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Abstract

Self-fulfilling expectations are commonly believed to play an important role in the transmission of currency crises across countries. However, existing models that use multiple equilibria to illustrate the importance of such expectations have many undesirable features. This paper presents a new mechanism, based on the incomplete information framework of Morris and Shin (AER, 1998), through which self-fulfilling expectations can generate contagion. If speculators expect contagion across markets to occur, they have an incentive to trade in both currencies to take advantage of this correlation. These actions, in turn, link the two markets in such a way that a sharp devaluation of one currency will be propagated to the other market and will fulfill the original expectations. Even though the resulting model has multiple equilibria, it places restrictions on observable variables that are broadly consistent with existing empirical evidence.

Key words: currency crises, contagion, speculative attacks, self-fulfilling prophesies

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1 Introduction

One of the most striking features of the currency crises of the 1990s was their contagiousness: a sharp devaluation of one currency was often followed by devaluations of other, sometimes seemingly unrelated, currencies. While many competing explanations have been offered for this phenomenon, no general consensus has emerged. One popular view is that at least some episodes of observed contagion were driven by the self-fulfilling beliefs of market participants.[1] Models formalizing this view, however, have been heavily criticized along several dimensions. This paper presents a mechanism through which self-fulfilling expectations can lead to contagion of a crisis, but which overcomes the standard criticisms of previous expectations-based approaches.

The argument that expectations are the driving force behind the spread of currency crises typically begins by focusing on the self-fulfilling nature of such crises. A currency market with a fixed exchange rate regime is, in this view, a coordination game that often has multiple equilibria (as in Obstfeld, 1996). In one equilibrium speculators attack the currency and a devaluation occurs, while in another no one attacks and no devaluation occurs; which equilibrium obtains depends entirely on the expectations of the market participants. This view leads naturally to a simple theory of contagion in which a devaluation of one currency acts as a signal that coordinates expectations on the crisis equilibrium in another currency market. In other words, if speculators believe that the Malaysian *ringgit* would experience a sharp devaluation were a currency crisis to occur first in, say, Thailand, then they will all choose to attack the *ringgit* after observing a crisis in Thailand and, as a result, contagion will occur.[2]

While many people find this view of contagion intuitively appealing and useful for thinking about the role of expectations in the spread of crises, it has been (rightly) criticized along several dimensions. First, this simple story is capable of explaining *any* correlation of outcomes across countries and hence has little or no empirical content. In other words, this story does not explain why the event "crisis in Thailand" should be the signal for speculators to attack in Malaysia and not the event "no crisis in Thailand" or "slight appreciation in Thailand". While one can argue that some correlations are more reasonable than others, the formal prediction of this simple the-

[1] See, for example, Calvo (1998), Krugman (1999), and Masson (1999, 2001).

[2] If two countries are highly integrated, of course, (through trade, etc.) it is not entirely surprising that a crisis in one would have strong effects on the other. The importance of expectations is, therefore, most often stressed in cases where the two currencies are, at least in principle, not closely related. The crises in Russia and Brazil in 1998 is another oft-cited example.

ory is that whatever correlation agents expect between the outcomes of the two markets will be self-fulfilling. A second line of criticism is that this view of contagion leaves no role for real connections, such as trade or financial-market linkages, in determining the country(ies) to which a crisis spreads. The empirical evidence, while far from conclusive, seems to indicate that fundamental links between countries are an important source of contagion.[3] Finally, recent work, beginning with Morris and Shin (1998), casts serious doubt on the multiple-equilibrium view of currency markets. Using the global games methodology of Carlsson and van Damme (1993), Morris and Shin show how the removal of the assumption that economic fundamentals are common knowledge can generate a *unique* equilibrium in an otherwise-standard currency crisis model. This result would seem to completely rule out self-fulfilling expectations as a driving force behind contagion, since the economic fundamentals in each currency market must uniquely determine whether or not a crisis occurs there.

This paper argues that even if individual currency markets have a unique equilibrium when viewed in isolation, expectations that contagion will occur across markets can be self-fulfilling. The model extends that of Morris and Shin (1998) by allowing speculators to trade on a second currency market. Because of the global-games structure, the domestic currency market, viewed in isolation, has a unique equilibrium. Whether or not a crisis occurs in this equilibrium depends only on the domestic economic fundamentals, including the willingness of the government to defend the currency and the ability of speculators to attack it. In order for contagion across markets to occur, therefore, some link between the economic fundamentals of the domestic market and the other currency market must arise.

The key observation of the paper is that when speculators expect the occurrence of a crisis to be correlated across countries, they have an incentive to engage in financial market transactions that create links between otherwise separate markets. In other words, if speculators expect that a crisis in Thailand would be followed by a crisis in Malaysia, they have an incentive to be active in both currency markets in order to benefit from this correlation. If a crisis then occurs in Thailand, it will change the wealth levels of these speculators and, therefore, change their actions in the Malaysian currency market in a way that increases the probability of a crisis there. The belief that contagion will occur is entirely self-fulfilling: if speculators expect there to be no correlation between the outcomes of the two markets, they will have no incentive to enter both markets and contagion will

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[3] See, for example, Glick and Rose (1999), Kaminsky and Reinhart (2000), and Van Rijckeghem and Weder (2001).

not occur.

The immediate source of equilibrium contagion when it occurs in this model is the fact that the same agents are active in both currency markets, which generates a wealth channel through which crises are transmitted. In this way, the analysis is related to a number of papers that study how financial interdependence can lead to the contagion of crises.[4] In the previous literature, the financial linkages arise for fundamental reasons. Most often, investors choose to operate in multiple markets for diversification purposes and these decisions create the link between markets that leads to contagion.

The point of the present paper, however, is that links between markets can also arise solely as a result of agents' expectation that contagion will occur. If agents did not expect contagion to occur, the set of links that would arise between markets might look very different. An immediate implication of these results is that empirical evidence of the importance of financial links as a source of contagion must be interpreted with caution. In particular, an empirical finding that common creditors or other financial links "explain" observed contagion does not demonstrate that expectations are unimportant or that contagion is an inevitable outcome. Rather, the observed links could themselves be a consequence of self-fulfilling beliefs in contagion.

Despite the fact that it can have multiple equilibria, one in which contagion occurs and another in which it does not, the model places strong restrictions on the equilibrium correlation between the outcomes of the two markets. In particular, there must either be zero correlation or the occurrence of a crisis in one market must raise the probability of a crisis in the other. In other words, the model presented here offers a genuine explanation of the *contagion* of crises, as opposed to an arbitrary correlation of outcomes across markets.

The model also places restrictions on the structure of the contagion equilibrium. The probability of a crisis in the second market, for example, is strictly increasing in the size of the devaluation in the first. The model predicts, therefore, that contagion should be most frequently observed following large devaluations. The model also predicts that a currency crisis is more likely to occur when domestic fundamentals are weak than when they are strong. Both of these predictions match what is observed in the data. Note that neither relationship need hold in the simple view described above where events in one market act only as a signal to coordinate expectations in the

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[4] Examples include Allen and Gale (2000), Dasgupta (2004), Goldstein and Pauzner (2004), Kodres and Pritsker (2002), Kyle and Xiong (2001), and Lizarazo (2005).

other. The predictions derive from the use of the global-games approach, which ties expectations to fundamentals by requiring some real link to arise between the two markets in order for equilibrium contagion to occur.

The remainder of the paper is organized as follows. The next section presents a modified version of the model of Morris and Shin (1998) in which trading on a second currency market is possible. Section 3 analyzes equilibrium in this model under different assumptions about the information structure in the second market and presents the main results of the paper. Section 4 contains a discussion of the results, including an analysis of the relationship between the model presented here and the classic sunspots model of Cass and Shell (1983).

2 The model

There are two time periods, with a different currency market meeting in each period. In the second period, the domestic currency is traded for a real numeraire (called "dollars"); this is the market to which a crises elsewhere may potentially spread. In the first period, a "foreign" currency is traded, also for dollars. Economic fundamentals are uncorrelated between the two markets. From the standpoint of the domestic market, the foreign currency represents any market that agents perceive as a potential source of a contagious crisis. While the two currency markets are unrelated in terms of fundamentals, speculators have the ability to attack each of the currencies by selling it short. The actions of these speculators may, therefore, generate a link between the markets. The outcome in each market is stochastic. The question of interest is whether (and how) the occurrence of a crisis in the foreign market affects the likelihood of a crisis in the domestic market.

The model is presented in two steps. First I describe the market for the domestic currency. Viewed in isolation, this market has a unique equilibrium; some properties of this equilibrium are discussed below. I then add the first-period market to the model so that the possibility of contagion across markets can be studied.

2.1 The domestic currency market

The model of the domestic currency market is nearly identical to that in Morris and Shin (1998); therefore, the description here is brief. The agents in this market are the domestic government and a $[0,1]$ continuum of identical speculators. The government has pegged the exchange rate at e^* dollars per unit of local currency. The economy is characterized by a fundamental "strength"

$\theta \in [0,1]$, which determines what the exchange rate would be in the absence of government intervention in the currency market. The variable θ captures the demand for the domestic currency for international trade, foreign direct investment, and other purposes. The exchange rate if the government takes no action will be given by $f(\theta)$, where f is continuous and strictly increasing. It is assumed that $e^* = f(1)$ holds. In other words, at the pegged value of the exchange rate, the domestic currency is overvalued in almost all possible states of the economy. The government must decide whether to take the actions necessary to maintain the peg or to abandon the peg and let the exchange rate fall to the market value $f(\theta)$.

A speculator has the ability to attack the domestic currency by selling it short. Short sales are limited by the speculator's wealth, which is denominated in dollars. In particular, each unit of wealth allows a speculator to short-sell one unit of the domestic currency.[5] Each speculator has w units of wealth available. Morris and Shin (1998) set $w = 1$ and provided an informal discussion of the comparative static results with respect to the level of wealth. Wealth enters the model here in a way that corresponds to their discussions. Speculators are risk neutral and, hence, their choice set is essentially binary: either a speculator will attack with all of her wealth or she will not attack at all. There is a cost t for each unit of the currency a speculator sells short; one can think of this cost as the interest rate differential between the domestic currency and dollars. If a speculator chooses to attack the currency, her net gain will be $w(e^* - f(\theta) - t)$ if the government abandons the peg and $(-wt)$ if the peg is maintained.

The government receives a value $v > 0$ if the peg is maintained. It will choose to maintain the peg if and only if this benefit is greater than the cost of doing so. The cost of maintaining the peg depends on on two things: the state of the economy and the size of the attack against the currency. This cost is represented by the function $c(\theta, z)$, where z is the size of the attack (*i.e.,* the number of units of domestic currency sold short). The function c is continuous, strictly increasing in z, and strictly decreasing in θ. Furthermore, $c(0,0) > v$ and $c(1,w) > v$ are assumed to hold. The first condition says that in the worst state of fundamentals, the peg will be abandoned even if there is no attack against the currency. The second says that even in the best state of fundamentals, the peg will be abandoned if the size of the attack is equal to the initial wealth of all speculators. See Morris and Shin (1998) for a detailed discussion of the role of these conditions in the analysis.

The timing of events within this market is as follows. Each speculator begins with a belief about

5 This one-for-one property is only a choice of units.

θ that is represented by a uniform distribution on $[0,1]$. Nature then draws the true value of θ from this distribution. Speculators do not observe the true state. Rather, speculator i observes a signal x_i which is drawn from a uniform distribution over the interval $[\theta-\varepsilon,\theta+\varepsilon]$, where ε is a small but positive number. A law of large numbers implies that the distribution of signals received by the different speculators is also uniform on $[\theta-\varepsilon,\theta+\varepsilon]$. Based on her signal, each speculator decides whether or not to attack the currency. Next, the government observes the true value of θ and the size of the attack $z = w\alpha$, where α is the fraction of speculators who chose to attack. The government then decides whether to abandon or maintain the peg, and payoffs are realized.

The model presented so far is identical to that in Morris and Shin (1998), except that the wealth level of each speculator is treated parametrically, and hence their results continue to hold. In particular, the game has a unique Bayesian Nash equilibrium for any $\varepsilon > 0$. In this equilibrium, there exists a cutoff value θ^* such that the peg is abandoned if $\theta \leq \theta^*$ and is maintained if $\theta > \theta^*$. In the limit as ε goes to zero, the cutoff value can be characterized in the following way. Define $a(\theta,w)$ to be the smallest fraction of speculators (each with wealth w) whose attack would lead the government to abandon the peg when θ is the true state of the economy. That is, the function $a(\theta,w)$ is implicitly defined by

$$c\left(\theta, w\cdot a\left(\theta,w\right)\right) \equiv v.$$

It is straightforward to show that a is increasing in θ and decreasing in w. Define $g(\alpha,\theta,w)$ to be the net benefit of attacking when a fraction α of the other agents attack. Then we have

$$g\left(\alpha,\theta,w\right) = \left\{ \begin{array}{c} w\left(e^* - f\left(\theta\right) - t\right) \\ -wt \end{array} \right\} \quad \text{if} \quad \left\{ \begin{array}{c} \alpha \geq a\left(\theta,w\right) \\ \alpha < a\left(\theta,w\right) \end{array} \right\}. \tag{1}$$

More recent work by Morris and Shin (2003) shows that the cutoff value θ^* must satisfy

$$\int_0^1 g\left(\alpha,\theta^*,w\right) d\alpha = 0.$$

In other words, an agent whose belief about the actions of other agents can be represented by a uniform distribution for α on $[0,1]$ must be indifferent between attacking and not attacking at θ^*. This condition can be viewed as a definition of *risk dominance* for symmetric binary-action games with a continuum of players.[6] The results of Morris and Shin can therefore be interpreted as saying that, in the limit as the noise on the individual signals goes to zero, the global games approach

[6] See Harsanyi and Selten (1988) for a detailed discussion of risk dominance in finite-player games.

"selects" the risk-dominant equilibrium of the common-knowledge game.

Using expression (1), the equation above can be rewritten as

$$[1 - a(\theta, w)]\, w\, (e^* - f(\theta) - t) + a(\theta, w)\, (-wt) = 0,$$

and therefore the equilibrium value of θ^* will solve

$$(1 - a(\theta^*, w))\, (e^* - f(\theta^*)) = t. \tag{2}$$

This expression implicitly defines a function $\theta^*(w)$ with the following property: in the unique equilibrium of this model, a devaluation will occur if $\theta \leq \theta^*(w)$ and will not occur if $\theta > \theta^*(w)$. In the analysis that follows, the properties of this function play a critical role. The proposition below, which was first shown by Heinemann (2000), states that when speculators have more wealth, the set of states in which a devaluation occurs becomes strictly larger.[7]

Proposition 1 *The equilibrium cutoff value θ^* is strictly increasing in the wealth level of speculators w.*

A simple proof of this result can be obtained by implicitly differentiating (2).

Continuing to focus on the limiting case as ε goes to zero, consider the equilibrium expected utility of a speculator who enters this market with wealth w_i (which in principle could be different from the wealth level w of other speculators). If the realization of θ is less than $\theta^*(w)$, a devaluation occurs and the speculator will gain the amount $(e^* - f(\theta) - t)$ for each unit of wealth that she has available. Her final wealth level, and hence her consumption, will then be

$$w_i\, (1 + e^* - f(\theta) - t)\,.$$

If the realization of θ is higher than $\theta^*(w)$, the speculator takes no action and simply consumes her wealth.[8] Before θ is realized, therefore, the speculator's marginal utility of wealth is equal to

$$1 + \int_0^{\theta^*(w)} (e^* - f(\theta) - t)\, d\theta \equiv \mu(w)\,. \tag{3}$$

[7] In their informal discussion, Morris and Shin (1998) stated that the cutoff θ^* is increasing in the wealth of speculators when ε is large, but that this effect disappears as ε goes to zero. Heinemann (2000) corrected their calculations and showed that the effect remains present in the limit.

[8] When ε is arbitrarily small, the speculator's signal about θ is very accurate and hence she is able to attack in precisely the set of states in which a devaluation occurs.

It is important to keep in mind that this marginal utility is independent of her own wealth level w_i because she is risk neutral. However, it depends on the wealth level w of the other speculators in the market because w determines the set of states in which a devaluation occurs. In fact, using Proposition 1, it is easy to see that the function μ defined in (3) is strictly increasing in w. The more wealth the other speculators have, the higher is the marginal value of wealth for an individual speculator. This result is crucial to the analysis that follows.

Proposition 2 *A speculator's marginal utility of wealth μ is strictly increasing in the average wealth w.*

In other words, in addition to the usual complementarity in actions (*i.e.*, attacking is more attractive when other agents attack), this model also exhibits a complementarity in *wealth levels*: wealth is more valuable in equilibrium when others are wealthier.

2.2 Speculation on another currency

In the first period, speculators have an opportunity to attack another currency. This "foreign" currency can be interpreted as any market that agents perceive as a potential source of a contagious crisis. The foreign currency market meets before any of the activity described above takes place, including speculators receiving signals about the strength of the domestic economy. Each speculator begins the first period with one unit of wealth (a normalization).

The basic structure of the foreign currency market is the same as that described above. The foreign government has pegged its exchange rate at e_F^* dollars per unit of foreign currency and must decide whether to maintain this peg or abandon it. The fundamental state of the foreign economy is denoted $\theta_F \in [0,1]$, and speculators' initial belief about this variable is represented by a uniform distribution on $[0,1]$. As above, in this model the variable θ_F includes the effects of all influences on the value of the foreign currency other than the actions of the agents in this model. If the peg is abandoned, the value of the foreign currency will be given by $f(\theta_F)$, where f is again continuous and strictly increasing with $e_F^* = f(1)$. The government receives a value v_F from maintaining the peg, and will do so if this value is greater than the cost $c_F(\theta_F, z_F)$, where z_F is the size of the speculative attack against the foreign currency. The variables θ and θ_F are uncorrelated; in other words, the economic fundamentals in the two countries are assumed to be completely unrelated.

As in the domestic market, each speculator can choose to attack the foreign currency by selling it short, and short sales are restricted by a speculator's wealth. Because she is risk neutral, she will either attack the foreign currency with all of her wealth or not at all. There is a cost t_F for each unit of foreign currency sold short, so that the net gain of attacking per unit of wealth is $(e_F^* - f(\theta_F) - t_F)$ if the peg is abandoned and $(-t_F)$ if it is maintained. The conditions $c_F(0,0) > v_F$ and $c_F(1,1) > v_F$ are assumed to hold; their interpretations are the same as those for the analogous conditions in the domestic market.

The foreign currency market may differ from the domestic market in one key respect: the precision of the signals received by speculators about the state of the fundamentals θ_F. As described in Morris and Shin (2003), the prior belief about θ_F (or θ in the domestic market) represents information that is *public*, as opposed to the *private* information in a speculator's idiosyncratic signal. The precision of speculators' signals can, therefore, be thought of as measuring the importance of private information relative to public information.

Two cases are studied in Section 3 below. In the first, signals about θ_F are assumed to be arbitrarily precise (exactly as in the domestic market) and only private information matters. In the second case, signals are completely uninformative and only public information matters. The extreme nature of these assumptions serves only to simplify the analysis; the important thing is the relative precision of private information. I argue below that the latter case, where private information is less important, is the more interesting one for studying contagion across markets that are, at least in principle, unrelated.

A *strategy* in the two-market game specifies (*i*) an action (attack or not attack) in the foreign market as a function of the speculator's signal in that market and (*ii*) an action in the domestic market as a function of both her signal and the realized fundamentals in the foreign market, as well as her signal in the domestic market. The analysis focuses on symmetric subgame-perfect equilibria, in which all speculators adopt the same strategy. In each of the cases studied below, all speculators have (essentially) the same information and, therefore, take the same action in the foreign currency market. Each thus carries the same wealth w, which may depend on the realized θ_F, to the domestic currency market. The unique equilibrium of this subgame, characterized by $\theta^*(w(\theta_F))$, must then be played.

The question of interest in this paper is under what conditions a crisis in the foreign currency market is transmitted to the domestic market. I will say that *contagion* occurs in a given equilibrium if the occurrence of a devaluation of the foreign currency raises the equilibrium probability of a devaluation in the domestic market; this statement is equivalent to the following definition.

Definition: *Contagion* is said to occur if a devaluation of the foreign currency strictly increases the equilibrium cutoff value θ^* in the domestic market.

Before moving to the analysis of equilibrium in the full model, it is useful to note what would happen if speculators were simply not allowed to trade in the foreign currency market. In this case, solving the model is completely straightforward. Each speculator will enter the domestic currency market with her original wealth level of one and, hence, the game played in this market will be exactly that of Morris and Shin (1998). There is a unique equilibrium in this market and, for the limiting case as ε goes to zero, the cutoff level $\theta^*(1)$ is implicitly defined in (2). In particular, this cutoff level is necessarily independent of events in the foreign currency market. There is no contagion: regardless of the realization of foreign fundamentals θ_F, the unique equilibrium in the domestic currency market is characterized by the same cutoff level. This simple result will be a useful benchmark in what follows.

Proposition 3 *If speculators cannot attack the foreign currency, there is a unique equilibrium and no contagion occurs.*

This proposition shows that the outcome of another currency market cannot act as a pure coordination device in this model.[9] If there are no links between the two currency markets, the unique equilibrium in the domestic market must be played regardless of what happens in the foreign market. The outcome in the foreign market is also easy to determine, since the only active agent is the foreign government. A devaluation occurs in this market if and only if the fundamentals are poor enough that the cost of defending the peg when no one attacks is larger than the value of doing so, that is, if

$$c_F(\theta_F, 0) \geq v_F$$

holds. Let $\underline{\theta}_F$ be the state at which the above expression holds with equality, so that a devaluation of the foreign currency occurs whenever $\theta_F \leq \underline{\theta}_F$ holds.

[9] See Heinemann and Illing (2002) on this point.

3 Equilibrium Contagion

This section presents the analysis of equilibrium under two different assumptions regarding speculators' information about the foreign currency market. In the first case, speculators receive very precise idiosyncratic signals about foreign economic fundamentals, exactly as in the domestic market. In the second (and more interesting) case, speculators receive uninformative signals, or no signals at all, about foreign economic fundamentals. In each case, the question of interest is whether or not contagion occurs in equilibrium and, if it does, what form it takes.

3.1 Informed speculation

Having speculators receive arbitrarily-precise idiosyncratic signals about θ_F represents a situation where private information about the state of foreign economic fundamentals is relatively important compared to the public information contained in the prior belief. Note that the very precise signals also imply that attacking the foreign currency is not a risky undertaking: in equilibrium, a speculator knows θ_F arbitrarily well and will therefore be able to attack in only those states where a devaluation will occur. The following proposition shows that, in this case, contagion *must* occur.

Proposition 4 *With precise idiosyncratic signals about θ_F, contagion must occur in equilibrium.*

Proof: The proof is simple. When speculators have precise signals about the state of foreign fundamentals, they will each be able to attack the foreign currency in exactly the set of states where a devaluation occurs. (Note that attacking is a dominant strategy for $\theta_F \leq \underline{\theta}_F$, so this set is not empty.) Wealth levels following a devaluation are given by

$$w = 1 + e_F^* - f(\theta_F) - t_F > 1.$$

In states where a devaluation does not occur, speculators do not attack and the wealth level each takes to the domestic market is $w = 1$. (Note that not attacking is a dominant strategy for values of θ_F close enough to one, because in such cases the gain from a devaluation if it occurs would be less than the cost t_F of attacking.) It then follows immediately from Proposition 1 that contagion occurs.[10] ■

[10] Note that no claim about uniqueness or multiplicity of equilibrium is made here. As in Goldstein and Pauzner (2004), the presence of a second market makes this a difficult issue (see their footnote 7, p. 153). For the present paper, however, the relevant point is that, with precise idiosyncratic signals about θ_F, *any* equilibrium will exhibit contagion.

This result simply reflects the fact that when speculators are well informed about foreign economic fundamentals, they will necessarily be active in the foreign currency market in some states of the world. Their gains (if any) in that market will then affect their behavior in subsequent markets. In particular, a profit in the foreign market will make speculators more aggressive in the domestic market and will thus raise the equilibrium probability of a domestic crisis. This aspect of the model is reminiscent of media reports during the East Asian crisis in 1997 that each devaluation "emboldened" speculators to leverage their gains and attack another currency.

In a sense, the type of contagion identified in Proposition 4 is not surprising. The result is very similar in spirit to those in Allen and Gale (2000), Goldstein and Pauzner (2004), Lizarazo (2005) and others. Those papers rely on a diversification motive for investors to operate in multiple markets, while the result in this section focuses instead on the role of information; however, in both approaches the two markets are fundamentally linked because the same set of agents naturally operates in each of them. In such a case, contagion of crises across markets must occur. Proposition 4, therefore, applies more readily to currencies that are closely related than to ones that are, at least in principle, unrelated. The question of interest in this paper is what forces might drive contagion across unrelated markets. Addressing this question requires changing the model so that speculators are not naturally active in the foreign market.

3.2 Uninformed speculation

One way to capture the idea that the speculators in this model are not naturally active in the foreign currency market is to assume that the signals they receive about foreign fundamentals are not very precise. In other words, the foreign currency in question is one in which these speculators have no particular expertise. To keep the model tractable, suppose the idiosyncratic signal a speculator receives is completely uninformative; it is drawn from a uniform distribution on $[0, 1]$ and, hence, is uncorrelated with the true state θ_F. In this case, the decision of whether or not to attack the foreign currency will be made based entirely on the prior belief about θ_F. Notice that attacking the foreign currency is no longer a riskless undertaking. In other words, this case differs from the previous one in two key respects: private information about θ_F is relatively unimportant *and* a speculator's belief about θ_F at the time the decision must be made is less precise.

Suppose further that parameter values are such that attacking the foreign currency is relatively unattractive to these speculators. To begin with, assume

$$\int_0^{\underline{\theta}_F} \left(e_F^* - f\left(\theta_F\right)\right) d\theta_F < t_F. \tag{4}$$

This condition says that the expected value of attacking the foreign currency when no one else attacks (and one's belief about θ_F is uniform on $[0,1]$) is negative. Everything on the left-hand side of this inequality (including $\underline{\theta}_F$) is independent of t_F and, therefore, this condition simply requires that t_F not be too small. If this inequality were reversed, risk neutral speculators would always want to gamble in the foreign currency market. In such a case, links between the two markets would arise simply because both offer attractive trading opportunities to the same set of speculators and contagion between these markets would necessarily arise as in Proposition 4. Instead, condition (4) requires that, *a priori*, the foreign currency market represent an unattractive gamble to the speculators in the model.

Under this condition, there is an equilibrium in which contagion does not occur. To see this, consider the problem of an individual speculator who believes that no other speculators will attack the foreign currency. If she were to attack, her expected wealth entering the domestic currency market would be

$$1 + \int_0^{\underline{\theta}_F} \left(e_F^* - f\left(\theta_F\right)\right) d\theta_F - t_F < 1.$$

Her marginal utility of wealth in the domestic market will equal $\mu\left(1\right)$, as defined in (3), regardless of the realization of θ_F. She would, therefore, choose not to attack and hence there is an equilibrium identical to the one described in Proposition 3: none of the speculators attack the foreign currency and all enter the domestic currency market with a wealth level of one. The equilibrium cutoff in the domestic market is given by $\theta^*\left(1\right)$, independent of the realization of θ_F, and no contagion occurs. This discussion is summarized in the following proposition.

Proposition 5 *If speculators are uninformed about θ_F and* (4) *holds, there exists an equilibrium in which no speculator attacks the foreign currency and no contagion occurs.*

There may, however, be other equilibria. Define

$$\underline{t}_F \equiv \int_0^{\underline{\theta}_F} \left(e_F^* - f\left(\theta_F\right)\right) d\theta_F,$$

so that condition (4) can be rewritten as $t_F > \underline{t}_F$. The next proposition is the main result of the paper. It shows that, for values of t_F that are not too large, there also exists an equilibrium in which contagion occurs.

Proposition 6 *If speculators are uninformed about θ_F and* (4) *holds, there exists $\bar{t}_F > \underline{t}_F$ such that $t_F < \bar{t}_F$ implies the existence of an equilibrium in which all speculators attack the foreign currency and contagion occurs.*

Proof: Suppose all speculators but one are attacking the foreign currency and consider the decision problem of the remaining speculator. The total size of the attack against the foreign currency in this case will be $z_F = 1$ and, therefore, the foreign government will defend the peg if and only if

$$c_F(\theta_F, 1) < v_F.$$

Let $\widehat{\theta}_F$ denote the unique value of θ_F for which the above relationship holds with equality; the condition $c_F(1,1) > v_F$ implies $\widehat{\theta}_F < 1$. Then a devaluation will occur for $\theta_F \leq \widehat{\theta}_F$ but not for $\theta_F > \widehat{\theta}_F$. Define $w(\theta_F)$ to be the wealth level of each speculator who attacks the foreign currency, measured after payoffs in the foreign market are realized. Then we have

$$w(\theta_F) = \left\{ \begin{array}{ccc} 1 + e^* - f(\theta_F) - t_F & \text{if} & \theta_F \leq \widehat{\theta}_F \\ 1 - t_F & & \theta_F > \widehat{\theta}_F \end{array} \right\}. \tag{5}$$

If the speculator in question attacks the foreign currency, her wealth entering the domestic currency market will also be equal to $w(\theta_F)$. If she does not attack, her wealth level will be equal to one. Recalling that the marginal utility of wealth in the domestic market is given by $\mu(w)$ as defined in (3), her total expected utility if she attacks the foreign currency is

$$\int_0^1 \mu(w(\theta_F))\, w(\theta_F)\, d\theta_F$$

and her total expected utility if she does not attack is

$$\int_0^1 \mu(w(\theta_F))\, d\theta_F.$$

The expected gain from attacking can therefore be written as

$$\int_0^{\widehat{\theta}_F} \mu(w(\theta_F))\,(e_F^* - f(\theta_F) - t_F)\, d\theta_F + \int_{\widehat{\theta}_F}^1 \mu(w(\theta_F))\,(-t_F)\, d\theta_F. \tag{6}$$

Suppose we evaluate this expression at $\underline{t}_F$. Recall that $\underline{t}_F$ satisfies

$$\int_0^{\underline{\theta}_F} \left(e_F^* - f\left(\theta_F\right) - \underline{t}_F\right) d\theta_F + \int_{\underline{\theta}_F}^1 \left(-\underline{t}_F\right) d\theta_F = 0.$$

Using the definitions

$$c_F\left(\widehat{\theta}_F, 1\right) = v_F \quad \text{and} \quad c_F\left(\underline{\theta}_F, 0\right) = v_F,$$

we clearly have $\widehat{\theta}_F > \underline{\theta}_F$. Furthermore, we know from (5) that $w\left(\theta_F\right) > 1$ holds for $\theta_F \leq \widehat{\theta}_F$ and $w\left(\theta_F\right) < 1$ holds for $\theta_F > \widehat{\theta}_F$. Together, these relationships imply that the value of (6) evaluated at $t_F = \underline{t}_F$ is strictly positive. Since (6) is continuous and monotone in t_F, there exists a value $\overline{t}_F > \underline{t}_F$ such that for any $t_F < \overline{t}_F$ there is an equilibrium in which all speculators attack the foreign currency.

The fact that contagion occurs in this equilibrium follows directly from (5) and Proposition 1. When all speculators are attacking the foreign currency, the wealth levels they carry into the domestic market depend on the realization of θ_F, particularly whether θ_F is low enough that a devaluation occurs. Since the equilibrium cutoff value θ^* is strictly increasing in this wealth level, contagion must occur. ■

To see why this contagion equilibrium exists, notice that other speculators attacking the foreign currency makes attacking more attractive to an individual in two ways. First, the attack makes a devaluation of the foreign currency more likely, as reflected in the above calculations by the relationship $\widehat{\theta}_F > \underline{\theta}_F$. This effect represents the complementarity in actions that is standard in coordination games. The second, and more interesting, effect is that the attack by others will induce a correlation between the returns in the two markets, and this correlation will make attacking more attractive to an individual. We can isolate this second effect, which derives from the complementarity in wealth levels, by imposing a stronger condition than (4) on the cost t_F. Define

$$\widehat{t}_F \equiv \int_0^{\widehat{\theta}_F} \left(e_F^* - f\left(\theta_F\right)\right) d\theta_F. \tag{7}$$

When $t_F > \widehat{t}_F$ holds, the expected return to attacking the foreign currency is negative even when all other speculators are attacking. Nevertheless, equilibrium contagion can still occur. To see this, rewrite (7) as

$$\int_0^{\widehat{\theta}_F} \left(e_F^* - f\left(\theta_F\right) - \widehat{t}_F\right) d\theta_F + \int_{\widehat{\theta}_F}^1 \left(-\widehat{t}_F\right) d\theta_F \equiv 0 \tag{8}$$

and compare this equation to the expected utility gain from attacking the foreign currency in (6). We know that $w(\theta_F) > 1$ holds when a devaluation occurs in the foreign market (i.e., for $\theta_F \leq \widehat{\theta}_F$) and $w(\theta_F) < 1$ holds when it does not. Since $\mu(w)$ is a strictly increasing function, the expression in (6) puts more weight on the positive term and less weight on the negative term, relative to (8), and hence must be strictly positive when evaluated at $\widehat{t}_F$. By continuity, therefore, the expected utility gain from attacking the foreign currency will be positive for an open interval of values of t_F above $\widehat{t}_F$, even though the expected *return* from attacking in these cases is negative. In other words, $\widehat{t}_F$ is strictly less than $\bar{t}_F$. This discussion is summarized in the following corollary.

Corollary 1 *Contagion can occur even when, in equilibrium, the expected return to attacking the foreign currency is negative.*

This result clearly highlights the implications of the complementarity in wealth levels. An individual speculator's marginal utility of wealth is high in states where the wealth levels of other speculators are high. She would, therefore, like to make the same risky trade(s) that the others are making, even if that trade would not be attractive when evaluated in isolation. In other words, the complementarity in wealth levels entering the domestic currency market generates a complementarity in "outside" trading activity, which can lead to an equilibrium in which each speculator places a seemingly unattractive bet in another currency market simply because everyone else is doing so.[11]

When Propositions 5 and 6 both apply, contagion, if it occurs, is clearly driven by self-fulfilling expectations. Speculators find it attractive to attack the foreign currency if (and only if) they expect the occurrence of a crisis to be correlated across markets, and the ensuing attack (or lack thereof) leads to the original expectations being fulfilled. Note that if $t_F > \widehat{t}_F$ holds, not attacking the foreign currency would be a dominant strategy if the effects of the domestic market (summarized by the μ function) were ignored. In other words, expectations-driven contagion can occur even if each currency market has a unique equilibrium when viewed in isolation.

[11] This effect is similar in spirit to that studied by Hellwig and Veldkamp (2006), who show how a complementarity in actions can lead to a complementarity in the choice of what information to obtain.

4 Discussion

The model presented here differs fundamentally from the simple multiple-equilibrium view of contagion described in the introduction, most notably by placing strong restrictions on the correlations between equilibrium variables. In the simple view, the domestic currency market (in isolation) has multiple equilibria. The outcome of some foreign currency market can then act as a signal that coordinates the actions of speculators in the domestic market, even if they do not (or cannot) trade in this other currency. There can be a domestic crisis if the foreign currency devalues and not if it does not, or the reverse, or a crisis in both cases, or a crisis in neither case. In other words, there is a large number of equilibria, one for each possible correlation between the outcomes of the two markets.

The present model, in contrast, has a small number of equilibria. It is fairly easy to see that the equilibria identified in Propositions 5 and 6 are the only symmetric, pure-strategy equilibria of the model for the case where speculators are uninformed about θ_F.[12] In other words, it is not the case that contagion of crises is merely one of many possible correlations between the equilibrium outcomes of the two markets. In this model, either there is no correlation or a crisis in the foreign market will *raise* the equilibrium probability of a domestic crisis.

The global games approach generates this restriction by tying agents' expectations to market fundamentals; any transmission of crises must occur through changes in the wealth of market participants. The next subsection presents two additional predictions generated by this approach. The following subsection then relates this approach to the sunspots model of Cass and Shell (1983).

4.1 Properties of equilibrium contagion

The contagion equilibrium, whether of the type identified in Proposition 6 or in Proposition 4, has several properties that are both intuitively appealing and broadly consistent with correlations observed in the data. First, a larger devaluation of the foreign currency leads to a larger increase in the probability of a domestic currency crisis. In other words, a large devaluation is more likely to prove contagious than a smaller one. This result follows immediately from Proposition 1 and

[12] When both of these equilibria exist, there is also a mixed strategy equilibrium (or, equivalently, an asymmetric pure strategy equilibrium) in which speculators are indifferent between attacking and not attacking the foreign currency. While this equilibrium is not studied here, it is easy to see that contagion occurs in the equilibrium through the same mechanism: some speculators will bring more wealth to the domestic market following a devaluation of the foreign currency.

expression (5).

Corollary 2 *In the contagion equilibrium, the probability of a domestic currency crisis is strictly increasing in the size of the devaluation of the foreign currency.*

Next, the occurrence of a domestic currency crisis will be negatively correlated with *domestic* economic fundamentals. When θ is higher, a larger devaluation of the foreign currency is required in order to provoke a domestic devaluation. Hence crises will occur less frequently when domestic fundamentals are strong and more frequently when fundamentals are weak.

Corollary 3 *In the contagion equilibrium, the probability of a domestic currency crisis is strictly decreasing in θ, the state of domestic fundamentals.*

It is important to bear in mind that neither of these relationships need hold under the simple multiple-equilibrium view of contagion. When the outcome of the foreign currency market acts only as a signal, a small devaluation could serve as the signal to attack the domestic currency just as well as a large one could. In addition, for all values of θ in the multiple equilibrium region, the likelihood of a domestic currency crisis would be independent of domestic fundamentals, as a domestic crisis would occur if and only if the appropriate signal is received.[13] The model here, in contrast, yields clear predictions that are consistent with the correlations observed in the data: a more severe currency crisis is more likely to prove contagious, and a domestic crisis is more likely to occur when domestic economic fundamentals are weak.

The model also predicts that existing financial links between markets are necessary for contagion to occur; this prediction is broadly consistent with the results in the empirical literature. Importantly, the model shows that empirical evidence on the sources of contagion should be interpreted with care. An empirical finding that financial links are useful predictors of the spread of crises does not imply that contagion is driven by underlying economic fundamentals nor that it is an inevitable outcome. Rather, these links could arise solely as a result of the (self-fulfilling) expectation that contagion will occur. Further research is needed to investigate appropriate methodologies for determining how important a role expectations have played in driving the observed spread of currency crises across countries.

[13] This fact is commonly used to criticize multiple-equilibrium models. See, however, Ennis (2003) and Ennis and Keister (2005).

4.2 A sunspots interpretation

The results in this paper can be interpreted in a way that closely mirrors the classic paper of Cass and Shell (1983). Cass and Shell studied a standard Walrasian economy augmented to include "sunspots," a random variable that is completely *extrinsic* in the sense that it has no effect on economic fundamentals. They showed that when the underlying economy (without sunspots) has a unique equilibrium and agents cannot trade sunspot-contingent assets, sunspots cannot affect equilibrium allocations. This results follows from the fact that in every sunspot state, the economy is exactly the same and hence the unique equilibrium must obtain.

They then showed two ways in which sunspots can matter, one fairly obvious and the other much less so. First, suppose that the underlying economy has multiple equilibria. Then different equilibria might obtain in different sunspot states; that is, the realization of the sunspot variable might act as a signal that coordinates agents on one equilibrium or another. The sunspot equilibrium constructed this way is a randomization over the equilibria of the underlying economy. The second, and more interesting, case is when agents can trade assets whose payoffs depend on the realization of the sunspot variable. In this case, Cass and Shell showed that even when the underlying economy has a unique equilibrium, there can (under some conditions) be equilibria where sunspots affect allocations. If agents believe that the relative prices of commodities will depend on the sunspot state, they may want to use the asset market to transfer wealth across states. In some cases, this reallocation of wealth can cause the original expectations about prices to be fulfilled.

In the model presented in this paper, one can interpret the devaluation state of the foreign currency as a "sunspot-like" variable.[14] Whether or not this currency devalues has no effect on the preferences of agents in the model nor on the fundamental state θ of the domestic economy. If speculators cannot trade in the foreign currency market, contagion can only occur in equilibrium if the domestic currency market (in isolation) has multiple equilibria. Events in the foreign market can then act as a signal that selects an equilibrium in the domestic market; this simple view is analogous to the first type of sunspot equilibrium described in the previous paragraph. Discussion (and criticism) of expectations-based theories of contagion has focused on this particular view.

The global-games approach, however, generates a unique equilibrium in the domestic currency market and hence rules out this type of sunspot-like equilibrium. The model of contagion presented

[14] See Spear (1989) for an interesting model in which the equilibrium price in each of two markets acts as a "sunspot-like" variable for the other market.

in this paper instead resembles the second, richer type of sunspot equilibrium. There is a unique equilibrium if trade on the foreign (or "sunspot") market is not allowed. Once such trade is introduced, however, beliefs that the probability of a domestic devaluation will differ across "sunspot" states can be self-fulfilling because agents will trade in the foreign currency market in order to transfer wealth across these states. As a result, contagion driven by self-fulfilling expectations can occur in precisely the same way that sunspot equilibria can exist even when the underlying Walrasian economy has a unique equilibrium.[15]

5 Concluding Remarks

While self-fulfilling expectations are commonly believed to play an important role in the spread of currency crises across countries, existing multiple-equilibrium explanations of contagion have many undesirable properties. This paper has shown how the incomplete-information approach of Morris and Shin (1998) can be used to generate a model in which contagion is driven by self-fulfilling expectations, but which nevertheless places restrictions on observable variables that are broadly consistent with existing empirical evidence. These restrictions derive from the fact that, under the global-games approach, contagion can only occur when speculators' beliefs lead them to be active in both markets.

The key observation in the paper is that if speculators expect contagion to occur, they have an incentive to enter both markets in order to benefit from the correlation in outcomes. These actions then create a wealth channel through which a crisis is propagated across markets, fulfilling the original expectations. Although the immediate cause of contagion is this financial link between markets, the role of expectations is paramount; speculators only choose to enter both markets if they expect contagion to occur.

Many features of the model studied here are fairly special, but these do not seem essential for the results. For example, the assumption that speculators are risk neutral simplifies the analysis by generating boundary solutions to the speculators' portfolio-choice problem. Introducing risk aversion would complicate matters technically, but the effects highlighted here would clearly remain.

[15] The analogy is not exact, of course. Cass and Shell (1983) require that some agents be restricted from trading before the sunspot state is realized; otherwise the first welfare theorem would guarantee that sunspots do not matter. In the present model, all agents can be granted access to the foreign market. In addition, Cass and Shell need heterogeneous agents for sunspot-contingent trade to occur in equilibrium. Here the homogeneous speculators are implicitly trading with the foreign central bank in the "sunspot" market.

Speculators would still have a higher marginal value of wealth in states where a devaluation of the domestic currency is more likely. If a speculator expects contagion to occur, therefore, she would still desire to transfer wealth into these states by short-selling the foreign currency. Other minor modifications of the model also seem unlikely to affect the main results.

The model does (following Morris and Shin, 1998, and others) take a particular view of the nature of currency crises: they are caused by speculative attacks. Real world currency crises are complex phenomena and many of their features are obviously not captured by this model. For example, during times of crisis investors who hold real assets in a country tend to pull out, selling these local-currency denominated assets.[16] Studying the behavior of speculators, and the role of their expectations, in an environment where other such features of currency crises are present seems a promising area for future research.

[16] See Goldstein and Pauzner (2004) and Guimaraes and Morris (2004).

REFERENCES

Allen, Franklin and Douglas Gale (2000) Financial Contagion, *Journal of Political Economy* 108, pp. 1-33.

Carlsson, Hans and Eric van Damme (1993) Global Games and Equilibrium Selection, *Econometrica* 61. pp. 989-1018.

Calvo, Guillermo A. (1998) Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops, *Journal of Applied Economics* 1, pp. 35-54.

Cass, David and Karl Shell (1983) Do Sunspots Matter? *Journal of Political Economy* 91, pp. 193-227.

Dasgupta, Amil (2004) Financial Contagion through Capital Connections: A Model of the Origin and Spread of Bank Panics, *Journal of the European Economic Association* 2, pp. 1049-1084.

Ennis, Huberto M. (2003) Economic Fundamentals and Bank Runs, Federal Reserve Bank of Richmond *Economic Quarterly* 89, pp. 55-71.

Ennis, Huberto M. and Todd Keister (2005) Optimal Fiscal Policy with Multiple Equilibria, *Journal of Monetary Economics* 52, pp. 1359-1377.

Gerlach, Stefan and Frank Smets (1995) Contagious Speculative Attacks, *European Journal of Political Economy* 11, pp. 45-63.

Glick, Reuven and Andrew K. Rose (1999) Contagion and Trade: Why are Currency Crises Regional? *Journal of International Money and Finance* 18, pp. 603-617.

Goldstein, Itay and Ady Pauzner (2004) Contagion of Self-Fulfilling Financial Crises Due to Diversification of Investment Portfolios, *Journal of Economic Theory* 119, pp. 51-183.

Guimaraes, Bernardo and Stephen Morris (2004) Risk and Wealth in a Model of Self-Fulfilling Currency Attacks, Cowles Foundation Discussion Paper No. 1433R, November.

Harsanyi, J. and R. Selten (1988) *A General Theory of Equilibrium Selection in Games*, MIT Press, Cambridge, MA.

Heinemann, Frank (2000) Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks: Comment. *American Economic Review* 90, pp. 316-318.

Heinemann, Frank and Gerhard Illing (2002) Speculative Attacks: Unique Equilibrium and Transparency, *Journal of International Economics* 58, pp. 429-450.

Hellwig, Christian and Laura Veldkamp (2006) Knowing What Others Know: Coordination Motives in Information Acquisition, mimeo., UCLA and NYU.

Kaminsky, Graciela L. and Carmen M. Reinhart (2000) On Crises, Contagion, and Confusion, *Journal of International Economics* 51, pp. 145-168.

Kodres, Laura E. and Matthew Pritsker (2002) A Rational Expectations Model of Financial Contagion, *Journal of Finance* 57, pp. 769-799.

Krugman, Paul (1999) Balance Sheets, the Transfer Problem, and Financial Crises, *International Tax and Public Finance* 6, pp. 459-472.

Kyle, Albert S. and Wei Xiong (2001) Contagion as a Wealth Effect, *Journal of Finance* 56, pp. 1401-1440.

Lizarazo, Sandra V. (2005) Contagion of Financial Crises in Sovereign Debt Markets, mimeo., ITAM.

Masson, Paul R. (1999) Contagion: macroeconomic models with multiple equilibria, *Journal of International Money and Finance* 18, pp. 587-602.

Masson, Paul R. (2001) Multiple Equilibria, Contagion, and the Emerging Market Crises, *in* Financial Crises in Emerging Markets, edited by R. Glick, R. Moreno and M. M. Spiegel. Cambridge, England: Cambridge University Press.

Morris, Stephen and Hyun Song Shin (1998), Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks, *American Economic Review* 88, pp. 587-597.

Morris, Stephen and Hyun Song Shin (2003), Global Games: Theory and Applications, *in* Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society), edited by M. Dewatripont, L. Hansen and S. Turnovsky. Cambridge, England: Cambridge University Press.

Obstfeld, Maurice (1996) Models of Currency Crises with Self-fulfilling Features, *European Economic Review* 40, pp. 1037-1047.

Spear, Stephen E. (1989) Are Sunspots Necessary? *Journal of Political Economy* 97, pp. 965-973.

Van Rijckeghem, Caroline and Beatrice Weder (2001) Sources of Contagion: Is it Finance or Trade? *Journal of International Economics* 54, pp. 293-308.