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Abstract

Cutting government spending can increase the budget deficit at zero interest rates according to a standard New Keynesian model, calibrated with Bayesian methods. Similarly, increasing sales taxes can increase the budget deficit rather than reduce it. Both results suggest limitations of "austerity measures." At zero interest rates, running budget deficits can be either expansionary or contractionary depending on how they interact with expectations about long-run taxes and spending. The effect of fiscal policy action is thus highly dependent on the policy regime. A successful stimulus, therefore, needs to specify how the budget is managed, not only in the short but also in the medium and long runs.

Key words: fiscal policy, liquidity trap

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1 Introduction

What is the effect of government spending cuts or tax hikes on the budget deficit? What is the effect of the budget deficit itself on short-run and long-run outcomes? Does the answer to these questions depend upon the state of the economy? Does it matter, for example, if the short-term nominal interest rate is close to zero and the economy is experiencing a recession? These are basic and fundamental questions in macroeconomics that have received increasing attention recently. Following the crisis of 2008, many governments implemented somewhat expansionary fiscal policy, but were soon confronted with large increases in public debt. That gave rise to calls for "austerity", i.e. government spending cuts and tax hikes – aimed to decrease government debt – a policy many claimed was necessary to restore "confidence." It follows that a model of the economy that makes sense of the policy discussion during this time has to account for the crisis and provide a role for fiscal policy, while also explicitly accounting for public debt dynamics. That is the objective of this paper.

The goal of this paper is to analyze public debt dynamics in a standard New Keynesian dynamic stochastic general equilibrium (DSGE) model in a low interest rate environment. One of our main findings is that the rules for budget management change once the short-term nominal interest rate approaches zero in a way that is important for the debate on "austerity" and "confidence." This shows up in our model in at least two ways. First, we show that once the short-term nominal interest rate hits zero, then cutting government spending or raising sales taxes has very different effects on deficits than under regular circumstances. Under regular circumstances, these austerity policies reduce the deficit roughly one-to-one. Once the nominal interest rate reaches zero, however, their effect becomes much smaller on the deficit. These policies may even increase rather than reduce the deficit. Second, we find that the economy is extraordinarily sensitive to expectations about the long-run at a zero interest rate. In particular, expectations about the future size of the government and future sales and/or labor taxes can have strong effects on short-run demand. This is, again, in contrast to an economy where the nominal interest rate is positive and the central bank targets zero inflation. In that environment, we show that long-run expectations about fiscal policy have no effect on short-run demand. An important implication of this is that in a low interest rate environment budget deficits can either increase or reduce aggregate demand in the short run, depending on how they influence expectations of future taxes, spending, and monetary policy. Hence the effect of deficit spending when the nominal interest rate is zero *depends critically on the policy* *regime.*

At a basic level, our paper highlights a general theme already emphasized in Eggertsson (2010) that once the nominal interest rate collapses close to zero, then the economy is demand-determined, i.e. the amount of stuff produced is entirely determined by how much stuff people want to buy. Thus, according to this framework, all emphasis should be on policies that increase aggregate spending in the short run. A key point this paper highlights is that short-run demand at a zero interest rate is not only determined by short-run fiscal policy, but also by expectations about the long run. The budget deficit is plausibly going to play a large role in how those long-run expectations are determined. More importantly, the way in which the budget deficit pins down those expectations depends critically upon the policy regime and can substantially affect estimates of various policy relevant issues, such as the computation of fiscal multipliers currently common in the literature. The outline of the paper is detailed next.

After laying out and parameterizing the model (Section 2), we first confront it (Section 3) with the following thought experiment: Suppose there are economic conditions such that the nominal interest rate is close to zero and the central bank wants to cut rates further, but cannot. Suppose sales and labor tax *rates* are held constant. How does the budget deficit change if the government tries to balance the budget by cutting government spending, i.e. implementing "austerity measures"? The model suggests that, under reasonable parameters, the deficit declines only by a modest amount and may even increase, rather than decrease. This occurs because the cut in government spending leads to a reduction in aggregate output, thus reducing the tax base and subsequently reducing tax revenues. We derive simple analytical conditions under which the deficit increases as a result of cuts in government spending. When we conduct the same experiment with sales taxes, we obtain a similar result. To a keen observer of the current economic turmoil, then, it may seem somewhat disturbing that expenditure cuts and sales tax increases were two quite popular "austerity measures" in response to the deficits following the crisis of 2008.

While the first set of results points against the popular call for "austerity," we have a second set of results that puts these calls, perhaps, in a bit more sympathetic light. We next consider (Section 4) the following question: How does demand in the short run react to expectations about long-run taxes, long-run productivity and the long-run size of the government? One motivation for this question is that we often hear discussion about the importance of "confidence" in the current economic environment and this is given as a rationale for reducing deficits. For example, Jean-Claude Trichet, then President of the European Central Bank, said in June 2010, "Everything that helps to increase the confidence of households, firms and investors in the sustainability of public finances is good for the consolidation of growth and job creation. We firmly believe that in the current circumstances confidence-inspiring policies will foster and not hamper economic recovery, because confidence is the key factor today." How does current demand, via "confidence," depend on future long-run policy? To be clear, we interpret "confidence" as referring to effects on current demand that come about due to expectations about long-run policy.

To get our second set of results, we consider how short-run demand depends upon expectations about long-run policy. We first look at the effect of long-run taxes and the long-run size of the government on short-run demand if the central bank is not constrained by the zero interest rate bound and successfully targets constant inflation. In this case, we show that expectations of future fiscal policy are irrelevant for aggregate demand. What happens in the model is that, if the central bank successfully targets inflation, it "replicates" the solution of the model that would take place if all prices were perfectly flexible, i.e. the "Real Business Cycle" (RBC) solution. Further, if all prices were flexible in the model, then aggregate demand would play no role in the first place. We then study the effect of fiscal policy expectations when there are large enough shocks so that the zero interest rate bound is binding and the central bank is unable to replicate the flexible price RBC solution. When this happens, the results are much more interesting: Output is completely demand-determined, i.e., the amount produced depends on how much people want to buy. Most importantly, expectations about future economic conditions start having an important effect on short-run demand and, thus, output. In turn, future economic conditions depend on long-run policy. Our key findings are that a commitment to reduce the size of the government in the long run or to reduce future labor taxes increases short-run demand. This is because both policies imply higher future private consumption, and thus will tend to raise consumption demanded in the short run. It is worth noting that any policy that tends to increase expectations of future output will also be expansionary in the same way. Meanwhile, a commitment to lower long-run sales taxes has the opposite effect, i.e., it reduces short-run demand. This is because lower future sales taxes induce people to delay short-run consumption to take advantage of lower future prices.

In Section 5, we analyze how debt dynamics may affect short-run demand. Taking short-run deficits as given, we ask: What are their effects on short-run demand given that they need to be paid off in the long run? In this case, we show that the effect of deficits depends – as a general matter – on the policy regime. If the deficits are paid off by a reduction in the long-run size of the government spending or higher long-run sale taxes, then the budget deficits are expansionary. If they are paid off by higher long-run labor taxes, then the budget deficits are contractionary. In Section 6, we look at some numerical examples. Section 7 shows how the picture changes once we account for the possibility of the government defaulting on its debt. Finally, we review that deficits are expansionary if they trigger expectations of medium-term inflation.

1.1 Related literature

The paper builds on Eggertsson (2010), who addresses the effects of taxes and spending on the margin, and a relatively large amount of literature on the zero interest rate bound (see in particular Christiano, Eichenbaum and Rebelo (2011) and Woodford (2011) for related analysis and more recently Rendahl (2012)). The contribution of this paper to the existing literature is that we study public debt dynamics and the interactions between output and debt, and between taxes and spending. An additional feature of the current paper is the greater attention to the short-run demand consequences of long-run taxes and spending. While there is some discussion in Eggertsson (2010) on the effect of permanent changes in fiscal policy, we here make some additional, but plausible assumptions, that allow us to illustrate the result in a much cleaner form and additionally illustrate some new effects. We consider the simple closed-form solution as a key contribution to the relatively large recent literature that studies the interaction of monetary and fiscal policy at the zero bound (see e.g. Leeper, Traum, Walker (2011) and references therein).

Our focus here is not on optimal policy. Instead we look at the effect of incremental adjustments in various tax and spending instruments at the margin. The hope is, of course, that these partial results give some insights in the study of optimal policy. A challenge for thoroughly studying optimal policy with a rich set of taxes (such as here) is that, in principle, the first best allocation can often be replicated with flexible enough taxes, as illustrated in Eggertsson and Woodford (2004) and Correia et al. (2011). Yet, as the current crisis makes clear, governments are quite far away from exploiting fiscal instruments to this extent. Most probably this reflects some unmodeled constraints on fiscal policy that prevent their optimal application. But even with these limitations, we think it is still useful to understand the answer to more specific questions, such as "What happens to output and the deficit if you do X?" The answer to this question often drives policy decisions. A politician, for example, may ask: "Can I reduce the budget deficit by doing A, B or C?"

The result that cuts in government spending can increase the deficit is close to the finding in Erceg and Linde (2010) that government spending can be self-financing in a liquidity trap. Relative to that paper, the main contribution is that we, first, show closed-form solutions for deficit multipliers and, second, we model how expectations of long-run policy may change short-run demand. The demand effect of the long-run labor tax policy is similar to that documented by Fernandez-Villaverde, Guerron-Quintana and Rubio-Ramirez (2011) and the permanent policies in Eggertsson (2010). The fact that a commitment to smaller government in the future can increase demand at the zero interest rate bound is illustrated in Eggertsson (2001) and Werning (2011). Eggertsson (2006) analyzes, in more detail, how deficits can trigger inflation expectations.

2 A simple New Keynesian model

We only briefly review the microfoundations of the model here, for a more complete treatment see Eggertsson (2010). The main difference between our model and the model from Eggertsson (2010) is that we are more explicit about the government budget constraint. There is a continuum of households of measure 1. The representative household maximizes

$$E_t \sum_{T=t}^{\infty} \beta^{T-t} \xi_T \left[u(C_T) + g(G_T) - \int_0^1 v(l_T(j))dj \right],$$

where β is a discount factor, $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ is the Dixit-Stiglitz aggregate of consumption of each of a continuum of differentiated goods with an elasticity of substitution equal to $\theta > 1$, $P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ is the Dixit-Stiglitz price index, and $l_t(j)$ is the quantity supplied of labor of type j. Each industry j employs an industry-specific type of labor, with its own real wage $W_t(j)$. The disturbance ξ_t is a preference shock, and $u(\cdot)$ and $g(\cdot)$ are increasing concave functions, while $v(\cdot)$ is an increasing convex function. G_T is the government spending and is also defined as a Dixit-Stiglitz aggregate analogous to private consumption. For simplicity, we assume that the only assets traded are one-period riskless bonds, B_t. The period budget constraint can then be written as

$$\begin{aligned} (1+\tau_t^s)P_t C_t + B_t &= (1+i_{t-1})B_{t-1} + (1-\tau_t^I)\left[\int_0^1 Z_t(i)di \right. \\ &\quad \left. + P_t \int_0^1 W_t(j) l_t(j) dj \right] - P_t T_t, \end{aligned} \tag{1}$$

where $Z_t(i)$ stands for profits that are distributed lump-sum to the households. There are three types of taxes in the baseline model: a sales tax τ_t^s on consumption purchases, a lump-sum tax T_t and an income tax τ_t^I (levied on income from both labor and the household's claim on firms profits).[1] The household maximizes the utility subject to the budget constraint, taking the wage rate as given. It is possible to include some resource cost of the lump-sum taxes, for example that collecting T_t taxes consumes $s(T_t)$ resources as in Eggertsson (2006) and total government spending is then defined as $F_t = G_t + s(T_t)$. Since we will not focus on the optimal policy, this alternative interpretation does not change any of the results.

There is a continuum of firms of measure 1. Firm i sets its price and then hires labor inputs necessary to meet realized demand, taking industry wages as given. A unit of labor produces one unit of output. The preferences of households and the assumption that the government distributes its spending on varieties in the same way as households imply a demand for good i of the form $y_t(i) = Y_t(\frac{p_t(i)}{P_t})^{-\theta}$, where $Y_t \equiv C_t + G_t$ is aggregate output. We assume that all profits are paid out as dividends and that firms seek to maximize profits. Profits can be written as $Z_t(i) = p_t(i)Y_t(p_t(i)/P_t)^{-\theta} - W_t(j)Y_t(p_t(i)/P_t)^{-\theta}$, where i indexes the firm and j indexes the industry in which the firm operates. Following Calvo (1983), let us suppose that each industry has an equal probability of reconsidering its price in each period. Let $0 < \alpha < 1$ be the fraction of industries with prices that remain unchanged in each period. In any industry that revises its prices in period t, the new price p_t^* will be the same. The maximization problem that each firm faces at the time it revises its price is then to choose a price p_t^* to maximize

$$\max_{p_t^*} E_t \left\{ \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T (1 - \tau_T^P)[p_t^* Y_T (p_t^*/P_T)^{-\theta} - W_T(j) Y_T (p_t^*/P_T)^{-\theta}] \right\},$$

where λ_T is the marginal utility of the nominal income for the representative household. An important assumption is that the price that the firm sets is *exclusive* of the sales tax. This means that if the government cuts sales taxes, then consumers face a lower store price by exactly the amount of the tax cuts for firms that have not reset their prices.

Without going into details about how the central bank implements a desired path for

[1] In an earlier variation of this paper, we assumed instead that only wages were subject to the income tax. Since wages are flexible, in this model, wages drop by more than output in a recession. This leads to a disproportionate drop in tax revenues in a recession that we felt exaggerated our results and relied too much on the complete flexibility of wages in the model. Under this alternative benchmark assumption, which is more conservative, income tax is proportional to aggregate output (any drop in real wages will be reflected by an increase in profits, and taxing profit and wages at the same rate means we abstract from this redistribution aspect of the model).

nominal interest rates, we assume that it cannot be negative so that[2]

$$i_t \geq 0.$$

The government's budget constraint can now be written as[3]

$$b_t = (1 + i_{t-1})b_{t-1}\Pi_t^{-1} + (1 + \tau_t^s)G_t + (1 - \tau_t^I)Y_t - T_t - (1 + \tau_t^s)Y_t,$$

where $b_t \equiv \frac{B_t}{P_t}$ is the real value of the government debt and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation. Note that we take into account that the government both pays the consumption tax τ_t^s and receives this tax back as revenues. If we wrote the budget constraint in terms of private consumption, C_t, this would net out.

The model is solved by an approximation around a steady state and we linearize it around a constant solution with positive government debt $\bar{b} > 0$ and zero inflation[4]. The consumption Euler equation of the representative household combined with the resource constraint can be approximated to yield

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t^e) + (\hat{G}_t - E_t\hat{G}_{t+1}) + \sigma\chi^s E_t(\hat{\tau}_{t+1}^s - \hat{\tau}_t^s), \tag{2}$$

where i_t is the one-period risk-free nominal interest rate, π_t is inflation, E_t is an expectation operator, the coefficients are $\sigma, \chi^s > 0$, $\hat{Y}_t \equiv \log Y_t/\bar{Y}$, $\hat{G}_t \equiv \log G_t/\bar{Y}$, while $\hat{\tau}_t^s \equiv \tau_t^s - \bar{\tau}^s$, and r_t^e is an exogenous disturbance that is only a function of the shock ξ_t (for details see footnote on the rationale for this notation).[5] Aggregate supply (AS) is

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^I\hat{\tau}_t^I + \chi^s\hat{\tau}_t^s - \sigma^{-1}\hat{G}_t) + \beta E_t\pi_{t+1}, \tag{3}$$

where the coefficients $\kappa, \psi > 0$ and $0 < \beta < 1$ and the zero bound is $i_t \geq 0$.

[2] See e.g. Eggertsson and Woodford (2003) for further discussion.

[3] To derive this, note that since profits by firm i are given by $Z_t(i) = p_t(i)Y_t(p_t(i)/P_t)^{-\theta} - P_tW_t(i)l_t(i)$ where we have used the Dixit-Stiglitz demand to substitute for $y_t(i)$. Then, we can aggregate to obtain $Y_tP_t = \int_0^1 Z_t(i)di + P_t\int_0^1 W_t(j)l_t(i)di$. This can be substituted into the representative household budget constraint. Then, using equation (1), we obtain the expression in the text.

[4] The steady state level of debt, $\bar{b}$, is not pinned down by our theory and thus we could pick various values for $\bar{b}$.

[5] The coefficients of the model are defined as follows $\sigma \equiv -\frac{\bar{u}_c}{\bar{u}_{cc}\bar{Y}}$, $\omega \equiv \frac{\bar{v}_{ll}\bar{L}}{\bar{v}_l}$, $\psi \equiv \frac{1}{\sigma^{-1}+\omega}$, $\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\sigma^{-1}+\omega}{1+\omega\theta}$, where a bar denotes that the variable is defined in steady state. The shock is defined as $r_t^e \equiv \bar{r} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$, where $\hat{\xi}_t \equiv \log \xi_t/\bar{\xi}$ and $\bar{r} \equiv \log \beta^{-1}$. Finally we define $\chi^I \equiv \frac{1}{1-\bar{\tau}^I}$ and $\chi^s \equiv \frac{1}{1+\bar{\tau}^s}$. In terms of our previous notation, i_t now actually refers to $log(1 + i_t)$ in the log-linear model. Observe also that this variable, unlike the others, is not defined in deviations from steady state. We do this so that we can still express the zero bound simply as the requirement that i_t is nonnegative. For further discussion of this notation, see Eggertsson (2010).

The government budget constraint can be approximated to yield

$$\frac{\bar{b}}{\bar{Y}}\hat{b}_t - \frac{\bar{b}}{\bar{Y}}(1+\bar{r})\hat{b}_{t-1} = \frac{\bar{b}}{\bar{Y}}(1+\bar{r})[\hat{\imath}_{t-1} - \pi_t] + (1+\bar{\tau}^s)\hat{G}_t - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_t - \hat{\tau}_t^I - \frac{\bar{C}}{\bar{Y}}\hat{\tau}_t^s - \hat{T}_t, \quad (4)$$

where $\hat{b}_t \equiv \log B_t/P_t - \log \bar{b}$ and $\hat{T}_t \equiv \log T_t/\bar{Y}$. What remains to be specified is government policy, i.e. how the government sets taxes, spending and monetary policy. We will be specific about this element of the model once we set up the shocks perturbing the economy.

2.1 The long run and short run: Output, inflation, budget deficits

To solve the model and take the zero bound explicitly into account, we make use of a simple assumption now common in the literature based on Eggertsson and Woodford (2003).

A1 In period 0, there is a shock $r_S^e < \bar{r}$ which reverts to a steady state with a probability $1-\mu$ in every period. We call the stochastic period in which the shock reverts to steady state t_S and assume that $(1-\mu)(1-\beta\mu) - \mu\sigma\kappa > 0$.

As discussed in Eggertsson (2010), we need to impose a bound on μ to avoid multiplicity which is stipulated at the end of A1.[6] For fiscal policy, we assume that

A2 $\hat{\tau}_t^I = \hat{\tau}_t^s = \hat{G}_t = 0$ for $\forall\ t$ and future lump sum taxes $\hat{T}_t$ are set so that the government budget constraint is satisfied, while $\hat{T}_t = 0$ for $\forall\ t < t_S$.

For monetary policy we assume that

A3 Short-term nominal interest rates are set so that $\pi_t = 0$. If this results in $i_t < 0$, we assume $i_t = 0$ and π_t is endogenously determined.

By assumption 3, we focus on the equilibrium in which inflation is zero if it can be achieved taking the zero bound into account. In this paper, we do not address how this equilibrium is implemented, e.g. via which interest rate policy and fiscal policy commitment, but there are several ways of doing this. What we are primarily interested in here is comparative statics for fiscal policy in the short run when the zero bound is binding and the central bank is *unable* to target zero inflation so that inflation becomes an endogenous object. Given assumptions A1 and A2, the policy commitment in A3 implies that

[6] See Mertens and Ravn (2010) for a discussion of multiple equilibria in this setting.

$\pi_t = \hat{Y}_t = 0$ for $t \geq t_S$. In the short run, either $\pi_t = \hat{Y}_t = 0$ (as long as the zero bound is not binding, i.e. $i_t = r_t^e > 0$) or π_t and $\hat{Y}_t$ are determined by the two equations

$$\pi_t^S = \kappa \hat{Y}_t^S + \beta\mu E_t \pi_{t+1}^S \quad (5)$$

$$\hat{Y}_t^S = (1-\mu) E_t \hat{Y}_{t+1}^S + \sigma(1-\mu) E_t \pi_{t+1}^S + \sigma r_S^e, \quad (6)$$

where S denotes the short run and we have substituted for $i_t^S = 0$. These equations can be solved to yield the first proposition.

Proposition 1 *Suppose A1, A2, and A3 hold and that $r_t^e < 0$. Then there is a unique bounded solution for output and inflation at zero short-term interest rates given by*

$$\pi_t = \pi_S = \frac{1}{(1-\mu)(1-\beta\mu) - \mu\sigma\kappa} \kappa\sigma r_S^e < 0 \text{ for } 0 \leq t < t_S \quad (7)$$

$$\hat{Y}_t = \hat{Y}_S = \frac{1-\beta\mu}{(1-\mu)(1-\beta\mu) - \mu\sigma\kappa} \sigma r_S^e < 0 \text{ for } 0 \leq t < t_S. \quad (8)$$

The proof of this proposition follows from the fact that one eigenvalue of the system (5)-(6) has to be outside of the unit circle and the other inside it so the proof follows from Blanchard and Kahn (1983).[7] Given this unique bounded solution, we will subsequently suppress the subscript t in the short run (when possible) and instead simply write π_S and $\hat{Y}_S$ to denote the endogenous variables in the time periods $0 \leq t < t_S$.

We can also derive a short-run evolution of the deficit. Recall that according to A2 we assume that the lump-sum taxes are at their steady state in the short run, i.e. $\hat{T}_t = 0$ for $0 \leq t < t_S$. Hence all adjustments will need to take place with long-run lump-sum taxes, while tax *rates* stay constant throughout. Under these assumptions, we obtain the following proposition for short-run deficits.

Proposition 2 *Suppose A1, A2 and A3 hold. Then, the deficit in the short run is given by*

$$\hat{D}_S = \frac{\bar{b}}{\bar{Y}} \hat{b}_t - \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})\hat{b}_{t-1} = \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})[\hat{\imath}_S - \pi_S] - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_S$$

$$= \begin{cases} 0 & \text{if } r_S^e > 0 \\ -\frac{\bar{b}}{\bar{Y}} r - \frac{[\frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})\kappa + (\bar{\tau}^I + \bar{\tau}^s)(1-\beta\mu)]}{(1-\mu)(1-\beta\mu) - \mu\sigma\kappa} \sigma r_S^e > 0 & \text{if } r_S^e < 0 \end{cases}$$

where $\hat{D}_S$ is the deficit.

[7] See Eggertsson (2010) for a more detailed proof in a similar context where the analytical expressions of this equation system are derived.

This proposition follows directly from the government budget constraint in equation (4), the policy specification and the last proposition. Observe that as output goes down the deficit automatically increases, since income and sales tax rates are at their steady state. Then, it follows that less will be collected from these taxes, which will be discussed in more detail in the following sections.

It is worth commenting briefly on Assumption 2, since it is driving the deficit. The basic idea is to assume that taxes that are proportional to the aggregate variables, such as sales and income taxes, stay constant at the pre-crisis *rate* and explore what happens to the government debt under this assumption. We think that this is a reasonable characterization of fiscal policy in practice, at least for the purpose of comparative statics. The way in which fiscal policy is discussed in the political spectrum is typically in the context of tax rates. Thus, a temporary increase in the tax rate *is* a tax increase and vice versa because this is typically – at least in very broad terms – the decision variable of the government. We are assuming that, in order to pay for current or future short-run deficits, there is an adjustment in future lump-sum taxes (that may have welfare effects due to resource costs). This assumption, however, is not made for the sake of realism, but to clarify the different channels through which current and future taxes can change debt dynamics and short-run demand. It is a natural first step to assume that future lump-sum taxes adjust, since they are neutral due to Ricardian equivalence. We elaborate on this more in coming sections when we move away from this assumption and instead start to assume – more realistically – that the future tax burden is financed via distortionary taxes.

2.2 Calibration

In the next section, we consider several policy experiments and will derive all of them in closed form. Before getting there, however, it is helpful to parameterize the model in order to translate our closed-form solutions into numerical examples. To do this, we parameterize the model using Bayesian methods described in more detail in Denes and Eggertsson (2009). We illustrate two baseline examples and we choose the parameters and the shock to match two "scenarios." The first is an extreme recession that corresponds to the Great Depression, that is, a 30 percent drop in output and 10 percent deflation. The other scenario is less extreme with a 10 percent drop in output and a 2 percent drop in inflation. We call the first numerical example the "Great Depression scenario" and the second the "Great Recession scenario" (abbreviated GD and GR for the rest of the paper). The parameters and the shock are chosen to match these scenarios exactly, while at the same time matching as closely as possible the priors we choose for both the parameters and the shocks shown in Table 1. Recall that the shock, r_L^e, is only driven by a shift in the preference parameter ξ_t and has the interpretation of being the short-term real interest rate if all prices were flexible. We use the same priors as in Denes and Eggertsson (2009). The posterior is approximated numerically by the Metropolis algorithm and is derived explicitly in Denes and Eggertsson (2009). Tables 2 and 3 show the posterior distribution for the two scenarios. We calibrate fiscal parameters $\bar{\tau}^s$ and $\bar{\tau}^I$ to 0.1 and 0.3, respectively. We calibrate the steady state debt-output ratio to correspond to 75 percent of annual output and the real government spending to output ratio to 20 percent of annual output. As the tables suggest, the most important difference across the two scenarios is that the shock is more persistent in the case of the GD scenario, thus leading to a more severe output contraction and deflation.

Table 1: Priors for the structural parameters and the shocks

	distribution	prior 5%	prior 50%	prior 95%
α	beta	0.5757	0.6612	0.7402
β	beta	0.9949	0.9968	0.9981
$1-\mu$	beta	0.0198	0.0740	0.1788
σ^{-1}	gamma	1.2545	1.9585	2.8871
ω	gamma	0.1519	0.8200	2.4631
θ	gamma	3.7817	7.6283	13.4871
r_L	gamma	-0.0036	-0.0094	-0.0196

Tables 2.Posterior for the Great Recession (GR) calibration

	post 5%	post 50%	post 95%	posterior mode
α	0.6958	0.7575	0.8141	0.784
β	0.9949	0.9968	0.9981	0.997
$1-\mu$	0.0970	0.1512	0.2229	0.143
σ^{-1}	0.9166	1.3881	2.0343	1.22
ω	0.8371	1.9156	3.8815	1.69
θ	8.0595	12.9533	19.8411	13.23
r_L	-0.0246	-0.0147	-0.0073	-0.0129

Tables 3.Posterior for the Great Depression (GD) calibration

	post 5%	post 50%	post 95%	posterior mode
α	0.7010	0.7610	0.8134	0.77
β	0.9950	0.9968	0.9981	0.997
$1-\mu$	0.0713	0.1056	0.1557	0.098
σ^{-1}	0.7951	1.2026	1.7641	1.153
ω	0.8612	1.9330	4.0484	1.53
θ	8.2119	13.0927	19.5951	12.70
r_L	-0.0269	-0.0146	-0.0067	-0.0107

3 Austerity plans

3.1 Deficits in a liquidity trap

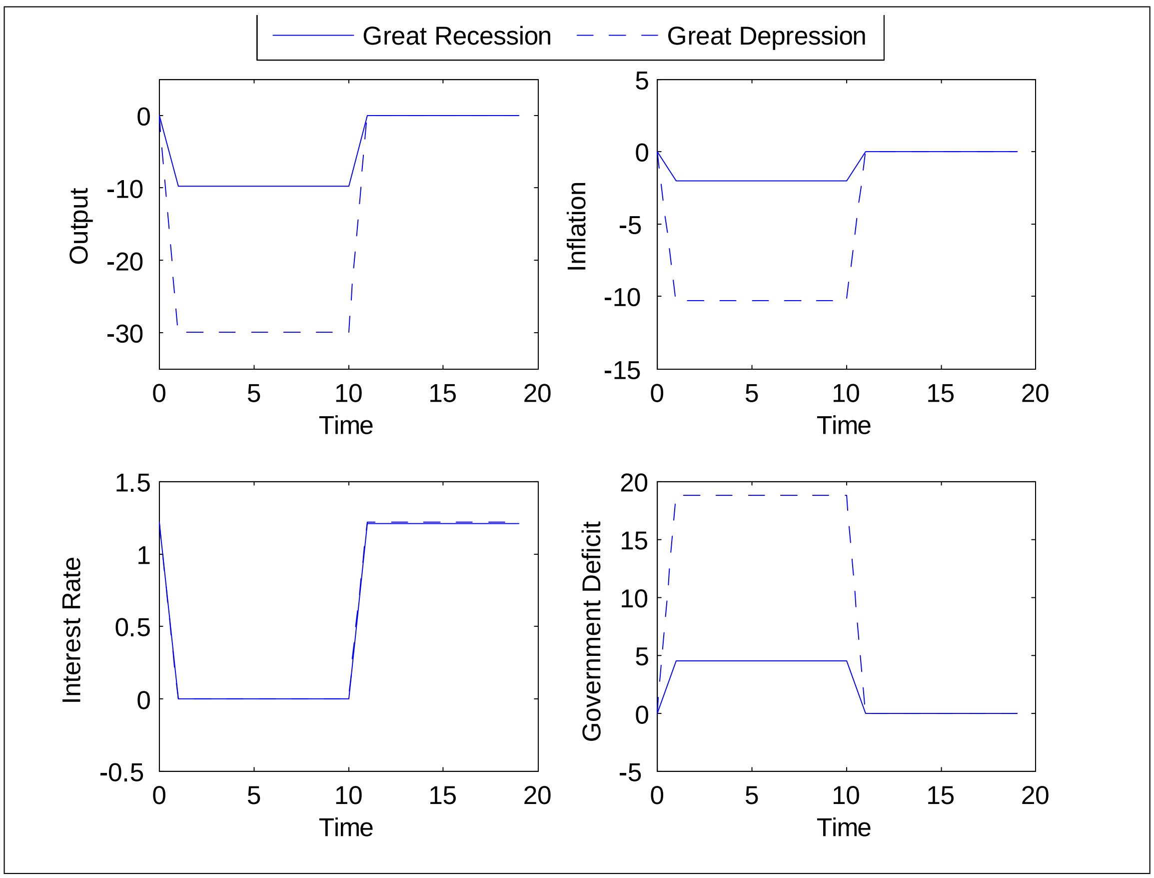

Figure 1: The Great Depression and the Great Recession in the model

Figure 1 shows the evolution of output, inflation and the nominal interest rate under our baseline parameterization. Recall that the parameters were chosen to replicate the "Great Depression" scenario and the "Great Recession" scenario in terms of the drop in output and inflation. The figure shows one realization of the shock, i.e. when it lasts for 10 quarters. Output and inflation drop due to the shock (panel (a) and (b)), and the nominal interest rate collapses to zero (panel (c)). Panel (d) is of the most interest, relative to previous work, as it shows the increase in the deficit of the government due to the crisis given by

$$\hat{D}_S = \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})[\hat{\imath}_S - \pi_S] - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_S. \tag{9}$$

As we see in panel (d) the deficit increases by 4.5 percent of GDP in the GR scenario and 18.8 percent in the GD scenario. One unit of deficit corresponds to the shortfall in the government's finances between spending and taxation as a fraction of GDP. The increase in the deficit is from two main sources. The first term reflects the contribution of the interest rates to the deficit and the revaluation of the nominal debt due to changes in the price level (i.e. deflation will increase the real value of the debt). The second term represents the drop in sale and income tax revenues due to the fact that overall output is reduced and, hence, tax collection drops. Of these two channels, it is the second that is driving most of the action, i.e. the deficit is increasing mostly because the tax base (output) is shrinking.

A key assumption in this model is that all income (from either profits or wages) is taxed at the same rate τ_t^I. The implication of this can be seen in equation (9) which says that every percentage deviation of output from steady state results in an increase in the deficit by a factor of $(\bar{\tau}^I + \bar{\tau}^s)$. Given our calibration, this implies that a percentage deviation of output from steady state would increase the deficit by 0.4 percent, which is consistent with the numbers reported for U.S. data by Follette and Lutz (2010) (they report 0.45 for the sum of federal and local tax revenues and 0.35 for federal taxes only). One alternative specification for income tax would be to assume that it is only levied on wages and not on profits. In this case, this elasticity would be much larger, since wages are flexible in the model and tend to drop by more than output.

A natural question from the point of view of a policymaker is: How can we balance the budget in the face of deficits? Here we illustrate three austerity plans which aim for short-run stabilization of the deficit. The first plan is to cut government spending, the second is to increase sales taxes and the third is to increase labor taxes. Of these, the first two plans are much less successful in reducing deficits in the zero bound environment while the last one is more effective. More explicitly, we study the following policies which replace A2.

A4 Let $(\hat{\tau}_t^I, \hat{\tau}_t^s, \hat{G}_t) = (\hat{\tau}_L^I, \hat{\tau}_L^s, \hat{G}_L) = 0$ for $\forall\ t \geq t_S$ and $(\hat{\tau}_t^I, \hat{\tau}_t^s, \hat{G}_t) = (\hat{\tau}_S^I, \hat{\tau}_S^s, \hat{G}_S)$ for $\forall$ $0 \leq t < t_S$. Lump-sum taxes $\hat{T}_t$ at dates $t \geq t_S$ are set so that the government budget constraint is satisfied, while $\hat{T}_t = 0$ for $\forall\ 0 \leq t < t_S$.

For any $0 \leq t < t_S$, the model satisfies the following equations (taking into account the existence of a unique bounded solution in the short run and the solution in the periods $t \geq t_S$ using the same argument as in Proposition 1):

$$(1-\mu)\hat{Y}_S = -\sigma i_S + \sigma\mu\pi_S + \sigma r_S^e + (1-\mu)\hat{G}_S - \sigma\chi^s(1-\mu)\hat{\tau}_S^s \tag{10}$$

$$\pi_S = \kappa\hat{Y}_S + \kappa\psi(\chi^I\hat{\tau}_S^I + \chi^s\hat{\tau}_S^s - \sigma^{-1}\hat{G}_S) + \beta\mu\pi_S \tag{11}$$

$$\hat{D}_S = \frac{\bar{b}}{Y}(1+\bar{\imath})[\hat{\imath}_S - \pi_S] + (1+\bar{\tau}^s)\hat{G}_S - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_S - \hat{\tau}_S^I - \frac{\bar{C}}{\bar{Y}}\hat{\tau}_S^s \tag{12}$$

For preliminaries and reference, Proposition 3 and 4 show the effects government spending and taxes have on output at positive and zero interest rates. Table 4 uses these propositions to compute tax and spending multipliers using our numerical example. These propositions provide closed-form solutions for the effect of a unit change in each of the fiscal instruments on output (these statistics are discussed in more detail in Eggertsson (2010) under a slightly different policy rule). We see that government spending increases output at the zero interest rate bound more than one-to-one. Sales tax cuts are also expansionary, though labor tax cuts are contractionary. Let us denote $\triangle\hat{x}_S = \hat{x}_S^{intervention} - \hat{x}_S$ as the percentage change in variable $\hat{x}_S$ due to a particular policy intervention. Thus the statistic $\frac{\triangle\hat{y}_S}{\triangle\hat{x}_S}$, where $\triangle\hat{y}_S$ is an endogenous variable, measures a policy multiplier. The following propositions are derived using equations (10) and (11) and summarize the output multipliers of the policy instruments at zero and positive interest rates.

Proposition 3 *Suppose A1, A3 and A4 hold. The output multiplier for government spending at positive and zero interest rates is:*

$$\frac{\Delta\hat{Y}_S}{\Delta\hat{G}_S} = \begin{cases} \psi\sigma^{-1} > 0 & \text{if } i_S > 0 \quad (i.e.\ r_L^e > 0) \\ \frac{(1-\mu)(1-\beta\mu)-\mu\kappa\psi}{(1-\mu)(1-\beta\mu)-\sigma\mu\kappa} > 1 & \text{if } i_S = 0 \quad (i.e.\ r_S^e < 0) \end{cases}.$$

The multipliers of sale tax cuts are identical but scaled by a factor of $-\sigma\chi^s$.

Proposition 4 *Suppose A1, A3 and A4 hold. The output multiplier for a labor tax increase is negative when the interest rate is positive. At the zero interest rate bound, it flips sign and becomes positive:*

$$\frac{\Delta\hat{Y}_S}{\Delta\hat{\tau}_S^I} = \begin{cases} -\chi^I\psi < 0 & \text{if } i_S > 0 \quad (i.e.\ r_S^e > 0) \\ \chi^I\frac{\mu\kappa\sigma\psi}{(1-\mu)(1-\beta\mu)-\mu\sigma\kappa} > 0 & \text{if } i_S = 0 \quad (i.e\ r_S^e < 0) \end{cases}.$$

Table 4: Tax and spending output multipliers at positive and zero interest rates

	$i > 0$		$i = 0$	
	GD	GR	GD	GR
	[5% , 95%]	[5% , 95%]	[5% , 95%]	[5% , 95%]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{G}_S}$	**0.4** [0.2 , 0.6]	**0.4** [0.3 , 0.6]	**2.2** [1.4 , 3.2]	**1.2** [1.1 , 1.5]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^s_S}$	**-0.3** [-0.5 , -0.2]	**-0.3** [-0.5 , -0.2]	**-1.8** [-3, -0.9]	**-0.9** [-1.3, -0.5]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}^I_S}$	**-0.5** [-0.8 , -0.3]	**-0.5** [-0.7 , -0.3]	**0.4** [0.2 , 0.5]	**0.1** [0.06 , 0.3]

These propositions and Table 4 illustrate that the government spending multiplier is much higher at a zero interest rate than at a positive interest rate, and this is true under both baseline calibrations. The reason for this is outlined in some detail in Eggertsson (2010) and is summarized below. At a positive interest rate, any increase in demand due to government spending will be offset, to some extent, by an increase in the nominal interest rate, as the central bank seeks to keep inflation at a target rate. At a zero interest rate, however, inflation is below the central bank target due to the shock r^e_t. Hence, any increase in demand via government spending is perfectly accommodated by the central bank and the nominal interest rate stays the same. But there is a second force at work here – the effect of the expectations. Even if the central bank perfectly accommodates government spending in the short run, this by itself will only result in a one-to-one increase in output.[8] Yet, as the table shows, the multiplier is much higher than one. The reason for this is that government spending increases output not only because of an increase in current spending, but also through expectations about higher future government spending in future states when the zero bound is binding. This channel is particularly important in the GD calibration since the probability of staying in the crisis state (the "short run") is higher than in the GR calibration.

Much of the recent literature on fiscal multipliers emphasizes the large range for the multipliers one can get out of the models (see e.g. Leeper et al (2011)). Perhaps somewhat surprisingly we see that, for each of the calibrations, the 5 to 95 percent range of the posterior of each of the multipliers is not very large. For the GD calibration of the gov-

[8] To see this, consider the case in which the shock goes back to zero in the next period so that $\mu = 0$. Then equation (10) shows that output increases one-to-one with spending.

ernment spending multiplier when the zero bound is binding, the 5 to 95 percent posterior band is 1.4 to 3.2. While this range is not trivial, it is relatively narrow in comparison to many other studies. We see that this is the case, despite the fact that the priors chosen for the calibration are not very tight (see Table 1). The main reason for this tight interval is that the "scenario" we choose, i.e. the Great Depression and the Great Recession, puts relatively strong restriction on the model, so that conditional on matching these scenarios, the prediction of the model is relatively sharp as further discussed in Denes and Eggertsson (2009). This is in contrast to many other studies that do not impose as strong of a restriction on what the model is supposed to generate as a benchmark. Meanwhile, we see that there is a considerable difference between the multipliers in the GD and GR scenarios at a zero interest rate. In fact, the 5 to 95 percent posterior intervals of the zero bound government spending multiplier barely overlap. This highlights that the main source of the difference in our estimate for the multiplier is the scenario the model is supposed to match. As the output drop in a given "scenario" becomes larger, the multiplier increases. As already noted, the key difference between the two scenarios is that the shock is more persistent in the GD case. Hence, it is the expectational channel of government spending that is driving the difference here, i.e., the *expectation* of spending in *future states of the world* in which the zero bound is binding.

3.2 The effect of cutting government spending on deficits

We now turn to the idea of cutting government spending to reduce the deficit. By equation (4), we see that this results in

$$\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S} = (1+\tau^s)\frac{\triangle \hat{G}_S}{\triangle \hat{G}_S} + \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})\frac{\triangle[\hat{\imath}_S - \pi_S]}{\triangle \hat{G}_S} - (\bar{\tau}^I + \bar{\tau}^s)\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S}.$$

For the budget to be balanced via this "austerity" policy, this number needs to be positive.

Consider first what happens at a positive interest rate. One can confirm that $\frac{\triangle \hat{G}_S}{\triangle \hat{G}_S} = 1$, $\frac{\triangle[\hat{\imath}_S - \pi_S]}{\triangle \hat{G}_S} = (1-\mu)\sigma^{-1}\omega\psi$ and $\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S} = \sigma^{-1}\psi$, yielding the next proposition.

Proposition 5 *Suppose A1, A3 and A4 hold. At a positive interest rate, cutting government spending always reduces the deficit. This reduction is given by*

$$\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S} = 1 + \bar{\tau}^s + \frac{\bar{b}}{Y}(1+\bar{\imath})(1-\mu)\sigma^{-1}\omega\psi - (\bar{\tau}^I + \bar{\tau}^s)\sigma^{-1}\psi > 0 \;\; \textit{if} \;\; i_S > 0 \;\; (\textit{i.e.} \;\; r_S^e > 0).$$

The proof of this proposition follows from the expression above and the fact that $0 < (\bar{\tau}^s + \bar{\tau}^I)\sigma^{-1}\psi = (\bar{\tau}^s + \bar{\tau}^I)\frac{\sigma^{-1}}{\sigma^{-1}+\omega} < 1$ and hence, at a positive interest rate, cutting government spending will always reduce the deficit (or create a surplus) given our specification for monetary policy. For our numerical example, this can be seen in Table 5. Observe that this statistic is close to $(1+\tau^s)$, but can be smaller than that number for two reasons that arise due to general equilibrium effects. First, a cut in government spending reduces real interest rates and, thus, the interest rate burden of debt. Second, it reduces output, which suppresses sales and income tax revenues. Putting the two pieces together, we see that, at a positive interest rate, a dollar cut in government spending will improve the budget close to one-to-one.

Table 5 Tax and spending deficit multipliers at positive and zero interest rates

	$i > 0$		$i = 0$	
	GD	GR	GD	GR
	[5% , 95%]	[5% , 95%]	[5% , 95%]	[5% , 95%]
$\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S}$	**1.1** [1.03 , 1.3]	**1.2** [1.09 , 1.5]	**-0.3** [-1 , 0.3]	**0.5** [0.2 , 0.6]
$\frac{\triangle \hat{D}_S}{\triangle \hat{\tau}_S^s}$	**-1.1** [-1.2 , -1]	**-1.2** [-1.3 , -1]	**0.3** [-0.3 , 1.1]	**-0.4** [-0.5 , -0.1]
$\frac{\triangle \hat{D}_S}{\triangle \hat{\tau}_S^I}$	**-0.6** [-0.8 , -0.4]	**-0.6** [-0.7 , -0.3]	**-1.4** [-1.6 , -1.2]	**-1.2** [-1.3 , -1.1]

Let us now consider a more interesting case when the nominal interest rate is zero. Can this overturn the result? The next proposition shows that the answer is yes.

Proposition 6 *Suppose A1, A3 and A4 hold. At a zero interest rate, cutting government spending can either increase or reduce the deficit at the following rate:*

$$\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S} = (1+\bar{\tau}^s) - \frac{\bar{b}}{Y}(1+\bar{\imath})\frac{\kappa}{1-\beta\mu}\left[\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S} - \sigma^{-1}\psi\right] - (\bar{\tau}^I + \bar{\tau}^s)\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S} \quad \text{if } i_S = 0,$$

where $\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S} > 1$ *is given by proposition (3) at the zero bound.*

Proof. This is most easily derived by first noting that at zero interest rates by equation (12) $\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S} = -\frac{\bar{b}}{Y}(1+\bar{\imath})\frac{\triangle \pi_S}{\triangle \hat{G}_S} + [1+\bar{\tau}^s] - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_t\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S}$. Now use the expression for the output multiplier in equation (3), note that $\frac{\triangle \pi_S}{\triangle \hat{G}_S} = \frac{\kappa}{1-\beta\mu}\frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S} - \frac{\kappa}{1-\beta\mu}\sigma^{-1}\psi$, substitute and solve. ■

Let us now interpret this. The first term is the same as in the prior proposition, namely $1 + \bar{\tau}^s$, which means that a dollar cut in government spending will result in a reduction of the deficit by the same amount. In partial equilibrium, thus, any drop in spending reduces the deficit by that amount. The other terms, as before, come about due to the general equilibrium effects but now they have much more power. A cut in government spending does not only lead to a drop in government expenditures. It will also result in a reduction in government revenues, due to the fact that it leads to a reduction in the overall level of economic activity and wages and through that, a change in the price level which may raise the real value of the outstanding nominal debt. All of these general equilibrium effects are captured in the expression above. The first term captures the increase in the real value of debt if the government cuts spending through deflation. This term is much bigger than before. The second term measures the reduction in sales tax and income taxes due to the drop in output.

Table 5 computes the value of the deficit multiplier for the two scenarios. As we can see, the deficit increases more than one-for-one for the GD scenario, i.e. a dollar cut in the spending increases the deficit by 30 cents. The GR scenario is much less extreme and a cut in government spending does in fact reduce the deficit. But it does so by less than one-to-one, a one dollar cut in the government spending reduces the deficit by only about 50 cents. Flipping this around, we see that, in the GD case, government spending is self-financing, while in the GR case the endogenous increase in output fills about half the gap created in the budget, i.e. for a dollar increase in spending, the deficit only increases by half that much. The table also reports 5 to 95 percent posterior bands. We see that, given our priors for the parameters, this posterior band is most of the time in the self-defeating austerity region for the GD case while in the GR calibration mostly in the other way around.

The main reason for the discrepancy between the GD and the GR calibration is that the government spending multiplier is much larger under the Great Depression (2.2) scenario than in the Great Recession scenario (1.2). To see this note that the proposition implies that

$$\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S} < 0 \text{ if } \frac{\triangle \hat{Y}_S}{\triangle \hat{G}_S} > \Gamma = \frac{1 + \bar{\tau}^s + \frac{\bar{b}}{Y}(1+\bar{\imath})\frac{\kappa}{1-\beta\mu}\sigma^{-1}\psi}{\bar{\tau}^I + \bar{\tau}^s + \frac{\bar{b}}{Y}(1+\bar{\imath})\frac{\kappa}{1-\beta\mu}}.$$

In other words, if the multiplier of government spending is larger than Γ, then the deficit will always increase when the government cuts spending at a zero interest rate. Also, note that the outcome in the GD scenario was more extreme, due to a more persistent fundamental shock, and thus there is more room for government spending to step in and trigger an increase in output and prices.

3.3 Sales taxes with Laffer-type properties

Let us now consider an increase in the sales tax. Here the two forces are as follows i) an increase in the sales tax rate will tend to increase revenues for a given level of production and ii) an increase in the rate will reduce overall production. Following exactly the same steps as in our previous proposition we obtain:

Proposition 7 *Suppose A1, A3 and A4 hold. At a positive interest rate, increasing sales tax always reduces the deficit. This reduction is given by*

$$\frac{\triangle\hat{D}_S}{\triangle\hat{\tau}^s_S} = -\left[\frac{\bar{C}}{\bar{Y}} + \frac{\bar{b}}{Y}(1+\bar{\imath})\chi^s(1-\mu)\omega\psi - (\bar{\tau}^I + \bar{\tau}^s)\psi\chi^s\right] < 0 \quad \textit{if} \;\; i > 0.$$

At a zero interest rate, increasing the sales tax rate can either increase or reduce the deficit by:

$$\frac{\triangle\hat{D}_S}{\triangle\hat{\tau}^s_S} = -\frac{\bar{C}}{\bar{Y}} - \frac{\bar{b}}{Y}(1+\bar{\imath})\frac{\kappa}{1-\beta\mu}\left[\frac{\triangle\hat{Y}_S}{\triangle\hat{\tau}^s_S} + \psi\chi^s\right] - (\bar{\tau}^I + \bar{\tau}^s)\frac{\triangle\hat{Y}_S}{\triangle\hat{\tau}^s_S} \quad \textit{if} \;\; i_S = 0.$$

As we can see in Table 5, the results in our numerical examples show that the contractionary force is dominating in the GD case. An obvious implication of this is that the effect of an "austerity measure" that involves increasing sales taxes is similar to cutting government spending. It may increase the deficit rather than reduce it. Conversely, cutting sales taxes may increase tax revenues, but this is akin to being on the wrong side of the "Laffer curve." In the GR case, increasing taxes does reduce the deficit. Again the reason here is that the output collapse is less extreme in the GR case and, as in the case of government spending, this means that the negative "multiplier" of sales tax increases is smaller.

3.4 The effect of income taxes increase on the deficit

We now consider the effect of increasing taxes on income, summarizing the result in the next two propositions (the calculation follows the same steps as in Proposition 6).

Proposition 8 *Suppose A1, A3 and A4 hold. At a positive interest rate, increasing income taxes has the following effect on the deficit:*

$$\frac{\triangle\hat{D}_S}{\triangle\hat{\tau}^I_S} = -1 + \left[(\bar{\tau}^I + \bar{\tau}^s) + \frac{\bar{b}}{Y}(1+\bar{\imath})\sigma^{-1}(1-\mu)\right]\psi\chi^I.$$

At a zero interest rate, this expression is given by:

$$\begin{aligned}\frac{\triangle\hat{D}_S}{\triangle\hat{\tau}^I_S} &= -1 - \frac{\bar{b}}{Y}(1+\bar{\imath})\frac{\kappa}{1-\beta\mu}\left[\frac{\triangle\hat{Y}_S}{\triangle\hat{\tau}^I_S} + \psi\chi^I\right] - (\bar{\tau}^I + \bar{\tau}^s)\frac{\triangle\hat{Y}_S}{\triangle\hat{\tau}^I_S} \\ &< 0.\end{aligned}$$

As seen in Table 5, this is a large negative number for both scenarios, i.e. increasing labor taxes cuts down the deficit considerably. The reason for this is described in Eggertsson (2010). In the model, an increase in labor taxes actually increases output in the short run, via changing deflationary expectations to inflationary ones. This is due to a number of special features discussed in Eggertsson (2010). Hence we do not wish to push this short-run property of the model too far.

4 Confidence and the long run

So far we have seen that two popular policies intended to balance the budget, namely cutting government spending or increasing sales taxes, are likely to increase the deficit rather than decrease it at a zero nominal interest rate. Both lead to a reduction in output, as documented in Table 3, which contracts the tax base. The call for "austerity" has usually been motivated by emphasizing the importance of creating a credible long-run economic environment. The deficit, then, is often pointed to as one element that may create havoc in the future. In some respects, the results above do not undercut that basic message of "austerity," but merely suggest that the short-run effects of government spending cuts or larger sales taxes may increase the budget deficit rather than decrease it. In general, they are not very effective in closing the deficit gap and subsequently may not be very effective in restoring "confidence," at least to the extent that "confidence" is tied to reducing budget deficits.

But what is "confidence" more explicitly and how is it tied to the long run? We now explore whether the standard New Keynesian model supports the popular discussion of the importance of "confidence" in the current crisis and find that in certain respects the answer is a qualified "yes" – if this "confidence" is taken to mean the effect of long-run expectations on current demand. This answer is specific to the environment of a zero interest rate and this gives one rationale for why confidence has been so high on the agenda following the crisis of 2008.[9]

In this section, we do not model directly how deficits influence expectations of the "long run." We will address that in the next section. Instead we first want to clarify the role of long-run expectations for taxes and government spending on current demand by using the

[9] An important abstraction in the model is the lack of endogenous investment, which is a strongly forward-looking variable. Additionally, there are no frictions in the labor market which could also make hiring a forward-looking variable. Incorporating these elements are important extensions that could have an effect on the results.

following assumption that replaces A4.

A5 Let $(\hat{\tau}^I_t, \hat{\tau}^s_t, \hat{G}_t) = (\hat{\tau}^I_L, \hat{\tau}^s_L, \hat{G}_L) \neq 0$ for $\forall\ t \geq t_S$ and $\hat{\tau}^I_t = \hat{\tau}^s_t = \hat{G}_t = 0$ for $\forall\ t < t_S$. Current and future lump-sum taxes $\hat{T}_t$ are set so that the government budget constraint is satisfied.

Let us again classify the economy in terms of the "long run" and the "short run." According to our policy rule and A3, in the long run, inflation is zero, i.e. $\pi_L = 0$. Then, output is given by the following proposition using equation (3).

Proposition 9 *Suppose that A1, A3 and A5 hold. Then*

$$\hat{Y}_L = -\psi\chi^I\hat{\tau}^I_L - \psi\chi^s\hat{\tau}^s_L + \psi\sigma^{-1}\hat{G}_L \quad \textit{for}\ \ t \geq t_S,$$

where $\hat{\tau}^I_L, \hat{\tau}^s_L$ and $\hat{G}^N_L$ are given by the policy rule and, then, long-run multipliers are given by $\left(\frac{\Delta\hat{Y}_L}{\Delta\hat{G}_L}, \frac{\Delta\hat{Y}_L}{\Delta\hat{\tau}^s_L}, \frac{\Delta\hat{Y}_L}{\Delta\hat{\tau}^I_L}\right) = \left(\psi\sigma^{-1}, -\psi\chi^s, -\psi\chi^I\right)$.

Since equation (2) only pins down the nominal interest rate, output is determined by equation (3). The proposition shows that higher long-run taxes reduce long-run output. Similarly, more long-run government spending increases long-run output. The reasons here are standard: higher labor and consumption taxes reduce labor supply and thus contract output, while larger government spending increases the labor supply (by increasing marginal utility of consumption). Table 6 shows the value of long-run multipliers given in the proposition and we see that the values of these are very similar in our two numerical examples.

Table 6: The effect of long-run taxes and spending on long-run output

	$i > 0$	
	GD	GR
	[5% , 95%]	[5% , 95%]
$\frac{\Delta\hat{Y}_L}{\Delta\hat{G}_L}$	**0.4** [0.2 , 0.6]	**0.4** [0.3 , 0.6]
$\frac{\Delta\hat{Y}_L}{\Delta\hat{\tau}^s_L}$	**-0.3** [-0.5 , -0.2]	**-0.3** [-0.5 , -0.3]
$\frac{\Delta\hat{Y}_L}{\Delta\hat{\tau}^I_L}$	**-0.5** [-0.8 , -0.3]	**-0.5** [-0.7 , -0.3]

Let us now explore under what condition the short-run output depends on these long-run variables. Let us write the equilibrium relationships (2) and (3) for the short run, taking into account that $\pi_L = 0$ according to our policy rule and now allow for the possibility that $\hat{\tau}_L^I, \hat{\tau}_L^s$ and $\hat{G}_L^N$ may be non-zero. We get

$$\begin{aligned}(1-\mu)\hat{Y}_S &= (1-\mu)\hat{Y}_L - \sigma i_S + \sigma\mu\pi_S + \sigma r_S^e + (1-\mu)\hat{G}_S \\ &\quad -(1-\mu)\hat{G}_L - \sigma\chi^s(1-\mu)\mu\hat{\tau}_S^s + \sigma\chi^s(1-\mu)\hat{\tau}_L^s \end{aligned} \tag{13}$$

and

$$(1-\beta\mu)\pi_S = \kappa\hat{Y}_S + \kappa\psi\chi^I\hat{\tau}_S^I + \kappa\chi^s\hat{\tau}_S^s - \sigma^{-1}\kappa\psi\hat{G}_S. \tag{14}$$

According to our specification for monetary policy, if the zero bound is not binding, the central bank will set the nominal interest rate so that inflation is $\pi_S = 0$. Equation (13) then simply determines the nominal interest rate, i.e.

$$\begin{aligned} i_S &= \sigma^{-1}(1-\mu)\hat{Y}_L - \sigma(1-\mu)\hat{Y}_S + \mu\pi_S + r_S^e + \sigma^{-1}(1-\mu)\hat{G}_S \\ &\quad -\sigma^{-1}(1-\mu)\hat{G}_L - \chi^s(1-\mu)\mu\hat{\tau}_S^s + \chi^s(1-\mu)\hat{\tau}_L^s. \end{aligned} \tag{15}$$

Hence if we plot these two relationships, taking monetary policy into account, in the $(\pi_S, \hat{Y}_S)$ space, we get a horizontal AD curve that is fixed at $\pi_S = 0$ and the equilibrium is given by point A, which is the intersection of the AD curve and the AS curve in equation (14). Observe now that movement in the long-run variables, i.e. $(\hat{Y}_L, \hat{G}_L, \hat{\tau}_L^s)$, only shift the AD curve but have no effect on the equilibrium that remains in A because the AD curve is horizontal. What is going on is that the central bank will offset any movements in these variables, hence the only effect will be to change the level of the nominal interest rate without any effect on output and prices. In this sense the level of "confidence," at least as measured by expectations for long-run variables, is completely irrelevant.[10]

[10] This stark result is special to the strong assumption that the central bank targets zero inflation and thus replicates the flexible price allocation. Under flexible prices, "demand" plays no role. More generally, if the central bank follows a Taylor rule, there is some short-run effect on long-run expectations, but they are quantitatively small which the current specification highlights sharply.

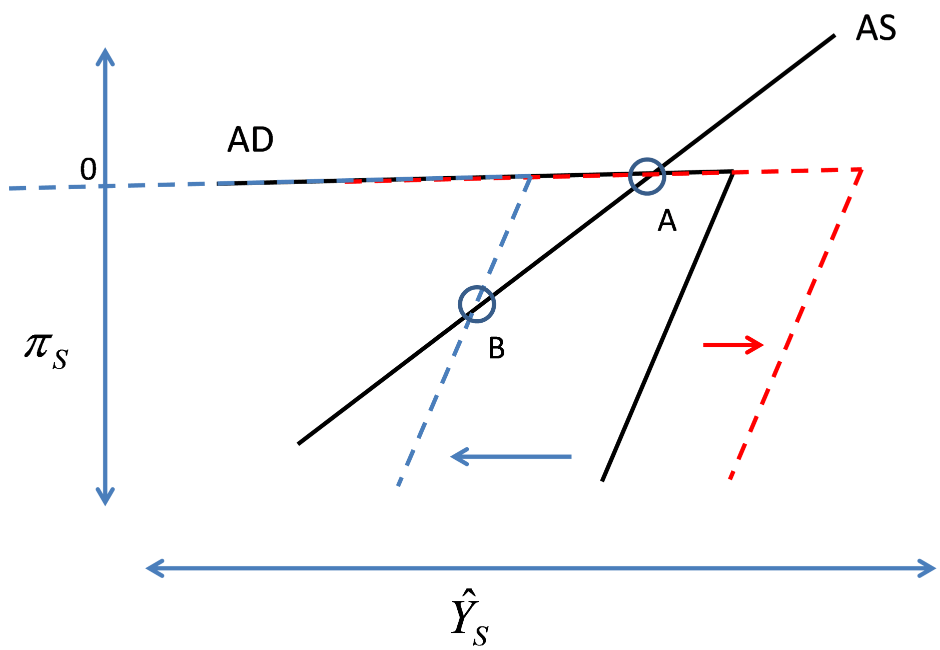

Figure 2: A large enough shock moves the AD curve to the left.

What does matter for determining the equilibrium point A is the movements in the AS curve, i.e. movement in short-term taxes and spending $(\tau_S^s, \tau_S^I, G_S)$. Local to point A the model behaves exactly like the model if it had perfectly flexible prices and then aggregate demand – or what we interpret as "confidence" – plays no role.

Consider now a shock to r_S^e. This shifts the AD curve to the left. The central bank will try to accommodate this shift via cuts in interest rates – that is why the AD curve is horizontal – however it will be unable to do so beyond the point at which the zero bound becomes binding. At this point, equation (15) becomes binding and now output is demand-determined and given by:

$$\begin{aligned}\hat{Y}_S &= \hat{Y}_L + \frac{\sigma\mu}{1-\mu}\pi_S + \sigma\pi_L + \frac{\sigma}{1-\mu}r_S^e \\ &\quad + \hat{G}_S - \hat{G}_L - \sigma\chi^s\mu\hat{\tau}_S^s + \sigma\chi^s\hat{\tau}_L^s,\end{aligned}$$

which is depicted by point B in figure 2. Observe that, at this point, the AD equation is upward-sloping in short-run inflation. The reason for this is that the central bank has the nominal interest rate fixed at zero – while previously the central bank would adjust the nominal interest rate freely to make sure inflation would stay at zero. The key thing here is that the central bank no longer can do this due to the zero bound. What is the implication? It is that any increase in short-run inflation will lead to a reduction in the real interest rate due to the fact that it triggers higher expected inflation (since the short run will last with a probability μ). The real interest rate will determine how much people want to spend in the short run. If inflation expectations rise, then spending today becomes cheaper and people spend more. But this has another important implication.

At this point *any* change in $(\hat{Y}_L, \hat{G}_L, \hat{\tau}^s_L, \hat{\tau}^I_L)$ will shift the AD curve and this will be reflected in movements in output and inflation since the central bank will not change the interest rate to offset the movements in demand – either because it is unable to or because inflation is below target (and hence it will accommodate any increase in inflation). Now "confidence" starts playing a key role. In particular, expectations about lower future income taxes matter or, more generally, higher future long-run output starts to matter. Cuts in income tax rates increase $\hat{Y}_L$ by a factor of $\psi\chi^I$, as shown in Proposition 9. Lower long-run government spending will also have an effect, first by reducing $\hat{Y}_L$ by a factor of σ^{-1} (see Proposition 9) and also decreasing short-term demand by the same amount. This also increases long-run demand via the term $\hat{G}_L$. This latter effect arises because consumption demand is affected by the price of goods today relative to the future and this price is affected by government consumption in the short run *relative to long-run consumption.* Long-run sales tax works via the same mechanism as government spending, since they enter the AS and AD equation exactly the same way except for being multiplied by $\sigma\chi_s$. Intuitively, higher expected long-run sales tax increases short-term demand, since it gives people incentive to spend today rather than in the future, since consumption today is taxed at a lower rate than future consumption. The following proposition proceeds directly from manipulating the expression above:

Proposition 10 *Suppose that A1, A3 and A5 hold. Then short-run output at a positive interest rate is given by*

$$\hat{Y}_S = 0,$$

while output at zero interest rate is given by

$$\begin{aligned}\hat{Y}_S &= \frac{\sigma(1-\beta\mu)}{(1-\beta\mu)(1-\mu)-\sigma\mu\kappa} r^e_S - \frac{(1-\mu)(1-\beta\mu)\omega\psi}{(1-\beta\mu)(1-\mu)-\sigma\mu\kappa}\hat{G}_L \\ &+ \frac{\chi^s(1-\mu)(1-\beta\mu)}{(1-\beta\mu)(1-\mu)-\sigma\mu\kappa}\sigma\omega\psi\hat{\tau}^s_L - \frac{(1-\beta\mu)(1-\mu)\psi\chi^I}{(1-\beta\mu)(1-\mu)-\sigma\mu\kappa}\hat{\tau}^I_L.\end{aligned}$$

The multipliers $\left(\frac{\Delta\hat{Y}_s}{\Delta\hat{G}_L}, \frac{\Delta\hat{Y}_s}{\Delta\hat{\tau}^s_L}, \frac{\Delta\hat{Y}_s}{\Delta\hat{\tau}^I_L}\right)$ *are given by the second, third and fourth coefficients, respectively.*

The numerical values for each of these multipliers are shown in Table 7. We see that expectations of one percent higher long-run labor taxes will reduce output in the short run by 0.7 percent in the GR scenario. One can think of a variety of stories which may trigger higher expected long-run income taxes and thus contract demand in the short run. Similarly, expectations of higher long-run sales taxes has a multiplier of 0.6, and that of government spending -0.8. We thus see that the effect of committing to lower long-run spending is almost as effective as increasing it in the short run.

Table 7: The effect of long-run taxes and spending on short-run output

	$i = 0$	
	GD	GR
	[5% , 95%]	[5% , 95%]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{G}_L}$	**-1.8** [-2.9 , -0.9]	**-0.8** [-1.2 , -0.5]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_L^S}$	**1.4** [0.2 , 0.9]	**0.6** [0.3 , 1.0]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{\tau}_L^I}$	**-1.7** [-0.5 , -0.1]	**-0.7** [-1.1 , -0.4]

5 Short-run fiscal crisis, debt dynamics, and confidence

5.1 A one-time increase in real debt

How does a deficit today change expectations about future taxes and spending? And, how do these expectations change short-run demand? There is no simple answer to these questions because it depends on how the deficit will be financed in the future, which depends upon the policy regime. In this section, we show that if the budget deficits are financed by future increases in long-run sales tax or reductions in long-run government spending, they increase short-run demand. If they are financed by increases in long-run labor tax, they are contractionary. This follows directly from the last section and below we relate long-run expectations more closely to short-run deficits. This is made explicit in Assumption 6.

A6 Fiscal policy in the short and long run is given by

$$
\begin{aligned}
i) &\quad \hat{b}_t = \hat{b}_{t-1} + \epsilon_0 \text{ for } t < t_S \\
ii) &\quad \hat{\tau}_t^s = \hat{\tau}_t^I = \hat{G}_t = 0 \text{ for } t < t_S \\
iii) &\quad \hat{b}_t = \delta \hat{b}_{t-1} \text{ for } t \geq t_S\text{, where } 0 < \delta < 1 \\
iv) &\quad \hat{T}_t = \frac{\bar{b}}{\bar{T}}(1+\bar{\imath})\hat{\imath}_{t-1} \text{ for } t \geq t_S.
\end{aligned} \tag{16}
$$

The key assumption is that, while distortionary taxes (or government spending) are not the direct source of the short-run deficit (see A6 (ii.)), they will need to adjust in the long run to bring down debt to its pre-crisis level (note that $\hat{b}_t$ is the deviation of debt from the steady state that we linearize around). The simplest way to interpret deficits or surpluses in equation (16) is that they come about through variations in lump-sum taxes. More generally, we think of these types of fiscal shocks as coming about due to shocks that do not directly affect sales tax, income tax or the overall size of the government, but instead affect some fiscal transfers that occur via other means. A banking crisis, for example, typically puts taxpayers on the hook for large amounts of money, yet this increase in debt is not driven by sharp cuts in some tax *rates* or increases in government spending on goods and services.

If short-run variations in $\hat{b}_t$ in equation (16) are met by increases in long-run lump-sum taxes, then that would be the end of the story since then the model would satisfy Ricardian equivalence. Instead, we suppose that long-run lump-sum taxes stay close to their steady state at time $t \geq t_S$ and thus other taxes or spending need to adjust to bring public debt back to steady state.[11] This is made explicit in A6 (iii.), where we assume that long-run fiscal policy adjusts to stabilize the debt level at its original level and that this adjustment takes place over some period of time at a rate δ. This means that while the government may run up deficits in the short run, in the long run debt is stabilized at whatever level it was prior to the crisis (i.e. prior to the shock r_S^e hitting).

Using the budget constraint in equation (4), we can see that by assumptions A6 (iii.) and A6 (iv.), it follows that

$$
\begin{aligned}
-(1+\bar{\imath}-\delta)\frac{\bar{b}}{\bar{Y}}\delta^{t+1-t_s}\hat{b}_{t_S-1} \quad = \quad & \left[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)\right]\hat{G}_t - \left[1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I\right]\hat{\tau}_t^I \\
& -\left[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s\right]\hat{\tau}_t^s \text{ for } t \geq t_S.
\end{aligned}
$$

[11] To be precise, A6 (iv.) actually assumes that lump-sum taxes pay for the excess interest rate cost of debt in the transition phase. This has quantitatively small effects, but simplifies the algebra somewhat.

This equation says that, in the long run, interest rate payments for debt need to be financed by either a cut in government spending or an increase in labor or sales taxes. However, the coefficient in front of each fiscal variable will, in general, be positive. Let us define the right-hand side of equation (17) as the budget deficit of the government, $\hat{D}_t$, which is thus determined at any time $t \geq t_S$ by

$$\hat{D}_t = -\frac{b}{Y}(1 + i - \rho)\delta^{t+1-t_s}\hat{b}_{t_S-1} \text{ for } t \geq t_S.$$

It follows that, if this is a negative number, the government is running a budget surplus. How is this surplus financed? By tax increases and spending cuts. We assume that each fiscal instrument is adjusted in fixed proportions. Let γ_I be the share of $\hat{D}_{t+j}$ financed by labor tax, γ_s be the share financed with sales tax, and $\gamma_G = 1 - \gamma_I - \gamma_s$ be the share financed by a reduction in government spending. Note that this assumption implies that the deviation of taxes and government spending from steady state will decline at the same rate as the debt, i.e. at the rate δ. Under this assumption, a full specification of long-run fiscal policy is now given by the choice of the weights γ_G, γ_I and γ_s. This is summarized by assumption A7:

A7 The sequence of $\{D_t\}$ at $t \geq t_S$, as implied by A6, is financed by $\hat{\tau}_t^I$, $\hat{\tau}_t^s$ and $\hat{G}_t$ in the fixed proportions

$$\gamma_{G,t} \equiv \frac{\left[1 + \bar{\tau}^s - \sigma^{-1}\psi(\bar{\tau}^I + \bar{\tau}^s)\right]\hat{G}_t}{\hat{D}_t} = \gamma_G$$

$$\gamma_{I,t} \equiv -\frac{\left[1 - (\bar{\tau}^I + \bar{\tau}^s)\psi\chi^I\right]\hat{\tau}_t^I}{\hat{D}_t} = \gamma_I$$

$$\gamma_{s,t} \equiv -\frac{\left[\frac{\bar{C}}{\bar{Y}} - (\bar{\tau}^I + \bar{\tau}^s)\psi\chi^s\right]\hat{\tau}_t^s}{\hat{D}_t} = \gamma_s.$$

Given this policy specification we can now do the following thought experiment: What is the effect of an increase in the deficit on aggregate demand? In this thought experiment, recall that the deficit is driven by a cut in lump-sum taxes and, hence, it has no direct effect on current demand. The effect, then, only comes about due to the effect it has on expectations about *long-run taxes and the long-run size of the government.* This effect will critically depend upon how the deficit is financed, which can, using assumption A7, be written as a function of how the debt is paid off, i.e. via tax or spending cuts. The next proposition summarizes this.

Proposition 11 *Suppose that A1, A3, A6 and A7 hold. Then, the multiplier of deficit spending on short-run output at positive interest rate (if* $r_S^e > 0$*) is given by*

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{b}_S} = 0,$$

while output at zero interest rate (if $r_S^e < 0$*) is given by*

$$\begin{aligned}\frac{\Delta \hat{Y}_S}{\Delta \hat{b}_S} &= \frac{(1-\beta\mu)(1-\mu)\psi\omega}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]}\frac{\frac{b}{Y}(1+i-\rho)\delta}{[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)]}\gamma_G \\ &+\frac{(1-\beta\mu)(1-\mu)\chi^s\sigma\psi\omega}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]}\frac{\frac{b}{Y}(1+i-\rho)\delta}{[(\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s]}\gamma_s \\ &-\frac{(1-\beta\mu)(1-\mu)\psi\chi^I}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]}\frac{\frac{b}{Y}(1+i-\rho)\delta}{1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I}\gamma_I\end{aligned}$$

Proof: See Appendix.

The first part of the proposition follows directly from equation (14) because short-run output at a positive interest rate does not depend on long-run fiscal policy. The second part is detailed in the Appendix. Hence, we see that deficit spending is irrelevant at a positive interest rate, but can be either contractionary or expansionary at a zero interest rate. Table 8 illustrates this using numerical examples.

Table 8 shows the effect of one dollar of deficit on short-run demand under three different assumptions about how this long-run adjustment takes place, i.e. via an increase in long-run sales taxes, a cut in the long-run size of the government or an increase in labor taxes. Therefore, this experiment assumes that either γ_G is 1 (and $\gamma_I = \gamma_s = 0$), $\gamma_I = 1$ or $\gamma_s = 1$. We use the same values for the parameters as before, but now there is one additional parameter, δ, which determines the speed at which debt is paid back to its original level. We assume that the half-life of debt is 5 years, which implies that $\delta = 0.9659$.

The results suggest that running budget deficits is expansionary if the deficit is financed by a reduction in the long-run size of the government or by an increase in sales tax. Meanwhile, the effect of budget deficits is negative on short-run demand, if it is financed by increases in long-term labor tax. The reason for this is exactly the same as already analyzed in the previous section. In this part, we make more explicit the way in which debt dynamics have an impact on those long-run variables. Quantitatively, we see that these "multipliers" are smaller than those that depend directly on short-run taxes and spending. We see that a dollar in deficit in period t will result in a 20 cent (GD scenario) or 10 cent (GR scenario) increase in output if the deficit is financed by a reduction in the long-run size of the government spending or an increase in sales tax. Similarly, it will reduce output by 20 and 10 cents, respectively, if financed by an increase in long-run labor taxes. This smaller size, however, may be somewhat misleading, as the next section makes clear.

Table 8: The effect of an increase in short-run debt on short-run output under three assumption about how the debt is financed

	$i = 0$	
	GD	GR
	[5% , 95%]	[5% , 95%]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{b}_S / \hat{G}_{L,t} > 0}$	**0.2** [0.11 , 0.32]	**0.1** [0.06 , 0.14]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{b}_S / \hat{\tau}^s_{L,t} > 0}$	**0.2** [0.1 , 0.4]	**0.1** [0.0 , 0.2]
$\frac{\Delta \hat{Y}_S}{\Delta \hat{b}_S / \hat{\tau}^I_{L,t} > 0}$	**-0.2** [-0.4 , -0.1]	**-0.1** [-0.2 , 0.0]

5.2 A constant increase in short-run deficits

The previous thought experiment considered the effect of a one-time increase in government debt. The rationale for this was mainly that it helped us get simple closed-form solutions, but it also seemed natural if one wants to consider a large one-time event that may create a fiscal burden, such as a banking crisis. Now consider a constant increase of the deficit in the short run. This assumption is natural in the context of our experiments in section 3.2, 3.3 and 3.4, where we considered fiscal interventions that generated a constant deficit or surplus in each period in the short run. This will also be useful to study policy regimes in the next subsection. To be more precise, let us replace A6 (i.) with the following assumption:

A8 Fiscal policy in the short and long run is given by the same rule as A6 (ii.) through A6 (iv.), but A6 (i.) is replaced with

$$i) \quad \hat{D}_t = \hat{D}_S \ \text{ for } t < t_S.$$

The solution in the long run is the same as in last subsection. In the short run, however, output and inflation are no longer some constants. Now, they are functions of the debt, the value of the deficit, $\hat{D}_S$, and the shock, r^e_S. Accordingly, we look for a solution of the form

$$\hat{Y}_{S,t} = Y^b b_{S,t-1} + Y^D \hat{D}_S + Y^r r_S \tag{17}$$

for output and an analogous solution for the other endogenous variables. The model can then be solved numerically using the method of undermined coefficients. Table 9 reports the numerical value of the coefficient Y^D in (17), comparable to Table 8, under three different assumptions of how the deficit is paid of in the long run. We thus have $Y^D = \frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{G}_{L,t} > 0}$ if the short-term deficit is paid off with a reduction in the long-run size of the government (so $\gamma_G = 1$), $Y^D = \frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{\tau}^I_{L,t} > 0}$ if it is paid off with increases in long-term income taxes (so $\gamma_I = 1$) and $Y^D = \frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{\tau}^s_{L,t} > 0}$ if $\gamma_s = 1$. This coefficient has the following interpretation: If the deficit increases by one dollar in every period t in the short run, by how many dollars will output go up or down in that period? Hence, this number is better comparable with the multipliers of spending and taxes than we explored in earlier sections.

As shown in the table, the effect of the deficit depends on how it will be paid off in the future. Consider the GR calibration. A one dollar increase in the deficit will increase output by 30 cents if the government pays it down by cutting long-run government spending. In contrast, if a one dollar deficit is financed by an increase in long-term income taxes, then output will decrease by 30 cents. Overall the multiplier increases relative to our last experiment, and even in a more extreme way in the GD calibration.

Table 9: The effect of an increase in short-run deficits on short-run output under three assumptions about how the debt is financed

	$i = 0$	
	GD	GR
	[5% , 95%]	[5% , 95%]
$\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{G}_{L,t} > 0}$	**1.8** [0.5 , 4.2]	**0.3** [0.1 , 0.7]
$\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{\tau}^s_{L,t} > 0}$	**2.2** [0.6 , 5.0]	**0.3** [0.1 , 0.8]
$\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{\tau}^I_{L,t} > 0}$	**-1.9** [-4.1 , -0.4]	**-0.3** [-0.7 , -0.1]

6 Policy regime matters

We are now in a position to study the effect of various types of fiscal policy, accounting for debt dynamics. The effect of government spending can approximately be split up into three pieces. First, a direct effect comes about even if taxes are lump-sum. Second, there is an indirect effect on the deficit. Finally, a third effect is how those deficits will influence expectations and thereby current output. By now, we have already explored each of these effects separately and now we combine the individual pieces. Consider the following policy regime at a zero interest rate:

Regime 1 In the short run ($t < t_S$), $G_t = G_S$, $\hat{\tau}^I_t = \hat{\tau}^s_t = \hat{T}_t = 0$ and $D_t = D_S$. In the long run, the government behaves according to A6 (iii.), A6(iv.) and A7 with $\gamma_G = 1$.

This policy regime suggests that, in the short run, the government will increase spending and that this spending will be associated with constant short-run deficits (or surpluses) D_S. Those deficits will be paid off at time $t \geq t_S$. The net effect of increasing government spending in this policy regime, in the short run, can then be approximated by

$$\underbrace{\frac{\Delta \hat{Y}_s}{\Delta \hat{G}_s}_{(from\ Table\ 4)}} + \underbrace{\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S}_{(from\ Table\ 5)}} * \underbrace{\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{G}_{L,t} > 0}_{(from\ Table\ 9)}}$$

GR (mode)	**1.2**	**0.5**	**0.3**	= **1.35**
GD (mode)	**2.2**	**-0.3**	**1.8**	= **1.66**.

For the GR calibration, we see that the effect of increasing government spending is higher in policy Regime 1 than when spending is financed just by current or future lump-sum taxes as in Table 4. Meanwhile, we see that the multiplier is smaller in the GD calibration. What is the intuition? Let us start with the GR case. The fact that government spending creates deficits triggers expectations of a future reduction in the size of the government. These long-run expectations also increase demand. The reason is that lower long-run government spending leaves more room for private consumption in the long run and thus, by the permanent income hypothesis, this means that the consumers want to consume more today (since they are trying to smooth consumption). In the GD calibration, however, the spending multiplier is reduced. The reason is that now government spending in the short run decreases the budget deficit and, thus triggers expectation of higher long-run government spending, which is contractionary in the short run.

Consider the following policy regime with the assumption that deficits are paid off using future income taxes:

Regime 2 In the short run, $t < t_S$, $G_t = G_S$, $\hat{\tau}^I_t = \hat{\tau}^s_t = \hat{T}_t = 0$ and $D_t = D_S$. In the long run, the government behaves according to A6 (iii.), A6 (iv.) and A7 with $\gamma_{\tau^I} = 1$.

We can now again compute the marginal effect of government spending as before by:

$$\underbrace{\frac{\Delta \hat{Y}_s}{\Delta \hat{G}_s}_{(from\ Table\ 4)}} + \underbrace{\frac{\triangle \hat{D}_S}{\triangle \hat{G}_S}_{(from\ Table\ 5)}} * \underbrace{\frac{\Delta \hat{Y}_{S,t}}{\Delta \hat{D}_S / \hat{\tau}^I_{L,t} > 0}_{(from\ Table\ 9)}}$$

GR (mode)	**1.2**	**0.5**	**-0.3**	= **1.05**
GD (mode)	**2.2**	**-0.3**	**-1.9**	= **2.77**

Here we see that the marginal effect of government spending in Regime 2 is now lower than if government spending is financed by lump-sum taxation in the GR scenario. The intuition is straightforward: Since government spending generates deficits, this implies that long-run income taxes need to be higher, which reduces output in the long run. This reduces demand in the short run. In the GD scenario, we see that the marginal effect of government spending is even higher. This is because government spending generates a budget surplus rather than a deficit, and thus implies lower labor taxes in the future.

Overall, the message of these examples is that how a fiscal expansion is financed, through adjustment of future taxes and spending, can have an important effect on demand in the short run. Thus, a given government stimulus at a zero interest rate should be complemented with a plan about how short-run budget deficits or surpluses will be met in the future.

7 Extension: Government default risk

The issue of government default is a subject for a different paper, as it involves several important considerations. Nevertheless we can still make a few useful observations, even within the context of our simple framework. Consider now an asset that has an interest rate i_t^v but with probability $(1-\rho_{t+1})$ it will not be repaid in the next period. This asset will satisfy the asset-pricing equation

$$u_c(C_t) = (1+i_t^v)\beta E_t(1-\rho_{t+1}(.))\frac{\xi_{t+1}u_c(C_{t+1})(1+\tau_t^s)P_t}{\xi_t(1+\tau_{t+1}^s)P_{t+1}}.$$

To a first-order approximation, the relationship between $\hat{\imath}_t$ and $\hat{\imath}_t^v$ is

$$\hat{\imath}_t^v = \hat{\imath}_t + E_t\rho_{t+1}.$$

The existence of an asset of this type does not change anything else in the model, since we maintain the assumption that the central bank sets the risk-free nominal interest rate. If we assume that government debt, however, is priced in accordance to this relationship (e.g. due to the fact that people believe that it may default), the government budget constraint is given by

$$\begin{aligned}\frac{\bar{b}}{\bar{Y}}\hat{b}_t - \frac{\bar{b}}{\bar{Y}}(1+\bar{r})\hat{b}_{t-1} &= \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})[\hat{\imath}_{t-1}^v - \pi_t] + (1+\bar{\tau}^s)\hat{G}_t - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_t - \hat{\tau}_t^I - \frac{\bar{C}}{\bar{Y}}\hat{\tau}_t^s - \hat{T}_t \\ &= \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})[\hat{\imath}_{t-1} - \pi_t] + (1+\bar{\tau}^s)\hat{G}_t - (\bar{\tau}^I + \bar{\tau}^s)\hat{Y}_t \\ &\quad -\hat{\tau}_t^I - \frac{\bar{C}}{\bar{Y}}\hat{\tau}_t^s - \hat{T}_t - \frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})E_{t-1}\rho_t,\end{aligned}$$

where, in the last line, we substituted $\hat{\imath}_t^v$ for the risk-free rate. The equilibrium will be determined as before, with this budget constraint replacing the previous one. As we can see, the only new element in the model is the existence of default risk $\frac{\bar{b}}{\bar{Y}}(1+\bar{\imath})E_{t-1}\rho_t$, which implies that the government needs to pay more to service its debt. Note that this risk enters exactly like any fiscal cost, as a negative lump-sum tax, and then our analysis from the previous section applies: The effect of the increase in the risk premium critically depends on how people expect the higher implied debt burden to be paid in the future.

8 Conclusion

In this paper, we have studied how various fiscal policies affect budget deficits and how those budget deficits may affect short-run demand via expectations. We have left several important aspects of this issue off the table, and let us bring up only one here.

A key assumption maintained throughout the paper was that inflation in the long run is zero. Hence there was no explicit interaction between fiscal policy today and inflation expectations over the medium or long term. An important consideration, however, is that fiscal deficits today and high nominal debt levels may very well create expectations of higher inflation tomorrow. In the face of high nominal debt, in fact, the government has an incentive to inflate. If higher deficits trigger expectations of higher future inflation, they are expansionary at a zero interest rate, since this reduces the real interest rate and then increases demand. This is documented more explicitly in Eggertsson (2006), where those links are modeled in an infinitely repeated game between the government and the private sector. Quantitatively, Eggertsson (2006) shows that these effects can be very big. Interestingly, however, that mechanism assumes that monetary and fiscal policy are coordinated, an assumption that seems inappropriate in large monetary unions. It is difficult to imagine, for example, that large amounts of Greek debt creates quantitatively significant inflation incentives for the European Central Bank, albeit this remains a topic for further study.

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10 Appendix: Proofs of propositions

Proposition 12 *Suppose that A1, A3, A6 and A7 hold. Then, the multiplier of deficit spending on short-run output at a positive interest rate (if $r_S^e > 0$) is given by*

$$\frac{\Delta \hat{Y}_S}{\Delta \hat{D}_S} = 0,$$

while output at a zero interest rate (if $r_S^e < 0$) is given by

$$\begin{aligned}\frac{\Delta \hat{Y}_S}{\Delta \hat{D}_S} \quad = \quad & \frac{(1-\beta\mu)(1-\mu)\psi\omega}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]}\frac{\frac{b}{Y}(1+i-\rho)\delta}{[1-\sigma^{-1}\psi[\bar{\tau}^s+\bar{\tau}^I\bar{w}]]}\gamma_G \\ & +\frac{(1-\beta\mu)(1-\mu)\chi^s\sigma\psi\omega}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]}\frac{\frac{b}{Y}(1+i-\rho)\delta}{[1-\psi\chi^s[\bar{\tau}^s+\bar{\tau}^I\bar{w}]]}\gamma_s \\ & -\frac{(1-\beta\mu)(1-\mu)\psi\chi^I}{[(1-\mu)(1-\beta\mu)-\mu\sigma\kappa]}\frac{\frac{b}{Y}(1+i-\rho)\delta}{([\bar{w}-\psi\chi^I[\bar{\tau}^s+\bar{\tau}^I\bar{w}])}\gamma_I.\end{aligned}$$

Proof: The first part of the proof follows directly from equation (14). The second part involves a few additional steps.

At date $t \geq t_S$, our policy commitment in assumption A3 implies that $\pi_t = 0$. By equation (14), then

$$\hat{Y}_{t_S} = -\psi\chi^I\hat{\tau}^I_{t_S} - \psi\chi^s\hat{\tau}^s_{t_S} + \psi\sigma^{-1}\hat{G}_{t_S}. \tag{18}$$

Then, in order to determine $\hat{Y}_{t_S}$, we need to find $(\hat{\tau}^I_{t_S}, \hat{\tau}^s_{t_S}, \hat{G}_{t_S})$. As stated in the paper, according to assumption A6:

$$-\frac{b}{Y}(1+i-\rho)\delta b_{t_S} = \left[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)\right]\hat{G}_{t_S} - \left[1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I\right]\hat{\tau}^I_{t_S} - \left[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s\right]\hat{\tau}^s_{t_S}.$$

Assumption A7 states that

$$\begin{aligned}\gamma_{G,t} \quad &\equiv \quad \frac{[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)]\hat{G}_t}{\hat{D}_t} = \gamma_G \qquad (19)\\ \gamma_{I,t} \quad &\equiv \quad -\frac{(1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I)\hat{\tau}^I_t}{\hat{D}_t} = \gamma_I \\ \gamma_{s,t} \quad &\equiv \quad -\frac{[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s]\hat{\tau}^s_t}{\hat{D}_t} = \gamma_s\end{aligned}$$

Using equations (18) and (19), we can write

$$
\begin{aligned}
\hat{Y}_{t_S} &= -\psi\sigma^{-1}\frac{\frac{b}{Y}(1+i-\delta)\delta}{[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)]}\gamma_G - \psi\chi^I\frac{\frac{b}{Y}(1+i-\delta)\delta}{(1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I)}\gamma_I \qquad (20)\\
&\quad -\psi\chi^s\frac{\frac{b}{Y}(1+i-\delta)\delta}{[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s]}\gamma_s\}b_{t_S-1}\\
&= \psi\sigma^{-1}\phi_G - \psi\chi^I\phi_{\tau^I} - \psi\chi^s\phi_{\tau^s}\}b_{t_S-1}\\
&= \phi_y b_{t_S-1},
\end{aligned}
$$

where $\phi_G \equiv -\gamma_G\frac{\frac{b}{Y}(1+i-\delta)\delta}{[1+\bar{\tau}^s-\sigma^{-1}\psi(\bar{\tau}^I+\bar{\tau}^s)]} < 0$, $\phi_{\tau_s} \equiv \gamma_s\frac{\frac{b}{Y}(1+i-\delta)\delta}{[\frac{\bar{C}}{\bar{Y}}-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^s]} > 0$, $\phi_{\tau_I} \equiv \gamma_I\frac{\frac{b}{Y}(1+i-\delta)\delta}{1-(\bar{\tau}^I+\bar{\tau}^s)\psi\chi^I}$, and $\phi_y \equiv \psi\sigma^{-1}\phi_G - \psi\chi^I\phi_{\tau^I} - \psi\chi^s\phi_{\tau^s}$.

Now consider the solution at time $t < t_s$. Let us again denote by subscript S the period in which the shock $r_t^e = r_S^e$ and let L denote the periods $t \geq t_S$. Recall that in the short run, by A6 (iii.), the tax and spending instruments are at steady state and that inflation is zero in the L state. Then, in periods $t < t_s$, we can then write

$$
\begin{aligned}
\hat{Y}_{S,t} &= \mu E_t\hat{Y}_{S,t+1} + (1-\mu)E_t\hat{Y}_{L,t+1} + \sigma\mu E_t\pi_{S,t+1} + \sigma r_S^e \qquad (21)\\
&\quad -(1-\mu)E_t\hat{G}_{L,t+1} + \sigma\chi^s(1-\mu)E_t\hat{\tau}^s_{L,t+1}\\
\pi_{S,t} &= \kappa\hat{Y}_{S,t} + \beta\mu E_t\pi_{S,t+1}. \qquad (22)
\end{aligned}
$$

First, observe that $E_t\hat{Y}_{L,t+1}$ is given by equation (20), which is a linear function of $\hat{b}_t$. Then, we can write $E_t\hat{Y}_{L,t+1} = \phi_y\hat{b}_t$. Similarly, $E_t\hat{G}_{L,t+1} = \phi_G b_t$ and $E_t\tau^s_{L,t+1} = \phi_\tau\hat{b}_t$, where ϕ_G and ϕ_τ are given by equation (20). A solution is then a collection of stochastic processes for $\{\hat{b}_t, Y_{S,t}, \pi_{S,t}\}$ that solve A6 (i.), equation (21) and equation (22). Because $\hat{b}_t$ is a random walk, then $\hat{b}_t$ and r_S^e are the state variables of this system. Then, for a unique bounded solution, we can write $\hat{Y}_{S,t} = E_t\hat{Y}_{S,t+1} = Y_r r_S^e + Y_b\hat{b}_t$, where Y_r and Y_b are unknown coefficients and similarly $\pi_{S,t} = E_t\pi_{S,t+1} = \pi_r r_S^e + \pi_b\hat{b}_t$. Substitute this into equations (21) and (22), along with $E_t\hat{Y}_{L,t+1} = \phi b_t$ to obtain the following two equations:

$$
\begin{aligned}
Y_b\hat{b}_t &= \mu Y_b\hat{b}_t + (1-\mu)\phi_y\hat{b}_t + \sigma\mu\pi_b\hat{b}_t + \sigma r_S^e\\
&\quad -(1-\mu)\phi_G b_t + \sigma\chi^s(1-\mu)\phi_{\tau_s} b_t\\
\pi_b b_t &= \kappa Y_b b_t + \beta\mu\pi_b b_t.
\end{aligned}
$$

By matching coefficients, the proposition is obtained.