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Abstract

This paper studies banks' incentives for choosing the timing of their payment submissions in a collateral-based real-time gross settlement payment system and the way in which these incentives change with the introduction of a liquidity-saving mechanism (LSM). We show that an LSM allows banks to economize on collateral while also providing incentives to submit payments earlier. The reason is that, in our model, an LSM allows payments to be matched and offset, helping to settle payment cycles in which each bank must receive a payment that provides sufficient funds to allow the settlement of its own payment. In contrast to fee-based systems, for which Martin and McAndrews (2008a) show that introducing an LSM can lead to lower welfare, in our model welfare is always higher with an LSM in a collateral-based system.

Key words: liquidity-saving mechanism, intraday liquidity, payments

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1 Introduction

A growing recognition of the key role played by payments systems in modern economies has lead to increasing interest in the behavior of such systems. Research on payment systems has also been motivated by the important design changes that have occurred in the last thirty years from delayed net settlement system, to real-time gross settlement (RTGS) system, to the introduction of liquidity-saving mechanisms in many countries more recently. This research has shown that the incentives embedded in a payment system are sensitive to its design, highlighting the importance of a better understanding of these incentives.[1]

There are two main types of RTGS payment systems that differ in the way banks can obtain access to intra-day liquidity from the central bank. In a collateral-based system, such as TARGET 2 (European Central Bank), CHAPS (Bank of England), or SIC (Swiss National Bank), banks can obtain intra-day liquidity at no fee against collateral. In contrast, in a fee-based system such as Fedwire (Federal Reserve) banks can obtain intra-day liquidity without collateral but at a fee.[2]

This paper studies the effect of introducing a liquidity-saving mechanism (LSM) in a collateral-based RTGS system. Our model is closely related to the model proposed by (Martin & McAndrews, 2008a), which studies a fee-based settlement system. The similarity allows us to compare and contrast our results. We show that, without an LSM, banks face a trade-off between the cost of collateral and the cost of delay. By increasing their initial collateral, banks face lower expected cost of delays. Introducing an LSM allows banks to reduce their need for collateral while providing incentives for payments to be submitted early. A reduced need for collateral is beneficial because tying up collateral in the payment system can be costly for the bank, as this collateral cannot be used in other markets. Early submission of payments is also beneficial as it reduces the risk associated with operational failures when payments are concentrated late in the day.

We characterize the optimal allocation obtained by a planner in our economy. Without an LSM, the equilibrium allocation may be different from the planner's allocation as the planner takes into account the effect of a bank's actions on other banks. For some parameter values, however, the equilibrium and the planner's allocation are the

[1]See (Martin & McAndrews, 2008b) for example.

[2]Note that the Federal Reserve has adopted a new policy that will allow banks to choose between collateralized overdrafts at no fee or uncollateralized overdrafts for a fee. For more details, see http://www.federalreserve.gov/paymentsystems/psr/default.htm.

same. When they are not, there is too much delay in equilibrium. In contrast, the equilibrium and the planner's allocation are the same for all parameter values with an LSM.

The incentives of banks are different in fee-based and collateral-based systems. In a fee-based system, banks choose whether to submit or delay a payment by comparing the cost of borrowing from the central bank with the cost of delaying the payment. The marginal cost of borrowing is a fixed fee per unit of liquidity borrowed, so the terms of the trade-off will depend on the amount each bank expects to borrow. In a collateral-based system, banks choose their initial level of collateral at the beginning of the day, before they must make decisions about whether to submit or delay payments. A bank that is below its collateral limit will face no marginal cost of sending a payment. Because increasing collateral during the day is costly, banks are likely to prefer to delay payments rather than obtain more collateral, if the collateral limit binds. Hence, we can think of banks as belonging to two groups. Banks that have sufficient collateral for settlement face no cost of submitting payments. Banks that have insufficient collateral risk hitting their collateral constraint if they submit a payment.

This difference in incentives between the two systems results in differences in outcome. A notable feature of fee-based RTGS systems is that they exhibit multiple equilibria[3]. The intuition is that if many banks send their payments early, the probability of receiving a payment early is high so that the expected cost of borrowing is low. A low expected cost of borrowing gives incentives for banks to send their payments early. A similar argument applies in reverse to the case where few banks send their payments early. Multiple equilibria can occur in collateral-based RTGS systems as well, but are less important. In particular, the multiplicity disappears if all payments form bilaterally offsetting pairs. The intuition is as follows: if a bank has sufficient liquidity, then it will submit its payment early regardless of what its counterparty does. Since if the bank has insufficient liquidity, then its payment can settle only if it receives a payment from its counterparty. This can happen only if the counterparty has sufficient collateral. Hence, there is no strategic interaction between banks that may have an incentive to delay; namely those with insufficient liquidity. Strategic interactions reappear when some payments are not bilaterally offsetting. Nevertheless, the number of possible equilibria is higher in a fee-based system than in a collateral-based system.

In a collateral-based system, the equilibrium allocation without an LSM can be the

[3]In this paper we limit our attention to symmetric pure strategy equilibria.

same as the planner's allocation, for some parameter values. This is in contrast to the results in (Atalay, Martin, & McAndrews, 2008), which show that there is always too much delay in equilibrium, so that the planner's allocation cannot be achieved.

In fee-based RTGS systems, (Martin & McAndrews, 2008a) show that introducing an LSM can lead to a decrease in welfare, for some parameter values. In contrast, in our model an LSM always leads to higher welfare in a collateral-based system. Indeed, we show that with an LSM, the equilibrium and the planner's allocation are always the same. Introducing an LSM increases welfare in a collateral-based system in two ways: it allows offsetting of payments and allows banks to economize on their collateral. Offsetting of payments prevents situations where a group of banks form a cycle and each bank needs to receive a payment from its counterparty to have enough liquidity for its own payment to settle. (Atalay et al., 2008) show that in a fee-based RTGS system, for some, but not all, parameter values the equilibrium allocation with an LSM can be the same as the planner's allocation.

The remainder of the paper is structured as follows. In Section 2 we review the literature. In Section 3 we develop a benchmark theoretical model for a collateralized RTGS payment system and characterize the equilibria. We introduce an LSM in Section 4 and compare the payment system with and without an LSM. In section 5, we study the planner's allocation with and without an LSM, and contrast the results with the equilibrium allocations. Section 6 concludes.

2 Literature review

The incentive properties of RTGS payment systems are well analyzed in the literature. (Angelini, 1998, 2000) and (Bech & Garratt, 2003) provide theoretical explanations for why banks may find it optimal to delay payments in RTGS systems. This is not only a theoretical possibility, but also an actual feature of some payment systems. (Armantier, Arnold, & McAndrews, 2008) show that a large proportion of payments in Fedwire are settled late in the day with a peak around 17:11 in 2006. Significant intra-day payment delays carry a non-pecuniary cost of "delay" (ie customer dissatisfaction), but most importantly it can exacerbate the costs of an operational failure or costs due to the default of a payment system participant.

This paper is closely related to (Martin & McAndrews, 2008a) and (Atalay et al., 2008). The two papers analyze the effects of introducing LSMs in a real-time gross settlement system with fee-based intra-day credit. (Martin & McAndrews, 2008a) classify possible equilibria that could result from introducing LSMs. They show that apart from matching and offsetting, queuing arrangements allow banks to condition the settlement of their payments on the receipt of other banks' payments.

The benefits of an LSM are also analyzed by (Roberds, 1999), (Kahn & Roberds, 2001), (Willison, 2005), (Ercevik & Jackson, 2007) and (Galbiati & Soramäki, 2009). We extend these studies on different dimensions. Most importantly we consider the effect of liquidity shocks on payment behavior.

3 Model

The economy lasts for two periods, morning and afternoon. There are infinitely many identical agents, called banks, and a non-optimizing agent, called the settlement systems. Banks make payments to each other and to the settlement systems.

Bank may receive three types of payment orders. A bank may be required to transfer funds to the settlement systems. We refer to such payments as "liquidity shocks" as they cannot be delayed and must be executed immediately. Such payments represent a contractual obligation to be settled immediately and any delay constitutes a default. An example of such payments are margin calls in securities settlement systems or foreign exchange settlement. A bank may also be required to transfer funds to another bank. In this case we distinguish between urgent payments, having the property that the bank suffers a delay cost, $\gamma > 0$, if the payment is not executed immediately, and non-urgent payments, which can be delayed without any cost.

By the end of the day, each bank must send, and will receive, one payment of size $\mu \in [1/2, 1]$ from another bank. At the beginning of the morning period, each bank learns if it must send a payment to, or receive a payment from, the settlement systems. These payments determine the bank's liquidity shock, denoted by λ. If a bank must send a payment then $\lambda = -1$, if it receives a payment, $\lambda = 1$, otherwise $\lambda = 0$. We assume that the probability of $\lambda = 1$ is equal to the probability of $\lambda = -1$, and is denoted by $\overline{\pi} \in [0, 0.5]$. The probability of $\lambda = 0$ is $1 - 2\overline{\pi}$. The size of payments to and from the settlements systems is $1 - \mu$.

At the beginning of the morning period, banks also learn whether the payment they must send to another bank is urgent, which occurs with probability θ, or non-urgent, which occurs with probability $1 - \theta$. Banks know the urgency of the payment they must send, but not the payment they receive. For example, if a payment is made on behalf of a customer, the sending bank will know how quickly the customer wants the payment to be sent but the receiving bank may not even be aware of the fact that a payment is forthcoming for one of its customers.

The combination of the urgency of the payment a bank must make to another bank and its liquidity shock determine a bank's type. Hence, banks can be of six types: a bank may have to send an urgent or a non-urgent payment and may receive a negative, a positive, or no liquidity shock. We assume that a bank's liquidity shock is uncorrelated with the urgency of the payment it must make to another bank. Banks do not know the type of their counterparties, but only the distribution of types in the population. Also, since the number of banks is large, individual bank cannot influence equilibrium variables.

Banks must have enough liquidity on their central bank account for the payments they send to settle. If necessary, banks can borrow reserves from the central bank at a net interest rate of zero, against collateral. However, posting collateral is costly. Banks choose an initial collateral level, at a cost κ per unit, before learning their type. κ corresponds to the opportunity cost of the collateral as well as the cost of bringing collateral from the securities settlement system to the payments system. At the end of the morning period, payments are delayed if available collateral is insufficient. Payments must be settled by the end of the day, however, so delay is not an option at the end of the afternoon period. Additional collateral can be obtained at any time during the day at a cost of $\Psi > \kappa$.

In modeling the need for collateral, we abstract from two considerations: (i) banks may also post collateral to satisfy any prudential liquidity requirement, which we ignore, and (ii) banks usually start the day with a positive settlement account balance to satisfy reserve requirements. We assume that initial settlement balances are zero for all banks in the model and focus only on the incentive to hold collateral due to payment flows. Hence, the liquidity available for a bank's payment to settle are given by the collateral posted to the central bank and incoming payments only.

The timing of events during the day is as follows:

- Beginning of morning period:
  - Banks choose the level of collateral L_0 to be posted at the central bank (cost κ per unit).
  - Banks learn their type. If L_0 is insufficient to absorb the liquidity shock, additional collateral must be posted (cost Ψ per unit).[4]

[4]Liquidity shocks cannot be settled using the funds that a bank accumulates due to incoming payments.

  - Banks decide to send their payment to other banks or delay them until the afternoon.

- End of morning period:
  - Incoming morning payments are observed.
  - If available collateral is insufficient payments are delayed unless additional collateral is posted (cost Ψ per unit).

- Afternoon period:
  - All unpaid payment orders are executed. If collateral is insufficient, addition collateral must be posted (cost Ψ per unit).

If a bank submits a payment for settlement but fails to settle it due to insufficient reserves, it incurs a resubmission or reputational cost of $R > 0$. For example, payments that bounce back typically require human intervention for rescheduling and resubmission, which is costly for the bank. Banks that do not have sufficient reserves to settle a payment may nevertheless decide to submit the payment in the hope that an offsetting payment will be received. If a payment is received, then the payment that was sent can settle.

We make two parameter restrictions concerning the cost of adding collateral during the day, Ψ. First, we assume that $\overline{\pi}\Psi \geq \kappa$, so banks choose a level of initial collateral of at least $1 - \mu$, which implies that they have enough collateral to settle a negative liquidity shock. Second, we assume that banks always prefer to delay a payment at the end of the morning period, rather than pay the cost of increasing collateral.[5] The formal expression is provided below. It is not possible to avoid that cost at the end of the afternoon period, since all payments must be settled before the end of the day.

Banks that receive a negative liquidity shock need to obtain liquidity so their settlement account is non-negative at the end of the day. We assume that an overnight money market in which banks can obtain such reserves opens at the end of the day. Since this represents a fixed cost, it does not influence the intra-day behavior of banks and we ignore it in the remainder of the paper. In other words, we assume that the intra-day and overnight reserve management of banks are independent.

To facilitate the comparison with fee-based systems, our model is closely related to the model developed in (Martin & McAndrews, 2008a). In both models there are 6

In other words, we assume that not making payments arising from liquidity shocks is very costly to banks.

[5] Available data suggests that banks very rarely increase their collateral during the day.

types of banks, as banks can receive a positive, a negative, or no liquidity shock, and banks may have to send a time-critical payment. In (Martin & McAndrews, 2008a) banks that need to borrow at the central bank face a fee. In contrast, borrowing from the central bank is free in our model, provided the bank has enough collateral. Despite the similarities between the model, our results differ from (Martin & McAndrews, 2008a) in interesting ways.

3.1 A bank's problem

A bank needs to choose an initial level of collateral, L_0, as well as whether to send or delay its payment to another bank, to minimize its expected cost. In this section, we provide the notation and derive the expressions needed to solve that problem. In particular, we derive expressions for the expected cost of a bank in the morning period and in the afternoon period.

Let $P = 1$ if a bank sends its payment in the morning period. Note that sending a payment in the morning does not guarantee that the payment will settle during that period. Similarly, $P = 0$ if the bank delays its payment until the afternoon. The amount of collateral, available to a bank after it observes its liquidity shock, but before it chooses whether to send or delay its payment to another bank, is the sum of the initial collateral posted by the bank and its liquidity shock. It is given by

$$L_1 = \max\{L_0 + \lambda(1 - \mu), 0\}.$$

If the bank does not have sufficient collateral to meet the liquidity shock, that is $L_0 + \lambda(1 - \mu) < 0$, then it must obtain additional collateral. But we assume that $\overline{\pi}\Psi \geq \kappa$ so that banks post enough collateral to meet a negative liquidity shock.

We use ϕ as an indicator variable for a bank's payment activity with other banks. If a bank sends a payment to another bank in the morning, the payment settles, and the bank does not receive an offsetting payment, then $\phi = -1$. If the bank does not send a payment to, but receives a payment from, another bank in the morning, then $\phi = 1$. If a payment sent to another bank settles in the same period as the payment received from another bank, then $\phi = 0$.

We can derive expressions for the probability of each of these events. These probabilities depend on whether a bank sends a payment in the morning and, if the bank sends a payment, whether it settles. The probability that a payment settles in the morning depends on the amount of collateral the bank has.

Let ω^s denote the belief regarding the probability of receiving a payment conditional on sending a payment and having enough collateral for the payment to settle, even if a payment from another bank is not received. The superscript 's' indicates that the bank has 'sufficient' collateral. We use ω^i to denote the belief regarding the probability of receiving a payment conditional on sending a payment that can settle only if a payment is received. The superscript 'i' indicates that the bank has 'insufficient' collateral. Note that the probability of receiving a payment if the bank delays is also equal to ω^i. The beliefs $\omega = \{\omega^i, \omega^s\}$ that banks form about receiving payments in the morning must be equal to their true value in equilibrium.

First, we derive the probability that a bank has $\phi = -1$. This occurs if the bank submits a payment, $P = 1$, and has enough collateral for the payment to settle, $L_1 \geq \mu$, despite the fact that it does not receive a payment from another bank, $1 - \omega^s$. We use I to denote the indicator function that takes value 1 if the expression in parenthesis is true and zero otherwise. Hence,

$$Prob(\phi = -1) = PI(L_1 \geq \mu)(1 - \omega^s). \tag{3.1}$$

A bank has $\phi = 0$ either if it does not send a payment and does not receive one, or if it sends a payment that does not settle, or if it sends a payment that settles and receives a payment. This last case occurs with a different probability depending on whether the bank has sufficient collateral. We can write

$$Prob(\phi = 0) = (1 - P)(1 - \omega^i) + PI(L_1 < \mu) + PI(L_1 \geq \mu)\omega^s. \tag{3.2}$$

The first term indicates that the bank does not send a payment and does not receive one. The second term corresponds to the case where a bank submits a payment without sufficient collateral. Regardless of the incoming payments the net balance will be zero (either no payment is received and the outgoing payment cannot be settled, or an incoming payment is received and the outgoing payment is settled). The third term corresponds to a bank with sufficient collateral that sends and receives a payment. Finally, a bank has $\phi = 1$ if it receives, but does not send, a payment:

$$Prob(\phi = 1) = (1 - P)\omega^i. \tag{3.3}$$

Now we can write the expression for the cost faced by a bank at the end of the morning period. If a bank sends a payment that fails to settle, it incurs a reputational or resubmission cost of $R > 0$. The cost function for the morning period for a bank with an urgent payment is:

$$\begin{aligned} C_1 &= \kappa L_0 - \Psi \min\{L_0 + \lambda(1-\mu), 0\} \\ &\quad + PI(L_1 < \mu)(1-\omega^i)(R+\gamma) + (1-P)\gamma. \end{aligned} \tag{3.4}$$

The bank pays a cost κ per unit for its initial choice of collateral. If $0 \leq L_1 < \mu$, a payment sent does not settle if the bank does not receive an offsetting payment, which occurs with probability $(1-\omega^i)$. In such a case the bank faces a resubmission cost R, and a delay cost γ. If the bank does not send a payment, it faces a delay cost. The expression for a bank with a non-urgent payment is similar, but γ is replaced by zero, since there is no delay cost.

The amount of collateral available to the bank in the afternoon is denoted by L_2:

$$L_2 = L_1 + \phi\mu.$$

Note, that $L_2 \geq 0$ as $\phi = -1$ only if $L_1 \geq \mu$.

We have assumed that payments sent to and received from other banks offset. Nevertheless, payments may not settle without additional collateral in some cases. Consider a bank that is part of a chain of banks, indexed by $i \in \{1, 2, ..., N\}$, forming a cycle. Bank 1 sends a payment to bank 2, bank 2 sends a payment to bank 3, ..., and bank N sends a payment to bank 1. If at least one of these banks has sufficient collateral, so that its payment can settle even if it does not receive an offsetting payment, then this payment triggers the settlement of all other payments in the cycle. If, instead, none of the banks have sufficient collateral, then all payments are stuck. In such a case, we assume that one of the banks must obtain additional collateral at cost Ψ per unit. The probability that a given bank in the cycle needs to add collateral is $1/N$. We denote the expected cost of having to add collateral at the end of the day with Γ. Note, that if the payment sent by a bank settles in the morning, then the bank will face no cost in the afternoon. A bank risks being stuck in a payment cycle, exposing it to the need to add collateral, only if the bank's payment needs to settle in the afternoon. Hence, the cost function for the afternoon period is given by:

$$C_2 = [(1-P)(1-\omega^i) + PI(L_1 < \mu)(1-\omega^i)] \max\{\mu - L_1, 0\}\Gamma. \tag{3.5}$$

The first term in the square brackets corresponds to the case where a bank did not send and did not receive a payment in the morning. The second term corresponds to the case where the bank did send a payment in the morning, but the payment did not settle because the bank did not have enough collateral and did not receive an offsetting payment. Thus the problem solved by each bank is as follows:

$$\min_{L_0} \underset{\lambda,\gamma}{E} \left[\min_{P} \underset{\phi(\omega)}{E} (C_1 + C_2) \right].$$

Each bank i chooses a strategy $\{L_0^i, P_i(\lambda_i, \gamma_i; L_0^i)\}$, where $L_0^i \in \mathbb{R}_+$ is the amount of collateral to be posted by bank i at $t = 0$, and $P(\lambda_i, \gamma_i; L_0^i) \in \{0, 1\}$ is a discrete payment choice of bank i conditional on observed liquidity shock and the type of payment to be made at $t = 1$. $P = 1$ means 'pay early' while $P = 0$ means 'delay'.

Definition. *Let $\boldsymbol{\omega} = \{\boldsymbol{\omega^s}, \boldsymbol{\omega^i}\}$ be the distribution of beliefs that banks hold about the probability to receive a payment given 'sufficient' (superscript s) and 'insufficient' (superscript i) collateral. Define $\Omega : (\boldsymbol{L_0}, \boldsymbol{\omega}) \to \{\Omega^s, \Omega^i\}$ to be a mapping between true equilibrium probabilities for a bank to receive a payment, given (i) it does have sufficient liquidity and submits a payment, Ω^s; (ii) it does not have sufficient liquidity, Ω^i, and the distribution of collateral postings $\boldsymbol{L_0}$ and beliefs $\boldsymbol{\omega}$ regarding the value of Ω across the banks.*

As we are interested in a symmetric Nash equilibrium we limit our attention to cases where in equilibrium all banks form the same beliefs ω and post the same amount of collateral L_0. Clearly, if we want to expand the set of possible equilibria to include asymmetric equilibria, we should keep track of the distribution of beliefs and collateral postings.

Definition. *A strategy $\{L_0^*, P^*(\lambda, \gamma; L_0)\}$ is a symmetric subgame perfect Nash equilibrium strategy, if there exists a set of beliefs $\omega = \{\omega^s, \omega^i\}$ such that:*

$$\begin{aligned}
P^*(\lambda, \gamma; L_0) &= \arg \min_{P(\lambda,\gamma;L_0)} C(L_0, P(\lambda, \gamma; L_0), \omega) \ \forall \, \lambda, \gamma, L_0 \\
L_0^* &= \arg \min_{L_0} \underset{\lambda,\gamma}{E} \left[C(L_0, P^*(\lambda, \gamma; L_0), \omega) \right] \\
\omega &= \Omega(L_0^*, \omega)
\end{aligned}$$

Thus an equilibrium is characterized by the strategy $\{L_0^*, P^*(\lambda, \gamma; L_0)\}$ and a set of beliefs ω that are true in equilibrium.

3.2 Banks' behavior

In the next two lemmas we show that only a few values of the banks' choice of initial collateral are consistent with an equilibrium and derive equilibrium $P^*(\lambda, \gamma, L_0)$.

Lemma 1. *Any value of* L_0 *different from* $L_0 \in \{1-\mu, 2\mu-1, \mu, 1\}$ *cannot support an equilibrium*

Proof. Inspection of equations 3.4 and 3.5 show that L_0 affects the expected cost of banks directly and through ω^i. The value of ω^i depends on the fraction of banks such that $L_0 < \mu + \lambda(1-\mu)$, for $\lambda \in \{-1, 0, 1\}$. Hence, the value of ω^i can only change at the thresholds values $L_0 \in \{2\mu - 1, \mu, 1\}$. For a given value of ω^i, equations 3.4 and 3.5 show that costs decrease with respect to L_0. We have assumed that banks never choose $L_0 < 1-\mu$ as the cost Ψ is too high compared to κ. Hence, banks will choose $L_0 \in \{1-\mu, 2\mu-1, \mu, 1\}$ □

The decision to pay or to delay a payment depends on λ and γ, as banks make a decision after posting collateral and observing the realization of liquidity shock and the type of payments to be made.

Proposition 2. *All banks submit payments early if* $L_1 \geq \mu$

Proof. If after receiving a liquidity shock a bank has enough liquidity to make a payment it will do so. The expected cost of making a payment, EC_P, is:

$$EC_P = L_0\kappa$$

while the cost of delaying a payment, EC_D, is:

$$EC_D = L_0\kappa + \gamma.$$

Therefore, unless $\gamma \leq 0$ a bank is strictly better off by making a payment early. We assume that if a bank is indifferent between sending a payment or delaying, it sends the payment. □

Proposition 3. *Banks with insufficient collateral,* $L_1 < \mu$*, and a time-critical payment delay if* $(1-\omega^i)(R+\gamma) > \gamma$

Proof. Since $L_1 < \mu$, the bank does not have sufficient liquidity to make a payment, unless a payment is received. If a bank send its payment in the morning, then with probability $1-\omega^i$ it does not receive payment in the morning and suffers a cost $R+\gamma$. Instead, if the bank chooses to delay it suffers the delay cost γ with certainty. The expected costs are

$$\begin{aligned} EC_P &= L_0\kappa + [...] + (R+\gamma)(1-\omega^i), \text{ and} \\ EC_D &= L_0\kappa + [...] + \gamma, \end{aligned}$$

where the terms in the brackets represent the cost of obtaining additional collateral during the day and depend on the choice of L_0. For our purposes it suffices to know that these terms do not depend on the bank's decision to pay early or to delay. □

Lemma 4. *In equilibrium,* $L_0 < 1$ *and* $\omega^i < 1$*.*

Proof. Suppose, instead, that $L_0 = 1$. In this case, all banks have sufficient collateral after they receive their liquidity shock. This implies that a deviating bank with insufficient collateral would face $\omega^i = 1$ and find it optimal to choose $L_0 < 1$. Given $L_0 < 1$, banks with a negative liquidity shock have 'insufficient' collateral and choose to delay time critical payments. Since there is a positive mass of such banks, $\omega^i < 1$. □

It follows from the Lemmas 1 and 4 that we can restrict our attention to $L_0 \in \{1-\mu, 2\mu-1, \mu\}$.

Proposition 5. *If* $L_1 < \mu$ *banks with a non-time critical payment delay.*

Proof. Banks with $\gamma = 0$ find it optimal to delay if $R > \omega^i R$. This is true unless $\omega^i = 1$, which cannot hold in equilibrium, as shown in Lemma 4. □

Table 1 specifies $P^*(\lambda, \gamma, L_0)$ for all possible values of initial collateral, L_0 and liquidity shock, λ, given the belief of ω^i and observed payment type γ. The notation $R_1, ..., R_6$ indicates the region corresponding to each case in Figure 1.

3.3 Equilibrium

Given banks' behavior described in Table 1 we can derive the equilibrium probability of receiving a payment in the morning and the expected cost associated with a given choice of initial collateral. We study symmetric Nash equilibria in pure strategies. We focus on the case where $\mu \geq 1/2$, so the size of liquidity shocks, $1-\mu$, is relatively small. This is empirically plausible and simplifies the analysis.

In the morning, after banks observe their liquidity shock, we can distinguish three groups: Some banks may delay their payment until the afternoon; we denote the proportion of banks that have this characteristic by τ_d. Some banks submit their payment and have enough collateral for their payment to settle even if they do not receive an offsetting payment; we denote the share of these banks τ_s. Finally, some banks submit their payment but do not have enough collateral for their payment to settle if they do not receive an offsetting payment; we denote the share of these banks

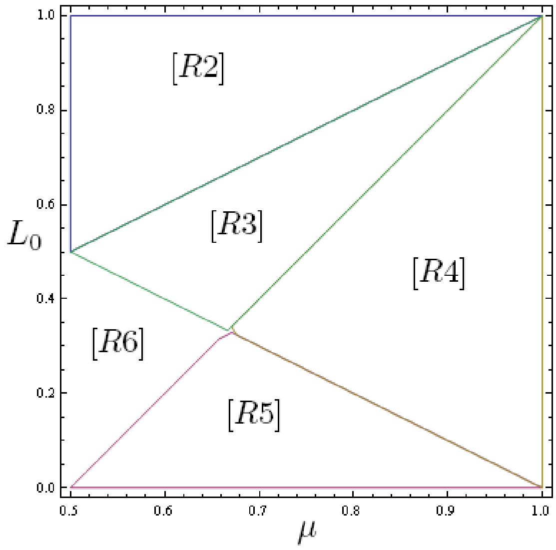

Figure 1: L_0 and μ corresponding to different regions of Table 1

Table 1: Equilibrium payment behavior $P^*(\lambda, \gamma, L_0)$ given beliefs ω.

L_0 region	λ	$P^*(\lambda, \gamma, L_0) = 0$ if
	-1	$\gamma < 0$ (never)
$L_0 \geq 1$	0	$\gamma < 0$ (never)
[$R1$]	1	$\gamma < 0$ (never)
	-1	$(1-\omega^i)(R+\gamma) > \gamma$
$\mu \leq L_0 < 1$	0	$\gamma < 0$ (never)
[$R2$]	1	$\gamma < 0$ (never)
$2\mu - 1 \leq L_0 < \mu$	-1	$(1-\omega^i)(R+\gamma) > \gamma$
and $1 - \mu \leq L_0$	0	$(1-\omega^i)(R+\gamma) > \gamma$
[$R3$]	1	$\gamma < 0$ (never)
	-1	$(1-\omega^i)(R+\gamma) > \gamma$
$1 - \mu \leq L_0 < 2\mu - 1$	0	$(1-\omega^i)(R+\gamma) > \gamma$
[$R4$]	1	$(1-\omega^i)(R+\gamma) > \gamma$
$0 \leq L_0 < 1 - \mu$	-1	$(1-\omega^i)(R+\gamma) > \gamma$
and $L_0 < 2\mu - 1$	0	$(1-\omega^i)(R+\gamma) > \gamma$
[$R5$]	1	$(1-\omega^i)(R+\gamma) > \gamma$
	-1	$(1-\omega^i)(R+\gamma) > \gamma$
$2\mu - 1 \leq L_0 < 1 - \mu$	0	$(1-\omega^i)(R+\gamma) > \gamma$
[$R6$]	1	$\gamma < 0$ (never)

τ_i. Note that $\tau_d + \tau_s + \tau_i = 1$. Abusing terminology, we say that a bank is of type τ_j if it is part of the group of bank of size τ_j, $\{j = d, s, i\}$.

The payments that banks make to each other form cycles, an example of which was provided above. For simplicity, we assume that all such cycles have the same length. In particular, we will focus on two cases: cycles of length two, in which case the payments of a pair of banks are bilaterally offsetting, and the case where all payments form a unique cycle.

In equilibrium, the probability Ω^s depends on the length of the payment cycle. If $n = 2$, $\Omega^s = 1 - \tau_d = \tau_s + \tau_i$. In that case, it is enough that the bank's counterparty sends a payment, regardless of the amount of collateral the counterparty has. For $n = 3$, $\Omega^s = \tau_s + \tau_i(1 - \tau_d) = \tau_s + \tau_i(\tau_i + \tau_s)$. The first term corresponds to the case where bank 1 receives a payment from bank 2 and bank 2 has sufficient collateral. The second term corresponds to the case where bank 1 receives a payment from bank 2 and bank 2 has insufficient collateral. However, bank 3 sends a payment to bank 2 and may or may not have sufficient collateral. This is enough since, by assumption, bank 1 sends a payment to bank 3. Extending the same argument for a cycle of length n, we obtain:

$$\Omega^s = \tau_i^{n-1} + \sum_{k=0}^{n-2} \tau_s \tau_i^k. \tag{3.6}$$

If $\tau_i < 1$ and the payment cycle is very long then

$$\lim_{n\to\infty} \Omega^s = \frac{\tau_s}{\tau_s + \tau_d}. \tag{3.7}$$

Similarly, in equilibrium $\Omega^i = \tau_s$ if $n = 2$, since a bank with 'insufficient' collateral can only receive a payment if its counterparty sends a payment and has 'sufficient' collateral. If $n = 3$, then $\Omega^i = \tau_s + \tau_s\tau_i$. In that case, either the bank's counterparty has sufficient collateral, or the counterparty has insufficient collateral but receives a payment from a bank with sufficient collateral. For any n,

$$\Omega^i = \sum_{k=0}^{n-2} \tau_s \tau_i^k. \tag{3.8}$$

If $\tau_i < 1$ and $n \to \infty$:

$$\lim_{n\to\infty} \Omega^i = \frac{\tau_s}{\tau_s + \tau_d}. \tag{3.9}$$

In equilibrium the beliefs regarding the probability to receive a payment must be true, $\omega = \Omega$. Therefore in the following we only use ω.

We can also derive an equilibrium value for Γ. First we need to find the probability that none of the other banks have made a payment. Given the payment cycle length of n this happens with the probability of $(1-\tau_s)^{n-1}$. Therefore

$$\Gamma = \frac{(1-\tau_s)^{n-1}}{n}\Psi < \Psi. \tag{3.10}$$

Based on Table 1 and on equations 3.4 and 3.5, we can write the expected cost faced by banks for different values of L_0 and on the relative values of γ and $(1-\omega^i)(R+\gamma)$. Consider the case where $\mu \leq L_0 < 1$ and $(1-\omega^i)(R+\gamma) \leq \gamma$, for example. For such parameters, banks with a positive or no liquidity shock have sufficient collateral and send their payment in the morning, as indicated by Table 1. These banks face no cost, other than the cost of the initial collateral. Banks with a negative liquidity shock delay non-time critical payments. These banks face no cost in the morning but may suffer a cost at the end of the day. Banks with a negative liquidity shock send time-critical payments in the morning. With probability $1-\omega^i$, the payment does not settle and the banks face a cost $R+\gamma$. They face no cost in the afternoon if they receive a payment in the morning but they may have to add an amount $\mu - L_1 = 1 - L_0$ of collateral in the afternoon, at a cost Γ, if they don't. Hence, the expected afternoon cost is $(1-\omega^i)(1-L_0)\Gamma$. Using similar reasoning, we obtain the following expected costs: if $(1-\omega^i)(R+\gamma) > \gamma$, then

$$\begin{aligned}
EC_{R2} &= L_0\kappa + \gamma\theta\overline{\pi} + \Gamma(1-\omega^i)(1-L_0)\overline{\pi}, \\
EC_{R3} &= L_0\kappa + \gamma\theta(1-\overline{\pi}) + \Gamma(1-\omega^i)(\mu - (2\mu-1)\overline{\pi} - L_0(1-\overline{\pi})), \\
EC_{R4} &= L_0\kappa + \gamma\theta + \Gamma(1-\overline{\pi})(\mu - L_0),
\end{aligned}$$

and if $(1-\omega^i)(R+\gamma) \leq \gamma$, then

$$\begin{aligned}
EC_{R2} &= L_0\kappa + (1-\omega^i)(R+\gamma)\theta\overline{\pi} + \Gamma(1-\omega^i)(1-L_0)\overline{\pi}, \\
EC_{R3} &= L_0\kappa + (1-\omega^i)(R+\gamma)\theta(1-\overline{\pi}) + \Gamma(1-\omega^i)(\mu - (2\mu-1)\overline{\pi} - L_0(1-\overline{\pi})), \\
EC_{R4} &= L_0\kappa + (1-\omega^i)(R+\gamma)\theta + \Gamma(1-\overline{\pi})(\mu - L_0).
\end{aligned}$$

As noted earlier, we assume that the cost of obtaining collateral during the day, Ψ, is large compared to the initial cost of positing collateral, κ, so that banks always choose $L_0 \geq 1-\mu$. This eliminates regions $R5$ and $R6$.

3.4 A long payment cycle

We focus our analysis on the case where all payments form a unique payment cycle. This special case is more tractable analytically and is representative of the problems associated with multilateral settlement. The case of short payment cycles is discussed in the Appendix.

Assuming a unique payment cycle it is easy to see from Equation 3.10 that as $n \to \infty$ $\Gamma \to 0$. We also demonstrated in Equations 3.7 and 3.9 that the equilibrium probabilities of receiving a payment in the morning are:

$$\omega^i = \omega^s = \frac{\tau_s}{1-\tau_i}. \tag{3.11}$$

Notice that ω^i depends on τ_i, so there are strategic interactions between banks with 'insufficient' collateral. These strategic interactions are responsible for the presence of multiple equilibria.

We derive the expected costs under different assumptions about the parameter values. First we choose a value for L_0 and we derive the value of τ_d, τ_i, and τ_s depending on the relative value of γ and $(1-\omega^i)(R+\gamma)$, based on Table 1. This allows us to find the value of ω^i, which we can then use to compute the expected cost faced by banks.

In the next 3 propositions we describe possible equilibria conditional on L_0^*.[6]

Proposition 6. *If parameters are such that* $L_0^* = \mu$ *and* $1-\overline{\pi} \leq \frac{R}{R+\gamma} \leq \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$, *then multiple equilibria in payment behavior are possible:*

(i) $L_0^* = \mu$, $\omega^i = \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$, *and* $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 0,1; \text{ or } \lambda = -1 \text{ and } \gamma > 0 \\ 0, & \text{if } \lambda = -1 \text{ and } \gamma = 0. \end{cases}$

(ii) $L_0^* = \mu$, $\omega^i = 1-\overline{\pi}$, *and* $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 0,1; \\ 0, & \text{if } \lambda = -1. \end{cases}$

(i) is the unique equilibrium, if $1-\overline{\pi} > \frac{R}{R+\gamma}$, *while (ii) is the unique equilibrium if* $\frac{R}{R+\gamma} > \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$.

The expected costs of these cases are given in equations 7.13 and 7.14.

Proposition 7. *If parameters are such that* $L_0^* = 2\mu - 1$ *and* $\overline{\pi} \leq \frac{R}{R+\gamma} \leq \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$, *then multiple equilibria in payment behavior are possible:*

(i) $L_0^* = 2\mu-1$, $\omega^i = \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$, *and* $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 1; \text{ or } \lambda = -1,0 \text{ and } \gamma > 0 \\ 0, & \text{if } \lambda = -1,0 \text{ and } \gamma = 0. \end{cases}$

(ii) $L_0^* = 2\mu - 1$, $\omega^i = 1-\overline{\pi}$, *and* $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 1; \\ 0, & \text{if } \lambda = -1,0. \end{cases}$

(i) is the unique equilibrium, if $\overline{\pi} > \frac{R}{R+\gamma}$, *while (ii) is the unique equilibrium if* $\frac{R}{R+\gamma} > \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$.

[6] For proofs and derivation see the appendix.

The expected costs of these cases are given in Equations 7.16 and 7.17.

Proposition 8. *If parameters are such that $L_0^* = 1-\mu$, then the unique equilibrium is characterized by: $L_0^* = 1-\mu$, $\omega^i = 0$, and $P^*(\lambda,\gamma,L_0^*) = 0$.*

The expected costs are given in Equation 7.18.

The expected costs described in Equations 7.13, 7.14, 7.16, 7.17, and 7.18 each consist of two terms: the first term expresses the cost of initial collateral, the second term is the expected cost of delay and, potentially, the resubmission cost R. Holding more collateral allows banks to reduce their expected delay and resubmission cost.

For a given set of parameters, we can determine the equilibrium level of initial collateral, $L_0 \in \{1-\mu, 2\mu-1, \mu\}$, by comparing the relevant expected costs. Determining the equilibrium in each region of the parameter space is straightforward but tedious. Instead, we consider some illustrative examples to provide some intuition.

Consider the region where

$$(1-\frac{1-\overline{\pi}}{1-\theta\overline{\pi}})(R+\gamma) > \gamma. \tag{3.12}$$

Intuitively, in this region even in the best case scenario (equilibrium *(i)* of Propositions 6 and 7) banks with insufficient funds and a time critical payment find it optimal to delay payment (at a cost of γ) than to submit a payment for settlement (expected cost equal to (or larger than) the left hand side of inequality 3.12). In this region, we investigate equilibrium candidates $L_0 \in \{1-\mu, 2\mu-1, \mu\}$ by comparing the expected costs given in Equations 7.14, 7.17, and 7.18 in the Appendix. Depending on the parameter values we characterize the set of possible equilibria[7]:

Proposition 9. *If $\frac{R}{R+\gamma} > \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}$ a subgame perfect Nash equilibrium strategy is:*

(i) $L_0^* = \mu$, $\omega^i = 1-\overline{\pi}$, $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 0,1; \\ 0, & \text{if } \lambda = -1. \end{cases}$
if $(1-\mu)\kappa < \gamma\theta(1-2\overline{\pi})$ *and* $(2\mu-1)\kappa < \gamma\theta(1-\overline{\pi})$.

(ii) $L_0^* = 2\mu-1$, $\omega^i = 1-\overline{\pi}$, $P^*(\lambda,\gamma,L_0^*) = \begin{cases} 1, & \text{if } \lambda = 1; \\ 0, & \text{if } \lambda = -1,0. \end{cases}$
if $(1-\mu)\kappa > \gamma\theta(1-2\overline{\pi})$ *and* $(3\mu-2)\kappa < \overline{\pi}\gamma\theta$.

(iii) $L_0^* = 1-\mu$, $\omega^i = 0$, *and* $P^*(\lambda,\gamma,L_0^*) = 0$.
if $(3\mu-2)\kappa > \overline{\pi}\gamma\theta$ *and* $(2\mu-1)\kappa > \gamma\theta(1-\overline{\pi})$.

[7]For proofs and derivation see the appendix.

For exposition, let $W(L_0 = x)$ denote the welfare associated with $L_0 = x$. We find that

$$W(L_0 = \mu) > W(L_0 = 2\mu - 1) \quad \Leftrightarrow \quad (1 - 2\overline{\pi})\theta\gamma > (1 - \mu)\kappa, \tag{3.13}$$

$$W(L_0 = 2\mu - 1) > W(L_0 = 1 - \mu) \quad \Leftrightarrow \quad \overline{\pi}\theta\gamma > (3\mu - 2)\kappa, \tag{3.14}$$

$$W(L_0 = \mu) > W(L_0 = 1 - \mu) \quad \Leftrightarrow \quad (1 - \overline{\pi})\theta\gamma > (2\mu - 1)\kappa. \tag{3.15}$$

These equations show that a higher value of κ, makes a high value of L_0 less desirable. In contrast, a higher value of $\theta\gamma$ (the product of the probability of having a time-critical payment and the cost of delay) makes a high value of L_0 more desirable.

The effect of the probability of liquidity shocks, $\overline{\pi}$, is more complicated. Consider the effect of $\overline{\pi}$: a higher value of $\overline{\pi}$ makes it more likely that a $L_0 = 1 - \mu$ is preferred to $L_0 = \mu$. In this case, less collateral is better. However, it also makes it more likely that $L_0 = 2\mu - 1$ is preferred to $L_0 = 1 - \mu$. In this case more collateral is better. To get some intuition, notice that if $\overline{\pi} \to 1/2$, the probability of having zero shock vanishes, and banks are almost certain to experience either a positive or a negative liquidity shock. When comparing $L_0 = \mu$ with $L_0 = 2\mu - 1$, a bank realizes that in either case it has 'sufficient' collateral with probability close to a half (and it has 'insufficient' collateral with probability close to a half). Hence the expected cost of delay is almost the same in both cases. Therefore, the primary consideration becomes the cost of initial collateral and the bank prefers a low level of collateral.

Now consider the comparison between $L_0 = \mu$ and $L_0 = 1 - \mu$. With $L_0 = 1 - \mu$, the bank does not have sufficient collateral even when it receives a positive shock. As $\overline{\pi}$ increases, the probability that a bank with $L_0 = \mu$ has 'insufficient' collateral increases. So the advantage of having high collateral shrinks as $\overline{\pi}$ increases, while the cost of having high collateral does not change.

When comparing $L_0 = 2\mu - 1$ and $L_0 = 1 - \mu$, similar reasoning applies, but in favor of more collateral. If $L_0 = 2\mu - 1$ the probability of having 'sufficient' collateral increases with $\overline{\pi}$. So the benefit of high collateral increases while the cost is unchanged.

In the appendix, we provide examples showing that depending on parameter values, any ranking of $W(L_0 = \mu)$, $W(L_0 = 2\mu - 1)$, and $W(L_0 = \mu)$ can occur.

3.5 Comparison with fee-based RTGS system

It is interesting to compare our results for an RTGS system where collateral is required to obtain intra-day overdrafts from the central bank with the results from (Martin &

McAndrews, 2008a) for and RTGS system where uncollateralized overdraft are available for a fee. In principle, it would be possible to use this model to determine which set of institutions provides higher welfare. Unfortunately, the results would depend crucially on the value of variables that are hard to measure such as the cost of collateral or the cost of delay. Nevertheless, we can look at how incentives to send or delay payments change in the two systems.

In a collateral-based system, banks with 'sufficient' collateral face no disincentive to send a payment, since once the cost of initial collateral is sunk, overdrafts have no costs. Hence, the key payment decision is made by the subset of banks that have 'insufficient' collateral but a time-critical payment to send. These banks can delay the payment, suffering the cost of delay with probability 1, or send their payment and risk suffering the resubmission cost R, in addition to the cost of delay. In fee-based system, all banks face a trade-off between the cost of delay and the expected cost of borrowing. Banks that receive a positive liquidity shock have higher reserves and thus their expected cost of borrowing is smaller.

An important aspect of a fee based RTGS system is that there are strategic interactions regarding the payment decision of banks, which leads to multiple equilibria. In contrast, multiple equilibria do not occur in a collateral based RTGS system in the case of short cycles, but they do occur with longer cycles. Even when they occur, there are only two equilibria in the collateral based RTGS system, while there can be as many as 4 equilibria in a similar model of a fee based RTGS system.

In a fee-based RTGS multiple equilibria arise because different types of banks may or may not submit payments early. This affects the probability to receive a payment and in turn changes incentives to submit a payment early. In a collateral-based RTGS the subset of banks with sufficient funds to settle payments, after receiving a liquidity shock, have no incentive to delay. Interestingly, in case of a small payments cycle, the probability to receive a payment does not depend on the payment activity of the group of banks with insufficient funds. Therefore multiple equilibria do not arise in the case of a small payments cycle. If payment cycles are not bilateral, actions of the banks with insufficient funds do affect the probability to receive a payment and therefore multiple equilibria arise. But since the subset of banks with sufficient funds has no incentives to delay there are only two possible equilibria that differ in the equilibrium payment actions of the banks with insufficient funds and time critical payments.

4 Liquidity-Saving Mechanism

In this section we introduce a liquidity-saving mechanism (LSM) that allows payments to offset and settle without using reserves. Banks can either submit a payment for settlement through the RTGS stream, in which case the payment settles immediately provided the bank has enough reserves, or submit a payment to the queue where it settles if an offsetting payment becomes available.

In a typical LSM, matched payments do not need to be offset perfectly, thus the LSM requires the use of some reserves from a bank's reserve account to improve its efficiency. If there is no limit to the amount of liquidity the LSM can use, however, banks may be reluctant to use it as the LSM could drain a bank's reserve account, leaving insufficient reserves for payments that the bank may want to settle through the RTGS stream.

Figure 2 represents the two extreme cases that our model can accommodate. The 'big box' illustrates the case where the LSM can use all of the reserves available in a bank's reserve account. In other words a payment is released when there is an incoming payment, regardless of the channel through which the incoming payment is submitted. The same assumption is made in (Martin & McAndrews, 2008a). The 'small box' in the figure illustrates the case where the LSM cannot use any reserves from the bank's reserve account. In other words, a payment in the queue can only be released by an incoming payment that is itself within the queue. (Galbiati & Soramäki, 2009) make a similar assumption.

Our model is too stylized to effectively capture the trade-off between making the LSM more efficient by allowing the use of reserves from banks' reserve accounts and reducing the incentives to use the LSM. Indeed, the assumption represented by the 'big box' in Figure 2 always leads to higher welfare (see appendix). This is the assumption we adopt in this paper. A richer model that captures this trade-off could allow banks to choose a limit above which reserves cannot be used by the LSM, for example.

4.1 A bank's problem

In this section, we describe a bank's problem when an LSM is available. Banks now face the following problem

$$\min_{L_0} \underset{\lambda,\theta}{E} \left[\min_{P,Q} \underset{\phi(\omega)}{E} \, (C1 + C2) \right].$$

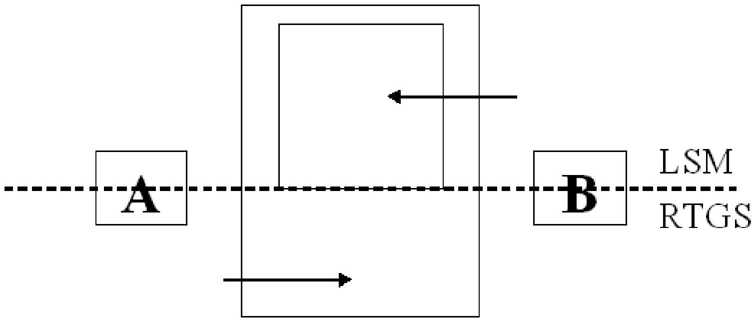

Figure 2: If two banks send offsetting payments to each other via RTGS (bank A) and LSM (bank B) the two payments could be either offset (the big box approach) or not (the small box approach).

$Q \in \{0,1\}$ is introduced to account for an additional choice, to queue, that banks have in the presence of an LSM. A bank chooses L_0, the level of collateral to be posted at the beginning of the day, to minimize the expect cost of its payment activity. This cost depends on whether the bank chooses to (i) submit its payment for settlement, ($P = 1$ and $Q = 0$); (ii) queue the payment, ($Q = 1$ and $P = 0$); or (iii) to delay it $P = Q = 0$. The decision to submit, queue, or delay is made after the bank observes its liquidity shock λ and whether it must make a time critical payment.

Banks have an additional choice of action with LSM, since they can queue. Hence, the set of beliefs is expanded to include the probability of receiving a payment in the morning if a payment is queued, ω^q. The set of beliefs is now $\omega = \{\omega^s, \omega^i, \omega^q\}$ and the corresponding set of true equilibrium probabilities is $\Omega = \{\Omega^s, \Omega^i, \Omega^q\}$.

As in the model without an LSM, we can derive the probability of observing different net payment balance in the morning, conditional on the beliefs ω:

$$\begin{aligned} Prob(\phi = -1) &= P(1-Q)I(L_1 \geq \mu)(1-\omega^s), & (4.1)\\ Prob(\phi = 0) &= (1-Q)\left[PI(L_1 < \mu) + PI(L_1 \geq \mu)\omega^s + (1-P)(1-\omega^i)\right] & \\ &\quad +Q(1-P), & (4.2)\\ Prob(\phi = 1) &= (1-Q)(1-P)\omega^i. & (4.3) \end{aligned}$$

The cost function for the first and second periods are:

$$\begin{aligned} C_1 &= (1-Q)\left[PI(L_1 < \mu)(1-\omega^i)(R+\gamma) + (1-P)\gamma\right] \\ &\quad +Q(1-P)(1-\omega^q)\gamma - \Psi \min(L_0 + \lambda(1-\mu), 0) + \kappa L_0 \end{aligned}$$

and

$$
\begin{aligned}
C_2 &= -\left\{(1-Q)(1-\omega^i)\left[(1-P)+PI(L_1<\mu)\right]+Q(1-P)(1-\omega^q)\right\} \\
&\quad \times \min(\mu - L_1, 0)\Gamma,
\end{aligned}
$$

respectively.

Definition. *A strategy $\{L_0^*, P^*(\lambda,\gamma;L_0), Q^*(\lambda,\gamma;L_0)\}$ is a symmetric subgame perfect Nash equilibrium strategy, if there exists a set of beliefs $\omega = \{\omega^s, \omega^i, \omega^q\}$ such that:*

$$
\begin{aligned}
\{P^*(\lambda,\gamma;L_0), Q^*(\lambda,\gamma;L_0)\} &= \arg\min_{P,Q} C(L_0, P(\lambda,\gamma;L_0), Q(\lambda,\gamma;L_0), \omega)\ \forall\, \lambda,\gamma,L_0 \\
L_0^* &= \arg\min_{L_0} \underset{\lambda,\gamma}{E}\left[C(L_0, P^*(\lambda,\gamma;L_0),\omega)\right] \\
\omega &= \Omega(L_0^*, \omega)
\end{aligned}
$$

Thus an equilibrium is characterized by the strategy $\{L_0^*, P^*(\lambda,\gamma;L_0), Q^*(\lambda,\gamma;L_0)\}$ and a set of beliefs ω that are true in equilibrium.

4.2 Banks' behavior with an LSM

To simplify the exposition we assume that if banks are indifferent between delaying and paying early, they pay early. If they are indifferent between queuing and paying early they queue. If they are indifferent between queuing and delaying, they queue.

Proposition 10. *If $L_1 \geq \mu$, then banks choose to pay early, unless $\omega^q = 1$, in which case they queue.*

Proof. If $L_1 \geq \mu$ the expected cost of delaying, paying early and queuing is, respectively:

$$
\begin{aligned}
EC_D &= L_0\kappa + \gamma \\
EC_P &= L_0\kappa \\
EC_Q &= L_0\kappa + (1-\omega^q)\gamma
\end{aligned}
$$

Clearly, paying early is the best option unless $\omega^q = 1$ in which case we assume that the bank would rather queue. □

Proposition 11. *If $L_1 \leq \mu$, then banks find it optimal to queue.*

Proof. If $\mu \leq L_0 < 1$ banks will have $L_1 < \mu$ only if they get a negative liquidity shock. The expected cost of delaying, paying early and queuing in that case is:

$$
\begin{aligned}
EC_{R2,D} &= L_0\kappa + \gamma + (1-\omega^i)(1-L_0)\Gamma, \\
EC_{R2,P} &= L_0\kappa + (R+\gamma)(1-\omega^i) + (1-\omega^i)(1-L_0)\Gamma, \\
EC_{R2,Q} &= L_0\kappa + (1-\omega^q)\gamma + (1-\omega^q)(1-L_0)\Gamma.
\end{aligned}
$$

Note that $EC_{R2,P} - EC_{R2,Q} = R(1-\omega^i) + (\omega^q - \omega^i)(\gamma + (1-L_0)\Gamma) \geq 0$, since $\omega^q \geq \omega^i$ (as shown in Section 6.1). Similarly, $EC_{R2,D} - EC_{R2,Q} = \omega^q\gamma + (\omega^q - \omega^i)\Gamma(1-L_0) \geq 0$. Therefore if $\mu \leq L_0 < 1$ and a bank receives a negative liquidity shock it will queue its payment.

If $2\mu - 1 \leq L_0 < \mu$ and a bank receives a negative liquidity shock the expected costs given different payment decisions are the same as above. If instead a positive liquidity shock is received, $L_1 > \mu$. In that case it is optimal to pay early unless $\omega^q = 1$, in which case banks queue as shown in Proposition 10. If a bank receive no liquidity shock the expected costs are:

$$
\begin{aligned}
EC_{R3,D} &= L_0\kappa + \gamma + (1-\omega^i)(\mu - L_0) \\
EC_{R3,P} &= L_0\kappa + (R+\gamma)(1-\omega^i) + (1-\omega^i)(\mu - L_0) \\
EC_{R3,Q} &= L_0\kappa + (1-\omega^q)\gamma + (1-\omega^q)(\mu - L_0)
\end{aligned}
$$

Again, as $EC_{3,P} - EC_{3,Q} = EC_{2,P} - EC_{2,Q} \geq 0$ and $EC_{2,D} - EC_{2,Q} = \gamma\omega^q + \Gamma(\omega^q - \omega^i)(\mu - L_0) \geq 0$ it is optimal to queue. Therefore if $2\mu - 1 \leq L_0 < \mu$ and a bank receives a negative or no liquidity shock it will queue its payment, while a bank with a positive liquidity shock will pay early.

Similar steps show that the expected cost of queuing is smaller than delaying or paying early if $1 - \mu \leq L_0 < 2\mu - 1$ or $0 \leq L_0 < 1 - \mu$ in which case regardless of the liquidity shock banks will have insufficient reserves to make a payment. $\square$

Intuitively, the probability of receiving a payment if an outgoing payment is queued is larger than, or the same as, the probability of receiving the payment if it is delayed. In addition, banks can avoid the resubmission cost by queuing. Therefore, if banks have insufficient funds queuing is better than delaying a payment or paying early.

Table 2 specifies the optimal decision for different types of the banks as a function of λ, γ, L_0 and beliefs ω.

Table 2: Equilibrium payment behavior $L_0^*, P^*(\lambda, \gamma; L_0)$ and $Q^*(\lambda, \gamma; L_0)$ given beliefs ω.

L_0 region	λ	Decision
	-1	P (or Q if $\omega^q = 1$)
$L_0 \geq 1$	0	P (or Q if $\omega^q = 1$)
$[R1]$	1	P (or Q if $\omega^q = 1$)
	-1	Q
$\mu \leq L_0 < 1$	0	P (or Q if $\omega^q = 1$)
$[R2]$	1	P (or Q if $\omega^q = 1$)
$2\mu - 1 \leq L_0 < \mu$	-1	Q
$L_0 \geq 1 - \mu$	0	Q
$[R3]$	1	P (or Q if $\omega^q = 1$)
	-1	Q
$1 - \mu \leq L_0 < 2\mu - 1$	0	Q
$[R4]$	1	Q
	-1	Q
$0 \leq L_0 < 1 - \mu$	0	Q
$[R5]$	1	Q
	-1	Q
$2\mu - 1 \leq L_0 < 1 - \mu$	0	Q
$[R6]$	1	P (or Q if $\omega^q = 1$)

4.3 Equilibrium with LSM

Given the optimal behavior in Table 2 we can derive τ_s, τ_i, τ_q, and τ_d, where τ_s is the fraction of banks that submit payment, $P = 1$, and have 'sufficient' collateral, $L_1 \geq \mu$, τ_i is the fraction of banks that submit a payment and have 'insufficient' collateral, $0 \leq L_1 < \mu$, τ_q is the fraction of banks that queue their payments, and τ_d the fraction of banks that delay their payment, $P = 0$. Note, that $\tau_s + \tau_i + \tau_q + \tau_d = 1$. Table 2 shows that $\tau_i = \tau_d = 0$.

With the expressions for τ_s and τ_q, we can obtain the probabilities of receiving a payment conditional on the bank's collateral and payment decision, ω^s, ω^i, ω^q, and π^d. We again consider the two extreme cases of $n = 2$ and $n \to \infty$.

We start with ω^i. If the payment cycle is of the length $n = 2$, then $\omega^i = \tau_s$. This means, that a bank receives a payment only if its counterparty is of type τ_s. If a payment cycle is of length $n = 3$, then there are two cases to be considered: (i) the bank sending a payment is of type τ_s, or (ii) it is of type τ_i or τ_q *and* the bank receiving the payment is of type τ_s. Therefore, for the cycle of length n:

$$\omega^i = \tau_s + \tau_s(\tau_i + \tau_q) + \tau_s(\tau_i + \tau_q)^2 + ...\tau_s(\tau_i + \tau_q)^{n-2} = \sum_{k=0}^{n-2} \tau_s(\tau_i + \tau_q)^k$$

Clearly, if $\tau_i + \tau_q < 1$ and $n \to \infty$

$$\omega^i \to \frac{\tau_s}{\tau_s + \tau_d}$$

The probability of receiving a payment, given that $L_1 \geq \mu$ and $P = 1$ also depends on the length of the payment cycle. If the payment cycle is of length $n = 2$, $\omega^s = 1 - \tau_d$. In other words, a payment is received unless the counterparty chooses to delay. If $n = 3$, assume a cycle consists of banks A, B, and C, with A sending a payment to B, B to C, and C to A. Consider the situation from the perspective of bank C. Bank C receives a payment if (i) bank B is of type τ_s, or if (ii) bank B is of type τ_i or τ_q *and* bank A is not delaying: $\omega^s = \tau_s + (\tau_i + \tau_q)(1 - \tau_d) = \tau_s + (\tau_i + \tau_q)\tau_s + (\tau_i + \tau_q)^2$. For the cycle of length n:

$$\omega^s = (\tau_i + \tau_q)^{n-1} + \sum_{k=0}^{n-2} \tau_s(\tau_i + \tau_q)^k$$

Again, if $\tau_i + \tau_q < 1$ and $n \to \infty$

$$\omega^s \to \frac{\tau_s}{\tau_s + \tau_d}$$

Similarly, we can derive the probability of receiving a payment, given that a bank chooses to delay ($P = 0$), π^d. If $n = 2$, $\pi^d = \tau_s$. For a cycle of length $n = 3$ this probability is $\pi^d = \tau_s + (\tau_i + \tau_q)\tau_s$. For the cycle of length n:

$$\pi^d = \omega^i = \sum_{k=0}^{n-2} \tau_s(\tau_i + \tau_q)^k$$

If a payment cycle is of the length $n = 2$, then $\omega^q = \tau_s + \tau_q$. A bank that queues its payment receives a payment if its counterparty also queues or has sufficient collateral. If $n = 3$, then $\omega^q = \tau_s + (\tau_i + \tau_q)\tau_s + \tau_q^2$. Again, consider the situation from the perspective of bank C, in the example introduced above. Either (i) bank B is of type τ_s, or (ii) bank B is of type τ_i or τ_q and bank A is of type τ_s, or (iii) all banks queues. For a payment cycle of the length n:

$$\omega^q = \tau_q^{n-1} + \sum_{k=0}^{n-2} \tau_s(\tau_i + \tau_q)^k$$

If some banks do not queue, $\tau_q < 1$, and the payment cycles is very long, $n \to \infty$, then

$$\omega^q \to \frac{\tau_s}{\tau_s + \tau_d}$$

We can derive an equilibrium value for Γ. First we need to find the probability that none of the other banks have made a payment. Given the payment cycle length of n

this happens with the probability of $(\tau_d + \tau_i + \tau_q)^{n-1}$. Therefore

$$\Gamma = \frac{(\tau_d + \tau_i + \tau_q)^{n-1}}{n} \Psi < \Psi. \tag{4.4}$$

Now we can write the expected cost of banks:

$$\begin{aligned}
EC_{R1} &= L_0 \kappa \\
EC_{R2} &= L_0 \kappa + (1 - \omega^q)\gamma\theta\overline{\pi} \\
EC_{R3} &= L_0 \kappa + (1 - \omega^q)(1 - \overline{\pi})\gamma\theta + (1 - \omega^q)\Gamma(\mu - \overline{\pi}(2\mu - 1) - L_0(1 - \overline{\pi})) \\
EC_{R4} &= L_0 \kappa + (1 - \omega^q)\gamma\theta + (1 - \omega^q)\Gamma(\mu - L_0) \\
EC_{R5} &= (1 - \omega^q)(\gamma\theta + \Gamma\mu) + (1 - \mu)\overline{\pi}\Psi + L_0(\kappa - (1 - \omega^q)\Gamma - \overline{\pi}\Psi)
\end{aligned}$$

4.4 Long payment cycle

In this section we consider the case of a unique long cycle, $n = \infty$. The case of many short cycles is considered in the appendix.

Proposition 12. *With LSM the equilibrium strategy is $L_0^* = 1 - \mu$, $P^*(\lambda, \gamma, L_0) = 0$, $Q^*(\lambda, \gamma, L_0) = 1 \ \forall \lambda, \gamma$ and $\omega^q = 1$.*

Proof. With a unique long payment cycle all banks queue, so $\omega^q = 1$. Indeed, $\omega^q = \frac{\tau_s}{\tau_s + \tau_d}$ and we know from Table 2 that banks choose not to delay, so that $\tau_d = 0$ and $\omega^q = 1$. Plugging $\omega^q = 1$ into the expressions for expected cost, we get

$$\begin{aligned}
EC_{R1,R2,R3,R4} &= L_0 \kappa, \\
EC_{R5} &= L_0 \kappa + (1 - \mu - L_0)\overline{\pi}\Psi.
\end{aligned}$$

When comparing $R1, R2, R3$ and$R4$ the only factor is the cost of collateral. Hence banks will prefer to choose the lowest level of initial collateral possible in these regions, $1 - \mu$. Next, notice that

$$EC_{R4} - EC_{R5} = (1 - \mu)(\kappa - \overline{\pi}\Psi).$$

We have assumed that $\kappa < \overline{\pi}\Psi$, so the expected cost associated with $R4$ is smaller than the expected cost associated with $R5$ or $R6$, and banks choose $L_0^* = 1 - \mu$. $\square$

4.5 Discussion

Our results show that an LSM allows banks to reduce their initial level of collateral, in a collateral-based system. In that sense, we could also call it a collateral-saving mechanism. In our model, an LSM prevents the situation where payments are stuck in a cycle where every bank needs to receive a payment in order to have enough collateral for its own payment to settle. In addition, the LSM allows banks to avoid the resubmission cost R. This benefit is hardwired into our model as we assume that payments that cannot settle through the RTGS stream have a cost that is different from payments that do not settle in the LSM. This is meant to capture the fact that if a bank reaches its collateral limit, it is unable to settle an unexpected urgent payment, which can be costly. An LSM allows banks to settle payments while leaving reserves available to settle unexpected payments. A richer model that captures this intuition should deliver results that are similar to the results from our model.

In a fee-based system, an LSM provides banks with a form of insurance against having to borrow from the CB. This consideration is less relevant here, because borrowing from the CB is free once the fixed cost of the initial level of collateral is sunk. Another interesting point to note is that an LSM completely eliminates delay in a collateral based system. This is not the case in a fee based-system. In a fee-based system, banks with a negative liquidity shock may prefer to delay in the hope that they receive a payment in the morning which allows them to avoid borrowing from the CB. Because the marginal cost of borrowing is zero in a collateral-based system, no bank has such incentives.

5 The planner's problem

In this section, we study the planner's problem. The planner can direct banks to choose an action, conditional on a bank's type, and aims to minimize the expected cost of settlement. The planner's allocation may differ from the equilibrium allocation because the planner takes into account the consequences of a bank's action on the expected settlement cost of other banks. Since the social planner can enforce a chosen behavior on all banks it knows the true probabilities for a bank to receive a payment, Ω.

5.1 Without LSM

With a long cycle the planner chooses either a relatively high level of collateral, and makes all banks submit their payment early, or a relatively low level of collateral and makes all banks delay their payment.[8]

[8] Case of short payment cycle is discussed in the Appendix.

Proposition 13. *Without LSM social planner's optimal behavior is characterized by* $L_0^* = 1 - \mu$ *and* $P^*(\lambda, \gamma, L_0) = 0 \;\; \forall \lambda, \gamma$ *if* $(3\mu - 2)\kappa > \gamma\theta$*, otherwise* $L_0^* = 2\mu - 1$ *and* $P^*(\lambda, \gamma, L_0) = 1 \;\; \forall \lambda, \gamma$.

If the planner chooses $L_0 = 2\mu - 1$ and $P = 1$ for all banks, then all payments settle in the morning. Indeed, banks with a positive liquidity shock have sufficient collateral and the settlement of their payment allows the payments of banks with insufficient collateral to settle as well. Hence, $\omega^i \to 1$ with a long payment cycle and therefore the expected cost of banks is $W(L_0 = 2\mu - 1) = (2\mu - 1)\kappa$.

If the planner chooses $L_0 < 2\mu - 1$, then we know that $\omega^i = 0$, irrespective of the actions of the banks. The best that can be achieved in this case is to delay all payments, so that $W(L_0 = 1 - \mu) = (1 - \mu)\kappa + \gamma\theta$. Thus the planner chooses a low level of collateral, $L_0 = 1 - \mu$, and all payments are delayed if

$$(3\mu - 2)\kappa > \gamma\theta.$$

Otherwise, $L_0 = 2\mu - 1$ and no payments are delayed.

With a long cycle, the expected cost of having to increase collateral during the day becomes infinitesimally small. For this reason, the planner focuses only on the initial cost of collateral and on the cost of delay. In contrast to the equilibrium allocation, the planner can direct a bank to submit a non-time-critical payment. This allows the planner to achieve $\omega^i = 1$ with only $L_0 = 2\mu - 1$ in initial collateral.

For some parameter values, the equilibrium and the planner's allocation will be the same. For example, we have seen in section 3.5 that banks will choose $L_0 = 1 - \mu$ if $\overline{\pi} = 1/2$, $R(1 - \theta) > \gamma$ and $(3\mu - 2)\kappa > \frac{1}{2}\theta\gamma$ and the planner chooses $L_0 = 1 - \mu$ if $(3\mu - 2)\kappa > \theta\gamma$. In contrast, (Atalay et al., 2008) report that for fee-based RTGS systems the planner's allocation cannot be achieved. In both collateral and fee-based systems, excessive delay is responsible for any differences between the equilibrium and the planner's allocation.

5.2 LSM

Proposition 14. *With an LSM social planner's optimal behavior is characterized by* $L_0^* = 1 - \mu$*,* $P^*(\lambda, \gamma, L_0) = 0$*,* $Q^*(\lambda, \gamma, L_0) = 1 \;\; \forall \lambda, \gamma$.

In a payment system with an LSM, the planner chooses $L_0 = 1 - \mu$ and required banks to queue their payment, irrespective of the length of the cycle. This yields

$W(L_0 = 1 - \mu) = (1 - \mu)\kappa$. We can see that the planner's allocation in a payment system with an LSM dominates planner's allocation in a payment system without LSM.

Note that the planner's allocation and the equilibrium allocation are the same with an LSM. This is a stronger result than what (Atalay et al., 2008) obtained for fee-based systems. They show that the equilibrium allocation is the same as the planner's allocation for some, but not all parameter values.

6 Conclusion

This paper investigates the effects of introducing a liquidity-saving mechanism in collateral-based RTGS payment systems. We characterize the equilibrium allocations and compare them to the allocation achieved by a planner. We develop a model closely related to (Martin & McAndrews, 2008a), who consider fee-based RTGS systems, to facilitate the comparison between our results and theirs.

In a collateral-based system without an LSM, the allocation in equilibrium can differ from the planner's allocation. However, in contrast to fee-based systems described by (Martin & McAndrews, 2008a), we find that for some parameter values the equilibrium and the planner's allocation are the same. When the cost of initial collateral is sufficiently high, compared to the cost of delay, the planner chooses to minimize the amount of initial collateral and makes all banks delay. In equilibrium, banks make the same choice. When the equilibrium and the planner's allocation differ, there is too much delay in both fee- and collateral-based system.

We show that introducing an LSM always improves welfare in a collateral-based system. This is in contrast to fee-based systems where (Atalay et al., 2008) show that an LSM can reduce welfare. In a fee-based system, an LSM can undo incentives to send payments early because it offers banks a way to insure themselves against the cost of borrowing at the central bank. In contrast, in a collateral-based system, the cost of borrowing is sunk at the time banks choose whether to submit, delay, or queue their payment. The LSM does not affect the incentives of banks that have sufficient collateral and it encourages banks with insufficient collateral to queue, instead of delay.

In a collateral-based system with an LSM, the equilibrium and the planner's allocations are the same for all parameter values. The LSM allows banks to save on their initial collateral while providing incentives to queue their payment. The LSM allows offsetting payment to settle with less collateral. This aligns the bank's incentives with that of the planner. (Atalay et al., 2008) show that in fee-based systems the equilibrium and the planner's allocation may differ, for some parameter values. In a fee-based system, if liquidity shocks are large, the planner may want banks with negative liquidity shocks to delay, rather than queue their payments. This is beneficial if the cost of the induced delay is smaller than the reduction in borrowing cost from the central bank. In a collateral-based system, the planner does not face this type of incentive because the cost of collateral is sunk at the time banks learn their liquidity shock.

We believe that, although highly stylized, the model developed in this paper accounts for the key trade-offs that banks face in making payments. Still there are several ways our analysis could be expanded. Greater uncertainty regarding payment flows over the day and multiple payments would be a desirable extension to our model.

7 Appendix

7.1 Alternative LSM

How things would be different with an alternative LSM definition (small box in Figure 2)?

$$\omega^s = \tau_i^{n-1} + \sum_{k=0}^{n-2} \tau_s \tau_i^k$$

$$\omega^s_{n\to\infty} \to \frac{\tau_s}{\tau_s + \tau_q + \tau_d}$$

$$\pi = \sum_{k=0}^{n-2} \tau_s \tau_i^k$$

$$\pi_{n\to\infty} \to \frac{\tau_s}{\tau_s + \tau_q + \tau_d}$$

$$\omega^q = \tau_q^{n-1}$$

$$\pi_{q,n\to\infty} \to 0, \text{ unless } \tau_q = 1, \text{ in which case } \omega^q = 1$$

$$\pi^d = \sum_{k=0}^{n-2} \tau_s \tau_i^k$$

$$\pi_{0,n\to\infty} \to \frac{\tau_s}{\tau_s + \tau_q + \tau_d}$$

Comparing the two alternatives, we can see that "big box" LSM leads to higher (or same) probability to receive a payment compared to "small box" LSM for all bank types!

Note, that $\omega^s \geq \omega^q \geq \pi$ for the "big box" approach.

7.2 Short payment cycles

In the following we assume that payments form bilaterally offsetting cycles.

7.2.1 Short payment cycles in RTGS

Assuming a payment cycle length of $n = 2$, the equilibrium probabilities of receiving a payment in the morning are:

$$\begin{aligned} \omega^s &= 1 - \tau_d \text{ and} \\ \omega^i &= \tau_s. \end{aligned}$$

Notice that ω^i depends only on the fraction of banks that have sufficient collateral. This fraction is independent of the relative values of γ and $(1-\omega^i)(R+\gamma)$. Hence, there are no strategic interactions between banks with 'insufficient' collateral and the equilibrium is unique for any set of parameters.

As in the previous section, we first pick a value of L_0 and derive the expressions for τ_d, τ_i, and τ_s depending on the relative value of γ and $(1-\omega^i)(R+\gamma)$. This allows us to derive ω^i and also Γ. With these expressions, we can obtain the expected cost faced by banks. If $L_0 = \mu$, and $(1-\omega^i)(R+\gamma) \leq \gamma$, then banks with insufficient liquidity delay non-time critical payments and

- $\tau_d = \overline{\pi}(1-\theta)$,
- $\tau_i = \overline{\pi}\theta$,
- $\tau_s = 1 - \overline{\pi} = \omega^i$.

From equation 3.10, we have $\Gamma = \frac{\Psi\overline{\pi}}{2}$, so the expected cost in this case is

$$EC\left(L_0 = \mu, \overline{\pi}(R+\gamma) \leq \gamma\right) = \mu\kappa + (R+\gamma)\theta\overline{\pi}^2 + \frac{1}{2}(1-\mu)\overline{\pi}^3\Psi. \tag{7.1}$$

If, instead, $(1-\omega^i)(R+\gamma) > \gamma$, then all banks with insufficient liquidity delay their payment, Γ is the same as above, and

- $\tau_d = \overline{\pi}$,
- $\tau_i = 0$,
- $\tau_s = 1 - \overline{\pi} = \omega^i$.

The expected cost in this case is

$$EC\left(L_0 = \mu, \overline{\pi}(R+\gamma) > \gamma\right) = \mu\kappa + \gamma\theta\overline{\pi} + \frac{1}{2}(1-\mu)\overline{\pi}^3\Psi. \tag{7.2}$$

Note that ω^i is the same whether $(1-\omega^i)(R+\gamma) > \gamma$ or $(1-\omega^i)(R+\gamma) \leq \gamma$, so we do not have multiple equilibria, in contrast to the previous section.

Similarly, if $L_0 = 2\mu - 1$, and $(1-\omega^i)(R+\gamma) \leq \gamma$, then

- $\tau_d = (1-\overline{\pi})(1-\theta)$,
- $\tau_i = (1-\overline{\pi})\theta$,
- $\tau_s = \overline{\pi} = \omega^i$,

Hence, $\Gamma = \frac{\Psi(1-\overline{\pi})}{2}$ and

$$\begin{aligned} &EC\left(L_0 = 2\mu - 1, (1-\overline{\pi})(R+\gamma) \leq \gamma\right) \\ &= (2\mu-1)\kappa + (R+\gamma)\theta(1-\overline{\pi})^2 + \frac{1}{2}(1-\overline{\pi})^2(1-\mu)\Psi. \end{aligned} \tag{7.3}$$

If, instead, $(1-\omega^i)(R+\gamma) > \gamma$, then

- $\tau_d = (1-\overline{\pi})$,
- $\tau_i = 0$,
- $\tau_s = \overline{\pi} = \omega^i$,

Γ is the same and

$$\begin{aligned} &EC\left(L_0 = 2\mu - 1, (1-\overline{\pi})(R+\gamma) > \gamma\right) \\ &= (2\mu-1)\kappa + \gamma\theta(1-\overline{\pi}) + \frac{1}{2}(1-\overline{\pi})^2(1-\mu)\Psi. \end{aligned} \tag{7.4}$$

Finally, if $L_0 = 1-\mu$, then $\omega^i = 0$ and the only relevant case is $R+\gamma > \gamma$.

- $\tau_d = 1$,
- $\tau_i = 0$,
- $\tau_s = 0 = \omega^i$,

$\Gamma = \frac{\Psi}{2}$, so the expected cost is

$$EC(L_0 = 1-\mu, R > 0) = (1-\mu)\kappa + \gamma\theta + \frac{1}{2}(2\mu-1)\Psi. \tag{7.5}$$

The expected costs described in equations 7.1, 7.2, 7.3, 7.4, and 7.5 each consist of three terms: the first term expresses the cost of initial collateral, the second term is the expected cost of delay and, potentially, the resubmission cost R, the third term describes the expected cost of having to obtain additional collateral at the end of the day.

We need to consider three different parameter regions. If $\overline{\pi}(R+\gamma) > \gamma$, then banks choose the initial level of collateral by comparing the expected cost in equations 7.2, 7.4, and 7.5. If $\overline{\pi}(R+\gamma) \geq \gamma > (1-\overline{\pi})(R+\gamma)$, then banks choose the initial level of collateral by comparing the expected costs in equations 7.1, 7.4, and 7.5. Finally, if $\gamma \geq (1-\overline{\pi})(R+\gamma)$, then banks choose the initial level of collateral by comparing the expected costs in equations 7.1, 7.3, and 7.5. The equilibrium choice of initial collateral is the one that minimizes expected cost in a given region of the parameter space.

The presence of an extra term representing the cost of having to obtain additional collateral at the end of the day gives banks incentive to choose a higher level of L_0. Otherwise, the effects of parameters on the choice of L_0 are similar to the ones described in the previous section and we do not expand on these further.

7.2.2 Short payment cycles in LSM

When $n = 2$, we find $\omega^q = \tau_s + \tau_q$. Since all banks either queue or pay early, as we saw from Table 2, the equilibrium probability is $\omega^q = 1$. Therefore, banks with sufficient collateral pay early and those with insufficient collateral queue.

As in the case of a long payment cycle the equilibrium collateral level is $L_0^* = 1-\mu$.

7.3 Social planner solution in case of short cycles

We now turn to the case with short cycles. We first prove the following lemma.

Lemma 15. *If a positive mass of banks has insufficient collateral, then the planner always chooses to delay the non-time-critical payments of such banks.*

Proof. First, we show that submitting these payments has no benefit. Consider a bank, called bank A, with insufficient collateral. If bank A's counterparty has sufficient collateral, then its payment will settle regardless of what bank A does. If bank A's counterparty has insufficient collateral, then its payment cannot settle, regardless of that bank A does. Indeed, even if bank A submits its payment, the payment will not be released as neither bank has sufficient collateral. Next, we observe that submitting bank A's payment has a positive expected cost, since there is a positive probability the payment will not settle. □

Now we consider the planner's actions for different values of $L_0 \in \{1-\mu, 2\mu-1, \mu, 1\}$.

If the planner chooses $L_0 = 1$, then all banks have sufficient collateral and the expected cost is κ.

If the planner chooses $L_0 = \mu$, then a fraction $\overline{\pi}$ of banks has insufficient collateral and $\Gamma = \overline{\pi}\Psi/2$. If the planner chooses to make banks with a negative liquidity shock submit their payments, then the expected cost is

$$EC\,(L_0 = \mu, \text{neg. shock submit}) = \mu\kappa + \overline{\pi}^3(1-\mu)\Psi/2 + \overline{\pi}^2\theta(R+\gamma). \tag{7.6}$$

If, instead, banks with a negative liquidity shock delay their time-critical payments, then the expected cost is

$$EC\,(L_0 = \mu, \text{neg. shock delay}) = \mu\kappa + \overline{\pi}^3(1-\mu)\Psi/2 + \overline{\pi}\theta\gamma. \tag{7.7}$$

Note that $EC\,(L_0 = \mu, \text{neg. shock submit}) > EC\,(L_0 = \mu, \text{neg. shock delay})$ if and only if $\overline{\pi}(R+\gamma) > \gamma$. This is the same condition as is faced by banks in equilibrium. This is not surprising since we saw that with short cycles there are no strategic interactions between banks with insufficient collateral.

If the planner chooses $L_0 = 2\mu - 1$, then a fraction $1 - \overline{\pi}$ of banks has insufficient collateral and $\Gamma = (1-\overline{\pi})\Psi/2$. The planner can choose to make all banks, only banks with positive or no liquidity shocks, or only banks with positive liquidity shocks, submit their time-critical payments in the morning. If all banks submit their time-critical payments early, the expected cost is given by

$$EC\,(L_0 = 2\mu - 1, \text{no delay}) = (2\mu-1)\kappa + (1-\overline{\pi})^2(1-\mu)\Psi/2 + (1-\overline{\pi})^2\theta(R+\gamma). \tag{7.8}$$

If, instead, banks with a positive or no liquidity shock submit their time-critical payments, then the expected cost is

$$\begin{aligned} &EC\,(L_0 = 2\mu - 1, \text{neg. shock delay}) \\ &= (2\mu-1)\kappa + (1-\mu)(1-\overline{\pi})^2\Psi/2 + \overline{\pi}\theta\gamma + (1-2\overline{\pi})(1-\overline{\pi})\theta(R+\gamma). \end{aligned} \tag{7.9}$$

Finally, if only banks with a positive liquidity shock submit their time-critical payments, then the expected cost is

$$EC\,(L_0 = 2\mu - 1, \text{neg. and no shock delay}) = (2\mu-1)\kappa + (1-\mu)(1-\overline{\pi})^2\Psi/2 + (1-\overline{\pi})\theta\gamma. \tag{7.10}$$

Here again, the choice made by the planner between submitting and delaying is the same as the choice made by banks.

If the planner chooses $L_0 = 1 - \mu$, then all banks have insufficient collateral and $\Gamma = \Psi/2$. In this case, the planner chooses to delay all payments and the expected cost is

$$EC(L_0 = 1 - \mu) = (1-\mu)\kappa + \gamma\theta + (2\mu - 1)\Psi/2. \tag{7.11}$$

Depending on parameter values, the planner may choose any of these actions. While the case with short cycles has more cases than the case with a long cycle, the main conclusions are the same. For some parameter values, the planner's and the equilibrium allocations are the same, as is the case when $L_0 = 1 - \mu$ in both cases.

In a payment system with an LSM, the planner chooses $L_0 = 1 - \mu$ and required banks to queue their payment, irrespective of the length of the cycle.

7.4 Proofs

7.4.1 Proof of Proposition 6

If $L_0 = \mu$ and $(1-\omega^i)(R+\gamma) \leq \gamma$, then based on Table 1

- $\tau_d = \overline{\pi}(1-\theta)$,
- $\tau_i = \overline{\pi}\theta$,
- $\tau_s = 1-\overline{\pi}$.

Using these values for τ_i and τ_s, we find

$$\omega^i = \frac{1-\overline{\pi}}{1-\overline{\pi}\theta} \geq 1-\overline{\pi}. \tag{7.12}$$

Hence, if $L_0 = \mu$ and $(1-\frac{1-\overline{\pi}}{1-\overline{\pi}\theta})(R+\gamma) \leq \gamma$, the expected cost is

$$EC\left(L_0 = \mu, (1-\frac{1-\overline{\pi}}{1-\overline{\pi}\theta})(R+\gamma) \leq \gamma\right) = \mu\kappa + (R+\gamma)\theta(1-\theta)\frac{\overline{\pi}^2}{1-\theta\overline{\pi}}. \tag{7.13}$$

If, instead, $(1-\omega^i)(R+\gamma) > \gamma$, then

- $\tau_d = \overline{\pi}$,
- $\tau_i = 0$,
- $\tau_s = 1-\overline{\pi} = \omega^i$.

Notice that the value of ω^i in this case is different from what it was above, unless $\theta = 0$. Hence, if $L_0 = \mu$ and $\overline{\pi}(R+\gamma) > \gamma$, the expected cost is

$$EC\left(L_0 = \mu, \overline{\pi}(R+\gamma) > \gamma\right) = \mu\kappa + \gamma\theta\overline{\pi}. \tag{7.14}$$

If

$$1-\overline{\pi} \leq \frac{R}{R+\gamma} \leq \frac{1-\overline{\pi}}{1-\overline{\pi}\theta},$$

there are multiple equilibria. Intuitively, if banks with 'insufficient' collateral submit their payments in the morning, rather than delay them, then the probability ω^i of receiving a payment is high: $\omega^i = \frac{1-\overline{\pi}}{1-\overline{\pi}\theta}$ and $(1-\frac{1-\overline{\pi}}{1-\overline{\pi}\theta})(R+\gamma) \leq \gamma$. This provides incentives for such banks to submit their payments early. In contrast, if banks with 'insufficient' collateral delay their payments, then the probability ω^i of receiving a payment is low: $\omega^i = 1-\overline{\pi}$ and $\overline{\pi}(R+\gamma) > \gamma$. This provides incentives for such banks to delay their payments.

7.4.2 Proof of Proposition 7

We use similar steps in the other cases. If $L_0 = 2\mu - 1$ and $(1-\omega^i)(R+\gamma) \leq \gamma$, then

- $\tau_d = (1-\overline{\pi})(1-\theta)$,
- $\tau_i = (1-\overline{\pi})\theta$,
- $\tau_s = \overline{\pi}$.

Hence,

$$\omega^i = \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta} \geq \overline{\pi} \tag{7.15}$$

and

$$\begin{aligned} EC & \left(L_0 = 2\mu - 1, (1 - \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta})(R+\gamma) \leq \gamma \right) \\ & = (2\mu-1)\kappa + (R+\gamma)\frac{(1-\overline{\pi})^2(1-\theta)\theta}{1-(1-\overline{\pi})\theta}. \end{aligned} \tag{7.16}$$

If, instead, $(1-\omega^i)(R+\gamma) > \gamma$, then

- $\tau_d = (1-\overline{\pi})$,
- $\tau_i = 0$,
- $\tau_s = \overline{\pi} = \omega^i$,

and

$$EC\left(L_0 = 2\mu - 1, (1-\overline{\pi})(R+\gamma) > \gamma\right) = (2\mu-1)\kappa + \gamma\theta(1-\overline{\pi}). \tag{7.17}$$

There are multiple equilibria if

$$\overline{\pi} \leq \frac{R}{R+\gamma} \leq \frac{\overline{\pi}}{1-(1-\overline{\pi})\theta}.$$

7.4.3 Proof of Proposition 8

If $L_0 = 1 - \mu$, then $\omega^i = 0$. So, the only relevant case is $R + \gamma > \gamma$ and the expected cost is

$$EC(L_0 = 1-\mu, R > 0) = (1-\mu)\kappa + \gamma\theta. \tag{7.18}$$

7.4.4 Proof of Proposition 9

Note that if condition 3.12 is satisfied, all banks with insufficient liquidity find it optimal to delay payments since in the best possible scenario, when the probability to receive a payment is the highest, it is not optimal to submit a payment with insufficient funds. One possible interpretation of condition 3.12 is that probability to receive liquidity shock is very low or resubmission cost is very high. Thus the expected costs for a particular bank do not depend on the equilibrium behavior of the other banks and are given in Equations 7.14, 7.17, and 7.18 corresponding for each candidate equilibrium level of collateral.

Comparing equations 7.14, 7.17, and 7.18 it can be seen that optimal level of collateral is (i) $L_0 = \mu$ if $(1-\mu)\kappa < \gamma\theta(1-2\overline{\pi})$ and $(2\mu-1)\kappa < \gamma\theta(1-\overline{\pi})$; (ii) $L_0 = 2\mu-1$ if $(1-\mu)\kappa > \gamma\theta(1-2\overline{\pi})$ and $(3\mu-2)\kappa < \overline{\pi}\gamma\theta$; and (iii) $L_0 = 1-\mu$ if $(3\mu-2)\kappa > \overline{\pi}\gamma\theta$ and $(2\mu-1)\kappa > \gamma\theta(1-\overline{\pi})$.

7.5 Examples: any ranking of $W(L_0 = 1-\mu)$, $W(L_0 = 2\mu - 1)$, and $W(L_0 = \mu)$ can occur.

We start by assuming that $\overline{\pi} \to 1/2$. In this case, condition 3.13 is violated, so $W(L_0 = 2\mu - 1) > W(L_0 = \mu)$ always holds. Notice also that $2\mu - 1 \geq 3\mu - 2$, since $1 \geq \mu$. It follows that if

$$\frac{1}{2}\theta\gamma > (2\mu - 1)\kappa > (3\mu - 2)\kappa,$$

then

$$W(L_0 = 2\mu - 1) > W(L_0 = \mu) > W(L_0 = 1 - \mu).$$

If

$$(2\mu - 1)\kappa > \frac{1}{2}\theta\gamma > (3\mu - 2)\kappa,$$

then

$$W(L_0 = 2\mu - 1) > W(L_0 = 1 - \mu) > W(L_0 = \mu).$$

If

$$(3\mu - 2)\kappa > \frac{1}{2}\theta\gamma,$$

then

$$W(L_0 = 1 - \mu) > W(L_0 = 2\mu - 1) > W(L_0 = \mu).$$

Note that the condition $(1 - \frac{1-\overline{\pi}}{1-\theta\overline{\pi}})(R+\gamma) > \gamma$ becomes $R(1-\theta) > \gamma$, so we can choose R large enough so that this condition is verified.

Finally, if κ is sufficiently small, then

$$W(L_0 = \mu) > W(L_0 = 2\mu - 1) > W(L_0 = 1 - \mu).$$

We assume that $\overline{\pi}$ is so small that condition 3.14 is violated. Specifically, we assume that $\overline{\pi}\theta\gamma = (3\mu - 2)\kappa - \varepsilon$, where $\varepsilon > 0$ and is close to zero. So $W(L_0 = 1 - \mu) > W(L_0 = 2\mu - 1)$. Note that this assumption constrains γ not to be too large. We can rewrite conditions 3.13 and 3.15 as

$$\theta\gamma > (5\mu - 3)\kappa - 2\varepsilon \text{ and} \tag{7.19}$$

$$\theta\gamma > (5\mu - 3)\kappa - \varepsilon, \tag{7.20}$$

respectively. Now if we choose

$$\theta\gamma > (5\mu - 3)\kappa - \varepsilon > (5\mu - 3)\kappa - 2\varepsilon,$$

then

$$W(L_0 = \mu) > W(L_0 = 1 - \mu) > W(L_0 = 2\mu - 1).$$

In contrast, if

$$(5\mu - 3)\kappa - \varepsilon > \theta\gamma > (5\mu - 3)\kappa - 2\varepsilon,$$

then

$$W(L_0 = 1 - \mu) > W(L_0 = \mu) > W(L_0 = 2\mu - 1).$$

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