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CoVaR

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Abstract

We propose a measure for systemic risk, ΔCoVaR, defined as the difference between the conditional value at risk (CoVaR) of the financial system conditional on an institution being in distress and the CoVaR conditional on the median state of the institution. Our ΔCoVaR estimates show that characteristics such as leverage, size, maturity mismatch, and asset price booms significantly predict systemic risk contribution. We provide out-of-sample forecasts of a countercyclical, forward-looking measure of systemic risk and show that the 2006:Q4 value of this measure would have predicted more than one-third of realized ΔCoVaR during the financial crisis.

Key words: value at risk, systemic risk, risk spillovers, financial architecture

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1 Introduction

In times of financial crisis, losses spread across financial institutions, threatening the financial system as a whole.[1] The spreading of distress gives rise to systemic risk—the risk that the intermediation capacity of the entire financial system is impaired, with potentially adverse consequences for the supply of credit to the real economy. In systemic financial events, spillovers across institutions can arise from direct contractual links and heightened counterparty credit risk, or can occur indirectly through price effects and liquidity spirals. As a result of these spillovers, the measured comovement of institutions' assets and liabilities tends to rise above and beyond levels purely justified by fundamentals. Systemic risk measures capture the potential for the spreading of financial distress across institutions by gauging this increase in tail comovement.

The most common measure of risk used by financial institutions—the value at risk (VaR)—focuses on the risk of an individual institution *in isolation*. For example, the $q\%$-VaR^i is the maximum loss of institution i at the $q\%$-confidence level.[2] However, a single institution's risk measure does not necessarily reflect its contribution to overall *systemic* risk, for several reasons. Some institutions are *individually systemic*—they are so interconnected and large that they can cause negative risk spillover effects on others. Similarly, several smaller institutions may be *systemic as part of a herd*. In addition to the cross-sectional dimension, systemic risk also has a time-series dimension. Systemic risks typically build in times of low volatility, and materialize during crises. A good systemic risk measure should capture this build-up. This means that high-frequency risk measures that rely mostly on contemporaneous price movements are potentially misleading.

In this paper, we propose a new reduced-form measure of contributions to systemic risk—the $\Delta CoVaR$. This new measure captures tail dependency and includes negative spillover dynamics in times of crises. We also project the $\Delta CoVaR$ on lagged institutional characteristics (in particular size, leverage, and maturity mismatch) and conditioning variables (in particular market volatility and fixed income spreads). This procedure gives a forward-looking systemic risk contribution measure—the *forward*-$\Delta CoVaR$— which captures the build-up of systemic risk in tranquil times.

To emphasize the systemic nature of our risk measure, we add to existing risk measures the

[1] Examples include the 1987 equity market crash, which was started by portfolio hedging of pension funds and led to substantial losses of investment banks; the 1998 crisis, which was started with losses of hedge funds and spilled over to the trading floors of commercial and investment banks; and the 2007-09 crisis, which spread from SIVs to commercial banks and on to investment banks and hedge funds. See e.g. Brady (1988), Rubin, Greenspan, Levitt, and Born (1999), Brunnermeier (2009), and Adrian and Shin (2010a).

[2] See Kupiec (2002) and Jorion (2006) for detailed overviews.

prefix "*Co*," for *co*nditional. We focus primarily on $CoVaR$, where institution i's $CoVaR$ relative to the system is defined as the VaR of the whole financial sector conditional on institution i being in a particular state, such as distress or the median state.[3] The difference between the $CoVaR$ conditional on the distress of an institution and the $CoVaR$ conditional on the "normal" state of the institution, $\Delta CoVaR$, captures the contribution of a particular institution, in a non-causal sense, to the overall systemic risk. $\Delta CoVaR$ is a statistical tail dependency measure, and so is best viewed as a useful reduced-form analytical tool capturing (tail) comovements.

The systemic risk contribution measure $\Delta CoVaR$ differs from an individual institution's risk measure VaR. Figure 1 shows this for the leading financial institutions in the US. Hence, it is not sufficient to regulate financial institutions purely based on the risk of institutions in isolation as it can lead to excessive risk-taking along systemic risk dimension.

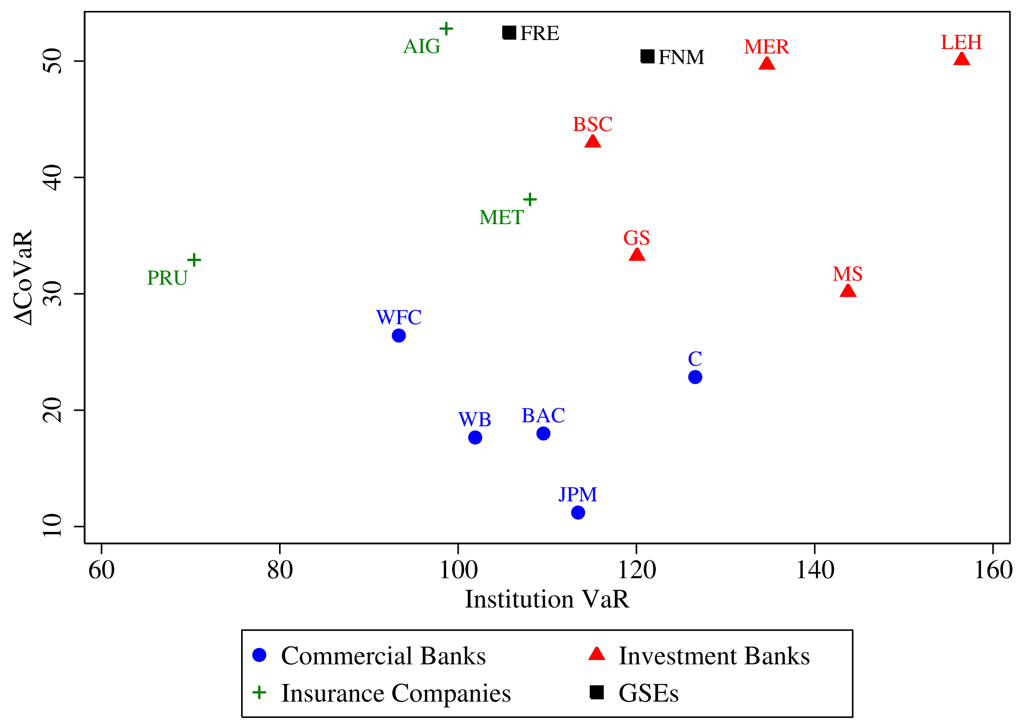

Figure 1: VaR and $\Delta CoVaR$. The scatter plot shows the weak link between institutions' risk in isolation, measured by VaR^i (x-axis), and institutions' contribution to system risk, measured by $\Delta CoVaR^i$ (y-axis). The VaR^i and $\Delta CoVaR^i$ are unconditional 99% measures estimated as of 2006Q4 and are reported in quarterly percent returns for merger adjusted entities. $\Delta CoVaR^i$ is the difference between the financial system's VaR conditional on firm $i's$ distress and the financial system's VaR conditional on firm $i's$ median state. The institution names are listed in Appendix D.

[3] Under many distributional assumptions (such as the assumption that shocks are conditionally Gaussian), the VaR of an institution is proportional to the variance of the institution, and the $CoVaR$ of an institution is proportional to the covariance of the financial system and the individual institution.

$\Delta CoVaR$ is directional. Reverting the conditioning allows one to answer which institutions are most at risk should a financial crisis occur (as opposed to which institutions contributes to it the most). $\Delta CoVaR$ is general enough to capture directional tail dependency from institution to institution across the whole financial network.

So far, we have deliberately not specified how to estimate $\Delta CoVaR$, since there are many possible ways. In this paper, we primarily use quantile regressions, which are appealing for their simplicity. Since we want to capture all forms of risk, including not only the risk of adverse asset price movements, but also funding liquidity risk, our estimates of $\Delta CoVaR$ are based on weekly equity returns of all publicly traded financial institutions. However, $\Delta CoVaR$ can also be estimated using methods such as $GARCH$ models, as we show in the appendix.

We calculate unconditional and conditional measures of $\Delta CoVaR$ using the full length of available data. We use weekly data from 1971Q1 to 2013Q2 for all publicly traded commercial banks, broker-dealers, insurance companies, and real estate companies. We also verify for financial firms that are listed since 1926 that a longer estimation window does not materially alter the systemic risk contribution estimates. We model the variation of $\Delta CoVaR$ as a function of state variables that capture the evolution of tail risk dependence over time. These state variables include the slope of the yield curve, the aggregate credit spread, and realized equity market volatility. We first estimate $\Delta CoVaR$ conditional on the state variables. In a second step we use panel regressions, and relate these time-varying $\Delta CoVaR$s—in a predictive, Granger causal sense—to measures of each institution's characteristics like maturity mismatch, leverage, size, and asset valuations. We find relationships that are in line with theoretical predictions: higher leverage, more maturity mismatch, larger size, and higher valuations forecast higher systemic risk contributions across financial institutions.

Systemic risk monitoring should be based on forward-looking risk measures. We propose such a forward-looking measure—the *forward*-$\Delta CoVaR$. This *forward*-$\Delta CoVaR$ has countercyclical features, reflecting the buildup of systemic risk in good times, and the realization of systemic risk in crises. Crucially, the countercyclicality of our forward measure is a *result*, not an *assumption*. Econometrically, we construct the *forward*-$\Delta CoVaR$ by regressing time-varying $\Delta CoVaR$s on lagged institutional characteristics and common risk factors. We estimate *forward*-$\Delta CoVaR$ out-of-sample. Consistent with the "volatility paradox"—the notion that low volatility environments breed the build up of systemic risk—the *forward*-$\Delta CoVaR$ is negatively correlated with the contemporaneous $\Delta CoVaR$. We also demonstrate that the *forward*-$\Delta CoVaR$ has out of sample predictive power for realized $\Delta CoVaR$ in tail events. In particular, the *forward*-$\Delta CoVaR$ estimated using data through the end of 2006 predicts a substantial fraction of the cross-sectional dispersion in realized $\Delta CoVaR$ during the financial crisis of 2007-08. The *forward*-$\Delta CoVaR$ can thus be used to monitor the buildup of systemic risk in a forward-looking manner. It remains, however, a reduced-form measure, and so does not *causally* allocate systemic risk contributions to different financial institutions.

Outline. The remainder of the paper is organized in five sections. We first present a review of the related literature. Then, in Section 3, we present the methodology, define $\Delta CoVaR$ and discuss its properties. In Section 4, we outline the estimation method via quantile regressions. We allow for time variation in the $\Delta CoVaR$s by modeling them as a function of various state variables and present estimates of these time varying $\Delta CoVaR$s. Section 5 then introduces the *forward*-$\Delta CoVaR$, illustrates its countercyclicality and demonstrates that institutional characteristics such as size, leverage, and maturity mismatch can predict systemic risk contribution in the cross section of institutions. We conclude in Section 6.

2 Literature Review

Our *co-risk measure* is motivated by theoretical research on externalities across financial institutions that give rise to amplifying liquidity spirals and persistent distortions. It also relates closely to recent econometric work on contagion and spillover effects.

2.1 Theoretical Background on Systemic Risk

Spillovers can give rise to excessive risk taking and leverage in the run-up phase to a crisis. Spillovers in the form of externalities arise when individual institutions take potential fire-sale prices as given, while fire-sale prices are determined jointly by all institutions. In an incomplete market setting, this pecuniary externality leads to an outcome that is not even constrained Pareto efficient. This result was derived in a banking context in Bhattacharya and Gale (1987) and a general equilibrium incomplete market setting by Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). Prices can also affect borrowing constraints. These externality effects are studied within an international finance context by Caballero and Krishnamurthy (2004), and most recently shown in Lorenzoni (2008), Acharya (2009), Stein (2009), and Korinek (2010). Runs on financial institutions are dynamic co-opetition games and lead to externalities, as does banks' liquidity hoarding. While hoarding might be microprudent from a single bank's perspective it need not be macroprudent (fallacy of the commons). Finally, network effects can also lead to spillover effects, as emphasized by Allen, Babus, and Carletti (2010).

Procyclicality occurs because risk measures tend to be low in booms and high in crises. Brunnermeier and Sannikov (2014) coined the term "volatility paradox". The margin/haircut spiral and precautionary hoarding behavior, outlined in Brunnermeier and Pedersen (2009) and Adrian and Boyarchenko (2012), lead financial institutions to shed assets at fire-sale prices. Adrian and Shin (2010b), Gorton and Metrick (2010), and Adrian, Etula, and Muir (2010) provide empirical evidence for the margin/haircut spiral. Borio (2004) is an early contribution that discusses a policy framework to address margin/haircut spirals and procyclicality.

2.2 Other Systemic Risk Measures

$\Delta CoVaR$, of course, is not the only systemic risk measure. Huang, Zhou, and Zhu (2010) develop a systemic risk indicator that measures the price of insurance against systemic financial distress from credit default swap (CDS) prices. Acharya, Pedersen, Philippon, and Richardson (2010) focus on high-frequency marginal expected shortfall as a systemic risk measure. Like our "*Exposure-*$\Delta CoVaR$"—to be defined later—they switch the conditioning and address the question of which institutions are most exposed to a financial crisis as opposed to which institution contributes most to a crisis. Importantly, their analysis focuses on a cross-sectional comparison of financial institutions and does not address the problem of procyclicality that arises from contemporaneuous risk measurement. In other words, they do not address the stylized fact that risk builds up in the background during boom phases characterized by low volatility and materializes only in crisis times. Acharya, Engle, and Richardson (2012) develop the closely related *SRISK* measure which calculates capital shortfall of individual institutions conditional on market stress. Billio, Getmansky, Lo, and Pelizzon (2010) propose a systemic risk measure that relies on Granger causality among firms. Giglio (2011) uses a nonparametric approach to derive bounds of systemic risk from CDS prices. A number of recent papers have extended the $\Delta CoVaR$ method and applied it to additional financial sectors. For example, Adams, Füss, and Gropp (2010) study risk spillovers among financial sectors; Wong and Fong (2010) estimate $\Delta CoVaR$ for the CDS of Asia-Pacific banks; Gauthier, Lehar, and Souissi (2012) estimate systemic risk exposures for the Canadian banking system. Another important strand of the literature, initiated by Lehar (2005) and Gray, Merton, and Bodie (2007), uses contingent claims analysis to measure systemic risk. Bodie, Gray, and Merton (2007) develop a policy framework based on the contingent claims. Segoviano and Goodhart (2009) use a related approach to measure risk in the global banking system.

2.3 The Econometrics of Tail Risk and Contagion

The $\Delta CoVaR$ measure is also related to the literature on volatility models and tail risk. In a seminal contribution, Engle and Manganelli (2004) develop *CAViaR*, which uses quantile regressions in combination with a *GARCH* model to capture the time varying tail behavior of asset returns. Manganelli, Kim, and White (2011) study a multivariate extension of *CAViaR*, which can be used to generate a dynamic version of *CoVaR*. Brownlees and Engle (2010) propose methodologies to estimate systemic risk measures using *GARCH* models.

The $\Delta CoVaR$ measure can additionally be related to an earlier literature on contagion and volatility spillovers (see Claessens and Forbes (2001) for an overview). The most common method to test for volatility spillovers is to estimate multivariate *GARCH* processes. Another approach is to use multivariate extreme value theory. Hartmann, Straetmans, and de Vries (2004) develop a contagion measure that focuses on extreme events. Danielsson and de Vries (2000) argue that extreme value theory works well only for very low quantiles.

Since the current paper was first circulated in 2008, a literature on alternative estimation approaches for *CoVaR* has been emerging. *CoVaR* is estimated using multivariate *GARCH* by Girardi and Tolga Ergün (2013) (see also our Appendix B). Mainik and Schaanning (2012) and Oh and Patton (2013) use copulas. Bayesian inference for *CoVaR* estimation is proposed by Bernardi, Gayraud, and Petrella (2013). Bernardi, Maruotti, and Petrella (2013) and Cao (2013) make distributional assumptions about shocks and employ maximum likelihood estimators. Extensions of the quantile regression approach for *CoVaR* can be found in Castro and Ferrari (2014).

3 *CoVaR* Methodology

3.1 Definition of Δ*CoVaR*

Recall that VaR_q^i is implicitly defined as the $q\%$ quantile, i.e.,

$$\Pr\left(X^i \leq VaR_q^i\right) = q\%,$$

where X^i is the loss of institution i for which the VaR_q^i is defined. Defined like this, VaR_q^i is typically a positive number when $q > 50$, in line with the commonly used sign convention. Hence more risk corresponds to a greater VaR_q^i. We define X^i as the "return loss".

Definition 1 *We denote by* $CoVaR_q^{j|\mathbb{C}(X^i)}$ *the VaR of institution* j *(or the financial system) conditional on some event* $\mathbb{C}\left(X^i\right)$ *of institution* i*. That is,* $CoVaR_q^{j|\mathbb{C}(X^i)}$ *is implicitly defined by the* $q\%$*-quantile of the conditional probability distribution:*

$$\Pr\left(X^{\mathrm{j}}|\mathbb{C}\left(X^i\right) \leq CoVaR_q^{j|\mathbb{C}(X^i)}\right) = q\%.$$

We denote institution i*'s contribution to* j *by*

$$\Delta CoVaR_q^{j|i} = CoVaR_q^{j|X^i=\mathrm{VaR}_q^i} - CoVaR_q^{j|X^i=\mathrm{VaR}_{50}^i},$$

and in dollar terms

$$\Delta^{\$}CoVaR_q^{j|i} = {}^{\$}\mathrm{Size}^i \cdot \Delta CoVaR_q^{j|i}.$$

In our benchmark specification j will be the losses of the portfolio of all financial institutions.

Conditioning. To obtain *CoVaR* we typically condition on an event $\mathbb{C}$ that is equally likely across institutions. Usually $\mathbb{C}$ is institution i's loss being at or above its VaR_q^i level, which—by definition—occurs with likelihood $(1-q)\,\%$. Importantly, this implies that the likelihood of the conditioning event is independent of the riskiness of i's strategy. If we were to condition on a particular return level (instead of a quantile), then more conservative (i.e., less risky) institutions could have a higher *CoVaR* simply because the conditioning event would be a more extreme event for less risky institutions.

Δ***CoVaR*** captures the increase in $CoVaR$ as one shifts the conditioning event from the median return of institution i to the adverse VaR_q^i (with equality). $\Delta CoVaR$ measures the "tail dependency" between two random variables. Note that, for jointly normally distributed random variables, $\Delta CoVaR$ is related to the correlation coefficient, while $CoVaR$ corresponds to a conditional variance. Conditioning by itself reduces the variance, while conditioning on adverse events increases expected losses.

$\Delta^{\$}$***CoVaR*** takes the size of institution i into account in order to compare the systemic risk contributions across institutions. For the purpose of this paper we capture the size with the market equity of the institution. Financial regulators (and in an earlier draft of our paper we) use total assets for the return as well as the size definition.[4]

CoES. One attractive feature of $CoVaR$ is that it can be easily adapted for other "*co*risk-measures." An example of this is the co-expected shortfall, *CoES*. Expected shortfall, the expected loss conditional on a VaR event, has a number of advantages relative to VaR[5], and these considerations extend to *CoES*. $CoES_q^{j|i}$ may be defined as the expected loss for institution j conditional on its losses exceeding $CoVaR_q^{j|i}$, and $\Delta CoES_q^{j|i}$ analogously is just $CoES_q^{j|i} - CoES_{50}^{j|i}$.

3.2 The Economics of Systemic Risk

Systemic risk has a time-series and a cross-sectional dimension. In the time-series, systemic risk builds up during credit booms when contemporaneously measured risk is low. This buildup of systemic risk during times of low measured risk gives rise to a "volatility paradox." Hence, contemporaneous systemic risk measures are not suited to capture the buildup component of systemic risk. In Section 5, we construct a "*forward-*$\Delta CoVaR$" that avoids the "procyclicality pitfall" by estimating the relationship between current firm characteristics and future tail dependency, as proxied by $\Delta CoVaR_{q,t}^{j|i}$.

The cross-sectional component of systemic risk relates to the spillover effects that amplify initial adverse shocks in times of crises. The contemporaneous $\Delta CoVaR^i$ measures the tail dependency and

[4]For multi-strategy institutions and funds, it might make sense to calculate the $\Delta CoVaR$ for each strategy s separately and obtain $\Delta^{\$}CoVaR_q^{j|i} = \sum_s Size^{s,i} \cdot \Delta CoVaR_q^{j|s}$. This ensures that mergers and carve-outs of strategies do not impact their overall systemic risk contribution measure, and also improves the cross-sectional comparison.

[5]In particular, the VaR is not subadditive and does not take distributional aspects within the tail into account. However, these concerns are mostly theoretical in nature as the exact distribution within the tails is difficult to estimate.

captures both spillover and common exposure effects. It captures how much an institution adds to the overall risk of the financial system. The spillover effects can be direct, through contractual links among financial institutions. Indirect spillover effects, however, are quantitatively more important. Selling off assets can lead to mark-to-market losses for all market participants who hold a similar exposure. Moreover, the increase in volatility might tighten margins and haircuts forcing other market participants to delever as well (margin spiral). This can lead to crowded trades which increases the price impact even further, see e.g. Brunnermeier and Pedersen (2009). Many of these spillovers are externalities. That is, when taking on the initial position with low market liquidity funded with short-term liabilities—i.e. with high liquidity mismatch— individual market participants do not internalize that subsequent individually optimal response in times of crises will impose (pecuniary) externalities on others. As a consequence, the initial risk taking is often excessive in the run-up phase, which generates the first component of systemic risk.

3.3 Tail Dependency versus Causality

$\Delta CoVaR_q^{j|i}$ is a statistical tail-dependency measure and does not necessarily correctly capture externalities or spillover effects, for several reasons. First, the externalities are typically not fully observable in equilibrium, since other institutions might reposition themselves in order to reduce the impact of the externalities. Second, $\Delta CoVaR_q^{j|i}$ also captures the common exposure to exogenous aggregate macroeconomic risk factors.

More generally, causal statements can only be made within a specific model. Here, we consider for illustrative purposes a simple stylized financial system that can be split into two groups, institutions of type i and of type j. There are two latent independent risk factors, ΔZ^i and ΔZ^j. We conjecture that institutions of type i are directly exposed to the sector specific shock ΔZ^i, and indirectly exposed to ΔZ^j due to spillover effects. The assumed data generating process of returns for type i institutions $-X_{t+1}^i = \Delta N_{t+1}^i / N_t^i$ is

$$-X_{t+1}^i = \overline{\mu}^i(\cdot) + \overline{\sigma}^{ii}(\cdot)\,\Delta Z_{t+1}^i + \overline{\sigma}^{ij}(\cdot)\,\Delta Z_{t+1}^j, \quad (1)$$

where the short-hand notation $(\cdot)$ indicates that the (geometric) drift and volatility loadings are functions of the following state variables $\left(M_t, L_t^i, L_t^j, N_t^i, N_t^j\right)$: the state of the macro-economy, M_t, the leverage and liquidity mismatch of type i institutions, L_t^i, and of type j institutions, L_t^j, as well as the net worth levels N_t^i and N_t^j. Leverage L_t^i is a choice variable and presumably, for i type institutions, increases the loading to the own latent risk factor ΔZ_{t+1}^i. One would also presume that the exposure of i type institutions to ΔZ_{t+1}^j due to spillovers, $\overline{\sigma}^{ij}(\cdot)$, is increasing in the own leverage, L_t^i, and others' leverage, L_t^j.

Analogously, for institutions of type j, we propose the following data generating process:

$$-X_{t+1}^j = \overline{\mu}^j(\cdot) + \overline{\sigma}^{jj}(\cdot)\,\Delta Z_{t+1}^j + \overline{\sigma}^{ji}(\cdot)\,\Delta Z_{t+1}^i. \tag{2}$$

As the two latent shock processes ΔZ_{t+1}^i and ΔZ_{t+1}^j are unobservable, the empirical analysis starts with the following two of reduced form equations:[6]

$$-X_{t+1}^i = \mu^i(\cdot) - \sigma^{ij}(\cdot)\,X_{t+1}^j + \sigma^{ii}(\cdot)\,\Delta Z_{t+1}^i, \tag{3}$$

$$-X_{t+1}^j = \mu^j(\cdot) - \sigma^{ji}(\cdot)\,X_{t+1}^i + \sigma^{jj}(\cdot)\,\Delta Z_{t+1}^j. \tag{4}$$

Consider an adverse shock $\Delta Z_{t+1}^i < 0$. This shock lowers $-X_{t+1}^i$ by $\sigma_t^{ii}\Delta Z_{t+1}^i$. First round spillover effects also reduce the others' return $-\Delta X_{t+1}^j$ by $\sigma_t^{ji}\sigma_t^{ii}\Delta Z_{t+1}^i$. Lower $-\Delta X_{t+1}^j$, in turn, lowers $-\Delta X_{t+1}^i$ by $\sigma_t^{ij}\sigma_t^{ji}\sigma_t^{ii}\Delta Z_{t+1}^i$ due to second round spillover effects. The argument goes on through third, fourth etc. round effects. When a fixed point is ultimately reached, we obtain the volatility loadings of the initially proposed data generating process $\bar{\sigma}_t^{ii} = \sum_{n=0}^{\infty}(\sigma_t^{ij}\sigma_t^{ji})^n\sigma_t^{ii} = \frac{\sigma_t^{ii}}{1-\sigma_t^{ij}\sigma_t^{ji}}$. Similarly, $\bar{\sigma}_t^{ij} = \sum_{n=0}^{\infty}(\sigma_t^{ij}\sigma_t^{ji})^n\sigma_t^{ij}\sigma_t^{jj} = \frac{\sigma_t^{ij}\sigma_t^{jj}}{1-\sigma_t^{ij}\sigma_t^{ji}}$. Analogously, replacing i with j and vice versa, we obtain $\bar{\sigma}_t^{jj}$ and $\bar{\sigma}_t^{ji}$. That is, this reasoning allows one to link reduced form σs to primitive $\bar{\sigma}$s.

Gaussian case. An explicit formula can be derived for the special case in which all innovations ΔZ_{t+1}^i and ΔZ_{t+1}^j are jointly Gaussian distributed. In this case

$$\Delta CoVaR_{q,t}^{j|i} = \Delta VaR_{q_t}^i \cdot \beta_t^{ij} \tag{5}$$

$$= -\left(\Phi^{-1}(q)\right)^2 \frac{Cov_t\left[X_{t+1}^i, X_{t+1}^j\right]}{\Delta VaR_{q,t}^i} = -\Phi^{-1}(q)\sigma_t^j\rho_t^{ij}, \tag{6}$$

[6]The location scale model outlined in Appendix A falls in this category, with $\mu^j(M_t)$, $\sigma^{ji} = const.$, $\sigma^{jj}(M_t, X_{t+1}^i)$, and the error term distributed i.i.d. with zero mean and unit variance. Another difference relative to this model are losses in return space (not net worth in return space) as the dependent variable.

where $\beta_t^{ij} = \frac{Cov_t\left[X_{t+1}^i, X_{t+1}^j\right]}{Var_t\left[X_{t+1}^i\right]} = \frac{\bar{\sigma}_t^{ii}\bar{\sigma}_t^{ji}+\bar{\sigma}_t^{ij}\bar{\sigma}_t^{jj}}{\bar{\sigma}_t^{ii}\bar{\sigma}_t^{ii}+\bar{\sigma}_t^{ij}\bar{\sigma}_t^{ij}}$ is the OLS regression coefficient of reduced form Equation (4). Note that in the Gaussian case the OLS and median quantile regression coefficient are the same. $\Phi(\cdot)$ is the standard Gaussian cdf, σ_t^j is the standard deviation of N_{t+1}^j/N_t^j, and ρ_t^{ij} the correlation coefficient between N_{t+1}^i/N_t^i and N_{t+1}^j/N_t^j. The Gaussian setting results in a nice analytical solution, but its tail properties are less desirable than those of more general distributional specifications.

3.4 CoVaR, Exposure-CoVaR, Network-CoVaR

The superscripts j or i can refer to losses of individual institutions or of a portfolio of institutions. $\Delta CoVaR_q^{j|i}$ is directional. That is, the $\Delta CoVaR_q^{system|i}$ of the system conditional on institution i is not equal to $\Delta CoVaR_q^{i|system}$ of institution i conditional on the financial system being in crisis. The conditioning radically changes the interpretation of the systemic risk contribution measure. In this paper we consider primarily the direction of conditioning $\Delta CoVaR_q^{system|i}$. It stresses how much more risky the system is in states of the world in which institution i is in distress relative to its normal times. Specifically,

$$\Delta CoVaR_q^{system|i} = CoVaR_q^{system|X^i=VaR_q^i} - CoVaR_q^{system|X^i=VaR_{50}^i}.$$

Exposure-Δ*CoVaR*. For risk management questions, it is sometimes useful to compute the opposite conditioning. We can derive $CoVaR^{j|system}$, which answers the question of which institutions are most at risk should a financial crisis occur. $\Delta CoVaR^{j|system}$, which we label "*Exposure*-$\Delta CoVaR$", reports institution j's increase in value at risk in the case of a financial crisis. In other words, the *Exposure*-$\Delta CoVaR$ is a measure of an individual institution's exposure to system wide distress, and is similar to the stress tests performed by individual institutions or regulators.

The importance of the direction of the conditioning is probably best illustrated with the following example. Consider a financial institution (perhaps a venture capitalist) with returns subject to substantial idiosyncratic noise. If the financial system overall is in significant distress, then this institution is also likely to face difficulties, so its *Exposure*-$\Delta CoVaR$ is high. At the same time, conditioning on this particular institution being in distress does not materially impact the probability that the wider financial system is in distress (due to the large idiosyncratic component of the returns), and so $\Delta CoVaR$ is low. In this example the *Exposure*-$\Delta CoVaR$ would send the wrong signal about systemicness, were it to be viewed as an indicator of such.

Network-$\Delta CoVaR$. Finally, whenever j and i in $CoVaR^{j|i}$ refer to individual institutions, we talk of a "*Network-*$\Delta CoVaR$". In this case we can study of tail dependency across a whole financial network.

To simplify notation we sometimes drop the subscript q when it is not necessary to specify the confidence level of the risk measures. Also, for the benchmark $\Delta CoVaR^{system|i}$ we often write only $\Delta CoVaR^{i}$. Later we will also introduce a time varying systemic risk contribution measure and add a subscript t to denote time $\Delta CoVaR_{q,t}^{system|i}$.

3.5 Properties of $\Delta CoVaR$

Clone Property. Our $\Delta CoVaR$ definition satisfies the desired property that, after splitting one large *individually systemic* institution into n smaller clones, the $CoVaR$ of the large institution (in return space) is exactly the same as the $CoVaR$s of the n clones. Put differently, conditioning on the distress of a large systemic institution is the same as conditioning on one of the n clones. This property of course also holds for the Gaussian case, as can be seen from Equation (6). Both the covariance and the ΔVaR are divided by n, leaving $\Delta CoVaR_{q,t}^{j|i}$ unchanged.

Systemic as Part of a Herd. Consider a large number of small financial institutions which hold similar positions, and are funded in a similar way—in short, they are exposed to the same factors. Now, if only one of these institutions falls into distress, then this will not necessarily *cause* a systemic crisis. However, if the distress is due to a common factor, then the other institutions will also be in distress. Overall, the institutions are *systemic as part of a herd.* Each individual institution's co-risk measure should capture this notion of being "systemic as part of a herd", even in the absence of a direct causal link. The $\Delta CoVaR$ measure achieves exactly that. Moreover, when we estimate $\Delta CoVaR$, we control for lagged state variables that capture variation in tail risk not directly related to the financial system risk exposure. This discussion connects naturally with the clone property: If we split a systemically important institution into n clones, then each clone is systemic as part of the herd. The $\Delta CoVaR$ of each clone is the same as that of the original institution, capturing the intuition of systemic risk in a herd.

Endogeneity of Systemic Risk. Note that each institution's $\Delta CoVaR$ is endogenous and depends on other institutions' risk taking. Hence, imposing a regulatory framework that forces institutions to lower their leverage and liquidity mismatch, L^i, lowers reduced form $\sigma^{i\cdot}(\cdot)$ in Equations (1, 2), and spillover effects captured in primitive $\bar{\sigma}^{i\cdot}(\cdot)$ in Equations (3, 4).

To the extent that a regulatory framework tries to internalize externalities, $\Delta CoVaR$ measures change. $\Delta CoVaR$ is an equilibrium concept which adapts to changing environments and provides incentives for institutions to reduce their exposure to risk if other institutions load excessively on it. Overall, we believe that $\Delta CoVaR$ can be a useful reduced-form analytical tool, but should neither serve as an explicit target for regulators, nor guide the setting of systemic taxes.[7]

4 $\Delta CoVaR$ Estimation

In this section we outline the estimation of $\Delta CoVaR$. In Section 4.1 we start with a discussion of alternative estimation approaches and then in Section 4.2 present the quantile regression based estimation method that we use in this paper. We go on to describe estimation of the time-varying, conditional $\Delta CoVaR$ in Section 4.3. Details on the econometrics are given in Appendix A and robustness checks including the $GARCH$ estimation of $\Delta CoVaR$ are provided in Appendix B. Section 4.4 provides estimates of $\Delta CoVaR$ and discusses properties of the estimates.

4.1 Alternative Empirical Approaches

The $CoVaR$ measure can be computed in various ways. Our main estimation approach relies on quantile regressions, as we explain in Sections 4.2 and 4.3. Quantile regressions are a numerically efficient way to estimate $CoVaR$. Bassett and Koenker (1978) and Koenker and Bassett (1978) are the first to derive the statistical properties of quantile regressions. Chernozhukov (2005) provides statistical properties for extremal quantile regressions, and Chernozhukov and Umantsev (2001) and Chernozhukov and Du (2008) discuss VaR applications.

It should be emphasized that quantile regressions are by no means the only way to estimate $CoVaR$. There is an emerging literature that prooposes alternative ways to estimate $CoVaR$. It can be computed from models with time-varying second moments, from measures of extreme events, using Bayesian methods, or using maximum likelihood estimation. We will now briefly discuss the

[7] The virtues and limitations of the $\Delta CoVaR$ thus aren't in conflict with Goodhart's law (see Goodhart (1975)).

most common alternative estimation procedures.

A particularly popular approach to estimating *CoVaR* is from multivariate *GARCH* models. We provide such alternative estimates using bivariate *GARCH* models in Appendix B. Girardi and Tolga Ergün (2013) also provide estimates of *CoVaR* from multivariate *GARCH* models. An advantage of the *GARCH* estimation is that it captures the dynamic evolution of systemic risk contributions explicitly.

CoVaR can also be calculated from copulas. Mainik and Schaanning (2012) present analytical results for *CoVaR* using copulas, and compare the properties to alternative systemic risk measures. Oh and Patton (2013) present estimates of *CoVaR* and related systemic risk measures from CDS spreads using copulas. An advantage of the copula methodology is that it allows estimation of the whole joint distribution including fat tails and heteroskedasticity.

Bayesian inference can also be used for *CoVaR* estimation. Bernardi, Gayraud, and Petrella (2013) present a Bayesian quantile regression framework based on a Markov chain Monte Carlo algorithm exploiting the Asymmetric Laplace distribution and its representation as a location-scale mixture of Normals.

A number of recent papers make distributional assumptions and use maximum likelihood techniques to estimate *CoVaR*. For example, Bernardi, Maruotti, and Petrella (2013) estimate *CoVaR* using a multivariate Markov switching model with a student-t distribution accounting for heavy tails and nonlinear dependence. Cao (2013) estimates a multivariate student-t distribution to calculate the joint distribution of *CoVaR* across firms "*Multi-CoVaR*". The maximum likelihood methodology has efficiency advantages relative to the quantile regressions if the distributional assumptions are correct.

In addition, there is a growing literature that develops the econometrics of quantile regressions for *CoVaR* estimation. Castro and Ferrari (2014) derive test statistics for *CoVaR* which can be used to rank firms according to systemic importance. Manganelli, Kim, and White (2011) propose a dynamic *CoVaR* estimation using a combination of quantile regressions and *GARCH*.

4.2 Estimation Method: Quantile Regression

We use quantile regressions to estimate *CoVaR*. In this section, the model underlying our discussion of the estimation procedure is an extremely stylized version of the reduced-form model discussed in Section 3. A more general version will be used in Section 4.3, and a full discussion is relegated to Appendix A.

To see the attractiveness of quantile regressions, consider the predicted value of a quantile regression of the financial sector losses X_q^{system} on the losses of a particular institution i for the $q\%$-quantile:

$$\hat{X}_q^{system|X^i} = \hat{\alpha}_q^i + \hat{\beta}_q^i X^i, \tag{7}$$

where $\hat{X}_q^{system|X^i}$ denotes the predicted value for a $q\%$-quantile of the system conditional on a return realization X^i of institution i.[8] From the definition of value at risk, it follows directly that

$$CoVaR_q^{system|X^i} = \hat{X}_q^{system|X^i}. \tag{8}$$

That is, the predicted value from the quantile regression of system return losses on the losses institution i gives the value at risk of the financial system conditional on X^i. The $CoVaR_q^{system|i}$ given X^i is just the conditional quantile. Using the particular predicted value of $X^i = VaR_q^i$ yields our $CoVaR_q^i$ measure ($CoVaR_q^{system|X^i=VaR_q^i}$). More formally, within the quantile regression framework, our specific $CoVaR_q^i$ measure is simply given by

$$CoVaR_q^i = VaR_q^{system|X^i=VaR_q^i} = \hat{\alpha}_q^i + \hat{\beta}_q^i VaR_q^i. \tag{9}$$

VaR^i can be obtained simply as the $q\%$-quantile of institution i's losses. So $\Delta CoVaR_q^i$ is

$$\Delta CoVaR_q^i = CoVaR_q^i - CoVaR_q^{system|VaR_{50}^i} = \hat{\beta}_q^i \left(VaR_q^i - VaR_{50}^i \right). \tag{10}$$

As explained in Section 3, we are referring to the conditional VaR expressed in percentage loss rates. The unconditional VaR_q^i and $\Delta CoVaR_q^i$ estimates for Figure *1* are based on Equation (10).

Measuring Losses. Our analysis relies on publicly available data and focuses on return losses to market equity, $X_{t+1}^i = -\Delta N_{t+1}^i / N_t^i$. Alternatively one could also conduct the analysis with book equity data, defined as the residual between total assets and liabilities. Systemic risk supervisors

[8] Note that a median regression is the special case of a quantile regression where $q = 50$.We provide a short synopsis of quantile regressions in the context of linear factor models in Appendix A. Koenker (2005) provides a more detailed overview of many econometric issues.

While quantile regressions are used regularly in many applied fields of economics, their applications to financial economics are limited.

have a larger set of data at their disposal. Hence they could also compute the VaR^i and $\Delta CoVaR^i$ from a broader definition of equity which would include equity in off-balance-sheet items, exposures from derivative contracts, and other claims that are not properly captured by publicly traded equity values. A more complete description would potentially improve the measurement of systemic risk and systemic risk contribution. The analysis could also be extended to compute the risk measures for assets or liabilities, separately. For example, the $\Delta CoVaR^i$ for liabilities captures the extent to which financial institutions rely on debt funding—such as repos or commercial paper—that can collapse during systemic risk events. Total assets are most closely related to the supply of credit to the real economy, and risk measures for regulatory purposes are typically computed for total assets (earlier versions of this paper used market valued total assets as a basis for the systemic risk contribution calculations).

Financial Institution Data. We focus on publicly traded financial institutions, consisting of four financial sectors: commercial banks, security broker-dealers (including the investment banks), insurance companies, and real estate companies. We start our sample in 1971Q1 and end it in 2013Q2. The data thus cover six recessions (1974-75, 1980, 1981, 1990-91, 2001, and 2007-09) and several financial crises (including 1987, 1994, 1997, 1998, 2000, 2008, and 2011). We also present a robustness check where we use financial institution data going back to 1926Q3. We obtain daily market equity data from CRSP and quarterly balance sheet data from COMPUSTAT. We have a total of 1823 institutions in our sample. For bank holding companies, we use additional asset and liability variables from the FR Y9-C reports. Overall the main part of our empirical analysis is carried out with weekly observations, allowing reasonable inference even with the relatively short samples available. Appendix C provides a detailed description of the data.

4.3 Time Variation Associated with Systematic State Variables

The previous section presented a methodology for estimating $\Delta CoVaR$ that is constant over time. To capture time variation in the joint distribution of X^{system} and X^i, we estimate $VaRs$ and $\Delta CoVaRs$ as a function of state variables, allowing us to model the evolution of the conditional distributions over time. We indicate time-varying $CoVaR^i_{q,t}$ and $VaR^i_{q,t}$ with a subscript t and estimate the time variation conditional on a vector of lagged state variables M_{t-1}. We run the following quantile regressions in the weekly data (where i is an institution):

$$X_t^i = \alpha_q^i + \gamma_q^i M_{t-1} + \varepsilon_{q,t}^i, \tag{11a}$$

$$X_t^{system|i} = \alpha_q^{system|i} + \gamma_q^{system|i} M_{t-1} + \beta_q^{system|i} X_t^i + \varepsilon_{q,t}^{system|i}. \tag{11b}$$

We then generate the predicted values from these regressions to obtain

$$VaR_{q,t}^i = \hat{\alpha}_q^i + \hat{\gamma}_q^i M_{t-1}, \tag{12a}$$

$$CoVaR_{q,t}^i = \hat{\alpha}_q^{system|i} + \hat{\gamma}_q^{system|i} M_{t-1} + \hat{\beta}_q^{system|i} VaR_{q,t}^i. \tag{12b}$$

Finally, we compute $\Delta CoVaR_{q,t}^i$ for each institution:

$$\Delta CoVaR_{q,t}^i = CoVaR_{q,t}^i - CoVaR_{50,t}^i \tag{13}$$

$$= \hat{\beta}_q^{system|i} \left(VaR_{q,t}^i - VaR_{50,t}^i \right). \tag{14}$$

From these regressions, we obtain a panel of weekly $\Delta CoVaR_{q,t}^i$. For the forecasting regressions in Section 5, we generate a weekly time series of the $\Delta^{\$}CoVaR_{q,t}^i$ by multiplying $\Delta CoVaR_{q,t}^i$ by the respective market equity ME_t^i. We then obtain a quarterly time series of $\Delta^{\$}CoVaR_{q,t}^i$ by averaging the weekly risk measures within each quarter. In order to obtain stationary variables, we furthermore divide each $\Delta^{\$}CoVaR_{q,t}^i$ by the cross-sectional average of market equity N_t^i.

State variables. To estimate the time-varying $\Delta CoVaR_t$ and VaR_t, we include a set of state variables M_t that are (i) well known to capture time variation in conditional moments of asset returns, and (ii) liquid and easily tractable. The systematic state variables M_{t-1} are lagged. They should not be interpreted as systematic risk factors, but rather as conditioning variables that are shifting the conditional mean and the conditional volatility of the risk measures. Note that different firms can load on these risk factors in different directions, so that particular correlations of the risk measures across firms—or correlations of the different risk measures for the same firm—are not imposed by construction. We restrict ourselves to a small set of risk factors to avoid overfitting the data. Our factors are:

(i) The *change in the three-month yield* from the Federal Reserve Board's H.15. We use the change in the three-month Treasury bill rate because we find that the change, not the level, is most significant in explaining the tails of financial sector market-valued asset returns.

(ii) The *change in the slope of the yield curve*, measured by the yield spread between the long term bond composite and the three-month bill rate obtained from the Federal Reserve Board's H.15 release.

(iii) A short term *"TED spread,"* defined as the difference between the three-month Libor rate and the three-month secondary market bill rate. This liquidity spread measures short-term funding liquidity risk. We use the three-month Libor rate that is available from the British Bankers Association, and obtain the three-month Treasury rate from the Federal Reserve Bank of New York.

(iv) The *change in the credit spread* between Moody's *Baa*-rated bonds and the ten year Treasury rate from the Federal Reserve Board's H.15 release.

(v) The weekly *market return* computed from the S&P500.

(vi) The weekly *real estate sector return* in excess of the market financial sector return (from the real estate companies with SIC code 65-66).

(vii) *Equity volatility*, which is computed as the 22 day rolling standard deviation of the daily CRSP equity market return.

Table 1 provides summary statistics of the state variables. The *1%*-stress level is the level of each respective variable during the *1%* worst weeks for financial system asset returns. For example, the average of the equity volatility during the stress periods is 2.27, as the worst times for the financial system include the times when the equity volatility was highest. Similarly, the stress level corresponds to a high level of the liquidity spread, a sharp decline in the Treasury bill rate, sharp increases of the term and credit spreads, and large negative market return realizations.

[Table 1 here]

4.4 $\Delta CoVaR$ Summary Statistics

Table 2 provides the estimates of our weekly conditional $\Delta CoVaR^{i}_{99,t}$ measures that we obtain from using quantile regressions. Each of the summary statistics constitutes the universe of financial institutions.

[Table 2 here]

Line (1) of Table 2 give the summary statistics for the market equity loss rates; line (2) gives the summary statistics for the $VaR^i_{99,t}$ for each institution; line (3) gives the summary statistics for $\Delta CoVaR^i_{99,t}$; line (4) gives the summary statistics for the *stress*-$\Delta CoVaR^i_{99,t}$; and line (5) gives the summary statistics for the financial system value at risk, $VaR^{system}_{99,t}$. The *stress*-$\Delta CoVaR^i_{99,t}$ is estimated by substituting the worst 1% of state variable realizations into the fitted model for $\Delta CoVaR^i_{99,t}$ (see equations 12a and 12b).

Recall that $\Delta^{\%} CoVaR^i_t$ measures the marginal contribution of institution i to overall systemic risk and reflects the difference between the value at risk of the financial universe conditional on the stressed and the median state of institution i. We report the mean, standard deviation, and number of observations for each of the items in Table 2. We have a total of *1823* institutions in the sample, with an average length of *736* weeks. The institution with the longest history spans all *2209* weeks of the 1971Q1-2013Q2 sample period. We require institutions to have at least *260* weeks of equity return data in order to be included in the panel. In the following analysis, we focus primarily on the 99% and the 95% quantiles, corresponding to the worst *22* weeks and the worst *110* weeks over the sample horizon, respectively. It is straightforward to estimate more extreme tails following Chernozhukov and Du (2008) by extrapolating the quantile estimates using extreme value theory, an analysis that we leave for future research. In the following analysis, we largely find results to be qualitatively similar for the 99% and the 95% quantiles.

[Table 3 here]

We obtain time variation of the risk measures by running quantile regressions of equity losses on the lagged state variables. We report average $t-$stats of these regressions in Table 3. A higher equity volatility, higher TED spread, and lower market return tend to be associated with a larger risk measures. In addition, increases in the three-month yield, increases in the term spread, and increases the credit spread tend to be associated with larger risk. Overall, the average significance of the conditioning variables reported in Table 3 show that the state variables do indeed proxy for the time variation in the quantiles and particularly in $CoVaR$.

4.5 $\Delta CoVaR$ versus VaR

Figure 1 shows that, *across institutions*, there is only a very loose link between an institution's VaR^i and its contribution to systemic risk as measured by $\Delta CoVaR^i$. Hence, imposing financial regulation solely based on the risk of an institution in isolation might not be sufficient to insulate the financial sector against systemic risk. Figure 2 repeats the scatter plot the time series average of $\Delta CoVaR_t^i$ against the time series average of VaR_t^i for all institutions in our sample, for each of the four financial industries. While there is only a weak link between $\Delta CoVaR_t^i$ and VaR_t^i in the cross section, there is a strong time series relationship. This can be seen in Figure 3, which plots the time series of the $\Delta CoVaR_t^i$ and VaR_t^i for a sample of the largest firms over time.

[Figure 2 here]

[Figure 3 here]

4.6 Out of Sample Estimates of $\Delta CoVaR$

Figure 4 shows the weekly $\Delta CoVaR$ of Lehman Brothers, Bank of America, JP Morgan, and Goldman Sachs for the crisis period 2007-08. The three vertical bars indicate the onset of the crisis when BNP reported funding problems (August 7, 2007), the Bear Stearns crisis (March 14, 2008) and the Lehman Bankruptcy (September 15, 2008). Each of the plots shows both the in-sample and the out-of sample estimate of $\Delta CoVaR$ using expanding windows.

[Figure 4 here]

Among these four figures, Lehman Brothers clearly stands out as its $\Delta CoVaR$ rises sharply with the onset of the financial crisis in the summer of 2007, and remains elevated throughout the middle of 2008. While the $\Delta CoVaR$ for Lehman declined following the Bear Stearns crisis, it is steadily increasing from mid-2008. It is also noteworthy that the level of $\Delta CoVaR$ for Goldman Sachs and Lehman is materially larger than those for Bank of America and JP Morgan, reflecting the fact that those were stand alone dealers, at least until October 2008 when Goldman Sachs was transformed into a Bank Holding Company with access to government backstops.

4.7 Historical $\Delta CoVaR$

It is potentially challenging to estimate systemic risk, as major financial crises occur rarely, making the estimation of tail dependence between individual institutions and the financial system statistically challenging. In order to understand the extent to which $\Delta CoVaR$ estimates are sensitive to the length of the estimation horizon, we select a subset of financial firms with equity market returns that extend back as far as 1926Q3.[9] We then compare the estimated $\Delta CoVaR_t^i$ time series since 1926Q3 to the one estimated since 1971Q1, as shown in Figure 5.

[Figure 5 here]

The comparison of the $\Delta CoVaR$s estimated over very long time horizons reveal two things. Firstly, systemic risk measures were not as high in the Great Depression as they were during the recent financial crisis. This could be an artifact of the composition of the firms, as the four firms with a very long time series are not necessarily a good proxy for the type of risks that emerged during the Great Depression.[10] Secondly, the longer time series does exhibit fatter tails, and in fact generates a slightly higher measure of systemic risk over the whole time horizon. Tail risk thus appears to be biased downwards in the shorter sample. However, the correlation between the shorter and longer time series is 96 percent. We conclude that the shorter time span for the estimation since 1971 provides an adequate estimate of systemic risk contributions in comparison to the longer estimation since 1926.

5 *Forward-$\Delta CoVaR$*

In this section we link $\Delta CoVaR$ to financial institutions' characteristics to address two key issues: procyclicality and measurement accuracy. Procyclicality refers to the time series component of systemic risk. Systemic risk builds in the background during seemingly quiet times, when volatility is low (the volatility paradox). Any regulation that relies on contemporaneous risk measure estimates would be unnecessarily loose in periods when imbalances are building up and unnecessarily tight after crises erupt. In other words, such regulation would amplify the adverse impacts after bad shocks, while also amplifying balance sheet growth and risk taking in expansions.[11] We propose to focus on variables that predict *future*, rather than *contemporaneous*, $\Delta^{\$} CoVaR$. In this section, we calculate a forward-looking systemic risk contribution measure that can serve as a useful analytical

[9] The four financial firms that we use in the basket are Adams Express Co (ADX), Century Business Credit Inc (CTY), Lee National Corp (LR), and Power REIT (PW).

[10] Bank equity was generally not traded in public equity markets until the 1960s.

[11] See Estrella (2004), Kashyap and Stein (2004), and Gordy and Howells (2006) for studies of the procyclical nature of capital regulation.

tool for financial stability monitoring, and may provide some guidance for (countercyclical) macroprudential policy. We first present the dependence of $\Delta CoVaR$ on lagged characteristics. We then use the characteristics to construct the *forward*-$\Delta CoVaR$.

Second, any tail risk measure, estimated at a high frequency, is by its very nature imprecise. Quantifying the relationship between $\Delta CoVaR$ and more easily observable institution-specific variables, such as size, leverage, and maturity mismatch, deals with the measurement inaccuracy in direct estimation of $\Delta CoVaR$, at least to some extent. For this purpose, we project $\Delta CoVaR$ onto explanatory variables. Since the analysis involves the comparison of $\Delta CoVaR$ across firms, we use $\Delta^{\$} CoVaR$, as the dollar valued number takes the size of firms into account.

For each firm we regress $\Delta^{\$} CoVaR$ on the institution i's characteristics, as well as the conditioning macro-variables. More specifically, for a forecast horizon $h = 1, 4, 8$ quarters, we run regressions

$$\Delta^{\$}CoVaR^i_{q,t} = a + cM_{t-h} + bX^i_{t-h} + \eta^i_t, \tag{15}$$

where X^i_{t-h} are the vector of characteristics for institution i, M_{t-h} is the vector of macro state variables lagged h quarters, and η^i_t is an error term.

We label the h quarters predicted value *forward*-$\Delta^{\$} CoVaR$,

$$\Delta^{\text{Fwd}}_h CoVaR^i_{q,t} = \hat{a} + \hat{c}M_{t-h} + \hat{b}X^i_{t-h}. \tag{16}$$

5.1 $\Delta CoVaR$ Predictors

As previously, the macro-state variables are the change in the three-month yield, the change in the slope of the yield curve, the TED spread, the change in the credit spread, the market return, the real estate sector return, and equity volatility.

Institutions' Characteristics. The main characteristics that we consider are the following:
(i) *Leverage.* For this, we use the ratio of market value assets to market equity.
(ii) The *maturity mismatch.* This is defined as the ratio of book assets to short term debt less short term investments less cash.
(iii) *Size.* As a proxy for size, we use the log of total market equity for each firm divided by the log

of the cross sectional average of market equity.

(iv) A *boom* indicator. Specifically, this indicator gives (for each firm) the number of consecutive quarters of being in the top decile of the market-to-book ratio across firms.

[Table 4 here]

[Table 5 here]

Table 4 provides the summary statistics for $\Delta^{\$} CoVaR_t^i$ at the quarterly frequency, and the quarterly firm characteristics. In Table 5, we ask whether systemic risk contribution can be forecast cross-sectionally by lagged characteristics at different time horizons. Table 5 shows that firms with higher leverage, more maturity mismatch, larger size, and higher equity valuation according to the boom variable tend to be associated with larger systemic risk contributions one quarter, one year, and two years later. These results hold for the 99% $\Delta^{\$} CoVaR$ and the 95% $\Delta^{\$} CoVaR$. The coefficients in Table 5 are sensitivities of $\Delta^{\$} CoVaR_t^i$ with respect to the characteristics expressed in units of basis points of systemic risk contribution. For example, the coefficient of 14.5 on the leverage forecast at the two-year horizon implies that an increase in leverage (say, from 15 to 16) of an institution is associated with an increase in systemic risk contribution as measured by $\Delta CoVaR$ of 14.5 basis points of quarterly market equity losses at the 95% systemic risk level. Columns (1)-(3) and (4)-(6) of Table 5 can be understood as a "term structure" of systemic risk contribution if read from right to left. The comparison of Panels A and B provide a gauge of the "tailness" of systemic risk contribution.

Importantly, these results allow us to connect $\Delta CoVaR$ with frequently and reliably measured institution-level characteristics. $\Delta^{\$} CoVaR$—like any tail risk measure—relies on relatively few extreme-crises data points. Hence, adverse movements, especially followed by periods of stability, can lead to sizable increases in tail risk measures. In contrast, measurement of characteristics such as size are very robust, and they can be measured more reliably at higher frequencies. The debate surrounding "too big to fail" suggests that size is considered by some to be the all-dominating variable, and, subsequently, that large institutions should face more stringent regulations then smaller institutions. As mentioned above, focusing on size alone fails to acknowledge that many small institutions can be systemic as part of a herd. Our solution to this problem is to combine the virtues of both types of measures by projecting the systemic risk contribution measure $\Delta^{\$} CoVaR$ on multiple, more frequently observable variables, providing a tool that might prove useful in identifying "systemically important financial institutions." The regression coefficients of Table 5 can be used to weigh the relative importance of various firm characteristics. For example, the trade-off between size and leverage is given by the ratio of the two respective coefficients of our forecasting regressions. Of course, in order to achieve a given level of systemic risk contribution per units of total assets, instead of lowering the size, the bank could also reduce its maturity mismatch or improve its systemic risk profile along other dimensions. In fact, in determining systemic importance of global banks for regulatory purposes, the Basel Committee on Bank Supervision BCBS (2013) relies on frequently observed firms characteristics.

Additional Characteristics for Bank Holding Companies. Ideally, one would like to link the systemic risk contribution measure to more institutional characteristics than simply size, leverage, maturity mismatch etc. If one restricts the sample to bank holding companies, we have more granular balance sheet items. On the asset side of banks' balance sheets, we use loans, loan-loss allowances, intangible loss allowances, intangible assets, and trading assets. Each of these asset composition variables is expressed as a percentage of total book assets. The cross-sectional regressions with these asset composition variables are reported in Panel A of Table 6. In order to capture the liability side of banks' balance sheets, we use interest-bearing core deposits, non-interest-bearing deposits, large time deposits, and demand deposits. Again, each of these variables is expressed as a percentage of total book assets. The variables can be interpreted as refinements of the maturity mismatch variable used earlier. The cross-sectional regressions with the liability aggregates are reported in Panel B of Table 6.

[Table 6 here]

Panel A of Table 6 shows which types of liability variables are significantly increasing or decreasing systemic risk contribution. Bank holding companies with a higher fraction of non-interest-bearing deposits have a significantly higher systemic risk contribution, while interest bearing core deposits and large time deposits are decreasing the forward estimate of $\Delta^{\$}CoVaR$. Non-interest-bearing deposits are typically held by nonfinancial corporations and households, and can be quickly reallocated across banks conditional on stress in a particular institution. Interest-bearing core deposits and large time deposits, on the other hand, are more stable sources of funding and are thus decreasing the systemic tail risk contribution (i.e., they have a negative sign). The maturity mismatch variable that we constructed for the universe of financial institutions is no longer significant once we include the more refined liability measures for the bank holding companies.

Panel B of Table 6 shows that the fraction of trading assets is a particularly good predictor for systemic risk contribution, with the positive sign indicating that increased trading activity is associated with greater systemicness (as gauged by $\Delta CoVaR$) of bank holding companies. Larger shares of loans also tend to increase banks' contribution to aggregate systemic risk, while intangible assets do not have much predictive power. Finally, loan loss reserves do not appear significant, likely because they do not have a strong forward-looking component.

In summary, the results of Table 6, in comparison to Table 5, show that more information about the balance sheet characteristics of financial institutions can potentially improve the estimated *forward*-$\Delta^{\$}CoVaR$. We expect additional data that capture particular activities of financial institutions, as well as supervisory data, to lead to further improvements in the estimation precision of forward systemic risk contribution.

5.2 *Forward*-$\Delta CoVaR$

The predicted values of the Regression (15) yields a time-series of *forward*-$\Delta CoVaR$ for each institution i. In Figure 6 we plot the $\Delta CoVaR$ together with the two-year *forward*-$\Delta CoVaR$ for the average of the largest 50 financial institutions, where the size is computed as of 2007Q1. The *forward*-$\Delta CoVaR$ is estimated in-sample through the end of 2001, and out-of-sample since 2002Q1. The figure clearly shows the strong negative correlation of the contemporaneous $\Delta CoVaR$ and the *forward*-$\Delta CoVaR$. In particular, during the credit boom of 2003-06, the contemporaneous $\Delta CoVaR$ is estimated to be small, while the forward $\Delta CoVaR$ is large. Macroprudential regulation based on the *forward*-$\Delta CoVaR$ is thus countercyclical.

[Figure 6 here]

From an economic perspective, the countercyclicality of the forward measure reflects the fact that risk taking of intermediaries is endogenously high in expansions, which makes them vulnerable to adverse economic shocks. For example, in the equilibrium model of Adrian and Boyarchenko (2012), contemporaneous volatility is low in booms, which relaxes risk management constraints on intermediaries, allowing them to increase risk taking, and making them more vulnerable to shocks.

Similarly, in Brunnermeier and Sannikov (2014), credit booms foreshadow episodes of increased financial fragility.

5.3 Cross-Sectional Predictive Power of *Forward-*$\Delta CoVaR$

Next, we test the extent to which the *forward-*$\Delta CoVaR^i$ predicts realized $\Delta CoVaR^i$ across institutions during the financial crisis. To do so, we calculate *forward-*$\Delta CoVaR^i$ for each firm up to 2006Q4. We also calculate the crisis $\Delta CoVaR^i$ for each firm for the 2007Q2-2009Q2 period. In order to show the out-of-sample forecasting performance of *forward-*$\Delta CoVaR^i$, regress the *crisis-*$\Delta CoVaR^i_{95}$ (computed for 2007Q1 -2008Q4) on the *forward-*$\Delta CoVaR^i_{95}$ (as of 2006Q4). We report the 95% level, though we found that the 99% gives very similar results.

[Table 7 here]

Table 7 shows that the two year ahead *forward-*$\Delta CoVaR$ as of the end of 2006Q4 was able to explain over one third of the cross sectional variation of realized $\Delta CoVaR$ during the crisis. The one year ahead forecast of 2008Q4 using data as of 2007Q4 only predicts one fifth of the cross sectional dispersion, while the one quarter ahead forecast for 2008Q4 as of 2008Q3 predicts over three quarters of the cross section of systemic risk. The last two columns of Table 7 also show the one year and one quarter ahead forecasts of realized $\Delta CoVaR$ as of 2006Q4. We view these findings as very strong ones, indicating that the systemic risk measures have significant forecasting power for the cross section of realized systemic risk.

6 Conclusion

During financial crises or periods of financial intermediary distress, tail events tend to spill across financial institutions. Such spillovers are preceded by a risk-buildup phase. Both elements are important contributors to financial system risk. $\Delta CoVaR$ is a parsimonious measure of systemic risk contribution that complements measures designed for individual financial institutions. $\Delta CoVaR$ broadens risk measurement to allow a macroprudential perspective. The *forward-*$\Delta CoVaR$ is a forward-looking measure of systemic risk contribution. It is constructed by projecting $\Delta CoVaR$ on lagged firm characteristics such as size, leverage, maturity mismatch, and industry dummies. This forward-looking measure can potentially be used in macroprudential policy applications.

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Appendices

A *CoVaR* Estimation via Quantile Regressions

This appendix explains how to use quantile regressions to estimate *VaR* and *CoVaR*. As discussed in footnote 5, the model considered here is a special case of the stylized financial system analyzed in Section 3, with particularly simple expressions for $\mu^j(\cdot)$, $\sigma^{ji}(\cdot)$ and $\sigma^{jj}(\cdot)$. Specifically, we assume that losses X_t^i have the following linear factor structure

$$X_{t+1}^j = \phi_0 + M_t\phi_1 + X_{t+1}^i\phi_2 + (\phi_3 + M_t\phi_4)\,\Delta Z_{t+1}^j, \tag{17}$$

where M_t is a vector of state variables. The error term ΔZ_{t+1}^j is assumed to be i.i.d. with zero mean and unit variance, and $E\left[\Delta Z_{t+1}^j | M_t, X_{t+1}^i\right] = 0$. The conditional expected return $\mu^j\left[X_{t+1}^j | M_t, X_{t+1}^i\right] = \phi_0 + M_t\phi_1 + X_{t+1}^i\phi_2$ depends on the set of state variables M_t and on X_{t+1}^i, and the conditional volatility $\sigma_t^{jj}\left[X_{t+1}^j | M_t, X_{t+1}^i\right] = (\phi_3 + M_t\phi_4)$ is a direct function of the state variables M_t.[12] The coefficients ϕ_0, ϕ_1, and ϕ_2 could be estimated consistently via OLS of X_{t+1}^i on M_t and X_{t+1}^i. The predicted value of such an OLS regression would be the mean of X_{t+1}^j conditional on M_t and X_{t+1}^i. In order to compute the *VaR* and *CoVaR* from OLS regressions, one would have to also estimate ϕ_3, ϕ_4, and ϕ_5, and then make distributional assumptions about ΔZ_{t+1}^j.[13] The quantile regressions incorporate estimates of the conditional mean and the conditional volatility to produce conditional quantiles, without the distributional assumptions that would be needed for estimation via OLS.

Instead of using OLS regressions, we use quantile regressions to estimate model (17) for different percentiles. We denote the cumulative distribution function (cdf) of ΔZ^j by $F_{\Delta Z^j}(\cdot)$, and its inverse cdf by $F_{\Delta Z^j}^{-1}(q)$ for the q%-quantile. It follows immediately that the inverse cdf of X_{t+1}^j is

$$F_{X_{t+1}^j}^{-1}\left(q | M_t, X_{t+1}^i\right) = \alpha_q + M_t\gamma_q + X_{t+1}^i\beta_q, \tag{18}$$

where $\alpha_q = \phi_0 + \phi_3 F_{\Delta Z^j}^{-1}(q)$, $\gamma_q = \phi_1 + \phi_4 F_{\Delta Z^j}^{-1}(q)$, and $\beta_q = \phi_2$ for quantiles $q \in (0, 100)$. We call

[12] Alternatively, X_{t+1}^i could have also been introduced as a direct determinant of the volatility. The model would then just be $X_{t+1}^j = \phi_0 + M_t\phi_1 + X_{t+1}^i\phi_2 + \left(\phi_3 + M_t\phi_4 + X_{t+1}^i\phi_5\right)\Delta Z_{t+1}^j$.

[13] The model (17) could alternatively be estimated via maximum likelihood if distributional assumptions about ΔZ are made.

$F^{-1}_{X^j_{t+1}}\left(q|M_t, X^i_{t+1}\right)$ the conditional quantile function. From the definition of *VaR,* we obtain

$$VaR^j_{q,t+1} = \inf_{VaR^j_{q,t+1}} \left\{\Pr\left(X^j_{t+1}|\left\{M_t, X^i_{t+1}\right\} \leq VaR^j_{q,t+1}\right) \geq q\%\right\} = F^{-1}_{X^j_{t+1}}\left(q|M_t, X^i_{t+1}\right).$$

The conditional quantile function $F^{-1}_{X^j_{t+1}}\left(q|M_t, X^i_{t+1}\right)$ is the $VaR^j_{q,t+1}$ conditional on M_t and X^i_{t+1}. By conditioning on $X^i_{t+1} = VaR^i_{q,t+1}$, we obtain the $CoVaR^{j|i}_{q,t+1}$ from the quantile function:

$$\begin{aligned} CoVaR^{j|i}_{q,t+1} &= \inf_{VaR^j_{q,t+1}} \left\{\Pr\left(X_{t+1}|\left\{M_t, X^i_{t+1} = VaR^i_{q,t+1}\right\} \leq VaR^j_{q,t+1}\right) \geq q\%\right\} \\ &= F^{-1}_{X^j_{t+1}}\left(q|M_t, X^i_{t+1} = VaR^i_{q,t+1}\right). \end{aligned} \quad (19)$$

We estimate the quantile function as the predicted value of the $q\%$-quantile regression of X^i_{t+1} on M_t and X^j_{t+1} by solving

$$\min_{\alpha_q,\beta_q,\gamma_q} \sum_t \begin{cases} q\% \left|X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q\right| & \text{if } \left(X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q\right) \geq 0 \\ (1-q\%) \left|X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q\right| & \text{if } \left(X^j_{t+1} - \alpha_q - M_t\beta_q - X^i_{t+1}\gamma_q\right) < 0 \end{cases}.$$

Bassett and Koenker (1978) and Koenker and Bassett (1978) provide statistical properties of quantile regressions. Chernozhukov and Umantsev (2001) and Chernozhukov and Du (2008) discuss *VaR* applications of quantile regressions.

B Robustness Checks

B.1 GARCH $\Delta CoVaR$

One potential shortcoming of the quantile estimation procedure described in Section 4 is that it models time varying moments only as a function of aggregate state variables. An alternative approach is to estimate bivariate *GARCH* models to obtain the time-varying covariance between institutions and the financial system. As a robustness check, we estimate $\Delta CoVaR$ using a bivariate diagonal *GARCH* model (*DVECH*) and find that this method produces estimates quite similar to the quantile regression method, leading us to the conclusion that the quantile regression framework is sufficiently flexible to estimate $\Delta CoVaR$. We begin by outlining a simple Gaussian framework under which $\Delta CoVaR$ has a closed-form expression, and then present the estimation results. The Gaussian framework is a special case of the stylized financial system we develop in Section 3, with deterministic mean and covariance terms, and jointly normally distributed latent shock processes.

Gaussian Model Assume firm and system losses follow a bivariate normal distribution:

$$\left(X^i_t, X^{system}_t\right) \sim N\left(\left(\begin{array}{c} 0 \\ 0 \end{array}\right), \left(\begin{array}{cc} \left(\sigma^i_t\right)^2 & \rho^i_t\sigma^i_t\sigma^{system}_t \\ \rho^i_t\sigma^i_t\sigma^{system}_t & \left(\sigma^{system}_t\right)^2 \end{array}\right)\right) \quad (20)$$

By properties of the multivariate normal distribution, the distribution of system losses conditional on firm losses is also normally distributed:

$$X_t^{system}|X_t^i \sim N\left(\frac{X_i^t \sigma_t^{system} \rho_t^i}{\sigma_t^i}, (1-\left(\rho_t^i\right)^2)\left(\sigma_t^{system}\right)^2\right) \tag{21}$$

Using the definition of $CoVaR_{q,t}^i$

$$\Pr\left(X_t^{system}|X_t^i = VaR_{q,t}^i \leq CoVaR_{q,t}^i\right) = q\%, \tag{22}$$

we find

$$\Pr\left(\left[\frac{X_t^{system} - X_t^i \rho_t^i \sigma_t^{system}/\sigma_t^i}{\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2}}\middle| X_t^i = VaR_{q,t}^i\right] \leq \frac{CoVaR_{q,t}^i - X_t^i \rho_t \sigma_t^{system}/\sigma_t^i}{\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2}}\right) = q\%. \tag{23}$$

Note that $\left[\frac{X_t^{system} - X_t^i \rho_t^i \sigma_t^{system}/\sigma_t^i}{\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2}}\right] \sim N(0,1)$. Also, the firm value-at-risk is given by $VaR_{q,t}^i = \Phi^{-1}(q\%)\,\sigma_t^i$. Combining the two, and using the simple expression for VaR if losses are distributed as in Equation (21), we can write:

$$CoVaR_{q,t}^i = \Phi^{-1}(q\%)\,\sigma_t^{system}\sqrt{1-\left(\rho_t^i\right)^2} + \Phi^{-1}(q\%)\,\rho_t^i \sigma_t^{system} \tag{24}$$

Because $\Phi^{-1}(50\%) = 0$, solving for $\Delta CoVaR$ gives:

$$\Delta CoVaR_{q,t}^i = \Phi^{-1}(q\%)\,\rho_t^i \sigma_t^{system} \tag{25}$$

In the Gaussian framework, $\Delta CoVaR$ is thus pinned down by three determinants: the correlation, the volatility of the financial system, and the Gaussian quantile. Cross-sectionally, the only ingredient that varies is the correlation of firms with the system, while over time, both the correlation and the system volatility are changing. While the time variation of $\Delta CoVaR$ is a function of the state variables M_t from section 4.3 in the quantile regression approach, it is only a function of the time varying variances and covariances in the $GARCH$ approach. Despite these very different computations, we will see that the resulting $\Delta CoVaR$ estimates are—perhaps surprisingly—similar.

Estimation We estimate a bivariate diagonal vech $GARCH$(1,1) for each institution in our sample.[14] As a robustness check, we estimated the panel regressions of Section 5 on a matched sample of 1035 institutions which our $GARCH$ estimates converged.

[Table 8 here]

The results in Table 8 show the coefficients on size, leverage, maturity mismatch, and boom are qualitatively similar between the $GARCH$ and quantile estimation methods. Hence the economic determinants of systemic risk contributions across firms does not appear to be dependent on the particular estimation method that is used to compute $CoVaR$. Figure 7 shows the GARCH and quantile estimates of $\Delta CoVaR$ for Citibank, Goldman Sachs, Metlife, and Wells Fargo showing close similarity across firms and over time.

[14] We were able to get convergence of the Garch model for 56% of firms. We found that convergence of the models in our data is very sensitive to both missing values and extreme returns. Truncation of returns generally, but not consistently, resulted in an increase in the fraction of the models that converged.

[Figure 7 here]

Specification Tests In order to compare the performance of quantile estimates and the *GARCH* model more formally, we perform the conditional specification tests for value at risk measures proposed by Christoffersen (1998). Table 10 shows the fraction of firms whose *VaR* estimates have correct conditional coverage, i.e. the fraction of firms for which the probability of losses that exceed the *VaR* is below the 95% or 99% values. The conditional coverage is computed via a likelihood ratio test whose alternative hypothesis is that the probability the *VaR* forecast is correct conditional on past observations and is equal to the specified probability. The table then presents the fraction of firms for which the null hypothesis is rejected.

[Table 10 here]

Table 10 shows that *GARCH* estimates perform better for the 95th percentile, while the quantile estimates perform considerably better for the 99^{th} percentile. For example, at the 5% level, the conditional coverage of the quantile model is .46 for the 99^{th} percentile and .52 for the 95^{th} percentile, while, for the *GARCH* model, the respective fractions are .20 and .70.

B.2 Alternative Financial System Losses

The financial system loss variable X_t^{system} used in the paper is the weekly loss on the market equity of the financial system, as proxied by the universe of financial institutions. This measure is generated by taking average market equity losses, weighted by lagged market eqiuty. One concern with this methodology is that it might introduce a mechanical correlation between each institution and the financial system proportional to the relative size of the financial institution. We check to see if such a mechanical correlation is driving our results by reestimating institutions' $\Delta CoVaR$ using system return variables formed from the value weighted returns of all other institutions in the sample, leaving out the institution for which $\Delta CoVaR$ is being estimated.

[Table 9 here]

We find a very strong correlation across institutions, and across time, for the two different systemic risk contribution measures. In fact, even for the largest institutions we find a very strong correlation between the baseline system return variable and the modified system return, with correlation coefficients over 99%. Table 9 reports the *forward*-$\Delta CoVaR$ regressions for the 95% level using both specifications. The coefficients under the two specifications are statistically indistinguishable, indicating that this mechanical correlation is not driving our results.

C Data Description

C.1 CRSP and COMPUSTAT Data

As discussed in the paper, we estimate $\Delta CoVaR$ for market equity losses of financial institutions. We start with daily equity data from CRSP for all financial institutions with two-digit COMPUSTAT SIC codes between 60 and 67 inclusive, indexed by PERMNO. Banks correspond to SIC codes 60, 61, and 6712; insurance companies correspond to SIC codes 63-64, real estate companies correspond to SIC codes 65-6, and broker-dealers are SIC code 67 (except for the bank holding companies, 6712). All other financial firms in our initial sample are placed in an "other" category. We manually adjust the COMPUSTAT SIC codes to account for the conversions of several large institutions into bank holding companies in late 2008, but otherwise do not find time varying industry classifications. Following the asset pricing literature, we keep only ordinary common shares (which exclude certificates, ADRs, SBIS, REITs, etc.) and drop daily equity observations with missing or negative prices or missing returns. Our keeping only ordinary common shares excludes several large international institutions, such as Credit Suisse and Barclays, which are listed in the United States as American Depository Receipts.

The daily data are collapsed to a weekly frequency and merged with quarterly balance sheet data from the CRSP/COMPUSTAT quarterly dataset. The quarterly data are filtered to remove leverage and book-to-market ratios less than zero and greater than 100. We also apply 1% and 99% truncation to maturity mismatch.

Market equity and balance sheet data are adjusted for mergers and acquisitions using the CRSP daily dataset. We use a recursive algorithm to traverse the CRSP DELIST file to find the full acquisition history of all institutions in our sample. The history of acquired firms is collapsed into the history of their acquirers. For example, we account for the possibility that firm A was acquired by firm B, which was then acquired by firm C, etc. Our final panel therefore does not include any firms that we are able to identify as having been ultimately acquired by another firm in our universe. The final estimation sample is restricted to include firms with at least 260 weeks of non-missing market equity returns. To construct the overall financial system portfolio (for $j = system$), we simply compute the average market equity-valued returns of all financial institutions, weighted by the (lagged) market value of their equity.

C.2 Bank Holding Company Y9-C Data

Balance sheet data from the FR Y-9C reports are incorporated into our panel data set using a mapping maintained by the Federal Reserve Bank of New York.[15] We are able to match data for 732 U.S. bank holding companies for a total of 40,241 bank-quarter observations. The link is constructed by matching PERMCOs in the linking table to RSSD9001 in the Y9-C data. We then match to the last available PERMCO of each institution in our CRSP/COMPUSTAT sample. It is important to note that our main panel of CRSP and COMPUSTAT data are historically merger-adjusted, but the Y9-C data is not.

In the forecasting regressions of Table 6, these variables are expressed as a percentage of total book assets. All ratios are truncated at the 1% and 99% level across the panel. Detailed descriptions of the Y9-C variables listed above can be found in the Federal Reserve Board of Governors Micro Data Reference Manual.[16]

[15] The mapping is available at `http://www.ny.frb.org/research/banking_research/datasets.html`.

[16] http://www.federalreserve.gov/reportforms/mdrm

	Date Range	FR Y-9C Series Name
Trading Assets	1986:Q1–1994:Q4	bhck2146
	1995:Q1–2013:Q2	bhck3545
Loans Net Loan-Loss Reserves	1986:Q1–2013:Q2	bhck2122-bhck3123
Loan-Loss Reserve	1986:Q1–2013:Q2	bhck3123
Intangible Assets	1986:Q1–1991:Q4	bhck3163+bhck3165
	1992:Q1–2000:Q4	bhck3163+bhck3164 +bhck5506+bhck5507
	2001:Q1–2013:Q2	bhck3163+bhck0426
Interest-Bearing Core Deposits	1986:Q1–2013:Q2	bhcb2210+bhcb3187+bhch6648 +bhdma164+bhcb2389
Non-Interest-Bearing Deposits	1986:Q1–2013:Q2	bhdm6631+bhfn6631
Large Time Deposits	1986:Q1–2013:Q2	bhcb2604
Demand Deposits	1986:Q1–2013:Q2	bhcb2210

D List of Financial Institutions for Figure 1[17]

Banks and Thrifts: Bank of America (BAC), Citigroup (C), JPMorgan Chase (JPM), Wachovia (WB), Wells Fargo (WFC)

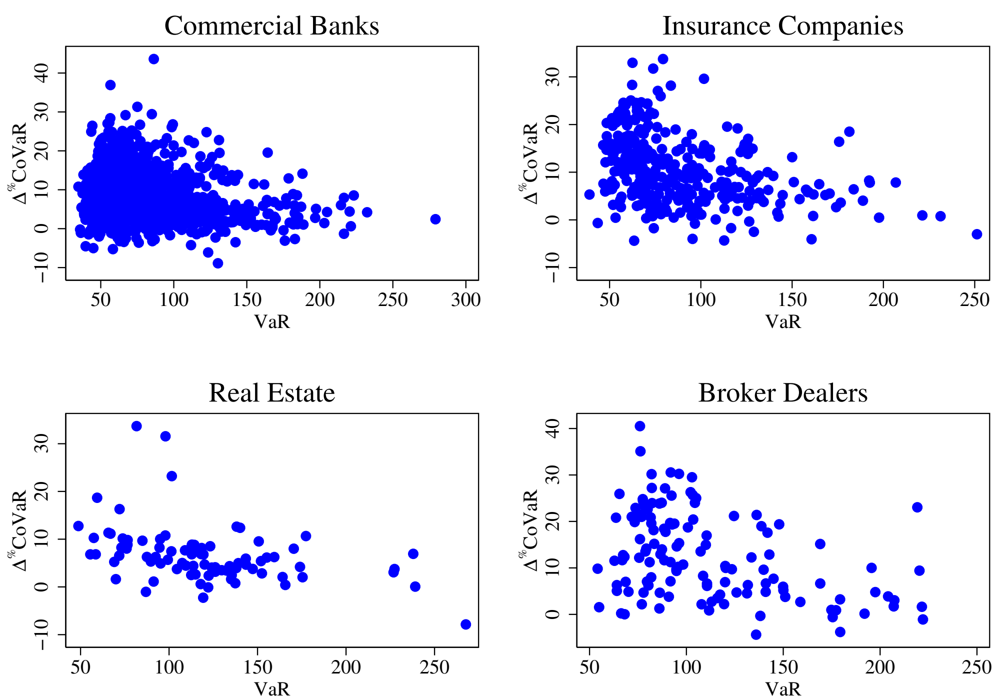
Investment Banks: Bear Stearns (BSC), Goldman Sachs (GS), Lehman Brothers (LEH), Merrill Lynch (MER), Morgan Stanley (MS)

GSEs: Fannie Mae (FNM), Freddie Mac (FRE)

Insurance Companies: American International Group (AIG), Metlife (MET), Prudential (PRU)

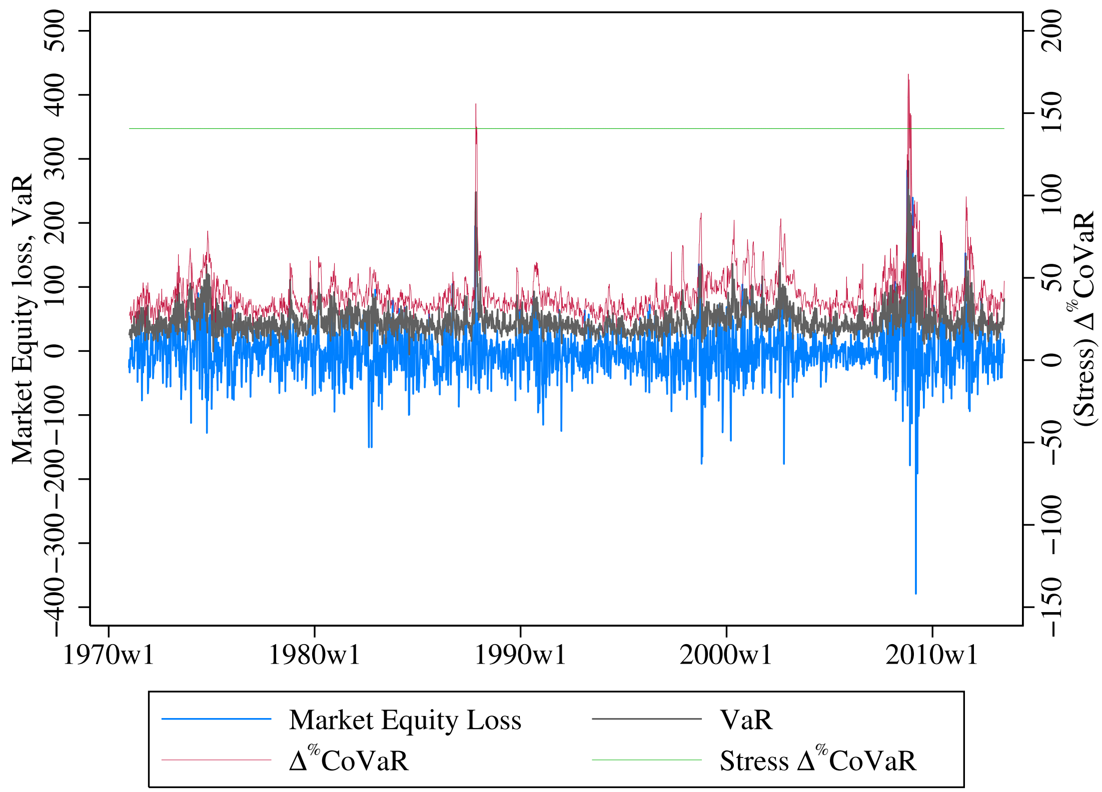
[17]Industry classifications are as of 2006Q4.

Figure 2: **Cross-section of $\Delta^{\%}CoVaR$ and VaR**



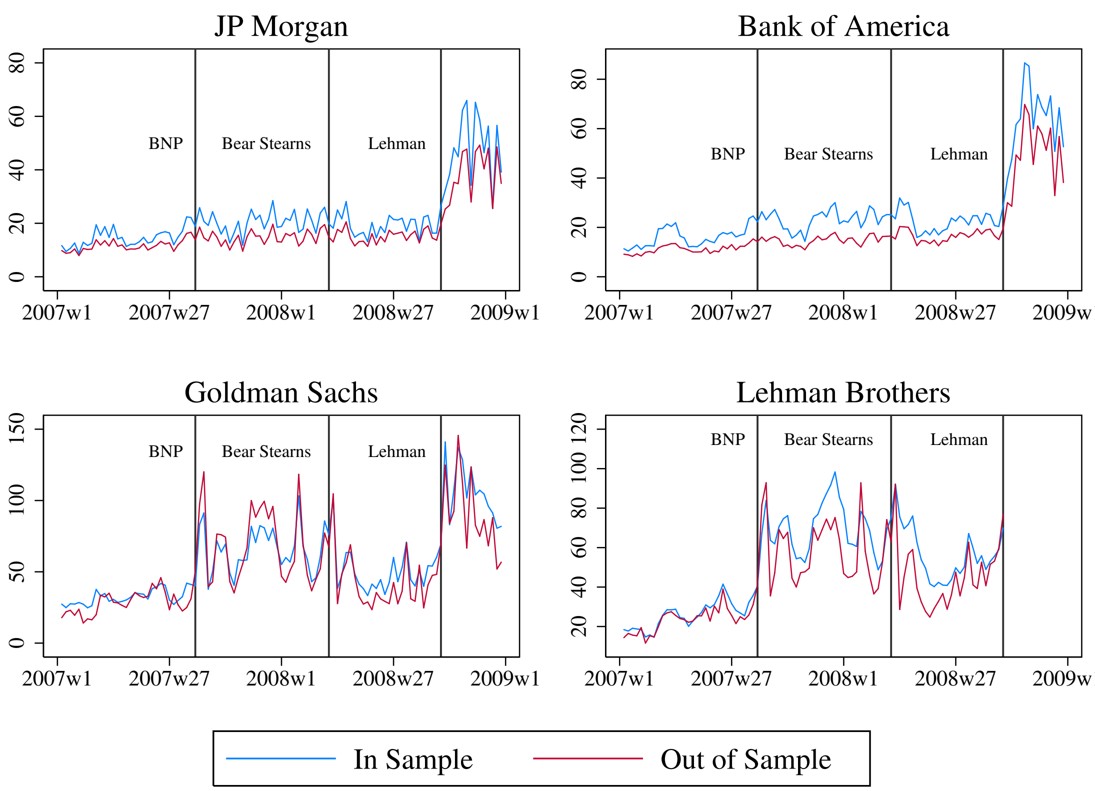
The scatter plot shows the weak cross-sectional link between the time-series average of a portfolio's risk in isolation, measured by $VaR^{i}_{95,t}$ (x-axis), and the time-series average of a portfolio's contribution to system risk, measured by $\Delta CoVaR^{i}_{95,t}$ (y-axis). The $VaR^{i}_{95,t}$ and $\Delta CoVaR^{i}_{95,t}$ are in units of quarterly percent of total market equity loss rates.

Figure 3: **Time-series of Δ CoVaR and VaR for Large Financial Institutions**



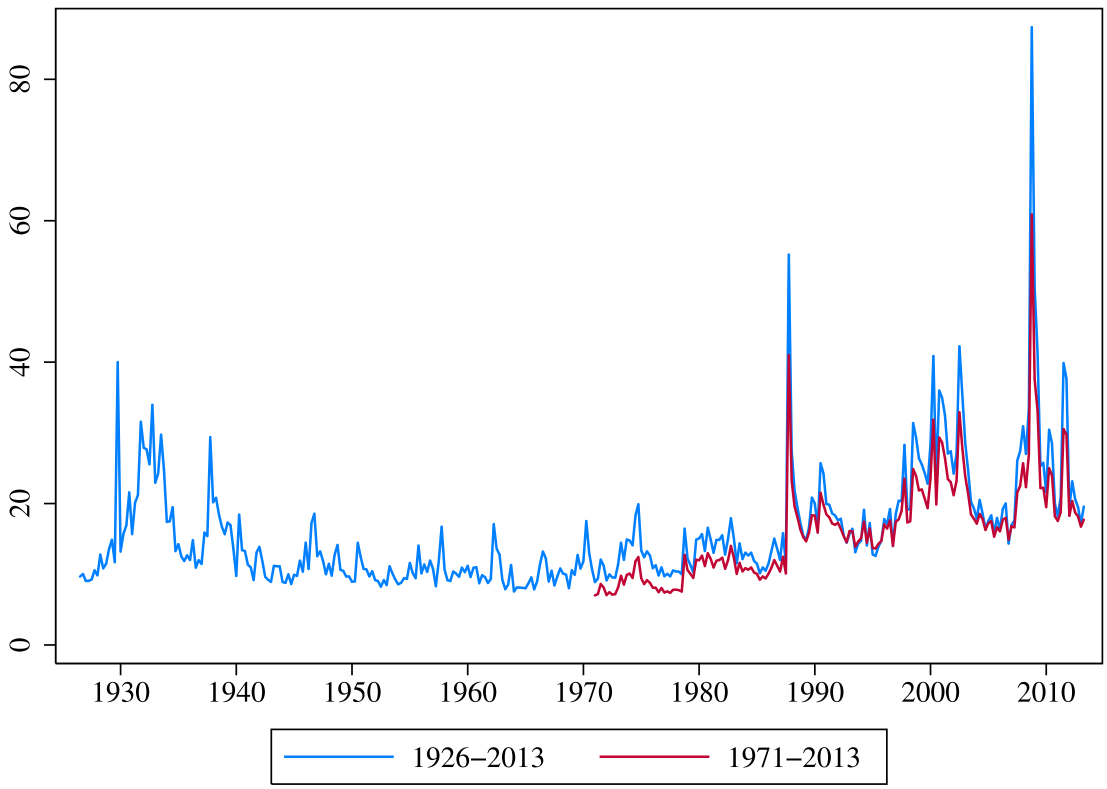
This figure shows the market equity losses (blue), the $VaR^i_{95,t}$ (gray), and the $\Delta CoVaR^i_{95,t}$ (red) for a sample of the 50 largest financial institutions as of the beginning of 2007. The *stress*-$\Delta CoVaR^i_{95,t}$ is also plotted. All variables are quarterly percent of market equity loss rates.

Figure 4: **Time-series of Δ CoVaR of four large Financial Institutions**



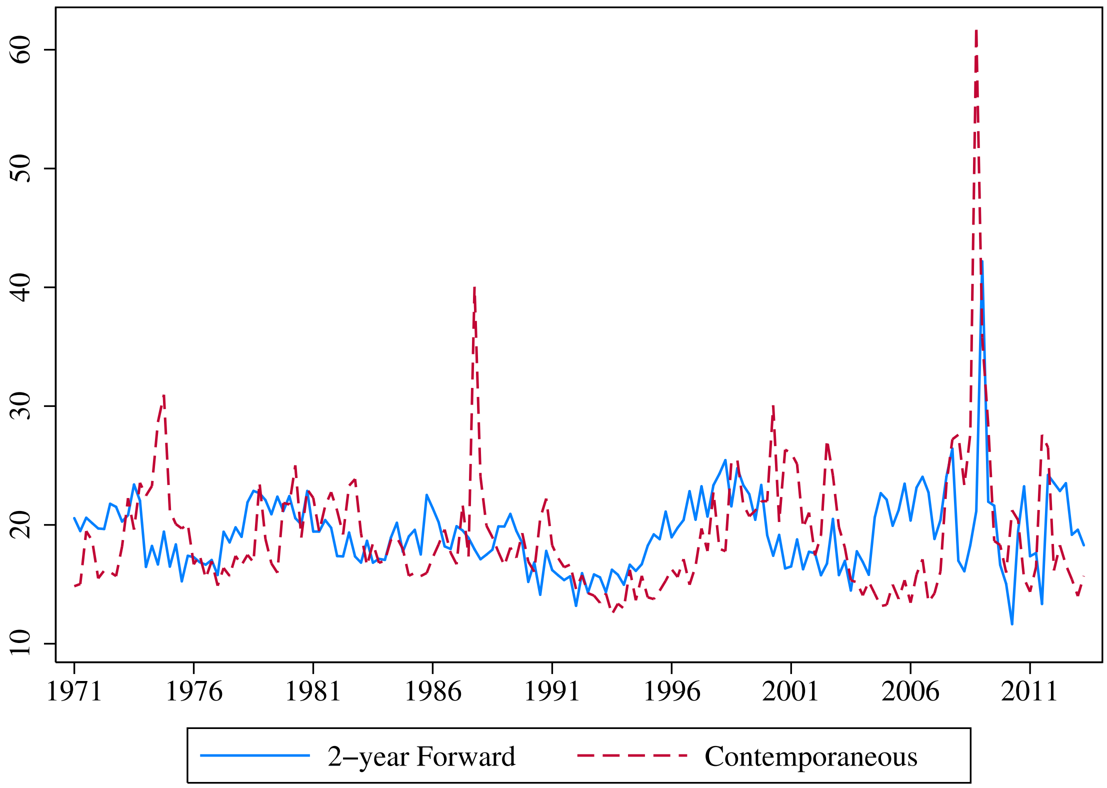
This figure shows the time series of weekly $\Delta^{\%} CoVaR^{i}_{95,t}$ estimated in sample (blue) and out of sample (red). All variables are quarterly percent of market equity loss rates. The first vertical line refers to the week of August 7, 2007, when BNP experienced funding shortages. The second vertical line corresponds to the week of March 15, 2008, when Bear Stearns was distressed. The third vertical line corresponds to the week of September 15, 2008, when Lehman Brothers filed for bankruptcy.

Figure 5: **Historical Δ CoVaR**



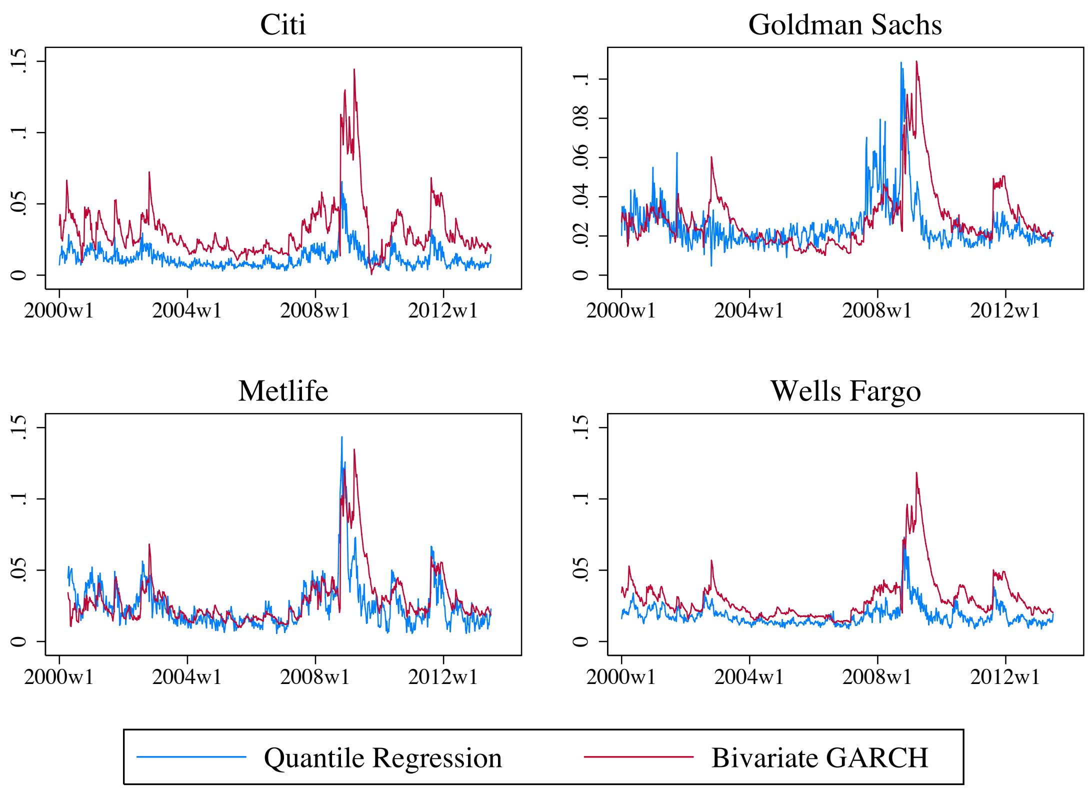
This figure shows the $\Delta CoVaR^i_{95,t}$ for a portfolio of four firms estimated in two ways from weekly data, shown as average within quarters. The red line shows the estimated $\Delta CoVaR^i_{95,t}$ since 1971, while the blue line shows the estimated $\Delta CoVaR^i_{95,t}$ since 1926Q3. The $\Delta CoVaR$s are estimated with respect to the value-weighted CRSP market return. The risk measures are in percent quarterly equity losses.

Figure 6: **Countercyclicality of Forward-$\Delta CoVaR$**



This figure shows average forward and contemporaneous $\Delta CoVaR^i_{95,t}$ estimated out-of-sample since 2002Q1 for the top 50 financial institutions, and in-sample prior to 2002. *Forward*-$\Delta CoVaR^i_{95,t}$ is estimated as described in the main body of the text. The *forward*-$\Delta CoVaR^i_{95,t}$ at a given date uses the data available at that time to predict $\Delta CoVaR^i_{95,t}$ two years in the future. All units are percent quarterly market equity losses.

Figure 7: Δ***CoVaR*** **via GARCH and Quantile Regressions**



The plots show a comparison of $\Delta CoVaR$ estimates using Quantile Regressions and using GARCH for four large financial firms.

Table 1: **State Variable Summary Statistics**. The spreads and spread changes are expressed in weekly basis points, and returns are in weekly percent.

	Mean	Std. Dev.	Skewness	Min	Max	1% Stress Level
Three month yield change	-0.22	21.76	-0.68	-182	192	-8.89
Term spread change	0.09	19.11	0.16	-168	146	5.83
TED spread	103.98	91.09	1.86	6.34	591	138.59
Credit spread change	-0.04	8.41	0.80	-48	60	7.61
Market return	0.15	2.29	-0.23	-15.35	13.83	-7.41
Real estate excess return	-0.03	2.58	0.27	-14.49	21.25	-3.01
Equity volatility	0.89	0.53	3.40	0.28	5.12	2.27

Table 2: **Summary Statistics for Estimated Risk Measures**. The table reports summary statistics for the market equity losses and 99% risk measures of the 1823 financial firms for weekly data from 1971Q1-2013Q2. X^i denotes the weekly market equity losses. The individual firm risk measures $VaR^i_{99,t}$ and the system risk measure $VaR^{system}_{99,t}$ are obtained by running 99% quantile regressions of returns on the one-week lag of the state variables and by computing the predicted value of the regression. $\Delta CoVaR^i_{99,t}$ is the difference between $CoVaR^i_{99,t}$ and the $CoVaR^i_{50,t}$, where $CoVaR^i_{q,t}$ is the predicted value from a q% quantile regression of the financial system equity losses on the institution equity losses and on the lagged state variables. The stress-$\Delta CoVaR^i_{99,t}$ is the $\Delta CoVaR^i_{99,t}$ computed with the worst 1% of state variable realizations and the worst 1% financial system returns replaced in the quantile regression. All quantities are expressed in units of weekly percent returns.

		Mean	Std. Dev.	Obs.
(1)	X^i_t	-0.286	6.111	1342547
(2)	$VaR^i_{99,t}$	11.136	6.868	1342449
(3)	$\Delta CoVaR^i_{99,t}$	1.172	1.021	1342449
(4)	Stress-$\Delta CoVaR^i_{99,t}$	3.357	4.405	1823
(5)	$VaR^{system}_{99,t}$	4.768	2.49	2209

Table 3: **Average t-Statistics of State Variable Exposures**. The table reports average t-statistics from 99%-quantile regressions. For the risk measures $VaR^i_{99,t}$ and the system risk measure $VaR^{system}_{99,t}$, 99-% quantile regressions of losses are run on the state variables. For $CoVaR^i_{99,t}$, 99-% quantile regressions of the financial system equity losses are run on the state variables and firm i's market equity losses.

	VaR^{system}	VaR^i	$\Delta CoVaR^i$
Three month yield change (lag)	(1.95)	(-0.26)	(2.10)
Term spread change (lag)	(1.73)	(-0.04)	(1.72)
TED spread (lag)	(6.87)	(1.97)	(8.86)
Credit spread change (lag)	(5.08)	(-0.28)	(4.08)
Market return (lag)	(-16.98)	(-3.87)	(-18.78)
Real estate excess return (lag)	(-3.78)	(-1.86)	(-4.41)
Equity volatility (lag)	(12.81)	(7.47)	(15.81)
Market equity oss X^i			(7.38)
Pseudo-R^2	39.94%	21.23%	43.42%

Table 4: **Quarterly Summary Statistics**. The table reports summary statistics for the quarterly variables in the *forward*-$\Delta CoVaR$ regressions. The data are from 1971Q1-2013Q2, covering 1823 financial institutions. $VaR^i_{q,t}$ is expressed in units of quarterly percent. $\Delta^{\$}CoVaR^i_{q,t}$ is normalized by the cross sectional average of market equity for each quarter and is expressed in quarterly basis points. The institution characteristics are described in section 5.1.

		Mean	Std. Dev.	Obs.
$\Delta^{\$}CoVaR^i_{95,t}$	792.93	3514.15	106531	
$\Delta^{\$}CoVaR^i_{99,t}$	1023.58	4030.08	106531	
$VaR^i_{95,t}$	84.84	44.79	106889	
$VaR^i_{99,t}$	145.46	80.01	106889	
Leverage	9.12	10.43	94772	
Size	-2.70	1.97	96738	
Maturity mismatch	3.51	11.17	96738	
Boom	0.30	1.34	116366	

Table 5: $\Delta \boldsymbol{CoVaR^i}$ **Forecasts for All Publicly Traded Financial Institutions.** This table reports the coefficients from panel forecasting regressions of the $\Delta^{\$} CoVaR^i_{95,t}$ on the quarterly, one-year, and two-year lags of firm characteristics in Panel A and for the $\Delta^{\$} CoVaR^i_{99,t}$ in Panel B. Each regression has a panel of firms. FE denotes fixed effect dummies. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

	Panel A: $\Delta^{\$} CoVaR^i_{95,t}$			Panel B: $\Delta^{\$} CoVaR^i_{99,t}$		
	2 Year	1 Year	1 Quarter	2 Year	1 Year	1 Quarter
VaR	7.760***	8.559***	9.070***	2.728***	3.448***	4.078***
	(9.626)	(10.566)	(11.220)	(7.484)	(9.443)	(10.225)
Leverage	14.573***	13.398***	13.272***	17.504***	15.958***	15.890***
	(5.946)	(6.164)	(6.317)	(5.854)	(6.299)	(6.627)
Size	1,054.993***	1,014.396***	990.862***	1,238.674***	1,195.072***	1,170.075***
	(22.994)	(23.420)	(23.630)	(27.549)	(28.243)	(28.603)
Maturity mismatch	7.306**	5.779**	4.559*	9.349***	7.918**	6.358**
	(2.187)	(1.968)	(1.760)	(2.725)	(2.537)	(2.225)
Boom	154.863***	160.391***	151.389***	155.184***	169.315***	165.592***
	(4.161)	(4.431)	(4.414)	(3.653)	(3.962)	(3.908)
Equity volatility	74.284	67.707	135.484***	212.860***	203.569***	286.300***
	(1.317)	(1.346)	(2.889)	(3.211)	(3.478)	(4.960)
Three month yield	-111.225***	-144.750***	-127.052***	-75.877***	-123.081***	-111.545***
change	(-5.549)	(-6.686)	(-7.059)	(-3.390)	(-5.518)	(-5.748)
TED spread	-431.094***	-187.730**	-232.091***	-436.214***	-175.884**	-196.513**
	(-6.989)	(-2.403)	(-3.207)	(-6.282)	(-2.163)	(-2.488)
Credit spread	-145.121***	-165.345***	-78.447**	-147.680***	-172.642***	-102.399**
change	(-3.319)	(-3.959)	(-2.036)	(-2.659)	(-3.336)	(-2.023)
Term spread	-275.593***	-243.509***	-187.183***	-251.197***	-236.494***	-179.935***
change	(-7.570)	(-9.017)	(-8.500)	(-6.678)	(-8.094)	(-7.216)
Market return	88.971***	30.799	-97.783***	97.187***	33.065	-111.336***
	(3.881)	(1.401)	(-4.327)	(3.735)	(1.381)	(-4.469)
Housing	27.373*	32.940**	17.800	4.517	15.249	6.644
	(1.845)	(2.479)	(1.156)	(0.269)	(0.992)	(0.390)
Foreign FE	-439.424**	-424.325**	-405.148**	-828.557***	-811.836***	-788.096***
	(-2.376)	(-2.378)	(-2.295)	(-4.492)	(-4.579)	(-4.532)
Insurance FE	-724.971***	-681.143***	-649.836***	-435.109***	-408.868***	-391.193***
	(-7.610)	(-7.629)	(-7.639)	(-4.086)	(-4.114)	(-4.138)
Real Estate FE	-50.644	-42.466	-24.328	66.136	80.733	98.908
	(-0.701)	(-0.647)	(-0.395)	(0.794)	(1.067)	(1.387)
Broker Dealer FE	128.640	99.435	84.613	396.346**	343.386**	310.082**
	(0.850)	(0.695)	(0.612)	(2.500)	(2.311)	(2.187)
Others FE	-373.424***	-388.902***	-381.562***	-209.309***	-235.844***	-240.875***
	(-5.304)	(-5.934)	(-6.121)	(-2.727)	(-3.338)	(-3.586)
Constant	4,608.697***	4,348.786***	3,843.332***	5,175.855***	4,970.410***	4,443.515***
	(16.292)	(17.542)	(19.802)	(18.229)	(19.768)	(21.566)
Observations	79,317	86,474	91,750	79,317	86,474	91,750
Adjusted R^2	24.53%	24.36%	24.35%	26.89%	26.75%	26.76%

Table 6: **$\Delta CoVaR^i$ Forecasts For Bank Holding Companies.** This table reports the coefficients from panel predictive regressions of $\Delta CoVaR_t^i$ on the quarterly, one year, and two year lags of liability and firm characteristics in Panel A, and for asset and firm characteristics in Panel B. The methodologies for computing the risk measures VaR_t^i and $\Delta CoVaR_t^i$ are given in the captions of Tables 2 and 3. The risk measures are caluclated for the 95% quantile. Newey-West standard errors allowing for up to 5 periods of autocorrelation are displayed in patentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

	Panel A: BHC Liability Variables			Panel B: BHC Asset Variables		
	2 Year	1 Year	1 Quarter	2 Year	1 Year	1 Quarter
VaR	9.995***	11.068***	11.699***	4.251**	6.402***	7.715***
	(4.774)	(5.099)	(5.310)	(2.211)	(3.305)	(3.977)
Leverage	56.614***	44.044***	38.639***	40.779***	29.967***	23.919***
	(9.654)	(9.683)	(9.340)	(6.476)	(6.103)	(5.532)
Size	1,457.875***	1,394.324***	1,360.318***	1,203.344***	1,141.656***	1,107.145***
	(13.539)	(13.781)	(13.850)	(13.069)	(13.352)	(13.591)
Boom	88.923	108.619	90.283	124.161*	143.028**	123.243**
	(1.238)	(1.617)	(1.560)	(1.758)	(2.179)	(2.209)
Equity volatility	92.400	-55.501	48.969	292.221***	90.937	165.272
	(0.821)	(-0.527)	(0.436)	(2.672)	(0.887)	(1.521)
Three month yield change	-541.105***	-512.923***	-442.339***	-357.807***	-343.193***	-278.257***
	(-6.815)	(-6.442)	(-7.344)	(-5.567)	(-5.145)	(-5.585)
TED Spread	-709.255***	-230.846	-448.387*	-622.720***	-142.805	-369.578*
	(-3.784)	(-0.830)	(-1.926)	(-3.688)	(-0.548)	(-1.716)
Credit spread change	-338.997***	-268.226**	-136.785	-208.571*	-113.161	35.318
	(-2.860)	(-2.082)	(-1.132)	(-1.879)	(-0.935)	(0.297)
Term spread change	-838.872***	-681.318***	-556.294***	-640.835***	-505.287***	-391.500***
	(-6.863)	(-7.575)	(-8.060)	(-6.521)	(-6.886)	(-6.718)
Market return	-16.289	-61.080	-205.718***	25.380	-34.474	-188.620***
	(-0.329)	(-1.156)	(-3.698)	(0.541)	(-0.706)	(-3.601)
Housing	88.442***	116.009***	69.066**	64.023**	95.015***	51.096*
	(2.853)	(4.091)	(2.297)	(2.236)	(3.687)	(1.795)
Core deposits	-50.915***	-53.888***	-53.439***			
	(-8.005)	(-8.217)	(-8.244)			
Non-interest deposits	51.610***	46.680***	43.844***			
	(3.840)	(4.278)	(4.519)			
Time deposits	-68.087***	-65.152***	-62.553***			
	(-8.347)	(-8.123)	(-8.064)			
Demand deposits	-13.782	-14.811	-16.195			
	(-0.929)	(-1.254)	(-1.511)			
Total loans				9.419**	6.568*	3.811
				(2.285)	(1.792)	(1.106)
Loan loss reserves				-133.352	-131.401	-72.177
				(-1.086)	(-1.263)	(-0.776)
Intanglible assets				48.721	43.285	41.082
				(0.884)	(0.874)	(0.915)
Trading assets				576.060***	565.476***	549.158***
				(5.332)	(5.636)	(6.088)
Constant	10,820.323***	10,077.831***	9,222.897***	5,825.307***	5,127.386***	4,404.582***
	(10.244)	(10.388)	(11.572)	(7.899)	(7.849)	(8.573)
Observations	25,578	28,156	30,128	25,481	28,060	30,030
Adjusted R^2	28.94%	28.17%	28.13%	36.29%	35.47%	35.41%

Table 7: $\Delta \boldsymbol{CoVaR}^i$ **Forecasts For Bank Holding Companies.** This table reports a regression of the $\Delta CoVaR$ during the financial crisis of 2007-2009 on *forward*-$\Delta CoVaR$ for the universe of bank holding companies. The columns correspond to different forecasting horizons at different dates. The first column is a two year forecast as of 2006Q4, the second column is a one year forecast as of 2007Q4, the third column is a one month forecast as of 2008Q3, the fourth column is a one year forecast as of 2006Q4, and the last column is a one quarter forecast as of 2006Q4. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

	Crisis $\Delta CoVaR$				
	2008Q4	2008Q4	2008Q4	2007Q4	2007Q1
2Y *Forward*-$\Delta CoVaR$ (2006Q4)	1.206***				
1Y *Forward*-$\Delta CoVaR$ (2007Q4)		0.664***			
1Q *Forward*-$\Delta CoVaR$ (2008Q3)			1.708***		
1Y *Forward*-$\Delta CoVaR$ (2006Q4)				0.848***	
1Q *Forward*-$\Delta CoVaR$ (2006Q4)					0.541***
Constant	13.08***	18.51***	2.409***	4.505***	2.528***
Observations	378	418	430	428	461
R^2	36.6 %	17.8 %	78.9 %	49.6 %	55.5%

Table 8: Δ ***CoVaR***^{i} **Forecasts using GARCH estimation.** This table reports the coefficients from panel forecasting regressions of the two estimation methods of $\Delta^{\$}CoVaR^i_{95,t}$ on the quarterly, one-year, and two-year lag of firm characteristics. FE denotes fixed effect dummies. The *GARCH*-$\Delta^{\$}CoVaR^i_{95,t}$ is computed by estimating the covariance structure of a bivariate diagnonal *VECH GARCH* model. Newey-West standard errors allowing for up to five periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

	2 Year		1 Year		1 Quarter	
	Quantile	GARCH	Quantile	GARCH	Quantile	GARCH
VaR	10.655***	22.126***	10.462***	22.290***	10.760***	22.953***
	(7.146)	(6.446)	(8.292)	(6.762)	(9.025)	(7.725)
Size	2,171.414***	3,650.023***	2,088.345***	3,495.286***	2,044.432***	3,432.149***
	(21.404)	(16.656)	(21.865)	(16.792)	(22.064)	(17.000)
Maturity mismatch	22.292***	40.001**	18.708**	30.449**	15.859**	28.336*
	(2.709)	(2.207)	(2.540)	(1.975)	(2.405)	(1.898)
Boom	224.188***	371.613***	250.597***	459.495***	243.163***	440.275***
	(3.060)	(2.934)	(3.417)	(3.683)	(3.448)	(3.599)
Equity volatility	355.404***	681.617***	421.286***	375.124	638.354***	1,676.640***
	(2.772)	(2.710)	(3.493)	(1.477)	(5.375)	(7.103)
Leverage	17.133***	19.563*	17.383***	18.788*	18.257***	22.212**
	(2.655)	(1.693)	(2.975)	(1.654)	(3.249)	(2.170)
Housing	16.228	82.042	3.684	-63.677	-29.998	-246.555***
	(0.460)	(1.192)	(0.116)	(-1.018)	(-0.784)	(-3.338)
Three month yield	-204.962***	-441.897***	-282.786***	-649.486***	-248.337***	-444.962***
change	(-4.476)	(-5.124)	(-5.584)	(-6.105)	(-5.854)	(-6.116)
TED spread	-683.690***	-1,119.422***	-171.649	250.885	-249.432	-567.982*
	(-4.966)	(-4.386)	(-0.937)	(0.659)	(-1.482)	(-1.881)
Credit spread	-341.279***	-715.071***	-392.110***	-616.055***	-215.385**	-45.730
change	(-3.295)	(-3.572)	(-3.959)	(-3.461)	(-2.319)	(-0.239)
Term spread	-554.797***	-1,174.171***	-493.187***	-847.419***	-369.603***	-557.334***
change	(-6.508)	(-6.821)	(-7.809)	(-7.419)	(-7.159)	(-6.241)
Market return	97.262*	154.923	-36.331	-162.681	-329.304***	-490.855***
	(1.875)	(1.580)	(-0.680)	(-1.295)	(-5.717)	(-4.905)
Foreign FE	-600.936	-110.671	-608.110	-275.112	-580.752	-445.307
	(-1.253)	(-0.144)	(-1.311)	(-0.377)	(-1.265)	(-0.652)
Insurance FE	-1,550.564***	-3,500.083***	-1,421.556***	-3,254.618***	-1,339.724***	-3,054.498***
	(-7.233)	(-8.064)	(-7.129)	(-7.974)	(-7.093)	(-8.030)
Real Estate FE	63.919	-34.037	156.418	68.235	223.824	234.309
	(0.369)	(-0.116)	(0.982)	(0.250)	(1.483)	(0.928)
Broker Dealer FE	202.435	-775.272	253.854	-708.925	281.495	-664.763
	(0.626)	(-1.527)	(0.820)	(-1.442)	(0.930)	(-1.424)
Others FE	-682.642***	-1,535.849***	-656.751***	-1,546.880***	-626.059***	-1,418.808***
	(-4.026)	(-5.076)	(-4.198)	(-5.226)	(-4.212)	(-5.196)
Constant	7,920.183***	14,203.481***	7,553.186***	13,032.912***	6,519.919***	9,495.516***
	(13.759)	(11.613)	(14.828)	(12.660)	(16.862)	(13.664)
Observations	51,294	51,286	55,347	55,343	58,355	58,352
Adjusted R^2	27.10%	21.23%	26.94%	21.20%	27.00%	21.93%

Table 9: $\Delta \boldsymbol{CoVaR}^i$ **Forecasts using alternative system returns variable.** This table reports the coefficients from forecasting regressions of the two estimation methods of $\Delta^{\$} CoVaR^i_{95,t}$ on the quarterly, one-year, and two-year lag of firm characteristics. In the columns labeled X^{system}, $\Delta^{\$} CoVaR^i_{95}$ is estimated using the regular system returns variable described in Section 3, while in columns labeled $X^{system-i}$, $\Delta^{\$} CoVaR^i_{95}$ is estimated using a system return variable that does not include the firm for which $\Delta^{\$} CoVaR^i_{95}$ is being estimated. FE denotes fixed effect dummies. Newey−West standard errors allowing for five periods of autocorrelation are displayed in parentheses. One, two, and three stars denote significance at the 10, 5, and 1 percent levels, respectively.

	2 Year		1 Year		1 Quarter	
	X^{system}	$X^{system-i}$	X^{system}	$X^{system-i}$	X^{system}	$X^{system-i}$
VaR	7.590***	7.079***	8.416***	7.901***	8.952***	8.432***
	(9.375)	(9.534)	(10.326)	(10.573)	(10.974)	(11.292)
Leverage	14.586***	14.150***	13.380***	12.910***	13.241***	12.743***
	(5.988)	(6.093)	(6.178)	(6.271)	(6.316)	(6.388)
Size	1,045.383***	1,003.729***	1,005.236***	965.690***	982.123***	943.777***
	(22.846)	(23.639)	(23.263)	(24.102)	(23.461)	(24.318)
Maturity mismatch	7.203**	6.349**	5.636*	4.970*	4.417*	3.900*
	(2.149)	(2.138)	(1.920)	(1.902)	(1.710)	(1.684)
Boom	155.288***	150.837***	160.661***	156.523***	151.023***	147.602***
	(4.203)	(4.266)	(4.470)	(4.543)	(4.445)	(4.533)
Equity volatility	77.476	77.382	67.143	67.784	135.018***	135.387***
	(1.389)	(1.474)	(1.337)	(1.428)	(2.897)	(3.062)
Three month yield change	-110.309***	-104.495***	-143.529***	-137.320***	-125.988***	-120.507***
	(-5.543)	(-5.535)	(-6.673)	(-6.717)	(-7.050)	(-7.102)
TED spread	-425.174***	-406.246***	-185.526**	-172.485**	-228.606***	-214.126***
	(-6.953)	(-7.103)	(-2.387)	(-2.361)	(-3.175)	(-3.181)
Credit spread change	-144.973***	-137.867***	-160.705***	-154.783***	-75.179**	-72.486**
	(-3.391)	(-3.416)	(-3.929)	(-3.987)	(-1.984)	(-2.006)
Term spread change	-274.462***	-262.126***	-242.640***	-231.757***	-185.570***	-176.715***
	(-7.606)	(-7.673)	(-9.052)	(-9.161)	(-8.492)	(-8.605)
Market return	88.292***	84.502***	29.547	27.823	-98.583***	-95.902***
	(3.848)	(3.948)	(1.349)	(1.347)	(-4.367)	(-4.496)
Housing	27.667*	25.551*	33.085**	30.527**	18.043	15.607
	(1.860)	(1.827)	(2.491)	(2.447)	(1.168)	(1.077)
Foreign FE	-433.032**	-420.804**	-418.956**	-407.989**	-400.882**	-391.199**
	(-2.336)	(-2.366)	(-2.343)	(-2.378)	(-2.266)	(-2.304)
Insurance FE	-712.369***	-656.614***	-669.666***	-617.652***	-639.305***	-589.511***
	(-7.496)	(-7.395)	(-7.516)	(-7.428)	(-7.527)	(-7.439)
Real Estate FE	-46.567	-37.370	-39.152	-31.187	-21.467	-14.662
	(-0.649)	(-0.546)	(-0.601)	(-0.503)	(-0.351)	(-0.252)
Broker Dealer FE	113.499	139.780	84.969	110.399	69.907	94.869
	(0.757)	(0.973)	(0.599)	(0.813)	(0.510)	(0.724)
Others FE	-362.552***	-341.190***	-379.304***	-358.435***	-373.640***	-353.344***
	(-5.182)	(-5.130)	(-5.815)	(-5.798)	(-6.015)	(-6.012)
Constant	4,573.181***	4,393.954***	4,310.730***	4,146.631***	3,805.569***	3,658.264***
	(16.248)	(16.599)	(17.495)	(17.884)	(19.746)	(20.250)
Observations	79,317	79,317	86,474	86,474	91,750	91,750
Adjusted R-Squared	24.41%	25.13%	24.24%	24.96%	24.24%	24.96%

Table 10: **Fraction of firms whose VaR estimates have correct conditional coverage.** Per definition of the VaR_t, losses at t should be less than the VaR_t with probability 95% or 99% depending on the percentile. The table reports the fraction of firms that have correct conditional coverage at the 5% and 10% confidence levels based on Christoffersen's (1998) likelihood ratio test.

	Quantile		GARCH	
	5%-Level	10%-Level	5%-Level	10%-Level
$95\% - VaR$	0.52	0.61	0.70	0.78
$99\% - VaR$	0.46	0.59	0.20	0.26