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Roc Armenter
Amartya Lahiri

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Abstract

Cross-country data reveal that the per capita incomes of the richest countries exceed those of the poorest countries by a factor of thirty-five. We formalize a model with embodied technical change in which newer, more productive vintages of capital coexist with older, less productive vintages. A reduction in the cost of investment raises both the quantity and productivity of capital simultaneously. The model induces a simple relationship between the relative price of investment goods and per capita income. Using cross-country data on the prices of investment goods, we find that the model does fairly well in quantitatively accounting for the observed dispersion in world income. For our baseline parameterization, the model generates thirty-five-fold income gaps and six-fold productivity differences between the richest and poorest countries in our sample.

Key words: cross-country income, productivity, vintage capital

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Armenter: Federal Reserve Bank of New York (e-mail: roc.armenter@ny.frb.org). Lahiri: University of British Columbia (e-mail: alahiri@interchange.ubc.ca). The authors thank Paul Beaudry, Jonathan Eaton, Esteban Rossi-Hansberg, Kei-Mu Yi, and seminar participants at the Federal Reserve Bank of New York, the Federal Reserve Bank of Philadelphia, Banco de España, the Society for Economic Dynamics 2006 annual meeting, the University of British Columbia, and the Bank of Canada–British Columbia Macroeconomics conference for helpful comments. The authors also thank Eleanor Dillon for excellent research assistance.

1 Introduction

Cross-country data reveals that the per capita incomes of the richest countries in the world exceed those in the poorest countries by a factor of 35. In this paper we formalize a model in which new, more productive vintages of capital coexist with older and less productive vintages. In such an environment, a lower relative price of investment induces a higher steady state capital stock as well as a higher level of average productivity. We quantify a calibrated version of the model using cross-country data on the relative price of investment goods. The model can generate almost as much variation in cross-country relative income as is observed in the data. Moreover, under our baseline parameterization, the model generates 35-fold income gaps along with 6-fold productivity differences between the richest and poorest countries in the sample.

There is a large literature which examines the sources of differences in incomes across countries. There are two basic views. One school of thought holds that most of the differences in incomes across nations is due to differences in productivity across nations. The most well known expressions of this view are Hall and Jones (1999) and Parente and Prescott (1994, 1999). A second view holds that differences in measured inputs can account for a significant component of the differences in incomes (e.g., see Chari, Kehoe and McGrattan (1997), Mankiw, Romer and Weil (1992), Kumar and Russell (2002), Young (1995)). In related work Klenow and Rodriguez (1998) attempt a systematic and careful decomposition of the data and conclude that productivity differences account for upwards of 60% of the income dispersion across nations with measured inputs accounting for the balance.

To put the issue in perspective, consider the standard one-sector neoclassical growth model with a production function $Y = AK^{\alpha}L^{1-\alpha}$ where K is capital, L is labor and A is a measure of total factor productivity (TFP). Letting $y = Y/L$ it is well known that this production function can be rewritten as

$$y = A^{\frac{1}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}$$

According to Hall and Jones (1999), K/Y in the richest countries is about 3.6 times K/Y in the poorest countries. With a capital share of 1/3, this implies that A in the richest countries must be about 7 times the A in the poorest countries in order to explain income gaps of 35. Hence, explaining the source of potentially large productivity differences between the richest and poorest countries appears to be key to explaining the large income gaps observed in the data. This paper is an attempt to endogeneize productivity differences across countries.

The key starting point for our work is the well documented relationship between the relative price of investment and relative per capita income: poorer countries are also the countries where the price of capital goods (relative to the price of consumption goods) is higher (see, among others, Jones (1994), and Hall and Jones (1999)). However, the documented importance of productivity differences across countries suggests that the standard view of investment prices impacting income through their effect on capital accumulation (or more generally, measured inputs) can at best be a partial explanation for the observed income disparity across countries. The primary goal of our work is to formalize an environment wherein the price of investment affects the productivity of an economy over and above its standard effect on measured capital.

The main idea behind our work is that productivity and measured inputs are often determined jointly and they respond to the same set of economic decisions and incentives. In order to highlight this, we write down an exogenous growth model with embodied capital. We use a very simplified version of Hopenhayn (1992) in which investment occurs through entry. In every period, potential producers of intermediate goods face a choice of different types of capital (or machines) that they can invest in. Capital goods are tradeable and the available list of capital goods from which the intermediate goods producer chooses at any date includes all vintages of capital goods produced till that date. The labor productivity of the firm is pinned down by the technology vintage of the machine that the intermediate goods producer chooses. The productivity of the latest vintage of capital good (the frontier capital good) grows at an exogenous rate that is common to all countries. Different types of new capital goods are distinct in their productivities and price, with the newer/later vintages being more productive and more expensive. At any given time, the overall productivity of the economy reflects the mix of old and new capital as well as the mix of the types of new capital. Changes in the relative price of new capital induce changes in not only the stock of new capital but also in the average productivity of the economy due to the changing mix of new (high productivity) and older (low productivity) capital.

While the underlying structure of the model is complicated, we show that the behavior of the aggregate variables of the model along a balanced growth path can be summarized by two variables: the average price of capital goods in the economy and the price of the latest capital good (which we call the frontier capital good). Hence, these two prices serve as summary statistics for the model. We show that the productivity gap between countries depends on the cross-country gap in one relative price: the price of frontier capital goods relative to the average price of capital goods. We also show that the per capita income gap across countries depends only on the cross-country gap in the price of frontier capital goods relative to the price of consumption. Lastly, we map these two key variables to available data on consumption and investment prices and quantify the model.

The paper has four main findings – one analytical and three quantitative. The analytical result is that the introduction of embodied capital, by itself, cannot generate steady state differences in productivity. We show that in a model where new capital is embodied with the latest technology but where all new capital is identical across countries, steady state productivity is identical across countries. In order to generate cross-country productivity differences one also needs to introduce variation in the productivity of new capital goods. In the model we accomplish this by enlarging the menu of investment choices at every date.

The other three findings are essentially quantitative. The model generates a steady-state distribution of relative incomes across countries as a function of the relative price of new capital.

Using the observed relative price of capital from the PWT dataset, we generate a model-specific cross-country income distribution and compare its properties with the actual distribution in the data. This yields three principal results. First, for our baseline parameterization, the model induces a cross-country distribution in which the per capita income of the richest countries exceeds that of the poorest countries of our sample by a factor of 35 which is almost the same as that in the data. Second, the model-induced relative income series tracks the actual data quite closely with the correlation between the two series being 0.75. Lastly, the model generates a 6-fold productivity difference between the richest and poorest countries in our sample. Based on these results, we consider the model to be a qualified success.

We would like to clarify that the reasons behind the cross-country variation of relative investment prices, while undoubtedly important to understand, are beyond the remit of this paper. Here, we simply ask whether the observed variation in investment prices, when passed through the lens of our model, can generate income variations along the lines observed in the data.

Since we calibrate the model using the relative price of investment goods series from the PWT dataset, one key observation is in order before we proceed. Hsieh and Klenow (2003) have argued that most of the observed variation in the relative price of investment goods in the PWT dataset is due to variations in the price of consumption across countries rather than variations in the price of investment goods. They interpret this result as suggesting that explanations of the world income dispersion that hinge on investment distortions in the form of import tariffs, taxes etc., are unlikely to be true. Instead, they argue the challenge is to explain the reasons for the low productivity of the investment goods sector in the poorer countries. Our model does not require a specific stand on whether the dispersion in the relative price of investment goods across countries is due to taxes or due to technology. All that is required for our results to go through is that there be observed variation in the cost of investment when expressed in terms of the domestic consumption good.

Our paper is related to previous work on models with vintage capital that were used to address cross-country data facts. Pessoa and Rob (2002) have a motivation which is very similar to our's. They write down a model of vintage capital with embodied technology and use it to show that given variations in investment distortions across countries create larger income differences than in the standard model. However, their model has a much richer but more complicated structure than our model. They choose a production function from a class of CES functions by estimating the parameters of the function. Their model allows firms to destroy old technology, adopt new technology, and to choose the quantity of the new capital to buy. This richness of structure comes at a significant cost of tractability and simplicity. Our model, while missing some of these features, provides a much simpler environment to solve and quantify. Gilchrist and Williams (2001) consider a model where technological change is embodied in new capital and at any point in time different vintages of capital coexist. However, in their model all steady state income differences are due to measured capital not productivity.[1]

Two other papers that are related to our work are Caselli and Wilson (2004) and Eaton and Kortum (2002). Caselli and Wilson note that there is huge variation in the composition of capital goods imports across countries. They then formalize a model in which capital composition in a country is linked to the productivity of different types of capital in that country. In their model the composition of capital provides a quality adjustment to the capital stock; hence it affects productivity. They use regressions to link these country-specific productivities of different types of capital to country characteristics such as education, property rights etc.. Using the estimated productivities they find that their model can account for a significantly larger share of the cross-country variation in relative incomes compared to the standard model with disembodied capital. There are two important differences between Caselli-Wilson and us. The first is an analytical difference. Caselli-Wilson focus on the productivity differences between different varieties of capital

[1] Our work is also related to Parente (1995) who develops a model of technology adoption. The key difference is that our framework formalizes environments with embodied technology while his work focuses on disembodied technology.

goods at a point in time while our focus is on productivity variations in capital goods over time; hence our focus is specifically on capital vintages while their's is on the cross-sectional capital composition at a point in time. The second differences concerns measurement. We measure cross-country differences in productivity by using the model dictated relationship between productivity and the price of investment goods. Caselli-Wilson measure cross-country productivity differences using regression estimates which link these to country characteristics. For both these reasons, we view our work as being complementary to the work of Caselli-Wilson since the papers emphasize different aspects of the data.

Eaton and Kortum (2002) develop a model with trade in capital goods. Their model predicts capital goods imports as a function of import prices of capital goods as well as other frictions to trade. They then use data on capital goods imports to derive a model implied series for the price of capital goods. Using this generated price series they show that the model can explain 25 percent of the cross-country variation in per capita income. The main difference of Eaton and Kortum's work from our's is that they do not focus on the cross-country differences in total factor productivity. While they allow productivity differences in the production technology for capital goods, these differences map into the price of capital goods, not the quality of the capital goods themselves. Thus, in their model a capital good which is cheaper to produce is used more. However, the output produced by a given combination of that capital good and other factors remains unaffected.

The rest of the paper is organized as follows: In the next section we lay out the model while Section 3 characterizes the steady state of the model. In Section 4 we calibrate the model and present the quantitative results while the last section concludes.

2 Model

We consider a world economy with many small open economies. We first describe one of these small open economies and then proceed to discuss the cross-country implications of the model.

Time is discrete $t = 0, 1, ...$ The environment is characterized by perfect foresight: all agents know past, present, and future realizations of exogenous variables with probability one. At any time t, the economy is inhabited by L_t identical households who consume a final good and supply labor inelastically. We let the final good be the numeraire good so that all prices within an economy are in terms of the final good.

The final good is produced by a perfectly competitive representative firm by combining a list of differentiated intermediate goods. Each intermediate good is provided by a monopolistically competitive firm. Intermediate goods are produced by combining labor input with a capital good (which we call a "machine").

Investment is realized through entry in the intermediate goods sector. Entering firms have a menu of investment options. They can either invest in the state of the art machine which embodies the frontier technology available; else they can invest in any older machine with the corresponding vintage technology. The "technology" of the machine determines the labor productivity of the firm. Machines with superior technology come at a higher cost. Once a machine is bought/installed, its productivity remains fixed for the duration of the life of the machine. Lastly, productivity of the frontier technology is assumed to grow at an exogenous rate which is common to all economies of the world.

Capital goods are produced by a sector of perfectly competitive firms. They are also the only tradeable goods in the economy. We also assume that trade is balanced in every period. Differences in the capital good production technology are the only source of variation across countries.

2.1 Households

The representative household maximizes the present discounted value of lifetime utility

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

subject to

$$c_t + q_t b_t \leq w_t + d_t + b_{t-1} + \tau_t$$

for all $t \geq 0$, where $\theta > 0$ and c_t is consumption of the representative household and b_t are one-period bonds contracted at date t that pay one unit of the final good next period.[2] Bonds are sold at discount at price q_t. Wages are given by w_t, and d_t and τ_t are dividends from firms and transfers from the government respectively. The representative household inelastically supplies one unit of labor every period.

The first order condition for the household problem leads to the standard Euler equation

$$q_t = \beta \left(\frac{c_{t+1}}{c_t} \right)^{-\theta} \tag{1}$$

which prices the bond. Let $q_t^j = q_t q_{t+1} ... q_{t+j}$ for $j \geq 1$.

2.2 Final Goods Sector

The final good is produced by combining a set Ω_t of distinct intermediate goods according to

$$Y_t = \left[\int_{\Omega_t} [y_t(\omega)]^\rho \, d\omega \right]^{\frac{1}{\rho}}$$

where $0 < \rho < 1$.

A perfectly competitive final good firm chooses inputs $y_t(\omega)$ to maximize profits

$$\pi_t^f = Y_t - \int_{\Omega_t} p_t(\omega) y_t(\omega) d\omega$$

subject to the posted prices, $p_t(\omega)$, for each intermediate good $\omega \in \Omega_t$. The implied demand function for intermediate good ω is

$$y_t(\omega) = Y_t [p_t(\omega)]^{-\sigma}$$

where $\sigma = \frac{1}{1-\rho}$ denotes the elasticity of demand for good ω.

[2] Under our assumption of balanced trade, households do not have access to international capital.

We index intermediate goods by their technology as given by their labor productivity $\varphi \in \Re^{+}$. This turns out to be convenient as technology differences are the source of all the relevant firm heterogeneity in the model. In other words, all goods/firms ω which share the same technology φ are indeed identical in their price and production decisions.

Let $M_t(\varphi)$ be the measure of goods/firms with technology φ. We can then rewrite the final good production function as

$$Y_t = \left[\int \left[y_t(\varphi) \right]^{\rho} dM_t(\varphi) \right]^{\frac{1}{\rho}} \tag{2}$$

and the implied demand

$$y_t(\varphi) = Y_t \left[p_t(\varphi) \right]^{-\sigma}. \tag{3}$$

Since this sector is perfectly competitive, the representative final good firm must be making zero profits. Hence, at each date we have

$$Y_t - \int p_t(\varphi) y_t(\varphi) dM_t(\varphi) = 0,$$

and substituting in (3)

$$\int \left[p_t(\varphi) \right]^{1-\sigma} dM_t(\varphi) = 1. \tag{4}$$

2.3 Intermediate goods firms

Intermediate goods firms in this economy produce output using a production technology that is linear in labor. Specifically, the production function is:

$$y_t(\varphi) = \varphi l_t(\varphi)$$

where φ is the productivity of the firm and $l_t(\varphi)$ its labor demand.[3] Hence, higher productivity is labor saving in that it lowers the labor required to produce the same unit of output.

[3] We describe intermediate firms by their technology for expositional convenience. But it is important to keep in mind that every firm produces a distinct good even if they share the technology level.

Intermediate goods firms are monopolistically competitive and maximize profits at every date t by choosing the price of their good subject to the inverse demand function (equation (3)). Profits of firm φ at date t are given by

$$\pi_t(\varphi) = p_t(\varphi)y_t(\varphi) - (1-s)w_t l_t(\varphi)$$

where w_t is the wage and s is a per unit labor wage subsidy. The intermediate firm's problem implies an optimal pricing rule given by

$$p_t(\varphi) = \frac{(1-s)\,w_t}{\rho\varphi}.$$

We set the wage subsidy such that $1 - s = \rho$.[4] This eliminates the monopoly distortion. The optimal pricing rule then reduces to

$$p_t(\varphi) = \frac{w_t}{\varphi}. \tag{5}$$

Note that the pricing rule implies that higher productivity firms will charge a lower price and thus have higher sales.

Using the optimal pricing rule (5), it is straightforward to check that

$$\pi_t(\varphi) = \frac{1}{\sigma}p_t(\varphi)y_t(\varphi)$$

so profits are a share $\frac{1}{\sigma}$ of revenues. Note that relative profits are scaled by the level technology:

$$\frac{\pi_t(\varphi)}{\pi_t(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^{\sigma-1}.$$

Hence higher productivity firms have higher profits.

2.4 Entry and Exit of Intermediate Good Firms

At every date there is a infinite pool of entrants. An entrant into the industry needs to purchase a machine/capital good in order to produce a new intermediate good. There are many different

[4] This has literally no other consequence than to simplify the algebra.

vintages of capital goods to choose from: the entering firm's investment decision determines its labor productivity φ.

At every date there is a state-of-art or frontier machine which is embodied with labor productivity φ_t. We assume that the productivity of the frontier machine evolves at an exogenous rate $\gamma > 1$,

$$\frac{\varphi_{t+1}}{\varphi_t} = \gamma. \tag{6}$$

A machine of vintage j at date t is embodied with a labor productivity φ_{t-j}.

We also assume that every period there is an exogenous exit rate δ of existing intermediate goods firms. Specifically, at the end of each period a fraction of δ of the existing stock of machines being used by intermediate goods firms in that period breaks down. Let $N_t(\varphi)$ be the measure of entrants who invest in a machine with embodied technology φ; the resulting law of motion for $M_t(\varphi)$ is then

$$M_t(\varphi) = N_t(\varphi) + (1-\delta) M_{t-1}(\varphi).$$

Let $v_t(\varphi)$ be the present value of a intermediate good firm with productivity φ operating at date t, net of entry costs,

$$v_t(\varphi) = \sum_{j=0}^{\infty} (1-\delta)^j q_t^j \pi_{t+j}(\varphi).$$

It is assumed that every intermediate good firm is owned by the representative household and hence profits in future periods are discounted according to q_t^j.

Let F_t^j be the total cost of a machine of vintage $j \leq t$. We assume that, for all vintages j, F_t^j is proportional to the size of the economy as measured by the labor force. In particular, we assume that $F_t^j = f_t^j L_t$ for all t, where f_t^j is the price of a machine.[5]

[5] This assumption formalizes the idea that a larger economy with more labor needs machines with bigger capacity (or equivalently, it needs a larger machine). Hence, the same productivity machine costs proportionately more in an economy with a larger labor force. This assumption ensures that the model does not generate any scale effects on development.

An entering firm at date t chooses the capital good j which solves

$$\max_{j\geq 0}\left\{v_t\left(\varphi_{t-j}\right)-F_t^j\right\}.$$

There will be positive entry in the intermediate good sector as long as it is profitable using any capital good

$$\max_{j\geq 0}\left\{v_t\left(\varphi_{t-j}\right)-F_t^j\right\}\geq 0.$$

Entry will continue until there are no positive rents left from entry. Thus, the free entry condition is that

$$\max_{j\geq 0}\left\{v_t\left(\varphi_{t-j}\right)-F_t^j\right\}\leq 0 \tag{7}$$

with strict equality if there is positive entry, $N_t > 0$. We can write a free entry condition for each vintage $j \geq 0$,

$$v_t\left(\varphi_{t-j}\right)\leq F_t^j \tag{8}$$

with strict equality if there is positive entry with a machine of vintage j, $N_t\left(\varphi_{t-j}\right) > 0$.

We will use a vintage notation as follows

$$M_t^j = M_t\left(\varphi_{t-j}\right)$$

and similarly for N_t^j, p_t^j, and π_t^j.

2.5 Capital Goods

Capital goods are the only tradeable goods in the economy. Since the economy is small, it takes as given the world prices for capital goods, denoted ϕ_t^j. We abstract from trade frictions, and therefore we have the following law of one price

$$f_t^j = \varepsilon_t \phi_t^j$$

for all $j \geq 0$, where ε_t is the real exchange rate defined in terms of the final good.

Capital goods locally produced are provided by perfectly competitive firms. In order to produce a machine of vintage $j \geq 0$ at date t, the representative capital good firm uses $g_t^j \left(x_t^j \right) > 0$ units of the final good, where g_t^j is a continuous and increasing function and x_t^j is the local production of capital goods. The assumption of an upward sloping cost curve reflects the presence of some factor in limited supply.

Perfect competition equates price to marginal cost

$$f_t^j = g_t^j \left(x_t^j \right) \tag{9}$$

if $x_t^j > 0$. Net exports of vintage j capital good are $\left(x_t^j - N_t^j L_t \right) f_t^j$.

We want to guarantee that all available capital goods are produced in all countries along the balanced growth path. For this we postulate that $g_t^j(0)$ is low enough such that $v_t \left(\varphi_{t-j} \right) / L_t > g_t^j(0)$ for all j along the balanced growth path.

2.6 Government

The government is this economy is assumed to follow a balanced budget period by period so that

$$\tau_t = s w_t L_t$$

Hence, the government finances its wage subsidy to intermediate firms through lump-sum taxes on households.

2.7 Market Clearing Conditions and Equilibrium Definition

Before defining a competitive equilibrium, we need to state the market clearing conditions. First, the labor market requires that we have

$$\int l_t(\varphi) dM_t(\varphi) = L_t \qquad \text{for all } t. \tag{10}$$

Second, balanced trade implies that

$$\sum_{j=0}^{\infty}\left(x_t^j - N_t^j L_t\right) f_t^j = 0 \tag{11}$$

for all t. Finally, we can use equation (11) to write the resource constraint for this economy is

$$c_t + \sum_{j=0}^{\infty} N_t^j f_t^j = Y_t / L_t. \tag{12}$$

Definition 1 *A small open economy equilibrium* Γ *is a sequence of prices*

$$\left\{\left\{p_t^j, f_t^j\right\}_{j\geq 0}, q_t, w_t, \varepsilon_t\right\}_{t\geq 0}$$

and quantities

$$\left\{\left\{M_t^j, N_t^j, x_t^j, y_t^j, l_t^j\right\}_{j\geq 0}, c_t, Y_t\right\}_{t\geq 0}$$

such that for all $t \geq 0$

1. *The household problem is solved, i.e., (1) holds.*

2. *All firms maximize profits.*

3. *The free entry conditions (8) are satisfied.*

4. *All markets clear.*

2.8 A World Equilibrium

Let C be the set of countries in the economy. In the world equilibrium, each country constitutes a small open economy equilibrium and world prices clear the international market of each capital good.

Definition 2 *A world equilibrium is a system of small open economy equilibria* $\{\Gamma_c : c \in C\}$ *and world prices* $\left\{\left\{\phi_t^j\right\}_{j\geq 0}\right\}_{t\geq 0}$ *such that*

$$\sum_{c\in C}\left(x_{ct}^j - N_{ct}^j L_{ct}\right) = 0$$

for all j *and* t.

Of course, only $N-1$ prices are pinned down in equilibrium as one country's consumption acts as the numeraire in the world markets.

2.9 Solving for Equilibrium

We start by noting that zero profits for final goods firms implies that

$$Y_t = \int p_t(\varphi) y_t(\varphi) dM_t(\varphi).$$

Substituting the production technology for intermediate goods and the optimal pricing equation (5) gives

$$Y_t = w_t L_t.$$

Hence the wage is also the income per person in this economy.[6]

Next, we can solve for equilibrium wages by substituting the optimal intermediate goods pricing equation (5) into equation (4)

$$\begin{aligned} 1 &= \left[\int \left(\frac{w_t}{\varphi}\right)^{1-\sigma} dM_t(\varphi)\right], \\ w_t^{\sigma-1} &= \left[\int \varphi^{\sigma-1} dM_t(\varphi)\right], \end{aligned}$$

and factoring out M_t,

$$w_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}} \tag{13}$$

where we define the average technology $\tilde{\varphi}_t$ at date t as

$$\tilde{\varphi}_t = \left[\int \varphi^{\sigma-1} \frac{dM_t(\varphi)}{M_t}\right]^{\frac{1}{\sigma-1}}.$$

[6] Note that the relation $w_t = Y_t/L_t$ arises in this economy due to the wage subsidy which eliminates the monopolistic distortion. This implies that the total value of intermediate output equals the value of labor, i.e., $w_t L_t$. However, firms do make profits since their effective wage bill (which equals $(1-\rho) w_t L_t$) is lower than the payments to labor due to the wage subsidy. The lump-sum tax finance of the subsidy implies that there is no additional distortion.

Expression (13) determines income per capita since $w_t = Y_t/L_t$.

We can use equations (3) and (5) to rewrite revenues of intermediate goods firms as

$$p_t(\varphi)\, y_t(\varphi) = \varphi^{\sigma-1} w_t^{2-\sigma} L_t.$$

Substituting this expression for revenues into the expression for intermediate firms' profits gives

$$\pi_t(\varphi) = (1-\rho)\, \varphi^{\sigma-1} w_t^{2-\sigma} L_t. \tag{14}$$

Since the free entry conditions rule out positive rents from entry, intermediate good firms use their profits to finance the initial investment. Hence, $\frac{1}{\sigma}$, which is the ratio of profits to revenues, can be equated to the share of capital in this model.

Using equation (13) one can re-arrange the expression for profits and write it as

$$\pi_t(\varphi) = (1-\rho)\, \varphi^{\sigma-1} \tilde{\varphi}_t^{1-\sigma} \left(\tilde{\varphi}_t M_t^{\frac{2-\sigma}{\sigma-1}} L_t \right).$$

In order to have a bounded economy, we need profits to fall with entry. Hence we impose the restriction $\sigma > 2$. The term in parenthesis is also the ratio of output to capital, Y_t/M_t. Note that if $\sigma < 2$, this ratio would be increasing in the stock of capital.

3 Balanced Growth Path

We now characterize a balanced growth path for this economy. In particular, we look for paths along which M_t, Y_t, c_t and $\left\{ f_t^j \right\}_{j \geq 0}$ grow at a constant rate. In the following we shall use γ_j to denote the constant, steady state rate of growth of variable $j = M, Y, y, L...$ Recall that both the frontier technology φ_t and the labor force L_t grow at an exogenously given constant growth rate.

Another possible source of growth is a downward trend in the cost of capital goods. We abstract from this possibility by assuming that the price of a capital good of a certain vintage is constant over time along a balanced growth path: $f_t^j = f_{t+1}^j$ for all j. Hence, f^j is independent of time.

Note that this assumption does not imply that the price of a given capital good is constant. As we show below, in equilibrium the price of a vintage declines as it gets older: $f_t^j > f_{t+1}^{j+1}$.[7]

We now proceed to derive several results for the balanced growth path. First, along the balanced growth path, the price of the bonds will be constant,

$$\tilde{q} = \beta \gamma_c^{-\theta}$$

as derived from (1).

Second, we want to solve for the net present value of profits at any given date. Using our characterization of profits, it follows that along the balanced growth path

$$\frac{\pi_{t+1}(\varphi)}{\pi_t(\varphi)} = \frac{\gamma_y \gamma_L}{\gamma_M \gamma_{\tilde{\varphi}}^{\sigma-1}}.$$

Therefore we can write the free entry condition for vintage j (8) as

$$\pi_t(\varphi_{t-j}) \sum_{i=0}^{\infty} \left((1-\delta) \tilde{q} \frac{\gamma_y \gamma_L}{\gamma_M \gamma_{\tilde{\varphi}}^{\sigma-1}} \right)^i \leq F_t^j.$$

It suffices to assume a high value of δ to guarantee that the left hand side is finite. The CES demand specification implies that

$$\frac{v_t(\varphi_{t-j})}{v_t(\varphi_t)} = \frac{\pi_t(\varphi_{t-j})}{\pi_t(\varphi_t)} = \left(\frac{\varphi_{t-j}}{\varphi_t}\right)^{\sigma-1}.$$

With positive entry in every vintage, we then have

$$\frac{f_t^j}{f_t^0} = \left(\frac{\varphi_{t-j}}{\varphi_t}\right)^{\sigma-1} \tag{15}$$

from combining any two free entry conditions (8) with strict equality. Condition (15) is key to this paper. As long as there is positive entry, the relative price of a vintage capital good is given by

[7] The assumption, of course, is in terms of the process underlying the cost functions g_t^j. Like any model with investment and consumption sectors, the investment price is only constant if the productivity growth rates in both sectors satisfy a point condition. We address this issue in greater detail in section 3.1 below.

the technology path. Hence, in equilibrium, capital goods price inherit the balanced growth path properties of technology. Specifically, the price of a capital good is falling at rate $\gamma^{1-\sigma}$.

Condition (15) also helps us to solve for growth rates. From the expression for income (13) we get

$$\gamma_w = \gamma_{\tilde{\varphi}} \gamma_M^{\frac{1}{\sigma-1}}.$$

Recalling that $w = Y/L$, it trivially follows that $\tilde{\varphi}_t$ must be growing at a constant rate if both Y and M grow at constant rates. The binding entry condition (8) for the same capital good taken across two adjacent time periods gives

$$\frac{v_{t+1}\left(\varphi_{t-j}\right)}{v_t\left(\varphi_{t-j}\right)} = \frac{F_{t+1}^{j+1}}{F_t^j}.$$

The right hand side of this expression is the ratio of the entry cost of the same capital good across period. This can be written as

$$F_{t+1}^{j+1}/F_t^j = \gamma_L f_{t+1}^{j+1}/f_t^j.$$

Since vintage prices are constant over time, i.e., $f_t^0 = f_{t+1}^0$, it follows that

$$\begin{aligned} F_{t+1}^{j+1}/F_t^j &= \gamma_L \left(\frac{f_{t+1}^{j+1}}{f_{t+1}^0}\right)\left(\frac{f_t^0}{f_t^j}\right) \\ &= \gamma_L \left(\frac{\varphi_t}{\varphi_{t+1}}\right)^{\sigma-1} \\ &= \gamma_L \gamma^{1-\sigma}. \end{aligned}$$

Following the same steps that were followed to derive (15) we can establish that

$$\frac{v_{t+1}(\varphi)}{v_t(\varphi)} = \frac{\gamma_w \gamma_L}{\gamma_M \gamma_{\tilde{\varphi}}^{\sigma-1}}.$$

Combining both these results yields

$$\frac{\gamma_w \gamma_L}{\gamma_M \gamma_{\tilde{\varphi}}^{\sigma-1}} = \gamma_L \gamma^{1-\sigma}.$$

Rearranging and using $\gamma_w = \gamma_{\tilde{\varphi}} \gamma_M^{\frac{1}{\sigma-1}}$, we get

$$\gamma_w = \gamma^{\frac{\sigma-1}{\sigma-2}}.$$

From the resource constraint (12), it follows that the ratio M_t/w_t must be constant. Otherwise, either consumption contracts or explodes as a share of output. Hence, $\gamma_M = \gamma_w$. This implies that $\tilde{\varphi}_t$ grows at rate γ along a balanced growth path. Since $\tilde{\varphi}$ and φ grow at the same rate it follows that along a balanced growth path the average technology is at a fixed distance from the technological frontier.

Finally, we show that the distribution of capital vintages is constant along a balanced growth path. From the definition of $\tilde{\varphi}_t$, we have

$$\tilde{\varphi}_t^{\sigma-1} = \varphi_t^{\sigma-1} \sum_{j=0}^{\infty} \left(\frac{M_t^j}{M_t} \right) \gamma^{(1-\sigma)j}.$$

The discussion above concluded that $\tilde{\varphi}_t/\varphi_t$ is constant along a balanced growth path. It follows that the distribution of vintages $\left\{ M_t^j \right\}_{j \geq 0}$ is invariant once scaled by total capital M_t, i.e.,

$$\mu^j \equiv \frac{M_t^j}{M_t} = \frac{M_{t+1}^j}{M_{t+1}}.$$

Otherwise, the sum $\sum_{j=0}^{\infty} \left(\frac{M_t^j}{M_t} \right) \gamma^{(1-\sigma)j}$ would not be constant.

Recapping, we have established a key relationship between capital goods prices as captured by equation (15). We then solved for the growth rates of output, capital and average productivity. We showed that these growth rates were functions of the exogenous growth rate of the technology frontier and do not depend on the cost of investment. Crucially, we have said nothing about the actual distribution of capital goods $\left\{ \mu^j \right\}_{j \geq 0}$ other than it is invariant along the balanced growth path.

3.1 An Example

We now provide a simple example to illustrate the equilibrium behavior of the distribution of capital vintages and the distribution of vintage prices along a balanced growth path. To keep things simple we abstract from trade issue for the purposes of this example.

Recall that along a balanced growth path (BGP) the distribution of vintages is constant, i.e., $\mu_t^j = \frac{M_t^j}{M_t} = \mu^j$. Hence, it follows that the stock of each vintage grows at the same rate as aggregate capital, i.e.,

$$\frac{M_{t+1}^j}{M_t^j} = \gamma_M. \tag{16}$$

As we have imposed the condition that capital goods prices must be constant along a balanced growth path (BGP), i.e., $f_t^j = f_{t+1}^j$, we must have $g_t^j \left(x_t^j \right) = g_{t+1}^j \left(x_{t+1}^j \right)$. Since $x_t^j = M_t^j$, this last condition implies that

$$g_t^j \left(\eta^j M_t \right) = g_{t+1}^j \left(\eta^j M_{t+1} \right)$$

where $\eta^j = \frac{N_t^j}{M_t}$. It is easy to check that under our conditions, η^j is constant along a BGP. Hence, if g_t^j is homogenous of some degree $\zeta > 0$ then we can always ensure the existence of a constant price path along any BGP. The homogeneity of g_t^j implies

$$g_t^j \left(\eta^j \right) = \gamma_M^\zeta g_{t+1}^j \left(\eta^j \right).$$

Note this implies that the cost of a fixed amount of capital goods is falling over time, $g_t^j \left(\eta^j \right) > g_{t+1}^j \left(\eta^j \right)$. Clearly, we can generalize our definition of a balanced growth path to accommodate any exogenous technological change on the production of capital goods.

The above properties map into an obvious choice for the g function:

$$g_t^j \left(x_t^j \right) = A_t^j \left(x_t^j \right)^\zeta$$

where $\zeta > 0$, $A_t^j > 0$. As discussed above, the constant prices implies a point condition on the growth rate of technology,

$$\frac{A_t^j}{A_{t-1}^j} = \gamma_M^{-\zeta}.$$

We solve for the distribution of vintages among total and new machines for a given sequence of $\left\{ A_t^j \right\}$. The following is the baseline choice of parameters. We work with a process for A_t^j of the form

$$A_t^j = \gamma_A A_t^{j-1}$$

with $\gamma_A = 1.02$. Hence, on any given date and for *constant* levels of demand, each vintage is 2 percent cheaper to produce than its previous version. Note that the specified process for A^j above implies that the marginal cost schedule for vintage j lies above the marginal cost schedule of newer vintages.

The information above is sufficient to pin down the entire steady state distribution of prices and shares of capital vintages. For the shape of the distribution we do not need to specify any level A_t^0. The remaining parameters are as follows: $\delta = .05$, $\gamma = 1.02$, $\sigma = 2.6$, $\zeta = .5$.

Figures 1-3 plot the frequency distributions of capital good prices, new capital goods and existing capital goods along the BGP. Figure 1 demonstrates that a capital good becomes cheaper as it becomes older. Figure 2 shows that the newer the vintage of a capital good the greater is its share in new investment. Lastly, Figure 3 shows the overall distribution of all different vintages along a BGP. The hump-shaped distribution of all capital goods is due to the fact that entry is occuring in not just the latest vintage but also older vintages. For our chosen parameterization, 10-year old machines have the highest share of total machines in the stationary steady state distribution.

We also plot the frequency distributions for two countries with different technologies. The blue country has $\gamma_A = 1.02$, the red country $\gamma_A = 1$ (constant tech). All remaining parameters are equal to the baseline choices for both countries. Figure 4 shows that amongst new capital goods bought at any date along the BGP, the blue country (which has positive technology growth in capital goods) has a higher share of newer vintages younger than age 11 than the red country and a smaller share of vintages older than 12 years. Correspondingly, Figure 5 shows that amongst all capital goods (old and new) in existence at any date along a BGP, the blue country has a larger share of capital goods younger than 20 years. Clearly, the blue country, in which newer vintages are cheaper to produce than older vintages on every date, ends up with a larger share of newer, more productive capital goods. Hence, its average productivity has to be higher as well relative to the red country

3.2 Cross-country comparisons

We now turn to quantifying the model. We posit that differences in the capital good production technology as the sole source of cross-country variation. For everything else, all countries are identical.

From the discussion in the previous section, it follows that all countries share the same growth rate. Differences in the capital good cost functions g_t^j map into different capital good distributions $\left\{\mu^j\right\}_{j\geq 0}$ as well as different capital-to-output ratios M_t/Y_t. Both map into cross-country variation in income. To see this, recall that income per capita is given by (13)

$$w_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}}.$$

Different distributions $\left\{\mu^j\right\}_{j\geq 0}$ shift the average productivity term $\tilde{\varphi}_t$, depending whether the majority of capital goods are close to the technological frontier or not. The cost of capital also changes the capital intensity of output,

$$\frac{w_t}{M_t} = \tilde{\varphi}_t M_t^{\frac{\sigma-2}{\sigma-1}}$$

by jointly determining $\tilde{\varphi}_t$ and M_t.

3.3 Special Case: One Investment Good

A central feature of our model is the technology margin in the investment decision. Yet the basics linking investment with technology seem to be already present in the classic embodied growth model of Solow (1960). Does our approach add to this literature in any way? To answer this question we now study a version of the model where entrants into the intermediate goods sector can only purchase the machine embodied with the frontier technology. We show that this version fails to generate any productivity differences across countries along the balanced growth path—despite variation in the cost of capital.

Since there is no entry in older vintages in this version, they depreciate at a constant rate δ, $M_t^j = (1-\delta) M_{t-1}^{j-1}$ for $j \geq 1$. All entry is on the frontier technology, $N_t = N_t^0$. We can then write a simple law of motion for the average technology $\tilde{\varphi}_t$,

$$\tilde{\varphi}_t^{\sigma-1} = (1-\delta) \frac{M_{t-1}}{M_t} \tilde{\varphi}_{t-1}^{\sigma-1} + \left[\frac{N_t}{M_t} \right] \varphi_t^{\sigma-1}.$$

Along a balanced growth path this reduces to

$$\left(\frac{\tilde{\varphi}_t}{\varphi_t} \right)^{\sigma-1} = \frac{N_t}{M_t} \frac{1}{1 - \frac{1-\delta}{\gamma_M \gamma^{\sigma-1}}}.$$

where we have used the fact that M grows at a constant rate γ_M along a balanced growth path. The evolution of firms is given by $M_t = (1-\delta) M_{t-1} + N_t$. Hence, along a balanced growth path this can be written as

$$\frac{N_t}{M_t} = 1 - \frac{1-\delta}{\gamma_M},$$

which shows that N/M is constant in steady state. Moreover, since γ is identical across countries and γ_M is a function of γ, N/M must also be identical across countries. This says that the share of new firms in total firms is identical across countries in this model. Hence, $\frac{\tilde{\varphi}_t}{\varphi_t}$ must be the same in all countries in steady state. Clearly, this version of the model resembles the standard neoclassical growth model with disembodied technology.

The intuition behind this result is that in the one capital good case, entrants make only one decision: enter or not enter. Contingent on entering, all new intermediate firms have the frontier machine. Hence, the quality of new machines is identical across countries. All that can vary across countries is the quantity of new machines as countries with higher costs of machines have fewer entrants and thereby fewer new machines at each date. In steady state this implies that countries with higher costs have both fewer new machines and fewer total machines. The average productivity of a country is the weighted average quality of new machines past and present with the weights being the shares of new firms in total firms at each date. Since neither the quality of new machines nor the share of new firms is different across countries, there is no difference in steady state productivity across countries.[8]

This result is actually quite general and is not specific to our model. In Appendix A we sketch the generality of the result for the interested reader. We highlight the fact that in order to have permanent productivity differences across countries to arise endogenously in models with embodied capital, one needs differences in the average quality of the new machines. Alternatively, one needs to have endogenous differences in depreciation rates (rates of firm exit in our model) such that the share of new firms may be different across countries in steady state. Without at least one of these two margins, these models cannot generate steady state cross-country productivity differences.

4 A Quantitative Evaluation

At this point, our model is a complex one. In order to solve for cross-country income differences, it seems we would have to first posit a theory of the cost of capital. A quantitative evaluation appears a daunting task: it appears to be necessary to know the distribution of labor productivity across existing firms, as well as have access to disaggregated data on capital good prices. These estimates are hard to come by especially on a cross-country basis.

However, we show below that all aggregate variables in the model can be expressed as functions of only two prices: the average price of capital goods and the price of the frontier capital good. Moreover, both the average and the frontier capital good price are readily identified from cross-country data on consumption and investment prices. Hence, we can evaluate the role of capital

[8] The steady state share of new machines in total machines is akin to the steady state investment-capital ratio (I/K) in the neoclassical model. Just as I/K in the neoclassical model depends only on the exogenous depreciation rate and the exogenous growth rate, here the steady state share of new machines in total machines, (N/M) depends only on the exogenous exit rate δ and the exogenous growth rate γ. Moreover, since both δ and γ are common to all countries, the steady state N/M is identical across countries. Note though that this is a steady state result. Even in this one capital good case, there will be productivity differences across countries along the transition path.

good prices in explaining income differences using readily available data.

The remainder of this section proceeds as follows. First we prove our claim that the average and the frontier's capital good price are summary statistics for aggregate income and productivity. Second we solve for cross-country income differences, highlighting the variation in both average productivity and capital intensity as a function of both the average and frontier's price. Lastly, we use the cross-country data on the prices of consumption and investment goods to identify the cross-country variation in the two summary statistics.

4.1 Just Two Moments

Consider a machine whose embodied technology is equal to the average productivity of the economy at the present date, $\tilde{\varphi}_t$. We will call this artificial construct the "average" machine. We deduce a price for the average machine, denoted $\tilde{f}_t$, from the free entry condition

$$\tilde{F}_t = v_t\left(\tilde{\varphi}_t\right)$$

where $\tilde{F}_t = \tilde{f}_t L_t$.

Like in the computation of the relative price of capital good vintages (15), we can combine the entry condition of the average machine with the frontier machine,

$$\left(\frac{\tilde{\varphi}_t}{\varphi_t}\right)^{\sigma-1} = \frac{\tilde{f}_t}{f_t^0}. \tag{17}$$

Expression (17) allows us to solve for the price of the average machine as the central first moment of the capital good price distribution,

$$\begin{aligned}\tilde{f}_t &= \left(\frac{\tilde{\varphi}_t}{\varphi_t}\right)^{\sigma-1} f_t^0 \\ &= \sum_{j=0}^{\infty} \mu^j \left(\frac{\varphi_{t-j}}{\varphi_t}\right)^{\sigma-1} f_t^0 \\ &= \sum_{j=0}^{\infty} \mu^j f_t^j.\end{aligned}$$

Hence, the price of the average machine is the average price among existing machines. Note that the weights are given by the vintage distribution $\mu^j = \frac{M_t^j}{M_t}$.

Now that we view the price of the average machine as just the average price, the relationship (17) is quite revealing. Average productivity is just a function of the ratio of the average to the frontier capital good price. That is, we only need two moments of the capital good price distribution: the average price $\tilde{f}_t$ and the maximum f_t^0, which also corresponds to the price of the frontier machine.

What about total capital and income per capita? It turns out these can also be expressed as functions of f_t^0 and $\tilde{f}_t$. The free entry condition for the frontier machine, along the balanced growth path, can be written as

$$\pi_t(\varphi_t) \sum_{i=0}^{\infty} \left((1-\delta)\, \tilde{q} \frac{\gamma_y \gamma_L}{\gamma_M \gamma_{\tilde{\varphi}}^{\sigma-1}} \right)^i = F_t^0.$$

Using some algebra on the profits we get

$$\left(\frac{\varphi_t}{\tilde{\varphi}_t}\right)^{\sigma-1} \left(\frac{w_t}{M_t}\right) A = f_t^0$$

where $A = (1-\rho)\sum_{i=0}^{\infty} \left((1-\delta)\, \tilde{q} \frac{\gamma_y \gamma_L}{\gamma_M \gamma_{\tilde{\varphi}}^{\sigma-1}} \right)^i$. Using equation (17), it follows that the output-to-capital ratio is

$$\frac{w_t}{M_t} = A^{-1} \tilde{f}_t. \tag{18}$$

Since income per capita is given by

$$w_t = \tilde{\varphi}_t M_t^{\frac{1}{\sigma-1}}$$

it is trivial to solve for M_t and w_t as functions of $\left\{ \tilde{f}_t, f_t^0 \right\}$ and parameters.

4.2 Cross-country Income Differences

There are three main variables of interest for our cross-country comparisons: income per capita (w_t/L_t), capital (M_t), and average productivity $\tilde{\varphi}_t$. We seek to express these in terms of the differences in $\left\{\tilde{f}_t, f_t^0\right\}$. In the following, we shall compare two countries by following the notational convention of denoting the second country variables with primes.

Since the process for φ_t is common, equation (17) implies that

$$\frac{\tilde{\varphi}_t}{\tilde{\varphi}'_t} = \left(\frac{f^{0\prime}}{\tilde{f}'}\frac{\tilde{f}}{f^0}\right)^{\frac{1}{\sigma-1}} \tag{19}$$

which shows that the productivity gap between countries depends on the difference in the relative cost of frontier to average machines across countries. The higher the relative price of frontier machines the lower is the relative productivity level of the country.

The free entry conditions for the notional average machine at home and abroad are given by

$$\begin{aligned} v_t\left(\tilde{\varphi}_t\right) &= \tilde{F}_t, \\ v_t\left(\tilde{\varphi}'_t\right) &= \tilde{F}'_t. \end{aligned}$$

Combining the two conditions gives

$$\frac{\tilde{\varphi}_t}{\tilde{\varphi}'_t}\left(\frac{M_t}{M'_t}\right)^{\frac{2-\sigma}{\sigma-1}} = \frac{\tilde{f}}{\tilde{f}'}. \tag{20}$$

Substituting equation (19) in (20) then gives

$$\frac{M_t}{M'_t} = \left(\frac{f^{0\prime}}{f^0}\right)^{\frac{1}{\sigma-2}}\left(\frac{\tilde{f}'}{\tilde{f}}\right) \tag{21}$$

This expression gives the ratio of machines at any given date along a balanced growth path. The ratio of machines depends in an obvious way on the cost of investing in both old and new machines – the higher the cost of a new machine (both f^0 and $\tilde{f}$) the lower is M/M'.

Next, recall that per capita output is given by $Y/L = w = \tilde{\varphi}M^{\frac{1}{\sigma-1}}$. Hence,

$$\frac{w}{w'} = \frac{\tilde{\varphi}}{\tilde{\varphi}'}\left(\frac{M}{M'}\right)^{\frac{1}{\sigma-1}}.$$

Using equations (19) and (21), this can be rewritten as

$$\frac{w}{w'} = \left(\frac{f^{0\prime}}{f^0}\right)^{\frac{1}{\sigma-2}}. \tag{22}$$

Hence, the income gap across countries depends on the relative cost of frontier machines. In particular, the higher the relative cost of the frontier machine in a country the lower is its relative per capita income.[9]

4.3 Using Price Data

The model allows us to generate income differences from readily available data on consumption and investment prices. We first deduce the two summary statistics $\left\{f^0, \tilde{f}\right\}$ of the model from consumption and investment goods prices and then discuss an estimate for the elasticity of substitution σ.

The frontier machine, like all the other capital goods, is tradeable. We use the law of one price

$$f_t^0 = \varepsilon_t \phi_t^0$$

to equate the ratio of the cost of a frontier machine to the real exchange rate

$$\frac{f^0}{f^{0\prime}} = \frac{p_c'}{p_c}.$$

In other words, the nominal price (say in dollars) of a frontier machine is roughly constant across countries.

The average capital good price $\tilde{f}_t$ presents a challenge. The previous section makes clear that the appropriate measure of $\tilde{f}$ is the average cost of the stock of machines, i.e., $\tilde{f}_t = \sum_{j=0}^{\infty} \mu^j f_t^j$. Available investment prices average over new machines, whose distribution may or may not coincide with the one of existing machines. However, this turns out **not** to be a problem due to the following

[9] It is instructive to note that the ratio of per capita steady state incomes can also be written as $\frac{w}{w'} = \left(\frac{\varphi}{\varphi'}\right)^{\frac{\sigma-1}{\sigma-2}} \left(\frac{M/Y}{M'/Y'}\right)^{\frac{1}{\sigma-2}} \left(\frac{L}{L'}\right)^{\frac{1}{\sigma-2}}$. This expression looks very similar to the standard expression for the income ratio under the Solow model with a Cobb-Douglas production technology. The only difference is that in our case the last two terms on the right hand side (which are measured inputs) are raised to the power $(\sigma-2)^{-1}$ while in the Solow model they are raised to a power which is the ratio of the capital share to the labor share. Hence a $\sigma = 2.5$ would generate a fit for our model analogous to the fit of the neoclassical model with a capital share of 2/3.

result:

$$\tilde{f}_t = (1+a) \sum_{j=0}^{\infty} \left(\frac{N_t^j}{N_t} \right) f_t^j \tag{23}$$

where

$$a = \left[\frac{(1-\delta)\left(1-\gamma^{1-\sigma}\right)}{\gamma^{\frac{\sigma-1}{\sigma-2}} - (1-\delta)} \right].$$

Since a is country-invariant, the ratio of the investment price of two countries will be equal to the ratio of average capital good prices

$$\frac{\tilde{f}}{\tilde{f}'} = \frac{p_i}{p_i'}.$$

In order to derive (23), consider the definition of $\tilde{\varphi}_t$. Along the balanced growth path we can write

$$\tilde{\varphi}_t^{\sigma-1} = (1-\delta) \frac{M_{t-1}}{M_t} \tilde{\varphi}_t^{\sigma-1} + \frac{N_t}{M_t} \left(\sum_{j=0}^{\infty} \left(\frac{N_t^j}{N_t} \right) \varphi_{t-j}^{\sigma-1} \right).$$

Re-writing the last term using the ratio of each vintage to the average $\tilde{\varphi}_t$ gives

$$\tilde{\varphi}_t^{\sigma-1} = (1-\delta) \frac{M_{t-1}}{M_t} \tilde{\varphi}_t^{\sigma-1} + \frac{N_t}{M_t} \frac{\tilde{\varphi}_t^{\sigma-1}}{\tilde{f}_t} \left(\sum_{j=0}^{\infty} \left(\frac{N_t^j}{N_t} \right) f_t^j \right).$$

Dividing both sides by $\tilde{\varphi}_t^{\sigma-1}$ and using the balanced growth path relationships $\frac{N_t}{M_t} = 1 - \frac{1-\delta}{\gamma_M}$ and $\frac{\tilde{\varphi}_t}{\tilde{\varphi}_{t-1}} = \gamma$, yields

$$\frac{\sum_{j=0}^{\infty} \left(\frac{N_t^j}{N_t} \right) f_t^j}{\tilde{f}_t} = \frac{1 - \left(\frac{1-\delta}{\gamma_M} \right) \gamma^{1-\sigma}}{1 - \frac{1-\delta}{\gamma_M}}.$$

The right hand side of this last expression does not depend on the capital good prices. Hence, in the model the average price of new investment is proportional to the average price of existing capital goods.

The final step is to calibrate the elasticity of substitution σ. This is the key parameter for our cross-country results: the other parameters have no impact on income dispersion as long as they are constant across countries. For our baseline quantification of the model we set $\sigma = 2.6$ which is the value for the elasticity of demand for intermediate goods used by Acemoglu and Ventura

(2003). We should note that since the capital income share in this model is σ^{-1}, setting $\sigma = 2.6$ implies a capital share of 0.38 which is close to the numbers reported by Gollin (2002).[10]

4.4 Results

We take data from year 2000. We measure income differences by using data on output per worker. Every country's income is expressed relative to the United States. The resulting estimates for income dispersion are reported in Table 1.

TABLE 1. **Predicted Values: GDP per worker**

Data: Penn World Tables, *Year*: 2000, $\sigma = 2.6$

	Std Dev		**Max/Min**		**Mean/Median**	
	Data	Model	Data	Model	Data	Model
Full Data	.28	.36	104	92	1.40	1.85
5 % censored	.23	.27	28	29	1.30	1.64
10 % censored	.20	.22	23	16	1.21	1.49
20 % censored	.12	.13	9	7	1.05	1.25

The first row of numbers in Table 1 shows the results for the full sample of 163 countries in our dataset. In the data the standard deviation of relative income per worker is 0.28 while the ratio of incomes of the richest (Luxembourg) to the poorest country (Zaire) in the sample is 104. The corresponding numbers generated by our model are 0.36 and 92. The second, third and fourth rows of the table show the results after dropping the richest and the poorest 5, 10 and

[10] Our model implies that the cross-country relative income ratio is given by $w/w' = (f^d/f^{d'})^{\frac{1}{\sigma-2}}$. Using this relationship, we also ran a simple linear regression

$$\log\left(\frac{y_{it}}{y_{jt}}\right) = b\log\left(\frac{f_i^d}{f_j^d}\right) + \varepsilon$$

and then use $b = \frac{1}{2-\sigma}$. The estimate is around $\sigma = 2.5$ which is very close to our baseline parameterization.

20 percent of countries from the sample, respectively. As the table makes clear, the results are surprisingly strong. The model reproduces almost exactly the income gap between the highest and the lowest income countries. On the income dispersion across countries as measured by the standard deviation, if anything, the model overshoots the data a little. We view these results as being supportive of the model.

As was pointed out above, the key parameter for our model is the elasticity of substitution between intermediate goods, σ. In Table 2 we report some robustness checks on our baseline results for GDP per worker for two different values: $\sigma = 2.5$, and 3. Table 2 shows two basic features. First, the ability of the model to reproduce the cross-country income dispersion is relatively robust to alternative values of σ. Even with $\sigma = 3$, the model generates a standard deviation of income which is almost the same as in the data. Contrarily, the fit of the model with respect to the income ratio of the richest to the poorest country in the sample declines as one increases the value of σ. Thus, for the 5 % percent censored sample, with $\sigma = 3$ the predicted max/min ratio of relative incomes from the model is 8 whereas in the data it is 28.[11]

We view the sensitivity of the relative income gap predictions with mixed feelings. Clearly, the fact that the relative income numbers move a lot with changes in σ suggest that it would be hard to identify exactly how much of the observed income gap the model is actually generating. That is a negative. On the positive side however, there are two ways to view this "excess" sensitivity result. First, note that $\sigma = 2.5$ implies a capital income share of 0.4 while $\sigma = 3$ implies a capital income share of 1/3. In the standard neoclassical model a capital income income share of 1/3 and $\frac{(K/Y)_{rich}}{(K/Y)_{poor}} = 3.6$, generates an income gap of only 1.9 while a capital share of 0.4 does only marginally better with an implied income gap of 2.4. Hence, in this range for the capital share,

[11] This is easy to see from equation (22) which says that $\frac{w}{w'} = \left(\frac{f^{d\prime}}{f^d}\right)^{\frac{1}{\sigma-2}}$. Hence, for $\sigma = 2.5$, the estimated relative price of frontier machines across countries is being raised to the power 2 whereas for $\sigma = 3$ the same relative price is only being raised to the power 1. Thus, the predicted income ratio under $\sigma = 3$ is only going to be the square root of the corresponding ratio under $\sigma = 2.5$.

the standard model generates very small income gaps. In contrast, our model generates an income gap of 8 even with $\sigma = 3$ (recall that $\sigma = 3$ implies an capital share of 1/3). This is four times as large as the standard model. We see this as an improvement.

Second, our model takes an extreme stance in that all differences across countries are assumed to be captured through differences in relative investment goods prices. This is clearly an oversimplification since we are not accounting for factors such as human capital, institutions, preferences etc., etc.. In as much as these factors are important in accounting for cross-country differences, our quantitative results leaves room for these explanations as well.

TABLE 2. **Robustness: GDP per worker**

Data: Penn World Tables, *Year*: 2000

$\sigma = 2.5$	**Std Dev**		**Max/Min**		**Mean/Median**	
	Data	Model	Data	Model	Data	Model
Full Data	.28	.38	104	225	1.40	2.34
5 % censored	.23	.27	28	56	1.30	1.96
10 % censored	.20	.21	23	28	1.21	1.73
20 % censored	.12	.12	9	11	1.05	1.36

$\sigma = 3$	**Std Dev**		**Max/Min**		**Mean/Median**	
	Data	Model	Data	Model	Data	Model
Full Data	.28	.30	104	15	1.40	1.29
5 % censored	.23	.24	28	8	1.30	1.23
10 % censored	.20	.21	23	5	1.21	1.18
20 % censored	.12	.14	9	3	1.05	1.10

We study the fit of the induced world income distribution from the model in two additional ways. First, the last column of both Tables 1 and 2 report the ratio of the mean to the median of the relative income series in the data and from the model. The tables show that the fit of the model is good for almost all sub-samples for our baseline calibration as well as being robust to changes in the elasticity parameter σ. Essentially, the mean of the distribution is greater than the median both in the data and in the model with the magnitudes being pretty close.

The second method of evaluating the fit of the induced income distribution is to plot the relative income per person in the data against the predicted series from the model. Figure 6 shows the fit: the scatter points are pretty tightly concentrated around the 45-degree line. The correlation between the two series is 0.75. We conclude that the model fits the data quite well along this dimension as well.

4.5 TFP differences

An additional variable of interest to us is the predicted relationship between productivity and income. Figure 7 show the implied relationship. The predicted relative productivity is increasing with predicted relative income. Given the positive correlation between predicted and actual relative incomes, this clearly indicates that the model is generating higher relative productivities for countries with higher relative incomes.

Given that a key motivating factor for this paper was the 7-fold productivity difference between the richest and poorest countries implied by the standard one-sector growth model, what does our model imply about productivity differences between the richest and poorest countries of the world. As can be deduced from Figure 7, the implied productivity gap between the richest and poorest five countries in our sample is almost 6. We consider this quite promising given the highly simplified structure that we chose to work with.

5 Conclusion

In this paper we have formalized a model of embodied technology adoption which allows us to endogeneize total factor productivity (TFP). The main advantage of this approach is that it is able to generate larger cross-country income differences for the same given level of investment distortions. The primary mechanism is simple. A higher relative price of new capital goods reduces purchases of new capital goods. This margin is the same as in the standard disembodied technology model. The larger effect on income differences comes from the fact that a smaller share of new capital goods also implies a lower quality of the average capital in the economy. This reduces average productivity and hence, per capita income. Intuitively, the mechanism of the model reduces per capita income both along the intensive margin (the number of capital goods) as well as the quality margin (the average productivity of installed capital).

Crucially, we find that this "embodied capital" channel, by itself, cannot generate any differences in cross-country productivity levels in steady state. Thus, if technology is embodied in new capital but the productivity of new capital is the same in all countries at any point in time, in steady state, a higher price of new capital reduces the number of firms adopting new capital and the total number of firms equiproportionately. This leaves the share of new capital goods identical across countries which implies that the quality of the average capital stock is identical across countries. To generate productivity differences one also needs differences in the types of new capital goods that are bought by different countries. In particular, higher capital goods prices need to not only reduce new investment but also bias new investment towards less productive technologies.

Based on the measured prices of investment goods from the PWT, we find that the predicted relative income series from the model fits the data quite well. The model replicates both the cross-country variation in relative incomes as well as the income disparity between the richest and the poorest countries of our sample. We also find that the model generates productivity differences of the order of 6 between the richest and poorest five countries in our sample. We consider the quantitative results to be a qualified endorsement of the model.

In closing two comments are in order. First, we have taken an extreme position regarding the sources of productivity and income differences across countries; we have linked them exclusively to differences in physical capital stocks across countries. This clearly is too strong a position since one can easily imagine compelling reasons why differences in human capital or institutional quality may be important for cross-country productivity and income differences. From a theoretical perspective, it is straightforward to expand our formalization of capital or machines to also incorporate human capital. The data implementation of this augmented structure would be more complicated since one would now require a different measure of investment goods prices which also incorporates the cost of acquiring human capital. However, in as much as differences in the relative price of investment goods across countries also reflect the cross-country variation in institutional quality and/or the stocks of human capital (so that better institutions and higher stocks of human capital reduce the cost of investment), our results do capture these elements as well. Second, we have been silent on the reasons behind differences in investment prices across countries. There may be multiple reasons for these differences ranging from technology to policy-induced distortions. This is an important issue which we hope to address in future work.

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Appendix

A Technology and Investment Price

We have argued that embodied technological change, by itself, does not link TFP with investment prices along the balanced growth path. We show here that the result is quite general.

Capital is heterogeneous. Let k_t be the distribution of capital types, characterized as the sequence $\left\{k_t^j : j \in \mathbb{Z}\right\}$. Superindex j is the birth date of the capital, so $t - j$ is the vintage.

Aggregate production is given by a differentiable function

$$y_t = F\left(k_t\right). \tag{24}$$

We assume that F is homogenous of degree α, i.e.,

$$F\left(ak_t\right) = a^{\alpha} F\left(k_t\right) \tag{25}$$

for all $k_t > 0$ and $a \geq 0$, with $\alpha \in (0, 1]$.[12] Denote the scalar product as $ak_t = \left\{ak_t^j : j \in \mathbb{Z}\right\}$.

Given a consumption sequence $\{c_t : t \in \mathbb{Z}\}$, the distribution of capital types k_t is as follows:

- for all $j > t$, $k_t^j = 0$,
- for $j = t$,

$$k_t^t = \frac{y_t - c_t}{p}, \tag{26}$$

  where $p > 0$, and

- for $j < t$

$$k_t^j = (1 - \delta)\, k_{t-1}^j \tag{27}$$

  where $\delta \in [0, 1]$ is the exogenous depreciation rate.

[12] Of course, (24) must be thought in terms of "per worker" output.

We proceed to define a balanced growth path as a join property of the output, consumption and capital sequences.

Definition 3 *A balanced growth path (BGP) consists of sequences* $\{y_t, c_t, k_t : t \in \mathbb{Z}\}$ *such that*

1. *Conditions (24), (26) and (27) hold for all* $t \in \mathbb{Z}$*,*

2. *Consumption and output grow at a constant rate* $\gamma > 1$*.*

The parameters of this economy are the depreciation rate δ, the production function F and the investment price p. We will entertain variation only on the latter and ask whether it implies differences in TFP.

We need to first define aggregate capital. Let aggregate capital x_t be given by a weighted sum of capital types

$$x_t = \sum_{j \in \mathbb{Z}} \omega_j k_t^j \tag{28}$$

where $\omega_j > 0$ for all $j \in \mathbb{Z}$. The weights $\{\omega_j : j \in \mathbb{Z}\}$ are a function only of the capital type j and reflect the productivity of the vintage.[13] Additionally, we want aggregate capital to inherit the BGP properties. Hence, we assume $\omega_t = (1+\rho)\,\omega_{t-1}$, i.e., the productivity of the frontier vintage is growing at an exogenous and constant rate ρ.

Next we derive the resource constraint. The constant depreciation rate δ implies that

$$x_t = \omega_t k_t^t + (1-\delta)\,x_{t-1}. \tag{29}$$

Using (26), this gives

$$y_t = c_t + p\left[\frac{x_t}{\omega_t} - (1-\delta)\,\frac{x_{t-1}}{\omega_t}\right]. \tag{30}$$

[13] Conceptually, no other weighting scheme seems reasonable. The production F is invariant: the same capital distribution delivers the same output at any date or location. Why would two economies with identical capital distribution and output have a different aggregate capital?

Since the consumption to output ratio is constant along the BGP, the investment to output ratio must be constant too

$$\frac{p\left(\frac{x_t}{\omega_t}-\frac{(1-\delta)}{1+\rho}\frac{x_{t-1}}{\omega_{t-1}}\right)}{y_t}=const.$$

It follows that the ratio $\frac{x_t}{\omega_t y_t}$ is constant as well. Hence, aggregate capital x_t grows at the rate $(1+\rho)\gamma$ along the BGP.

From (29), the share of type t capital to aggregate capital is given by

$$\frac{\omega_t k_t^t}{x_t}=1-\frac{(1-\delta)}{(1+\rho)\gamma}.$$

Using (27) we can determine the shares of any type $j<t$ of capital

$$\frac{\omega_j k_t^j}{x_t}=\frac{(1-\delta)}{\gamma}\frac{\omega_j k_{t-1}^j}{x_{t-1}}.$$

The key observation is that these capital shares are constant along the BGP. We proceed then to use vintage notation. Let $\psi_i=\frac{\omega_{t-i}k_t^{t-i}}{x_t}$ denote the share of vintage $t-i$ capital in total capital. Correspondingly, let $\psi=\{\psi_i\}_{i=0}^{\infty}$. Use these expressions

$$\psi_0=1-\frac{(1-\delta)}{(1+\rho)\gamma}$$

and

$$\psi_i=\frac{1-\delta}{\gamma}\psi_{i-1}$$

for $i\geq 1$. Note the investment price does not show up in the characterization of ψ.

By construction, $k_t=\psi x_t$. Since F is homogeneous of degree α,

$$\alpha F(k_t)=\sum_{i=0}^{\infty}F_{t-i}(k_t)\,k_t^{t-i}$$

and each of its partial derivatives is honogeneous of degree $\alpha-1$. We can then express the j-derivative of F at date t as

$$F_j(k_t)=F_j(\psi)\,x_t^{\alpha-1}. \tag{31}$$

Combining

$$\begin{aligned}\alpha F\left(k_t\right) &= \sum_{i=0}^{\infty} F_{t-i}\left(\psi\right)\frac{k_t^{t-i}}{x_t}x_t^{\alpha} \\ &= \left(\sum_{i=0}^{\infty}\omega_{t-i}F_{t-i}\left(\psi\right)\psi_i\right)x_t^{\alpha}.\end{aligned}$$

The term $\tilde{\varphi}_t = \sum_{i=0}^{\infty}\omega_{t-i}F_{t-i}\left(\psi\right)\psi_i$ is the TFP: none of its terms depends on the investment price p.

Figure 1: **Distribution of Capital Good Prices**

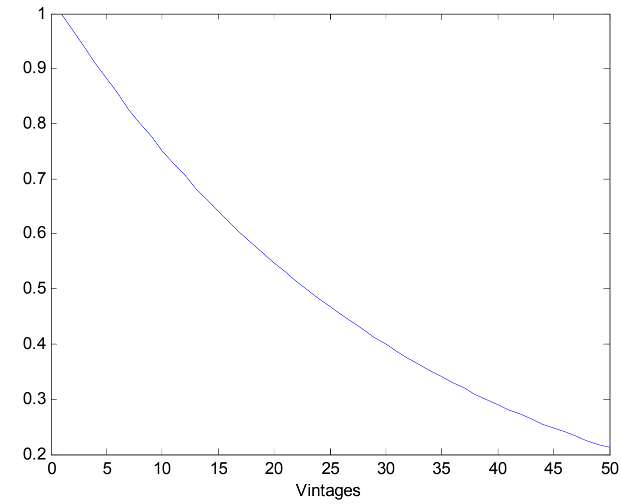

Figure 2: **Distribution of new capital goods along a balanced growth path**

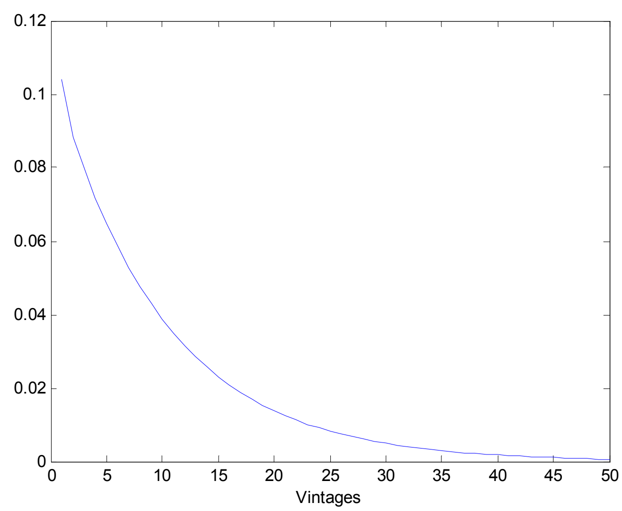

Figure 3: **Distribution of Existing Capital Goods**

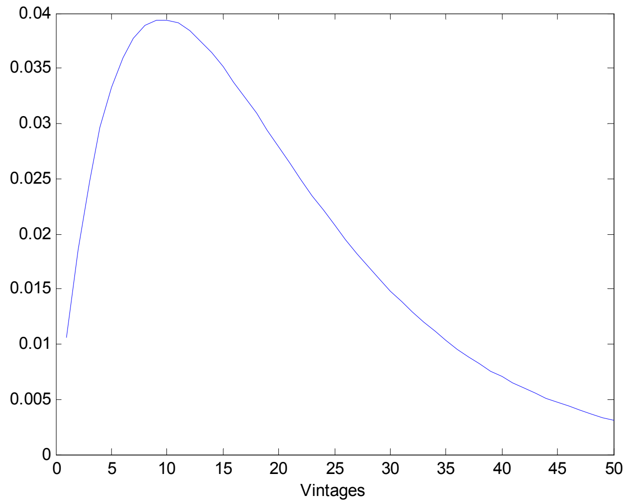

Figure 4: **Distribution of New Capital Goods** Blue line $\gamma_A = 1.02$, red line $\gamma_B = 1$.

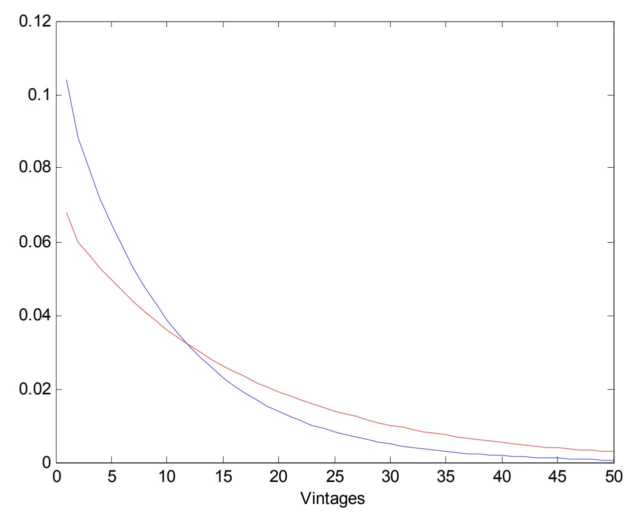

Figure 5: **Distribution of Existing Capital Goods**

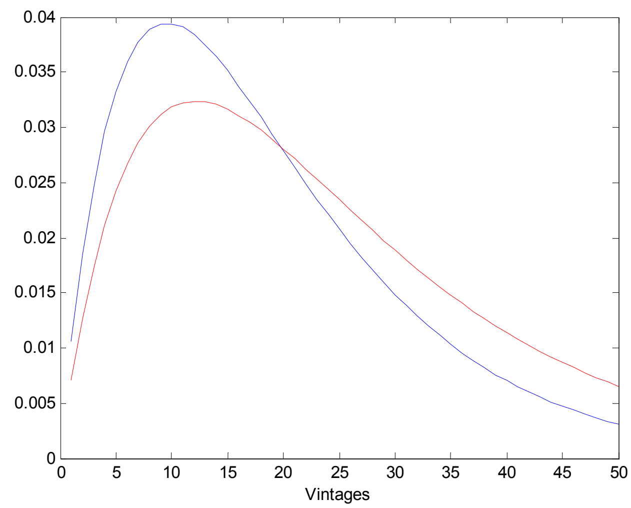

Figure 6: **Relative income per worker: Predicted vs data**

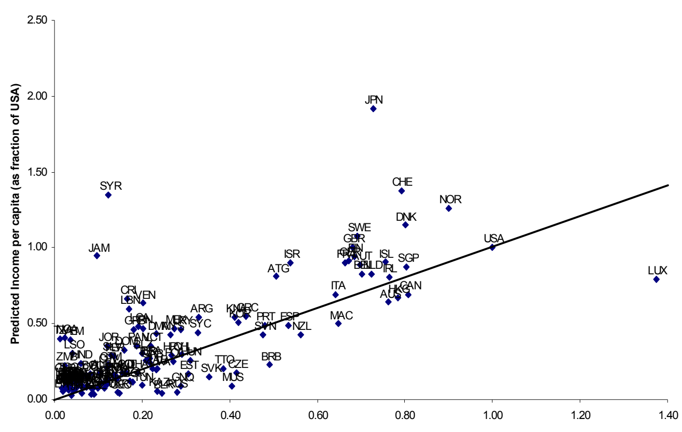

Figure 7: **Predicted relative TFP vs predicted relative income per worker**