A Model of Liquidity Hoarding and Term Premia in Inter-Bank Markets

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Abstract

Financial crises are associated with reduced volumes and extreme levels of rates for term inter-bank loans, reflected in the one-month and three-month Libor. We explain such stress by modeling leveraged banks' precautionary demand for liquidity. Asset shocks impair a bank's ability to roll over debt because of agency problems associated with high leverage. In turn, banks hoard liquidity and decrease term lending as their rollover risk increases over the term of the loan. High levels of short-term leverage and illiquidity of assets lead to low volumes and high rates for term borrowing. In extremis, inter-bank markets can completely freeze.

Keywords: inter-bank lending, financial crisis, precautionary demand, rollover risk, Libor-OIS spread

1 Introduction

Extreme levels of inter-bank lending rates, particularly at longer maturities, were seen as a principal market friction of the financial crisis of 2007-09. Figure 1 shows that the spreads between London Interbank Offer Rate (LIBOR) and Overnight Indexed Swap (OIS) rate for 1-month, 3-month, and 6-month terms increased to over 300 bps at the peak of the crisis compared to less than 10 bps before the crisis.[1] Figure 2 shows the weighted-average maturity of inter-bank term lending estimated by Kuo, Skeie, Youle and Vickery (2011).[2] This work suggest that lending maturities fell from a peak average term of over 40 days before August 2007 to less than 20 days after the bankruptcy of Lehman Brothers in September 2008.[3] Such rising inter-bank term rates have been widely interpreted as a manifestation of rising counterparty risk of borrowing banks. However, during the crisis, banks with even the best credit quality borrowed in term markets at extremely high spreads to the risk-free rate, suggesting that lenders demanded heightened compensation for term lending even from relatively safe borrowers.

We provide an explanation of this rise in spreads and a collapse in maturities in the term inter-bank market by building a model of lending banks' precautionary demand for liquidity. Our key insight is that each bank's willingness to provide term lending (for a given counterparty risk of its borrower) is determined by its own *rollover risk*, which is the risk that it will be unable to roll over its debt that matures before the term of the loan. If adverse asset shocks materialize in the interim, debt overhang can prevent highly leveraged banks from being able to raise financing required to pay off creditors. Thus, during times of heightened rollover

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[1]The LIBOR-OIS spread is a measure of the credit and liquidity term spread to the risk-free rate for inter-bank loans. LIBOR is a measure of banks' unsecured term wholesale borrowing rates. OIS is a measure of banks' expected unsecured overnight wholesale borrowing rates for the period of the fixed-for-floating interest rate swap settled at maturity, where the floating rate is the effective (average) fed funds rate for the term of the swap.

[2]A caveat with estimates of inter-bank activity (both overnight and term) is that their quality is not currently known.

[3]Consistent with this, Ashcraft, McAndrews and Skeie (2010) and Afonso, Kovner and Schoar (2010) document that volumes in overnight inter-bank markets did not fall much during the crisis in contrast to the collapse in term lending volumes.
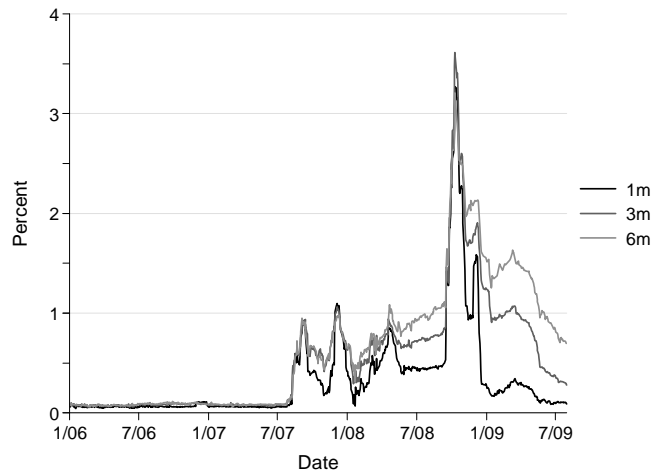
Figure 1: LIBOR-OIS spread, measured on the y-axis for 1-month, 3-month, and 6-month maturities. The spread was a primary measure of banking stress, increasing to over 300 bps at the peak of the crisis, in comparison to spreads of less than 10 bps before the crisis. Source: British Banker's Association and Bloomberg.
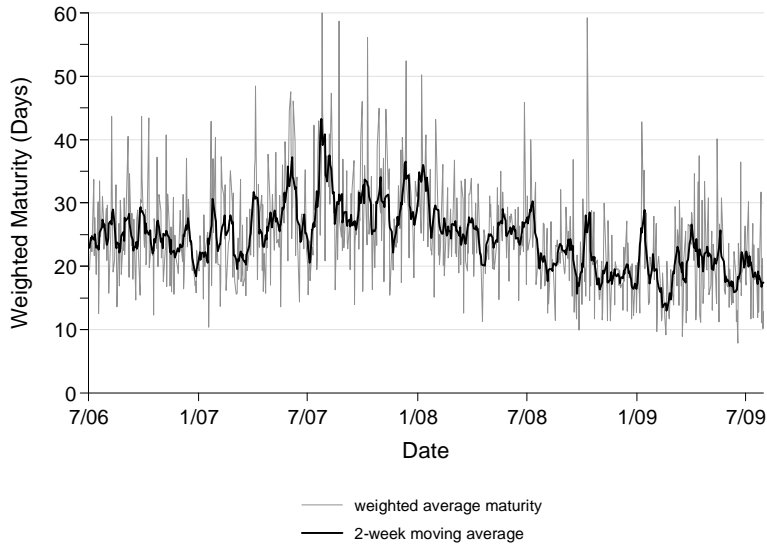
Figure 2: Weighted Average Maturity of Term Inter-Bank Lending, source: Kuo, Skeie, Youle and Vickery (2011). This work suggests that average lending maturities fell from a peak of over 40 days before the start of the crisis in August 2007 to less than 20 days after the bankruptcy of Lehman Brothers in September 2008. Weighted average maturity is the average maturity of estimated term inter-bank issuance, weighted by maturity for terms between one week and one year.

risk, banks "hoard" liquidity by lending less and more expensively at longer term maturities.[4] Elevated rates for term borrowing, in turn, aggravate the debt overhang and rollover risk problems of borrowing banks. Even strong banks are forced to cut back on borrowing term and potentially bypass profitable investments such as real-sector lending for long-term and illiquid projects.[5]

In the extremis, there can be a complete freeze in the inter-bank market, in the sense that there is no interest rate at which inter-bank lending will occur. In particular, even when banks with profitable investment opportunities do not have solvency or liquidity risk, they may be unable to access liquidity on the inter-bank market if the lending banks have high enough short-term leverage and asset illiquidity. In these cases, paying lending banks their opportunity cost of liquidity renders borrowers' investments unprofitable. More generally, when the banking sector is weak (fewer profitable investments, high uncertainty about asset quality and high short-term leverage), lenders' precautionary demand for liquidity manifests as low volumes and high rates in term inter-bank lending.

The key feature of our model – rollover risk of banks central to the inter-bank markets – was key to the ignition of the financial crisis of 2007-09. Acharya, Schnabl and Suarez (2009) show empirically that the onset of the crisis in August 2007 was due to commercial bank exposures to off-balance sheet vehicles (conduits and SIVs). These vehicles held securities (primarily, sub-prime mortgage backed) that were funded with extremely short-dated asset-backed commercial paper (ABCP). Hit by worsening house prices and the BNP Paribas' public announcement on August 8,

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[4]The Global Head of Loan Syndications & Trading for BNP Paribas explains that "many lenders intermix cost of risk and cost of funding whereas clearly they reflect two very different elements. First, their own risk as perceived by their own lenders, and second the risk of the borrower – put another way, a poor quality borrower and a poor quality lender should result in a very high overall cost to the borrower, conversely the opposite would also apply." (van Kan, 2010)

[5]We develop these ideas in a model that builds upon the asset-substitution or risk-shifting model of Jensen and Meckling (1976), Stiglitz and Weiss (1981), Diamond (1989, 1991), and more recently, Acharya and Viswanathan (2011). In essence, these papers provide a micro-economic foundation for the funding constraints of a leveraged financial firm: the firm can switch to a riskier, negative net present value investment ("loan") after borrowing from financiers. In anticipation, the financiers are willing to lend to the firm only up to a threshold level of funding so as to ensure there is enough equity to keep the firm's risk-shifting incentives in check.

2007 that sub-prime assets had become practically illiquid, the off-balance sheet vehicles – and, in turn, the commercial banks facing draw downs on credit lines provided to the vehicles – incurred significant rollover risk.[6] Acharya, Schnabl and Suarez (2009) also document that outstanding short-term ABCP fell by the end of 2008 to half of its level of $1.2 billion in August 2007. Our model suggests that this rollover risk induced precautionary demand for liquidity in banks, causing term inter-bank spreads to rise – and volumes to shrink – dramatically.

It is interesting to contrast our analysis to the traditional view of banking panics and runs. In the canonical model of Diamond and Dybvig (1983), banks fail because of liquidity demand by retail depositors. These depositors receive exogenous liquidity shocks and demand immediacy of payments precipitating bank runs. Our model can be viewed as one in which the liquidity needs of wholesale financiers play a crucial role. These providers, such as banks and money market funds, are funded with short-term debt themselves and face rollover risk when adverse asset shocks materialize. In Diamond and Dybvig style runs, there is an inefficient liquidation of banking assets. In our model, liquidity demand by wholesale financiers leads to bypassing of profitable investment opportunities, such as lending to the real sector.

It is equally interesting to contrast our analysis with the alternative view that borrowing rates and volumes in the inter-bank market are determined primarily by the credit risk of borrowers. Under this view, an increase in the credit risk of borrowing banks increases lending rates to compensate lenders for the higher risk and could also lead to reduced volumes. Our analytical framework predicts that spreads in term inter-bank markets can be larger, and volumes smaller, than ones based purely on borrower credit risk. At a minimum, this suggests caution in interpreting the entire rise in term inter-bank rates as being attributable to counterparty risk concerns.[7] Further, our model's most important and novel implication is that a

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[6] This risk of being unable to acquire short-term funding, in order to honor draw downs on credit lines backing the illiquid assets funded by ABCP, is equivalent to the more general risk for banks of being unable to roll over short-term borrowing against illiquid long-term assets.

[7] Current research appears to have been unable to rationalize the extreme spreads in the inter-bank market as per this borrower-risk channel alone. While Taylor and Williams (2008a, 2008b)

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bank's borrowing rate for a particular maturity in the inter-bank market increases with the credit risk and liquidity risk of its lender, *controlling for the borrower's own credit risk*. In the same vein, bilateral inter-bank markets can freeze even for some healthy borrowers when most other banks are leveraged, especially at short maturities, and are holding riskier and more illiquid assets. These implications are inconsistent with a pure borrower-risk view of inter-bank rates and volumes.

The remainder of the paper is organized as follows. Section 2 sets up our benchmark model of inter-bank lending supply and examines how asset illiquidity can lead to decreases in inter-bank lending. Section 3 extends the model to consider rollover risk and precautionary behavior on both supply and demand sides of the inter-bank market. Section 4 relates the results to existing empirical evidence, derives new empirical and policy implications, and discusses the related literature on liquidity hoarding by banks. Section 5 concludes. Details of proofs and analysis of the borrowing bank demand are contained in the appendix.

2 Liquidity hoarding

We build a model in which a bank with surplus liquidity either hoards it for its own future liquidity needs or lends the liquidity in the term inter-bank market to another bank that has additional capacity for investment in a long-term, illiquid asset. In Section 2.1, we introduce the benchmark model, in which the lending bank has existing risky assets and short-term debt in place. We show as a benchmark that absent any frictions, the lending bank's own credit risk and leverage do not lead the bank to hold any liquidity. The bank supplies its full liquidity for term inter-bank lending. In Section 2.2, we add a moral hazard problem for the lending bank aimed at capturing opacity and illiquidity of banking assets and activities. The illiquidity of the lending bank's assets decreases its ability to roll over its short-term debt.

attribute the large one-month and three-month LIBOR-OIS spreads (as shown in Figure 1) primarily to counterparty credit risk, McAndrews, Sarkar and Wang (2008), Michaud and Upper (2008) and Schwarz (2009) attribute most of the spread to liquidity risk, whereas Smith (2010) argues that time-varying risk premia explain half of the variation in spreads.

Because of this, the lending bank retains liquidity to pay off short-term debt rather than lend on the term inter-bank market. Term inter-bank lending may thus fall relative to the benchmark amount even in the absence of credit risk of the borrowing bank.

2.1 Benchmark model of the term inter-bank market

There are three periods, dates $t = 0, 1, 2$, and two types of banks $i = B, L$. At date 0, each bank has in place investment in one unit of a long-term asset that pays at date 2 y with probability θ, corresponding to success of the investment, and zero otherwise, corresponding to failure of the investment. Thus, the expected return on the asset is $\theta y \geq 1$. (In Section 3, we will allow θ to be a random variable that is realized at date 1 to allow for the partial revelation of information ex-interim). Bank $i = B$ is called the "borrowing bank" because it has an opportunity at date 0 for additional investment of up to one unit into the long-term asset but has no additional liquid goods, which we call "liquidity," required for the investment. The bank also does not have any additional borrowing sources from outside depositors (at least not in the very short term).

To start with, the focus of the model is on bank $i = L$, called the "lending bank." At date 0, this bank has an additional unit of liquidity but has no opportunity for additional investment in the long-term asset. The bank lends $l \leq 1$, which is a two-period term inter-bank loan, to the borrowing bank at a term interest rate of r and stores liquidity $(1 - l)$ for a rate of return of one for one period. The borrowing bank invests the two-period term inter-bank loan amount that is borrowed into the long-term asset and repays the loan at date 2 with probability θ.

At date 1, the lending bank has to repay short-term debt $\rho^L \in [1, 2]$ held by depositors.[8] The bank can repay the debt with liquidity it holds, $(1 - l)$, and by

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[8]The amount ρ^L reflects the lending bank's effective short-term leverage in place. This leverage can alternatively be thought of as a broader type of liquidity need, such as the draw-down of the bank's extended credit lines to corporations or special purpose vehicles. At the minimum value of ρ^L, the bank has sufficient liquidity to repay all short-term debt at date 1. At the maximum value of ρ^L, the bank's one unit of the long-term asset in place is entirely financed by short-term debt.

issuing new debt to depositors with a face amount f^L due at date 2. The bank defaults if it cannot repay or roll over its debt, in which case the proceeds of the bank's asset-in-place and inter-bank loan have no salvage value to either the bank's depositors nor itself.[9] Depositors must break even on their expected return θf^L to be willing to roll over debt $\rho^L - (1 - l)$ according to their individual rationality constraint

$$\rho^L - (1 - l) \leq \theta f^L. \tag{1}$$

For a given term inter-bank market rate r, the bank chooses (i) date 0 term lending l and (ii) date 1 borrowing with face amount f^L (due at date 2) in order to maximize its expected profit

$$\pi^L = \theta(y + lr - f^L), \tag{2}$$

subject to constraint (1). The optimal values l^* and f^{L*} satisfy the first order condition and the depositor rationality constraint, respectively. Substituting from (1), expected profit can be written as $\pi^L = \theta y + l(\theta r - 1) - \rho^L + 1$. The lending bank's solution is to lend fully, $l^* = 1$, at any term rate $r \geq \frac{1}{\theta}$. This gives the bank a risk-adjusted expected rate of return at least that of storage, $\theta r \geq 1$. The bank lends nothing otherwise, $l^* = 0$ for $r < \frac{1}{\theta}$.

Next, we consider a simplified term inter-bank loan demand by a borrowing bank. The borrowing bank maximizes expected profits by borrowing a full unit at any term rate below the rate of return on investment, so as to increase investment in the asset in place. In other words, the borrowing bank has a perfectly elastic demand to borrow at any term rate that is below the rate of return that investment pays, $b^*(r) = 1$ for $r \leq y$, and does not borrow otherwise, $b^*(r) = 0$ for $r > y$. To summarize, with no frictions, the lending bank in equilibrium fully lends its liquidity at any positive expected rate of return. For $\theta < 1$, credit risk of the borrower is reflected in the term lending rate in the inter-bank market of $r^* = \frac{1}{\theta} > 1$ but does not affect term inter-bank lending volume.

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[9] For instance, these assets are rendered worthless by disintermediation of the bank or illiquid for a while due to its bankruptcy.

t = 0 t = 1 t = 2

risk-shifting
θ $k^L + rl$
1-θ k^L
rollover
θ $y + rl - f^L$
no risk-shifting

Chooses inter-bank loan l
Holds liquidity $(1 - l)$ and
asset-in-place
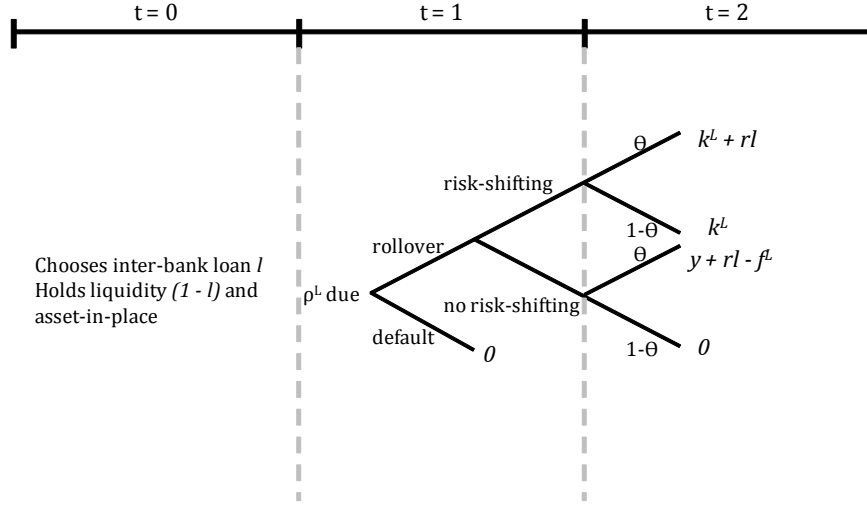
ρ^L due

default
0

1-θ 0

Figure 3: Lending Bank Timeline. The timeline shows for the benchmark model with agency problems the lending bank's choice of inter-bank loan l at date $t = 0$; choice of whether to risk-shift, conditional on short-term debt ρ^L being successfully rolled over at date t=1; and payoffs at date t=2. With probability θ, the inter-bank loan pays rl, and if there is no risk-shifting the asset-in-place pays y and the bank pays short-term debt f^L. If there is risk-shifting, there is an expected return of k^L.

2.2 Illiquidity of assets

The timeline of the benchmark model with illiquid assets is shown in Figure 3.

Specifically, we add two agency problems for the surplus bank. One, we assume that the lending bank can risk-shift its assets. After the bank rolls over its short term debt at date 1, the bank can costlessly and unverifiably increase the risk, while decreasing the expected return, of the asset in place. Specifically, the bank can receive a bank-specific, higher payoff $y_R^L > y$, which occurs when the asset investment is a success and has a positive payoff. This higher payoff comes at the cost of a lower probability of payoff $\theta_R^L < \theta$ that is uncorrelated with θ. Risk-shifting decreases the expected return: $\theta_R^L y_R^L \leq \theta y$.[10] The common payoff y reflects systematic risk; the bank-specific payoff from risk-shifting reflects idiosyncratic risk.

Two, we assume that the lending bank cannot pledge any returns of the inter-bank loan. In particular, we assume that depositors have limits on the information

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[10] An interpretation of the risk-shifting problem is that the bank decreases its risk management or monitoring of the asset.

they can verify about the lending bank's three types of assets, depending on their opacity. First, inter-bank loans are the most opaque assets to depositors. The lending bank itself can verify returns on its inter-bank loan, reflecting its ability for peer monitoring. However, the depositors of the lending bank cannot verify any information about the returns of the inter-bank loan because they are the furthest removed from the borrowing bank's assets that ultimately back the inter-bank loan. Second, the asset in place is held directly by the lending bank and is less opaque to the bank's depositors than inter-bank loan is. The depositors can verify whether the return on the asset in place is positive or zero, but cannot distinguish between whether a positive return is y or y_R^L.[11] Third, liquidity held by the bank is perfectly verifiable by depositors, and can be paid out to depositors at date 1.

An increase in term lending by the bank decreases the liquidity $(1 - l)$ that is available to pay depositors at date 1. However, since returns on inter-bank loans are fully internalized by the lending bank, it would not attempt to increase the risk of these returns. This, however, is not the case for the bank's incentives to increase the risk of the asset in place. Consider the four possible states for the lending bank under risk-shifting on the asset in place, conditional on the bank first rolling over its short-term debt. For each state, the date 2 payoffs of the asset in place and term inter-bank loan, the state probability, and the lending bank's profit are as follows:

Asset in place	Inter-bank loan	State probability	Profit
y_R^L	lr	$\theta_R^L \theta$	$y_R^L - f^L + lr$
y_R^L	0	$\theta_R^L(1 - \theta)$	$y_R^L - f^L$
0	lr	$(1 - \theta_R^L)\theta$	lr
0	0	$(1 - \theta_R^L)(1 - \theta)$	0

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[11]See Acharya and Viswanathan (2011) for a related model showing that debt with liquidation rights is the optimal contract in presence of risk-shifting and coarseness of verifiable information on asset payoffs. Calomiris and Kahn (1991) and Diamond and Rajan (2001) also explain optimality of demandable debt in models with moral hazard and hold-up problems.

The expected profit from risk shifting conditional on rolling over debt is thus[12]

$$\pi_R^L = \theta_R^L(y_R^L - f^L) + \theta l r. \tag{3}$$

The lending bank's incentive compatibility constraint requires that profits without risk-shifting given by equation (2) are greater than profits with risk-shifting given by equation (3). This incentive constraint can be written as

$$\theta(y - f^L) \geq \theta_R^L(y_R^L - f^L), \tag{4}$$

and it holds if and only if the probability of asset payoff θ is large enough that new debt is $f^L \leq \frac{\theta y - \theta_R^L y_R^L}{\theta - \theta_R^L}$. For tractability, we assume the risk-shifting payout y_R^L to the lending bank increases as the probability of success θ_R^L decreases. In the limit, as $\theta_R^L \to 0$, we assume that $y_R^L \to \infty$ and $\theta_R^L y_R^L \to k^L$, where k^L equals the expected return of the risk-shifting assets. The value k^L is our measure of the illiquidity of the asset in place. We also refer to k^L as the severity of the moral hazard problem. It is equivalent to an amount of expected profits at date 2 that cannot be pledged at date 1. Under this limiting case, the incentive constraint (4) can be rewritten as

$$\theta f^L \leq \theta y - k^L. \tag{5}$$

The bank's rollover constraint requires both the incentive constraint (5) and individual rationality constraint (1) to hold, which can be combined into a constraint on the amount of term inter-bank lending:

$$l \leq \theta y - k^L - \rho^L + 1. \tag{6}$$

Hence, the bank's optimal choice of new debt f^L must be small enough to satisfy the incentive constraint (5), which according to the depositor rationality constraint

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[12]Recall that the payoff to the lending bank on the inter-bank loan lr that occurs with probability θ is unverifiable to depositors and is not used to repay depositors. Hence, the lender only repays f^L from the payoff y_R^L of its assets in place.

(1), limits the amount of term lending such that the bank does not risk-shift. It follows that

Lemma 1. *Term inter-bank lending is constrained to be less than one unit, $l^* < 1$, if leverage plus moral hazard costs are greater than the expected return on investment: $\rho^L + k^L > \theta y$.*

Now, the rollover condition requires the bank inelastically to hold liquidity $(1-l^*)$ in order to avoid interim liquidation and loss of payoffs at date 2. This is true for all term inter-bank lending rates $r^* \geq \frac{1}{\theta}$ that are on a risk-adjusted basis greater than or equal to the rate on storage. Indeed, the rollover condition may imply complete withdrawal by the lender from the term inter-bank market, that is, $l^* = 0$, regardless of the term inter-bank rate.[13] In this extreme case, the lending bank has to hold all of its liquidity to repay short-term debt, according to equation (6): For parameters such that $\rho^L + k^L > 1 + \theta y$, a *term lending freeze* occurs.

2.3   The alternative view: Counterparty risk

The traditional view of spreads in term inter-bank rates over the risk-free rate is that of compensation to lenders for counterparty (borrower) credit risk. Throughout the crisis, however, many empirical studies (see footnote 7) questioned whether counterparty credit risk alone could explain spreads in the term inter-bank market and suggested liquidity factors were at play. Our model explains these liquidity spreads as arising due to lenders' demand driven by their own credit risk for holding liquidity to meet their future obligations.

Specifically, consider in our model the case of borrower credit risk, with no other

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[13]It is worth observing which of the two agency problems plays a critical role in the model. We show in the online appendix that the minimum assumption necessary is that returns on inter-bank loans are not fully verifiable, which captures the illiquidity of inter-bank lending. If instead, the lending bank could fully borrow against its inter-bank loans when rolling over debt, then increasing the inter-bank loan would help to relax the bank's rollover constraint and to increase its profits. This holds for any profitable rate $r > \frac{1}{\theta}$, regardless of the extent of moral hazard k^L. Moral hazard can, however, increase the threshold rate r necessary for the bank to meet the rollover constraint. Additionally, when there is opacity of the inter-bank loan, the decrease in inter-bank lending is exacerbated by moral hazard.

frictions. In this setting, the lender lends its full liquidity unit, $l^* = 1$, at a term rate $r = \frac{1}{\theta}$ to cover the credit risk of the borrower, which is represented by $(1 - \theta)$. Hence, the term lending rate adjusts to the borrower's credit risk, but there is no effect on the quantity of term inter-bank lending. In contrast, our model shows that if the lender's leverage (ρ^L) and asset illiquidity (moral hazard k^L) are large enough, then the lender may decrease the volume of term lending to less than a full unit. Indeed, the lender may hold all of its liquidity to pay off short-term debt because it cannot roll over any amount of it. But provided that the lender lends, the rate is $r = \frac{1}{\theta}$. In other words, the term lending rate adjusts to the borrower's credit risk, but the *quantity of term lending is determined by the lender's credit risk.*

3 Precautionary behavior

In this section, we consider banks' precautionary demand for liquidity. We enrich the model with partial information revelation regarding the credit risk of assets in place, ex-interim at the time that short-term debt is due. This creates rollover *risk*, or in other words, uncertainty about the ability of the bank to roll over its short-term debt. Under the richer model, the liquidity demand of the lending bank takes a precautionary nature, in that the bank holds liquidity to reduce its rollover risk. Precautionary liquidity not only decreases term inter-bank lending, even in the absence of credit risk of the borrower, but also increases the term equilibrium inter-bank rate spread above the credit risk spread. Also, there is lower term inter-bank lending in equilibrium compared to the benchmark.

3.1 Rollover risk

We continue to use l to denote the term lending volume and r to denote the term rate, respectively. For brevity, at times we omit the word 'term' and simply refer to l as inter-bank lending and r as the inter-bank rate. To introduce rollover risk, we now assume that the asset payoff probability θ is a random variable that is realized

at date 1, where θ has a distribution $G(\theta)$ and density $g(\theta) > 0$ over $[\underline{\theta}, \bar{\theta}]$. We define $\hat{\theta}^L(l)$ as the bankruptcy cutoff value for the lending bank such that for a large enough realization $\theta \geq \hat{\theta}^L(l)$, the rollover constraint (6) holds:

$$\hat{\theta}^L(l) = \frac{\rho^L - (1 - l) + k^L}{y}, \tag{7}$$

where $k^L \leq \underline{\theta} y$. The following lemma shows that the rollover risk for the bank increases in leverage ρ^L and the severity of moral hazard k^L.

Lemma 2. *The lending bank cannot roll over its debt at date 1 if its probability of default is less than its bankruptcy cutoff value: $\theta < \hat{\theta}^L(l)$. The cutoff value $\hat{\theta}^L$, and hence the bank's rollover risk, is increasing in leverage ρ^L, the severity of moral hazard k^L, and the inter-bank lending amount l, and is decreasing in the payoff of the asset y and liquidity held $(1 - l)$.*

Subject to the rollover constraint, the bank's optimization is to maximize expected profits, which are given by[14]

$$\pi^L \equiv \int_{\hat{\theta}^L(l)}^{\bar{\theta}} [\theta(y + lr) - (\rho^L - (1 - l))]g(\theta)d\theta. \tag{8}$$

For an interior solution $l^*(r) \in (0, 1)$, the first order condition is

$$\int_{\hat{\theta}^L(l)}^{\bar{\theta}} (\theta r - 1)g(\theta)d\theta = (k^L + \hat{\theta}^L lr)g(\hat{\theta}^L)\frac{\partial \hat{\theta}^L}{\partial l}. \tag{9}$$

The left-hand side of this condition gives the benefit of a marginal increase in lending, which is the expected rate of return to the bank on the inter-bank loan whenever the bank survives the asset shock at date 1. The right-hand side of the condition gives the cost of lending at the margin, which is the marginal increase in bankruptcy risk, $g(\hat{\theta}^L)\frac{\partial \hat{\theta}^L}{\partial l}$, applied to the illiquidity of the asset in place, k^L, and the expected

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[14]We can confine our analysis to considering $\hat{\theta}^L \leq \bar{\theta}$. The lending bank would not choose $l(r) > 0$ such that $\hat{\theta}^L(l > 0) > \bar{\theta}$. For the case of $\hat{\theta}^L < \underline{\theta}$, we define $g(\theta < \underline{\theta}) \equiv 0$.

gross lending return lr at the bankruptcy cutoff $\hat{\theta}^L$.

Assumption 1. *We assume that the second order condition holds for an interior lending solution. We show in the appendix that a uniform distribution $g(\cdot)$ and sizable enough values for leverage ρ^L and moral hazard k^L are sufficient for this.*

Then, it can be shown that

Lemma 3. *The lending bank's supply on the inter-bank market $l^*(r) \in [0,1]$ is increasing in r and is decreasing in leverage ρ^L and the severity of moral hazard k^L.*

Intuitively, the lending bank holds precautionary liquidity to reduce the rollover risk on its short-term leverage arising from illiquidity of the asset in place. The rollover risk materializes whenever there is adverse information revelation in the ex-interim period about the asset quality. The induced precautionary demand for liquidity reduces the bank's supply of inter-bank lending.[15]

3.2   Inter-bank market freeze

In the appendix, we derive the borrowing bank's demand in the inter-bank market, $b^*(r)$, when also facing a risk-shifting problem with short-term debt, analogous to the lending bank's optimization for $l^*(r)$. For the analysis in this section, we simply assume the properties of $b^*(r)$ that are derived in the appendix: The borrowing demand $b^*(r) \in (0,1)$ is decreasing in the inter-bank rate r, the borrowing bank's short-term leverage ρ^B, and the severity of its moral hazard k^B. Also, the bank does not borrow at an interest rate greater than the return on the asset: $b^*(r > y) = 0$.

Consider first the case of no leverage or moral hazard costs for the borrowing bank, $\rho^B = k^B = 0$. In this case, the borrowing bank has a perfectly elastic demand to borrow at a rate that is not greater than the rate of return that investment pays: $b^*(r) = 1$ for $r \leq y$, and $b^*(r) = 0$ for $r > y$.

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[15]Recall that when there are no agency problems or rollover risk as in the benchmark model, inter-bank lending *helps* to meet the lending bank's leverage payments since a part of the borrowing bank's investment opportunity is pledgeable to the lending bank's creditors. However, with agency problems and rollover risk, an increase in inter-bank lending at a given rate *exacerbates* the lending bank's difficulty in meeting its leverage payments.

Consider also for sake of illustration a uniform distribution of $g(\theta)$ on the interval $[\underline{\theta}, \bar{\theta}]$. Then, there is an explicit solution for the lending bank's problem:

$$r(l^*) = \frac{2[y(\bar{\theta} - \hat{\theta}^L) + k^L]}{y[\bar{\theta}^2 - (\hat{\theta}^L)^2] - 2\hat{\theta}^L l}.$$

(10)

For parameters such that $r(0) > y$, a *lending freeze* occurs. In a lending freeze, the lending bank does not supply any amount of the inter-bank loan at an interest rate below the rate of return on the investment assets because the lending bank prefers to hoard liquidity for precautionary reasons rather than to lend. Formally, equation (10) implies that $r(0) > y$ iff

$$(\bar{\theta}y - 1)^2 < (\rho^L + k^L - 2)^2 + k^L,$$

(11)

which is satisfied whenever the lending bank's leverage ρ^L and moral hazard cost k^L are sufficiently large. Figure 4 illustrates such an inter-bank lending freeze.

Next, consider the general case including rollover risk and moral hazard costs for the borrowing bank, that is, with positive ρ^B and k^B. The condition for a freeze in the inter-bank market in this case is

$$\max r(b = 0) \leq \min r(l = 0),$$

(12)

under which there is no interest rate at which inter-bank lending will occur. Such a freeze can also arise at interest rates $r < y$ since the borrowing bank too is concerned about its own ability to roll over short-term debt and reduces borrowing as the interest rate in the inter-bank market rises.

3.3 Inter-bank market stress

More broadly, in an interior equilibrium the amount of inter-bank lending is less than a full unit and is lower if the lending bank or the borrowing bank is more
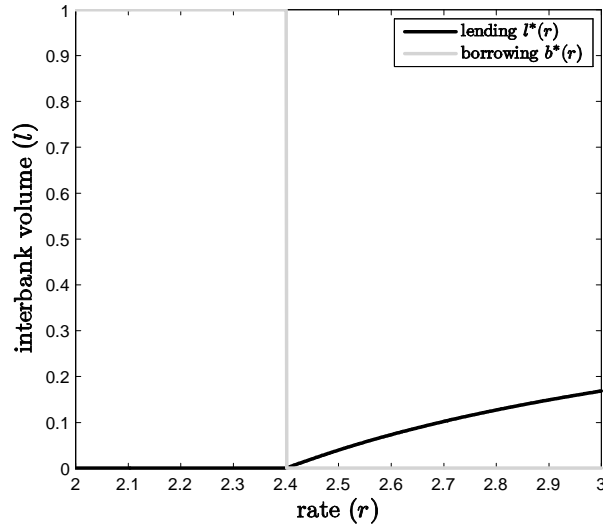
Figure 4: Inter-bank Lending Freeze. Large leverage ρ^L and moral hazard k^L for the lending bank can cause a lending freeze in the inter-bank market, where the lending bank does not supply any amount of the inter-bank loans at an interest rate below the return on the investment assets: $l^*(r \leq y) = 0$. The borrowing bank never borrows at $r > y$. Parameters are $y = 2.4$, $\rho^L = 1.5$, $\rho^B = 0$, $k^L = 0$, $k^B = 0.5$, and $\theta \tilde{} U[0.4, 1]$.

leveraged or if their moral hazard problems are more severe. Neither bank risk-shifts in equilibrium, but the possibility for risk-shifting in the future creates rollover risk and leads the lending bank to hoard liquidity in advance and the borrowing bank to reduce borrowing at high rates.

Proposition 1. Stress in inter-bank lending. *The equilibrium inter-bank lending quantity l^* is decreasing in the lending and borrowing banks' leverage ρ^i and moral hazard k^i: $\frac{dl^*}{d\rho^i} \leq 0$ and $\frac{dl^*}{dk^i} \leq 0$ for $i = B, L$.*

It is also the case that as moral hazard or leverage increases for the lending bank, the equilibrium inter-bank rate increases. Conversely, as the moral hazard problem becomes more severe or leverage increases for the borrowing bank, its demand for term borrowing decreases, which drives the inter-bank rate down.

Proposition 2. Stress in inter-bank rates. *The equilibrium inter-bank rate r^* is increasing in the lending bank's leverage ρ^L and moral hazard k^L, $\frac{dr^*}{d\rho^L} \geq 0$ and*
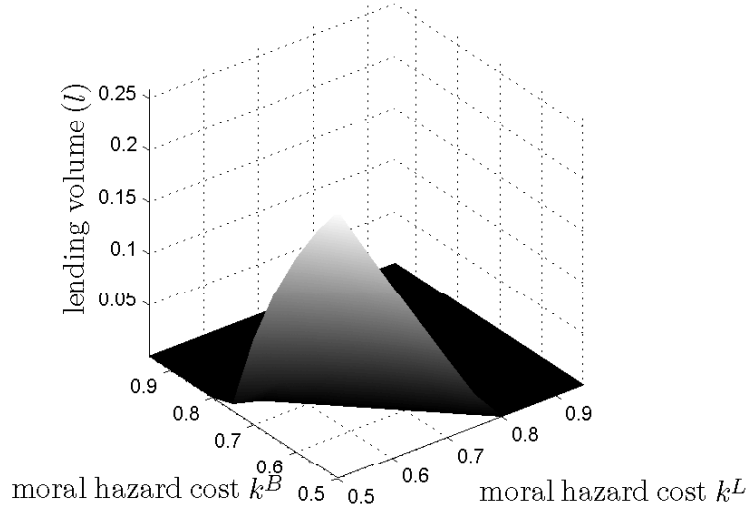
16

Figure 5: Equilibrium Lending Decreases in Banks' Moral Hazard. The figure illustrates the decrease in equilibrium lending l^* with increasing moral hazard costs by the lending bank, k^L, and the borrowing bank, k^B. Lending freezes entirely, $l^* = 0$, for large enough moral hazard. Parameters are $y = 2.4$, $\rho^L = 1.3$, and $\rho^B = 0$, $\theta \tilde{} U[0.4, 1]$.

$\frac{dr^*}{dk^L} \geq 0$, and decreasing in the the borrowing bank's leverage ρ^B and moral hazard k^B, $\frac{dr^*}{d\rho^B} \leq 0$ and $\frac{dr^*}{dk^B} \leq 0$.

Figure 5 illustrates the sharp decrease in equilibrium lending with increasing moral hazard costs (or illiquidity of the asset in place) k^B and k^L. Lending freezes entirely for severe enough moral hazard, and similar illustrations are obtained for large enough short-term leverage (ρ^B and ρ^L).

3.4 The alternative view: Counterparty risk

Consider again the contrast of our model with the traditional view of spreads in interbank rates as based solely on counterparty (borrower) credit risk. With rollover risk and induced precautionary desire to hold liquidity, the lender requires an interest rate spread *above* the rate adjusted for the borrower's credit risk: $r > \frac{1}{E[\theta]}$. The lender requires this spread to compensate it for the risk of not being able to roll

over its short-term debt. This spread increases in the lender's leverage, its asset illiquidity, and uncertainty about its asset quality.

The increased spread charged by the lender induces the borrowing bank to reduce its demand for inter-bank borrowing, as it faces its own rollover risk. At higher rates, the borrower finds it less worthwhile to make long-term investments at the expense of taking on rollover risk and cuts back borrowing. Viewed another way, the borrower too exhibits precautionary hoarding since it effectively maintains more liquidity for itself by borrowing less to fund long-term projects. As in the case of the lender, the borrower's precautionary behavior is stronger as its leverage, asset illiquidity, and asset risk rise.

4   Empirical predictions and relevance of results

Our analytical results on hoarding of liquidity by banks and its effect on inter-bank rates are corroborated by empirical findings in the extant literature. Further, the model also provides testable implications for teasing out borrower (demand) and lender (supply) effects in inter-bank markets. The precautionary liquidity view provides ground for new policy insights. Moreover, the precautionary basis for liquidity hoarding sits in contrast to the conventional basis for liquidity hoarding based on strategic reasons. We discuss these points in turn.

4.1   Extant empirical evidence

The empirical literature on the inter-bank market predominantly examines the overnight market, which we first examine. We then contrast this literature with the emerging empirical literature on term inter-bank markets.

4.1.1   Liquidity hoarding and inter-bank markets

Acharya and Merrouche (2009) show empirically that during the first year of the crisis (August 2007 to June 2008), some settlement banks in the United Kingdom

voluntarily revised upward their reserve balance targets with the Bank of England. Such revisions followed critical dates of the crisis, such as the freeze in the asset-backed commercial paper market of August 8, 2007, the collapse of Northern Rock in mid-September 2007 and that of Bear Stearns in mid-March of 2008. In the cross-section of banks, hoarding of liquidity – as measured by increases in reserve balance targets – was greater for banks that had suffered greater equity losses in the crisis (low θ realization in the model) and had greater reliance on overnight wholesale financing (greater ρ). They also document that this hoarding caused increases in inter-bank lending rates for all other banks, in line with our Proposition 2.

Ashcraft, McAndrews and Skeie (2010) argue with similar evidence for the United States during the crisis: to insure themselves against intraday liquidity shocks, weaker banks facing heightened rollover risk held larger reserve balances. In particular, they report that banks sponsoring ABCP conduits witnessed increased payments shocks, and that greater payments shocks led to an increase in bank's reserves. In addition, banks appear to have responded to higher uncertainty about payments during the crisis by becoming more reluctant to lend excess reserves to other banks when reserves were high. These results are also suggestive of a precautionary demand for liquidity and are entirely consistent with Lemma 3.

4.1.2   Overnight versus term inter-bank markets

Ashcraft, McAndrews and Skeie (2010) and Afonso, Kovner and Schoar (2010) argue that overnight inter-bank lending in the fed funds and Eurodollar market increased through much of the early crisis and held up well even after the Lehman bankruptcy. However, Kuo, Skeie, Youle and Vickery (2011) suggest this was *not* the case for maturities longer than overnight, especially one-month onwards. They argue that inter-bank lending volumes generally fell during the heart of the crisis while term spreads increased. Kuo, Skeie, Youle and Vickery (2011) claim a decline in the maturity-structure (as illustrated in Figure 2), in line with our Proposition 1. The claim of a decrease in term lending provides an explanation for the steady and even

increasing levels of overnight inter-bank lending: borrowing banks, facing heightened costs of term borrowing, shifted from term to overnight markets. Focusing exclusively on the resilience of overnight inter-bank volumes can thus paint too rosy a picture of inter-bank markets as it ignores the underlying stress felt in term inter-bank markets. There is a potential caveat with the empirical evidence presented in Section 4.1. Estimates of U.S. inter-bank activity are obtained by applying an algorithm based on Furfine (1999) to Fedwire Funds data. Preliminary attempts by NYFRB researchers (MaPS function) were unable to assess how large are the type I and II errors generated by the algorithm.

4.2 Novel empirical predictions

Our model also offers several new testable implications:

First, Lemma 3 shows that a bank's lending rate for a particular maturity in the inter-bank market increases with its own credit risk (e.g., balance-sheet leverage), illiquidity of assets (e.g., holding of complex assets) and rollover risk (e.g., nature of leverage – uninsured or wholesale deposits relative to insured or retail deposits), controlling for the credit risk of the counterparties that borrow. More uniquely to our model, Proposition 2 shows that a bank's borrowing rate for a particular maturity in the inter-bank market increases with credit risk, illiquidity and rollover risk of the lender, controlling for the borrower's own credit risk.[16]

Second, our model suggests tests that combine inter-bank rates and volumes. An increase in the lender or borrower bank's leverage drives down the bank's lending supply or borrowing demand for loans, respectively, which decreases the amount of inter-bank lending (Proposition 1). An increase in the lender leverage, however, *increases* the equilibrium inter-bank rate; in sharp contrast, an increase in the bor-

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[16]In the model, we assume that banks act competitively, yet we recognize that inter-bank markets are often segmented and that banks may not interact with all others. We model this by considering individual borrower-lender pairs, in which both banks act as price-takers. An equilibrium inter-bank rate thus depends on the lender's supply curve and hence the lender's characteristics. Empirically, our model suggests that a rate at which a bank borrows depends on the characteristics of the lending bank.

rower leverage *decreases* the equilibrium inter-bank rate (Proposition 2). A joint analysis of inter-bank rates and volumes can thus help tease out the effects of lender supply versus borrower demand shifts. To the best of our knowledge, such joint analysis of inter-bank rates and volumes with borrower and lender fixed effects (or characteristics) has not yet been conducted. All of these tests should hold if we replaced borrower or lender leverage with asset illiquidity.

Third, our model suggests that an increase in the risk of asset-level shocks increases term inter-bank rates, and reduces term inter-bank volumes. This is not just due to an increase in the borrower's credit risk. It is *also* due to an increase in the rollover risk of the lender, as in Propositions 1 and 2. That is, term inter-bank rates and volumes should contain an interaction effect between risk (e.g., realized or implied market volatility, as reflected in the VIX) and both borrower and lender leverage and rollover risk.[17]

Finally, our results indicate that measures of inter-bank market rates such as LIBOR do not necessarily indicate the full breakdown that may occur in the inter-bank market since there is no coincident provision of information on volumes. When there is a complete breakdown of terms between some borrowers and lenders, the inter-bank rate between some parties is not even well-defined. Rates based on actual or quoted transactions may mask the breakdown in some parts of the market, understating the market average rate. Hence, measurement and reporting of *volumes* in term inter-bank markets is crucial for understanding the stress and collapse in these markets.[18]

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[17]Furfine (2010) finds that the LIBOR-OIS spread is related to VIX over time, supporting our prediction, and results could be further tested by examining separate borrower and lender effects.

[18]Documenting transaction volumes that go with one, three and six month LIBOR rates is potentially also important as they are used to index over $360 trillion of notional financial contracts, as estimated by the British Bankers' Association (BBA), ranging from interest rate swaps and other derivatives to floating-rate residential and commercial mortgages.

4.3 Policy conjectures

It is important to consider cases where the lending banks' precautionary demand for liquidity may be excessive relative to its socially efficient level. In turn, this would suggest possible policy interventions to address excessive hoarding of liquidity by highly leveraged banks. Is it desirable to have an unconditional (traditional) lender of last resort (LOLR) in which a central bank provides liquidity to strong as well as weak banks? Or would it be better to have a solvency-contingent LOLR in which the central bank provides liquidity only to sufficiently strong banks? And should there instead (or in addition) be a resolution authority that forces weak banks to reduce their rollover risk? We conjecture that (i) a resolution authority to address weak banks' excess leverage and rollover risk, and (ii) a solvency-contingent LOLR by a central bank (that itself has lower credit and rollover risk than its banks), are likely to be more effective interventions than the traditional, unconditional LOLR.

Such welfare analysis can also help understand the impact of interventions that were put in place during the crisis of 2007-09, such as the Term Auction Facility (TAF) by the Federal Reserve for 28-day and later 84-day loans. We conjecture that through these facilities, the Federal Reserve, acting as a relatively risk-free intermediary, intermediated liquidity hoardings (reserves) of riskier banks to banks that participated in the facilities. This intervention should have increased volumes of lending to the real sector by the participating banks. Note that under the alternative view, that stress in inter-bank markets is caused purely by borrower credit-risk problems, lending by central bank to banks at lower than market rates would effectively constitute a "bail out" of these banks and engender severe moral hazard. However, a resolution authority to address weak banks' leverage and rollover risk would be a robust intervention also under this alternative view.

4.4 Related literature

There is a growing body of theoretical literature on inter-bank markets. Our focus is on the positive implications for the terms (quantity and interest rates) of liquid-

ity transfers in inter-bank markets when banks have short-term leverage and face attendant agency problems. Hence, we restrict discussion of the related literature on this theme.[19]

Rollover risk in our model induces banks to hold liquidity and raise inter-bank rates or withdraw liquidity altogether from inter-bank markets. The literature has also explored other motives for banks' desires to hold liquidity in crises. Acharya, Shin and Yorulmazer (2008) derive a strategic motive for holding cash. When banks' ability to raise external financing is low, they anticipate fire sales of assets by troubled banks and as a result hoard liquidity and forego profitable but illiquid investments. Diamond and Rajan (2009) also study long-term credit contraction that operates through a channel of asset fire sales. During a crisis, banks delay asset sales as part of their efforts to stay alive (a version of the risk-shifting problem). In turn, high rates are required ex ante on term loans to the real sector. Finally, Caballero and Krishnamurthy (2008) derive a propensity for firms to hoard liquid assets and reduce risk-sharing when there is Knightian uncertainty about their risks.

While these papers focus on aggregate liquidity shortages and strategic or behavioral demand for liquidity by bank(er)s, we derive instead a precautionary demand for liquidity by (weak) banks as contributing to heightened borrowing costs for (even safe) banks. In a contemporary paper, Gale and Yorulmazer (2010) model both the precautionary and the strategic motive for holding cash and show that banks may hoard liquidity and lend less than the maximum possible amount, as in our model. In our paper as well as in these other papers, a common theme is that the increase in bank propensity to hold liquidity is in *anticipation* of crises, rather than (just) upon their incidence. Diamond and Rajan (2005) also show how asset liquidations

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[19]Goodfriend and King (1988) provide the benchmark result that with complete markets, inter-bank lending allows for the efficient provision of lending among banks. To obtain deviations from this benchmark, Flannery (1996), Freixas and Jorge (2007), Freixas and Holthausen (2005) and Heider, Hoerova and Holthausen (2009) consider asymmetric information among banks, whereas Donaldson (1992) and Acharya, Gromb and Yorulmazer (2007) consider strategic behavior by relationship-specific lenders. Skeie (2004, 2008) and Freixas, Martin and Skeie (2011) study the effects of nominal contracts and monetary policy, respectively, on inter-bank markets and banking fragility. Finally, Acharya, Gale and Yorulmazer (2008) show how rollover risk can arise upon adverse news even in absence of agency problems.

by some banks can ex post reduce the endogenous amount of aggregate liquid resources available to even fundamentally healthy banks. The contagion in their paper also operates through an increase in inter-bank market rates and results in a decrease in lending to the real sector. This is, however, an *ex post* contagion rather than an *ex ante* one that is in anticipation of insolvency or rollover risk (as in our model).

5   Concluding remarks

Stress and freezes in term inter-bank lending markets can be explained by rollover risk of highly leveraged lenders and illiquidity of assets underlying term loans. We showed that the term inter-bank lending rates and volumes are jointly determined, reflecting the precautionary demand for liquidity of lenders and aversion of borrowers to trade at high rates of interest, both induced by their respective rollover risks. The model's implications are consistent with several empirical papers that study inter-bank activity during financial crises.

The primary message from our analysis is that heightened rates and reduced volumes in inter-bank markets should not necessarily be interpreted as being caused solely by counterparty credit risk. These phenomena may reflect the reluctance of banks to give up liquidity for long maturities, or conversely, their precautionary demand for liquidity.

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Appendix: Proofs

Analysis following Lemma 1. Consider the case where the lending bank can fully pledge returns on the inter-bank loan, but the bank still faces the moral hazard problem on the asset in place. The bank's expected profit under no risk shifting is $\theta(y + lr - f^L)$ and under risk shifting is $\theta_R^L(y^L + \theta lr - f^L)$. The incentive constraint not to risk shift is $\theta f^L \le \theta(y^L + lr) - k^L$. The individual rationality constraint for the bank's depositors is $\rho^L - (1 - l) \le \theta f^L$. Together, the two constraints imply a rollover constraint of

$$l \le \frac{\theta y^L - k^L - \rho^L + 1}{1 - \theta r}.$$

For any lending rate $r \ge \frac{1}{\theta}$, an increased quantity of lending l relaxes the rollover constraint. The bank prefers to lend its full liquidity. For parameters such that the rollover constraint does not hold for $l = 1$ at a given lending rate r, the bank will default at date 1 regardless of its inter-bank lending. The rate requirement for the bank to to be able to roll over its debt is $r \ge \frac{k^L + \rho^L}{\theta} - y^L$, which is increasing in the severity of the bank's moral hazard, k^L.

In contrast, consider the case where the lending bank cannot pledge returns on the inter-bank loan. Instead, suppose there is no moral hazard problem: $k^L = 0$. The rollover constraint is given by $l \le \theta y - \rho^L + 1$ and is tightened by increased lending l. The bank does not lend its full liquidity if $\rho^L > \theta y$. Moreover, for the case of moral hazard case, when $k^L > 0$, the quantity of lending declines linearly in k^L, as seen by the rollover constraint (6).

Assumption 1. We make two assumptions that ensure that the second order condition is satisfied. First, we assume a uniform distribution for $g(\cdot)$, which is

always sufficient to satisfy the condition needed for $g'(\hat{\theta}^L)$ to be not too small. This ensures that the lending bank has a minimal enough increase in its marginal bankruptcy risk for marginal increases in its bankruptcy cutoff value $\hat{\theta}^L$. Second, we assume large enough parameters for k^L and ρ^L relative to y such that

$$l > \frac{y}{2r} - \frac{1}{2}(\rho^L + k^L - 1). \tag{13}$$

Proof of Lemma 3. To study the second order condition of lender's optimization problem, note that

$$\frac{\partial^2 \pi^L}{\partial l^2} = -\frac{1}{y}(2\hat{\theta}^L r + \frac{lr}{y} - 1)g(\hat{\theta}^L) - \frac{1}{y^2}(\hat{\theta}^L lr + k^L)g'(\hat{\theta}^L) \tag{14}$$

$$= -\frac{g(\hat{\theta}^L)}{y}\left[2\hat{\theta}^L r + \frac{lr}{y} - 1 + \frac{1}{y}(\hat{\theta}^L lr + k^L)\frac{g'(\hat{\theta}^L)}{g(\hat{\theta}^L)}\right]. \tag{15}$$

For $g'(\hat{\theta}^L) \geq 0$, which is satisfied by a uniform distribution for $g(\cdot)$, condition (13) is sufficient for $\frac{\partial^2 \pi^L}{\partial l^2} < 0$. For $l \in [0, 1]$, we can see that lending is increasing in r, since

$$\frac{\partial^2 \pi^L}{\partial l \partial r} = \int_{\hat{\theta}^L (l)}^{\bar{\theta}} \theta g(\theta)d\theta - \hat{\theta}^L \frac{l}{y}g(\hat{\theta}^L) \tag{16}$$

$$\geq \hat{\theta}^L g(\hat{\theta}^L)(1 - \frac{l}{y}) \tag{17}$$

$$\geq 0, \tag{18}$$

where the last inequality holds since $l \leq 1 < y$. Lending is decreasing in ρ^L, since

$$\frac{\partial^2 \pi^L}{\partial l \partial \rho} = -(\hat{\theta}^L r - 1)g(\hat{\theta}^L)\frac{1}{y} - \frac{lr}{y}g(\hat{\theta}^L)\frac{1}{y} \leq 0, \tag{19}$$

which is satisfied by condition (13). Lending is also decreasing in k^L, since

$$\frac{\partial^2 \pi^L}{\partial l \partial k^L} = -(\hat{\theta}^L r - 1)g(\hat{\theta}^L)\frac{1}{y} - (1 + \frac{lr}{y})g(\hat{\theta}^L)\frac{1}{y} \leq 0, \tag{20}$$

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which is always satisfied for $l \leq 1$ and $r \leq y$; the borrowing bank demand is never positive for $r > y$, which can be excluded. ∎

Proof of Proposition 2 and 3. In equilibrium, $l^*(r^*, x) = b^*(r^*, x)$ and hence $\frac{dl^*}{dx} = \frac{db^*}{dx}$ for $x \in \{k^B, k^L, \rho^B, \rho^L\}$. Thus,

$$\frac{\partial l^*}{\partial x} + \frac{\partial l^*}{\partial r^*}\frac{dr^*}{dx} = \frac{\partial b^*}{\partial x} + \frac{\partial b^*}{\partial r^*}\frac{dr^*}{dx}, \tag{21}$$

so we have

$$\frac{dr^*}{dx} = -\frac{[\frac{\partial b^*}{\partial x} - \frac{\partial l^*}{\partial x}]}{[\frac{\partial b^*}{\partial r^*} - \frac{\partial l^*}{\partial r^*}]}. \tag{22}$$

Now, $\frac{\partial b^*}{\partial r^*} \leq 0$ and $\frac{\partial l^*}{\partial r^*} \geq 0$, therefore $\text{sign}(\frac{dr^*}{dx}) = \text{sign}(\frac{\partial b^*}{\partial x} - \frac{\partial l^*}{\partial x})$. For $x = k^B$, $\frac{\partial l^*}{\partial x} = 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dk^B} \leq 0$. For $x = k^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{dk^L} \geq 0$. For $x = \rho^B$, $\frac{\partial l^*}{\partial x} = 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{d\rho^B} \leq 0$. For $x = \rho^L$, $\frac{\partial l^*}{\partial x} \leq 0$ and $\frac{\partial b^*}{\partial x} \leq 0$, thus $\frac{dr^*}{d\rho^L} \geq 0$.

Consider

$$\frac{dl^*}{dx} = \frac{\partial l^*}{\partial x} + \frac{\partial l^*}{\partial r^*}\frac{dr^*}{dx}. \tag{23}$$

For $x = k^L$, as shown above,

$$\frac{dr^*}{dk^L} = \frac{\frac{\partial l^*}{\partial k^L}}{\frac{\partial b^*}{\partial r^*} - \frac{\partial l^*}{\partial r^*}}, \tag{24}$$

therefore

$$\frac{dl^*}{dk^L} = \frac{\partial l^*}{\partial k^L}\left[1 - \frac{\frac{\partial l^*}{\partial r^*}}{\frac{\partial l^*}{\partial r^*} - \frac{\partial b^*}{\partial r^*}}\right]. \tag{25}$$

Now

$$\frac{\frac{\partial l^*}{\partial r^*}}{\frac{\partial l^*}{\partial r^*} - \frac{\partial b^*}{\partial r^*}} \leq 1 \tag{26}$$

as $\frac{\partial b^*}{\partial r^*} \leq 0$; hence, $\frac{dl^*}{dk^L} \leq 0$. Similarly, as $\frac{\partial l^*}{\partial \rho^L} \leq 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{d\rho^L} \geq 0$, we have $\frac{dl^*}{d\rho^L} \leq 0$. As $\frac{\partial l^*}{\partial k^B} = 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{dk^B} \leq 0$, we have $\frac{dl^*}{dk^B} \leq 0$. Finally, as $\frac{\partial l^*}{\partial \rho^B} \leq 0$, $\frac{\partial l^*}{\partial r^*} \geq 0$, and $\frac{dr^*}{d\rho^B} \leq 0$, we have $\frac{dl^*}{d\rho^B} \leq 0$. ∎

Appendix: Borrowing bank

At date 1, the borrowing bank needs to roll over short-term debt ρ^B by issuing new debt to depositors with a face amount f^B due at date 2. Depositors of the borrowing bank can verify whether the quantity $(1+b)$ invested in the asset has a positive payoff, but the cannot distinguish the level of the positive payoff or whether the bank risk-shifts. The bank's incentive constraint not to risk shift is

$$\theta[(1+b)y - br - f^B] \geq \theta_R^B[(1+b)y_R^B - br - f^B]. \tag{27}$$

Greater amounts of inter-bank borrowing and additional investment into the asset increase the borrowing bank's moral hazard problem and tighten the incentive constraint.

Similar to the lending bank, the borrowing bank's incentive constraint (27) holds if and only if the realization of θ is large enough that

$$f^B \leq \frac{(1+b)(\theta y - \theta_R^B y_R^B)}{\theta - \theta_R^B} - br. \tag{28}$$

In the limit as $\theta_R^B \to 0$ and $\theta_R^B y_R^B \to k^B$, the incentive constraint (27) is

$$\theta f^B \leq (1+b)(\theta y - k^B) - \theta br. \tag{29}$$

Subject to the incentive constraint holding, the depositors' individual rationality constraint for rolling over the short-term debt amount ρ^B is

$$\rho^B \leq \theta f^B. \tag{30}$$

The rollover constraint depends on both the incentive constraint (27) and individual rationality constraint (30) holding:

$$\rho^B \leq \theta[(1+b)y - br] - (1+b)k^B. \tag{31}$$

We define $\hat{\theta}^B(b)$ as the bankruptcy cutoff value for the lending bank such that for

a large enough realization $\theta \geq \hat{\theta}^B(\cdot)$, the rollover constraint (31) holds. The cutoff $\hat{\theta}^B(b)$ is thus given by the rollover constraint (31) binding:

$$\hat{\theta}^B(b) = \frac{\rho^B + k^B(1+b)}{(1+b)y - br}. \tag{32}$$

The rollover risk for the borrowing bank increases in leverage ρ^B and the severity of moral hazard k^B.

Assumption B-1. *We assume large enough moral hazard k^B and not too large leverage ρ^B such that the bankruptcy cutoff is increasing in borrowing, $\frac{\partial \hat{\theta}^B}{\partial b} \geq 0$. Writing*

$$\frac{\partial \hat{\theta}^B}{\partial b} = \frac{(k^B + \rho^B)r - \rho^B y}{[(1+b)y - br]^2}, \tag{33}$$

$\frac{\partial \hat{\theta}^B}{\partial b} > 0$ *iff k^B is large enough and ρ^B is not too large relative to y such that*

$$r > \frac{\rho^B}{\rho^B + k^B} y. \tag{34}$$

Lemma B-1. *The borrowing bank cannot rollover its debt at date 1 if its probability of default is less than its bankruptcy cutoff value:$\theta < \hat{\theta}^B(b)$. The bank's cutoff value $\hat{\theta}^B$, and hence rollover risk, is increasing in the severity of moral hazard k^B, the inter-bank borrowing amount b, and the interest rate r, and is decreasing in leverage ρ^B and the payoff of the asset y.*

The borrowing bank's optimization is $\max_b \pi^B$, where π^B expected profits:

$$\pi^B \equiv \int_{\hat{\theta}^B(b)}^{\bar{\theta}} \{\theta[y + b(y - r)] - \rho^B\} g(\theta) d\theta. \tag{35}$$

For an interior solution $b^*(r) \in (0, 1)$, the first order condition is

$$\int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta(y - r)g(\theta)d\theta = k^B(1+b)g(\hat{\theta}^B)\frac{\partial \hat{\theta}^B}{\partial b}. \tag{36}$$

The LHS of the FOC gives the benefit of a marginal increase in borrowing, which is

equals the expected rate of return on investing in the asset minus the rate of return on borrowing, conditional on the borrowing bank meeting its liquidity rollover needs at date 1. The RHS of the FOC gives the cost, which is the increase in bankruptcy risk, $g(\hat{\theta}^B)\frac{\partial\hat{\theta}^B}{\partial b}$, multiplied by the moral hazard cost k^B and the amount of all assets $(1+b)$.

Remark B-1. *The borrowing bank does not borrow at an interest rate greater than the return on the asset:* $b^*(r > y) = 0$.

Proof. We will show that $b^*(r > y) = 0$. To prove by contradiction, suppose instead that $b(r > y) > 0$. Positive borrowing $b^*(r) > 0$ requires that $\frac{\partial\pi^B}{\partial b} \geq 0$. However, with $r > y$, both terms in the RHS of equation (36) are negative, which implies $\frac{\partial\pi^B}{\partial b} < 0$, a contradiction. Thus, $b^*(r > y) = 0$. ∎

Lemma B-2. *The borrowing bank's demand on the inter-bank market* $b^*(r) \in (0,1)$ *is decreasing in the inter-bank rate* r, *leverage* ρ^B, *and the severity of moral hazard* k^B.

Proof. To study the second order condition, for $g'(\cdot) = 0$,

$$\frac{\partial^2\pi^B}{\partial b^2} = \frac{-g(\hat{\theta}^B)[(k^B r)^2 - (\rho^B)^2(y-r)^2]}{[(1+b)y - br]^3}, \tag{37}$$

For $r \leq y$, $\frac{\partial^2\pi^B}{\partial b^2} < 0$ iff $r > \frac{\rho^B}{\rho^B + k^B}y$, which holds by Assumption 2. Continuing assuming $g'(\cdot) = 0$ and $r > \frac{\rho^B}{\rho^B + k^B}y$, we can see that borrowing is decreasing in r, since

$$\frac{\partial^2\pi^B}{\partial b\partial r} = -\int_{\hat{\theta}^B(b)}^{\bar{\theta}} \theta g(\theta)d\theta - \hat{\theta}^B g(\hat{\theta}^B)(y-r)\frac{\partial\hat{\theta}^B}{\partial r} - \frac{k^B(1+b)g(\hat{\theta}^B)(k^B + \rho^B)}{[(1+b)y - br]^2} \tag{38}$$

$$- \frac{2k^B(1+b)g(\hat{\theta}^B)(k^B + \rho^B)r - \rho^B y)b}{[(1+b)y - br]^3} \leq 0. \tag{39}$$

Borrowing is decreasing in ρ^B since

$$\frac{\partial^2\pi^B}{\partial b\partial\rho^B} = -\frac{\rho^B(y-r)g(\hat{\theta}^B)}{[(1+b)y - br]^2} \leq 0. \tag{40}$$

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Borrowing is decreasing in k^B since

$$\frac{\partial^2 \pi^B}{\partial b \partial k^B} = -\frac{(1+b)[k^B(1+b)y + k^B(1-b)r](\hat{\theta}^B)}{[(1+b)y - br]^2} \leq 0. \tag{41}$$

Also, note that when Assumption B-1 does not hold, $r < \frac{\rho^B}{\rho^B + k^B}y$, then $\frac{\partial \pi^B}{\partial db} > 0$ and $b^*(r) = 1$. ∎