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Abstract

We explore the connection between optimal monetary policy and heterogeneity among agents. We utilize a standard monetary economy with two types of agents that differ in the marginal utility they derive from real money balances—a framework that produces a nondegenerate stationary distribution of money holdings. Without type-specific fiscal policy, we show that the zero-nominal-interest-rate policy (the Friedman rule) does not maximize type-specific welfare; further, it may not maximize aggregate ex ante social welfare. Indeed one or, more surprisingly, *both* types of agents may benefit if the central bank deviates from the Friedman rule.

Key words: Friedman rule, monetary policy, heterogeneous agents

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1 Introduction

In almost every standard monetary economy populated by *representative* infinitely-lived agents, the optimal long run monetary policy is one in which nominal interest rates are zero, also known as the Friedman rule (Friedman, 1969). Researchers have demonstrated that this result is robust to a wide variety of modifications.[1] Starting with the seminal work of Levine (1991), a new burgeoning literature has emerged that studies environments with *heterogeneity* in which the Friedman rule is not optimal (see, for example, Edmonds (2002), Green and Zhou (2002), Albanesi (2003), Ireland (2004), Paal and Smith (2000), among others). [2]This paper adds to this literature by characterizing the set of optimal monetary policies that is favored by heterogenous agent-types in a standard monetary economy. The novel punchline is that it is possible for *every* agent-type to dislike the Friedman rule.

A major part of our analysis is conducted in a fairly standard pure exchange money-in-the-utility function (MIUF) economy modified to include the presence of two types of agents, distinguished by their different marginal utilities from real money balances.[3] The introduction of this heterogeneity produces a nondegenerate stationary distribution of money holdings. Put simply, in a steady state equilibrium, one type holds more money balances than the other. In this setting, faster money growth affects the welfare of each type through two channels. First, there is the rate-of-return effect: both types reduce their money holdings in the face of a higher opportunity cost of holding money. Second, if the central bank is restricted to making (imposing) the *same* lump-sum transfer(tax) on both types, a (general equilibrium) transfer effect emerges that alters agents' budget sets, affects their demand for money, and creates a divergence in their consumptions.[4] Indeed, for positive money growth rates, the type that holds more money contributes more to seigniorage than the other type but receives the same transfer, in effect causing a redistribution of income from the former to the latter. For negative money growth rates, the direction of the redistribution is reversed: now, the type that holds more money pays a smaller tax, in effect engineering a income transfer from the type that holds less money to the type that holds more money.

[1]See, for instance, Woodford (1990) and Ljungqvist and Sargent (2000). Chari, Christiano and Kehoe (1996) and Correia and Teles (1996) extend this to the case in which other distortionary taxes are present. More recently, da Costa and Werning (2003) examine a model with hidden actions, finding that the optimal policy is one with zero nominal interest rates.

[2]Levine (1991) considers an environment in which there are two types of infinitely-lived agents who randomly become buyers or sellers and information on agents' type is private. If buyers value consumption sufficiently more than sellers do, and if there is some randomness in the economy, Levine shows that the optimal monetary policy is *expansionary* and not contractionary as the Friedman rule would suggest.

[3]An advantage of using the MIUF setup is that it encompasses a wide array of different rationales for valuing money (Feenstra,1986) and produces a perfectly "flexible" money demand function. Additionally, many of the important papers on this topic use this or a similar formulation. Many of our assertions carry over into other monetary models with infinitely-lived agents (for example, a cash-in-advance or a shopping time model).

[4]Following Pigou and Patinkin, Ireland (2004) calls it the "real balance effect". If, the government is allowed to make type-specific transfers, then the Friedman rule will again be optimal, as shown by Gahvari (1988).

It is possible for the redistributive effect of an increase in the money growth rate to dominate the rate-of-return effect for some types of agents. In that case, an increase in the money growth rate may even be welfare enhancing. We are able to show that at least one of the types always dislikes the Friedman rule (locally), i.e., they are better off in a lifetime welfare sense if the money growth rate increases locally around the Friedman rule money growth rate.[5] In most settings, the type that holds less money dislikes the Friedman rule (locally) but in special circumstances which we discuss below, even the type that holds more money balances may join the other type in their shared distaste of the Friedman rule. We go on to show that if the type that holds more money dislikes the Friedman rule locally, their welfare is never maximized globally at a *non-negative* money growth rate. Interestingly, a parallel result for the type that holds less money is that even if they like the Friedman rule locally, they may be globally better off at (possibly) a *positive* money growth rate. Perhaps most surprisingly, welfare of each type may be maximized away from the Friedman rule. In other words, it is possible for *everyone* to prefer positive nominal interest rates over Friedman's zero-nominal-interest-rate prescription.[6]

An intuitive explanation for these results is in order. Recall that the type that holds more money contributes more to seigniorage than the other type but receives the same transfer. As a result, she receives net transfers when the money growth rate (i.e., inflation tax rate) is negative. The net transfer is simply the product of the inflation tax rate and the *difference* in money holdings of the two types. As the money growth increases starting from the Friedman rule money growth rate, the inflation tax rate rises; this rate-of-return effect lowers the net transfer, and therefore, always hurts the type that holds more money. The effect coming from the changes in agents' money holdings is more complicated. Much depends on the rate at which each type adjusts their money balances in response to an increase in the money growth rate, i.e., on the elasticity of money demand. If both types reduce their money balances at similar rates in response to an increase in the inflation tax rate, then the aforementioned rate-of-return effect dominates; in this case the type that holds more money likes the Friedman rule. Precisely for the same reason, the type that holds less money will not like the Friedman rule.

On the other hand, if the type that holds less money changes her money holdings at a faster rate than the other type, then the difference in money holdings grows as the money growth rate is raised. In such a setting, the type that holds more money would increase its net transfers and therefore dislike the Friedman rule; indeed their welfare may be maximized at a much higher money growth rate. Under certain parameter sets, we find that the difference in money holdings responds non-monotonically to the money growth

[5] In Levine (1991), as in our setting, lump-sum taxes that fund the contraction are imposed symmetrically on both the types. As such, a contraction hurts "an unlucky buyer" and because buyers value consumption sufficiently more than sellers do, this monetary action hurts buyers more than it benefits sellers and hence reduces overall welfare.

[6] In a political economy context, it follows that if the central bank is not in any position to (in the words of current Fed chairman, Alan Greenspan) "shut off the political pressure valve", and if the median voter is the type that likes inflation, the central bank may pursue a positive nominal interest rate policy.

rate; near the Friedman rule it rises for a while and then starts to fall again. This makes the size of the redistribution respond non-monotonically to the money growth rate. This explains why money growth rates higher than that implied by the Friedman rule, including positive money growth rates, may be welfare maximizing for one or both types. What is novel here is that while all agents may prefer some deviation from the Friedman rule, different types may want deviations of different sizes.

Thus far we have deliberated on the effects of an increase in the money growth rate on type-specific welfare. What about societal welfare, a population-weighted aggregate welfare of both types? We are able to show that a sufficient (but not necessary) condition for societal welfare to not be maximized at the Friedman rule is that the type that holds less money locally dislikes the Friedman rule. This is because at the Friedman rule money growth rate, the rate of return distortion is absent and all agents are optimally satiated with real balances; however, the type that holds more money has the higher consumption but values it marginally less. As such, it may become efficient to redistribute some income away from these people and this benefits the type that holds less money (hence their "local dislike" of the Friedman rule). Somewhat interestingly, we can prove that the societal-welfare-maximizing money growth rate is *non-positive.* The intuition here is straightforward. Both types increase their money holdings as the money growth rate falls. Additionally, a zero money growth rate is preferred to a positive money growth rate because at the former, the transfer effect is absent and consumption is efficiently equalized across the types. At the other extreme of the Friedman rule money growth rate, as discussed above, it may become efficient to redistribute some income away from those who hold more money. This redistribution is achieved by choosing a money growth rate at which the transfer effect reallocates consumption such that the combined gain in utility from consumption dominates the combined loss of utility from the holding of smaller money balances. The novelty here is that the Friedman rule, contrary to received wisdom from many representative infinitely-lived agent models, is not necessarily welfare maximizing.

A version of our result that the Friedman rule may not appeal to all types appears in Bhattacharya, Haslag, and Martin (2005). There they show that is quite possible (in a wide range of monetary environments) that one type may not like the Friedman rule. Unlike Bhattacharya, Haslag, and Martin (2005), we conduct our analysis in a standard representative infinitely-lived agent model and go much further and characterize the set of monetary policies that each type likes. We show that it is possible that both types dislike the Friedman rule (something that is not possible in Levine, 1991) and that the rule may not even maximize *ex ante* social welfare. Indeed our analysis highlights several crucial components of the underlying political economy dimension of the larger question of the optimal monetary policy. It bears emphasis here that while the MIUF environment permits "closed form" characterization of these results, many of the insights themselves are not specific to the chosen environment; indeed, they are applicable in standard cash-in-advance, turnpike, and shopping-time models of money.

The rest of the paper proceeds as follows. Section 2 presents the model economy while

Section 3 studies whether the Friedman rule is optimal for both types of agents. In Section 4 we study the optimal money growth rule that would be chosen by a social planner, while Section 5 studies the money growth rates that maximize type-specific welfare. Section 6 concludes. Proofs of many of the results are relegated to the appendices.

2 The model

In this section, we modify the standard representative-agent money-in-the-utility function economy to include two types of agents distinguished by their preference for real money balances. The economy is populated by a continuum of unit mass of infinitely-lived agents. Time is discrete and denoted by $t = 0, 1, 2, .., \infty$. Let μ be the fraction of agents that place a relatively high value on the services from real money holdings, a notion that will be made precise below.

2.1 The environment

There is a single consumption good which is perishable. Every period both types of households are endowed with constant $\bar{y} > 0$ units of this good.[7] Money is the only asset in the economy. All agents maximize the discounted sum of momentary utilities over an infinite horizon. Agents who place a relatively high (low) value on the services of real money balances are referred to as type $H\,(L)$. The preferences of the type-i where $i = H, L$ agents are represented by

$$W^i \equiv \sum_{t=0}^{\infty} \beta^t U^i\left(c_t^i, m_t^i\right) \qquad\qquad i = H, L, \tag{1}$$

where $0 < \beta < 1$ is the agent's subjective rate of time preference; for a type-i agent, c^i is the quantity of the consumption good, and $m_t^i \equiv \frac{M_t^i}{p_t}$ denotes the quantity of real money balances carried over from period t to $t+1$. We assume that $U_j^i > 0$ and $U_{jj}^i < 0$, $i = L, H$, $j = m, c$, where $U_j^i \equiv \frac{\partial U^i}{\partial j}$ and $U_{jj}^i \equiv \frac{\partial^2 U^i}{\partial j^2}$. Also, as is standard, we posit there exists a satiation level of real money balances such that $U_m^L\left(c_L, m^{*L}\right) = U_m^H\left(c^H, m^{*H}\right) = 0$ with m^{*H} not less than m^{*L}. Finally, we assume $U_m^H\left(\hat{c}, \hat{m}\right) > U_m^L\left(\hat{c}, \hat{m}\right)$, $\forall \hat{m} \leq m^{*H}$, for $i = L, H$. In words, for the same values of consumption and real balances, the type-H derives greater *marginal* utility from the services associated with money than does a type-L agent.[8]

[7] The assumption of an endowment economy is harmless. It will be easy to see, in what follows, that introducing capital and endowing households with a production technology will yield a steady state capital stock that is independent of monetary policy.

[8] As is well known, the MIUF formulation captures some underlying technology for decentralized exchange through which money saves on transactions costs or shopping time. Differences in their preferences over real balances across agent types (with similar incomes) may then be thought of as proxying differences in the transactions technologies they have access to. Presumably, these differences in preferences reflect unmodelled differences in familiarity and comfort with the use of various payments systems. We use the

Every period, an agent allocates its real balances from last period, current endowment, and transfers received from the government between current consumption and money balances to be carried over to the next period. Formally, the budget set of an agent i is defined by

$$\bar{y} + \frac{m_{t-1}^i}{(1+z_t)} + \tau_t \geq c_t^i + m_t^i. \tag{2}$$

where $1 + z_t = \frac{p_{t-1}}{p_t}$, p_t is the price level in period t, and τ denotes transfers from the government. There are two maximization problems, one for each type of agent. The optimal choice for the type-i agents, $i = L, H$ is characterized by a sequence $\left\{c_t^i, m_t^i\right\}_0^\infty$ that maximizes W^i as given by (1) subject to its sequence of budget constraints, (2). It is easy to check that the relevant first order condition is given by

$$U_c^i\left(c_t^i, m_t^i\right) = U_m^i\left(c_t^i, m_t^i\right) + \frac{\beta U_c^i\left(c_{t+1}^i, m_{t+1}^i\right)}{(1+z_t)}. \tag{3}$$

Equation (3) has a standard interpretation. At the margin, an agent is indifferent between consuming a unit this period versus carrying it over and consuming next period. The factor $1 + z_t$ in the denominator of the second term captures the notion that carrying over a unit of nominal balance this period is worth $\frac{1}{1+z_t}$ in the next.

The government runs a balanced budget period by period. At each date $t \geq 0$, the government finances a lump-sum tax or transfer, denoted τ, by altering the money supply. Formally, the date-t government budget constraint is: $\tau_t = \frac{M_t - M_{t-1}}{p_t}$, where M_t denotes the per-capita quantity of nominal money at date t. We assume the government follows a constant money growth rule given by $M_t = (1+z)\, M_{t-1}$, where $z > -1$. The money supply expands if $z > 0$, so that $\tau_t > 0$ is a transfer. Conversely, the money supply contracts if $-1 < z < 0$, so that $\tau_t < 0$ is a tax.

2.2 Stationary equilibrium

In a stationary environment, the price level increases at the same rate as the money supply. Hence $p_t = (1+z)\, p_{t-1}$ obtains. Thus, the money market clearing condition can be represented as follows:

$$m_t = \mu\, m_t^H + (1-\mu)\, m_t^L \tag{4}$$

where $m_t \equiv \frac{M_t}{p_t}$ is the economywide stock of real balances. Further, in steady state, consumption and real money balances are constant over time so that $c_t^i = \bar{c}^i$, $m_t^i = \bar{m}^i$, and $m_t = \bar{m}$ for all t. Notice that $\tau_t = \frac{zM_{t-1}}{p_t} = \frac{z}{(1+z)} \frac{M_t}{p_t}$ which in steady states reduces

MIUF formulation precisely because we wish to remain agnostic on the issue of what the exact form the transaction technology takes. What is important is that there be heterogeneity in equilibrium money holdings; whether the source of that heterogeneity is income or innate preferences is not crucial. Indeed, in Section 3.2 below, we study a setting in which every agent-type encounters the same transactions technology but their equilibrium money holdings are unequal due to differences in productivities.

to $\tau = \frac{z}{1+z}\bar{m}$. We assume that the amount of tax or transfer τ must be the same for both types of agents. This is the precise sense in which type-specific tax/transfer schemes are disallowed in our model. We justify this assumption by appealing to the implausibility of a tax/transfer scheme that attempts to identify people on the basis of their marginal preference for money, an object that is almost impossible for the government to observe.

Imposing steady state on (3) yields

$$\frac{U_m^i\left(\bar{c}^i, \bar{m}^i\right)}{U_c^i\left(\bar{c}^i, \bar{m}^i\right)} = 1 - \frac{\beta}{1+z} \equiv \pi\left(z\right). \tag{5}$$

where $\pi\left(z\right)$, by definition, is the opportunity cost of holding real balances.[9] For future reference, note that as $1+z \to \beta$, or $\pi\left(z\right) \to 0$, i.e., when the money growth rate approaches the Friedman rule, the money holdings of each type reach their satiation levels. Finally, note that (5) implies that, given z, a higher level of consumption is associated with a higher level of real money balances.

Using the agents' budget constraints (2), the government's budget constraint $\tau = \frac{z}{1+z}\bar{m}$, and noting that (4) in steady state implies $\bar{m} = \mu\,\bar{m}^H + (1-\mu)\,\bar{m}^L$, the agents' steady-state consumption is given by

$$\bar{c}^L = \bar{y} + \mu\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right), \tag{6a}$$

$$\bar{c}^H = \bar{y} - (1-\mu)\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right). \tag{6b}$$

Thus, $\bar{m}^L, \bar{m}^H, \bar{c}^L$, and $\bar{c}^H$ solve (5) - (6b) simultaneously. Furthermore, it is easy to see that all the allocations can be implicitly represented as functions of z.

Notice from equations (6a) and (6b) that heterogeneity in money balances affects consumption of each type. This is because an agent pays a type-specific seigniorage, $\frac{z}{1+z}\bar{m}^i$, whereas the transfer rebated by the government, $\frac{z}{1+z}\bar{m}$, is type-independent. Thus $\frac{z}{1+z}\left(\bar{m} - \bar{m}^i\right)$, which is the second term in both equations, is the *net transfer* to an agent i. In the absence of any heterogeneity, this net transfer would be zero. Henceforth, we identify the second terms in (6a) and (6b) as capturing the *transfer effect*.[10] Evidently, the transfer effect depends on the money growth rate and the difference between the real balances held by the two types.

[9] Note that the gross nominal interest rate $1+i = \beta^{-1}(1+z)$. Thus $\pi = \frac{i}{1+i}$.

[10] An alternative explanation of the transfer effect is the following. Suppose there is no heterogeneity, and all agents were identically L types. As all seigniorage is rebated back to the agents, the net transfer will trivially be zero. Suppose instead that a fraction μ of agents hold "excess real balances", $\bar{m}^H - \bar{m}^L \geq 0$. As the excess seigniorage $\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)$ raised from them is equally redistributed to all, it transpires that each agents (of both types) receive $\mu\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)$ as 'excess rebate', which equals the net transfer to an L type as in equation (6a). On the other hand, each H type pays $\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)$ but receives only $\mu\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)$. As a result, each H type's loss of income equals $(1-\mu)\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)$. The above interpretation assumed $z > 0$. It is easy to argue that $z < 0$ simply reverses the direction of income redistribution.

Below we will establish sufficient conditions under which the H types hold more money than the L types, i.e., $\bar{m}^H \geq \bar{m}^L$ will obtain. We will further specify conditions under which both $\bar{m}^H$ and $\bar{m}^L$ monotonically decrease with z. The reason why we are unable to obtain condition-free results is the following. On the one hand, depending on whether the inflation tax rate is positive or negative, one or the other type is getting a net income transfer; the type that gets the transfer can afford to hold more money. However, the different marginal utilities from holding money also dictates whether they *actually* hold more money or not.

2.3 Money growth rate and allocations

For analytical convenience, we assume a separable utility form given by

$$U^i(c,m) = u(c) + v^i(m); \qquad i \equiv L, H,$$

where $v^i(m) \equiv \lambda^i \left[w(m) - mw'\left(m^{*i}\right)\right]$, and both u and w have CES forms, $c^{1-\sigma}/1-\sigma$. To conform to our assumptions made in Section 2.1, we assume $\lambda^H > \lambda^L$ and $m^{*H} \geq m^{*L}$ hold. Then, for any $\hat{m}$,

$$\frac{U_m^H(\hat{c},\hat{m})}{U_m^L(\hat{c},\hat{m})} = \frac{\lambda^H\left[w'(\hat{m}) - w'\left(m^{*H}\right)\right]}{\lambda^L\left[w'(\hat{m}) - w'\left(m^{*L}\right)\right]} > 1. \tag{7}$$

We are then able to show the following.

Lemma 1 *Suppose*

$$\bar{y} > \bar{y}^*, \tag{A.1}$$

where $\bar{y}^*$ *is implicitly determined by* $\frac{u_c\left(\bar{y}^*+\mu(1-\beta)m^{*L}\right)}{u_c\left(\bar{y}^*-(1-\mu)(1-\beta)m^{*L}\right)} = \frac{\lambda^L}{\lambda^H}$. *Then given the assumptions on preferences and endowments,* $\bar{m}^H > \bar{m}^L$ *for all* $m^{*H} > m^{*L}$, *i.e., the* H *types hold more money than the* L *types. If* $m^{*H} = m^{*L}$, $\bar{m}^H > \bar{m}^L$ $\forall$ $z > \beta - 1$ *and* $\bar{m}^H = \bar{m}^L$ *at* $z = \beta - 1$.

When $z \geq 0$, the intuition behind why the H types hold more money than the L types is straightforward. If it were otherwise, there would be a net income transfer away *from* the L types. A lower income in addition to a lower marginal utility from money would imply that they are holding lower real balances than the H types, thus contradicting our initial supposition. For $z < 0$, suppose contrary to Lemma 1 that L types hold more money and thus receive net income transfers. Now the income effect and the relatively lower preference for real balances work in opposite directions. If (A.1) is satisfied, the income effect from transfers is dominated and L types always hold relatively smaller real balances.

An immediate implication of Lemma 1 is that

Corollary 2

$$\bar{c}^H \gtreqless \bar{c}^L \Longleftrightarrow z \lesseqgtr 0. \tag{8}$$

The type that holds more money gets the higher consumption if and only if there is deflation.

Further, differentiating (6a) and (6b) yields

$$\frac{1}{\mu}\frac{dc^L}{dz} = -\frac{1}{1-\mu}\frac{dc^H}{dz} \tag{9}$$

$$= \frac{z}{1+z}\left(\frac{dm^H}{dz} - \frac{dm^L}{dz}\right) + \frac{1}{(1+z)^2}\underbrace{\left(\bar{m}^H - \bar{m}^L\right)}_{\geq 0}, \tag{10}$$

Notice first that a change in z affects income transfers *between* the two types, and thus, changes in consumption have opposite signs (see eq. (9)). Lemma 1 ensures that the second term in (10) is positive. Thus, a higher z brings more (less) income transfers for the L (H) types. The first term, on the other hand, depends on the differential rate of change of real balances of the two types. In general, away from the Friedman rule, it turns out that the second term in (10) dominates the first, and thus consumption of L (H) types increases (decreases) with z. However, near the Friedman rule, as both types adjust their real balances relatively sharply towards satiation, the direction of consumption changes may depend on their rates of real balance adjustment relative to each other. If these adjustment rates are similar, the second term in (10) still dominates and consumption of L (H) types increases (decreases) with z. However, with a specific set of parameters, we find that the difference in money holdings responds non-monotonically with the money growth rate; near the Friedman rule it rises for a while and then starts to fall again. Then the direction of the changes in consumption is reversed.

Thus, in order to further study changes in allocations with respect to z, we need to first understand how real balances of both types change with z.

Lemma 3 *At the Friedman rule, real money balances of both types are decreasing in the money growth rate. Furthermore, suppose*

$$\bar{y} > \bar{y}^{**}, \tag{A.2}$$

where $\bar{y}^{**} \equiv \left(\varphi - \frac{1-\beta}{\lambda^H}\right) m^{*H}$*, and where* $\varphi \equiv \begin{cases} \mu\sigma(1-\beta) & \text{if } \sigma\beta < 1 \\ \mu(\sigma-1) & \text{if } \sigma\beta > 1 \end{cases}$. *Then, real money balances for both types are decreasing in the money growth rate for all* $z \geq 0$.

From (5), it follows that if consumption remained the same, real balances would simply decrease with z, a pure price effect. However, as is clear from (6a) and (6b), consumption of both types changes with z. Moreover, (6a) and (6b) imply a) if the difference $\bar{m}^H - \bar{m}^L$ remained same, a higher z will bring more (less) income for the L (H) types, b) $\bar{m}^H - \bar{m}^L$

changes with z, which also impacts their income. The two income effects of z may combine or oppose each other but, in general, the first component dominates. As a result, as z increases the total income of H (L) types decreases (increases). Thus, for H types a higher z not only increases the opportunity cost of money, but also decreases their income. As a result, $\bar{m}^H$ is decreasing in z. On the other hand, the income of L types is increasing in z. Assumption (A.2) ensures that the income effect is dominated by the price effect of a higher z. Thus, $\bar{m}^L$ is also decreasing in z.[11]

Note in passing that Assumptions (A.1) and (A.2) are sufficient but not *necessary*. Further, both can be combined as $\frac{\bar{y}}{1-\beta} > \max\{\bar{y}^*, \bar{y}^{**}\}$.

3 Who does not like the Friedman rule?

In this section, we first show that for a general class of MIUF models it is never the case that the Friedman rule is optimal for both types of agents. To verify whether this result holds under model specifications in which monetary policy has an output effect, we then study a cash-in-advance economy with production.

3.1 One type always dislikes the Friedman rule

We start by proving that for all the utility functions that incorporate satiation the Friedman rule is disliked by one type. The marginal rate of substitution between consumption and real balances are given by (5), which is repeated below for convenience:

$$\frac{U_m^i\left(\bar{c}^i, \bar{m}^i\right)}{U_c\left(\bar{c}^i, \bar{m}^i\right)} = 1 - \frac{\beta}{1+z} \equiv \pi(z). \tag{5}$$

Note that by assumption $U_m^L\left(\bar{c}^L, m^{*L}\right) = U_m^H\left(\bar{c}^H, m^{*H}\right) = 0$. Therefore, at the Friedman rule, $\bar{m}^i = m^{*i}$.

The analysis in Section 2 implies that the equilibrium steady state utilities of agents can be expressed as function of the money growth rate z. Further, using (5), it follows that

$$\begin{aligned}(1-\beta)\frac{dW^i}{dz} &= U_c^i\frac{d\bar{c}^i}{dz} + U_m^i\frac{d\bar{m}^i}{dz} \\ &= U_c^i\left[\underbrace{\frac{d\bar{c}^i}{dz}}_{\text{Transfer effect}} + \underbrace{\pi(z)\frac{d\bar{m}^i}{dz}}_{\text{Rate-of-return effect}}\right]\end{aligned} \tag{11}$$

[11] Lemma 3 asserts that real money balances for both types are decreasing in the money growth rate both locally near the Friedman rule and globally for all non-negative money growth rates. While it does not claim a similar behavior for the allowable range of negative money growth rates, such behavior is infact true. Numerical examples confirm it; additionally a messier analog of a sufficient condition like (A.2) can easily be written.

Notice that the first term within brackets represents the *transfer* effect of changes in z, while the second term denotes its *rate-of-return* effect. Since real balances are decreasing in z, the rate-of-return effect hurts both types when z is increased. Note from (5) that at the Friedman rule, the second term in (11) vanishes. Thus, at the Friedman rule, a change in utility takes place solely through a change in consumption. From (9), we know that the change in consumption for the two types have opposite signs. Thus, using (9), it follows that

$$\frac{dW^L}{dz} = -\frac{U_c^L}{U_c^H}\frac{\mu}{1-\mu}\frac{dW^H}{dz} \tag{12}$$

Hence, increasing z at the Friedman rule is always a local improvement for one type of agents.[12] We summarize the above discussion in the following proposition.

Proposition 4 *Given our assumptions, the Friedman rule is always (locally) disliked by one type.*

Notice that at the Friedman rule, both types are optimally satiated with real balances. Hence, a small change in z (engineered via changes in real balances) has no rate-of-return effect on their welfare. However, changes in real balances do affect net transfers between agents; indeed equation (11) makes clear that the direct rate-of-return effect of an increase in z is washed out leaving only the indirect transfer effect. As eq. (12) highlights, the transfer effect hurts one and benefits the other; as such, it can never be, that locally near the Friedman rule, both types will want money growth rates unchanged. Recall from (6a) and (6b) that the transfer effect depends on the gap between real balances held by the two types. If this gap shrinks as z increases, net transfers to (from) H (L) types decreases. In that case, L (H) types will be made better (worse) off by a *local* deviation in z. On the other hand, if the aforementioned gap widens, net transfers will depend on changes in the product $\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)$, which in turn will depend on the preference specification. Nevertheless, the change will hurt one type at the cost of the other.[13]

The following Lemma 5 establishes necessary and sufficient conditions to identify the agent type that would benefit from a marginal increase in z at the Friedman Rule.

Lemma 5 *Given agents' preferences, L (H) types will prefer an increase in z at the Friedman rule, if and only if*

$$\frac{u_c\left(c^{*H}\right)}{\lambda^H w_{mm}\left(m^{*H}\right)} - \frac{u_c\left(c^{*L}\right)}{\lambda^L w_{mm}\left(m^{*L}\right)} < (>) \, \frac{m^{*H} - m^{*L}}{1-\beta}, \tag{13}$$

[12] With homogeneous agents, at the Friedman rule all agents are satiated with real balances; the envelope theorem implies that a small increase in money growth will have at most a second-order impact on utility through the familiar inflation-tax channel. When the two agent types hold different levels of real balances, this change in the rate of money growth has first-order distributional effects. But these distributional effects are necessarily zero-sum: one type of agent benefits at the expense of the other.

[13] Notice that the assumption of separability is not required for the result stated in Proposition 4.

where c^{*L} and c^{*H} denote consumptions of L type and H type respectively, at the Friedman rule.

We can explain the condition (13) as follows. Suppose z is increased infinitesimally at the Friedman rule. Then there will be a change in the net transfer between the two types attributable to two effects: a) a change in inflation tax rate $\frac{z}{1+z}$ and b) a change in the difference between the real balances of the two types $\bar{m}^H - \bar{m}^L$. Increasing z reduces $\frac{z}{1+z}$ in absolute value and thus reduces (increases) transfers to the H (L) types. However, if the difference between real balances widens, i.e., $\frac{dm^H}{dz} - \frac{dm^L}{dz} > 0$, then H (L) types are better (worse) off by a larger transfer. The right hand side of condition (13) in Lemma 5 represents the tax rate effect, while the left hand side represents the effect of changes in real balances. If the widening of real balances dominates the tax rate change, the H (L) types will (will not) prefer a deviation from the Friedman rule. The situation is reversed if the widening of real balances is smaller, or if it shrinks instead, i.e., $\frac{dm^H}{dz} - \frac{dm^L}{dz} < 0$.

It is instructive to work through a special case. To that end, start by assuming that $m^{*H} = m^{*L}$ holds; then it is obvious that $c^{*L} = c^{*H}$ holds. In this case, notice that condition (13) in Lemma 5 reduces to

$$\frac{1}{\lambda^H w_{mm}\left(m^{*H}\right)} - \frac{1}{\lambda^L w_{mm}\left(m^{*L}\right)} < (>)\, 0 \tag{14}$$

Since $\lambda^H > \lambda^L$ and $w_{mm} < 0$ holds, eq. (14) implies that the L types like the Friedman rule, but the H types would prefer a higher money growth rate.[14] Thus, in this case, even the H types (who always hold higher real balances relative to L types, and with $z < 0$, are the *net receivers* of income) dislike the Friedman rule. This can happen because of the following reason. Notice that while the Friedman rule obtains the agents a satiation level of real balances, it does not maximize their income from net transfers. Now as z rises, faced with a positive opportunity cost, both types reduce their real balances. However, the decrease in L types' real balances is sharper relative to that of the H types. Thus with a marginal increase in z, the H types can obtain bigger transfers (which to them has a positive worth in terms of marginal utility of consumption), whereas losing real balances at the margin is costless to them since they are already satiated with real balances.

The same logic implies that L types will not prefer a local deviation from the Friedman rule. Note however, it is not clear from the above condition if the Friedman rule is *globally* preferred by L types. Finally, suppose that the condition stated in Lemma 5 holds in a way such that L types prefer a higher money growth rate than the Friedman rule. Again, even though now the H types do not prefer a local increase in z, it is not clear if the Friedman rule maximizes their welfare.

The above discussion raises two key policy questions. First, what are the most-preferred type-specific money growth rules? And, more importantly, what is the socially optimal

[14] By continuity, same holds true even for cases where $m^{*H} > m^{*L}$, but λ^H is sufficiently larger than λ^L.

money growth rate? While the answer to the first question is postponed until Section 5, the socially optimal level of z is studied next in Section 4.

3.2 Models in which superneutrality fails

Is Proposition 4 simply an artifact of the assumptions in the model that yields superneutrality? If changes in the money growth rate distort output, do our results disappear? Below we first present a simple extension of our model that adds a labor-leisure choice and which reaffirms the results stated in Proposition 4. Next, we contrast our results with a cash-in-advance set up where monetary growth additionally creates an intertemporal price distortion that depresses output. Both extensions prove that the presence of superneutrality is not needed for the flavor of Proposition 4 to survive.

3.2.1 MIUF with labor-leisure choice

Here, each agent has a unit of time that it can divide between labor and leisure. Let agents' momentary utility be given by $U^i(c, l, m)$ and let each type have access to an identical production technology described by $f(l)$, where f has the standard properties of a production function. It is straightforward to show that the marginal rate of substitution between consumption and labor is given by

$$-\frac{U_l^i}{U_c^i} = f_l\left(l^i\right) \tag{15}$$

Now that each agent's output is given by $f\left(l^i\right)$, using (6a) and (6b), their consumption is given by

$$\begin{aligned}
\bar{c}^L &= f\left(\bar{l}^L\right) + \frac{z}{1+z}\mu\left(\bar{m}^H - \bar{m}^L\right) \\
\bar{c}^H &= f\left(\bar{l}^H\right) - \frac{z}{1+z}(1-\mu)\left(\bar{m}^H - \bar{m}^L\right)
\end{aligned}$$

As before, each agent's allocations and utility can be implicitly expressed as a function of z. Differentiating the L types' utility with respect to z yields

$$\frac{dU^L}{dz} = U_l^L\frac{dl^L}{dz} + U_c^L f_l\left(l^L\right)\frac{dl^L}{dz} + U_c^L\frac{d}{dz}\left[\frac{z}{1+z}\mu\left(\bar{m}^H - \bar{m}^L\right)\right] + U_m^L\frac{dm^L}{dz},$$

which using (15) reduces to

$$\frac{dU^L}{dz} = U_c^L\frac{d}{dz}\left[\mu\frac{z}{1+z}\left(\bar{m}^H - \bar{m}^L\right)\right] + U_m^L\frac{dm^L}{dz}$$

Again, the second term in the above equation vanishes at the Friedman rule. Combining the above with a similar equation for the H types replicates (12).[15]

[15] As discussed in footnote 12 above, the envelope theorem applies in considering small departures from the Friedman rule. Whether this means that the marginal utility of real money balances equals zero or

3.2.2 A cash-in-advance economy

The agents' heterogeneity now stems from their differential abilities to produce and therefore accumulate unequal real balances from the sale of their produce. Here, both types of agents have identical preferences in consumption (c) and leisure ($1-l$), represented by a standard utility function $u(c, 1-l)$. Agents produce consumption goods by using the following technology

$$y^i = \alpha^i f\left(l^i\right), \ \alpha^H > \alpha^L, \ f' > 0.$$

As is standard in these models, we assume that a household consists of a shopper-seller pair, who separate at the beginning of each period and then reunite in the end. While the seller works at the mill and sells the output, the shopper goes to the mills (other than her own) with cash to purchase goods. Note that the money accumulated through sales can only be used for purchases during the next period. Thus, once the inflation is taken into account, a unit of labor that earns $\alpha^i f'\left(l^i\right)$ units of goods today is worth only $\frac{\alpha^i}{1+z} f'\left(l^i\right)$ units tomorrow. At the optimum, an agent is indifferent between enjoying a unit of leisure today, or working in the market and consuming $\frac{\alpha^i}{1+z} f'\left(l^i\right)$ units of goods tomorrow. Thus, a household's optimal labor leisure choice is given by

$$-u_l^i = \frac{\alpha^i f'\left(l^i\right)}{(1+z)} \beta u_c^i, \tag{17}$$

where $u_j^i \equiv u_j\left(\bar{c}^i, \bar{l}^i\right)$. Alternatively, (17) equates the marginal rate of substitution between consumption and leisure $\frac{u_l^i}{u_c^i}$ to its marginal rate of transformation $\alpha^i f'\left(l^i\right)$ discounted by the gross nominal interest rate $(1+z)\beta^{-1}$. Were the labor earnings consumed during the same period, the relative price of earnings to consumption would identically equal 1. Thus, the cash-in-advance constraint lowers the price of earnings relative to consumption by $1 - \frac{\beta}{1+z}$, which discourages work relative to the case in which earnings are consumed contemporaneously.

Further, in the steady state, agents' consumption is given by [see (42a) and (42b) in Appendix E]

$$\bar{c}^L = \alpha^L f\left(\bar{l}^L\right) + \frac{z}{1+z} \mu \left(\bar{m}^H - \bar{m}^L\right) \tag{18a}$$

$$\bar{c}^H = \alpha^H f\left(\bar{l}^H\right) - \frac{z}{1+z} (1-\mu) \left(\bar{m}^H - \bar{m}^L\right) \tag{18b}$$

Observe that the terms in the above expressions are identical to those in (6a) and (6b), except that agents' output now depends on their optimal choice of labor which in turn depends on the money growth rate z.

that the MRS between consumption and leisure equals the marginal product of labor at the Friedman rule is not crucial; either way, the fact that these efficiency conditions hold implies that the allocative effects of a small departure from the Friedman rule will have at most a second-order impact on welfare.

Once again, agents' steady state utilities can be expressed as functions of z. Then,

$$\frac{du^i}{dz} = u_c^i \frac{d\bar{c}^i}{dz} + u_l^i \frac{d\bar{l}^i}{dz},$$

which, using (17) - (18b), yields

$$\frac{du^L}{dz} = \underbrace{u_c^L \alpha^L f'\left(\bar{l}^L\right)\left(1 - \frac{\beta}{1+z}\right)\frac{d\bar{l}^L}{dz}}_{\text{Rate-of-return effect}} + \underbrace{u_c^L \frac{d}{dz}\left[\frac{z}{1+z}\mu\left(\bar{m}^H - \bar{m}^L\right)\right]}_{\text{Transfer effect}}, \tag{19}$$

$$\frac{du^H}{dz} = u_c^H \alpha^H f'\left(\bar{l}^H\right)\left(1 - \frac{\beta}{1+z}\right)\frac{d\bar{l}^H}{dz} - u_c^H \frac{d}{dz}\left[\frac{z}{1+z}\left(1-\mu\right)\left(\bar{m}^H - \bar{m}^L\right)\right].$$

Notice that the first term on the right hand side of (19) captures the rate-of-return effect, while the second term represents the transfer effect. As discussed above, the rate-of-return effect now stems from the intertemporal price wedge introduced by the cash-in-advance constraint. Under some mild restrictions on preferences, it can be shown that a higher rate of inflation z discourages work.[16] Then, as in our MIUF version, the rate-of-return effect implies that both types are hurt by an increase in z, while the net transfer effect benefits one type at the cost of the other.

Notice also that the intertemporal price wedge $1 - \frac{\beta}{1+z}$, and thus the rate-of-return effect, vanishes at the Friedman rule. The change in welfare can be attributed solely to the transfers and, once again, the result is identical to (12) obtained for the MIUF version, i.e.,

$$\frac{dW^L}{dz} = -\frac{u_c^L}{u_c^H}\frac{\mu}{1-\mu}\frac{dW^H}{dz}.$$

Thus, as before, one type dislikes the Friedman rule.

4 Social Welfare

The preceding analysis showed that precisely one type of agents will prefer a local deviation from the Friedman rule. That is, the type-specific welfare of one of the types is not maximized at the Friedman rule money growth rate. Is the Friedman rule "socially optimal" in this case? In order to answer this question, we first define social welfare W as

[16] For example, preferences of the form

$$u(c, l) = u\left[c - \frac{l^\nu}{\nu}\right],\ \nu > 1$$

will readily generate this result.

a population-weighted sum of type-specific utilities.[17] Formally:

$$W \equiv (1-\mu)\, W^{L} + \mu W^{H},$$

where W^H and W^L are as given by (1). A benevolent central bank chooses z to maximize W (where $\tilde{z} \equiv \arg\max_{z} W$), i.e., pick the z that solves $\frac{dW}{dz} \leq 0$. [18]

4.1 When is the Friedman rule socially optimal?

Differentiating W with respect to z and using (12) it can be shown that at the Friedman rule, i.e., at $z^{\mathrm{FR}} \equiv \beta - 1$,

$$\left.\frac{dW}{dz}\right|_{z^{\mathrm{FR}}} = \mu \frac{du^{H}}{dz} + (1-\mu)\frac{du^{L}}{dz} = (1-\mu)\left(1 - \frac{u_c^H}{u_c^L}\right)\frac{du^L}{dz}$$

holds. Notice that

$$\left.\frac{dW}{dz}\right|_{z^{\mathrm{FR}}} \begin{cases} = 0, \text{ if } m^{*H} = m^{*L} \\ \gtreqless 0 \text{ iff } \frac{du^L}{dz} \gtreqless 0, \text{ if } m^{*H} > m^{*L}\ . \end{cases} \tag{20}$$

If $m^{*H} = m^{*L}$ holds, then $\bar{c}^H = \bar{c}^L$, and $u_c^L = u_c^H$ holds; here the Friedman rule is also globally optimal as it allocates consumption efficiently while simultaneously allowing both types to hold their satiation level of real balances. On the other hand, if $m^{*H} > m^{*L}$, at the Friedman rule $\bar{c}^H > \bar{c}^L$, and thus $u_c^L > u_c^H$. The following proposition is then immediate from an examination of (20).

Proposition 6 *If $m^{*H} > m^{*L}$, the Friedman rule is socially optimal only if the L types do not prefer a higher money growth rate.*

Proposition 6 states that for the Friedman rule to be socially optimal it is *necessary* that the L types locally like it. Conversely, it is implied that the Friedman rule can not be socially optimal if increasing z yields a higher utility for the L types. At the Friedman rule, all agents are optimally satiated with real balances. Therefore, a marginal increase in z which cause real balance holdings to decline marginally is costless in terms of lost marginal utility. However, since $m^{*H} > m^{*L}$, at the Friedman rule $\bar{c}^H > \bar{c}^L$ (see (8)), and therefore, the L types value a unit of consumption more than the H types do. So it is efficient to redistribute some income from the H to the L types in order to allocate consumption more efficiently. This would make the L types better off and render the Friedman rule socially sub-optimal.

[17] Our notion of social optimality is identical to the parallel concept of ex-ante optimality; in the latter, agents "pick their preferred monetary policy under a "veil of ignorance", before knowing their true identity" [Ljungqvist and Sargent (2000)]; hence they attach a probability μ of being the H type.

[18] The inequality accounts for the case in which the Friedman rule money growth rate happens to be a corner solution.

On the other hand, if L types prefer the Friedman rule to any marginal increase, then $\frac{dW}{dz}\big|_{z^{\mathrm{FR}}} < 0$. But, it does not ensure that the Friedman rule is also globally optimal. In Section 5 we show that even when the L types prefer the Friedman rule locally, their type-specific optimal choice may turn out to be $z > 0$. Arguably, under such a scenario, a social planner may choose a $\tilde{z} > \beta - 1$.

4.2 Can a positive money growth rate ever be socially optimal?

Clearly, if the L types do not like the Friedman rule, the planner's choice is $\tilde{z} > \beta - 1$. Even otherwise, the planner may choose $\tilde{z} > \beta - 1$. But can $\tilde{z}$ ever be positive? The following proposition asserts that $\tilde{z}$ must be negative.

Proposition 7 *The socially optimal money growth rate is negative, i.e.,* $\beta - 1 \leq \tilde{z} < 0$.

The intuition behind Proposition 7 is quite straightforward. By choosing $z > 0$, the planner imposes a needless opportunity cost on all agents' stock of real balances; additionally, as argued above, by making $\bar{c}^H < \bar{c}^L$, the planner engineers an inefficient income redistribution. If the money supply is constant, i.e., $z = 0$, there is no income redistribution and $\bar{c}^H = \bar{c}^L$. The marginal social cost of reallocating consumption at $z = 0$ is essentially zero. Thus, both types can gain by holding marginally higher real balances; this can be achieved by marginally cutting z from $z = 0$.

5 Type-specific optimal rules

We go on to study the question: which money growth rate is globally liked by each type? In particular, is it possible that both types would like money growth rates that are higher than that implied by the Friedman rule? Can they each prefer positive money growth rates? Our analysis below shows that the type-specific welfare maximizing values of z for both types, denoted as $\tilde{z}^L$ and $\tilde{z}^H$, crucially depend on their relative preference for real balances, particularly the money demand elasticities.

First, we specialize to a special functional form first popularized by Greenwood, Hercowitz, and Huffman (1988). Let utility be defined as follows:

$$U^i\left(c^i, m^i\right) = u\left[c^i + \lambda^i\left(\ln m^i - \frac{m^i}{m^{*i}}\right)\right];\ i \equiv H, L,\ \lambda^H > \lambda^L \tag{21}$$

We choose this form for two reasons. First, it enables us to make analytical progress and compute a closed form solution for the optimal z that is liked by each type. Second, it differentiates between the rate-of-return and transfer effects with changes in z more sharply. Note that the basic dispute between the two types over the choice of z arises from the fact that their unequal real balances lead to unequal net transfers from the government, which in turn generates income effects for the both types. With a more general utility form, the income effect will affect agents' real balances as well as consumption. With (21), real balances are insulated from the income effect and the changes in income are completely absorbed by the changes in consumption. As a result, the choice of real balances solely depend on the rate of money growth z.

Using (5), the optimal demand for real balances is given by

$$\bar{m}^i = \left(\frac{\pi(z)}{\lambda^i} + \frac{1}{m^{*i}}\right)^{-1} = \frac{m^{*i}}{1 + \frac{m^{*i}}{\lambda^i}\pi(z)}, \tag{22}$$

where $\pi(z) \equiv 1 - \frac{\beta}{1+z}$. It is clear from equation (22) that both types are satiated with real balances at the Friedman rule. Further, real balances of both types decrease as the money growth rate is raised implying that the flavor of Lemma 1 continues to hold. We maintain our assumption that $m^{*H} \geq m^{*L}$ and $\lambda^H > \lambda^L$ hold. In addition, if we further assume that

$$\frac{\lambda^H}{\lambda^L} \geq \frac{m^{*H}}{m^{*L}} \tag{A.3}$$

hold, then as evident from (22), a stronger version of the result in Lemma 3 also holds; indeed, under (A.3), the H type's preference for real balances are uniformly stronger than the L type at all z. Both "$=$" and "$>$" in the above assumption are studied below.

5.1 Equal elasticities of money demand

We further assume that the money demand elasticities of the two types with respect to z, denoted as $\zeta_{\bar{m}^i,z}$, are equal.[19] First, note that

$$\zeta_{\bar{m}^i,z} \equiv \left|\frac{\frac{d\bar{m}^i}{\bar{m}^i}}{\frac{dz}{z}}\right| = \left|\frac{\kappa^i}{1+\kappa^i\pi(z)}\frac{\beta z}{(1+z)^2}\right|, \tag{23}$$

where $\kappa^i \equiv \frac{m^{*i}}{\lambda^i}$. Then for the money demand elasticities of the two types to be equal, it is required that

$$\frac{m^{*i}}{\lambda^i} = \kappa^i = \kappa, \ \forall\ i \tag{24}$$

hold. Further, notice that since $\lambda^H > \lambda^L$ holds, it is implied that $m^{*H} > m^{*L}$. It directly follows from (22) that

$$\bar{m}^H - \bar{m}^L = \frac{m^{*H} - m^{*L}}{1 + \kappa\pi(z)} \tag{25}$$

From equation (25) it is obvious that $\bar{m}^H - \bar{m}^L$ increases as money growth rate is lowered. In particular, this difference peaks at the Friedman rule.

[19] The optimal value of z for the H type is critically affected by this assumption. In the next subsection, we allow the types to have different elasticities of money demand.

Note from (6b) that the net transfer to H types, which equals $-(1-\mu)\frac{z}{1+z}\frac{m^{*H}-m^{*L}}{1+\kappa\pi(z)}$, is positive when $z<0$. A simple differentiation verifies that these transfers decrease as z increases. Clearly, at the Friedman rule, the H types enjoy the maximum consumption feasible at any $z\geq\beta-1$, in addition to satiating themselves with real balances. Thus, the Friedman rule is the best rule for the H types, i.e., $\tilde{z}^H=\beta-1$.

The net transfer to the L types, on the other hand, is negative as long as z is negative. However, they do enjoy the benefits of a lower inflation by holding a higher stock of real balances. The optimal z for them, thus, depends on the trade-off between these two effects. At the Friedman rule, the rate of return effect vanishes as discussed in Section 3. However, both the seigniorage tax rate $\frac{z}{1+z}$ and the difference between the real balances of the two types $\bar{m}^H-\bar{m}^L$ decrease in absolute value at the Friedman rule, as z is increased. Thus, L types would benefit from an increase in the money growth rate as the absolute value of net transfers *from* them decreases . Then the question is what is the optimal money growth rate for the L types? In particular, is a positive z ever optimal for them? To compute $\tilde{z}^L$, we first obtain the consumption of L types by substituting (25) in (6a):

$$\bar{c}^L=\bar{y}+\mu\frac{z}{1+z}\frac{m^{*H}-m^{*L}}{1+\kappa\pi(z)}, \tag{26}$$

Thus $\tilde{z}^L$ is obtained by maximizing L types' utility, i.e., as a solution to

$$\frac{du^L}{dz}=\left(u^L\right)'\frac{d}{dz}\left[\bar{c}^L+\lambda^L\left(\ln\bar{m}^L-\frac{\bar{m}^L}{m^{*L}}\right)\right]=0.$$

Substituting (22) and (26) into the above equation implies that $\tilde{z}^L$ solves

$$\underbrace{\mu\left(\frac{1}{1+z}\right)^2\frac{m^{*H}-m^{*L}}{1+\kappa\pi(z)}-\mu\frac{z}{1+z}\frac{m^{*H}-m^{*L}}{\left(1+\kappa\pi(z)\right)^2}\kappa\pi'(z)}_{\text{Transfer effect}}=\underbrace{\pi(z)\frac{m^{*L}}{\left(1+\kappa\pi(z)\right)^2}\kappa\pi'(z)}_{\text{Rate-of-return effect}}$$

where $\pi'(z)=\beta\left(\frac{1}{1+z}\right)^2$. The above equation simply states that at the optimum, the marginal cost of raising z in terms of its rate-of-return effect, equals the marginal benefit of a higher z in terms of its transfer effect. Some algebra yields

$$\tilde{z}^L=\frac{\beta^2m^{*L}}{\beta m^{*L}-\mu\left(m^{*H}-m^{*L}\right)\left((1-\beta)+1/\kappa\right)}-1 \tag{27}$$

The following Lemma establishes the necessary and sufficient conditions which determine when $\tilde{z}^L$ is positive.

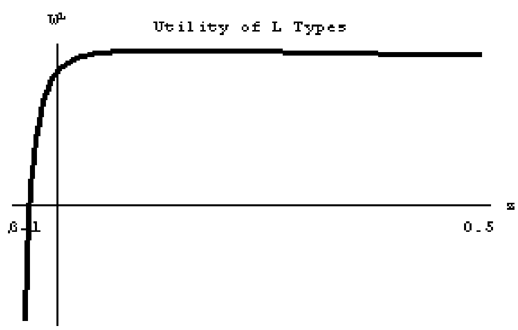
Lemma 8 *The L types prefer a positive money growth rate if and only if*

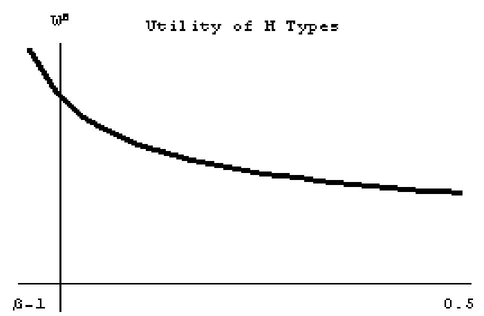
$$\frac{m^{*H}}{m^{*L}}>1+\frac{\beta}{\mu}\left(1+\frac{1}{(1-\beta)\kappa}\right)$$

The higher the ratio $\frac{m^{*H}}{m^{*L}}$, and higher the fraction of H types in the population, μ, the higher is the transfer to the L types under a positive money growth rate. Then it may be optimal for the L types to sacrifice utility from real balances in favor of higher income transfers. As an example, for $\beta = 0.96$, $\mu = 0.5, \lambda^H = 1, \lambda^L = 0.1$, $m^{*H} = 100$, and $m^{*L} = 10$ the above condition is satisfied. Substituting these values in (27) yields an optimal value $\tilde{z}^L = 0.2539$.

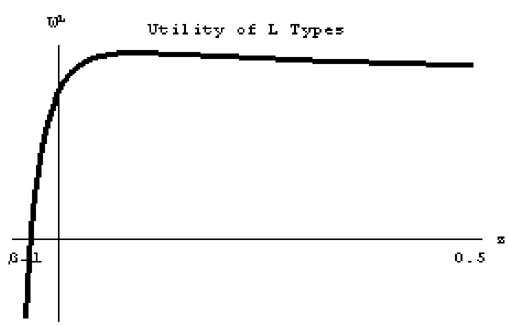
It is not possible to make any analytical progress toward the issue of globally optimal z, even using common functional forms like the CRRA or the ln. Below we will present the results of several numerical exercises using these common functional forms that will shed light on the questions that motivated this section. For each of these examples below, we set $\bar{y} \doteq 2.28$, $\beta = 0.96$ and $\mu = 0.5$.

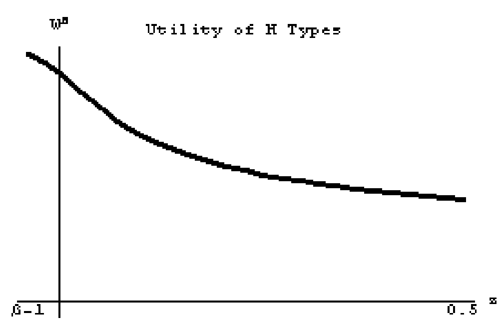
Example 9 (Logarithmic utility) *Suppose* $u^i\left(\bar{c}^i, \bar{m}^i\right) = \ln \bar{c}^i + \lambda^i \left(\ln \bar{m}^i - \frac{\bar{m}^i}{m^{*i}}\right)$ *where* $\lambda^H = 1 > \lambda^L = 0.1$. *Assume* $m^{*H} = 100$ *and* $m^{*L} = 10$. *Then as illustrated below, the L types like a positive value of z while the H types like the Friedman rule.*





Example 10 (CRRA utility) *Suppose* $u^i\left(c^i, m^i\right) = \ln c^i + \lambda^i \left(\frac{\left(m^i\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{m^i}{\left(m^{*i}\right)^{\frac{1}{\sigma}}}\right)$, *where* $\lambda^H = 1 > \lambda^L = 0.1$. *Assume* $m^{*H} = 100$ *and* $m^{*L} = 10$. *Then for* $\sigma = 2$, *as illustrated below, the L types like a positive value of z while the H types like the Friedman rule.*





The exact story as told by these examples is fairly robust to numerous changes in the parametric specifications.

5.2 Unequal elasticities

In this section, we show that it is possible that neither type likes the Friedman rule. For this purpose, we drop the assumption (24) and allow the elasticities of money demand to be unequal across the two types. In particular, we assume that

$$\frac{m^{*H}}{\lambda^H} = \kappa^H < \frac{m^{*L}}{\lambda^L} = \kappa^L. \tag{28}$$

For simplicity, we assume that the satiation level of real balances is same for the both types, i.e., $m^{*H} = m^{*L} = m^*$. However, we maintain our earlier assumption that $\lambda^H > \lambda^L$.[20] Thus, (22) can be rewritten as

$$\bar{m}^i = \frac{m^*}{(1+ \kappa^i \pi(z))} \tag{29}$$

Thus, $\bar{m}^H > \bar{m}^L$ for all $z > \beta - 1$. Assumption (28) implies that close to the Friedman rule the elasticity of money demand for the L types exceeds that of the H types. Indeed, note that

$$\left| \zeta_{\bar{m}^i,z}\big|_{z=\beta-1} \right| = \kappa^i \frac{1-\beta}{\beta}$$

Thus, our assumptions on preferences essentially imply that although the H types always hold a higher stock of real balances relative to the L types, the closer is the z to the Friedman rule, the faster is the rate of adjustment of real balances (to changes in z) of the L types relative to the H types.

Next, using (6a) and (6b) with (29), the steady state consumptions can be rewritten as

$$\bar{c}^L = \bar{y} + \mu \frac{z\ m^*}{1+z}\left[\frac{1}{(1+ \kappa^H \pi(z))} - \frac{1}{(1+\kappa^L \pi(z))}\right], \tag{30}$$

$$\bar{c}^H = \bar{y} - (1-\mu)\frac{z\ m^*}{1+z}\left[\frac{1}{(1+ \kappa^H \pi(z))} - \frac{1}{(1+\kappa^L \pi(z))}\right]. \tag{31}$$

We know from (12) that one of the types would benefit if the central bank deviates from the Friedman rule. The following Lemma clarifies that it is now the H types that dislike the Friedman rule. Indeed, as we show below, it is even possible for both types to disfavor the Friedman rule.

Lemma 11 *The Friedman rule is disliked by the H types; indeed, they would prefer a positive nominal interest rate. However, $\tilde{z}^H \in (\beta - 1, 0)$.*

[20] The equality of satiation levels is not necessary and our results hold even if we allow $m^{*H} > m^{*L}$. If so, a λ^H sufficiently larger than λ^L will generate the results that follow. See the discussion that follows Lemma 5.

The result that $\tilde{z}^H > \beta - 1$ has the following intuition. Recall from (6b) that the net transfer to the H types depends on the gap between real balances of the two types. Although this gap is always positive, it may shrink or widen as z is decreased depending on the relative elasticities of the two types at any given z. Since the L types have a relatively higher elasticity of money demand close to the Friedman rule, the gap shrinks as z gets closer to the Friedman rule. Thus, it turns out that the net transfer to the H types becomes smaller as z gets closer to the Friedman rule. As the rate-of-return effect vanishes at the Friedman rule, $\tilde{z}^H > \beta - 1$. On the other hand, it is clear that $\tilde{z}^H < 0$. At such a money growth rate, the H types gain on both dimensions: they receive positive net transfers from the L types, and also benefit from the rate-of-return effect.

Also from (12), it is clear that the L types would dislike a small deviation from the Friedman rule; hence the L types like the Friedman rule locally. It remains to be checked whether the Friedman rule is also their *global* optimum. Below we show that under certain parameter restrictions, the L types will be better off at some $z > \beta - 1$. The following Lemma asserts that $\tilde{z}^L$ either equals z^{FR} or is positive. In addition, it establishes sufficient conditions when $\tilde{z}^L > 0$.

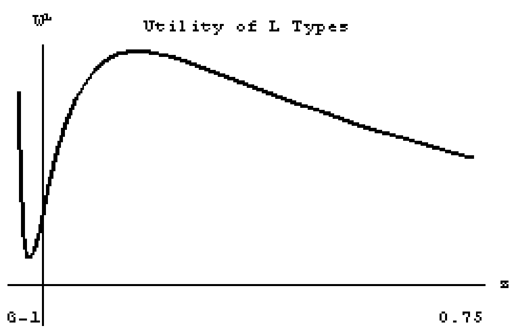
Lemma 12 $z \in (\beta - 1, 0)$ *can never be optimal for the L types. Furthermore, if* $\lambda^H >> \lambda^L$*, i.e., if the preference of H types for real balances is sufficiently stronger than for the L types,* $\tilde{z}^L > 0$ *holds.*

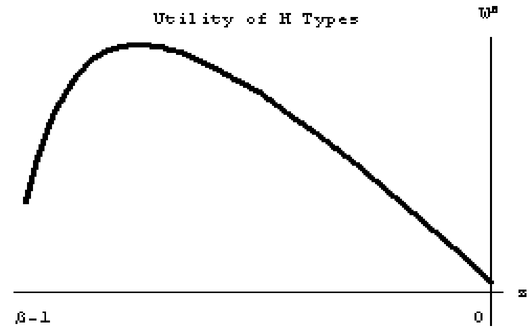
The intuition behind this result is quite obvious. At the Friedman rule, not only the L types consume their total endowment, but they also satiate themselves with real balances. The only way they can be induced to like any other z is if there is a net income and consumption gain that compensates for them for their resultant loss of real balances. When λ^H is sufficiently large, the H type will hold a sufficiently large amount of money *even* when $z > 0$. As a result, at some $z > 0$, L types receive a level of net transfers that gives them a higher welfare than that available at z^{FR}. We collect the punchline of the above discussion in the next Proposition.

Proposition 13 *If* $\lambda^H >> \lambda^L$*, both types dislike the Friedman rule.*

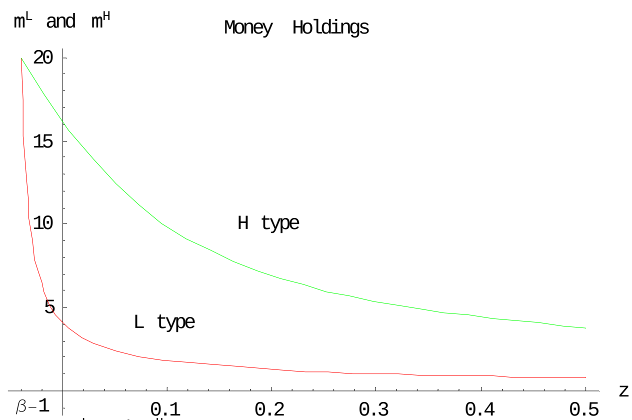
Following the derivation in Appendix G, assume $m^* = e$, $\lambda^L = 0.1$. Then for any $\lambda^H > 0.7699$ even though the L types dislike a local increase in z at the Friedman rule, their global optimum now is $\tilde{z}^L = 0.25$. We verify the robustness of our result via the following CRRA example.

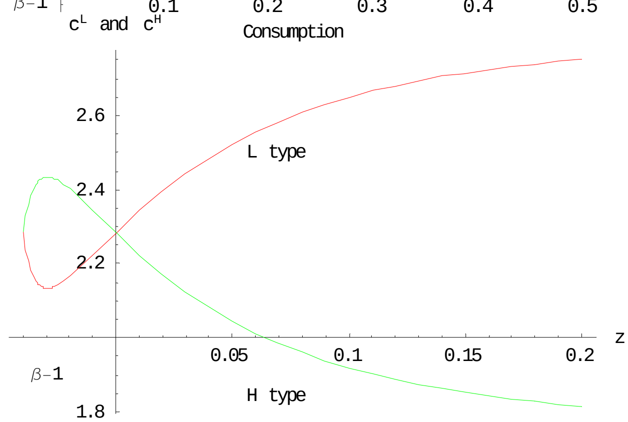
Example 14 *Suppose* $U^i\left(c^i, m^i\right) = \ln c^i + \lambda^i \left(\frac{\left(m^i\right)^{1-\frac{1}{\sigma^i}}}{1-\frac{1}{\sigma^i}} - \frac{m^i}{\left(m^{*i}\right)^{\frac{1}{\sigma^i}}} \right)$*, where* $\lambda^H = 0.7$ *and* $\lambda^L = 0.1$*. Assume* $m^{*H} = m^{*L} = 20$*, and* $\mu = 0.5$*. Suppose* $\sigma^H = 2$ *and* $\sigma^L = 0.5$*. Then as illustrated below, both the types dislike the Friedman rule.*





Thus, for the set of parameter values derived above, the Friedman rule does not maximize welfare for any of the types. The following set of figures tell the same story as has been laid out above.





6 Concluding remarks

By construction, monetary policy cannot have redistributive effects in representative-agent models. Yet these effects are known to be quantitatively significant and important (see, for example, Erosa and Ventura, 2002). The purpose of this paper is to examine whether optimal monetary policy is sensitive to heterogeneity. To that end, we develop a model economy in which the equilibrium distribution of money holdings is non-degenerate. The analysis essentially plays off the two effects of an increase in the money growth rate. There is the rate-of-return effect which cause both types to reduce their money holdings in the face of a higher opportunity cost. In the absence of type-specific taxes and transfers, a transfer/redistributive effect emerges. For example, in the case of positive money growth rates, the type that holds more money contributes more to seigniorage than the other type but receives the same transfer, in effect causing a redistribution of income from the former to the latter.

The possible benefits of a net transfer of income may easily overwhelm the negative rate-of-return effect for some types of agents. In that case, an increase in the money growth rate may even be welfare enhancing for some. Much depends on the rate at which each type adjusts their money balances in response to an increase in the money growth rate. We show that at least one of the types always dislikes the Friedman rule (locally). We go on to show that if the type that holds more money dislikes the Friedman rule locally, their welfare is never maximized globally at a *non-negative* money growth rate. Interestingly, it is possible for everyone to prefer positive nominal interest rates over Friedman's zero-nominal-interest-rate prescription. In terms of the question posed by the title of this paper then the answer may be that everyone is "afraid" of the Friedman rule. Societal welfare, defined as the population-weighted aggregate welfare of both types in our model, is almost never maximized at the Friedman rule. The upshot is that unlike in models with representative agents, here the prescription for "optimal" monetary policy depends on whether welfare of the individual or that of society is being maximized. Our analysis highlights some crucial components of the inevitable political economy dimensions of the larger question of the optimal monetary policy.

Appendix

A Proof of Lemma 1 : $\bar{m}^H > \bar{m}^L$

First, for $z > 0$, we prove that $\bar{m}^H > \bar{m}^L$ by contradiction. Choose any $z > 0$. Suppose $\bar{m}^L \geq \bar{m}^H$. Then (7) holds. Further, from (5), it is implied that $\frac{u_c(\bar{c}^L)}{u_c(\bar{c}^H)} < 1$, which in turn implies $\bar{c}^L > \bar{c}^H$. But, given (6a) and (6b), this violates our assumption. Hence, $\bar{m}^H > \bar{m}^L$ for all $z > 0$.

Now, choose any $z < 0$. A sufficient condition for $\bar{m}^L < \bar{m}^H$ is that $\frac{v'(\bar{m}^L) - v'(m^{*i})}{v'(\bar{m}^H) - v'(m^{*i})} > 1$. Notice that for all $z > \beta - 1$ an upper bound for the consumption of L types is $\bar{y} + \mu (1-\beta) m^{*L}$. Alternatively, a lower bound for the consumption of the H types is $\bar{y} - (1-\mu)(1-\beta) m^{*L}$. Thus, a lower bound for $\frac{u_c(\bar{c}^L)}{u_c(\bar{c}^H)} \frac{\lambda^H}{\lambda^L}$ equals $\frac{u_c(\bar{y}+\mu(1-\beta)m^{*L})}{u_c(\bar{y}-(1-\mu)(1-\beta)m^{*L})} \frac{\lambda^H}{\lambda^L}$. Hence, $\frac{u_c(\bar{y}+\mu(1-\beta)m^{*L})}{u_c(\bar{y}-(1-\mu)(1-\beta)m^{*L})} \frac{\lambda^H}{\lambda^L} > 1$ implies that $\frac{v'(\bar{m}^L) - v'(m^{*i})}{v'(\bar{m}^H) - v'(m^{*i})} > 1$. Note that

$$
\begin{aligned}
& \frac{d}{d\bar{y}} \left[\frac{u_c\left(\bar{y} + \mu(1-\beta) m^{*L}\right)}{u_c\left(\bar{y} - (1-\mu)(1-\beta) m^{*L}\right)} \right] \\
= \; & \frac{u_{cc}\left(\bar{y}+\mu(1-\beta)m^{*L}\right) u_c\left(\bar{y}-(1-\mu)(1-\beta)m^{*L}\right) - u_c\left(\bar{y}+\mu(1-\beta)m^{*L}\right) u_{cc}\left(\bar{y}-(1-\mu)(1-\beta)m^{*L}\right)}{\left[u_c(\bar{y}-(1-\mu)(1-\beta)m^{*L})\right]^2} > 0
\end{aligned}
$$

where we have used the fact that $u_{cc} < 0$ and $u_{ccc} > 0$ for any CES form. Define $\bar{y}^*$ as the value of $\bar{y}$ that obtains $\frac{u_c(\bar{y}^*+\mu(1-\beta)m^{*L})}{u_c(\bar{y}^*-(1-\mu)(1-\beta)m^{*L})} = \frac{\lambda^L}{\lambda^H}$. Thus, a sufficient condition for $\bar{m}^L < \bar{m}^H$ is that $\bar{y} > \bar{y}^*$. For $u(.) \equiv \ln(.)$, it is easy to show that $\bar{y}^* = (1-\beta) \frac{m^{*L}}{\lambda^H - \lambda^L} \left(\lambda^H (1-\mu) + \lambda^L \mu\right)$. Finally, note that when $m^{*H} = m^{*L}$, $\bar{m}^H = \bar{m}^L$ holds at the Friedman Rule trivially.

B Proof of Lemma 3: $\frac{d\bar{m}^L}{dz}, \frac{d\bar{m}^H}{dz} \leq 0$

As we have assumed that the consumption utility has a CES form, let $u(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}}$, where σ is the intertemporal elasticity of substitution $\sigma = 1$ represents the logarithmic case. Note that equations (5) - (6b) simultaneously determine consumption and real money balances in steady state for type-i agents. Totally differentiating them together yields

$$
\left| \begin{array}{cc} \frac{v^H_{mm}}{u^H_c} + \frac{u^H_{cc}}{u^H_c} Z\pi(z)(1-\mu) & -\frac{u^H_{cc}}{u^H_c} Z\pi(z)(1-\mu) \\ -\frac{u^L_{cc}}{u^L_c} Z\pi(z)\mu & \frac{v^L_{mm}}{u^L_c} + \frac{u^L_{cc}}{u^L_c} Z\pi(z)\mu \end{array} \right| \left| \begin{array}{c} \frac{dm^H}{dz} \\ \frac{dm^L}{dz} \end{array} \right| = \left| \begin{array}{c} \frac{1}{(1+z)^2} \left[\beta - (1-\mu)\left(m^H - m^L\right) \frac{u^H_{cc}}{u^H_c} \pi(z)\right] \\ \frac{1}{(1+z)^2} \left[\beta + \mu\left(m^H - m^L\right) \frac{u^L_{cc}}{u^L_c} \pi(z)\right] \end{array} \right|
$$

where $Z \equiv \frac{1}{1+z}$. Note that $u_c^i \equiv u_c\left(\bar{c}^i\right)$ and $u_{cc}^i \equiv u_c\left(\bar{c}^i\right)$.[21] Using Kramer's rule, and after some algebra, obtain

$$\frac{dm^L}{dz} = \frac{1}{(1+z)^2} \frac{\beta\left[\frac{v_{mm}^H}{u_c^H} - \sigma \frac{\bar{y}}{\bar{c}^H \bar{c}^L} Z\pi(z)\right] - \sigma\mu \frac{\bar{m}^H - \bar{m}^L}{\bar{c}^L} \frac{v_{mm}^H}{u_c^H} \pi(z)}{\frac{v_{mm}^H}{u_c^H}\frac{v_{mm}^L}{u_c^L} - \sigma\left[\mu \frac{v_{mm}^H}{u_c^H}\frac{1}{\bar{c}^L} + (1-\mu)\frac{v_{mm}^L}{u_c^L}\frac{1}{\bar{c}^H}\right] Z\pi(z)} \tag{32a}$$

$$\frac{dm^H}{dz} = \frac{1}{(1+z)^2} \frac{\beta\left[\frac{v_{mm}^L}{u_c^L} - \sigma \frac{\bar{y}}{\bar{c}^H \bar{c}^L} Z\pi(z)\right] + \sigma(1-\mu) \frac{\bar{m}^H - \bar{m}^L}{\bar{c}^H} \frac{v_{mm}^L}{u_c^L} \pi(z)}{\frac{v_{mm}^H}{u_c^H}\frac{v_{mm}^L}{u_c^L} - \sigma\left[\mu \frac{v_{mm}^H}{u_c^H}\frac{1}{\bar{c}^L} + (1-\mu)\frac{v_{mm}^L}{u_c^L}\frac{1}{\bar{c}^H}\right] Z\pi(z)} \tag{32b}$$

Below, we evaluate the above derivatives for the entire range of z in two steps:

STEP I: $z \geq 0$: Note first that $\frac{dm^H}{dz} < 0$, since all terms on the numerators are negative, while all in the denominator are positive. However, a sufficient condition for $\frac{dm^L}{dz} < 0$ is

$$1 - \mu\sigma \frac{\bar{m}^H - \bar{m}^L}{\bar{c}^L}\pi(z) > 0$$

For $\sigma = 1$, i.e., the log case, the above inequality holds if

$$\begin{aligned} c^L &= \bar{y} + \mu Z\left(\bar{m}^H - \bar{m}^L\right) > \mu\sigma\left(\bar{m}^H - \bar{m}^L\right)\pi(z), \text{ i.e.,} \\ \bar{y} &> \mu\left(\bar{m}^H - \bar{m}^L\right)\left(\sigma\pi(z) - Z\right) \end{aligned}$$

As $\frac{dm^H}{dz} < 0$, an upper bound for the RHS equals $\max_{z \geq 0}\{(\sigma\pi(z) - Z)\}\,\mu\,\bar{m}^H\big|_{z=0}$. Thus a sufficient condition for $\frac{d\bar{m}^L}{dz} < 0$ is that $\bar{y} > \varphi\,\bar{m}^H\big|_{z=0}$, where $\varphi = \mu\sigma(1-\beta)$ if $\sigma\beta < 1$, $\varphi = \mu(\sigma - 1)$ if $\sigma\beta > 1$. Note that $\bar{m}^H\big|_{z=0} = \frac{\lambda^H \bar{y} m^{*H}}{(1-\beta)m^{*H} + \lambda^H \bar{y}}$. Hence,

$$\bar{y} > \bar{y}^{**} = m^{*H}\left(\varphi - \frac{1-\beta}{\lambda^H}\right) \Rightarrow \frac{dm^L}{dz} < 0$$

which we have assumed in the main text.

STEP II: $z = \beta - 1$: Finally, at the Friedman rule $z = \beta - 1$, $\pi(z) = 0$, and then

$$\begin{aligned} \frac{d\bar{m}^L}{dz} &= \frac{1}{\beta}\frac{u_c^L}{u_{mm}^L} < 0 \text{ and} \\ \frac{d\bar{m}^H}{dz} &= \frac{1}{\beta}\frac{u_c^H}{u_{mm}^H} < 0 \end{aligned}$$

[21] Recall from our assumption in Section ?? that the functional form of the consumption utility is identical for both types.

C Proof of Lemma 3

At the Friedman rule $\pi(z) = 0$. Then, using (32a) and (32b), we obtain

$$\left.\frac{dm^H}{dz} - \frac{dm^L}{dz}\right|_{\pi(z)=0} = \frac{1}{\beta}\left(\frac{u_c^H}{v_{mm}^H} - \frac{u_c^L}{v_{mm}^L}\right)$$

Thus, following (11) and (9), a type$-L$ (H) agent's utility will increase (decrease) with z at the Friedman rule, if and only if

$$\frac{u_c\left(c^{*H}\right)}{\lambda^H w_{mm}\left(m^{*H}\right)} - \frac{u_c^L\left(c^{*L}\right)}{\lambda^L w_{mm}\left(m^{*L}\right)} - \frac{m^{*H} - m^{*L}}{1-\beta} < 0$$

where, from (6a) and (6b), $c^{*L} = \bar{y} + \mu\frac{1-\beta}{\beta}\left(m^{*H} - m^{*L}\right)$ and $c^{*H} = \bar{y} - (1-\mu)\frac{1-\beta}{\beta}\left(m^{*H} - m^{*L}\right)$.

D Proof of Proposition 7

Differentiating the social welfare function yields

$$\frac{dW}{dz} = \mu\frac{dW^H}{dz} + (1-\mu)\frac{dW^L}{dz}$$

Using (6a), (6b), and (9) in (11), and simplifying yields

$$\begin{aligned}(1-\beta)\frac{dW}{dz} \quad = \quad & \mu\, u_c^H\left[\pi(z) - Z(1-\mu)\left(1 - \frac{u_c^L}{u_c^H}\right)\right]\frac{d\bar{m}^H}{dz} - \frac{\mu(1-\mu)\left(u_c^H - u_c^L\right)}{(1+z)^2}\left(\bar{m}^H - \bar{m}^L\right) \\ & + (1-\mu)\pi(z)\, u_c^L\frac{d\bar{m}^L}{dz} + \mu(1-\mu)\left(u_c^H - u_c^L\right) Z\frac{d\bar{m}^L}{dz}\end{aligned} \tag{34}$$

From Corollary 2, we know that for any $z \geq 0$, $\bar{c}^L \geq \bar{c}^H$, $u_c^L \leq u_c^H$. Further, from Lemma 3 $\frac{d\bar{m}^H}{dz} < 0$ and $\frac{d\bar{m}^L}{dz} < 0$, and $\left(\bar{m}^H - \bar{m}^L\right) > 0$ by Lemma 1. Hence, all the terms on the right hand side of (34) are nonpositive. Thus for all $z \geq 0$, $\frac{dW}{dz} < 0$ holds. Hence, $z \geq 0$ can not be socially optimal.

E A cash-in-advance economy in which money growth affects output

Let the agents be endowed with a unit of labor. Their period utility functions are identical

$$u^i(c,l) \equiv u(c, 1-l), \quad i \equiv L, H, \tag{35}$$

where $1-l$ is the amount of leisure they enjoy, and the function $u(.,.)$ has the standard properties. The agents are differentially endowed with technologies

$$y^i = \alpha^i f\left(l^i\right), \ \alpha^H > \alpha^L, \tag{36}$$

and the $f(.)$ has the standard properties of a production function. Each household consists of a shopper seller pair. While the shopper goes to the market with cash to buy consumption good, seller works and sells output to the buyers who arrive at the factory outlet. Thus, at the end of period t, the seller accumulates the following money balances:

$$M_t^i = p_t y_t^i = p_t \alpha^i f\left(l_t^i\right) \tag{37}$$

In steady state, (37) can be rewritten as

$$m_t^i = y_t^i = \alpha^i f\left(l_t^i\right) \tag{38}$$

The shopper, on the other hand, inherits nominal balances from the previous period, receives transfers from the government, and then goes out to shop. Thus,

$$p_t c_t^i \leq M_{t-1}^i + T_t \tag{39}$$

Note that

$$\begin{aligned} T_t &= M_t - M_{t-1}, \text{ or} \\ \tau_t &= \frac{M_t - M_{t-1}}{p_t} = \frac{z M_{t-1}}{p_t} = \frac{z}{1+z} m_{t-1} \end{aligned}$$

The cash-in-advance constraint, (39), can be rewritten as

$$c_t^i \leq m_{t-1}^i \frac{p_{t-1}}{p_t} + \tau_t \tag{40}$$

Clearly, (40) binds with equality in the steady state. Otherwise exchanging excess real balances with consumption will be a strict improvement. The optimization problem maximizes

$$\sum_{s=t}^{\infty} \beta^{s-t} u\left(c_s^i, 1 - l_s^i\right) = u\left(\alpha^i f\left(l_{t-1}^i\right) \frac{p_{t-1}}{p_t} + \tau_t, 1 - l_t^i\right) + \beta u\left(\alpha^i f\left(l_t^i\right) \frac{p_t}{p_{t+1}} + \tau_{t+1}, 1 - l_{t+1}\right) + ..$$

subject to constraints (37) and (40). The optimum is characterized by the following first-order-condition:

$$-u_l\left(c_t^i, l_t^i\right) = \alpha^i f'\left(l_t^i\right) \frac{\beta}{(1+z)} u_c\left(c_{t+1}^i, l_{t+1}^i\right); \text{ for } i = L, H \tag{41}$$

Steady state In steady state, (41) yields

$$-u_l^i = \alpha^i f'\left(\bar{l}^i\right) \frac{\beta}{(1+z)} u_c^i$$

which is equation (17) in the main text. Further, equation (40) can be rewritten as

$$\begin{aligned} \bar{c}^i &= \frac{\bar{m}^i}{1+z} + \frac{z}{1+z} \bar{m} \\ \bar{c}^i &= \bar{m}^i + \frac{z}{1+z}\left(\bar{m} - \bar{m}^i\right) \\ &= \alpha^i f\left(\bar{l}^i\right) + \frac{z}{1+z}\left(\bar{m} - \bar{m}^i\right) \end{aligned}$$

where the last step makes use of (38). Since $\bar{m} = \mu\bar{m}^H + (1-\mu)\bar{m}^L$, the steady state consumption is given by

$$\begin{aligned} \bar{c}^L &= \alpha^L f\left(l^L\right) + \frac{z}{1+z}\mu\left(\bar{m}^H - \bar{m}^L\right), \qquad (42a) \\ \bar{c}^H &= \alpha^H f\left(l^H\right) - \frac{z}{1+z}(1-\mu)\left(\bar{m}^H - \bar{m}^L\right). \qquad (42b) \end{aligned}$$

which are presented as (18a) and (18b) in the main text.

F Proof of Lemma 11

To check if the H types would like to deviate, we differentiate H type's utility aggregate with respect to z, and use (29) with (31) to obtain

$$\begin{aligned} \frac{d}{dz}\left[\bar{c}^H + \lambda^H\left(\ln\bar{m}^H - \frac{\bar{m}^H}{m^{*H}}\right)\right] &= -(1-\mu)\frac{m^*}{1+z}\left[\frac{1}{(1+\ \kappa^H\pi(z))} - \frac{1}{(1+\kappa^L\pi(z))}\right] \\ &+ (1-\mu)\frac{z\ (m^*)^2}{1+z}\frac{\beta}{(1+z)^2}\left[\frac{\frac{1}{\lambda^H}}{(1+\ \kappa^H\pi(z))^2} - \frac{\frac{1}{\lambda^L}}{(1+\kappa^L\pi(z))^2}\right] \\ &+ \pi(z)\frac{d\bar{m}^H}{dz}. \end{aligned}$$

Clearly, at the FR $\pi(z) = 0$, the first and the last term vanish, while the second term is positive. Now, unlike the previous case, the H types would prefer $z > \beta - 1$. But can it be that $\tilde{z}^H > 0$? The answer is negative, as seen from the derivative above. At $z = 0$, i.e., $\pi(z) = 1-\beta$, the second term vanishes and the first and the third term are negative. Thus, there lies a maximum for $\tilde{z}^H \in (\beta - 1, 0)$. It is easy to see that this must be the H types' global maximum, as for any $z > 0$ they incur both a loss of consumption as well as a loss of real balances.

G Proof of Lemma 12

Using (30) in (21), we obtain

$$(1-\beta)\left(U^L\big|_{z^{\mathrm{FR}}} - U^L\big|_z\right) = u\left[\bar{y} + \lambda^L(\ln m^* - 1)\right] - u\left[\bar{y} + \mu Z\left(\bar{m}^H - \bar{m}^L\right) + \lambda^L\left(\ln\bar{m}^L - \frac{\bar{m}^L}{m^*}\right)\right]$$

Thus $U^L\big|_{z^{\mathrm{FR}}} - U^L\big|_z \lesseqgtr 0$ if and only if

$$\bar{y} + \lambda^L(\ln m^* - 1) \lesseqgtr \bar{y} + \mu Z\left(\bar{m}^H - \bar{m}^L\right) + \lambda^L\left(\ln\bar{m}^L - \frac{\bar{m}^L}{m^*}\right)$$

Using (29), the above condition can be rewritten as

$$\mu\kappa^L Z\pi(z)\left[\frac{\kappa^L - \kappa^H}{(1+\kappa^L\pi(z))(1+\kappa^H\pi(z))}\right] \lesseqgtr \left[\ln\left(1+\kappa^L\pi(z)\right) - \frac{\kappa^L\pi(z)}{1+\kappa^L\pi(z)}\right]$$

Note that the RHS is always positive. But for any $z \leq 0$, the LHS is nonpositive. Hence $U^L\big|_{z^{\mathrm{FR}}} > U^L\big|_z$ for all $z \leq 0$.

For the second part, suppose $\kappa^H = 0$. Then, $U^L\big|_{z>0} > U^L\big|_{z^{\mathrm{FR}}}$ if and only if

$$\frac{\kappa^L \pi(z)}{1 + \kappa^L \pi(z)} \left(1 + \frac{\mu Z}{\pi(z)} \kappa^L \pi(z)\right) > \ln\left(1 + \kappa^L \pi(z)\right) \tag{43}$$

Fix any $z = \tilde{z} > 0$. It is easily shown that the above condition holds for all $\kappa^L > \hat{\kappa}^L$, where $\hat{\kappa}^L$ is obtained as an implicit solution of

$$\frac{\hat{\kappa}^L \pi(\tilde{z})}{1 + \hat{\kappa}^L \pi(\tilde{z})} \left(1 + \frac{\mu \tilde{Z}}{\pi(\tilde{z})} \hat{\kappa}^L \pi(\tilde{z})\right) = \ln\left(1 + \hat{\kappa}^L \pi(\tilde{z})\right).$$

By continuity, (43) should hold for $\kappa^H > 0$, provided κ^L is then sufficiently larger than κ^H.

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