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Repo Runs

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Abstract

The recent financial crisis has shown that short-term collateralized borrowing may be highly unstable in times of stress. The present paper develops a dynamic equilibrium model and shows that this instability can be a consequence of market-wide changes in expectations, but does not have to be. We derive a liquidity constraint and a collateral constraint that determine whether such expectations-driven runs are possible and show that they depend crucially on the microstructure of particular funding markets that we examine in detail. In particular, our model provides insights into the differences between the tri-party repo market and the bilateral repo market, which were both at the heart of the recent financial crisis.

Key words: investment banking, securities dealers, repurchase agreements, tri-party repo, bilateral repo, money market mutual funds, asset-backed commercial paper, runs, financial fragility.

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1 Introduction

The sudden collapse of highly levered financial institutions was a key factor in the financial crisis of 2007-09. We develop a model of short-term collateralized funding and of such collapses as expectation-driven runs, and study when runs can occur in various short-term funding markets. In a run, a leveraged financial institutions may not be able to roll over its short-term borrowing, despite it being collateralized. We derive two conditions, one related to the institution's liquidity, one to its collateral, that characterize when such a run can occur. Both constraints depend crucially on the microstructure of the particular short-term funding market. We model the differences between various repo and other related funding markets and examine the consequences of these differences.

The framework of this paper is general and can be applied to various types of financial institutions that suffered from losses in short term funding during the financial crisis of 2007-09. Such institutions include money market mutual funds (MMMFs), hedge funds, off-balance sheet vehicles including asset-backed commercial paper (ABCP) conduits, and structured investment vehicles (SIVs). The primary application of our model is to large securities dealers who use the tri-party repo market as a main source of financing. In that market dealers borrow from institutional investors, such as MMMFs, against collateral that is held by a third-party clearing bank. Dealers' borrowing in the tri-party repo market reached over \$2.8 trillion outstanding in aggregate at its peak in 2008; individual dealer borrowing reached \$400 billion, most of which with overnight maturity.

Our model is motivated by the observation that the collapses of Bear Sterns and Lehman Brothers were triggered by a precipitous decrease in funding from the tri-party repo market. As noted by Bernanke (2009), these sudden stops were surprising because tri-party repo borrowing is collateralized by securities. The Task Force on Tri-Party Repo Infrastructure (2009), a private sector body sponsored by the Federal Reserve Bank of New York, noted that "tri-party repo arrangements were at the center of the liquidity pressures faced by securities firms at the height of the financial crisis."[2] As

[2]See http://www.newyorkfed.org/tripartyrepo/.

a response, the creation of the primary dealer credit facility (PDCF) was an attempt to provide a backstop for the tri-party repo market.

Given the importance of repo markets for some key events in the crisis, we compare the organization of the tri-party repo market, which is the primary repo market for borrowing by dealers, with the bilateral repo market, which is the primary repo market for lending by dealers. Comparing tri-party repos and bilateral repos is particularly interesting because the two markets behaved very differently in the crisis. As documented by Gorton and Metrick (2011), haircuts in bilateral repos increased dramatically during the crisis, consistent with the margin spirals described in Brunnermeier and Pedersen (2009). In contrast, Copeland, Martin, and Walker (2010) show that haircuts in the tri-party repo market barely moved and document large differences in haircuts between the two markets for comparable asset classes. Our model clarifies the distinction between increasing margins, which is a potentially equilibrating phenomenon, and runs, which can happen if margins do not increase sufficiently to provide protection to investors. Furthermore, our analysis shows that a particular institutional feature of the tri-party repo market, the early settlement of repos by clearing banks called the "unwind", can have a destabilizing effect on the market. This finding lends theoretical support to the recent reform proposals by the Tri-Party Repo Infrastructure Reform Task Force to eliminate the unwind procedure. A general lesson of our analysis, therefore, is that the market microstructure of the shadow banking system plays a critical role for the system's fragility.

Krishnamurthy, Nagel and Orlov (2011) document that prior to the crisis, the two main providers of funds to the shadow banking system, MMMFs and securities lenders, invested heavily in ABCP and the corresponding conduits. As shown by Covitz, Liang, and Suarez (2009), ABCP has very short maturities that shortened even further during the crisis. ABCP conduits therefore are an important case in point for our theory. And indeed, Covitz, Liang, and Suarez (2009) argue that the precipitous drop in outstanding ABCP of roughly $190 billion in August 2007 had many characteristics of a traditional bank run.

Our theory of fragility differs from the classic literature on commercial bank runs in several ways. First, we model collateral and the incentives that it provides in a collective refusal to continue lending. Second, we do not model bank contracts as insurance arrangements for risk-averse investors and place no constraints on investor preferences. And third, we distinguish between collateral and liquidity concerns by endogenizing banks' liquidity. In our model, borrowers have the choice between funding securities with their own cash or with short-term debt. We derive a dynamic participation constraint under which borrowers will prefer to fund their operations with short-term debt and show that this condition implies that borrowers make positive profits in equilibrium. These profits can be used to forestall a run and thus serve as a systemic buffer.

Our model is a simple dynamic rational expectations model with multiple equilibria. However, unlike in conventional models of multiple equilibria, not "everything goes" in our model. The theory pins down under what conditions individual institutions are subject to potential self-fulfilling runs, and when they are immune to such expectations. The intermediaries in our model are heterogenous, and the liquidity and collateral constraints are specific to each institution. The equilibrium is therefore consistent with observations of some institutions failing and others surviving in case of changing market expectations. In particular, our theory is consistent with the observation by Krishnamurthy, Nagel, and Orlov (2011) that "the effects of the run on repo seem most important for a select few dealer banks who were heavy funders of private collateral in the repo market" (p.6). While our theory focuses on multiple equilibria, the history of the 2007-09 crisis clearly also has a fundamental component. Our choice of model is motivated, on the one hand, by the wish to simplify the exposition and, on the other hand, by the belief that illiquidity was an important issue at some key turning points during the crisis.

The remainder of the paper proceeds as follows. Section 2 describes our model. Section 3 characterizes its steady states. In particular, we derive the borrowers' dynamic participation constraint in this section and show that equilibrium profits are positive. Section 4 studies the dealers' ability to withstand runs in terms of liquidity. Section 5 considers the fragility of different market microstructures and derives collateral constraints. Section 6 concludes.

2 The Model

The economy is populated by two classes of agents, "investors" and "borrowers". We keep the terminology deliberately general in order to be able to consider different markets later on. The time horizon is infinite and there is no initial date. Investors live for three dates, and at each date t a continuum of mass N of them is born. On the first date of their lives, "young" investors receive an endowment of 1 unit of goods that they can invest at date t. Investors have no endowment in the following two periods when they are "middle-aged" and "old".

Investors' want to consume either in middle or in old age, but do not know when at the time they are born. The timing of the investors' needs of cash is uncertain because of "liquidity" shocks. Investors learn their type in middle age: "Impatient" investors need cash immediately, while "patient" investors do not need cash until their old age. The information about the investors' type and age is private, i.e. cannot be observed by the market. Ex ante, the probability of being impatient is α. We assume that the fraction of impatient agents in each generation is also α (the Law of Large Numbers).

In practice, investors such as MMMFs may learn about longer term investment opportunities and wish to redeploy their cash or they may need to generate cash to satisfy sudden outflows from their own investors. We do not model explicitly what investors do with their cash in the event of a liquidity shock and, for the remainder of the paper, simply assume that they value it sufficiently highly to want to use it at the given point in time. As in the seminal paper by Diamond and Dybvig (1983), their utility from getting payments (r_1, r_2) over the two-period horizon then is given as

$$U(r_1, r_2) = \begin{cases} u_1(r_1) & \text{with prob. } \alpha \\ u_2(r_2) & \text{with prob. } 1 - \alpha \end{cases} \tag{1}$$

with u_1 and u_2 strictly increasing.[3]

The economy is also populated by M infinitely-lived risk-neutral agents, called "borrowers", who have profitable investment opportunities, but no

[3] We do not assume the traditional consumption-smoothing motive of the Diamond-Dybvig literature (concave u_t), which would make little sense in our context.

endowments. Investors and borrowers have access to a one-period storage technology, which can be thought of as cash and returns 1 for each unit invested. In addition, borrowers have access to an investment technology, which we think of as investment in, and possibly the creation of, securities. These investments are illiquid in the sense that they cannot be liquidated instantaneously. They yield constant returns to scale up to a capacity constraint, which we interpret as the firm's size and which is exogenous. Specifically, investing I^t units at date t yields

$$\begin{cases} R_i I^t & \text{if } I^t \leq \overline{I_i} \\ R_i \overline{I_i} & \text{if } I^t \geq \overline{I_i} \end{cases} \tag{2}$$

with $R_i > 1$ at date $t+2$ and yields nothing at date $t+1$. To simplify things, we assume that the return on these investments is riskless. Yet, the returns are not verifiable, which creates a role for collateral in our model. Indeed, investors cannot be sure that a borrower has indeed realized $R_i I^t$ from his past investment. Although this is a probability zero event, a borrower who has received funds from investors could claim that he is unable repay.

Investment returns can only be realized by the borrower who has invested in the asset, because borrowers have a comparative advantage in managing their security portfolio. Other market participants only realize a smaller return. Investors could realize a return of $\gamma_i^t R_i$ from these assets, with $\gamma_i^t < 1$, where the discount reflects different skills in valuing or managing the assets, possible restrictions on the outsider's portfolio composition, transactions and timing costs, and similar asymmetries.[4] We allow γ_i^t to depend on the borrower, reflecting potential differences in the portfolio of collateral that different borrowers seek to finance, and on time, reflecting changing market environments.

Borrowers use the endowment of young investors to invest in securities. The total investment capacity $\overline{I} = \sum \overline{I_i}$ strictly exceeds the investors' amount of cash available for investment, N, so borrowers must compete for investors'

[4]For T-bills, γ_i should be very close to 1. But many market participants also finance large volumes of less liquid securities. Simplifying somewhat, the main categories of collateral in repo markets are (i) US treasuries and strips, (ii) Agency debentures, (iii) Agency ABS/MBS, (iv) Non-Agency ABS/MBS, (v) corporate bonds. We could have different γ_i for each class of collateral without changing the analysis.

funds. As usual, all quantities are expressed per unit mass of investors. The alternative assumption that investors compete for the right to lend to borrowers is neither realistic not analytically interesting, because then borrowers could extract all the surplus from investors, by simply offering to pay the storage return of 1 each period, and there would be no instabilities or runs. Instead of the condition $\overline{I} > N$, we assume the slightly stronger condition

$$\sum_{j \neq i} \overline{I_j} > N \tag{3}$$

for all i. Hence no borrower is pivotal, and even if one borrower fails, there will still be competition for investor funds.

If borrower i in period t invests I_i^t, holds c_i^t in cash, receives b_i^t from young investors, repays r_{1i}^t after one period or r_{2i}^t after two periods, impatient investors do not roll over their funding when middle-aged, but patient investors do, then the borrower's expected cash flow, which we also refer to as profits, is

$$\pi_i^\tau = R_i I_i^{\tau-2} + c_i^{\tau-1} + b_i^\tau - \alpha r_{1i}^{\tau-1} b_i^{\tau-1} - (1-\alpha) r_{2i}^{\tau-2} b_i^{\tau-2} - I_i^\tau - c_i^\tau. \tag{4}$$

At each date, borrowers consume their profits (or pay them out as dividends). The borrower's objective at each date t then is to maximize the sum of discounted expected cash flows $\sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi_i^\tau$, where $\beta < 1$. In order to make the problem interesting, we assume that borrowers are sufficiently patient and their long-term investment is sufficiently profitable:

$$\beta^2 R_i > 1. \tag{5}$$

Given the corner preferences of investors in (1), there is no scope for rescheduling the financing from impatient middle-aged or old investors. Hence, if $\pi_i^t < 0$ at any date t, the borrower is bankrupt. Of course, his assets may be taken over by other borrowers to realize the borrower's going concern value, as in the case of Bear Sterns, but from the point of view of the individual borrower the relevant bankruptcy condition is $\pi_i^t < 0$.

3 Steady-states

In steady-state, young investors fund borrowers and withdraw their funds precisely at the time of their liquidity shocks. If either of these two properties did not hold, the free capacity of investors in (3) would permit a profitable deviation. We assume that the Law of Large Numbers also holds at the borrower level: each period the realized fraction of impatient investors at each borrower is α. Hence, in every period, each borrower obtains funds from young investors, and repays a fraction α of middle-aged investors and all remaining old investors. Thus there is no uncertainty about borrowers' profits, and each borrower's realized profit is equal to his expected profit (4).

Each period, borrowers compete for investors' funds. Since borrowers have a fixed investment capacity, they cannot make unconditional interest rate offers, but must condition their offers on the amount of funds they receive. The simplest market interaction with this feature is as follows. At each date $t \in (-\infty, \infty)$:

1. borrowers offer contracts $(r_{1i}^t, r_{2i}^t, Q_i^t, k_i^t) \in \mathbb{R}_+^4$, $i = 1, ..., M$.

2. New and patient middle-aged investors decide whether to finance the borrower.

3. If the borrower is unable to repay all investors who demand repayment, he must declare bankruptcy. Otherwise, the borrower invests I_i^t and continues.

Here, $r_{\tau i}^t$ is the (gross) interest payment offered by borrower i on τ-period funding, Q_i^t the maximum amount for which this offer is valid, and k_i^t is the amount of collateral posted per unit borrowed. Total new borrowing by the M borrowers then is $(b_1^t, ..., b_M^t) \in \mathbb{R}_+^M$, with $b_i^t \leq Q_i^t$ for $i = 1, ..., M$ and $\sum b_i^t \leq N$. Since investment returns are non-verifiable, the collateral posted must be sufficient to incentivize borrowers to repay, i.e. to honor the repurchase leg of the repo transaction. Since borrowers face investors of different age, the collateral constraints must be age-specific. At the time of the contract offer to middle-aged investors, the borrower needs r_{1i}^t in cash and offers that the investor keeps the collateral that he has received in the previous period and that matures one period later. Hence, at that time the borrower will prefer to repay instead of keeping his cash if

$$R_i k_i^t r_{1i}^t \geq r_{2i}^t \tag{6}$$

In order to obtain cash from young investors, the borrower offers to put up the assets he creates with these funds as collateral. One period later, he will want to repay instead of giving up the assets if

$$\beta R_i k_i^t \geq r_{1i}^t \tag{7}$$

We will abstract from more complicated considerations of default and ex post bargaining, and simply assume that collateral must satisfy the two repayment constraints (6) and (7).[5] A steady state equilibrium is a collection of $(r_{1i}, r_{2i}, k_i, b_i, I_i, c_i)$ for each borrower i, where b_i is new funding, k_i collateral, c_i cash holding, and $I_i \leq \overline{I_i}$ investment per borrower, such that no borrower and investor would prefer another funding and investment policy, given the behavior of all others.[6]

We now characterize the steady states in which borrowers invest by a sequence of simple observations.

Lemma 1 *For each borrower i with $b_i > 0$, $r_{2i} = r_{1i}^2$.*

Proof. Clearly, $r_{2i} \geq r_{1i}^2$, because otherwise investors would strictly prefer to never roll over their funding, regardless of their type. Patient middle-aged investors would withdraw their funds and then invest again together with young investors. Suppose that this inequality is strict. In this case, an

[5]See, e.g., Hart and Moore (1998) or von Thadden, Berglöf and Roland (2010) for more complex models of default and renegotiation. We also abstract from reputational or other dynamic concerns, which would trade off the possible loss of future access to investor funds against current cash gains. Note that (6) and (7) are consistent with observed practice in the repo market in the sense that in the (rare) cases in which repos are not repaid investors usually choose to extend them for another night.

[6]For simplicity, we can ignore the bound Q_i in the description of the steady state, where it can be thought of as being set to $Q_i = b_i$. The bound plays no substantive role in steady state, but is important for runs in later sections.

impatient middle-aged investor will optimally extend her funding and at the same time borrow and consume the amount $r_{1i}+\varepsilon$ on the market at interest rate $r_{1i}-1$. He can then claim back r_{2i} from the borrower one period later and repay his one-period loan $(r_{1i}+\varepsilon)r_{1i}$ which is feasible and profitable if $\varepsilon > 0$ is sufficiently small. ■

The proof is based on a simple no-arbitrage argument. It is different from the classical argument by Jacklin (1987) in the context of the Diamond-Dybvig (1983) model, because investors in our context do not have access to the long-term investment technology. It is also different from the argument by Qi (1994), who assumes and uses strict concavity of the investors' utility. In our market context, the no-arbitrage argument is natural and sufficient. Note that although Lemma 1 forces the yield curve to be flat, borrowers still provide maturity transformation if $r_{1i} > 1$.

Lemma 2 $r_{1i} = r_{1j}$ *for all borrowers* i, j *with* $b_i, b_j > 0$.

Proof. Suppose that $r_{1i} < r_{1j}$ for some i, j with $b_i, b_j > 0$. Let $\mathcal{J}_i$ be the set of all borrowers k with $r_{1k} > r_{1i}$ and $b_k > 0$. $\mathcal{J}_i$ is not empty because $j \in \mathcal{J}_i$. All $k \in \mathcal{J}_i$ must be saturated, i.e. have $b_k = Q_k$ (otherwise investors from i would deviate). Hence, any borrower $k \in \mathcal{J}_i$ can deviate to $r_{1k} - \varepsilon$ for $0 < \varepsilon < r_{1k} - r_{1i}$ and strictly increase his profit. ■

By Lemma 2 the Law of One Price holds, and we can denote the single one-period interest rate quoted by all active borrowers by $r = r_1$. Then the steady-state budget identity of borrower i is

$$R_i I_i + b_i = I_i + \alpha r b_i + (1-\alpha) r^2 b_i + \pi_i \tag{8}$$

where the left-hand side are the total inflows per period and the right-hand side total outflows.

Lemma 3 *In steady-state borrowers do not hold cash:* $c_i = 0$ *for all* i.

Proof. Since $\beta < 1$, and $c_i > 0$ does not affect the borrower's budget constraint (8), each borrower does strictly better by consuming c_i. ■

Lemma 4 *If $r > 1$, total steady-state funding by investors is maximal: $\sum_{i=1}^{M} b_i = N$.*

Proof. The total supply of funds is inelastically equal to N in each period if $r > 1$. The scarcity constraint (3) implies that there is a borrower who invests less than full capacity, $I_i < \overline{I_i}$. Suppose that $\sum_{i=1}^{M} b_i < N$. If i makes strictly positive profits, he strictly increases his profits by setting $Q_i = \overline{I_i}$ and thus attracting more funds. If i makes zero profits, he can make strictly positive profits by reducing his interest rate marginally, setting $Q_i = \overline{I_i}$, and attracting the previously idle supply of funds. ■

Lemma 5 *If $\pi_i > 0$, steady-state investment of borrower i is maximal: $I_i = \overline{I_i}$.*

Proof. Suppose the lemma is wrong. The borrower can then increase investment slightly at any date t by using his own cash. By condition (5), this yields a strict increase in discounted profits. ■

Lemma 6 *If there exists a borrower i with $\pi_i > 0$ and $b_i > 0$ then steady-state interest rate satisfies*

$$(1-\alpha)\beta^2 r^2 + \alpha\beta r = 1 \tag{9}$$

Proof. For each unit of cash that borrower i receives and invests at date t, he pays back αr in $t+1$, generates returns R_i in $t+2$, and pays back $(1-\alpha)r^2$ in $t+2$. Hence, his expected discounted profits on this one unit are $\beta^2(R_i - (1-\alpha)r^2) - \beta\alpha r$. Alternatively he could invest his own cash. The discounted profits from not using the one unit of outside funds and rather investing his own money is $\beta^2 R_i - 1$. If the borrower receives funds from investors in steady state ($b_i > 0$) and has funds of his own ($\pi_i > 0$), this cannot be strictly better, which implies $(1-\alpha)\beta^2 r^2 + \alpha\beta r \leq 1$.

Suppose that this inequality is strict. For an arbitrary borrower j, this means that

$$\beta^2(R_j - (1-\alpha)r^2) - \beta\alpha r > \beta^2 R_j - 1 \tag{10}$$

which is strictly positive by (5). Hence, all borrowers strictly prefer $b_i = \overline{I_i}$. This contradicts (3), because the demand for funds would exceed supply. ■

Lemma 6 is surprisingly strong: the existence of one active borrower with strictly positive profits pins down the equilibrium interest rate. We call condition (9) the borrowers' "dynamic participation constraint." Basic algebra shows that its solution is

$$\overline{r} \equiv 1/\beta > 1.$$

This makes sense: at the margin, borrowers discount profits with the market interest rate. But it is interesting to note that $\overline{r}$ does not depend on other supply and demand characteristics such as R_i and α. In steady-state, the cost of funds, $\overline{r} - 1$, is determined exclusively by the borrowers' discount factor. This makes them indifferent at the margin between attracting more cash from investors, which increases current borrower consumption, or attracting less and using their own cash to finance investments, which increases future borrower consumption. Consequently, the dynamic participation constraint implies that the marginal profit from outside funds is strictly positive if the profitability of investment is sufficiently high to make an interest rate of $r = \overline{r}$ feasible. In this case, since the profits from outside funds and from investing own funds must be equal by (9), borrowers make positive profits in equilibrium.

Proposition 1 *If*

$$\frac{(1+\beta)\beta^3}{1-\alpha+\beta} \sum_i R_i \overline{I_i} \geq N \tag{11}$$

then steady states exist in which

- *investors roll over their loans according to their liquidity needs,*

- *all borrowers make strictly positive profits,*
- $I_i = \overline{I_i}$, $c_i = 0$, *and* $r = \overline{r} = 1/\beta$,
- *borrowing satisfies* $\sum_i b_i = N$,

$$b_i \leq \frac{(1+\beta)\beta^3 R_i}{1-\alpha+\beta}\overline{I_i}, \tag{12}$$

and is otherwise indeterminate,

- *collateral* k_i *satisfies*

$$\frac{1}{\beta^2 R_i} \leq k_i \leq \frac{(1+\beta)\beta\overline{I_i}}{(1-\alpha+\beta)b_i} \tag{13}$$

and is otherwise indeterminate.

Proof. First, it is easy see that there is no steady state equilibrium without outside funding ($b_1 = ...b_M = 0$).

Next, suppose there is an equilibrium in which at least one active borrower makes $\pi_i > 0$. Lemma 6 implies that $r = \overline{r}$. Simple algebra shows that (12) and (5) then imply

$$b_i < \frac{(R_i - 1)\overline{I_i}}{(1-\alpha)\overline{r}^2 + \alpha\overline{r} - 1}$$

This implies that borrower equilibrium profits are strictly positive for all i. Hence, $I_i = \overline{I_i}$ for all i by Lemma 5. At the interest rate $\overline{r}$, every borrower i is indifferent at any date t between using outside funds and using his own cash π_i for investment, and thus finds it indeed optimal to borrow any positive amount b_i satisfying (12). Because $\overline{r} > 1$ and all borrowers pay the same interest rate, patient middle-aged investors find it indeed optimal to roll over their funding and young investors find it optimal to invest all their endowment. By (11), there exist borrowing levels $(b_1, ..., b_M)$ satisfying (12) such that $\sum_i b_i = N$. This establishes the existence of equilibria with the first four features of Proposition 1.

The repayment condition (7) is equivalent to the first inequality in (13) and implies (6). For the second inequality in (13), note that in steady state the borrower has two types of securities to offer as collateral, those maturing at $t+1$ or maturing at $t+2$. Because $r = 1/\beta$, both borrowers and investors value both types of securities identically. Hence, the maximum amount of collateral a borrower can pledge in steady state is $\bar{I}_i(1+\beta)$, in terms of securities maturing at $t+1$. The total amount of funds provided by investors per period is $b_i\left[1+(1-\alpha)\bar{r}\right] = b_i\left[1-\alpha+\beta\right]/\beta$. It follows that the maximum amount of collateral per unit that the borrower can offer is

$$\kappa_i \equiv \frac{\beta \bar{I}_i(1+\beta)}{b_i\left[1-\alpha+\beta\right]}. \tag{14}$$

The second inequality in (13) is the condition $k_i \leq \kappa_i$. Both inequalities in (13) are compatible because of (12) ■

The steady states identified in Proposition 1 will serve as a benchmark for the rest of the analysis. An important and novel feature of these equilibria is that condition (9) prevents competition from driving up interest rates to levels at which borrowers make zero profits. Zero-profit equilibria can exist under some parameter conditions,[7] but they are clearly uninteresting to describe investment banking activity. The reason why borrower profits are positive in the equilibria we consider is intuitive (but not trivial): borrowers must have an incentive to use their investment opportunities on behalf of investors instead of using internal funds to reap those profits for themselves. This rationale of positive intermediation profits is different from the traditional banking argument of positive franchise values (e.g., Bhattacharya, Boot, and Thakor (1998), or Hellmann, Murdock and Stiglitz, (2000)), as it explicitly recognizes the difference between internal and external funds. Hence, the coexistence of internal and external funds and the internalization of all cash flows arising from them implies that financial intermediaries make positive profits.[8]

The steady states of Proposition 1 all feature maximum investment and the same interest rate $\bar{r}$, but borrowers can differ in their reliance on outside funds b_i and the collateral k_i they post. In fact, in steady state the exact

[7] A complete characterization of steady states is available upon request from the authors.

[8] This is different from Acharya, Myers, and Rajan (2011) where overlapping generations of bankers try to pass on the externality of debt.

amount of collateral, subject to constraint (13), plays no role because investors never consume it. It is important nevertheless, because it makes sure that each period the cash changes hands as specified.

In steady state, the funding level b_i is only limited by the requirement that the borrower has sufficiently profitable investment opportunities (12). This by itself implies that the borrower's steady state asset base is sufficient to collateralize his funding. It is important to realize that in steady state borrowers have no incentive to change their exposure, but that they may prefer other steady states. Hence, Proposition 1 is consistent with the notion that borrowers can be "trapped" in an equilibrium with high short-term funding and low profits. In fact, borrower profits are strictly decreasing in b_i. Therefore, to the extent that period profits act as a buffer against adverse shocks, as we show in the following sections, borrowers with larger exposure to short-term funding will be more fragile.

4 Runs without asset sales

In this section, we study the stability of borrowers in the face of possible runs. We analyze this problem under the assumption that behavior until date t is as in Proposition 1 and ask whether a given borrower can withstand the collective refusal of all middle-aged investors to extend their funding and of young investors to provide fresh funds.[9] In the next section we will describe the specific microstructure of the tri-party repo market and other institutions that can make such collective behavior of investors optimal and thus imply that the corresponding individual expectations are self-fulfilling.

The key question is how much cash the borrower can mobilize to meet the repayment demands by middle-aged investors in such a situation. At the beginning of the period, a borrower, on the asset side of his balance sheet, holds $R_i\overline{I_i}$ units of cash from investments at date $t-2$, as well as securities that will yield $R_i\overline{I_i}$ units of cash at date $t+1$. The borrower holds investor

[9] Note that in our infinite-horizon model, there are two sources of instability: middle-aged investors may not roll over their funding and new investors may not provide fresh funds. The former corresponds to the classical Diamond-Dybvig problem, the latter arises only in fully dynamic models.

claims for dates t and $t+1$ on the liability side of his balance sheet. In this section, we assume that the borrower cannot sell his assets.

The borrower's repayment obligations in case of a run are $(\overline{r}+(1-\alpha)\overline{r}^2)b_i$. If there is no fresh funding in the run and new investment is maintained at the steady-state level $\overline{I_i}$, the run demand can be satisfied by the individual borrower if

$$(R_i - 1)\overline{I_i} \geq (\overline{r} + (1-\alpha)\overline{r}^2)b_i. \tag{15}$$

If (15) holds, a run would have no consequence whatsoever and all out-of-equilibrium investor demand would be buffered by the borrower's profits. Anticipating this, investors have no reason to run. But more is possible. In the event of a run at date t, the cash position of the individual borrower who satisfies the run demand is

$$I_0 = R_i\overline{I_i} - (\overline{r} + (1-\alpha)\overline{r}^2)b_i. \tag{16}$$

Clearly, if $I_0 < 0$ the borrower does not have the liquidity to stave off the run and is bankrupt. If $I_0 \geq 0$, but (15) does not hold, the borrower must adjust his funding or investment in order to survive the run. Since after a run in $t+1$ the borrower will have $R_i\overline{I_i}$ in cash and nothing to repay, he can resume his operations by investing $\overline{I_i}$ at date $t+1$ and save and invest thereafter. Whether he can attract fresh funds after t depends on the market, but this is immaterial for his survival.

The liquidity constraint, (17) in the following proposition, is obtained by simply writing out the condition $I_0 \geq 0$ from (16).

Proposition 2 *In steady state, a run on borrower i who cannot sell her assets can be accommodated if and only if the borrower's liquidity constraint holds, i.e. if*

$$\beta^2 R_i\overline{I_i} \geq (1-\alpha+\beta)b_i. \tag{17}$$

Condition (17) is independent of the funding restriction $b_i \leq \overline{I_i}$ of Proposition 1, in the sense that (17) can hold or fail in steady state, depending on the parameters. Hence, a borrower who makes positive profits in steady state may still fail in a run. The comparative statics of the liquidity constraint are simple and we collect them in the following proposition.

Proposition 3 *The liquidity constraint (17) is the tighter,*

- *the higher is the borrower's short-term exposure* $b_i/\overline{I}_i$,
- *the smaller is the borrower's size* $\overline{I}_i$,
- *the lower is the borrower's profitability* R_i.

Proposition 3 shows that if borrowers have sufficient access to profitable investment ($\overline{I}_i$ large), if these investment opportunities are sufficiently profitable (R_i large), or if they have sufficiently low exposure to short-term outside funding ($b_i/\overline{I}_i$ small), then borrowers are more likely to be able to stave off runs individually, only by reducing their investment temporarily. In this case, unexpected runs cannot bring down borrowers out of equilibrium. If condition (17) is violated, a run bankrupts the individual borrower if it occurs.

It is interesting to note that the pecking order found in Proposition 3 corresponds closely to the classification of the, then, 5 big investment banks in 2007. The two weakest of them according to the criteria of the proposition went bankrupt in 2008.

5 Fragility

This section examines different microstructures that are associated with repo markets or other money markets. We ask whether runs can occur in each of the institutional environments considered. The focus is on the tri-party repo market, but we also examine bilateral repos, MMMFs, ABS-backed conduits, and traditional bank deposits. We derive a collateral constraint for each market and show that if and only if the liquidity constraint and the collateral constraint are violated, then a run can occur for the particular market structure.

We study unanticipated runs that arise from pure coordination failures. As noted in the previous section, in a run at date t all investors believe that i) no middle-aged investors renew their funding to borrower i, so the borrower must pay $[\overline{r} + (1-\alpha)\overline{r}^2]\, b_i$ to middle-aged and old investors, and ii) no new young investors lend to the borrower. The question is whether such beliefs can be self-fulfilling in a collective deviation from the steady state.

Since the Law of One Price holds in steady state by Lemma 2, a trivial coordination failure may induce all investors of a given borrower to switch to another borrower out of indifference. This looks like a "run", but is completely arbitrary. We will therefore assume that investors if indifferent lend to the borrower they are financing in steady state. Hence, in order for a collective deviation from the steady state to occur we impose the stronger requirement that the individual incentives to do so must be strict.

The first insight, which applies to all institutional environments considered in this section, is simple but useful to state explicitly: a run cannot occur if a borrower is liquid in the sense of Proposition 2.

Lemma 7 *If a borrower satisfies the liquidity constraint (17), there are no strict incentives to run on this borrower.*

The proof is simple. In a run on this borrower, all middle-aged patient investors would be repaid in full regardless of what young investors do and without affecting the borrower's asset position. Hence, patient middle-aged and young investors are indifferent between lending to the borrower or to another one. By our assumption about the resolution of indifference, there is thus no reason to run in the first place. Intuitively, patient middle-aged investors would just "check on their money" before it is re-invested. Since the borrower has the money, such a check does not cause any real disruption, and the borrower may as well keep it until he invests in new securities.

5.1 The US tri-party repo market

This section briefly reviews the microstructure of the tri-party repo market and the key role played by the clearing bank.[10] In particular, we show that a practice called the "unwind" of repos increases fragility in this market.

The clearing banks play many roles in the tri-party repo market. They take custody of collateral, so that a cash investor can have access to the collateral in case of a borrower default, they value the securities that serve as collateral, they make sure the specified margin is applied, they settle the repos on their books, and importantly, they provide intraday credit to borrowers.[11]

In the US tri-party repo market, new repos are organized each morning, between 8 and 10 AM. These repos are then settled in the afternoon, around 5 PM, on the books of the clearing banks. For operational simplicity, because borrowers need access to their securities during the day to conduct their business, and because some cash investors want their funds early in the day, the clearing banks "unwind" all repos in the morning. Specifically, the clearing banks send the cash from the borrowers' to the investors' account and the securities from the investors' to the borrowers' account. They also finance the borrowers' securities during the day, extending large amounts of intraday credit. At the time when repos are settled in the evening, the cash from the overnight investors extinguishes the clearing bank's intraday loan.

From the perspective of our theory, we can model the clearing bank as an agent endowed with a large amount of cash. By assumption, the clearing bank can finance the borrower only intraday. At each date, the clearing bank finances borrowers according to the following intra-period timing, which complements the timing considered in the previous section:

1. The clearing bank "unwinds" the previous evening's repos. For a specific borrower i this works as follows:

[10] More details about the microstructure of the tri-party repo market can be found in Task Force (2010) and Copeland, Martin, and Walker (2010). The description of the market corresponds to the practice before the implementation of the 2010 reforms.

[11] The reform proposed by the Task Force would limit considerably the ability of the clearing banks to extend intraday credit (Task Force 2010).

(a) The clearing banks sends the cash amount $b_i\left[\overline{r}+(1-\alpha)\overline{r}^2\right]$ to all investors of borrower i, extinguishing the investors' exposure to the borrower they have invested in.

(b) At the same time, the clearing bank takes possession of the assets the borrower has pledged as collateral.

(c) In the process, the clearing bank finances the borrowers temporarily, holding the assets as collateral for its loan.

2. $\bar{I}_i$ assets of a borrower mature (yielding $R_i\overline{I_i}$ in cash), allowing the borrower to repay some of its debt to the clearing bank.

3. Possibly a sunspot occurs.

4. The borrower offers a new repo contract $(\widehat{r}_i, \widehat{Q}_i, \widehat{k}_i)$.

5. New and patient middle-aged investors decide whether to engage in new repos with the borrower.

6. If the borrower is unable to repay its debt to the clearing bank, he must declare bankruptcy. Otherwise, the borrower continues.

This time line explicitly takes into account the sunspot that may cause a change of investor expectations. This is a zero-probability event that allows investors to coordinate on a run, if such out-of-equilibrium behavior is optimal for them.[12] For simplicity, we assume that the clearing bank extends the intraday loan to the borrower at a zero net interest rate. Also, since runs are zero probability events the clearing banks has no reason not to unwind repos.[13]

In the tri-party repo market, traders choose only the interest rate applicable to the repo. The haircut for each collateral class is included in the custodial undertaking agreement between the investor, the borrower, and the

[12] The sunspot also allows the dealer to react to the run. This adds realism to the model and makes runs more difficult (because the dealer's contract offer in stage 4 can now be different from the steady-state offer $(\overline{r}, Q_i, b_i, k_i)$).

[13] In the appendix, we consider the coordination problem between the clearing bank and the investors.

clearing bank, and is not negotiated trade by trade. It is possible to change haircuts by amending the custodial agreement but this takes time. In practice, these changes appear to occur only rarely. We therefore assume that the contract offered in response to a sunspot must leave collateral unchanged from its steady state value, $\widehat{k}_i = k_i$, from Proposition 1.[14]

In response to the contract offer by the borrower, individual investors must compare their payoff from investing with the borrower in question to that from investing with another borrower. The latter decision yields the common market return $\overline{r}$,[15] the return from the former depends on what the other investors do. Table 1 shows the payoffs of the two decisions for the individual investor (rows) as a function of what the other investors do (columns), if the borrower is potentially illiquid (i.e. if the liquidity constraint (17) is violated). If the investor re-invests her funds with the borrower, the clearing bank will accept the cash, since it reduces its intraday exposure to the borrower, and give the investor assets that mature at date $t+1$. These are the only assets available in case of a run since the clearing bank will not let the borrower invest in new securities unless it obtains enough funding. Hence, in case of a run, an investor who agrees to provide financing receives securities that yield $\gamma_i^t R_i k_i$ at date $t+1$ if the borrower defaults.

	other investors	
	invest	don't
invest	$\widehat{r}_i$	$\gamma_i^t R_i k_i$
don't	$\overline{r}$	$\overline{r}$

Table 1: Payoffs in tri-party repo with unwind

[14] Copeland, Martin, and Walker (2010) provide more details about haircuts in the tri-party repo market. In particular, they document that haircuts hardly moved, even at the peak of the crisis.

[15] This is obvious if the investor is the only one to deviate, because then he is negligible. If all investors of the dealer in question deviate, this follows from the slack in assumption (3).

Hence, investors will finance the borrower in case of a run iff[16]

$$\overline{r} \leq \gamma_i^t R_i k_i \tag{18}$$

Note that the investors' decision-making is completely dichotomous. If they anticipate a run, only collateral matters; if they anticipate no run, only interest matters. If condition (18) does not hold, the collective decision not to lend to the borrower in question is self-enforcing. In this case, the yield from the securities pledged as collateral is so low that an investor who believes that nobody will invest with borrower i would also choose not to invest. In our model, steady state collateral is not unique, but clearly, if constraint (18) is violated for the maximum possible amount of collateral κ_i in (14), then it cannot hold in any case.

Combining the above results with those of the previous section and writing out condition (18) for $k_i = \kappa_i$, the maximum amount of collateral per unit borrowed, yields the following prediction about the stability of the tri-party repo market.

Proposition 4 *In the tri-party repo market, a run on a borrower i can occur and bankrupt the borrower if and only if the borrower's liquidity constraint (17) and his collateral constraint*

$$\beta^2 R_i \overline{I_i} \geq \frac{1-\alpha+\beta}{\gamma_i^t(1+\beta)} b_i \tag{19}$$

are both violated.

Condition (19) is implied by the steady-state borrowing constraint (12) of Proposition 1 if γ_i^t is close to 1 and stronger than that constraint if γ_i^t is small. Hence, if investors can use the collateral almost as efficiently as borrowers ("good" collateral in "normal" times), the collateral constraint is slack, and the borrower is run-proof. The collateral constraint becomes relevant only

[16] The weak inequality is due to the assumption that investors do not switch dealers if indifferent. If $\overline{r} = \gamma_i R_i \kappa_i$, there exists the trivial run equilibrium discussed at the beginning of this section.

when the perceived value of collateral falls such that there are larger differences in valuation between investors and borrowers. Furthermore, condition (19) is independent of the liquidity constraint (17). The comparative statics of the collateral constraint for the tri-party model are again simple and we collect them in the following proposition.

Proposition 5 *The collateral constraint (19) is the tighter,*

- *the lower is the value γ_i^t of collateral to investors*
- *the higher is the borrower's short-term leverage $b_i/\overline{I}_i$,*
- *the smaller is the borrower's size $\overline{I_i}$,*
- *the lower is the borrower's productivity R_i.*

Hence, the comparative statics with respect to b_i, $\overline{I_i}$, and R_i are identical for the two constraints (17) and (19). Both constraints are relaxed if borrowers have sufficient access to profitable investment ($\overline{I_i}$ large), if these investment opportunities are sufficiently profitable (R_i large), or if they have sufficiently low leverage ($b_i/\overline{I}_i$ small). In this case, there is no reason for unexpected runs to occur on the investor side, and they cannot bring down borrowers if they occur out of equilibrium. In the opposite case, a run can be a self-fulfilling prophecy and bankrupt the borrower.

5.2 Tri-party repo without unwind

To highlight the importance of the unwind mechanism for the fragility of the tri-party repo market, it is interesting to consider what would happen to the game described in the previous section if there were no unwind.[17] This case is similar to the tri-party repo markets in Europe. It is also similar to what

[17] In this paper, we do not model why the unwind may be necessary. As described in Task Force (2010) and Copeland, Martin, and Walker (2010), the unwind makes it easier for dealers to trade their securities during the day. Automatic substitution of collateral, as is currently available in the European tri-party repo market and is being introduced in the US, allows dealers to have access to their securities even as investors remain collateralized.

the US tri-party repo market should become once the recommendation of the Task Force will be implemented.[18]

When there is no unwind, the timing of events intraday is as follows:

1. Possibly a sunspot occurs.

2. The borrower offers a new repo contract $(\widehat{r}_i, \widehat{Q}_i, k_i)$.

3. New and patient middle-aged investors decide whether to engage in new repos with a borrower.

4. If the borrower is unable to repay his debt to last period's repo investors, he must declare bankruptcy. Otherwise, the borrower continues.

From Lemma 7 it is again enough to consider the case in which the borrower is illiquid after a run. The situation without the unwind differs in two important respects from the one with unwind. First, without the unwind, an individual investor is repaid $\overline{r}$ if and only if the borrower can repay everybody - otherwise the borrower is bankrupt and repays everybody less than the contractual payment. Second, in contrast to the case with unwind, young and middle-aged investors are in a different situation when there is no unwind. Young investors hold cash while middle-aged investors hold a repo with the borrower, until the borrower is able to repay.

In case of a run, an illiquid borrower is bankrupt. All middle-aged investors then keep their collateral and may obtain additional cash as unsecured creditors depending on the bankruptcy rules. This payment is independent of whether an individual investor has demanded to be repaid or has agreed to roll over his loan. Hence, middle-aged investors are indifferent whether to run or not. Given the tie-braking rule assumed throughout this section, patient middle-aged investors therefore reinvest. This in turn induces young investors to invest with the borrower:

[18] More information about the proposed change to settlement in the tri-party repo market can be found at http://www.newyorkfed.org/tripartyrepo/task_force_proposal.html.

Lemma 8 *If middle-aged patient investors reinvest, investing is a (weakly) dominant strategy for new investors.*

Proof. If middle-aged patient investors do not withdraw their funds, the borrower is liquid, because

$$R_i\overline{I_i} - \left(\frac{\alpha}{\beta} + \frac{1-\alpha}{\beta^2}\right) b_i > 0$$

by (12). The borrower therefore has enough assets that will mature in the future to satisfy all future claims by young agents who invest today. ■

Hence, when there is no unwind, the incentives of investors are modified so that they never have a strict incentive to run. In essence, this is because the overnight repo market is an institution that creates simultaneity: if a sufficiently large number of investors do not re-invest, there is bankruptcy and all current creditors (the middle-aged investors) are treated equally, regardless of their intention to withdraw funding. This eliminates fragility due to pure coordination failures.

Proposition 6 *In the tri-party repo market without unwind, there are no strict incentives to run on borrowers.*

5.3 Bilateral repos

In this section, we apply our model to bilateral repos. Typically, bilateral repos have a longer term than tri-party repos. Hence, one period in our model should be thought of as representing a few days to a few weeks.[19] In terms of our assumptions this means that borrowers can adjust the whole contract offer in response to a sunspot.

To simplify the exposition of institutional details, we consider a borrower that funds "Fed-eligible" securities; securities that can be settled using the

[19] Also, a dealer may choose to stagger the terms of its repos, so that only a small portion of these repos are due on any given day. Because of the distribution of investor liquidity needs, this cannot happen in our model. He and Xiong (2010) analyze the consequences of (exogenously determined) staggered short-term debt for the stability of financial institutions.

Fedwire Securities Service®. Fedwire Securities is a delivery versus payment settlement mechanism, meaning that the transfer of the securities and the funds happen simultaneously. The settlement is triggered by the sender of securities and reserves are automatically deducted from the Fed account of the institutions receiving the securities and credited to the Fed account of the institution sending the securities.

This procedure creates a "first come first serve" constraint. In the case of a run, investors who send the securities they hold as collateral early are more likely to receive cash than investors who send their securities late. With bilateral repos, the timing is as follows:

1. Possibly a sunspot occurs.
2. The borrower offers a new repo contract $(\widehat{r}_i, \widehat{Q}_i, \widehat{k}_i)$.
3. New and patient middle-aged investors decide whether to engage in new repos with a borrower.
4. Patient middle-aged investors are repaid in the order in which they send back their collateral, until the borrower runs out of cash. From that point on, investors receive their collateral and any investor who chooses to invest receives his collateral.

The total amount of collateral available is as before. Yet, borrowers can now reduce their borrowing level by changing $\widehat{Q}_i$, which effectively allows them to increase the collateral per unit borrowed. In order to withstand the run, the borrower must at least cover the missing amount

$$m_i \equiv (\overline{r} + (1-\alpha)\overline{r}^2)b_i - R_i\overline{I_i} \tag{20}$$

At the time when he must pledge the collateral the borrower has $\overline{I_i}$ units, which will mature in $t+1$. Hence, the maximum possible value of collateral per unit borrowed is

$$\overline{k}_i = \overline{I_i}/m_i. \tag{21}$$

Again, there are two different investor groups the borrower can borrow from, young investors who hold cash and middle-aged investors who hold a repo with the borrower that may be rolled over.

	other investors	
	invest	don't
invest	$\widehat{r}_i$	$\gamma_i^t R_i \widehat{k}_i$
don't	$\overline{r}$	$\overline{r}$

Table 2: Payoffs to young investors in bilateral repos

Table 2 gives the payoff to an individual young investor as a function of the collective behavior of all other investors. The payoffs are as in Table 1, with the exception that the promised collateral can differ from the steady-state value. Hence, the run outcome (don't, don't) is not a strict equilibrium if and only if

$$\gamma_i^t R_i \widehat{k}_i \geq \overline{r} \tag{22}$$

Now, if the funding shortfall m_i is small, the borrower can increase his collateralization beyond κ_i, and this condition is weaker than (18) in the tri-party context.

Note that the borrower can attract as many young investors as necessary to fund the shortfall m_i if he has the collateral, because he can compete away investors from other borrowers if his offer is sufficiently attractive. Inserting m_i from (20) into (21) yields the collateral constraint of the following proposition.

Proposition 7 *In bilateral repo markets, a run on a borrower i can occur and bankrupt the borrower if and only if the borrower's collateral constraint*

$$\beta^2 R_i \overline{I}_i \geq \frac{1-\alpha+\beta}{1+\gamma_i^t \beta} b_i \tag{23}$$

is violated.

Proof. Condition (23) is (22) evaluated at $\overline{k}_i = \overline{I}_i / m_i$. We already have shown that this condition is sufficient to prevent a run, because young investors will fund the shortfall if it holds. In order to prove necessity, we must examine the incentives of middle-aged patient investors to roll over their existing repos.

Suppose therefore that condition (23) is violated. From (20), only a fraction

$$\varphi \equiv \frac{R_i \bar{I}_i}{b_i \left[\overline{r} + (1-\alpha)\overline{r}^2\right]} \in (0,1) \tag{24}$$

of middle-aged investors can stop renewing their repos before the borrower becomes illiquid. With probability $1-\varphi$, patient middle-aged investors who run are forced to keep their collateral. Investors who are able to obtain their cash back can invest it with another borrower. The payoffs of patient middle-aged investors (per unit of funds) are therefore as in the following table.

	other investors: invest	other investors: don't
invest	$\widehat{r}_i$	$\gamma_i^t R_i \widehat{k}_i$
don't	$\overline{r}$	$\varphi\overline{r} + (1-\varphi)\gamma_i^t R_i k_i$

Table 3: Payoffs to middle-aged patient investors in bilateral repos

Table 3 differs from Table 2 in the lower right cell, which reflects the different positions of young and patient middle-aged investors. The outcome (don't, don't) is strictly optimal for the individual patient middle-aged investor if and only if

$$\gamma_i^t R_i \widehat{k}_i < \varphi\overline{r} + (1-\varphi)\gamma_i^t R_i k_i \tag{25}$$

This condition holds for all k_i and $\widehat{k}_i$ iff it holds for $\widehat{k}_i = \overline{k}_i$ from (21) and $k_i = 1/\beta^2 R_i$ from (13). Re-writing (25) for these two extreme values and setting $d_i = \overline{I}_i / b_i$ yields

$$\gamma_i^t R_i \frac{d_i}{\overline{r} + (1-\alpha)\overline{r}^2 - R_i d_i} < \varphi\overline{r} + (1-\varphi)\gamma_i^t R_i \frac{1}{\beta^2 R_i} \tag{26}$$

$$\Leftrightarrow \qquad \frac{\gamma_i^t \beta^4 d_i R_i}{1-\alpha+\beta-\beta^2 d_i R_i} < \frac{\beta^3 d_i R_i}{1-\alpha+\beta} + \gamma_i^t \frac{1-\alpha+\beta-\beta^2 d_i R_i}{1-\alpha+\beta} \tag{27}$$

Since (23) is violated, we have

$$1-\alpha+\beta-\beta^2 d_i R_i > \gamma_i^t \beta^3 d_i R_i \tag{28}$$

Hence, (27) is equivalent to

$$(1-\alpha+\beta)\gamma_i^t\beta^4 d_i R_i < \left[\beta^3 d_i R_i + \gamma_i^t(1-\alpha+\beta-\beta^2 d_i R_i)\right](1-\alpha+\beta-\beta^2 d_i R_i) \tag{29}$$

Suppose first that $\gamma_i^t > \beta$. By (28), it is enough to show that

$$\begin{aligned} & \beta(1-\alpha+\beta) \leq \beta^3 d_i R_i + \gamma_i^t(1-\alpha+\beta-\beta^2 d_i R_i) \\ \Leftrightarrow \quad & (\beta-\gamma_i^t)(1-\alpha+\beta) \leq (\beta-\gamma_i^t)\beta^2 d_i R_i \end{aligned}$$

which is implied by (28).

Now suppose that $\gamma_i^t \leq \beta$. (29) is linear in γ_i and holds for $\gamma_i^t = 0$ and for $\gamma_i^t = \beta$. Hence, it holds for all $\gamma_i^t \leq \beta$.

Finally, note that condition (23) is strictly weaker than the liquidity constraint (17). Hence, if it is violated, (17) is violated as well, and (23) is necessary and sufficient for the stability of bilateral repos. ■

As condition (19) in the tri-party case, condition (23) is implied by the steady-state borrowing restriction (12) if γ_i^t is close to 1 and stronger if γ_i^t is small. Hence, for "good" collateral in "normal" times, the collateral constraint is slack, and it becomes relevant only in "stress" times.

Furthermore, and differently from the tri-party case, condition (23) is strictly weaker than the liquidity constraint (17). Hence, if it is violated, (17) is violated as well. This means that (23) is necessary and sufficient for the stability of bilateral repos.

Finally, the bilateral collateral constraint is strictly weaker than the tri-party constraint (19). This implies that there are borrowers who are run-proof in the bilateral repo market but can fail in the tri-party market. In this sense, the tri-party market is more fragile than the bilateral market. This problem is exacerbated by the fact that cash investors in the tri-party market are generally considered to be less sophisticated and more restricted in processing collateral than those in the bilateral market, hence have a lower γ_i^t.[20]

Our analysis of the bilateral market has assumed that collateral can adjust in response to a run and has shown that this can be achieved by reducing

[20] See, e.g., Krishnamurthy, Nagel and Orlov (2011, pp. 9-10).

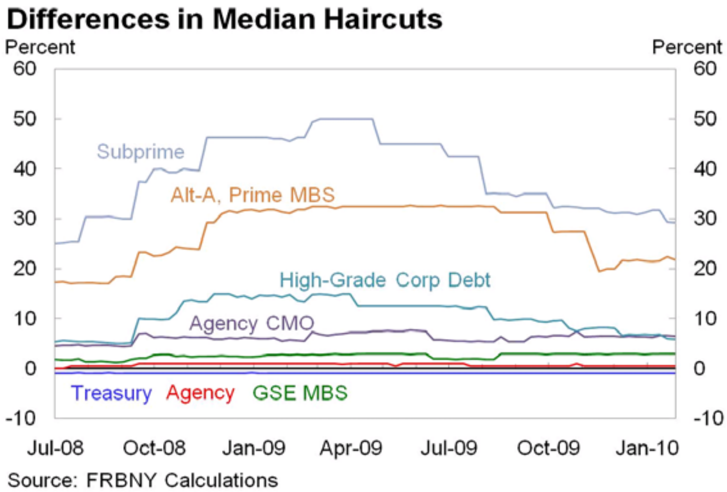

Figure 1: Differences in median haircut between bilateral and tri-party repos per asset class

borrowing and is indeed optimal. This is consistent with the evidence in Gorton and Metrick (2011) of sharply rising haircuts during the crisis of 2008.[21] However, the behavior of haircuts was very different in the tri-party and bilateral repo markets. Figure 1 provides some graphical evidence of this striking difference, taken from Copeland, Martin, and Walker (2010). In the tri-party repo market, haircuts barely moved (this information is not in the figure) while there were large increases in haircuts in some bilateral repo markets. Lehman experienced a sudden reduction of funding in the tri-party repo market that led to its downfall with hardly any adjustment in haircuts. We are not aware of similar sudden losses of funding in the bilateral repo market. Instead, all institutions in this market saw a gradual increase in haircuts that reduced the amount of funding they could obtain (Gorton and Metrick, 2011). Our results in Sections 5.1 and 5.3 are consistent with these two different developments in the bilateral and tri-party repo markets.

[21] If the price of the collateral (the loan size) is p and the market value of collateral is v, then the haircut is $(v-p)/v$.

5.4 Money market mutual funds

In this section, we adapt our model to the case of money market mutual funds (MMMFs) that can offer shares at a fixed net asset value (NAV). These funds are also known as 2a-7 funds, named after SEC rule 2a-7. MMMFs offer their investors shares that can be redeemed at a fixed price, typically $1. Positive returns by the fund increase the number of shares, without affecting the shares' price. If the fund loses value, however, the number of shares cannot decrease. In such a case, the fund is said to have "broken the buck" and is liquidated. Investors' shares give them a pro-rata claim on the proceeds from the liquidation of the assets.

The fixed NAV makes MMMFs similar to banks since, under most circumstances, investors can obtain their funds on demand at a fixed price. However, MMMFs do not hold a capital buffer and do not have access to the discount window. MMMFs invest mainly in marketable safe assets, such as ABCP-backed special investment vehicles, in ABCP directly, and other short-term notes. As a percentage of their balance sheet, MMMFs have invested relatively little in tri-party repos backed by non-Agency MBS/ABS (and hardly anything in bilateral repos), although overall they were an important source of funds to the tri-party repo market.[22] In contrast to repo investors, MMMF investors do not have a claim on a specific piece of collateral.

In our framework, a MMMF can be thought of as an agent who invests $I_i = b_i \leq \overline{I_i}$ and offers to pay investors a short-term "interest rate" r obtained by increasing their shareholdings by $100(r-1)$ percent. Since MMMFs do not invest capital of their own, the argument used to establish the dynamic participation constraint (9) cannot be applied in this context. However, this characterization ignores the important role played by MMMFs' parent institutions. A MMMF is typically part of a larger financial institution that provides start-up funding, is the claimant to returns on the form of fees, and even provides discretionary financial support if the MMMF experiences difficulties. Support by parent institutions has been an important source of stability for MMMFs during the recent financial crisis and earlier episodes,

[22] See Krishnamurthy, Nagel and Orlov (2011).

as documented by Shilling, Serrao, Ernst, and Kerle (2010).

When applied to the parent institution, the same argument as in Lemma 6 shows that the MMMF's implied interest rate in steady state equilibrium must be $\overline{r} = 1/\beta$. Hence, Proposition 1 applies, with the exception that investment $I_i = b_i \leq \overline{I_i}$ is required to equal borrowing.

Abusing our terminology slightly and recognizing the important role of the parent institution, we can describe the run scenario for a MMMF by the following extensive form.

1. Possibly a sunspot occurs.
2. The MMMF offers a new contract $(\hat{r}_i, \hat{Q}_i)$.
3. New and patient middle-aged investors decide whether to withdraw from the MMMF.
4. The parent institution decides whether to inject liquidity into the MMMF.
5. Investors who redeem their shares get cash until the MMMF runs out. At that time, the MMMF has broken the buck and the remaining investors get a pro-rata claim on the fund's illiquid assets.

In our simple framework, the parent company will always inject liquidity in stage 4 if the fund is illiquid, because the fund is in principle profitable. The only reason why the parent may not do so in our model is that the parent, too, does not have sufficient liquidity. This was indeed the case in 2008 and threatened to bring down the whole money market fund industry in September.[23]

[23] Perhaps the most prominent case was that of the Reserve Primary Fund, which broke the buck on September 16. "Despite efforts to calm share holders in the Primary Fund, Bruce Bent II reported to the board that morning that redemption requests as of 9 A.M. stood at $24.6 billion. He also told the board that Reserve Management had not arranged any credit facility or injected any capital to maintain the one-dollar net asset value. And State Street had refused to extend additional overdraft privileges to the fund. The parent company, Reserve, did not have adequate capital to buy the Lehman assets at par. The Bents were unable to inject any of their own personal funds, contrary to representations they had made the previous day" (James Stewart, *New Yorker*, 9/21/2009).

Compared to our lead example of Section 2, the liquidity of MMMFs therefore differs in two respects. First, MMMFs do not invest beyond the level of their short-term funding b_i. This reduces their liquidity and thus tightens their liquidity constraint (17). Second, however, MMMFs can obtain liquidity support from their parent, which loosens their liquidity constraint. If the parent is expected to inject sufficient liquidity in stage 4 of the game, the fund is expected to be liquid, and there is no run in stage 3. In order to analyze the run scenario, we therefore assume that the liquidity constraint is violated and that the parent does not inject liquidity.[24] Since the liquidity constraint is violated, the withdrawals $b_i\left[\bar{r}+(1-\alpha)\bar{r}^2\right]$ exceed the fund's cash $R_i b_i$, which implies

$$\beta^2 R_i < 1 - \alpha + \beta.$$

As in (24), the probability that a withdrawing investor is able to obtain cash therefore is

$$\varphi = \frac{R_i}{\bar{r} + (1-\alpha)\bar{r}^2} \in (0,1).$$

With probability $1-\varphi$, the investor is unable to withdraw quickly enough to obtain cash. The investor thus gets a claim on the fund's assets. The amount of these assets divided by the total claims outstanding is

$$\mu_i \equiv \frac{I_i}{I_i\left[\bar{r}+(1-\alpha)\bar{r}^2\right] - R_i I_i}.$$

Note that the denominator is again $m_i \equiv (\bar{r}+(1-\alpha)\bar{r}^2-R_i)I_i$. The payoffs to middle-aged patient investors as a function of how the other middle-aged patient investors behave are therefore given by the following matrix.

	other investors: invest	other investors: don't
invest	$\widehat{r}_i$	$\gamma_i^t R_i \mu_i$
don't	$\bar{r}$	$\varphi\bar{r} + (1-\varphi)\gamma_i^t R_i \mu_i$

Table 4: Payoffs to middle-aged patient investors in MMMFs

[24] More generally, the parent may be able to inject some cash, but not enough to plug the liquidity hole m_i. In this case, the parent will optimally not inject any cash at all, because the fund will not survive anyhow and the cash will go to the investors.

If $\mu_i \geq \bar{r}$, then investors do not have a strict incentive to run on an MMMF. Rewriting this condition we get

$$\beta^2 R_i \geq \frac{1+\beta-\alpha}{1+\gamma_i^t \beta}. \tag{30}$$

Interestingly, this condition is the same as (23), evaluated at $b_i = \overline{I}_i$. Note, however, that condition (30) is independent of fund size (which is equal to outside funding). This is consistent with the observation that the crisis of MMMFs in the wake of the Lehman bankruptcy hit funds across the board, regardless of their size. Again, if (30) is violated, so is the liquidity constraint. Hence, if (30) is violated, the survival of the fund depends on whether the parent company has the cash m_i necessary to stabilize the fund. As (20) shows, this cash shortfall depends on the size of the fund.

5.5 Asset-backed commercial paper conduits

In this subsection we briefly describe the structure of ABCP conduits that Acharya, Schnabl, and Suarez (2009) and Krishnamurthy, Nagel, and Orlov (2011) have identified as an important destination of funds in the shadow banking system and as a main mechanism of contraction during the crisis. While Krishnamurthy, Nagel, and Orlov (2011) consider the evolution of funding from 2007 to 2009 more broadly, Covitz, Liang, and Suarez (2009) focus on the turmoil of the ABCP market in the second half of 2007, which marked the onset of the Great Financial Crisis.

ABCP conduits are institutions that are "sponsored" (i.e., set up, managed, and guaranteed) by banks mainly for the purpose of regulatory arbitrage (or to "optimize yield"). They mostly invest in relatively short-term assets such as receivables or notes and are funded by commercial paper that is of very short maturity. Covitz, Liang, and Suarez (2009) report that "more than half of ABCP daily issuance has maturities of 1 to 4 days, and the average maturity of outstanding paper is about 30 days" (p. 7). ABCP can be liquidated daily, and ABCP conduits are usually opaque. However, unlike traditional banks they are not insured by the government and rather rely on the liquidity support by their sponsoring bank, very much like MMMFs.

We do not provide a formal model of ABCP conduits, which would be similar to that of MMMFs sketched previously, and only report the findings of Covitz, Liang, and Suarez (2009) about the precipitous fall in ABCP finance in August 2007. They find a decrease of outstanding ABCP of $187 billion, almost 20 percent, in August alone, which moreover was mostly concentrated in the two weeks following August 9. More importantly, they analyze the incidence of runs, defined as weeks in which a conduit has more than 10 percent of its outstanding paper maturing but does not issue new paper. Their most important econometric finding, corroborated by various robustness checks, is that "runs are related importantly to program fundamentals, but there is strong evidence that programs that would be sound in more stable market conditions were also subject to runs in the early weeks of the financial crisis" (p. 19).

5.6 Traditional banks

The investors in traditional banks, depositors, are different from the money market participants whom we have considered up to now. Although much of the analysis in Sections 2 - 4 does not change in substance for the case of banks, the no-arbitrage argument underlying Lemma 1 does not apply to depositors. However, in this case it is appropriate to assume that the utility functions u_1 and u_2 are strictly concave, which again implies a flat yield curve, as shown by Qi (1994). Apart from that the analysis for traditional banks is similar to the analysis for MMMFs. With $b_i < \bar{I}_i$, the assets $\left(\bar{I}_i - b_i\right)(1+\beta)$ can be thought of as the equity of the bank. Like MMMF investors, bank depositors do not get a claim to a specific piece of collateral, but rather a claim on the bank's assets in case of bankruptcy. The major difference between a MMMF and a bank is that banks hold largely nonmarketable assets. Hence, the outside value of assets γ_i^t is low in the case of a bank.

The timing of bank funding in our model structure is as follows.

1. The bank offers a new deposit contract (r_i, Q_i).

2. New and patient middle-aged investors decide whether to deposit (again) with the bank.

3. Investors can withdraw cash until the bank runs out. At that time, the bank is bankrupt and the remaining investors get a claim on the remaining assets.

The analysis and the payoff table is as in the case of a MMMF, with the exception that the bank (hopefully) has equity, i.e. that $b_i < \bar{I}_i$. The collateral constraint therefore becomes

$$\beta^2 R_i \overline{I_i} \geq \frac{1-\alpha+\beta}{1+\gamma_i^t \beta} b_i$$

which is identical to the bilateral constraint (23). The main difference here is that the collateral value γ_i^t of assets of a failing bank is likely to be very low. Hence, the collateral constraint is unlikely to be satisfied and the liquidity constraint (17) thus crucial for bank stability.

Our work therefore nests the classic literature on bank stability which emphasizes the importance of liquidity. It adds to this literature by endogenizing the profits that can serve as liquidity buffers and therefore can make predictions which banks are likely to be subject to runs if investor sentiment changes.

6 Conclusion

We have developed a dynamic equilibrium model to study how the fragility of short-term funding markets depends upon the particular microstructures, liquidity, and collateral arrangements that may lead to runs at various types of financial institutions. The value of collateral and the endogenous liquidity of intermediaries then become crucial for their stability. Runs can be forestalled by mobilizing sufficient liquidity and having sufficiently valuable collateral. It is therefore tempting to augment Brunnermeier and Pedersen's (2009) distinction between market liquidity and funding liquidity by the notion of "balance sheet liquidity".

Our model sheds light on the panic in the ABCP market in August 2007 that triggered the Great Financial Crisis and on the puzzling behavior of margins in different repo markets in the crisis of 2008. We can account for the difference between the bilateral repo market, where haircuts increased dramatically during the crisis, and the tri-party repo market, where the haircuts barely moved. The model also clarifies the distinction between increasing margins, which is a potentially equilibrating phenomenon, and runs, which can happen if margins do not increase sufficiently to reassure investors. The model also shows that the practice of early settlement of tri-party repos, called the "unwind", can increase fragility in the market; this result lends support to reforms currently underway to eliminate the unwind. Our results on the particular fragility of the tri-party repo market show how a lack of increase in haircuts and the practice of "unwind", each of which may appear to provide additional liquidity for borrowers in normal times, actually can explain the sudden collapse of securitized lending that contributed to the runs on Bear Stearns and Lehman Brothers.

Our framework can be used to consider a number of policy questions related to the fragility of short-term funding markets. For the tri-party repo mechanism, for example, Lehman's demise highlighted the problem that there is no process to unwind the positions of any large bank that deals in repo should it fail. Lehman required large loans from the Federal Reserve Bank of New York to settle its repo transactions. Our framework can be used to study a liquidation agent, as suggested in the Task Force on Tri-Party Repo Infrastructure (2009), with the objective to unwind the positions of a defaulting borrower. Similarly, our analysis sheds light on the role of institutional features such as the unwind mechanism in the tri-party market or the difference between bilateral and tri-party repo lending and thus should contribute to a better understanding of the fragility of wholesale banking markets.

7 Appendix: Coordination problem between the clearing bank and investors

The tri-party repo market is also vulnerable to another coordination problem, this time between the clearing bank and the investors. Suppose that, in the timing described in section 5.1, just before step 1 the clearing bank comes to believe that at step 5 all investors will refuse to engage in repos with borrower

i. In this case, the clearing bank will refuse to unwind if the loan it makes to the borrower, $b_i\left[\bar{r}+(1-\alpha)\bar{r}^2\right]$, exceeds the proceeds it could obtain from the assets, $R\bar{I}(1+\beta\gamma)$.[25] This condition can be written as

$$\beta^2 R_i \bar{I}_i \geq \frac{1+\beta-\alpha}{1+\gamma\beta} b_i. \tag{31}$$

This condition is the same as the collateral condition for bilateral repos, (23).

The flip side of this coordination problem is that investors may choose not to invest with borrower i if they believe that the clearing bank will refuse to unwind that borrower's repos the next morning.[26] In this case, the condition for investors to have a strict incentive to run is the same as in the case where investors believe other investors may not engage in repos.

[25] Here we assume that the clearing bank faces the same γ as the investors.

[26] Clearing banks have the contractual right not to unwind a dealer's repos. Failure to unwind the repos would almost certainly force the dealer into bankruptcy.

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