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Enghin Atalay
Antoine Martin
James McAndrews

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Abstract

This paper considers the welfare effects of introducing a liquidity-saving mechanism (LSM) in a real-time gross settlement (RTGS) payment system. We study the planner's problem to get a better understanding of the economic role of an LSM and find that an LSM can achieve the planner's allocation for some parameter values. The planner's allocation cannot be achieved without an LSM, as long as some payments can be delayed without cost. In equilibrium with an LSM, we show that there can be either too few or too many payments settled early compared with the planner's allocation, depending on the parameter values.

Key words: liquidity-saving mechanisms, real-time gross settlement, large-value payment systems

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Atalay: University of Chicago (e-mail: atalay@uchicago.edu). Martin: Federal Reserve Bank of New York (e-mail: antoine.martin@ny.frb.org). McAndrews: Federal Reserve Bank of New York (e-mail: jamie.mcandrews@ny.frb.org). Part of this research was done while Antoine Martin was visiting the University of Bern and the University of Lausanne. The authors thank Marco Galbiati, Thor Koeppl, Tomohiro Ota, Jean-Charles Rochet, Matthew Willison, and seminar participants at the University of Western Ontario, the University of Mannheim, the Higher School of Economics, the Bank of England, the Banque de France, the Moncasca workshop (Rome 2008), and the Payments and Networks conference (Santa Barbara 2008) for useful comments. They also thank Dana Lane for editorial assistance.

1 Introduction

This paper studies the planner's allocation in a model of a large-value payment system, with and without a liquidity saving mechanism, and compares it with the equilibrium allocation(s). Studying the planner's problem allows us to deepen our understanding of the economic role of liquidity-saving mechanisms.

In contrast to the standard theory of intertemporal trade, where agents borrow and lend at a market price that they take as given, there is typically no market for short-term (intraday) credit in interbank settlement systems. Instead, under typical arrangements, banks borrow reserves from the central bank at a fixed price.

Concerns about systemic and operational risk have made managing the flow of interbank payments an important concern of central banks in the past 25 years. Over this period, most central banks have abandoned settlement systems based on the netting of interbank claims and replaced them with systems in which all payments are settled on a gross basis (see Bech and Hobijn 2007). Net settlement systems were viewed as representing too big a risk of cascades of defaults or unexpected settlement demands on banks. Real-time gross settlement (RTGS) systems have reduced the risk of cascades of defaults, but at the cost of increased need for liquidity and incentives for banks to delay their payments. In recent years, a number of countries have introduced, or are planning to introduce, liquidity-saving mechanisms (LSMs), as a complement to their RTGS systems.[1] LSMs aim to combine the strengths of net settlement and RTGS systems.[2] The Federal Reserve is also studying the possibility of implementing an LSM for Fedwire, its large-value payment system.

In an RTGS system, banks have the choice between sending a payment early or delaying it. While banks are required to hold positive reserves, they can borrow reserves from the central bank. Banks have an incentive to delay their payment if they expect the cost of borrowing reserves to exceed the cost of delaying their payment. A bank's expected cost of sending a payment early increases as other banks delay more of their payments, leading to undesirable surges of payments late in the day.

[1] For example, the European Central Bank launched TARGET 2 in November 2007 and the Bank of Japan introduced liquidity-saving features to its large-value payment system in October 2008.

[2] Martin and McAndrews (2008) provide a descriptive overview of RTGS systems and LSMs. See also McAndrews and Trundle (2001) and Bank for International Settlements (2005) for a review and extensive descriptive material on LSMs.

With an LSM, banks still have the option to send or delay a payment. In addition, banks can put a payment in a queue. The payment will be released from the queue according to some prespecified rules. Typically, a payment is released from the queue if the bank has sufficient reserves in its account or if the payment is part of a bilaterally or multilaterally offsetting group of payments in the queue.

We first study the planner's allocation, with and without an LSM, in the model of Martin and McAndrews (2008). In that model, all banks are ex ante identical. Banks are exposed to shocks and can be of six different types: They either have to make a time-critical payment, which is costly to delay, or a non–time-critical payment, which can be delayed at no cost; in addition, banks receive either a positive, a negative, or no liquidity shock. We assume that the planner can choose the action of banks: delay or send in the case of RTGS; delay, queue, or send in the case of an LSM.

The planner maximizes the ex ante welfare of all banks. In particular, the planner can direct banks to take a feasible action even if, ex post, it is not in the best interest of that particular bank, given its type. We can think of the planner as having a commitment technology since it can achieve allocations banks would like to commit to ex ante. The planner knows the distribution of payments in the economy but does not know the identity or the type of the bank that will receive a specific payment. Hence, the planner cannot choose the action of a sending bank conditional on the type or the identity of the receiving bank.

Unless liquidity shocks are large, the planner chooses the same allocation whether an LSM is available or not. With small liquidity shocks, the planner wants all banks to send their payments early. Hence, the ability to commit could make an LSM inessential. When liquidity shocks are large, however, the planner is able to achieve higher welfare with an LSM than without. Here, the LSM goes beyond the commitment technology by helping the planner find receiving banks that can reciprocate in delivering payments.

For some parameter values, the planner's allocation, where all banks make their payment early, can be achieved in equilibrium with an LSM. However, the planner's allocation cannot be achieved with an RTGS system only, if some payments can be delayed without cost. With RTGS, banks always delay their non–time-critical payments.

When liquidity shocks are large, the planner can improve welfare by making banks with a negative liquidity shock delay their payments. Some of these banks will receive a payment that will offset the liquidity shock and reduce their borrowing cost. When some banks delay, an LSM can help the planner by allowing the release of some payments conditional on the receipt of an offsetting payment. This conditionality, which cannot be achieved with an RTGS system, has value in addition to the kind of commitment technology available to the planner.

These results show how the conditionality that an LSM provides can partially substitute for commitment, when commitment is unavailable, and can complement the commitment, when commitment is available. In addition, an LSM can improve welfare by offsetting payments inside the queue.

Our paper is part of a growing literature concerned with settlement systems in general and liquidity-saving mechanisms in particular. Roberds (1999) compares gross and net payment systems with systems offering an LSM. He examines the incentives participants have to engage in risk-taking behavior in the different systems. Kahn and Roberds (2001) consider the benefits of coordination from an LSM in the case of Continuous Linked Settlement (CLS). Willison (2005) examines the behavior of participants in an LSM. Martin and McAndrews (forthcoming) study two different designs for an LSM. Our paper uses the framework first presented in Martin and McAndrews (2008); they derive the equilibrium allocations used in this paper.

The remainder of the paper proceeds as follows. Section 2 introduces the environment. Section 3 describes the planner's problem. Section 4 characterizes the solution to the planner's problem when an LSM is not available, while section 5 characterizes the solution when an LSM is available. Section 6 concludes.

2 The environment

The environment is similar to the one in Martin and McAndrews (2008). The economy is populated by a continuum of mass 1 of risk-neutral agents. These agents are called payment-system participants or banks. A nonstrategic agent is identified with settlement institutions.[3]

The economy lasts two periods, morning and afternoon. Each bank makes two payments and receives two payments each day. One payment is sent to another bank and is called the core payment. The other payment is sent to the nonstrategic agent and affects the bank's liquidity shock. Similarly, one payment is received from another bank, and one is received from the nonstrategic agent. Core payments have size $\mu \geq \frac{1}{2}$, while payments to and from the nonstrategic agent have size $1 - \mu$.

Three factors influence the banks' payoff of sending, queuing, or delaying their core payment. First, banks must pay a cost to borrow from the central bank. Second, banks may need to send a time-critical payment. Third, banks may receive a liquidity shock.

Each bank starts the day with zero reserves. Reserves can be borrowed from the central bank at an interest cost of R.[4] In our welfare analysis, we think of R as representing both the private and the social cost of borrowing reserves. Banks that receive more payments than they send in the morning have excess reserves. It is assumed that these reserves cannot be lent to other banks, so that banks receive no benefit from excess reserves. Payments received and sent in the same period offset each other. Hence, a bank needs to borrow from the central bank only if the payments it sends in the morning exceed the payments it receives in the morning.

Banks learn in the morning whether the payment they must make to another bank is time critical. We assume that banks know the time criticality of the payments that they must make but not the payments that they receive. For example, a customer of bank A may want to make a time-critical payment to one of its counterparties, which is a customer of bank B. In this case, bank A would know that the payment is time critical but bank B may not. The probability that a payment is time critical is denoted by θ.[5] If a bank fails to make a time-critical payment in the morning, a cost γ is incurred. Delaying non–time-critical payments until the afternoon has no cost. Banks choose whether to send their payment in

[3] One can think of the nonstrategic agent as aggregating several distinct institutions such as the CLS bank, the Clearing House Interbank Payment System (CHIPS), and the Depository Trust Company (DTC), as well as the payment side of securities transactions.

[4] Evidence discussed in Mills and Nesmith (2008) suggests that the cost of intraday reserves can influence banks payment behavior.

[5] Throughout the paper, it is assumed that if x represents the probability that an event will occur for a bank, then the fraction of banks for whom this event occurs is x as well. Hence, a fraction θ of banks must make a time-critical payment.

the morning before they know if they will receive a payment from another bank in the same period. Banks form rational expectations about the probability of receiving a payment from some other bank in the morning. Let π denote this expectation.

In the morning, banks learn when they receive a payment from the nonstrategic agent, and when they must send an offsetting payment. The probability of receiving the payment in the morning is $\bar{\pi}$ and so is the probability of having to send the payment in the morning. The events are independent of each other. Payments to the nonstrategic agent cannot be delayed. Let $\sigma \equiv \bar{\pi}(1-\bar{\pi})$. A fraction σ of banks receive a payment from the nonstrategic agent in the morning and do not need to make a payment until the afternoon; these are the banks that receive a positive liquidity shock. A fraction σ of banks must make a payment to the nonstrategic agent in the morning but do not receive an offsetting payment until the afternoon; these are the banks that receive a negative liquidity shock. The remaining banks, a fraction $1-2\sigma$, make and receive a payment from the strategic agent in the same period, either in the morning or in the afternoon. These banks do not receive a liquidity shock.

In summary, banks can be of six different types: A bank may receive a positive, a negative, or no liquidity shock; they may or may not have to make a time-critical payment. Table 1 contains the definition of all parameters.

$\mu \in [0.5, 1]$	Size of payment to other banks
$R > 0$	Cost of borrowing
$\theta \in [0, 1]$	Probability of having to make a time-critical payment
$\gamma > 0$	Cost of delay
$\sigma \in [0, 0.25]$	Probability of a liquidity shock

Table 1: Parameters of the model

The timing of events is summarized in figure 1. First, nature chooses the banks that receive a liquidity shock and the banks that must make time-critical payments. Next, morning payments to and from the nonstrategic agent are made, and banks make the choice to send their payment to another bank, delay their payment, or–if an LSM is available–put their payment in queue. At the end of the morning period, banks that must borrow from the central bank incur a borrowing cost, while banks that did not send time-critical payments incur a delay cost. All remaining payments are made in the afternoon.

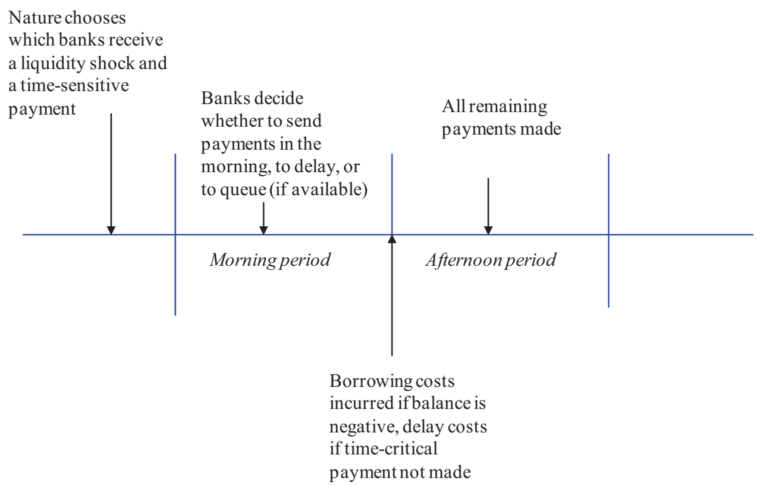

Figure 1: Timeline for shocks and for sending and receiving payments.

The role played by the different frictions in the model can be summarized as follows: The cost of borrowing provides an incentive to bunch payments. Indeed, absent liquidity shocks, banks could avoid borrowing if either all payments are sent in the morning or all payments are delayed. Time-critical payments provide an incentive for some banks to send their payment early to avoid the delay cost. In contrast, banks have an incentive to delay their payment to avoid having to borrow from the central bank.

We abstract from settlement failure in this model as it does not play an important role in the comparison between RTGS systems with or without an LSM. Indeed, the cost of a settlement failure in RTGS systems does not depend on whether there is an LSM. In contrast, the cost of a settlement failure is much higher in a delayed net settlement system than in an RTGS system. Lester (2009) models the difference between a delayed net settlement system and an RTGS system in a model with settlement risk.

2.1 The settlement system design

We consider two types of settlement system designs: real-time gross settlement (RTGS) and an RTGS augmented by a liquidity-saving mechanism (LSM). We consider the settlement system as part of the physical environment.

With an RTGS system, banks have the choice between sending their core payment in the morning or delaying that payment until the afternoon. With an LSM, banks get an

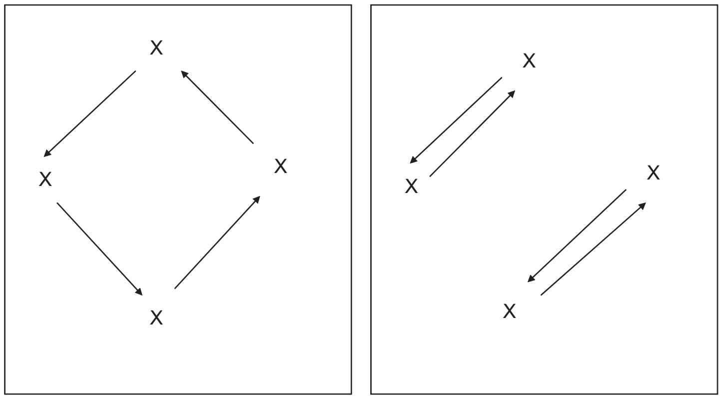

Figure 2: Pattern of payments. Left panel: one cycle. Right panel: multiple unique cycles. Xs denote banks; arrows denote payments from one bank to another.

additional option: They can put their payment in a queue, and the payment will be settled under prespecified conditions.

With an LSM, the number of payments settled depends on the underlying pattern of those payments. This pattern affects the number of payments that are released from the queue. In this paper, we consider two cases, illustrated by figure 2. The Xs in figure 2 denote banks, and the arrows denote payments from one bank to another. In the left panel of figure 2, all payments form a unique cycle. In the right panel of figure 2, payments are matched in bilaterally offsetting pairs.

2.1.1 The queue

By assumption, every payment is part of a cycle. In the left panel of figure 3, all payments in a cycle are also in the queue. Otherwise, at least one payment in the cycle is not in the queue, as illustrated in the right panel of figure 3. In the former case, all the payments in that cycle are released by the queue since they offset multilaterally (or bilaterally if the cycle is of length 2). In the latter case, the payment belongs to a path (within the queue).[6]

Since payments in a path cannot offset multilaterally, they may not be released from the queue. These payments will be released from the queue if the bank that must make the "first" payment in the path receives a payment from outside the queue. In that case, the

[6] A queue can contain payments in cycles and payments in paths.

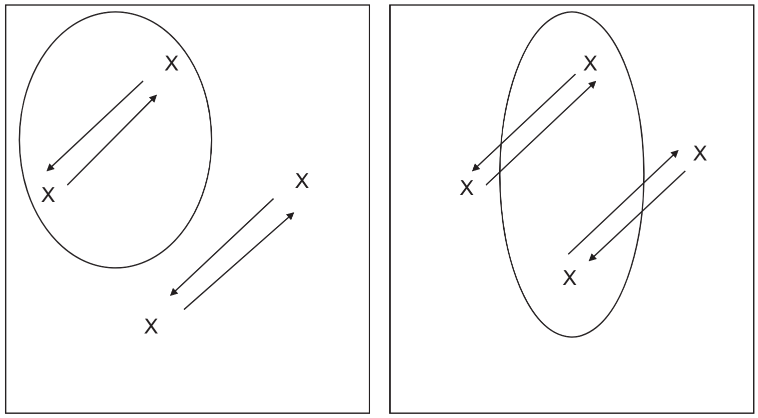

Figure 3: Left panel: queued payments in a cycle. Right panel: queued payments in paths.

first payment in the path is released, creating a cascade of settlements until, eventually, a payment is made to a bank outside the queue. In the right panel of figure 3, for example, if a bank outside the queue sends its payment in the morning, then the payment of the receiving bank, which is in the queue, will also be released in the morning. Otherwise, the queued payment will be released in the afternoon.[7]

We denote by χ the probability that a payment in the queue is part of a cycle and $1 - \chi$ the probability that it is part of a path. We consider the value of χ for the two extreme cases described above. We use λ_e to denote the fraction of banks that sends payments early, λ_q to denote the fraction that puts its payments in the queue, and λ_d to denote the fraction that delays its payments. Clearly, $\lambda_e + \lambda_q + \lambda_d = 1$.

If all payments form a unique cycle, then the probability that a payment in the queue is in a cycle is zero unless all banks put their payment in the queue. Formally, $\chi = 0$ if $\lambda_q < 1$ and $\chi = 1$ if $\lambda_q = 1$. Under this assumption, the queue releases the fewest payments. This case is also interesting because the role of the queue is only to allow banks to send their payment conditional on receiving another payment. The queue no longer plays the role of settling multilaterally offsetting payments. If all payments are in cycles of length 2, then the

[7] If payments form a unique long cycle, any proper subset of payments in the queue cannot be multilaterally offsetting. With this pattern of payments, no offsetting occurs in the queue unless all payments are queued. In contrast, when payments form many cycles of length 2, the amount of offsetting that occurs in the queue in maximized. The pattern of payments only matters for the amount of offsetting occurring in the queue. Other patterns would imply more offsetting in the queue than the unique cycle but less than the cycles of length 2. The results would be qualitatively similar to the two polar cases we consider.

probability that a payment in the queue is in a cycle is λ_q.

The probability of receiving a payment conditional on not putting the payment in the queue, π^o, and the probability of receiving a payment conditionally on putting a payment in the queue, π^q, are given by

$$\pi^o \equiv \frac{\lambda_e}{\lambda_e + \lambda_d} = \frac{\lambda_e}{1 - \lambda_q}, \tag{1}$$

$$\pi^q \equiv \chi + (1-\chi)\frac{\lambda_e}{\lambda_e + \lambda_d} = \chi + (1-\chi)\pi^o. \tag{2}$$

The derivation of these expressions is provided in the appendix. Note that under the "long-cycle" assumption, $\chi = 0$ if $\lambda_q < 1$ so that $\pi^o = \pi^q = \lambda_e/(\lambda_e + \lambda_d)$. If $\lambda_q = 1$, then $\pi^o = 0$ and $\pi^q = 1$, since all the payments are put in the queue. Under the "short-cycles" assumption, $\chi = \lambda_q$ so that

$$\pi^q = \lambda_q + (1-\lambda_q)\frac{\lambda_e}{\lambda_e + \lambda_d} = \lambda_q + (1-\lambda_q)\pi^o.$$

3 The planner's problem

In this section, we describe a planner's problem. The planner assigns an action to each bank. Feasible actions are pay early, delay, and, in the case of an LSM, queue. An assigned action can depend on the type of the sending bank but not on the type of the receiving bank. However, the planner knows the distribution of types of potential receiving banks. The planner's objective is to maximize a weighted average of the welfare of all banks in the economy, where the weights are given by the population sizes. Equivalently, we can interpret the planner's objective function as the expected utility of a representative bank before the bank's type is known.

The planner's allocation corresponds to the allocation that banks would choose if they could commit to a type-conditional action before they know their type. Absent commitment, banks may have an incentive to deviate from the action prescribed by the planner because they do not take into account the effect of their actions on the probability that other banks will receive a payment early.

Let the set of types be $I = \{s+, s0, s-, r+, r0, r-\}$. We use s to denote banks with a time-critical (or sensitive) payment and r to denote banks with a non–time-critical (or regular) payment. We use $+$ to denote banks with a positive liquidity shock, 0 to denote banks with no liquidity shock, and $-$ to denote banks with a negative liquidity shock. Let λ_i^j denote the fraction of participants of type $i \in I$ who choose action $j \in J = \{e, q, d\}$, where e means that the payment is sent early, q means that the payment is queued, and d means that the payment is delayed. We have the following restriction:

$$\lambda_i^e + \lambda_i^q + \lambda_i^d = 1, \forall i \in I. \tag{3}$$

In addition, $\lambda_i^q \equiv 0$, for all i, if the settlement system is RTGS.

The planner's objective, W, is given by

$$\begin{aligned}
W = & -\sigma \left[(\theta\lambda_{s+}^e + (1-\theta)\lambda_{r+}^e)(1-\pi^o)(2\mu - 1)R\right] \\
& - \sigma\theta\lambda_{s+}^q(1-\pi^q)\gamma \\
& - \sigma\theta\lambda_{s+}^d\gamma \\
& - (1-2\sigma)\left[(\theta\lambda_{s0}^e + (1-\theta)\lambda_{r0}^e)(1-\pi^o)\mu R\right] \\
& - (1-2\sigma)\theta\lambda_{s0}^q(1-\pi^q)\gamma \\
& - (1-2\sigma)\theta\lambda_{s0}^d\gamma \\
& - \sigma\left[(\theta\lambda_{s-}^e + (1-\theta)\lambda_{r-}^e)\left(1-\mu\pi^o\right)R\right] \\
& - \sigma\left[\theta\lambda_{s-}^q(1-\pi^q)\gamma + (\theta\lambda_{s-}^q + (1-\theta)\lambda_{r-}^q)\left(1-\mu\right)R\right] \\
& - \sigma\left[\theta\lambda_{s-}^d\gamma + \left(\theta\lambda_{s-}^d + (1-\theta)\lambda_{r-}^d\right)(1-\pi^o)(1-\mu)R\right],
\end{aligned} \tag{4}$$

where π^o is defined as

$$\pi^o \equiv \frac{\sigma\left[\theta\left(\lambda_{s+}^e + \lambda_{s-}^e\right) + (1-\theta)\left(\lambda_{r+}^e + \lambda_{r-}^e\right)\right] + (1-2\sigma)[\theta\lambda_{s0}^e + (1-\theta)\lambda_{r0}^e]}{\sigma\Gamma + (1-2\sigma)\Sigma}, \tag{5}$$

with

$$\Gamma \equiv \theta \left(\lambda_{s+}^{d} + \lambda_{s-}^{d} + \lambda_{s+}^{e} + \lambda_{s-}^{e}\right) + (1-\theta) \left(\lambda_{r+}^{d} + \lambda_{r-}^{d} + \lambda_{r+}^{e} + \lambda_{r-}^{e}\right), \tag{6}$$

$$\Sigma \equiv \theta \left(\lambda_{s0}^{d} + \lambda_{s0}^{e}\right) + (1-\theta) \left(\lambda_{r0}^{d} + \lambda_{r0}^{e}\right). \tag{7}$$

In addition, π^q is given by equation (1). We can write χ as

$$\chi = \sigma \left[\theta \left(\lambda_{s+}^{q} + \lambda_{s-}^{q}\right) + (1-\theta) \left(\lambda_{r+}^{q} + \lambda_{r-}^{q}\right)\right] + (1-2\sigma) \left(\theta\lambda_{s0}^{q} + (1-\theta)\lambda_{r0}^{q}\right),$$

in the short-cycles case. In the long-cycle case, $\chi = 0$ if any of the banks do not put their payments into queue, i.e.,

$$\sigma \left[\theta \left(\lambda_{s+}^{q} + \lambda_{s-}^{q}\right) + (1-\theta) \left(\lambda_{r+}^{q} + \lambda_{r-}^{q}\right)\right] + (1-2\sigma) \left(\theta\lambda_{s0}^{q} + (1-\theta)\lambda_{r0}^{q}\right) < 1,$$

and $\chi = 1$ otherwise.

To get an idea of how equation (4) is constructed, note that the first three lines give the welfare cost associated with banks that receive a positive liquidity shock, multiplied by the fraction of such banks. The first line gives the welfare of the banks that send their payments early, the second line gives the welfare of the banks that queue their payments, and the third line gives the welfare of the banks that delay. The next three lines give the welfare cost of banks that receive no liquidity shock, and the last three lines give the welfare cost of banks that receive a negative liquidity shock.

If the settlement system is RTGS, χ is trivially equal to zero. The expression for the probability of receiving a payment in the morning is given by equation (5).

Lemma 1 *W is convex in λ_i^j, for all $i \in I, j \in J$.*

All proofs are provided in the appendix. A consequence of lemma 1 is that the planner always assigns the same action to all banks of the same type. Hence, when solving the planner's problem, we can limit our attention to action profiles of the type $\{a_i\}_{i \in I}$, with $a_i \in J$ in the case of an LSM and $a_i \in \{e, d\}$ in the case of an RTGS system.

The function W can be interpreted as social welfare if R represents the social cost of borrowing and if γ represents the social cost of delay. In particular, these variables must adequately capture the cost to the payment systems for households, which are not modeled in this paper. Otherwise, the function W represents the welfare of the banking system more narrowly.

4 The planner's solution in the case of RTGS

This section characterizes the solution to the planner's problem with an RTGS settlement system. Depending on the parameter values, the planner may choose to make all banks, all banks except those of type $r-$, or all banks except those of type $r-$ and $s-$ make their core payment early.

The next lemma compares the benefit of delay depending on a bank's liquidity shock.

Lemma 2 *The benefit of delay, relative to sending a payment early, is greater for a bank that receives a negative liquidity shock than for a bank that receives no liquidity shock. It is greater for a bank that receives no liquidity shock than for a bank that receives a positive liquidity shock.*

Comparing banks with the same liquidity shock, the planner always prefers to delay the payment of a bank that must make a non–time-critical payment. The expected cost of delay to other banks, however, is independent of the delaying bank's type because we have assumed that the type of the recipient is not correlated with the sender's type.

Proposition 1 *Depending on parameter values, the solution to the planner's problem can take one of three forms: (1) all banks pay early; (2) only bank of type $r-$ delay; (3) only banks of types $r-$ and $s-$ delay.*

The parameter configurations that determine the planner's choice are provided in the proof in the Appendix. Here we provide some intuition. Inspection of the function W shows that, everything else being equal, a reduction in the fraction of payments settled in the morning, π^o, reduces welfare. This effect provides incentives for the planner to make banks pay early. However, there can be welfare gains from delaying a payment, to the extent that such delay helps redistribute liquidity between banks. Consider a bank that receives a negative liquidity shock. If this bank delays its core payment, it will receive a payment with probability $1 - \pi^o$, in which case it will not need to borrow. Even if that bank does not receive a payment, the amount it needs to borrow is only $(1 - \mu)$ if it delays its core payment instead of 1 if it sends the payment early. The delay of this bank's payment will have a cost for the bank that was supposed to receive the payment. That cost will be lower than the benefit when the bank not receiving the payment has a positive liquidity shock. In contrast, if a bank that has a positive liquidity shock sends its payment early and does not receive an offsetting payment, it will have to borrow only the difference between the core payment it sent and the liquidity shock it received.

Gains from delay are large when the shocks are large (μ is small) because banks that receive positive liquidity shock need not borrow much if they send a payment and do not receive an offsetting payment. In addition, the cost of not delaying payments for banks that have a negative liquidity shock is large. The gains are also large when σ is large, so that more banks receive either a positive or a negative liquidity shock. Whether the planner decides to delay the time-critical payments of banks that receive a negative shock depends on the size of $\frac{\gamma}{R}$. If the cost of delay is small, relative to the cost of borrowing, delaying those payments will not be very costly, compared to the benefits from reduced borrowing.

4.1 Comparison with equilibria

Equilibria for this model are characterized in Martin and McAndrews (2008).

Proposition 2 *Four equilibria can exist. For all equilibria, non–time-critical payments are delayed. In addition,*

1. *If $\gamma \geq [\mu - \theta(2\mu - 1)]\, R$, it is an equilibrium for all time-critical payments to be sent early.*

2. *If* $\{\mu - \theta(1-\sigma)(2\mu - 1)\} R > \gamma \geq [1 - \theta(1-\sigma)] \mu R$, *then it is an equilibrium for banks of type* $s-$ *to delay time-critical payments while other banks pay time-critical payments early.*

3. *If* $(1-\sigma\theta)\mu R > \gamma \geq (1-\sigma\theta)(2\mu - 1)R$, *it is an equilibrium for only banks of type* $s+$ *to send time-critical payments early.*

4. *If* $(2\mu - 1)R > \gamma$, *then it is an equilibrium for all banks to delay.*

These equilibria can coexist, as shown in Martin and McAndrews (2007).

The next two propositions show that the planner's allocation typically cannot be achieved as an equilibrium and that one reason is excessive delay.

Proposition 3 *The planner's allocation cannot be attained with an RTGS settlement system as long as some payments can be delayed without cost.*

To better understand this result, note that, according to proposition 2, all banks with a non–time-critical payment will delay their payments under the equilibrium solution. Indeed, the expected cost of sending early is positive while the cost of delay is 0 for such banks. However, the planner would like to use the liquidity of banks with a positive or zero liquidity shock to ensure that banks with a negative liquidity shock will not have to borrow too much, as is shown in proposition 1.

Proposition 4 *With RTGS, there are at least as many payments settled early under the planner's allocation as in equilibrium.*

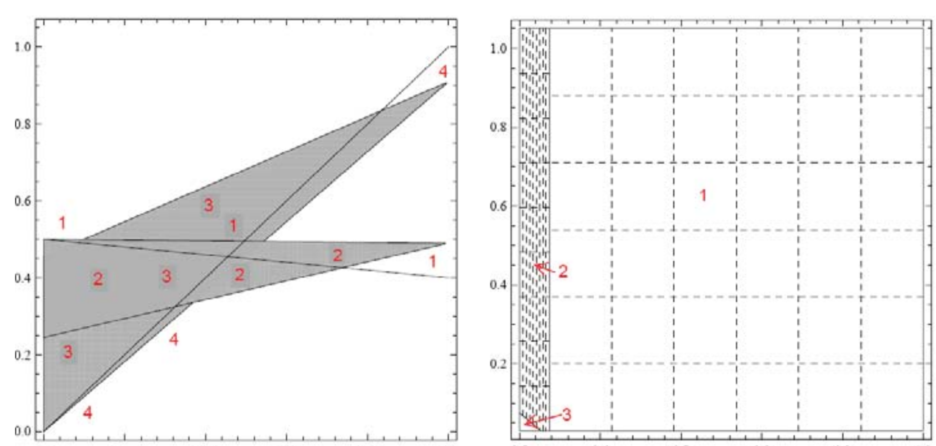

Figure 4: Comparing the equilibrium and planner's allocations, when an LSM is available, under the long-cycle assumption. μ is given on the X-axis and γ / R is given on the Y-axis. For these graphs we used $\sigma = 0.15$ and $\theta = 0.6$. Left panel: equilibrium allocations. Right panel: planner's allocations. The numbers correspond to those in propositions 1 and 2

The result that too few payments are settled early in equilibrium is important because it shows that our model captures the main policy concern associated with RTGS systems.[8] Several central banks have adopted measures aimed at providing incentives for early submission of payments. For example, the Bank of England imposes "throughput requirements" on the Clearing House Automated Payment System (CHAPS), its RTGS payment system. Under such requirements, banks must send a certain fraction of their daily payments before a given time, which limits the banks' ability to delay. The Swiss National Bank provides incentives by charging a higher price for payments sent late than payments sent early. Policies of this type could improve welfare in our model.

5 Welfare in the case of an LSM

In this section, we characterize the solution to the planner's problem when an LSM is available. The planner may find it beneficial to use the LSM because it allows the settlement of a payment to be made conditional on the receipt of another payment. This conditionality reduces the amount, and thus the total cost, of borrowing.

As in the RTGS case, the planner always chooses the same action for all banks of the same type. However, since each of the six types of banks can, in principle, be assigned any of three actions, this leaves $6^3 = 216$ cases to check. The next four lemmas allow us to eliminate some cases.

Lemma 3 *The planner never chooses to make banks with a positive or zero liquidity shock delay their payments.*

Since the planner aims to redistribute liquidity to banks that have negative liquidity shocks, it is important that other banks do not delay. In particular, the planner can exploit

[8] Several authors have studied RTGS environment with excessive delays, including Angelini (1998, 2000), Bech and Garratt (2003), and Martin (2004).

the conditionality of the LSM and will prefer to have banks with positive or zero liquidity shock queue rather than delay.

Lemma 4 *The planner never chooses to make banks of type $r-$ queue or send early, unless all other payments are being sent early.*

Banks of type $r-$ are the banks that benefit the most from delay and for which delay is least costly. Hence, if any type of bank delays its payment, this type should.

Lemma 5 *The cost of sending a payment early, relative to queuing or delaying, is smaller for banks with a positive liquidity shock than for banks with no liquidity shock. It is smaller for banks with no liquidity shock than for banks with a negative liquidity shock.*

An interpretation of this result is that there are decreasing returns to holding reserves. Banks that have a negative liquidity shock need reserves the most and find it particularly costly to send a payment. Banks that have a positive liquidity shock need reserves the least and suffer the smallest cost from paying early.

Lemma 6 *The planner will never choose to have banks of type $s+$ delay or queue payments while other banks send payments early.*

Banks of type $s+$ have the smallest expected borrowing cost of any type of banks sending a payment early. In addition, they suffer a cost of delay if the payment is not settled in the morning. Hence, if any type of bank pays early, this type should.

Lemma 6 shows that the planner always chooses for banks of type $s+$ to make their payment early.[9] Lemma 4 shows that unless all banks pay early, the planner makes banks of type $r-$ delay. Lemma 3 shows that banks of types $s+$, $s0$, $r+$, and $r0$ never delay. This brings the number of options the planner will consider down to 25 $(1 + 2 \cdot 2 \cdot 2 \cdot 3)$.

[9] All action profiles that have all banks queueing or sending their payments early are equivalent to the action profile in which all banks send early. For simplicity, we assume that the planner always chooses for all banks to send their payment early instead of an alternative action profile with the same outcome. In addition, the planner would never choose for all banks to delay, since this allocation would imply the same borrowing cost as the allocation in which all banks pay early, but a higher cost of delay.

We can use lemma 5 to eliminate some more action profiles, as shown in the appendix. For example, we know that the planner would not choose for banks of type $s0$ to queue, while banks of type $s-$ pay early. After such profiles are discarded, we are left with 14 possible profiles, which are listed in table 7 in the appendix. The next proposition shows which strategy profiles can be chosen by the planner.

Proposition 5 *Let E denote that a payment is sent early, Q that a payment is queued, and D that a payment is delayed. Both for the long-cycle and for the short-cycles assumption, depending on parameter values, the planner will choose one of four profiles:*

Type	*s+*	*s0*	*s-*	*r+*	*r0*	*r-*
1	*E*	*E*	*E*	*E*	*E*	*E*
2	*E*	*E*	*E*	*E*	*Q*	*D*
3	*E*	*Q*	*Q*	*E*	*Q*	*D*
4	*E*	*Q*	*D*	*E*	*Q*	*D*

Table 2: Equilibria for proposition 5

The proof characterizes the boundaries between profiles 1, 2, 3, and 4 in the parameter space. The planner's allocation for $\theta = 0.6$, $\sigma = 0.15$, and different values of $\frac{\gamma}{R}$ and μ is given in the right panel of figure 5. As in the RTGS case, the planner chooses for all banks to send early if the size of the liquidity shock is not too large. The benefits from redistributing liquidity between banks are large enough only if liquidity shocks are large. In that case, and if γ, cost of delaying time-critical payments, is also large, then the planner may choose profile 2. This profile allows some redistribution of liquidity but limits the cost of delay since all time-critical payments are sent early. Profile 2 is chosen when there are few time-critical payments (θ is small) and/or many banks receive shocks (σ is large). When γ is small, and for moderate values of θ, profiles 3 or 4 may be chosen. In these cases, some time-critical payments are queued or delayed. Profile 4 will be chosen for smaller values of the cost of delay, γ, since it implies delaying some time-critical payments.

Proposition 6 gives a bound for how small shocks need to be for the planner to make all banks send payments early, regardless of the value of other parameters.

Proposition 6 *In both the RTGS and LSM case, for $\mu > \frac{2}{3}$, the planner will choose to have all payments sent early. This implies that, for $\mu > \frac{2}{3}$, the planner will do no better with an LSM, relative to RTGS.*

Proposition 7 shows that the planner is more likely to delay payments under the long-cycle rather than under the short-cycles assumption.

Proposition 7 *For a given set of parameters, if the planner chooses to delay some payments in the long-cycle case, then the planner will also choose to delay some payments in the short-cycles case*

With a long cycle, no strict subset of payments in the queue is offsetting. Hence, the benefit of queuing under the long-cycle case is smaller than under the short-cycles case, where many payments in the queue can offset bilaterally. For that reason, the planner is more likely to make banks queue rather than delay under the short-cycles case, compared to the long-cycle case.

5.1 Comparison with equilibria

As in the RTGS case, the equilibria with an LSM are characterized in Martin and McAndrews (2008).

Proposition 8 *Under the long-cycle assumption, the following equilibria exist:*

1. *If $\gamma < (2\mu - 1)R$, then all banks queue their payment.*
2. *If $\gamma \geq (2\mu - 1)R$ and $\mu \geq 2/3$, then*
   (a) *If $\gamma \geq \mu R$, then all time-critical payments are sent early. Banks with a negative liquidity shock delay non–time-critical payments and non–time-critical payments from other types of banks are queued.*
   (b) *If $\mu R > \gamma \geq (2\mu - 1)R$, then only banks with a positive liquidity shock send time-critical payment early. Banks with a negative liquidity shock delay non–time-critical payments, and all others queue their payment.*

3. If $\gamma \geq (2\mu - 1)R$ and $\mu < 2/3$, then

   (a) If $\gamma \geq \mu R$, the equilibrium is the same as under 2a.

   (b) If $\mu R > \gamma \geq (1 - \mu)R$, the equilibrium is the same as under 2b.

   (c) If $(1-\mu)R > \gamma \geq (2\mu-1)R$, then banks that receive a negative liquidity shock delay their payment. Banks that receive a positive liquidity shock send their time-critical payment early. All other payments are queued.

The same types of equilibria arise in the short-cycles case but for different parameter values. Also, while it is always an equilibrium for all banks to queue their payments, this equilibrium is not robust when other equilibria exist, as shown in Martin and McAndrews (2008).

To facilitate the comparison with proposition 5, we display these equilibria in table 3.

Type	s+	s0	s-	r+	r0	r-
1	E	E	E	E	E	E
2	E	E	E	Q	Q	D
3	E	Q	Q	Q	Q	D
4	E	Q	D	Q	Q	D

Table 3: Equilibria for proposition 8

Contrary to the RTGS case where the planner's allocation cannot be achieved in equilibrium, the planner's allocation can be achieved in the LSM case for some parameter values, as can be seen by looking at the first row of tables 2 and 3. If $\frac{\gamma}{R}$ is not too large, all banks queue in equilibrium, leading to all payments being settled early. This is the allocation chosen by the planner when the liquidity shock is not too large.

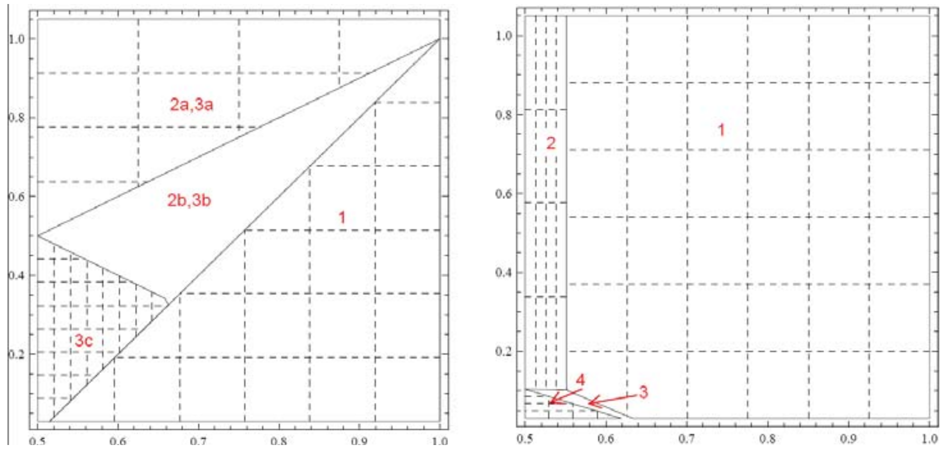

Figure 5: Comparing the equilibrium and planner's allocations, when an LSM is available, under the long-cycle assumption. μ is given on the X-axis and γ / R is given on the Y-axis. For these graphs we used $\sigma = 0.15$ and $\theta = 0.6$. Left panel: equilibrium allocations. Right panel: planner's allocations. The numbers correspond to those in propositions 5 and 8

The other rows of tables 2 and 3 show that no other equilibria correspond to an allocation chosen by the planner. One problem is that the planner would like banks with a positive liquidity shock to send their non–time-critical payment early. In equilibrium these banks always prefer to queue, unless all other banks send their payment early, because they do not want to take the risk of having to borrow.

Another difference with the case of RTGS, where there can never be too much early settlement in equilibrium, is that with an LSM there can be too little or too much early settlement in equilibrium. That situation arises because the planner cares about the size of the liquidity shocks and the opportunity to redistribute reserves between banks. In contrast, banks care only about the relative costs of borrowing and delaying when deciding whether or not to send a payment early. Hence, we find that there is too little early settlement if the liquidity shock is small and the cost of delay, relative to the cost of borrowing, is large. In that case, the planner would like all core payments to settle in the morning, but this will not occur in equilibrium. Banks with a time-critical payment will want to send payments early to avoid the cost of delay, and this action will induce banks with non–time-critical payments to delay their payments. In contrast, if the liquidity shock is large but the relative cost of delay is small, then there can be too much early settlement with an LSM. Because $\frac{\gamma}{R}$ is small, all banks queue in equilibrium and all core payments are settled early. However, if the liquidity shock is large, the planner would like to redistribute some liquidity between banks with positive and negative liquidity shock. Thus, the planner would prefer if not all payments were settled in the morning. For instance, when $\mu = 0.55, \frac{\gamma}{R} = 0.1, \theta = 0.4$ and $0.25 > \sigma > 0.1$, all payments are settled in the morning period, in equilibrium. Under the planner's solution, $r-$ banks delay their payment, $s-$ banks either queue or delay, and $s0$ and $r0$ banks queue their payment.

6 Conclusion

In this paper, we compare the planner's allocation and the equilibrium allocation of a model of a large-value payment system, with and without an LSM. The planner's allocation is interesting because it provides us with a benchmark and because it corresponds to the allocation that could be obtained in equilibrium with commitment.

Our analysis yields a number of interesting results. First, we show that in an RTGS equilibrium, there is always too much delay. This finding confirms that our model captures the key concern of policy makers with respect to RTGS systems. It suggests that policies designed to provide incentives for banks to settle their payments early, such as throughput requirements in CHAPS, or fees for late payments in SIC, could improve welfare.

We also show that an LSM can improve upon an RTGS system and can, in some cases, implement the planner's allocation. In other cases, there can be either too much or too little delay in equilibrium. Banks focus on the cost of delay relative to the cost of borrowing in deciding when to send a payment. In contrast, the planner takes into account the fact that when liquidity shocks are large, the benefits from redistributing liquidity are also large. Hence, there can be too much delay if the liquidity shocks are small and too little delay when they are large.

For Fedwire–the payment system operated by the Federal Reserve–do we observe too much or too little delay? Would the welfare effects of including an LSM in Fedwire be economically meaningful? In a companion paper, we calibrate the model of Martin and McAndews (2008) to answer these questions; see Atalay et al. (2010).

7 Appendix

Derivation of π^o and π^q

Recall, π^o is the probability of receiving a payment conditionally on not putting the payment in the queue, and π^q the probability of receiving a payment conditionally on putting a payment in the queue. The latter probability is equivalent to the probability that a payment in the queue is released.

Suppose that there are no payments in the queue. Then, the probability of receiving a payment is given by the mass of banks who send a payment outright divided by the total mass of banks. Formally, $\pi^o = \lambda_e/(\lambda_e + \lambda_d)$. It turns out that the expression for π^o does not change when there are payments in the queue. Indeed, note that every payment made early by some bank outside the queue to a bank inside the queue must ultimately trigger a payment from a bank inside the queue to a bank whose payment is outside the queue. From the perspective of banks outside the queue, this is the same as if the payment had been made directly from a bank outside the queue. For this reason, we can ignore the queue.

If a bank puts a payment in the queue, the payment will be in a cycle with probability χ, in which case it is released for sure. With probability $1 - \chi$, the payment is in a path. The probability that a payment in a path is released is equal to the probability of receiving a payment from outside the queue. This probability is equal to π^o. So the expression for π^q is given by

$$\pi^q \equiv \chi + (1-\chi)\frac{\lambda_e}{\lambda_e + \lambda_d} = \chi + (1-\chi)\pi^o. \tag{8}$$

Proof of lemma 1

The proof is an application of the fact that the product of two strictly increasing and weakly convex functions is convex. First, we can use equation (3) to eliminate λ_i^d, for all i. Notice that the denominator of π^o becomes

$$\Omega = 1 - \sigma\left[\theta(\lambda_{s+}^q + \lambda_{s-}^q) + (1-\theta)(\lambda_{r+}^q + \lambda_{r-}^q)\right] + (1-2\sigma)(\theta\lambda_{s0}^q + (1-\theta)\lambda_{s0}^q),$$

so that π^o is linear in the λ_i^es. Inspection of W reveals that the only terms that are not linear in λ_i^e are of the form $\lambda_i^e\pi^o$. Since this is the product of two increasing and weakly convex functions of λ_i^e, it must be a convex function of λ_i^e. A similar argument can be used to show that W is convex in λ_i^d, for all i.

To show that W is convex in λ_i^q, for all i, we need to show that π^q is convex in λ_i^q. First note that the derivative of Ω with respect to λ_i^q is a constant, for all i. The first and second derivatives of π^o, with respect to the λ_i^qs are given by

$$\frac{\partial \pi^o}{\partial \lambda_i^q} = -\pi^o \frac{\partial \Omega}{\partial \lambda_i^q}\frac{1}{\Omega} \text{and} \frac{\partial^2 \pi^o}{\partial \lambda_i^{q2}} = 2\pi^o(\frac{\partial \Omega}{\partial \lambda_i^q})^2(\frac{1}{\Omega})^2.$$

Now we can take the partial derivative of π^q with respect to λ_i^q. Note that the partial derivative of χ with respect to λ_i^q is a constant. We get

$$\frac{\partial \pi^q}{\partial \lambda_i^q} = \frac{\partial \chi}{\partial \lambda_i^q} + (1-\chi)\frac{\partial \pi^o}{\partial \lambda_i^q} - \frac{\partial \chi}{\partial \lambda_i^q}\pi^o,$$
$$\frac{\partial^2 \pi^q}{\partial \lambda_i^{q2}} = (1-\chi)\frac{\partial^2 \pi^o}{\partial (\lambda_i^q)^2} - 2\frac{\partial \chi}{\partial \lambda_i^q}\frac{\partial \pi^o}{\partial \lambda_i^q}.$$

Since $\partial \pi^o/\partial \lambda_i^q < 0$, $\partial^2 \pi^q/\partial \lambda_i^{q2} > 0$, which completes the proof.

Proof of lemma 2

A bank with a positive liquidity shock has an expected borrowing cost of $(1-\pi^o)(2\mu-1)R$ if it does not delay, and does not need to borrow if it delays. So the expected benefit from delaying is $(1-\pi^o)(2\mu-1)R$. A bank with no liquidity shock has an expected borrowing cost of $(1-\pi^o)\mu R$ if it does not delay, and does not need to borrow if it delays. So the expected benefit from delaying is $(1-\pi^o)\mu R$. A bank with a negative liquidity shock has an expected borrowing cost of $(1-\pi^o)R+\pi^o(1-\mu)R$ if it does not delay, and an expected cost of $(1-\pi^o)(1-\mu)R$ if it delays. So the expected benefit from delaying is $(1-\pi^o)\mu R+\pi^o(1-\mu)R$. It can easily be verified that

$$(1-\pi^o)(2\mu-1)R \leq (1-\pi^o)\mu R \leq (1-\pi^o)\mu R + \pi^o(1-\mu)R.$$

Proof of lemma 1

Let W_0 denote welfare when all banks make their core payment early, W_{r-} denote welfare when only banks of type $r-$ delay their core payment, and W_- denote welfare when banks with a negative liquidity shock delay their core payments.

$$W_0 = -\sigma(1-\mu)R, \tag{9}$$
$$W_{r-} = -(1-\theta)\sigma\left[\sigma(2\mu-1)+(1-2\sigma)\mu+(1-\theta)\sigma(1-\mu)\right]R \tag{10}$$
$$-\sigma\theta\left[(1-\theta)\sigma+(1-(1-\theta)\sigma)(1-\mu)\right]R, \tag{11}$$
$$W_- = -\sigma\left[\sigma(2\mu-1)+(1-2\sigma)\mu+\sigma(1-\mu)\right]R - \theta\sigma\gamma. \tag{12}$$

Depending on the parameter values, either W_0, or W_{r-}, or W_- can be largest. However, the planner never chooses to make a bank with no liquidity shock delay. Since the benefit of delay is even lower for banks with a positive liquidity shock, the planner will not make such banks delay.

First, we compare W_0 and W_{r-}:

$$W_0 - W_{r-} = \left[\mu(2+2\theta\sigma-\sigma)-1-\sigma\theta\right](1-\theta)\sigma R,$$

so $W_0 < W_{r-}$ if and only if

$$\mu(2+2\theta\sigma-\sigma) < \sigma\theta+1.$$

Since $\theta \geq 0$ and $\sigma \leq \frac{1}{4}$, this inequality can hold only if $\mu < \frac{4}{7}$.

Next, we compare W_{r-} and W_-:

$$W_{r-} - W_- = \theta\sigma\gamma + \theta\sigma R\left[(2\mu-1)(1-\sigma)-\mu\sigma(1-\theta)-\theta\sigma(1-\mu)\right].$$

This expression is positive if $\frac{\gamma}{R}$ is large enough. Indeed, if the cost of delay is large, the planner will never choose to make banks with time-critical payment delay. However, if μ is close enough to $\frac{1}{2}$ and $\frac{\gamma}{R}$ is small, this expression is negative. We already know that if μ is small then W_{r-} can be larger than W_0.

We check that the planner would never choose to delay banks of type $r0$. The welfare when all banks of type $r-$ and $r0$ delay is given by

$$\begin{aligned} W_{r0} &= -(1-\theta)(1-\sigma)\left[\sigma(2\mu-1)+\theta(1-2\sigma)\mu\right]R \\ &\quad -\theta\sigma\left[1-\mu(\theta+\sigma-\theta\sigma)\right]R \\ &\quad -(1-\theta)\sigma\left[(1-\theta)(1-\sigma)\right](1-\mu)R. \end{aligned}$$

Since

$$W_{r-} - W_{r0} = (1-2\sigma)(\mu-\sigma+\mu\sigma)(1-\theta)\theta R \geq 0,$$

the planner never chooses to delay banks of type $r0$.

Finally, we check that the planner would never choose to delay banks of type $s0$ and $r0$. The welfare when all banks of type $s-$, $s0$, and $r-$ and $r0$ delay is given by:

$$W_{s0,r0} = -\sigma(1-\sigma)\mu R - (1-\sigma)\theta\gamma$$

Since

$$W_{-} - W_{s0,r0} = \theta\gamma(1-2\sigma) \geq 0,$$

the planner will never choose to delay banks of type $r0$ and $s0$.

Proof of proposition 3

If some payments are non–time-critical, then the planner never chooses to make banks of type $r+$ or $r0$ delay. If all payments are time-critical, then the planner's allocation can be achieved if the cost of delay is high enough. For a high cost, neither the planner, nor the banks in equilibrium, wish to delay.

Proof of proposition 4

The planner will always have $s+$ and $s0$ banks send their payments early. If, in equilibrium, $s-$ banks delay their payments then there will be no more payments sent early than in the planner's allocation.

If $s-$ banks send their payments early in the equilibrium allocation, then θ payments will be sent early. This is the largest fraction of payments that can possibly be sent early in the equilibrium allocation. The condition for $s-$ banks to send their payments early, in equilibrium, is

$$\frac{\gamma}{R} \geq \mu - \theta(2\mu-1). \tag{13}$$

Under the planner's allocation, at least $1-\sigma$ payments are sent early. These represent the payments of all banks of types $s+$, $s0$, $r+$, and $r0$.

So we need to show that if $\theta \geq 1-\sigma$, then the planner will prefer to make all banks pay early rather than only banks of types $s+$, $s0$, $r+$, and $r0$.

The condition for $W_0 \geq W_-$ is

$$\frac{\gamma}{R} \geq \frac{\sigma\mu - (2\mu - 1}{\theta}. \tag{14}$$

We need to show that the condition given by equation (13) implies the condition given by equation (14) when $\theta \geq 1 - \sigma$. This amounts to showing that

$$\theta\mu + (1 - \theta^2)(2\mu - 1) \geq \sigma\mu,$$

whenever $\theta \geq 1 - \sigma$. This must be true since $1 - \sigma > \sigma$.

Proof of lemma 3

Queuing is always preferable to delay for such banks. Indeed, if a bank with positive or zero liquidity shock does not receive a payment in the morning, queueing a payment will be equivalent to delaying. If a bank with positive or zero liquidity shock does receive a payment, then the queue will release the outgoing payment, but the bank's balance will remain nonnegative.

Proof of lemma 4

Suppose that banks of type $r-$ are queueing or sending their payments early while some other banks are delaying their payments. Let Δ denote the set of these other banks. Recall that the benefit of receiving a payment is independent of the type of the sending bank. If the planner switches the actions of some of the banks of type $r-$ that were not delaying and an equal mass of the banks in Δ, this leaves the welfare of all other banks unchanged, since the mass of banks that delay, queue, or send early has not changed. However, because the benefit of delay is greater for banks of type $r-$ than for any other banks, from lemma 2, welfare must have increased.

Proof of lemma 5

Banks that have a positive or no liquidity shock never need to borrow if they queue or delay their payment. If they pay early, banks with a positive liquidity shock face an expected cost of $(1 - \pi^o)(2\mu - 1)$, while banks with no liquidity shock face an expected cost of $(1 - \pi^o)\mu \geq (1 - \pi^o)(2\mu - 1)$. Banks with a negative liquidity shock face an expected cost of $1 - \pi^o\mu$ if they pay early, $1 - \mu$ if they queue, and $(1 - \pi^o)(1 - \mu)$ if they delay. Hence the cost of sending a payment early, relative to queuing of delaying, is at least as large or greater for banks with a negative liquidity shock than for banks with no liquidity shock.

Proof of lemma 6

Let us suppose banks of type $s+$ are queueing or delaying payments while some other banks are sending payments early. Let Δ denote the set of these other banks. As in the proof of the previous lemma, the planner can switch the actions of some of the banks of type $s+$ and some of the banks in Δ, leaving all other banks unaffected. However, because the cost of sending a payment early is smaller for banks of type $s+$ than for any other banks, from lemma 5, welfare must have increased.

Ruling out more action profiles

Of the 25 action profiles that cannot be eliminated by lemmas 3, 4, and 6, 14 are in table 2. Here we show how the remaining 11 profiles can be ruled out. We use the notation of table 2 to describe a profile. Some profiles can be ruled out by the observation that, everything else being equal, it is more costly to queue or delay a time-critical payment than a non–time-critical payment. These profiles are:

Type	s+	s0	s-	r+	r0	r-
	E	Q	E	E	E	D
	E	Q	E	Q	E	D
	E	Q	Q	E	E	D
	E	Q	Q	Q	E	D
	E	Q	D	E	E	D

Other profiles can be eliminated using lemma 5:

Type	s+	s0	s-	r+	r0	r-
	E	E	E	Q	E	D
	E	E	Q	Q	E	D
	E	Q	E	E	Q	D
	E	Q	E	Q	Q	D
	E	E	D	Q	E	D
	E	Q	D	Q	E	D

In each of these profiles, a type of bank for which it is relatively costly to send payments early sends early, while a type of bank for which it is relatively less costly to send early queues.

Proof of proposition 5

The following table shows the 14 strategy profiles left to consider. The remainder of the proof shows that only profiles 1 to 4 can be chosen by the planner. It also provides the parameter boundaries between different profiles.

Type	s+	s0	s-	r+	r0	r-
1	E	E	E	E	E	E
2	E	E	E	E	Q	D
3	E	Q	Q	E	Q	D
4	E	Q	D	E	Q	D
A	E	E	E	E	E	D
B	E	E	E	Q	Q	D
C	E	E	Q	E	E	D
D	E	E	Q	E	Q	D
E	E	E	Q	Q	Q	D
F	E	E	D	E	E	D
G	E	E	D	E	Q	D
H	E	E	D	Q	Q	D
I	E	Q	Q	Q	Q	D
J	E	Q	D	Q	Q	D

Table 4: Fourteen strategy profiles

Long Cycles Case:

Let W_i, $i \in \{1, ..., 4, A, ..., J\}$, denote welfare when banks adopt the action corresponding to their type in row i of table 2. Then we have

$$W_1 = -(1-\mu)R\sigma, \tag{15}$$

$$W_2 = \frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[-\theta+\mu(2\theta-1)\right]}{2\sigma(1-\theta)+\theta}, \tag{16}$$

$$W_3 = -\frac{\mu R\sigma(1-2\theta)+\theta\left[R\sigma+\gamma(1-\sigma)(1-\theta)\right]}{2-\theta}, \tag{17}$$

$$W_4 = -\frac{1}{2}(\mu R\sigma+\gamma\theta), \tag{18}$$

$$W_A = -R\sigma\left[1+\sigma(\theta-1)\right]\left[\theta+\mu(1-2\theta)\right], \tag{19}$$

$$W_B = -\frac{R\sigma\left[(1-\mu)\sigma(1-\theta)^2+\theta(\mu+\theta-2\mu\theta)\right]}{\sigma(1-\theta)+\theta}, \tag{20}$$

$$W_C = -\frac{\sigma\left\{\left[R(1-\sigma)+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta}, \tag{21}$$

$$W_D = -\frac{\sigma\left\{\mu R(2\theta-1)\left[\sigma(2\theta-1)-\theta\right]+\theta\left[R(\sigma+\theta(1-2\sigma)+\gamma\sigma(1-\theta)\right]\right\}}{\theta+\sigma(2-3\theta)}, \tag{22}$$

$$W_E = -\frac{\sigma\left\{\gamma\sigma(1-\theta)\theta+R\left[\theta(\mu+\theta-2\mu\theta)+\sigma(\mu(\theta+\theta^2-1)+1-2\theta)\right]\right\}}{\theta+\sigma(1-2\theta)}, \tag{23}$$

$$W_F = -\sigma(\mu R(1-\sigma)+\theta\gamma), \tag{24}$$

$$W_G = -\sigma\left[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})\right], \tag{25}$$

$$W_H = -\frac{\sigma\left\{\theta\gamma(\sigma+\theta-\sigma\theta)+R\left[\mu\theta-\sigma(-1+\mu+\theta)\right]\right\}}{\sigma(1-\theta)+\theta}, \tag{26}$$

$$W_I = -R\sigma\left[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)\right]-\gamma\theta(1-\sigma)(1-\theta), \tag{27}$$

$$W_J = -\frac{R\sigma(1-\mu-\theta+2\theta\mu)+\gamma\theta(1-\sigma+\theta\sigma)}{1+\theta}. \tag{28}$$

Now we show that the planner will never choose action profiles A to J in table 2.

A) The planner will never choose profile A, because aggregate welfare under profile 2 is always higher than profile A. $W_2 \geq W_A$ holds if

$$-\frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[\theta+\mu(1-2\theta)\right]}{2\sigma(1-\theta)+\theta}+R\sigma\left[1+\sigma(\theta-1)\right]\left[\theta+\mu(1-2\theta)\right] \geq 0.$$

This expression can be simplified to yield

$$\frac{(\theta+\mu-2\mu\theta)(1-\theta)^2(1-2\sigma)R\sigma^2}{2\sigma(1-\theta)+\theta} \geq 0.$$

This inequality always holds since $\theta+\mu-2\mu\theta \geq 0$ is equivalent to $1 \geq \mu$.

B) The planner will never choose profile B, because either the welfare under profile 1 is higher than the welfare under profile B or the welfare under profile 2 is higher. If the shock is small, the planner prefers to send all core payments early. First we show that $W_1 \geq W_B$

if $\mu \geq \frac{1+\sigma}{2+\sigma}$. $W_1 \geq W_B$ holds if

$$-(1-\mu)R\sigma \geq -\frac{R\sigma\left[(1-\mu)\sigma(1-\theta)^2+\theta(\mu+\theta-2\mu\theta)\right]}{\sigma(1-\theta)+\theta},$$

which can be simplified to $\frac{R\sigma[-1-\sigma+\mu(2+\sigma)](1-\theta)\theta}{\sigma(1-\theta)+\theta} \geq 0$. The numerator is positive whenever $\mu \geq \frac{1+\sigma}{2+\sigma}$.

Next we show that $W_2 \geq W_B$ if $\mu \leq \frac{1+\sigma}{2+\sigma}$. $W_2 \geq W_B$ holds if

$$\frac{R\sigma^2(1-\theta)^2\left\{\theta-\mu\theta+\sigma\left[2-3\theta+\mu(4\theta-3)\right]\right\}}{2\sigma^2(1-\theta)^2+3\theta\sigma(1-\theta)+\theta^2} \geq 0.$$

The numerator is positive if $\mu \leq \frac{\theta+2\sigma-3\sigma\theta}{\theta+3\sigma-4\sigma\theta}$. It can be checked that $\mu \leq \frac{1+\sigma}{2+\sigma}$ implies $\mu \leq \frac{\theta+2\sigma-3\sigma\theta}{\theta+3\sigma-4\sigma\theta}$.

C) The planner will never choose profile C. If $\gamma\theta \leq R\theta(1-2\mu)+\mu R$ then the planner will choose profile D over profile C. Otherwise, the planner will choose profile A over profile C. $W_D \geq W_C$ holds if

$$\begin{aligned}&-\frac{\sigma\left\{\mu R(2\theta-1)\left[-\theta+\sigma(-1+2\theta)\right]+\theta\left[R(\sigma+\theta-2\theta\sigma)+\gamma(\sigma-\sigma\theta)\right]\right\}}{\theta+\sigma(2-3\theta)}\\&+\frac{\sigma\left\{\left[R-R\sigma+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta} \geq 0.\end{aligned}$$

This can be simplified to yield

$$\frac{\sigma^2(1-\theta)^2(1-2\sigma)[R\theta-\gamma\theta+\mu R(1-2\theta)]}{[\theta+\sigma(2-3\theta)](1-\sigma\theta)} \geq 0.$$

The numerator will be positive if $R\theta-\gamma\theta+\mu R(1-2\theta) \geq 0$, which is equivalent to $\gamma\theta \leq R\theta(1-2\mu)+\mu R$.

$W_A \geq W_C$ is equivalent to

$$\begin{aligned}&-R\sigma\left[1+\sigma(\theta-1)\right]\left[\theta+\mu(1-2\theta)\right]\\&+\frac{\sigma\left\{\left[R-R\sigma+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta} \geq 0.\end{aligned}$$

Simplifying gives us

$$\frac{\sigma^2(1-\theta)\theta[\gamma-R\sigma(\theta+\mu-2\mu\theta)]}{1-\sigma\theta} \geq 0.$$

The numerator is positive if $\gamma\theta \geq R\sigma\theta(\theta+\mu-2\mu\theta)$, which occurs provided $\gamma\theta \geq R\theta(1-2\mu)+\mu R$.

D) The planner never chooses action profile D as it provides lower welfare than either 2 or 3, depending on parameter values. $W_3 \geq W_D$ if $\gamma[2\sigma(-1+\theta)-\theta]+R\sigma(\mu+\theta-2\mu\theta) \geq 0$.

$W_3 \geq W_D$ if

$$-\frac{\mu R\sigma(1-2\theta)+\theta\left[R\sigma+\gamma(1-\sigma)(1-\theta)\right]}{2-\theta} + \frac{\sigma\left\{\mu R(2\theta-1)\left[-\theta+\sigma(-1+2\theta)\right]+\theta\left[R(\sigma+\theta-2\theta\sigma)+\gamma(\sigma-\sigma\theta)\right]\right\}}{\theta+\sigma(2-3\theta)} \geq 0.$$

This can be simplified to

$$\frac{(1-2\sigma)(1-\theta)\theta\left[\gamma(2\sigma(-1+\theta)-\theta)+R\sigma(\mu+\theta-2\mu\theta)\right]}{(2-\theta)\left[\theta+\sigma(2-3\theta)\right]} \geq 0,$$

which must hold if $\gamma\left[2\sigma(-1+\theta)-\theta\right]+R\sigma(\mu+\theta-2\mu\theta) \geq 0$.

Next we show that $W_2 \geq W_D$ if $\gamma(2\sigma(-1+\theta)-\theta)+R\sigma(\mu+\theta-2\mu\theta) \leq 0$. $W_2 \geq W_D$ if

$$-\frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[\theta+\mu(1-2\theta)\right]}{2\sigma(1-\theta)+\theta} + \frac{\sigma\left\{\mu R(2\theta-1)(-\theta+\sigma(-1+2\theta))+\theta\left[R(\sigma+\theta-2\theta\sigma)+\gamma(\sigma-\sigma\theta)\right]\right\}}{\theta+\sigma(2-3\theta)} \geq 0.$$

This can be simplified to

$$-\frac{(1-\theta)\sigma^2\theta\left[\gamma(2\sigma(-1+\theta)-\theta)+R\sigma(\mu+\theta-2\mu\theta)\right]}{\left[2\sigma(1-\theta)+\theta\right]\left[\theta+\sigma(2-3\theta)\right]} \geq 0,$$

which must hold if $\gamma\left[2\sigma(-1+\theta)-\theta\right]+R\sigma(\mu+\theta-2\mu\theta) \leq 0$.

E) The planner will never choose action profile E because if E provides higher welfare than action profile 1, then welfare under action profile C is higher than under E. $W_1 \geq W_E$ if $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \leq 0$. $W_1 \geq W_E$ holds if

$$-(1-\mu)R\sigma + \frac{\sigma\left(\gamma\sigma(1-\theta)\theta+R\left\{\theta(\mu+\theta-2\mu\theta)+\sigma\left[1-2\theta+\mu(-1+\theta+\theta^2)\right]\right\}\right)}{\theta+\sigma(1-2\theta)} \geq 0.$$

This simplifies to

$$-\frac{\sigma(1-\theta)\theta\left\{R\left[1-\mu(2-\sigma)\right]-\gamma\sigma\right\}}{\theta+\sigma(1-2\theta)} \geq 0,$$

which must hold if $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \leq 0$.

$W_C \geq W_E$ if $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \geq 0$. $W_C \geq W_E$ holds if

$$\frac{\sigma\left\{\left[R(1-\sigma)+\gamma\sigma(1-\theta)\right]\theta+\mu R(1-\sigma)(1-2\theta)\right\}}{1-\sigma\theta} + \frac{\sigma\left(\gamma\sigma(1-\theta)\theta+R\left\{\theta(\mu+\theta-2\mu\theta)+\sigma\left[1-2\theta+\mu(-1+\theta+\theta^2)\right]\right\}\right)}{\theta+\sigma(1-2\theta)} \geq 0.$$

This can be simplified to

$$\frac{\sigma^2(1-\theta)^2\left\{R\left[1-\mu(2-\sigma)\right](1-\theta)+\gamma(1-\sigma)\theta\right\}}{(1-\sigma\theta)\left[\theta+\sigma(1-2\theta)\right]} \geq 0,$$

which must hold since $R\left[1-\mu(2-\sigma)\right]-\gamma\sigma \geq 0$ implies $R\left[1-\mu(2-\sigma)\right](1-\theta)+\gamma(1-\sigma)\theta \geq$

0.

F) The planner will never choose action profile F because action profile G always provides higher welfare. $W_G \geq W_F$ holds if

$$-\sigma\left[\gamma\theta + \mu R(1 - \frac{\sigma}{2\sigma + \theta - 2\theta\sigma})\right] + \sigma\left[\mu R(1-\sigma) + \theta\gamma\right] \geq 0,$$

which must hold since

$$\frac{\mu R\sigma^2(1-2\sigma)(1-\theta)}{2\sigma(1-\theta)+\theta} \geq 0.$$

G) The planner will never choose action profile G. Action profile 4 provides higher welfare if $\mu R\sigma \geq \gamma(2\sigma(1-\theta)+\theta)$ and action profile 2 provides higher welfare otherwise. $W_4 \geq W_G$ holds if

$$-\frac{1}{2}(\mu R\sigma + \gamma\theta) + \sigma\left[\gamma\theta + \mu R(1 - \frac{\sigma}{2\sigma + \theta - 2\theta\sigma})\right] \geq 0.$$

This simplifies to

$$\frac{(1-2\sigma)\theta\{\mu R\sigma - \gamma[2\sigma(1-\theta)+\theta]\}}{4\sigma(1-\theta)+2\theta} \geq 0,$$

which must hold when $\mu R\sigma \geq \gamma(2\sigma(1-\theta)+\theta)$.

$W_2 \geq W_G$ holds if

$$-\frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[\theta+\mu(1-2\theta)\right]}{2\sigma(1-\theta)+\theta} + \sigma\left[\gamma\theta + \mu R(1 - \frac{\sigma}{2\sigma + \theta - 2\theta\sigma})\right] \geq 0.$$

This simplifies to

$$-\frac{\theta\sigma\left[(1-\mu)R\sigma + (1-2\mu)R(1-\sigma)\theta - \gamma(2\sigma(1-\theta)+\theta)\right]}{2\sigma(1-\theta)+\theta} \geq 0.$$

$(1-\mu)R\sigma + (1-2\mu)R(1-\sigma)\theta \leq \gamma(2\sigma(1-\theta)+\theta)$ will be true if $(1-\mu)R\sigma \leq \gamma(2\sigma(1-\theta)+\theta)$. This inequality must hold when $\mu R\sigma \leq \gamma(2\sigma(1-\theta)+\theta)$ since $\mu \geq 1-\mu$.

H) The planner will never choose profile H. Either profile 1 will produce higher welfare or profile G will. If the shock is small, the planner will always want all payments to be made early. $W_1 \geq W_H$ holds if

$$-(1-\mu)R\sigma + \frac{\sigma\{\theta\gamma(\sigma+\theta-\sigma\theta) + R[\mu\theta - \sigma(-1+\mu+\theta)]\}}{\sigma(1-\theta)+\theta} \geq 0.$$

This simplifies to

$$\frac{\sigma\theta[R(1-2\mu+\mu\sigma) - \gamma\sigma - \gamma\theta + \gamma\theta\sigma]}{\sigma(-1+\theta)-\theta} \geq 0$$

$R(1-2\mu+\mu\sigma) - \gamma\sigma - \gamma\theta + \gamma\theta\sigma \leq 0$ will be true if $(1-2\mu+\mu\sigma) \leq 0$ which will occur if $\mu \geq \frac{2}{3}$.

$W_G \geq W_H$ holds if

$$-\sigma[\gamma\theta + \mu R(1 - \frac{\sigma}{2\sigma+\theta-2\theta\sigma})] + \frac{\sigma\{\theta\gamma(\sigma+\theta-\sigma\theta) + R[\mu\theta - \sigma(-1+\mu+\theta)]\}}{\sigma(1-\theta)+\theta} \geq 0.$$

This simplifies to

$$\frac{R\sigma^2(1-\theta)[\theta(1-\mu)(1-2\sigma)+\sigma(2-3\mu)]}{2\sigma^2(1-\theta)^2+3\sigma(1-\theta)\theta+\theta^2} \geq 0.$$

This occurs if $\theta(1-\mu)(1-2\sigma)+\sigma(2-3\mu) \geq 0$, which holds if $\mu \leq \frac{2}{3}$.

I) The planner will never choose profile I. If $\mu \geq \frac{2}{3}$, profile 1 will produce higher welfare. Otherwise, profile 3 will. $W_1 \geq W_I$ holds if

$$\begin{aligned}&-(1-\mu)R\sigma+\gamma\theta(1-\sigma-\theta+\sigma\theta)+\\&+R\sigma[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]+\gamma\theta(1-\sigma-\theta+\sigma\theta)] \geq 0.\end{aligned}$$

This simplifies to $((3\mu-2)R\sigma+\gamma(1-\sigma))(1-\theta)\theta \geq 0$, which holds if $\mu \geq \frac{2}{3}$.

$W_3 \geq W_I$ is true if

$$\begin{aligned}&-\frac{\mu R\sigma(1-2\theta)+\theta[R\sigma+\gamma(1-\sigma)(1-\theta)]}{2-\theta}\\&+R\sigma[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]+\gamma\theta(1-\sigma-\theta+\sigma\theta)] \geq 0.\end{aligned}$$

This simplifies to

$$\frac{(1-\theta)^2[(2-3\mu)R\sigma(1-\theta)+\gamma(1-\sigma)\theta]}{2-\theta} \geq 0,$$

which occurs if $(2-3\mu)R\sigma(1-\theta)+\gamma(1-\sigma)\theta \geq 0$. This holds if $\mu \leq \frac{2}{3}$.

J) The planner will never choose profile J. Either profile 1 will produce higher welfare or profile 4 will. $W_1 \geq W_J$ if

$$-(1-\mu)R\sigma+\frac{R\sigma(1-\mu-\theta+2\theta\mu)+\gamma\theta(1-\sigma+\theta\sigma)}{1+\theta} \geq 0.$$

This inequality is true if

$$\frac{[(3\mu-2)R\sigma+\gamma(1-\sigma+\sigma\theta)]\theta}{1+\theta} \geq 0,$$

which is true for $\mu \geq \frac{2}{3}$.

$W_4 \geq W_J$ if

$$-\frac{1}{2}(\mu R\sigma+\gamma\theta)+\frac{R\sigma(1-\mu-\theta+2\theta\mu)+\gamma\theta(1-\sigma+\theta\sigma)}{1+\theta} \geq 0,$$

which simplifies to

$$\frac{(1-\theta)[(2-3\mu)R\sigma+\gamma(1-2\sigma)\theta]}{2(1+\theta)} \geq 0.$$

This occurs if $(2-3\mu)R\sigma+\gamma(1-2\sigma)\theta \geq 0$, which is true if $\mu \leq \frac{2}{3}$.

Short Cycles Case:

Let W_i, $i \in \{1,...,4,A,...,J\}$, denote welfare when banks adopt the action in row i of table 2, this time for the short cycles scenario. We have

$$W_1 = -(1-\mu)R\sigma, \tag{29}$$

$$W_2 = \frac{R\sigma\left[\sigma(1-\theta)+\theta\right]\left[-\theta+\mu(2\theta-1)\right]}{2\sigma(1-\theta)+\theta}, \tag{30}$$

$$W_3 = -\frac{\sigma\{[R+\gamma(1-\sigma)(2-\theta)(1-\theta)]\theta+\mu R(1-2\theta)\}}{2-\theta}, \tag{31}$$

$$W_4 = -\frac{1}{2}\mu R\sigma - 2\gamma(1-\sigma)\sigma\theta, \tag{32}$$

$$W_A = -R\sigma[1+\sigma(\theta-1)][\theta+\mu(1-2\theta)], \tag{33}$$

$$W_B = -\frac{R\sigma\left[(1-\mu)\sigma(1-\theta)^2+\theta(\mu+\theta-2\mu\theta)\right]}{\sigma(1-\theta)+\theta}, \tag{34}$$

$$W_C = -\frac{\sigma\{\mu R(1-\sigma)(1-2\theta)+\theta[R-R\sigma+\gamma\sigma(1-\theta)(1-\sigma\theta)]\}}{1-\sigma\theta}, \tag{35}$$

$$W_D = -\frac{\sigma\{\mu R(2\theta-1)[\sigma(2\theta-1)-\theta]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)}, \tag{36}$$

$$W_E = -\frac{\sigma\{\gamma\sigma(1-\theta)\theta[\theta+\sigma(1-2\theta)]+R[\theta(\mu+\theta-2\mu\theta)+\sigma(1-2\theta+\mu(-1+\theta+\theta^2))]\}}{\theta+\sigma(1-2\theta)}, \tag{37}$$

$$W_F = -\sigma[\mu R(1-\sigma)+\theta\gamma)], \tag{38}$$

$$W_G = -\sigma[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})], \tag{39}$$

$$W_H = -\frac{\sigma\{\theta\gamma(\sigma+\theta-\sigma\theta)+R[\mu\theta-\sigma(-1+\mu+\theta)]\}}{\sigma(1-\theta)+\theta}, \tag{40}$$

$$W_I = -\sigma\{\gamma\theta(1-\sigma-\theta+\sigma\theta)+R[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]\}, \tag{41}$$

$$W_J = -\frac{\sigma[2\gamma(1-\sigma)\theta(1+\theta)+R(1-\mu-\theta+2\mu\theta)]}{1+\theta}. \tag{42}$$

Notice that, for profiles such that no banks with time-critical payments are queueing their payments, the welfare under the long cycles scenario is the same as the welfare in the short cycles scenario. Thus W_1, W_2, W_A, W_B, W_F, W_G, and W_H, are the same for the long and short cycles scenarios. Hence action profiles A, B, F, G, and H will not be chosen by the planner in the short cycles case.

C) The planner will never choose profile C. The welfare under profile D is always higher than the welfare under profile C. $W_D \geq W_C$ is equivalent to

$$-\frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)]+\theta[R(\sigma+\theta-2\theta\sigma)+\gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)}$$
$$+\frac{\sigma\{\mu R(1-\sigma)(1-2\theta)+\theta[R-R\sigma+\gamma\sigma(1-\theta)(1-\sigma\theta)]\}}{1-\sigma\theta} \geq 0.$$

This simplifies to $\frac{R\sigma^2(1-2\sigma)(1-\theta)^2(\mu+\theta-2\mu\theta)}{(1-\theta\sigma)(\theta+2\sigma-3\theta\sigma)} \geq 0$, which is always true since $\mu+\theta-2\mu\theta \geq 0$.

D) Likewise, the planner will never choose profile D. Either profile 2 or profile 3 will

produce higher welfare than the welfare under profile D.
$W_3 \geq W_D$ holds if

$$
\begin{aligned}
&- \frac{\sigma\{[R+\gamma(1-\sigma)(2-\theta)(1-\theta)]\theta + \mu R(1-2\theta)\}}{2-\theta} \\
&+ \frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)] + \theta[R(\sigma+\theta-2\theta\sigma) + \gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)} \geq 0.
\end{aligned}
$$

This simplifies to

$$\frac{[-R\theta + \mu R(-1+2\theta) + \gamma(2-\theta)(2\sigma+\theta-3\sigma\theta)]\theta(1-\theta)(1-2\sigma)\sigma}{(2-\theta)(\theta+\sigma(2-3\theta))} \leq 0.$$

The numerator will be negative provided $-R\theta + \mu R(-1+2\theta) + \gamma(2-\theta)(2\sigma+\theta-3\sigma\theta) \leq 0$.

$W_2 \geq W_D$ is equivalent to

$$
\begin{aligned}
&- \frac{R\sigma[\sigma(-1+\theta)-\theta][-\theta+\mu(2\theta-1)]}{2\sigma(1-\theta)+\theta} \\
&+ \frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)] + \theta[R(\sigma+\theta-2\theta\sigma) + \gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)} \geq 0.
\end{aligned}
$$

This simplifies to

$$\frac{\sigma^2(1-\theta)\theta(-R\sigma\theta + \mu R\sigma(-1+2\theta) + \gamma(2\sigma(1-\theta)+\theta)(2\sigma+\theta-3\sigma\theta))}{(2\sigma(1-\theta)+\theta)(\theta+\sigma(2-3\theta))} \geq 0.$$

The numerator is positive provided $-R\theta\sigma + \mu R\sigma(-1+2\theta) + \gamma(2\sigma-2\theta\sigma+\theta)(2\sigma+\theta-3\sigma\theta) \geq 0$. Since $-2\theta\sigma+\theta \geq -\theta\sigma$, the inequality from the last sentence is true if $-R\theta\sigma + \mu R\sigma(-1+2\theta) + \gamma(2\sigma-\theta\sigma)(2\sigma+\theta-3\sigma\theta) \geq 0$. Dividing by σ, we see that the last inequality is equivalent to $-R\theta + \mu R(-1+2\theta) + \gamma(2-\theta)(2\sigma+\theta-3\sigma\theta) \geq 0$.

E) The planner will never choose profile E. If the size of the shock is small, the planner will choose to have all payments sent early. Otherwise, the planner would choose profile D over profile E. $W_D \geq W_E$ holds if

$$
\begin{aligned}
&- \frac{\sigma\{\mu R(2\theta-1)[-\theta+\sigma(-1+2\theta)] + \theta[R(\sigma+\theta-2\theta\sigma) + \gamma\sigma(1-\theta)(2\sigma+\theta-3\sigma\theta)]\}}{\theta+\sigma(2-3\theta)} \\
&+ \frac{\sigma\{\gamma\sigma(1-\theta)\theta[\theta+\sigma(1-2\theta)] + R[\theta(\mu+\theta-2\mu\theta) + \sigma(1-2\theta+\mu(-1+\theta+\theta^2))]\}}{\theta+\sigma(1-2\theta)} \geq 0.
\end{aligned}
$$

This simplifies to $\frac{R\sigma^2(1-\theta)^2(\theta-\mu\theta+\sigma(2-4\theta+\mu(5\theta-3)))}{\sigma(3-5\theta)\theta+\theta^2+\sigma^2(2-7\theta+6\theta^2)} \geq 0$. Now $\sigma(3-5\theta)\theta+\theta^2+\sigma^2(2-7\theta+6\theta^2)$ will be nonnegative for $\theta \in [0,1]$ and $\sigma \in [0, 0.25]$. The numerator is nonnegative provided $\theta - \mu\theta + \sigma(2-4\theta+\mu(5\theta-3)) \geq 0$, which is equivalent to $\theta + 2\sigma - 4\sigma\theta \geq \mu(\theta - 5\theta\sigma + 3\sigma)$. Since $\frac{2}{3} \leq \frac{.5}{.75-.25\theta} \leq \frac{\theta+2\sigma-4\theta\sigma}{\theta+3\sigma-5\theta\sigma}$, $W_D \geq W_E$ will hold for all $\mu \leq \frac{2}{3}$.

$W_1 \geq W_E$ occurs if

$$
\begin{aligned}
&-(1-\mu)R\sigma \\
&+\frac{\sigma\{\gamma\sigma(1-\theta)\theta[\theta+\sigma(1-2\theta)]+R[\theta(\mu+\theta-2\mu\theta)+\sigma(1-2\theta+\mu(-1+\theta+\theta^2))]\}}{\theta+\sigma(1-2\theta)} \geq 0.
\end{aligned}
$$

This inequality simplifies to

$$
\frac{\sigma\theta(-1+\theta)\{R[1+\mu(-2+\sigma)]+\gamma\sigma[-\theta+\sigma(-1+2\theta)]\}}{\theta+\sigma(1-2\theta)} \geq 0.
$$

The numerator will be positive provided $R[1+\mu(-2+\sigma)]+\gamma\sigma[-\theta+\sigma(-1+2\theta)] \leq 0$, which will occur for $\mu \geq \frac{1}{2-\sigma} - \frac{\gamma\sigma(\theta+\sigma(1-2\theta))}{2R-\sigma R}$. This inequality holds true for $\mu \geq \frac{2}{3}$.

G) The planner will never choose profile G. If the size of the shock is small and the penalty for delaying time-critical payments is large the planner will choose profile 2. Otherwise the planner will choose profile 4 over profile G. The only difference between these two profiles is that zero liquidity shock banks with time-critical payments are putting their payments in queue under profile 4, but sending them outright under profile G. $W_4 \geq W_G$ is equivalent to

$$
-\frac{1}{2}\mu R\sigma - 2\gamma\theta\sigma(1-\sigma) + \sigma[\gamma\theta + \mu R(1 - \frac{\sigma}{2\sigma+\theta-2\theta\sigma})]) \geq 0,
$$

which can be written as $\frac{\sigma(1-2\sigma)[\mu R - 2\gamma(2\sigma(1-\theta)+\theta)]\theta}{4\sigma(1-\theta)+2\theta} \geq 0$. The numerator is positive if $\frac{\mu R}{2} \geq \gamma(2\sigma(1-\theta)+\theta)$.

$W_2 \geq W_G$ is equivalent to

$$
-\frac{R\sigma[\sigma(-1+\theta)-\theta][-\theta+\mu(2\theta-1)]}{2\sigma(1-\theta)+\theta}\sigma[\gamma\theta+\mu R(1-\frac{\sigma}{2\sigma+\theta-2\theta\sigma})] \geq 0
$$

Combining terms we get

$$
\frac{\theta\sigma((1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta-\gamma(2\sigma(1-\theta)+\theta))}{2\sigma(-1+\theta)-\theta} \geq 0,
$$

which occurs if $(1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta-\gamma(2\sigma(1-\theta)+\theta) \leq 0$. This is equivalent to $(1-\mu)R\sigma+(1-2\mu)R(1-\sigma)\theta \leq \gamma(2\sigma(1-\theta)+\theta)$ which occurs if $(1-\mu)R\sigma \leq \gamma(2\sigma(1-\theta)+\theta)$. This is true if $\frac{\mu R}{2} \leq \gamma(2\sigma(1-\theta)+\theta)$ since $(1-\mu) \leq \mu$ and $\sigma \leq \frac{1}{4} < \frac{1}{2}$.

I) The planner will never choose profile I. If the size of the shock is small the planner would rather have everyone pay early rather than choose profile I. If the size of the shock is large profile 3 is preferable to profile I. $W_1 \geq W_I$ if

$$
\begin{aligned}
&-(1-\mu)R\sigma \\
&+\sigma\{\gamma\theta(1-\sigma-\theta+\sigma\theta)+R[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]\} \geq 0.
\end{aligned}
$$

This occurs provided $[(3\mu-2)R+\gamma(1-\sigma)]\sigma(1-\theta)\theta \geq 0$, which holds for $\mu \geq \frac{2}{3}$.

$W_3 \geq W_I$ is true when

$$-\frac{\sigma\{[R+\gamma(1-\sigma)(2-\theta)(1-\theta)]\theta+\mu R(1-2\theta)\}}{2-\theta} + \sigma\{\gamma\theta(1-\sigma-\theta+\sigma\theta)+R[1-2(1-\theta)\theta-\mu(1+3(-1+\theta)\theta)]\} \geq 0.$$

holds. This is true if $\frac{(1-\theta)^3 R\sigma(2-3\mu)}{2-\theta} \geq 0$, which is true for $\mu \leq \frac{2}{3}$.

J) As in the previous case, when the size of the shock is small profile 1 leads to higher welfare than profile J. Otherwise, profile 4 is preferable to profile J. $W_1 \geq W_J$ is equivalent to

$$-(1-\mu)R\sigma + \frac{\sigma[2\gamma(1-\sigma)\theta(1+\theta)+R(1-\mu-\theta+2\mu\theta)]}{1+\theta} \geq 0,$$

which simplifies to

$$\frac{\sigma\theta((3\mu-2)R+2\gamma(1-\sigma)(1+\theta))}{1+\theta} \geq 0.$$

The numerator is positive provided $\mu \geq \frac{2}{3}$.

$W_4 \geq W_J$ is positive if

$$-\frac{1}{2}\mu R\sigma - 2\gamma\theta\sigma(1-\sigma) + \frac{\sigma[2\gamma(1-\sigma)\theta(1+\theta)+R(1-\mu-\theta+2\mu\theta)]}{1+\theta} \geq 0.$$

This simplifies to

$$\frac{(2-3\mu)R\sigma(1-\theta)}{2(1+\theta)} \geq 0.$$

The numerator is positive if $\mu \leq \frac{2}{3}$

Conditions on Planner's Choices with an LSM

Long Cycles Case:

$W_1 \geq W_2$

$\iff -(1-\mu)R\sigma \geq -\frac{R\sigma[\sigma(1-\theta)+\theta][-\theta+\mu(2\theta-1)]}{2\sigma(1-\theta)+\theta}$

$\iff \frac{R\sigma(1-\theta)[(3\mu-2)\sigma+(2\mu-1)(1-\sigma)\theta]}{2\sigma(1-\theta)+\theta} \geq 0$

$\iff (3\mu-2)\sigma+(2\mu-1)(1-\sigma)\theta \geq 0$

$\iff \mu[3\sigma+2\theta(1-\sigma)] \geq 2\sigma+(1-\sigma)\theta$

$\iff \mu \geq \frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$

$W_1 \geq W_3$

$\iff -(1-\mu)R\sigma \geq -\frac{\mu R\sigma(1-2\theta)+\theta[R\sigma+\gamma(1-\sigma)(1-\theta)]}{2-\theta}$

$\iff (-1+\theta)[(-2+3\mu)R\sigma-\gamma(-1+\sigma)\theta] \leq 0$

$\iff (-2+3\mu)R\sigma-\gamma(-1+\sigma)\theta \geq 0$

$\iff 3\mu R\sigma \geq 2R\sigma+\gamma(-1+\sigma)\theta$

$\iff \mu \geq \frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma}$.

$W_1 \geq W_4$

$\iff -(1-\mu)R\sigma \geq -\frac{1}{2}(\mu R\sigma+\gamma\theta)$

$\iff R\sigma - \frac{1}{2}\gamma\theta \leq \frac{3}{2}\mu R\sigma$

$\iff \mu \geq \frac{2R\sigma-\gamma\theta}{3R\sigma}$

Note: Since $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \geq \frac{2R\sigma-\gamma\theta}{3R\sigma}$, $W_1 \geq W_3 \Longrightarrow W_1 \geq W_4$

$W_2 \geq W_3$

$\iff -\frac{R\sigma(\sigma(1-\theta)+\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta} \geq -\frac{\mu R\sigma(1-2\theta)+\theta(R\sigma+\gamma(1-\sigma)(1-\theta))}{2-\theta}$

$\iff \frac{(1-\sigma)(1-\theta)\theta\{\gamma[2\sigma(-1+\theta)-\theta]+R\sigma(\mu+\theta-2\theta\mu)\}}{[2\sigma(1-\theta)+\theta](2-\theta)} \leq 0$
$\iff (1-\sigma)(1-\theta)\theta\{\gamma[2\sigma(-1+\theta)-\theta]+R\sigma(\mu+\theta-2\theta\mu)\} \leq 0$
$\iff R\sigma(\mu-2\theta\mu) \leq \gamma[2\sigma(1-\theta)+\theta]-R\sigma\theta$
$\iff \mu \leq \frac{\gamma[2\sigma(1-\theta)+\theta]-R\sigma\theta}{R\sigma(1-2\theta)}$ if $\theta < \frac{1}{2}$
$\mu \geq \frac{\gamma[2\sigma(1-\theta)+\theta]-R\sigma\theta}{R\sigma(1-2\theta)}$ if $\theta > \frac{1}{2}$

$W_2 \geq W_4$
$\iff -\frac{R\sigma(\sigma(1-\theta)+\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta} \geq -\frac{1}{2}(\mu R\sigma+\gamma\theta)$
$\iff \theta\gamma[2\sigma(-1+\theta)-\theta]+R\sigma\theta[\mu+2\sigma-4\mu\sigma+2(2\mu-1)(-1+\sigma)\theta] \leq 0$
$\iff R\sigma\theta[\mu+2\sigma-4\mu\sigma+2(2\mu-1)(-1+\sigma)\theta] \leq \theta\gamma(2\sigma(1-\theta)+\theta)$
$\iff R\sigma\mu(1-4\sigma+4(-1+\sigma)\theta) \leq \gamma(2\sigma(1-\theta)+\theta)+2(-1+\sigma)R\sigma\theta-2R\sigma^2$
$\iff \mu \leq \frac{\gamma[2\sigma(1-\theta)+\theta]+2(-1+\sigma)R\sigma\theta-2R\sigma^2}{R\sigma(1-4\sigma+4(-1+\sigma)\theta)}$ if $1-4\sigma+4(-1+\sigma)\theta > 0$
$\mu \geq \frac{\gamma[2\sigma(1-\theta)+\theta]+2(-1+\sigma)R\sigma\theta-2R\sigma^2}{R\sigma(1-4\sigma+4(-1+\sigma)\theta)}$ if $1-4\sigma+4(-1+\sigma)\theta < 0$

$W_3 \geq W_4$
$\iff -\frac{\mu R\sigma(1-2\theta)+\theta(R\sigma+\gamma(1-\sigma)(1-\theta))}{2-\theta} \geq -\frac{1}{2}(\mu R\sigma+\gamma\theta)$
$\iff \frac{\theta\{(3\mu-2)R\sigma+\gamma[2\sigma(1-\theta)+\theta]\}}{2(-2+\theta)} \leq 0$
$\iff (3\mu-2)R\sigma+\gamma[2\sigma(1-\theta)+\theta] \geq 0$
$\iff 3\mu R\sigma \geq 2R\sigma-\gamma[2\sigma(1-\theta)+\theta]$
$\iff \mu \geq \frac{2R\sigma-\gamma[2\sigma(1-\theta)+\theta]}{3R\sigma}$

Short Cycles Case:

$W_1 \geq W_2$
Same as long cycles case.

$W_1 \geq W_3$
$\iff -(1-\mu)R\sigma+\frac{\sigma((R+\gamma(1-\sigma)(2-\theta)(1-\theta)\theta)+\mu(R-2R\theta))}{2-\theta} \geq 0$
$\iff \frac{\sigma(1-\theta)((3\mu-2)R+\gamma\theta(1-\sigma)(2-\theta))}{2-\theta} \geq 0$
$\iff (3\mu-2)R+\gamma\theta(1-\sigma)(2-\theta) \geq 0$
$\iff (3\mu-2) \geq -\frac{\gamma\theta}{R}(1-\sigma)(2-\theta)$
$\iff \mu \geq \frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$

$W_1 \geq W_4$
$\iff -(1-\mu)R\sigma+\frac{1}{2}\mu R\sigma+2\gamma(1-\sigma)\sigma\theta \geq 0$
$\iff -\frac{1}{2}\sigma((2-3\mu)R-4\gamma\theta(1-\sigma)) \geq 0$
$\iff (2-3\mu)R-4\gamma\theta(1-\sigma) \leq 0$
$\iff 3\mu R \geq 2R-4\gamma\theta(1-\sigma)$
$\iff \mu \geq \frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma)$

$W_2 \geq W_3$
$\iff -\frac{R\sigma(\sigma(-1+\theta)-\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta}+\frac{\sigma((R+\gamma(1-\sigma)(2-\theta)(1-\theta)\theta)+\mu(R-2R\theta))}{2-\theta} \geq 0$
$\iff \frac{(1-\sigma)\sigma(1-\theta)\theta(\gamma(2\sigma(-1+\theta)-\theta)(-2+\theta)-R\theta+\mu R(-1+2\theta))}{(2\sigma(1-\theta)+\theta)(2-\theta)} \geq 0$
$\iff \gamma(2\sigma(-1+\theta)-\theta)(-2+\theta)-R\theta+\mu R(-1+2\theta) \geq 0$
$\iff \mu R(2\theta-1) \geq \gamma(2\sigma(-1+\theta)-\theta)(2-\theta)+R\theta$
$\iff \mu \geq \frac{\gamma(2\sigma(-1+\theta)-\theta)(2-\theta)+R\theta}{R(2\theta-1)}$ if $\theta > \frac{1}{2}$
$\mu \leq \frac{\gamma(2\sigma(-1+\theta)-\theta)(2-\theta)+R\theta}{R(2\theta-1)}$ if $\theta < \frac{1}{2}$

$W_2 \geq W_4$
$\iff -\frac{R\sigma(\sigma(-1+\theta)-\theta)(-\theta+\mu(2\theta-1))}{2\sigma(1-\theta)+\theta}+\frac{1}{2}\mu R\sigma+2\gamma(1-\sigma)\sigma\theta \geq 0$
$\iff \frac{1}{2}\mu R\sigma+\mu\frac{R\sigma(\sigma(1-\theta)+\theta)(2\theta-1)}{2\sigma(1-\theta)+\theta} \geq \frac{R\sigma\theta(\sigma(1-\theta)+\theta)}{2\sigma(1-\theta)+\theta}-2\gamma(1-\sigma)\sigma\theta$

$\iff \frac{1}{2}\mu R\sigma[2\sigma(1-\theta)+\theta]+\mu\{R\sigma[\sigma(1-\theta)+\theta](2\theta-1)\} \geq R\sigma\theta(\sigma(1-\theta)+\theta)-2\gamma(1-\sigma)\sigma\theta(2\sigma(1-\theta)+\theta)$

$\iff \mu(\frac{1}{2}R\theta\sigma(4\theta-1+4\sigma(1-\theta))) \geq \sigma\theta((R+4\gamma(-1+\sigma))\sigma+(1-\sigma)(R+\gamma(-2+4\sigma))\theta)$

$\iff \mu \geq \frac{2((R+4\gamma(-1+\sigma))\sigma+(1-\sigma)(R+\gamma(-2+4\sigma))\theta)}{R(4\theta-1+4\sigma(1-\theta))}$ if $4\theta-1+4\sigma(1-\theta)>0$

$\mu \leq \frac{2((R+4\gamma(-1+\sigma))\sigma+(1-\sigma)(R+\gamma(-2+4\sigma))\theta)}{R(4\theta-1+4\sigma(1-\theta))}$ if $4\theta-1+4\sigma(1-\theta)<0$

$W_3 \geq W_4$

$\iff -\frac{\sigma((R+\gamma(1-\sigma)(2-\theta)(1-\theta)\theta)+\mu(R-2R\theta))}{2-\theta}+\frac{1}{2}\mu R\sigma+2\gamma(1-\sigma)\sigma\theta \geq 0$

$\iff \frac{\sigma\theta((3\mu-2)R+2\gamma(1-\sigma)(2-\theta)(1+\theta))}{2(2-\theta)} \geq 0$

$\iff (3\mu-2)R+2\gamma(1-\sigma)(2-\theta)(1+\theta) \geq 0$

$\iff 3\mu R \geq 2R-2\gamma(1-\sigma)(2-\theta)(1+\theta)$

$\iff \mu \geq \frac{2}{3}-\frac{2\gamma}{3R}(1-\sigma)(2-\theta)(1+\theta)$

Proof of proposition 6

Proof. We need to show that if the parameters are such that the planner does not choose profile 1, then $\mu < \frac{2}{3}$. Formally, for the BRLSM long cycle case, we need to show that $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$, $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma}$, and $\frac{2R\sigma-\gamma\theta}{3R\sigma}$ are all less than or equal to $\frac{2}{3}$. For the BRLSM short cycles case we need to show that $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$, $\frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$, and $\frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma)$ are all less than or equal to $\frac{2}{3}$. For the RTGS case, we will have to show that $\frac{(1+\sigma\theta)}{2(1+\sigma\theta)-\sigma}$ and $\frac{R-\gamma\theta}{R(2-\sigma)}$ are both less than $\frac{2}{3}$.

1) , 4) $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)} \leq \frac{2}{3}$ holds if $6\sigma+4\theta(1-\sigma) \geq 6\sigma+3\theta(1-\sigma)$, which is true since $\theta(1-\sigma) \geq 0$.

2) $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \leq \frac{2}{3}$ holds if $6R\sigma-3\gamma\theta(1-\sigma) \leq 6R\sigma$, which is true since $-3\gamma\theta(1-\sigma) \leq 0$.

3) $\frac{2R\sigma-\gamma\theta}{3R\sigma} \leq \frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma}$.

5) $\frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta) \leq \frac{2}{3}$ since $\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta) \geq 0$.

6) $\frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma) \leq \frac{2}{3}$ since $\frac{4\gamma\theta}{3R}(1-\sigma) \geq 0$.

7) $\frac{(1+\sigma\theta)}{2(1+\sigma\theta)-\sigma} \leq \frac{2}{3}$ holds if $3+3\sigma\theta \leq 4+4\sigma\theta-2\sigma$, which must be true since $2\sigma \leq 1+\sigma\theta$.

8) $\frac{R-\gamma\theta}{R(2-\sigma)} \leq \frac{2}{3}$ holds if $3R-3\gamma\theta \leq 4R-2\sigma R$, which must be true since $-3\gamma\theta \leq 0 \leq R(1-2\sigma)$. ■

Proof of proposition 7

Proof. We will have proven is proposition if we can show that $W_1 \geq Wi$ for the short cycles case implies $W_1 \geq Wi$ for the long cycles case, for $i \in \{2,3,4\}$ To do this we just check that 1) $\frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)} \leq \frac{2\sigma+\theta(1-\sigma)}{3\sigma+2\theta(1-\sigma)}$, 2) $\frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \leq \frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$, and 3) $\frac{2R\sigma-\gamma\theta}{3R\sigma} \leq \frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma)$:

1) Trivial.

2) $(2-\theta) \leq 2 \leq \frac{1}{\sigma}$

$\implies (2-\theta)\frac{\gamma\theta(1-\sigma)}{3R} \leq \frac{\gamma\theta(1-\sigma)}{3R\sigma}$

$\implies \frac{2R\sigma-\gamma\theta(1-\sigma)}{3R\sigma} \leq \frac{2}{3}-\frac{\gamma\theta}{3R}(1-\sigma)(2-\theta)$

3) $4(1-\sigma) \leq \frac{1}{\sigma}$

$\implies \frac{4\gamma\theta}{3R}(1-\sigma) \leq \frac{\gamma\theta}{3R\sigma}$

$\implies \frac{2}{3}-\frac{4\gamma\theta}{3R}(1-\sigma) \geq \frac{2R\sigma-\gamma\theta}{3R\sigma}$ ■

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