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Abstract

This paper addresses the effectiveness of fiscal policy at zero nominal interest rates. I analyze a stochastic general equilibrium model with sticky prices and rational expectations and assume that the government cannot commit to future policy. Real government spending increases demand by increasing public consumption. Deficit spending increases demand by generating inflation expectations. I derive fiscal spending multipliers that calculate how much output increases for each dollar of government spending (real or deficit). Under monetary and fiscal policy coordination, the real spending multiplier is 3.4 and the deficit spending multiplier is 3.8. However, when there is no policy coordination, that is, when the central bank is "goal independent," the real spending multiplier is unchanged but the deficit spending multiplier is zero. Coordination failure may explain why fiscal policy in Japan has been relatively less effective in recent years than during the Great Depression.

Key words: policy coordination, fiscal multiplier, zero interest rates, deflation

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1 Introduction

"It is important to recognize that the role of an independent central bank is different in inflationary and deflationary environments. In the face of inflation, which is often associated with excessive monetization of government debt, the virtue of an independent central bank is its ability to say "no" to the government. With protracted deflation, however, excessive monetary creation is unlikely to be the problem, and a more cooperative stance on the part of the central bank may be called for."

- Ben Bernanke, Chairman of the Board of Governors of the Federal Reserve, before the Japan Society of Monetary Economics, Tokyo, Japan, May 31, 2003.

"Coordinate, Coordinate

If monetary policy lacks sufficient power on its own to end deflation, the solution is not to give up but to try a coordinated monetary and fiscal stimulus."

– The Economist, June 2003, editorial on Japan's fiscal and monetary policy

The conventional wisdom about monetary and fiscal policy is as follows[1]: "The first line of defence against an economic slump is monetary policy: the ability of the central bank – the Federal Reserve, the European central bank, the Bank of Japan – to cut interest rates. Lower real interest rates persuade businesses and consumers to borrow and spend, which creates new jobs, encourages people to spend more, and so on. Since the 1930's this strategy has worked. Specifically, interest rate cuts have pulled the US out of each of its big recessions in the past 30 years – in 1975, 1982 and 1991. The second line of defense is fiscal policy: If cutting interest rates is not enough to support the economy, the government can pump up demand by cutting taxes or its own spending. The conventional wisdom among economic analysts is that fiscal policy is not necessary to deal with most recessions, that interest rate policy is enough. But the possibility of fiscal action always stands in reserve."

When the central bank has cut the short-term nominal interest rate to zero, the second line of defense may be needed, especially if the economy faces excessive deflation. Many economists believe it was wartime government spending that finally pulled the US out of the Great Depression, a period in which the short-term nominal interest rate had been close to zero for several years. Recent events in Japan, however, raise questions about the effectiveness of fiscal policy. The Bank of Japan (BOJ) cut the short-term nominal interest rate to zero in 1998 and since then the budget deficit has ballooned with the gross public debt exceeding 150 percent of GDP today. Yet deflation persisted and unemployment remained high despite several "fiscal stimulus" programs (although

[1] The paragraph in the quotation mark is a summary of Krugman's (2001) account of the conventional wisdom.

recent data indicates that the Japenese economy is finally improving, see e.g. Eggertsson and Ostry (2005) for discussion of the recovery and the role of policy in supporting it). Is standard fiscal and monetary policy insufficient to curb deflation and increase demand at low interest rates? Should we overturn the conventional Keynesian wisdom, based on the Japanese experience, as some economist have argued?[2]

This paper addresses these questions from a theoretical perspective by analyzing a stochastic general equilibrium model with sticky prices. The interest rate decline is due to temporary demand shocks that make the natural rate of interest – i.e. the real interest rate consistent with zero output gap – temporarily negative, resulting in excessive deflation and an output collapse. I assume that the government, the treasury, and the central bank, cannot commit to future policy apart through the issuance of bonds (following Lucas and Stokey (1983) I assume that the treasury can commit to pay back the nominal value of its debt). The equilibrium concept used throughout the paper is that of Markov Perfect Equilibria (MPE), a relatively standard solution concept in game theory, formally defined by Maskin and Tirole (2001).

I analyze two fiscal policy options to increase demand at low interest rates. The first is increasing real government spending, i.e. raising government consumption (holding the budget balanced). The second is increasing deficit spending, i.e. cutting taxes and accumulating debt (holding real government spending constant). The central conclusion of the paper is that either deficit spending or real government spending can be used to eliminate deflation and increase demand when the short-term nominal interest rate is zero. Of the two options I find deficit spending is more effective, both in terms of reducing deflation/increasing output in equilibrium, and improving aggregate welfare. This conclusion may seem to vindicate the conventional wisdom.

There are, however, at least three twists to the conventional Keynesian wisdom. First, deficit spending is only effective if fiscal and monetary policy are coordinated. If there is no coordination, deficit spending has no effect. This may help explain the weak response of the Japanese economy to the deficits in recent years. I argue that this is because the deficit spending has not been accumulated in the context of a coordinated reflation program by the Ministry of Finance and the Bank of Japan, in contrast to the coordinated reflation policy in the US and Japan during the Great Depression. Second, real government spending does not only work through current spending as the conventional wisdom maintains. It also works through expectations about future spending. Indeed, under optimal fiscal policy, expectations about future spending are even more important than current spending, contrary to the old fashion IS-LM model where expectations are fixed. Third, as described in better detail below, the quantitative effectiveness of fiscal policy is *much larger* than found by the traditional literature, especially when monetary and fiscal policy are coordinated.

[2]See e.g. Krugman (2001).

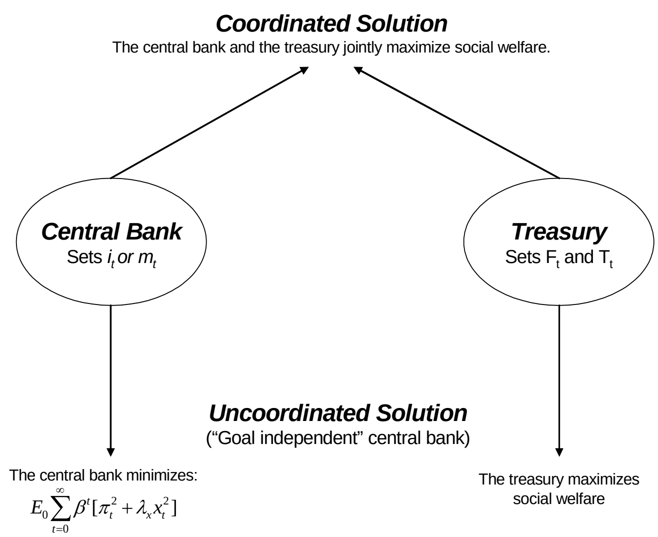

Diagram 1: The central bank and the treasury can act together or separately when setting their policy instruments.

What does coordination of monetary and fiscal policy mean in this paper? The institutional arrangement considered is illustrated in diagram 1. There are two government agencies, the central bank and the treasury. The central bank sets the interest rate, i_t, (or alternatively the money supply M_t). The treasury decides on government consumption spending F_t, and taxes T_t. Policy is coordinated when the treasury and the central bank join forces to maximize social welfare. Policy is uncoordinated when each agency pursues its own objectives. The example I consider for uncoordinated policy is when the treasury maximizes social welfare but the central bank pursues a more narrow objective. I refer to this institutional arrangement as a case in which the central bank is "goal independent". I assume that the goal independent central bank minimizes the quadratic deviation of inflation and output from a target, a relatively standard objective in the literature. The main difference between coordinated and uncoordinated solution in the model is that the goal independent central bank does not take into account the fiscal consequences of its actions.

As emphasized by Ben Bernanke, Chairman of the Federal Reserve (quoted above), one of the standard arguments for an independent central bank is its ability to say "no" to the government's wishes to "monetize its debt". For standard dynamic inconsistency reasons it is good for a goal independent central bank to ignore the fiscal consequences of its actions, because the presence of nominal government debt gives it an inefficient bias to inflate (see e.g. Calvo (1977)). This remains true in this paper so that in normal circumstances (i.e. in the absence of deflationary shocks) it is optimal to endow a goal independent central bank with a more narrow mandate than social welfare. In a deflationary environment, however, this is no longer true so that at least "temporarily" coordination of monetary and fiscal policy is beneficial, which requires a common objective of maximizing social welfare, as suggested by Bernanke (2003). Indeed, conditional on deflationary shocks, coordination is crucial to ensure the effectiveness of fiscal policy as shown in Table 1 and 2 which summarize the central quantitative results of the paper.

Table 1. Fiscal Multipliers for Coordinated Policy

	$i = 0$	$i > 0$
Real Spending Multiplier	3.37	0.50
Deficit Spending Multiplier	3.76	0.81

Table 2. Fiscal Multipliers for Uncoordinated Policy

	$i = 0$	$i > 0$
Real Spending Multiplier	3.37	0.50
Deficit Spending Multiplier	0	0

Table 1 and 2 summarize the power of fiscal spending by computing dynamic multipliers. The first column of Table 1 shows the multipliers under coordination conditional on shocks that make the zero bound binding. The thought experiment is to compare equilibrium output and fiscal spending under two scenarios: one when fiscal policy is used for stabilization (either real or deficit spending), and a second when fiscal policy is inactive. By comparing these two equilibria I can compute a dynamic multipliers. The multipliers answer the question: by how many dollars does each dollar of fiscal spending (real or deficit) increase output moving from one equilibria to the other? In computing the multiplier I calculate spending and output in expected present value. I find that under coordination each dollar of real government spending increases output by 3.37 dollars and each dollar of deficit spending increases output by 3.76 dollars. These multipliers are much bigger than have been found in the traditional Keynesian literature. The most cited paper on fiscal policy during the Great Depression, for example, is Brown (1956). In his baseline calibration the real spending multiplier is 0.5 and the deficit spending multiplier is 2.[3] The reason for this large difference is that the old models ignore the expectations channel. Modelling expectations is the key to understand the large effect of government spending. The expectation channel is that the fiscal expansion increases expectations about future inflation, which reduces real interest rates thus stimulating spending, and also increases expectations about future income which further stimulates spending.

[3] See Table 1 in Brown (1956). Column 14 is his baseline calibration where he assumes: a="marginal propensity to spend disposable income and profits"=0.8 and b="marginal propensity to spend national product"=0.6. The real spending multiplier in his model is $\frac{1-a}{1-b}$ and the deficit spending multiplier is $\frac{a}{1-b}$ which give the numbers cited above.

In the second column of Table 1 I also compute the same multipliers assuming no shocks (so that interest rates are positive) but that fiscal spending follows the same path *as if* the shocks occurred. In this case the multipliers are much smaller. This illustrates that fiscal policy is unusually powerful when interest rates are zero and there is inefficient deflation. The reason is that when interest rates are zero the central bank will accommodate any increase in demand because inflation and output are below their desired social optimum. At positive interest rates, in contrast, the central bank will counteract the fiscal expansion to some extent.

Table 2 computes the multipliers when monetary and fiscal policy are uncoordinated. In this case the multiplier is unchanged for real spending but there is a big change in the multiplier of deficit spending. Without coordination deficit spending has *no effect* so that the multiplier is zero. The reason is that deficit spending works entirely through expectations about future interest rate policy (i.e. through the expectation of higher future money supply). Under coordinated policy deficit spending implies higher nominal debt, and optimal monetary policy under discretion implies that this will increase inflation expectations, because higher nominal debt makes a permanent increase in the money supply incentive compatible. Without coordination, however, this link is broken because the central bank has a narrow objective that does not take into account the fiscal consequences of its actions. Instead there is strong deflation bias of discretionary monetary policy which is severely suboptimal when there are deflationary shocks. This indicates that a more "cooperative stance" is required by the central bank when there is deflation, as suggested by Bernanke (2003).

In a companion paper (Eggertsson (2006)) I study a similar model but with two important differences. That paper does not study the effect of increasing real government spending and does not analyze the role of coordination of monetary and fiscal policy. In a related paper (Eggertsson (2005)) I apply a simplified version of the theoretical framework presented here (but without any analysis of coordination) to study the US recovery from the Great Depression.

Two lines of research have emerged on the zero bound. The first attributes zero interest rates to a suboptimal policy rule and views the liquidity trap as an example of a self-fulfilling "bad equilibrium" that is not driven by real shocks. The solution is for the government to commit to a different policy rule that eliminates the self-fulfilling "bad" equilibria (leading examples of this approach include Benhabib et al (2002) and Buiter (2003)). The other line of research attributes deflation and the zero bound to an inefficient policy response to real disturbances. In this case the zero bound can either be binding because of an inefficient policy rule (see e.g. Eggertsson and Woodford (2003)) or because of the government's inability to commit to future policy (see Eggertsson (2006)). This paper follows the second line of research so that the zero bound is binding due temporary real disturbances and the resulting equilibrium may be suboptimal due to the government's policy constraints and inability to commit to future policy. As emphasized by Krugman (1998), Eggertsson and Woodford (2003), Jung et al (2006), Adam and Billi (2006), and Nakov (2006), the optimal policy is to commit to higher future inflation but this policy may not be credible if the government has no explicit commitment mechanism. In this paper deficit spending is mainly useful because it helps the government to solve this commitment problem. Real government spending is mainly effective because it reduces the potency of negative shocks by increasing aggregate spending when the zero bound is binding. Jeanne and Svensson (2004) and Eggertsson (2006) consider some alternatives to fiscal policy, such as exchange rate interventions, to solve this commitment problem.

A body of literature has emerged in recent years emphasizing the connection between the price level and fiscal policy. This literature is often referred to as the Fiscal Theory of the Price Level (FTPL) (see e.g. Leeper (1992), Sims (1994), Woodford (1996), and Sargent and Wallace (1981) for an early contribution). A key difference between the approach in this paper and the FTPL is the way the government is modelled. Papers applying the FTPL often model the central bank as committing to a (possibly suboptimal) interest rate feedback rule, and fiscal policy is modelled as a (possibly suboptimal) exogenous path of real government surpluses (typically abstracting away from any variations in real government spending). Under these assumptions innovations in real government surpluses may influence the price level since prices may have to move for the government's budget constraint to be satisfied (because any changes in the policy choices of the government are ruled out by assumption, i.e. by the assumed policy commitments of the government). In contrast, in my setting, fiscal policy can only affect the price level because it changes the government's future inflation incentive or because real government spending directly increases demand.

2 The Model

Here I outline a simple sticky price general equilibrium model and define the set of feasible equilibrium allocations that are consistent with the private sector maximization problems and the technology constraints the government faces.

2.1 The private sector

2.1.1 Households

I assume that there is a representative household that maximizes expected utility over the infinite horizon:

$$E_t \sum_{T=t}^{\infty} \beta^T U_T = E_t \left\{ \sum_{T=t}^{\infty} \beta^T [u(C_T, \frac{M_T}{P_T}, \xi_T) + g(G_T, \xi_T) - \int_0^1 v(h_T(i), \xi_T) di] \right\} \tag{1}$$

where C_t is a Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods

$$C_t \equiv [\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}}$$

with elasticity of substitution equal to $\theta > 1$, G_t is is a Dixit-Stiglitz aggregate of government consumption, ξ_t is a vector of exogenous shocks, M_t is end-of-period money balances, P_t is the Dixit-Stiglitz price index,

$$P_t \equiv [\int_0^1 p_t(i)^{1-\theta}]^{\frac{1}{1-\theta}}$$

and $h_t(i)$ is the quantity supplied of labor of type i. $u(.)$ is assumed to be concave and strictly increasing in C_t for any possible value of ξ. The utility of holding real money balances is assumed to be increasing in $\frac{M_t}{P_t}$ for any possible value of ξ up to a satiation point at some finite level of real money balances as in Friedman (1969).[4] $g(.)$ is the utility of government consumption and is assumed to be concave and strictly increasing in G_t for any possible value of ξ. $v(.)$ is the disutility of supplying labor of type i and is assumed to be an increasing and convex in $h_t(i)$ for any possible value of ξ. E_t denotes mathematical expectation conditional on information available in period t. ξ_t is a vector of r exogenous shocks. I assume that ξ_t follows a Markov process so that:[5]

A1 *(i)* $pr(\xi_{t+j}|\xi_t) = pr(\xi_{t+j}|\xi_t, \xi_{t-1}, ...)$ *for* $j \geq 1$ *where* $pr(.)$ *is the conditional probability density function of* ξ_{t+j}.

For simplicity I assume complete financial markets and no limit on borrowing against future income. As a consequence, a household faces an intertemporal budget constraint of the form:

$$E_t \sum_{T=t}^{\infty} Q_{t,T}[P_T C_T + \frac{i_T - i^m}{1+i_T} M_T] \leq W_t + E_t \sum_{T=t}^{\infty} Q_{t,T}[\int_0^1 Z_T(i)di + \int_0^1 n_T(j)h_T(j)dj - P_T T_T] \quad (2)$$

looking forward from any period t. Here $Q_{t,T}$ is the stochastic discount factor that financial markets use to value random nominal income at date T in monetary units at date t; i_t is the riskless nominal interest rate on one-period obligations purchased in period t, i^m is the nominal interest rate paid on money balances held at the end of period t, W_t is the beginning of period nominal wealth at time t (note that its composition is determined at time $t-1$ so that it is

[4] The idea is that real money balances enter the utility because they facilitate transactions. At some finite level of real money balances, e.g. when the representative household holds enough cash to pay for all consumption purchases in that period, holding more real money balances will not facilitate transaction any further and thereby add nothing to utility. This is at the "satiation" point of real money balances. We assume that there is no storage cost of holding money so increasing money holding can never reduce utility directly through $u(.)$. A satiation level in real money balances is also implied by several cash-in-advance models such as Lucas and Stokey (1987) or Woodford (1998).

[5] Assumption A1 is the Markov property. Since ξ_t is a vector of shocks this assumption is not very restrictive since I can always augment this vector by lagged values of a particular shock.

equal to the sum of monetary holdings from period $t-1$ and return on non-monetary assets), $Z_t(i)$ is the time t nominal profit of firm i, $n_t(i)$ is the nominal wage rate for labor of type i, T_t is net real tax collections by the government. The problem of the household is: at every time t the household takes W_t and $\{Q_{t,T}, n_T(i), P_T, T_T, Z_T(i), \xi_T; T \geq t\}$ as exogenously given and maximizes (1) subject to (2) by choice of $\{M_T, h_T(i), C_T; T \geq t\}$.

2.1.2 Firms

The production function of the representative firm that produces good i is:

$$y_t(i) = f(h_t(i), \xi_t) \tag{3}$$

where f is an increasing concave function for any ξ and ξ is again the vector of shocks defined above (that may include productivity shocks). I abstract from capital dynamics. As Rotemberg (1983), I assume that firms face a cost of price changes given by the function $d(\frac{p_t(i)}{p_{t-1}(i)})$[6] but I can derive exactly the same result assuming that firms adjust their prices at stochastic intervals as assumed by Calvo (1983).[7] Price variations have a welfare cost that is separate from the cost of expected inflation due to real money balances in utility. The Dixit-Stiglitz preferences of the household imply a demand function for the product of firm i given by

$$y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$$

The firm maximizes

$$E_t \sum_{T=t}^{\infty} Q_{t,T} Z_T(i) \tag{4}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{u_c(C_T, \frac{M_T}{P_T}, \xi_T)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_T} \tag{5}$$

I can write firms period profits as:

$$Z_t(i) = (1+s) Y_t P_t^{\theta} p_t(i)^{1-\theta} - n_t(i) f^{-1}(Y_t P_t^{\theta} p_t^{-\theta}) - P_t d(\frac{p_t(i)}{p_{t-1}(i)}) \tag{6}$$

where s is an exogenously given production subsidy that I introduce for algebraic convenience (for reasons described below).[8] The problem of the firm is: at every time t the firm takes

[6] I assume that $d'(\Pi) > 0$ if $\Pi > 1$ and $d'(\Pi) < 0$ if $\Pi < 1$. Thus both inflation and deflation are costly. $d(1) = 0$ so that the optimal inflation rate is zero (consistent with the interepretation that this represent a cost of changing prices). Finally, $d'(1) = 0$ so that in the neighborhood of the zero inflation the cost of price changes is of second order.

[7] The reason I do not assume Calvo prices is that it complicates to solution by introducing an additional state variable, i.e. price dispersion. This state variable, however, has only second order effects local to the steady state I approximate around and the resulting equilibrium is to first order exactly the same as derived here.

[8] I introduce it so that I can calibrate an inflationary bias that is independent of the other structural parameters, and this allows me to define a steady state at the fully efficient equilibrium allocation. I abstract from any tax costs that the financing of this subsidy may create.

$\{n_T(i), Q_{t,T}, P_T, Y_T, C_T, \frac{M_T}{P_T}, \xi_T; T \geq t\}$ as exogenously given and maximizes (4) by choice of $\{p_T(i); T \geq t\}$.

2.1.3 Private Sector Equilibrium Conditions: AS, IS and LM Equations

This subsection illustrates the necessary conditions for equilibrium that stem from the maximization problems of the private sector. These conditions must hold for *any* government policy. The first order conditions of the household maximization imply an Euler equation of the form:

$$\frac{1}{1+i_t} = E_t\{\frac{\beta u_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}, \xi_{t+1})}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \frac{P_t}{P_{t+1}}\} \tag{7}$$

where i_t is the nominal interest rate on a one period riskless bond. This equation is often referred to as the IS equation. Optimal money holding implies:

$$\frac{u_{\frac{M}{P}}(C_t, \frac{M_t}{P_t}, \xi_t)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} = \frac{i_t - i^m}{1+i_t} \tag{8}$$

This equation defines money demand and is often referred to as the "LM" equation. Utility is increasing in real money balances. At some finite level of real money balances, further holdings of money add nothing to utility so that $u_{\frac{M}{P}} = 0$. The left hand side of (8) is therefore weakly positive. Thus there is bound on the short-term nominal interest rate given by:

$$i_t \geq i^m \tag{9}$$

In most economic discussions it is assumed that the interest paid on the monetary base is zero so that (9) becomes $i_t \geq 0$.[9]

The optimal consumption plan of the representative household must also satisfy the transversality condition[10]

$$\lim_{T\to\infty} E_t Q_{t,T} \frac{W_T}{P_t} = 0 \tag{10}$$

to ensure that the household exhausts its intertemporal budget constraint. I assume that workers are wage takers so that the households optimal choice of labor supplied of type j satisfies

$$n_t(j) = \frac{P_t v_h(h_t(j); \xi_t)}{u_c(C_t, \frac{M_t}{P_t}, \xi_t)} \tag{11}$$

I restrict my attention to a symmetric equilibria where all firms charge the same price and produce the same level of output so that

$$p_t(i) = p_t(j) = P_t; \quad y_t(i) = y_t(j) = Y_t; \quad n_t(i) = n_t(j) = n_t; \quad h_t(i) = h_t(j) = h_t \quad \text{for} \quad \forall\ j, i \tag{12}$$

[9] The intuition for this bound is simple. There is no storage cost of holding money in the model and money can be held as an asset. It follows that i_t cannot be a negative number. No one would lend 100 dollars if he or she would get less than 100 dollars in return.

[10] For a detailed discussion of how this transversality condition is derived see Woodford (2003).

Given the wage demanded by households I can derive the aggregate supply function from the first order conditions of the representative firm, assuming competitive labor market so that each firm takes its wage as given. I obtain the equilibrium condition often referred to as the AS or the "New Keynesian" Phillips curve:

$$\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(C_t,\frac{M_t}{P_t},\xi_t)-\tilde{v}_y(Y_t,\xi_t)]+u_c(C_t,\frac{M_t}{P_t},\xi_t)\frac{P_t}{P_{t-1}}d'(\frac{P_t}{P_{t-1}}) \\ -E_t\beta u_c(C_{t+1},\frac{M_{t+1}}{P_{t+1}},\xi_{t+1})\frac{P_{t+1}}{P_t}d'(\frac{P_{t+1}}{P_t})=0 \tag{13}$$

where for notational simplicity I have defined the function:

$$\tilde{v}(y_t(i),\xi_t)\equiv v(f^{-1}(y_t(i)),\xi_t) \tag{14}$$

2.2 The Government

There is an output cost of taxation (e.g. due to tax collection costs as in Barro (1979)) captured by the function $s(T_t)$.[11] For every dollar collected in taxes $s(T_t)$ units of output are wasted without contributing anything to utility. Government real spending is then given by:

$$F_t = G_t + s(T_t) \tag{15}$$

I could also define the tax cost that would result from distortionary taxes on income or consumption and obtain similar results.[12] I assume a representative household so that in a symmetric equilibrium, all nominal claims held are issued by the government. It follows that the government flow budget constraint is:

$$B_t + M_t = W_t + P_t(F_t - T_t) \tag{16}$$

where B_t is the end-of-period nominal value of bonds issued by the government. Having defined both private and public spending I can verify that market clearing implies that aggregate demand satisfies:

$$Y_t = C_t + d(\frac{P_t}{P_{t-1}}) + F_t \tag{17}$$

[11] The function $s(T)$ is assumed to be differentiable with $s'(T)>0$ and $s''(T)>0$ for $T>0$.

[12] The specification used here, however, gives very clear result that clarifies the main channel of taxations that I am interested in. This is because for a constant F_t the level of taxes has no effect on the private sector equilibrium conditions (see equations above) but will only affect the equilibrium by reducing the utility of the households because a higher tax costs mean lower government consumption G_t. This allows me to isolate the effect current tax cuts will have on expectation about future monetary and fiscal policy, abstracting away from any effect on relative prices that those tax cuts may have. It is thus they key behind the proposition that deficit spending has no effect when the central bank is goal independent. There is no doubt the effect of tax policies on relative prices is important, but that issue is quite separate from the main focus of this paper. Eggertsson and Woodford (2004) consider how taxes that change relative prices can be used to affect the equilibrium allocations assuming full commitment. They find that this channel can be quite important.

I now define the set of possible equilibria that are consistent with the private sector equilibrium conditions and the technological constraints on government policy.

Definition 1 *Private Sector Equilibrium (PSE) is a collection of stochastic processes*

$\{P_t, Y_t, W_{t+1}, B_t, M_t, i_t, F_t, T_t, Q_t, Z_t, G_t, C_t, n_t, h_t, \xi_t\}$ *for* $t \geq t_0$ *that satisfy equations (2)-(17) for each* $t \geq t_0$*, given* W_{t_0} *and* $P_{t_{0-1}}$ *and the exogenous stochastic process* $\{\xi_t\}$ *that satisfies A1 for* $t \geq t_0$.

2.3 Recursive representation

It is useful to rewrite the model in a recursive form so that I can identify the endogenous state variables at each date. When the government only issues one period nominal debt I can write the total nominal liabilities of the government (which in equilibrium are equal to the total nominal wealth of the representative household) as:

$$W_{t+1} = (1 + i_t)B_t + (1 + i^m)M_t$$

Substituting this into (16) and defining the variables $w_t \equiv \frac{W_{t+1}}{P_t}$, $m_t \equiv \frac{M_t}{P_{t-1}}$ and $\Pi_t = \frac{P_t}{P_{t-1}}$ I can write the government budget constraint as:

$$w_t = (1 + i_t)(w_{t-1}\Pi_t^{-1} + (F_t - T_t) - \frac{i_t - i^m}{1 + i_t} m_t \Pi_t^{-1}) \tag{18}$$

Note that I use the time subscript t on w_t (even if it denotes the real claims on the government at the beginning of time $t+1$) to emphasize that this variable is determined at time t. I impose a borrowing limit on the government that rules out Ponzi schemes:

$$u_c w_t \leq \bar{w} < \infty \tag{19}$$

where $\bar{w}$ is an arbitrarily high finite number. This condition can be justified by the constraint that that the government can never borrow more than the equivalent of the expected discounted value of its maximum tax base (e.g. discounted future value of all future output).[13] It is easy to show that this limit ensures that the representative household's transversality condition is satisfied at all times.

The treasury's policy instruments are taxation, T_t, that determines the end-of-period government debt which is equal to $B_t + M_t$, and real government spending F_t. The central bank determines how the end-of-period debt is split between bonds and money by open market operations. Thus the central bank's policy instrument is M_t. Since P_{t-1} is determined in the previous period I alternatively consider $m_t \equiv \frac{M_t}{P_{t-1}}$ as the instrument of monetary policy at any time t.

[13] Since this constraint will never be binding in equilibrium and $\bar{w}$ can be any arbitrarily high number to derive the results in the paper. For this reason I do not model in detail the endogenous value of the debt limit.

It is useful to note that I can reduce the number of equations that are necessary and sufficient for a private sector equilibrium substantially from those listed in Definition 1. First, note that the equations that determine $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ are redundant, i.e. each of them is only useful to determine one particular variable that has no effect on any of the other variables. Thus I can define the necessary and sufficient condition for a private sector equilibrium without specifying the stochastic process for $\{Q_t,\ Z_t, G_t, C_t, n_t, h_t\}$ and do not need to consider equations (3), (5), (6), (11), (15) and I use (17) to substitute out for C_t in the remaining conditions. Furthermore condition (19) ensures that the transversality condition of the representative household is satisfied at all times so I do not need to include (10) in the list of necessary and sufficient conditions.

It is useful to define the expectation variable

$$f_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F_t, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1}^{-1} \tag{20}$$

as the part of the nominal interest rate that is determined by the expectations of the private sector formed at time t. Here I have used (17) to substitute for consumption in the utility function. The IS equation can then be written as:

$$1 + i_t = \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{\beta f_t^e} \tag{21}$$

Similarly it is useful to define the expectation variable

$$S_t^e \equiv E_t u_c(Y_{t+1} - d(\Pi_{t+1}) - F_t, m_{t+1}\Pi_{t+1}^{-1}, \xi_{t+1})\Pi_{t+1} d'(\Pi_{t+1}) \tag{22}$$

The AS equation can now be written as:

$$\theta Y_t[\frac{\theta - 1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] + u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e = 0 \tag{23}$$

Finally the money demand equation (8) can be written in terms of m_t and Π_t as

$$\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, \xi_t)} = \frac{i_t - i^m}{1 + i_t} \tag{24}$$

The next two propositions are useful to characterize equilibrium outcomes. Proposition 1 follows directly from our discussion above:

Proposition 1 *A necessary and sufficient condition for the set of variables* $(\Pi_t, Y_t, F_t, w_t, m_t, i_t, T_t)$ *in a PSE at each time* $t \geq t_0$ *is that they satisfy : (i) conditions (9), (18),(19), (21), (23) and (24) given* w_{t-1} *and the expectations* f_t^e *and* S_t^e. *(ii) in each period* $t \geq t_0$, *expectations are rational so that* f_t^e *is given by (20) and* S_t^e *by (22).*

Proposition 2 *The possible PSE equilibrium for the variables* $(\Pi_t, Y_t, F_t, w_t, m_t, i_t, T_t)$ *defined by the necessary and sufficient conditions for any date* $t \geq t_0$ *onwards depends only on* w_{t_0-1} *and* ξ_{t_0}.

The second proposition follows from observing that w_{t-1} is the only endogenous variable that enters with a lag in the necessary conditions specified in (i) of Proposition 1 and using the assumption that ξ_t is Markovian (i.e. using A1) so that the conditional probability distribution of ξ_t for $t > t_0$ only depends on ξ_{t_0}. It follows from this proposition (w_{t-1}, ξ_t) are the only state variables at any time t that directly affects the PSE.

2.4 Policy Objectives and Policy Games

To define equilibrium I need to specify policy objectives for the government, i.e. the treasury and the central bank. Throughout this paper I assume that the treasury maximizes social welfare, which is given by the utility of the representative household. Furthermore, following Lucas and Stokey (1983), I assume that the treasury can commit to paying the future face value of debt, which is assumed to be issued in nominal terms. The treasury cannot commit to any other future policy action and I only consider Markovian strategies that will be more precisely defined in the next section. Whereas fiscal policy maximizes social welfare at all times, I consider monetary policy under two institutional arrangement. Under the first arrangement, which I call "coordination", monetary and fiscal policy are coordinated to maximize social welfare. I define the maximization problem in the next two sections when I define the Markov Perfect Equilibrium.

A2 Coordinated Fiscal and Monetary Policy. *The government, i.e. the treasury and the central bank, determine F_t, T_t and m_t to maximize the utility of the representative household.*

Under the second institutional arrangement, I assume that monetary policy is delegated to satisfy goals that are different than social welfare. This is what Svensson (1999) calls a flexible inflation target and I refer to as a "goal independent" central bank. In this case the central bank seeks to minimize the criterion $L_t = [\pi_t^2 + \lambda_x x_t^2]$ where x_t is the output gap, defined as the percentage difference between actual output, Y_t, and the natural rate of output, Y_t^n, i.e. $x_t \equiv Y_t/Y_t^n - 1$. The natural rate of output is the output that would be produced if prices where completely flexible, i.e. it is the output that solves the equation

$$v_y(Y_t^n, \xi_t) = \frac{\theta - 1}{\theta}(1 + s)u_c(Y_t^n, \xi_t). \tag{25}$$

There is a long tradition in the literature of assuming that this loss function describes the the behavior of independent central banks. Under the flexible inflation target the central bank minimizes its loss function and the treasury sets taxes and real spending to maximize social welfare.

A3 Goal Independent Central Bank. *The central bank sets m_t to maximize $-E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda_x x_t^2]$. The treasury sets T_t and F_t to maximize the utility of the representative household.*

The motivation for A3 is that in several industrial countries monetary policy has been separated from fiscal policy and given to independent central bankers. It is common practice to give the central bank a fairly narrow mandate such as aiming for "price stability" and protecting employment. The central banks mandate almost never includes any considerations of fiscal variables. Indeed the move towards central bank independence has often involved explicitly excluding fiscal policy from the bank's goals. In the case of Japan, for example, the Diet explicitly forbade the BOJ from underwriting government bonds after the experience of hyperinflation in World War II. Similarly the Federal Reserve's role in government finances was substantially reduced in the 1950s. I argue later in the paper that these institutional reforms may make some sense under normal circumstances (especially when inflation is a problem). They can, however, limit the effectiveness of fiscal and monetary policy when the economy is plagued by deflation. I argue that cooperation (at least temporarily) between the treasury and the central bank, as defined in A2, may be useful to fight deflation. Note that A3, i.e. the goal independent central bank, is consistent with Rogoff's (1985) conservative central banker and is also consistent with Dixit and Lambertini (2003) institutional framework, but the latter authors also assume that the treasury maximizes social welfare but the central bank has more narrow goals.[14]

Coordination does not necessarily mean that central bank independence is reduced if one thinks of "independence" as meaning the ability of the central bank to set its own policy instruments. Indeed, as Bernanke (2003) argues, cooperation between the central bank and the treasury need not imply the elimination of the central bank's independence "any more than cooperation between two independent nations in pursuit of a common objective is inconsistent with the principle of national sovereignty." Bernanke's interpretation of "cooperation" as a "pursuit of common objective" is consistent with A2 where this common objective is simply social welfare. Thus although I will refer to A3 as "goal independence", in practice a move towards coordination of policy would not imply that the instrumental independence of the central bank is reduced. No particular institutional changes are needed to move from the institutional framework defined in A3 to A2, the central bank can itself simply state the fiscal health of the government as one of its policy concerns and act accordingly (see further discussion with historical examples in section 6)

[14] There are two key differences between this analysis and Dixit and Lambertini (2003). First, in their model fiscal policy is a choice of the optimal subsidy/tax on the private sector thus changing the equilibrium markup of firms. Here I abstract from any effect fiscal policy can have on relative prices and instead focus on deficit spending and real spending as the principal tools of policy (and these policy instruments have no effect on the markup of firms). Second, and perhaps more obviously, their paper does not address the questions posed by the zero bound.

3 Markov Perfect Equilibrium under Coordinated Monetary and Fiscal Policy: A Definition and an Approximation Method

3.1 Defining a Markov Equilibrium under Coordination

In this section I define a Markov Perfect Equilibrium (MPE) under A2. I defer to section 5 to discuss the case when the central bank is goal independent. A MPE is formally defined by Maskin and Tirole (2001) and has been extensively applied in the monetary literature. The basic idea behind this equilibrium concept is to restrict attention to equilibria that only depend on the minimum set of variables that directly affect market conditions. The assumption of a MPE has several advantages. The first is that it allows us to use modern game theory to analyze an infinite game between the private sector and the government. A common criticism of policy proposals, e.g. for the BOJ, is that they are not credible. A MPE is subgame perfect, so that the government has no incentive to deviate from it's policy. Since no one has an incentive to deviate in a MPE the policies analyzed are, by construction, fully credible. The second advantage of assuming no commitment is that it gives a rigorous theory of expectations. As emphasized by Eggertsson and Woodford (2003), expectations about future policy are crucial to understanding the effect of different policy alternatives. Analyzing a MPE provides a clear theory of how expectations about future policy are formed: Agents are rational so they anticipate future actions of the government. The government's future policy actions, on the other hand, are determined by its *incentives* from that period onwards. The third advantage of assuming no commitment is that it is rare for a central bank or a treasury to announce future policies that cannot be reversed in the light of new circumstances (apart from paying back debt issued). Furthermore, since most governments are elected for short periods of time future regimes may not regard their predecessors announcements as binding.[15]

The timing of events in the game is: at the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector and the government. The monetary and fiscal authorities choose policy for period t given the state and the private sector forms expectations f_t^e and S_t^e. I assume that the private sector may condition its expectation at time t on w_t, i.e. it observes the

[15] I do not mean to claim, however, that government agencies cannot make any binding commitments under any circumstances. But the assumption about imperfect commitment is particularly appealing when the zero bound is binding. As emphasized by Krugman (1998) (and shown in Eggertsson and Woodford (2003) in fully stochastic dynamic general equilibrium model) when the zero bound is binding, the optimal commitment by the government is to increase inflation expectations. This type of commitment, however, may be unusually hard to achieve in a deflationary environment. One reason is that it requires no actions. Since the short-term nominal interest rate is already at zero the central bank cannot use it's standard policy tool to make this commitment visible to the private sector. The second is that most central banks have required reputation for fighting inflation. Announcing a positive inflation target without direct actions to achieve it, therefore, may not be very effective to change expectations.

policy actions of the government in that period so that expectations are determined jointly with the other endogenous variables. This is important because w_t is the relevant endogenous state variable at date $t+1$. The set of feasible equilibria that can be achieved by the policy decisions of the government are those that satisfy the equations given in Propositions 1 given the values of w_{t-1}, ξ_t and the expectation f_t^e and S_t^e.

I may economize on notation by introducing vector notation. I define vectors

$$\Lambda_t \equiv \begin{bmatrix} \Pi_t & Y_t & i_t & m_t & F_t & T_t \end{bmatrix}^T, \text{ and } e_t \equiv \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix}.$$

Since Proposition 2 indicates that w_t is the only endogenous state variable I prefer not to include it in either vector but keep track of it separately. Proposition 2 also indicates that a Markov equilibrium requires that the variables (Λ_t, w_t) only depend on (w_{t-1}, ξ_t), since this is the minimum set of state variables that affect the private sector equilibrium. Thus, in a Markov equilibrium, there must exist policy functions $\bar{\Pi}(.), \bar{Y}(.), \bar{\imath}(.), \bar{m}(.), \bar{F}(.), \bar{T}(.), \bar{w}(.)$ that I denote by the vector valued function $\bar{\Lambda}(.)$ and the function $\bar{w}(.)$ such that each period:

$$\begin{matrix} \Lambda_t \\ w_t \end{matrix} \equiv \begin{matrix} \bar{\Lambda}(w_{t-1}, \xi_t) \\ \bar{w}(w_{t-1}, \xi_t) \end{matrix} \tag{26}$$

Note that the functions $\bar{\Lambda}(.)$ and $\bar{w}(.)$ also defines a set of functions of (w_{t-1}, ξ_t) for $(Q_t, Z_t, G_t, C_t, n_t, h_t)$ by the redundant equations from Definition 1. Using $\bar{\Lambda}(.)$ I may also use (20) and (22) to define a function $\bar{e}(.)$ so so that

$$e_t = \begin{bmatrix} f_t^e \\ S_t^e \end{bmatrix} = \begin{bmatrix} \bar{f}^e(w_t, \xi_t) \\ \bar{S}^e(w_t, \xi_t) \end{bmatrix} = \bar{e}(w_t, \xi_t) \tag{27}$$

Rational expectations imply that the function $\bar{e}$ satisfies:

$$\bar{e}(w_t, \xi_t) = \begin{bmatrix} E_t u_c(\bar{C}(w_t, \xi_{t+1}), \bar{m}(w_t, \xi_{t+1})\bar{\Pi}(w_t, \xi_{t+1})^{-1}; \xi_{t+1})\bar{\Pi}(w_t, \xi_{t+1})^{-1} \\ E_t u_c(\bar{C}(w_t, \xi_{t+1}), \bar{m}(w_t, \xi_{t+1})\bar{\Pi}(w_t, \xi_{t+1})^{-1}; \xi_{t+1})\bar{\Pi}(w_t, \xi_{t+1})d'(\bar{\Pi}(w_t, \xi_{t+1})) \end{bmatrix} \tag{28}$$

To economize on notation I can write the utility function as the function $U : \mathbb{R}^{6+r} \to \mathbb{R}$

$$U_t = U(\Lambda_t, \xi_t)$$

using (15) to solve for G_t as a function of F and T_t, along with (12) and (14) to solve for $h_t(i)$ as a function of Y_t. I define a value function $J(w_{t-1}, \xi_t)$ as the expected discounted value of the utility of the representative household, looking forward from period t, given the evolution of the endogenous variable from period t onwards that is determined by $\bar{\Lambda}(.)$ and $\{\xi_t\}$. Thus I define:

$$J(w_{t-1}, \xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T [U(\bar{\Lambda}(w_{T-1}, \xi_T), \xi_T] \right\} \tag{29}$$

The optimizing problem of the government is as follows. Given w_{t-1} and ξ_t the government chooses the values for (Λ_t, w_t) (by its choice of the policy instruments m_t, F_t and T_t) to maximize the utility of the representative household subject to the constraints in Proposition 1. Thus its problem can be written as:

$$\max_{m_t, F_t, T_t} [U(\Lambda_t, \xi_t) + \beta E_t J(w_t, \xi_{t+1})] \tag{30}$$

s.t. (9), (18),(19), (21), (23), (24) and (27).

I can now define a Markov Equilibrium under coordination

Definition 2 *A Markov Equilibrium under coordination is a collection of functions $\bar{\Lambda}(.)$,$\bar{w}(.)$,$J(.)$,$\bar{e}(.)$, such that (i) given the function $J(w_{t-1}, \xi_t)$ and the vector function $\bar{e}(w_t, \xi_t)$ the solution to the policy maker's optimization problem (30) is given by $\Lambda_t = \bar{\Lambda}(w_{t-1}, \xi_t)$ and $\bar{w}(.)$ for each possible state $(w_{t-1}$,$\xi_t)$ (ii) given the vector functions $\bar{\Lambda}(w_{t-1}, \xi_t)$ and $\bar{w}(w_{t-1}, \xi_t)$ then $e_t = \bar{e}(w_t, \xi_t)$ is formed under rational expectations (see equation (28)). (iii) given the vector function $\bar{\Lambda}(w_{t-1}, \xi_t)$ and $\bar{w}(w_{t-1}, \xi_t)$ the function $J(w_{t-1}, \xi_t)$ satisfies (29).*

I will only look for a Markov equilibrium in which the functions $\bar{\Lambda}(.), \bar{w}(), J(.), \bar{e}(.)$ are continuous and have well defined derivatives. The value function satisfies the Bellman equation:

$$J(w_{t-1}, \xi_t) = \max_{m_t, F_t, T_t, w_t} [U(\Lambda_t, \xi_t) + E_t \beta J(w_t, \xi_{t+1})] \tag{31}$$

s.t. (9), (18),(19), (21), (23), (24) and (27).

The solution can now be characterized by using a Lagrangian method for the maximization problem on the right hand side of (31). In addition the solution satisfies an envelope condition. The Lagrangian, the associated appropriate first order conditions, and the envelope condition, are shown in the Technical Appendix.

3.2 Approximation method

I define a steady state as a solution in the absence of shocks where each of the variables $(\Pi_t, Y_t, m_t, i_t, T_t, w_t, f_t^e, S_t^e) = (\Pi, Y, m, i, T, w, f^e, S^e)$ are constants. I define this steady state in a cashless limit at the efficient equilibrium allocation so that (see Technical Appendix for further discussion):

A4 *Steady state assumptions. (i)* $\bar{m} \to 0$, *(ii)* $1 + s = \frac{\theta}{\theta - 1}$ *(iii)* $i^m = 1/\beta - 1$

A4 (ii) implies that there is no inflation bias in steady state. In Eggertsson (2006) I relax this assumption and illustrate that the basic issue addressed here (i.e. inefficient deflation) is still a problem, provided that the shocks the economy is subject to (that I will define in A5) are correspondingly larger.

Using A2 I prove in the Technical Appendix that there exists a steady state given by $(\Pi, Y, \frac{m}{\tilde{m}}, i, F, T, w, f^e, S^e) = (1, \bar{Y}, \tilde{m}, \frac{1}{\beta} - 1, \bar{F}, \bar{T}, 0, u_c(\bar{Y} - \bar{F}), 0)$ and give the equations that the values $\bar{T}$, $\bar{F}$, $\bar{T}$ and $\tilde{m}$ must satisfy. I discuss how this result relates to the work of Albanesi et al (2003), Dedola (2002) and King and Wolman (2003) in the Technical Appendix. The solution can now be approximated by a linearization around this steady state, keeping explicit track of Kuhn-Tucker conditions that arise due to the inequality constraints. The resulting equilibrium is accurate to the order $o(||\xi||^2)$. A complication is introduced by the presence of the inequality constraints due to the Kuhn-Tucker conditions and I apply a solution method discussed in the Technical Appendix to solve this problem. As discussed in the Technical Appendix the approximate solution is also valid for $i^m = 0$ which I assume in the following sections, and the resulting solution is accurate to the order $o(||\xi, \delta||^2)$ where $\delta \equiv \frac{i - i^m}{1+i}$. A further complication arises because of the expectation function $\bar{e}(w_t, \xi_t)$ is unknown. The method to approximate this function is shown in the Technical Appendix, where I also discuss how my solution method relates to Klein et al (2003). Matlab codes implementing these solution methods are further discussed in the Technical Appendix.

4 Markov Perfect Equilibrium under Coordinated Monetary and Fiscal Policy: Results

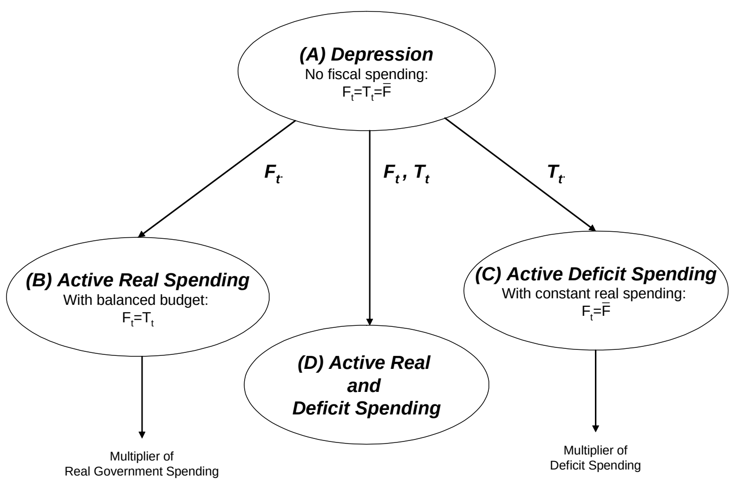

Diagram 2: Roadmap for results under coordination. The presentation of the results when the central bank is goal independent has the same structure.

This section shows results for optimal policy in a MPE under coordination, applying the definition and approximation methods described in the last section. To clarify the organization of the results diagram 2 shows a road map for the results. The goal is to analyze the power of fiscal policy to stimulate demand when interest rate are zero. To identify the power of different fiscal policy option I analyze the results in four steps (I follow exactly the same steps in the next section when I assume the central bank is goal independent). I first show the equilibrium under the condition that fiscal policy is completely inactive and the economy is subject to deflationary shocks (equilibrium A in the diagram 2). I then analyze the consequences of optimally increasing real government spending, F_t, but holding the budget balanced (so that $F_t = T_t$), which is equilibrium B in diagram 2. By comparing equilibrium A and B I can compute the multiplier of real government spending which calculates by how many dollars output increases per dollar of real government spending, moving from equilibrium A to B. In equilibrium C the government optimally use deficit spending T_t to stimulate demand but real government spending is kept constant at its steady state ($F_t = \bar{F}$). By comparing C and A I can similarly compute the multiplier of deficit spending. Finally equilibrium D considers the effect of using both deficit and real spending optimally to stimulate demand.

4.1 Equilibrium A: Pushing on a string

Consider first optimal monetary policy assuming real spending, taxes and debt are held constant, denoted equilibrium A in diagram 2, that is

$$F_t = \bar{F},\ T_t = \bar{F} = \bar{T} \text{ and } w_t = 0. \tag{32}$$

Equation (32) simply imposes additional conditions on the private sector equilibrium that the government faces relative to my previous definition of MPE. Thus I can substitute these conditions into the constraints of the maximization problem (30) and then my definition of a MPE is the same as in Definition 1 (even though in this case ξ_t is now the only relevant state variable since w_t is constant at zero).

To gain insights into the solution in an approximate equilibrium, it is useful to consider the linear approximation of the private sector equilibrium constraints. The AS equation is:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \qquad \text{AS} \tag{33}$$

where $\kappa \equiv \theta \frac{(\sigma^{-1}+\omega)}{d''}$ and $\sigma \equiv -\frac{\bar{u}_{cc} Y}{\bar{u}_c}$ and $\omega = \frac{\bar{v}_y}{\bar{v}_{yy}\bar{Y}}$ and the bar denotes that the functions are evaluated at steady state. Here $\pi_t \equiv \Pi_t - 1$ is the inflation rate and $x_t \equiv \hat{Y}_t - \hat{Y}_t^n$ is the output gap, where the hat denotes that the variables are measured as percentage deviation from steady state. The natural rate of output can be approximated by

$$\hat{Y}_t^n = \frac{\sigma^{-1}}{\sigma^{-1}+\omega} g_t + \frac{\omega}{\sigma^{-1}+\omega} q_t + \frac{\sigma^{-1}}{\sigma^{-1}+\omega} \hat{F}_t \tag{34}$$

where there two terms $g_t \equiv -\frac{\bar{u}_{c\xi}}{\bar{Y}\bar{u}_{cc}} \xi_t$ and $q_t \equiv \frac{\bar{v}_{y\xi}}{\bar{Y}\bar{v}_{yy}} \xi_t$ are exogenous shocks. The "Phillips curve" (33) has become close to standard in the literature.

The IS equation is given by:

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n) \qquad \text{IS} \tag{35}$$

where

$$r_t^n \equiv \frac{1-\beta}{\beta} + \frac{\sigma^{-1}\omega\beta^{-1}}{\sigma^{-1}+\omega}(g_t - E_t g_{t+1}) + \frac{\sigma^{-1}\omega\beta^{-1}}{\sigma^{-1}+\omega}(q_t - E_t q_{t+1}) + \frac{\sigma^{-1}\omega\beta^{-1}}{\sigma^{-1}+\omega}(\hat{F}_t - E_t\hat{F}_{t+1}) \tag{36}$$

is a linear approximation of the natural rate of interest, i.e. the real interest rate that is consistent with the natural rate of output. In the linear approximation in (35) i_t is again the short term nominal interest rate.[16] For simplicity I do not express i_t as deviation from steady state so that the zero bound is simply the requirement that i_t is positive.

As in Eggertsson and Woodford (2003) and Eggertsson (2005,6) I limit my attention to stochastic shocks that make the natural rate of interest temporarily negative. I denote the part of the natural rate of interest that is exogenous in my model (i.e. the natural rate of interest if government spending are held constant) as r_t^{nF}. The following assumption allows for a simple characterization of the equilibrium when the zero bound is binding:

A5 $r_t^{nF} = r_L^n < i^m$ *at* $t = 0$ *and* $r_t^{nF} = r_{ss}^n = \frac{1}{\beta} - 1$ *at all* $0 < t < K$ *with probability* α *if* $r_{t-1}^{nF} = r_L^n$ *and probability 1 if* $r_{t-1}^n = r_{ss}^n$ *at all* $t > 0$*. There is an arbitrarily large number* K *so that* $r_t^n = r_{ss}^n$ *with probability 1 for all* $t \geq K$*.*

According to assumption A5 the natural rate of interest becomes temporarily negative in period 0 and reverts back to steady state with constant probability α in the following periods. In the limit as $K \to \infty$ the natural rate reverts back with a fixed probability α in all remaining periods so that the expected duration of the shock is $\frac{1}{\alpha}$. For simplicity I assume that the natural level of output that is determined by the exogenous shocks, denoted Y_t^{nF} is constant so that $\hat{Y}_t^{nF} = 0$. As shown in Eggertsson (2006) the first best allocation would be achieved if the government could set $i_t = r_t^n$ at all times. In this case the government can achieve $x_t = 0$ and $\pi_t = 0$ at all times. This maximizes the utility of the representative consumer because output is at the natural rate of output at all times and inflation is zero (and as shown by Eggertsson (2006) the utility of the representative household in this model can be approximated by quadratic deviation of each of the these variables from zero under A4). This solution however, cannot be attained if r_t^n is lower than 0, since this implies a negative nominal interest rate that violates the zero bound.

I now consider the solution under A5. Observe first that for all $t \geq K$ then $\pi_t = x_t = 0$ (this is formally proven in Eggertsson (2006) in the nonlinear model). Intuitively this can be seen by noting that the objectives of the government, under the restriction imposed in (32), can be approximated by the quadratic objectives $-\pi_t^2 - \lambda_x x_t^2$ in each period. Thus once the natural rate

[16] It corresponds to i_t in our previos notation in the nonlinear model times β^{-1}.

of interest becomes positive (i.e. for all $t \geq K$) those objectives can be minimized in each period from then on by $\pi_t = x_t = 0$. Since the government is Markovian it will immediately achieve this equilibrium, even if the optimal commitment solution may involve a different outcome as I discuss further below. I first consider the most simple case when $K = 1$. In this case the first best allocation cannot be achieved in period zero and the zero bound will be binding. Since I know how the the solution looks like in period $t \geq 1$ I can write $E_0\pi_1 = E_0 x_1 = 0$ and then observe from (33) and (35) that since $i_0 = 0$ the solution the takes the form:[17]

$$x_0 = \sigma r_L^n < 0$$

$$\pi_0 = \kappa\sigma r_L^n < 0$$

This solution illustrates that the presence of the zero bound creates deflation and output gap if the natural rate of interest is negative. What if the natural rate can be negative for more than one period? Consider first the case $K = 2$. In this case the natural rate of interest can either be r_L (with prob. $1-\alpha$) or r_{ss} (with prob. α) in period 1. If $r_1^n = r_L$ the solution is the same as above in period 1. If $r_1^n = r_{ss}^n$ then $x_1 = \pi_1 = 0$. Then one observes from (35) that the solution in period 0 is:

$$x_0 = E_0 x_1 - \sigma(i_t - E_t\pi_{t+1} - r_t^n) = (1-\alpha)\sigma r_L^n + \sigma\kappa(1-\alpha)\sigma r_L^n + \sigma r_L^n < \sigma r_L^n < 0 \quad (37)$$

Note that this expression indicates that the output gap is larger if the private sector puts a positive probability of the zero bound to be binding for more than one period. This is due to the first two terms in the right hand side of (37). The logic is simple: The expectation of lower output in period 1 (the first term) reduces demand by the permanent income hypothesis. The expectation of future deflation (the second term) increases the real rate of return thus depressing demand. These two forces, that come about through expectation about future slump, have significant effect on demand in period 0 . One can similarly use (33) to solve for the deflation in period 0.

Equation (37) indicates that expectations about future slumps can make the current slump even worse. I can similarly solve for inflation and output by the same backward induction for the case when K is arbitrarily high. In the limit as $K \to \infty$ it is easy to show that the solution is:

$$x_t = \frac{1-\beta(1-\alpha)}{\alpha(1-\beta(1-\alpha)) - \sigma\kappa(1-\alpha)}\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } x_t = 0 \text{ otherwise}$$

$$\pi_t = \frac{1}{\alpha(1-\beta(1-\alpha)) - \sigma\kappa(1-\alpha)}\kappa\sigma r_L^n \text{ if } r_t^n = r_L^n \text{ and } \pi_t = 0 \text{ otherwise}$$

To ensure that the solution is bounded I need to assume that α satisfies the inequalities $\beta\alpha^2 + (1 + \sigma\kappa - \beta)\alpha - \sigma\kappa > 0$ and $0 < \alpha < 1$. If this condition is not satisfied the solution explodes and a linear

[17] One can proof that i_0 must be equal to zero by the first order condition conditions of the government maximization problem. See Eggertsson (2004) for details.

approximation of the IS and the AS equation is not valid for shocks of any order of magnitude. Thus I would need to use other nonlinear solution methods to solve for the equilibrium if the value of α does not satisfy these bounds. Here I simply assume parameters so that these two inequalities are satisfied and a linear approximation of the IS and AS is feasible and the solution is accurate of order $o(||\xi, \delta||^2)$ (see Technical Appendix). This solution illustrates that the associated output gap and deflation can be substantial if the natural rate of interest is expected to stay negative for a long time. In particular, the higher probability of the natural rate of interest staying low for long, the more negative the output gap and the deflation. Thus even if the natural rate of interest is only modestly negative, the effect can be dramatic, if it is expected to stay there for an extended period. It follows that small shocks can have very bad consequences when the zero bound is binding and especially if one assumes, as I do in condition (32), that fiscal policy cannot be used to fight the problem.

Figure 1 shows the state-contingent path of output gap and inflation for a numerical example. In the figure we assume that in period 0 that the natural rate of interest becomes -2 percent per annum and then reverts back to the steady-state value of $+4$ percent per annum with a probability 0.1 each quarter. Thus the natural rate of interest is expected to be negative for 10 quarters on average at the time the shock occurs. The numerical values assumed for this exercise are the same as in Eggertsson and Woodford (2003) and Eggertsson (2006) (see the Technical Appendix for the nonlinear solution method and the numerical values assumed). The first line shows the equilibrium if the natural rate of interest returns back to steady state in period 1, the next line if it returns in period 2, and so on. The inability of the central bank to set a negative nominal interest rate results in roughly 15 percent output gap and 11 percent deflation. Expectations of future slumps make the outcome much worse than if the trap were only to last for a single period. Since there is a 90 percent chance of the natural rate of interest remaining negative next quarter, expectations of future deflation and negative output gap create even further deflation.

Open market operations, i.e. printing money and buying government bonds, does nothing to increase either output or prices. As stressed by Eggertsson (2006), when the zero bound is binding the private sector will regard any increase in the money supply as temporary because the government has an incentive to contract the money supply to its previous level once deflationary pressures have subsided. This can explain why BOJ has more than doubled the monetary base in recent years without any apparent effect on prices or inflation expectations. If the government could commit to permanently increasing the money supply this would indeed increase inflation expectation and stimulate demand – which is optimal. As I have shown in this section, however, this commitment is not feasible in a MPE under the constraints imposed in (32).[18]

[18] This explains an important difference between my result and the one obtained by Auerbach and Obstfeld (2003) who argue that open market operations are effective. They assume that open market operations automatically increase expectations about future money supply. In a MPE however, expectations about future money supply are

4.2 Equilibrium B: The Power of Real Government Spending under Coordination

In this section I explore the power of real government spending to close the output gap and curb deflation, equilibrium B in diagram 2., when monetary and fiscal policy are coordinated. To focus on the effects of real government spending I assume that the budget is balanced at all times so that $F_t = T_t$ and then relax this assumption in the next section. To be precise I assume

$$F_t = T_t \text{ and } w_t = 0 \tag{38}$$

If the zero bound is never binding, the government's maximization problem (30) implies a FOC condition that equates marginal utility of spending to its marginal cost

$$g_G(F_t - s(F_t), \xi_t) = u_c(Y_t - d(\pi_t) - F_t, \xi_t) + g_G(F - s(F_t), \xi_t)s'(F_t) \tag{39}$$

This condition says that the marginal utility of increasing government spending (the left hand side) should be equal to the marginal cost (the right hand side). Note that the marginal cost of increasing government spending is the sum of private consumption forgone by additional spending and the cost of taxation due to the higher tax rates. The first order condition (52) in the Technical Appendix indicates that the only reason the treasury may deviate from this rule is if the zero bound is binding. The zero bound gives the treasury a reason to use fiscal spending for stabilization purposes.

Variation in the optimal size of the government, i.e. the value of F_t, depends on how the marginal utility of private and public consumption shifts with the vector of shocks ξ_t. For simplicity I assume that these shocks shift $u_c(., \xi)$ and $g_G(., \xi)$ so that the optimal size of the government, in the absence of the zero bound, is constant over time so that there is a unique value $F_t = \bar{F}$ that solves (39). This assumption is useful for interpreting the results below because it implies that all variation in fiscal spending away from $\bar{F}$ are due to the zero bound.

To understand the importance of real spending when the zero bound is binding let us again do the simple experiment I conducted in the last section: suppose the natural rate of interest is unexpectedly negative in period 0 and reverts back to the steady state with a fixed probability in every period. Figure 2 shows the same numerical experiment as in the last section, but now the treasury can increase fiscal spending to eliminate deflation. I use the approximation method shown in the Technical Appendix to solve the model numerically. Figure 2 indicates that the treasury increases government spending by 3.4 percent (as a fraction of GDP) when the zero bound is binding. This eliminates about 70-80 percent of the deflation and the output gap.

unaffected by open market operations.

4.2.1 The Keynesian Channel vs the RBC channel of government spending

Government spending works through two separate channels when the zero bound is binding. Real spending increases the natural level of output through the first. This channel has been extensively documented in the RBC literature (see e.g. Baxter and King (1993) and references therein). In the context of our model, just as in Baxter and King, the natural rate of output increases if government expenditures increase. Recall that a first order approximation of the natural rate of output (the output that would be produced if prices are flexible) yields:

$$\hat{Y}_t^n = \frac{\sigma^{-1}}{\omega + \sigma^{-1}} g_t + \frac{\omega}{\omega + \sigma^{-1}} q_t + \frac{\sigma^{-1}}{\omega + \sigma^{-1}} \hat{F}_t \tag{40}$$

Thus the model predicts that an increase in fiscal spending increases the natural rate of output. This increase is due to an increase in the willingness of people to work. Higher government spending increases the marginal utility of consumption (for given level of employment) which in turn induces people to work more to equate the marginal utility of private consumption and the disutility of working.

Government spending influences output in the model thought another channel. I call this the *Keynesian channel* of government spending. The Keynesian channel only works if prices are sticky, i.e. if the real rate can be different from the natural rate of interest (which is the real interest rate if prices are perfectly flexible). To see the Keynesian channel note that by equation (72) an increase in government spending (holding everything else constant) increases the natural rate of interest. Then if the nominal interest rate is held fixed and expectations about future inflation are held constant, a wedge opens between the real interest rate and the natural rate of interest. By the IS equation (holding expectation about future output gap constant) a positive wedge between $r_t = i_t - E_t\pi_{t+1}$ and r_t^n stimulates demand. This is the Keynesian channel for government spending. In the next paragraph, I make this statement more precise in order to compare the effects of the two channels.

I now do the following thought experiment: Suppose the central bank in period t and successive government agencies follow optimal strategies. What is the marginal effect of the treasury increasing F_t above its steady state? I can calculate this marginal effect by substituting for x_t into to IS equation and taking a partial derivative with respect to F_t. This yields:

$$\frac{\partial Y_t}{\partial F_t} = \frac{\partial Y_t^n}{\partial F_t} - \sigma\left(\frac{\partial i_t}{\partial F_t} - \frac{\partial r_t^n}{\partial F_t}\right) \tag{41}$$

where the derivative with respect to π_{t+1} and x_{t+1} is zero because these variables are determined by a successive government (since there is no state variable in the game under condition (38) it follows that $\frac{\partial x_{t+1}}{\partial F_t} = \frac{\partial \pi_{t+1}}{\partial F_t} = 0$). The first term of the derivative in (41) is $\frac{\partial Y_t^n}{\partial F_t} = \frac{\sigma^{-1}}{\omega+\sigma^{-1}}$. This is the RBC channel for fiscal policy. The second term of this derivative is $-\sigma(\frac{\partial i_t}{\partial F_t} - \frac{\partial r_t^n}{\partial F_t})$. This is the Keynesian channel of real government spending. Note that if the zero bound is not binding and the central bank maximized social welfare under condition (38) then $i_t = r_t^n$ at all times and this remains true regardless of the value of F_t. It follows that the Keynesian channel is zero in the absence of the zero bound: The central bank offsets any increase/decrease in the natural rate of interest. In contrast, if the natural rate of interest is negative and the zero bound is binding, it is easy to verify that $\frac{\partial i_t}{\partial F_t} = 0$. In this case (by equation (72)) the value of the second derivative is $-\sigma(\frac{\partial i_t}{\partial F_t} - \frac{\partial r_t^n}{\partial F_t}) = \frac{\omega}{\sigma^{-1}+\omega}$.[19] In sum, then, the marginal effect of increasing government spending on output is $\frac{\sigma^{-1}}{\omega+\sigma^{-1}} + \frac{\omega}{\sigma^{-1}+\omega} = 1$. This is exactly what Krugman (1998) notes. He argues that real government spending is not very effective in fighting deflation because the "multiplier" is small – only 1! Incidentally this number is also equal to the "balanced budget" multiplier in the old fashion IS-LM model. This is however a misleading observation since this partial derivative does not take account of the expectations channel. In the next section I derive an alternative definition of the multiplier that takes expectations into account.

4.3 The Multiplier of Real Government Spending

One aspect of figure 2 that may be surprising is that only 4 percent of government spending in each period (when the zero bound is binding) eliminates about 70-80 percent of the output gap and the deflation. The may be particularly surprising given the small value of the partial derivative discussed in the last paragraph. This large effect of small government spending is due to the expectation channel. As I discuss in the last section, the main cause of the large decline in output and prices is the expectation of a future slump and deflation. Consider the outcome from the perspective of period 0. If the private sector expects even only a small increase in future government spending when the zero bound is binding, deflation expectations are changed in all periods when the zero bound is binding; thus having a large effect on spending in period 0. This illustrates that an analysis of partial derivatives – of the type I discussed in the last section – is very misleading when uncovering the general equilibrium effect of real government spending in a liquidity trap.

A useful summary statistic that accounts for the expectation channel is what I define as the

[19] It may be surprising that the value of this derivative due to the Keynesian channel of real government spending, $\frac{\sigma^{-1}}{\sigma^{-1}+\omega}$, does not rely on the degree of price stickiness. After all, the ability of the government to set the real rate of interest above/below the natural rate of interest depends on prices being sticky. The reason for this is that output is completely determined by the IS equation when the zero bound is binding. In this equation expectations are fixed by the expectation about the actions of future governments. The IS equation does not in any way depend on price stickiness and the same applies therefore for this derivative. The price adjustment that must take place to accommodate the change in government spending when the zero bound is binding, however, is highly dependent on the stickiness of prices. This can be seen by the linear approximation of the AS equation. Since the output gap is determined by the IS equation when the zero bound is binding (and expectations are fixed), we can see from this equation that the level of inflation/deflation depends on the value of κ. This coefficient depends on d_J which reflects the cost of adjusting prices.

multiplier of real government spending. This measure answers the question: How much does each dollar of real spending increase output moving from the equilibrium in which $F_t = F$ (equilibrium A in diagram 2) to the one where F_t is optimally set (equilibrium B in diagram 2)? I measure each variable in net present value. This statistic is well defined because the only difference between the two equilibria (A and B) is that in the latter real government spending can be increased. This statistic can be analytically derived, yielding the following result

$$MP_{A,B} \equiv \frac{E_0 \sum_{t=0}^{\infty} \beta^t (\hat{Y}_t^A - \hat{Y}_t^B)}{E_0 \sum_{t=0}^{\infty} \beta^t (\hat{F}_t^A - \hat{F}_t^B)} = \frac{[\frac{1}{1-\alpha} - \beta]\sigma^{-1} - \alpha^{-1}\kappa \frac{\sigma^{-1}}{\sigma^{-1}+\omega}}{[\frac{1}{1-\alpha} - \beta]\sigma^{-1} - \alpha^{-1}\kappa} > 1$$

Figure 7 shows that this multiplier is 3.3 percent under the baseline calibration. It also decomposes the multiplier into the part that is due to the RBC channel and the one that is due to the Keynesian channel. The Keynesian channel accounts for about 80 percent of the size of the multiplier. As a comparison figure 7 also shows the multiplier when government spending is increased by the same amount but there is no shock. In this case the multiplier is much smaller (0.5 percent) and is only due to the RBC channel. The reason is that in this case inflation and output are not below the central bank's optimal inflation target and so it counteract the demand effect of higher government spending by increasing the interest rate, thus containing demand to some extent.

4.4 Equilibrium C: The Power of Deficit Spending under Coordination

In this subsection I explore the ability of deficit spending to close the output gap and curb deflation under coordination. This is equilibrium C in diagram 2. Deficit spending is the difference between real spending and current taxes i.e. $d_t = F_t - T_t$. To contrast the power of deficit spending to real government spending I assume that the latter is constant i.e.

$$F_t = \bar{F}$$

When government uses deficit spending, the value of the real debt becomes a state variable. This allows the government to change deflationary expectations into inflationary ones by increasing nominal debt. This is exactly what is needed when the zero bound is binding. To see this consider the IS equation. This equation illustrates that the output gap depends on an expected future path or real interest rate, i.e. $i_t - E_t\pi_{t+1}$. Even if demand cannot be increased by lowering the nominal interest rate, it can still be increased by raising inflation expectations. This is not possible if the only instrument of monetary policy is open market operations because even if the central bank has an incentive to promise future inflation when zero bound is binding, it has an incentive to renege on this promise once deflationary pressures have subsided (since there is cost of inflation in the model). Thus a discretionary central bank cannot increase inflation expectations when the zero bound is binding and the result is excessive deflation. This is what Eggertsson (2006) calls the deflation bias of discretionary policy. When monetary and fiscal policy are coordinated, however, the government can credibly commit to future inflation by increasing government debt. This is exactly why deficit spending is effective when the zero bound is binding, it increases inflation expectations.

The channel is: Budget deficits generate nominal debt. Nominal debt in turn makes a higher inflation target in the future credible (i.e. higher money supply in the future) because the real value of the debt increases if the government reneges on the target. Higher debt is undesirable for the government if there are some tax distortions. Higher inflation expectations lower the real rate of interest and thus stimulate aggregate demand.

Figures 3 and 4 show the equilibrium when the central bank uses deficit spending (see the Technical Appendix for the numerical values assumed and the approximation method). As can be seen by this figure the ability of the government to use deficit spending to raise inflation expectations substantially improves the equilibrium. Deficit spending eliminates 93 percent of the deflation and 86 percent of the output gap. The price of this improvement during the trap, is an increase in inflation once out of the trap.

4.5 The Multiplier of Deficit Spending under Coordination

Again it is useful to summarize the effect of the deficit spending on output through a multiplier. Some adjustment to the definition of the multiplier is needed, however, for it to be useful. What I consider instead is a variable $\tilde{T}_t$ that has the defined as $\tilde{T}_t = \hat{T}_t$ if $r_t^n = r_t^L$ and $\tilde{T}_t = 0$ if $r_t^n = r_{ss}^n$. (The results derived for $\hat{F}_t$ would have been unchanged if I had defined $\tilde{F}_t$ in this way because $\hat{F}_t = 0$ if $\tilde{r}_t^n = 0$). This variable captures the deficit spending used in the depression state.[20] Hence I define the multiplier of deficit spending as

$$MP_{A,C}(\tilde{T}) = -\frac{E_0 \sum_{t=0}^{\infty} \beta^t (Y_t^A - Y_t^C)}{E_0 \sum_{t=0}^{\infty} \beta^t (\tilde{T}_t^A - \tilde{T}_t^C)}$$

The value of this multiplier answers the following question: By how much does each dollar spent on deficit spending in a liquidity trap increase output? In our baseline calibration the answer is 3.7. Figure 7 decomposes the size multiplier between the RBC channel and the New Keynesian channel. No part of the multiplier can be explained by the RBC channel. Effectiveness of deficit spending comes entirely through increasing inflation expectations, and this is only valuable if one assumes sticky prices. Since prices are flexible in an RBC model deficit spending has no role in that model. Figure 7 also shows the size of the multiplier when interest rates are positive in which case it is much smaller. The reason is that when interest rates are positive the central

[20]To the first order the net present value of taxes will alway be equal to zero for the transversality condition of the representative household to be satisfied. A summary statistic like the one introduced earlier will then not be defined.

bank's actions are not constrained by the zero bound and the inflation rate is not below its desired optimum. This implies that the bank will seek to offset the increase in inflation expectations due to the deficit spending by raising interest rates.[21] In contrast the central bank will keep interest rate low when the zero bound is binding due to deflationary shocks because in that case inflation is below the bank's desired inflation target.

4.6 Equilibrium D: Deficit and real government spending under coordination

How important is deficit spending versus real spending in equilibrium when the government has access to both instruments? Figures 5 and 6 compare the equilibrium under the two policies derived in the last two sections with the optimal policy if the government can use both deficit and real spending. This is equilibrium D in diagram 2. These figures show the same numerical experiment as was done in past section but to reduce the number of lines shown in the graph I only report the path for each variable in the case the natural rate of interest returns back to steady state in period 0,4,7 and so on (thus not graphing the contingencies in between to avoid cluttering the pictures). As can be seen by the figure the government will use both real and government spending in a liquidity trap. Of the two instruments deficit spending is more effective, at least in terms of eliminating deflation and the output gap when the zero bound is binding. The figure indicates that if deficit spending is the only policy instrument, about 93 percent of the deflation is eliminated when the zero bound is binding compared to about 70 percent if the government can only use real spending. Similarly deficit spending eliminates about 86 percent of the output gap compared to 79 percent if the government can only use real government spending.

An even more instructive measure of the effectiveness of each policy instruments is the utility of the representative household under the different policy regimes. Table 3 lists the welfare in the four equilibria discussed above and compares them with the optimal policy if the government could commit to future policy (the Ramsey/Commitment solution). The commitment equilibrium is the fully efficient allocation (it is solved in Eggertsson (2006)) and is thus the best the government could ever hope to achieve. The table expresses utility in terms of consumption equivalence units. This measure expresses the expected utility flow in units of a constant consumption endowment. The first number show how much the representative agent would give up in constant consumption endowment stream to avoid the shocks that give rise to the zero bound altogether. This number is small if the government can commit, only 0.014 percentage. The table shows that if the government coordinates monetary and fiscal policy and uses both real and deficit spending as policy instruments, the value of commitment is not very large or less than 0.01 percent. Deficit spending discretion (i.e. if the government is unable to commit but can use deficit spending as

[21] In computing the multiplier at positive interest rate I assumed the same shock but that the central bank was not constrained by the zero bound.

a commitment device) yields higher utility than if the government can use only real government spending discretion. This indicates that of these two instrument deficit spending is more important to improve economic welfare. If the government cannot use either deficit or real spending to battle deflation there are sizable welfare losses. The representative agent would give up 4.27 percent of its constant consumption endowment stream every single period to avoid the shocks. Put in net present value this is a very large number or equivalent to about one years worth of production.

Table 3

	Welfare loss in Consumption Equivalence Units
Commitment Equilibrium	-0.0143
Full Discretion (D)	-0.0201
Deficit Spending Discretion (C)	-0.0268
Real Spending Discretion (B)	-0.5975
Constrained Discretion (A)	-4.2715

One interesting aspect of deficit spending versus real spending that is worth noting (see figure 6) is the different time path of these policy variables. While the real spending solution involves a permanent increase in real spending during all periods in which the zero bound is binding, deficit spending is only temporarily high. Deficit spending is thus more consistent with the old Keynesian idea that a quick jolt of spending can "jump start" the economy. The reason is that it is the level of government debt that is the important state variable, because it increases inflation expectations. Only temporary deficit spending is needed to permanently increase government debt. In contrast, stimulating demand by real government spending requires a sustained increase in government spending in all periods in which the zero bound is binding.

5 The Markov Equilibrium when the Central Bank is Goal Independent: A Definition and Results

In the preceding section I assumed that monetary and fiscal policy are coordinated to maximize social welfare. This assumption may be questionable. In many countries the central bank has been assigned more narrow goals than social welfare. This institutional framework was made precise in A3, which is what I called a "goal independent" central bank. I now show how the results change if I assume that the central bank is goal independent, i.e. if its goals are as assumed in A3. The organization of this section is the same as in the last section which was illustrated in diagram 2. After formally defining the equilibrium, I first explore the power real government spending (equilibrium B) and then deficit spending (equilibrium C). The main conclusion is that the power of real spending is unchanged even if the central bank is goal independent (equilibrium

B is unchanged) but that deficit spending has no effect (equilibrium C is equivalent to equilibrium A in the diagram, and equilibrium B and D are also equivalent).

5.1 Defining a Markov Equilibrium when the central bank is goal independent

The timing of events in the game is as follows: At the beginning of each period t, w_{t-1} is a predetermined state variable. At the beginning of the period, the vector of exogenous disturbances ξ_t is realized and observed by the private sector, the treasury and the central bank. The monetary and fiscal authorities simultaneously choose policy at time t given the state. The private sector forms expectations e_t and I assume that the private sector may condition its expectation at time t on w_t, as in the previous section. The policy function of the treasury can then be written as:

$$Tr_t = \begin{bmatrix} F_t \\ T_t \end{bmatrix} = \begin{bmatrix} \bar{F}(w_{t-1}, \xi_t) \\ \bar{T}(w_{t-1}, \xi_t) \end{bmatrix} = \bar{T}r(w_{t-1}, \xi_t) \tag{42}$$

and the policy function of the central bank as:

$$m_t = \bar{m}(w_{t-1}, \xi_t) \tag{43}$$

This implies that in equilibrium I can once again write a function $\Lambda_t = \bar{\Lambda}(w_{t-1}, \xi_t)$ and define a function $\bar{e}(.)$ of the form (27). I define the value functions for the treasury, J^{Tr}, and the central bank,J^{Cb}, as:

$$J^{Tr}(w_{t-1}, \xi_t) \equiv E_t \left\{ \sum_{T=t}^{\infty} \beta^T [U(\bar{\Lambda}(w_{T-1}, \xi_T), \xi_T] \right\} \tag{44}$$

$$J^{Cb}(w_{t-1}, \xi_t) \equiv -E_t \left\{ \sum_{T=t}^{\infty} \beta^T [(\bar{\Pi}(w_{t-1}, \xi_t) - 1)^2 + \lambda(\frac{\bar{Y}(w_{t-1}, \xi_t)}{Y_t^n} - 1)^2] \right\} \tag{45}$$

Given $\bar{m}(.)$, w_{t-1} and ξ_t the treasury maximizes the utility of the representative household subject to the constraints in Proposition 1. Thus its problem can be written as:

$$\max_{F_t, T_t} [U(\Lambda_t, \xi_t) + \beta E_t J^{Tr}(w_t, \xi_{t+1})] \tag{46}$$

s.t. (9), (18),(19), (21), (23), (24), (27) and (43).

Given $\bar{T}r(.)$, w_{t-1} and ξ_t the central bank maximizes its objective subject to the constraints in Proposition 1. Thus its problem can be written as:

$$\max_{m_t} [-(\Pi_t - 1)^2 - \lambda(\frac{Y_t}{Y_t^n} - 1)^2 + \beta E_t J^{Cb}(w_t, \xi_{t+1})] \tag{47}$$

s.t. (9), (18), (21), (23), (24), (27) and (42).

The conditions that constrain the actions of the treasury and the central bank in the maximization problems (46) and (47) are the private sector equilibrium conditions and the strategy functions of the other government agency.[22] Apart from the other players strategy function these constraints are the same for both players but with one important exception. The borrowing constraint of the treasury is *only a restriction on the treasuries taxing and borrowing strategies*, it does not impose any constraint on the central bank. To see why this is important suppose the contrary was true. In this case there would be a much more complicated strategic game between the treasury and the central bank. The treasury could for example accumulate large amounts of debt up to its debt limit $\bar{w}$ and then cut taxes further. In this case, in order to no to violate the borrowing constraint, the central bank would need to inflate away the some of the existing debt. The definition of an independent central bank proposed below is that the central bank has its own objective and is furthermore not responsible for that the treasury satisfies its debt limit. Satisfying the debt limit is solely the responsibility of the treasury so the treasury cannot force the central bank's hand brute force.

I can now define a Markov Equilibrium when the central bank is goal independent.

Definition 3 *A Markov Equilibrium when the central bank is goal independent is a collection of functions $\bar{\Lambda}(.), \bar{T}r(.), \bar{m}(.), J^{Tr}(.), J^{Cb}, \bar{e}(.)$, such that: (i) Treasury maximization. Given the functions $J^{Tr}(w_{t-1}, \xi_t)$, $\bar{e}(w_t, \xi_t)$ and $\bar{m}(.)$, the solution to the treasury optimization problem (46) is given by $Tr_t = \bar{T}r(w_{t-1}, \xi_t)$ for each possible state $(w_{t-1,}\xi_t)$. (ii) Central Bank maximization. Given the functions $J^{Tr}(w_{t-1}, \xi_t)$ $\bar{e}(w_t, \xi_t)$ and $\bar{T}r(.)$, the solution to the central bank maker's optimization problem (45) is given by $m_t = \bar{m}(w_{t-1}, \xi_t)$ for each possible state $(w_{t-1,}\xi_t)$. (iii) $\bar{m}(.)$ and $\bar{T}r(.)$ are as subset of the vector function $\bar{\Lambda}(.)$ and $\bar{\Lambda}(.)$ is PSE (iv) given the vector function $\bar{\Lambda}(w_{t-1}, \xi_t)$ then $e_t = \bar{e}(w_t, \xi_t)$ is formed under rational expectations. (v) given the vector function $\bar{\Lambda}(w_{t-1}, \xi_t)$ the functions $J^{Tr}(w_{t-1}, \xi_t)$ and $J^{Cb}(w_{t-1}, \xi_t)$ satisfy (44) and (45).*

5.2 Real government spending when the central bank is goal independent

I first consider the power of real government spending when the central bank is goal independent. In order to isolate the effect of real government spending I constraint the budget to be balanced at all times so that $F_t = T_t$ (corresponding to equilibrium B in diagram 2 when the central bank is goal independent) and

$$w_t = 0 \tag{48}$$

How does the solution look like? It turns out that the solution – at least to first order – does not depend on whether the central bank is goal independent or not. To be more precise:

[22] Note that the government budget constraint can equivalently be interpretet as the budget constraint of the household and it thus belong in both maximization problems as a private sector equilibrium constraint.

Proposition 3 *The power of real government spending does not depend on policy coordination, i.e. equilibrium B in diagram 2 (equation (48) holds) does not depend on whether or not the central bank is goal independent.*

Proof: See Technical Appendix.

Proposition 3 indicates that the power of real government spending is not affected by whether or not the central bank is goal independent. The intuition for the proposition is as follows: Observe first that the solution when the natural rate of interest becomes positive (and the zero bound is no longer binding) is the same under either coordination or goal independence because the central bank will target zero inflation and zero output gap at that time (and the treasury will then set $F_t = F$). Consider now the solution when the zero bound is binding. Since monetary policy is constrained by the zero bound at this time, its different objective is irrelevant during this period as long as it implies a zero interest rate. The central bank interest rate policy, therefore, only matters in period $t \geq K$ and I have just argued that its policy will be the same in those periods as under coordination. Turning to the the treasury, according to A3 it is maximizing social welfare, and it follows that the path for government spending will be exactly the same as analyzed in section 4.2 during the trap. It follows that the solution is the same under coordination and goal independence if I assume (48).

5.3 Deficit spending and when the central bank is goal independent

I now turn to the case of deficit spending when the central bank is goal independent. I assume that

$$F_t = \bar{F} \tag{49}$$

to focus on the effect of deficit spending (corresponding to equilibrium C in diagram 2 when the central bank is goal independent). In contrast to the last section, I find that there is now a dramatic difference in the power of deficit spending depending on whether the central bank is goal independent. If the central bank is goal independent, as defined in A3, deficit spending has *no effect* on inflation or output.

Proposition 4 *If the central bank is goal independent deficit spending has no effect, i.e. in equilibrium C in diagram 2 (equation (49) holds) deficit spending has no effect on inflation, output or interest rates when the central bank is goal independent.*

Proof: See Technical Appendix

The reason for this is as follows: For a given path of F_t Ricardian equivalence holds in the model so that debt does not enter into any of the equilibrium conditions of the private sector apart from the budget constraint of the private sector (18). Under A3 monetary policy is set to minimize $(\Pi_t - 1)^2 + \lambda_x x_t^2$. Government debt or deficits do not enter this objective or the private sector equilibrium constraints other than the budget constraint. It follows that in the central bank maximization problem the Lagrangian multiplier on the budget constraint is zero and debt has no effect on the equilibrium determination of inflation, output and interest rates which – to a linear approximation – is determined by exactly the same set of equations as if fiscal policy was completely inactive (i.e. in equilibrium C in diagram 2 when (32) holds). It follows that if I set $\{F_t\}_{t=0}^{\infty}$ to be exogenously given, deficit spending has no effect on the equilibrium outcome when the central bank is goal independent. The central bank will determine inflation and the output gap without any reference to deficits or debt.[23]

The effect of fiscal policy when coordinated with monetary policy is thus fundamentally different from its effects if the central bank is goal independent. This can be of potential importance in practice. Thus Krugman (2001) raises the question of why deficit spending in Japan has failed to lift Japan out of its recent recession (which finally looks like its on the mend) while some economists believe that deficit spending helped Japan avoiding the Great Depression and that the WWII deficit spending jolted the US economy out of the Great Depression. One critical difference between deficit spending of that period and now is that the Bank of Japan is independent today unlike during the Great Depression (and in the US the Fed and the treasury cooperated in various ways during the Great Depression). This paper thus points towards an important channel of fiscal and monetary policy that may have been at work in Japan and US in the Great Depression and the US but is not present in Japan today. When monetary and fiscal policies are coordinated, deficit spending increases inflation expectations, which in turn lowers the real rate of return and stimulates aggregate demand.

6 Coordination in the Great Depression in Japan and the US

Suppose a central bank is goal independent. Is it straight forward to change expectations by changing the overall goals of a central bank and increasing deficit spending? Are such regime shifts credible? From a theoretical standpoint the answer to this question is unambiguous. Since the cooperation between the treasury and the central bank, as I define it, involves a maximization of social welfare, it is always credible. The main challenge then, is not really whether or not such policy is credible, but how to make it visible and verifiable by the private sector. One way of doing this is for the central bank to announce its intention to support fiscal policy and then buy government bonds. In principle, such policy should have no effect, because money and bonds are perfect substitutes. But if such operations are accompanied by explicit announcements that the

[23] Note that if the treasury chooses F_t in each period, deficit spending can in principle have effect by influencing the expectations about future spending F_{t+j}.It can be verified, however, that in this model this effect is only of second order.

bank is attempting to support fiscal policy, for example by announcing that the debt bought by the central bank would not be collected from the treasury when due, this could have large effect on inflation expectations. The effect follows, not from the purchases themselves, but from the way in which they are interpreted. Thus open market operations can be used to *signal* a change in the central bank's objective and a determination to support fiscal policy to end deflation. A key element of such policy, therefore, is for the bank to be transparent about its policy objectives and how it want to move expectations.

Have regime changes and coordination been effective in the past to curb deflation? There is an interesting historical precedent from Japan for a cooperative solution. During the late 1920's Japan was slipping into a depression. Growth had slowed down considerably, GNP rose by only 0.5 percent in 1929, 1.1 in 1930 and 0.4 percent in 1931. At the same time deflation was crippling the economy. This was registered by several macroeconomic indicators as is illustrated in table 4. In December 1931 Korekiyo Takahasi was appointed the Finance Minister of Japan. Takahasi took three immediate actions. First, he abolished the gold standard. Secondly, he subordinated monetary policy to fiscal policy by having the BOJ underwrite government bonds. Third, he ran large budget deficits. These actions had dramatic effects as can be seen in table 4. All the macroeconomic indicators changed in the direction predicted by our model. As the budget deficit increased, GNP rose and deflation was halted. During the same period, interest rates were at a historical low and rates on government bonds were close to zero during the 30's. In addition to the nominal interest rate cuts our model indicates that the other actions taken, i.e. aggressive deficit spending that was financed by underwriting of government bounds, could have had considerable effects on the real rate of return through increasing *expected inflation*. This channel can be of potential importance in explaining the success of these policy measures in Japan in the Great Depression. In 1936 Takahasi was assassinated and the government finances subjugated to military objectives. The following military expansion eventually led to excessive government debt and hyperinflation. Until Takahasi was assassinated, however, the economic policies in Japan during the 1930's were remarkably successful.

Another notable example of coordination of monetary and fiscal policy is from the US in the Great Depression. This episode is discussed in some detail in Eggertsson (2005). What emerges from that paper is that the end of the Great Depression in the US is to a large extent explained by a shift in expectation that can be largely explained by a regime change due to monetary and fiscal coordination.

	Change in GNP deflator	Change in CPI	Change in WPI	Change in GNP	Government surplus over GNP
1929	-	-2.3%	-2.8%	0.5%	-1.0%
1930	-	-10.2%	-17.7%	1.1%	2.0%
1931	-12.6%	-11.5%	-15.5%	0.4%	0.4%
1932	3.3%	1.1%	11.0%	4.4%	-3.5%
1933	5.4%	3.1%	14.6%	10.1%	-3.0%
1934	-1.0%	1.4%	2.0%	8.7%	-3.5%
1935	4.1%	2.5%	2.5%	5.4%	-3.3%
1936	3.0%	2.3%	4.2%	2.2%	-2.0%

Table 4: Coordination of Fiscal and Monetary Policy in the Great Depression in Japan.

A topic for further research that carries considerable promise is to study the relative importance of deficit versus real spending in periods in which the government has aggressively increases both. Japan's recent experience is one case worth studying in a calibrated model. As I argue above, I doubt that deficit spending has done much do increase inflation expectation in Japan in recent years, given the ongoing deflation and continuing deflationary expectation (that most surveys indicate still remain subdued). But it may well be that increases in real government purchases have been effective in preventing the Japanese slowdown from being even worse. The model I presented showed that in the absence of any increases in real government spending the resulting deflation and output slump would have been even worse than what has been observed in Japan in recent years. The model indicates that the active increases in real government spending that have been observed in Japan in recent years (in a response to the slump) may have played an important role in preventing an even more acute slump (although it is an open question if more should have been done on that front). It should be noted, however, that there is no agreement on how aggressive the Japanese government has been in using real government spending to increase demand. Kuttner and Posen (2001), for example, argue that cyclically adjusted real government spending increases have been modest at best. In addition they have not been implemented on a sustained basis as would be required by the Markov solution shown here (i.e. real government spending should be increased in all states of the world in which the zero bound is binding). This is important, because our model predicts that it is not the current increase in real government spending that is of principal importance, but the expectation that it will also be increased in all future states of the world in which the zero bound is binding. Thus the government needs to announce that it will increase real government spending *until deflationary pressures have subsided* and this is a credible announcement as shown by the analysis of a Markov equilibrium.

7 Conclusion

Inflation has been considered the main threat to monetary stability for several decades. In the aftermath of the double digit inflation of the 70's, there was a movement to separate monetary policy from fiscal policy and vest it in the hands of "independent" central bankers whose primary responsibility was to prevent inflation. This development was reinforced by important contributions on the theoretical level, most notably by Kydland/Prescott (1977) and Barro/Gordon's (1983) illustration of the "inflation bias" of a discretionary government. It is easy to forget that in the aftermath of the Great Depression, when deflation was the norm, the discussion at the political and theoretical level was quite the opposite. Paul Samuelson claimed that the Federal Reserve was "the prisoner of its own independence" during the Great Depression, exaggerating the slump by its inability to fight deflation.[24] Similarly Milton Friedman claimed that "monetary policy is much too serious a matter to be left to the central bankers".[25] This paper shows that in a deflationary situation there may be some benefit to fiscal and monetary coordination. The exact nature of this coordination is certainly an interesting topic of further research. It is worth pointing out that this paper's solution suggests that it may only need to be temporary to be effective, as the solution illustrated that the coordination solution converges to the same one that would result in the absence of coordination.

One may argue that the central bank could, without any coordination with the treasury, engage in various activities to stimulate the prices and output, such as purchasing foreign exchange or private assets. An independent central bank may use its own balance sheet to achieve a similar commitment to higher future prices as was illustrated for deficit spending under coordination in this paper (i.e. it can increase inflation expectation by open market operations in private assets or foreign exchange see e.g. Eggertsson (2006) and Svensson and Jeanne (2004) for further discussion). The idea is that an independent central bank is typically very concerned about the value of its balance sheet since it would need to finance any capital losses by either printing money (which may lead inflation than higher than is optimal) or a bailout from the treasury (that may lead to loss of independence).The snag is, however, that if the bank is too concerned about its own balance sheet it may find itself as "the prisoner of its own independence" that prevents it from taking these actions, even if they in principle allow it to commit it to future inflation, much as suggested by Paul Samuelson. The reason is that any asset bought in a non-standard open market operations has uncertain returns, and there are always some states of the world in which the central bank may need to trade-off excessive balance sheet losses to excessive inflation. Thus even if one considers additional policy instruments there may still be an persuasive case for temporary coordination of monetary and fiscal policy.

[24] See Mayer, Thomas (1990) p. 6.

[25] Although he suggested rules to solve the problem rather than coordinated discretion as I do here.

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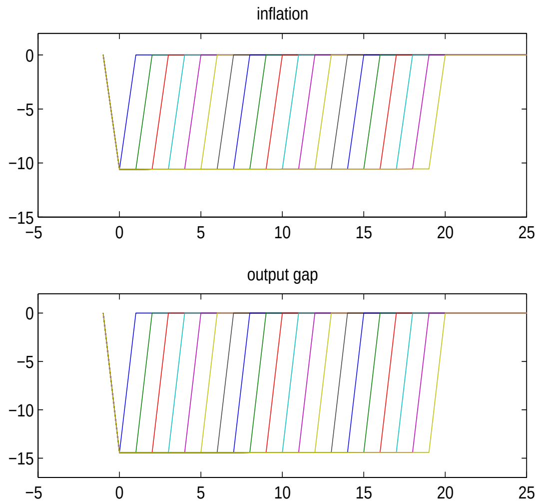

Figure 1: A Markov equilibrium in the absence of active fiscal policy.

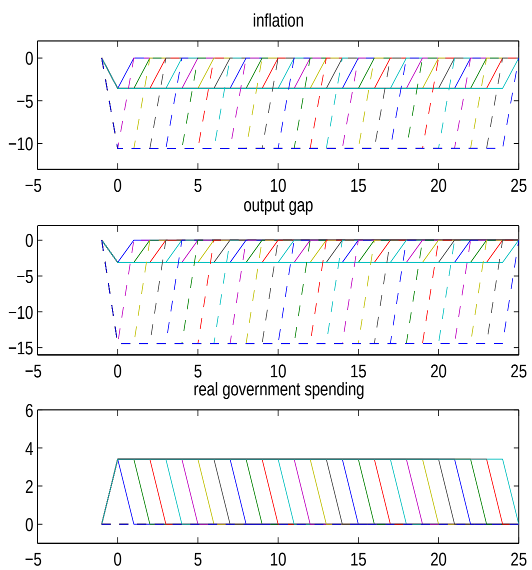

Figure 2: A Markov equilibrium when the government uses discretionary real spending.

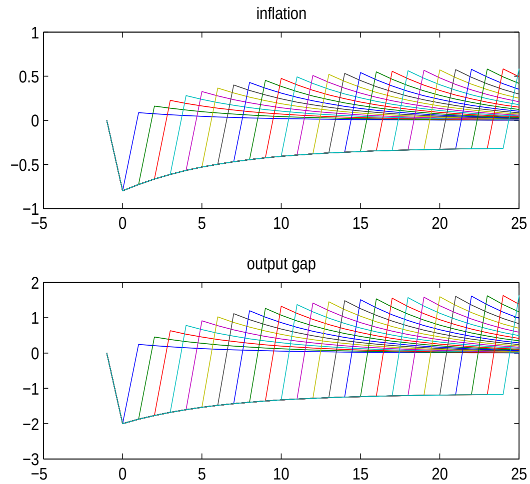

Figure 3: A Markov equilibrium for inflation and the output gap when the government uses discretionary deficit spending.

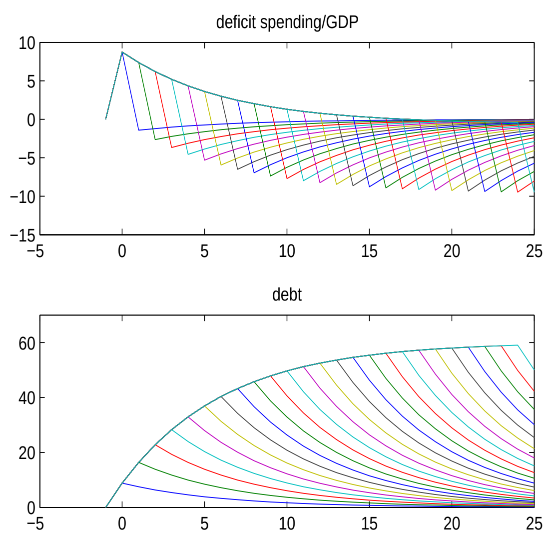

Figure 4: A Markov equilibrium for deficit spending and nominal debt when the government uses discretionary deficit spending.

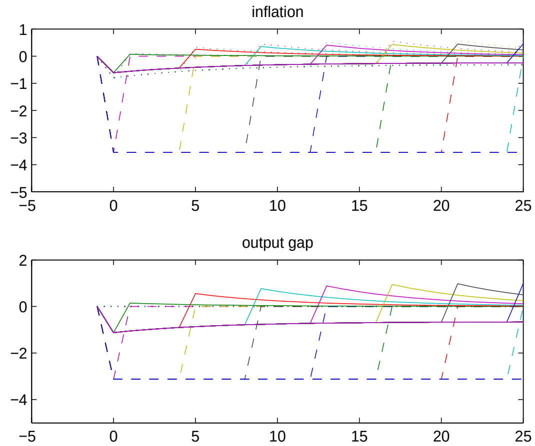

Figure 5: Comparison of inflation and the output gap in Markov equilibrium when the government only utilizes discretionary real spending (dashed line), only uses discretionary deficit spending (dotted line) and when it takes advantage of both (solid line).

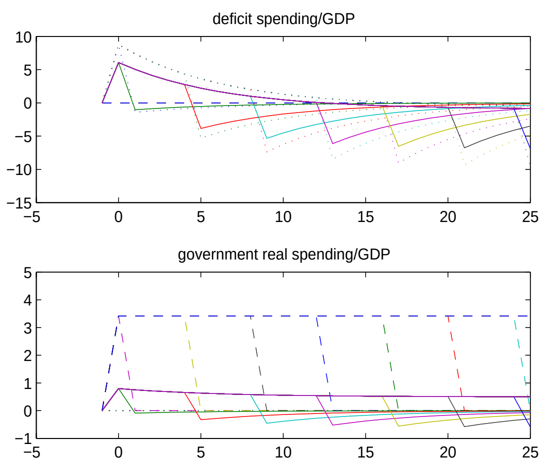

Figure 6: Comparison of deficit spending and debt in Markov equilibrium when the government only utilizes discretionary real spending (dashed line), only uses discretionary deficit spending (dotted line) and when it takes advantage of both (solid line).

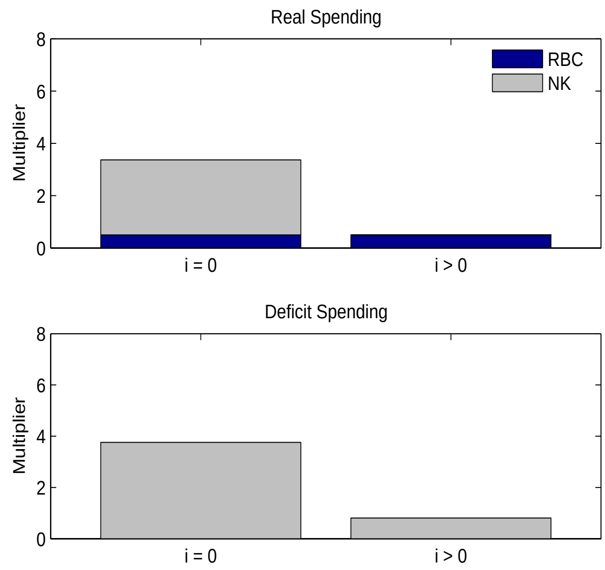

Figure 7: Multiplier Effects Under Real and Deficit Spending

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A Technical Appendix

This Technical Appendix details the numerical solution methods used and some further details for the proofs, for readers interested in the technical details. Some of this material is also contained in the Technical Appendix of a companion paper Eggertsson (2006) and the computation method shown in section (C.6) is also applied in Eggertsson and Woodford (2003) with appropriate modifications.

B Explicit first order conditions

This section shows the first order conditions of the government maximization problem.

The period Lagrangian is:

$$
\begin{aligned}
L_t &= u(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)) + g(F_t - s(T_t), \xi_t) - \tilde{v}(Y_t) + E_t\beta J(w_t, \xi_{t+1}) \\
&+ \phi_{1t}\left(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1+i_t}\right) \\
&+ \phi_{2t}(w_t - (1+i_t)\Pi_t^{-1}w_{t-1} - (1+i_t)F_t + (1+i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) + \\
&+ \phi_{3t}\left(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1+i_t}\right) \\
&+ \phi_{4t}(\theta Y_t[\frac{\theta-1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
&+ u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
&+ \psi_{1t}(f_t^e - \bar{f}^e(w_t, \xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(w_t, \xi_t)) + \gamma_{1t}(i_t - i^m) + \gamma_{2t}(\bar{w} - w_t)
\end{aligned}
$$

FOC (all the derivative should be equated to zero)

$$
\begin{aligned}
\frac{\delta L_t}{\delta \Pi_t} &= -u_c d'(\Pi_t) - u_m m_t \Pi_t^{-2} \\
&+\phi_{1t}[-\frac{u_{mc}d'\Pi_t^{-1}}{u_c} - \frac{u_{mm}m_t\Pi_t^{-3}}{u_c} - \frac{u_m\Pi_t^{-2}}{u_c} + \frac{u_m u_{cc} d'\Pi_t^{-1}}{u_c^2} + \frac{u_m u_{cm} m_t \Pi_t^{-2}}{u_c^2}] \\
&+[\phi_{2t}(1+i_t)w_{t-1}\Pi_t^{-2} - (i_t - i^m)m_t\Pi_t^{-2}] + \phi_{3t}[\frac{u_{cc}d'}{(1+i_t)} + \frac{u_{cm}m_t\Pi_t^{-2}}{(1+i_t)}] \\
&+\phi_{4t}[-Y_t(\theta-1)(1+s)(u_{cc}d' + m_t\Pi_t^{-2}u_{cm}) - u_{cc}\Pi_t d'^2 - u_{cm}m_t\Pi_t^{-1}d' + u_c\Pi_t d'' + u_c d']
\end{aligned} \tag{50}
$$

$$
\frac{\delta L_t}{\delta Y_t} = u_c - \tilde{v}_y + \phi_{1t}[\frac{u_{mc}}{u_c} - \frac{u_m}{u_c^2}]\Pi_t^{-1} - \phi_{3t}\frac{u_{cc}}{1+i_t} + \phi_{4t}[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) + \theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) + u_{cc}\Pi_t d'] \tag{51}
$$

$$
\frac{\delta L_t}{\delta F_t} = -u_c + g_G + \phi_{1t}[-\frac{u_{mc}}{u_c} + \frac{u_m}{u_c^2}]\Pi_t^{-1} + (1+i_t)\phi_{2t} + \phi_{3t}\frac{u_{cc}}{1+i_t} - \phi_{4t}[Y_t(\theta-1)(1+s)u_{cc} + u_{cc}\Pi_t d'] \tag{52}
$$

$$\frac{\delta L_t}{\delta i_t} = -\phi_{1t}\frac{1+i^m}{(1+i_t)^2} + \phi_{2t}(m_t\Pi_t^{-1} + T_t - w_{t-1}\Pi_t^{-1} - F) + \phi_{3t}\frac{u_c}{(1+i_t)^2} + \gamma_{1t} \tag{53}$$

$$\frac{\delta L_t}{\delta m_t} = u_m\Pi_t^{-1} + \phi_{1t}[\frac{u_{mm}}{u_c} - \frac{u_m}{u_c^2}u_{cm}\Pi_t^{-1}]\Pi_t^{-1} + \phi_{2t}(i_t - i^m)\Pi_t^{-1} - \phi_{3t}\frac{u_{cm}}{1+i_t}\Pi_t^{-1} - \phi_{4t}[Y_t(\theta-1)(1+s)u_{cm}\Pi_t^{-1} - u_{cm}d'] \tag{54}$$

$$\frac{\delta L_t}{\delta T_t} = -g_G s'(T_t) + \phi_{2t}(1+i_t) \tag{55}$$

$$\frac{\delta L_t}{\delta w_t} = \beta E_t J_w(w_t, \xi_{t+1}) - \psi_{1t}f_w^e - \psi_{2t}S_w^e + \phi_{2t} - \gamma_{2t} \tag{56}$$

$$\frac{\delta L_t}{\delta f_t^e} = \beta\phi_{3t} + \psi_{1t} \tag{57}$$

$$\frac{\delta L_t}{\delta S_t^e} = -\beta\phi_{4t} + \psi_{2t} \tag{58}$$

The complementary slackness conditions are:

$$\gamma_{1t} \geq 0,\ i_t \geq i^m,\ \ \gamma_{1t}(i_t - i^m) = 0 \tag{59}$$

$$\gamma_{2t} \geq 0,\ \ \bar{w} - w_t \geq 0,\ \ \gamma_{2t}(\ \bar{w} - w_t) = 0 \tag{60}$$

The optimal plan under discretion also satisfies an envelope condition:

$$J_w(w_{t-1}, \xi_t) = -\phi_{2t}(1+i_t)\Pi_t^{-1} \tag{61}$$

Necessary and sufficient condition for a Markov equilibrium thus are given by the first order conditions (50) to (61) along with the constraints (8), (18), (21), (23) and the definitions (20) and (22). Note that the first order conditions imply restrictions on the unknown vector function Λ_t and the expectation functions.

C Approximation Method

This section show the approximation method used to approximate the Markov equilibrium.

C.1 Equilibrium in the absence of seigniorage revenues

As discussed in the text it simplifies the discussion to assume that the equilibrium base money small, i.e. that m_t is a small number (see Woodford (2003), chapter 2, for a detailed treatment). This simplifies the algebra and my presentation of the results. I discuss in the footnote some reasons for why I conjecture that this abstraction has no significant effect.[26]

[26] First, as shown by Woodford (2003), for a realistic calibration parameters, this abstraction has trivial effect on the AS and the IS equation under normal circustances. Furthermore, at zero nominal interest rate, increasing

To analyze an equilibrium with a small monetary base I parameterize the utility function by the parameter $\bar{m}$ and assume that the preferences are of the form:

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \qquad (62)$$

As the parameter $\bar{m}$ approaches zero the equilibrium value of m_t approaches zero as well. At the same time it is possible for the value of u_m to be a nontrivial positive number, so that money demand is well defined and the government's control over the short-term nominal interest rate is still well defined (see discussion in the proofs of Propositions 5 in section D). I can define $\tilde{m}_t = \frac{m_t}{\bar{m}}$ as the policy instrument of the government, and this quantity can be positive even as $\bar{m}$ and m_t approach zero. Note that even as the real monetary base approaches the cashless limit the *growth rate* of the nominal stock of money associated with different equilibria is still well defined. I can then still discuss the implied path of money supply for different policy options. To see this note that

$$\frac{\tilde{m}_t}{\tilde{m}_{t-1}} = \frac{\frac{M_t}{P_{t-1}\bar{m}}}{\frac{M_{t-1}}{P_{t-2}\bar{m}}} = \frac{M_t}{M_{t-1}}\Pi_{t-1}^{-1} \qquad (63)$$

which is independent of the size of $\bar{m}$. For a given equilibrium path of inflation and $\tilde{m}_t$ I can infer the growth rate of the nominal stock of money that is required to implement this equilibrium by the money demand equation. By assuming $\bar{m} \rightarrow 0$ I only abstract from the effect this adjustment has on the marginal utility of consumption and seigniorage revenues, both of which would be trivial in a realistic calibration (see footnote 26).

C.2 Steady state discussion and relation to literature on Markov Equilibrium

In general a steady-state of a Markov equilibrium is non-trivial to compute, as emphasized by Klein et al (2003). This is because each of the steady state variables depend on the mapping between the endogenous state (i.e. debt) and the unknown functions $J(.)$ and $\bar{e}(.)$, so that one needs to know the derivative of these functions with respect to the endogenous policy state variable to calculate the steady state. Klein

money balances further *does nothing* to facilitate transactions since consumer are already satiated in liquidity. This was one of the key insights of Eggertsson and Woodford (2003), which showed that at zero nominal interest rate increasing money supply has no effect if expectations about future money supply do not change. It is thus of even *less* interest to consider this additional channel for monetary policy at zero nominal interest rates than if the short-term nominal interest rate was positive. Second, assuming m_t is a very small number is likely to change the government budget constraint very little in a realistic calibration. By assuming the cashless limit I am assuming no seignorage revenues so that the term $\frac{i_t - i^m}{1+i_t}m_t\Pi_t^{-1}$ in the budget constraint has no effect on the equilibrium. Given the low level of seignorage revenues in industrialized countries I do not think this is a bad assumption. Furthermore, in the case the bound on the interest rate is binding, this term is zero, making it of even *less* interest when the zero bound is binding than under normal circumstances.

et al suggest an approximation method by which one may approximate this steady state numerically by using perturbation methods. In this paper I take a different approach. Proposition 5 shows that a steady state may be calculated under assumptions that are fairly common in the monetary literature, without any further assumptions about the unknown functions $J(.)$ and $e(.)$.

Proposition 5 *If $\xi = 0$ at all times and (i)-(iii) in A4 hold there is a Markov equilibrium steady state that is given by $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ and $Y = \bar{Y}$ where $\bar{Y}$ and $\bar{F}$ are the unique solution to the equations: $u_c(Y - F) = v_y(Y)$ and $u_c(Y - F) + g_G(F - s(F))s'(F) = g_G(F - s(F))$*

To proof the proposition about the steady state I look at the algebraic expressions of the first order conditions of the government maximization problem. The proof is in section (D). A noteworthy feature of the proof is that the mapping between the endogenous state and the functions $J(.)$ and $e(.)$ does not matter (i.e. the derivatives of these functions cancel out). The reason is that the Lagrangian multipliers associated with the expectation functions are zero in steady state and I may use the envelope condition to substitute for the derivative of the value function. The intuition for why these Lagrangian multipliers are zero in equilibrium is simple. At the steady state the distortions associated with monopolistic competition are zero (because of A2 (ii)). This implies that there is no gain of increasing output from steady state. In the steady the real debt is zero and according to assumption (i) seigniorage revenues are zero as well. This implies that even if there is cost of taxation in the steady state, increasing inflation does not reduce taxes. It follows that all the Lagrangian multipliers are zero in the steady state apart from the one on the government budget constraint. That multiplier, i.e. ϕ_2, is positive because there are steady state tax costs. Hence it would be beneficial (in terms of utility) to relax this constraint.

There is by now a rich literature studying the question whether there can be multiple Markov equilibria in monetary models that are similar in many respects to the one I have described here (see e.g. Albanesi et al (2003), Dedola (2002) and King and Wolman (2003)). I do not proof the global uniqueness of the steady state in Proposition 5 but show that it is locally unique.[27] I conjecture, however, that the steady state is globally unique under A2.[28] But even if I would have written the model so that it had more than one

[27] See Woodford (2003) Appendix A3 for definition and discussion of local uniqueness in stochastic general equilibrium models of this kind.

[28] The reason for this conjecture is that in this model, as opposed to Albanesi et al and Dedola work, I assume in A2 that there are no monetary frictions. The source of the multiple equilibria in those papers, however, is the payment technology they assume. The key difference between the present model and that of King and Wolman, on the other hand, is that they assume that some firms set prices at different points in time. I assume a representative

steady state, the one studied here would still be the one of principal interest as discussed in the footnote.[29]

C.3 Approximate system and order of accuracy

The conditions that characterize equilibrium are given by the constraints of the model and the first order conditions of the governments problem. A linearization of this system is complicated by the Kuhn-Tucker inequalities (59) and (60). I look for a solution in which the bound on government debt is never binding, and then verify that this bound is never binding in the equilibrium I calculate. Under this conjectured the solution to the inequalities (59) and (60) can be simplified into two cases:

$$\text{Case 1}: \gamma_t^1 = 0 \;\; \text{if } i_t > i^m \tag{64}$$

$$\text{Case 2}: i_t = i^m \;\; \text{otherwise} \tag{65}$$

Thus in both Case 1 and 2 I have equalities characterizing equilibrium. These equations are (9), (18),(19), (21), (23), (24), (20), (22) and (50)-(58) and either (64) when $i_t > i^m$ or (65) otherwise. Under the condition A1(i) and A1(ii) but $i^m < \frac{1}{\beta} - 1$ then $i_t > i^m$ and Case 1 applies in the absence of shocks. In the knife edge case when $i^m = \frac{1}{\beta} - 1$,however, the equations that solve the two cases (in the absence of shocks) are identical since then both $\gamma_{1t} = 0$ and $i_t = i^m$. Thus both Case 1 and Case 2 have the same steady state in the knife edge case $i_t = i^m$. If I linearize around this steady state (which I show exists in Proposition 5) I obtain a solution that is accurate up to a residual $(||\xi||^2)$ for both Case 1 and Case 2. As a result I have one set of linear equations when the bound is binding, and another set of equations when it

firm, thus abstacting from the main channel they emphasize in generating multiple equilibria. Finally the present model is different from all the papers cited above in that I introduce nominal debt as a state variable. Even if the model I have illustrated above would be augmented to incorporate additional elements such as montary frictions and staggering prices, I conjecture that the steady state would remain unique due to the ability of the government to use nominal debt to change its future inflation incentive. That is, however, a topic for future reasearch and there is work in progress by Eggertsson and Swanson that studies this question.

[29] Even if I had written a model in which the equilibria proofed above is not the unique global equilibria the one I illustrate here would still be the one of principal interest. Furthermore a local analysis would still be useful. The reason is twofold. First, the equilibria analyzed is identical to the commitment equilibrium (in the absence of shocks) and is thus a natural candidate for investigation. But even more importantly the work of Albanesi et al (2002) indicates that if there are non-trivial monetary frictions there are in general only two steady states.There are also two steady states in King and Wolman's model. (In Dedola's model there are three steady states, but the same point applies.) The first is a low inflation equilibria (analogues to the one in Proposition 1) and the other is a high inflation equilibria which they calibrate to be associated with double digit inflation. In the high inflation equilibria, however, the zero bound is very unlikely ever to be binding as a result of real shocks of the type I consider in this paper (since in this equilibria the nominal interest rate is very high as I will show in the next section). And it is the distortions created by the zero bound that are the central focus of this paper, and thus even if the model had a high inflation steady state, that equilibria would be of little interest in the context of the zero bound.

is not. The challenge, then, is to find a solution method that, for a given stochastic process for $\{\xi_t\}$, finds in which states of the world the interest rate bound is binding and the equilibrium has to satisfy the linear equations of Case 1, and in which states of the world it is not binding and the equilibrium has to satisfy the linear equations in Case 2. Since each of these solution are accurate to a residual $(||\xi||^2)$ the solutions can be made arbitrarily accurate by reducing the amplitude of the shocks. The next subsection show a solution method, assuming as simple process for the natural rate of interest, that numerically calculates when Case 1 applies and when Case 2 applies.

Note that I may also consider solutions when i^m is below the steady state nominal interest rate. A linear approximation of the equations around the steady state in Proposition 5 is still valid if the opportunity cost of holding money, i.e. $\bar{\delta} \equiv (i - i^m)/(1+i)$, is small enough. Specifically, the result will be exact up to a residual of order $(||\xi, \bar{\delta}||^2)$. In the numerical example in the text I suppose that $i^m = 0$ (see Eggertsson and Woodford (2003) for further discussion about the accuracy of this approach when the zero bound is binding). A nontrivial complication of approximating the Markov equilibrium is that I do not know the unknown expectation functions $\bar{e}(.)$. I illustrate a simple way of matching coefficients to approximate this function in section (C.5).

C.4 Linearized solution

I here linearize the first order conditions and the constraints around the steady state in Propositions 5. I assume the form of the utility discussed in section C.1. I allow for deviations in the vector of shocks ξ_t and in i^m so that the equations are accurate of order $o(||\xi, \bar{\delta}||^2)$. I abstract from the effect of the shocks on the disutility of labor. Here $dz_t = z_t - z_{ss}$ The economic constraints are:

$$\bar{u}_c d'' d\Pi_t + \theta(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \theta \bar{u}_{c\xi} d\xi_t - \bar{u}_c d'' \beta E_t d\Pi_{t+1} = 0 \tag{66}$$

$$\bar{u}_{cc} dY_t + \bar{u}_{c\xi} d\xi_t - \beta \bar{u}_{cc} E_t dY_{t+1} - \beta \bar{u}_{c\xi} E_t d\xi_{t+1} - \beta \bar{u}_c di_t + \beta \bar{u}_c E_t d\Pi_{t+1} = 0 \tag{67}$$

$$dw_t - \frac{1}{\beta} dw_{t-1} + \frac{1}{\beta} dT_t = 0 \tag{68}$$

$$dS_t^e - \bar{u}_c d'' E_t d\Pi_{t+1} = 0 \tag{69}$$

$$df_t^e + \bar{u}_c E_t d\Pi_{t+1} - \bar{u}_{cc} E_t dY_{t+1} - \bar{u}_{c\xi} E_t \xi_{t+1} = 0 \tag{70}$$

The equation determining the natural rate of output is:

$$(v_{yy} - u_{cc})dY_t^n + (v_{y\xi} - u_{c\xi})d\xi_t - \frac{(\theta - 1)}{\theta} u_c ds = 0 \tag{71}$$

The equation determining the natural rate of interest is:

$$\beta E_t(\bar{u}_{cc}dY^n_{t+1} - \bar{u}_{c\xi}E_t d\xi_{t+1}) - (\bar{u}_{cc}dY^n_t - \bar{u}_{c\xi}d\xi_t) + \beta\bar{u}_{cc}dr^n_t = 0 \tag{72}$$

Note that the real money balances deflated by $\bar{m}$, i.e. $\tilde{m}_t$, are well defined in the cashless limit so that equation 63 is

$$d\tilde{m}_t - d\tilde{m}_{t-1} - d\frac{M_t}{M_{t-1}} + d\pi_{t-1} = 0$$

and money demand is approximated by

$$\frac{\bar{\chi}_{mm}}{u_c}d\tilde{m}_t - \frac{\bar{\chi}_{mm}}{u_c}\tilde{m}d\Pi_t - \frac{\bar{\chi}_{mm}}{u_c}\tilde{m}dY_t - \beta di_t + \beta di^m = 0$$

The Kuhn Tucker conditions imply that

Case 1 when $i_t > i^m$

$$d\gamma_{1t} = 0 \tag{73}$$

Case 2 when $i_t = i^m$

$$di_t = 0 \tag{74}$$

I look for a solution in which case the debt limit is never binding so that $d\gamma_{2t} = 0$ at all times and verify that this is satisfied in equilibrium. Linearized FOC in a Markov Equilibrium

$$-d''\bar{u}_c d\Pi_t + \bar{\phi}_2\beta^{-1}dw_{t-1} + d''\bar{u}_c d\phi_{4t} = 0 \tag{75}$$

$$(\bar{u}_{cc} - \bar{v}_{yy})dY_t + \bar{u}_{c\xi}d\xi_t - \bar{v}_{y\xi}d\xi_t - \bar{u}_{cc}\beta d\phi_{3t} + \theta(\bar{u}_{cc} - \bar{v}_{yy})d\phi_{4t} = 0 \tag{76}$$

$$\begin{aligned}&-\bar{u}_{cc}dY_t + \bar{u}_{cc}d'd\Pi_t + (\bar{u}_{cc} + \bar{g}_{GG})dF_t + (\bar{g}_{GG} - \bar{u}_{cc})d\xi_t - \bar{g}_{GG}s'dT_t \\ &+(1+\bar{\imath})d\phi_{2t} + \bar{\phi}_2 di_t + \frac{\bar{u}_{cc}}{1+\bar{\imath}}d\phi_{3t} - [\bar{Y}(\theta-1)(1+s)\bar{u}_{cc} + \bar{u}_{cc}d']d\phi_{4t} = 0\end{aligned} \tag{77}$$

$$\bar{\phi}_2 dT_t - \bar{\phi}_2 dw_{t-1} + \bar{u}_c\beta^2 d\phi_{3t} + d\gamma_{1t} = 0 \tag{78}$$

$$\bar{g}_{GG}(s')^2 dT_t - \bar{g}_G s'' dT_t - \bar{g}_{G\xi}d\xi_t + \beta^{-1}d\phi_{2t} + \bar{\phi}_2 di_t = 0 \tag{79}$$

$$d\phi_{2t} - E_t d\phi_{2t+1} - \beta\bar{\phi}_2 E_t di_{t+1} + \bar{\phi}_2 E_t d\Pi_{t+1} + \beta f_w d\phi_{3t} - \beta S_w d\phi_{4t} - d\gamma_{2t} = 0 \tag{80}$$

Note that the first order condition with respect to m_t does not play any role in the cashless limit so that it is omitted above. Also note that the two derivatives f_w and S_w are in general not known. In the next section I show how these derivatives can be found

C.5 Approximating f_w and S_w

I show how the two derivatives f_w and S_w can be approximated under A5. At time $t \geq \tau$ the system is deterministic. Then I can approximate these functions to yield $w_t = w^1 w_{t-1}$ and $d\Lambda_t = \Lambda^1 w_{t-1}$, where the first element of the vector $d\Lambda_t$ is $d\pi_t = \pi^1 w_{t-1}$, the second $dY_t = Y^1 w_{t-1}$ and so on and $w_t = w^1 w_{t-1}$ where the vector Λ^1 and the number w^1 are some unknown constants. To find the value of each of these coefficients I substitute this solution into the system (66)-(70) and (75)-(80) and match coefficients. For example equation (66) implies that

$$\bar{u}_c d'' \pi^1 w_{t-1} + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 w_{t-1} - \bar{u}_c d'' \beta \pi^1 w^1 w_{t-1} = 0 \tag{81}$$

where I have substituted for $d\pi_t = \pi^1 w_{t-1}$ and for $d\pi_{t+1} = \pi^1 w_t = \pi^1 w^1 w_{t-1}$. Note that I assume that $t \geq \tau$ so that there is perfect foresight and I may ignore the expectation symbol. This equation implies that the coefficients π^1, y^1 and w^1 must satisfy the equation:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_c d'' \beta \pi^1 w^1 = 0 \tag{82}$$

I may similarly substitute the solution into each of the equation (66)-(70) and (75)-(80) to obtain a system of equation that the coefficients must satisfy:

$$\bar{u}_c d'' \pi^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_c d'' \beta \pi^1 w^1 = 0 \tag{83}$$

$$\bar{u}_{cc} Y^1 - \beta \bar{u}_{cc} Y^1 w^1 - \beta \bar{u}_c i^1 + \beta \bar{u}_c \pi^1 w^1 = 0 \tag{84}$$

$$w^1 - \frac{1}{\beta} + \frac{1}{\beta} T^1 = 0 \tag{85}$$

$$S^1 - \bar{u}_c d'' \pi^1 w^1 = 0 \tag{86}$$

$$f^1 + \bar{u}_c \pi^1 w^1 - \bar{u}_{cc} Y^1 w^1 = 0 \tag{87}$$

$$-d_{\mathrm{J}} \bar{u}_c \pi^1 + \frac{s' \bar{g}_G}{\beta} + d'' \bar{u}_c \phi_4^1 = 0 \tag{88}$$

$$(\bar{u}_{cc} - \bar{v}_{yy}) Y^1 - \bar{u}_{cc} \beta \phi_3^1 + \theta(\bar{u}_{cc} - \bar{v}_{yy}) \phi_4^1 = 0 \tag{89}$$

$$s' \bar{g}_G T^1 - s' \bar{g}_G + \bar{u}_c \beta^2 \phi_3^1 = 0 \tag{90}$$

$$\bar{g}_{GG} (s')^2 T^1 - \bar{g}_G s'' T^1 + \beta^{-1} \phi_2^1 + \bar{g}_G s' i^1 = 0 \tag{91}$$

$$\phi_2^1 - \phi_2^1 w^1 - \beta \bar{g}_G s' i^1 w^1 + \bar{g}_G s' \pi^1 w^1 + \beta f^1 \phi_3^1 - \beta S^1 \phi_4^1 = 0 \tag{92}$$

There are 11 unknown coefficients in this system i.e. $\pi^1, Y^1, i^1, F^1, S^1, f^1, T^1, \phi_2^1, \phi_3^1, \phi_4^1, w^1$. For a given value of w^1, (83)-(91) is a linear system of 10 equations with 10 unknowns, and thus there is a unique value given for each of the coefficients as long as the system is non-singular (which can be verified to be the case for standard functional forms for the utility and technology functions). The value of w^1 is in general not unique, but in the calibrated model there is always a unique bounded solution in the examples I have studied (and the unbounded solutions will violate the debt limit). In a simplified version of the model it can be proofed that there is a unique solution for w^1 that satisfies all the necessary conditions, but I have not managed to proof it in this model (see discussion in Eggertsson (2006)).

C.6 Computational method

Here I illustrate a solution method for the optimal commitment solution. This method can also be applied, with appropriate modification of each of the steps, to find the Markov solution. I assume shocks so that the natural rate of interest becomes unexpectedly negative in period 0 and the reverts back to normal with probability α_t in every period t as in A5 (one may use (71) and (72) to find what a given negative number for the natural rate of interest implies for the underlying exogenous shocks). I assume that there is a final date K in which the natural rate becomes positive with probability one (this date can be arbitrarily far into the future).

The solution takes the form:

$$\begin{array}{lcccc} \text{Case 2 } i_t & = & 0 & \forall & t & 0 \leq t < \tau + k \\ \text{Case 1 } i_t & > & 0 & \forall & t & t \geq \tau + k \end{array}$$

Here τ is he stochastic date at which the natural rate of interest returns to steady state. I assume that τ can take any value between 1 and the terminal date K that can be arbitrarily far into the future. The number $\tau + k_\tau$ is the period in which the zero bound stops being binding in the contingency when the natural rate of interest becomes positive in period τ. Note that the value of k_τ can depend on the value of τ. I first show the solution for the problem as if I knew the sequence $\{k_\tau\}_{\tau=1}^S$. I then describe a numerical method to find the sequence $\{k_\tau\}_{\tau=1}^S$.

C.6.1 The solution for $t \geq \tau + k_\tau$

The system of linearized equations (75)-(80), (66)-(70), and (73) can be written in the form:

$$\begin{bmatrix} E_t Z_{t+1} \\ P_t \end{bmatrix} = M \begin{bmatrix} Z_t \\ P_{t-1} \end{bmatrix}$$

where $Z_t \equiv \begin{bmatrix} \Lambda_t & e_t & \phi_t & \psi_t & \gamma_t^1 \end{bmatrix}^T$ and $P_t \equiv w_t$. If there are fifteen eigenvalues of the matrix M outside the unit circle this system has a unique bounded solution of the form:

$$P_t = \Omega^0 P_{t-1} \tag{93}$$

$$Z_t = \Lambda^0 P_{t-1} \tag{94}$$

C.6.2 The solution for $\tau \leq t < \tau + k$

Again this is a perfect foresight solution but with the zero bound binding. The solution now satisfies the equations (75)-(80), (66)-(70) but (74) instead of (73).The system can be written on the form:

$$\begin{bmatrix} P_t \\ Z_t \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_{t-1} \\ Z_{t+1} \end{bmatrix} + \begin{bmatrix} M \\ V \end{bmatrix}$$

This system has a solution of the form:

$$P_{\tau+j} = \Omega^{k_\tau - j} P_{\tau+j-1} + \Phi^{k_\tau - j} \tag{95}$$

$$Z_{\tau+j} = \Lambda^{k_\tau - j} P_{\tau+j-1} + \Theta^{k_\tau - j} \tag{96}$$

where $j = 0, 1, 2, ..., k$. Here $\Omega^{k_\tau - j}$ is the coefficient in the solution when there are $k_\tau - j$ periods until the zero bound stops being binding (i.e. when $j - k_\tau = 0$ the zero bound is not binding anymore and the solution is equivalent to (93)-(94)). We can find the numbers $\Lambda^j, \Omega^j, \Theta^j$ and Φ^j for $j = 1, 2, 3, ..., k$ by solving the equations below using the initial conditions $\Phi^0 = \Theta^0 = 0$ for $j = 0$ and the initial conditions for Λ^j and Ω^j given in (93)-(94):

$$\Omega^j = [I - B\Lambda^{j-1}]^{-1} A$$

$$\Lambda^j = C + D\Lambda^{j-1}\Omega^j$$

$$\Phi^j = (I - B\Lambda^{j-1})^{-1}[B\Theta^{j-1} + M]$$

$$\Theta^j = D\Lambda^{j-1}\Phi^j + D\Theta^{j-1} + V$$

C.6.3 The solution for $t < \tau$

The solution satisfies (75)-(80), (66)-(70), and (74). Note that each of the expectation variables can be written as $\tilde{x}_t = E_t x_{t+1} = \alpha_{t+1}\tilde{x}_{t+1} + (1 - \alpha_{t+1})x_{t+1}$ where α_{t+1} is the probability that the natural rate of interest becomes positive in period $t + 1$. Here hat on the variables refers to the value of each variable contingent on that the natural rate of interest is negative. I may now use the solution for Z_{t+1} in 96 to substitute for Z_{t+1}, i.e. the value of each variable contingent on that the natural rate becomes positive again, in terms of the hatted variables. The value of x_{t+1}, for example, can be written as $x_{t+1} = \Lambda_{21}^{k_{t+1}}\tilde{\phi}_{1t} + \Lambda_{22}^{k_{t+1}}\tilde{\phi}_{2t} + \Theta_2^{k_{t+1}}$ where $\Lambda_{ij}^{k_{t+1}}$ is the ijth element of the matrix $\Lambda^{k_{t+1}}$ and the value k_{t+1} depends on the number of additional periods that the zero bound is binding (recall that I am solving the equilibrium on the assumption that I know the value of the sequence $\{k_\tau\}_{\tau=1}^{S}$) . Hence I can write the system as:

$$\begin{bmatrix} \tilde{P}_t \\ \tilde{Z}_t \end{bmatrix} = \begin{bmatrix} A_t & B_t \\ C_t & D_t \end{bmatrix} \begin{bmatrix} \tilde{P}_{t-1} \\ \tilde{Z}_{t+1} \end{bmatrix} + \begin{bmatrix} M_t \\ V_t \end{bmatrix}$$

I can solve this backwards from the date K in which the natural rate returns back to normal with probability one. I can then calculate the path for each variable to date 0. Note that.

$$B_{K-1} = D_{K-1} = 0$$

By recursive substitution I can find a solution of the form:

$$\tilde{P}_t = \Omega_t \tilde{P}_{t-1} + \Phi_t \tag{97}$$

$$\tilde{Z}_t = \Lambda_t \tilde{P}_{t-1} + \Theta_t \tag{98}$$

where the coefficients are time dependent. To find the numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t consider the solution of the system in period $K-1$ when $B_{K-1} = D_{K-1} = 0$. I have:

$$\Omega_{K-1} = A_{K-1}$$

$$\Phi_{K-1} = M_{K-1}$$

$$\Lambda_{K-1} = C_{K-1}$$

$$\Theta_{K-1} = V_{K-1}$$

I can find of numbers $\Lambda_t, \Omega_t, \Theta_t$ and Φ_t for period 0 to $K-2$ by solving the system below (using the initial conditions shown above for $S-1$):

$$\Omega_t = [I - B_t \Lambda_{t+1}]^{-1} A_t$$

$$\Lambda_t = C_t + D_t \Lambda_{t+1} \Omega_t$$

$$\Phi_t = (I - B_t \Lambda_{t+1})^{-1} [B_t \Theta_{t+1} + M_t]$$

$$\Theta_t = D_t \Lambda_{t+1} \Phi_t + D_t \Theta_{t+1} + V_t$$

Using the initial condition $\tilde{P}_{-1} = 0$ I can solve for each of the endogenous variables under the contingency that the trap last to period K by (97) and (98). I then use the solution from (93)-(96) to solve for each of the variables when the natural rate reverts back to steady state.

C.6.4 Solving for $\{k_\tau\}_{t=0}^{\infty}$

A simple way to find the value for $\{k_\tau\}_{\tau=1}^{\infty}$ is to first assume that k_τ is the same for all τ and find the k so that the zero bound is never violated. Suppose that the system has converged at $t = 25$ (i.e. the response of each of the variables is the same). Then I can move to 24 and see if $k_\tau = 4$ for $\tau = 1, 2, ...24$ is a solution that never violates the zero bound. If not move to 23 and try the same thing and so on. For preparing this paper I wrote a routine in MATLAB that applied this method to find the optimal solution and verified that the results satisfied all the necessary conditions. It turned out that in the Markov equilibrium the zero bound stopped being binding as soon as the natural rate of interest is positive again (the same is not true for the commitment equilibrium as shown in Eggertsson (2006) and Eggertsson and Woodford (2003)).

C.7 Calibration for numerical results

In the numerical examples I assume the following functional forms for preferences and technology:

$$u(C, \xi) = \frac{C^{1-\tilde{\sigma}^{-1}} \bar{C}^{\tilde{\sigma}^{-1}}}{1 - \tilde{\sigma}^{-1}}$$

where $\bar{C}$ is a preference shock assumed to be 1 in steady state.

$$g(G, \xi) = g_1 \frac{G^{1-\tilde{\sigma}^{-1}} \bar{G}^{\tilde{\sigma}^{-1}}}{1 - \tilde{\sigma}^{-1}}$$

where $\bar{G}$ is a preference shock assumed to be 1 in steady state

$$v(H, \xi) = \frac{\lambda_1}{1 + \lambda_2} H^{1+\lambda_2} \bar{H}^{-\lambda_2}$$

where $\bar{H}$ is a preference shock assumed to be 1 in steady state

$$y = Ah^{\epsilon}$$

where A is a technology shock assumed to be 1 in steady state. I may substitute the production function into the disutility of working to obtain (assuming A=1):

$$\tilde{v}(Y, \xi_t) = \frac{\lambda_1}{1 + \omega} Y^{1+\omega} \bar{H}^{-\lambda_2}$$

When calibrating the shocks that generate the temporarily negative natural rate of interest I assume that it is the shock $\bar{\text{C}}$ that is driving the natural rate of interest negative (as opposed to A) since otherwise a negative natural rate of interest would be associated with a higher natural rate of output which does not seem to be the most economically interesting case. I assume that the shock $\bar{G}$ is such that the F_t would be constant in the absence of the zero bound, in order to keep the optimal size of the government (in absence of the zero bound) constant as discussed in the text. The cost of price adjustment is assumed to take the form:

$$d(\Pi) = d_1\Pi^2$$

The cost of taxes is assumed to take to form:

$$s(T) = s_1T^2$$

Aggregate demand implies$Y = C + F = C + G + s(F)$. I normalize $Y = 1$ in steady state and assume that the share of the government in production is $F = 0.3$. Tax collection as a share of government spending is assumed to be $\gamma = 5\%$ of government spending. This implies

$$0.05 = \frac{s(F)}{F} = s_1F$$

so that $s_1 = \frac{\gamma}{F}$. The result for the inflation and output gap response are not very sensitive to varying γ under either commitment or discretion. The size of the public debt issued in the Markov equilibrium, however, crucially depends on this variable. In particular if γ is reduced the size of the debt issued rises substantially. For example if $\gamma = 0.5\%$ the public debt issued is about ten times bigger than reported in the figure in the paper. I assume that government spending are set at their optimal level in steady state as discussed in the text:

$$g_2 = \frac{u_c}{g_G - s'g_G} = \frac{C^{-\tilde{\sigma}^{-1}}}{G^{-\tilde{\sigma}^{-1}}(1-s')} = (\frac{G}{C})^{\tilde{\sigma}^{-1}}\frac{1}{1-s'} = (\frac{G}{C})^{\tilde{\sigma}^{-1}}\frac{1}{1-2s_1F}$$

The IS equation and the AS equation are

$$x_t = E_tx_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^n)$$

$$\pi_t = k\pi_t + \beta E_t\pi_{t+1}$$

I assume, as Eggertsson and Woodford, that the interest rate elasticity, $\tilde{\sigma}$, is 0.5. The relationship between σ and $\tilde{\sigma}$ is

$$\tilde{\sigma} = \sigma\frac{Y}{C}$$

I assume that κ is 0.02 as in Eggertsson and Woodford (2003). The relationship between κ and the other parameters of the model is $\kappa = \theta\frac{(\sigma^{-1}+\lambda_2)}{d''}$. I scale hours worked so that $Y = 1$ in steady state which implies

$v_y = \lambda_1$ and using the $u_c = v_y$ one obtains $\lambda_1 = 1$.Finally I assume that $\theta = 7.87$ as in Rotemberg and Woodford and that $\omega = 2$. For a linear production function ω is the inverse of the Frisch elasticity of labor supply. The calibration value for the parameters are summarized in the table below:

Table 5

σ	0.71
g_1	1.11
λ_1	1
ω	2
d_1	787
s_1	0.17
θ	7.87

D Proofs

D.1 Proof of Proposition 5

Proposition 5 *If $\xi = 0$ at all times and (i)-(iii) in A4 hold there is a Markov equilibrium steady state that is given by $i = 1/\beta - 1$, $w = S^e = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0$, $\Pi = 1$, $\phi_2 = g_G(\bar{F} - s(\bar{F}))s'(\bar{F})$, $f^e = u_c(\bar{Y}), F = \bar{F} = G = T + s(T)$ and $Y = \bar{Y}$ where $\bar{Y}$ and $\bar{F}$ are the unique solution to the equations: $u_c(Y - F) = v_y(Y)$ and $u_c(Y - F) + g_G(F - s(F))s'(F) = g_G(F - s(F))$*

I only proof existence of this steady state here but do not discuss uniqueness (see Eggertsson (2006) for discussion about uniqueness of a Markov equilibrium in this model). In the assumption made in the proposition I assume the cashless limit and the form of the utility:

$$u(C_t, m_t\Pi_t^{-1}, \xi_t) = \tilde{u}(C_t, \xi_t) + \chi(\frac{m_t}{\bar{m}}\Pi_t^{-1}C_t^{-1}, \xi_t) \tag{99}$$

The partial derivatives with respect to each variable are given by

$$u_c = \tilde{u}_c - \chi'\frac{m}{\bar{m}}C^{-2}\Pi^{-1} \tag{100}$$

$$u_m = \frac{\chi'}{\bar{m}}C^{-1}\Pi^{-1} \tag{101}$$

$$u_{mm} = \frac{\chi''}{\bar{m}^2}C^{-2}\Pi^{-2} < 0 \tag{102}$$

$$u_{cm} = -\chi''\frac{m}{\bar{m}^2}C^{-3}\Pi^{-2} - \frac{\chi'}{\bar{m}}C^{-2}\Pi^{-1} \tag{103}$$

As $\bar{m}- > 0$ I assume that for $\tilde{m} = \frac{m}{\bar{m}} > 0$ I have

$$lim_{\bar{m}\to 0}\frac{\chi\prime}{\bar{m}} \equiv \bar{\chi}' \geq 0 \tag{104}$$

$$lim_{\bar{m}\to 0}\frac{\chi\prime\prime}{\bar{m}^2} \equiv \bar{\chi}'' > 0 \tag{105}$$

This implies that there is a well defined money demand function, even as money held in equilibrium approaches zero, given by

$$\frac{\bar{\chi}\prime(\tilde{m}C_t^{-1}\Pi_t^{-1}, \xi_t)C_t^{-1}\Pi_t^{-1}}{\bar{u}_c(C_t, \xi_t)} = \frac{i_t - i^m}{1 + i_t}$$

so that $\bar{\chi}' = 0$ when $i_t = i_t^m$. From the assumptions (104)-(105) it follows that:

$$lim_{\bar{m}\to 0}\chi\prime = 0$$

$$lim_{\bar{m}\to 0}\chi\prime\prime = 0$$

Then the derivatives u_c and u_{cm} in the cashless limit are:

$$\lim_{\bar{m}\to 0} u_c = \tilde{u}_c$$

and

$$\lim_{\bar{m}\to 0} u_{cm} = \lim_{\bar{m}\to 0}[-\bar{m}\frac{\chi\prime\prime}{\bar{m}^2}\frac{m}{\bar{m}}C^{-3}\Pi^{-1} - \frac{\chi\prime}{\bar{m}}C^{-2}] = -\bar{\chi}\prime C^{-2}$$

Hence in a steady state in which $\bar{m} \to 0$ and $i_t = i^m$ I have that $\bar{\chi}\prime = 0$ so that at the steady state

$$\lim_{\bar{m}\to 0} u_{cm} = 0. \tag{106}$$

Note that this does not imply that the satiation point of holding real balances is independent of consumption. To see this note that the satiation point of real money balances is is given by some finite number $S^* = \frac{m}{\bar{m}}Y$ which implies that $\chi(S \geq S^*) = \tilde{v}(S^*)$. The value of the satiation point as $\bar{m} \to 0$ is:

$$lim_{\bar{m}\to 0}S^* \equiv \bar{S} = \tilde{m}C$$

The value of this number still depends on C even as $\bar{m} \to 0$ and even if $u_{cm} = 0$ at the satiation point.

I now show that the steady state stated in Proposition 3 satisfies all the first order conditions and the constraints. The steady state candidate solution in is:

$$i = \frac{1}{\beta} - 1, w = \phi_1 = \phi_3 = \phi_4 = \psi_1 = \psi_2 = \gamma_1 = \gamma_2 = 0, \Pi = 1, \phi_2 = g_G s',\ T = F \tag{107}$$

and Y and F are the unique solution to the equations stated in the proposition. Note that (107) and the functional assumption about d imply that:

$$d' = 0 \tag{108}$$

Let us first consider the constraints. In the steady state the AS equation is

$$\theta Y[\frac{\theta - 1}{\theta}(1+s)u_c - \tilde{v}_y] - u_c \Pi d'(\Pi) + \beta u_c \Pi d'(\Pi) = 0$$

Since by (108) d'=0, and according to assumption (ii) of the propositions $\frac{\theta-1}{\theta}(1+s) = 1$ the AS equation is only satisfied in the candidate solution if

$$u_c = v_y \tag{109}$$

Evaluated in the candidate solution the IS equation is:

$$\frac{1}{1+i} = \frac{\beta u_c}{u_c}\Pi^{-1} = \beta$$

which is always satisfied at because it simply states that $i = 1 - 1/\beta$ which is consistent with the steady state I propose in the propositions and assumption (iii). The budget constraint is:

$$w - (1+i)\Pi^{-1}w - (1+i)F + (1+i)T + (1+i)\bar{m}\tilde{m}\Pi_t^{-1} = 0$$

which is also always satisfied in our candidate solution since it states that $F = T$, $w = 0$ and $\bar{m} \to 0$. The money demand equation indicates that the candidate solutions is satisfied if

$$u_m = \Pi u_c \frac{i - i^m}{1+i} = 0 \tag{110}$$

By (20) and (22) the expectation variables in steady state are

$$S^e = u_c \Pi d'$$

$$f^e = u_c \Pi$$

Since $\Pi = 1$ and $d' = 0$ by (108) these equations are satisfied in the candidate solution. Finally both the inequalities (9) and (19) are satisfied since $\bar{w} > w = 0$ in the candidate solution and $i = i^m$.

I now show that the first order conditions, i.e. the commitment and the Markov equilibrium first order conditions, that are given by (50)-(61), are also consistent with the steady state suggested.

I start with (50). It is

$$-u_c d' - u_m \bar{m}\tilde{m}\Pi^{-2} + \phi_1[-\frac{u_{mc} d'\Pi^{-1}}{u_c} - \frac{u_{mm}\bar{m}\tilde{m}\Pi^{-3}}{u_c} - \frac{u_m \Pi^{-2}}{u_c} + \frac{u_m u_{cc} d'\Pi^{-1}}{u_c^2} + \frac{u_m u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{u_c^2}] \quad (111)$$

$$+[\phi_2(1+i)w\Pi^{-2} - (i-i^m)\bar{m}\tilde{m}\Pi^{-2}] + \phi_3[\frac{u_{cc}d'}{1+i} + \frac{u_{cm}\bar{m}\tilde{m}\Pi^{-2}}{(1+i)}]$$

$$+\phi_4[-Y(\theta-1)(1+s)(u_{cc}d' + \bar{m}\tilde{m}\Pi^{-2}u_{cm}) - u_{cc}\Pi d'^2 - u_{cm}\bar{m}\tilde{m}\Pi^{-1}d' + u_c\Pi d'' + u_c d']$$

By (108) and (110) the first two terms are zero. The constraints that are multiplied by ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 are also zero because each of these variables are zero in our candidate solution (107). Finally, the term that is multiplied by ϕ_2 (which is positive) is also zero because $w = 0$ in our candidate solution (107) and so is $i - i^m$. Thus I have shown that the candidate solution (107) satisfies (50).

Let us now turn to (51). It is

$$u_c - \tilde{v}_y + \phi_1[\frac{u_{mc}}{u_c} - \frac{u_m}{u_c^2}]\Pi^{-1} - \phi_3\frac{u_{cc}}{1+i} + \phi_4[\theta(\frac{\theta-1}{\theta}(1+s)u_c - \tilde{v}_y) - \theta Y(\frac{\theta-1}{\theta}(1+s)u_{cc} - \tilde{v}_{yy}) - u_{cc}\Pi d'] = 0 \quad (112)$$

The first two terms $u_c - v_y$ are equal to zero by (109). The next terms are also all zero because they are multiplied by the terms ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 which are all zero in our candidate solution (107). Hence this equation is also satisfied in our candidate solution. Let us then consider (53). It is:

$$-\phi_1\frac{1+i^m}{(1+i)^2} + \phi_2(\bar{m}\tilde{m} + T - w\Pi^{-1} - F) + \phi_3\frac{u_c}{(1+i)^2} + \gamma_1 = 0$$

Again this equation is satisfied in our candidate solution because $\phi_1 = \phi_3 = w = 0$, $F = T$ and $\bar{m} \to 0$ in the candidate solution. Conditions (54) in steady state is:

$$\bar{m}\tilde{m}u_m\Pi^{-1} + \phi_1[\frac{u_{mm}}{u_c} - \frac{u_m}{u_c^2}u_{cm}\Pi^{-1}] + \phi_2(i-i^m)\bar{m}\tilde{m} - \phi_3\frac{u_{cm}}{1+i}\Pi^{-1} - \phi_4[Y(\theta-1)(1+s)u_{cm}\Pi^{-1} - u_{cm}d'] = 0 \quad (113)$$

The first term is zero by (110). All the other terms are also zero because ϕ_1, ϕ_3, ϕ_4, ψ_1 and ψ_2 are all zero in our candidate solution (107). Finally $i = i^m$ in our candidate solution so that the third term is zero as well. Conditions (55)and (52) in steady state are:

$$-g_G s'(T) + \phi_2(1+i) = 0 \quad (114)$$

$$-u_c + g_G + \phi_{1t}[-\frac{u_{mc}}{u_c} + \frac{u_m}{u_c^2}]\Pi_t^{-1} + (1+i)\phi_2 + \phi_{3t}\frac{u_{cc}}{1+i_t} - \phi_{4t}[\theta Y_t(\frac{\theta-1}{\theta}(1+s)u_{cc} + u_{cc}\Pi_t d'] \quad (115)$$

Using our candidate solution (107) I obtain:

$$u_c(Y-F) = g_G(F - s'(F)) + g_G s'(F)$$

which along with (109) is the equation that determine $\bar{Y}$ and $\bar{F}$ that was stated in the propositions 3 and 4. Using our assumption on s and standard Inada boundary conditions one may show that these equations have unique solution for $\bar{Y}$ and $\bar{F}$.

Let us now turn to (56). This equation involves three unknown functions, J_w, f_w^e and S_w^e.I can use (61) to substitute for J_w obtaining

$$-\beta\phi_2(1+i)\Pi^{-1} - \psi_1\beta f_w^e - \psi_2\beta S_w^e + \phi_2 - \gamma_2 = 0 \tag{116}$$

In general I cannot know if this equation is satisfied without making further assumption about f_w^e and S_w^e. But note that in my candidate solution $\psi_1 = \psi_2 = 0$. Thus the terms involving these two derivatives in this equation are zero. Since $\gamma_2 = 0$, this equation is satisfied if $(1+i)\Pi^{-1} = 1/\beta$. This is indeed the case in our candidate solution. Finally (57) and (58) are satisfied since $\phi_3 = \phi_4 = \psi_1 = \psi_2 = 0$ in the candidate solution. Thus I have shown that all the necessary and sufficient conditions of a Markov equilibrium are satisfied by our candidate solution (107). QED

D.2 Proof of Proposition 3 and 4

Proposition 3 *The power of real government spending does not depend on policy coordination, i.e. equilibrium B in diagram 2 (equation (48) holds) does not depend on whether or not the central bank is goal independent.*

The proof is simple but tedious. The central bank takes $F_t = T_t$ as exogenously given and its maximization problem can be characterized by the Lagrangian:

$$\begin{aligned}
L_t &= (\Pi_t - 1)^2 + \lambda_x(Y_t - Y_t^n)^2 \\
&+ \phi_{1t}\left(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}\right) \\
&+ \phi_{2t}(w_t - (1+i_t)\Pi_t^{-1}w_{t-1} - (1+i_t)F_t + (1+i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) \\
&+ \phi_{3t}\left(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t}\right) \\
&+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1+s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
&+ u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
&+ \psi_{1t}(f_t^e - \bar{f}^e(\xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(\xi_t)) + \gamma_{1t}(i_t - i^m)
\end{aligned}$$

taking F_t as given.

The treasury takes m_t as exogenously given and its maximization problem is:

$$
\begin{aligned}
L_t &= u(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)) + g(F_t - s(T_t), \xi_t) - \tilde{v}(Y_t) \\
&+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}) \\
&+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1}w_{t-1} - (1 + i_t)F_t + (1 + i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) + \\
&+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t}) \\
&+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
&+ u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) + \gamma_{1t}(i_t - i^m) \\
&+ \gamma_{2t}(\bar{w} - w_t)
\end{aligned}
$$

The proof is obtained by writing the first order condition of each of these maximization problems, linearizing them around the steady state in Proposition 3 and showing that the resulting equilibrium conditions are identical to the equilibrium conditions under coordination (detailed derivation is available upon request).

Proposition 4 *If the central bank is goal independent deficit spending has no effect, i.e. in equilibrium C in diagram 2 (equation (49) holds) deficit spending has no effect on inflation, output or interest rates when the central bank is goal independent.*

The proof here is similar to the last proof and a little tedious. The central bank takes $T_t = T(w_{t-1}, \xi)$ as exogenously given and its maximization problem can be characterized by the Lagrangian:

$$
\begin{aligned}
L_t &= (\Pi_t - 1)^2 + \lambda_x(Y_t - Y_t^n)^2 \\
&+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}) \\
&+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1}w_{t-1} - (1 + i_t)F_t + (1 + i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) \\
&+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t}) \\
&+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
&+ u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
&+ \psi_{1t}(f_t^e - \bar{f}^e(\xi_t)) + \psi_{2t}(S_t^e - \bar{S}^e(\xi_t)) + \gamma_{1t}(i_t - i^m)
\end{aligned}
$$

taking $T(w_{t-1}, \xi)$ as given.

The treasury takes m_t as exogenously given and its maximization problem is:

$$
\begin{aligned}
L_t &= u(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)) + g(F_t - s(T_t), \xi_t) - \tilde{v}(Y_t) \\
&+ \phi_{1t}(\frac{u_m(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t^{-1}}{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)} - \frac{i_t - i^m}{1 + i_t}) \\
&+ \phi_{2t}(w_t - (1 + i_t)\Pi_t^{-1}w_{t-1} - (1 + i_t)F_t + (1 + i_t)T_t + (i_t - i^m)m_t\Pi_t^{-1}) + \\
&+ \phi_{3t}(\beta f_t^e - \frac{u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)}{1 + i_t}) \\
&+ \phi_{4t}(\theta Y_t[\frac{\theta - 1}{\theta}(1 + s)u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t) - \tilde{v}_y(Y_t, \xi_t)] \\
&+ u_c(Y_t - d(\Pi_t) - F_t, m_t\Pi_t^{-1}, \xi_t)\Pi_t d'(\Pi_t) - \beta S_t^e) \\
&+ \gamma_{2t}(\bar{w} - w_t)
\end{aligned}
$$

The proof is obtained by writing the first order condition of each of these maximization problems, linearizing them around the steady state in Proposition 3 and showing that the resulting equilibrium conditions of the central bank are identical to the first order conditions of the government when there is no deficit spending. These conditions are sufficient to characterize the solution for output, inflation and interest rates. The first order conditions for the treasury then determine the equilibrium path for taxes and debt (detailed derivation is available upon request)..