical problem that the IP-IQ method ends up not using a large part of the available price quotes in markets where new and obsolete models make up the bulk of models traded. This is particularly a problem for computers and related equipment.

The believe is that by taking the price data for the obsolete and new models into account and relating them directly to quality characteristics, hedonic price indexes more properly adjust for quality and are less subject to quality bias. This seems to be confirmed by the fact that hedonic price indexes tend to find less inflation for most of the goods to which they are applied[3] than standard matched model indexes, which are said to overestimate inflation.

Is it true that hedonic price indexes have a smaller quality bias than matched model indexes? Not necessarily. In order to see why not, consider Figure 3. Which prices are imputed in a hedonic price index depends on the price index formula applied. The two panels of Figure 3 depict the two most common cases.

The top panel considers a hedonic Laspeyres index, which intends to measure the percentage change in the cost of the models sold in period *0*. The Laspeyres index requires the use of the prices of the models that became obsolete in period *1*. Therefore, a hedonic regression model is used to impute these prices and the price schedule in period *1* is extended by the imputed part D_1-E_1. The Laspeyres index then basically approximates the change in the overall price levels implied by the curves A_0-D_0 and B_1-E_1. The overall price level implied by A_0-D_0 coincides with the actual price level in period *0*, i.e. P_0^T. The price level implied by B_1-D_1, denoted by P_1^{HL} in the figure, is lower than the actual price level in period *1*. The reason is that for the calculation of the Laspeyres index the above average prices per quality unit in the part A_1-B_1 are ignored. Moreover, the imputation adds below average prices per quality unit in the section D_1-E_1. Hence, the inferred price level in period *1* is below the actual price level and inflation is underestimated. In fact, because the A_0-D_0 schedule is above the B_1-E_1 schedule everywhere, in this example the hedonic price index would find spurious price deflation.

The bottom panel depicts the calculation of a hedonic Paasche index. It is meant to approximate the change in the cost of the models sold in period *1*. Therefore it requires the imputation of the D_0-E_0 part of the price schedule in period *0* and ignores the part A_0-B_0 in its calculation. The imputed part D_0-E_0 consists of above average prices per quality unit and the ignored part A_0-B_0 of below average prices per quality unit.

[3] See for example Gordon's (1990) hedonic price indexes.

This leads to the hedonic method overestimating the price level in period *0*. This estimate is denoted by $P^{HP}{}_{0}$. Again, the hedonic method will find spurious deflation. This time because it overestimates the price level in period *0*, rather than underestimates the price level in period *1*.

Thus, my tentative graphical example illustrates why matched model and hedonic methods might actually result in estimates of inflation that are *too low* rather than *too high*. However, this simple graphical example can only be used for illustration purposes, it does not prove that such biases might occur in the data. In order to show that these biases are likely to occur, I introduce a fairly standard theoretical model in the next section and show how the equilibrium outcome of the model gives rise to biases of the same kind discussed here.

4. Theoretical model

The aim of this section is to introduce a simple theoretical framework that generates the kind of bias that I discussed in the section above. The theoretical framework introduced here is based on the CES model first considered by Anderson, de Palma, and Thisse (1992). Feenstra (1995) applied this model to hedonic price indexes. I will introduce the theoretical model in three subsections. The first explains the demand side of the market, while the second focuses on the supply side of the market. In the final subsection, I will prove existence and uniqueness of the Pure Strategy Nash equilibrium that determines prices and quantities in the market and will derive some of the relevant comparative statics for this equilibrium.

Demand side of the market

The aggregate demand in this market can be represented as generated by a representative agent choosing the demand $\{X_{i,t}\}_{i\in C_t}$ to maximize the aggregate utility function in equation (1). This utility function is maximized subject to the budget constraint

$$Y_t = X_{0,t} + \sum_{i\in C_t^*} p_{i,t} X_{i,t} \tag{4}$$

where Y_t denotes real income in terms of the numeraire commodity.

The maximization of this utility function yields the demand functions

$$X_{i,t} = \left(\frac{\widetilde{Y}_t}{p_{i,t}}\right)\left(\frac{(p_{i,t}/a_i)^{-1/\lambda}}{\sum_{j\in C_t^*}(p_{j,t}/a_j)^{-1/\lambda}}\right) = \left(\frac{\widetilde{Y}_t}{p_{i,t}}\right)\left(\frac{(p_{i,t}/a_i)^{-1/\lambda}}{(P_t^T)^{-1/\lambda}}\right) \tag{5}$$

for the non-numeraire commodity, i.e. $i\in C^*{}_t$. The variable $\widetilde{Y}_t = Y_t/(1+\alpha)$ is the level of total expenditures on these models. These demand functions are very similar to the ones implied by standard CES preferences where the level of quality for all goods is the same, i.e. $a_i=1$ for all $i\in C^*{}_t$. The main difference is that the relevant relative price of each good that determines its market share is its price per unit of quality, that is p_{it}/a_i, rather than its unit price, p_{it}.

Supply side of the market

The next concern is the supply side of the market for $i \in C^*_t$. I will assume that the producer of model i at each point in time, t, faces a constant unit production cost c_{it}. I will consider pure strategy Nash equilibria in prices for a market with a fixed set of models, $\mathbf{a}_t = \{a_i\}_{i \in C^*_t}$. Such Nash equilibria imply that the supplier of model i takes as given the prices p_{jt} for $j \in C^*_t \backslash \{i\}$ and chooses its price p_{it} to maximize its profits

$$(p_{it} - c_{it})X_{it} \tag{6}$$

subject to the demand function (5). The profit maximizing choice of price p_{it} in this case satisfies the following first order condition.

$$\frac{c_{it}}{p_{it}} = 1 + \left[\frac{\partial X_{it} / \partial p_{it}}{X_{it} / p_{it}}\right]^{-1} \tag{7}$$

This condition implies that the supplier of each model chooses its price such that its cost-price ratio equals one plus the inverse of the own price elasticity of demand for good i.

Since the own price elasticity of demand for good i is negative, this implies that $c_{it}/p_{it}<1$. That is, price exceeds marginal and average cost and the firm charges a markup. For the price elasticity of demand, we obtain that

$$\frac{\partial X_{it} / \partial p_{it}}{X_{it} / p_{it}} = -\left(1 + \frac{1}{\lambda}\left(1 - \frac{(p_{it}/a_{it})^{-1/\lambda}}{\sum_{j \in C^*_t}(p_{jt}/a_{jt})^{-1/\lambda}}\right)\right) = -\theta_i(\mathbf{p}_t, \mathbf{a}_t) \tag{8}$$

where $\theta_i(\mathbf{p}_t, \mathbf{a}_t) > 0$ is the negative of the price elasticity of demand for good i and $\mathbf{p}_t = \{p_{it}\}_{i \in C^*_t}$ is the sequence of prices charged in the market. Essential for the results that are to follow is that this elasticity is specific to good i. This is contrary to the setup of monopolistic competition that is often used to model imperfect competition in models with price rigidities, like in Hornstein (1993). These models generally consider a symmetric equilibrium in which each monopolistic competitor is too small to affect the aggregate price level and its own price elasticity of demand.

Using the notation above, the supplier of good i will set its price such that

$$\frac{p_{it}}{c_{it}} = \frac{1}{1 - \theta_i(\mathbf{p}_t, \mathbf{a}_t)^{-1}} = \frac{\theta_i(\mathbf{p}_t, \mathbf{a}_t)}{\theta_i(\mathbf{p}_t, \mathbf{a}_t) - 1} = \mu_i(\mathbf{p}_t, \mathbf{a}_t) \tag{9}$$

where $\mu_i(\mathbf{p}_t, \mathbf{a}_t) > 1$ is the markup charged by the firm. Solving for this markup yields that

$$\mu_i(\mathbf{p}_t, \mathbf{a}_t) = (1 + \lambda) + \lambda \frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j \in C^*_t \backslash \{i\}}(p_{jt}/a_j)^{-1/\lambda}} \tag{10}$$

This implies that the pure strategy Nash equilibrium in this market satisfies the following system of equations

$$p_{it} = \left[(1+\lambda) + \lambda \frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j \in C_t^* \backslash \{i\}} (p_{jt}/a_j)^{-1/\lambda}} \right] c_{it} \text{ for all } i \in C^*_t \tag{11}$$

This system of equations will be the center of attention in what is to follow.

Equilibrium

Now that I have derived the conditions for a Pure Strategy Nash equilibrium in equation (11), the question that remains is whether there exists a set of prices the satisfies this equation. In this section I will not only show that this is the case, but also prove the uniqueness of this equilibrium price schedule. I will then proceed by deriving some of its comparative statics that are relevant for the price index measurement results that I will prove later on.

First and foremost though, it is important to realize that the Pure Strategy Nash equilibrium in prices that I consider actually exists and is unique. This is what I prove in the following proposition.

Proposition 1: ***Existence and uniqueness of equilibrium***

For any $\lambda>0$ and sequences $\mathbf{a}_t = \{a_i\}_{i \in C_t^*}$ and $\mathbf{c}_t = \{c_{it}\}_{i \in C_t^*}$ where $a_i, c_{it}>0$ for all $i \in C^*_t$ there exists a unique pure strategy Nash equilibrium in prices.

Proof: (*existence*) Existence of the pure strategy Nash equilibrium follows from the application of Brouwer's fixed point theorem. In order to see how Brouwer's fixed point theorem applies here, it is most convenient to define $c^*_{it}=c_{it}/a_i$. Rewrite the system of equations that defines the equilibrium, i.e. (11), in the form

$$\left(\frac{p_{it}}{c^*_{it} a_i} \right) = \left[1 + \lambda \frac{\sum_{j \in C_t^*} (c^*_{jt})^{-1/\lambda} (p_{jt}/c^*_{jt} a_j)^{-1/\lambda}}{\sum_{j \in C_t^*} (c^*_{jt})^{-1/\lambda} (p_{jt}/c^*_{jt} a_j)^{-1/\lambda} - (c^*_{it})^{-1/\lambda} (p_{it}/c^*_{it} a_i)^{-1/\lambda}} \right] \text{ for all } i \in C^*_t \tag{12}$$

which implies that

$$\left(\frac{p_{it}}{c_{it}} \right)^{-1/\lambda} = \left[1 + \lambda \frac{\sum_{j \in C_t^*} (c^*_{jt})^{-1/\lambda} (p_{jt}/c_{jt})^{-1/\lambda}}{\sum_{j \in C_t^*} (c^*_{jt})^{-1/\lambda} (p_{jt}/c_{jt})^{-1/\lambda} - (c^*_{it})^{-1/\lambda} (p_{it}/c_{it})^{-1/\lambda}} \right]^{-1/\lambda} \text{ for all } i \in C^*_t \tag{13}$$

Let N_t be the number of elements of C^*_t, i.e. the number of competing models in the market at time t. 0Define

$$v_{it} = \left(p_{it} / c_{it}\right)^{-1/\lambda} \text{ and the space } V_t = \left\{ \{v_{it}\}_{i \in C_t^*} \in \mathbf{R}^{N_t} \middle| 0 \le v_{it} \le 1 \right\} \tag{14}$$

then (13) defines a continuous mapping from V_t to V_t and thus, according to Brouwer's fixed point theorem must have a fixed point. Hence, there must exist an equilibrium.

(*uniqueness)* Define

$$z_{it} = \left(c_{it}^*\right)^{-1/\lambda}, \; w_{it} = z_{it} \left(\frac{p_{it}}{c_{it}} \right)^{-1/\lambda}, \text{ and } W_t = \sum_{i \in C_t^*} w_{it} \tag{15}$$

then (13) can we rewritten as

$$w_{it} = z_{it} \left[1 + \lambda \frac{W_t}{W_t - w_{it}} \right]^{-1/\lambda} \quad \text{for all } i \in C^*_t \tag{16}$$

Given W_t, for all $i \in C^*_t$, there is one unique $w_{it} \in \mathbf{R}_+$ that solves (16). This follows from a straightforward application of the intermediate value theorem to (16). Define the function $f: [0, W_t] \to \mathbf{R}_+$ as

$$f(w_{it}) = w_{it} - z_{it} \left[1 + \lambda \frac{W_t}{W_t - w_{it}} \right]^{-1/\lambda} \tag{17}$$

then $f(w_{it})$ is continuous and strictly increasing. Furthermore, $f(0) = -z_{it}[1+\lambda]^{-1/\lambda}$ and $f(W_t) = W_t$. Hence, the intermediate value theorem implies that there must be a unique $w_{it,} \in [0, W_t]$ for which $f(w_{it}) = 0$.

Suppose the equilibrium is not unique, then there exist W_t and W'_t such that $W_t > W'_t = (1+\delta) W_t$, where $\delta > 0$, such that

$$W_t = \sum_{i \in C_t^*} w_{it} \text{ and } W_t' = \sum_{i \in C_t^*} w_{it}' \tag{18}$$

and W_t and w_{it} for all $i \in C^*_t$ satisfy (16), which is also true for W'_t and w'_{it} for all $i \in C^*_t$.

Note that the reason that W_t and W'_t can not be the same is because I have shown above that the same W_t will lead to the same best response by the suppliers of all models and thus to the same equilibrium.

What I will show in the following is that if (16) holds for W_t and w_{it} for all $i \in C^*_t$, then for all $i \in C^*_t$ it must be the case that the w'_{it} that satisfies (16) given W'_t has to satisfy $w'_{it} < (1+\delta) w_{it}$. However, this would imply that $W'_t < (1+\delta) W_t = W'_t$ which is a contradiction.

In order to see this, suppose that $w'_{it} \ge (1+\delta) w_{it}$. In that case, equation (16) implies that

$$\begin{aligned} w_{it}' &= z_{it} \left[1 + \lambda \frac{W_t'}{W_t' - w_{it}'} \right]^{-1/\lambda} = z_{it} \left[1 + \lambda \frac{(1+\delta) W_t}{(1+\delta) W_t - w_{it}'} \right]^{-1/\lambda} \\ &\le z_{it} \left[1 + \lambda \frac{(1+\delta) W_t}{(1+\delta) W_t - (1+\delta) w_{it}} \right]^{-1/\lambda} = z_{it} \left[1 + \lambda \frac{W_t}{W_t - w_{it}} \right]^{-1/\lambda} = w_{it} \\ &< (1+\delta) w_{it} \le w_{it}' \end{aligned} \tag{19}$$

which is a contradiction. Hence, there can only be one equilibrium. γ

The benchmark case, and as it turns out the only one in which standard price index methods do not generate a bias, is the case in which each supplier faces the same unit production cost per quality unit. As I show in the proposition below, the price per quality unit is the same for all models in the market in that case.

Proposition 2: ***Symmetric equilibrium***

The market has a symmetric equilibrium in which the price per quality unit is constant across models, i.e. $p_{it}=p^*_t a_i$ *for all* $i\in C^*_t$*, if and only if the producer of each model faces the same marginal unit production cost per quality unit, i.e.* $c_{it}=c^*_t a_i$*.*

Proof: (⇒) If $c_{it}/a_i=c^*_t$ and does not depend on i, then (11) reduces to

$$\left(\frac{p_{it}}{a_i}\right)=\left[1+\lambda+\lambda\frac{\left(p_{it}/a_i\right)^{-1/\lambda}}{\sum_{j\in C_t^*\setminus\{i\}}\left(p_{jt}/a_j\right)^{-1/\lambda}}\right]c_t^* \text{ for all } i\in C^*_t \tag{20}$$

When we choose $p_{it}/a_t=p^*_t$ for all $i\in C^*_t$ and substitute it in the system of equations (20) we obtain that for all $i\in C^*_t$

$$\begin{aligned} p_t^* &= \left[1+\lambda+\lambda\frac{p_t^{*-1/\lambda}}{\sum_{j\in C_t^*\setminus\{i\}} p_t^{*-1/\lambda}}\right]c_t^* \\ &= \left[1+\lambda\frac{N_t}{N_t-1}\right]c_t^* \end{aligned} \tag{21}$$

which does not depend on i. Hence, if $c_{it}/a_i=c^*_t$, then $p^*_t=(1+\lambda N_t/(N_t-1))c^*_t$ is the symmetric pure strategy Nash equilibrium in which all suppliers charge the same price per quality unit and all have an equal market share.

(⇐) If there is a symmetric equilibrium, then for all $i\in C^*_t$

$$\left(\frac{p_{it}}{a_i}\right)=p_t^*=\left[1+\lambda\frac{N_t}{N_t-1}\right]\left(\frac{c_{it}}{a_i}\right) \tag{22}$$

which implies that

$$\left(\frac{c_{it}}{a_i}\right)=p_t^*\Big/\left[1+\lambda\frac{N_t}{N_t-1}\right]=c_t^* \tag{23}$$

and does not depend on $i\in C^*_t$. γ

In the previous section I argued that the bias that I illustrated graphically was the result of the price per quality unit not being constant across models sold in the market. In fact, in the example, the price per quality unit was higher for better models. In the symmetric equilibrium derived above the price per quality unit is constant and it is thus unlikely that this equilibrium will yield a bias of the sort described before. However, if the marginal production cost per quality unit is not the same across models sold in the market, then neither is the price charged per quality unit. In that case the market equilibrium will be asymmetric in the sense the models will have different market shares. As I show in the following proposition, the suppliers that produce the models with the higher marginal production cost per quality unit will charge a higher price per quality unit and will have a lower market share.

Proposition 3: ***Properties of asymmetric equilibrium***

In the asymmetric equilibrium, producers with higher marginal production costs per efficiency units, i.e. c_{it}/a_i, (i) charge a higher price per efficiency unit, p_{it}/a_i, and (ii) a lower markup, p_{it}/c_{it}.

Proof: (*i*) Equation (20) implies that, when we define

$$W_t = \sum_{j\in C_t^*} (p_{it}/a_i)^{-1/\lambda}\ , \tag{24}$$

then for all $i \in C^*_t$ it must be in equilibrium that

$$\left(\frac{p_{it}}{a_i}\right) = \left[1+\lambda\frac{W_t}{W_t-(p_{it}/a_i)^{-1/\lambda}}\right]\left(\frac{c_{it}}{a_i}\right) \tag{25}$$

Applying the implicit function theorem to the above equation yields

$$\frac{\partial(p_{it}/a_i)}{\partial(c_{it}/a_i)} > 0 \tag{26}$$

Furthermore, because (p_{it}/a_i) is higher, equation (5) implies that the market share of the model must be lower.

(*ii*) In order to prove this part, it is easiest to reconsider (13), which reads

$$\left(\frac{p_{it}}{c_{it}}\right)^{-1/\lambda} = \left[1+\lambda\frac{\sum_{j\in C_t^*}(c^*_{jt})^{-1/\lambda}(p_{jt}/c_{jt})^{-1/\lambda}}{\sum_{j\in C_t^*}(c^*_{jt})^{-1/\lambda}(p_{jt}/c_{jt})^{-1/\lambda}-(c^*_{it})^{-1/\lambda}(p_{it}/c_{it})^{-1/\lambda}}\right]^{-1/\lambda} \tag{27}$$

Again, redefining

$$v_{it} = (p_{it}/c_{it})^{-1/\lambda} \text{ and defining } \widetilde{V}_t = \sum_{i\in C_t^*}(c^*_{it})^{-1/\lambda} v_{it} \tag{28}$$

equation (27) boils down to

$$v_{it} = \left[1 + \lambda \frac{\widetilde{V}_t}{\widetilde{V}_t - \left(c_{it}^*\right)^{-1/\lambda} v_{it}}\right]^{-1/\lambda} \quad \text{for all } i \in C^*_t \tag{29}$$

It is straightforward to show that the v_{it} that solves this equation is increasing in c^*_{it}. Since the markup, p_{it}/c_{it}, is decreasing in v_{it}, this implies that the equilibrium markup is decreasing in c^*_{it}, which is what is claimed. γ

The above result is important because it suggests that any asymmetric equilibrium exhibits prices per quality unit that are unequal across the models sold in the market and thus has the potential of generating the bias described in the previous section.

5. Price index bias in the theoretical model

Now that I have developed the theoretical model of this market, it is time to consider what conventional price index methods would measure in this market. In order to illustrate the quality bias it is essential to consider dynamic universes of goods such that

$$C_0^* \neq C_1^* \text{ and } C_0^* \cap C_1^* \neq \varnothing \tag{30}$$

In principle, there are many ways in which the set of models traded in the market can change and each of these changes might have a different effect in the theoretical example considered here. Because it is simply impossible to consider all of these different cases, I will limit myself to one specific example. In the first subsection, I will describe the parameterization of this example in detail. Then, in the second subsection, I will consider what happens when standard price index methods are applied in this example.

Parameterization of example

The example that I will consider is one where the model at 'the bottom of the line' in period $t=0$ becomes obsolete in period $t=1$ and in which in period $t=1$ a new 'top of the line' model is introduced. The 'bottom of the line' model at $t=0$ is the lowest-quality model, i.e. the one with the smallest a_i among all $i \in C^*_0$. The 'top of the line' model introduced in period $t=1$ is such that its quality exceeds that of all models traded in period $t=0$.

Consequently, in both periods the same number of models is sold. I will denote this number by $N=N_0=N_1$. I will index the models as $i=1,\ldots,N+1$, where model 1 is the 'bottom of the line' model that becomes obsolete at time $t=1$ and model $N+1$ is the new 'top of the line' model introduced at $t=1$. This indexation implies that $C^*_0=\{1,\ldots,N\}$ and $C^*_1=\{2,\ldots,N+1\}$.

Two things are still to be defined. The first is the parameterization of the quality levels $\{a_i\}_{i=1}^{N+1}$. I will assume that quality is increasing in i such that

$$a_i = (1+g)^{i-1} \tag{31}$$

where $g>0$ represents the quality growth rate across models.

The second is the parameterization of the unit production costs $\{c_{it}\}_{i=1}^{N+1}$. The parameterization that I will choose for these unit production costs is

$$c_{it} = c_t^* a_i^{1+\gamma} \tag{32}$$

This parameterization is such that if $\gamma=0$ then the production costs per quality unit are identical across models and the equilibrium is symmetric. If $\gamma<0$ then the production costs per quality unit are lower for better models and their suppliers will charge a lower price per quality unit and a higher markup in equilibrium, as shown in proposition 3. Similarly, if $\gamma>0$ then production costs per quality unit are higher for better models and, as in the graphical example, the price per quality unit is higher for better models. Hence, γ represents the steepness of the cross-model production costs per quality unit schedule. Because of proposition 3, this implies that γ also represents the steepness of the cross-sectional price per quality unit schedule.

I will parameterize the change of c^*_t over time as follows. Let $\overline{a}_t = \prod_{i \in C_t^*} a_t^{1/N}$ then I will assume that

$$c_t^* = \frac{\widetilde{c}_t}{\overline{a}_t^{\gamma}} \text{ where } \widetilde{c}_1 = (1+\pi)\widetilde{c}_0 \tag{33}$$

The reason that I parameterize c^*_t like this is because, in equilibrium, the structural parameter π has a specific interpretation. This is proven in the following proposition.

Proposition 4: ***Interpretation of structural parameter π***

In equilibrium, the structural parameter π equals the theoretical inflation rate, i.e. $\pi=\pi^T$.

Proof: The basis of this proof is the equilibrium equation

$$p_{it} = \left[(1+\lambda) + \lambda \frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j \in C_t^* \setminus \{i\}} (p_{jt}/a_j)^{-1/\lambda}} \right] c_{it} \text{ for all } i \in C^*_t \tag{34}$$

when we define

$$\widetilde{p}_{it} = \frac{p_{it}}{a_i \widetilde{c}_t} \tag{35}$$

then (34) can be rewritten as

$$\widetilde{p}_{it} = \left[(1+\lambda) + \lambda \frac{(\widetilde{p}_{it})^{-1/\lambda}}{\sum_{j \in C_t^* \setminus \{i\}} (\widetilde{p}_{jt})^{-1/\lambda}} \right] \left(\frac{a_i}{\overline{a}_t} \right)^{\gamma} \text{ for all } i \in C^*_t \tag{36}$$

what I will show is that

$$a_i / \overline{a}_t = a_{i+1} / \overline{a}_{t+1} \tag{37}$$

If this is the case, then it must be that $\widetilde{p}_{it} = \widetilde{p}_{(i+1)(t+1)}$. We know that

$$C_0^* = \{1, \ldots, N\} \text{ and } C_1^* = \{2, \ldots, N+1\} \tag{38}$$

Given these sets, (31) implies that

$$\overline{a}_0 = (1+g)^{\frac{1}{N}\sum_{i=1}^{N}(i-1)}, \text{ while } \overline{a}_1 = (1+g)^{\frac{1}{N}\sum_{i=2}^{N+1}(i-1)} = (1+g)\left[(1+g)^{\frac{1}{N}\sum_{i=1}^{N}(i-1)}\right] = (1+g)\overline{a}_0 \tag{39}$$

Since $a_{i+1}=(1+g)a_i$, this implies that

$$a_{i+1} / \overline{a}_{t+1} = (1+g)a_i / (1+g)\overline{a}_t = a_i / \overline{a}_t \tag{40}$$

Therefore, $\widetilde{p}_{it} = \widetilde{p}_{(i+1)(t+1)}$. This implies that

$$P_0^T = \left[\sum_{i=1}^{N} (p_{i0}/a_i)^{-1/\lambda}\right]^{-\lambda} = \widetilde{c}_0 \left[\sum_{i=1}^{N} \widetilde{p}_{i0}^{-1/\lambda}\right]^{-\lambda} \tag{41}$$

and

$$P_1^T = \left[\sum_{i=2}^{N+1} (p_{i1}/a_i)^{-1/\lambda}\right]^{-\lambda} = \widetilde{c}_1 \left[\sum_{i=2}^{N+1} \widetilde{p}_{i1}^{-1/\lambda}\right]^{-\lambda} = \widetilde{c}_1 \left[\sum_{i=1}^{N} \widetilde{p}_{(i+1)1}^{-1/\lambda}\right]^{-\lambda} = \frac{\widetilde{c}_1}{\widetilde{c}_0} P_0^T \tag{42}$$

Hence,

$$\pi^T = \frac{P_1^T - P_0^T}{P_0^T} = \frac{\widetilde{c}_1 - \widetilde{c}_0}{\widetilde{c}_0} = \pi \tag{43}$$

Thus, π represents the theoretical inflation rate that is supposed to be approximated by the empirical price index methods. γ

Given this parameterization, the question is how estimated inflation on the basis of the various price index methods depends on the underlying structural parameters, N, g, π, γ, and λ and how it compares to the actual level of inflation, π^T. This question is addressed in the next subsection.

Quality bias

Just like in the graphical example of section 3, I will first address the bias induced by matched model indexes and then consider hedonic price indexes in this theoretical model.

For the matched model price indexes I will solely consider the, most frequently used, IP-IQ method. The following proposition states the properties of the IP-IQ linked matched model indexes in this example.

Proposition 5: ***Matched model index properties***

An IP-IQ linked matched model index yields an estimate of inflation, π^M, that has the following properties:

(i) $\pi^M = \pi^T$ *if* $\gamma=0$.

(ii) $\pi^M > \pi^T$ *if* $\gamma<0$.

(iii) $\pi^M < \pi^T$ *if* $\gamma>0$.

This result does not depend on which of the price index formulas (except the Feenstra (1994) index which is exact) is applied.

Proof: *(iii):* Note that the inflation rate of good i between $t=0$ and $t=1$ is given by

$$\begin{aligned}\pi_i &= \frac{p_{i1}-p_{i0}}{p_{i0}} = \frac{\tilde{c}_1\tilde{p}_{i1}-\tilde{c}_0\tilde{p}_{i0}}{\tilde{c}_0\tilde{p}_{i0}} \\ &= \frac{\tilde{c}_1-\tilde{c}_0}{\tilde{c}_0}+\frac{\tilde{c}_1}{\tilde{c}_0}\left[\tilde{p}_{i1}-\tilde{p}_{i0}\right] \\ &= \pi^T+\left(1+\pi^T\right)\left(\tilde{p}_{i1}-\tilde{p}_{i0}\right)\end{aligned} \tag{44}$$

Hence, what will be essential in the rest of this proof is the property of $\tilde{p}_{i1}-\tilde{p}_{i0}$. It turns out that $\tilde{p}_{it}$ is increasing in $\left(a_i/\bar{a}_t\right)^\gamma$. In order to see why, it is useful to rewrite (36) as

$$\tilde{p}_{it} = \left[1+\lambda\frac{\tilde{P}_t}{\tilde{P}_t-\left(\tilde{p}_{it}\right)^{-1/\lambda}}\right]\left(\frac{a_i}{\bar{a}_t}\right)^\gamma \text{ for all } i\in C^*_t \tag{45}$$

where

$$\tilde{P}_t = \sum_{i\in C_t^*}\tilde{p}_{it}^{-1/\lambda} \tag{46}$$

Applying the implicit function theorem to the above two equations yields in a straightforward manner that

$$\frac{\partial\tilde{p}_{it}}{\partial\left(a_i/\bar{a}_t\right)^\gamma} > 0 \tag{47}$$

Therefore $\tilde{p}_{it}$ is strictly increasing in $\left(a_i/\bar{a}_t\right)^\gamma$. Furthermore, note that if $\gamma=0$, then the equilibrium is symmetric and $\tilde{p}_{it}$ is equal for all $i\in C^*_t$.

Consequently, if $\gamma>0$ then $\left(a_i/\bar{a}_t\right)^\gamma$ is increasing in i and models of higher quality have a higher $\tilde{p}_{it}$. This implies that if $\gamma>0$ then

$$\tilde{p}_{(i+1)0} > \tilde{p}_{i0} = \tilde{p}_{(i+1)1} \tag{48}$$

where the second equality follows from the proof of proposition 4. Hence, if $\gamma>0$ then for all $i\in C^*_t$,

$$\pi_i = \pi^T + \left(1+\pi^T\right)\left(\tilde{p}_{i1} - \tilde{p}_{i0}\right) < \pi^T \qquad (49)$$

Because this inequality holds for all models sold in the market in periods $t=0$ and $t=1$, matched model price indexes calculated using the Laspeyres, Paasche, Geometric mean, Fischer, Tornqvist, and Sato formula will all underestimate inflation. The reason is that all these price index formula have the property that measured inflation is in the range of inflation rates of the individual models. Since the actual inflation rate is above the maximum inflation rate measured for the models it must be that the actual inflation rate is understated by the matched model indexes.

A reverse but similar argument yields that the matched model indexes overstate inflation whenever $\gamma<0$. γ

This proposition is the formal mathematical proof of the informal argument that I stated with respect to matched model price indexes for the graphical example in section 3. That is, the sign and magnitude of the quality bias in matched model price indexes *does not depend on the sign and magnitude of the overall inflation rate*. Instead, *it depends on the cross-sectional behavior of prices per quality unit for the models sold* in the market.

That the bias does not depend on the sign and magnitude of the overall inflation rate follows directly from the fact that the result in proposition 5 does not depend on the structural parameter π. The dependence of the bias on the steepness of the cross-sectional schedule of prices per quality unit across models is implied by the bias in the matched model indexes only depending on the parameter γ.

That is, if $\gamma>0$ then, according to proposition 3, the price per quality unit is increasing in the level of quality embodied in the model. This is the case depicted in the graphical example of section 3 and is the case that yields a *downward* bias in the measured inflation rate. If $\gamma<0$ then the reverse is true.

So, how do hedonic price methods behave in the theoretical model here? This question can only be answered conditional on the behavior of the imputed price levels. I do so in the next proposition.

Proposition 6: ***Hedonic price index properties***

A hedonic price index yields an estimate of inflation, π^H, that has the following properties:

(i) $\pi^H = \pi^T$ *if* $\gamma=0$.

(ii) $\pi^H > \pi^T$ *if* $\gamma<0$, *if the imputed prices satisfy the property of the equilibrium price schedule that prices per quality unit are decreasing in the quality embodied in the model.*

(iii) $\pi^H < \pi^T$ *if* $\gamma>0$, *if the imputed prices satisfy the property of the equilibrium price schedule that prices per quality unit are increasing in the quality embodied in the model.*

Just like in proposition 5, this result does not depend on which of the price index formulas (except the Feenstra (1994) index which is exact) is applied.

Proof: *(i):* If $\gamma=0$ then the equilibrium price schedule satisfies

$$\frac{p_{it}}{a_i} = p_t^{\;*} \text{ for all } i \in C^*_t \text{ and for } t=0,1 \tag{50}$$

If the imputed prices, $\hat{p}_{N+1,0}$ and $\hat{p}_{1,1}$,from the hedonic regression model also satisfy this property such that

$$\frac{\hat{p}_{N+1,0}}{a_{N+1}} = p_0^* \text{ as well as } \frac{\hat{p}_{1,1}}{a_1} = p_1^* \tag{51}$$

then we find that the observed and imputed inflation rates satisfy

$$\pi_i = \begin{cases} \dfrac{p_{i,1} - p_{i,0}}{p_{i,0}} = \dfrac{p_1^* - p_0^*}{p_0^*} = \pi^T & \text{for } i = 2, \ldots, N \\ \dfrac{p_{N+1,1} - \hat{p}_{N+1,0}}{\hat{p}_{N+1,0}} = \dfrac{p_1^* - p_0^*}{p_0^*} = \pi^T & \text{for } i = N+1 \\ \dfrac{\hat{p}_{1,1} - p_{1,0}}{p_{1,0}} = \dfrac{p_1^* - p_0^*}{p_0^*} = \pi^T & \text{for } i = 1 \end{cases} \tag{52}$$

Consequently, no matter what type of weighted average one takes of the observed and imputed inflation rates across models to calculate π^H, this average will always equal π^T.

(ii): If $\gamma<0$ then the equilibrium price schedule satisfies

$$\frac{p_{it}}{a_i} < \frac{p_{i-1t}}{a_i} \text{ for all } i, i\text{-}1 \in C^*_t \text{ and for } t=0,1 \tag{53}$$

If the imputed prices in the hedonic regression model also satisfy this property, such that

$$\frac{\hat{p}_{N+1,0}}{a_{N+1}} < \frac{p_{N,0}}{a_N} \text{ and } \frac{\hat{p}_{1,1}}{a_1} > \frac{p_{2,1}}{a_1} \tag{54}$$

then, in terms of the notation of proposition 5, the observed and imputed prices obey

$$\tilde{p}_{i,0} < \tilde{p}_{i-1,0} = \tilde{p}_{i,1} \text{ for } i=2,\ldots,N, \text{ as well as } \hat{\tilde{p}}_{N+1,0} < \tilde{p}_{N,0} = \tilde{p}_{N+1,1} \text{ and } \hat{\tilde{p}}_{1,1} > \tilde{p}_{2,1} = \tilde{p}_{1,0} \tag{55}$$

This means that the observed and imputed inflation rates satisfy

$$\pi_i = \begin{cases} \pi^T + (1+\pi^T)(\tilde{p}_{i,1} - \tilde{p}_{i,0}) > \pi^T & \text{for } i = 2, \ldots, N \\ \pi^T + (1+\pi^T)(\tilde{p}_{N+1,1} - \hat{\tilde{p}}_{N+1,0}) > \pi^T & \text{for } i = N+1 \\ \pi^T + (1+\pi^T)(\hat{\tilde{p}}_{1,1} - \tilde{p}_{1,0}) > \pi^T & \text{for } i = 1 \end{cases} \tag{56}$$

Hence, no matter what weighted average one takes of these inflation rates across models to calculate the hedonic inflation rate π^H, it will always yield $\pi^H>\pi^T$.

(iii): This follows in the same way as the proof of part *(ii)*. The only thing that is different is that in this case the equilibrium price schedule is such that the prices per quality unit are increasing in the quality levels of the models, which yields a reversal of the inequality signs. γ

The proof of the proposition above gives some interesting insights. First of all, the hedonic price indexes only yield an unbiased estimate of inflation whenever the equilibrium is such that the price per quality unit is constant across the models traded in the market. However, if the price per quality unit is constant across models, then matched model indexes will do just fine. In fact, if the price per quality unit is constant across the models sold in the market, then one can simply measure overall inflation by considering the percentage price change of a single model. That is, when the price per quality unit is constant across the models sold in the market quality bias is not an issue. This itself is an important observation.

Bils and Klenow (2001) for example use microdata from the Consumer Expenditure Survey to estimate the quality bias in the CPI for several durable consumption goods. They do so by estimating a structural model of durable goods consumption. In order to quantify quality growth in this model, however, they assume that independent of each household's expenditures on a particular durable consumption good, the price paid per quality unit is constant for all households. Hence, no matter what model the households are buying, they are assumed to pay the same price per quality unit. This means that Bils and Klenow (2001) implicitly assume that the price per quality unit is constant across models. However, if this identifying assumption would be true in the data then the BLS would have had no problem quantifying quality growth in the first place.

If the price per quality unit is constant, then relative prices represent relative quality differences. In that case the coefficients in the hedonic regression model will represent the marginal quality coefficients of the quality indicators. Feenstra (1995) shows that when these coefficients represent these marginal values, hedonic price indexes will work properly. In fact, for certain classes of preferences Feenstra (1995) derives exact hedonic indexes. However, when he considers the existence of markups he also observes that when this is not the case then the estimated hedonic regression coefficients might over- or underestimate the quality difference between the models.

This is the case when $\gamma>0$ and $\gamma<0$. In those cases hedonic regression coefficients do not only reflect the marginal quality differences between the models but also reflect the slope of the price per quality unit schedule. In order to illustrate this point in practice, I present the results of a numerical simulation of the theoretical model in the next section.

6. A numerical simulation

So far, I have presented the properties of the price indexes in my theoretical model in the form of several formal propositions. These propositions proved the existence of the quality bias that can be both upward as well as downward and exists for both matched model as well as hedonic price indices. However, it is worthwhile to see how big the bias is in the model when we actually put in some numbers. This is what I will do in the numerical simulation in this section.

The numbers to be put in are the structural parameters. These are N which is the number of models sold in each period, g the quality growth rate across models, λ the preference parameter that determines the elasticity of substitution, γ the slope of the average cost curve per quality unit, and π the theoretical inflation level.

Propositions 5 and 6 claim that their results do not depend on the price index formula applied, except the exact matched model index for CES-preferences by Feenstra (1994). For that reason, I will apply all the most commonly used price index formulas. These formulas are listed in Table 1. The table contains the names and definition of the formulas as well as a brief description where some of them are applied in practice.

Finally, in order to simulate the hedonic price indexes, I have to choose a particular model specification for the hedonic regressions. I will assume that the researcher observes a_i for each model but does not realize that it is the actual quality level of the model and uses it as a quality indicator in a hedonic regression. The hedonic regression model that I apply is of the following 'log-log' form

$$\ln p_{it} = \beta_{0t} + \beta_{1t} \ln a_i + \beta_{2t} \ln^2 a_i + \varepsilon_i \tag{57}$$

I allow the coefficients in the regression to be changing over time[4]. This means that I will perform separate cross-sectional hedonic regressions for $t=0$ and $t=1$.

I will present my simulation in two parts. In the first part, I will present a benchmark example that turns out to yield results that are similar to the graphical example that I gave in section 3. I discuss this example and these similarities in detail. Then, in the second part, I will present the results for a set of other examples that each deviate from the benchmark in the difference in one particular structural parameter.

Benchmark case

The benchmark case that I will consider is that of a market with ten models, i.e. $N=10$. The quality ranking of the models is such that each model is 5% better than the next best one, i.e. $g=0.05$. The elasticity of substitution between the various models is constant and assumed to equal two, such that $\lambda=1$. On the cost side, average production costs per quality unit are increasing in quality[5], such that its elasticity with respect to quality is 0.1. This implies that $\gamma=0.1$. Finally, the benchmark case is such that overall inflation is zero, i.e. $\pi^T=\pi=0$. Hence, all inflation or deflation that is measured by the price indexes is spurious.

The top panel of Figure 4 depicts the equilibrium price schedules for $t=0$ and $t=1$. What a researcher would observe in this market is that the model with the lowest price at $t=0$ drops out of the market, while the prices of the other models drop by about half a percent. The model that drops out of the market is replaced by a model with a price that is higher than that of the other models at $t=1$. Because of this higher price of the new model the average price paid per unit sold in the market increases between $t=0$ and $t=1$. However, the

[4] In practical applications of hedonic price indexes the regression coefficients turn out to fluctuate a lot over time. This has been a topic of discussion in the literature for a while. See Hulten (2002) for review of this discussion.

[5] One reason that production costs per quality unit might be increasing in the number of quality units is when the best models are the newest models and learning by doing reduces production costs over time at a higher rate than quality per model grows. That learning by doing might be a significant source of price declines has been argued for semiconductors by Irwin and Klenow (1994).

crucial question is how much of this increase is due to the superiority of the new model introduced at time $t=1$.

What matters for the overall price change is again not the levels of unit prices but the levels of prices per quality unit. The second panel of Figure 4 depicts the equilibrium schedules of prices per quality unit at times $t=0$ and $t=1$. Because $\gamma>0$ these schedules are monotonically increasing in the level of quality embodied in the various models. Finally, the bottom panel of Figure 4 depicts the log price per quality unit schedule as a function of the logarithm of the quality level. The reason that I included this panel is because it depicts the relationship estimated in the hedonic regression model (57). As can be seen from the panel, the logarithm of the price per quality unit seems to be virtually linear in the logarithm of the quality level. Therefore, (57) should yield a good fit.

So, what happens when an IP-IQ matched model index is applied in this benchmark case? Proposition 5 implies that an IP-IQ matched model index will underestimate inflation in this case. To see why this is the case, consider Figure 5, which is the numerical equivalent of Figure 2 discussed in section 3. The application of the IP-IQ method results in ignoring the unmatched model in $t=0$ which is obsolete in $t=1$. However, this model has the lowest price per quality unit. Therefore, ignoring this model raises the inferred overall price level at $t=0$. Similarly, the IP-IQ method also ignores the price of the new model in period $t=1$. However, this model has the highest price per quality unit and, thus, ignoring it will artificially lower the inferred overall price level in period $t=1$. Since the prices of all matched models fall by 0.46% between period $t=0$ and $t=1$, the IP-IQ method underestimates actual inflation by about half a percent. Since the price drop for all matched models is identical, it does not matter what index formula is applied. This can be seen in the column labeled (1) in Table 2. All matched model indexes, except the exact Feenstra (1994) index, yield the same measure of 0.46% deflation.

Another way of looking at this bias is to realize that the IP-IQ method is based on the identifying assumption that the change in the price per quality unit between the obsolete and the new model equals the price change of the matched models. However, in this case the price change for the matched models is -0.46% while the new model is more expensive then the obsolete one in terms of price per quality unit.

The only matched model index that works in this case is that by Feenstra (1994). Feenstra's index assumes that aggregate demand is generated by CES preferences and also requires that one knows, or at least has an accurate estimate of, the constant elasticity of substitution. If this is the case, then Feenstra's index is an exact matched model index here. Not surprisingly, the Feenstra index thus estimates the right inflation rate in this case, as can be seen again in column (1) of Table 2.

It turns out to be informative to consider what the Feenstra (1994) index does differently from the other matched model indexes. As can be seen in Table 1, Feenstra's index is similar to Sato's, except that it is multiplied by a correction factor. This correction factor is a function of the ratio of the shares of the matched models in period $t=1$ and in $t=0$. As can be seen from the demand function (5), the higher the price per efficiency unit of a model compared to the overall price level, the lower its market share. Hence, the ratio of the market shares of the matched models can be used to estimate difference between the price per quality unit of the obsolete model in period $t=0$ and that of the new model in period $t=1$ compared to the overall inflation rate. The IP-IQ method does not estimate this magnitude. It assumes it is one. Therefore, the Feenstra (1994) index corrects for the deviations from the identifying assumption that is made when applying the IP-IQ method.

It turns out that hedonic price indexes do not fare any better than the matched model indexes in this benchmark case. Figure 6, which is the equivalent of Figure 3 in the graphical example of section 3, depicts this point. The prices imputed based on the hedonic regressions are such that they add an above average price per quality unit for the new model in period $t=0$. Because of this the imputed price level in period $t=0$ is biased upward. The imputed price for the obsolete model in period $t=1$ implies a lower than average price per quality unit. Therefore the imputed price level in period $t=1$ is biased downward. Consequently, the hedonic price index method will underestimate inflation.

For this particular example, the imputed price schedule in period $t=0$ is 0.46% higher than that in period $t=1$ for all models. Therefore, the hedonic price index method yields 0.46% deflation, independent of the price index formula applied.

One thing is important to note. This is that this bias is not due to the hedonic regression model (57) not properly fitting the data. As I noted before, the logarithm of the price per quality unit is virtually linear in the logarithm of the quality level. This translates into a perfect fit of the regression model (57) in both periods, as can be seen from the R^2 equaling one for the hedonic regressions in column (1) of Table 2.

Other examples

Besides the benchmark example in column (1), Table 2 contains 14 more numerical examples. I will briefly discuss the main results implied by these examples here.

Only in 1 out of the 15 examples listed in Table 2 do the price index methods actually work. This is in case (11), in which, as suggested by the results of Propositions 5 and 6, $\gamma=0$. This is the case in which the prices per quality unit are constant across models in each period.

All 15 examples have four things in common. First of all, the Feenstra (1994) index is indeed exact and correctly estimates the inflation level in each of the 15 cases. Secondly, nowhere do the hedonic price indexes do much better than the matched model indexes. That is, for the numerical examples here, it does not seem to be the case that hedonic methods better correct for quality bias then matched model indexes do. Thirdly, the measurement bias in these examples is independent of which of the price index formulas from Table 1 are applied. Finally, in all cases the hedonic regression model (57) seems to provide a perfect fit of the equilibrium price schedule. Several things are worth pointing out by considering and comparing some of the specific examples.

When we compare cases (13) through (15) with the benchmark case (1), then we observe that the changes in the inflation rate imposed in (13)-(15) hardly matter for the size of the bias. The same is true for the number of models in the market, when we compare cases (2) and (3) with the benchmark case. I will discuss how these parameters influence the size of the bias in more detail in the next subsection when I consider a log-linear approximation to the bias.

In all cases the estimated hedonic regression coefficient, β_{1t}, is positive except for case (9). The reason that $\beta_{1t}<0$ in that case, even though quality is obviously increasing in a_i, is that marginal production costs per quality unit are falling rapidly as a function of the quality of the models. This is implied by the elasticity

γ=-2. This is a numerical example of Pakes' (2002) argument that one should not consider 'counterintuitive' signs of hedonic regression coefficients as a sign of misspecification of the hedonic regression. These coefficients are a complex function of the intersection of supply, in large part determined by the behavior of production costs, and demand.

Because the logarithm of the price per quality unit is approximately linear in the logarithm of the quality level, we can actually approximate the actual values of the regression coefficients in (57) by log-linearizing the equilibrium conditions. I do so in the next subsection in which I use this approximation to quantify the quality bias as function of the underlying structural parameters.

Log-linearized approximation of the bias

Log-linearization of equation (25) around the symmetric equilibrium derived in Proposition 2 yields

$$\begin{aligned}\ln p_{it} &\approx \ln \tilde{c}_t + \ln\left(1+\lambda\frac{N}{N-1}\right) + \ln a_i + \gamma\left[1-\frac{N}{1+(1+\lambda)N(N-1)}\right](\ln a_i - \ln \bar{a}_t)\\ &= \left[\ln \tilde{c}_t + \ln\left(1+\lambda\frac{N}{N-1}\right) - \gamma\left[1-\frac{N}{1+(1+\lambda)N(N-1)}\right]\ln \bar{a}_t\right] + \left[1+\gamma-\gamma\frac{N}{1+(1+\lambda)N(N-1)}\right]\ln a_i \\ &= \beta_{0t} + \beta_{1t}\ln a_i\end{aligned} \tag{58}$$

This implies that the logarithm of the price per quality unit satisfies

$$\ln(p_{it}/a_i) \approx \ln \mu_{it} + \ln \tilde{c}_t + \gamma(\ln a_i - \ln \bar{a}_t) \tag{59}$$

where $ln\mu_{it}$ is the logarithm of the markup charged on model i in period t which equals approximately

$$\ln \mu_{it} \approx \ln\left(1+\lambda\frac{N}{N-1}\right) - \gamma\left[\frac{N}{1+(1+\lambda)N(N-1)}\right](\ln a_i - \ln \bar{a}_t) \tag{60}$$

When we take the first difference, over time, of (59), then we obtain that the percentage price change in the price of model i between $t=0$ and $t=1$ can be approximated by

$$\begin{aligned}\pi_i &\approx \Delta \ln p_{i1} \approx \Delta \ln \tilde{c}_1 + \Delta \ln \mu_{i1} - \gamma\Delta \ln \bar{a}_1 \approx \pi^T + \Delta \ln \mu_{i1} - \gamma \Delta \ln \bar{a}_1 \\ &\approx \pi^T - \gamma\left[1-\frac{N}{1+(1+\lambda)N(N-1)}\right]g = \pi^T - \theta g\end{aligned} \tag{61}$$

Thus, we obtain that the inflation rates of each of the matched models deviate from the actual inflation rate by approximately the same amount, namely $-\theta g$. Because the hedonic regression extrapolates this approximately linear relationship for the imputation of the unobserved prices, it also finds that the imputed inflation levels $\pi_{N+1}=\pi_1=\pi^T-\theta g$. Therefore, the results for the hedonic price indexes do not differ much from the matched model indexes and both are biased by approximately $-\theta g$.

What constitutes this bias? The reason for this bias is that the price index methods can not distinguish between a movement in the price per quality unit schedule over time due to an actual change in the overall price level and a move in the schedule because the introduction of a new model shifts the relative competitive advantages (in production and the market) and thus prices of the models sold in the market.

In order to see this, consider Figure 7. It disentangles the various effects on the schedule of the logarithm of the price per quality as a function of the logarithm of the quality. That is, it graphically represents the first difference of (59) over time and the various things that influence it.

Consider model i at time $t=0$. It has price p_{i0}, such that the logarithm of its price per efficiency unit is $ln(p_{i0}/a_i)$. At time $t=0$ it is at point A on the log price per quality unit schedule. For expositional purposes, I have drawn this graph for $\pi<0$ and $\gamma>0$. The drop in the overall price level $\pi<0$ shifts the log of the price per quality unit of model i down from point A to point B. However, something else happens at the same time as well. That is the introduction of the new model $N+1$ and the exit of model 1.

Because of the introduction of the new, superior, model and the fact that production costs are increasing in the quality embodied in the model, the production costs of model i relative to those of its competitors will drop. This allows the supplier of model i to charge a lower price per quality unit than in period $t=0$. In fact, because of the setup of the model, model $i+1$ takes over model i's position in the relative quality ladder in period $t=1$. Therefore, in period $t=1$ model $i+1$ will be sold at the same relative price per quality unit that model i was sold at in period $t=0$. This is depicted in Figure 7 by the horizontal shift from B to C. The slope of the log price per quality unit schedule, i.e. θ, and the length of the horizontal shift, i.e. g, then jointly determine how far below $ln(p_{i0}/a_i)+\pi$ the logarithm of model i's price per quality unit in period $t=1$, i.e. $ln(p_{i1}/a_i)$, ends up.

Hence, in terms of this Figure 7, the problem of the price index methods is that they do not distinguish between the actual change in the overall price level, depicted by the shift from A-B, and the effect of the shift in the relative qualities of the models due to the introduction of a new model, depicted by the movement from B-D.

7. Conclusion

In this paper I argued that the quality bias in price indexes does not necessarily always bias them upwards. I illustrated how the sign and the magnitude of this bias depend on the cross-sectional behavior of prices per quality unit across the models sold in the market. I did so by introducing a theoretical model that generated a quality bias in inflation as measured using the most common price index methods. The three main points that can be taken away from the analysis here are.

First and foremost, the quality bias can be both positive and negative. The sign of the bias does *not* depend on the actual underlying overall inflation rate. Instead, it solely depends on the cross-sectional behavior of prices per quality unit.

Secondly, the bias does not depend on which of the many proposed price index formulas are used to calculate the index. Laspeyres, Paasche, Geometric mean, Fisher Ideal, Tornqvist, and Sato indexes all performed is a similar manner in the theoretical model in this paper.

Finally, hedonic price indexes do not necessarily reduce the quality bias. In the examples in this paper, hedonic methods did just as poorly as matched model indexes. However, other examples, like the one given in Hobijn (2001), suggest that they might actually do worse in some cases.

This result is important because the application of hedonic price indexes seems to gain momentum both with statistical agencies, see Moulton (2001) for example, as well as with researchers. In fact, an extensive recent research agenda, including Greenwood, Hercowitz, and Krusell (1997), Violante, Ohanian, Rios-Rull, and Krusell (2000), and Cummins and Violante (2002), has been using Gordon's (1990) hedonic equipment price index as a measure of the 'true' quality adjusted price change for equipment in the U.S.. However, the results here suggest that one has to be careful in using this hedonic price index as such a benchmark. Simply because it measures less equipment price inflation than price indexes published by the Bureau of Economic Analysis and Bureau of Labor Statistics, does not necessarily mean it adjusts better for quality.

The results in this paper provide additional insight in which type of competitive circumstances are suspect to generating a bias, up or down, in the price indexes we calculate. Future research could focus on empirical tests of these conditions and of identifying in which markets what bias is the most likely to occur. At least it seems that the conventional wisdom that the quality bias biases measured inflation upward deserves a more thorough empirical verification.

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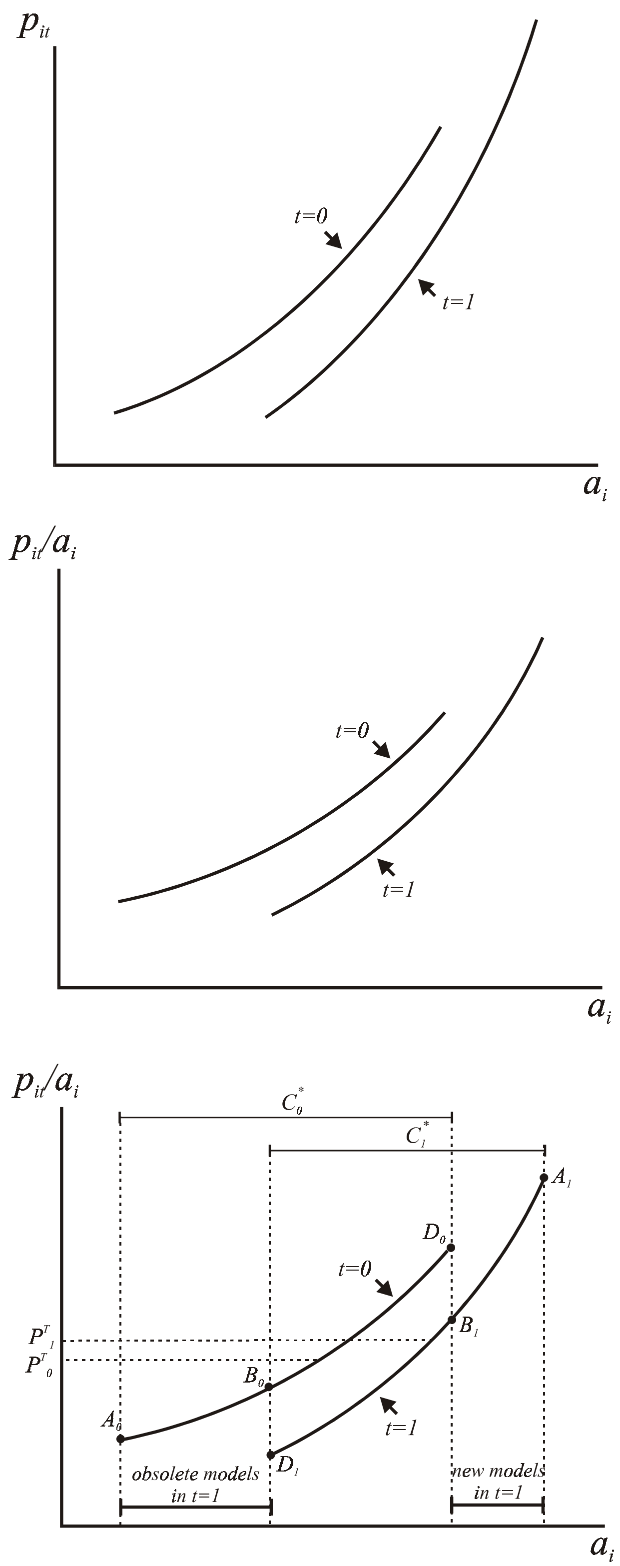
Table 1. Price index formulas applied in this paper

Index	Formula	Applied
Laspeyres	$\pi^L = \sum_{i \in C^*} s_{i0}\pi_i$	• Applied by Bureau of Labor Statistics for the calculation of the Consumer Price Index.
Paasche	$\pi^P = \left[\sum_{i \in C^*} s_{i1}(1+\pi_i)^{-1}\right]^{-1} - 1$	
Geometric mean (0)	$\pi^{G_0} = \left(\prod_{i \in C^*}(1+\pi_i)^{s_{io}}\right) - 1$	
Geometric mean (1)	$\pi^{G_1} = \left(\prod_{i \in C^*}(1+\pi_i)^{s_{i1}}\right) - 1$	
Fisher Ideal	$\pi^{FI} = \sqrt{(1+\pi^L)(1+\pi^P)} - 1$	• Applied by Bureau of Economic Analysis for the calculation of chained price indexes in the National Income and Product Accounts.
Tornqvist	$\pi^{TQ} = \sqrt{(1+\pi^{G_0})(1+\pi^{G_1})} - 1$	• Applied by Bureau of Labor Statistics (2002) in chained Consumer Price Index.
Sato	$\pi^S = \left(\prod_{i \in C^*}(1+\pi_i)^{\phi_i}\right) - 1$ where $\phi_i = \theta_i / \sum_{j \in C^*}\theta_j$ $\theta_i = \begin{cases} \dfrac{s_{i1}-s_{i0}}{\ln s_{i1} - \ln s_{i0}} & \text{if} \quad s_{i0} \neq s_{i1} \\ s_{i1} & \text{if} \quad s_{i0} = s_{i1} \end{cases}$	• Exact price index for CES preferences when same goods are sold in both periods.
Feenstra	$\pi^F = \left(\prod_{i \in (C_0^* \cap C_1^*)}(1+\pi_i)^{\phi_i}\right)\left(\dfrac{\varpi_1}{\varpi_0}\right)^{1/(1-\sigma)} - 1$ where ϕ_i is as in the Sato index $\varpi_t = \sum_{i \in (C_0^* \cap C_1^*)} s_{it}$ for $t=0,1$ σ elasticity of substitution ($\sigma=1+\lambda/\lambda$)	• Exact price index for CES with matched model correction in case not all goods are sold in both periods.

Note: All indexes are meant to measure percentage price change between t=0 and t=1. π_i denotes the percentage price change of item i between t=0 and t=1. s_{it} is the expenditure share of good i in period t. C^*_t is the set of items sold in period t. Whenever it is denoted without time index it is assumed that $C^*_0 = C^*_1 = C^*$. $\lambda>0$ is the parameter used in the CES specification of the theoretical model of section 3 and beyond.

Table 2. Inflation estimates and hedonic regression results for various price index methods and formulas

parameter	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
N	***10***	***5***	***50***	10	10	10	10	10	10	10	10	10	10	10	10
g	***0.05***	0.05	0.05	***0.025***	***0.10***	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
λ	***1***	1	1	1	1	***0.5***	***2***	***20***	1	1	1	1	1	1	1
γ	***0.1***	0.1	0.1	0.1	0.1	0.1	0.1	0.1	***-2***	***-0.1***	***0***	***2***	0.1	0.1	0.1
π *(in %)*	***0***	0	0	0	0	0	0	0	0	0	0	0	***-10%***	***10%***	***50%***
								Matched model indexes (IP-IQ linked)							
Laspeyres	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.622	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Paasche	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.621	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Geometric (G0)	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.621	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Geometric (G1)	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.621	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Fisher	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.621	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Tornqvist	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.621	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Sato	-0.460	-0.427	-0.482	-0.233	-0.896	-0.451	-0.469	-0.484	9.621	0.462	*0.000*	-8.777	-10.414	9.494	49.310
Feenstra	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	*0.000*	0.000	-10.000	10.000	50.000
								Hedonic price indexes							
Laspeyres	-0.460	-0.428	-0.482	-0.233	-0.897	-0.451	-0.469	-0.484	9.620	0.462	*0.000*	-8.838	-10.414	9.494	49.310
Paasche	-0.460	-0.428	-0.482	-0.233	-0.897	-0.451	-0.469	-0.484	9.550	0.462	*0.000*	-8.828	-10.414	9.494	49.310
Geometric (G0)	-0.460	-0.428	-0.482	-0.233	-0.897	-0.451	-0.469	-0.484	9.619	0.462	*0.000*	-8.838	-10.414	9.494	49.310
Geometric (G1)	-0.460	-0.428	-0.482	-0.233	-0.897	-0.451	-0.469	-0.484	9.550	0.462	*0.000*	-8.828	-10.414	9.494	49.310
Fisher	-0.460	-0.428	-0.482	-0.233	-0.897	-0.451	-0.469	-0.484	9.585	0.462	*0.000*	-8.833	-10.414	9.494	49.310
Tornqvist	-0.460	-0.428	-0.482	-0.233	-0.897	-0.451	-0.469	-0.484	9.585	0.462	*0.000*	-8.833	-10.414	9.494	49.310
								Hedonic regression results for t=0							
constant	0.726	0.802	0.585	0.737	0.707	0.422	1.149	3.123	1.166	0.768	*0.747*	0.336	0.726	0.726	0.726
$ln(a_i)$	1.094	1.088	1.099	1.094	1.094	1.092	1.096	1.099	-0.940	0.905	*1.000*	2.841	1.094	1.094	1.094
$ln(a_i)^2$	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.117	0.000	*0.000*	0.112	0.000	0.000	0.000
R^2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	*1.000*	1.000	1.000	1.000	1.000
								Hedonic regression results for t=1							
constant	0.722	0.798	0.580	0.734	0.698	0.417	1.144	3.118	1.261	0.773	*0.747*	0.246	0.617	0.817	1.127
$ln(a_i)$	1.094	1.088	1.099	1.094	1.094	1.092	1.096	1.099	-0.951	0.905	*1.000*	2.830	1.094	1.094	1.094
$ln(a_i)^2$	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.113	0.000	*0.000*	0.113	0.000	0.000	0.000
R^2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	*1.000*	1.000	1.000	1.000	1.000



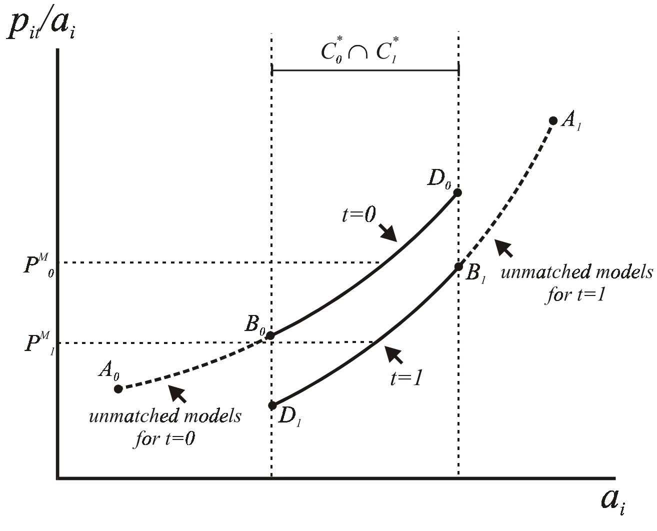
panel (a):
Even though the researcher does not observe the actual quality levels, a_i, the researcher does observe the actual price levels, p_{it}. This panel plots a hypothetical price schedule in two period $t=0,1$.

panel (b):
What is relevant for the price aggregate, the percentage change of which is to be measured, is actually the schedule of price per quality unit across models in the market.
This panel plots this schedule. It plots the implied relationship between p_{it}/a_i and a_i.

panel (c):
This panel is identical to panel *(b)*. What is added are:

C^*_t	set of models sold in market at time t
P^T_t	theoretical price level at time t
B_t-D_t	part of price schedule at time t for models that are sold in both periods
A_t-B_t	part of price schedule at time t for models that are only sold at time t
Note:	$P^T_1 > P^T_0$ such that the theoretical inflation level is positive.

Figure 1. Price per quality unit schedule and related notation.



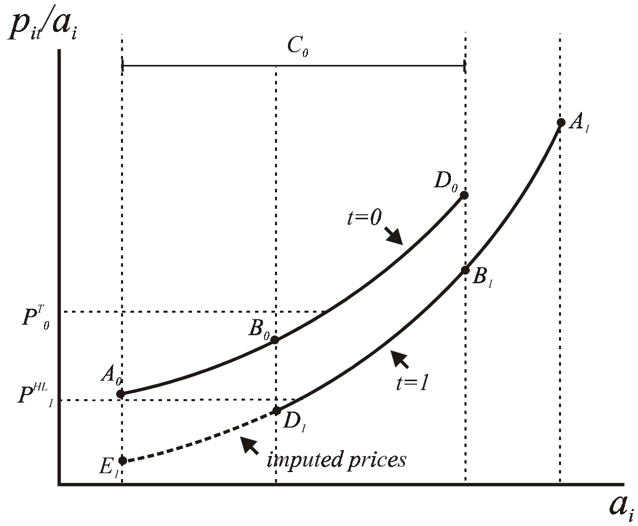
Matched model index: *IP-IQ linking method*

An IP-IQ linked matched model index only compares the prices of models that were sold in both periods and calculates a change in the price index based on these models:

- The set of models sold in both periods is $C^*_1 \cap C^*_2$.
- Calculates price increase between schedules B_0-D_0 and B_1-D_1.
- P^M_t is price level implied by schedule B_t-D_t.
- Measured inflation is change from P^M_0 to P^M_1.

Note: $P^M_1 < P^M_0$ such that the measured inflation level is negative, while the actual level of inflation is positive.

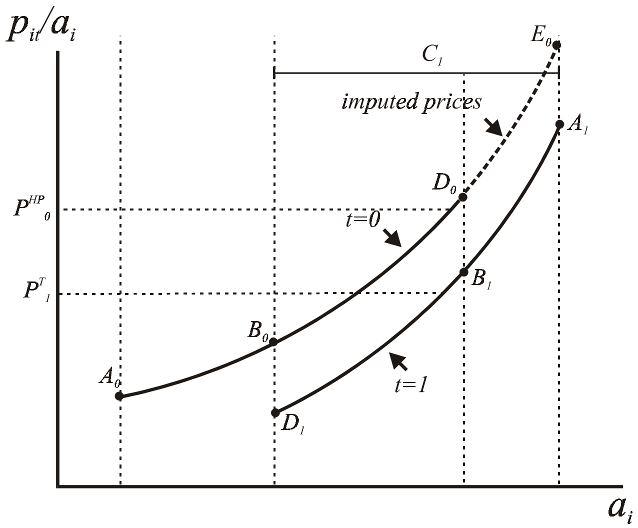
Figure 2. Downward bias in inflation measured using matched model index.



Hedonic index: *Laspeyres index*
A hedonic regression model is used to impute the prices of the unmatched models in period *t=0* for *t=1* in order to calculate a Laspeyres index with *t=0* as baseperiod.

- A Laspeyres index cannot be calculated because price schedule D_1-E_1 is not observed.
- Unobserved part, D_1-E_1, of period *t=1* price schedule is imputed using hedonic model.
- P^{HL}_1 is price level imputed for schedule B_1-E_1.
- Measured inflation is change from P^T_0 to P^{HL}_1.

Note: $P^{HL}_1 < P^T_0$ such that the measured inflation level is negative, while the actual level of inflation is positive.



Hedonic index: *Paasche index*
A hedonic regression model is used to impute the prices of the unmatched models in period *t=1* for *t=0* in order to calculate a Paasche index with *t=1* as measurement period.

- A Paasche index cannot be calculated because price schedule D_0-E_0 is not observed.
- Unobserved part, D_0-E_0, of period *t=0* price schedule is imputed using hedonic model.
- P^{HP}_0 is price level imputed for schedule B_0-E_0.
- Measured inflation is change from P^{HP}_0 to P^T_1.

Note: $P^T_1 < P^{HP}_0$ such that the measured inflation level is negative, while the actual level of inflation is positive.

Figure 3. Downward bias in inflation measured using hedonic price index methods.

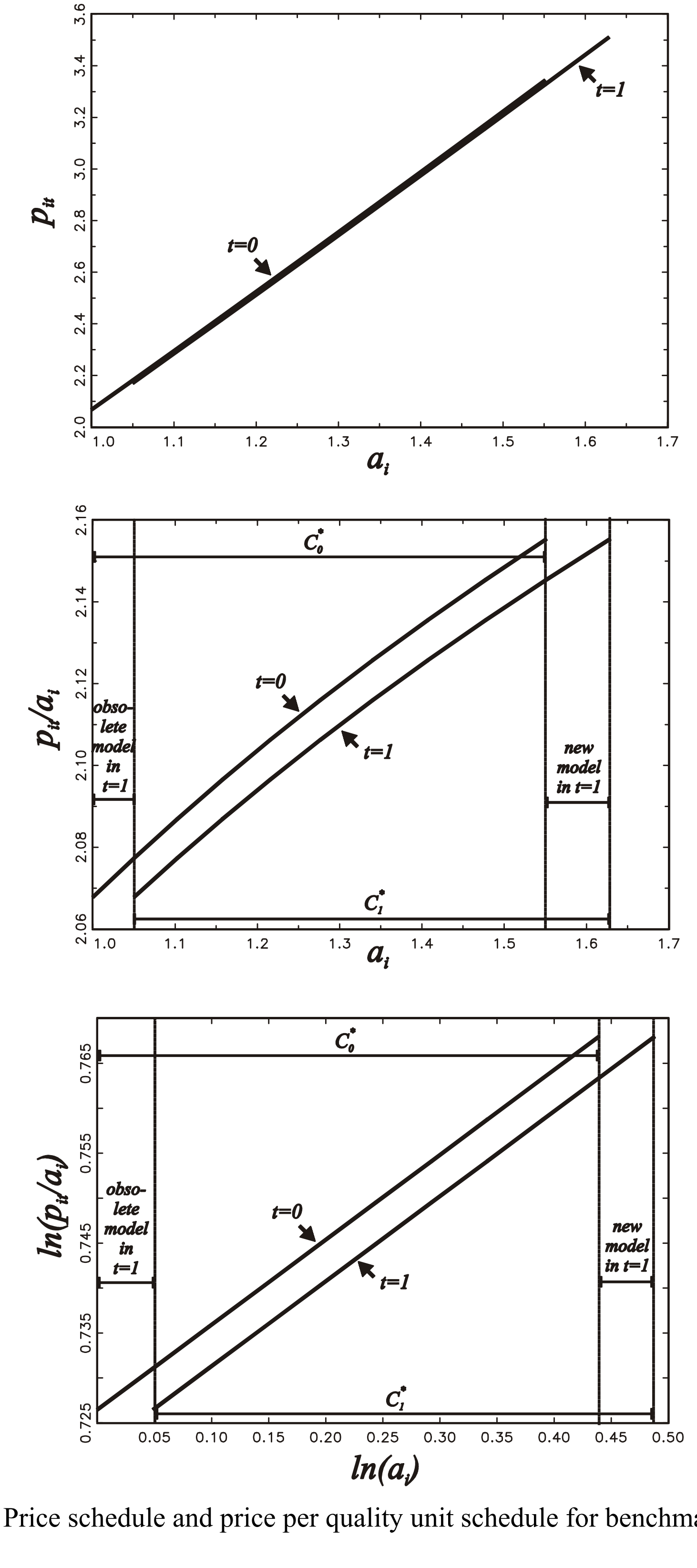


Figure 4. Price schedule and price per quality unit schedule for benchmark case (1).

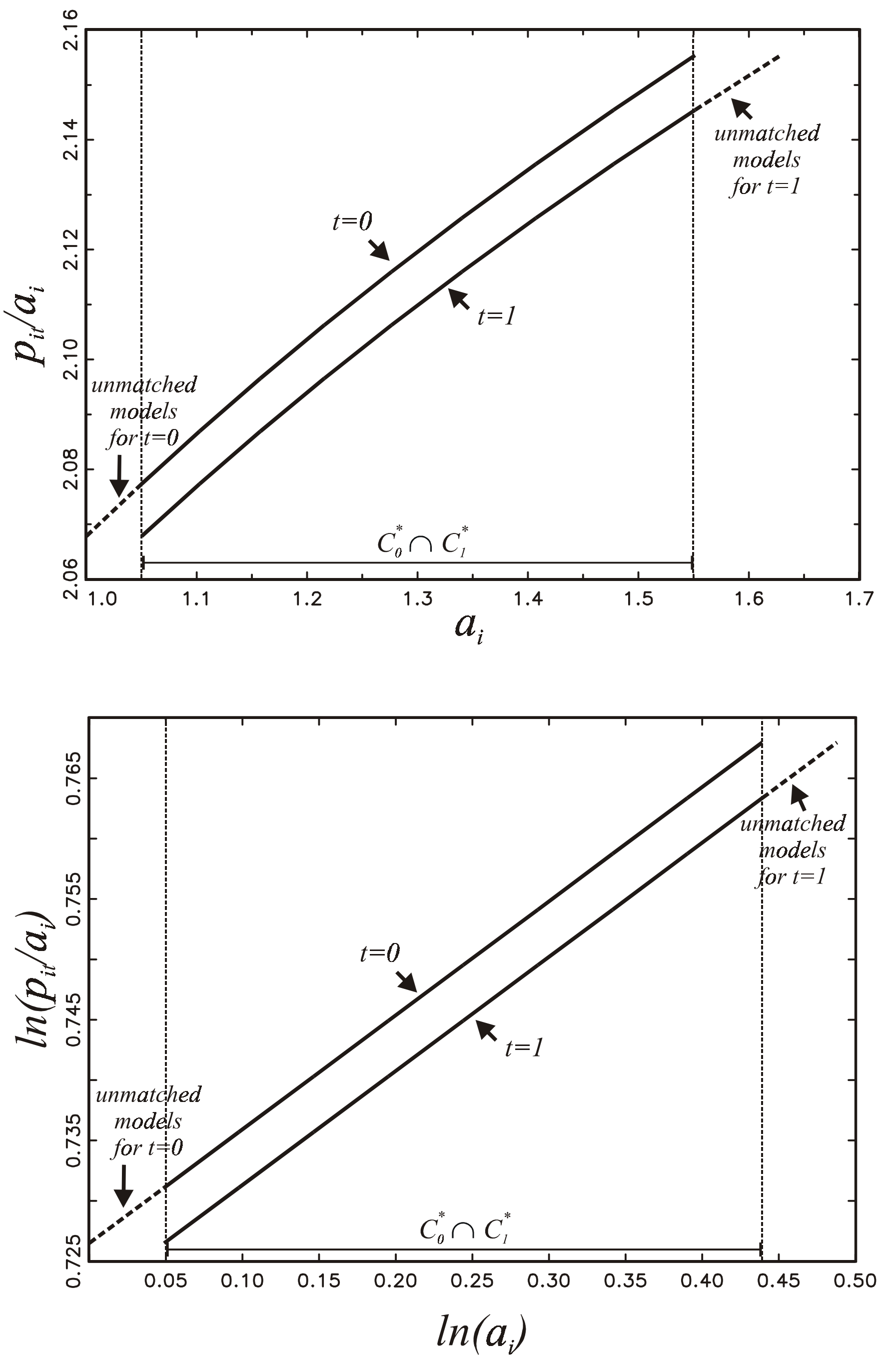


Figure 5. Application of matched model indexes in benchmark case (1).

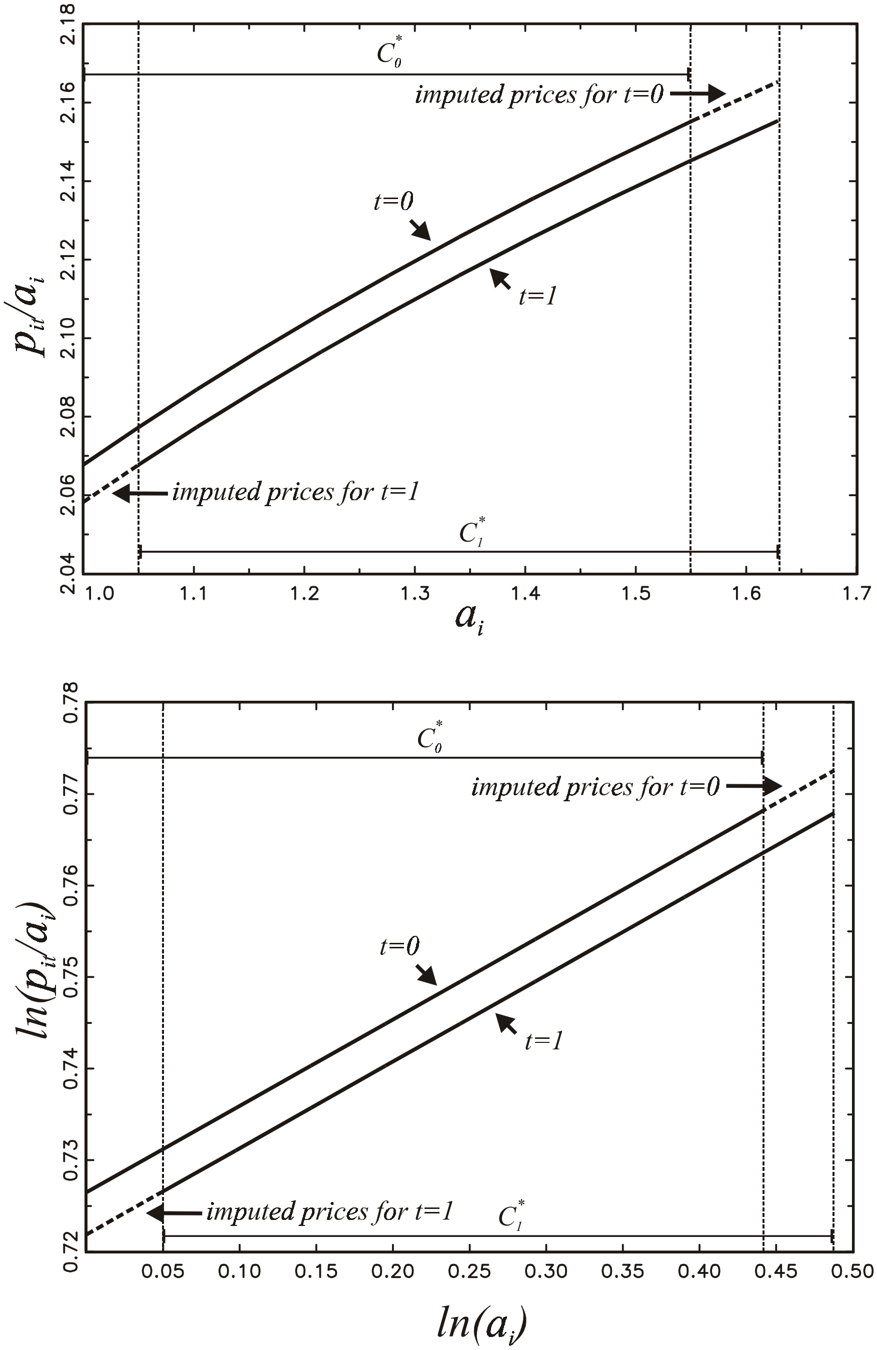


Figure 6. Application of hedonic price indexes in benchmark case (1).

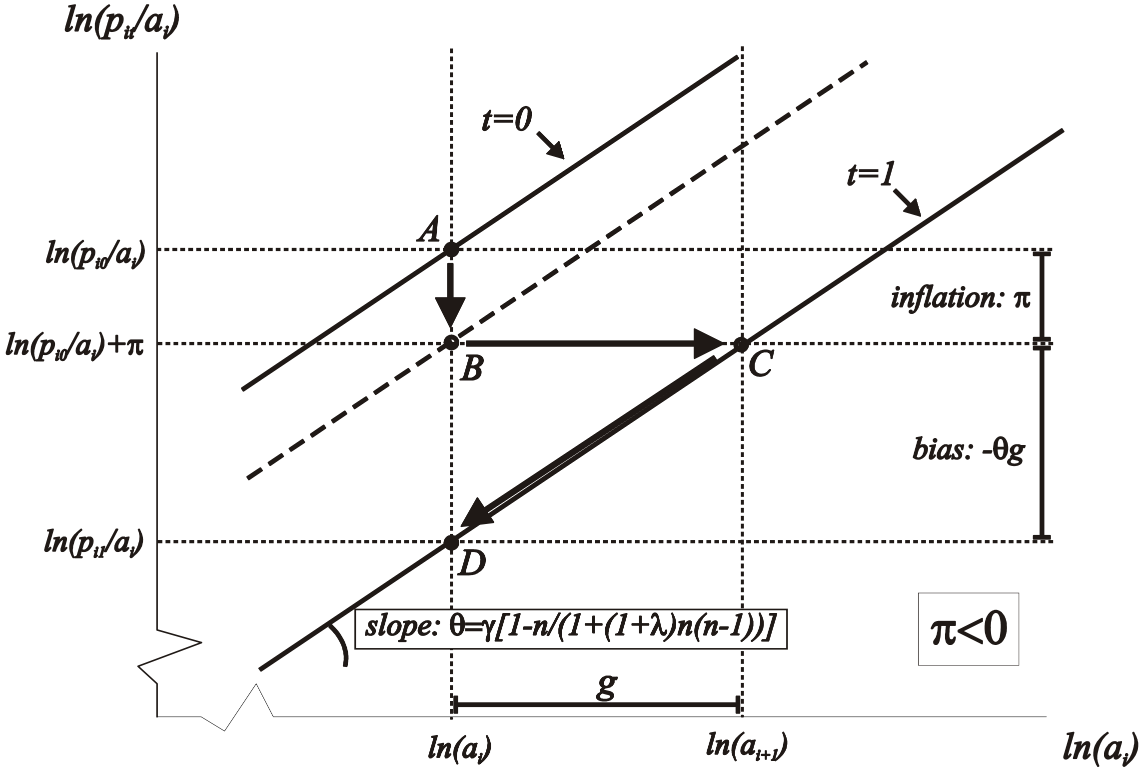


Figure 7. Graphical representation of log-linearization of bias in inflation measures