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Roc Armenter
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Abstract

Lack of commitment in monetary policy leads to the well known Barro-Gordon inflation bias. In this paper, we argue that two phenomena associated with the time inconsistency problem have been overlooked in the exchange rate debate. We show that, absent commitment, independent monetary policy can also induce expectation traps—that is, welfare-ranked multiple equilibria—and perverse policy responses to real shocks—that is, an equilibrium policy response that is welfare inferior to policy inaction. Both possibilities imply higher macroeconomic volatility under flexible exchange rates than under fixed exchange rates.

Key words: time inconsistency, independent monetary policy, exchange rate regimes

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Armenter: Federal Reserve Bank of New York (e-mail: roc.armenter@ny.frb.org). Bodenstein: Board of Governors of the Federal Reserve System (e-mail: martin.r.bodenstein@frb.gov).
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1 Introduction

The most influential case for fixed exchange rates has rested exclusively on the celebrated inflation bias of Barro and Gordon (1983).[1] A monetary authority lacking the credibility to commit to a policy, the logic goes, can peg its currency and import the monetary policy of another country with more credible institutions. Of course, this argument assumes that the exchange rate regime is a credible commitment even if monetary policy is not. The textbook argument against fixed exchange rates follows the lines of the classic Mundell-Flemming analysis. A fixed exchange rate means no independent monetary policy and therefore no ability to ease real macroeconomic volatility. This roughly summarizes the conventional wisdom about the costs and benefits of an exogenous commitment to a fixed exchange rate.[2]

We argue that two phenomena associated with the time inconsistency problem have been overlooked in the exchange rate debate. First, the lack of commitment can induce *expectation traps*, i.e.,welfare ranked multiple equilibria, even in finite horizon economies.[3] Second, we show that real shocks can exacerbate the time inconsistency problem. As a result, the equilibrium policy response to these shocks can be worse than policy inaction—we label this as *perverse policy response*.

Expectation traps and perverse policy responses are the reasons why we answer positively to the question posed in the title. Contrary to the standard view, flexible exchange rates may feature larger macroeconomic volatility than fixed exchange rates. First, in the presence of expectation traps, independent monetary policy may react unnecessarily to shifts in expectations. Second, the Mundell-Flemming argument does not necessarily hold if the monetary authority lacks credibility. The possibility of perverse policy responses implies that not only may an independent monetary policy fail to ease macroeconomic volatility, but it may even magnify it.

We also show that expectation traps can be ruled out by a soft exchange rate peg with appropriately chosen bands. However, in order to avoid perverse policy responses, a hard exchange rate peg is required.

We present a tractable model of a small open economy that builds upon Armenter and Bodenstein (2004). Nominal rigidities introduce a role for active monetary policy. Combined with the monopoly distortion, nominal rigidities also set the stage for optimal monetary

[1] See Obstfeld and Rogoff (1996) and references herein.

[2] Of course there are other persuasive macroeconomic arguments in favor of fixed exchange rates, as the well known "fear of floating" of Calvo and Reinhart (2002). See also Arellano and Heathcote (2003) who argues that dollarization can provide better access to financial markets.

[3] Chari, Christiano and Eichenbaum (1998) originally introduced the term. There is a growing literature on multiple equilibria with discretionary monetary policy, e.g., Albanesi, Chari and Christiano (2003), Armenter (2004), King and Wolman (2004), Armenter and Bodenstein (2004), and Siu (2004).

policy to be time inconsistent. We define three policy equilibrium concepts, where monetary policy is endogenously determined as the outcome of a benevolent policymaker. In the analysis of flexible exchange rates, we work with Markov equilibria where the monetary authority has full discretion in setting monetary policy.[4] We are also interested in the optimal monetary policy with commitment, which is formalized as the Ramsey equilibrium. Finally, we define policy equilibria under the constraint of an arbitrary exchange rate regime. The policymaker takes the exchange rate regime as given and it therefore constitutes an exogenous commitment device for monetary policy.

We show that there are expectation traps in an economy calibrated to match several stylized facts about inflation and openness.[5] We find two Markov equilibria, which we label low and high inflation equilibrium. Hence, under flexible exchange rates, the monetary authority can unwillingly be caught in a high inflation equilibrium for long periods, and shifts in expectations can induce unnecessary macroeconomic volatility.

Expectation traps increase the costs of the lack of commitment by a significant amount. In our calibrated economy, the welfare loss of a shift in expectations from the low to the high inflation equilibrium is about three times the welfare change from implementing the optimal monetary policy in place of the low inflation equilibrium. A soft exchange rate peg with appropriately chosen bands is sufficient to rule out expectation traps, without hindering the ability of the monetary authority to respond to macroeconomic shocks.

We illustrate the perverse policy response phenomenon with a negative terms of trade shock. The shock contracts the sector of tradeables, which makes the whole economy less competitive and therefore it increases the time inconsistency problem. The heightened monopoly distortion raises the incentives of the monetary authority to inflate. In equilibrium, private sector inflation expectations rise, leading monetary policy away from the optimal response to the shock.

In our calibrated economy, the policy response in a Markov equilibrium overshoots the optimal response by a factor of ten. Households prefer no policy response—the outcome of a fixed exchange rate—to the Markov equilibrium policy response. Hence, a flexible exchange rate fails to provide the macroeconomic stability which is presumed to be its main virtue. Due to concavity, a positive terms of trade shock does not outweigh the welfare losses of a negative shock under the flexible exchange rate regime.

We do not attempt to establish, theoretically or empirically, that fixed exchange rates are welfare superior. Indeed, a definitive welfare ranking of exchange rate regimes may be more elusive than ever. However, we see clear implications for the exchange rate policy

[4] We label this equilibrium Markov because we focus on equilibria sustainable in finite horizon economies. This rules out equilibria based in trigger strategies.

[5] Markov equilibrium multiplicity is quite robust. Indeed, Armenter (2004) shows that the conditions for the existence of expectation traps in monetary policy are very general.

debate. For example, the recent literature on dollarization has dealt with the time inconsistency problem.[6] However, to the best of our knowledge, none considered the possibility of expectation traps or perverse policy responses. This omission renders any welfare analysis incomplete.

The results of this paper also imply that we should treat with caution some of the arguments made lately in favor of flexible exchange rates. For example, the observed fall of inflation rates worldwide should not be taken as conclusive evidence that "the credibility consideration seems to be less compelling than it once was for emerging markets" as Chang and Velasco (2000) claims. All what is needed to be back into high inflation is a shift in the private sector expectations.

Moreover, larger real volatility does not necessarily make a stronger case for flexible exchange rate regimes. Summarizing the state of the debate, Frankel (1998) asserts that "if the country is subject to many external disturbances, [...] then it is more likely to want to float its currency." Chang and Velasco (2000) also concludes that the case for exchange-rate flexibility is "especially strong for countries that are often hit by large real shocks from abroad." In the light of our analysis, it is necessary to check that the relevant real shocks do not induce perverse policy responses. Otherwise, more real volatility makes the case for hard exchange rate pegs stronger.

The remainder of the paper is organized as follows. In Section 2 we present our model and define the equilibrium concepts. Section 3 discusses expectation traps and Section 4 takes upon the possibility of perverse policy responses. Section 5 concludes. An Appendix, containing several proofs as well as calibration details, is included.

2 The Economy

First, we characterize the private sector equilibrium, which includes a detailed description of the economy. Then we define the different policy equilibrium concepts considered: Markov equilibrium, Ramsey equilibrium and Exchange Rate policy equilibrium.

2.1 Private Sector Equilibrium

This infinite-horizon small open economy is populated by a representative household, a representative final good firm, a continuum of intermediate good firms and a monetary authority.

Figure 1 illustrates the timing of the model. Several of the decisions relevant for period t are made one period in advance. First, a fraction of the intermediate good firms—the sticky

[6] A small sample are Chang and Velasco (2003), Cooley and Quadrini (2001) and Mendoza (2001).

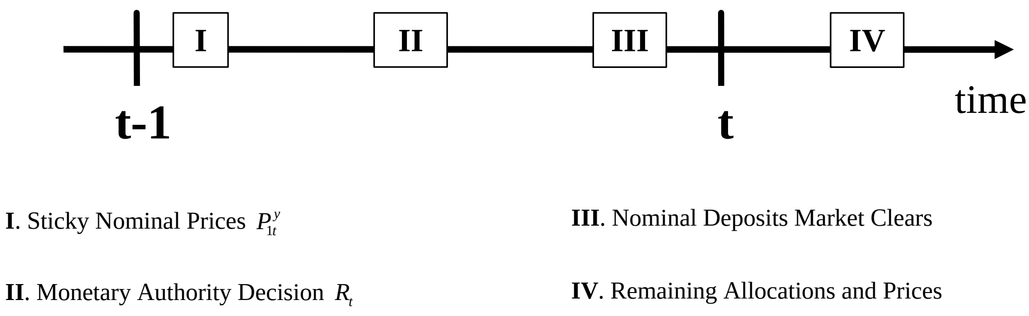

Figure 1: Timing of Relevant Decisions for Date t

price firms indexed by $i = 1$—set their nominal price for period t, P^y_{1t}, at the beginning of period $t-1$. The monetary authority then chooses the policy instrument to maximize the representative household's welfare taking P^y_{1t} as given. At the end of period $t-1$, the market for nominal deposits clears. The remaining prices and allocations are determined during period t.

We assume that the monetary policy instrument is the nominal interest rate, R_t, that is paid at date t on nominal deposits carried from period $t-1$. The nominal interest rate is implemented by means of a monetary transfer, X_{t-1}, such that the market for nominal deposits clears at the chosen rate R_t. We show below that the monetary authority can implement any inflation rate at date t, π_t, within some feasibility bounds.[7] From now on, we will think of inflation as the policy instrument.

The sticky price firms form a belief about inflation in period t, denoted $\hat{\pi}_t$, in order to set their nominal price P^y_{1t}. Following the literature, we commonly refer to $\hat{\pi}_t$ as private sector inflation expectations, although "beliefs" would be a more accurate term.

We show that real prices and allocations in a private sector equilibrium at date t are fully determined by the state of the economy $s_t = (\hat{\pi}_t, \pi_t)$. Neither past nor future policy decisions are relevant and there is no physical state variable in the economy. By focusing on Markov perfect equilibria, we can study the monetary authority's decision as a sequence of

[7]Inflation is bounded from above and below. The zero nominal interest bound implies a lower bound on inflation. There is also a upper bound given by feasibility, which we show is never binding.

static problems.

We do not model money directly. Implicitly, nominal deposits are as good as cash balances. This feature of the model allows us to abstract from money demand considerations and to focus on nominal frictions on the supply side of the economy.[8]

Finally, we normalize the last period's aggregate price index to 1 in order to resolve the nominal indeterminacy.

2.1.1 Households

Household preferences at date t are given by

$$\sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j, n_j\right)$$

with $0 < \beta < 1$. For tractability, we assume quasi-linear preferences

$$u\left(c, n\right) = c + h\left(1 - n\right)$$

where h is a strictly increasing, concave function that satisfies the usual Inada conditions.

We express the household problem in recursive form

$$v(D, s) = \max_{c,n,D'} u\left(c, n\right) + \beta v\left(D', s'\right) \tag{1}$$

subject to

$$\begin{aligned} c &\geq 0 \\ 0 &\leq n \leq 1 \end{aligned}$$

and

$$P\left(s\right) c + D' \leq R\left(s\right) D + W\left(s\right) n + T^f\left(s\right) \tag{2}$$

where D are nominal deposits, which pay a nominal interest rate R, and T^f are profits. Nominal deposits, D, are the unique asset holdings of the household and $s = (\hat{\pi}, \pi)$ is the economy-wide state. As both sticky prices and the actual policy choice are set one period in advance, next period's state s' is fully determined by the time households make their decisions. Time subscripts are dropped for the rest of the paper following the recursive formulation.

[8] This in the spirit of the cashless economies discussed in Woodford (2003).

Labor supply is characterized by the first order condition

$$-\frac{u^n(s)}{u^c(s)} = w(s)$$

which implies

$$h'(1-n(s)) = w(s) \tag{3}$$

where $w(s) = \frac{W}{P}$. The concavity of h implies that the labor supply has a positive slope everywhere.

Finally, the intertemporal Euler equation implies

$$R' = \frac{1}{\beta}\pi'$$

where $\pi' = \frac{P'}{P}$ and we used the envelope theorem to establish that $\frac{dv}{dR}(D', s') = R'$. This is the standard Fischer equation. All uncertainty with respect to the monetary authority's decision has been resolved before the nominal deposits market clears. Hence, next period's inflation π' is known by the time of the household's savings decision. As the policy choice for date t is made at date $t-1$, the relevant relationship for the date t private sector equilibrium is given by

$$R(s) = \frac{1}{\beta}\pi. \tag{4}$$

2.1.2 Firms

There is a continuum $I = [0,1]$ of intermediate goods. There is a representative final good firm which combines a continuum $[0, 1-\mu_x]$ of intermediate inputs $y_i(s)$ to produce the final good $y(s)$ according to

$$y(s) = \left[\int_0^{1-\mu_x} y_i(s)^\eta \, di\right]^{\frac{1}{\eta}} \tag{5}$$

with $\eta < 1$. Its profit-maximization problem is

$$\max_{c, \{y_i\}_0^{1-\mu_x}} P(s)\, c - \int_0^{1-\mu_x} P_i^y(s)\, y_i(s)\, di$$

subject to (5). Using the first order conditions, the demand good $y_i(s)$ is given by

$$p_i^y(s) = \left(\frac{y(s)}{y_i(s)}\right)^{1-\eta} \tag{6}$$

where $p_i^y(s) = \frac{P_i^y(s)}{P(s)}$.

There is a fraction α of non-tradeable intermediate goods and a fraction $(1-\alpha)$ of tradeable intermediate goods.

There is monopolistic competition in the non-tradeable intermediate good sector. Each good is produced by a single firm i according to a linear technology on labor,

$$y_i(s) = \theta_i n_i(s).$$

There are three types of intermediate good firms in the non-tradeable input sector. Let μ_i denote the measure of firms of type i, with $\mu_1 + \mu_2 + \mu_3 = \alpha$. We assume symmetry within each firm type.

Firms of type 1—the sticky price firms—set their nominal prices before the monetary authority's policy choice. As a consequence, their nominal price, $P_1^y(\hat{\pi})$, is a function of the private sector inflation expectations $\hat{\pi}$ but not of the actual inflation π. As all intermediate good firms, the sticky price firms take in account the demand function for its own good, $y_1(s)$. Given our specification for the demand of each good i, (6), profit maximization implies that the nominal price equals a constant markup over the expected marginal cost

$$P_1^y(\hat{\pi}) = \frac{1}{\eta}\frac{\hat{W}}{\theta_1}$$

where $\hat{W}$ is the expected nominal wage. Rational expectations require that $\hat{W}$ is the equilibrium nominal wage under the expectation that $\hat{\pi}$ is the actual policy choice, i.e.

$$P_1^y(\hat{\pi}) = \frac{1}{\eta}\frac{w(\hat{\pi},\hat{\pi})}{\theta_1}\hat{\pi} \tag{7}$$

where $w(\hat{\pi},\hat{\pi})\hat{\pi}$ is the nominal wage.

Firms of type 2 are flexible price setters, i.e.,they set the nominal price, $P_2^y(s)$, after the monetary authority's decision. Hence it is a function of both π and π^e. We assume that firms of type 2 are financially constrained and they must borrow the nominal wage bill Wn one period in advance at the nominal interest rate $R(s)$.[9] Their optimal pricing rule is

$$p_2^y(s) = \frac{1}{\eta}R(s)\frac{w(s)}{\theta_2}. \tag{8}$$

The fact that their marginal cost is augmented by $R(s)$ is reflected in the real price.

[9] These firms provide the demand side for the household deposits. Note that the deposit demand is also determined with knowledge of the actual policy choice π.

Finally, firms of type 3 are flexible price setters and financially unconstrained. Therefore we have

$$p_3^y(s) = \frac{1}{\eta} w(s). \tag{9}$$

Note that if the expectation and the actual inflation rate are the same, $\hat{\pi} = \pi$, (7) and (9) imply that prices and output are the same across sticky and non-financially constrained flexible price firms, i.e.,$p_1^y(\pi, \pi) = p_3^y(\pi, \pi)$ and $y_1(s) = y_3(s)$. Moreover, if $R(\pi, \pi) = 1$, all firms' prices and production are identical. Since the production function for the final good (5) is convex, symmetry across firm types is a necessary condition for production efficiency. In other words, $R(s) > 1$ and $\hat{\pi} \neq \pi$ introduce costly price distortions.

The **tradeable intermediate good sector** is composed of export and import firms. There is a measure μ_x of export firms, which produce domestically and they supply exclusively to the world markets. We assume that the country's export goods are not differentiated so the export price is determined in the world markets. Hence, export firms take the price as given.

The production function for export firms is

$$y_x(s) = \theta_x n_x(s).$$

The first order conditions associated with the profit-maximization problem implies

$$p_x(s) = \frac{w(s)}{\theta_x}. \tag{10}$$

In addition the law of one price equates the domestic price, in nominal terms, to the world market price for x, P_x^*,

$$P_x(s) = \varepsilon(s) P_x^*$$

where $\varepsilon(s)$ is the nominal exchange rate. In terms of real prices,

$$p_x(s) = q(s) p_x^* \tag{11}$$

where $q(s) = \frac{\varepsilon(s)P^*}{P(s)}$ is the real exchange rate and P^* is the world price for the final good. We set the last period world final good price equal to one. Then we can express the real exchange rate in terms of inflation rates

$$q(s) = \frac{\varepsilon(s)\pi^*}{\pi} \tag{12}$$

where π^* is the world rate of inflation.

Import firms do not produce domestically: they simply buy $y_m(s)$ from the world markets. Import prices are determined in the world market and taken as given by firms. Hence

$$p_m(s) = q(s)\, p_m^*. \tag{13}$$

Imports constitute a measure μ_m of total tradeable inputs, with $\mu_x + \mu_m = 1 - \alpha$.

Because there is no trade in intertemporal assets with the rest of the world, the value of imports and exports must be equated every period,

$$\mu_m y_m(s) = \mu_x tt y_x(s) \tag{14}$$

where $tt = \frac{P_x^*}{P_m^*}$ are the terms of trade.

2.1.3 Market Clearing Conditions and Private Sector Equilibrium Definition

The aggregate resource constraint is

$$c(s) = \left[\sum_{i=1}^{3} \mu_i (\theta_i n_i(s))^\eta + \mu_m (\theta_m n_m(s))^\eta\right]^{\frac{1}{\eta}} \tag{15}$$

where (5) has been combined with each intermediate good production technology. The market clearing condition for the labor market is

$$n(s) = \sum_{i=1}^{3} \mu_i n_i(s) + \mu_x n_x(s). \tag{16}$$

Equations (3)-(16) are sufficient to solve for all real prices and allocations as function of expected and actual inflation. This confirms our conjecture that $s = (\hat{\pi}, \pi)$ fully characterizes all allocations. We proceed now to define a Private Sector Equilibrium (PSE) given $\hat{\pi}$ as a collection of allocation and price functions defined over π and a sticky nominal price $P_1^y(\hat{\pi})$.

Definition 1 *Given an inflation rate expectation $\hat{\pi}$, a* ***Private Sector Equilibrium*** *is a number, $P_1^y(\hat{\pi})$, and a collection of functions, $\{p_i^y(s), y_i(s), n_i(s)\}_{i\in I}$, $R(s)$, $w(s)$, $n(s)$, $c(s)$, $\varepsilon(s)$, $q(s)$ and $y(s)$, such that*

1. *The household optimal conditions, (3) and (4), are satisfied.*

2. *Firm maximize profits, (7)-(10) are satisfied.*

3. *Markets clear, (5), (6) and (12)-(16) hold.*

A ***Private Sector Equilibrium outcome*** *in state* $s = (\hat{\pi}, \pi)$ *is the collection of allocations and prices which occur at a PSE given* $\hat{\pi}$ *evaluated at* π*.*

Our definition of the PSE is sufficient to characterize the monetary authority's problem. Note that nominal prices, deposits and monetary transfers are not included in the PSE. Now we show how to characterize these variables and why they are not relevant for the monetary authority's problem.

It is straightforward to recover all nominal prices, as under our normalization, $\pi = P(s)$. The nominal deposit market clearing condition is

$$D = W(s) \int_{I_2} n_i(s)\, di - X(D, s) \tag{17}$$

where $X(D, s)$ are monetary transfers by the monetary authority. For any level of nominal deposits D and state s, there is $X(D, s)$ that clears the nominal deposits market. Hence for any D and $\hat{\pi}$, the monetary authority can implement its policy decision in terms of inflation by setting $X(D, s)$ correspondingly.

Finally, the household budget constraint (2) gives a law of motion for nominal deposits, $D' = R(s) D$. Since $R(s) \geq 1$, the path for nominal deposits is strictly positive given $D_0 > 0$.

2.1.4 Solving for the PSE

In our model, the PSE can be solved for analytically. We start by taking P_1^y, a number, as given. Then we solve for the PSE functions that map the actual inflation rate π into allocations and prices. Using these PSE functions, we can characterize the sticky price firms decision as function of the expected inflation rate, $P_1^y(\hat{\pi})$.

From the Fischer equation (4), the nominal interest rate and inflation are simply linked by

$$R(s) = \frac{\pi}{\beta}.$$

The relative price of sticky price firm's goods is given by

$$p_1^y(s) = \frac{P_1^y(\hat{\pi})}{\pi}.$$

Next we solve for relative quantities,

$$\frac{y_i(s)}{y_j(s)} = \left[\frac{p_j^y(s)}{p_i^y(s)}\right]^{\frac{1}{1-\eta}}$$

combining the demand function (6) for two given goods of type i and j.

Using the pricing formulas (7)-(13),

$$
\begin{aligned}
\frac{y_1(s)}{y_3(s)} &= \left[\frac{1}{\eta\theta_3}\frac{w(s)}{p_1^y(s)}\right]^{\frac{1}{1-\eta}} \\
\frac{y_2(s)}{y_3(s)} &= \left[\frac{\theta_2}{\theta_3}R(s)^{-1}\right]^{\frac{1}{1-\eta}} \\
\frac{y_x(s)}{y_3(s)} &= \left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{1}{1-\eta}} \\
\frac{y_m(s)}{y_3(s)} &= \frac{\mu_x}{\mu_m}tt\frac{y_x(s)}{y_3(s)}
\end{aligned}
$$

where the latest equality is derived using (14). We combine these expressions with (5) to obtain

$$
\frac{y(s)}{y_3(s)} = \left[\mu_3 + \mu_2\left[\frac{\theta_2}{\theta_3 R(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_1\left[\frac{w(s)}{\eta\theta_3 p_1^y(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_m\left(\frac{\mu_x}{\mu_m}tt\right)^{\eta}\left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{\eta}{1-\eta}}\right]^{\frac{1}{\eta}}. \tag{18}
$$

Next, we use the pricing formula and demand for the intermediate good $i = 3$,

$$
\left[\frac{w(s)}{\eta\theta_3}\right]^{\frac{\eta}{1-\eta}} = \mu_3 + \mu_2\left[\frac{\theta_2}{\theta_3 R(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_1\left[\frac{w(s)}{\eta\theta_3 p_1^y(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_m\left(\frac{\mu_x}{\mu_m}tt\right)^{\eta}\left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{\eta}{1-\eta}}
$$

and the real wage rate can be explicitly solved for

$$
w(s) = \eta\left[\frac{\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{\eta}{\eta-1}} + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}}{1 - \mu_1 p_1^y(s)^{\frac{\eta}{\eta-1}}}\right]^{\frac{1-\eta}{\eta}} \tag{19}
$$

where

$$
\tilde{\mu}_i = \mu_i \theta_i^{\frac{\eta}{1-\eta}}
$$

for $i = 1, 2, 3$ and

$$
\begin{aligned}
\tilde{\mu}_m &= \mu_m\left(\frac{\mu_x}{\mu_m}tt\right)^{\eta}\theta_x^{\frac{\eta}{1-\eta}}, \\
\tilde{\mu}_x &= \mu_x\theta_x^{\frac{\eta}{1-\eta}}.
\end{aligned}
$$

This expression is the key to solve for the PSE. With knowledge of the real wage rate $w(s)$, the rest of equilibrium allocations and prices follow. Labor $n(s)$ is given by (3). To pin down output, use (16) to derive

$$\frac{n(s)}{y_3(s)} = \frac{\mu_3}{\theta_3} + \frac{\mu_2}{\theta_2}\left[\frac{\theta_2}{\theta_3 R(s)}\right]^{\frac{1}{1-\eta}} + \frac{\mu_1}{\theta_1}\left[\frac{w(s)}{\eta\theta_3 p_1^y(s)}\right]^{\frac{1}{1-\eta}} + \frac{\mu_x}{\theta_x}\left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{1}{1-\eta}}$$

and combining the last expression with (18)

$$y(s) = \varphi(s)\, n(s)$$

where

$$\varphi(s) = \frac{\left[\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{\eta}{\eta-1}} + \mu_1\left[\frac{w(s)}{\eta p_1^y(s)}\right]^{\frac{\eta}{1-\eta}} + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}\right]^{\frac{1}{\eta}}}{\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{1}{\eta-1}} + \frac{\mu_1}{\theta_1}\left[\frac{w(s)}{\eta p_1^y(s)}\right]^{\frac{1}{1-\eta}} + \tilde{\mu}_x \eta^{\frac{1}{\eta-1}}}.$$

The numerator is also equal to $\left[\frac{w(s)}{\eta}\right]^{\frac{1}{1-\eta}}$.

To close the PSE, it is still needed to solve for $P_1^y(\hat{\pi})$. Given expectations $\hat{\pi}$, (7) implies that $P_1^y(\hat{\pi})$ will satisfy $p_1^y(s) = \frac{\theta_3}{\theta 1} p_3^y(s)$. This allows to write the real wage rate when $\pi = \hat{\pi}$ as

$$w(\hat{\pi}, \hat{\pi}) = \eta\left[\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{\eta}{\eta-1}} + \tilde{\mu}_1 + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}\right]^{\frac{1-\eta}{\eta}} \tag{20}$$

and hence, using (7) again,

$$P_1^y(\hat{\pi}) = \hat{\pi}\frac{w(\hat{\pi}, \hat{\pi})}{\eta\theta_1}.$$

It can be easily shown that $P_1^y(\hat{\pi})$ is increasing in the expected inflation.

2.2 Policy Equilibria

We view the policy decision as an equilibrium object. We consider three different policy equilibrium concepts: the Markov equilibrium, the Ramsey equilibrium and the Exchange Rate Policy equilibrium.

In the Markov equilibrium, the monetary authority's problem is to choose the inflation rate which maximizes household welfare taking nominal prices P_1^y as given. Hence the monetary authority has no ability to manipulate the private sector inflation expectations.

The Ramsey equilibrium characterizes the optimal monetary policy with commitment. A formal definition is given below but the reader can think of the Ramsey equilibrium as the result of an alternative timing where the monetary policy is determined once and for all before sticky price are set.

Finally, the Exchange Rate Policy (ERP) equilibrium captures the possibility that the monetary authority's decision is constrained by an exchange rate policy.

2.2.1 The Markov Equilibrium

The monetary authority's problem is to choose the inflation rate that maximizes household welfare. The monetary authority takes the nominal price $P_1^y(\hat{\pi})$ as given.

The choice of the inflation rate is constrained as follows. First, the nominal interest rate is bounded below by one, i.e.,$R(s) \geq 1$. This bound is implied by the arbitrage condition between nominal bonds and cash balances. The latter are not explicitly modelled here, yet we can use (4) to establish that the lower bound for inflation equals the intertemporal discount rate, $\pi \geq \beta$.

Second, the existence of a PSE outcome also imposes an upper bound, $\bar{\pi}(\hat{\pi})$, on the inflation rate. This upper bound is an increasing function of the private sector inflation expectations. As π approaches the upper bound $\bar{\pi}$, the sticky price firms have unbounded losses.[10]

Proposition 2 *For any $\hat{\pi} \geq \beta$, a PSE outcome exists for all π such that*

$$\pi < \bar{\pi}(\hat{\pi}) = \hat{\pi} P_1^y(\hat{\pi}) \mu_1^{\frac{\eta-1}{\eta}}.$$

Proof. As long as we have a finite, strictly positive real wage rate, a PSE outcome exists. From (19), $B \geq w(s) > 0$ implies that

$$\left(1 - \mu_1 p_1^y(s)^{\frac{\eta}{\eta-1}}\right)^{\frac{\eta-1}{\eta}} > 0.$$

The above restriction can be rewritten

$$p_t^y(s) > \mu_1^{\frac{1-\eta}{\eta}},$$

or in terms of π and $\hat{\pi}$,

$$\pi < \bar{\pi}(\hat{\pi}) = \frac{P_1^y(\hat{\pi})}{\mu_1^{\frac{1-\eta}{\eta}}}$$

[10] It is possible to allow firms to shut down or re-set nominal prices if profits fall below some arbitrary level. A PSE would then exist for all $\pi \geq \beta$. Whether we allow for negative profits or not does not affect our results.

■

In Armenter and Bodenstein (2004), we show that the policy choice set can be defined without any loss of generality as

$$\beta \leq \pi \leq \bar{\pi}\left(\hat{\pi}\right) - \varepsilon$$

for an arbitrarily small $\varepsilon > 0$. First, the upper bound will never be binding. Second, we prove that the policy choice set is never empty as $\bar{\pi}\left(\hat{\pi}\right) > \beta$ for all $\hat{\pi} \geq \beta$.

Because a PSE outcome fully determines the household period welfare, we can state the monetary authority's problem as an intratemporal optimization problem

$$\max_{\beta \leq \pi < \bar{\pi}(\hat{\pi})} u\left(c\left(s\right), n\left(s\right)\right) \tag{21}$$

where $c\left(s\right)$ and $n\left(s\right)$ belong to a PSE given $\hat{\pi}$. Let $\pi^*\left(\hat{\pi}\right)$ be the best policy response function which solves (21) given any $\hat{\pi} \geq \beta$.[11]

All is set for the definition of a Markov equilibrium. The nomenclature emphasizes that equilibria based on trigger strategies are ruled out.

Definition 3 *A **Markov equilibrium** is a PSE given private sector expectations $\hat{\pi}$ and an inflation rate π such that the solution to (21) is*

$$\pi^*\left(\hat{\pi}\right) = \pi$$

and private sector expectations are rational

$$\hat{\pi} = \pi.$$

We will say that a policy π is time consistent if there exists a Markov equilibrium with $\hat{\pi} = \pi$. The definition is for an one-period economy. We will spare the reader from the corresponding definition for the infinite horizon economy.

2.2.2 The Ramsey Equilibrium

In the Ramsey equilibrium, the monetary authority pins down private sector expectations with its policy decision. The Ramsey equilibrium policy is also characterizes the optimal monetary policy with commitment.

[11] Existence of $\pi^*\left(\hat{\pi}\right)$ follows from $u\left(c, n\right)$ being bounded above and the closure of the policy choice set previously discussed. However, the solution of (21) can be a correspondence. Armenter and Bodenstein(2005) carefully explores this rare possibility.

Definition 4 *A Ramsey Equilibrium is an inflation rate π^r and a PSE given π^r such that for all π,*

$$u\left(c\left(\pi^r,\pi^r\right),n\left(\pi^r,\pi^r\right)\right)\geq u\left(c\left(\pi,\pi\right),n\left(\pi,\pi\right)\right)$$

where $c(s)$ and $n(s)$ are respective PSE functions.

Not surprisingly, the optimal monetary policy with commitment turns out to be the Friedman rule. All distortions associated with price dispersion are zeroed by setting the nominal interest rate to zero, $R(s)=1$. The distortion that arises from monopoly pricing remains. However, there is nothing monetary policy can do to curtail the market power of the intermediate good firms.[12] Hence, labor remains undersupplied.

Proposition 5 *The Ramsey equilibrium features $R(s)=1$.*

Proof. Consider functions $\tilde{\varphi}(\pi)=\varphi(\pi,\pi)$ and $\tilde{w}(\pi)=w(\pi,\pi)$. Simple algebra shows that $\tilde{\varphi}$ and $\tilde{w}$ are decreasing in π, and $\tilde{\varphi}(\pi)\geq\tilde{w}(\pi)$ for all $\pi\geq\beta$. Next we show that the household welfare is increasing in φ and w. Let

$$\tilde{u}\left(\varphi,w\right)=\varphi\tilde{n}\left(w\right)+h\left(1-\tilde{n}\left(w\right)\right)$$

where $\tilde{n}(w)$ is given by (3). It is clear that $\tilde{u}$ is increasing in $\tilde{\varphi}$. Moreover,

$$\frac{d\tilde{u}}{dw}=\left(\varphi-w\right)\frac{d\tilde{n}}{dw}$$

so given that $\varphi>w$ and the labor supply has an upward slope, household welfare is also increasing in the wage. Hence any policy choice $\pi>\beta$ is welfare dominated by $\pi=\beta$ ■

Does the Friedman Rule constitute a Markov Equilibrium? Assume private sector expectations are such that $R(\hat{\pi},\hat{\pi})=1$, i.e., sticky nominal prices are set under the belief that the Friedman Rule will be chosen by the monetary authority. Ex-post, the monetary authority can choose to inflate $\pi>\beta$ and cut the markup of the sticky price firms. However, such a move creates price distortions. The price difference between the sticky and flexible price firm goods is welcome as it reflects the improved efficiency in the sticky price firms' good production. However, there is an additional price distortion. The marginal cost of financially constrained firms are augmented by $R(s)$. This implies lower efficiency in the production of financially constrained firms. Hence, at least on the margin, whether the Friedman Rule is time consistent depends on the relative weight of each distortion which, in our model, are closely linked to each firm type.

[12] Dupor (2003) shows that optimal monetary policy with commitment may have a random component which can alleviate the monopoly distortion. This is not the case in this economy.

2.2.3 The Exchange Rate Policy Equilibrium

In the exchange rate policy equilibrium (ERP), the monetary authority takes the private sector expectations as given and maximizes household welfare—as in the Markov equilibrium. However, its policy choice is exogenously constrained by the exchange rate policy.

We formalize the exchange rate policy as a set of acceptable nominal exchange rates, Σ. A fixed exchange rate regime reduces Σ to a singleton $\bar{\varepsilon}$, $\Sigma = \{\bar{\varepsilon}\}$. A soft exchange rate peg, which specifies some bands $-\delta_0/+\delta_1$, would be formalized as $\Sigma = [\bar{\varepsilon} - \delta_0, \bar{\varepsilon} + \delta_1]$. As we focus our analysis in commonly observed exchange rate regimes, we do not consider the possibility that the set Σ is history dependent.

The monetary authority's problem in a ERP is

$$\max_{\beta \leq \pi < \bar{\pi}(\hat{\pi})} u\left(c\left(s\right), n\left(s\right)\right) \tag{22}$$

subject to

$$\varepsilon\left(s\right) \in \Sigma$$

where $c\left(s\right)$, $n\left(s\right)$ and $\varepsilon\left(s\right)$ belong to a PSE given $\hat{\pi}$.

Definition 6 *An* ***Exchange Rate Policy*** Σ ***equilibrium*** *is an inflation rate* π *and a PSE given private sector expectations* $\hat{\pi}$ *such that* π *solves (22) given* $\hat{\pi}$ *and private sector expectations are rational*

$$\hat{\pi} = \pi.$$

We have assumed implicitly that it is not possible to review the exchange rate policy. In other words, the policymaker is able to commit to an exchange rate regime but not to a given monetary policy.

3 Expectation Traps

In this section we show that the absence of commitment can lead to costly volatility due to self-fulfilling private sector expectations. The multiplicity of Markov equilibria is a robust property of this economy.[13]

[13] See Armenter and Bodenstein (2004) for an exhaustive exploration of expectation traps in a closed economy version of the model.

3.1 Understanding Expectation Traps

To understand expectation traps, we first discuss the monetary authority's decision for given private sector expectations. In this economy, the costs and benefits of inflation are driven by the heterogeneous impact of inflation across firm types. Inflation reduces overall efficiency by distorting relative prices and by increasing the cost of working capital for financially constrained firms. On the other hand, unexpected inflation erodes the markup of sticky price firms thereby improving efficiency. In a Markov equilibrium, the costs and benefits from unexpected inflation are balanced.

Expectations traps arise because expected inflation changes the composition of the intermediate good sector. While the measure of each firm type is constant, inflation alters the relative output of sticky price firms and financially constrained firms.

Sector composition is the key determinant of the monetary authority's decision. When expected inflation is low, each type of firm operates at similar scale. Efficiency gains from unexpected inflation are almost zero: any cut in the markup of the sticky price firms is roughly offset by an increase in the marginal cost of the financially constrained firms. As a result, there are little net efficiency gains to outweigh the costs of price distortion. A low inflation equilibrium exists where the marginal costs of price distortion are low.

When the private sector expects high inflation, financially constrained firms operate at reduced scale because of the large costs of nominal working capital. There are considerable net efficiency gains from unexpected inflation because the sticky price sector is relatively large compared to the financially constrained sector. These large efficiency gains can exceed the higher marginal cost of price distortion. Hence, the monetary authority will find it optimal to validate the high inflation expectations.

Figure 2 displays the policy best response π^* as a function of private sector expectations $\hat{\pi}$. The 45-degree line (dashed) is the set of points where actual inflation equals expected inflation, $\pi = \hat{\pi}$. This is the rational expectation locus. Crossings of the policy best response function with the 45-degree line indicate Markov equilibria. We calibrated the economy displayed in Figure 2 to match some stylized facts of the U.S. economy.[14] There are two Markov equilibria with inflation rates of 2% and 13.2%. As argued in Armenter and Bodenstein (2004), this matches the inflation experience in the U.S. before and after the 80s. An additional feature to note is that each Markov equilibrium is locally unique.

We have argued that the changes in the composition of the intermediate good sector are behind the expectation traps. This is illustrated in Figure 3. Output for sticky price firms and financially constrained firms is plotted along the rational expectations locus, i.e., $\hat{\pi} = \pi$, for different values of inflation π. Firms' output is similar across firm types when inflation

[14] In the Appendix we provide the details of our calibration.

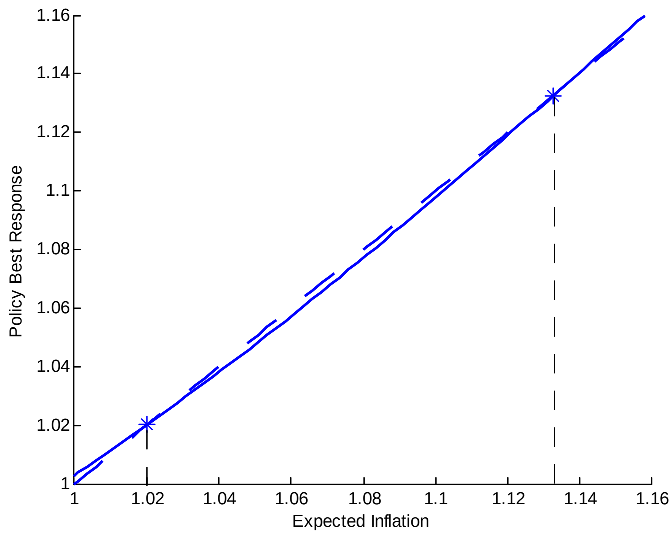

Figure 2: Expectation Traps in a Calibrated Economy

is low.[15] High inflation disproportionately reduces the production of financially constrained firms. Sticky price firm production also falls because aggregate demand is reduced by the price distortion.

The welfare implications of expectation traps dwarf the classic inflation bias analyzed by Barro and Gordon (1983). Table 1 documents this claim for several economies calibrated to match different equilibrium inflation rates. This is achieved by varying the measure of sticky price firms and financially constrained firms in the economy.

For each economy we compute the welfare implications of several experiments. First, we reduce inflation from the low inflation, π_1, to the Ramsey equilibrium, π^r. This is equivalent to correcting the classic inflation bias in an economy with a single equilibrium. Second, we evaluate the shift from the high inflation, π_2, to the low inflation equilibrium, π_1. The last two columns report the welfare change per period as given by the equivalent consumption change in percentage points evaluated at the low inflation equilibrium π_1.

In our baseline calibration, with a low inflation of 2%, the welfare impact of an equilibrium shift is about three times the welfare gains of removing the classic inflation bias. The overall magnitude of welfare losses is significant but not large. The situation is similar for alternative

[15] Indeed, firm production is identical when inflation is equal to β, the optimal monetary policy, as firms do not differ in productivity in this numerical illustration.

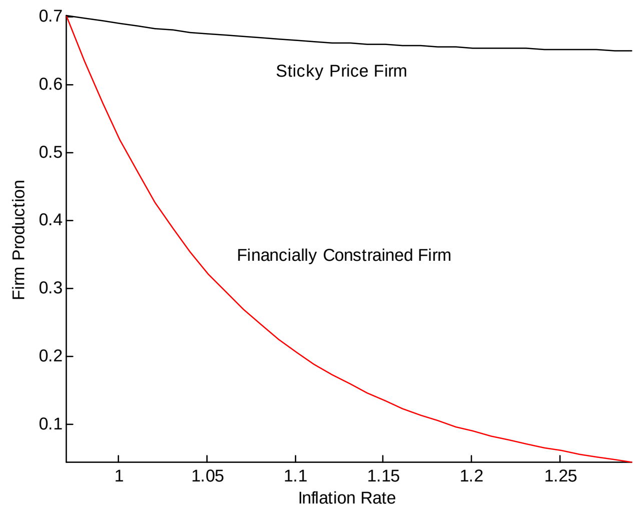

Figure 3: Intermediate Good Output for Firms $i = 1, 2$.

Low Inflation	High Inflation	Welfare Change per period	
π_1	π_2	*From* π_1 *to* π^r	*From* π_2 *to* π_1
1.5 %	14.4 %	.11 %	.43 %
2 %	13.2 %	.12 %	.36 %
2.5 %	12.2 %	.13 %	.31 %
3 %	11.5 %	.14 %	.27 %

Welfare changes computed as percentage points of consumption at equilibrium inflation π_1. See the Appendix for calibration details.

Table 1: **Welfare Implications: Several Calibrations.**

calibrations.

Armenter and Bodenstein (2004) perform an exhaustive characterization of expectation traps in a closed economy version. The main finding is that for all parametrizations with an equilibrium inflation rate between 2% and 2.5%, there is an additional Markov equilibrium with higher inflation. This property of the model is robust and it does not rely upon large nominal frictions.

3.2 The Case for Soft Exchange Rate Pegs

Expectation traps do not conflict with monetary policy flexibility. With respect to exchange rate policy, a soft peg with appropriately chosen bands is sufficient to rule out expectation traps, yet it allows to the monetary authority to react to real shocks.

To see this, consider an inflation cap $\bar{\pi}$ strictly below the high inflation equilibrium, $\bar{\pi} < \pi_2$, but strictly above the low inflation equilibrium rate, $\pi_1 < \bar{\pi}$. Such a cap exists because the Markov equilibria are locally unique. Assume the cap is an exogenous constraint: the monetary authority cannot validate high inflation expectations even if it would like to. Therefore the low inflation equilibrium π_1 becomes the unique Markov equilibrium of the economy.[16]

Next we show how to implement a given inflation cap $\bar{\pi}$ with an exchange rate policy Σ. Combining (10) with (12), we obtain

$$\frac{w(s)\pi}{\theta_x} = \varepsilon(s) P^*.$$

In any Markov equilibrium, $\hat{\pi} = \pi$. Using (20) and some algebra,

$$\frac{\eta}{\theta_x}\left[\left(\tilde{\mu}_1 + \tilde{\mu}_3 + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}\right)\pi^{\frac{\eta}{1-\eta}} + \tilde{\mu}_2 \beta^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}} = \varepsilon(s)\pi^* \tag{23}$$

where we use the normalization that $P^* = \pi^*$. The left hand side is an increasing function in inflation. Hence, there is a one-to-one relationship between inflation and the nominal exchange rate for given π^*. Thus, it is possible to implement any inflation cap $\bar{\pi}$ with the proper choice of the exchange rate policy $\Sigma = \{\varepsilon : \varepsilon \leq \bar{\varepsilon}\}$, where

$$\bar{\varepsilon} = \frac{\eta}{\pi^* \theta_x}\left[\left(\tilde{\mu}_1 + \tilde{\mu}_3 + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}\right)\bar{\pi}^{\frac{\eta}{1-\eta}} + \tilde{\mu}_2 \beta^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}}$$

[16] The inflation cap does not constitute a Markov equilibrium by itself because, for all $\hat{\pi} \in (\pi_1, \pi_2)$, $\pi^*(\hat{\pi}) < \hat{\pi}$, i.e., the policy best response is always below inflation expectations. This property is specific of a two Markov equilibria economy.

and $\pi_1 < \bar{\pi} < \pi_2$. Note there is a continuum of inflation caps that effectively rule out the high inflation equilibrium, so the soft exchange rate policy is not uniquely determined.

A soft exchange rate regime improves welfare even if it does not correct the classic inflation bias, i.e.,it does not implement the optimal monetary policy. First, the monetary authority cannot be caught in the high inflation equilibrium. Second, there will be no volatility arising from expectation shifts.

Moreover, the exchange rate bands can be wide enough so they allow considerable monetary policy flexibility. In our calibration, the difference between the low and high equilibrium inflation rates is about ten percentage points. This leaves plenty of room for policy responses to plausible real shocks. Hence, absent any other considerations and leaving the inflation bias unchanged, the classic textbook argument a la Mundell-Flemming favours broad bands to a hard exchange rate pegs. We challenge this view in the next section.

4 Perverse Policy Responses

The textbook argument against fixed exchange rates builds on the classic Mundell-Flemming analysis. A fixed exchange rate regime means no independent monetary policy. The monetary authority loses its ability to react to real shocks and ends up "importing" the foreign monetary policy. The loss of flexibility is often seen as the downside of the gains that the commitment to a fixed exchange rate can provide.

We argue that the Mundell-Flemming argument does not hold for the case of monetary policy without commitment. We show that the policy response to certain real shocks can be perverse, i.e.,worse than inaction, as shocks exacerbate the time inconsistency problem. Independent monetary policy is no guarantee for lower macroeconomic volatility.

The intuition behind a perverse policy response is quite general. A real shock can increase the welfare gains from unexpected inflation. Consequently, firms anticipate higher inflation. The monetary authority reacts, rightfully, to the real shock but also reacts, unnecessarily, to the induced change in private sector expectations. If the latter dominates, the equilibrium policy response leads to a worsening of the inflation bias and to welfare inferior allocations.

We focus on a negative terms of trade shock because of its appeal for developing economies, where the case for fixed exchange rates is often built upon time inconsistency issues.[17] A negative terms of trade shock contracts the open intermediate sector, which is characterized by perfect competition. As a result, the economy is less competitive, the distortion from monopolistic competition is larger and so is the temptation to cut markups with unexpected inflation.

[17] In our economy, a terms of trade shock is akin to a productivity shock in the tradeables sector.

To see this, we compute an "aggregate" markup κ by dividing the final good price by the aggregate marginal cost of production. In the Appendix, we detail the construction of the aggregate markup and show that

$$\kappa = \frac{\left[\left(\frac{\mu_1 y_1}{y} + \frac{\mu_2 y_2}{y} R^{\frac{1}{1-\eta}} + \frac{\mu_3 y_3}{y}\right)\left(\frac{1}{\eta}\right)^{\frac{1}{1-\eta}} + \frac{\mu_m y_m}{y}\right]^{1-\eta}}{\left[\left(\frac{\mu_1 y_1}{y} + \frac{\mu_2 y_2}{y} R^{\frac{1}{1-\eta}} + \frac{\mu_3 y_3}{y}\right) + \frac{\mu_m y_m}{y}\right]^{1-\eta}}.$$

For simplicity we assume that all firms have identical productivity. The aggregate markup is a geometric average of the monopolistic sectors, with markup $\frac{1}{\eta} > 1$, and the perfect competitive sectors, with no markup.

In response to a negative terms of trade shock, imports contract in relative terms, i.e., $\frac{y_m}{y}$ falls as the relative price of imports goes up.[18] The aggregate markup increases as the competitive sector is weighted less. In the Appendix we show that the markup is decreasing in $\frac{y_m}{y}$.

The assumption that the tradeable sector is competitive is important. One possible motivation is that the country's exports are not differentiated and hence export prices p_x are set in the world markets. This particularly suits a developing economy framework.

We illustrate the perverse policy phenomenon in the calibrated version of our model. We compare a fixed and flexible exchange rate regime in the event of a unanticipated and permanent negative terms of trade shock. The fixed exchange rate regime is modelled as an Exchange Rate Policy equilibrium with $\Sigma = \{\bar{\varepsilon}\}$. For the flexible exchange rate regime, we use our concept of Markov equilibrium. Since there are usually multiple Markov equilibria, we pick the one with lowest inflation.[19]

In order to abstract from the classic inflation bias, we set the world inflation rate π^* such that the flexible and the fixed exchange rate regime deliver the same allocations in the pre-shock economy. In other words, there are no "level" gains in terms of inflation under a fixed exchange rate regime as the world inflation rate is set equal to the inflation rate π_1 in the low inflation Markov equilibrium.

We model the terms of trade shock as unforeseen. This is the best scenario for active monetary policy. By adjusting inflation, the monetary authority can ease the impact of a real shock. The stabilization role ends after one period once all firms have had a chance to re-set their prices.

[18] The measure of firms μ_m is an exogenous parameter and stays constant. However, production y_m is endogenous and it adjusts to the shock.

[19] Alternatively, the reader can think of a comparison between a soft and a hard exchange rate regime. The former would be characterized by exchange rate bands chosen to rule out the high inflation equilibrium and to allow enough flexibility, as documented in the previous section.

We also assume that the shock is permanent. We then report all welfare computations per period. Hence the assumption that the shock is permanent has no impact beyond providing us with at least one period where the shock is unanticipated and one period where the shock is anticipated by the sticky price firms.

The timing of the shock is as follows. At date $t = 0$, the economy is in the original steady state. The terms of trade deteriorate by 1% after firms of type 1 have set their sticky price for date $t = 1$ but before the monetary authority policy decision. Hence, there is a stabilization role for monetary policy. At date $t = 2$, sticky price firms are aware that the shock is permanent and they set their prices accordingly. Prices and allocations reach the new steady state at date $t = 2$.[20]

Figure 4 displays the response of selected prices and allocations. The solid line corresponds to the Markov equilibrium and the dashed line to the ERP equilibrium with $\Sigma = \{\bar{\varepsilon}\}$. The most important graph is in the upper left corner and it displays the inflation rate. Under the fixed exchange rate inflation is constant. Under independent monetary policy, inflation increases in two steps. At date $t = 1$, there is a small inflation increase. This is the optimal response induced by the presence of nominal frictions.[21] However, at date $t = 2$ inflation jumps by a large amount in the Markov equilibrium, when there is no longer a role for monetary policy to ease the real shock. From date $t = 2$ onwards, high inflation only reflects higher sticky prices. This response is clearly welfare reducing.

Prices and allocations tell the same story. At date $t = 1$, the policy response in the Markov equilibrium keeps the wage and labor close to their steady state values despite the shock, while under the fixed exchange rate there is no smoothing. However, from date $t = 2$ onwards, the impact is more pronounced under flexible exchange rate regime. Higher expected inflation brings wage, labor and output below their counterparts under the fixed exchange rate regime.

Table 2 compares the welfare properties of both exchange rate regimes. We report the per period consumption compensation, in percentage points, for a shift from the fixed exchange rate regime to the Markov equilibrium. A negative number means that households are willing to pay to keep the fixed exchange rate regime for the given period. We include

[20] We need to be more precise about our terms of trade shock. Given a change in the *relative price* of exports and imports, there are many possible changes in the price levels. We pick the change in the price levels such that, given a constant monetary policy, the ratio of domestic to world inflation remains constant. In other words, we abstract from non-policy induced real exchange rate movements which may occur simultaneously with a terms of trade shock.

[21] This is the *ex-post* optimal response: the monetary authority is a benevolent policymaker. The Ramsey policy in a stochastic economy would not necessarily look alike. First, the response would be evaluated around the Friedman rule, which is the optimal level of inflation. Second, if the terms of trade shock had a positive probability of occurring, the Ramsey policy would have *ex-ante* considerations.

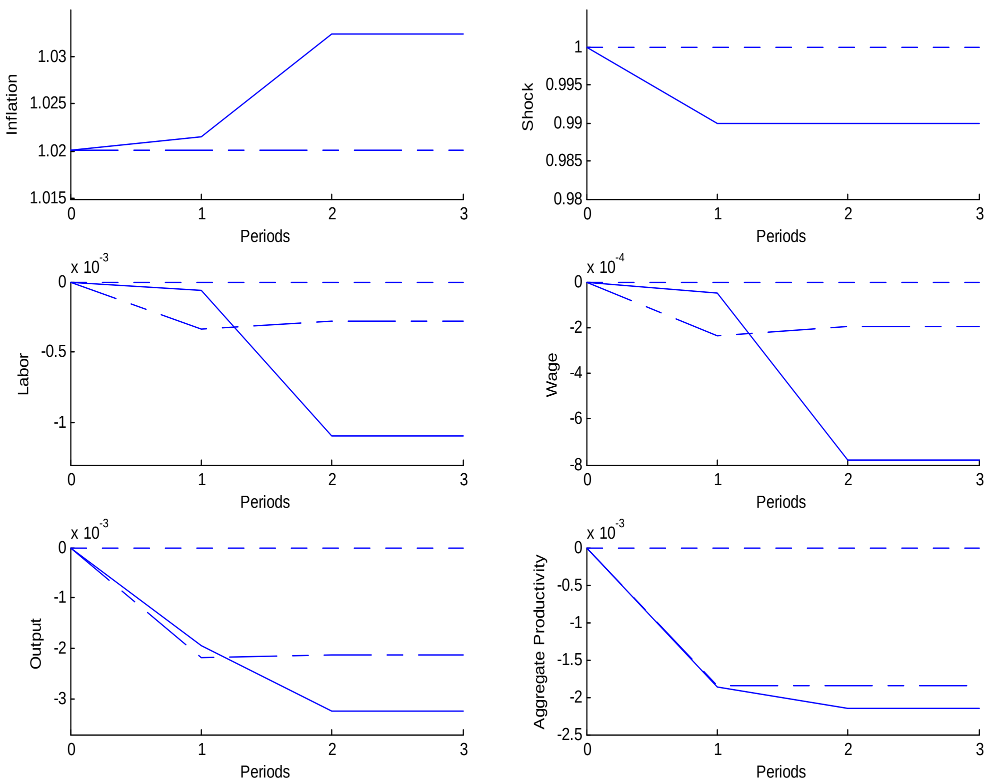

Figure 4: **Equilibrium Response to a Negative Terms of Trade Shock**. Solid line correspond to the low inflation Markov equilibrium. Dashed line corresponds to a fixed exchange rate regime. See text for details.

Period	*Date $t = 0$ Inflation Rate* $\pi = 2.0$	$\pi = 2.5$	$\pi = 3.0$
Date $t = 0$	0	0	0
Negative Shock			
Date $t = 1$	0.00015	0.00016	0.00017
Date $t = 2$	-0.0393	-0.0597	-0.0912
Positive Shock			
Date $t = 1$	0.00014	0.00016	0.00017
Date $t = 2$	0.0241	0.0296	0.0348

Welfare changes reported as percentage points of consumption under non-stochastic economy. Values per period.

Table 2: **Welfare Comparison: Markov equilibrium versus fixed exchange rate in the event of a terms of trade shock.**

several calibrations: we report the corresponding Markov equilibrium inflation rate in the pre-shock economy. In each calibration, the world inflation rate is set such that the flexible and the fixed exchange rate have the same welfare properties in the pre-shock economy.

At date $t = 1$, right after the negative terms of trade shock, the Markov equilibrium dominates the fixed exchange rate. The welfare difference, though, is quite small. From date $t = 2$ onwards, the fixed exchange rate equilibrium is welfare dominant. The welfare gains from a fixed exchange rate at date $t = 2$ are about three times the welfare losses at date $t = 1$. These are per period welfare changes. So even if the shock lasted only two periods, the fixed exchange rate would be preferred. Under the assumption of a permanent shock, we should multiply the welfare change at date $t = 2$ by $\frac{1}{1-\beta} \approx 34$.

The impact of a positive terms of trade shock is not symmetric. Table 2 also reports the welfare ranking in the aftermath of a positive shock to the terms of trade. In this event, the flexible exchange rate is welfare superior both at date $t = 1$ and $t = 2$. However, the welfare implications under positive and negative shocks do not cancel each other. As shown in Table 2, a positive terms of trade shock brings welfare gains which are about half the welfare loss under a negative terms of trade shock. This is a direct consequence of the concavity of the policy problem.

To summarize, Table 2 clearly speaks in favour of fixed exchange rates in the event of a real shock—a scenario usually associated with the costs of losing monetary independence.

Period	*Date* $t = 0$ *Inflation Rate* $\pi = 2.0$
Date $t = 0$	0
Negative Shock	
Date $t = 1$	0.00041
Date $t = 2$	-0.0705
Positive Shock	
Date $t = 1$	0.00047
Date $t = 2$	0.0388

Welfare changes reported as percentage points of consumption under non-stochastic economy. Values per period.

Table 3: **Welfare Comparison: Markov equilibrium versus fixed exchange rate in the event of a shock to financially constrained firms.**

4.1 Other Shocks

A terms of trade shock is not the sole instance of a perverse policy response. Consider a negative shock to the financially constrained firm's sector. We have in mind an exogenous tightening of financial constraints, perhaps due to a banking crisis.

Figure 5 shows the response of selected prices and allocations to a unexpected fall in the productivity of financially constrained firms of 1%. The monetary response in date $t = 1$, right after the shock, eases the impact on the wage rate and output. However, once sticky price firms adjust their prices, the resulting policy response leads to further, unnecessary inflation. Wage rate and output are significantly lower then.

The welfare properties of the two exchange rate regimes—shown in Table 3—are not surprising. In the event of a negative shock, a fixed exchange rate shock is clearly preferred: the welfare loss associated with independent monetary policy from date $t = 2$ onwards is much larger than any welfare gains from stabilization. Due to concavity, the fixed exchange rate retains its welfare dominance even if positive and negative shocks are equiprobable.

We do not claim that a fixed exchange rate is welfare superior in the event of any shock. In some cases, stabilization is very important or the shock leaves the monetary authority's incentives unchanged. Aggregate productivity shocks are an example of the latter.

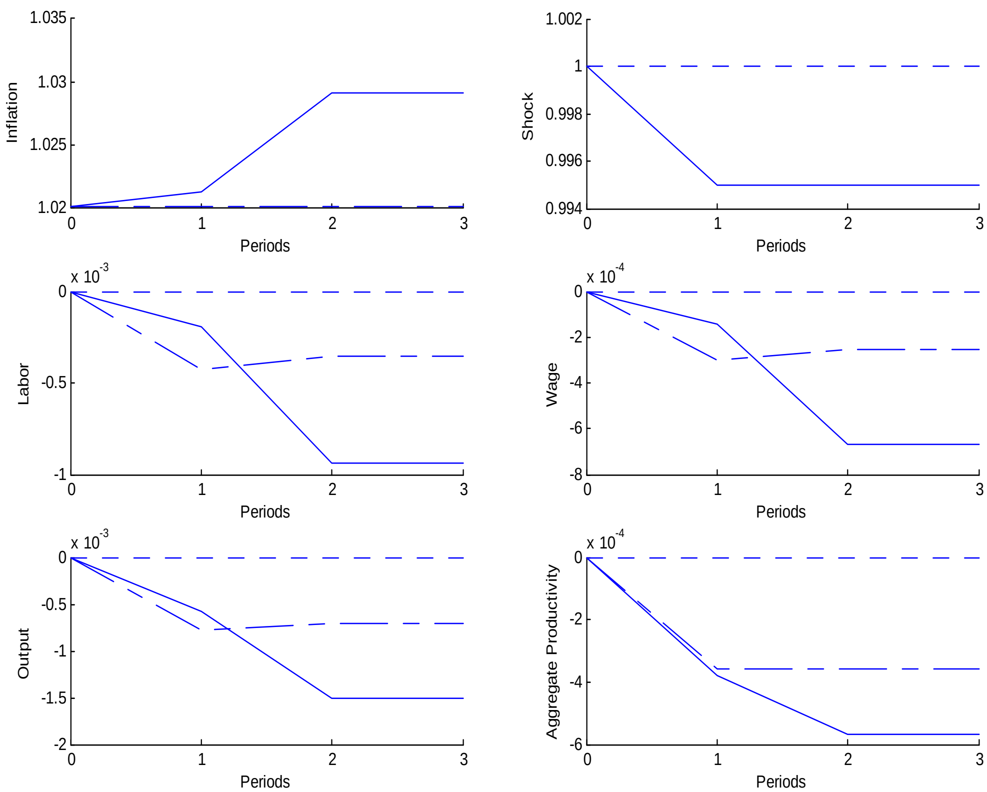

Figure 5: **Equilibrium Response to a Negative Productivity Shock in Financially Constrained Firms**. Solid line correspond to the low inflation Markov equilibrium. Dashed line corresponds to a fixed exchange rate regime. See text for details.

5 Conclusion

This paper contributes to the exchange rate debate but by no means settles it. Indeed, a definitive welfare ranking of exchange rate regimes seems more elusive than ever. Expectation traps and perverse policy responses increase the complexity of any welfare evaluation of exchange rate regimes—yet any such evaluation is incomplete without considering both phenomena.

For example, we should not conclude from the world-wide downward trend in inflation that commitment problems are a matter of the past and consequently that fixed exchange rate regimes are no longer appealing. A low inflation country may be only a shift in expectations away from high inflation. Moreover, large real volatility does not necessarily make a stronger case for a flexible exchange rate. We have to ask first what is the impact of the relevant real shocks on the time inconsistency problem and how likely it is that independent monetary policy reacts perversely.

In our analysis we have abstracted from the time consistency of the exchange rate policy itself. We did so in order to communicate our point clearly. We certainly do not think the exchange rate policy is free of credibility problems. A country may be left only with extreme and costly solutions such as dollarization. Trying to sustain a fixed exchange rate absent commitment can lead to self-fulfilling currency crises, as discussed in Obstfeld (1996)—although, as our paper points out, a flexible exchange rate can lead to self-fulfilling currency crises, too.

Yet, it is often the case that fixed exchange rates are brought down by fiscal rather than monetary crises. We do not view fixed exchange rates as a solution to fiscal problems. The ongoing skepticism about fixed exchange rate credibility arises very much from using the wrong tool for the wrong problem.

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Parameter	*Notation*	*Value*
Intertemporal Discount Rate	β	.9709
Leisure-Consumption	ψ_0	.5
Inverse Frisch Labor Elasticity	ψ	1
Share of Non Tradeables	α	.6
Inverse Markup	η	1.12^{-1}
Measure of Firms $i = 1$	μ_1	.12
Measure of Firms $i = 2$	μ_2	.0552
Measure of Firms $i = x$	μ_x	.12

Table 4: **Baseline Calibration.**

A Appendix

A.1 Calibration

We start by setting the preference parameters to standard values. The inverse of β is the real interest rate in our economy: we set it equal to 3%, $\beta = .9709$, which means evaluating the model at the annual frequency.

Our choice for leisure preferences is

$$h(1-n) = \psi_0 \frac{(1-n)^{1-\psi}}{1-\psi}.$$

Our parameters on the labor supply are set to match a Frisch labor elasticity of 1 and the Aristotelian proportion of leisure and work to $n = \frac{1}{2}$ in the first best.

We set the share of nontradeable goods at 60% and the share of export firms at 12%. The last of the pre-set parameters is η, which is set to replicate a 12% markup.

We calibrate the measure of firms of type 1 and 2 to match a $1.5\% - 3\%$ range in the low inflation equilibrium. In every case we find a second equilibrium with inflation in the range of $11\% - 15\%$. We find that we can match these numbers with reasonable parameter values. The share of firms with sticky prices is about 12%, and the measure of financially constrained firms is just above 5%.

Our parameter choices are summarized in Table 4.

A.2 The Aggregate Markup

In order to provide some insight on the perverse policy response phenomenon, we compute an aggregate markup. From the final good firm's profit maximization problem, we write the corresponding cost minimization problem:

$$\min_{\{y_i\}^{1-\mu_x}} \int_0^{1-\mu_x} p_i^y(s)\, y_i di$$

subject to

$$y \leq \left[\int_0^{1-\mu_x} y_i^\eta di\right]^{\frac{1}{\eta}}.$$

The first order condition is

$$p_i^y(s)^{\frac{1}{1-\eta}} \frac{y_i}{y} = \lambda^{\frac{1}{1-\eta}}$$

where λ is the Lagrangian multiplier associated with the technological constraint, which equals the marginal cost of one unit of final good. The previous condition is necessary for all $y_i > 0$. Hence,

$$\left[\int_0^{1-\mu_x} p_i^y(s)^{\frac{1}{1-\eta}} \frac{y_i}{y} di\right]^{1-\eta} = \lambda$$

gives the marginal cost as a geometric weighted average of each intermediate good price.

We compute the markup in a Markov equilibrium. We substitute the pricing formula for each price to obtain:

$$\lambda = \left[\mu_1 \frac{y_1}{y}\left(\frac{w}{\eta\theta_1}\right)^{\frac{1}{1-\eta}} + \mu_2 \frac{y_2}{y}\left(\frac{wR}{\eta\theta_2}\right)^{\frac{1}{1-\eta}} + \mu_3 \frac{y_3}{y}\left(\frac{w}{\eta\theta_3}\right)^{\frac{1}{1-\eta}} + \mu_m \frac{y_m}{y}\left(q(s)\, p_m^*\right)^{\frac{1}{1-\eta}}\right]^{1-\eta}.$$

Absent differences in productivity, i.e.,$\theta_i = 1$ for all goods, this simplifies to

$$\lambda = w\left[\left(\mu_1 \frac{y_1}{y} + \mu_2 \frac{y_2}{y} R^{\frac{1}{1-\eta}} + \mu_3 \frac{y_3}{y}\right)\left(\frac{1}{\eta}\right)^{\frac{1}{1-\eta}} + \mu_x \frac{y_m}{y}\right]^{1-\eta}.$$

We do not want to mix the price distortions and the markup distortion. The (social) marginal cost of producing one unit of the final good, given the current price distortions, is

$$\tilde{\lambda} = w\left[\left(\mu_1 \frac{y_1}{y} + \mu_2 \frac{y_2}{y} R^{\frac{1}{1-\eta}} + \mu_3 \frac{y_3}{y}\right) + \mu_x \frac{y_m}{y}\right]^{1-\eta}.$$

Since the final good producer is competitive, λ is also the final good price. We set our aggregate markup definition as $\kappa \equiv \lambda/\tilde{\lambda}$.

To see that κ is decreasing with y_m/y, note that $\left(\frac{1}{\eta}\right)^{\frac{1}{1-\eta}} > 1$ and apply simple differential calculus.