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Optimal Monetary and Fiscal Policy under Discretion in the New Keynesian Model: A Technical Appendix to "Great Expectations and the End of the Depression"

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Abstract

This paper details the microfoundations of the model presented in Staff Report no. 234, “Great Expectations and the End of the Depression.” It defines the Markov perfect equilibrium formally in the nonlinear model, discusses in some detail the approximation method used and the order of accuracy of this approximation, and gives proofs of two propositions not proved in Staff Report no. 234. In addition, this paper states a proposition that shows the equivalence between the linear quadratic approximation in Staff Report no. 234 and a first order approximation to the exact nonlinear conditions of the government in the Markov perfect equilibrium defined here.

Key words: deflation, Great Depression, regime change, zero interest rates, Markov perfect equilibrium

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1 A Microfounded model

Here I outline a simple sticky price general equilibrium model in more detail than in the Staff Report nr. 234. I assume that there is a representative household that maximizes expected utility over the infinite horizon:

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} U_T = E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} [u(C_T, \xi_T) + g(G_T, \xi_T) - \int_0^1 v(h_T(i), \xi_T) di] \right\} \tag{1}$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods $C_t \equiv [\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}}$ with elasticity of substituting equal to $\theta > 1$, G_t is is a Dixit-Stiglitz aggregate of government consumption, ξ_t is a vector of exogenous shocks, P_t is the Dixit-Stiglitz price index $P_t \equiv [\int_0^1 p_t(i)^{1-\theta}]^{\frac{1}{1-\theta}}$ and $h_t(i)$ is quantity supplied of labor of type i. $u(.)$ is assumed to be concave and strictly increasing in C_t for any possible value of ξ. $g(.)$ is the utility of government consumption and is assumed to be concave and strictly increasing in G_t for any possible value of ξ. $v(.)$ is the disutility of supplying labor of type i and is assumed to be an increasing and convex in $h_t(i)$ for any possible value of ξ. E_t denotes mathematical expectation conditional on information available in period t. ξ_t is a vector of r exogenous shocks. I assume that ξ_t follows a Markov process. For simplicity I assume complete financial markets and no limit on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form:

$$E_t \sum_{T=t}^{\infty} Q_{t,T} P_T C_T \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T} [\int_0^1 Z_T(i) di + \int_0^1 n_T(j) h_T(j) dj - P_T T_T] \tag{2}$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor that financial markets use to value random nominal income at date T in monetary units at date t; i_t is the riskless nominal interest rate on one-period obligations purchased in period t, W_t is the beginning of period nominal wealth at time t (note that its composition is determined at time $t-1$ so that it is equal to the sum of monetary holdings from period $t-1$ and returns on non-monetary assets), $Z_t(i)$ is the time t nominal profit of firm i, $n_t(i)$ is the nominal wage rate for labor of type i, T_t is net real tax collections by the government.

The first order conditions of the household maximization imply an Euler equation of the form:

$$\frac{1}{1+i_t} = E_t \{ \frac{\beta u_c(C_{t+1}, \xi_{t+1})}{u_c(C_t, \xi_t)} \Pi_{t+1}^{-1} \} \tag{3}$$

This equation is often referred to as the IS equation. There is bound on the short-term nominal interest rate given by:

$$i_t \geq i^m \tag{4}$$

I simply impose this bound here but for a detailed derivation see Eggertsson (2005). The optimal consumption plan of the representative household must also satisfy the transversality condition[1]

$$\lim_{T\to\infty} E_t Q_{t,T} \frac{W_T}{P_t} = 0$$

to ensure that the household exhausts its intertemporal budget constraint. I assume that workers are wage takers so that the households optimal choice of labor supplied of type j satisfies

$$n_t(j) = \frac{P_t v_h(h_t(j), \xi_t)}{u_c(C_t, \xi_t)}$$

The production function of the representative firm that produces good i is:

$$y_t(i) = f(h_t(i), \xi_t) \tag{5}$$

where f is an increasing concave function for any ξ and ξ is again the vector of shocks defined above (that may include productivity shocks). I abstract from capital dynamics. As Rotemberg (1982), I assume that firms face a cost of price changes given by the function $d(\frac{p_t(i)}{p_{t-1}(i)})$[2] but I can derive exactly the same results assuming that firms adjust their prices at stochastic intervals as assumed by Calvo (1983).[3] Price variations have a welfare cost that is different from the cost of expected inflation that can arise due to real money balances in utility. The Dixit-Stiglitz preferences of the household imply a demand function for the product of firm i given by $y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$. The firm maximizes

$$E_t \sum_{T=t}^{\infty} Q_{t,T} Z_T(i)$$

where $Q_{t,T} = \beta^{T-t} \frac{u_c(C_T,\xi_T)}{u_c(C_t,\xi_t)} \frac{P_t}{P_T}$ and $Z_t(i)$ is profits of firm i. I can write firms period profits as:

$$\begin{aligned} Z_t(i) &= y_t(i)p_t(i) - n_t(i)h_t(i) - P_t d(\frac{p_t(i)}{p_{t-1}(i)}) \\ &= (1+s)Y_t P_t^{\theta} p_t(i)^{1-\theta} - n_t(i) f^{-1}(Y_t P_t^{\theta} p_t(i)^{-\theta}) - P_t d(\frac{p_t(i)}{p_{t-1}(i)}) \end{aligned}$$

[1] For a detailed discussion of how this transversality condition is derived see Woodford (2003).

[2] I assume that $d'(\Pi) > 0$ if $\Pi > 1$ and $d'(\Pi) < 0$ if $\Pi < 1$. Thus both inflation and deflation are costly. $d(1) = 0$ so that the optimal inflation rate is zero (consistent with the interepretation that this represent a cost of changing prices). Finally, $d'(1) = 0$ so that in the neighborhood of the zero inflation the cost of price changes is of second order.

[3] The reason I do not assume Calvo prices is that it complicates to solution by introducing an additional state variable, i.e. price dispersion. This state variable, however, has only second order effects local to the steady state I approximate around and the resulting equilibrium is to first order exactly the same as derived here.

where s is an exogenously given production subsidy that I introduce for algebraic convenience (for reasons described below).[4]

I restrict my attention to a symmetric equilibria where all firms charge the same price and produce the same level of output so that

$$p_t(i) = p_t(j) = P_t; \quad y_t(i) = y_t(j) = Y_t; \quad n_t(i) = n_t(j) = n_t; \quad h_t(i) = h_t(j) = h_t \quad \text{for} \quad \forall\, j, i \tag{6}$$

Given the wage demanded by households I can derive the aggregate supply function from the first order conditions of the representative firm, assuming competitive labor market so that each firm takes its wage as given. I obtain the equilibrium condition often referred to as the AS or the "New Keynesian" Phillips curve:

$$\begin{aligned} &\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(C_t,\xi_t) - \tilde{v}_y(Y_t,\xi_t)] + u_c(C_t,\xi_t)\Pi_t d'(\Pi_t) \\ &- E_t \beta u_c(C_{t+1},\xi_{t+1})\Pi_{t+1}d'(\Pi_{t+1}) = 0 \end{aligned} \tag{7}$$

where for notational simplicity I have defined the function:

$$\tilde{v}(y_t(i),\xi_t) \equiv v(f^{-1}(y_t(i)),\xi_t)$$

which implies that $\tilde{v}_y = \frac{v_h}{f'}$.

There is an output cost of taxation (e.g. due to tax collection costs as in Barro (1979)) captured by the function $s(T_t)$.[5] For every real dollar collected in taxes $s(T_t)$ units of output are wasted without contributing anything to utility. Government real spending is then given by:

$$F_t = G_t + s(T_t) \tag{8}$$

I could also define the tax cost that would result from distortionary taxes on income or consumption and obtain similar results.[6]

[4] I introduce it so that I can calibrate an inflationary bias that is independent of the other structural parameters, and this allows me to define a steady state at the fully efficient equilibrium allocation. I abstract from any tax costs that the financing of this subsidy may create.

[5] The function $s(T)$ is assumed to be differentiable with $s'(T) > 0$ and $s''(T) > 0$ for $T > 0$.

[6] The specification used here, however, gives a very clear result that clarifies the main channel of taxation in which I am interested. This is because, for a constant F_t the level of taxes has no effect on the private sector equilibrium conditions (see equations above) but will only affect the equilibrium by reducing the utility of the households because a higher tax cost means lower government consumption G_t. This allows me to isolate the effect current tax cuts will have on expectation about future monetary and fiscal policy, abstracting away from any effect on relative prices that those tax cuts may have. Eggertsson and Woodford (2004) analyze the effect of a different form of taxation but assume that the government can commit to future

When the government only issues one period nominal debt I can write the total nominal claims of the government (which in equilibrium are equal to the total nominal wealth of the representative household) as:

$$W_{t+1} = (1 + i_t)B_t$$

where B_t is the end-of-period nominal value of bonds issued by the government. Defining the variable $w_t \equiv \frac{W_{t+1}}{P_t}$ as in the text I can write the government budget constraint as:

$$w_t = (1 + i_t)(w_{t-1}\Pi_t^{-1} + F_t - T_t) \tag{9}$$

Note that I use the time subscript t on w_t (even if it denotes the real claims on the government at the beginning of time $t + 1$) to emphasize that this variable is determined at time t. The policy instrument of the central bank is the nominal interest rate and the policy instrument of the treasury is real government spending F_t and taxes T_t. For simplicity I assume that the government must satisfy the borrowing constraint

$$w_t \leq \bar{w} \tag{10}$$

that can be rationalized by that the government can never borrow more than the maximum of its expected future taxbase (that is bounded by potential output). In the equilibria I analyze this bound will never be binding so that $\bar{w}$ can be arbitrarily high. This constraint guarantees that the transversality conditions of the household shown above is always satisfied.

Having defined both private and public spending I can verify that market clearing implies that aggregate demand satisfies:

$$Y_t = C_t + d(\Pi_t) + F_t \tag{11}$$

I can now define a private sector equilibrium in the model, that summarizes the list of equations that need to be satisfied for an equilibrium to be consistent with household and firm maximization and the technology constraints of the model.

Definition 1 *Private Sector Equilibrium (PSE) is a collection of stochastic processes* $\{\Pi_t, Y_t, C_t, w_t, i_t, F_t, T_t, G_t, \xi_t\}$ *for* $t \geq t_0$ *that satisfy equations (3)-(11) for each* $t \geq t_0$, *given* w_{t_0-1} *and the exogenous stochastic process* $\{\xi_t\}$.

My definition, and model above, abstracts from monetary fraction. This is only done to simplify the analysis. As shown by Eggertsson and Woodford (2003) adding a money in the utility function does not

policy.

alter the set of feasible equilibrium allocations at zero interest rates. If one adds a money in the utility-function that is additive separable (Eggertsson (2005) discusses the more general case) there is a money demand equation that determines real money balances, given the path for the other endogenous variables:

$$\frac{q_m(\frac{M_t}{P_t}, \xi_t)}{u_c(C_t, \xi_t)} = \frac{i_t - i^m}{1 + i_t} \tag{12}$$

where M_t is the nominal stock of money and $q(.)$ is utility of holding real money balances. This is the first order condition referred to in Staff Report nr. 234. If one incorporates the monetary base there is also an additional term in the budget constraints that captures seigniorage revenues as discussed in Staff Report nr. 234. Taking this term explicitly into account will only strengthen the result: It gives the government even further incentive to create inflation, the higher its outstanding nominal debt. It does, however, complicate the notation considerable (see Eggertsson (2005)).

1.1 Recursive representation

It is useful to rewrite the model in a recursive form so that I can identify the endogenous state variables at each date. The treasury's policy instruments is taxation, T_t, and government spending F_t, that determines the end-of-period government debt. The central banks policy instrument is i_t.

It is useful to note that I can reduce the number of equations that are necessary and sufficient for a private sector equilibrium substantially from those listed in Definition 1. First, note that the equations that determine $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ are redundant, i.e. each of them is only useful to determine one particular variable but has no effect on the any of the other variables. Thus I can define necessary and sufficient condition for a private sector equilibrium without specifying the stochastic process for $\{Q_t,$ $Z_t, G_t, C_t, n_t, h_t\}$ and only need a subset of the equations in the last section to do this. For the remaining conditions I use (11) to substitute out for C_t.

It is useful to define the expectation variable

$$f_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F_{t+1}, \xi_{t+1})\Pi_{t+1}^{-1} \tag{13}$$

as the part of the nominal interest rates that is determined by the expectations of the private sector formed at time t. The IS equation can then be written as

$$1 + i_t = \frac{u_c(Y_t - d(\Pi_t) - F_t, \xi_t)}{\beta f_t^e} \tag{14}$$

Similarly it is useful to define the expectation variable

$$S_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F_t, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1}) \tag{15}$$

The AS equation can be written as

$$\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F_t, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F_t, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e = 0. \tag{16}$$

The next two propositions are useful to characterize equilibrium outcomes. Proposition 1 follows directly from our discussion above:

Proposition 1 *A necessary and sufficient condition for a PSE at each time $t \geq t_0$ is that the variables $(\Pi_t, Y_t, w_t, F_t, i_t, T_t)$ satisfy: (i) conditions (4), (9),(10), (14), (16), given w_{t-1} and the expectations f_t^e and S_t^e. (ii) in each period $t \geq t_0$, expectations are rational so that f_t^e is given by (13) and S_t^e by (15).*

Proposition 2 *The possible PSE equilibrium defined by the necessary and sufficient conditions for any date $t \geq t_0$ onwards depend only on w_{t-1} and ξ_t.*

The second proposition follows from observing that w_{t-1} is the only endogenous variable that enters with a lag in the necessary and sufficient conditions in (i) of Proposition 1 and using the assumption that ξ_t is Markovian so that the conditional probability distribution of ξ_t for $t > t_0$ only depends on ξ_{t_0}. It follows from this proposition that (w_{t-1}, ξ_t) are the only state variables at time t that directly affect the PSE. I may economize on notation by introducing vector notation. I define vectors

$$\Lambda_t \equiv \begin{bmatrix} \Pi_t & Y_t & F_t & i_t & T_t \end{bmatrix}^T, \text{ and } e_t \equiv \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix}.$$

Since Proposition 2 indicates that w_t is the only relevant endogenous state variable, I prefer not to include it in either vector but keep track of it separately. It simplifies notation a bit to write the utility function as a function of Λ_t i.e. I define the function $U : \mathbb{R}^{5+r} \to \mathbb{R}$ (recall that r is the length of the vector ξ).

$$U_t = U(\Lambda_t, \xi_t)$$

using (8) and (11) to solve for G_t and C_t as a function of Λ_t, along with (5) and (6) to solve for $h_t(i)$ as a function of Y_t.

2 The Markov Perfect Equilibrium formally defined

Here I consider an equilibrium that occurs when policy is conducted under discretion so that the government is unable to commit to any future actions. To do this I solve for a Markov equilibrium (it is formally defined by Maskin and Tirole (2001)) that has been extensively applied in the monetary literature. The basic idea behind this equilibrium concept is to define a minimum set of state variables that directly affect market conditions and assume that the strategies of the government and the private sector expectations depend only on this minimum state. Proposition 3 indicates that a Markov equilibrium requires that the variables (Λ_t, w_t) only depend on (w_{t-1}, ξ_t), since this is the minimum set of state variables that affect the PSE.

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector and the government. The monetary and fiscal authorities choose policy for period t given the state and the private sector forms expectations e_t. Note that I assume that the private sector may condition its expectation at time t on w_t, i.e. it observes the policy actions of the government in that period so that Λ_t and e_t are jointly determined. This is important because w_t is the relevant endogenous state variable at date $t+1$. Since the state in this game is captured by (w_{t-1}, ξ_t) a Markov equilibrium requires that there exist policy functions $\bar{\Pi}_t(.), \bar{Y}_t(.), \bar{\imath}_t(.), \bar{T}_t(.), \bar{F}_t(.)$ that I denote by the vector valued function $\bar{\Lambda}_t(.)$ and a function $\bar{w}_t(.)$, such that each period:[7]

$$\begin{bmatrix} \Lambda_t \\ w_t \end{bmatrix} \equiv \begin{bmatrix} \bar{\Lambda}_t(w_{t-1,}\xi_t) \\ \bar{w}_t(w_{t-1}, \xi_t) \end{bmatrix} \tag{17}$$

Note that the functions $\bar{\Lambda}_t(.)$ and $\bar{w}_t(.)$ will also define a set of functions of (w_{t-1}, ξ_t) for $(Q_t, Z_t, G_t, C_t, n_t, h_t)$ by the redundant equations from Definition 1. Using $\bar{\Lambda}_t(.)$ I may also use (13) and (15) to define a function $\bar{e}_t(.)$ so so that

$$e_t = \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix} = \begin{bmatrix} \bar{f}_t^e(w_t, \xi_t) \\ \bar{S}_t^e(w_t, \xi_t) \end{bmatrix} = \bar{e}_t(w_{t,}\xi_t) \tag{18}$$

Rational expectations imply that the function $\bar{e}_t$ satisfies

$$\bar{e}_t(w_{t,}\xi_t) = \begin{bmatrix} E_t u_c(\bar{C}_t(w_t, \xi_{t+1}); \xi_{t+1}) \bar{\Pi}_t(w_t, \xi_{t+1})^{-1} \\ E_t u_c(\bar{C}_t(w_t, \xi_{t+1}), \bar{m}_t(w_t, \xi_{t+1}); \xi_{t+1}) \bar{\Pi}_t(w_t, \xi_{t+1}) d'(\bar{\Pi}_t(w_t, \xi_{t+1})) \end{bmatrix} \tag{19}$$

I define a value function $J_t(w_{t-1}, \xi_t)$ as the expected discounted value of the utility of the representative household, looking forward from period t, given the evolution of the endogenous variable from period t

[7] Note that if the conditional expectation of ξ_{t+1} at time t does not depend on calender time, these functions will be time invariant and one may drop the subscript t.

onwards that is determined by $\bar{\Lambda}_t(.)$, $\bar{w}_t(.)$ and $\{\xi_t\}$. Thus I define:

$$J_t(w_{t-1}, \xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} [U(\bar{\Lambda}_T(.), \xi_T] \right\} \tag{20}$$

The optimizing problem of the government is as follows. Given w_{t-1} and ξ_t, the government chooses the values for (Λ_t, w_t) (by its choice of the policy instruments i_t, F_t and T_t) to maximize the utility of the representative household subject to the conditions in Proposition 1 and (18). Thus its problem can be written as:

$$\max_{i_t, F_t, T_t} [U(\Lambda_t, \xi_t) + \beta E_t J_{t+1}(w_t, \xi_{t+1})] \tag{21}$$

s.t. (4), (9),(10), (14), (16), and (18)

I can now define a Markov equilibrium.

Definition 3 *A Markov equilibrium is a collection of functions* $\bar{\Lambda}_t(.)$,$\bar{w}_t()$,$J_t(.)$,$\bar{e}_t(.)$, *such that (i) given the function* $J_t(w_{t-1}, \xi_t)$ *and the vector function* $\bar{e}_t(w_t, \xi_t)$ *the solution to the policy maker's optimization problem (21) is given by* $\Lambda_t = \bar{\Lambda}_t(w_{t-1}, \xi_t)$ *and* $w_t = \bar{w}_t(w_{t-1}, \xi_t)$ *for each possible state* $(w_{t-1}$,$\xi_t)$ *(ii) given the vector function* $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ and $\bar{w}_t(w_{t-1}, \xi_t)$ *then* $e_t = \bar{e}_t(w_t, \xi_t)$ *is formed under rational expectations (see equation (19)). (iii) given the vector function* $\bar{\Lambda}_t(w_{t-1}, \xi_t)$ *and* $\bar{w}_t(w_{t-1}, \xi_t)$ *the function* $J_t(w_{t-1}, \xi_t)$ *satisfies (20).*

I will only look for a Markov equilibrium in which the functions $\bar{\Lambda}_t(.)$, $J_t(.)$, $\bar{e}_t(.)$ are continuous and have well defined derivatives. Then the value function satisfies the Bellman equation:

$$J_t(w_{t-1}, \xi_t) = \max_{i_t, F_t, T_t} [U(\Lambda_t, \xi_t) + E_t \beta J_{t+1}(w_t, \xi_{t+1})] \tag{22}$$

s.t. (4), (9),(10), (14), (16) and (18).

Necessary conditions for the Markov Equilibrium can now be characterized by using a Lagrangian method for the maximization problem on the right hand side of (22). In the next section I show these conditions explicitly. Below I show them using vector notation. The reason I write the first order conditions here in vector notation is that I use this notation for some of the Matlab codes used for the numerical solution. The explicit solution shown in the next section, however, is required for some of the analytical proofs. I obtain the necessary conditions for a Markov equilibrium by differentiating the Lagrangian

$$\mathcal{L}_t = U(\Lambda_t, \xi_t) + E_t \beta J_t(w_t, \xi_{t+1}) + \phi_t' \Gamma(e_t, \Lambda_t, w_{t,} w_{t-1}, \xi_t) + \psi_t'(e_t - \bar{e}(w_{t,} \xi_t)) + \gamma_t' \Upsilon(\Lambda_t, w_t, \xi_t) \tag{23}$$

where ϕ_t is a (3×1) vector, ψ_t is (2×1) and γ_t is (2×1). Here the vector function $\Gamma(e_t, \Lambda_t, w_{t,} w_{t-1}, \xi_t)$

summarizes (9), (14), (16). $\Upsilon(\Lambda_t, w_t, \xi_t)$ summarizes (4) and (10). I have already defined the function $\bar{e}(.)$ in (18). The first order conditions for $t \geq 0$ are (where each derivatives of $\mathcal{L}$ are equated to zero):

$$\frac{d\mathcal{L}}{d\Lambda_t} = \frac{dU(\Lambda_t, \xi_t)}{d\Lambda_t} + \phi_t' \frac{d\Gamma(e_t, \Lambda_t, w_t, w_{t-1}, \xi_t)}{d\Lambda_t} + \gamma_t' \frac{d\Upsilon(\Lambda_t, w_t, \xi_t)}{d\Lambda_t} \tag{24}$$

$$\frac{d\mathcal{L}}{de_t} = \phi_t' \frac{d\Gamma(e_t, \Lambda_t, w_t, w_{t-1}, \xi_t)}{de_t} + \psi_t \tag{25}$$

$$\frac{d\mathcal{L}}{dw_t} = \beta E_t J_w(w_t, \xi_{t+1}) + \phi_t' \frac{d\Gamma(e_t, \Lambda_t, w_t, w_{t-1}, \xi_t)}{dw_t} - \psi_t' \frac{d\bar{e}(w_t, \xi_t)}{dw_t} + \gamma_t' \frac{d\Upsilon(\Lambda_t, w_t, \xi_t)}{dw_t} \tag{26}$$

$$\gamma_t \geq 0, \quad \Upsilon(\Lambda_t, w_t, \xi_t) \geq, 0 \quad \gamma_t' \Upsilon(\Lambda_t, w_t, \xi_t) \tag{27}$$

The Markov equilibrium must also satisfy an envelope condition:

$$J_w(w_{t-1}, \xi_t) = \ \phi_t' \frac{d\Gamma(e_t, \Lambda_t, w_t, w_{t-1}, \xi_t)}{dw_{t-1}} \tag{28}$$

2.1 Explicit first order conditions of the government maximization problem

In this section I show the first order conditions of the government's problem (24)-(28) in explicit form. I need to do this to proof some of the propositions of Staff Report nr. 234. The period Lagrangian (23) is:

$$\begin{aligned}
L_t &= u(Y_t - d(\Pi_t) - F_t, \xi_t)) + g(F_t - s(T_t), \xi_t) - \tilde{v}(Y_t, \xi_t) + E_t \beta J(w_t, \xi_{t+1}) \\
&+ \phi_{2t}(w_t - (1+i_t)\Pi_t^{-1} w_{t-1} - (1+i_t)F_t + (1+i_t)T_t) + \\
&+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, \xi_t)}{1+i_t}) \\
&+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F_t, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F_t, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
&+ \psi_{1t}(f_t^e - \bar{f}^e(w_t, \xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(w_t, \xi_t)) + \gamma_{1t} i_t + \gamma_{2t}(\bar{w} - w_t)
\end{aligned}$$

The first order conditions (all the derivative should be equated to zero):

$$\begin{aligned}
\frac{\delta L_t}{\delta \Pi_t} \quad &= \quad -u_c d'(\Pi_t) + \phi_{2t}(1+i_t) w_{t-1} \Pi_t^{-2} + \phi_{3t} \frac{u_{cc} d'}{1+i_t} \\
&\quad\ + \phi_{4t}[-Y_t(\theta - 1)(1+s)u_{cc} d' - u_{cc}\Pi_t d'^2 + u_c \Pi_t d'' + u_c d']
\end{aligned} \tag{29}$$

$$\frac{\delta L_t}{\delta Y_t} = u_c - \tilde{v}_y - \phi_{3t}\frac{u_{cc}}{1+i_t} + \phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc}\Pi_t d'] \tag{30}$$

$$\frac{\delta L_t}{\delta F_t} = -u_c + g_G - (1+i_t)\phi_{2t} + \phi_{3t}\frac{u_{cc}}{1+i_t} - \phi_{4t}[\theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc}) + u_{cc}\Pi_t d'] \tag{31}$$

$$\frac{\delta L_t}{\delta i_t} = \phi_{2t}(T_t - w_{t-1}\Pi_t^{-1} - F_t) + \phi_{3t}\frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{32}$$

$$\frac{\delta L_t}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1+i_t) \tag{33}$$

$$\frac{\delta L_t}{\delta w_t} = \beta E_t J_w(w_t, \xi_{t+1}) - \psi_{1t}\bar{f}_w^e - \psi_{2t}\bar{S}_w^e + \phi_{2t} - \gamma_{2t} \tag{34}$$

$$\frac{\delta L_t}{\delta f_t^e} = \beta\phi_{3t} + \psi_{1t} \tag{35}$$

$$\frac{\delta L_t}{\delta S_t^e} = -\beta\phi_{4t} + \psi_{2t} \tag{36}$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0, \;\; i_t \geq i^m, \;\;\; \gamma_{1t}(i_t - i^m) = 0 \tag{37}$$

$$\gamma_{2t} \geq 0, \;\;\; \bar{w} - w_t \geq 0, \;\;\; \gamma_{2t}(\;\bar{w} - w_t) = 0 \tag{38}$$

The optimal plan under discretion also satisfies an envelope condition:

$$J_w(w_{t-1}, \xi_t) = -\phi_{2t}(1+i_t)\Pi_t^{-1} \tag{39}$$

Necessary and sufficient condition for a Markov equilibrium thus are given by the first order conditions (29) to (39) along with the constraints (9), (14), (16) and the definitions (13) and (15). Note that the first order conditions imply restrictions on the unknown vector function $\bar{\Lambda}_t$ and the expectation functions $\bar{e}_t$.

3 Approximation Method

This section show the approximation method used to approximate the Markov Perfect Equilibrium. The first sub section shows the steady state and relates the result to other literature. The second subsection discussed the order of accuracy of the proposed approximation method.

Following Woodford (2003), I define a steady state where monetary frictions are trivial as discussed above. Eggertsson (2005) makes this more explicit by parameterize the utility function by the technology parameter $\bar{m}$ so that as $\bar{m}$ is reduced the household will demand ever lower real money balances. The maintained assumption here is $\bar{m} \to 0$ (see Eggertsson (2005) for details). Furthermore I assume, following Woodford (2003), that the steady state is fully efficient so that $1 + s = \frac{\theta-1}{\theta}$. Finally I suppose that in steady state $i^m = 1/\beta - 1$. To summarize:

A2 Steady state assumptions. (i) $\bar{m} \to 0$, (ii) $1 + s = \frac{\theta}{\theta-1}$ (iii) $i^m = 1/\beta - 1$.

3.1 Steady state discussion and relation to literature on Markov Perfect Equilibrium

In general a steady-state of a Markov equilibrium is non-trivial to compute, as emphasized by Klein et al (2003). This is because each of the steady state variables depend on the mapping between the endogenous state (i.e. debt) and the unknown functions $J(.)$ and $\bar{e}(.)$, so that one needs to know the derivative of these functions with respect to the endogenous policy state variable to calculate the steady state. Klein et al suggest an approximation method by which one may approximate this steady state numerically by using perturbation methods. In this paper I take a different approach. Proposition (3) shows that a steady state may be calculated under assumptions that are fairly common in the monetary literature, without any further assumptions about the unknown functions $J(.)$ and $e(.)$.

Proposition 3 *If $\xi = 0$ at all times and A2(i)-(iii) hold there is a Markov equilibrium steady state that is given by $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})\beta$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ and $Y = \bar{Y}$ where $\bar{Y}$ and $\bar{F}$ are the unique solution to the equations: $u_c(Y - F) = v_y(Y)$ and $u_c(Y - F) + g_G(F - s(F))s'(F) = g_G(F - s(F))$*

This proposition in proofed in Eggertsson (2005). It is straight forward. I simply look at the algebraic expressions of the first order conditions of the government maximization problem shown in last section and show that the solution above solves all the conditions. A noteworthy feature of the proof is that the mapping between the endogenous state and the functions $J(.)$ and $e(.)$ does not matter (i.e. the derivatives of these functions cancel out). The reason is that the Lagrangian multipliers associated with the expectation functions are zero in steady state and I may use the envelope condition to substitute for the derivative of the value function. The intuition for why these Lagrangian multipliers are zero in equilibrium is that at the steady state the distortions associated with monopolistic competition are zero (because of

A2 (ii)). This implies that there is no gain of increasing output from steady state. In the steady the real debt is zero and according to assumption (i) seigniorage revenues are zero as well. This implies that even if there is cost of taxation in the steady state, increasing inflation does not reduce taxes. It follows that all the Lagrangian multipliers are zero in the steady state apart from the one on the government budget constraint. That multiplier, i.e. ϕ_2, is positive because there are steady state tax costs. Hence it would be beneficial (in terms of utility) to relax this constraint.

There is by now a rich literature studying the question whether there can be multiple Markov equilibria in monetary models that are similar in many respects to the one I have described here (see e.g. Albanesi et al (2003), Dedola (2002) and King and Wolman (2003)). I do not proof the global uniqueness of the steady state in Proposition 3 but show that it is locally unique.[8] I conjecture, however, that the steady state is globally unique under A2.[9] But even if I would have written the model so that it had more than one steady state, the one studied here would still be the one of principal interest as discussed in the footnote.[10]

3.2 Approximate system and order of accuracy

The conditions that characterize equilibrium are given by the constraints of the model and the first order conditions of the governments problem. A linearization of this system is complicated by the Kuhn-Tucker inequalities (37) and (38). I look for a solution in which the bound on government debt is never binding, and then verify that this bound is never binding in the equilibrium I calculate. Under this conjectured the solution to the inequalities (37) and (38) can be simplified into two cases:

$$\text{Case } 1 : \gamma_t^1 = 0 \ \text{ if } i_t > i^m \tag{40}$$

$$\text{Case } 2 : i_t = i^m \ \text{ otherwise} \tag{41}$$

[8] See Woodford (2003) Appendix A3 for definition and discussion of local uniqueness in stochastic general equilibrium models of this kind.

[9] The reason for this conjecture is that in this model, as opposed to Albanesi et al and Dedola work, I assume in A2 that there are no monetary frictions. The source of the multiple equilibria in those papers, however, is the payment technology they assume. The key difference between the present model and that of King and Wolman, on the other hand, is that they assume that some firms set prices at different points in time. I assume a representative firm, thus abstacting from the main channel they emphasize in generating multiple equilibria. Finally the present model is different from all the papers cited above in that I introduce nominal debt as a state variable. Even if the model I have illustrated above would be augmented to incorporate additional elements such as montary frictions and staggering prices, I conjecture that the steady state would remain unique due to the ability of the government to use nominal debt to change its future inflation incentive. That is, however, a topic for future reasearch and there is work in progress by Eggertsson and Swanson that studies this question.

[10] Even if I had written a model in which the equilibria proofed above is not the unique global equilibria the one I illustrate here would still be the one of principal interest. Furthermore a local analysis would still be useful. The reason is twofold. First, the equilibria analyzed is identical to the commitment equilibrium (in the absence of shocks) and is thus a natural candidate for investigation. But even more importantly the work of Albanesi et al (2002) indicates that if there are non-trivial monetary frictions there are in general only two steady states.There are also two steady states in King and Wolman's model. (In Dedola's model there are three steady states, but the same point applies.) The first is a low inflation equilibria (analogues to the one in Proposition 1) and the other is a high inflation equilibria which they calibrate to be associated with double digit inflation. In the high inflation equilibria, however, the zero bound is very unlikely ever to be binding as a result of real shocks of the type I consider in this paper (since in this equilibria the nominal interest rate is very high as I will show in the next section). And it is the distortions created by the zero bound that are the central focus of this paper, and thus even if the model had a high inflation steady state, that equilibria would be of little interest in the context of the zero bound.

Thus in both Case 1 and 2 I have equalities characterizing equilibrium. These equations are (4), (9),(10), (14), (13), (16), (15) and (29)-(37) and either (40) when $i_t > i^m$ or (41) otherwise. Under the condition A1(i) and A1(ii) but $i^m < \frac{1}{\beta} - 1$ then $i_t > i^m$ and Case 1 applies in the absence of shocks. In the knife edge case when $i^m = \frac{1}{\beta} - 1$, however, the equations that solve the two cases (in the absence of shocks) are identical since then both $\gamma_{1t} = 0$ and $i_t = i^m$. Thus both Case 1 and Case 2 have the same steady state in the knife edge case $i_t = i^m$. If I linearize around this steady state (which I show exists in Proposition 3) I obtain a solution that is accurate up to a residual $(||\xi||^2)$ for both Case 1 and Case 2. As a result I have one set of linear equations when the bound is binding, and another set of equations when it is not. The challenge, then, is to find a solution method that, for a given stochastic process for $\{\xi_t\}$, finds in which states of the world the interest rate bound is binding and the equilibrium has to satisfy the linear equations of Case 1, and in which states of the world it is not binding and the equilibrium has to satisfy the linear equations in Case 2. Since each of these solution are accurate to a residual $(||\xi||^2)$ the solutions can be made arbitrarily accurate by reducing the amplitude of the shocks. The next subsection shows a solution method, assuming the simple process for the natural rate of interest in the text, that numerically calculates when Case 1 applies and when Case 2 applies.

Note that I may also consider solutions when i^m is below the steady state nominal interest rate. A linear approximation of the equations around the steady state in Proposition 3 is still valid if the opportunity cost of holding money, i.e. $\bar{\delta} \equiv (i - i^m)/(1 + i)$, is small enough. Specifically, the result will be exact up to a residual of order $(||\xi, \bar{\delta}||^2)$. In the text I assume that $i^m = 0$ (see Eggertsson and Woodford (2003) for further discussion about the accuracy of this approach when the zero bound is binding). A non-trival complication of approximating the Markov equilibrium is that I do not know the unknown expectation functions $\bar{e}(.)$. I illustrate a simple way of matching coefficients to approximate this function in subsection 3.4.

3.3 Linearized solution

I here linearize the first order conditions and the constraints around the steady state in Propositions 3. I allow for deviations in the vector of shocks ξ_t and in i^m so that the equations are accurate of order $o(||\xi, \bar{\delta}||^2)$ as discussed in the last subsection.

3.3.1 Functional forms

I assume the following functional forms

$$u(C, \xi) = \frac{C^{1-\sigma^{-1}} u^{\sigma^{-1}}}{1 - \sigma^{-1}}$$

where u is a preference shock that I assume in steady state to be $u = \bar{C}$,

$$g(G, \xi) = \chi \frac{G^{1-\sigma^{-1}} g^{\sigma^{-1}}}{1 - \sigma^{-1}}$$

where g is a preference shock assumed to be $\bar{G}$ in steady state. For disutility of working I assume

$$v(H, \xi) = \frac{\lambda_1}{1+\omega} H^{1+\vartheta} q^{-\omega}$$

where q is a preference shock. Production is given by

$$y = H^{\epsilon}$$

I may substitute the production function into the disutility of working to obtain and can then write the disutility of working as a function of output

$$\tilde{v}(Y, \xi_t) = \frac{\lambda_1}{1+\omega} Y^{1+\omega} q^{-\omega}$$

where in steady state I normalize $q = \bar{Y}$ and where $1 + \omega = \frac{1}{\epsilon}(1 + \vartheta)$

I furthermore make the normalizing assumption that $\bar{Y} = 1$. This implies that $\lambda_1 = 1$. For notational simplicity I define the variable $\tilde{\sigma}^{-1} = \sigma^{-1} \frac{Y}{C}$.[11]

3.3.2 The natural rate of output and interest

To derive some of the equations in the text, we need to define the natural rate of interest and output in the model.

I define the natural rate of output as the output that would be produced in the absence of price frictions. It therefore solves the equation

$$v_y(Y_t^n, \xi_t) = \frac{\theta - 1}{\theta}(1+s) u_c(Y_t^n - F_t, \xi_t). \tag{42}$$

[11] For steady state we have
$u_c = C^{-\sigma^{-1}} u^{\sigma^{-1}} = 1$
$u_{c\xi} = \sigma^{-1} C^{-\sigma^{-1}} u^{\sigma^{-1}-1} = C^{-1}\sigma^{-1}$
$u_{cc} = -\sigma^{-1} C^{-\sigma^{-1}-1} u^{\sigma^{-1}} = -C^{-1}\sigma^{-1}$
$v_y = \lambda_1 Y^{\omega} q^{-\omega} = 1$
$v_{yy} = \omega \lambda_1 Y^{\omega-1} q^{-\omega} = \omega$
$v_{y\xi} = -\lambda_1 \omega Y^{\omega} q^{-\omega} = -\omega$
$g_G = \chi G^{-\sigma^{-1}} g^{\sigma^{-1}} = \chi$
$g_{GG} = -\sigma^{-1} \chi G^{-\sigma^{-1}-1} g^{\sigma^{-1}} = -\chi G^{-1}\sigma^{-1}$
$\mathrm{g}_{G\xi} = \sigma^{-1} \chi G^{-\sigma^{-1}} g^{\sigma^{-1}-1} = \chi G^{-1}\sigma^{-1}$

Linearizing this equation around the steady state one obtains

$$(\omega + \tilde{\sigma}^{-1})\hat{Y}_t^n - \omega\hat{q}_t - \tilde{\sigma}^{-1}\hat{u}_t - \tilde{\sigma}^{-1}\hat{F}_t = 0$$

where I have defined $\hat{Y}_t^n \equiv \frac{Y_t^n - \bar{Y}^n}{\bar{Y}^n}$. I define the variable F_t as $\hat{F}_t \equiv \frac{F_t - \bar{F}}{\bar{Y}}$. The shocks are defined as $\hat{q}_t \equiv \frac{q_t - \bar{q}}{\bar{Y}}$ and $\hat{u}_t \equiv \frac{u_t - \bar{u}}{\bar{Y}}$. I define the variable $\tilde{Y}_t^n$ as the part of the natural rate of output that is given by the exogenous disturbances $\hat{q}_t$ and $\hat{u}_t$. It is

$$\tilde{Y}_t^n \equiv \frac{\tilde{\sigma}^{-1}}{\tilde{\sigma}^{-1} + \omega}\hat{u}_t + \frac{\omega}{\tilde{\sigma}^{-1} + \omega}\hat{q}_t.$$

I define the natural rate of interest as the real interest rate that would result if prices were flexible. Thus it solves the equation

$$\frac{1}{1 + r_t^n} = E_t \frac{\beta u_c(Y_{t+1}^n - F_{t+1}, \xi_{t+1})}{u_c(Y_t^n - F_t, \xi_t)}$$

Linearizing this equation around the steady state one obtains

$$-\tilde{\sigma}^{-1}E_t\hat{Y}_{t+1}^n + \tilde{\sigma}^{-1}E_t\hat{u}_{t+1} + \tilde{\sigma}^{-1}E_t\hat{F}_{t+1} + \tilde{\sigma}^{-1}\hat{Y}_t^n - \tilde{\sigma}^{-1}\hat{u}_t - \tilde{\sigma}^{-1}\hat{F}_t + \hat{r}_t^n = 0$$

where $\hat{r}_t^n \equiv \frac{r_t^n - \bar{r}^n}{1 + \bar{r}^n}$. I define the variable $\tilde{r}_t^n$ as the part of the natural rate of interest that is given by the exogenous disturbances $\hat{q}_t$ and $\hat{u}_t$. It is

$$\tilde{r}_t^n \equiv \tilde{\sigma}^{-1}(\hat{u}_t - E_t\hat{u}_{t+1}) - \tilde{\sigma}^{-1}(\tilde{Y}_t^n - E_t\tilde{Y}_{t+1}^n) = \frac{\tilde{\sigma}^{-1}\omega}{\sigma^{-1} + \omega}(\hat{u}_t - E_t\hat{u}_{t+1}) - \frac{\tilde{\sigma}^{-1}\omega}{\sigma^{-1} + \omega}(\hat{q}_t - E_t\hat{q}_{t+1})$$

3.3.3 AS and IS equations

Here I show how to derive the IS and AS equation in the text. These two equations and the budget constraint of the government constitute the relevant constraints that are needed to characterize a private sector equilibrium.

I first derive the IS equation that is reported in Staff Report nr. 234. Equation (14)

$$-\tilde{\sigma}^{-1}\hat{Y}_t + \tilde{\sigma}^{-1}\hat{u}_t + \tilde{\sigma}^{-1}\hat{F}_t - \hat{\imath}_t - \hat{f}_t^e = 0 \tag{43}$$

and (13)

$$\hat{f}_t^e + E_t\pi_{t+1} + \tilde{\sigma}^{-1}E_t\hat{Y}_{t+1} - \tilde{\sigma}^{-1}E_t\hat{u}_{t+1} - \tilde{\sigma}^{-1}E_t\hat{F}_{t+1} = 0 \tag{44}$$

These two equations, together with the definitions of the natural rate of interest and output in last subsection, give the IS equation in the text.

I now derive the AS equation. Equation (16) can be written as.

$$d''\pi_t - \theta(\tilde{\sigma}^{-1} + \omega)(\hat{Y}_t - \tilde{Y}_t^n) + \theta\tilde{\sigma}^{-1}\hat{F}_t - \hat{S}_t^e = 0 \tag{45}$$

and (15)

$$S_t^e - d'' E_t \pi_{t+1} = 0. \tag{46}$$

These two equations, together with the definition of the natural rate of output, give the AS equation in the text.

Finally the budget constraint (9) can be written as

$$w_t - \beta^{-1} w_{t-1} + \beta^{-1}\hat{T}_t - \beta^{-1}\hat{F}_t = 0 \tag{47}$$

where $\hat{T}_t = \frac{T_t - \bar{T}}{\bar{Y}}$.

Equations (43) to (47) constituted a linear approximation to the conditions that are needed to characterize a private sector equilibrium. To close the model, to a linear approximation, we need to approximate the government decision rules. To do this we approximate the first order condition of the government maximization problem.

3.3.4 First order conditions

The Kuhn-Tucker conditions imply that

Case 1 when $i_t > i^m$

$$\gamma_{1t} = 0 \tag{48}$$

Case 2 when $i_t = i^m$

$$i_t = 0 \tag{49}$$

I look for a solution in which case the debt limit is never binding so that $\gamma_{2t} = 0$ at all times and verify that this is satisfied in equilibrium.

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$$-d''\pi_t + \bar{\phi}_2 \beta^{-1} w_{t-1} + d''\phi_{4t} = 0 \tag{50}$$

$$-(\tilde{\sigma}^{-1}+\omega)\hat{Y}_t+\tilde{\sigma}^{-1}\hat{u}_t+\omega\hat{q}_t+\tilde{\sigma}^{-1}\hat{F}_t+\tilde{\sigma}^{-1}\beta\phi_{3t}-\theta(\tilde{\sigma}^{-1}+\omega)\phi_{4t}=0 \tag{51}$$

$$\tilde{\sigma}^{-1}\hat{Y}_t-\tilde{\sigma}^{-1}\hat{u}_t-(\tilde{\sigma}^{-1}+\chi\tilde{\sigma}^{-1}\frac{C}{G})\hat{F}_t+\chi\tilde{\sigma}^{-1}\frac{C}{G}s'T_t+\chi\tilde{\sigma}^{-1}\frac{C}{G}\hat{g}_t-\bar{\phi}_2\beta_t^{-1}\hat{\imath}_t-\bar{\phi}_2\beta^{-1}\hat{\phi}_{2t}-\beta\tilde{\sigma}^{-1}\phi_{3t}+\theta\tilde{\sigma}^{-1}\phi_{4t}=0 \tag{52}$$

$$\bar{\phi}_2\hat{T}_t-\bar{\phi}_2\hat{F}_t-\bar{\phi}_2w_{t-1}+\beta^2\phi_{3t}+\gamma_{1t}=0 \tag{53}$$

$$\chi\tilde{\sigma}^{-1}\frac{C}{G}s'\hat{F}_t-\chi\tilde{\sigma}^{-1}\frac{C}{G}(s')^2\hat{T}_t-\chi s''\hat{T}_t-\chi\tilde{\sigma}^{-1}\frac{C}{G}s'\hat{g}_t+\beta^{-1}\bar{\phi}_2\hat{\phi}_{2t}+\beta^{-1}\bar{\phi}_2\hat{\imath}_t=0 \tag{54}$$

$$\bar{\phi}_2\hat{\phi}_{2t}-\bar{\phi}_2E_t\hat{\phi}_{2t+1}-\bar{\phi}_2E_t\hat{\imath}_{t+1}+\bar{\phi}_2E_t\pi_{t+1}+\beta f^1\phi_{3t}-\beta S^1\phi_{4t}=0 \tag{55}$$

where

$$\hat{g}_t=\frac{g_t-\bar{g}}{\bar{Y}}$$

Note that the two derivatives f^1 and S^1 are in general not known. In the next section I show how these derivatives can be found. The variable $\hat{\phi}_{2t}$ is defined in terms of deviation from steady state i.e. $\hat{\phi}_{2t}\equiv\frac{\phi_{2t}-\bar{\phi}_2}{\bar{\phi}_2}$. Note that the variables $w_t,\gamma_{1t},\phi_{3t},\phi_{4t}$ are zero in steady state, hence they are not defined in terms of deviation from steady state but simply correspond to a linear approximation of the actual value of these variables in the non-linear model.

3.4 Approximating f^1 and S^1

Here I show how the two derivatives f^1 and S^1 can be approximated under the assumption about the shock stated in the text (and restated in the next subsection). At time $t\geq\tau$ the system is deterministic. Then I can approximate these functions to yield $w_t=w^1w_{t-1}$ and $d\Lambda_t=\Lambda^1w_{t-1}$, where the first element of the vector $d\Lambda_t$ is $\pi_t=\pi^1w_{t-1}$, the second $Y_t=Y^1w_{t-1}$ and so on and $w_t=w^1w_{t-1}$ where the vector Λ^1 and the number w^1 are some unknown constants. To find the value of each of these coefficients I substitute this solution into the system (43)-(47) and (50)-(54) and match coefficients. For example equation (45) and (46) imply that

$$d''\pi^1w_{t-1}+\theta(\tilde{\sigma}^{-1}+\omega)Y^1w_{t-1}+\theta\tilde{\sigma}^{-1}F^1w_{t-1}-d''\beta\pi^1w^1w_{t-1}=0 \tag{56}$$

where I have substituted for $\pi_t=\pi^1w_{t-1}$ and for $\pi_{t+1}=\pi^1w_t=\pi^1w^1w_{t-1}$. Note that I assume that $t\geq\tau$ so that there is perfect foresight and I may ignore the expectation symbol. This equation implies that the coefficients π^1, Y^1 and w^1 must satisfy the equation:

$$d''\pi^1 - \theta(\tilde{\sigma}^{-1} + \omega)Y^1 + \theta\tilde{\sigma}^{-1}F^1 - d''\beta\pi^1 w^1 = 0 \tag{57}$$

I may similarly substitute the solution into each of the equation (43)-(47) and (50)-(55) to obtain a system of equation that the coefficients must satisfy:

$$d''\pi^1 - \theta(\tilde{\sigma}^{-1} + \omega)Y^1 + \theta\tilde{\sigma}^{-1}F^1 - d''\beta\pi^1 w^1 = 0 \tag{58}$$

$$-\tilde{\sigma}^{-1}Y^1 + \tilde{\sigma}^{-1}F^1 - i^1 - f^1 = 0 \tag{59}$$

$$w^1 - \frac{1}{\beta} + \frac{1}{\beta}T^1 - \frac{1}{\beta}F^1 = 0 \tag{60}$$

$$S^1 - d''\pi^1 w^1 = 0 \tag{61}$$

$$f^1 + \pi^1 w^1 + \tilde{\sigma}^{-1}Y^1 w^1 - \tilde{\sigma}^{-1}F^1 w^1 = 0 \tag{62}$$

$$-d''\pi^1 + \frac{\bar{\phi}_2}{\beta} + d''\phi_4^1 = 0 \tag{63}$$

$$-(\tilde{\sigma}^{-1} + \omega)Y^1 + \tilde{\sigma}^{-1}F^1 + \tilde{\sigma}^{-1}\beta\phi_3^1 - \theta(\tilde{\sigma}^{-1} + \omega)\phi_4^1 = 0 \tag{64}$$

$$\tilde{\sigma}^{-1}Y^1 - (\tilde{\sigma}^{-1} + \chi\tilde{\sigma}^{-1}\frac{C}{G})F^1 + \chi\tilde{\sigma}^{-1}\frac{C}{G}s'T^1 - \bar{\phi}_2\beta^{-1}i^1 - \bar{\phi}_2\beta^{-1}\phi_2^1 - \beta\tilde{\sigma}^{-1}\phi_3^1 + \theta\tilde{\sigma}^{-1}\phi_4^1 \tag{65}$$

$$\bar{\phi}_2 T^1 - \bar{\phi}_2 F^1 - \bar{\phi}_2 + \bar{u}_c\beta^2\phi_3^1 = 0 \tag{66}$$

$$\chi\tilde{\sigma}^{-1}\frac{C}{G}s'F^1 - \chi\tilde{\sigma}^{-1}\frac{C}{G}(s')^2T^1 - \chi s''T^1 + \beta^{-1}\bar{\phi}_2\phi_2^1 + \beta^{-1}\bar{\phi}_2 i^1 = 0 \tag{67}$$

$$\bar{\phi}_2\phi_2^1 - \bar{\phi}_2\phi_2^1 w^1 - \bar{\phi}_2 i^1 w^1 + \bar{\phi}_2\pi^1 w^1 + \beta f^1\phi_3^1 - \beta S^1\phi_4^1 = 0 \tag{68}$$

There are 11 unknown coefficients in this system i.e. $\pi^1, Y^1, i^1, F^1, S^1, f^1, T^1, \phi_2^1, \phi_3^1, \phi_4^1, w^1$. For a given value of w^1, (58)-(67) is a linear system of 10 equations with 10 unknowns, and thus there is a unique value given for each of the coefficients as long as the system is non-singular (which can be verified to be the case for standard functional forms for the utility and technology functions). The value of w^1 is in general not unique, but in the calibrated model there is always a unique bounded solution in the examples I have studied (and the unbounded solutions will violate the debt limit). In a simplified version of the model it can be proofed that there is a unique solution for w^1 that satisfies all the necessary conditions, but I have not managed to proof it in this model (see discussion in Eggertsson (2005)).

3.5 Shock Assumptions

In the past few sections I have shown a system of linear equations and derived the appropriate coefficients of this system. Before showing how to solve them it remains to be more explicit about the assumed path for the exogenous variables. As shown in previous sections the fundamental shocks of the model are summarized by the three disturbances $\hat{u}_t$, $\hat{q}_t$ and $\hat{g}_t$. In the text, instead, I make assumptions on the terms $\tilde{Y}_t^n$, $\tilde{r}_t^n$,$\hat{F}_t^n$ and $\hat{T}_t^n$. I now show what these assumption imply for the fundamental shocks.

Recall that Assumption 1 in Staff Report nr. 234 is:

A1: The Great Depression structural shocks $\tilde{r}_t^n = \tilde{r}_L^n < 0$ *at* $t = 0$. It returns back to steady state with probability α in each period. Furthermore, $\tilde{Y}_t^n = 0 \; \forall \; t$. The stochastic date the shock returns back to steady state is denoted τ.

Conditional on $\tilde{r}_t^n = \tilde{r}_L^n$ this implies that

$$r_L^n = \tilde{\sigma}^{-1}(\hat{u}_L - E_t\hat{u}_{t+1}) = \tilde{\sigma}^{-1}\alpha\hat{u}_L$$

This allows us to express $\hat{u}_L$ in terms of the assumption made about $\tilde{r}_L^n$ which I assume to be -4% in the numerical example. In terms of the other structural shocks this assumption implies that

$$\hat{q}_t = -\tilde{\sigma}^{-1}\omega^{-1}\hat{u}_t$$

Recall that Assumption 4 in Staff Report nr. 234 is:

A4 The natural rate of fiscal spending and taxation is such that $\hat{F}_t^n = -\frac{\lambda_T}{\lambda_F}\hat{T}_t^n$.

This assumption is made for simplicity and to ensure that the real government spending variations are not prompted by shifts in preferences. If this assumption holds, then in the absence of the zero bound government spending will be held constant at all times as discussed in the text. The term $\hat{F}_t^n$ is defined as

$$\hat{F}_t^n \equiv \frac{\chi G^{-1}}{C^{-1} + \chi G^{-1}}\hat{g}_t - \frac{C^{-1}}{C^{-1} + \chi G^{-1}}\hat{u}_t$$

and the term $\hat{T}_t^n$ is defined as

$$\hat{T}_t^n \equiv -\frac{G^{-1}\sigma^{-1}s'}{\sigma^{-1}G^{-1}(s')^2 + s''}g_t$$

Thus in terms of the fundamental shock, $\hat{g}_t$, this assumption implies that

$$\hat{g}_t = \frac{G}{C}\hat{u}_t$$

We have now completely characterized the path for the fundamental shocks $\hat{u}_t$,$\hat{q}_t$ and $\hat{g}_t$ and can turn to how the model can be solved based on the linearization discussed and the assumed path for the exogenous variables.

3.6 Computational method

Here I illustrate a solution method. A more general solution method is presented in Eggertsson (2005) which is required for the commitment solution referred to in the text. I assume shocks so that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t as in A5. I assume that there is a final date K in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future).

The solution takes the form:

$$
\begin{array}{lccccc}
\text{Case 2 } i_t & = & 0 & \forall & t & 0 \leq t < \tau \\
\text{Case 1 } i_t & > & 0 & \forall & t & t \geq \tau
\end{array}
$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. I assume that τ can take any value between 1 and the terminal date K that can be arbitrarily far into the future. The solution above is a conjectured solution and I verify that it does in fact solve the model in the computer codes written.

3.6.1 The solution for $t \geq \tau$

The system of linearized equations (50)-(54), (43)-(47), and (48) can be written in the form:

$$
\begin{bmatrix} E_t Z_{t+1} \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ P_{t-1} \end{bmatrix}
$$

where $Z_t \equiv \begin{bmatrix} \Lambda_t & e_t & \phi_t & \psi_t & \gamma_t^1 \end{bmatrix}^T$ and $P_t \equiv w_t$. If there are fifteen eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{69}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{70}$$

3.6.2 The solution for $t < \tau$

The solution satisfies (50)-(54), (43)-(47), and (49). Note that each of the expectation variables can be written as $\tilde{x}_t = E_t x_{t+1} = \alpha_{t+1}\tilde{x}_{t+1} + (1-\alpha_{t+1})x_{t+1}$ where α_{t+1} is the probability that the natural rate of interest becomes positive in period $t+1$. Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. I may now use the solution for Z_{t+1} in 70 to substitute for Z_{t+1}, i.e. the value of each variable contingent on that the natural rate becomes positive again, in terms of the hatted variables. Hence I can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ V_t \end{bmatrix}$$

I can solve this backwards from the date K in which the natural rate returns back to normal with probability one. I can then calculate the path for each variable to date 0. Note that.

$$B_{K-1} = D_{K-1} = 0$$

By recursive substitution I can find a solution of the form:

$$\tilde{P}_t = \Omega_t \tilde{P}_{t-1} + \Phi_t \tag{71}$$

$$\tilde{Z}_t = \Lambda_t \tilde{P}_{t-1} + \Theta_t \tag{72}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $K-1$ when $B_{K-1} = D_{K-1} = 0$. I have:

$$\Omega_{K-1} = A_{K-1}$$

$$\Phi_{K-1} = M_{K-1}$$

$$\Lambda_{K-1} = C_{K-1}$$

$$\Theta_{K-1} = V_{K-1}$$

I can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $K-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t \Lambda_{t+1}]^{-1} A_t$$

$$\Lambda_t = C_t + D_t \Lambda_{t+1} \Omega_t$$

$$\Phi_t = (I - B_t \Lambda_{t+1})^{-1} [B_t \Theta_{t+1} + M_t]$$

$$\Theta_t = D_t \Lambda_{t+1} \Phi_t + D_t \Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ I can solve for each of the endogenous variables under the contingency that the trap last to period K by (71) and (72). I then use the solution from (69)-(70) to solve for each of the variables when the natural rate reverts back to steady state.

4 Calibration of parameters

The cost of price adjustment is assumed to take the form:

$$d(\Pi) = d_1 \Pi^2$$

The cost of taxes is assumed to take to form:

$$s(T) = s_1 T^2$$

Aggregate demand implies$Y = C + F = C + G + s(F)$. I normalize $Y = 1$ in steady state and assume that the share of the government in production is $F = 0.10$. Tax collection as a share of government spending is assumed to be $\gamma = 5\%$ of government spending. This implies

$$\gamma = \frac{s(F)}{F} = s_1 F$$

so that $s_1 = \frac{\gamma}{F}$. The result for the inflation and output gap response are not very sensitive to varying γ under either commitment or discretion. The size of the public debt issued in the Markov equilibrium, however, crucially depends on this variable. In particular if γ is reduced the size of the debt issued rises substantially. For example if $\gamma = 0.5\%$ the public debt issued is about ten times bigger than reported in the figure in Staff Report nr. 234. I assume that government spending are set at their optimal level in steady state as shown in proposition (3) so that $u_c = (1-s')g_G$. This implies that

$$\chi = \frac{1}{1-s'} = \frac{1}{1-2s_1F}$$

The IS equation and the AS equation are

$$x_t = E_t x_{t+1} - \tilde{\sigma}(i_t - E_t\pi_{t+1} - r_t^n)$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1}$$

I assume, as Eggertsson and Woodford, that the interest rate elasticity, $\tilde{\sigma}$, is 0.5. The relationship between σ and $\tilde{\sigma}$ is

$$\sigma = \tilde{\sigma}\frac{Y}{C}$$

I assume that κ is 0.02 as in Eggertsson and Woodford (2003). The relationship between κ and the other parameters of the model is $\kappa = \theta\frac{(\tilde{\sigma}^{-1}+\omega)}{d''}$. I scale hours worked so that $Y = 1$ in steady state which implies $v_y = \lambda_1 = 1$. Finally I assume that $\theta = 7.87$ as in Rotemberg and Woodford and that $\omega = 2$. The calibration value for the parameters are summarized in the table below:

Table 1

σ	0.71
g_1	0.33
λ_2	2
d_1	787
s_1	0.17
θ	7.87

5 Linear Quadratic Approximation

In this section I show the validity of the remark in Staff Report nr. 234 that the linear quadratic presentation of the results is equivalent to a first order approximation to the non-linear conditions. The first subsection derives the second order approximation of the utility of the representative household. The second subsection proofs the remark.

5.1 Government Objective

Here I do a linear quadratic approximation of the utility of the representative household. The utility function of the household is

$$E_t \sum_{t=0}^{\infty} \beta^t \{u(Y_t - F_t - d(\pi_t), \xi_t) + g(F_t - s(T_t), \xi_t) - v(Y_t, \xi_t)\}$$

Note that in steady state we have

$u_c = C^{-\sigma^{-1}} u^{\sigma^{-1}} = 1$

$u_{c\xi} = \sigma^{-1} C^{-\sigma^{-1}} u^{\sigma^{-1}-1} = C^{-1}\sigma^{-1}$

$u_{cc} = -\sigma^{-1} C^{-\sigma^{-1}-1} u^{\sigma^{-1}} = -C^{-1}\sigma^{-1}$

$v_y = \lambda_1 Y^{\omega} q^{-\omega} = 1$

$v_{yy} = \omega\lambda_1 Y^{\omega-1} q^{-\omega} = \omega$

$v_{y\xi} = -\lambda_1 \omega Y^{\omega} q^{-\omega} = -\omega$

$g_G = \chi G^{-\sigma^{-1}} g^{\sigma^{-1}} = \chi$

$g_{GG} = -\sigma^{-1}\chi G^{-\sigma^{-1}-1} g^{\sigma^{-1}} = -\chi G^{-1}\sigma^{-1}$

$g_{G\xi} = \sigma^{-1}\chi G^{-\sigma^{-1}} g^{\sigma^{-1}-1} = \chi G^{-1}\sigma^{-1}$

$(1 - s\prime)\chi = 1$

Also recall that in steady state I normalize $Y = 1$.

The first piece of the utility is

$$\begin{aligned}
& u(Y_t - F_t - d(\pi_t), \xi_t) \\
= \quad & u + u_c dY_t - u_c dF_t - u_c d' d\pi_t + u_\xi d\xi_t \\
& + \frac{1}{2} u_{cc} dY_t^2 + u_{c\xi} d\xi_t dY_t - u_{c\xi} d\xi_t dF_t - u_{c\xi} d\xi_t d' d\pi_t - u_{cc} dY_t dF_t + u_{cc} d' dY_t d\pi_t + u_{cc} d' dF_t d\pi_t \\
& + \frac{1}{2} u_{cc} dF_t^2 - \frac{1}{2} u_c d'' d\pi_t^2 + \frac{1}{2} u_{cc} (d')^2 d\pi_t^2 + \frac{1}{2} \xi_t' u_{\xi\xi} \xi_t \\
= \quad & \hat{Y}_t - \hat{F}_t \\
& + [-\frac{1}{2}\sigma^{-1} C^{-1} \hat{Y}_t^2 + \sigma^{-1} C^{-1} \hat{Y}_t \hat{F}_t + \sigma^{-1} C^{-1} \hat{Y}_t \hat{u}_t - \sigma^{-1} C_t^{-1} \hat{F}_t \hat{u}_t - \frac{1}{2} d'' d\pi_t^2 - \frac{1}{2}\sigma^{-1} C^{-1} \hat{F}_t^2] \\
& + t.i.p.
\end{aligned}$$

where $t.i.p.$ stands for terms independent of policy. The second piece is:

$$\begin{aligned}
& g(F_t - s(T_t), \xi_t) \\
= \quad & \bar{g} + g_G dF_t - g_G s' dT_t + g_\xi d\xi_t + \frac{1}{2} g_{GG} dF_t^2 + \frac{1}{2} g_{GG} (s')^2 dT_t^2 - \frac{1}{2} g_G s'' dT_t^2 \\
& + g_{G\xi} d\xi_t dF_t - g_{G\xi} d\xi_t s' dT_t + \frac{1}{2} \xi_t' g_{\xi\xi} \xi_t \\
= \quad & \chi \hat{F}_t - s' \chi \hat{T}_t \\
& - \frac{1}{2}\chi\sigma^{-1} G^{-1} \hat{F}_t^2 - \frac{1}{2}\chi\sigma^{-1} G^{-1} (s')^2 \hat{T}_t^2 - \frac{1}{2} s'' \chi dT_t^2 + \chi G^{-1}\sigma^{-1} \hat{F}_t \hat{g}_t - \chi G^{-1}\sigma^{-1} s' \hat{T}_t \hat{g}_t] + t.i.p.
\end{aligned}$$

The final piece is

$$
\begin{aligned}
v(Y_t, \xi_t) &= v + v_y dY_t + v_y d\xi_t \\
&\quad + \frac{1}{2} v_{yy} dY_t^2 + v_{y\xi} d\xi_t dY_t \\
&\quad + \frac{1}{2} \xi_t' v_{\xi\xi} d\xi_t \\
&= \hat{Y}_t + \frac{1}{2}\omega \hat{Y}_t^2 - \omega \hat{Y}_t \hat{q}_t + t.i.p.
\end{aligned}
$$

Combine period utility to yield:

$$
\begin{aligned}
= \;& \hat{Y}_t - \hat{F}_t - \frac{1}{2}\sigma^{-1}C^{-1}\hat{Y}_t^2 + \sigma^{-1}C^{-1}\hat{Y}_t\hat{F}_t + \sigma^{-1}C^{-1}\hat{Y}_t\hat{u}_t - \sigma^{-1}C_t^{-1}\hat{F}_t\hat{u}_t - \frac{1}{2}d'' d\pi_t^2 - \frac{1}{2}\sigma^{-1}C^{-1}\hat{F}_t^2 \\
& + \chi\hat{F}_t - s'\chi\hat{T}_t - \frac{1}{2}\chi\sigma^{-1}G^{-1}\hat{F}_t^2 - \frac{1}{2}\chi\sigma^{-1}G^{-1}(s')^2\hat{T}_t^2 - \frac{1}{2}s''\chi\hat{T}_t^2 + \chi G^{-1}\sigma^{-1}\hat{F}_t\hat{g}_t - \chi G^{-1}\sigma^{-1}s'\hat{T}_t\hat{g}_t] \\
& - \hat{Y}_t - \frac{1}{2}\omega\hat{Y}_t^2 + \omega\hat{Y}_t\hat{q}_t \\
= \;& (\chi - 1)\hat{F}_t - s'\chi\hat{T}_t - \frac{1}{2}d''\pi_t^2 + [-\frac{1}{2}(\sigma^{-1}C^{-1} + \omega)\hat{Y}_t^2 + \sigma^{-1}C^{-1}\hat{Y}_t\hat{F}_t + \sigma^{-1}C^{-1}\hat{Y}_t\hat{u}_t + \omega\hat{Y}_t\hat{q}_t] \\
& + [-\frac{1}{2}\sigma^{-1}(C^{-1} + \chi G^{-1})\hat{F}_t^2 + \chi G^{-1}\sigma^{-1}\hat{F}_t\hat{g}_t - \sigma^{-1}C_t^{-1}\hat{F}_t\hat{u}_t] \\
& + [-\frac{1}{2}\chi(\sigma^{-1}G^{-1}(s')^2 + s'')\hat{T}_t^2 - \chi G^{-1}\sigma^{-1}s'\hat{T}_t\hat{g}_t]
\end{aligned}
$$

Welfare criterion can now be written as

$$
\begin{aligned}
& \sum_{t=0}^{\infty} \beta^t [-\frac{1}{2}d''\pi_t^2 - \frac{1}{2}(\sigma^{-1}C^{-1} + \omega)(\hat{Y}_t - \hat{Y}_t^n)^2 \\
& - \frac{1}{2}\sigma^{-1}(C^{-1} + \chi G^{-1})(\hat{F}_t - \hat{F}_t^n)^2 - \frac{1}{2}\chi(\sigma^{-1}G^{-1}(s')^2 + s'')(\hat{T}_t - \hat{T}_t^n)^2]
\end{aligned}
$$

where

$$
\hat{Y}_t^n \equiv \frac{\sigma^{-1}C^{-1}}{\sigma^{-1}C^{-1} + \omega}\hat{F}_t + \frac{\sigma^{-1}C^{-1}}{\sigma^{-1}C^{-1} + \omega}\hat{u}_t + \frac{\omega}{\sigma^{-1}C^{-1} + \omega}\hat{q}_t
$$

$$
\hat{F}_t^n \equiv \frac{\chi G^{-1}}{C^{-1} + \chi G^{-1}}\hat{g}_t - \frac{C^{-1}}{C^{-1} + \chi G^{-1}}\hat{u}_t
$$

$$
\hat{T}_t^n \equiv -\frac{G^{-1}\sigma^{-1}s'}{\sigma^{-1}G^{-1}(s')^2 + s''}g_t
$$

Because

$$\sum_{t=0}^{\infty} \beta^t [\chi - 1] dF_t - s' \chi dT_t = w_{-1} + \sum_{t=0}^{\infty} \beta^t [-1 + \chi(1 - s')] dF_t = w_{-1} = 0$$

5.2 A Proposition

Proposition 4 *Proposition 1-5 in the text of Staff Report nr. 234 can also be proved in the non-linear model*

Proof: To see this note that the first order conditions in the text are equivalent to the first order conditions (50)-(55) with suitable adjustment for the different cases considered in the text.

6 Proofs

Proposition 6

Proof: The proof of this propositions follows from Proposition 1 and 2 in this Technical Appendix and the approximation method discussed. The conditions the coefficients must satisfy are given by (43)-(55) and (58)-(68).

7 Programs and Data

The programs and data used by this paper are enumerated in the readme.txt file accompanying this appendix.

8 References

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