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Reconciling Bagehot with the Fed's Response to September 11

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Abstract

The nineteenth-century economist Walter Bagehot maintained that in order to prevent bank panics, a central bank should provide liquidity at a very high rate of interest. However, most of the theoretical literature on liquidity provision suggests that central banks should lend at an interest rate of zero. This latter recommendation is broadly consistent with the Federal Reserve's behavior in the days following September 11, 2001. This paper shows that Bagehot's recommendation can be reconciled with the Fed's policy if one recognizes that Bagehot had in mind a commodity money regime in which the amount of reserves available is limited. A high price for this liquidity allows banks that need it most to self-select. To the contrary, the Fed has a virtually unlimited ability to temporarily expand the money supply so that self-selection is unnecessary.

Key words: liquidity provision, lender of last resort, Bagehot, commodity money

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1 Introduction

This paper studies liquidity provision by a central bank with commodity money. In contrast to much of the theoretical literature on liquidity provision by a central bank, which focuses on fiat money, I show that lending at a high rate of interest may be necessary to prevent bank panics. Lending at a high rate is the policy advocated by Bagehot (1873). Hence this paper helps reconcile Bagehot's recommended policy and the Federal Reserve's response to September 11, which was consistent with the theoretical literature focusing on fiat money.

Bagehot (1873) states that a central bank (CB) can prevent panics by providing liquidity to banks.[1] Specifically, "there are two rules. First. That these loans should only be made at a very high rate of interest... Second. That at this rate these advances should be made on all good banking securities and as largely as the public ask for them."[2]

Bagehot's recommended policy stands in stark contrast with much of the theoretical literature. A number of authors have found that in a variety of environments a CB should lend at an interest rate of zero. For example, this policy arises in Allen and Gale (1998), Antinolfi, Huybens, and Keister (2001), Champ, Smith, and Williamson (1996), Freeman (1996), Green (1997), Martin (2006), Rochet and Vives (2002), Williamson (1998 and 2004), among others. I also show that the Federal Reserve's behavior after September 11, 2001, is broadly consistent with the recommendation of these studies. So was Bagehot wrong?

This paper proposes a reconciliation between the theoretical literature

[1] Throughout this paper I think of bank difficulties as arising because of a liquidity shortage and use the terms "panics" and "crises" interchangeably.

[2] Page 199.

and the Federal Reserve's policy, on the one hand, and Bagehot's recommendation, on the other. I consider a version of the model introduced by Diamond and Dybvig (1983). Specifically, I use the model of Cooper and Ross (1998) that was recently studied by Ennis and Keister (2006). In the context of that model I consider a liquidity provision policy with commodity money. With commodity money, the amount of reserves the CB can provide to banks is limited. For that reason, there are states of nature in which some banks may not have access to the reserves. This creates strategic interactions between banks. If CB reserves are available at a low cost, banks have an incentive to insure themselves by borrowing reserves before they know whether these reserves are needed. When banks borrow "too early," CB reserves will not be allocated properly as some banks will have reserves they do not need while other banks are not able to acquire reserves they need ex-post. The CB can eliminate the incentive for banks to withdraw too early by charging a high interest rate.

Understanding the role of high interest rates in Bagehot's recommendation is important because his ideas are still influential today. For example, Lacker (2004) uses Bagehot's recommendation as a benchmark in his study of the Fed's response to the events of September 11, 2001. Also, Peter Bernstein in his foreword to a 1999 reissue of *Lombard Street* notes that "After nearly 150 years, [Bagehot's] wise words are still the prescription of choice for containing financial crises, as well as a handbook for avoiding them... ."

The remainder of the paper is organized as follows. The next section provides some historical background. Section 3 presents the model. Section 4 considers liquidity provision in a commodity money regime. Section 5 concludes.

2 Some historical background

2.1 Bagehot's recommended policy

Although many of the ideas in *Lombard Street* had been expressed before, notably by Thornton (1802), Bagehot is often credited for exposing them in a systematic way.[3] Bagehot's proposed policy contains two main elements. In times of crisis:

1) The CB should lend freely and vigorously.

2) Loans should be made at a very high interest rate.

Bagehot credits the Bank of England for having prevented a panic in 1866 by following this policy. Subsequently, in 1878 when the City Bank of Glasgow failed, and in 1890 when Baring Bank failed, the same policy is credited for preventing widespread crisis. This is in contrast to the crises of 1847 and 1857, when the Bank of England initially refused to lend, leading to bank panics.

This paper focuses on the second element of Bagehot's proposed policy: the interest rate at which loans should be made. There are, in the literature, two main arguments to justify Bagehot's claim that the CB should lend at a high interest rate. First, under the gold standard, a high rate of interest prevents a drain of gold. Second, a high rate of interest helps prevent moral hazard.

The first argument can be found in Humphrey (1975) and Humphrey and Keleher (1984). They note that following Thornton (1802), Bagehot distinguishes between two types of shocks: internal (or domestic) and external (or foreign) cash drains. The former shock occurs when pessimistic depositors

[3] Laidler (2002) studies the differences and the similarities between the views of Bagehot and Thornton.

withdraw their deposits to hold cash and can, according to Bagehot, be countered if the CB lends vigorously. The latter shock occurs when gold flows out of England to be deposited in a foreign country. To counter such a shock the CB should raise its lending rate, so as to attract foreign gold and retain domestic gold. When the two shocks arise simultaneously, the CB should lend vigorously and at a high rate of interest.

The argument about moral hazard can be found in Sheng (1991) and Summers (1991), among others. The basic idea is that banks may take excessive risk if they know that they can borrow at a low rate during difficult times. Proponents of this view usually argue that the high interest rate Bagehot mentions is a penalty rate.

To justify his policy, Bagehot argues that "[a very high interest rate] will operate as a heavy fine on unreasonable timidity, and will prevent the greatest number of applications by persons who don't require it. The rate should be raised early in the panic, so that the fine may be paid early; that no one may borrow out of idle precaution without paying well for it; that the banking reserve may be protected as far as possible."[4]

No reference is made in this passage to an external cash drain or to moral hazard. Indeed, there are very few references to moral hazard in *Lombard Street*, and Bagehot has been criticized by Hirsch (1977) for not realizing that his proposed policy could create such a problem.[5] Instead, the quote points to the need to allocate the CB liquidity in an appropriate way. Thus, my paper argues that lending at a high rate of interest allows banks to self-select.[6]

[4]Page 199.

[5]The model in this paper does not consider moral hazard problems. Martin (2006) shows that a well-designed liquidity provision policy similar to the one considered here can prevent bank panics without moral hazard.

[6]It is interesting to note that Thornton, who writes at a time during which England is off the gold standard, does not mention the need to lend at a high interest rate. This

Bernanke (2008) seems to share this view, as does Fisher (1999) who notes that the high interest rate "limits the demand for credit by institutions that are not in trouble." This interpretation is also consistent with an argument by Goodhart (1999) that Bagehot does not propose a penalty rate.

The approach adopted by this paper is interesting for two reasons. First, from the perspective of history of thought, one wants to consider the internal consistency of Bagehot's argument. Hence, the case for a high interest rate should be made based on the type of economic mechanisms that Bagehot emphasizes, rather than on some other mechanism.[7] Second, this paper provides a formal analysis of the self-selection story which has not been studied yet.

2.2 The Fed's policy after 9-11-2001

The events of September 11 caused a breakdown in the usual means of communication between banks, and resulted in the temporary shutdown of the interbank market.[8] Some banks found themselves with high liquidity needs, while others had large excesses of liquidity. Because the interbank market was not functioning normally, the latter banks were not able to lend to the former. To alleviate the effects of the liquidity shortage and prevent a more generalized panic, the Federal Reserve provided unusually large amounts of reserves.

is consistent with the argument in this paper and is further support for the view that Bagehot's main concern is self-selection and not external cash drains. I am indebted to Tom Humphrey for pointing this out to me.

[7] I do not mean to suggest that moral hazard is not important in this case, only that it is not necessary to understand Bagehot. Moral hazard could provide an additional reason for a CB to charge a high rate of interest.

[8] See McAndrews and Potter (2002), Lacker (2004) for more information concerning the impact of the events of September 11 on the interbank market.

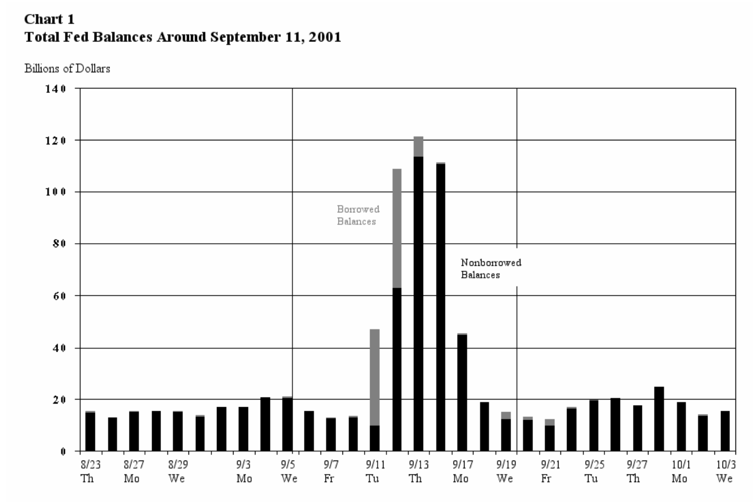
The Fed typically provides liquidity to markets through the discount window (DW) and through open market operations (OMOs).[9] In an OMO the Fed provides funds to primary security dealers through a repurchase agreement (RP). The dealers lend these funds to banks on the interbank market. Ordinarily, the Fed auctions off a fixed amount of reserves and does not engage in transactions at prices that would imply a lending rate lower than its target. The DW allows banks to obtain funds directly from the Fed. At the time, the interest rate at the DW was 50 basis point lower than the federal funds market target rate.[10] Banks were not allowed to lend these funds on the interbank market.

The following discussion details some of the actions of the Fed in the days following September 11. A good description of the Federal Reserve's policy after September 11 is provided by the Markets Group of the Federal Reserve Bank of New York (2002). Chart 1 shows borrowed balances (funds obtained through the DW) and nonborrowed balances (funds obtained through OMOs).[11] On September 11 and 12, large amounts of liquidity were provided through the DW because the interbank market was not functioning properly. On subsequent days, as interbank communications improved, OMOs provided much more liquidity than the DW. While the interest rate on DW loans did not change–until September 17, when the federal funds rate target was decreased by 50 basis points–banks were encouraged to borrow which made the effective cost of borrowing lower than usual. Around noon on September 11, the Board of Governors issued a press release stating: "The

[9] A third source of liquidity is float. Float is the length time between the moment a check is deposited and the moment it is available.

[10] It was 3 percent until 9/14, 2.5 percent between 9/17 and 10/1, and 2 percent after that.

[11] Charts 1, 2, and 3 come from Markets Group of the Federal Reserve Bank of New York (2002).

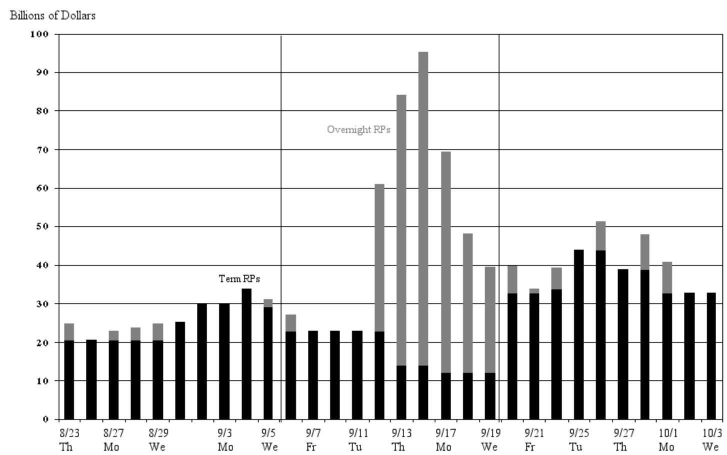


Federal Reserve is open and operating. The discount window is available to meet liquidity needs."

Chart 1 also shows that the Fed lent large amounts through OMOs. On September 13 and 14, the size of nonborrowed balances was more than 5 times as high as it had been in the days leading to September 11. The Fed's vigorous provision of liquidity would have satisfied Bagehot: "From Wednesday [9-12] through the following Monday [9-17], the size of open market operations were aimed at satisfying all the financing that dealers wished to arrange with the Desk, in order to mitigate to the extent possible the disruptions to normal trading and settlement arrangements."[12] Chart 2 shows overnight RPs and term RPs. Overnight RPs over this period can be associated with emergency lending. The size of these RPs between September 12 and 19 testifies to the large amount of liquidity the Fed provided to the interbank market.

[12]Markets Group of the Federal Reserve Bank of New York (2002), page 22.

Chart 2
Outstanding Term and Overnight RPs Around September 11, 2001

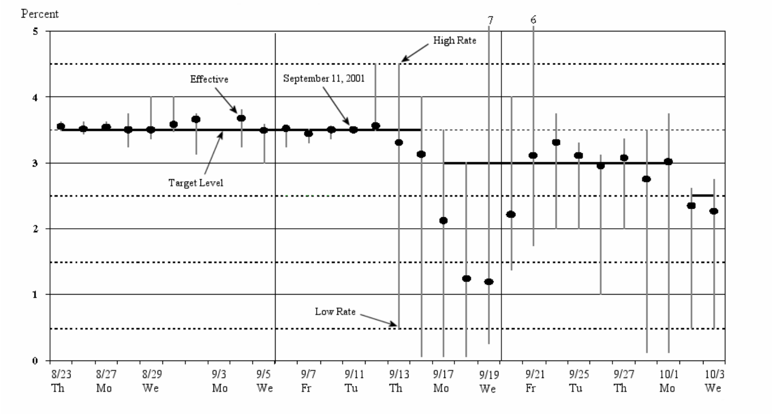


Contrary to what Bagehot would have advised, however, the Fed did not provide liquidity at a high rate: "[The Desk] had to accept the vast majority of propositions–even those offered at rates well below the new 3 percent target level–in order to arrange RPs of sufficient size."[13] The consequences of providing such large amounts of liquidity can be seen in Chart 3. The federal funds rate reached lows very close to zero on September 14, 17, and 18. The effective rate (a volume-weighted average of rates on trades arranged through the major brokers) was well below the target rate from September 17 to September 20.

The difference between Bagehot's recommended high rate of interest and the Fed's provision of liquidity at a low cost is striking and, on the face of it, puzzling. In the remainder of the paper, I argue that these differences

[13]Markets Group of the Federal Reserve Bank of New York (2002), page 24.

Chart 3
Federal Funds Rates Around September 11, 2001: High, Low, and Effective Rates



can be explained by the fact that Bagehot had in mind a commodity money environment while the Fed operates in a fiat money environment.

3 The model

In this section, I describe a model of banks operating in a commodity money environment. In this model a central bank can prevent bank panics if it lends at a high interest rate.[14]

The economy takes place at three dates, 0, 1, and 2. There is a continuum of agents called depositors, a continuum of banks, and a central bank. Each depositor is endowed with one unit of the economy's single consumption good

[14]I view the result that the central bank should lend at a very low cost in a fiat money environment as standard and do not present a model in this paper.

at date 0 and nothing at dates 1 and 2. The depositors can be thought of as residing inside a square of sides of length 1 while the banks reside on a line of length 1. Hence, each bank has a large number of depositors.

3.1 Technologies

There are two kinds of investment technologies. The *short-term* (storage) technology yields one unit of the consumption good at date t for each unit invested at date $t-1$, $t = 1, 2$. The *long-term* technology yields $R > 1$ units of the consumption good at date 2 for each unit invested at date 0. Liquidating the long-term technology at date 1 is assumed to carry a cost in terms of the consumption good and returns only $1-\tau$, where $\tau \geq 0$. For example, assume that a proportion θ of the unit invested is liquidated at date 1, then the technology has return $(1-\tau)\theta$ at date 1 and $(1-\theta)R$ at date 2. R and τ are known by all agents.

3.2 Preferences

Households can be of two types: impatient or patient. The impatient type only derives utility from consumption at date 1, and the patient type derives utility only from consumption at date 2. Types are learned at the beginning of date 1 and are private information. Each depositor has a probability $\pi > 0$ of being impatient and a law of large number is assumed to hold so the proportion of impatient depositors in the population is also π. To keep things as simple as possible, it is assumed that π is not a random variable. All agents know the value of π.

Let c_t denote the amount of goods consumed at date t. A depositor's expected utility is:

$$U(c_1, c_2, \pi) = \pi u(c_1) + (1 - \pi) u(c_2).$$

Patient agents can use the storage technology to store goods they obtain at date 1. Alternatively, it could be assumed that they derive utility from the sum of their subperiod 1 and subperiod 2 consumption. The function u is strictly increasing, strictly concave, and satisfies $u(0) = 0$.

3.3 Spatial and informational constraints

As in Wallace (1988), depositors are assumed to be spatially separated and unable to meet or trade with each other at date 1. At date 0, agents can deposit their endowment in any bank. Depositors can withdraw their funds either at date 1 or at date 2. Those depositors who withdraw at date 1 arrive at the bank in random order. A sequential service constraint is imposed so that the bank must pay depositors as they arrive, without knowing how many depositors will ultimately show up. Following Freeman (1988), I assume that the number of withdrawals at date 1 is not observable to depositors immediately. Depositors can only observe whether or not their bank runs out of funds. Hence, bank payments at date 1 cannot depend on the number of depositors who withdraw at that date or on the order in which depositors arrive at the bank. Depositors would not accept such pattern of bank payments because they would be unable to verify whether they receive the correct payment. Hence banks cannot offer contracts involving suspension of convertibility.

3.4 The deposit contract

Banks offer a contract that promises a fixed payment of c_1 goods at date 1. All depositors withdrawing at date 1 receive this amount unless the bank runs out of funds. In such a case, depositors arriving at the bank after it has run out of funds receive nothing. All depositors who withdraw at date 2 receive an equal share of the resources left in the bank. The payment promised to such depositors is denoted by c_2.

As is standard in this kind of model, if everyone believes that only impatient depositors will withdraw at date 1, then it is individually rational for patient depositors to withdraw at date 2. However, if everyone believes that all patient depositors will withdraw at date 1, then it is individually rational for them to do so, provided that $c_1 > 1 - i\tau$. Indeed, in that case liquidating the long-term technology does not provide enough resources for the bank to give c_1 to all depositors at date 1. The resulting allocation is associated with a bank run. This paper focuses on economies in which the above inequality holds.[15]

If the probability of a bank run is perceived to be strictly positive, the bank will take that into account when choosing its investment in the long-term and the storage technology.

[15] There exists a third equilibrium to the post-deposit game. If the right fraction of patient depositors withdraws at date 1, then all depositors receive c_1 whether they withdraw at date 1 or at date 2. This equilibrium can be associated with a partial bank run since some, but not all, patient depositors withdraw early. Consistent with most of the literature, I do not consider partial bank runs in this paper.

3.5 Sunspot

As in Ennis and Keister (2006) I assume that bank runs are triggered by a sunspot.[16] Depositors have the following beliefs: If a sunspot is observed, everyone believes patient depositors withdraw at date 1, provided it is individually rational for them to do so. Otherwise everyone believes they withdraw at date 2. As is the case with most of the literature, I assign a probability zero to partial bank runs.

Let $q > 0$ denote the expected probability that an individual bank will be affected by a sunspot. I allow for the possibility that the depositors of only a fraction of banks observe the sunspot when it occurs; for example, because banks are in different regions. One possible case is that with probability q a sunspot occurs and all banks are affected. It could also be the case that every period a sunspot is observed by a fraction q of all banks. Linear combinations of these two cases are possible as well. From the perspective of an individual bank, only the expected probability of a sunspot matters.

3.6 The banks' problem

Banks are assumed to maximize profits. Because of perfect competition, banks offer, in equilibrium, a deposit contract that maximizes the expected utility of depositors.[17] Depositors' beliefs are coordinated by a sunspot as described above and depositors choose when to withdraw so as to maximize their utility. Hence, impatient depositors always withdraw at date 1 since

[16] This is a common approach adopted, among others, by Benthal et al. (1990), Cooper and Ross (1998) and (2002), Freeman (1988), and Peck and Shell (2003). Also, Ennis (2003) argues that empirical evidence is not inconsistent with the idea that bank run can be triggered by sunspots.

[17] Allen and Gale (1998), Cooper and Ross (1998), Schreft and Smith (1998), among others, adopt this approach.

they get no utility from consuming later. Patient depositors will withdraw at date 1 if there is a sunspot and at date 2 otherwise. In other words, all consumption enjoyed by depositors who withdraw at date 2 comes from investment in a long-term technology.[18] The bank's problem can be written

$$\max(1-q)[\pi u(c_1) + (1-\pi)u(c_2)] + q\hat{\pi}u(c_1)$$

subject to

$$\begin{aligned} \pi c_1 &\leq 1-i, & (1)\\ (1-\pi)c_2 &\leq Ri, & (2)\\ \hat{\pi} &= \min\{\frac{1-i\tau}{c_1}, 1\}, & (3)\\ c_1, c_2, i &\geq 0, & (4) \end{aligned}$$

where i denotes the investment in the long-term technology. Hence, in case of a bank run, a depositor receives c_1 with probability $\hat{\pi}$ and nothing otherwise.

The bank can choose to offer a deposit contract such that bank runs never occur. As shown by Cooper and Ross (1998), if q is sufficiently small banks offer a deposit contract that allows runs. I focus on such cases in this paper. Ennis and Keister (2006) show that there exists a unique solution to the bank's problem.

3.7 The central bank

The CB can tax depositors but it is costly for the CB to raise funds. The CB obtains $\delta < 1$ reserves for each unit of good taxed. The CB is assumed to be unable to invest in the long-term technology. This reflects the fact that

[18]Ennis and Keister (2006) prove that banks do not invest in excess liquidity whenever $u(c) = c^{\alpha}/\alpha$, where $\alpha < 1$. I maintain this assumption throughout the paper.

the CB is not able to identify good projects as well as banks can.[19] I argue, in appendix A, that with these assumptions there is no loss of generality in restricting the CB to make loans to banks at some net interest rate r.

Assume that, with probability ε, all banks observe a sunspot. With probability $1-\varepsilon$, only a fraction q' of banks observe a sunspot. Let $q \equiv (1-\varepsilon)q' + \varepsilon$. I maintain the assumption that q is sufficiently small so that individual banks do not offer a run-preventing contract. Also, I assume that it is too costly for the CB to prevent a run at all banks (see appendix A). Since raising funds is costly, the CB will acquire enough reserves to prevent bank runs when only a fraction q' of banks are affected. When, with probability ε, all banks are affected, the CB is unable to prevent bank runs at some banks.[20]

I consider the case where $\varepsilon = 0$, so that the economy faces no aggregate uncertainty. This is the best case scenario for CB since it knows exactly the liquidity needs of the banking system. Even in that case, the CB may need to charge a high interest rate to provide the right incentives for banks. The case with $\varepsilon > 0$ is relegated to appendix C.

4 Liquidity provision with commodity money

This section considers a liquidity provision policy by a central bank (CB). The objective of the CB is to maximize the expected utility of depositors. In a commodity money economy the CB can choose how much gold it hold in reserves. Once these reserves are set, it is very costly, if not impossible,

[19] Assuming the CB is able to invest in a long-term technology with return $\tilde{R} < R$ would not modify the results.

[20] This appears to be the relevant case for the Bank of England historically.

to increase the CB's stock of reserves in the short run.[21] I assume that the CB cannot increase its reserves at date 1. In contrast, the central bank can increase the amount of reserves it supplies to banks at negligible cost in a fiat money economy.[22]

The CB can prevent sunspot-driven bank runs if it is able to provide enough goods to banks affected by a sunspot at date 1.[23] Indeed, bank runs occur because depositors are concerned that their bank may have to liquidate its long-term investment in order to pay depositors at date 1. If the bank can guarantee that this will not happen, then bank runs are avoided. When the CB charges a low interest rate, banks have an incentive to borrow before they know if they need reserves. This can lead to a misallocation of those scarce reserves. If the CB charges a high enough interest rate, this incentive disappears.

Why would there be a need for a CB to supply liquidity since banks can already choose run-proof contracts? The reason is that while banks face idiosyncratic risk the system may or may not face aggregate risk. Consider the two extreme cases: If all banks in the system are affected by a sunspot, then the liquidity need of an individual bank and of the system are the same. A CB can play no role in this case. However, if in every period a given fraction of banks are affected by a sunspot, then the system faces no aggregate risk

[21]Note that while the Bank of England had, in principle, the ability to expend its issue of notes and suspend their convertibility, the assumption that the CB does not hold enough reserves to prevent certain panics remains valid if the cost of suspension of convertibility (real or perceived) is high enough. There is some evidence that this cost was indeed high; for example the panics of 1847 and 1857 subsided only after the Chancellor of the Exchequer announced it would cover the cost of the Bank of England if its Issue Department expanded its note issue without gold backing and was sued.

[22]Martin (2006) considers a liquidity provision policy with fiat money in a related model.

[23]Assuming a fixed price of goods in terms of gold, it is equivalent to assume that the central bank supplies liquidity to banks in the form of gold or in the form of goods.

and a CB may be able to help.[24]

4.1 Central bank lending

The timing of events in every period is as follows: First, at date zero, the CB levies a lump-sum tax T from the endowment of all consumers. Next, consumers deposit their net-of-tax endowment in banks and banks decide how much to invest in the storage and the long-term technology. At the beginning of date 1, before the sunspot occurs, banks can choose to borrow from the CB. Then, banks' depositors may observe a sunspot and depositors learn whether they are impatient or not. Again, banks are able to borrow from the CB, this time knowing whether or not their depositors have observed a sunspot. Whether a sunspot occurs is observable to banks but not verifiable. Then withdrawals take place. At date 2, CB loans are repaid. Next, the CB distributes all its assets equally to banks that were not subject to a run.[25] Finally the assets of each bank are divided equally among all agents who did not withdraw at date 1. The CB does not observe the sunspot and does not know exactly when the sunspot occurs. Hence, the CB is unable to determine whether a bank that asks for a loan has been affected by a sunspot or not. For example, the sunspot may occur either early or late, with some probability, and the CB is unable to determine whether or not a bank has observed the

[24]It might be possible for a market to play the role that the CB plays in this paper (see, for example, Allen and Gale forthcoming). I do not consider this possibility as CB provided liquidity appears to be the historically relevant case. A number of arguments have been made to explain why a CB may be better at providing liquidity than an interbank market (see, for example, Goodhart 1988, or Rochet and Vives 2004).

[25]The assumption is that banks that were subject to a run are no longer in business at date 2. Finding the optimal redistribution policy is not necessary to establish that emergency lending is welfare enhancing.

early sunspot.[26]

I assume that CB loans are senior to deposits, so that banks must make sure that they have enough resources to repay their debt to the CB before patient depositors can withdraw.[27] In appendix B, I show that the CB makes loans of size

$$L = \frac{R\left[(1-i)\frac{1-\pi}{\pi} - (1-\tau)i\right]}{R - (1+r)(1-\tau) + R\frac{1-\pi}{\pi}\frac{q}{\delta}} \tag{5}$$

where r denote the (net) interest rate on the loan. This is the smallest size of BC loans that will prevent bank runs. I restrict my attention to parameters such that $L > 0$.

The largest loan a bank might need is $L_{max} \equiv (1-\pi)c_1$. In this case, the bank has resources $Ri = (1-\pi)c_2$ to repay the loan. It follows that if the CB charges an interest rate $1 + r \leq c_2/c_1$ then the bank can repay any loan of size less than or equal to L_{max}.

4.2 Bank incentives

Now I can write the expected utility of depositors at a bank under different assumptions about banks' behavior. In this section, I consider a bank's choice of borrowing early from the CB or waiting, taking i as given. Below I describe how a bank would choose i.

A bank can borrow from the CB before it knows whether its depositors

[26]It is likely that CBs today have better information about banks and the financial system than the Bank of England did in the XIXth century. If the CB is able to determine whether a sunspot has occurred, it can eliminate bank's incentives to borrow early by lending conditionally on the sunspot. This could offer an alternative explanation for the difference between Bagehot's recommendation and current central bank practice. I thank an anonymous referee for pointing this out.

[27]Note that if CB liquidity provision is feasible and desirable under this assumption, then it is also feasible and desirable if deposits are senior to CB loans.

observe a sunspot or it can choose to wait. The bank chooses the action that maximizes the expected utility of its depositors, given its belief about what other banks do. Since banks are ex-ante identical, they will either all try to borrow early, or all choose to wait, unless they are indifferent between the two. I focus on equilibria in pure strategies.

I denote by E the case where all banks choose to borrow early from the CB and by W the case where all banks wait. If all banks borrow early, the expected utility of depositors in a bank that is able to obtain reserves from the CB is denoted $EU_{B/E}$, while the expected utility of depositors in a bank that is unable to obtain reserves is denoted $EU_{NB/E}$. Similarly, in the case where banks wait, the expected utility of depositors in a bank that borrows from the CB is $EU_{B/W}$ while the expected utility of depositors in a bank that does not borrow is $EU_{NB/W}$.

Consider the case where banks only borrow if their depositor observes a sunspot. With probability q, the bank's depositors observe the sunspot and the bank borrows from the CB. Since banks that do not observe the sunspot do not borrow, the bank obtains enough reserves to prevent a panic with probability 1. Depositors' expected utility is given by

$$EU_{B/W}(i) = \pi u\left(c_1\right) + (1-\pi)u\left(c_2\right). \tag{6}$$

where

$$\pi c_1 = 1 - i - \frac{1}{\delta}qL, \tag{7}$$

$$(1-\pi)c_2 = Ri - rL + (1+r)qL. \tag{8}$$

The consumption of impatient depositors corresponds to the endowment minus the goods invested in the long-term technology and those taxed by the CB. The consumption of patient depositors is given by the return on goods invested in the long-term technology, minus the interest on the funds borrowed from the CB plus the bank's share of the goods the CB redistributes to all banks. Substituting for c_1 and c_2 in the expression for the expected utility, we can write

$$EU_{B/W}(i) = \pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi} + \frac{-rL+(1+r)qL}{1-\pi}\right). \tag{9}$$

With probability $1-q$, the bank's depositors do not observe the sunspot and the bank does not borrow from the CB. In this case, the depositor's expected utility is given by

$$EU_{NB/W}(i) = \pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi} + \frac{(1+r)qL}{1-\pi}\right). \tag{10}$$

Comparing equations (9) and (10), the consumption of impatient depositors is the same whether their bank borrows or not. The consumption of patient depositors is greater in banks that do not borrow if $r > 0$ since in such cases interest must be paid on CB loans. Hence, for any i, $EU_{NB/W}(i) \geq EU_{B/W}(i)$ for all $r \geq 0$, with a strict inequality if and only if $r > 0$. This immediately implies the following result.

Proposition 1 *It is a Nash equilibrium for all banks to wait until they know if their depositors observe a sunspot before borrowing from the CB.*

If $r > 0$, a bank strictly prefers to wait and borrow only if its depositors observe a sunspot, provided all banks wait. If $r = 0$, the bank is indifferent between waiting or borrowing early, since there is no cost of borrowing.[28]

[28]In appendix C, I show that if $\varepsilon > 0$, then banks strictly prefer to borrow early from the CB whenever $r = 0$. The extension of proposition 1 in this case states that there exists an $\bar{r} > 0$ such that it is a Nash equilibrium for all banks to wait if $r \geq \bar{r}$.

Now consider the case where all banks choose to borrow early. I assume that all banks have the same probability, q, of being able to obtain reserves from the CB. If a bank is able to borrow from the CB, it will not be affected by a bank run even if its depositors observe a sunspot. The depositor's expected utility in this case is

$$EU_{B/E}(i) = \pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right)+(1-\pi)u\left(\frac{Ri}{1-\pi}+\frac{-rL+(1+r)\frac{q}{1-q(1-q)}L}{1-\pi}\right). \tag{11}$$

The expression for the consumption of patient depositors is different than in the case where all banks wait. When all banks try to borrow early, a fraction $q(1-q)$ of banks are unable to obtain the funds they need from the CB. These banks suffer a run at date 1 and do not receive a share of the CB's resources at date 2. Hence the CB's resources are shared among a smaller set of banks. This implies $EU_{B/E}(i) > EU_{B/W}(i)$.

With probability $1-q$, the bank is unable to obtain reserves from the CB. The bank's depositors observe a sunspot with probability q, so their expected utility is given by

$$\begin{aligned} EU_{NB/E}(i) &= (1-q)\left\{\pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right)+(1-\pi)u\left(\frac{Ri}{1-\pi}+\frac{(1+r)\frac{q}{1-q(1-q)}L}{1-\pi}\right)\right\} \\ &\quad +q\hat{\pi}u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right). \end{aligned} \tag{12}$$

If $r=0$, then $EU_{NB/E} = (1-q)EU_{B/E} + q\hat{\pi}u[(1-i-\frac{1}{\delta}qL)/\pi]$ and $EU_{B/E} > EU_{B/W} = EU_{NB/W}$. Since $\hat{\pi}u[(1-i-\frac{1}{\delta}qL)/\pi] < EU_{NB/W}(i)$, it follows that $EU_{NB/E}(i) < EU_{B/E}(i)$ if r is small enough. This can be summarized in the next proposition.

Proposition 2 *It is a Nash equilibrium for all banks to try to borrow early from the CB, if the CB charges a small enough interest rate.*

Propositions 1 and 2 imply that there are multiple equilibria if r is low enough.[29] If all banks are expected to try to borrow early, then it is an equilibrium to do so, while if all banks expected to wait, then it is a equilibrium to wait.

The ex-ante expected utility of depositors when all banks wait is given by

$$EU_E = qEU_{B/E} + (1-q)EU_{NB/E} \tag{13}$$

and it is given by

$$EU_W = qEU_{B/W} + (1-q)EU_{NB/W} \tag{14}$$

when all banks wait. For a given i, the consumption at date 1 is the same whether banks wait or borrow early. The amount of resources available at date 2 is lower when banks try to borrow early, since in this case some goods invested in the long-term technology must be liquidated. Moreover, the available goods are distributed in a less equal way when banks try to borrow early. This variability in expected consumption is disliked by risk-averse depositors.

Note that banks may not choose the same value of i under each kind of equilibrium. Nevertheless, since the expected utility of depositors when banks wait is higher for all values of i, it will be higher at the value that maximizes the expected utility of depositors when banks try to borrow early. And it may be even higher at the value of i that maximizes the expected utility of depositors if all banks wait.

This argument can be summarized in the following proposition.

Proposition 3 *The ex-ante expected utility of depositors is higher if all banks choose to wait than if all banks try to borrow from the CB early.*

[29] In appendix C, I show that proposition 2 extends to the case where $\varepsilon > 0$.

4.3 Eliminating the bad equilibrium

In light of proposition 3, a CB may seek to avoid the equilibrium where all banks try to borrow early. In this section I show that a CB can achieve this goal by charging a high enough interest rate.

The problem faced by an individual bank is to choose i, and whether to try to borrow early or not, in order to maximize the expected utility of its depositors. When the equilibria of propositions 1 and 2 both exist, we can assume that banks' beliefs are coordinated by a sunspot. With probability q^B all banks try to borrow early and with probability $1 - q^B$ all banks wait. In this case, the expected utility of depositors is given by

$$EU(i) = q^B EU_E + (1 - q^B) EU_W, \tag{15}$$

where EU_E and EU_W are given by equations (13) and (14), respectively. Banks choose i to maximize equation (15).

Ideally, one would want to solve for the optimal choice of i by banks, given the parameters of the model and the CB policy. Then, it would be possible to find the optimal value of r, in principle. However, this problem is difficult. Even absent a CB liquidity provision policy, neither Cooper and Ross (1998) nor Ennis and Keister (2006) provide an expression for the optimal investment i.

It is possible, however, to show that the CB can eliminate the equilibrium of proposition 2 by setting a high interest rate r. It is a dominant strategy for banks to wait if $EU_{NB/E}(i^e) > EU_{B/E}(i^e)$, where i^e denotes equilibrium investment. Recall that

$$EU_{B/E}(i) = \pi u \left(\frac{1 - i - \frac{1}{\delta} qL}{\pi} \right) + (1 - \pi) u \left(\frac{Ri}{1 - \pi} + \frac{-rL + (1 + r) qL}{1 - \pi} \right) \tag{16}$$

and

$$
\begin{aligned}
EU_{NB/E}(i) \;=\; & (1-q)\left[\pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right)+(1-\pi)u\left(\frac{Ri}{1-\pi}+\frac{(1+r)qL}{1-\pi}\right)\right] \\
& +q\hat{\pi}u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right). \qquad (17)
\end{aligned}
$$

If $EU_{NB/E}(i) - EU_{B/E}(i) > 0$, then it is not an equilibrium for banks to borrow early. This condition is equivalent to

$$
\begin{aligned}
(1-\pi)\left[(1-q)u\left(\frac{Ri}{1-\pi}+\frac{(1+r)qL}{1-\pi}\right)-u\left(\frac{Ri}{1-\pi}+\frac{-rL+(1+r)qL}{1-\pi}\right)\right] \\
+\pi u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right)\left[q\frac{i(1-\tau)}{1-i}\right] > 0. \qquad (18)
\end{aligned}
$$

Since the second term is positive, a sufficient condition for $EU_{NB/E}(i) - EU_{B/E}(i) > 0$ is

$$
(1-q)u\left(\frac{Ri}{1-\pi}+\frac{(1+r)qL}{1-\pi}\right) > u\left(\frac{Ri}{1-\pi}+\frac{-rL+(1+r)qL}{1-\pi}\right) \qquad (19)
$$

Since L increases with r (see appendix B), it is possible to chose r large enough so that the condition holds, for any i.

There is a constraint on how high the CB can set r. It may be the case that r is so high that the consumption of depositors who withdraw at date 2 is lower that the consumption of depositors who withdraw at date 1, if the bank borrows. In such a case, the lender of last resort policy would not be able to prevent bank runs. Note, however, that the smaller q is, the lower r can be set without violating the constraint. We can summarize this argument in the following proposition.

Proposition 4 *There is an interest rate on CB loans high enough such that it is a a dominant strategy for all banks to wait before they borrow from the CB, provided q is not too big.*

The CB does not need to hit a particular rate of interest "just right" in order to provide incentive for banks to wait. It only needs to set the interest rate above some level. Of course, the CB would like to set the lowest rate that provides the proper incentives since depositors' expected utility decreases with r.

A few remarks are in order regarding the desirability of a liquidity provision policy. Introducing a liquidity provision policy and changing the interest rate r affects the level of investment i. If the interest rate $1 + r < R$, then banks have an incentive to invest more in the long-term technology and try to borrow from the CB to pay their impatient depositors. This could result in a large change in i and make the liquidity provision policy undesirable. One solution is to set $1 + r \geq R$, in which case this incentive disappears. I do this in the numerical example presented below. Alternatively, one could assume that capital requirements force banks to hold sufficiently many liquid assets. Another option would be to assume that the CB can observe whether a sunspot occurs, but not which banks are affected.[30] If the CB offers liquidity only if a sunspot occurs, and if this is a rare enough event, then the incentives to distort investment is reduced.

Since charging a high r makes consumption at date 2 more unequal, it may not be desirable to eliminate the multiplicity of equilibria. If q^B is sufficiently small, eliminating the bad equilibrium by charging a high r could reduce welfare. The analysis in this paper is relevant under the assumption that the probability q^B is large.[31]

[30]To be consistent with our other assumptions, the CB would not know precisely what time the sunspot is observed, just that a sunspot will occur.

[31]Note also that if $\varepsilon > 0$ then only the equilibrium of proposition 2 exists if r is small, as shown in the appendix.

4.4 An example

In this section I provide an example showing that the CB can improve welfare by making loans despite the fact that it must charge a high interest rate and that the consumption of some patient depositors comes from stored goods.[32]

Following Ennis and Keister (2006), I assume that the utility function is given by $u(c) = c^{\alpha}/\alpha$. Parameters are set at the following values: $\alpha = 0.9$, $\pi = 0.8$, $R = 1.15$, $q = 0.1$, $\delta = 0.95$, and $\tau \to 1$.[33] I also assume that $q^{B} = 1$ so that the bad equilibrium always occurs when multiple equilibria are possible.

Rather than jointly solve for the optimal i and the optimal r, I choose $r = 0.2$ and solve for i. I show that $EU_{NB/E} > EU_{B/E}$ in equilibrium, which implies that it is better not to borrow early from the CB even if all banks choose to borrow early. Further, for these parameters and the chosen interest rate, welfare is higher with CB lending at a high rate than without any CB lending. Finally, I show that welfare is higher with CB lending at a high rate than if the CB obtains enough goods to prevent runs at all banks and lends at a low interest rate.

Under the above assumptions, it can be verified that $L \to \frac{95}{381}(1-i)$, so that

$$\begin{aligned} EU_{E}(i) &= qEU_{B/E}(i) + (1-q)EU_{NB/E}(i) \\ &= \pi u\left(\frac{\frac{380}{381}(1-i)}{\pi}\right) \\ &\quad +(1-\pi)\left[qu\left(\frac{1.15\cdot i - 0.03L}{1-\pi}\right) + (1-q)^{2}\left(\frac{1.15\cdot i + 0.12L}{1-\pi}\right)\right]. \end{aligned}$$

[32] The Excel spreadsheet used for the numerical example is available from the author upon request.

[33] Note that $\tau \to 1$ implies that banks cannot offer run preventing contacts.

The expected utility EU_E is maximized at $i \approx 0.25$. Since $\tau \to 1$, then

$$EU_{NB/E}(i) > EU_{B/E}(i) \Leftrightarrow (1-q)u(\frac{Ri + (1+r)qL}{1-\pi}) > u(\frac{Ri + (1+r)qL - rL}{1-\pi}). \tag{20}$$

This inequality holds at the maximizing i, so it is not an equilibrium for all banks to borrow early.

From proposition 1, we know that $EU_{NB/W}(i) > EU_{B/W}(i)$ so it is an equilibrium for banks to wait until before they borrow. Given $r = 0.2$, the welfare maximizing choice of i for that equilibrium is $i^* \approx 0.465$ and welfare is given by

$$EU_W(i^*) = qEU_{B/W}(i^*) + (1-q)EU_{NB/W}(i^*) \approx 1.155. \tag{21}$$

At this level of investment, the interest rate on the CB loan is low enough that banks are able to repay. Also, date 2 consumption is higher than date 1 consumption, despite the need to repay the CB loan.

It remains to be verified that CB lending at rate $r = 0.2$ is preferred to the absence of CB intervention or to a policy in which the CB would hold enough reserves to prevent panics at all banks, and charge an interest rate of zero. Let EU_{NCB} denote the expected utility of depositors when the CB does not intervene.

$$EU_{NCB}(i) = (1-q)\left[\pi u\left(\frac{1-i}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi}\right)\right] + q\pi u\left(\frac{1-i}{\pi}\right). \tag{22}$$

In this case, the expected utility of depositors is maximized for $i \approx 0.235$ and $EU_{NCB}(i = 0.235) \approx 1.116 < EU_W(i^*)$. Hence, CB lending at $r = 0.2$ yields more expected utility that no CB lending.

As is shown in appendix A, the CB can prevent bank runs at all banks if it raises enough taxes. Let EU_{NR} denote the expected utility of depositors when the CB sets $T = 1 - i$ and lends at an interest rate of zero.

$$EU_{NR}(i) = \pi u\left(\frac{\delta(1-i)}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi}\right). \quad (23)$$

In this case, the expected utility of depositors is maximized for $i = 0.585$ and $EU_{NR}(i = 0.585) = 1.132 < EU_W(i^*)$. Hence, expected utility is higher if the CB holds enough reserves to prevent panics at only q banks, and charges a high interest rate for loans, than if the CB holds enough reserves to prevent panics at all banks and charges no interest rate.

Finally, note a central bank that only worried about the moral hazard associated with CB lending would not charge a rate high enough to eliminate the bad equilibrium. Such a CB would charge a rate $r = R = 0.15$. At that interest rate $EU_{NB/E} < EU_{B/E}$, at the investment level chosen by banks, and the bad equilibrium exists.

5 Conclusion

This paper developed a model of the lender of last resort policy of a CB using commodity money. I show that the bank may need to charge a high rate of interest in order to eliminate a bad equilibrium where all banks want to borrow before they know if they are affected by a sunspot. This is consistent with Bagehot's recommended policy.

This result stand in contrast to most of the literature on CB liquidity provision, which shows that CB should provide reserves at an interest rate of zero. This literature considers fiat money environments. In support of the idea that whether a CB operates in a fiat or commodity money environment may matter, it is interesting to note that Thornthon (1802) who wrote before Bagehot, at a time where the Bank of England was off the gold standard, does not recommend lending at a high interest rate.

Hence this paper helps reconcile Bagehot's recommended policy and the Federal Reserve's response to September 11. Bagehot's policy is consistent with a desirable policy in a commodity money world while the Federal Reserves's policy is consistent with the optimal policy in a fiat money world.

Finally, note that the logic of the argument presented in this paper should extend to the case of a CB operating with fiat money but trying to defend a exchange rate peg. In that case also, the reserves that the CB can lend to banks is limited and a high rate will serve as way to screen banks.

Appendix A: The CB loan contract

If the CB can invest in the long-term technology, and there were no cost of raising funds, then bank runs could be prevented in the following way: The CB invests all the resources it taxes in the long-term technology. Banks invest all the deposits they receive in the storage technology and the deposit contract promises $c_2 = 0$. All the assets in the CB are given to depositors who do not withdraw at date 1. There would be no bank runs in this case since all patient depositors know that the long-term projects held by the CB will not be liquidated.

If the CB cannot invest in the long-term technology, but raising funds is not costly, then bank runs can be prevented in a different way: The CB invests all the resources it taxes in the storage technology. Banks invest all the deposits they receive in the long-term technology and the deposit contract promises $c_1 = 0$. All assets in the CB are given to depositors who "withdraw" at date 1. Again, bank runs are prevented in this case because banks never need to liquidate the long-term technology.

Under the scheme proposed above, the bank does not need to set c_1 equal to zero. It is enough to set c_1 low enough that $c_1 \leq 1 - \tau i - T$. Recall that the bank's deposit contract is

$$c_1 = \frac{1 - i - T}{\pi}, \tag{24}$$

$$c_2 = \frac{Ri}{1 - \pi}. \tag{25}$$

It follows that $c_1 \leq 1 - \tau i - T$ if and only if

$$i \geq \frac{(1 - \pi)(1 - T)}{1 - \tau\pi}. \tag{26}$$

The expected utility of depositors under the CB scheme is given by

$$EU_{NR} = \pi u \left(\frac{1 - i - T(1-\delta)}{\pi} \right) + (1-\pi) u \left(\frac{Ri}{1-\pi} \right), \tag{27}$$

where T is implicitly defined by equation 26 at equality. The expected utility of depositors if the CB does no intervene is given by

$$EU_{NCB} = (1-q) \left[\pi u \left(\frac{1-i}{\pi} \right) + (1-\pi) u \left(\frac{Ri}{1-\pi} \right) \right] + q\hat{\pi} u \left(\frac{1-i}{\pi} \right). \tag{28}$$

Inspection of these two expressions reveals that for any $\delta < 1$, there exists a q small enough that $EU > EU^{CB}$. This paper focuses on the case where it is too costly for the CB to provide goods to all banks.

A transfer scheme between the CB and banks can be described as follows. At date 1, the CB transfers goods to banks that ask for it, until it runs out. At date 2, the transfers between the CB and banks is conditional on whether or not a bank received a transfer from the CB at date 1.[34] Note that the transfers can be positive or negative. It can be shown that any set of transfers can be rewritten as a combination of two transfers: First, a transfer from banks that obtained reserves from the CB at date 1 to the CB and, second, a transfer from the CB to all banks, regardless of whether they obtained reserves at date 1. Since all banks borrow the same amount from the CB, the transfers at date 1 and 2 between the CB and banks that obtain funds can be though of as a loan. The ratio of the date 2 transfer to the date 1 transfer, in absolute value, is the gross interest rate on the loan.

[34] In principle, the transfer at date 2 could also depend on whether a bank tried to obtain reserves from the CB but was not able to. This case can be ruled out using the following argument: If such a transfer was positive, all banks would have an incentive to borrow to receive the transfer. This cannot be optimal for the CB. If the transfer is negative, one can assume that banks observe that the CB runs out of reserves as soon as it does, and do not request reserves in this case. Hence, the CB is unable to distinguish between banks that intended to borrow but were not able to and banks that did not intend to borrow.

Appendix B: The size of CB loans

Let θ denote the fraction of the long term-technology that is liquidated at date 1, L denote the size of the CB loan. The loan must satisfy

$$\begin{aligned}(1-\pi)c_1 &\leq (1-\tau)\theta i + L, \quad (29)\\ (1+r)L &\leq Ri(1-\theta), \quad (30)\end{aligned}$$

and $L \geq 0$. The first equation indicates that the sum of the CB loan and the goods obtained from the partial liquidation of the long-term technology are enough to provide c_1 to all patient depositors withdrawing at date 1. The second equation assures that the bank has enough goods left to repay the CB loan. Note that a bank would never borrow funds from the CB if the gross interest rate $1+r$ is greater than $R/(1-\tau)$. Hence, $1+r < R/(1-\tau)$ is assumed throughout.

I focus on the case where raising funds is sufficiently costly so that the CB only taxes the minimum necessary. Hence, L denotes the smallest loan necessary to prevent bank runs. The CB holds qL reserves and makes a loan of L to each bank that wants to borrow until there are no reserves left. Each bank is assumed to have the same probability of arriving at the CB early enough to borrow.

To find L, I solve equations (29) and (30) at equality. Indeed, the CB will lower L until equation (29) binds. If equation (30) does not bind, then it is possible to reduce L a little, to force the bank to increase θ. The increase in θ relaxes equation (29) so it is not violated. L cannot be reduced further when both constraints bind. I also use $\pi c_1 = 1 - i - T$, to get

$$L = \max\left\{0, \frac{R\left[(1-i-T)\frac{1-\pi}{\pi} - (1-\tau)i\right]}{R-(1+r)(1-\tau)}\right\}. \quad (31)$$

To prevent panics at a mass q of banks, the CB must raise taxes $\delta T = qL$. Combining these two equations, I get

$$L = \max\left\{0, \frac{R\left[(1-i)\frac{1-\pi}{\pi} - (1-\tau)i\right]}{R-(1+r)(1-\tau)+R\frac{1-\pi}{\pi}\frac{q}{\delta}}\right\}. \tag{32}$$

Note that L increases in r. When the interest rate r is high, banks need more resources to repay their debt. For this reason, the CB must increase L, everything else constant.

Appendix C: The case with $\varepsilon > 0$

I use a superscript to distinguish the expressions for expected utility in this case. For example, $EU^{\varepsilon}_{B/W}$ to denote the expected utility of depositors in a bank that borrows when all banks wait and $\varepsilon > 0$. Similarly, I also use $EU^{\varepsilon}_{NB/W}$, $EU^{\varepsilon}_{B/E}$, $EU^{\varepsilon}_{NB/E}$.

First note that

$$EU^{\varepsilon}_{B/W} = \pi u\left(\frac{1-i-\frac{1}{\delta}q'L}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi} + \frac{-rL+(1+r)q'L}{1-\pi}\right). \tag{33}$$

This is the same as equation (9) with q' replacing q, since a bank that is able to borrow is unaffected by a sunspot when it occurs.

Next,

$$\begin{aligned} EU^{\varepsilon}_{NB/W} &= (1-\varepsilon)\left[\pi u\left(\frac{1-i-\frac{1}{\delta}q'L}{\pi}\right) + (1-\pi)u\left(\frac{Ri}{1-\pi} + \frac{(1+r)q'L}{1-\pi}\right)\right] \\ &\quad + \varepsilon\hat{\pi}u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right). \end{aligned} \tag{34}$$

With probability $1-\varepsilon$, only a fraction q' of banks observe the sunspot and the bank would not borrow if and only if it is not affected by the sunspot.

With probability ε, all banks are affected by the sunspot and if the bank was unable to borrow, a run occurs.

Also,

$$EU^{\varepsilon}_{B/E} = \pi u\left(\frac{1-i-\frac{1}{\delta}q'L}{\pi}\right)+(1-\pi)u\left(\frac{Ri}{1-\pi}+\frac{-rL+(1+r)\frac{q'}{1-q'(1-q')}L}{1-\pi}\right). \tag{35}$$

This is the same as equation (11) with q' replacing q, since a bank that is able to borrow is unaffected by a sunspot when it occurs.

Finally,

$$\begin{aligned} EU^{\varepsilon}_{NB/E} &= (1-\varepsilon)(1-q')\left\{\pi u\left(\frac{1-i-\frac{1}{\delta}q'L}{\pi}\right)+(1-\pi)u\left(\frac{Ri}{1-\pi}+\frac{\frac{(1+r)q'}{1-q'(1-q')}L}{1-\pi}\right)\right\} \\ &\quad +\left[(1-\varepsilon)q'+\varepsilon\hat{\pi}\right]u\left(\frac{1-i-\frac{1}{\delta}qL}{\pi}\right). \end{aligned} \tag{36}$$

With probability $(1-\varepsilon)$, only a fraction q' of banks are affected by a sunspot. If the bank is a member of that set, a bank run occurs. With probability ε, all banks are affected by a sunspot and bank run occurs since the bank was unable to borrow.

Inspection of these expressions reveal that $EU^{\varepsilon}_{B/W} > EU^{\varepsilon}_{NB/W}$ and $EU^{\varepsilon}_{B/E} > EU^{\varepsilon}_{B/E}$ if $r = 0$. Hence, the only equilibrium when $r = 0$ is for all banks to try to borrow early.

If ε and q' are small, there exists r large enough such that $EU^{\varepsilon}_{B/W} \leq EU^{\varepsilon}_{NB/W}$ and $EU^{\varepsilon}_{B/E} \leq EU^{\varepsilon}_{B/E}$. Hence, the only equilibrium when r is large enough is for all banks to wait. Multiple equilibria occur if r is neither to small nor too large.

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