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**Abstract**

The Capital Assistance Program (CAP) was created by the U.S. government in February 2009 to provide backup capital to large financial institutions unable to raise sufficient capital from private investors. Under the terms of the CAP, a participating bank receives contingent capital by issuing preferred shares to the Treasury combined with embedded options for both parties: the bank gets the option to redeem the shares or convert them to common equity, with conversion mandatory after seven years; the Treasury earns dividends on the preferred shares and gets warrants on the bank's common equity. We develop a contingent claims framework in which to estimate market values of these CAP securities. The interaction between the competing options held by the buyer and issuer of these securities creates a game between the two parties, and our approach captures this strategic element of the joint valuation problem and clarifies the incentives it creates. We apply our method to the eighteen publicly held bank holding companies that participated in the Supervisory Capital Assessment Program (the stress test) launched together with the CAP. On average, we estimate that, compared with a market transaction, the CAP securities carry a net value of approximately 30 percent of the capital invested for a bank participating to the maximum extent allowed under the terms of the program. We also find that the net value varies widely across banks. We compare our estimates with abnormal stock price returns for the stress test banks at the time the terms of the CAP were announced; we find correlations between 0.78 and 0.85, depending on the precise choice of period and set of banks included. These results suggest that our valuation aligns with shareholders' perception of the value of the program, prompting questions about industry reactions and the overall impact of the program.

Key words: financial crisis, banking, derivative pricing, convertible preferred, risk management, troubled asset relief program

# 1  Introduction

The Capital Assistance Program (CAP) was announced on February 10, 2009, in a joint statement by the Treasury Department, the Board of Governors of the Federal Reserve, the Federal Deposit Insurance Corporation, the Office of the Comptroller of the Currency, and the Office of Thrift Supervision outlining a financial stability plan. The first phase of the program called for an assessment of the capital needs of major U.S. banking institutions in a scenario of continuing decline of the economic environment — this was the widely publicized stress test applied to nineteen firms. In the second phase of the program, banks requiring additional capital and unable to raise sufficient private capital would sell convertible preferred securities and warrants on common shares to the Treasury. Our objective is to value these preferred securities and their embedded options and to examine the market response to the program.

The combination of the stress test, which provided information about the downside risk faced by the largest U.S. banks, and the CAP securities, which provided backup capital to mitigate this downside risk, was an unprecedented regulatory response to a financial crisis. The design of the backup capital was an important part of this response. Its combination of conversion and redemption options and warrants was designed to support banks facing deteriorating conditions while providing a built-in exit from government support and compensation to taxpayers as business conditions improved. This capital guarantee improved public confidence so that these banks were able to raise the necessary capital from the private sector. Valuation of the CAP is essential to quantifying its impact and useful for the design of future programs and similar hybrid securities.

The precise terms of the CAP's preferred securities were announced on February 25. The most distinctive feature of these securities — called *mandatorily convertible preferred* shares in the term sheet — is that they grant the *issuer* the option to convert the securities to common equity, with conversion becoming mandatory at the end of seven years. The conversion option makes the CAP securities a form of contingent capital — a source of common equity on which a bank can draw in the face of a dwindling capital ratio. Proposals for other forms of contingent capital have been put forward by Flannery [11, 12], Kashyap, Rajan, and Stein [17], and the Squam Lake Working Group [30]; remarks from regulators[1] indicate that some form of contingent capital may well be included in new capital requirements.

The full terms of the CAP include additional features and embedded options. The issuer may

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[1] See Chairman Bernanke's speech of October 23, 2009, and Fed Governor Tarullo's speech of November 9, 2009, transcripts of which are available at www.federalreserve.gov. The Dodd-Frank financial reform act includes a contingent capital requirement.

redeem the preferred shares at par within the first two years, and the Treasury gets warrants on the banks common equity as part of the transaction. The Treasury also earns dividends on the CAP securities. These features must all be taken into account in determining the net value of the securities. By *net value* we mean any value received by the issuer beyond what the absence of arbitrage would dictate for the transaction.

We adopt a contingent claims approach to valuing the CAP preferred security and its embedded options. Our objective is to gauge the price at which such a security would sell in a market transaction without government participation. We value the securities through arbitrage-free pricing in a frictionless market. This approach has some evident limitations: it does not address how additional capital and government involvement may change a bank's investment opportunities, nor does it reflect ways in which the Treasury's decision to exercise warrants might differ from those of a private investor. Nevertheless, we view the "arbitrage value" of these securities as an important reference point in evaluating the costs and benefits of the program.

From the perspective of contingent claims analysis, the CAP presents an interesting problem because of the combination of options it grants to the issuer and the buyer. The decisions of the two parties interact because both conversion (by the issuer) and warrant exercise (by the buyer) cause significant dilution of common shares. This design sets up a dynamic stochastic game between the two parties; valuing the full CAP contract — combining the preferred shares and the warrants — entails finding the value of this game.

We calculate the value of the game through an extended binomial model for the issuer's equity. The model is extended to capture dilution and the competing options of the two parties; we also consider further extensions to include sudden default and stochastic interest rates using the method of Das and Sundaram [7]. In addition to providing valuations, our approach finds optimal strategies for buyer and issuer; these shed light on the incentives created by the complex interaction of multiple features of the CAP.

The values we calculate indicate that the terms of the CAP provide substantial benefit to the issuer. For the eighteen publicly held banking institutions that participated in the stress test, we estimate that, at the February 25 announcement, the CAP securities carried an average net value of approximately 30% of the capital invested for a bank participating to the maximum extent allowed under the terms of the program. We also find that the net value varies widely across banks. We compare our estimates, normalized by market capitalization, with abnormal stock price returns for the stress test banks at the time the terms of the CAP were announced; we find correlations between 0.78 and 0.85, depending on the period over which returns are

calculated and the set of banks included. These results suggest that our valuation aligns with the market's perception of the value of the program. This in turn raises questions about banks' decisions not to participate in the program. The Treasury closed the program on November 9, 2009, with no banks having issued CAP preferred shares.

The Treasury's experience with the CAP and the closely related inter-agency stress test suggests potential lessons for the future. We discuss specific features of the program in greater detail in Sections 2 and 8 but make a general observation here. In many models of financial crises, the role of a central bank is to solve a coordination problem (as in Diamond and Dybvig [8] and Rochet and Vives [27]), but in a different line of work (including Caballero and Krishnamurthy [4] and Holmstrom and Tirole [14]) financial crises result from uncertainty about aggregate liquidity. More specifically, Caballero and Krishnamurthy [4] emphasize the impact of Knightian uncertainty about tail events and the role of central banks in reducing uncertainty. In early 2009, the world faced a great deal of uncertainty about the current condition and future disposition of many of its largest financial institutions. As discussed in Morgan et al. [24], the stress test differed from conventional regulatory actions through its role in providing information to the market. The terms of the CAP—particularly its conversion option—reduced uncertainty about subsequent outcomes by effectively ruling out the possibility that any of the largest banks would be allowed to fail or taken over by the government, at least in the near term. But at what potential cost? Our estimates put the size of the Treasury's potential investment in the 18 publicly held stress test banks at approximately $152 billion dollars with a potential net value to the banks of approximately $59 billion. Although the guarantee was never drawn, its availability may have contributed significantly to the increase in bank stock prices in mid-2009. Moreover, as stock prices increase, the net value to banks of the CAP securities decreases, creating an automatic exit feature for the program. Our valuations quantify this effect.

The rest of this paper is organized as follows. Section 2 reviews the terms of the CAP, and Section 3 describes our valuation approach. Section 4 formulates the valuation game, and Section 5 develops the binomial framework for calculation. Section 6 details our calibration procedure and reports results for the stress test banks. In Section 7, we compare our model valuations with the market's response, and in Section 8 we offer some general remarks on the program. Extensions and additional details are contained in an on-line appendix.

# 2 Terms of the Capital Assistance Program

The terms of the CAP are detailed in a term sheet [32] released by the Treasury on February 25, 2009, along with a white paper [33] providing additional information on the program. Both are available through the Treasury's website and through FinancialStability.gov.

Under the terms of the CAP, a qualifying financial institution[2] (QFI) issues mandatorily convertible preferred securities to the Treasury (UST) in an amount ordinarily between 1% and 2% of the QFI's risk-weighted assets. (The amount may be greater if the additional capital is used to repay other government investments made through the Troubled Asset Relief Program.) The CAP's preferred securities pay a dividend, set at 9% in the term sheet. They are bundled with warrants on the QFI's common stock granted to the UST. The redemption and conversion features of the CAP preferred securities are as follows:

*Redemption option:* At any time within the first two years, the QFI may redeem the preferred shares at par, together with the payment of any accrued but unpaid dividends.

*Conversion option:* At any time within the first seven years, the QFI may convert the preferred shares to common equity. The conversion price is 90% of the average closing price for the common stock over the 20 trading day period ending February 9, 2009. For example, if a preferred share has a par value of $100 and the average common stock price is $5.56, one preferred share would convert to $100/(.9 \cdot 5.56) = 20$ common shares.

*Mandatory conversion:* At the end of seven years, if the preferred shares have not previously been converted (or redeemed), conversion becomes mandatory at the same conversion price that applies to optional conversion.

The terms of the warrants granted to the UST under the CAP are as follows:

*Warrants:* At any time within the first ten years, the UST may exercise the warrants and buy shares of the QFI's common stock at a strike price equal to the conversion price that applies to the preferred shares. The number of warrants held by the UST is set so that the number of shares it can acquire through exercise of the warrants is equal to 20% of the ratio of the par value of the preferred shares divided by the conversion price. In other words, if the UST exercised the warrants at the issue date, its holdings in common shares would equal 20% of the par value of the preferred shares.

These descriptions of features of the CAP include some simplifications of the full terms detailed in the Treasury's term sheet. The option to redeem actually continues beyond the first

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[2]Qualifying financial institutions are bank holding companies, financial holding companies, insured depository institutions, and savings and loan holding companies that are "deemed viable" by the corresponding regulator and not owned by a foreign entity.

two years but is subsequently penalized at a cost at least as large as the conversion option; so, for purposes of valuation, we subsume the redemption option in the conversion option after the initial period. The warrants and the redemption and conversion options may each be exercised "in whole or in part," but we will assume that each is exercised fully or not at all.[3] The term sheet also includes penalties on the conversion and strike price if the QFI's shareholders delay approval of the terms of the contract; we exclude this provision from our analysis.

The CAP's preferred shares provide an additional buffer against losses and additional Tier 1 capital to meet regulatory requirements. The dividends paid by these securities compensate taxpayers and provide an incentive for banks to replace public funds with private capital, especially at the CAP's rate of 9%. The warrants allow taxpayers to share in the future profitability of a participating bank. The conversion option contained in the CAP provides a participating bank with contingent common equity that can be activated (at the bank's choosing) to boost public confidence in the bank's health. According to the Treasury's white paper [33], pp.3–4, "Market participants pay particular attention to common equity as a measure of health in stressed environments, and regulators have long believed that common equity should be the dominant component of a banking organization's highest quality forms of capital." This is consistent with reports through the crisis of investors attaching greater significance to tangible common equity (TCE) ratios and less to Tier 1 capital.

The Capital Assistance Program followed, but did not replace, the Capital Purchase Program created in October 2008. Preferred shares issued under the CPP pay a dividend of 5% for the first five years, increasing to 9% thereafter. Warrants issued under the CPP are limited to a notional value of 15% of the preferred shares, and the amount of preferred shares issued under the CPP may be up to 3% of a bank's risk-weighted assets. But the most significant qualitative differences between the CAP and the CPP are the conversion features (optional and mandatory) included in the CAP preferred shares.

As noted in the previous section, the conversion option makes the CAP shares a type of contingent capital — capital that becomes available when a bank nears insolvency. In this vein, Flannery [11] proposed *reverse convertible debentures* (called *contingent capital certificates* in Flannery [12]) in which conversion to common equity is triggered by a market measure of a bank's capital ratio. In October 2009, Lloyds Banking Group announced plans to swap existing debt for contingent capital that converts to common shares based on a Tier 1 capital ratio. In

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[3]Partial exercise, which is found in many energy contracts, could be incorporated using the method of Jaillet et al. [16] at the cost of additional complexity. A partial conversion option could only increase the CAP value to banks, and, at least for stand-alone warrants, partial exercise would never be optimal. As a practical matter, the approvals required for exercise of any of the options make it unlikely that either party would spread its exercise decisions over many dates.

the Squam Lake Working Group's [30] proposal and in McDonald [23], the conversion trigger has both a systemic and bank-specific feature. Landier and Ueda [19] and Philippon and Schnabl [26] analyze mechanisms to resolve the debt overhang problem faced by banks in a financial crisis; contingent capital is a potential mechanism to preempt the problem.

Mandatory convertible debt has long qualified as regulatory capital under both Basel I and Basel II rules, and in December 2008, an ailing UBS received government support by issuing mandatorily convertible notes to the Swiss national bank. These securities grant conversion options to both the issuer and the holder, and they force conversion at the end of thirty months. In the context of private investments, Woodson [37] devotes a full chapter to mandatory convertible securities. Chemmanur, Nandy, and Yan [6] give a theoretical and empirical analysis of mandatory convertibles, stressing the role of asymmetric information in the decision to issue such securities. Reverse convertibles in which the conversion option is held by the issuer, as in the CAP, are discussed in Doherty [9], pp.474–482.

## 3    A Contingent Claims Approach to Valuation

We will value the combination of mandatorily convertible preferred shares and warrants on common stock as a single, hybrid structured product, and we will refer to this composite security as the CAP. The fact that both the buyer and seller hold options under the CAP sets up a complicated interaction — the possibility of conversion (and thus dilution) changes the incentives for a value maximizing investor to exercise the warrants. We value the composite CAP security to capture this interaction.[4]

An essential premise of our valuation approach is that changes in capital structure resulting from the exercise of the CAP's embedded options have no effect on total firm value — only how this value is shared between the UST and the QFI's shareholders. One can certainly imagine departures from this assumption in practice; for example, a higher TCE ratio might allow a bank to enter new lines of business, borrow at lower cost, or command higher fees as a counterparty to swaps and other transactions. We do not model this type of phenomenon and will instead assume that the bank's equity exhibits constant returns to scale.

As a first step, we lay out the accounting we will follow to keep track of cashflows under the CAP and upon exercise of any of the CAP's options. We use the notation in Table 1, starting with the variables in the top half of the table.

The terms of the CAP include the initial capital $G$ paid by the UST, the dividends paid

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[4]Work on assessing and valuing TARP programs includes Bayazitova and Shivdasani [2], Duff and Phelps [10], Morgan et al. [24], Taliaferro [31], Veronesi and Zingales [34], and Wilson [36].

| Notation | Explanation |
|---|---|
| $S$ | Current stock price of the QFI |
| $n$ | Number of current outstanding shares |
| $G$ | Preferred equity (par value) |
| $R$ | Preferred dividend rate |
| $K$ | Conversion price |
| $p$ | Ratio of warrants to the preferred equity |
| $q$ | Number of shares at conversion $(= G/K)$ |
| $m$ | Number of warrants $(= pG/K)$ |
| $\sigma$ | Volatility of the QFI's stock price |
| $d$ | Dividend yield of the QFI's stock |
| $r$ | Interest rate |
| $T_r$ | Expiration date of redemption option |
| $T_c$ | Date of mandatory conversion |
| $T$ | Maturity date of warrants |

Table 1: Variable definitions

by the QFI, the warrants granted to the UST, and the conversion and redemption options held by the QFI. Rather than keep track of intermediate cashflows between the two parties, we will assume that all payments are deferred until a hypothetical liquidation.[5] Liquidation is triggered by the exercise of any of the CAP's three options: redemption, conversion, and warrant exercise. Until liquidation, we assume the initial capital $G$ accumulates interest at the risk-free rate, and dividends are also reinvested at the risk-free rate. The allocation of value at liquidation is as follows:

*Redemption:* At redemption, the QFI pays the UST the initial capital $G$, the cumulative value of the dividends, and the value of the warrants. The QFI keeps all $n$ shares of equity and the interest earned on the initial capital.

*Conversion:* The QFI pays the UST the cumulative value of the dividends and the value of the warrants, but it keeps the initial capital and the interest earned on the initial capital. The total equity value is divided in proportion to the number of shares held by each party: the UST gets $q$ shares or a fraction $q/(q+n)$ of the total, and the QFI keeps a fraction $n/(q+n)$. The increase in the number of shares dilutes the stock price. If we write $S_-$ and $S_+$ for the stock price just before and just after conversion, these must satisfy $(n+q)S_+ = nS_-$, and thus

$$S_+ = \frac{n}{q+n}S_-. \tag{1}$$

The dilution in the stock price lowers the value of the UST's warrants, and this will be reflected in the value the UST receives when liquidation is triggered by conversion.

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[5]Dividends on common equity affect the stock price and thus the value of options on the stock. We assume that deferring dividends on the CAP preferred securities until liquidation does not affect the stock price.

*Warrant exercise:* The UST pays in the exercise cost $mK$, with $m$ the number of warrants and $K$ the strike price. We model this as increasing the total equity value from $nS$ to $nS + mK$. This value is divided between the two parties in proportion to the number of shares, so the UST gets a fraction $m/(n+m)$. The stock prices just before and just after warrant exercise satisfy the standard relationship (as in, e.g., McDonald [22], p.512)

$$(n+m)S+ = nS_- + mK \quad \text{and} \quad S_+ = \frac{nS_- + mK}{n+m}, \tag{2}$$

The QFI continues to hold the option to convert or redeem the preferred shares, so it gets the value of a CAP security stripped of its warrants, valued at the diluted stock price (2).

In the absence of arbitrage and of any market frictions, the expected present value of what the QFI gets at liquidation should equal the initial value $nS$ of the equity; equivalently, the expected net present value to the QFI should be zero. We refer to any nonzero value received or paid by the issuer, after accounting for all terms of the contract, as the net value to the issuer. We will calculate this net value $C_0$, and if we find that $C_0 > 0$, we can think of the QFI as paying out a dividend of $C_0$ to shareholders at time zero. Alternatively, we can think of $C_0$ as the fair price for the issuer to pay the buyer in entering into the CAP transaction. In either interpretation, any net value is transferred at time zero, so conversion and redemption have no effect on the total equity value, and warrant exercise increases equity value only by the amount of the exercise cost paid by the UST.

In each of three possible liquidation scenarios, the party that did not trigger the liquidation continues to hold one or more options: if the QFI redeems or converts, the UST continues to hold warrants; if the UST exercises its warrants, the QFI continues to hold the option to redeem or convert. Thus, to determine the payoff to each party at liquidation, we need to value these remaining options.[6] The details of the valuation are given in the next section. As an input to that calculation, here we explain the accounting we use for the stock price dilution that results from exercise of these remaining options.

If liquidation is triggered by warrant exercise, we need to consider the value of the QFI's remaining option to convert. (Redemption does not dilute the share price.) Equation (1) assumes that conversion occurs before warrant exercise, so the number of shares just before conversion has the original value $n$. But if the warrants are exercised before conversion, the number of shares just before a subsequent conversion would be $n + m$, and we replace (1) with

$$S_+ = \frac{n+m}{q+n+m} S_-. \tag{3}$$

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[6]In contrast, in a callable convertible bond, exercise of either the call or conversion option terminates the other option as well.

Similarly, (2) assumes the warrants are exercised before conversion. But if liquidation is triggered by conversion, then the stock price dilution resulting from subsequent warrant exercise would be

$$S_+ = \frac{(n+q)S_- + mK}{n+q+m}.$$
(4)

We do not need to account for other potential changes in the number of shares because the terms of the CAP are protected by standard anti-dilutive provisions; thus, we may assume that conversion and warrant exercise are the only mechanisms that change the number of shares.

The liquidation scenarios described above and the accounting identities (1)–(4) set up the payoffs and dilution effects resulting from the terms of the CAP. We will combine these with a model of stock price dynamics to determine the CAP value. Our approach will be to posit a model of the risk-neutral dynamics of the stock price and to value the CAP as a derivative security. An interesting feature of the CAP is the game it sets up between the QFI and UST by granting embedded options to both parties.[7] In order to arrive at a market valuation of the CAP, we need to capture the strategic behavior of both parties and not, for example, try to anticipate policy considerations in the UST's behavior. Such considerations are important, but should be analyzed separately from a market valuation.

An alternative valuation approach would start by valuing the dynamics of the QFI's assets, rather than its stock price. This approach would entail modeling the firm's equity as a derivative of the firm's asset value and then the CAP as a derivative on this derivative. (See, for example, the valuation of government guarantees in Lucas and McDonald [20].) Such an approach would allow a more fine-grained model of the QFI's business activities and opportunities, but it would be difficult to capture optimal strategies for the CAP in a structural model of the firm. Even reproducing market values of a bank's equity and debt as options on firm value presents a challenge. The substantial added complexity of starting from the dynamics of the QFI's assets would not necessarily result in more accurate valuation of the CAP because a firm's asset value is difficult to measure whereas the market value of its equity is directly observable.

A further consideration in valuation is the lag between our valuation date (in February) and the November deadline for participation. This would appear to grant the QFI a further option — the option to time its entry into the program. However, this cannot be treated as an option for a QFI to issue the security because, if it was found to require additional capital through the stress test, the QFI would have to raise additional capital privately to avoid participating in the CAP. In this sense, the CAP should be viewed as mandatory rather than optional if the

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[7]Sirbu and Shreve [29] analyze a continuous-time game formulation of the pricing of callable convertible bonds and note that the game interpretation of that setting is implicit in earlier work.

CAP terms are designed to provide positive net value. Strictly speaking, our valuation should factor the alternative cost of raising equity. This is difficult if we consider the debt overhang of these over-leveraged banks as well as underwriting costs. However, on February 25, 2009, which was in the middle of the worst period of the crisis, raising additional capital through issuing equity was expected to be more expensive than obtaining the same amount of capital from the CAP. Therefore, treating the CAP as a security issued by the QFIs on February 25 should be a reasonable approximation for the value that the CAP transfers to the QFIs.

## 4  Recursive Game Formulation

To simplify the analysis, we work with a discrete-time formulation. We use a time step of size $h$ and let $S_t$ denote the risk-neutral stock price process at time $t \cdot h$, $t = 0, 1, \ldots, T$. We suppose the stock pays a dividend yield of $d$, and (for now) we use a constant interest rate of $r$. Letting $E_t$ denote conditional expectation under the risk-neutral measure given the history of all market data up to time $t$, we require

$$S_t = e^{(r-d)h} E_t[S_{t+1}]. \tag{5}$$

To simplify notation, we will assume that the stock price process $S_t$ is Markov and condition on $S_t$ rather than the full history up to time $t$; alternatively, we could record additional state variables to get a Markovian state description. We specialize to a binomial model in Section 5, but at this point we keep the discussion generic. In the setting of Appendix A.3, the state becomes Markovian when the curve of forward interest rates is included.

For the purpose of our no-arbitrage valuation, we may assume, as in the previous section, that the initial capital $G$ is invested at the risk-free rate $r$, so its value at step $t$ is

$$G_t = e^{rth}G, \quad t = 0, 1, \ldots.$$

Prior to redemption or conversion, the CAP pays a dividend at rate $R$, so the dividend in each period is $GRh$. If dividends are reinvested at the risk-free rate, the cumulative value of dividends to time $t$ is

$$D_t = GRh \sum_{j=1}^{t} e^{r(t-j)h}.$$

We denote by $C$ the value of the CAP security, writing $C_t$ to denote its value at time $t$ and $C_t(S)$ to denote its value at time $t$ with an underlying stock price of $S$. Our goal is to evaluate $C_0(S_0)$, but to accomplish this we will need to introduce some auxiliary variables which we

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interpret as securities closely related to the CAP. These are needed because of the strategic interactions between the two parties in the CAP:

$$C = \text{CAP value to QFI; } n \text{ shares outstanding;}$$

$$C^e = \text{CAP value to QFI if warrants were previously exercised; } n + m \text{ shares outstanding;}$$

$$W = \text{total value to UST if CAP not previously converted or redeemed; } n \text{ shares outstanding;}$$

$$W^c = \text{warrant value if CAP previously converted; } n + q \text{ shares outstanding;}$$

$$W^r = \text{ordinary warrant value without CAP; } n \text{ shares outstanding;}$$

The original variables are $C$ and $W$, representing the values to the QFI and UST, respectively. We will have $W = -C$, but it will be convenient to track the bank's optimal decisions through $C$ and the Treasury's optimal decisions through $W$. We need $C^e$, $W^c$, and $W^r$, to keep track of the remaining option value held by one party after the other party exercises an option. The variable $C^e$ records the value (to the issuer) of a security identical to the CAP but stripped of its warrants and with $n + m$ shares outstanding. Thus, if the UST exercises its warrants before the QFI converts or redeems, the UST is effectively replacing the original $C$ security with a new $C^e$ security. Similarly, when the QFI converts, it replaces the UST's original warrants with the new $W^c$ warrants, and when the QFI redeems, it replaces the original warrants with $W^r$ warrants. These steps will be made explicit in the equations below. Wang [35] used a simplified approach in which warrants can never be exercised prior to conversion in advising the Treasury on the design of the program.

It is possible, in principle, for the bank and the Treasury to exercise their options simultaneously, in which case the net effect would depend on the order in which their decisions are executed. To avoid this potential ambiguity and simplify the exposition, we will impose the requirement that the two parties take turns: they are granted the opportunity to exercise their options at alternating dates. (In our numerical results, we average over the two possible sequences of moves to reduce the influence of this convention.) Our valuation will start at the final maturity $T$, which is 10 years in the CAP term sheet. As in Table 1, $T_c$ denotes the mandatory conversion date (7 years), and $T_r$ denotes the lifetime of the redemption option (2 years). Consistent with the terms of the CAP, we assume $T_r \leq T_c \leq T$.

In the Appendix, we give a normal-form specification of the game between the QFI and the UST, defining the admissible strategies (stopping times for conversion, redemption, warrant exercise) and payoffs for the two players. We show that the game has an equivalent extensive-form specification in which the value can be found by backward induction. Here we proceed directly to the backward induction, specifying the terminal values for each player and then

finding their optimal moves at each step and each level of the stock price.

**Terminal Values:** For any level $S$ of the underlying stock price

$$W_T^r(S) = \frac{mn}{m+n} \max\{S - K, 0\} \tag{6}$$

$$W_T^c(S) = \frac{mn}{m+n+q} \max\{S - K, 0\}. \tag{7}$$

If $T_c < T$, then $W_t$, $C_t$, and $C_t^e$ are undefined for $T_c < t \le T$ because these variables cease to be meaningful after the mandatory conversion at $T_c$. Their terminal values are

$$C_{T_c}(S) = -\frac{qn}{n+q} S + G_{T_c} - D_{T_c} - W_{T_c}^c \left( \frac{n}{n+q} S \right) \tag{8}$$

$$C_{T_c}^e(S) = -\frac{qn}{n+q+m} S + G_{T_c} - D_{T_c}. \tag{9}$$

$$W_{T_c}(S) = -C_{T_c}(S). \tag{10}$$

Equation (6) is the standard expression for the payoff of a warrant at expiration, taking into account the dilution from $n$ to $m+n$ shares. In (7), we have a similar dilution from $n+q$ to $m+n+q$ shares; however, the value of $W^c$ presupposes that $q$ of these shares are already held by the Treasury (as a result of conversion), so the Treasury bears a further dilution cost as a result of the reduction in share price on the $q$ shares it held prior to warrant exercise. The net payoff to the Treasury from warrant exercise after conversion is, then,

$$\frac{m(n+q)}{m+n+q} \max\{S - K, 0\} - q \left( S - \frac{(n+q)S + mK}{n+q+m} \right);$$

the first term is the standard warrant payoff (with dilution from $n+q$ shares to $m+n+q$ shares) and the second term is the loss in value of $q$ shares that results from the price dilution in (4). This expression simplifies to (7).

Equation (8) reflects our description in Section 3 of liquidation resulting from conversion (in this case, the mandatory conversion at date $T_c$). The QFI grants $q$ shares to the UST at a per-share cost of $nS/(n+q)$, the diluted price (1). The QFI keeps the accumulated value of the initial capital $G$ and pays the accumulated value of the dividends to the UST. It also grants the UST post-conversion warrants $W^c$. The value of these post-conversion warrants is not the value they would have at a stock price of $S$, but rather their value at a stock price of $nS/(n+q)$, because of the dilution that results from the conversion.

The explanation of (9) is similar. The function value $C_{T_c}^e(S)$ answers the question, "If we reach the mandatory conversion date $T_c$ and the Treasury has already exercised its warrants and the stock price is $S$, what is the value of the CAP to the QFI at the moment of conversion?"

The first term in (9) is the cost of issuing the $q$ shares to the UST, now determined by the diluted price (3) because the warrants have already been exercised. For the same reason, the last term in (8) has no counterpart in (9): if the warrants were already exercised, no warrants are granted in the hypothetical liquidation at the mandatory conversion date. Finally, (10) records the fact that the value of the CAP to the UST is just its cost to the QFI.

At earlier dates, the function values for the CAP and the auxiliary variables are determined recursively, with the QFI and UST taking alternating turns. At the QFI's turn, it chooses to convert, redeem, or continue; at its turn, the UST chooses to exercise or continue. In what follows, we abbreviate conditional expectations of the form $E[\,\cdot\,|S_t = S]$ to $E[\,\cdot\,|S]$.

**QFI's Turn at time** $t$**:** For any level $S$ of the underlying stock price,

$$W_t(S) = e^{-rh}E[W_{t+1}|S] \tag{11}$$

$$W_t^r(S) = e^{-rh}E[W_{t+1}^r|S] \tag{12}$$

$$W_t^c(S) = e^{-rh}E[W_{t+1}^c|S]. \tag{13}$$

If conversion, redemption, and continuation are all permitted (i.e., $t \leq T_r$),

$$C_t(S) =$$
$$\max\left\{-\frac{qn}{n+q}S + G_t - D_t - W_t^c(\frac{nS}{n+q}), G_t - D_t - G - W_t^r(S), e^{-rh}E_t[C_{t+1}|S]\right\}. \tag{14}$$
$$C_t^e(S) =$$
$$\max\left\{-\frac{qn}{n+m+q}S + G_t - D_t, G_t - D_t - G, e^{-rh}E_t[C_{t+1}^e|S]\right\}. \tag{15}$$

If redemption is not allowed ($t > T_r$), remove the second argument from the max in both (14) and (15). If the maximum in (14) is attained by the first argument (conversion) or second argument (redemption), set $W_t(S) = -C_t(S)$.

Equations (11)–(13) reflect the UST's passive role during the QFI's turn: each of these values is just the expected presented value of the corresponding security one step later. Equation (14) reflects the QFI's option to convert (the first argument in the max), redeem (the second argument) or continue (the third argument). The first argument follows from exactly the same explanation as (8); indeed, (8) is just the special case of (14) in which we force conversion by removing the option to continue or redeem. If the QFI redeems the CAP, it keeps the gains on the initial capital, $G_t - G$, pays the cumulative dividends $D_t$, and grants the UST ordinary warrants with value $W_t^r(S)$. These are ordinary warrants precisely because the CAP has been redeemed; the QFI no longer holds a conversion option. A similar explanation applies to (15)

without the warrants. Finally, if the QFI converts or redeems, the total value to the UST is the cost to the QFI.

**UST's Turn at time** $t$**:** For any level $S$ of the underlying stock price,

$$
\begin{aligned}
C_t(S) &= e^{-rh} E_t[C_{t+1}|S] \\
C_t^e(S) &= e^{-rh} E_t[C_{t+1}^e|S],
\end{aligned}
$$

$$
W_t^r(S) = \max\left\{\frac{mn}{m+n}(S-K), e^{-rh} E_t[W_{t+1}^r|S]\right\} \tag{16}
$$

$$
W_t^c(S) = \max\left\{\frac{mn}{m+n+q}(S-K), e^{-rh} E_t[W_{t+1}^c|S]\right\} \tag{17}
$$

$$
W_t(S) = \max\left\{\frac{mn}{m+n}(S-K) - C_t^e\left(\frac{nS+mK}{n+m}\right), e^{-rh} E_t[W_{t+1}|S]\right\}. \tag{18}
$$

If the maximum in (18) is attained by the first argument (i.e., if the CAP warrants are exercised), set $C_t(S) = -W_t(S)$.

The first two equations reflect the QFI's passive role during the UST's turn. Equations (16) and (17) are standard expressions for the value of a warrant as the maximum of its immediate exercise value and its continuation value; the immediate exercise values in the two cases reflect the dilution costs discussed following (6) and (7). Both $W^r$ and $W^c$ are the values of stand-alone warrants, whereas $W$ reflects the value of embedded options for the QFI. Equation (18) reflects our description in Section 3 of liquidation resulting warrant exercise: The UST collects the immediate exercise value and grants the QFI a new security, $C^e$; the new security carries no embedded warrants, and it is valued at the diluted stock price $(nS+mK)/(n+m)$.

## 5   Extended Binomial Model

Solving equations (6)–(18) determines the values of the variables as functions of the time index $t$ and the stock price $S$. The calculation of one-step conditional expectations, as in (11)–(13), is greatly simplified in a binomial model of the underlying stock price, where each such conditional expectation is just a weighted average of two possible downstream values.

### 5.1   Price Transitions

In the version of the binomial model we use, over a time step of length $h$, the stock price moves up or down as illustrated below. A move up takes the stock price from $S$ to $Se^\delta$, where $\delta = \sigma\sqrt{h}$ and $\sigma$ is the stock's volatility. A move down takes the stock price from $S$ to $Se^{-\delta}$.

The risk-neutral probability that the price goes up is $\pi_{\text{up}}$, and the probability that the price goes down is $\pi_{\text{dn}}$.
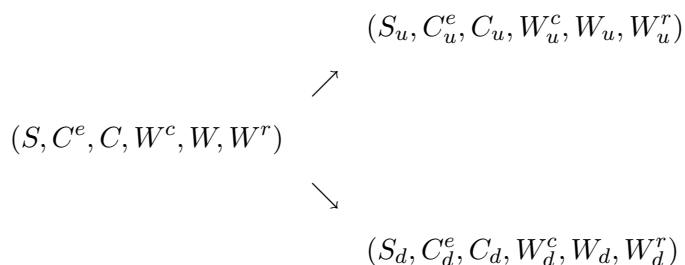


$$\pi_{\text{up}} = \frac{e^{(r-d)h} - e^{-\delta}}{e^{+\delta} - e^{-\delta}}$$

$$\pi_{\text{dn}} = 1 - \pi_{\text{up}}$$
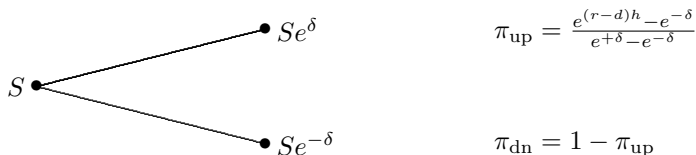
We can think of the process of solving (6)–(18) in a binomial tree as recording the values of all the variables at each node, as in this diagram:

$$(S_u, C_u^e, C_u, W_u^c, W_u, W_u^r)$$

$$\nearrow$$

$$(S, C^e, C, W^c, W, W^r)$$

$$\searrow$$

$$(S_d, C_d^e, C_d, W_d^c, W_d, W_d^r)$$

The subscripts $u$ and $d$ indicate values at the "up" and "down" nodes. We record the terminal values (6)–(10) at the terminal nodes (corresponding to times $T$ or $T_c$, depending on the variable), and work backwards. A one-step present value calculation, as in (11) is calculated as

$$W = e^{-rh} \left( \pi_{\text{up}} W_u + \pi_{\text{dn}} W_d \right).$$

In a standard binomial model, the stock price process is restricted to a lattice of values that can be reached through moves up and down. In our setting, additional transitions in the stock price process are possible through dilution. Moreover, the proportional dilution through conversion (in (1) and (3)) has a different effect than the affine dilution through warrant exercise (in (2)and (4)). The stock prices reachable from the original binomial lattice through the conversion dilution (1) form another binomial lattice of stock prices scaled down by $n/(n+q)$ from the original prices. In other words, dilution followed by a move up (or down) yields the same price as a move up (or down) followed by dilution.

This property is not shared by dilution through warrant exercise. The affine dilution in (2) leads, in effect, to a separate binomial lattice from each stock price. So, the theoretical state space of our stock price process is the set of values reachable from $S_0$ through up and down moves and (1) and (4) or (2) and (3), depending on the order of conversion and warrant exercise. This is still a finite state space, but it is not a conventional binomial lattice.

As a practical matter, it is never necessary to generate the full state space. Instead, on a single binomial lattice (starting at the diluted value of $nS_0/(n+q)$), we calculate $W^c$ using (7)

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and (17); this is the standard procedure for valuation through backward induction. In (8), (10), (14) and (18), we read the required value of $W^c$ from the diluted binomial lattice. Similarly, we use (6) and (16) to evaluate $W^r$ at every node of the original (undiluted) binomial lattice. As we work backwards in the original binomial lattice, at each turn of the UST we need to evaluate $C^e$, which we do by launching a separate binomial lattice from the diluted stock price $(nS + mK)/(n + m).$[8]

## 5.2 Example

To illustrate our method, we consider a simple example. The initial stock price is $S_0 = 20$ with volatility $\sigma = 0.6$ and a continuous dividend yield of $d = 0.002$. The CAP pays a dividend of 9% and the risk-free rate is 2%. The conversion and strike price is $K = 0.9S_0 = \$18$. The initial capital $G$ is 2% of risk-weighted assets, which we take to be 25 times the market capitalization; thus, $G = .02 \cdot 25nS_0 = nS_0/2$, and $n$, the number of shares outstanding, is 10 million. The number of warrants $m$ is 20% of $G/K = n/1.8 = 5,555,556$.

The optimal strategies of the QFI and the UST are illustrated in Figure 1 for a time step of 1/16 year. The horizontal axis represents time, and the scale is in years. The vertical axis shows the level of the stock price in the lattice, counting nodes above or below the root node, which corresponds to a stock price of \$20. The layer 50 nodes up from the root node corresponds to a stock price of $S_0 u^{50} = \$3,616$, and the layer 50 nodes down corresponds to a stock price of $S_0 u^{-50} = \$0.0111$. The middle band of the figure is of primary interest.

The figure shows the optimal exercise regions to be used by the QFI and UST so long as the other player has not yet exercised. (In other words, these are exercise regions for $C$ and $W$.) The figure is calculated on the assumption that the UST takes the first turn. If the stock price enters the lower shaded region, the QFI converts, and if the stock price reaches a level marked by a circle, the QFI redeems the CAP. If the stock price were to reach a node marked by a square before the QFI redeems or converts, the UST would exercise its warrants. In this example, it is impossible for the stock price to reach a square without first hitting a circle or cross, so the warrants will not be exercised before the QFI converts or redeems. Moreover, the QFI is guaranteed to convert or redeem by the end of the first two years (when the redemption option expires). The figure goes out to seven years because conversion would be mandatory at that point if the QFI had not previously converted or redeemed.[9] See the on-line appendix for

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[8]The time required to complete a backward induction through a standard binomial lattice with $T$ time steps is $O(T^2)$; in our procedure, at each time $t = 1, \ldots, T$, we launch a diluted binomial lattice with maturity $T - t$ at each of the $t + 1$ stock prices of the original lattice, so the total time required is $O(T^4)$. This could be accelerated by interpolating the diluted values $(nS + mK)/(n + m)$ from a single binomial lattice.

[9]On this scale, some of the boundaries look linear. For a magnified view, see Figure 3.
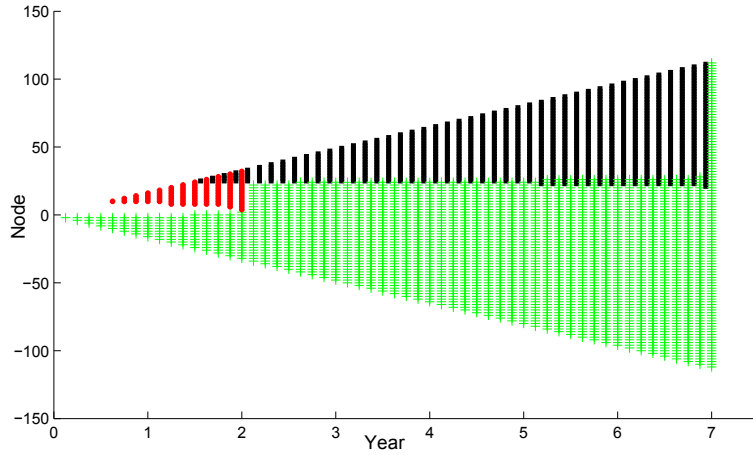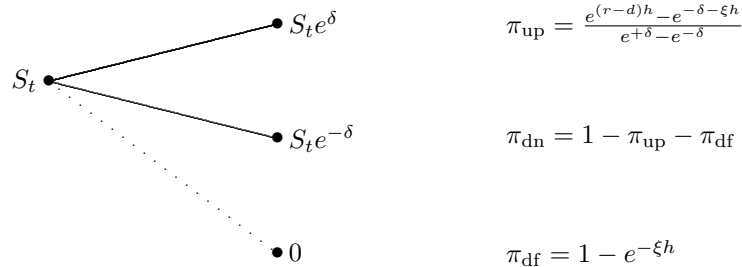
Figure 1: Exercise regions for the CAP: The cross marks show the region in which the QFI exercises its conversion option, the circles show where redemption is optimal, and the squares show the exercise region for the UST's warrants.

further discussion of the example.

## 5.3 Including Default

In a standard binomial model with multiplicative price changes, the stock price process can never reach zero. A simple and standard way to incorporate an unexpected sudden default in a binomial model is to include a possible jump to zero out of each node:



$$\pi_{\text{up}} = \frac{e^{(r-d)h} - e^{-\delta - \xi h}}{e^{+\delta} - e^{-\delta}}$$

$$\pi_{\text{dn}} = 1 - \pi_{\text{up}} - \pi_{\text{df}}$$

$$\pi_{\text{df}} = 1 - e^{-\xi h}$$

The stock price remains at zero following a jump to zero. With a default intensity of $\xi$, the probability of default is $\pi_{\text{df}} = 1 - \exp(-\xi h)$ over a time step of length $h$. The figure shows the parameterization in Hull [15], p.600; the probabilities of moves up and down are adjusted so that the expected growth rate of the stock remains $r - d$, even after including the jump to default, as it must be under the risk-neutral probabilities. In addition, the levels of the move up and down are adjusted by setting $\delta = \sqrt{(\sigma^2 - \xi)h}$. Adding the jump to zero increases the volatility of the stock; reducing $\delta$ in this way offsets this effect and keeps the volatility of the one-period return (approximately, for small $h$) unchanged.

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As noted by Das and Sundaram [7], Samuelson [28] showed that if a stock price follows a geometric Brownian motion, adding a jump to zero with constant intensity has the same effect on option prices as increasing the interest rate. But increasing the interest rate in the Black-Scholes formula increases the price of call options. Thus, we have the seemingly paradoxical result that adding the possibility of default actually makes call options more valuable. The paradox is resolved by noting the implications of leaving the initial stock price unchanged in comparing models with and without default. In leaving the stock price unchanged, we are implicitly assuming a higher expected rate of return conditional on survival. Thus, the real comparison we are making is between one stock with no jump to default and a lower rate of return and another stock with a higher rate of return but the possibility of default. The increased upside potential increases the value of a call option on the stock.

Similar observations apply in adding a jump to zero in a binomial model: with all other parameters held fixed, this increases the prices of call options. Rather than match the local volatility at a single node, we might therefore choose to reduce $\sigma$ to keep option prices (say at-the-money calls) unchanged. In our application, we will calibrate the volatility adjustment using credit default swap spreads and option prices.

To add some additional realism to the jump-to-default model, one can also make the default intensity a function of the stock price, so that default becomes more likely as the firm's equity loses value. Several authors (including Andersen and Buffum [1], Carr and Linetsky [5], and Madan and Unal [21]) have proposed models of this type. Das and Sundaram [7] use a flexible parameterization in which the default intensity is given by

$$\xi(t) = \frac{\exp[a_0 + a_1 r(t) + a_3 t]}{S(t)^{a_2}}, \tag{19}$$

with short rate $r(t)$, stock price $S(t)$, and parameters $a_0$–$a_3$. We will use the Das-Sundaram parameterization, but, because the options embedded in the CAP can change the number of shares, we replace the stock price in the denominator with the firm's market capitalization. Thus, dilution will not, by itself, affect the default intensity.

## 6  Calibration and Valuation

### 6.1  Data and First Results

Table 2 lists the QFI stock price data we need for our valuations. The table reports the stock price, the number of shares and the one-year implied volatility (from at-the-money call options as reported by OptionMetrics) for each financial institution, as of February 25, 2009. The table

| Name and Ticker | | Stock Price | Avg. Price | Shares (000s) | RWA (000s) | Vol. |
|---|---|---|---|---|---|---|
| American Express | AXP | 12.8007 | 15.2016 | 1,160,000 | 92,714,122 | 0.8283 |
| Bank of America | BAC | 5.1414 | 5.4139 | 6,358,168 | 1,694,704,842 | 1.2079 |
| BB&T | BBT | 16.1502 | 16.8211 | 559,248 | 111,620,088 | 0.8424 |
| BNY Mellon | BK | 23.7044 | 25.6225 | 1,148,467 | 117,412,000 | 0.8055 |
| Citigroup | C | 2.5200 | 3.2735 | 5,450,100 | 1,023,038,000 | 1.2289 |
| Capital One | COF | 12.2426 | 13.1512 | 389,200 | 128,159,657 | 1.0621 |
| Fifth Third | FITB | 1.9291 | 2.0002 | 577,387 | 109,087,385 | 1.8840 |
| Goldman Sachs | GS | 89.6893 | 88.0731 | 461,784 | 415,112,400 | 0.7503 |
| JPMorgan Chase | JPM | 21.6596 | 23.8649 | 3,732,800 | 1,277,105,800 | 0.8200 |
| KeyCorp | KEY | 7.6745 | 7.0507 | 495,002 | 101,076,763 | 1.0626 |
| MetLife | MET | 22.0500 | 27.0190 | 793,591 | 324,117,093 | 0.9527 |
| Morgan Stanley | MS | 21.6918 | 21.4127 | 1,074,498 | 288,262,300 | 0.9177 |
| PNC Financial | PNC | 29.1274 | 29.5599 | 427,956 | 244,746,397 | 0.8748 |
| Regions Financial | RF | 3.6551 | 3.2580 | 691,366 | 113,312,094 | 1.3680 |
| SunTrust | STI | 10.8175 | 9.9113 | 354,515 | 157,464,171 | 1.1215 |
| State Street | STT | 25.7563 | 25.4769 | 431,558 | 76,138,465 | 0.9221 |
| US Bancorp | USB | 12.6859 | 13.5007 | 1,755,000 | 232,043,110 | 0.8606 |
| Wells Fargo | WFC | 13.4111 | 15.9839 | 4,228,631 | 1,071,526,300 | 0.9372 |

Table 2: QFI parameters: From left to right, the numbers reported for each institution are the closing stock price on February 25, 2009, the average stock price over the twenty days ending February 9, the number of shares (in thousands) on February 25, risk-weighted assets (in thousands) at the end of the first quarter of 2009, and implied volatilities for one-year at-the-money call options as of February 25.

also reports the average stock price over the twenty day window ending February 9, 2009. This average (multiplied by 0.9) determines the conversion price and warrant strike price in the CAP under the terms released at the launch of the program. Each bank's risk-weighted assets (RWA) are listed as of the end of the first quarter[10] of 2009 as reported on the bank's form FR Y-9C of the Federal Financial Institutions Examination Council (www.ffiec.gov). In our valuations, we take the capital invested for each bank to be the maximum allowed under the terms of the program, which is 2% of RWA. The CAP limits dividends on common stock to a penny per share. Common dividends for participating institutions are thus too small to be material, and, as a simple rule, we use a dividend yield of 0.002 on all stocks. In Appendix A.3, we extend our valuation model to include stochastic interest rates; in our basic model we use a short rate of 0.24%, based on the overnight indexed swap (OIS) curve on February 25.

Figure 2 displays results from our basic extended binomial model using 32 time steps per year.[11] The figure shows the net CAP value for each QFI as a fraction of the initial capital

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[10]Risk-weighted assets are reported quarterly, and the end of the first quarter of 2009 is the reporting date closest to our valuation date of February 25. Some of the institutions listed became bank holding companies at the end of 2008 and did not report risk-weighted assets prior to 2009.

[11]We average the results obtained by letting the QFI or UST take the first turn. In numerical tests, averaging these two results appears to accelerate convergence to the common value reached by the two cases as the step size shrinks.
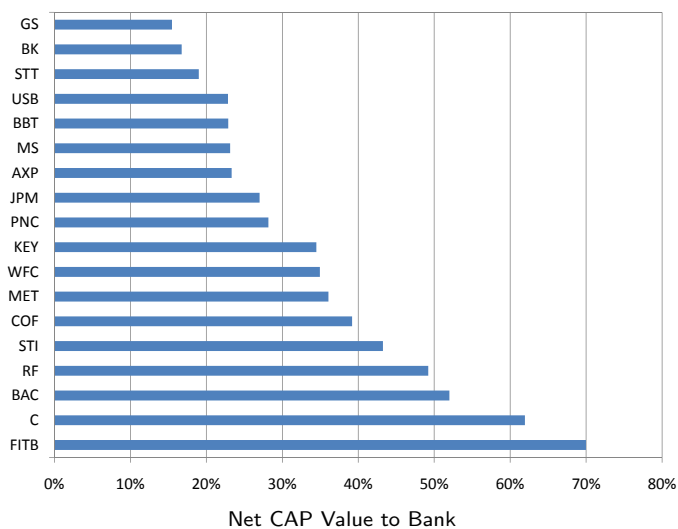
Figure 2: Net CAP values in the extended binomial model as a percentage of the capital invested. The mean is 34.4% and the median is 31.3%.

$G_0$. The mean and median across the eighteen banking institutions are 34.4% and 31.3%, respectively. The total potential investment under the program (2% of RWA) is \$152 billion, and the sum of the net value to the banks is \$59 billion. The figure also shows substantial variation across firms. (Numerical values are included in Table 3.) The factors causing this variation are the inputs in Table 2: stock volatility, risk-weighted assets, market capitalization, and the ratio of the starting stock price to the average used to set the conversion and strike price. In all eighteen cases, we find that it is optimal for the QFI to redeem or convert within the first two years (as will be the case in Figure 3). In Section 7, we compare the variation in CAP values with stock returns for the eighteen firms at the announcement of the CAP terms.

## 6.2 Values with Default

Table 3 reports our valuations using the extended binomial model with (third column) and without (second column) the possibility of a jump to default. (The fourth and fifth columns refer to the appendix.) Each CAP value is reported as $C_0/G_0$, the net value of the CAP to the QFI as a percentage of the initial capital $G_0$. We get lower CAP values when we include a potential jump to default; the mean and median values are 30.7% and 26.6% in this case. In calculating CAP values with a potential jump to default, we have assumed that all value is lost for both parties in the event of the jump to zero; introducing fractional recovery upon default would bring the values with and without default closer to each other.

For reference, Table 3 also includes the average CDS spread for each QFI, averaged over

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|  | Fixed Rates | | Stochastic Rates | | |
| Ticker | No Default | Default | No Default | Default | Avg. Spread |
| --- | --- | --- | --- | --- | --- |
| AXP | 23.3 | 17.1 | 22.6 | 14.1 | 328.78 |
| BAC | 52.0 | 49.0 | 51.5 | 47.4 | 234.72 |
| BBT | 22.9 | 21.1 | 22.2 | 19.5 | 130.94 |
| BK | 16.8 | 15.3 | 16.1 | 14.0 | 109.92 |
| C | 61.9 | 59.4 | 61.5 | 57.2 | 414.42 |
| COF | 39.2 | 33.9 | 38.8 | 31.6 | 334.29 |
| FITB | 70.0 | 69.4 | 69.4 | 68.6 | 70.81 |
| GS | 15.5 | 9.9 | 14.7 | 7.2 | 283.71 |
| JPM | 27.0 | 25.5 | 26.3 | 24.3 | 122.67 |
| KEY | 34.5 | 30.2 | 33.9 | 24.8 | 349.38 |
| MET | 36.1 | 27.6 | 35.6 | 24.0 | 558.12 |
| MS | 23.1 | 15.3 | 22.6 | 11.7 | 392.19 |
| PNC | 28.2 | 24.4 | 27.5 | 22.5 | 243.67 |
| RF | 49.2 | 43.1 | 48.8 | 40.1 | 509.47 |
| STI | 43.2 | 40.9 | 42.8 | 39.5 | 223.54 |
| STT | 19.0 | 17.7 | 18.4 | 16.1 | 156.20 |
| USB | 22.9 | 21.1 | 22.2 | 19.8 | 146.56 |
| WFC | 34.9 | 32.1 | 34.4 | 30.5 | 181.69 |
| mean | 34.4 | 30.7 | 33.9 | 28.5 | 266.17 |
| median | 31.3 | 26.6 | 30.7 | 24.1 | 239.20 |

Table 3: The table shows CAP values as a percentage of the initial capital invested in models with and without default and with fixed and stochastic interest rates. The last column reports the average CDS spread for each bank.

maturities from six months to ten years and averaged over the ten days ending on February 25. In general, the larger the spread the greater the difference in CAP values with and without default; see Figure 8 in Appendix A.2.

The difference in CAP values with and without a possible jump to default requires explanation. Ordinarily, when party A values a transaction with party B it uses B's creditworthiness to discount future payments that B may be required to make to A. In the over-the-counter derivatives market, this is called a credit value adjustment. In our setting, the net value of the CAP is positive for the QFI, and we are taking into account the QFI's *own* credit risk in valuing positive net payments it will receive from the UST. This is analogous to party A considering its own creditworthiness in valuing its transaction with party B, leading to a *bilateral* credit value adjustment, which is in fact the standard practice in the inter-dealer market. Introducing the QFI's credit risk in valuing a positive net payment to the QFI lowers the net value to the QFI, just as it would lower the cost to the QFI of an existing liability.

The effect can also be seen through our game formulation of the valuation problem. When we introduce a potential jump to default, we create an incentive for the QFI to convert earlier
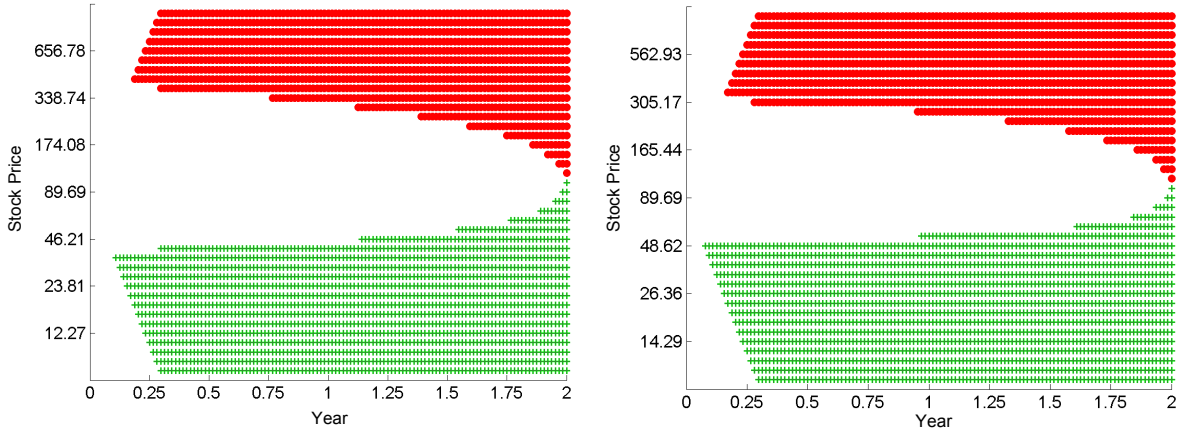
Figure 3: Exercise boundaries for Goldman Sachs without default (left) and with default (right). Circles mark the redemption region and crosses mark the conversion region. The vertical axis shows stock price levels and the horizontal axis shows time (in years). The vertical scale changes when we add default because the volatility adjustment changes the placement of nodes in the binomial lattice.

than it would otherwise, for fear that default will occur before the QFI can capture the value of exercising its option. Numerically, we find that the optimal conversion boundary rises; this is illustrated in Figure 3, which compares exercise regions for Goldman Sachs with and without default. This change in the exercise boundary produces behavior for the QFI which would be suboptimal without default and thus yields less value for the QFI. The UST also has an incentive to exercise earlier, but this effect is much less pronounced in the parameter ranges used in our valuations. The net effect is thus to reduce the value of the CAP to the QFI, just as a bilateral credit value adjustment would reflect the reduced likelihood of the QFI paying its liabilities.

Viewed from the perspective of the UST, the reduced CAP value reflects the possibility that the QFI may become bankrupt before the UST takes a larger stake in the firm. But here the limitation of the jump-to-default model becomes apparent: in reality, the UST could certainly intervene before default if it chose to. The jump model is thus perhaps best viewed as reflecting a sudden discovery of a massive fraud or some other event that radically changes the UST's disposition toward the QFI. Under this interpretation, the default intensity would be smaller than that extracted from CDS spreads.

These considerations lead us to the conclusion that the jump-to-default model is not necessarily preferable to the model without default. Nevertheless, we see both models as relevant and useful in understanding the value of the CAP securities. Even with the relatively large differences in some individual cases, the average valuations of 34.4% and 30.7% are fairly close and add to the robustness of the results.
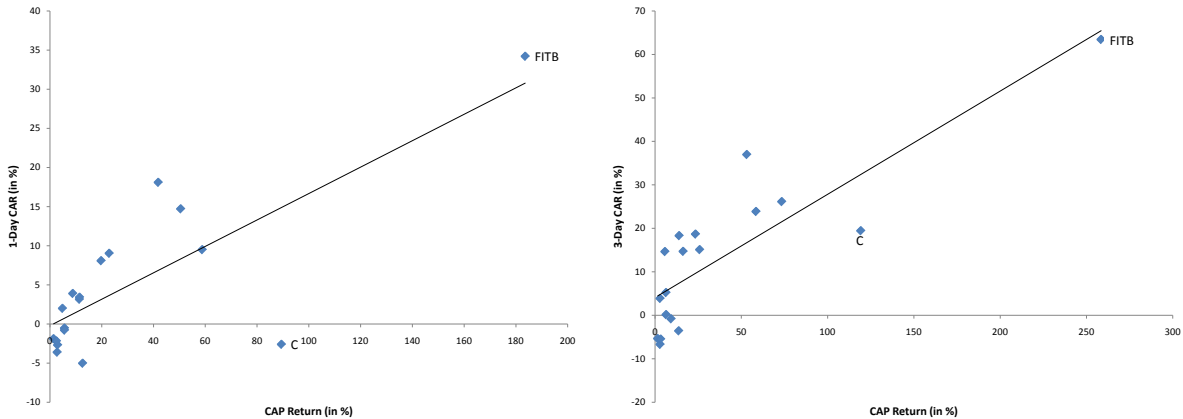
22

Figure 4: Cumulative abnormal returns over one day (left) and three days (right) plotted against CAP returns.

# 7    Market Response

In a perfectly efficient market, the value of an unexpected gain by a publicly held firm would be reflected in a change in its stock price when the gain becomes known. In this section, we therefore compare our CAP valuations with changes in stock prices of the same eighteen bank holding companies analyzed thus far. In an examination of the earlier CPP, Veronesi and Zingales [34] find significantly positive market responses to the first round of TARP funds at the time of the actual capital injections in October 2008; Bayazitova and Shivdasani [2] also find positive abnormal returns for banks participating in the CPP between October 2008 and April 2009.

The detailed terms of the CAP were announced on the morning of February 25, 2009, so we examine abnormal returns on that day. Some information[12] about the CAP structure was brought to the attention of the market on February 23, so we also consider three-day cumulative abnormal returns (CARs). Abnormal returns are calculated relative to the S&P Financial index using betas estimated from daily returns from August 2006 to July 2007, just prior to the onset of the financial crisis.

Figure 4 compares the CAR values (vertical axis) against CAP "returns" (horizontal axis) calculated by taking CAP values as a percentage of market capitalization in order to make the two axes comparable. The left panel uses one-day returns and the right panel uses three-day returns. In both cases, the scatter plots show a notably strong relationship between the

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[12]The Treasury and other financial regulators issued a joint statement on February 23 reiterating that government funds under the program would take the form of mandatorily convertible preferred shares. In answering questions from senators on February 24, Chairman Bernanke noted that banks (not the government) would hold the conversion option, and this appears to be the first public mention of this important feature of the program.
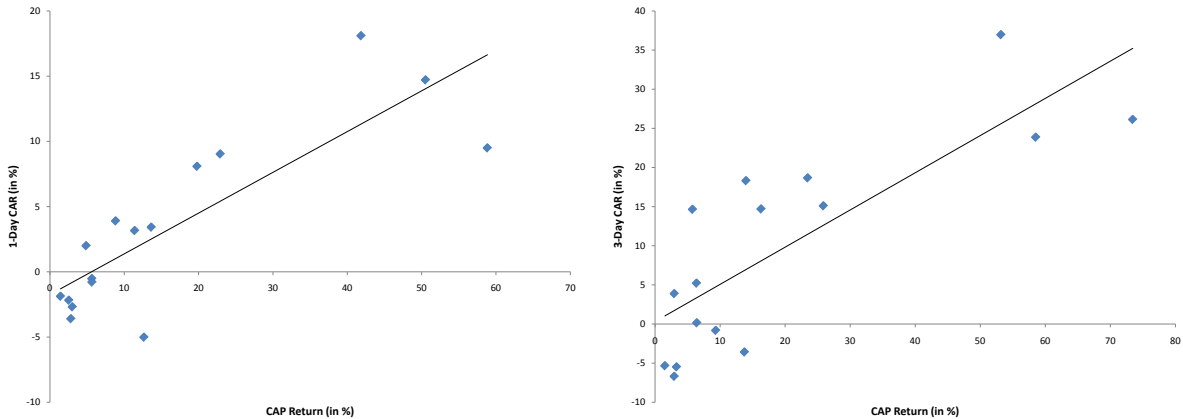
Figure 5: Cumulative abnormal returns over one day (left) and three days (right) plotted against CAP returns, excluding C and FITB.

market response and our estimated values; the correlations are 0.78 over one day and 0.85 over three days. The CAP returns differ across the two figures because the values on the left are normalized by February 24 stock prices, and the values on the right are normalized by prices on February 20 (a Friday).

As discussed in Section 6.1, our CAP values are based on the maximum size permitted under the program, which is 2% of risk-weighted assets. For banks with a low ratio of market capitalization to risk-weighted assets, the resulting CAP value can be a large fraction of the market value of the bank's equity. We have labeled FITB and C on the graphs in Figure 4 because they appear to be outliers, particularly over one day. Figure 5 displays the same comparison but with FITB and C omitted. The correlations over one day and three days are now 0.84 and 0.80, respectively.

The changes in stock prices as information about the CAP terms became available might be interpreted as the market's response to a signal that banks found to be weak through the stress test would be supported through additional government funds. Such a response could produce the positive correlations in Figures 4 and 5 if both the CAP returns and abnormal stock price returns are positively correlated with bank weakness as measured by the stress test.

To examine this possibility, we regress the CARs against several variables:

*LogT1CLev:* The logarithm of the ratio of risk-weighted assets to Tier 1 common capital at the end of the fourth quarter of 2008.

*SCAPGap:* The additional capital required for each bank by the results of the stress test.

*SCAPLoss:* The loss for each bank under the "more adverse scenario" in the stress test.

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| | One-Day CARs | | | | | Three-Day CARs | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Constant | −0.21 | 8.02 | −0.15 | 1.08 | −9.05 | 4.09 | −9.42 | 3.51 | 4.78 | −19.77* |
| | (1.80) | (10.68) | (1.91) | (2.05) | (7.40) | (2.72) | (16.51) | (2.87) | (3.28) | (9.67) |
| CAPreturn | 0.17*** | 0.18*** | 0.17*** | 0.18*** | 0.11* | 0.24*** | 0.22*** | 0.23*** | 0.24*** | 0.13** |
| | (0.03) | (0.04) | (0.04) | (0.03) | (0.06) | (0.04) | (0.04) | (0.04) | (0.04) | (0.05) |
| LogT1CLev | | −3.11 | | | | | 5.08 | | | |
| | | (3.98) | | | | | (6.13) | | | |
| SCAPGap | | | −0.03 | | | | | 0.22 | | |
| | | | (0.19) | | | | | (0.29) | | |
| SCAPLoss | | | | −0.05 | | | | | −0.02 | |
| | | | | (0.04) | | | | | (0.06) | |
| LogBMLev | | | | | 3.55 | | | | | 9.39** |
| | | | | | (2.88) | | | | | (3.69) |
| Adj $R^2$ | 0.59 | 0.58 | 0.56 | 0.60 | 0.60 | 0.70 | 0.69 | 0.69 | 0.68 | 0.77 |

Table 4: Regression of one-day CAR (left panel) and three-day CAR (right panel) against CAP returns and measures of bank weakness with all 18 banks included. Significance at the 10%, 5%, and 1% levels are indicated by one, two, and three asterisks, respectively.

*LogBMLev:* The logarithm of the ratio of risk-weighted assets to market capitalization.

The first three variables are taken from the results of the stress test released on May 7, 2009.[13] The report includes a summary sheet for each bank holding company, and our variables reflect the most important figures from these summaries. Our first variable is a baseline measure of how leveraged or undercapitalized a bank is from a regulatory perspective. The capital gap is the stress test's bottom-line number for each bank, and this is zero for the ten banks found not to need additional capital. For the loss variable, we have taken the total estimated losses in the stress test's more adverse scenario before any adjustments.[14] Our last variable may be viewed as a market perception of capital deficiency rather than one based on the stress test. In the denominator we use the market capitalization as of February 20th and 24th in regressing three-day and one-day CARs, respectively. In the numerator we use risk-weighted assets from the first quarter of 2009, as this is closer to the dates we use for the market capitalization.

Table 4 reports regression results for one-day (left) and three-day (right) CARs with all eighteen banks included. Table 5 reports corresponding results with C and FITB excluded. Overall, the stress-test-related measures do not affect the significance of CAP returns in explaining the stock returns. The market measure LogBMLev weakens and, in Table 5, eliminates the significance of the CAP returns. It should be emphasized, however, that plans for the CAP and the outline of the program were announced on February 10; thus, the CARs we observe are best explained as responses to new information about the CAP's detailed terms. To the extent

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[13] *The Supervisory Capital Assessment Program: Overview of Results*, Board of Governors of the Federal Reserve System.

[14] The values of SCAPGap and SCAPLoss were not known until May, so we are in effect using the eventual values as proxies for the market's anticipation of what these values would be.

| | One-Day CARs | | | | | Three-Day CARs | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Constant | −1.69 | −7.55 | −2.10* | −1.27 | −11.72* | 0.32 | −23.34 | −0.13 | 1.13 | −20.19* |
| | (1.32) | (7.42)) | (1.19) | (1.39) | (5.94) | (2.75) | (14.71) | (2.81) | (2.91) | (12.39) |
| CAPreturn | 0.31*** | 0.30*** | 0.40*** | 0.33*** | 0.14 | 0.48*** | 0.42*** | 0.55*** | 0.51*** | 0.19 |
| | (0.05) | (0.06) | (0.06) | (0.06) | (0.11) | (0.09) | (0.10) | (0.12) | (0.10) | (0.19) |
| LogT1CLev | | 2.26 | | | | | 9.16 | | | |
| | | (2.82) | | | | | (5.61) | | | |
| SCAPGap | | | −0.28* | | | | | −0.28 | | |
| | | | (0.13) | | | | | (0.32) | | |
| SCAPLoss | | | | −0.03 | | | | | −0.05 | |
| | | | | (0.03) | | | | | (0.06) | |
| LogBMLev | | | | | 4.62 | | | | | 9.36 |
| | | | | | (2.67) | | | | | (5.53) |
| Adj $R^2$ | 0.68 | 0.67 | 0.75 | 0.67 | 0.72 | 0.62 | 0.66 | 0.62 | 0.62 | 0.67 |

Table 5: Regression of one-day CAR (left panel) and three-day CAR (right panel) against CAP returns and measures of bank weakness with all C and FITB excluded. Significance at the 10%, 5%, and 1% levels are indicated by one, two, and three asterisks, respectively.

that LogBMLev predicts the market's response, it must do so by reflecting the value revealed by the CAP's terms. In other words, the changes in stock prices suggest that the market viewed the details of the CAP terms as most beneficial to banks with high ratios of risk-weighted assets to market capitalization, and this is consistent with our valuations.

# 8   Discussion

The Treasury announced the closing of the Capital Assistance Program on November 9. The only firm to issue securities under the terms of the CAP was GMAC[15], and these were issued under a different program, the Automobile Industry Financing Program. Citizens Republic Bancorp of Flint, Michigan, issued a press release stating that it was withdrawing its CAP application and citing, among other reasons, "the lack of activity surrounding CAP." We do not know if any other banks applied.

Our valuation results indicate that the terms of the CAP were attractive to banks and thus raise questions about the lack of participation, particularly in light of the importance attached to the program at its launch. The Treasury's February 25 press release announcing the terms of the CAP called it "a core element of the Administration's Financial Stability Plan." The program featured prominently in Ben Bernanke's testimony before the Senate Banking, Housing and Urban Affairs committee on February 24, especially in his answers to questions from senators (available through *Congressional Quarterly*). We therefore offer some possible explanations for the discrepancy between our valuations and the market's response.

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[15]We have excluded GMAC from our valuations because it does not have publicly traded equity. The GMAC agreement called for immediate exercise of the Treasury's warrants at a strike price of a penny per share.

It should first be noted that bank stock prices increased substantially in the months following the CAP announcement, and this lowered the net value of CAP securities to banks. The banks that were required to raise additional capital after the stress test had until Monday, June 8, to submit their plans, so Friday, June 5, is a relevant reference point. By then, the stocks of the eighteen stress test banks had increased by an average of 65%. Using our basic binomial valuation method, the average net value of the CAP across the eighteen banks falls to 2.8% on June 5, and for some banks the net value becomes negative. As discussed in the Introduction, the availability of CAP funds appears to have helped raise bank stock prices and, in so doing, reduced the net value potentially transferred to recovering banks (though our valuation results point to the risk of a high cost to the program had things gone differently). For several banks the CAP securities continue to have significant value as of June 5 (for example, 46% for Citi and KeyCorp, 38% for Regions Financial, 17% for Fifth Third and SunTrust), and the CAP was also available to banks outside the group of eighteen in the stress test, so we consider the effect of other aspects of the program as well.

Our arbitrage-based approach to valuation does not capture potential costs to a bank resulting from constraints attached to the Capital Assistance Program. The CAP imposes rules on executive compensation and hiring foreign workers that could put a participating firm at a disadvantage in competing for talent, and political pressure associated with partial government ownership could force a participating firm to have to forego certain profitable business opportunities.[16] These considerations, however, apply to participation in all TARP programs and are not unique to the CAP. The CAP does appear to carry greater controls than some earlier programs, requiring applicants to submit a plan for how they intend to use capital to expand lending and requiring recipients to detail their lending in monthly reports and comparing it with what their lending would have been without government support.

These considerations reflect potential direct costs of the CAP to firms, but potential agency costs are also relevant. Managers concerned about limits on compensation or removal by the government as influential shareholder may choose not to participate in a government program, even if participation would generate value for shareholders. The CAP program requires banks to undertake a review of their senior management, and this may also reduce its attractiveness.

Choosing to participate or not to participate may also carry important signaling value quite apart from direct costs and benefits to shareholders and managers. Much of the value of the CAP as measured by our analysis lies in the QFI's option to convert preferred shares to

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[16]Citigroup's decision to sell its profitable Phibro trading division to avoid the complications of an outsized compensation commitment to one of its traders appears to be an example of this phenomenon.

common equity at a favorable conversion ratio. This option is particularly valuable to a bank that has private information that its condition is likely to deteriorate. The market recognizes the firm's informational advantage and may therefore take a firm's decision regarding the CAP as a strong signal, punishing firms that participate and rewarding firms that do not. Thus, a value-maximizing firm may pass up an opportunity to participate in a positive NPV program if doing so lowers its cost of raising private capital.

This brings us to a further possible reason for the limited response to the CAP. The CAP's preferred shares can be redeemed "solely with the proceeds of one or more issuances of common stock for cash."In not including this requirement in our analysis, we are ignoring the cost of issuing new equity — this is part of the frictionless contingent claims approach that underlies our valuations. But if this cost is sufficiently large, the QFI effectively loses the redemption option. Without the redemption option, it is left to choose between continuing to carry the burden of the CAP's 9% dividend or converting the preferred shares to common shares at a potentially disadvantageous ratio.[17] Interestingly, the much more widely subscribed Capital Purchase Program carries no time limit on redemption,[18] and the CPP rewards banks for issuing new equity by canceling up to half the Treasury's warrants.

The CPP shares (which carry a 5% dividend for the first five years) do not contain the conversion option included in the CAP shares. Our analysis attaches a great deal of value to the conversion option, and this appears to have been the intention of regulators because they included a provision allowing banks to use CAP funds to repay CPP funds. But banks might perceive an implicit conversion option in the CPP — one that would be exercised if the bank's equity capital were sufficiently depleted — and they might therefore be reluctant to pay for the CAP's conversion option with a higher dividend. Indeed, on February 27, Citi announced[19] a negotiated conversion of its original TARP preferred shares to common equity, and there have been reports[20] that the Treasury considered similar arrangements with other banks. A May 27 press release from PNC Financial reporting on its capital raising efforts states "The company has no plans to convert preferred shares issued under the U.S. Treasury Department's Capital Purchase Program," suggesting that the bank considered conversion an option even though the CPP carries no such provision. The perception of an implicit option in the earlier program may

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[17]Caballero [3] discusses banks' fears of dilution under the CAP and proposes a government-guaranteed floor on bank stock prices as an alternative to the conversion option.

[18]When originally launched the CPP required a qualified equity offering for redemption within the first three years, but this requirement was dropped with the passage of the American Recovery and Reinvestment Act in January 2009; see http://www.financialstability.gov/docs/CPP/CPP-FAQs.pdf.

[19]http://www.citibank.com/citi/press/2009/090227a.htm

[20]For example, Andrews, E.L., "U.S. May Convert Banks' Bailouts to Equity Share," *New York Times*, April 19, 2009.

have worked against the value of the explicit conversion option in the CAP.

In the end, a case can be made that the CAP accomplished its intended purpose simply by providing a backup guarantee that reduced uncertainty about potentially adverse outcomes and helped boost confidence in the financial system. The nine public bank holding companies that were required to raise additional capital following the stress test met their targets by raising funds from private investors and taking other actions without drawing on the guarantee. As bank stock prices increased, the net value to banks of the CAP securities decreased, creating an almost automatic termination of the program. Our valuation results shed light on the potential risks and costs of this mechanism.

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On-Line Appendix to

# Valuing the Treasury's Capital Assistance Program

In this appendix, we expand the discussion of the example in Figure 1, we detail the calibration of the model with a jump to default, we extend the model to include stochastic interest rates, and we show that our recursive game formulation is equivalent to an extensive-form version.

## A.1  Example

This section continues and expands the discussion of the example in Section 5.2.

The top panel in Figure 6 repeats Figure 1. Following conversion or warrant exercise, the game is governed by the lower figure. The nodes with squares in the lower figure show the warrant exercise region for the UST *after* the QFI has converted (the exercise region for $W^c$). Similarly, the circles and crosses in the lower figure show the redemption and conversion regions the QFI would use if the UST had already exercised its warrants (the regions for $C^e$). Beyond the mandatory conversion date, the QFI has no remaining options, so we allow the UST to exercise at every time step between the seventh and tenth years.

One set of decisions remains, not covered by the figures: If the QFI redeems before the UST exercises its warrants, the UST is then left holding a standard warrant ($W^r$) on undiluted shares. The calculation of the exercise region for a standard warrant is straightforward, so we have not included a separate figure for that case.

In Figure 7, we illustrate how the exercise regions change as we vary the dividend rate on the CAP. The figures correspond to the top half of Figure 6, in the sense that they show exercise regions for $C$ and $W$, before either party has exercised. In the top left panel, the dividend rate is 2% and thus equal to the risk-free rate; this makes early redemption less attractive to the QFI, creating the possibility that the UST will exercise its warrants before the QFI converts or redeems. In the top right panel, we further reduce the CAP's dividend below the risk-free rate to 1%. Here, the benefit of terminating the dividend payments is reduced relative to the cost of conversion, and a gap opens in the QFI's exercise regions, allowing the CAP to continue beyond the initial two years. In the bottom panel, the CAP pays no dividend and the QFI never chooses early conversion. The QFI still has an incentive to redeem the CAP at the end of the initial two years if the stock price has increased in order to avoid mandatory conversion at the end of seven years at an unattractive conversion price. Indeed, the mandatory conversion feature creates a strong incentive for the QFI to redeem if the stock price increases.
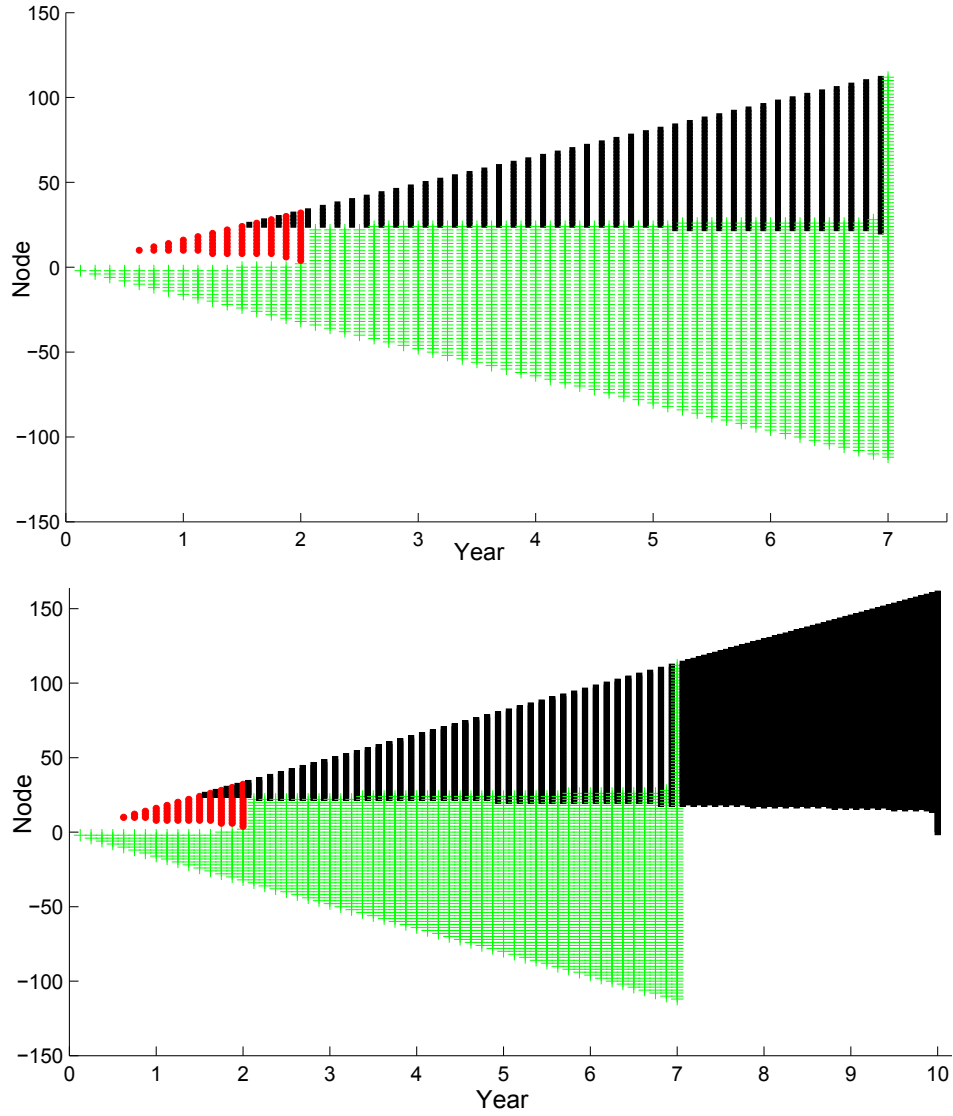
Figure 6: Exercise regions for the CAP: The cross marks show the region in which the QFI exercises its conversion option, the circles show where redemption is optimal, and the squares show the exercise region for the UST's warrants. The top figure applies before either party has exercised an option; once the QFI converts or the UST exercises, the other party's decisions are governed by the lower figure. After year seven, the UST is the only remaining player.
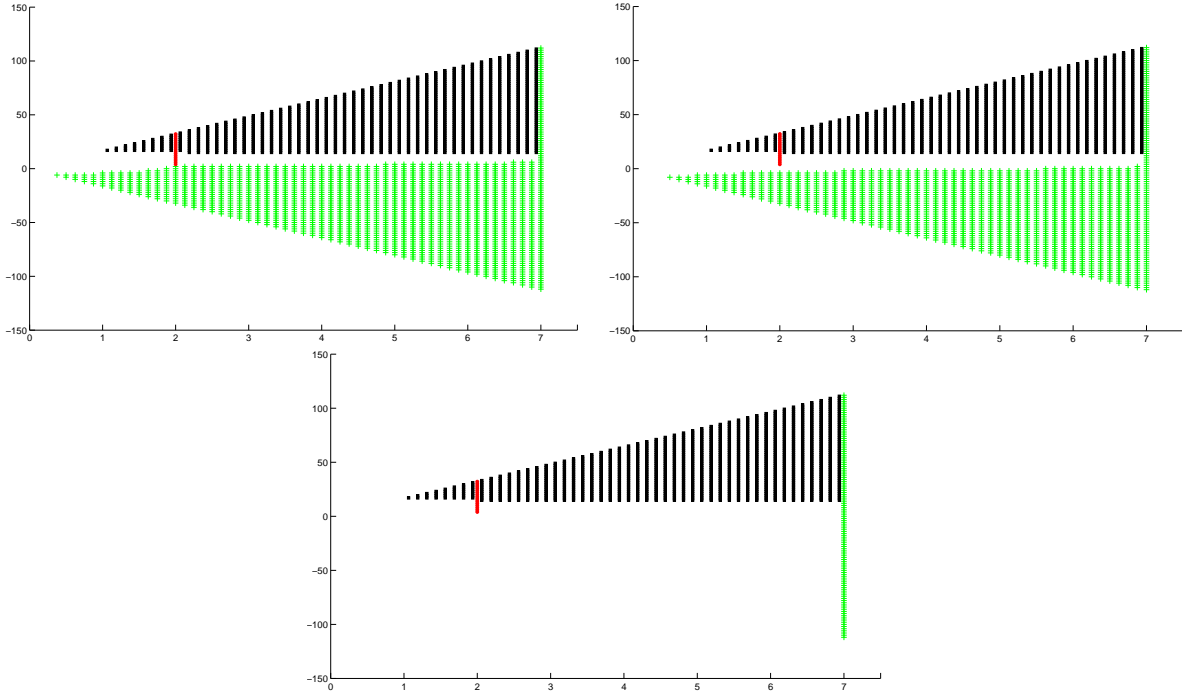
Figure 7: Exercise regions for the CAP with a dividend of 2% (top left), 1% (top right), and 0% (bottom).

One way to gauge the importance of the interaction between the two players is to compare the value of the CAP with the values of two separate securities granting options separately to the two players. In the example of Figure 6, we get a CAP value of $C_0 = \$25.67$ million, or 25.67% of the initial capital $G$. If we value the warrants in isolation within the same model, we get a value to the Treasury of $13.85 million, and if we value the CAP stripped of warrants (i.e., with $m = 0$), we get a value to the QFI of $34.14 million. Ignoring the interaction between the two contracts would yield a net value of $34.14 - \$13.85 = \$20.29$ million to the QFI. But the QFI captures more value (25.67 rather than 20.29) when the embedded options are combined. This is a consequence of the conversion option which lowers the value of the warrants through dilution and by shifting part of the cost of subsequent dilution from warrant exercise to the UST.

## A.2   Default Calibration

For the extension of the model that includes a potential jump to default, we calibrate parameters for each firm using its credit default swap (CDS) spreads. We use CDS spreads from Markit

at maturities of six months and 1, 2, 3, 4, 5, 7 and 10 years.[21] For consistency with our stock price model, we use spreads for the holding company rather than the bank in cases (like Bank of America) where both are available. We also use CDS recovery rates from Markit, which are 40% in all cases reported, except BAC for which it is 44.86%. We use 40% for BK and RF, for which no Markit recovery rates were available.

For six firms (BBT, BK, FITB, PNC, RF, and STT) some or all spreads were missing for the period of interest (February 2009). For BBT, BK, PNC, and STT just a few maturities were missing, so we imputed values through regression, using as regressors the average spread over all banks for each maturity. To impute, say, a missing 1-year spread at BBT, we regress BBT's 1-year spread against the averages for all maturities, estimating the model with daily data from January 2006 through May 2009. The imputed value on a given day is then the value fit by the model using the estimated coefficients and the average spreads on that day. For FITB and RF, we had too much missing data across maturities and days to apply this procedure, so we used only the 5-year contract (typically the most liquid), for which we had data for both banks, and simply assumed a constant spread across all maturities equal to the 5-year spread.

Using the CDS spreads, we calibrate an adjusted volatility and default intensity for each firm; the results are summarized in Table 6. Recall from the discussion in Section 5.3 that the volatility adjustment offsets the volatility increase that results from adding a jump to zero.[22] We calculate the volatility adjustment as follows. First, we calculate a constant default intensity $\xi_{const}$ calibrated to the 1-year CDS spread for each firm. Then, we solve numerically for the value of the adjusted volatility that yields the same price for a 1-year at-the-money call option in a binomial lattice with a jump intensity of $\xi_{const}$ as would be obtained with no jump to zero and the original volatility in Table 2. The choice of calibration to a 1-year horizon is a compromise between the greater liquidity of options at shorter maturities and the greater liquidity of CDS at somewhat longer maturities.

Once we have an adjusted volatility, we can calibrate the parameters $a_0$–$a_3$ of the default intensity (19). For now, we fix $a_1$ at zero and reintroduce this parameter when we include stochastic interest rates. The calibration proceeds iteratively. We start from initial values for the parameters[23] and calculate CDS spreads in the jump-to-default binomial lattice, as in

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[21]We use spreads for senior unsecured debt under document clause XR, which takes the most restrictive definition of a credit event, excluding restructuring.

[22]Whether such an adjustment is needed depends on whether one interprets volatility in the binomial tree as measuring market implied volatility, including the possibility of default, or as measuring stock price fluctuations in the absence of default. In our calibration to market data, we adopt the first interpretation.

[23]We used $a_0 = 0$, $a_2 = 1$, and $a_3 = .1$ as initial values, except in the case of USB for which we needed to start at $a_0 = 5$ to get a satisfactory fit. We compared results from multiple starting points.

| Ticker | Spread | $\xi_{\text{const}}$ | Adj. Vol. | $a_0$ | $a_2$ | $a_3$ | $\lambda_{\max}$ |
|---|---|---|---|---|---|---|---|
| AXP | 394.71 | 0.0673 | 0.7627 | 1.5584 | 0.181 | -0.1407 | 0.0500 |
| BAC | 285.06 | 0.0512 | 1.1643 | -2.8121 | 0.0078 | -0.0653 | 0.0500 |
| BBT | 152.53 | 0.0256 | 0.8182 | 14.508 | 0.8152 | -0.1971 | 0.0500 |
| BK | 74.57 | 0.0125 | 0.7936 | 9.9998 | 0.5943 | -0.1229 | 0.1063 |
| C | 523.17 | 0.0897 | 1.152 | -1.5994 | 0.0357 | -0.1159 | 0.0100 |
| COF | 410.06 | 0.0705 | 0.9988 | 1.3649 | 0.1773 | -0.2331 | 0.0719 |
| FITB | 70.81 | 0.0119 | 1.8758 | -2.9019 | 0.0739 | -0.1257 | 0.1492 |
| GS | 340.65 | 0.0577 | 0.6927 | 0.7806 | 0.1479 | -0.1247 | 0.0469 |
| JPM | 106.84 | 0.0179 | 0.803 | 11.503 | 0.6204 | -0.165 | 0.0500 |
| KEY | 298.13 | 0.0512 | 1.0169 | 1.4622 | 0.2117 | 0.1458 | 0.0080 |
| MET | 659.18 | 0.1122 | 0.8454 | -2.1008 | 0.0035 | -0.0406 | 0.0724 |
| MS | 499.94 | 0.0865 | 0.8351 | 0.7394 | 0.1331 | -0.1726 | 0.0500 |
| PNC | 242.90 | 0.0416 | 0.8355 | 0.429 | 0.1553 | -0.0321 | 0.0750 |
| RF | 509.47 | 0.0865 | 1.2972 | -2.453 | 0.0018 | 0.0926 | 0.0450 |
| STI | 192.20 | 0.0320 | 1.0937 | 1.16 | 0.2092 | -0.0288 | 0.0069 |
| STT | 127.71 | 0.0214 | 0.9024 | 24.5275 | 1.2986 | 0.077 | 0.0500 |
| USB | 93.75 | 0.0157 | 0.8459 | 9.919 | 0.5888 | -0.0608 | 0.0100 |
| WFC | 200.48 | 0.0352 | 0.9047 | -0.6811 | 0.1104 | -0.0675 | 0.0427 |

Table 6: Parameters for jump-to-default model. The spread column reports 1-year CDS spreads; these determine the value of $\lambda_{\text{const}}$, which we use to calculate the adjusted volatility. The last four columns report the parameters of the default intensity obtained through calibration to CDS spreads of maturities from six months to ten years.

equations (12)–(15) in Das and Sundaram [7]. We compare the model spreads with our observed (or imputed) spreads and adjust $a_0$, $a_2$ and $a_3$ to minimize the mean square error between the two. At sufficiently large values of the default intensity, the up or down probabilities in the binomial lattice may become negative. To avoid this problem, we cap the default probability at $\lambda_{\max}$ and include $\lambda_{\max}$ in the calibration process itself, starting at 0.075. The final values are included in Table 6.

For reference, Table 3 also includes the average CDS spread for each QFI, averaged over maturities from six months to ten years and averaged over the ten days ending on February 25. In general, the larger the spread the greater the difference in CAP values with and without default; see Figure 8. The correlation between the price differences and the average spread is 0.85. Although not included in the table, we have also calculated prices for the CAP warrants *in isolation* with and without a jump to default. The results are in most cases identical, and the difference is never more than half a percentage point. Recall that we calibrated the default model to keep the one-year implied volatility unchanged when we add jumps, so the agreement in warrant values between the two models is consistent with our objective in the calibration.
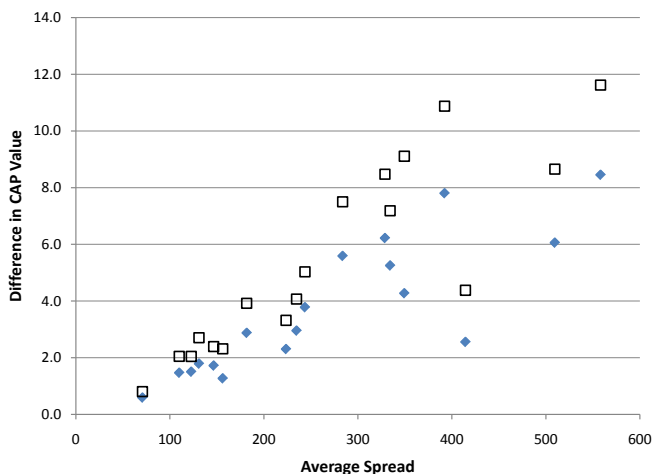
Figure 8: The figure shows differences in CAP values (with and without default) plotted against average CDS spreads. The diamonds mark differences with fixed interest rates, and the squares mark differences with stochastic rates. The correlations are 0.85 and 0.88, respectively.

## A.3 Stochastic Interest Rates

We now further extend our valuation model to allow for a stochastic term structure of interest rates. For this, we apply the implementation of the Heath-Jarrow-Morton (HJM) [13] framework developed by Das and Sundaram [7]. Their model allows for consistent valuation of equity and credit derivatives in a stochastic interest rate setting.

The central modeling element in the HJM setting is the forward rate curve. In a discrete-time setting, it is convenient to use the same time step $h$ both for calendar time and maturity. Thus, $f(t, T)$ denotes the forward rate fixed at time $t$ for borrowing or lending over the interval from $T$ to $T + h$, and the HJM framework describes the evolution of all such rates from time $t$ to time $t + h$. In the notation of Das and Sundaram [7], we have

$$f(t + h, T) = f(t, T) + \alpha(t, T)h + \sigma(t, T)X_f(t)\sqrt{h}, \tag{20}$$

where $\alpha$ is the drift coefficient, $\sigma$ is the diffusion coefficient, and $X_f(t)$ is a random variable taking the values $\pm 1$. This is a discrete-time approximation to a diffusion model, with $X_f(t)\sqrt{h}$ approximating a Brownian increment. It is implicit in Das-Sundaram [7] that $\sigma(t, T)$ depends only on $T$; given $\sigma$, the form of $\alpha$ is determined by the absence of arbitrage through the key result of HJM. The short rate at time $t$ is given by $r(t) = f(t, t)$.

In a binomial model of the evolution of the stock price $S$ and the forward curve $f$, each node records a pair $(S, f)$. The stock price may take a step up or down, and the forward curve may

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receive a positive or negative shock $X_f$, so there are four possible transitions out of each node corresponding to the four possible combinations of up and down moves. The special feature of the Das and Sundaram [7] model is that it recombines in the sense that a move up followed by a move down reaches the same node as a move down followed by a move up. This makes the model computationally tractable compared to a tree in which the number of nodes grows exponentially. It is, however, computationally much more demanding than a binomial model of the stock alone. Whereas a single-asset binomial model produces a triangular lattice of $O(T^2)$ nodes after $T$ steps, the bivariate model produces a pyramid of $O(T^3)$ nodes.[24]

The equations in Section 4 require some modification to apply to the current setting. The difficulty we face in a direct application of the previous equations is that when rates are stochastic, the value of cash becomes path-dependent. Suppose, for example, that the capital $G$ is invested at the risk-free rate at time zero. At some future node $(S, f)$, we would like to determine the liquidation value of the CAP contract, but the value of the initial investment of $G$ is not uniquely determined by the state information $(S, f)$. Different paths from the root node to $(S, f)$ would produce different paths of the short rate $r(t)$ and thus different values of the investment at the same ending node.

We circumvent this difficulty by working with bonds rather than money market accounts. At time zero, part of the capital $G$ is used to buy a strip of zero-coupon bonds whose cashflows match the CAP's dividend payments all the way to the mandatory conversion date $T_c$. This strip "defeases" the promised dividend payments. Let $B_t$ denote the value of the remaining cashflows from this strip as of time $t$; this value can be calculated at any node $(S, f)$ because the remaining cashflows can be discounted off the forward curve.

Next we suppose that, at time zero, the remaining cash $G - B_0$ (which could be negative) is invested in zero-coupon bonds maturing at $T_c$. If we let $P_t(T_c)$ denote the time-$t$ price of such a zero-coupon bond with a face value of 1, then the number of bonds $N$ satisfies

$$G - B_0 = N \cdot P_0(T_c).$$

If at some future time $t$ we liquidate the CAP (due to conversion or redemption, for example), the UST returns the bond strip $B_t$ to the QFI to stop the remaining dividend payments, and the QFI keeps the $N$ zero-coupon bonds, now worth $P_t(T_c)$ each. (In the case of redemption, the QFI also pays the par value $G$ to the UST.) Thus, the net effect of these transaction is that

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[24]This limits our numerical calculations to a time step of 1/8 of a year, compared with 1/32 for our basic binomial model.

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everywhere in equations (8)–(15) that we previously had $G_t - D_t$, we now have

$$B_t + N \cdot P_t(T_c), \quad N = \frac{G - B_0}{P_0(T_c)}.$$

The new expression has the advantage that the prices needed at each node can be computed from the forward curve at that node.

A simple calculation verifies that the expression we have derived here actually reduces to the previous expression in the case of a constant interest rate $r$. This follows from the fact that, with a fixed rate,

$$B_t = e^{-r(T-t)h}D_T - D_t \quad \text{and} \quad P_t(T_c) = e^{-rT_c h}$$

which yields $B_t + N \cdot P_t(T_c) = G_t - D_t$ after some algebraic simplification.

Details of the construction of the Das-Sundaram [7] model are given in their paper, so we do not repeat them here but simply note the parameter values we use. We fix the parameter $\gamma$ in their paper at 1, meaning that we continue to model the stock price process as a binomial approximation to a geometric Brownian motion. Their framework also allows correlation between the increments of the stock price and forward curve process; we fix this parameter at 0.3; however, we have found our results to be relatively insensitive to this parameter.

We extract a forward curve from overnight index swap (OIS) rates on February 25, 2009. OIS maturities range from one week to five years, extending the overnight Fed funds rates. We build zero-coupon bond prices from the OIS rates, interpolate the bonds, and then extract forward rates for any maturity time step $h$. Beyond five years, we assume the forward curve remains flat. For the diffusion coefficients $\sigma$, we start with implied volatilities for swaptions from Bloomberg on February 25. These are "Black volatilities" in the sense that they are based on the Black formula for swaptions, as in Hull [15], p.660. We use interpolation to get volatilities for forward rates at arbitrary maturities. As these are proportional or lognormal volatilities, to convert to the diffusion coefficients used in the Das-Sundaram [7] model, we multiply each lognormal volatility by the corresponding rate. In other words, we set $\sigma(0, T) = \sigma_L(0, T)f(0, T)$, where $\sigma_L(0, T)$ is a lognormal volatility; we then take $\sigma(t, T) = \sigma(0, T)$ for all $t$.

To include a possible jump to default, we need to recalibrate the default intensity to account for the addition of stochastic interest rates. The results are displayed in Table 7. CAP values with and without default in the presence of stochastic interest rates are recorded in Table 3. These results are in line with those we found with fixed rates, adding to the robustness of the results.

| Ticker | $\xi_{\text{const}}$ | Adj. Vol. | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $\lambda_{\max}$ |
|--------|------|-----------|---------|----------|--------|---------|---------|
| AXP | 0.0675 | 0.7013 | 1.5011 | -8.0150 | 0.1790 | -0.0241 | 0.1500 |
| BAC | 0.0530 | 1.0896 | -2.1610 | -29.7816 | 0.0324 | 0.1421 | 0.0700 |
| BBT | 0.0256 | 0.7923 | 14.4938 | 5.0000 | 0.8166 | -0.1392 | 0.1200 |
| BK | 0.0128 | 0.7798 | 10.8446 | 7.1339 | 0.6325 | -0.1142 | 0.1000 |
| C | 0.0901 | 1.0277 | -1.5604 | -18.7977 | 0.0370 | 0.0693 | 0.2000 |
| COF | 0.0707 | 0.9152 | 1.8990 | -14.4515 | 0.2015 | -0.0778 | 0.1500 |
| FITB | 0.0120 | 1.8432 | -2.9026 | -2.6515 | 0.0743 | -0.0670 | 0.1400 |
| GS | 0.0578 | 0.6447 | 0.7011 | -27.1054 | 0.1432 | 0.0990 | 0.0498 |
| JPM | 0.0176 | 0.7856 | 11.3953 | -1.0196 | 0.6172 | -0.0926 | 0.0500 |
| KEY | 0.0514 | 0.9553 | -2.1085 | -3.5350 | 0.0041 | 0.0422 | 0.0724 |
| MET | 0.1136 | 0.7239 | -2.1245 | -1.0055 | 0.0029 | 0.0515 | 0.0500 |
| MS | 0.0853 | 0.7513 | 0.6824 | -21.6812 | 0.1303 | 0.0392 | 0.0224 |
| PNC | 0.0401 | 0.7949 | 0.5101 | 3.7111 | 0.1613 | 0.0044 | 0.0500 |
| RF | 0.0869 | 1.1584 | -2.3995 | 1.3801 | 0.0043 | 0.1095 | 0.0500 |
| STI | 0.0353 | 1.0451 | 0.6813 | -3.3801 | 0.1886 | 0.0688 | 0.0700 |
| STT | 0.0216 | 0.8781 | 22.6242 | -4.3263 | 1.2148 | 0.2656 | 0.1200 |
| USB | 0.0157 | 0.8292 | 10.4310 | 1.6605 | 0.6125 | -0.0105 | 0.0700 |
| WFC | 0.0337 | 0.8701 | 0.2264 | -9.4252 | 0.1469 | 0.0210 | 0.0343 |

Table 7: Parameters for jump-to-default model with stochastic interest rates.

## A.4    From Normal Form to Extensive Form

In this appendix, we formulate the game between the QFI and the UST created by the CAP's options more precisely, and we justify the extensive form of the game that underlies the recursive equations in Section 4.

We start with a more generic formulation of the type used in Ohtsubo [25] and references therein. Recall our key modeling assumption that the exercise of the CAP's embedded options does not affect total firm value, only how value is allocated between the two players. Thus, any payoff to one party comes at the expense of the other. We focus (arbitrarily) on the payoff to the QFI and thus designate the QFI to be the maximizing player and the UST to be the minimizing player.

The strategies available to the two parties are stopping times with respect to the history of the underlying state process. Initially, we allow these stopping times (the admissible strategies) to take values in the set $\{0, 1, \dots \}$. We introduce payoffs $X_t$ and $Y_t$, for $t = 0, 1, \dots$. If the QFI exercises at $t$, it receives $X_t$; if the UST exercises at $t$, the QFI receives $Y_t$. In either case, the game stops when either party exercises. Thus, if the QFI and UST choose strategies $\tau_Q$ and

$\tau_U$, respectively, the payoff to the QFI is

$$\Pi(\tau_Q, \tau_U) = X_{\tau_Q} I_{\{\tau_Q \leq \tau_U\}} + Y_{\tau_U} I_{\{\tau_U < \tau_Q\}}.$$

We have arbitrarily chosen $X_t$ as the payoff if both players exercise at $t$, though we rule out this possibility shortly. The upper and lower values of the game are given by

$$v^+ = \inf_{\tau_U} \sup_{\tau_Q} E[e^{-r \min(\tau_Q, \tau_U)h} \Pi(\tau_Q, \tau_U)], \quad v^- = \sup_{\tau_Q} \inf_{\tau_U} E[e^{-r \min(\tau_Q, \tau_U)h} \Pi(\tau_Q, \tau_U)],$$

with obvious modifications in the case of stochastic interest rates. We will impose conditions ensuring the existence of the expectations in these expressions and thus ensuring that the values are well-defined. We will also establish that for the game created by the CAP's options, the upper and lower values are equal and can be calculated through backward induction.

We first need to fit the CAP game into this generic formulation. To have the players alternate turns, we can restrict the set of admissible strategies, limiting one player to exercise at even times and the other to exercise at odd times. We can similarly force exercise by a specific date (conversion at year seven, warrant exercise at year ten) by restricting the admissible strategies. In a discrete-time, discrete-state model (like our extended binomial model), we can alternatively impose these restrictions by adding sufficiently large penalties to payoffs at stopping times that violate the restrictions.

In the CAP game, the QFI holds two options (conversion and redemption) in the first two years. For purposes of valuation, we may reduce the QFI's three choices — convert, redeem, continue — to two choices — stop or continue — where the payoff upon stopping is the greater of the payoff from conversion and (when permissible) redemption. Also, under the terms of the CAP the game continues after one party exercises an option, though our generic formulation has the game terminate at that point. We accommodate this feature by defining the payoffs $X_t$ and $Y_t$ appropriately.

In more detail, if the QFI redeems at time $t$ (assumed to be the QFI's turn), its payoff is

$$G_t - D_t - G - W_t^r(S_t),$$

where

$$W_t^r(S) = \sup_{\tau \in \mathcal{T}_t} \frac{m}{m+n} E[e^{-r(\tau-t)h} \max(S_\tau - K, 0)|S_t = S],$$

and $\mathcal{T}_t$ is the set of stopping times taking values in $\{t, t+1, \ldots, T\}$. Here, $W_t^r(S_t)$ is just the value at time $t$ of a standard warrant for $m$ shares with strike price $K$ and expiration $T$. In this formulation, when the QFI redeems the game stops and the cash value of the warrants

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is deducted from the payoff to the QFI. Similarly, as explained following (14), in the case of conversion the payoff is

$$-\frac{qn}{n+q}S_t + G_t - D_t - W_t^c(\frac{nS_t}{n+q}),$$

in which the last term is the value of a warrant on $m$ shares with $n+q$ shares outstanding at a stock price of $nS_t/(n+q)$. Thus, we take

$$X_t = \max\{-\frac{qn}{n+q}S_t + G_t - D_t - W_t^c(\frac{nS_t}{n+q}), G_t - D_t - G - W_t^r(S_t)\},$$

omitting the second term inside the max once the redemption option expires. In the case of exercise by the UST, the payoff to the QFI is

$$Y_t = -\frac{mn}{m+n}(S_t - K) + C_t^e\left(\frac{nS_t + mK}{n+m}\right);$$

see the discussion following (18). The key point is that $C_t^e$ (like the warrant prices in the previous cases) is the value of a security in which only one player (in this case, the QFI) holds an option. In our formulation, the value of this security is paid to the QFI when the UST exercises and the game stops.

We now have the following:

*Proposition.* Suppose that $E[S_t] < \infty$ for $t = 1, 2, \ldots, T$. Then the upper and lower values of the game $v^+$ and $v^-$ are equal and given by $C_0(S_0)$, the value found recursively through (7)–(18).

*Proof.* This is a special case of Proposition 2.1 in Ohtsubo [25], except for some minor differences. Ohtsubo's [25] setting is infinite horizon, but his result applies to our simpler setting if we restrict the admissible strategies so that the players exercise their options in accordance with the CAP rules. Ohtsubo's [25] payoffs are not discounted; if we redefine our payoffs by setting $\tilde{X}_t = \exp(-rth)X_t$ and $\tilde{Y}_t = \exp(-rth)Y_t$ (in other words, if we denominate our payoffs in time-0 dollars) we put our problem in his setting. The discounted case is treated explicitly in Kifer [18]. Our equations (14) and (18) can be expressed as

$$C_t(S_t) = \max\{X_t, e^{-rh}E[C_{t+1}(S_{t+1})|S_t]\} \qquad \text{(QFI turn)} \qquad (21)$$

$$C_t(S_t) = \min\{Y_t, e^{-rh}E[C_{t+1}(S_{t+1})|S_t]\} \qquad \text{(UST turn)} \qquad (22)$$

because $W_t = -C_t$. Ohtsubo [25] requires $X_t \le Y_t$ for all $t$ and concludes (in our notation) that $C_t(S_t)$ is the median of the three values $X_t$, $Y_t$, and $e^{-rh}E[C_{t+1}(S_{t+1})|S_t]$. In our game, the players alternate turns; we do not need $X_t \le Y_t$, nor do (21)–(22) require or imply the median property. □