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Bank Liquidity, Interbank Markets, and Monetary Policy

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Abstract

A major lesson of the recent financial crisis is that the interbank lending market is crucial for banks that face uncertainty regarding their liquidity needs. This paper examines the efficiency of the interbank lending market in allocating funds and the optimal policy of a central bank in response to liquidity shocks. We show that, when confronted with a distributional liquidity-shock crisis that causes a large disparity in the liquidity held by different banks, a central bank should lower the interbank rate. This view implies that the traditional separation between prudential regulation and monetary policy should be rethought. In addition, we show that, during an aggregate liquidity crisis, central banks should manage the aggregate volume of liquidity. Therefore, two different instruments—interest rates and liquidity injection—are required to cope with the two different types of liquidity shocks. Finally, we show that failure to cut interest rates during a crisis erodes financial stability by increasing the probability of bank runs.

Key words: bank liquidity, interbank markets, central bank policy, financial fragility, bank runs

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1 Introduction

The appropriate response of a central bank's interest rate policy to banking crises is the subject of a continuing and important debate. A standard view is that monetary policy should play a role only if a financial disruption directly affects inflation or the real economy; that is, monetary policy should not be used to alleviate financial distress per se. Additionally, several studies on interlinkages between monetary policy and financial-stability policy recommend the complete separation of the two, citing evidence of higher and more volatile inflation rates in countries where the central bank is in charge of banking stability.[1]

This view of monetary policy is challenged by observations that, during a banking crisis, interbank interest rates often appear to be a key instrument used by central banks for limiting threats to the banking system and interbank markets. During the recent crisis, which began in August 2007, interest rate setting in both the U.S. and the E.U. appeared to be geared heavily toward alleviating stress in the banking system and in the interbank market in particular. Interest rate policy has been used similarly in previous financial disruptions, as Goodfriend (2002) indicates: "Consider the fact that the Fed cut interest rates sharply in response to two of the most serious financial crises in recent years: the October 1987 stock market break and the turmoil following the Russian default in 1998." The practice of reducing interbank rates during financial turmoil also challenges the long-debated view originated by Bagehot (1873) that central banks should provide liquidity to banks at high-penalty interest rates (see Martin 2009, for example).

We develop a model of the interbank market and show that the central bank's interest rate policy can directly improve liquidity conditions in the interbank lending market during a financial crisis. Consistent with central bank practice, the optimal policy in our model consists of reducing the interbank rate during a crisis. This view implies that the conventionally supported separation between prudential regulation and monetary policy should be abandoned during a systemic crisis.

Intuition for our results can be gained by understanding the role of the interbank market. The main purpose of this market is to redistribute the fixed amount of reserves that is held within the banking system. In our model, banks may face uncertainty regarding

[1] See Goodhart and Shoenmaker (1995) and Di Giorgio and Di Noia (1999).

their need for liquid assets, which we associate with reserves. The interbank market allows banks faced with distributional shocks to redistribute liquid assets among themselves. The interest rate will therefore play a key role in amplifying or reducing the losses of banks enduring liquidity shocks. Consequently, it will also influence the banks' precautionary holding of liquid securities. High interest rates in the interbank market during a liquidity crisis would partially inhibit the liquidity insurance role of banks, while low interest rates will decrease uncertainty and increase the efficiency of banks' contingent allocation of resources. Yet in order to make low interest rates during a crisis compatible with the higher return on banks' long-term assets, during normal times interbank interest rates must be higher than the return on long-term assets.

We allow for different states regarding the uncertainty faced by banks. We associate a state of high uncertainty with a crisis and a state of low uncertainty with normal times. We also permit the interbank market rate to be state dependent. A new result of our model is that there are multiple Pareto-ranked equilibria associated with different pairs of interbank market rates for normal and crisis times. The multiplicity of equilibria arises because the demand for and supply of funds in the interbank market are inelastic. This inelasticity is a key feature of our model and corresponds to the fundamentally inelastic nature of banks' short-term liquidity needs. By choosing the interbank rate appropriately, high in normal times and low in crisis times, a central bank can achieve the optimal allocation.

The interbank rate plays two roles in our model. From an ex-ante perspective, the expected rate influences the banks' portfolio decision for holding short-term liquid assets and long-term illiquid assets. Ex post, the rate determines the terms at which banks can borrow liquid assets in response to idiosyncratic shocks, so that a trade-off is present between the two roles. The optimal allocation can be achieved only with state-contingent interbank rates. The rate must be low in crisis times to achieve the efficient redistribution of liquid assets. Since the ex-ante expected rate must be high, to induce the optimal investment choice by banks, the interbank rate needs to be set high enough in normal times. As the conventional separation of prudential regulation and monetary policy implies that interest rates are set independently of prudential considerations, our result is a strong criticism of such separation.

Our framework yields several additional results. First, when aggregate liquidity shocks are considered, we show that the central banks should accommodate the shocks by injecting or withdrawing liquidity. Interest rates and liquidity injections should be used to address two different types of liquidity shocks: Interest rate management allows for coping with efficient liquidity reallocation in the interbank market, while quantitative easing allows for tackling aggregate liquidity shocks. Hence, when interbank markets are modeled as part of an optimal institutional arrangement, the central bank should respond to different types of shocks with different tools. Second, we show that the failure to implement a contingent interest rate policy, which will occur if the separation between monetary policy and prudential regulation prevails, will undermine financial stability by increasing the probability of bank runs.

In their seminal study, Bhattacharya and Gale (1987) examine banks with idiosyncratic liquidity shocks from a mechanism design perspective. In their model, when liquidity shocks are not observable, the interbank market is not efficient and the second-best allocation involves setting a limit on the size of individual loan contracts among banks. More recent work by Freixas and Holthausen (2005), Freixas and Jorge (2008), and Heider, Hoerova, and Holthausen (2008) assumes the existence of interbank markets even though they are not part of an optimal arrangement.

Both our paper and that of Allen, Carletti, and Gale (2008) develop frameworks in which interbank markets are efficient. In Allen, Carletti, and Gale (2008), the central bank responds to both idiosyncratic and aggregate shocks by buying and selling assets, using its balance sheet to achieve the efficient allocation. The modeling innovation of our paper is to introduce multiple states with different distributional liquidity shocks. With state-contingent interbank rates, the full-information efficient allocation can be achieved.

Goodfriend and King (1988) argue that central bank policy should respond to aggregate, but not idiosyncratic, liquidity shocks when interbank markets are efficient. In our model, their result does not hold, even though bank returns are known and speculative bank runs are ruled out. The reason is that the level of interest rates determines the banks' cost of being short of liquidity and, therefore, penalizes the long-term claim holders who have to bear this liquidity-related risk. The results of our paper are similar to those of Diamond and Rajan (2008), who show that interbank rates should be low during a crisis and high in normal times. Diamond and Rajan (2008) examine the limits of central bank influence over bank interest rates based on a Ricardian equivalence argument, whereas we find a new mechanism by which the central bank can adjust interest rates based on the inelasticity of banks' short-term supply of and demand for liquidity. Our paper also relates to Bolton, Santos and Scheinkman (2008), who consider the trade-off faced by financial intermediaries between holding liquidity versus acquiring liquidity supplied by a market after shocks occur. Efficiency depends on the timing of central bank intervention in Bolton et al. (2008), whereas in our paper the level of interest rate policy is the focus. Acharya and Yorulmazer (2008) consider interbank markets with imperfect competition. Gorton and Huang (2006) study interbank liquidity historically provided by banking coalitions through clearinghouses. Ashcraft, McAndrews, and Skeie (2008) examine a model of the interbank market with credit and participation frictions that can explain their empirical findings of reserves hoarding by banks and extreme interbank rate volatility.

Section 2 presents the model of distributional shocks. Section 3 gives the market results and central bank interest rate policy. Section 4 analyzes aggregate shocks, and Section 5 examines financial fragility. Available liquidity is endogenized in Section 6. Section 7 concludes.

2 Model

The model has three dates, denoted by $t = 0, 1, 2$, and a continuum of competitive banks, each with a unit continuum of consumers. Ex-ante identical consumers are endowed with one unit of good at date 0 and learn their private type at date 1. With a probability $\overline{\lambda} \in (0, 1)$, a consumer is "impatient" and needs to consume at date 1. With complementary probability $1 - \overline{\lambda}$, a consumer is "patient" and needs to consume at date 2. Throughout the paper, we disregard sunspot-triggered bank runs. At date 0, consumers deposit their unit good in their bank for a deposit contract that pays an amount when withdrawn at either date 1 or 2.

There are two possible technologies. The short-term liquid technology, also called liquid assets, allows for storing goods at date 0 or date 1 for a return of one in the following period. The long-term investment technology, also called long-term assets, allows for investing goods at date 0 for a return of $r > 1$ at date 2. Investment is illiquid and cannot be liquidated at date 1.[2]

[2] We extend the model to allow for liquidation at date 1 in Section 6.

Since the long-term technology is not risky in our model, we cannot consider issues related to counterparty risk. However, our model is well suited to think about the first part of the recent crisis, mid-2007 to mid-2008. During this period, many banks faced the liquidity risks of needing to pay billions of dollars for ABCP conduits, SIVs, and other credit lines; meanwhile, other banks received large inflows from financial investors who were fleeing AAA-rated securities, commercial paper, and money market funds in a flight to quality and liquidity.

We model distributional liquidity shocks within the banking system by assuming that each bank faces stochastic idiosyncratic withdrawals at date 1. There is no aggregate withdrawal risk for the banking system as a whole so. On average, each bank has $\overline{\lambda}$ withdrawals at date 1.[3]

The innovation that distinguishes our model from that of Bhattacharya and Gale (1987) and Allen, Carletti, and Gale (2008) is that we consider two states of the world regarding the idiosyncratic liquidity shocks. Let $i \in \mathcal{I} \equiv \{0, 1\}$, where

$$i = \left\{ \begin{array}{lll} 1 & \text{with prob } \rho & \text{(“crisis state”)} \\ 0 & \text{with prob } 1-\rho & \text{(“normal-times state”),} \end{array} \right.$$

and $\rho \in [0, 1]$ is the probability of the liquidity-shock state $i = 1$. We assume that state i is observable but not verifiable, which means that contracts cannot be written contingent on state i. Banks are ex-ante identical at date 0. At date 1, each bank learns its private type $j \in \mathcal{J} \equiv \{h, l\}$, where

$$j = \left\{ \begin{array}{lll} h & \text{with prob } \frac{1}{2} & \text{(“high type”)} \\ l & \text{with prob } \frac{1}{2} & \text{(“low type”).} \end{array} \right.$$

In aggregate, half of banks are type h and half are type l. Banks of type $j \in \mathcal{J}$ have a fraction of impatient depositors at date 1 equal to

$$\lambda^{ij} = \left\{ \begin{array}{lll} \overline{\lambda} + i\varepsilon & \text{for } j = h & \text{(“high withdrawals”)} \\ \overline{\lambda} - i\varepsilon & \text{for } j = l & \text{(“low withdrawals”),} \end{array} \right. \tag{1}$$

where $i \in \mathcal{I}$ and $\varepsilon > 0$ is the size of the bank-specific liquidity withdrawal shock. We assume that $0 < \lambda^{il} \leq \lambda^{ih} < 1$ for $i \in \mathcal{I}$.

To summarize, when state $i = 1$, a crisis occurs. Banks of type $j = h$ have relatively high liquidity withdrawals at date 1 and banks of type $j = l$ have relatively low liquidity

[3] We study a model with distributional and aggregate shocks in Section 4.

withdrawals. When state $i = 0$, there is no crisis and all banks have constant withdrawals of $\overline{\lambda}$ at date 1. At date 2, banks of type $j \in \mathcal{J}$ have a fraction of patient depositor withdrawals equal to $1 - \lambda^{ij}$, $i \in \mathcal{I}$.

A depositor receives consumption of either c_1 for withdrawal at date 1 or c_2^{ij}, an equal share of the remaining goods at the depositor's bank j, for withdrawal at date 2. Depositor utility is

$$U = \left\{ \begin{array}{lll} u(c_1) & \text{with prob } \overline{\lambda} & (\text{“impatient depositors”}) \\ u(c_2^{ij}) & \text{with prob } 1 - \overline{\lambda} & (\text{“patient depositors”}), \end{array} \right.$$

where u is increasing and concave. We define $c_2^0 \equiv c_2^{0j}$ for all $j \in \mathcal{J}$, since consumption for impatient depositors of each bank type is equal during normal-times state $i = 0$. A depositor's expected utility is

$$E[U] = \overline{\lambda} u(c_1) + (1-\rho)(1-\overline{\lambda})u(c_2^0) + \rho \left[\frac{1}{2}(1-\lambda^{1h})u(c_2^{1h}) + \frac{1}{2}(1-\lambda^{1l})u(c_2^{1l}) \right]. \quad (2)$$

Banks maximize profits. Because of competition, they must maximize the expected utility of their depositors. Banks invest $\alpha \in [0,1]$ in long-term assets and store $1-\alpha$ in liquid assets. At date 1, depositors and banks learn their private type. Bank j borrows $f^{ij} \in \mathbb{R}$ liquid assets on the interbank market (the notation f represents the federal funds market and $f^{ij} < 0$ represents a loan made in the interbank market) and impatient depositors withdraw c_1. At date 2, bank j repays the amount $f^{ij}\iota^i$ for its interbank loan and the bank's remaining depositors withdraw, where ι^i is the interbank interest rate. If $\iota^0 \neq \iota^1$, the interest rate is state contingent, whereas if $\iota^0 = \iota^1$, the interest rate is not state contingent. Since banks are able to store liquid assets for a return of one between dates 1 and 2, banks never lend for a return of less than one, so $\iota^i \geq 1$ for all $i \in \mathcal{I}$. A timeline is shown in Figure 1.

The bank budget constraints for bank j for dates 1 and 2 are

$$\lambda^{ij} c_1 = 1 - \alpha - \beta^{ij} + f^{ij} \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \quad (3)$$

$$(1-\lambda^{ij})c_2^{ij} = \alpha r + \beta^{ij} - f^{ij}\iota^i \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J}, \quad (4)$$

respectively, where $\beta^{ij} \in [0, 1-\alpha]$ is the amount of liquid assets that banks of type j store between dates 1 and 2. We assume that the coefficient of relative risk aversion for $u(c)$ is greater than one, which implies that banks provide risk-decreasing liquidity insurance. We also assume that banks lend liquid assets when indifferent between lending and storing.

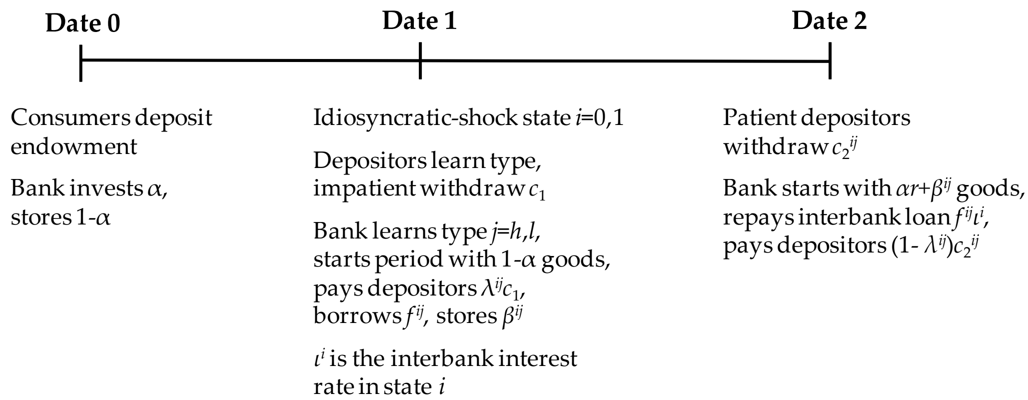

Figure 1: Timeline

We only consider parameters such that there are no bank defaults in equilibrium.[4] As such, we assume that incentive compatibility holds:

$$c_2^{ij} \geq c_1 \qquad \text{for all } i \in \mathcal{I},\ j \in \mathcal{J},$$

which rules out bank runs based on very large bank liquidity shocks.

The bank optimizes over α, c_1, $\{c_2^{ij}, \beta^{ij}, f^{ij}\}_{i\in\mathcal{I},\ j\in\mathcal{J}}$ to maximize its depositors' expected utility. From the date 1 budget constraint (3), we can solve for the quantity of interbank borrowing by bank j as

$$f^{ij}(\alpha, c_1, \beta^{ij}) = \lambda^{ij} c_1 - (1-\alpha) + \beta^{ij} \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J}. \tag{5}$$

Substituting this expression for f^{ij} into the date 2 budget constraint (4) and rearranging gives consumption by impatient depositors as

$$c_2^{ij}(\alpha, c_1, \beta^{ij}) = \frac{\alpha r + \beta^{ij} - [\lambda^{ij} c_1 - (1-\alpha) + \beta^{ij}]\iota^i}{(1-\lambda^{ij})}. \tag{6}$$

The bank's optimization can be written as

$$\max_{\alpha\in[0,1];c_1,\{\beta^{ij}\}_{i\in\mathcal{I},j\in\mathcal{J}}\geq 0} \quad E[U] \tag{7}$$

$$\text{s.t.} \quad \beta^{ij} \leq 1-\alpha \quad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \tag{8}$$

$$c_2^{ij}(\alpha, c_1, \beta^{ij}) = \tfrac{\alpha r + \beta^{ij} - [\lambda^{ij} c_1 - (1-\alpha) + \beta^{ij}]\iota^i}{(1-\lambda^{ij})} \quad \text{for } i \in \mathcal{I}, j \in \mathcal{J}, \tag{9}$$

[4] Bank defaults and insolvencies that cause bank runs are considered in Section 5.

where constraint (8) gives the maximum amount of liquid assets that can be stored between dates 1 and 2.

The clearing condition for the interbank market is

$$f^{ih} = -f^{il} \qquad \text{for } i \in \mathcal{I}. \tag{10}$$

An equilibrium consists of contingent interbank market interest rates and an allocation such that banks maximize profits, consumers make their withdrawal decisions to maximize their expected utility, and the interbank market clears.

3 Results and interest rate policy

In this section, we derive the optimal allocation and characterize equilibrium allocations. We start by showing that the optimal allocation is independent of the liquidity-shock state $i \in \mathcal{I}$ and bank types $j \in \mathcal{J}$. Next, we derive the Euler and no-arbitrage conditions. After that, we study the special cases in which a "crisis never occurs" when $\rho = 0$ and in which a "crisis always occurs" when $\rho = 1$. This allows us to build intuition for the general case where $\rho \in [0, 1]$.

3.1 First best allocation

To find the full-information first best allocation, we consider a planner who can observe consumer types. The planner can ignore the liquidity-shock state i, bank type j, and bank liquidity withdrawal shocks λ^{ij}. The planner maximizes the expected utility of depositors subject to feasibility constraints:

$$\begin{aligned}
\max_{\alpha \in [0,1]; c_1, \beta \geq 0} \quad & \overline{\lambda} u(c_1) + \left(1 - \overline{\lambda}\right) u(c_2) \\
\text{s.t.} \quad & \overline{\lambda} c_1 \leq 1 - \alpha - \beta \\
& \left(1 - \overline{\lambda}\right) c_2 \leq \alpha r + 1 - \alpha + \beta - \overline{\lambda} c_1 \\
& \beta \leq 1 - \alpha.
\end{aligned}$$

The constraints are the physical quantities of goods available for consumption at date 1 and 2, and available storage between dates 1 and 2, respectively. The first-order conditions and binding constraints give the well-known first best allocations, denoted with asterisks, as implicitly defined by

$$\begin{aligned} u'(c_1^*) &= ru'(c_2^*) && (11) \\ \overline{\lambda} c_1^* &= 1-\alpha^* && (12) \\ \left(1-\overline{\lambda}\right) c_2^* &= \alpha^* r && (13) \\ \beta^* &= 0. && (14) \end{aligned}$$

Equation (11) shows that the ratio of marginal utilities between dates 1 and 2 is equal to the marginal return on investment r.

3.2 First-order conditions

Next, we consider the optimization problem of a bank of type j given by equations (7) - (9) in order to find the Euler and no-arbitrage pricing equations.

Lemma 1. *First-order conditions with respect to c_1 and α are, respectively,*

$$\begin{aligned} u'(c_1) &= E[\frac{\lambda^{ij}}{\overline{\lambda}} \iota^i u'(c_2^{ij})] && (15) \\ E[\iota^i u'(c_2^{ij})] &= rE[u'(c_2^{ij})]. && (16) \end{aligned}$$

Proof. The Lagrange multiplier for constraint (8) is θ_β^{ij}. The first-order condition with respect to β^{ij} is

$$\begin{aligned} \tfrac{1}{2}\rho u'(c_2^{1j})(1-\iota^1) &\leq \theta_\beta^{1j} \qquad \text{for } j \in \mathcal{J} \qquad (= \text{ if } \beta^{1j} > 0) && (17) \\ (1-\rho)u'(c_2^0)(1-\iota^0) &\leq \theta_\beta^{0j} \qquad \text{for } j \in \mathcal{J} \qquad (= \text{ if } \beta^{0j} > 0). && (18) \end{aligned}$$

We first will show that $\theta_\beta^{ij} = 0$ for all $i \in \mathcal{I}$, $j \in \mathcal{J}$. Suppose not, that $\theta_\beta^{\widehat{i}\widehat{j}} > 0$ for some $\widehat{i} \in \mathcal{I}$, $\widehat{j} \in \mathcal{J}$. This implies that equation (17) or (18) corresponding to $\widehat{i}, \widehat{j}$, does not bind (since $\iota^i \geq 1$), which implies that $\beta^{\widehat{i}\widehat{j}} = 0$. Hence, equation (8) does not bind (since clearly $\alpha < 1$, otherwise $c_1 = 0$); thus, $\theta_\beta^{\widehat{i}\widehat{j}} = 0$ by complementary slackness, a contradiction. Therefore, $\theta_\beta^{ij} = 0$ for all $i \in \mathcal{I}$, $j \in \mathcal{J}$ can be substituted into the binding first order conditions (17) and (18), which can be written in expectation form to give equations (15) and (16). ■

Equation (15) is the Euler equation and determines the investment level α given ι^i for $i \in \mathcal{I}$. Equation (16), which corresponds to the first-order condition with respect to α, is the no-arbitrage pricing condition for the rate ι^i, which states that the expected marginal utility-weighted returns on storage and investment must be equal at date $t = 0$. The return on investment is r. The return on storage is the rate ι^i at which liquid assets can be lent at date 1, since banks can store liquid assets at date 0, lend them at date 1, and will receive ι^i at date 2. At the interest rates ι^1 and ι^0, banks are indifferent to holding liquid assets and long-term assets at date 0 according to the no-arbitrage condition. A corollary result shown in the proof of Lemma 1 is that banks do not store liquid assets at date 1:

$$\beta^{ij} = 0 \text{ for all } i \in \mathcal{I},\ j \in \mathcal{J}. \tag{19}$$

All liquid goods at date 1 are distributed by the banking system to impatient depositors.

The interbank market-clearing condition (10), together with the interbank market demand equation (5), determines $c_1^j(\alpha)$ and $f^{ij}(\alpha)$ as functions of α:

$$c_1(\alpha) = \frac{1-\alpha}{\overline{\lambda}} \tag{20}$$

$$\begin{aligned} f^{ij}(\alpha) &= (1-\alpha)\left(\frac{\lambda^{ij}}{\overline{\lambda}} - 1\right) \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \\ &= \left\{ \begin{array}{ll} i\varepsilon c_1 & \text{for } i \in \mathcal{I},\ j = h \\ -i\varepsilon c_1 & \text{for } i \in \mathcal{I},\ j = l. \end{array} \right. \end{aligned} \tag{21}$$

Since no liquid assets are stored between dates 1 and 2 for state $i = 0, 1$, patient depositors' consumption c_2^0 in state $i = 0$ equals the average of patient depositors' consumption c_2^{ij} in state $i = 1$ and equals total investment returns αr divided by the mass of impatient depositors $1 - \overline{\lambda}$:

$$\begin{aligned} c_2^0(\alpha) &= \frac{(1-\lambda^{1h})c_2^{1h} + (1-\lambda^{1l})c_2^{1l}}{1-\overline{\lambda}} \\ &= \frac{\alpha r}{1-\overline{\lambda}}. \end{aligned} \tag{22}$$

3.3 Single liquidity-shock state: $\rho \in \{0, 1\}$

We start by finding solutions to the special cases of $\rho \in \{0, 1\}$ in which there is certainty about the single state of the world i at date 1. These are particularly interesting benchmarks. In the case of $\rho = 0$, the state $i = 0$ is always realized. This case corresponds to the standard framework of Diamond and Dybvig (1983) and can be interpreted as a crisis never occurring. In the case of $\rho = 1$, the state $i = 1$ is always realized. This corresponds to the case studied by Bhattacharya and Gale (1987) and can be interpreted as a crisis always occurring. These boundary cases will then help to solve the general model $\rho \in [0,1]$.

With only a single possible state of the world at date 1, it is easy to show that the interbank rate must equal the return on long-term assets. First-order conditions (15) and (16) can be written more explicitly as

$$\begin{aligned} & \rho[\tfrac{1}{2}u'(c_2^{1h}) + \tfrac{1}{2}u'(c_2^{1l})]\iota^1 + (1-\rho)u'(c_2^0)\iota^0 \\ = \; & \rho[\tfrac{1}{2}u'(c_2^{1h}) + \tfrac{1}{2}u'(c_2^{1l})]r + (1-\rho)u'(c_2^0)r \end{aligned} \tag{23}$$

$$u'(c_1) = \rho[\tfrac{\lambda^{1h}}{2\overline{\lambda}}u'(c_2^{1h}) + \tfrac{\lambda^{1l}}{2\overline{\lambda}}u'(c_2^{1l})]\iota^1 + (1-\rho)u'(c_2^0)\iota^0. \tag{24}$$

As is intuitive, for $\rho = 0$, the value of ι^1 is indeterminate, and for $\rho = 1$, the value of ι^0 is indeterminate. In either case, there is an equilibrium with a unique allocation c_1, c_2^{ij}, and α. The indeterminate variable is of no consequence for the allocation. The allocation is determined by the two first-order equations, in the two unknowns α and ι^0 (for $\rho = 0$) or ι^1 (for $\rho = 1$). Equation (23) shows that the interbank lending rate equals the return on long-term assets: $\iota^0 = r$ (for $\rho = 0$) or $\iota^1 = r$ (for $\rho = 1$). With a single state of the world, the interbank lending rate must equal the return on long-term assets.

For $\rho = 0$, the crisis state never occurs. There is no need for banks to borrow on the interbank market. The banks' budget constraints imply that in equilibrium no interbank lending occurs, $f^{0j} = 0$ for $j \in \mathcal{J}$. However, the interbank lending rate ι^0 still plays the role of clearing markets: It is the lending rate at which each bank's excess demand is zero, which requires that the returns on liquidity and investment are equal. The result is $\iota^0 = r$, which is an important market price that ensures banks hold optimal liquidity. Our result—that the banks' portfolio decision is affected by a market price at which there is no trading—is similar to the effect of prices with no trading in equilibrium in standard portfolio theory and asset pricing with a representative agent. The Euler equation (24) is equivalent to equation (11) for the planner. Banks choose the optimal α^* and provide the first best allocation c_1^* and c_2^*.

Proposition 1. *For $\rho = 0$, the equilibrium is characterized by $\iota^0 = r$ and has a unique first best allocation c_1^*, c_2^*, α^*.*

Proof. For $\rho = 0$, equation (23) implies $\iota^0 = r$. Equation (24) simplifies to $u'(c_1) = u'(c_2^0)r$, and the bank's budget constraints bind and simplify to $c_1 = \frac{1-\alpha}{\lambda}$, $c_2^0 = \frac{\alpha r}{1-\lambda}$. These results are equivalent to the planner's results in equations (11) through (13), implying there is a unique equilibrium, where $c_1 = c_1^*$, $c_2^0 = c_2^*$, and $\alpha = \alpha^*$. ■

To interpret these results, note that banks provide liquidity at date 1 to impatient depositors by paying $c_1^* > 1$. This can be accomplished only by paying $c_2^* < r$ on withdrawals to patient depositors at date 2. The key for the bank being able to provide liquidity insurance to impatient depositors is that the bank can pay an implicit date 1 to date 2 intertemporal return on deposits of only $\frac{c_2^*}{c_1^*}$, which is less than the interbank market intertemporal rate ι^0, since $\frac{c_2^*}{c_1^*} < \iota^0 = r$. This contract is optimal because the ratio of intertemporal marginal utility equals the marginal return on long-term assets, $\frac{u'(c_2^*)}{u'(c_1^*)} = r$.

We now turn to the symmetric case of $\rho = 1$, where the crisis state $i = 1$ always occurs. We show that, in this case, the optimal allocation cannot be obtained, even though interbank lending provides redistribution of liquidity. Nevertheless, because the interbank rate is high, $\iota^1 = r$, patient depositors face inefficient consumption risk, and the liquidity provided to impatient depositors is reduced. The banks' borrowing demand from equation (21) shows that $f^{1h} = \varepsilon c_1$ and $f^{1l} = -\varepsilon c_1$.

First, consider the outcome at date 1 holding fixed $\alpha = \alpha^*$. With $\iota^1 = r$, patient depositors do not have optimal consumption since $c_2^{1h}(\alpha^*) < c_2^* < c_2^{1l}(\alpha^*)$. A bank of type h has to borrow at date 1 at the rate $\iota^1 = r$, higher than the optimal rate of $\frac{c_2^*}{c_1^*}$.

Second, consider the determination of α. Banks must compensate patient depositors for the risk they face. They can do so by increasing their expected consumption. Hence, in equilibrium, investment is $\alpha > \alpha^*$ and impatient depositors see a decease of their consumption. The results are illustrated in Figure 2. The difference of consumption c_2^0 for equilibrium α compared to $c_2^*(\alpha^*)$, $c_2^{1h}(\alpha^*)$, and $c_2^{1l}(\alpha)$ for a fixed $\alpha = \alpha^*$ is demonstrated by the arrows in Figure 2. The result is $c_1 < c_1^*$, $c_2^0 > c_2^*$, $c_2^{1h} > c_2^{1h}(\alpha^*)$, and $c_2^{1l} > c_2^{1l}(\alpha^*)$. For any $\varepsilon > 0$ shock, banks do not provide the optimal allocation.

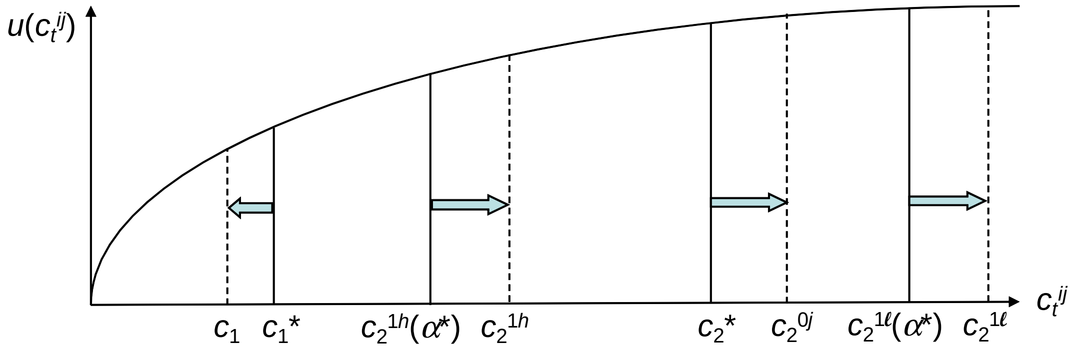

Figure 2: First best allocation and equilibrium allocation for $\rho = 1$

Proposition 2. *For $\rho = 1$, there exists an equilibrium characterized by $\iota^1 = r$ that has a unique suboptimal allocation*

$$\begin{aligned} c_1 &< c_1^* \\ c_2^{1h} &< c_2^* < c_2^{1l} \\ \alpha &> \alpha^*. \end{aligned}$$

Proof. For $\rho = 1$, equation (23) implies $\iota^1 = r$. By equation (6), $c_2^{1l} > c_2^{1h}$. From the bank's budget constraints and market clearing,

$$\frac{1 - \overline{\lambda} - \varepsilon}{2(1 - \overline{\lambda})} c_2^{1h} + \frac{1 - \overline{\lambda} + \varepsilon}{2(1 - \overline{\lambda})} c_2^{1l} = \frac{\alpha r}{1 - \overline{\lambda}} = c_2^0,$$

which implies $\frac{1}{2} c_2^{1h} + \frac{1}{2} c_2^{1l} < c_2^0$, since $c_2^{1l} > c_2^{1h}$. Because $u(\cdot)$ is concave, $\frac{1}{2} u'(c_2^{1h}) + \frac{1}{2} u'(c_2^{1l}) > u'(c_2^0)$. Further, $\frac{\lambda^{1h}}{2\overline{\lambda}} u'(c_2^{1h}) + \frac{\lambda^{1l}}{2\overline{\lambda}} u'(c_2^{1l}) > u'(c_2^0)$ since $\lambda^{1h} > \lambda^{1l}$, $\frac{\lambda^{1h}}{2\overline{\lambda}} + \frac{\lambda^{1l}}{2\overline{\lambda}} = 1$ and $c_2^{1h} < c_2^{1l}$. Thus,

$$\begin{aligned} u'(c_1(\alpha^*)) &= r u'(c_2^0(\alpha^*)) \\ &< r\Big[\frac{\lambda^{1h}}{2\overline{\lambda}} u'(c_2^{1h}(\alpha^*)) + \frac{\lambda^{1l}}{2\overline{\lambda}} u'(c_2^{1l}(\alpha^*))\Big]. \end{aligned}$$

Since $u'(c_1(\alpha))$ is increasing in α and $u'(c_2^{1j}(\alpha))$ for $j \in \mathcal{J}$ is decreasing in α, the Euler equation implies that, in equilibrium, $\alpha > \alpha^*$. Hence, $c_1 = \frac{1-\alpha}{\overline{\lambda}} < c_1^*$, $c_2^{1l} > c_2^0 = \frac{\alpha r}{1-\overline{\lambda}} > c_2^*$ and $c_2^{1h} < c_2^*$. ■

Notice that, for $\rho = 1$, the difference between our approach and that of Bhattacharya and Gale (1987) is that in our framework the market cannot impose any restriction on the size of the trades. This forces the interbank market to equal r and creates an inefficiency. The mechanism design approach of Bhattacharya and Gale (1987) yields a second best allocation that achieves higher welfare, but in that case the market cannot be anonymous anymore, as the size of the trade has to be observed and enforced.

3.4 Multiple liquidity-shock states: $\rho \in [0,1]$

We now apply our results for the special cases $\rho \in \{0,1\}$ to the general case $\rho \in [0,1]$. It is convenient to define an *ex-post equilibrium*, which refers to the interest rate that clears the interbank market in state i at date 1, conditional on a given α and c_1. For distinction, we use the term *ex-ante equilibrium* to refer to our equilibrium concept used above from the perspective of date 0. We first show that the supply and demand in the interbank market are inelastic, which creates an indeterminacy of the ex-post equilibrium interest rate. Next, we show that there is a real indeterminacy of the ex-ante equilibrium. There is a continuum of Pareto-ranked ex-ante equilibria with different values for c_1, c_2^{ij}, and α.

We first show the indeterminacy of the ex-post equilibrium interest rate. In state $i = 1$, bank type l has excess liquid assets that it supplies in the interbank market of

$$-f^{1l}(\iota^1) = \{ \begin{array}{ll} \varepsilon c_1 & \text{for } \iota^1 \geq 1 \\ 0 & \text{for } \iota^1 < 1. \end{array} \tag{25}$$

The liquid bank has an inelastic supply of liquid assets above a rate of one because its alternative to lending is storage, which gives a return of one. Bank type h has a demand for liquid assets of

$$f^{1h}(\iota^1) = \{ \begin{array}{ll} 0 & \text{for } \iota^1 > 1 + \frac{(1-\overline{\lambda})(c_2^0 - c_1)}{\varepsilon c_1} \\ \varepsilon c_1 & \text{for } \iota^1 \in [1, 1 + \frac{(1-\overline{\lambda})(c_2^0 - c_1)}{\varepsilon c_1}] \\ \infty & \text{for } \iota^1 < 1. \end{array} \tag{26}$$

The maximum rate ι^1 at which the illiquid bank type j can borrow, such that the incentive constraint $c_2^{1h} \geq c_1$ holds and patient depositors do not withdraw at date 1, is $1 + \frac{(1-\overline{\lambda})(c_2^0 - c_1)}{\varepsilon c_1}$. The illiquid bank has an inelastic demand for liquid assets below the rate ι^1 because its alternative to borrowing is to default on withdrawals to impatient depositors at date 1. The banks' supply and demand curves for date 1 are illustrated in Figure 3. In state $i = 0$, each bank has an inelastic net demand for liquid assets of

$$f^{0j}(\iota^0) = \{ \begin{array}{ll} 0 & \text{for } \iota^0 \geq 1 \\ \infty & \text{for } \iota^0 < 1. \end{array} \tag{27}$$

At a rate of $\iota^0 > 1$, banks do not have any liquid assets they can lend in the market. All such assets are needed to cover the withdrawals of impatient depositors. At a rate of $\iota^0 < 1$, a bank could store any amount of liquid assets borrowed for a return of one.

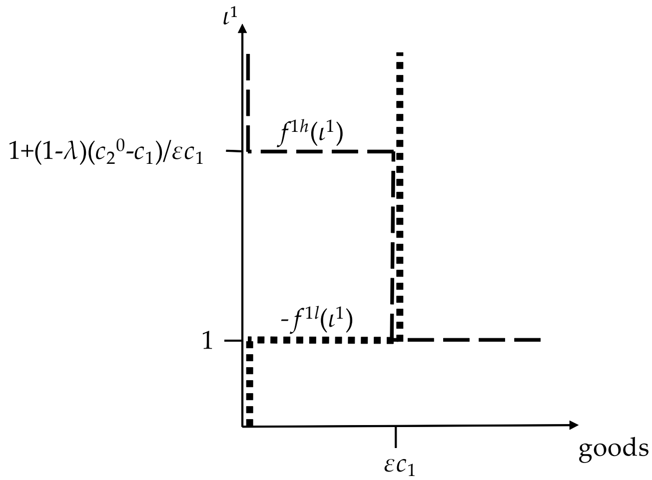

Figure 3: Interbank market in state $i = 1$

Lemma 2. The ex-post equilibrium rate ι^i in state i, for $i \in \mathcal{I}$, is indeterminate:

$$\begin{aligned} \iota^0 &\geq 1 \\ \iota^1 &\in [1, 1 + \frac{(1-\overline{\lambda})(c_2^0 - c_1)}{\varepsilon c_1}]. \end{aligned}$$

Proof. Substituting for $f^{0j}(\iota^0)$ from (27), for $j \in \mathcal{J}$, into market-clearing condition (10) and solving gives the condition for the equilibrium rate ι^0. Substituting for $f^{1l}(\iota^1)$ and $f^{1h}(\iota^l)$ from (25) and (26) into market-clearing condition (10) and solving gives the corresponding condition for the equilibrium rate ι^1. ■

This result highlights a key insight of our model: The supply and demand of short-term liquidity are fundamentally inelastic. By the nature of short-term financing, distributional liquidity shocks imply that liquidity held in excess of immediate needs is of low fundamental value to the bank that holds it, while demand for liquidity for immediate needs is of high fundamental value to the bank that requires it to prevent default. The interest rate ι^i determines how gains from trade are shared ex-post among banks. Low rates benefit illiquid banks and their claimants, and decrease impatient depositors' consumption risk, which increases ex-ante expected utility for all depositors.

Next, we show that there exists a continuum of Pareto-ranked ex-ante equilibria. Finding an equilibrium amounts to solving the two first-order conditions, equations (15) and

(16), in three unknowns, α, ι^1, and ι^0. This is a key difference with respect to the benchmark cases of $\rho = 0, 1$. For each of these cases, there is only one state that occurs with positive probability, and the corresponding state interest rate is the only ex-post equilibrium rate that is relevant. Hence, there are two relevant variables, α and ι^i, where i is the relevant state, that are uniquely determined by the two first-order conditions.

In the general two-states model, a bank faces a distribution of probabilities over two interest rates. A continuum of pairs (ι^1, ι^0) supports an ex-ante equilibrium. This result is novel in showing that, when there are two idiosyncratic liquidity states i at date 1, there exists a continuum of ex-ante equilibria.[5] Allen and Gale (2004) also show that a continuum of interbank rates can support an ex-post sunspot equilibrium. However, because they consider a model with a single state, the only rate that supports an ex-ante equilibrium is r, similar to our benchmark case of $\rho = 1$.

If the interbank rate is not state contingent, $\iota^1 = \iota^0 = r$ is the unique equilibrium, as is clear from equation (23). The allocation resembles a weighted average of the cases $\rho \in \{0, 1\}$ and is suboptimal, showing an important drawback of the separation between prudential regulation and monetary policy. In the case where $\iota^1 = \iota^0 = r$, equation (24) implies that $\alpha(\rho)$, $c_2^0(\rho)$, $c_2^{1h}(\rho)$, and $c_2^{1l}(\rho)$ are implicit functions of ρ. The cases of $\rho = 0$ and $\rho = 1$ provide bounds for the general case of $\rho \in [0, 1]$. Equilibrium consumption $c_1(\rho)$ and $c_2^{ij}(\rho)$ for $i \in \mathcal{I}$, $j \in \mathcal{J}$, written as functions of ρ, are displayed in Figure 4. This figure shows that $c_1(\rho)$ is decreasing in ρ while $c_2^{ij}(\rho)$ is increasing in ρ:

$$\begin{aligned} c_2^{ij}(0) &\leq c_2^{ij}(\rho) \leq c_2^{ij}(1) \qquad \text{for } \rho \in [0,1],\ i \in \mathcal{I},\ j \in \mathcal{J} \\ c_1(1) &\leq c_1(\rho) \leq c_1(0) \qquad \text{for } \rho \in [0,1]. \end{aligned}$$

In addition,

$$\begin{aligned} c_2^0(\rho = 0) &= c_2^* && \text{for } j \in \mathcal{J} \\ c_1(\rho = 0) &= c_1^* \\ c_2^{1j}(\rho = 0) &= c_2^{1j}(\alpha = \alpha^*) && \text{for } j \in \mathcal{J}. \end{aligned}$$

With interbank rates equal to r in all states, patient depositors face too much risk. To compensate them for this risk, their expected consumption must be increased to the detriment of impatient depositors.

[5] The results from this section generalize in a straightforward way to the case of N states, as shown in the Appendix A.

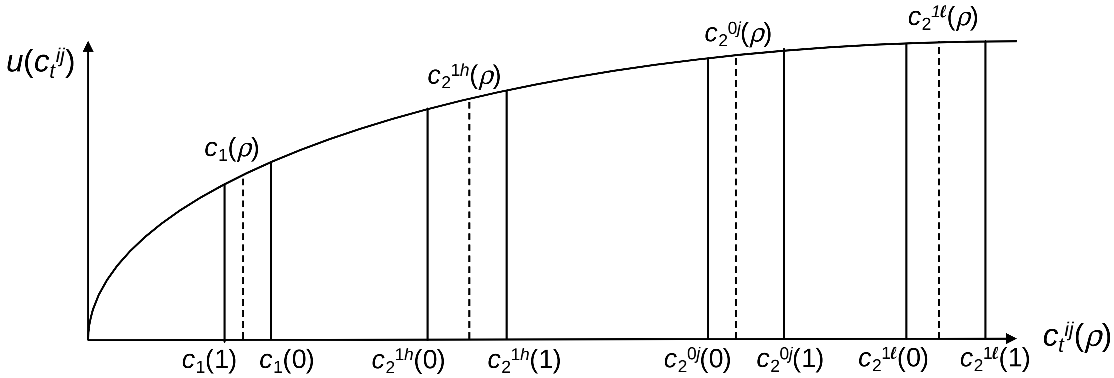

Figure 4: Equilibrium allocation for $\rho \in [0, 1]$

Finally, we show that there exists a first best ex-ante equilibrium with state contingent interest rates for $\rho < 1$. The interest rate must equal the optimal return on bank deposits during a crisis:

$$\iota^1 = \iota^{1^*} \equiv \frac{c_2^*}{c_1^*} < r. \tag{28}$$

To show this, first we substitute for ι^1, λ^{ij}, c_1, and β^{ij} from equations (28), (1), (20), and (19) into equation (6) and simplify, which for $i = 1$ and $j = h, l$ gives

$$c_2^{1h} = c_2^{1l} = \frac{\alpha r}{1 - \overline{\lambda}}. \tag{29}$$

This shows that, with ι^1 equal to the optimal intertemporal return on deposits between dates 1 and 2, there is optimal risk-sharing of the goods that are available at date 2. This implies that the interbank market rate has to be low for patient depositors to face no risk. Substituting for ι^1, c_2^{1j}, and c_2^0 from equations (28), (29), and (22), respectively, into equation (23) and rearranging gives the interest rate in state $i = 0$:

$$\iota^0 = r + \frac{\rho(r - \frac{c_2^0}{c_1})}{1 - \rho}, \tag{30}$$

and further substituting for these variables into equation (24) and rearranging gives $u'(c_1) = r'u'(c_2^0)$. This is the planner's condition and implies $\alpha = \alpha^*$, $c_1 = c_1^*$, and $c_2^0 = c_2^*$, a first best allocation.

Substituting these equilibrium values into equation (30) and simplifying shows that

$$\iota^0 = \iota^{0^*} \equiv r + \frac{\rho(r - \frac{c_2^*}{c_1^*})}{1 - \rho} > r. \tag{31}$$

The market rate ι^0 must be greater than r during the no-shock state, in order for the expected rate to equal r, such that banks are indifferent to holding liquid assets and investing at date 0. Equation (16) implies, then, that the expected market rate is $E[\iota^i] = r$.

Figure 5 illustrates the difference between the first best equilibrium (with ι^{1^*}, ι^{0^*}) and the suboptimal equilibrium (with $\iota^1 = \iota^0 = r$). Arrows indicate the change in consumption between the suboptimal and the first best equilibria.

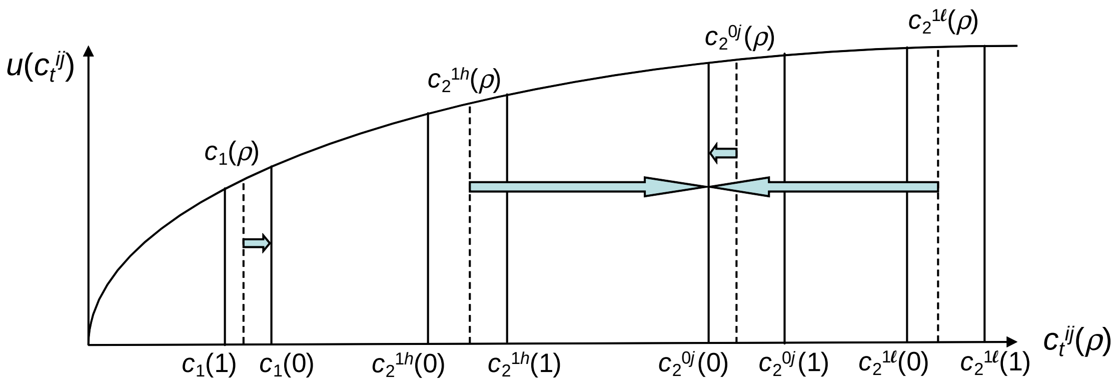

Figure 5: Difference between equilibrium allocation and first best allocation for $\rho \in [0, 1]$

Proposition 3. *For $\rho \in (0, 1)$, there exists a continuum of ex-ante equilibria with different Pareto-ranked allocations. In particular, there exists a suboptimal ex-ante equilibrium with*

$$\begin{aligned} \iota^1 &= \iota^0 = r \\ \alpha &> \alpha^* \\ c_1 &< c_1^* < c_2^* < c_2^0 \\ c_2^{1h} &< c_2^* < c_2^{1l}, \end{aligned}$$

corresponding to a noncontingent monetary policy and a first best ex-ante equilibrium with

$$\begin{aligned} \iota^1 &= \frac{c_2^*}{c_1^*} < r \\ \iota^0 &= \iota^{0^*} > r \\ \alpha &= \alpha^* \\ c_1 &= c_1^* \\ c_2^{ij} &= c_2^* \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J}. \end{aligned}$$

3.5 Central bank interest rate policy

The result of multiple Pareto-ranked equilibria and a need for a state-contingent interest rate in our model suggest a role for an institution that can select the best equilibrium. Since equilibria can be distinguished by the interest rate in the interbank market, a central bank is the natural candidate for this role. A central bank can select the optimal equilibrium and intervene by targeting the optimal market interest rate. We think of the interest rate ι^i at which banks lend in the interbank market as the unsecured interest rate that many central banks target for monetary policy. In the U.S., the Federal Reserve targets the overnight interest rate, also known as the federal funds rate.

We extend the model by adding a central bank that can offer to borrow an amount $\Delta > 0$ below ι^{i*} and lend an amount $\Delta > 0$ above ι^{i*} on the interbank market in order to target the interbank rate equal to ι^{i*}. The central bank's objective is to maximize the depositor's expected utility equation (2), subject to the bank's optimization equations (7) through (9), by submitting the following demand and supply functions, respectively, for the interbank market:

$$f^{iD}(\iota^i) = \begin{cases} 0 & \text{for } \iota^i \geq \iota^{i*}, \quad i \in \mathcal{I} \\ \Delta & \text{for } \iota^i < \iota^{i*} \quad i \in \mathcal{I} \end{cases} \tag{32}$$

$$f^{iS}(\iota^i) = \begin{cases} -\Delta & \text{for } \iota^i > \iota^{i*}, \quad i \in \mathcal{I} \\ 0 & \text{for } \iota^i \leq \iota^{i*}, \quad i \in \mathcal{I}, \end{cases} \tag{33}$$

for any $\Delta > 0$. The goods-clearing condition for the interbank market (10) is replaced by

$$f^{ih}(\iota^i) + f^{iD}(\iota^i) = -[f^{il}(\iota^i) + f^{iS}(\iota^i)] \qquad \text{for } i \in \mathcal{I}. \tag{34}$$

Substituting for the supply and demand functions, the market-clearing condition (34) can be written as

$$i\varepsilon c_1 + \mathbf{1}_{\iota^i < \iota^{i*}}\Delta = i\varepsilon c_1 + \mathbf{1}_{\iota^i > \iota^{i*}}\Delta \qquad \text{for } i \in \mathcal{I},$$

which, for any $\Delta > 0$, holds for the unique state i ex-post equilibrium rate $\iota^i = \iota^{i*}$, for $i \in \mathcal{I}$. The ex-post equilibrium rate in state $i = 1$ is shown in Figure 6. The figure illustrates how the central bank shifts the market supply and demand curves such that there is a unique equilibrium at ι^{1*}. At ι^{i*}, the equilibrium quantity that clears the market according to condition (34) is $i\varepsilon c_1$. The quantity Δ, with which the central bank intervenes out of equilibrium, is irrelevant. The state-conditional equilibrium rate is uniquely determined as ι^{i*} and the ex-ante equilibrium is uniquely determined as $(\alpha^*, \iota^{0*}, \iota^{1*})$, for any $\Delta > 0$.

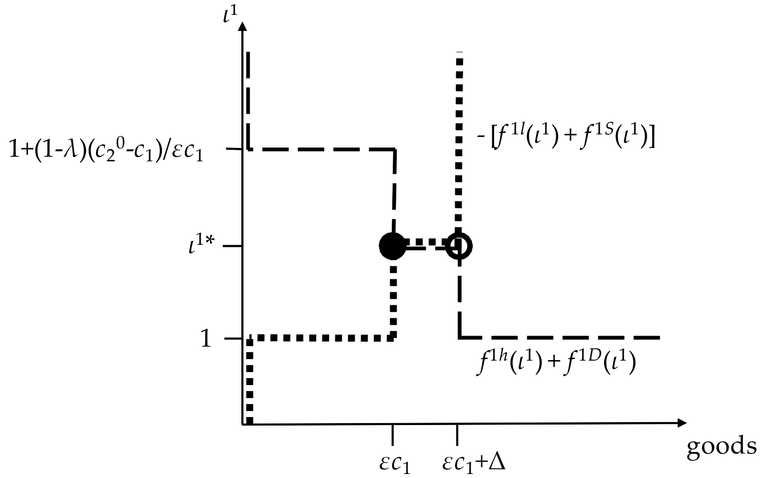

Figure 6: Interbank market in state $i = 1$ with optimal central bank interest rate policy

The central bank policy may also be interpreted as setting interest rates according to "open mouth operations," which refers to the concept of the central bank adjusting short-term market rates by announcing its intended rate target, without any trading or lending by the central bank in equilibrium. In the model, zero trading is required by the central bank in equilibrium, and the amount Δ of borrowing and lending offered by the central bank approaches zero in the limit. Open mouth operations have been used to describe how the Federal Reserve uses a very small amount of changes in liquidity reserves to effect interest rate changes after the target change has been announced.[6]

Proposition 4. *Under optimal central bank interest rate policy, the central bank sets* $\iota^1 = \iota^{1*} < r$ and $\iota^0 = \iota^{0*} > r$. *There exists a unique ex-ante equilibrium, which has a first best allocation* α^*, c_1^*, c_2^*.

This proposition provides the main policy result of our model. Several things are worth noting. First, the central bank should respond to pure distributional liquidity shocks, i.e., involving no aggregate liquidity shocks, by lowering the interbank rate. Second, the central bank must keep the interbank rate sufficiently high in normal times to provide banks with

[6]See also Guthrie and Wright (2000), who describe monetary policy implementation through open mouth operations in the case of New Zealand.

incentives to invest enough in liquid assets. Third, the policy rule should be announced in advance so that banks can anticipate the central bank's state-contingent actions.

All of our results hold in a version of our model where bank deposit contracts are expressed in nominal terms and fiat money is borrowed and lent at nominal rates in the interbank market, along the lines of Skeie (2008) and Martin (2006). In the nominal version of the model, the central bank targets the real interbank rate by offering to borrow and lend at a nominal rate in fiat central bank reserves rather than goods (see Appendix B). This type of policy resembles more closely the standard tools used by central banks.

4 Aggregate shocks and central bank quantitative policy

The standard view on aggregate liquidity shocks is that they should be dealt with through open market operations, as advocated by Goodfriend and King (1988), for example. Since our framework provides micro-foundations for the interbank market, and this has consequences for the overall allocation, it is worth revisiting the issue of aggregate liquidity shocks. Despite the apparent complexity, we verify that the central bank should use a quantitative liquidity-injection policy in the face of aggregate shocks. Thus, the central bank should respond to different kinds of shocks with different policy instruments: liquidity injection to deal with aggregate liquidity shocks and interest rate policy for distributional liquidity shocks.

We extend the model to allow the probability of a depositor being impatient—and, hence, the aggregate fraction of impatient depositors in the economy—to be stochastic. This probability is denoted by $\overline{\lambda}_a$, where $a \in \mathcal{A} \equiv \{H, L\}$ is the aggregate-shock state,

$$a = \{ \begin{array}{ll} H & \text{with prob } \pi \\ L & \text{with prob } 1-\pi, \end{array}$$

and $\pi \in [0,1]$. The state $a = H$ denotes a high aggregate liquidity shock, in which a high fraction of depositors are impatient, and state $a = L$ denotes a low aggregate liquidity shock, in which a low fraction of depositors are impatient. We assume that $\overline{\lambda}_H \geq \overline{\lambda}_L$ and $\pi\overline{\lambda}_H + (1-\pi)\overline{\lambda}_L = \overline{\lambda}$. Hence, $\overline{\lambda}$ remains the unconditional fraction of impatient depositors. The aggregate-state random variable a is independent of the idiosyncratic-state variable i. We assume that the central bank can tax the endowment of agents at date 0, store these goods, and return the taxes at date 1 or at date 2. We denote these transfers, which can be conditional on the aggregate shock, τ_0, τ_{1a}, τ_{2a}, $a \in \mathcal{A}$, respectively.

The depositor's expected utility (2) is replaced by

$$\begin{aligned}
E[U] &= \left[\pi\overline{\lambda}_H + (1-\pi)\overline{\lambda}_L\right] u(c_1) \\
&+(1-\rho)\left[\pi(1-\overline{\lambda}_H)u(c_{2H}^0) + (1-\pi)(1-\overline{\lambda}_L)u(c_{2L}^0)\right] \\
&+\rho\frac{\pi}{2}\left[(1-\lambda_H^{1h})u(c_{2H}^{1h}) + (1-\lambda_H^{1l})u(c_{2H}^{1l})\right] \\
&+\rho\frac{1-\pi}{2}\left[(1-\lambda_L^{1h})u(c_{2L}^{1h}) + (1-\lambda_L^{1l})u(c_{2L}^{1l})\right],
\end{aligned}$$

and the bank's budget constraints (3) and (4) are replaced by

$$\begin{aligned}
\lambda_a^{ij}c_1 &= 1-\tau_0-\alpha-\beta_a^{ij}+f_a^{ij}+\tau_{1a}, \qquad \text{for } a\in\mathcal{A},\ i\in\mathcal{I},\ j\in\mathcal{J} \\
(1-\lambda_a^{ij})c_{a2}^{ij} &= \alpha r+\beta_a^{ij}-f_a^{ij}\iota_a^i+\tau_{2a}, \qquad \text{for } a\in\mathcal{A},\ i\in\mathcal{I},\ j\in\mathcal{J},
\end{aligned}$$

respectively, where the subscript a in variables c_{2a}^{ij}, λ_a^{ij}, β_a^{ij}, and ι_a^i denotes that these variables are conditional on $a \in \mathcal{A}$ in addition to $i \in \mathcal{I}$ and $j \in \mathcal{J}$.

The planner's optimization with aggregate shocks is identical to the problem described in Allen, Carletti, and Gale (2008). They show that there exists a unique solution to this problem. Intuitively, the first best with aggregate shocks is constructed as follows. The planner stores just enough goods to provide consumption to all impatient agents in the state with many impatient agents, $a = H$. This implicitly defines c_1^*. In this state, patient agents consume only goods invested in the long-term technology. In the state with few impatient agents, $a = L$, the planner stores $(\overline{\lambda}_H - \overline{\lambda}_L)c_1^*$ goods in excess of what is needed for impatient agents. These goods are stored between dates 1 and 2 and given to patient agents.

Proposition 5. *If $\rho < 1$, a central bank can implement the first best allocation.*

Proof. We prove this proposition by constructing the allocation the central bank implements. The first-order conditions take the same form as in the case without aggregate risk and become

$$u'(c_1) = E\left[\frac{\lambda_a^{ij}}{\pi\overline{\lambda}_H + (1-\pi)\overline{\lambda}_L}\iota_a^i u'(c_{a2}^{ij})\right], \qquad \text{for } a\in\mathcal{A},\ i\in\mathcal{I},\ j\in\mathcal{J} \tag{35}$$

$$E[\iota_a^i u'(c_{a2}^{ij})] = rE[u'(c_{a2}^{ij})], \qquad \text{for } a\in\mathcal{A},\ i\in\mathcal{I},\ j\in\mathcal{J}. \tag{36}$$

Assume that the amount of stored goods that the central bank taxes is $\tau_0 = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$. Consider the economy with idiosyncratic shocks, $i = 1$. If there are many impatient depositors, the banks do not have enough stored goods, on aggregate, for these depositors. However, the central bank can return the taxes at date 1, setting $\tau_{1H} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$ (and $\tau_{2H} = 0$), so that banks have just enough stored goods on aggregate. The interbank market interest rate is indeterminate, since the supply and demand of stored goods are inelastic, so the central bank can choose the rate to be $\iota^1 = \frac{c_2^*}{c_1^*}$. If there are few impatient depositors, the central bank sets $\tau_{1L} = 0$ (with $\tau_{2L} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$) and $\iota^1 = \frac{c_2^*}{c_1^*}$.

Now consider the economy in the case where $i = 0$. If there are many impatient depositors, the banks will not have enough stored goods for their them. However, as in the previous case, the central bank can return the taxes at date 1, setting $\tau_{1H} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$ (and $\tau_{2H} = 0$), so that banks have enough stored goods. There is no activity in the interbank market, and the interbank market rate is indeterminate. If there are few impatient depositors and the central bank sets $\tau_{1L} = 0$ (with $\tau_{2L} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$), then banks have just enough goods for their impatient depositors at date 1. Again, there is no activity in the interbank market, and the interbank market rate is indeterminate. Hence, the interbank market rate can be chosen to make sure that equation (36) holds.

With interbank market rates set in that way, banks will choose the optimal investment. Indeed, since equation (36) holds, banks are willing to invest in both storage and the long-term technology. In states where there is no idiosyncratic shock, there is no interbank market lending, so any deviation from the optimal investment carries a cost. In states where there is an idiosyncratic shock, the rate on the interbank market is such that the expected utility of a bank's depositors cannot be higher than under the first best allocation, so there is no benefit from deviating from the optimal investment in these states. ■

The interest rate policy of the central bank is effective only if the inelastic parts of the supply and demand curves overlap. With aggregate liquidity shocks, this need not happen, which creates inefficiencies. Proposition 5 illustrates that the role of the quantitative policy is to modify the amount of liquid assets in the market so that the interest rate policy can be effective. Hence, the central bank uses different tools to deal with aggregate and idiosyncratic shocks. When an aggregate shock occurs, the central bank needs to inject liquidity in the form of stored goods. In contrast, when an idiosyncratic shock occurs, the central bank needs to lower interest rates. Both actions are necessary if both shocks occur simultaneously.

During the recent crisis, certain central banks have used tools that have been characterized as similar to fiscal policy. This is consistent with our model in that the central bank policy of taxing and redistributing goods in the case of aggregate shocks resembles fiscal policy. The model does not imply that the central bank should be the preferred institution to implement this kind of policy. For example, we could assume that different institutions are in charge of 1) setting the interbank rate, and 2) choosing τ_0, τ_{1a}, τ_{2a}, $a \in A$. Regardless of the choice of institutions, our model suggests that implementing a good allocation may require using tools that resemble fiscal policy in conjunction with more standard central bank tools.

5 Contingent interest rate setting and financial stability

Our model allows us to shed light on the role of the interbank market in coping with idiosyncratic liquidity shocks and the impact of interest rates on the ex-post redistribution of risks. In our framework, a contingent interest-rate-setting policy dominates a noncontingent one. This is a strong criticism of the conventional view supporting the separation of prudential regulation and monetary policy. We now proceed to compare contingent and noncontingent interest rate policy in terms of financial stability. We show that fundamental bank runs can occur for a noncontingent interest-rate-setting policy, whereas they cannot arise when a contingent interest rate setting is implemented. Thus, contingent interest-rate-setting policy, and the rejection of separation between prudential and monetary policies, fares better also in terms of financial stability.

To simplify the exposition, we assume that the probability of an aggregate liquidity shock is zero, such that the fraction of impatient depositors is always $\overline{\lambda}$, as in the basic idiosyncratic-shock-state model of Section 3. We now consider a wider range of parameters. We no longer require that $c_2^{ij} > c_1$; we now consider any parameters such that $c_2^{ij} > 0$. This allows us to consider fundamental bank runs, which we define as occurring to bank j in state i if $c_2^{ij} < c_1$. In this case, each impatient consumer prefers to withdraw at date 1 even if all other consumers withdraw at date 2. The origin of possible fundamental bank runs is that, in the state where $i = 1$, patient depositors of banks with many impatient agents will consume less if the central bank sets the interest rate higher than $\frac{c_2^*}{c_1^*}$. If ε is large, it may be the case that the consumption of patient depositors of banks with many impatient agents would be lower if they withdraw at date 2 than if they withdraw at date 1, which would trigger a bank run. Obviously, if the optimal contingent interest-rate-setting policy is applied, and $i^1 = \frac{c_2^*}{c_1^*}$, fundamental bank runs are ruled out.

Proposition 6. *If $\overline{\lambda} \geq 1/2$ and the central bank chooses to implement a noncontingent interest rate policy, for ρ sufficiently low there exist ε sufficiently large such that bank runs will occur in equilibrium.*

Proof. The central bank sets an interest rate $\iota^1 = \iota^0 = r > \frac{c_2^*}{c_1^*}$. For $\iota^i > 1$, $i \in \mathcal{I}$, the first-order conditions of the bank's optimization with respect to β^{ij}, equations (17) and (18), do not bind, implying $\beta^{ij} = 0$ for $i \in \mathcal{I}$, $j \in \mathcal{J}$. As ρ converges to 0, by continuity, equilibrium allocations converge to

$$c_2^{1h} = \frac{\alpha r - \varepsilon c_1 r}{1 - \overline{\lambda} - \varepsilon} = \frac{r}{1 - \overline{\lambda} - \varepsilon} \left[\alpha - \varepsilon c_1\right],$$

with $c_1 = \frac{1-\alpha}{\overline{\lambda}}$ and $c_2^0 = \frac{\alpha r}{1-\overline{\lambda}}$. A fundamental bank run will occur in state $i = 1$ if and only if $c_2^{1h} < c_1$. This is equivalent to

$$r\left[\alpha - \varepsilon c_1\right] < (1 - \overline{\lambda} - \varepsilon)c_1,$$

so that ε has to satisfy

$$\varepsilon > \frac{r\alpha - (1 - \overline{\lambda})c_1}{c_1(r-1)},$$

or equivalently

$$\varepsilon > \frac{(1 - \overline{\lambda})(\frac{c_2^0}{c_1} - 1)}{r - 1}.$$

Recall that $0 \leq \lambda^{ij} \leq 1$ implies $\varepsilon \leq \min\{\overline{\lambda}, 1 - \overline{\lambda}\}$. If $\overline{\lambda} \geq \frac{1}{2}$, then this condition becomes $\varepsilon \leq 1 - \overline{\lambda}$. The condition on parameters such that $c_2^{ij} > 0$ requires $\varepsilon < \frac{\alpha}{c_1}$, which is sufficient to ensure $\varepsilon \leq 1 - \overline{\lambda}$ since $\alpha = 1 - \overline{\lambda} c_1$ and $c_1 > 1$. So, bank runs will occur for $\varepsilon \in (\frac{(1-\overline{\lambda})(\frac{c_2^0}{c_1}-1)}{r-1}, \frac{\alpha}{c_1})$, which is a non-empty interval. Thus, there exist ε for which bank runs will occur. Now, since bank runs are anticipated, banks could choose a "run preventing" deposit contract, as suggested by Cooper and Ross (1998). However, following the argument in that paper, banks will not choose a run-preventing deposit contract if the probability of a bank run is sufficiently small. So for ρ sufficiently close to zero, there exist ε for which bank runs will occur in equilibrium. ■

6 Liquidation of the long-term technology

We endogenize the amount of liquid assets available in the interbank market at date 1 by extending the model to allow for premature liquidation of the investment. Allowing for liquidation also allows us to examine the robustness of the central bank's interest rate policy to banks' outside options for borrowing on the interbank market. Banks in need of liquidity may choose to liquidate investment if the interbank rate is too high. This can restrict the set of feasible real interbank rates and may preclude the first best equilibrium. Indeed, as banks have the alternative option of liquidating their assets, interbank market rates that are larger than the return on liquidation are not feasible, and this might restrict the central bank's policy options.

Again, to simplify the exposition, we assume that the fraction of impatient depositors is always $\overline{\lambda}$. At date 1, bank j can liquidate γ^{ij} of the investment for a salvage rate of return s at date 1 and no further return at date 2. The bank budget constraints (3) and (4) are replaced by

$$\begin{aligned} \lambda^{ij} c_1 &= 1 - \alpha - \beta^{ij} + \gamma^{ij} s + f^{ij} \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \qquad (37) \\ (1 - \lambda^{ij}) c_2^{ij} &= (\alpha - \gamma^{ij}) r + \beta^{ij} - f^{ij} \iota^i \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J}, \qquad (38) \end{aligned}$$

respectively, and the bank optimization (7) is replaced by

$$\begin{aligned} \max_{\alpha, c_1, \{\beta^{ij}, \gamma^{ij}\}_{i \in \mathcal{I}, j \in \mathcal{J}}} \quad & E[U] \\ \text{s.t.} \quad & \beta^{ij} \leq 1 - \alpha \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \\ & \gamma^{ij} \leq \alpha \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \\ & \text{equations (37) and (38).} \end{aligned}$$

The ability for banks to liquidate long-term assets for liquid assets and lend them on the interbank market restricts the ex-post equilibrium interest rate from being too high. This is the case because, for any state i, the ex-post equilibrium rate is restricted by $\iota^i \leq \frac{r}{s}$. Indeed, for $\iota^i > \frac{r}{s}$ banks would prefer to liquidate all investment and no banks would borrow. Consequently, the optimal equilibrium cannot be supported if the rate required to support the first best ex-ante equilibrium in state $i = 0$ is too high. Proposition 7 gives a more precise statement:

Proposition 7. *If* $\iota^{0*} = r + \frac{\rho(r - \frac{c_2^*}{c_1^*})}{1-\rho} > \frac{r}{s}$, *the first best cannot be achieved as a contingent interest-rate-setting market equilibrium.*

Proof. If $\iota^{0*} > \frac{r}{s}$, then the equilibrium rate is $\iota^0 < \frac{r}{s} < \iota^{0*}$; it is less than the equilibrium rate required to support a first best equilibrium. ■

If the probability of a crisis is low enough, then the first best equilibrium is always feasible. The limit of ι^{0*} as $\rho \longrightarrow 0$ is r. Moreover, for small ρ, ι^{0*} has to be only slightly greater than r for the interest rate in expectation to equal r, because the probability of the rate being low during a crisis is small. This result is expressed in the next proposition.

Proposition 8. *For any* $s \leq 1$, *there exists a* $\widehat{\rho} > 0$ *such that for all* $\rho \in (0, \widehat{\rho})$, $\iota^{0*} < \frac{r}{s}$ *and the first best ex-ante equilibrium exists.*

Proof. Consider $\widehat{s} < 1$ and define $\widehat{\rho} \equiv \frac{r(1-\widehat{s})}{1+\widehat{s}(r - \frac{c_2^*}{c_1^*})} > 0$. The first best equilibrium exists if

$$\begin{aligned} \iota^{0*} &< \frac{r}{s} \\ \Longleftrightarrow \quad & r + \frac{\rho(r - \frac{c_2^*}{c_1{}^*})}{1-\rho} < \frac{r}{s} \\ \Longleftrightarrow \quad & \rho < \widehat{\rho}. \blacksquare \end{aligned}$$

It is interesting to emphasize that, as s stands for salvage value of the investment, it can be interpreted as the liquidity of a market for the long-run technology. From that perspective, our result states that the higher the liquidity of the market for the long-term technology, the lower the ex-ante efficiency of the banking system. Our result is surprising in the context of central bank policy, but it is quite natural in the context of Diamond-Dybvig models, where the trading of deposits destroys the liquidity insurance function of banks.

7 Conclusion

Our paper provides micro-foundations for the interbank market role in allocating liquidity, which is important in order to understand how central banks should respond to liquidity shocks. Two types of liquidity shocks are considered: distributional shocks and aggregate shocks. The main insight is that, because of the inelasticity of the short-term market for the liquid asset, the central bank can pick an optimal equilibrium from a set of equilibria by setting the interest rate in the interbank market appropriately. Faced with a distributional shock, the central bank should lower the interbank rate to facilitate the reallocation of liquid assets between banks. However, in order to provide incentives for banks to hold enough liquid assets ex ante, the central bank must make sure that interbank rates are high enough when the distributional shock does not occur.

On the other hand, the central bank should respond to aggregate shocks with a quantitative policy of injecting liquid assets in the economy. The goal of this policy is twofold. First, it helps achieve the optimal distribution of consumption between patient and impatient depositors. Second, it sets the amount of liquid assets in the interbank market at the level at which the central bank's interest rate policy can be effective. Hence, the quantitative policy required in the face of aggregate shocks complements the interest rate policy that is optimal in the face of distributional shocks.

Our model also shows that a failure to implement the optimal interest rate policy can lead to bank runs. When the interbank market rate is not set appropriately, a distributional shock creates consumption risk for patient depositors. If the rate is high, banks that need to borrow in the interbank market will be left with few goods for their patient depositors. For some parameter values, and if the rate is high enough, the goods available to patient depositors will yield less consumption than the amount promised to impatient depositors. This will create a run as all patient depositors will have an incentive to claim to be impatient.

While our model does not consider risky long-term assets, and thus, prevents us from studying issues related to counterparty risk, it provides valuable insights into the optimal policy of central banks during the first part of the crisis, which occurred from mid-2007 to mid-2008. During that period, distributional shocks to the interbank market were important. The policies adopted by central banks in this instance resemble the ones our model suggests are optimal.

8 Appendix A: Generalization to N states

Consider a generalization of the baseline model (without runs or liquidation of long-term assets) with N idiosyncratic states $i_1, ..., i_N \geq 0$. We assume $i_1 = 0$, $\lambda^{i_n H} = \overline{\lambda} + i_n \varepsilon$, and $\lambda^{i_n L} = \overline{\lambda} - i_n \varepsilon$, where $i_n \in \{i_1, ..., i_N\}$. The probability of i_n is ρ_n, $\sum_{n=1}^{N} \rho_n = 1$.

A bank's problem is thus

$$\begin{aligned}
\max_{\alpha \in [0,1]; c_1, \{\beta^{ij}\}_{i \in \mathcal{I}, j \in \mathcal{J}} \geq 0} \quad & \overline{\lambda} u(c_1) + \sum_{n=1}^{N} \rho_n [\tfrac{1}{2}(1 - \lambda^{i_n H}) u(c_2^{i_n H}) + \tfrac{1}{2}(1 - \lambda^{i_n L}) u(c_2^{i_n L})] \\
\text{s.t.} \quad & \lambda^{i_n j} c_1 \leq 1 - \alpha + \beta^{i_n j} + f^{i_n j} \\
& (1 - \lambda^{i_n j}) c_2^{i_n j} \leq \alpha r - \beta^{i_n j} - f^{i_n j} \iota^{i_n} \\
& \text{for } i_n \in \{i_1, ..., i_N\},\ j \in \mathcal{J}.
\end{aligned}$$

The first-order conditions with respect to α and c_1 are, respectively,

$$\sum_{n=1}^{N} \rho_n [\tfrac{1}{2} u'(c_2^{i_n H}) + \tfrac{1}{2} u'(c_2^{i_n L})] \iota^{i_n} = \sum_{n=1}^{N} \rho_n [\tfrac{1}{2} u'(c_2^{i_n H}) + \tfrac{1}{2} u'(c_2^{i_n L})] r$$

$$u'(c_1) = \sum_{n=1}^{N} \rho_n [\tfrac{\lambda^{i_n H}}{2\overline{\lambda}} u'(c_2^{i_n H}) + \tfrac{\lambda^{i_n L}}{2\overline{\lambda}} u'(c_2^{i_n L})] \iota^{i_n}.$$

By the same logic as in the case with two states, the interest rate in the interbank market should be equal to $\frac{c_2^*}{c_1^*}$ whenever $i_n > 0$ in order to facilitate risk sharing between banks. Without loss of generality, assume that $i_n > 0$ for all $n \geq 2$. Then we have $\iota^{i_n} = \frac{c_2^*}{c_1^*}$ and $c_2^{i_n H} = c_2^{i_n L} = \frac{\alpha r}{1 - \overline{\lambda}}$ for all $n \geq 2$. Let $\rho = \sum_{n=2}^{N} i_n$, and then we can write interest rate ι^{i_1} as

$$\iota^{i_1} = r + \frac{\rho(r - \frac{c_2^*}{c_1^*})}{1 - \rho},$$

which is equal to $\iota^0 = \iota^{0^*}$ in the two-state baseline model.[7]

[7] We can show that if there is no state with a zero-size shock, then a first best equilibrium does not exist because an equilibrium requires an interest rate of $l^i > \frac{c_2^*}{c_1^*}$ for at least one idiosyncratic state i, which is then always distortionary. If the baseline model is modified such that with two idiosyncratic states $0 < i_0 < i_1$, we can show that there is a constrained-efficient equilbrium with $l^{1^*} < l^{i_1} < r < l^{i_0} < l^{0^*}$, which is chosen by the central bank.

9 Appendix B: Monetary policy with nominal rates

We expand the real model to allow for nominal interbank lending rates. With nominal fiat interest rates, the central bank can explicitly enforce its target for the interbank rate, in order to actively select the rational expectations equilibrium. The central bank offers to borrow and lend to banks any amount of nominal, fiat money at the central bank's policy rate at date 1, which ensures that the interbank market rate equals the central bank's policy rate. The equilibrium and allocation of the nominal rate model is equivalent to the real rate model.

9.1 Nominal rate model extension

The extension of the model to include nominal rates is based on Skeie (2008). A nominal unit of account, inside money and a goods market with firms are added to the model of banks with real deposits. To establish a fiat nominal unit of account, the central bank offers at date 0 to buy or sell goods to the extent feasible for fiat currency (equivalent to central bank reserves) at a fixed nominal price $P_0 = 1$. After date 0, the central bank does not set the price of goods and does not offer to buy or sell goods. At date 0, each bank makes a loan to a firm. The firm buys the good from the bank's unit continuum of consumers, and consumers deposit in the bank. All the transactions at date 0 are paid for in the amount of one nominal unit of account. These nominal payments can be called "inside money," and are payable simultaneously. Inside money payments are "settled" in currency, which is defined as inside money payments being netted and any outstanding inside money amount due being paid in currency. Since zero currency is held by each agent, the individual budget constraint for each of the banks, firms and consumers requires that the net inside money payments of each party must net to zero at date 0.

Again, to simplify the exposition, we assume that the fraction of impatient depositors is always $\overline{\lambda}$. Each bank lends to its firm for loan repayments of nominal amounts $(1-\alpha)\,K_1$ and αK_2^i payable in inside money at dates 1 and 2, respectively. Uppercase variables denote nominal values and lowercase variables denote real values. The firm buys the good from consumers for price $P_0 = 1$. The firm invests α and stores $1-\alpha$ of the good, where α is chosen and can be enforced by the bank.[8] Consumers deposit in their bank for a

[8] If the bank could not enforce the firm's storage, the bank could alternatively buy and store $1-\alpha$

demand deposit contract that pays in inside money a nominal consumption amount of either $C_1 \geq 1$ if withdrawn at date 1 or $C_2^{ij} \geq 1$ if withdrawn at date 2. Although no currency circulates, the central bank's offer to trade with currency establishes the nominal unit of account for transactions with bank inside money. This is equivalent to Skeie (2008), where currency rather than inside money transactions occur at date 0.

In each period $t = 1, 2$, payments are made simultaneously among banks with either currency or inside money that is settled with currency. At date 1, λ^{ij} early consumers of bank j withdraw to buy goods from a firm in the goods market. At date 2, $1 - \lambda^{ij}$ late consumers withdraw from banks to buy goods. The representative firm repays loans and banks borrow or lend inside money if needed on the interbank market or currency from the central bank.

The bank's budget constraints from the real model (3) and (4) are replaced by budget constraints for nominal payments:

$$\text{s.t.} \qquad \lambda^{ij} C_1 = (1-\alpha)K_1 + M_f^{ijD} + M_o^{ijD}, \qquad \forall\, i \in \mathcal{I},\ j \in \mathcal{J}, \tag{39}$$

$$(1-\lambda^{ij})C_2^{ij} = \alpha K_2^i - M_f^{ijD} R_f^i - M_o^{ijD} R_o^i, \qquad \forall\, i \in \mathcal{I},\ j \in \mathcal{J}, \tag{40}$$

respectively, where bank j's demand to borrow from other banks is M_f^{ijD} and from the central bank (in currency) is M_o^{ijD}, and where R_f^i and R_o^i are the returns on interbank loans and central bank loans, respectively. The notation 'M' represents money (inside money or currency), subscript 'f' represents the fed funds interbank market, and subscript 'O' represents open market operations. R_f^i is the interbank market rate, which is determined in equilibrium. At date 1, the central bank targets R_f^i by choosing its policy rate R_o^i at which it offers to borrow and lend to banks an unlimited amount. Specifically, the central offers to supply a loan of $M_o^{ijS}(R_o^i) \in (-\infty, \infty)$ to bank j at rate R_o^i, where $M_o^{ijS}(R_o^i)$ is a correspondence. The central does not have a budget constraint to equate its borrowing and lending of central bank currency, since it can create and destroy currency as needed. The central bank's lending supply is perfectly elastic at its chosen rate R_o^i. The way in which we model the central bank offering to borrow and lend at a single policy rate is similar to open market operations in practice. Many central banks in essence offer to borrow and lend a perfectly elastic amount of funds at a chosen rate to target the interbank rate at

goods, sell them at date, and lend α to the firm without any storage requirements. Results of the model would be unchanged.

which banks lend uncollateralized to each other. Open market operations lending is often collateralized in practice, as in the form of repos against government securities in the case of the Federal Reserve. We abstract from collateralization since there is no risk of loss or default.

Consumers buy goods from firms at date $t = 1, 2$ in a Walrasian market using inside money as numeraire. Consumption for early and late consumers is

$$c_1(P) = \frac{C_1}{P_1} \tag{41}$$

$$c_2^{ij}(P) = \frac{C_2^{ij}}{P_2^i}, \tag{42}$$

where P_t^i is the nominal price of goods at date $t = 1, 2$ and $P \equiv (P_1, P_2^i)$ is a vector. We consider only $P_t^i \in (0, \infty)$, which is for simplicity and does not effect the results. Consumers' aggregate demand is given by

$$q_1^D(P) = \frac{\frac{1}{2}(\lambda^{ih} + \lambda^{il})C_1}{P_1} \tag{43}$$

$$q_2^D(P) = \frac{\frac{1}{2}[(1 - \lambda^{ih}) + (1 - \lambda^{il})]C_2^{ij}}{P_2^i}. \tag{44}$$

The representative firm submits a supply schedule $q_t^{iS}(P_t^i)$ for the goods market. The firm's optimization is to maximize profits:

$$\max_{q_1^{iS}, q_2^{iS} \geq 0} \quad 1 - \alpha + \alpha r - q_1^{iS} - q_2^{iS} \tag{45a}$$

$$\text{s.t.} \quad q_1^{iS} \leq 1 - \alpha \tag{45b}$$

$$q_2^{iS} \leq 1 - \alpha + \alpha r - q_1^{iS} \tag{45c}$$

$$q_1^{iS} \geq \frac{(1 - \alpha) K_1}{P_1} \tag{45d}$$

$$q_2^{iS} \geq \frac{\alpha K_2^i}{P_2^i}. \tag{45e}$$

The objective function (45a) is the profit in goods that the firm consumes at date 2. Constraints (45b) and (45c) are the maximum amounts of goods that can be sold at dates 1 and 2, respectively. Constraints (45d) and (45e) are the firm's budget constraints to repay its loan at date 1 and date 2, respectively.

The bank's demand for borrowing on the interbank market can be solved for from equation (39) as

$$M_f^{ijD} = \lambda^{ij} C_1 - (1 - \alpha) K_1 - M_o^{ijD}. \tag{46}$$

Substituting for M_f^{ijD} from equation (46) into equation (40) and rearranging, we find that bank j pays withdrawals to late consumers the amount

$$C_2^{ij} = \frac{\alpha K_2^i - [\lambda^{ij} C_1 - (1-\alpha) K_1] R_f^i + (R_f^i - R_o^i) M_o^{ijD}}{1 - \lambda^{ij}}. \tag{47}$$

The bank's optimization problem (7) is replaced by

$$\begin{array}{cc} \max\limits_{\alpha\in[0,1];C_1\geq 0;\{M_o^{ijD}\}_{i,j}\in\mathbb{R}} & E[U], \\ \text{s.t.} & (47) \end{array} \tag{48}$$

where $c_1(P)$ and $c_2^{ij}(P)$ are given by (41) and (42), respectively.

An *equilibrium* is defined as goods market prices and quantities (P, q_1, q_2), deposit and loan returns and quantities $\{C_1, R_f^i, M_o^{ij}\}_{i,j}$, and investment ($\alpha$) that solve goods market clearing conditions

$$q_t^D(P) = \tfrac{1}{2}[q_t^{hS}(P) + q_t^{lS}(P)] \text{ for } t = 1, 2,$$

and interbank market clearing condition

$$M_f^{hD}(M_o^{hD}) + M_f^{lD}(M_o^{lD}) = 0, \tag{50}$$

where $\{\alpha, C_1, M_o^{ij}\}_{i,j}$ is a solution to bank j's optimization (48); $\{q_t^D(P)\}_{t=1,2}$ is given by the consumers' aggregate demand (43) and (44), and $(q_1^{iS}(P), q_2^{iS}(P))$ is a solution to the firm's optimization (45).

9.2 Nominal rate results

The results of the nominal model are equivalent to those of the real model, with the addition that the central bank can choose its policy rate to target the interbank rate. The first order conditions for bank j's optimization (48) with respect to α, c_1 and M_o^{ijD} are

$$E[\frac{K_2^i}{P_2^i} u'(c_2^{ij})] = E[\frac{K_1}{P_2^i} R_f^i u'(c_2^{ij})], \qquad \forall\ i \in \mathcal{I} \tag{51}$$

$$E[\frac{1}{P_1} u'(c_1)] = E[\frac{\lambda^{ij} R_f^i}{\overline{\lambda} P_2^i} u'(c_2^{ij})], \qquad \forall\ i \in \mathcal{I} \tag{52}$$

$$R_f^i = R_o^i, \qquad \forall\ i \in \mathcal{I}, \tag{53}$$

respectively. Loan returns are set according to a competitive loan market as

$$K_1 = P_1 \tag{54}$$

$$K_2^i = r P_2^i, \tag{55}$$

such that the real returns $\frac{K_1}{P_1} = 1$ and $\frac{K_2^i}{P_2^i} = r$ equal the marginal product of capital for their respective terms and firms make zero profits in equilibrium. Substituting for K_t^i from equations (54) and (55), conditions (51) and (52) can be written as

$$E[u'(c_2^{ij})] = E[\frac{R_f^i}{P_2^i/P_1} u'(c_2^{ij})] \tag{56}$$

$$E[u'(c_1)] = E[\frac{\lambda^{ij}}{\overline{\lambda}} \frac{R_f^i}{P_2^i/P_1} u'(c_2^{ij})]. \tag{57}$$

Condition (53) states that because of arbitrage, the interbank rate R_f^i equals the central bank's policy rate R_o^i. The real interbank rate equals the nominal rate divided by nominal goods price inflation between dates 1 and 2:

$$\iota^i = \frac{R_f^i}{P_2^i/P_1}, \tag{58}$$

which implies that the first order conditions for the nominal model, equations (56) and (57), and for the real model, equations (23) and (24), are equivalent. The central bank can target any real interbank lending rate ι^i at date 1 by choosing

$$R_o^i = \frac{P_2^i}{P_1} \iota^i,$$

subject to satisfying the date 0 first order conditions for ι^0 and ι^1. In particular, the central bank can implement the first best allocation by choosing

$$R_o^i = \overline{R}_o^i \equiv \frac{P_2^i}{P_1} \overline{\iota}^i. \tag{59}$$

Proposition 5. *The central bank can choose $R_o^i = \overline{R}_o^i$, and there exists a unique equilibrium with first best allocation $\alpha = \alpha^*$, $c_1 = c_1^*$ and $c_2^{ij} = c_2^*$.*

Proof. Equilibrium prices and quantities satisfy

$$P_1 = \frac{\overline{\lambda} C_1}{q_1} \tag{60}$$

$$P_2^i = \frac{(1-\overline{\lambda}) C_2^{ij}}{q_2}. \tag{61}$$

The constraints in the firm's optimization (45) bind, which gives

$$q_1 = 1 - \alpha \tag{62}$$

$$q_2 = \alpha r. \tag{63}$$

Substitution for quantities and prices from (60) - (63) into (41) and (42),

$$c_1 = \frac{1-\alpha}{\overline{\lambda}} \tag{64}$$

$$c_2^{0j} = \frac{\alpha r}{1-\overline{\lambda}}. \tag{65}$$

To find C_1, substituting for M_f^{ijD} from (46) into the market clearing condition (50) and simplifying gives

$$C_1 = \frac{(1-\alpha)\,K_1}{\overline{\lambda}} + \frac{M_o^{ihD} + M_o^{ilD}}{2\overline{\lambda}}. \tag{66}$$

Substituting from (66) for C_1 into (46) and simplifying gives the demand for interbank borrowing by bank j as

$$M_f^{ijD} = (\frac{\lambda^{ij}}{\overline{\lambda}} - 1)\,(1-\alpha)\,K_1 + \frac{\lambda^{ij}}{\overline{\lambda}}(M_o^{ihD} + M_o^{ilD}) - M_o^{ijD}.$$

Rearranging, aggregate bank borrowing is

$$M_f^{ijD} + M_o^{ijD} = (1-\alpha)\,K_1(\frac{\lambda^{ij}}{\overline{\lambda}} - 1) + \frac{\lambda^{ij}}{\overline{\lambda}}(M_o^{ihD} + M_o^{ilD}), \tag{67}$$

Using (67), we can show that

$$(M_f^{ihD} + M_f^{ilD}) + (M_o^{ihD} + M_o^{ilD}) = 2(M_o^{ihD} + M_o^{ilD}). \tag{68}$$

By market clearing equation (50), aggregate net interbank borrowing is zero, $M_f^{ihD} + M_f^{ilD} = 0$, which by equation (67) implies $(M_o^{ihD} + M_o^{ilD}) = 2(M_o^{ihD} + M_o^{ilD})$. Hence, $(M_o^{ihD} + M_o^{ilD}) = 0$. Aggregate net borrowing from the central bank is zero in equilibrium. The central bank lends zero net supply of currency to the market. While bank j aggregate net borrowing from the interbank market and the central bank is determined by equation (67) as $M_f^{ijD} + M_o^{ijD} = (1-\alpha)\,K_1(\frac{\lambda^{ij}}{\overline{\lambda}} - 1)$, the individual components M_f^{ijD} and M_o^{ijD} are not determined. The central bank does not need to lend to any banks in equilibrium. Lending by the central bank is equivalent and a substitute for interbank lending.

Substition into (47) for K_t from (54) and (55), for R_o^i from (53), for R_o^i from (59), for $1-\alpha = \overline{\lambda} c_1$ from (64), and for $C_1 = \frac{P_1 q_1}{\overline{\lambda}} = \frac{P_1(1-\alpha)}{\overline{\lambda}}$ from (60) and (62), and rearranging gives

$$c_2^{ij} = \frac{C_2^{ij}}{P_2^i} = \frac{\alpha r - (\lambda^{ij} - \overline{\lambda})c_1 \overline{\iota}^i}{1-\lambda^{ij}},$$

which is identical to c_2^{ij} in the real model given by equation (**??**). The bank has an optimization identical to that in the real model and chooses $\alpha = \alpha^*$. Hence, the equilibrium is identical to that of the real model and the allocation is $c_1 = c_1^*$ and $c_2^{ij} = c_2^*$. ■

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