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Abstract

With short-term interest rates at the zero lower bound, forward guidance has become a key tool for central bankers, and yet we know little about its effectiveness. This paper first empirically documents the impact of forward guidance announcements on a broad cross section of financial markets data and professional forecasts. We find that Federal Open Market Committee (FOMC) announcements containing forward guidance had heterogeneous effects depending on the other content of the statement. We show that once we control for these other elements, forward guidance had, on average, positive and meaningful effects on output and inflation expectations. Using this benchmark, we then show that standard medium-scale DSGE models tend to grossly overestimate the impact of forward guidance on the macroeconomy, a phenomenon we call the "forward guidance puzzle." We explain why this is the case and show that incorporating a perpetual youth structure into the benchmark provides a possible resolution to the puzzle.

Key words: unconventional monetary policies, forward guidance, DSGE models, perpetual youth models

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1 Introduction

For decades, macroeconomists have attempted to quantify the effects of monetary policy actions on the economy. By now, a large literature has documented the transmission mechanism of surprise changes in short-term interest rates onto the economy, using either VARs or DSGE models (e.g., Sims (1980), Christiano et al. (1999), Christiano et al. (2005)). While we thus have some understanding of the effects of short-term interest rates, this tool has been constrained by the zero lower bound (ZLB) during the financial crisis in most developed economies. Instead, many central banks have used other tools such as announcements about the future path of the policy rate ("forward guidance"), or "quantitative easing" measures involving a change in the size and the composition of the central bank balance sheet. Forward guidance has been used extensively and explicitly by the Federal Reserve since the FOMC meeting of December 16, 2008, so as to affect long-term bond yields and stimulate aggregate expenditures (see Woodford (2012) and Campbell et al. (2012a)).[1] Moreover, Woodford (2012), building on results by Krishnamurthy and Vissing-Jorgensen (2011) and Bauer and Rudebusch (2011), emphasizes the "signaling channel" of the Fed's asset purchases — that is, he argues that quantitative easing itself can at least in part be interpreted as implicit forward guidance.

While the literature has provided strong theoretical justifications for the use of such forward guidance (e.g., Eggertsson and Woodford (2003)), evidence on the quantitative effects of this policy tool on the macroeconomy is still limited. This paper is intended to help fill that gap. The lack of evidence in the literature may not be surprising in light of the fact that there are significant identification challenges in the case of shocks that are anticipated.[2]An announcement by policymakers that they will maintain the policy rate at the ZLB for longer

[1]At that meeting, the FOMC's statement mentioned that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate for some time." Three months later, the FOMC reinforced its forward guidance by stating that the exceptionally low levels of the federal funds rate would likely be warranted "for an extended period." This sentence was reiterated in each subsequent FOMC statement until August 9, 2011, when the FOMC argued that economic conditions "are likely to warrant exceptionally low levels of the federal funds rate at least through mid-2013." That sentence was maintained in subsequent statements until January 25, 2012, when the date was pushed forward to "late 2014."

[2]Gertler and Karadi (2015) use a VAR to provide some evidence on forward guidance shocks, which suggests that these shocks have larger effects on activity and inflation than standard contemporaneous shocks. D'Amico and King (2015) perform a similar analysis using sign-restrictions.

than initially anticipated by market participants may have two types of effects. On the one hand, it could be interpreted as more monetary stimulus: it should lower the market's expectation of future federal funds rate (FFR), which contributes to lower longer term yields, hence stimulates economic activity and puts upward pressure on inflation. On the other hand, such an announcement could be interpreted by market participants as revealing negative news about the state of the economy, if they believe that the FOMC has access to information not shared by market participants. In this case, such an announcement would be associated with lower long-term yields and lower projections of economic activity. The interpretation chosen by the market participants could thus depend in very subtle ways on the FOMC communication.[3]

Empirically, Gürkaynak et al. (2005b) and more recently Campbell et al. (2012a) find strong evidence that FOMC announcements move asset prices. However, Campbell et al. (2012a) illuminate the challenge in using time series to identify the effect of forward guidance. Specifically, when they assess the impact of exogenous anticipated changes in monetary policy on private expectations, they find that the opposite sign from what theory predicts, showing that the public's interpretation of the policy depends crucially on other components of the communication. Moreover, even if it were possible to identify the impact of, say, four quarters-ahead forward guidance, its effect would not necessarily be the same as, say, that of eight-quarters ahead forward guidance (Campbell et al. (2012a) consider one through four quarters ahead forward guidance; forward guidance communicated in early 2012 was going through the end of 2014, and hence amounted to approximately eight quarters). Given that policymakers have seldom experimented with forward guidance this far in the future, there is little data to guide them.

New Keynesian DSGE models following the work of Christiano et al. (2005) and Smets and Wouters (2007) are in principle well suited to study the effects of forward guidance. Such models have been found to fit the data reasonably well and provide a good forecasting performance relative to reduced form models such as VARs, private forecasters, or the Greenbook (see Smets and Wouters (2007), Del Negro et al. (2007), Edge and Gürkaynak (2010), and Del Negro and Schorfheide (2013)). Most importantly, being laboratory economies, they can be used to study the impact of policy experiments never performed before. As shown

[3]Woodford (2012) argues that several announcements made prior to August 2012 about the future path of policy rates did not indicate a clear commitment to maintaining short-term rates low, so that they ran the risk of being interpreted as reflecting a deteriorating forecast for output and or inflation.

by Laseen and Svensson (2011), forward guidance can be captured in DSGE models using anticipated policy shocks. Such shocks reflect deviations of the short-term interest rate from the historical policy rule that are anticipated by the public. They can be affected by policymakers' announcements about their intentions regarding the future path of the policy rate. Milani and Treadwell (2011) study the impulse responses to anticipated policy shocks using a simple three-equations New Keynesian DSGE model. Campbell et al. (2012b) go further and investigate the impact of forward guidance on the macroeconomy by estimating a medium scale DSGE model broadly similar to the one in this paper using data on market expectations for the federal funds rate, in addition to a standard set of macro variables, for the sample 1987-2007. They find that forward guidance explains about 9 percent of output and hours fluctuations at the business cycle frequency, and more than 50 percent of the movements in the federal funds rate. Their results indicate that even in the pre-Great Recession period, forward guidance played a large role in monetary policy — a finding that echoes that of Gürkaynak et al. (2005b) — and a significant role in terms of business cycle fluctuations.

The problem with DSGE models, however, is that they appear to deliver unreasonably large responses of key macroeconomic variables to central bank announcements about future interest rates — a phenomenon we can call the "forward guidance puzzle". Carlstrom et al. (2012b) show that the Smets and Wouters model would predict an explosive inflation and output if the short-term interest rate were pegged at the ZLB between eight and nine quarters. This is an unsettling finding given that the horizon of forward guidance by the FOMC has at times been of at least eight quarters. This "forward guidance puzzle" results from the interaction of many features of DSGE models. These include the excess sensitivity of consumption to interest rate changes and the front-loading associated with the New Keynesian Phillips curve, which have long been criticized for being counterfactual.[4] In addition, we stress the excessive response of consumption to interest rate changes far in the future implied by the standard consumption Euler equation, emphasized also in a recent paper by McKay et al. (2015). As we explain in Section 3.3, what is novel about the forward guidance policy experiment is that it compounds all these implications, bringing to the fore the limitations of typical medium scale DSGE models used for policy analysis.

[4] Campbell and Mankiw (1989) and Reis (2006) for instance discuss the excess sensitivity of consumption to interest rates, while Ball (1994) and Mankiw and Reis (2002) criticize the New Keynesian Phillips curve.

This paper makes three contributions. First, we document empirically the response of financial market variables and forecasts of key macroeconomic variables to actual FOMC communications involving changes in calendar-based forward guidance. We both provide a detailed description of the heterogeneity in past forward guidance episodes, as well as a realistic benchmark against which our model's predictions can be compared. We find that the estimated effect of forward guidance on output and inflation expectations are positive and non trivial.

Second, we characterize the quantitative implications of forward guidance using an arguably realistic experiment. Specifically, we focus on the September 2012 episode and compute the FFR expected path before and after the announcement, as implied by OIS rates. We show that the model-implied response of macroeconomic variables to this rather modest change in expectations is unrealistically large.[5]

The third contribution of the paper is to point to the source of the problem and suggest a solution. Specifically, we adopt Blanchard (1985)'s and Yaari (1965)'s perpetual youth model, which has been incorporated recently in simple New Keynesian models by Piergallini (2006), Nisticò (2012), and Castelnuovo and Nisticò (2010), and embed it in the full fledged medium scale model of Smets and Wouters (2007), yielding what we will refer to as the SWBY model. We assume, as in these papers, that agents face each period a constant probability of dying and being replaced by a new agent, an assumption that we believe adds realism. While each individual in this model behaves similarly to the representative agent, the cohort structure results in an aggregate economy that discounts the future more heavily. This implies that announcements of policy changes in the future generate smaller effects on current aggregate variables than is the case in models with infinitely lived agents.

Much interesting recent work, some of which appeared after the first draft of our paper, provides possible resolutions to the forward guidance puzzle. McKay et al. (2015) show that aggregate consumption in a model with heterogeneity and borrowing constraints does not suffer the same pitfall as in standard representative agent models, where the consumption response to current real rate cuts is just as large as that to interest rate cuts very far in the future. Precautionary savings considerations limit the individual response to future shocks because agents are leery to draw down their buffer for a long period of time. They also show

[5] An earlier version of Carlstrom et al. (2012b), Carlstrom et al. (2012a), featured a rather extreme forward guidance experiment.

that discounting expected future consumption in the aggregate Euler equation can generate responses in a medium scale DSGE model that are similar in magnitude to those in the heterogeneous agents' economy.[6] Caballero and Farhi (2014) show that forward guidance at the zero lower bound has limited effects (in their model, no effect at all) on the real economy but sizable effects on risk premia. This is because forward guidance promises stimulus when the economy has already recovered and the marginal utility of consumption is low.[7] Carlstrom et al. (2012b), Chung et al. (2014), and Kiley (2014) focus on the other ingredient in the forward guidance puzzle, namely the front loading implicit in the New Keynesian Phillips curve. These papers argue that sticky information models à la Mankiw and Reis (2002) are less susceptible to the forward guidance puzzle. Andrade et al. (2015) also build on information heterogeneity and present a model in which agents disagree on the nature of the forward guidance announcements, whether *Delphic* or *Odyssean*, making these announcements less effective. Using the terminology of Campbell et al. (2012a), announcements are *Delphic* when they mainly reveal bad news about the economy, while they are *Odyssean* when they are interpreted as announced deviations from the policy rule (anticipated shocks) or possible changes in the rule itself (as in Engen et al. (2014)). In the empirical part of the paper we show that the public may have indeed interpreted some forward guidance announcements as *Delphic*. Finally, in Gabaix (2015) bounded rationality yields a discounting of future consumption in the aggregate Euler equation similar to that arising in our model.[8]

We think that many of these explanations are plausible and contribute to the solution of the puzzle, and to the construction of models that can provide credible answers to forward guidance experiments. Our proposed solution is closest to that of McKay et al. (2015), in that it mainly affects the aggregate Euler equation, but contains some flavor of the sticky information literature, in that it results in more discounting of in the New Keynesian Phillips curve than in a similarly parameterized representative agent model. Importantly, our pro-

[6]Werning (2015) however argues that market incompleteness does not by itself lead to an attenuation of the effects of forward guidance.

[7]Hanson and Stein (2015) provide evidence that forward guidance, and monetary policy in general, mainly manifest itself through its effects on risk premia, and term premia in particular.

[8]Several other recent papers discuss forward guidance. For instance, Keen et al. (2015) explore forward guidance using a non-linear representative agent model. Harrison (2014) argues that forward guidance policy experiments are not "modest" in the sense of Leeper and Zha (2003), and hence it is not surprising that the DSGE models yield implausible answers, while Bodenstein et al. (2012) and Haberis et al. (2014) present the view that forward guidance is ineffective because it is imperfectly credible.

posed solution has the advantage than it can be easily embedded in a standard medium-scale DSGE model.

The paper proceeds as follows. Section 2 provides some empirical evidence on the effects of forward guidance in the US. It documents the responses of financial market data as well as private sector forecasts of key macroeconomic variables to FOMC announcements. Section 3 briefly describes the DSGE model used, its estimation, and how we formalize the introduction of a fixed interest-rate path. It then presents the implications of interest rate announcements in this model, and reports that the model generates an excessive response to announcements about changes in the expected future interest rate path, a phenomenon we call the forward guidance puzzle. Section 4 proposes incorporating a perpetual-youth structure into a DSGE model as a potential solution to this puzzle. Section 5 concludes.

2 Empirical Evidence on the Effects of Forward Guidance

In this section, we provide empirical evidence on the effect of forward guidance announcements. Our goal is twofold – first, we aim to provide a comprehensive description of forward guidance episodes, and second, we aim to provide a benchmark against which we can compare the predictions of our model. Empirically documenting the effects of forward guidance on the macroeconomy is challenging due to both data constraints and the nature of forward guidance announcements, which confound negative news about the economy, forward guidance and in some cases quantitative easing. Since we model forward guidance as purely stimulative in our theoretical exercises, it is important for our benchmark to disentangle the direct effect of forward guidance from these other elements.

2.1 Descriptive Evidence of Forward Guidance Episodes

We begin by documenting the impact of the three FOMC announcements in which there were calendar-based announcements about future interest rate paths: August 2011, January 2012, and September 2012. We focus on calendar-based forward guidance events as they provide, at least in principle, a quantifiable degree of expected policy accommodation. We first document the impact of these announcements on a broad cross section of daily financial markets data, following the event study approach of Krishnamurthy and Vissing-Jorgensen (2011). Since we are mostly interested in the macroeconomic effects of forward guidance, we complement this evidence by documenting the impact of FOMC announcements on Blue Chip forecasters' expectations. In principle we would like to perform an event study exercise using survey forecasts, comparing the projections the day before and the day after the announcement. Because survey forecasts are available at a monthly, as opposed to daily, frequency, we document the change in the forecasts in between survey dates "sandwiching" the event, trying to control for other news and asset price changes that may have affected the forecast. Since the survey also contains projections for financial variables that are part of the event study. we can cross-validate our survey-based approach with the outcome of the more standard event study. Table 1 summarizes the important elements of each of these three events.[9] While all statements are similar in their forward guidance announcements, the table below shows that they differ significantly in their language about the economy as well as their QE announcements. As we will discuss further in Section 2.1.2, the various components of the statement each play an important role.

Table 1: Summary of Forward Guidance FOMC Statement Language

	Economic Conditions		Policy Action	
	Growth	Inflation	Forward Guidance	Quantitative Easing
August 2011	"considerably slower"	"picked up"	"low levels warranted at least through mid 2013"	–
January 2012	"expanding moderately"	"at or below mandate"	"low levels warranted at least through late 2014"	–
September 2012	"moderate pace"	"subdued"	"until after the recovery strengthens... at least through mid 2015"	announce QE3

[9]See Table A1 and A2 in the Appendix for additional detail on the FOMC statement language.

2.1.1 Evidence from Financial Markets

Table 2 reports changes in several asset prices (measured in basis points, unless otherwise noted) in the two day window following each forward guidance event, as in Krishnamurthy and Vissing-Jorgensen (2011).[10] Most of these asset prices coincide with those reported in Krishnamurthy and Vissing-Jorgensen (2011), and are constructed using the same methodology and data sources. Our main findings are as follows. First, the top panel of Table 2 shows that nominal rates declined substantially at most horizons following the January 2012, and especially the August 2011, announcements. In the two days following the August 2011 meeting, constant maturity Treasury yields fell by 23 basis points. Similarly, Figure 1 reports changes in federal funds rate futures, swap basis-adjusted Eurodollar futures, and forward rates extracted from the Treasury yield curve. The futures and forward interest rate curves flattened substantially not only at the short horizons, consistent with the announcements, but also at longer horizons. We can show that the shift in forward rates appears to be entirely consistent with the central bank's announcement.

Second, in August 2011, corporate yields fell less than Treasuries for high credit quality bonds, but actually rose substantially for low credit quality bonds, suggesting an increase in the safety premium. Caballero and Farhi (2014) provide a model where forward guidance has no effect on economic activity but results in increased safety premia, and this evidence appears to be consistent with such a theory. Evidence shown in the next section, however, shows that the August 2011 statement may have been interpreted by forecasters as conveying bad news about the economy, and many of the asset price movements shown here (the decrease in real rate, the stall in the stock market in spite of significant lower real rates, and the increase in safety premia) are also consistent with this explanation. Conversely, in January 2012, yields on corporate bonds fell by roughly the same amount as Treasuries, regardless of the credit rating, suggesting little variation in the safety premium following the announcement.

Third, the response of financial markets to the September 2012 announcement was altogether different from the others. Nominal yields on government securities rose, instead of falling. Agency debt yield also rose, but less than for corresponding Treasuries, while agency MBS yields fell. Real yields fell substantially. Note that the rise in nominal yields

[10] A subset of these data has already been analyzed by Filardo and Hoffman (2014), who limit their analysis to forward rates and nominal long term Treasuries, and Femia et al. (2013).

Table 2: Evidence from Financial Markets

Maturity (years)	**Treasury Yields** (constant maturity) 30	10	5	3	1	**Agency Yields** (Fannie/Freddie) 30	10	5	3	**MBS Yields** 30	15
8/9/2011	-14	-23	-18	-12	-3	-19	-23	-27	-25	-24	-26
1/25/2012	-5	-12	-15	-8	0	-10	-13	-18	-14	-16	-18
9/13/2012	17	11	2	2	0	10	5	0	1	-13	-11

Maturity (years)	**TIPS** (constant maturity) 30	20	10	7	5	**Implied Vol.**	**SP 500** (% change)	**DJ IA** (% change)	**FX** USD/EUR (% change)
8/9/2011	-26	-16	-33	-52	-39	-8.11	0.12	-0.83	-0.01
1/25/2012	-8	-11	-15	-18	-20	-4.21	0.29	0.46	0.56
9/13/2012	-9	-8	-15	-19	-25	-1.13	2.03	1.95	1.78

Maturity (years)	**Breakevens** 20	10	5	**Inflation Swaps** 30	20	10	5	1	**TIPS Spread** 20	10	5
8/9/2011	-7	10	21	8	9	14	13	-3	16	4	-8
1/25/2012	3	3	5	3	3	4	8	12	0	1	3
9/13/2012	24	26	27	26	27	21	28	23	3	-5	1

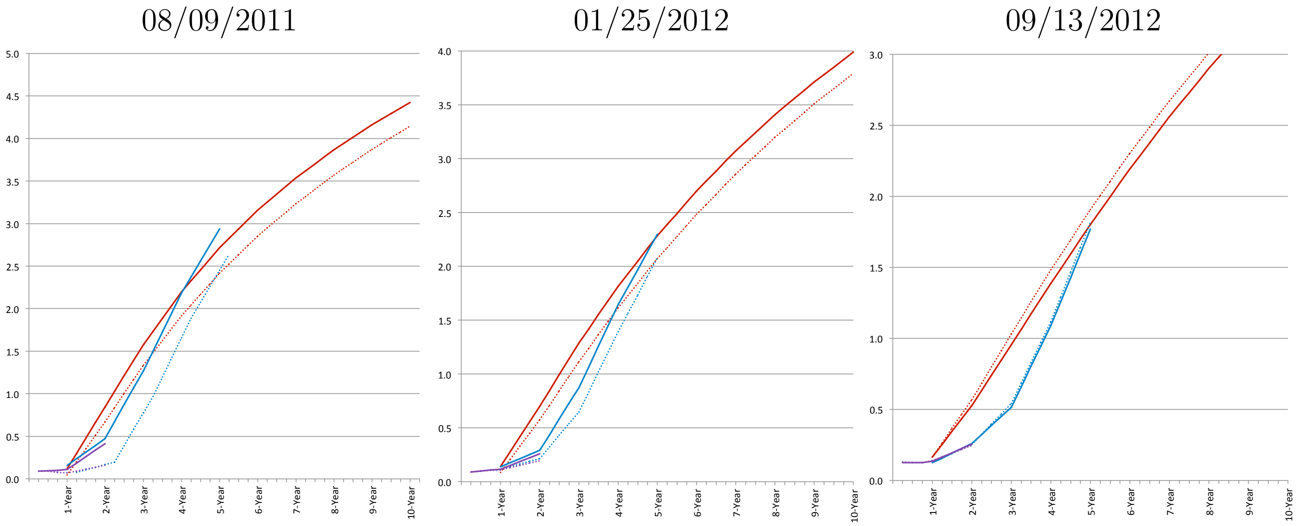
Corporate Yields	Intermediate term Aaa	Aa	A	Baa	Ba	B	Long term Aaa	Aa	A	Baa	Ba	B
8/9/2011	-8	-6	-8	-8	2	16	-11	-9	-5	-5	26	33
1/25/2012	-10	-13	-11	-16	-9	-13	-12	-15	-17	-13	-16	-10
9/13/2012	11	10	7	-2	-8	-15	0	-1	-1	5	-12	-18

Notes: All figures are in basis points unless otherwise noted.

makes the search-for-yield explanation (e.g., Hanson and Stein (2015)) for the fall in real yields arguably less plausible. Breakeven and inflation swap rates rose at both short and long horizons, and the dollar depreciated. The stock market rose by about two percent. Most notably, while high credit quality corporate bond yields rose in line with Treasuries, the yield on low credit quality fell, indicating a sizable decrease in the safety premium.

One explanation for the heterogeneity across episodes is related to the start of QE3: as

Figure 1: Change in the Nominal Futures and Forward Curve following FOMC Announcements



Notes: Federal funds futures, swap basis-adjusted Eurodollar futures, and forward rates extracted from the Treasury yield curve are in purple, blue, and red, respectively. Pre- and post-announcement curves are solid and dotted, respectively.

suggested in Table 1, the central bank announced in September 2012 that it intended to "increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month." According to this explanation, the announced acquisition of agency MBS lowered safety premia, affecting the yield of low credit quality corporate bonds, while the increase in nominal Treasury was then the result of disappointed expectations that QE3 would also include long-term Treasuries. A complementary explanation is that this announcement, together with other parts of the statement, also signaled that monetary policy was going to be more accommodative than previously thought, resulting in a decline in real rates, and increased inflation and growth expectations. Evidence presented in the next section, based on the revision of Blue Chip forecasts, is consistent with this explanation. The more optimistic outlook, in turn, resulted in a flattening of the safety premium. We will explore this heterogeneity further in Section 2.2.[11]

[11] De Graeve et al. (2014) stress that forward guidance does not necessarily yield lower nominal long term rates, as the main source of accommodation consists in lower expected real rates instead.

2.1.2 Evidence from Surveys of Forecasts

In order to quantify the effect of forward guidance on other aspects of economic activity, we use an event study framework and the Blue Chip Financial Forecasts. If information on expectations for economic activity, inflation, and financial market variables were available the day before and two days after the event, we would simply report the two day window change in expectations and attribute it to the news about monetary policy. Unfortunately, the expectations survey is available only once a month. Instead, using the exact collection dates of the survey, we control for any macroeconomic news and changes in asset prices in between surveys and attribute the residual change in expectations during the (approximate) one-month window to the FOMC announcement.[12] Specifically, we use the panel of forecasters i to run the following panel regression separately for each predicted variable (k) and h-quarter ahead forecast (h):

$$\Delta f_{(k,h)_{i,t}} = \gamma_{(k,h)_0} + \gamma_{(k,h)_1}' \ \Delta M_t + \gamma_{(k,h)_2}' \ \Delta AP_t + \gamma_{(k,h)_3}' \ Z_{(k,h)_{t,i}} + \beta_{(k,h)^e} \ D_t^e + \varepsilon_{(k,h)_{i,t}}, \ \text{for } t = 1,..,T, \ i = 1,..,n, \quad (1)$$

where $\Delta f_{(k,h)_{i,t}}$ is the change in the h-quarters ahead forecast of participant i for variable k (e.g., GDP growth) between periods t and $t-1$, ΔM_t and ΔAP_t are vectors of macroeconomic surprises (e.g., payroll report) and changes in asset prices (e.g., stock prices) in the one-month window, respectively, $Z_{(k,h)_{t,i}}$ is a vector of participant-specific variables (e.g., the lagged change in the forecast), and D_t^e is the event dummy which is equal to one if the forward guidance event takes place in the one-month window (that is, $t = t^e$), and is zero otherwise. The γ and β coefficients are indexed by (k,h) to stress the fact that these vary

[12] Romer and Romer (2000) follow a very similar approach in their investigation of whether the Federal Reserve actions reveal information that private forecasters do not have. They regress essentially the change in private forecasts on the change in the Federal Reserve projections (to control for other news) and a dummy variable for central bank actions. We differ from them in that we use i) a panel regression, and ii) actual news and asset price changes as controls, as opposed to the change in Federal Reserve staff projections (which are not yet public for this period.) Altavilla and Giannone (2014) follow a related approach to assess the impact of unconventional monetary policy on forecasters' projections for bond yields. These authors use the Survey of Professional Forecasters however, which is available only once a quarter, which makes it harder to control for other developments in the economy. Our approach also bears similarities with that of Campbell et al. (2012a) in that it looks at the effect of policy announcements on private forecasts, but it differs in two important ways. First, we use the individual-level forecast, rather than the consensus. Second, we adopt an event study framework.

across variables k and forecast horizons h. They are therefore estimated by running separate regressions for each h and k.[13]

The vector ΔM_t contains surprises in macroeconomic releases occurring between $t-1$ and t, where t is the date of the forecast collection.[14] These surprises, which are listed in Table A3 in the Appendix, are computed as the difference between the actual release and the median forecast in the Bloomberg survey and are expressed in the units of the release.[15] The vector ΔAP_t contains changes in asset prices that occurred between the time the $t-1$ and t surveys were collected.[16] These are a subset of the variables discussed in the previous section, one for each category. Appendix Table A3 lists these variables, along with the units in which the change is measured (basis points or log change). Part of these asset price changes may be due to the forward guidance announcement itself. In computing ΔAP_t we therefore subtract from the overall change the change in asset prices that occurred in the two-day window after the FOMC announcement. Finally, the vector of participant-specific variables $Z(k,h)_{t,i}$ includes the lagged value of the change in forecast $\Delta f(k,h)_{i,t-1}$, the previous period's forecast itself $f(k,h)_{i,t-1}$, and participant specific fixed-effects. These variables are included to control for mean reversion and the possibility that forecasters are not rational, and hence do not respond to current period news only (see Coibion and Gorodnichenko (2012)).

We use OLS to obtain the estimates of all coefficients in the spirit of White (1982), but we take into account both heteroskedasticity and correlation across i in computing the

[13]This implies that we ignore the potential correlation of $\varepsilon(k,h)_{i,t}$ across k and h, leading to a loss of efficiency.

[14]We assume that forecasters incorporate news occurring in the two-day collection period, as data is usually released in the morning, allowing forecasters to adjust their Blue Chip submission in light of relevant information released on the last day of collection of the survey.

[15]This specification is inspired by the literature computing the effect of economic news on financial variables (e.g., Fleming and Remolona (1999) and Gürkaynak et al. (2005a))

[16]Specifically, this window ranges from the day after the last collection day of the $t-1$ release and the last collection day of the t release.

standard errors.[17] Specifically, we assume a factor structure for $\varepsilon(k,h)_{i,t}$:

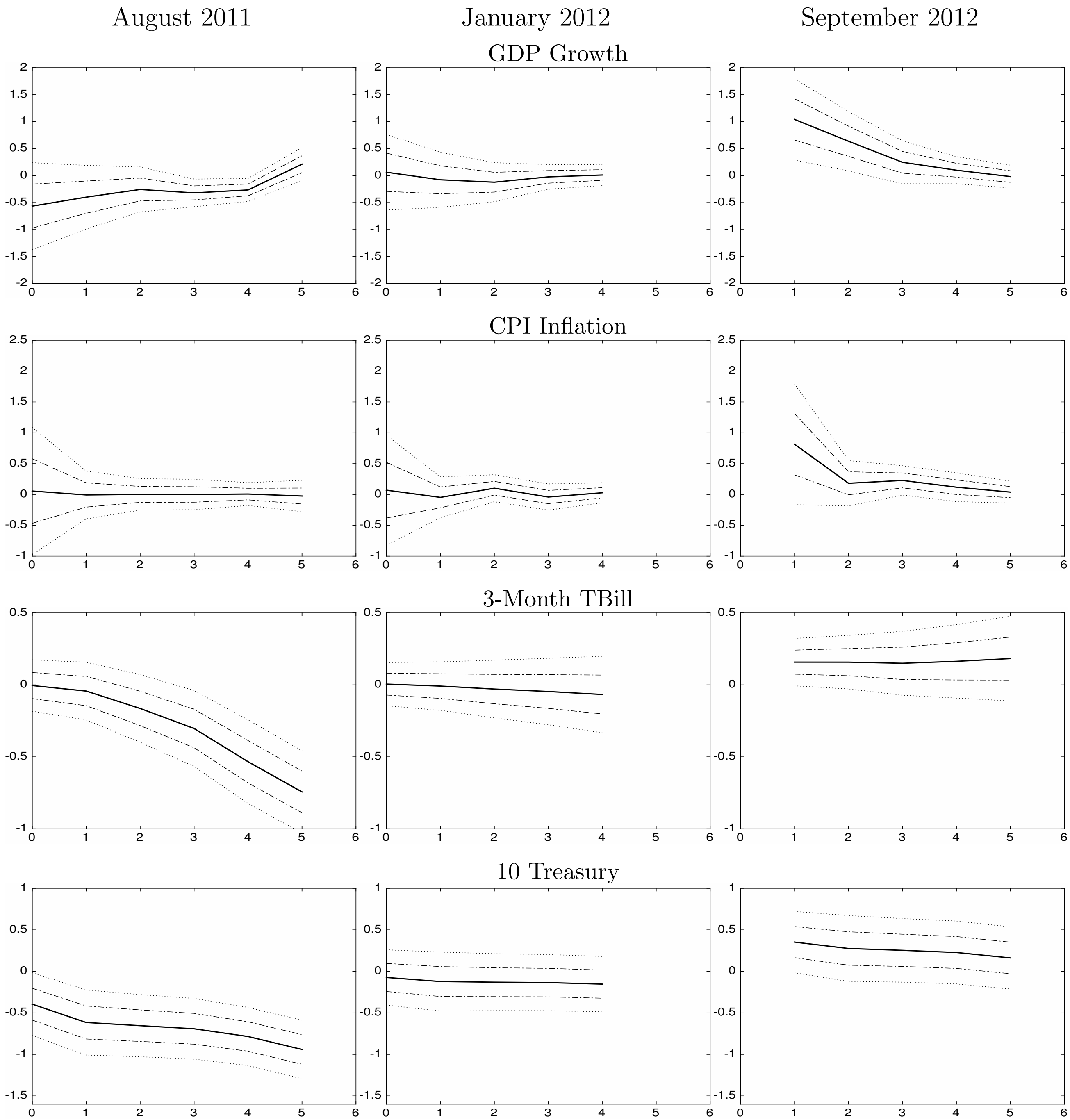
$$
\begin{aligned}
\varepsilon(k,h)_{i,t} &= e(k,h)_t + \nu(k,h)_{i,t}, \\
E[e(k,h)_t^2] &= \sigma^2(k,h)_0, \\
E[\nu(k,h)_{i,t}\nu(k,h)_{j,t}] &= \begin{cases} \sigma^2(k,h)_i & \text{for } i=j \\ 0 & \text{otherwise,} \end{cases} \\
E[e(k,h)_t e(k,h)_s] &= E[\nu(k,h)_{i,t}\nu(k,h)_{i,s}] = 0, \ \text{for } t \neq s. \quad (2)
\end{aligned}
$$

We estimate $\sigma^2(k,h)_0$ as the sample variance of $\hat{e}(k,h)_t = \frac{1}{n}\sum_i \hat{\varepsilon}(k,h)_{i,t}$, and $\sigma^2(k,h)_i$ as the sample variance of $\hat{\nu}(k,h)_{i,t} = \hat{\varepsilon}(k,h)_{i,t} - \hat{e}(k,h)_t$. We then use the $n \times n$ variance covariance matrix $\hat{\Sigma}(k,h)$, whose diagonal and off-diagonal elements are $\hat{\sigma}^2(k,h)_0 + \hat{\sigma}^2(k,h)_i$ and $\hat{\sigma}^2(k,h)_0$, respectively, to construct the White (1980)-robust standard errors. Using the first principal component yields similar results.

We can properly recover the effect of the announcement on expectations under the following assumptions: 1) the γ coefficients are stable over time and are consistently estimated using the panel regression, 2) the regressors capture all public (common) news over the one-month event window, 3) the effect of the announcement on financial markets is fully captured by the two-day window change in asset prices. Assumption (1) is standard, and we use only post-2008 data (that is, data that are mostly in the zero lower bound regime) to account for the fact that the γs may have been different in the pre-zero lower bound period. We should stress that one important caveat to this assumption is that the non-linearities associated with the zero lower bound may result in time variation in γ. Assumption (2) states that the average residual $\varepsilon(k,h)_{i,t^e}$ only captures the reaction to the event. This assumption can be defended by including all possible sources of common information, especially asset price movements. Despite our many macroeconomic controls, it is unlikely that we cover all public information, but asset prices should be a sufficient statistic for all relevant public information, and therefore, their inclusion should help cover gaps. Finally, assumption (3) is taken from the event-study literature (Krishnamurthy and Vissing-Jorgensen (2011), Krishnamurthy and Vissing-Jorgensen (2013), Gagnon et al. (2010)). If it is violated because the impact of the event on asset prices extends beyond the two-day window, then some of

[17] This is particularly important in this setting as forecasters likely speak to each other over the forecast period. Indeed, as we show in Table A4, standard errors adjusted in this way are much larger than those only taking into account heteroskedasticity.

Figure 2: The Effect of Forward Guidance Announcements on Expectations



Notes: The panels in Figure 2 show the estimates of $\beta(k,h)^e$ for three different events, August 2011, January 2012, and September 2012, and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot. For each triplet (e,k,h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is $t = 2008.06, .., 2015.02$.

the movements in ΔAP_{t^e} are attributable to the event. Vice versa, if the movements in the two-day window reflect information other than the event, then this information is not adequately captured in ΔAP_{t^e}.[18]

Under these admittedly heroic assumptions, the mapping between the forecast horizon h and $\beta(k,h)^e$ can be interpreted as an impulse response, as it shows the change in the projection for variable k following the FOMC announcement, $E[f(h,k)|\Omega_{t-1} \cup e] - E[f(h,k)|\Omega_{t-1}]$, over horizon h, where Ω_{t-1} represents the time $t-1$ information set of survey forecasters.

The panels in Figure 2 show the estimates of $\beta(k,h)^e$ for three different events, August 2011, January 2012, and September 2012, and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. The sample for each regression is $t = 2008.06, .., 2015.02$. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.[19] For each triplet (e, k, h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using the previously described robust standard errors.[20]

As with the financial variables, Figure 2 shows that the response of expectations was very different across events. In August 2011, the response of the expected 3-month TBill is muted at short horizons, since rates were already expected to stay at the zero lower bound, but is very strong at longer horizon. This response has the same sign and pattern as those in Section 2.1.1, although the magnitude is larger in terms of the point estimates.[21]

[18]We check the robustness of this assumption in Table A4. One important issue is that of anticipation – that is, the fact that part of the language (whether bad news or forward guidance) may be expected. In this regard, we stress that: 1) we look at changes in the forecast, so that any news/announcements that occurred in the previous period should have already been incorporated, 2) to the extent that the anticipated component is reflected in the asset price changes included as controls, we should in principle just capture the novel part of the language.

[19]Note that for August 2011 and January 2012 we report the change in the nowcast ($h = 0$), that is, the forecast for the quarter in which the event takes place, for September 2012 this information is not available given that the first survey after the event is the October survey, for which the nowcast is the first quarter after the event. Hence the estimate for $\beta(k,h)^e$ begin with the first quarter ($h = 1$).

[20]For the results in Figure 2 we run separate regressions for each event (that is, we use one dummy at the time), but the results are nearly identical if we use all dummies at the same time. See Table A4.

[21]We should note that the actual forecasts revisions $\Delta f(k,h)_{i,t}$, 5-quarters ahead, are larger than the adjusted revisions $\beta(k,h)^e + \varepsilon(k,h)_{i,t}$. As we would expect, the controls attribute some of the downward revisions to factors other than the forward guidance announcement.

Similarly, the response of the nominal 10-year Treasury rate is significantly negative and very persistent. The January 2012 announcement seems to produce little significant movement in expectations for either the 3-month TBill or the 10-year Treasury rates.[22] Lastly, in September 2012, expectations for the short-term rate increase by a small amount (less than 20 basis points) while those for the 10-year rate increase more substantially, in line with the reaction of financial markets to the announcement, but the change in expectations is quite imprecisely estimated.

The fact that both short and long-term nominal rates *increased* in September 2012 may seem puzzling: How can policy accommodation lead to an increase in rates? To understand this, it is useful to look at that response of inflation and output growth forecasts following the FOMC announcements. The August 2011 event lowers the GDP growth nowcast by about 0.6 percent, although the estimates are quite uncertain and the 90 percent bands include zero while the September 2012 event raised expectations by one percentage point and the January 2012 episode had no effect.[23] The effects in August 2011 and September 2012 were largest in the beginning, but the effect is quite persistent.[24] The response of CPI inflation to the August 2011 and January 2012 announcements was close to zero and insignificant, but the September 2012 episode increased inflation expectations by almost 1 percentage point.[25]

One likely explanation for the large difference in the reaction of expectations and asset prices to the announcements has to do with the language used in the FOMC statements. This interpretation is similar to that in Campbell et al. (2012a), who distinguish between types of

[22]This is slightly at odds with the evidence on forward rates and nominal long term rates presented in the previous section, which show both as declining, although the extent of this decline is indeed much smaller than in August 2011.

[23]Note that the September 2012 response implies that the impulse response on the *level* of output is hump-shaped, with a peak at least 5 quarters after the shock, in line with the VAR evidence on the impact of policy shocks.

[24]Note that the first forecast pertains to the first quarter ($h = 1$) *after* the announcement in September 2012 because we do not have information about the change in the nowcast from September to October.

[25]This response is – at least qualitatively – consistent with the one implied by DSGE models of the kind presented below. For instance, this pattern could be generated with change in the policy rule that would increase the response coefficient on the output gap. The announced change in the reaction function increases inflationary expectations more than nominal interest rates increase, thereby reducing the real rate and stimulating the economy. The increase in nominal rates is then simply the equilibrium response to a much improved state of the economy. However, as discussed below, such rational expectations DSGE models tend to exaggerate the impact of the change in the policy, as the expectational channel is very strong.

forward guidance based on the public's interpretation of the language within the statement. Table 1 shows a substantial change in the language associated with the interest rate announcement. The August 2011 FOMC statement emphasized negative news about economic activity, while the September 2012 language was more positive on economic progress.[26] Importantly, the September 2012 statement was also more clearly stimulative in its message that monetary policy would need to remain highly accommodative "for a considerable time after the economy strengthens." Additionally, as we discussed in Section 2.1.1, the safety premium rose following the August 2011 announcement while it fell after September 2012, again suggesting that news played a different role across these episodes. Together, using the language of Campbell et al. (2012a), these observations suggest that forward guidance in August 2011 was interpreted as *Delphic*, in that is was interpreted as conveying bad news about the economy, while forward guidance in September 2012 was interpreted as *Odyssean*, in that it was interpreted as a commitment to more monetary policy stimulus.

The September 2012 statement also included an announcement of QE3, which could account for the positive reaction of output and interest rate forecasts. While it is hard to separate the effect of the QE3 announcement from that of forward guidance, we make two points. First, to the extent that the QE3 announcement had a positive effect, the effect of forward guidance on output and inflation were *less* than those reported in Figure 2. Second, the next section tries to disentangle (albeit at the cost of additional assumptions) the effects of QE from those of forward guidance.

2.2 Benchmarking Forward Guidance in Our Model

While the heterogeneity in the effect of various statements containing forward guidance is descriptively interesting, in order to move to an estimate that we can more directly compare to the impulse responses generated by our models in Section 3 and 4, we need to isolate the effect of stimulative forward guidance from the news that is associated with it. In order to do this, we make more rigorous use of the language in the entire set of FOMC statements, in the spirit of Romer and Romer (1989). While this method relies on additional assumptions and therefore does not provide fully satisfactory estimates, it suggests that we should expect

[26] See Table A1 and A2 in the Appendix for the precise language. Romer and Romer (2000) also provide ample evidence that the Federal Reserve possesses information that private forecasters do not have. It is therefore plausible to think that the language used in the statement can influence the Blue Chip's projections.

purely stimulative forward guidance to have positive, yet not unrealistically large, effects on the economy.

Rather than comparing calendar dates in which there was a forward guidance announcement (D_t^e), we instead include a set of dummies that control for different aspects of the FOMC statement that have the potential to move expectations. Specifically, similar to equation (1), we estimate the following panel regression:

$$\Delta f(k,h)_{i,t} = \gamma(k,h)_0 + \gamma(k,h)_1' \ \Delta M_t + \gamma(k,h)_2' \ \Delta AP_t + \gamma(k,h)_3' \ Z(k,h)_{t,i} + \beta(k,h)^e \ POLICY_t^e + \varepsilon(k,h)_{i,t}, \text{ for } t = 1,..,T, \ i = 1,..,n, \quad (3)$$

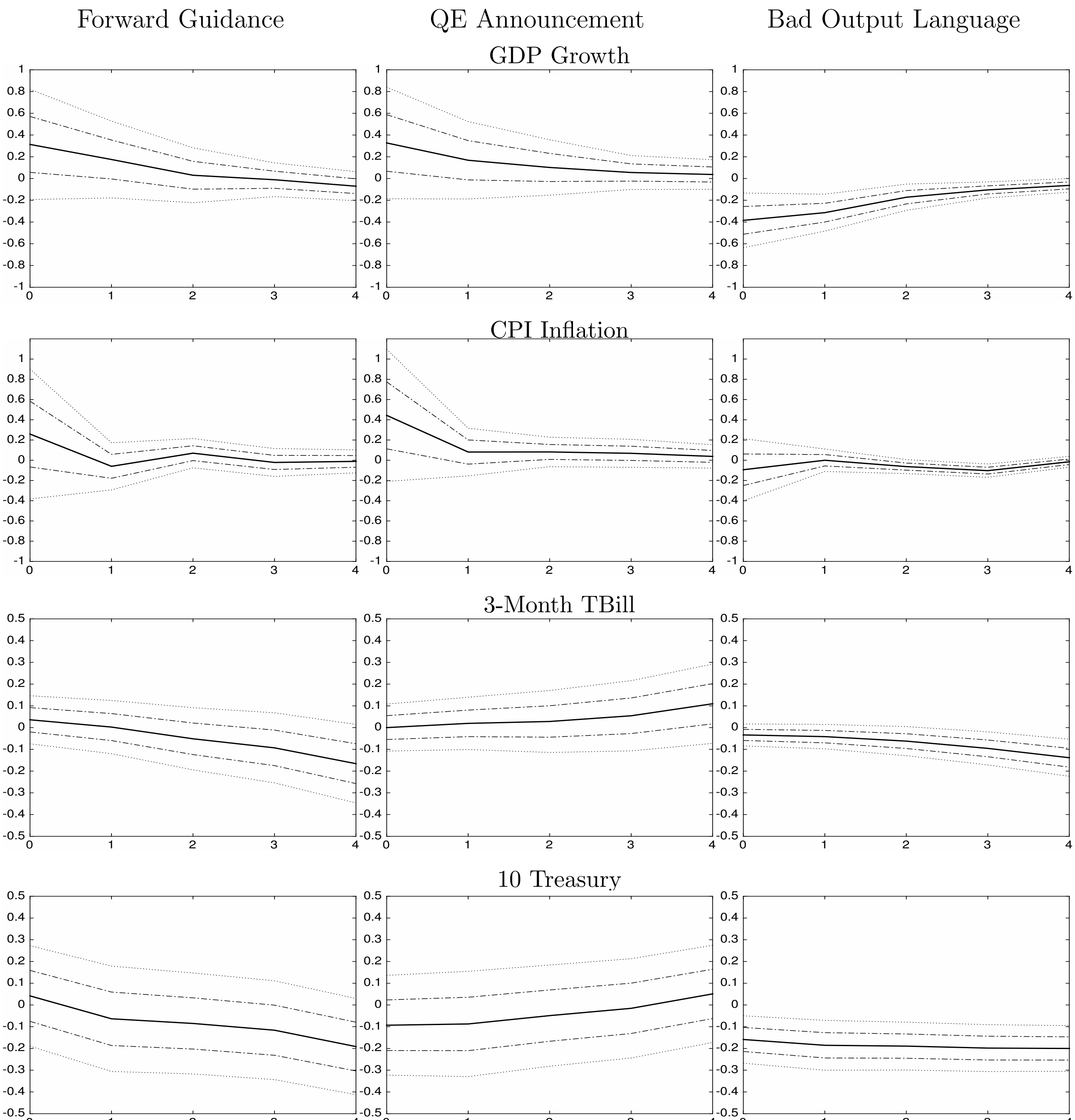
where $POLICY_t$ includes the following series of dummies: a dummy for the announcement of a calendar-based forward guidance episode, a dummy for a quantitative easing announcement, a dummy for an announced continuation of quantitative easing, a measure of the language for output conditions, and a measure of the language for inflation conditions. See the Appendix for additional details on the construction of these dummies.[27] By utilizing other FOMC statements in which similar language was used or policies were announced, we can separately estimate the effect of the components of the statements and decompose the heterogeneity in the forward guidance episodes studied in Section 2.1. In addition to the necessary modeling assumptions discussed in Section 2.1.2, this will accurately characterize the effect of Odyssean forward guidance as long as 1) the constructed statement dummies capture all other relevant aspects of the statement, 2) the effect of FOMC language and policies are constant over time, and 3) the effects of the statements are additive. We can attempt to check the validity of the first assumptions by exploring alternate specifications. The third assumption is more heroic – if it is the case that quantitative easing is seen as a commitment device for forward guidance, then forward guidance that is announced with quantitative easing may be more effective than when it is announced alone. Similarly, if forward guidance that is announced with bad news is discounted more than forward guidance that is announced with better news, then this equation would be misspecified.

Figure 3 shows the impulse response functions implied by the β coefficients on the forward guidance dummy, the quantitative easing announcement dummy and the negative GDP language dummy.[28] We see here that, while the estimates are noisy, the coefficient relating

[27]In the future, we can improve this by using machine learning techniques that may better quantify these categories.

[28]See the Appendix Table A1 for the estimated coefficients on all policy dummies.

Figure 3: Decomposing FOMC Statements: The Effect of Forward Guidance, QE Announcements and Bad GDP Language



Notes: The panels in Figure 3 show the estimates of $\beta(k,h)^e$ for three different elements of the FOMC statement – forward guidance announcements, QE announcements and bad GDP language – and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.For each triplet (e,k,h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is $t = 2008.06, .., 2015.02$.

both forward guidance and quantitative easing to GDP growth are slightly positive, with the point estimate indicating a 0.3 percent increase in output growth.[29] The coefficient for bad language regarding GDP is instead significantly negative. The coefficient relating both forward guidance and quantitative easing to inflation are also slightly positive while the coefficient for bad language regarding GDP is almost zero, perhaps reflecting the fact that news about inflation enters separately.

Although forward guidance announcements have little contemporaneous effect on the short-term rate, reflecting the fact that the announcements considered have been made while near the zero lower bound, the typical calendar-based forward guidance announcement lowers the short-term rate four quarters ahead by about 15 basis points. While the long-term rate does not move much on impact either, it is also expected to be nearly 20 basis points lower after four quarters. These estimates suggest that forward guidance announcements have had economically meaningful effects, with GDP growth and inflation increasing by about 0.3 percentage points in response to a reduction of about 15 basis points in the expectation of short-term rates one year ahead. QE announcements have similar positive effects on GDP and inflation. One key difference with respect to forward guidance though, is that QE announcements tend to lower the long-term rate on impact. Language suggesting bad news about GDP has negative and persistent effects on the 3-month and 10-year Treasury rate, consistent with the event-based analysis discussed in Section 2.1.1.

This analysis lends additional support for the *Delphic* and *Odyssean* interpretation of the heterogeneity in the effect of forward guidance across episodes as discussed in Campbell et al. (2012a). We find that language about the economy does matter, with negative language inducing lower forecasters' expectations. The results also suggest that the effect of Odyssean forward guidance involving a 15 basis point decrease in short-term rates 4 quarters ahead is to produce positive effects on output growth and inflation of about 0.3 percentage points.

[29]Note that the forward guidance dummy here can be interpreted as the average residual change in the forecast in the three forward guidance episodes explored in Section 2.1 *after* controlling for the other aspects of the statement.

3 The Macroeconomic Implications of Interest Rate Announcements

We now proceed with an evaluation of the effects of extending the forward guidance focusing on the stimulative effects of policy, and abstracting from the possible effects of information conveyed by the FOMC regarding the assessment the state of the economy. In this section, we first briefly describe the DSGE model, its estimation, and the baseline forecasts. In particular we discuss the modification of the standard feedback rule describing monetary policy to allow for anticipated policy shocks, and how we incorporate current FFR market expectations into the forecast. Next, we show that when we condition the forecast on a specific interest-rate path the model produces results that are hardly credible, and explain why this is the case.

3.1 Model and Baseline Forecast

The FRBNY DSGE model is a medium-scale, one-sector, dynamic stochastic general equilibrium model. It builds on the neoclassical growth model by adding nominal wage and price rigidities, variable capital utilization, costs of adjusting investment, and habit formation in consumption. The model follows the work of Christiano et al. (2005) and Smets and Wouters (2007), but also includes credit frictions, as in the financial accelerator model developed by Bernanke et al. (1999). The actual implementation of the credit frictions closely follows Christiano et al. (2009). Detailed information about the equilibrium, the data, and the priors used in the Bayesian estimation of this model are contained in Del Negro et al. (2013). The appendix to this paper also includes the list of log linearized equilibrium conditions, as well as the priors and posteriors for the estimated parameters. In this section we focus on the features of the model that are needed to properly describe this exercise. In particular, we discuss: i) the state-space representation of the linearized DSGE model, ii) anticipated policy shocks, iii) incorporating market's FFR expectations into the baseline forecast.

The solution to the log-linear approximation of the model's equilibrium conditions around the deterministic steady state (obtained using the method in Sims (2002)) yields the following transition equation:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \tag{4}$$

where s_t is the model's vector of "state" variables, the matrices Φ_1 and Φ_ϵ are functions of the vector of all model parameters θ, and ϵ_t is the vector of structural shocks. The vector of observables y_t described below is in turn related to the states according to the system of measurement equations:

$$y_t = \Psi_1(\theta) + \Psi_2(\theta)s_t. \tag{5}$$

The variables included in y_t are: 1) annualized real GDP per capita growth, where the real gross domestic product is computed as the ratio of nominal GDP (SAAR) to the chain-type price index from the BEA;[30] 2) the log of labor hours, measured as per capita hours in non-farm payroll; 3) the log of labor share, computed as the ratio of compensation of employees to nominal GDP, from the BEA; 4) the annualized rate of change of the core PCE deflator (PCE excluding food and energy, but including purchased meals and beverages), seasonally adjusted; 5) the effective federal funds rate, percent annualized, computed from daily data; and 6) the spread between the Baa rate and the rate on 10 year Treasuries. We estimate the vector of model parameters θ using data from 1984Q1 to 2012Q3 using Bayesian methods as described in Del Negro and Schorfheide (2010), applied to the state-space representation of the linearized DSGE model provided by equations (4) and (5).

Starting in 2008Q3 (one period before the implementation of the zero lower bound) we incorporate FFR market expectations, as measured by OIS rates, into our outlook following the approach described in Section 5.4 of Del Negro and Schorfheide (2013).[31] These data contain valuable information for the estimation of the state of the economy, as the market expectations of continued low interest rates reflect both a relatively weak economy as well as an accommodative monetary policy.

These market expectations are assumed to be driven by the policy rule that the Cental Bank is expected to follow as well as on the deviations from that rule that the Central Bank has already communicated in its forward guidance. Specifically, we assume that the Central Bank sets the short-term interest rate according to the following feedback rule

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \psi_y \sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R, \tag{6}$$

[30]Per capita variables are obtained by dividing through the civilian non-institutionalized population over 16. We HP-filter the population series in order to smooth out the impact of Census revisions.

[31]Specifically, we take FFR expectations up to K quarters ahead into account by augmenting the measurement equation (5) with the expectations for the policy rate: $FFR^e_{t,t+k} = 400\left(\mathbb{E}_t \widehat{R}_{t+k} + \ln R_*\right)$, where $FFR^e_{t,t+k}$ denotes data on market's expectations for the FFR k quarters ahead.

where $\sum_{j=0}^{3} \hat{\pi}_{t-j}$ is 4-quarter inflation expressed in deviation from the Central Bank's objective π_* (which corresponds to steady state inflation), $\sum_{j=0}^{3}(\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$ is 4-quarter growth rate in real GDP expressed in deviation from steady state growth, and $\epsilon_{R,t}$ is the standard contemporaneous policy shock, where $\epsilon_t^R \sim N(0, \sigma_r^2)$, $i.i.d.$.[32] The last term captures forward guidance following Laseen and Svensson (2011), where $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume that $\epsilon_{k,t-k}^R \sim N(0, \sigma_{k,r}^2)$, $i.i.d.$ Note that we make the – arguably counterfactual – assumption that the anticipated shocks are independent from one another. Campbell et al. (2012b) argue, based on their own findings as well as Gürkaynak et al. (2005b)'s, that anticipated shocks follow a factor structure. It would be important to relax the independence assumption if we were to estimate the model with forward guidance shocks. However, this assumption bears no implications in the policy exercise described in sections 3.2 and 3.3.[33]

For simplicity we estimate the model parameters assuming no forward guidance — that is setting the last term in (6) to zero —, and without data on expected future policy rates. Implicitly we are assuming that forward guidance has little impact on the estimated model parameters. We are however recognizing that it has a potentially large impact on our inference about the state of the economy s_t in the 2008Q3-2012Q3 period (conditional on the estimated parameters), and hence on the model's forecasts. We are therefore re-estimating the state s_t in that period using expectations of future federal funds rates.[34] Our baseline forecast, which is described in Table 3 and Figure 4, is therefore obtained using data released through 2012Q2 augmented for 2012Q3 with observations on the federal funds rate and the Baa corporate bond spread, and with market's FFR expectations through mid-2015 (hence $K = 11$ in equation (6)) as measured by OIS rates on August 28, 2012.[35]

[32]The economy displays a stochastic trend, so if $\hat{y}_{t-j}$ is output in deviation from this trend and $\hat{z}_t$ corresponds to the growth rate of technology in deviations from steady state, then the growth rate of output in period t is $\hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$.

[33] In this log-linearized model the variance-covariance matrix of the shocks does not affect the equilibrium conditions.

[34]The only extra parameters introduced by the forward guidance are the standard deviations $\sigma_{k,r}$ of the anticipated shocks. Since we do not have estimates for these parameters, we assume that these shocks have the same standard deviation as the contemporaneous shock: $\sigma_{k,r} = \sigma_r$. Importantly, note that the parameters $\sigma_{k,r}$ do not enter *any* of the policy experiments described below.

[35]As 2012Q3 observations for the the FFR and the Baa corporate bond spread we are using the average

Table 3: The model-implied consequences of forward guidance

	2012 (Q4/Q4)	2013 (Q4/Q4)	2014 (Q4/Q4)	2015 (Q4/Q4)
GDP growth				
Baseline	1.9	2.2	1.7	1.3
Forward guidance	3.5	4.9	1.5	0.3
Core PCE inflation				
Baseline	1.6	1.2	1.5	1.6
Forward guidance	1.8	1.9	1.8	1.7
Federal funds rate				
Baseline	0.10	0.20	0.40	1.32
Forward guidance	0.25	0.25	0.25	1.09

Notes: The table reports the model's predictions conditional on alternative assumptions regarding the federal funds rate: the baseline forecast, and a counterfactual policy experiment in which the federal funds rate is maintained at 25 basis points until 2015Q2.

Figure 4 shows the model's predictions for real GDP growth, core PCE inflation and the federal funds rate, conditional on alternative assumptions regarding the federal funds rate. These forecasts are obtained using the mode of the posterior distribution for θ and s_t, although these modal forecasts in the baseline case essentially coincide with the mean of the forecast distribution obtained by drawing from the full posterior of θ and s_t. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. In this forecast, GDP growth is 1.9 percent in 2012 (Q4/Q4), rises to 2.2 percent in 2013 but remains mostly below 2 percent throughout the rest of the forecast horizon (see the first row in each of the three panels of Table 3). Core PCE inflation is predicted to be at 1.6 percent in 2012 and is also expected to remain below 2 percent throughout the forecast horizon.

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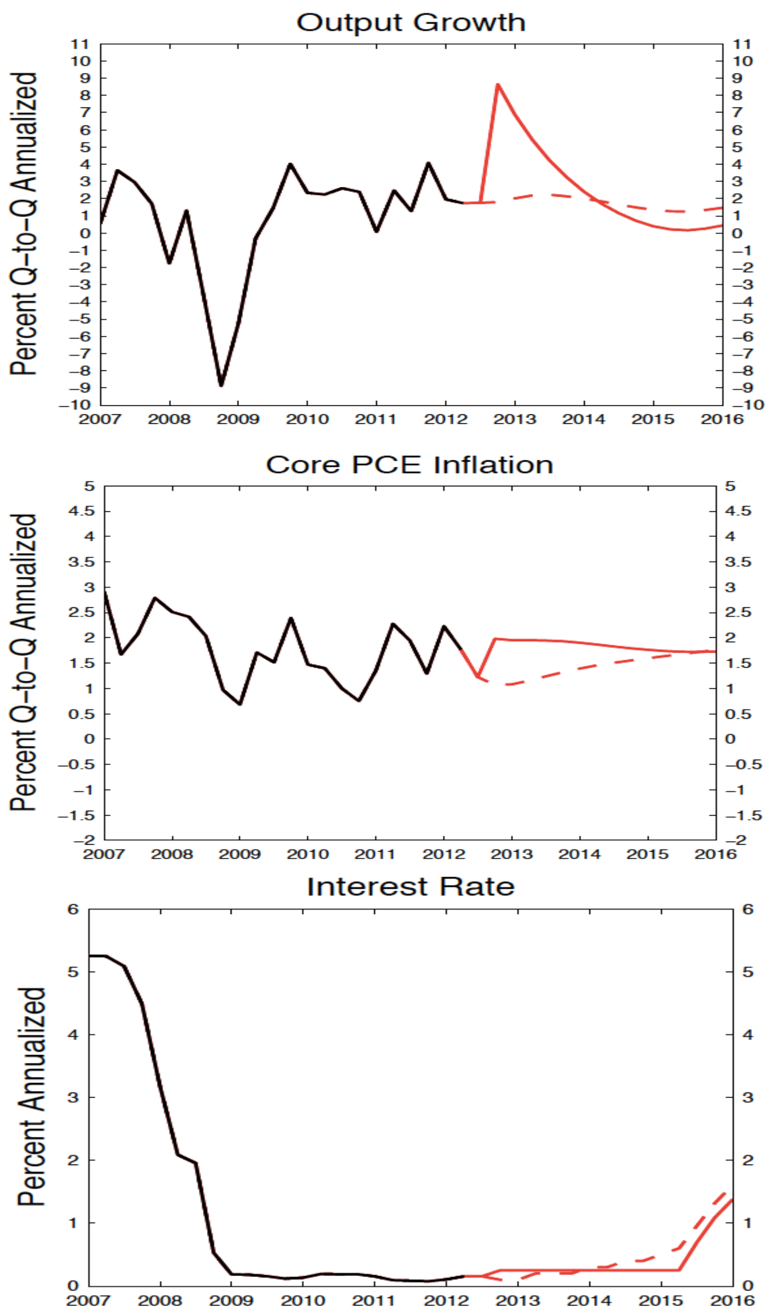
of daily rates during the quarter up to this date.

Figure 4: The model-implied consequences of forward guidance



Notes: The figure shows the model's predictions conditional on alternative assumptions regarding the federal funds rate. The black solid lines show the historical data. The dashed red lines show the FRBNY DSGE model's baseline forecast. The solid red lines show in turn the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2.

3.2 A Forward Guidance Experiment

We now proceed with our counterfactual policy experiment in which the central bank announces at the end of 2012Q3 that it will maintain the federal funds rate at 25 basis points until 2015Q2, and that it follows the historical policy rule after that.[36] We first summarize the procedure used to condition the model's predictions on a given interest-rate path, which is taken from section 6.3 of Del Negro and Schorfheide (2013), and then describe the outcome of the experiment.

Suppose that at the end of period T, after time T shocks are realized, the central bank announces its intention to commit to a given interest-rate path: $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$. For the agents, the announcement is a one-time surprise in period $T+1$. This corresponds to the realization of a single unanticipated monetary policy shock ϵ^R_{T+1} and a sequence of anticipated shocks $\{\epsilon^R_{1,T+1}, \epsilon^R_{2,T+1}, \ \ldots, \ \epsilon^R_{\bar{H}-1,T+1}\}$. Notice that all policy shocks that are used to implement the interest rate path are dated $T+1$. The Appendix describes an algorithm that determines the time $T+1$ monetary policy shocks needed to generate predictions conditional on a given interest rate path, so that the announced interest rate path will be attained in expectation.

The solid red lines in Figures 4 show the model's predictions in our counterfactual policy experiment. Such a policy change would imply a reduction in the expected federal funds rate of 15 basis points at the end of 2014 compared to the baseline forecast. According to the model, this alternative policy assumption generates a massive stimulus in 2012 and 2013. Indeed, in this alternative scenario, quarterly annualized real GDP growth surges to almost 9 percent in the quarter following the forward guidance announcement. On a year-over-year basis real GDP growth is forecast to jump to 3.5 percent in 2012 (Q4/Q4), and to 4.9 percent in 2013. GDP growth is however lower than under the baseline scenario in the subsequent years (i.e., 2014 and 2015), as the effects of the policy stimulus fade over time and the GDP *level* returns to the level it would have had without the stimulus (see the second row in each

[36] At the time we wrote the first version of this paper, this was one policy option discussed by market commentary for the upcoming FOMC meeting, see the September 10, 2012 WSJ "MarketBeat" Blog at `blogs.wsj.com/marketbeat//qe3-what-everybody-that-matters-on-wall-street-expects/`. We chose 25 basis points for simplicity as it coincides with the interest paid on excess reserves, but of course choosing any lower rate would make the results even stronger as the policy would be even more accommodative.

of the three panels of Table 3). The stimulative effect of policy also raises inflation in 2012 and 2013 to respectively 1.8 percent (Q4/Q4) and 1.9 percent, but inflation is also forecast to remain below 2 percent in 2014 and 2015.

Compared to the empirical estimates of the effects of forward guidance reported in Section 2.2, the model generates an implausibly large response of real GDP growth and inflation to a relatively small change in the federal funds rate. This is what we call the "Forward Guidance Puzzle." The fact that the model captures so poorly the effects of forward guidance is particularly puzzling given that such estimated medium-scale models have been found to fit the data quite well, that they generate reasonable responses to a variety of contemporaneous shocks, and that they perform reasonably well for forecasting key macroeconomic variables.

3.3 What Is the Excessive Response Due to?

To understand why the model's response to forward guidance is excessive, consider a simplified version of the FRBNY DSGE model in which there is no habit persistence, the intertemporal elasticity is one, and there are no shocks other than monetary policy shocks. In this case, the consumption Euler equation reduces to the conventional expression

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - (\hat{R}_t \; - \; \mathbb{E}_t[\hat{\pi}_{t+1}]), \tag{7}$$

where $\hat{c}_t$ denotes consumption deviations from steady state. Iterating this equation forward to eliminate expected future consumption, we obtain

$$\hat{c}_t = -\sum_{k=0}^{\infty} \mathbb{E}_t[\hat{R}_{t+k} \; - \; \hat{\pi}_{t+1+k}], \tag{8}$$

so that contemporaneous consumption is directly negatively related to the long-term real interest rate (at infinite maturity), which is given by $\hat{r}_t^L = \sum_{k=0}^{\infty} \mathbb{E}_t[\hat{R}_{t+k} \; - \; \hat{\pi}_{t+1+k}]$. It follows that anticipated future changes to the short-term real rate affect consumption today and in all periods before the change takes effect. To see this, suppose for now that prices are fixed so that the nominal and real interest rates move by the same amount, and suppose that the short-term rates declines by $\hat{R}_t = -\Delta$ contemporaneously but reverts to steady state ($\hat{R}_{t+k} = 0$) after that. In this case, the long run real rate declines also by Δ in period t and reverts to steady state after that, so that consumption increases temporarily by Δ

in period t. Consider now an the announcement of a temporary decline in the short-term rate at date $t + k$ of $\hat{R}_{t+k} = -\Delta$. In that case, the long-term real rate $\hat{r}_t^L$ declines from periods t to $t + k$, before reverting to steady state from $t + k + 1$ onward. This implies that consumption increases by Δ in period t and remains at that level in every period until $t + k$, before reverting to steady state. McKay et al. (2015) emphasize the fact that this persistent consumption response does not take place in an economy where precautionary motives are important, as households are unwilling to draw their wealth down to very low levels.

Now let prices adjust. A simplified version of the New Keynesian Phillips curve used in the medium-scale model (when abstracting from all non-monetary shocks) implies that

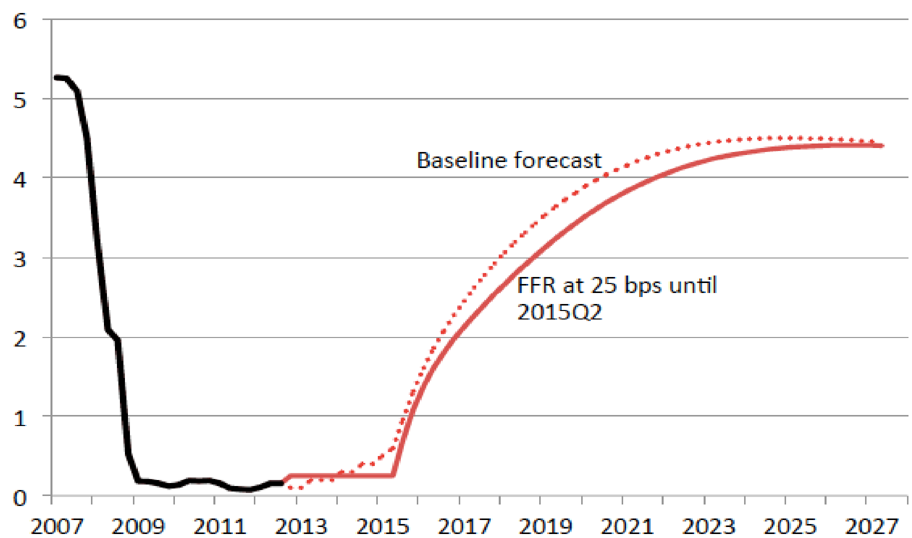
$$\hat{\pi}_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t[\hat{y}_{t+k}]. \tag{9}$$

where κ captures the inflation response to movements in aggregate demand. Inflation thus depends on the discounted *sum* of all future deviations of output from its steady state. Given that aggregate demand is expected to be above steady state from period t to $t+k$ following a forward guidance announcement of a drop in rates in $t+k$, the rise inflation tends to be larger the farther in the future will the drop in interest rate take place. The rise in inflation induces in turn a further reduction in current and expected future real interest rates, which amplifies even more the stimulus provided by the forward guidance announcement. Carlstrom et al. (2012b), Kiley (2014), and Chung et al. (2014) argue that the economy's excessive response to forward guidance is due to this amplification mechanism implied by the New Keynesian Phillips curve. In addition, the output response in the medium-scale model also depends on the behavior of real investment, which also relates to the long-term real interest rate.

Finally, all these effects are made stronger by the fact that in this model the long-term interest rate responds very strongly to the policy announcements. Figure 5 shows the paths of short-term interest rates under the baseline projection (red dashed lines), and the counterfactual policy (red solid line) until 2027Q4. This figure reveals that while the expected short-term rate is only 15 basis points lower in the counterfactual than in the baseline at the end of 2014, the difference between the two interest-rate paths is expected to be much larger farther in the future, in particular between 5 and 10 years.

To see how all effects combine in the medium scale model, Figure 6 shows the impulse response functions to contemporaneous and anticipated policy shocks. Specifically, the figure

Figure 5: Interest-rate projections farther into the future

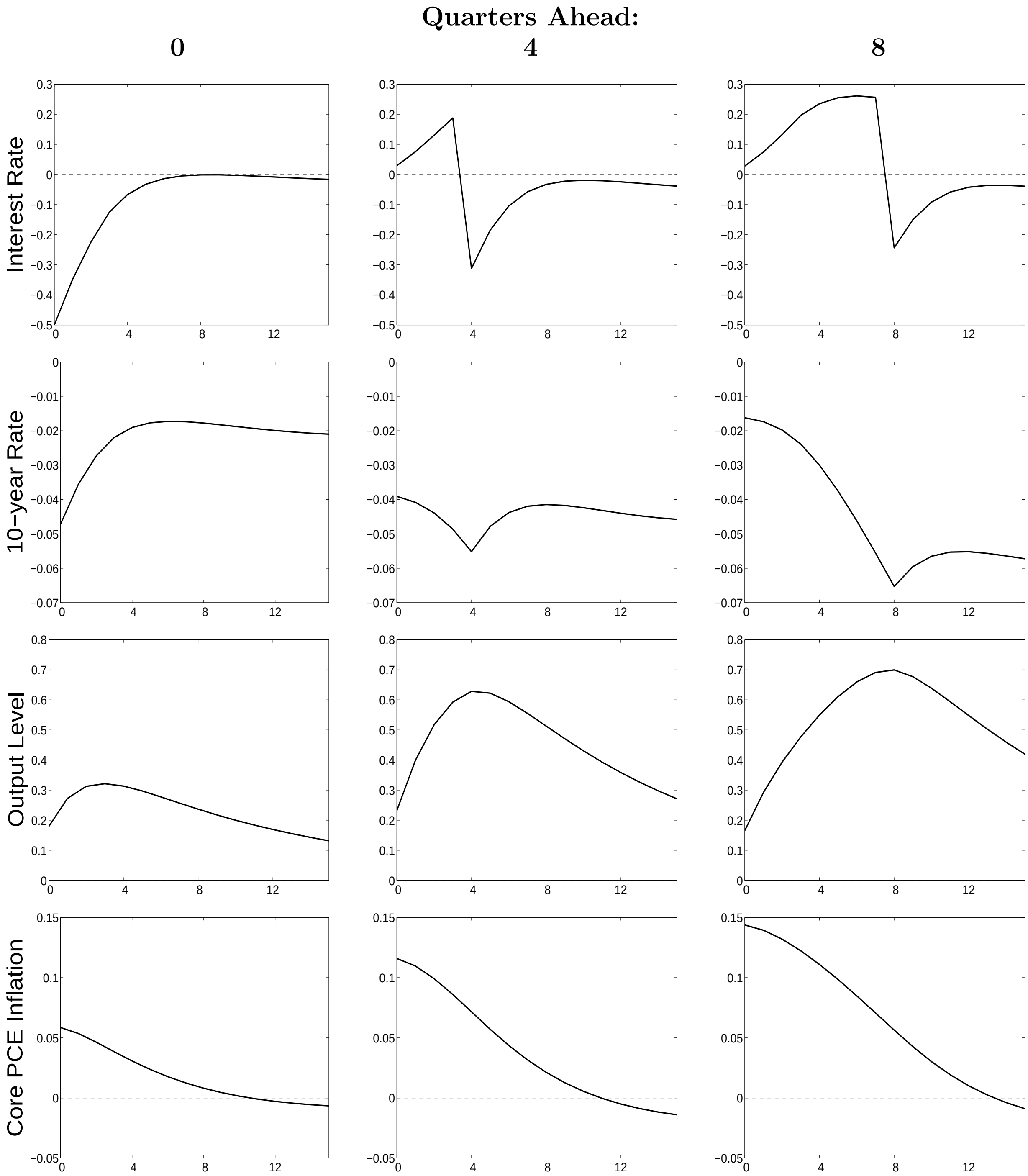


Notes: The figure shows the model's predictions for the federal funds rate farther into the future. The black solid line shows the historical data. The dashed red line shows the FRBNY DSGE model's baseline forecast. The solid red line shows the model's predictions in a counterfactual policy experiment in which the federal funds rate is set to 0.25 percent until 2015Q2.

shows the response of the short term interest rate, the 10-year nominal rate, the level of output, and inflation to a contemporaneous 50 basis points drop in the short-term interest rate (left column), as well as to announcements of a similar drop 4 and 8 periods ahead (middle and right columns, respectively). Four features stand out: i) Since the anticipated expansionary shock leads to higher inflation and output *before the shock takes place*, and since the policy authorities are bound to follow the rule before that date, the interest rate follows a zig-zag pattern, where it first rises and then falls. If this pattern of interest rates appears awkward, bear in mind that we are unlikely to see an eight periods-ahead shock *in isolation* (e.g., Campbell et al. (2012b)). ii) The response of the 10-year rate, quite understandably, reaches its lowest point at the time the shock takes place, and the trough decreases monotonically with the anticipation horizon. iii) The peaks in the response of output roughly coincide (with a slight delay) with the peaks in the response of the 10-year rate, in agreement with equation (8). In addition, the effect on output increases monotonically with the horizon. The delay is due to features like habit persistence. iv) The impact on inflation also increases monotonically with the horizon, which is not surprising given the output responses.

The responses in Figure 6 provide some economic intuition behind the finding of Carlstrom et al. (2012b) that the response of macroeconomic variables to an interest rate peg is

Figure 6: Impulse response functions to contemporaneous and anticipated policy shocks



Notes: The figure shows the percent change over a 12 quarter horizon of the short term interest rate, the 10-year nominal rate, the level of output, and Core PCE inflation in response to a contemporaneous, 4 quarter and 8 quarter ahead negative 50 basis points policy shock.

a convex function of the horizon of the peg. Imagine that the policymakers want to lower interest rates by 50 basis points for 7 periods. This can be implemented with a sequence of contemporaneous and anticipated shocks up to 7 periods ahead. Now imagine they decide to extend the peg one extra period. Because of the zig-zag feature of the 8th period impulse response, that decision will tend to lift the short-term rate in quarters 0 to 7 and so requires a cascade of shocks over that period to push the interest rate back down. In light of these impulse responses (and the related impact on the long rate) it is not surprising that even a modest amount of forward guidance produces very large effects in the model, as long as it extends far enough into the future.

4 A Proposed Resolution

The potential resolution to the forward guidance puzzle that we propose here builds on the overlapping generations structure, as in Blanchard (1985)'s and Yaari (1965)'s perpetual youth model, which has been incorporated recently in simple New Keynesian models by Piergallini (2006), Nisticò (2012), and Castelnuovo and Nisticò (2010).[37] We show that our model addresses the counterfactual implications of the standard consumption Euler equation discussed in Section 3.3 and McKay et al. (2015). However, it will also imply a higher discounting of future economic conditions for price-setting decisions, as well as for the choice

[37]While our explanation of the forward guidance puzzle retains a rational expectations framework, other authors have recently proposed a resolution to this puzzle by assuming boundedly rational agents. In Gabaix (2015), agents use a simplified model of the world as a default and consider parsimonious enrichments to this model paying attention only to those enrichments that matter enough for their decisions. In making consumption and saving decisions, these agents pay limited attention to the interest rate and income. This yields a more muted response of agents' consumption and hence of inflation to announcements about future interest rate changes. Gabaix's application to a simple New Keynesian model implies a discounted Euler equation similar to the one presented here, where the rate of discounting depends on the degree of attention that households give to movements in interest rates and income, instead of the death probability. García-Schmidt and Woodford (2015) also propose a resolution to the forward guidance puzzle by considering deviations from rational expectations. They assume an explicit cognitive process for the formation of expectations according to which agents beliefs are adjusted in the direction of the discrepancy between the model prediction and the conjectured beliefs. They show that in the simple New Keynesian model, a commitment to keep interest rates low should raise inflation and output but by less than under perfect foresight as long as agents' expectations havent fully adjusted to the perfect foresight expectations.

of wages and investment, thereby mitigating the concerns raised by Carlstrom et al. (2012b), Chung et al. (2014), and Kiley (2014).

In order to explore the effect of this additional feature on the economy's response to forward guidance, we first present a simple model in which we can clearly discuss the intuition. Then, in order to quantify the empirical relevance of this mechanism, we use an estimated medium-scale DSGE model and show that for a reasonable calibration of the death probability, the perpetual-youth modification yields more reasonable estimates of the effect of forward guidance.

4.1 Intuition

We use the discrete time version of Blanchard (1985)'s model to illustrate the effect that a perpetual youth structure has on forward guidance. We assume that in every period j a new cohort is born with mass p, and each cohort has a constant probability p of dying which does not depend on j. Hence, in period t, the mass of survivors from cohort j is $p\,(1-p)^{t-j}$.[38] Household's utility is given by:

$$\sum_{s=0}^{\infty}(\beta(1-p))^s \log(C_{j,t+s}), \tag{10}$$

where $C_{j,t}$ denotes consumption of cohort j, and the future is discounted at the rate $\beta(1-p)$ to take into account the probability of dying. Since death is random, households will generally die with positive wealth. We follow Blanchard (1985) and assume that households buy fair annuities, which they will want to do because there is no bequest motive. Under this contract, agents receive a fraction p of their current wealth in each period in which they are alive in exchange from paying their entire wealth upon their death. Therefore, the budget constraint for cohort j is:

$$\mathcal{S}_{j,t+1} \leq \frac{R_t}{1-p}(\mathcal{S}_{j,t} + Y_t - C_{j,t}), \tag{11}$$

where $\mathcal{S}_{j,t}$ is the period t wealth for cohort j, Y_t is income (which is the same across cohorts) and $R_t/(1-p)$ is the gross rate of return for the household, where the $1/(1-p)$ term comes

[38]Note that the total population is normalized to one, since $\sum_{j=-\infty}^{t} p\,(1-p)^{t-j} = p\sum_{k=0}^{\infty}(1-p)^k = 1$, where $k = t - j$.

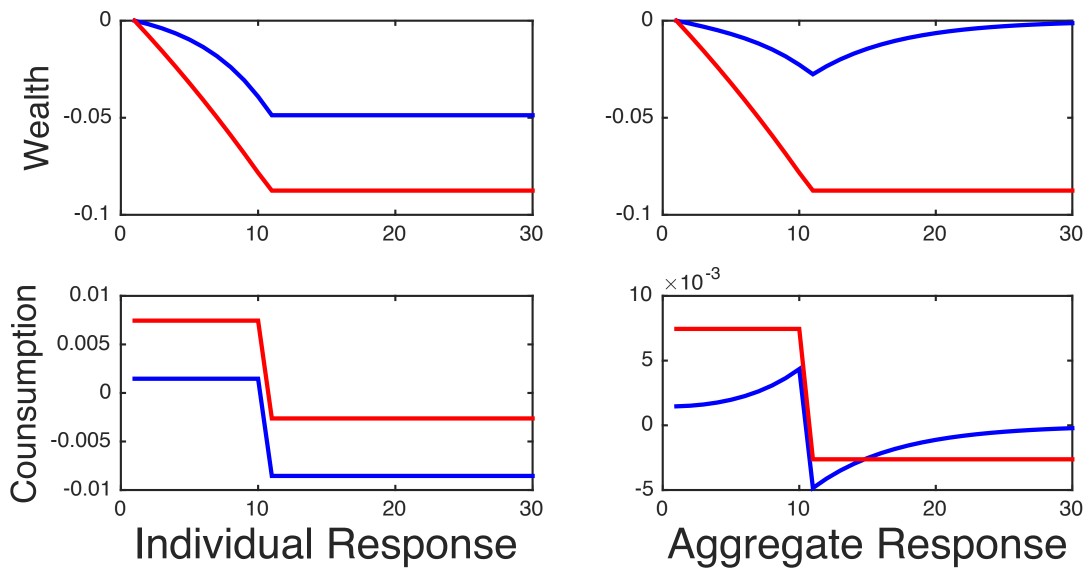
from the annuity contract.[39]

The first-order condition yields the familiar consumption Euler equation:

$$\beta \frac{C_{j,t}}{C_{j,t+1}} R_t = 1, \tag{12}$$

which is the same as in the standard representative agent model. The probability p does not show up directly in this equation, as the increased discounting at the individual level cancels out with the increased wealth from the annuity contract.

Figure 7: Forward Guidance in a Blanchard-Yaari Model: Individual vs. Aggregate Response



Notes: The figure shows impulse response functions to an anticipated drop in interest rates 10 quarters in the future. The red line shows the simulation with $p = 0.0001$ and the blue line shows the impulse responses with $p = 0.15$.

Assume that we are in a small open economy where $R_t = R = \beta^{-1}$, and where income Y is constant over time and across cohorts. If $p > 0$, wealth must be zero in steady state, and $C_{j,t} = Y$.[40] Consider a forward guidance experiment where the path of the interest rate is

[39]To understand why, note that $1/(1-p) = 1+p/(1-p)$ where for any unit of wealth carried over from the previous period, agent j receives in addition $p/(1-p)$ units from the life insurance which collects a fraction p of the financial wealth each cohort (given that a fraction p of the agents in each cohort dies) and divides it up among the $(1-p)$ remaining agents.

[40]Intuitively, since new cohorts are born with no wealth but receive a steady stream of income Y, they have to consume Y in each period if they are to keep consumption constant over time. In contrast, in the representative agent model, consumption is given by $C_{j,t} = Y + (R-1)S$ for some indeterminate level of wealth $\mathcal{S}$.

temporarily reduced at some future date T ($R_T < R$) but remains unchanged ($R_t = R$) at all other dates. The left panel of Figure 7 shows the simulated impulse responses to this future change in the interest rate for the individual when $T = 10$. The blue lines show the impulse responses for $p = 0.15$ while the red lines assume $p = 0.0001$ (essentially the representative agent model). The individual Euler equation (12) implies that each cohort's consumption level rises immediately, remains constant until period T and decreases to a constant lower level after T (bottom-left panel). Intuitively, agents want to take advantage of the lower borrowing rates, so they start de-cumulating wealth right away (top left panel).[41]

However, unborn cohorts cannot react to the interest rate announcement, at least until they are born. When that happens, they will have a higher consumption than the rest of the population, because their human capital is exactly the same as everyone else's but their financial wealth is higher (they are less in debt). The constant influx of new cohorts with zero wealth makes aggregate wealth decline at a slower pace than individual wealth (top right panel). At the same time, it makes the aggregate consumption response grow over time (bottom right panel), as newborn cohorts can react to the announcement and borrow just as the rest of the population. For the representative agent model, of course, individual and aggregate responses coincide (red lines).

This simple model illustrates a key mechanism underlying this perpetual youth structure which helps mediate the forward guidance puzzle. While each existing cohort acts according to a standard Euler equation, those who are not yet born cannot already increase their consumption to benefit from the future interest rate drop. This leads to a dampening of the initial consumption response and to a compositional effect resulting in a progressively larger aggregate consumption response as time approaches the date of the interest-rate drop T.[42]

[41] As is well known, substituting (12) into the budget constraint we obtain that cohorts consume a constant fraction of their financial ($\mathcal{S}_{j,t}$) and human (H_t) capital: $C_{j,t} = (1 - \beta(1-p))(\mathcal{S}_{j,t} + H_t)$, where H_t is the present discounted value of future income. Consumption remains constant in periods t to T because the decline in wealth is exactly compensated by the rise in the present discounted value of future income resulting from the lower discount rate. After date T, consumption is lowered due to lower wealth.

[42] Note that higher p also induces an "attenuation" of the consumption response for the individual agent: the blue line in the bottom left panel is below the red line. This is due to the fact that the rate of return for the household, $R_t/(1-p)$, is higher the larger p, which implies a lower present discounted value of future income H_t. Nonetheless, the household's consumption profile (blue line) is still flat until time T, which implies that it is the compositional effect that generates the increasing aggregate consumption until T in the lower right panel.

The simple model also highlights similarities and differences between the perpetual youth model and frameworks where a constant fraction of the population is hand-to-mouth (e.g., Campbell and Mankiw (1989)). The similarity is that unborn cohorts cannot react to changes in interest rates, just like hand-to-mouth household. The difference is that while hand-to-mouth households remain hand-to-mouth, unborn cohorts at some point can join financial markets. This dynamics generates a different response between contemporaneous and future interest rate changes. Recent literature however has shown that i) the notion of hand-to-mouth households encompasses so-called wealthy hand-to-mouth (households with illiquid wealth, Kaplan and Violante (2014)); ii) there is a transition in and out of hand-to-mouth status, similar to that in perpetual youth models. Kaplan et al. (2014) show that in the United States the expected length of non-hand-to-mouth status is 11 years. If we think of perpetual youth model as very loosely capturing some of the features of this transition, this would translate into a (quarterly) value of p of roughly 2.3 percent.[43]

4.2 The Smets and Wouters (2007) Model with Perpetual Youth

We now insert the perpetual youth households into the framework of Smets and Wouters (2007) and Christiano et al. (2005). For convenience, we simplify the household's problem by separating the consumption/saving decision from the portfolio allocation decision. The latter is made by "mutual funds" on behalf of the households. This device allows us to retain the familiar setup for the rest of the model. We describe the problem of the households, that of the mutual funds managers and, since this model is non-Ricardian, tax policy. The remainder of the model is identical to Smets and Wouters (2007).

4.2.1 Households

In every period j a new cohort is born with mass p, and each cohort has a constant probability of dying p which does not depend on j.

The budget constraint for members of cohort j is

$$C_{j,t} + Q_t S_{j,t} \leq \frac{(Q_t + D_t)}{1-p} S_{j,t-1} + \frac{W_t^N}{P_t} L_{j,t} - \mathcal{T}_t, \tag{13}$$

[43] Nisticò (2014) indeed interprets a perpetual youth model as one where agents transition in and out of hand-to-mouth status.

where $S_{j,t}$ represents the number of shares of cohort j in a "mutual fund" offering dividends D_t; Q_t is the value of the shares in terms of consumption goods; $C_{j,t}$ and $L_{j,t}$ are consumption and labor supply for cohort j, respectively; W_t^N, $\mathcal{T}_t$, and P_t are nominal wages, lump sum taxes (which are the same across cohorts), and the price level in period t. The return on the mutual fund for each member of cohort j still alive is given by $\frac{(Q_t + D_t)}{(1-p)Q_{t-1}}$ because, as in Blanchard (1985), they have entered an annuity contract in which the fraction p of members dying in each periods leaves its wealth to those who remain alive.

Household's utility is

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta(1-p))^s \left[\log(C_{j,t+s} - hC_{t+s-1}) + \nu \log(1 - L_{j,t+s})\right], \tag{14}$$

where the future is discounted at the rate $\beta(1-p)$ to take into account the probability of dying, and where the parameter h captures (external) habits. The household's consumption Euler equation can be expressed as

$$\mathbb{E}_t \left[\frac{Q_t}{C_{j,t} - hC_{t-1}} - \beta \frac{(Q_{t+1} + D_{t+1})}{C_{j,t+1} - hC_t}\right] = 0, \tag{15}$$

and the intratemporal condition for the optimal choice of consumption and hours worked is

$$\frac{W_t^h}{C_{j,t} - hC_{t-1}} = \nu \frac{1}{1 - L_{j,t}}, \tag{16}$$

where $W_t^h = \frac{W_t^N}{P_t}$ is the real wage paid to households. Define cohort j's stochastic discount factor, conditional on surviving, as:

$$\mathcal{F}_{j,t,t+s} = \beta^s \frac{C_{j,t} - hC_{t-1}}{C_{j,t+s} - hC_{t+s-1}}. \tag{17}$$

Following Piergallini (2006), Nisticò (2012), and Castelnuovo and Nisticò (2010), one can show that the policy function for consumption amounts to

$$C_{j,t} - hC_{t-1} = \frac{1 - \beta(1-p)}{(1+\nu)} \left(\frac{(Q_t + D_t)}{1-p} S_{j,t-1} + H_{j,t}\right), \tag{18}$$

where

$$H_{j,t} = \mathbb{E}_t \left[\sum_{s=0}^{\infty} (1-p)^s \mathcal{F}_{j,t,t+s} (W_{t+s}^h - \mathcal{T}_{t+s} - hC_{t+s-1})\right] \tag{19}$$

is the present discounted value of future disposable labor income, when working the maximum number of hours, adjusted for past consumption.

Aggregation

Consider any variable $X_{j,t}$ for cohort born at time j and define the aggregate variable

$$X_t \equiv \sum_{j=-\infty}^{t} p\,(1-p)^{t-j}\,X_{j,t}. \tag{20}$$

Define $\mathcal{F}_{t,t+s}$ as the population weighted average of the cohort-specific stochastic discount factors:

$$\mathcal{F}_{t,t+s} = \sum_{j=-\infty}^{t} p\,(1-p)^{t-j}\,\mathcal{F}_{j,t,t+s}, \tag{21}$$

and note that the Euler equation (15) can be rewritten as

$$I\!E_t\left[\mathcal{F}_{t,t+1}\frac{(Q_{t+1}+D_{t+1})}{Q_t}\right] = 1. \tag{22}$$

Since (18) is linear in cohort-specific variables, we obtain

$$C_t - hC_{t-1} = \frac{1-\beta(1-p)}{(1+\nu)}\left(\mathcal{S}_t + H_t\right), \tag{23}$$

where total financial wealth at the beginning of period t is defined as

$$\mathcal{S}_t = (Q_t + D_t)\,S_{t-1} \tag{24}$$

and

$$H_t = I\!E_t\left[\sum_{s=0}^{\infty}(1-p)^s\mathcal{F}_{t,t+s}(W^h_{t+s} - \mathcal{T}_{t+s} - hC_{t+s-1})\right]. \tag{25}$$

Aggregating the household's budget constraint and combining it with the Euler condition (22) and the intratemporal condition (16), we obtain a law of motion for financial wealth

$$(1+\nu)C_t - \nu hC_{t-1} + I\!E_t\left[\mathcal{F}_{t,t+1}\mathcal{S}_{t+1}\right] = \mathcal{S}_t + W^h_t - \mathcal{T}_t, \tag{26}$$

Equations (23) and (26) describe the evolution of aggregate consumption and wealth in this economy for given W^h_t and $\mathcal{T}_t$, together with the Euler equation (22), and the law of motion for H_t, namely

$$H_t = W^h_t - \mathcal{T}_t - hC_{t-1} + (1-p)I\!E_t\left[\mathcal{F}_{t,t+1}H_{t+1}\right]. \tag{27}$$

In fact, H_t can be eliminated from the equilibrium conditions by replacing (23) with

$$C_t - hC_{t-1} = \frac{p}{1-p}\frac{(1-\beta(1-p))}{\beta(1+\nu)}I\!E_t\left[\mathcal{F}_{t,t+1}\mathcal{S}_{t+1}\right] + I\!E_t\left[\beta^{-1}\mathcal{F}_{t,t+1}(C_{t+1}-hC_t)\right], \tag{28}$$

which obtains by combining (23), (26), and (27). The aggregate version of the intratemporal condition

$$W_t^h(1 - L_t) = \nu(C_t - hC_{t-1}), \tag{29}$$

yields labor supply.

4.2.2 Mutual Funds

There is a continuum of competitive, identical mutual funds, who are placeholders for the households' portfolio problem. This portfolio problem differs depending on whether we are considering the Smets and Wouters (2007) version of the model, or the version with financial frictions. In the Smets and Wouters (2007) version of the model, the mutual funds' budget constraint is:

$$D_t S_{t-1} = \frac{B_{t-1}}{P_t} + \left(\frac{R_t^k}{P_t} u_t \bar{K}_{t-1} - a(u_t)\bar{K}_{t-1}\right) + \Pi_t - I_t - \frac{B_t}{b_t R_t P_t} + Q_t(S_t - S_{t-1}), \tag{30}$$

where I_t is investment, B_t is holdings of government bonds, R_t is the gross nominal interest rate paid on government bonds, b_t is a "risk premium shock", and Π_t is the profit the mutual fund gets from owning firms. The first term within parenthesis represents the return to owning $\bar{K}_t$ units of capital. Mutual funds managers choose the utilization rate of their own capital, u_t, and end up renting to firms in period t an amount of "effective" capital equal to

$$K_t = u_t \bar{K}_{t-1}, \tag{31}$$

and getting $R_t^k u_t \bar{K}_{t-1}$ in return. However, they have to pay a cost of utilization in terms of the consumption good which is equal to $a(u_t)\bar{K}_{t-1}$. Mutual funds accumulate capital according to the equation:

$$\bar{K}_t = (1 - \delta)\bar{K}_{t-1} + \mu_t \left(1 - \phi(\frac{I_t}{I_{t-1}})\right) I_t, \tag{32}$$

where δ is the rate of depreciation, and $\phi(\cdot)$ is the cost of adjusting investment, with $\phi'(\cdot) > 0, \phi''(\cdot) > 0$. The term μ_t is a stochastic disturbance to the price of investment relative to consumption. Note that expression (30) comes from the aggregation of the transaction between the mutual funds and households of cohort j, which includes the annuity:

$$\frac{(D_t + Q_t)}{1 - p} S_{j,t-1} = \frac{B_{t-1}}{P_t} + \left(\frac{R_t^k}{P_t} u_t \bar{K}_{t-1} - a(u_t)\bar{K}_{t-1}\right) + \Pi_t - I_t - \frac{B_t}{b_t R_t P_t} + Q_t S_{j,t}. \tag{33}$$

Each mutual fund supplies a constant number of shares equal to one, which, since the continuum of mutual funds has mass one, is also the aggregate number of shares in the economy. The fund's manager objective function is to maximize the present discounted value of dividends:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \mathcal{F}_{t,t+s} D_{t+s}. \tag{34}$$

subject to the constraints (30)–(33).

The first-order conditions for bonds imply:

$$1 = R_t b_t \mathbb{E}_t \left[\frac{\mathcal{F}_{t,t+1}}{\pi_{t+1}} \right]. \tag{35}$$

Call q_t^k the Lagrange multiplier associated with constraint (32) expressed in units of the multiplier on the resource constraint (30). q_t^k can be interpreted as the value of installed capital relative to consumption goods (i.e., Tobin's Q). The first-order conditions with respect to investment, capital, and capital utilization are:

$$q_t^k \mu_t \left(1 - \phi(\frac{I_t}{I_{t-1}}) - \phi'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}} \right) + \mathbb{E}_t [q_{t+1}^k \mathcal{F}_{t,t+1} \mu_{t+1} \phi'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2] = 1, \tag{36}$$

$$q_t^k = \mathbb{E}_t \left[\mathcal{F}_{t,t+1} \left(\frac{R_{t+1}^k}{P_{t+1}} u_{t+1} - a(u_{t+1}) + q_{t+1}^k (1 - \delta) \right) \right], \tag{37}$$

$$\frac{R_t^k}{P_t} = a'(u_t). \tag{38}$$

4.2.3 Other Private Agents

The problems of the other private agents are identical to those in Smets and Wouters (2007). There are final goods producers who buy intermediate goods, and package them into a final good Y_t, which they resell to consumers, the mutual funds, and the government, on a perfectly competitive market. Intermediate goods producers use labor and capital to produce intermediate goods on a monopolistic competitive market, facing price stickiness as in Calvo (1983) and partial indexation. Whenever they can reoptimize their price, they do so by maximizing their present value of future profits, where they discount future profits using the stochastic discount factor $\mathcal{F}_{t,t+s}$. Labor unions combine the homogeneous labor supplied by households and differentiate the labor services to firms, setting wages according to a Calvo (1983) mechanism with indexation, and discounting the future using again the stochastic discount factor $\mathcal{F}_{t,t+s}$.

4.2.4 Fiscal Policy

The government budget constraint is of the form

$$P_tG_t + B_{t-1} = P_t\mathcal{T}_t + \frac{B_t}{b_tR_t}, \tag{39}$$

where $\mathcal{T}_t$ are nominal lump-sum taxes (or subsidies) that also appear in household's budget constraint. As this model is non-Ricardian,, we need to specify a tax policy. For simplicity, we will assume that government debt is constant (in detrended real terms), so that the transfer policy equals government spending plus interest payments.

4.2.5 Equilibrium

Adding up the budget constraint of the households, mutual funds, and the government, we obtain the resource constraint

$$C_t + I_t + a(u_t)\bar{K}_{t-1} + G_t = Y_t \tag{40}$$

where Y_t denotes the aggregate level of output.

We detrend all non-stationary variables by $Z_t = e^{\gamma t + \frac{1}{1-\alpha}\tilde{z}_t}$, where γ is the steady-state growth rate of the economy, and $\tilde{z}_t$ is the linearly detrended log productivity process that follows the autoregressive law of motion

$$\tilde{z}_t = \rho_z\tilde{z}_{t-1} + \sigma_z\varepsilon_{z,t}. \tag{41}$$

The growth rate of Z_t in deviations from γ, denoted by z_t, follows the process:

$$z_t = \ln(Z_t/Z_{t-1}) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z\epsilon_{z,t}. \tag{42}$$

We determine the steady state and perform log-linear approximations of all equilibrium conditions around that steady state.

4.2.6 Log-Linear Equilibrium Conditions of the SWBY Model

All variables in the following equations are expressed in log deviations from their non-stochastic steady state. Steady-state values are denoted by $*$-subscripts and steady-state formulas are provided in Appendix A.7. The consumption Euler equation is given by:

$$c_t = -\frac{(1-\tilde{h})}{(1+\tilde{h}\eta)}\left(R_t - \mathbb{E}_t[\pi_{t+1} + z_{t+1}] + b_t\right) + \frac{\tilde{h}}{(1+\tilde{h}\eta)}\left(c_{t-1} - z_t\right) + \frac{\eta}{(1+\tilde{h}\eta)}\mathbb{E}_t\left[c_{t+1} + \tilde{h}z_{t+1}\right] + \frac{(1-\tilde{h})(1-\eta)}{(1+\tilde{h}\eta)}\mathbb{E}_t[s_{t+1}], \quad (43)$$

where c_t is consumption, R_t is the nominal interest rate, π_t is inflation, s_t is total household financial wealth at the beginning of period t. The exogenous "risk premium shock" b_t is assumed to and follow an AR(1) process with parameters ρ_b and σ_b. The parameter $\tilde{h} \equiv he^{-\gamma}$ measures the habit persistence adjusted for the economy's steady state growth rate. The parameter

$$\eta \equiv \left(1 + \frac{p}{1-p}\frac{(1-\beta(1-p))}{(1+\nu)(1-\tilde{h})}\frac{s_*}{c_*}\right)^{-1} \quad (44)$$

plays an important role. In the conventional case that $p = 0$, or if households have no net wealth in steady state ($s_*/c_* = 0$), then we have $\eta = 1$. When $p > 0$, and household net wealth is positive, $\eta < 1$. Furthermore, η is a decreasing function of the degree of habit persistence $\tilde{h}$. Looking at the linearized consumption Euler equation, we note that current consumption depends on expected future consumption with a coefficient of $\eta/(1+\tilde{h}\eta)$. Therefore the higher the death probability p, the lower η and thus the less future conditions affect current consumption.

This is particularly clear in the case of no habit persistence ($\tilde{h} = 0$); indeed, by integrating forward (43), we obtain

$$\begin{aligned} c_t &= -\left(R_t - \mathbb{E}_t[\pi_{t+1} + z_{t+1}] + b_t\right) + \eta\mathbb{E}_t\left[c_{t+1}\right] + (1-\eta)\mathbb{E}_t[s_{t+1}] \\ &= \mathbb{E}_t\sum_{k=0}^{\infty}\eta^k\left[(1-\eta)\,s_{t+k+1} - \left(R_{t+k} - \pi_{t+1+k} - z_{t+k+1} + b_{t+k}\right)\right]. \end{aligned} \quad (45)$$

The integrated Euler equation (45) shows that consumption depends negatively on all future real interest rates $(R_{t+j} - \pi_{t+1+j})$, however with weights η^k that are declining with the horizon. This will be the key mechanism to address the forward guidance puzzle. With infinitely lived agents, i.e. when $p = 0$, these weights are constant at $\eta = 1$, so that current output depends on the sum of all future deviations of the short-term real interest rate $(R_{t+j} - \pi_{t+1+j})$ from its steady state. In the hypothetical case that the real interest rate would be brought permanently below its steady state, consumption, and hence output, would jump to $+\infty$. When $p > 0$ instead, the response of output to a similar permanent drop in the real interest rate is finite.

Turning to investment, the log-linearized approximation to the first-order condition (36) can be expressed as

$$i_t = \frac{1}{\phi'' e^{2\gamma}(1+\tilde{\beta})} q_t^k + \frac{1}{1+\tilde{\beta}}(i_{t-1} - z_t) + \frac{\tilde{\beta}}{1+\tilde{\beta}} \mathbb{E}_t [i_{t+1} + z_{t+1}] + \mu_t, \tag{46}$$

indicating that investment, i_t, depends on the value of capital in terms of consumption q_t^k, the investment adjustment cost (ϕ'' is the second derivative of the adjustment cost function) and the exogenous shock μ_t, which follows an AR(1) process with parameters ρ_μ and σ_μ.

The parameter $\tilde{\beta}$ is the steady-state value of the detrended stochastic discount factor. It relates to the steady-state gross real interest rate $r_* \equiv R_*/\pi_*$ and the economy's steady state growth rate according to $r_* = \tilde{\beta} e^\gamma$. As we show in the Appendix, $\tilde{\beta}$ relates to the underlying time preference parameter β according to

$$\tilde{\beta} = \beta\eta. \tag{47}$$

This implies that for positive death probability p, the steady-state value of the stochastic discount factor $\tilde{\beta}$ is smaller than the time preference parameter β, reflecting the fact that household discount future streams of income at a higher rate than given by their time preference. For instance, the response of current investment to expected future investment (which is given by $\tilde{\beta}/(1+\tilde{\beta})$) in (46) is less than with infinitely lived households ($p = 0$). This is another channel through which a positive p mitigates the effect of future movements in interest rates.

The capital stock, $\bar{k}_t$, evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} \phi'' e^{2\gamma}(1+\tilde{\beta})\mu_t, \tag{48}$$

where $i_*/\bar{k}_*$ is the steady-state ratio of investment to capital. The arbitrage condition between the return to capital and the riskless rate is:

$$\frac{r_*^k}{r_*^k + (1-\delta)} \mathbb{E}_t[r_{t+1}^k] + \frac{1-\delta}{r_*^k + (1-\delta)} \mathbb{E}_t[q_{t+1}^k] - q_t^k = R_t + b_t - \mathbb{E}_t[\pi_{t+1}], \tag{49}$$

where r_t^k is the rental rate of capital, r_*^k its steady-state value, and δ the depreciation rate. Given that capital is subject to variable capacity utilization u_t, the relationship between $\bar{k}_t$ and the amount of capital effectively rented out to firms k_t is

$$k_t \;=\; u_t - z_t + \bar{k}_{t-1}. \tag{50}$$

The optimality condition determining the rate of utilization is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \tag{51}$$

where ψ captures the utilization costs in terms of forgone consumption. Real marginal costs for firms are given by

$$mc_t \;=\; w_t + \alpha l_t - \alpha k_t, \tag{52}$$

where w_t is the real wage, l_t denotes hours worked, and α is the income share of capital (after paying markups and fixed costs) in the production function. From the optimality conditions of goods producers, it follows that all firms have the same capital-labor ratio:

$$k_t \;=\; w_t - r_t^k + l_t. \tag{53}$$

The production function is:

$$y_t \;=\; \Phi_p\left(\alpha k_t + (1-\alpha) l_t\right) + \mathcal{I}\{\rho_z < 1\}(\Phi_p - 1)\frac{1}{1-\alpha}\tilde{z}_t, \tag{54}$$

if the log productivity is trend stationary. The last term $(\Phi_p - 1)\frac{1}{1-\alpha}\tilde{z}_t$ drops out if technology has a stochastic trend, because in this case one has to assume that the fixed costs are proportional to the trend. Similarly, the resource constraint is:

$$y_t = g_* g_t + \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r_*^k k_*}{y_*} u_t - g_* \mathcal{I}\{\rho_z < 1\}\frac{1}{1-\alpha}\tilde{z}_t, \tag{55}$$

where the last term disappears if technology follows a unit root process. Government spending g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz}\sigma_z \varepsilon_{z,t}.$$

The price Phillips curve takes the conventional form

$$\pi_t = \kappa \; mc_t + \frac{\iota_p}{1+\iota_p\tilde{\beta}}\pi_{t-1} + \frac{\tilde{\beta}}{1+\iota_p\tilde{\beta}}\mathbb{E}_t[\pi_{t+1}] + \lambda_{f,t}, \tag{56}$$

where

$$\kappa = \frac{(1-\zeta_p\tilde{\beta})(1-\zeta_p)}{(1+\iota_p\tilde{\beta})\zeta_p((\Phi_p-1)\epsilon_p+1)},$$

the parameters ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices. Again, the key difference with respect to the Smets and Wouters (2007) is that the coefficient on expected future inflation is replaced by a smaller coefficient ($\tilde{\beta}/(1+\iota_p\tilde{\beta})$) when p is positive. Integrating this equation forward, we note that this mechanism attenuates the response of current inflation to fluctuations in expected future marginal costs.

The wage Phillips curve is in turn of the usual form

$$\begin{aligned} w_t \;=\; & \frac{(1-\zeta_w\tilde{\beta})(1-\zeta_w)}{(1+\tilde{\beta})\zeta_w((\lambda_w-1)\epsilon_w+1)}\left(w_t^h - w_t\right) - \frac{1+\iota_w\tilde{\beta}}{1+\tilde{\beta}}\pi_t \\ & + \frac{1}{1+\tilde{\beta}}\left(w_{t-1} - z_t + \iota_w\pi_{t-1}\right) + \frac{\tilde{\beta}}{1+\tilde{\beta}}I\!\!E_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t}, \qquad (57) \end{aligned}$$

where ζ_w, ι_w, and ϵ_w are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for wages. Similarly to the price Phillips curve, the coefficient on expected future real wages in the SW model is replaced by a smaller coefficient ($\tilde{\beta}/(1+\tilde{\beta})$) when p is positive. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{\nu c_*}{(1-L_*)w_*}\left(c_t - \tilde{h}c_{t-1} + \tilde{h}z_t\right) + \frac{L_*}{1-L_*}l_t, \qquad (58)$$

where $L_*/(1-L_*)$ would equal the inverse of the Frisch elasticity in the absence of wage rigidities. The markups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes

$$\lambda_{f,t} = \rho_{\lambda_f}\lambda_{f,t-1} + \sigma_{\lambda_f}\varepsilon_{\lambda_f,t} + \eta_{\lambda_f}\sigma_{\lambda_f}\varepsilon_{\lambda_f,t-1}, \text{ and}$$

$$\lambda_{w,t} = \rho_{\lambda_w}\lambda_{w,t-1} + \sigma_{\lambda_w}\varepsilon_{\lambda_w,t} + \eta_{\lambda_w}\sigma_{\lambda_w}\varepsilon_{\lambda_w,t-1},$$

respectively.

Given that household wealth enters the consumption Euler equation, we need to take into account its law of motion, which is given by

$$\begin{aligned} \tilde{\beta}\frac{s_*}{c_*}I\!\!E_t\left[s_{t+1}\right] = \tilde{\beta}\frac{s_*}{c_*}\left(R_t + b_t - I\!\!E_t[\pi_{t+1} + z_{t+1}]\right) + \frac{s_*}{c_*}s_t \\ + \frac{w_*}{c_*}L_*w_t^h + \frac{w_*}{c_*}L_*L_t - \frac{\mathtt{t}_*}{c_*}\mathtt{t}_t - c_t. \qquad (59) \end{aligned}$$

Since the model is non-Ricardian, we further need to include the linearized government budget constraint

$$\mathtt{t}_*\mathtt{t}_t = y_* g_* g_t - y_* g_* \mathcal{I}\{\rho_z < 1\}\frac{1}{1-\alpha}\tilde{z}_t - \frac{b_*^g e^{-\gamma}}{\pi_*}(z_{t-1} + \pi_t) + \frac{b_*^g}{R_*}(b_t^g + R_t), \tag{60}$$

where $\mathtt{t}_t$ denotes taxes, and b_t^g represents government debt. Finally, the monetary authority follows a generalized feedback rule:

$$\begin{aligned} R_t &= \rho_R R_{t-1} + (1-\rho_R)\left(\psi_1 \pi_t + \psi_2 (y_t - y_t^f)\right) \\ &\quad + \psi_3\left((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\right) + r_t^m, \end{aligned} \tag{61}$$

where the flexible price/wage output y_t^f is obtained from solving the version of the model without nominal rigidities (that is, Equations (43) through (55) and (58)), and the residual r_t^m follows the process:

$$r_t^m = \rho_{r^m} r_{t-1}^m + \sigma_r \varepsilon_{r^m,t} + \sum_{k=1}^{K} \sigma_{k,r} \varepsilon_{k,t-k}^R, \tag{62}$$

where $\varepsilon_{R,t}$ is the usual contemporaneous policy shock and $\varepsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t-k$, but affects the policy rule k periods later, that is, at time t. We assume as usual that $\varepsilon_{k,t-k}^R \sim N(0,1)$, *i.i.d.*.

4.3 Quantitative Assessment

As discussed in the previous section, introducing a positive death probability p in the model of Smets and Wouters (2007) results in a very similar model except that consumption tends to respond less to fluctuations in expected future variables such as interest rates, investment depends less on future investment, price inflation depends less on expected future marginal costs, and the real wage is less responsive to expected future real wages. To assess whether such a SW model with Blanchard-Yaari households (i.e., the SWBY model) includes mechanisms that are sufficiently relevant to address the forward guidance puzzle, we take the original SW model, as described and estimated in Smets and Wouters (2007), and analyze the effects of changing the parameter p. The parameters of the model are the modal estimates reported in Table 1A-1B of Smets and Wouters (2007). The SWBY model differs from the original SW model in that it assumes a utility function that is nonseparable in consumption and hours worked, while we assume a separable utility function (14).[44] Despite these differences our SWBY model with p set to 0 approximates quite well the original SW model. Figure A2 in the Appendix shows the effect of a contemporaneous monetary policy shock, that is, an unanticipated decline of 25 basis points in the short-term nominal interest rate, for both the SW and the SWBY models. Clearly both models reveal very similar responses, showing that the difference in utility functions does not appear to play an important role, at least in terms of the responses to monetary shocks.

4.3.1 Calibration of p

To have a sense the magnitude of the death probability p, we use the actuarial life table from the Social Security Administration and construct the death probability for individuals of ages 20 and above, weighting by the population at each age. This yields an average probability of dying before the next quarter of $p = 0.0042$. These calculations treat the wealth to consumption ratio and the probability of death as independent from each other. In the data, however, both the probability of death and wealth are rising with age. As an alternative calibration, we weight the probability of death at each age by both the fraction of the population at each age and by the share of wealth in each age category.[45] The resulting weighted average probability of dying before the next quarter is $p = 0.0076$.

While our model refers formally to the probability of dying, such an event amounts to re-setting the initial wealth of the household to zero. In this light, one can think of death in the model as representing also default on the part of households. Based on the Quarterly Report on Household Debt and Credit from the Federal Reserve Bank of New York, the fraction of consumers with new bankruptcies in a given quarter has been on average 0.2 percent from 2003 to 2014. Focusing on mortgage owners, the probability of transitioning from being current on mortgage payments to being 90+ days late on that payment within a quarter has been on average 0.0034.[46] Taking into account both the probability of a new

[44] We set the weight on leisure in (14), ν, such that the Frisch elasticity of labor supply $(1 - L_*)/L*$ equals the SW estimate.

[45] For wealth, we use data from the Federal Reserve's Survey of Consumer Finances for 2010, and use median wealth by age category.

[46] In addition, the probability of transitioning from being current to being 30-60 days late on the mortgage payment in substantially larger, averaging 1.7 percent per quarter. This provides a signal of incoming default

bankruptcy and the probability of being 90+ days late on a mortgage payment, and noting that the two events can be correlated, we set the probability of defaulting in a given quarter to at least 0.005. Adding this to the probability of dying mentioned above suggests that our calibrated value for p should be at least 0.0126.

Alternatively, as mentioned in Section 4.1, the perpetual youth model shares similarities with a model in which households transition in and out of hand-to-mouth status. To the extent that our SWBY model captures some elements of this transition, the (quarterly) value of p would be roughly 0.0227, based on Kaplan et al. (2014)'s estimate of the expected length of non-hand-to-mouth status which is 11 years. Adding this to the probability of dying implies a value for p of around 0.030.

We set our benchmark value of p to 0.03 and raise it to 0.06 to include other forms of wealth "re-setting" that we might have omitted. Such values are still well below the posterior mean of $p = 0.1292$. obtained by Castelnuovo and Nisticò (2010) based on a Bayesian estimation using only aggregate data.

4.3.2 Results

Table 2 shows how different assumptions about the death probability p affect some of the key parameters. The implied value for the discounting coefficient entering equations (45), (46), (56), and (57) varies significantly with changes in p. Our lower bound $p = 0$ implies no discounting in the consumption Euler equation as $\eta = 1$. This coefficient is lowered to 0.96 when $p = 0.06$. Similarly, with such a value of p, the fluctuations in expected future variables are also discounted more heavily in the equations determining investment, price inflation, and real wages. The slope of the Phillips curve κ is also affected, rising with the

as the probability of transitioning form this category to being 90+ days late on a mortgage payment is 24 percent. It follows that the probability in any given quarter of being 90+ days late on the mortgage payment within the next 6 months is $0.0034 + 0.017 * 0.24 = 0.00748$.

death probability p.

Table 2. Implied Coefficients for Alternative Death Probabilities

Death probability	p	0	0.03	0.06
Implied coefficients				
Discounting in consumption	η	1	0.987	0.960
Discounting in investment	$\tilde{\beta}/(1+\tilde{\beta})$	0.500	0.496	0.490
Discounting in price inflation	$\tilde{\beta}/(1+\iota_p\tilde{\beta})$	0.817	0.809	0.791
Discounting in real wage	$\tilde{\beta}/(1+\tilde{\beta})$	0.500	0.496	0.490
Slope of Phillips Curve	κ	0.022	0.022	0.024

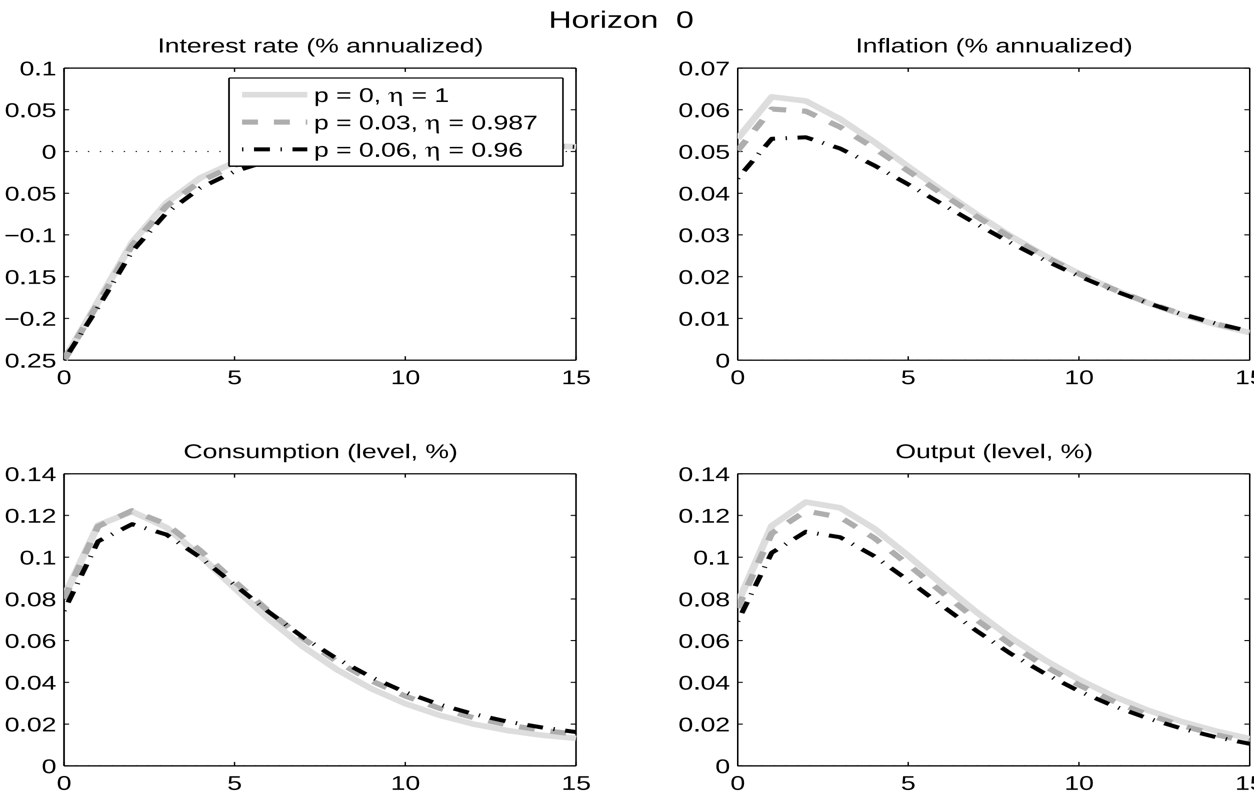
As shown in Figure 8, increasing the death probability has little effect on the economy's response to *contemporaneous* monetary shocks. This suggests that the model has standard implications in response to contemporaneous shocks.

However, in response to *anticipated* shocks, the effects of a positive p become much more significant. Figure 9 shows the effect of an announcement that the short-term nominal interest rate will be lowered by 25 basis points 4 quarter into the future. This experiment also assumes that this interest rate will remain constant at its initial value in all periods until the announced drop takes place.[47] Figure 10 shows the same, but for an announcement that the nominal interest rate will be lowered by 25 basis points 8 quarters in to the future. The latter experiment relates closely to the one presented in Section 3.2.

For $p = 0$, the model-implied effects of such announcements are again very large. Indeed, as shown in Figure 10, the announcement of a interest-rate drop 8 quarters ahead causes output to increase by almost 4 percent on impact and by about 7 percent after a year. Inflation also jumps by more than 4 percentage points over the subsequent year. This experiment reproduces again the forward guidance puzzle documented in Section 3.2. However, when $p = 0.03$, the effects are significantly muted with the dynamic responses of consumption, output and inflation rising by less than half: output increases by about 1.5 percent on impact and reaches a peak of 2.5 percent about a year after the announcement. Inflation also increases, but by less than 2 percentage points. This confirms that with realistic values for p,

[47]Specifically, this is modeled by having an unanticipated monetary shock as well as anticipated monetary policy shocks in periods 1 through 4 that implement that path of the short-term policy rate.

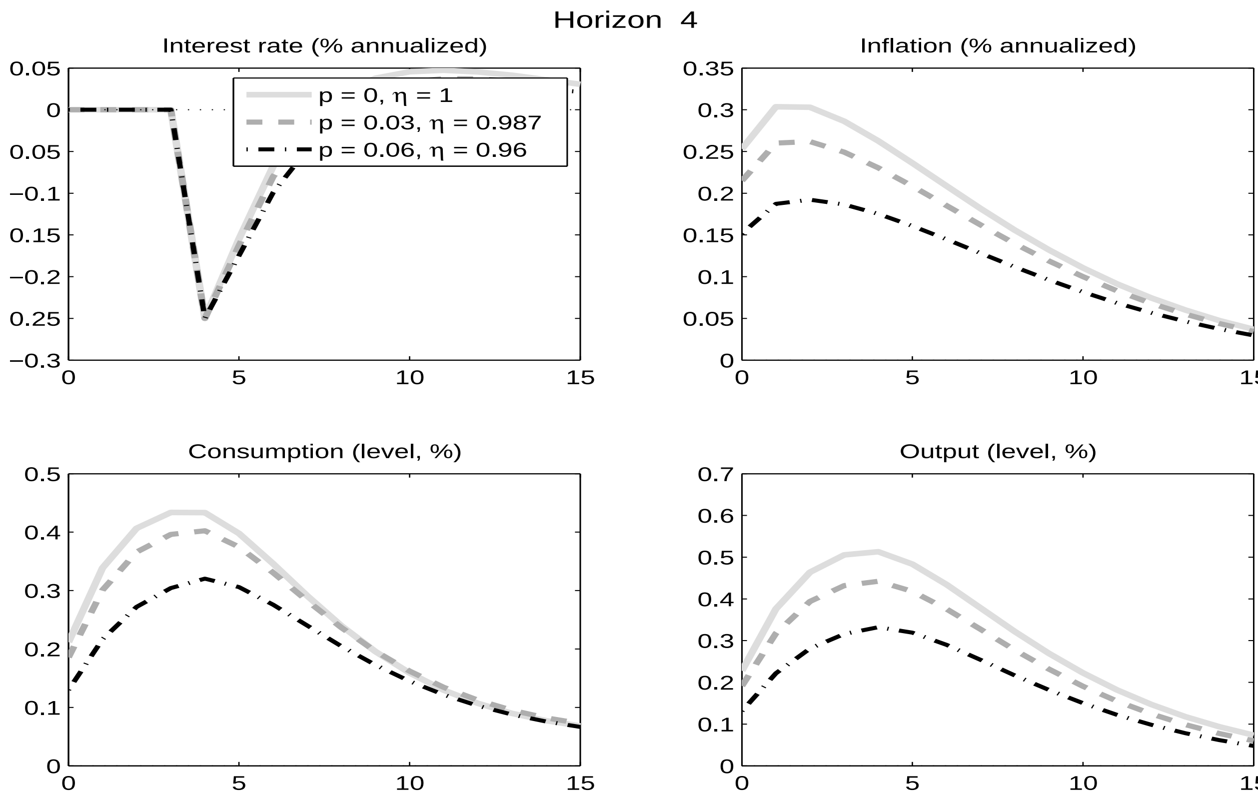
Figure 8: Effect of a Contemporaneous Nominal Interest Rate Cut in SWBY Model for Different Values of p



the model implies much more reasonable responses of inflation, output and real interest rates to forward guidance announcements. When we consider a death probability of $p = 0.06$, the dampening effects of the perpetual youth model are even stronger, with consumption, output and inflation responding by less than 1 percent to the announcement, as indicated by black dashed-dotted lines in Figure 10. The latter responses seem quite realistic when compared to the evidence reported in Section 2.2.

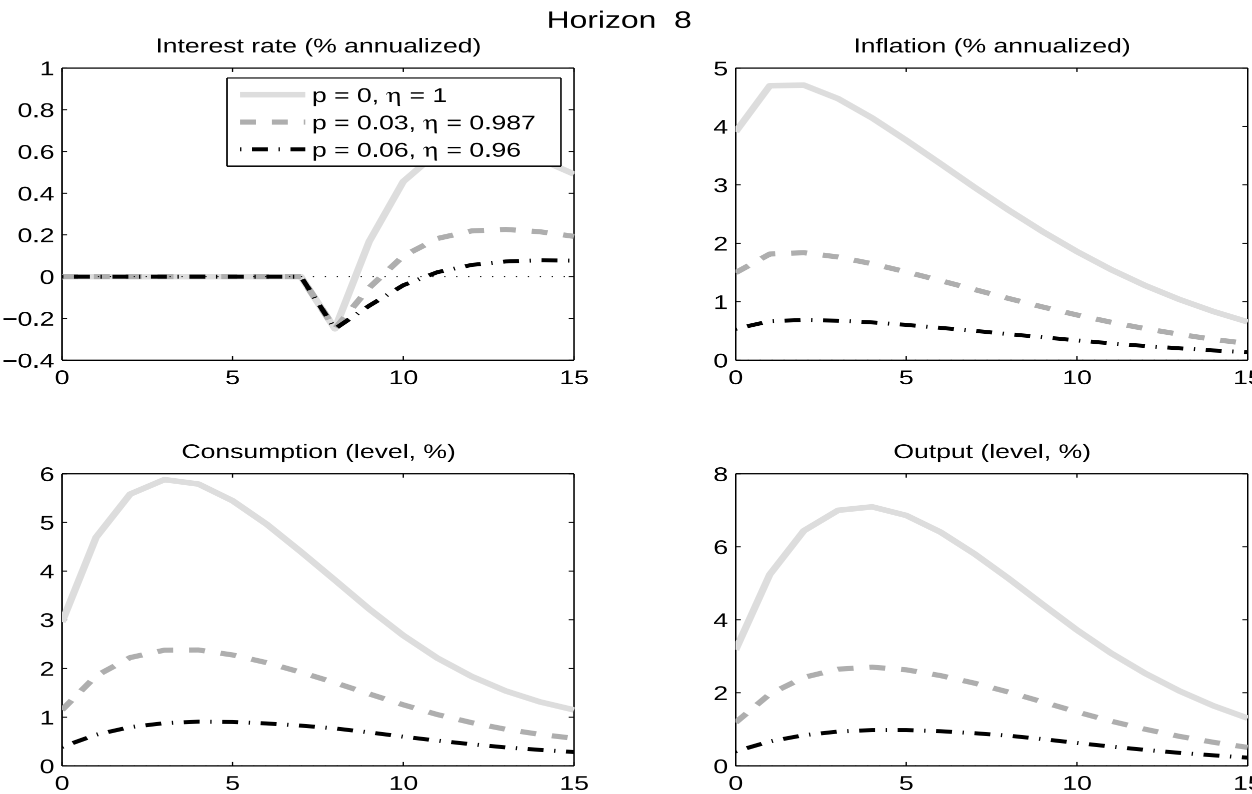
In order to capture the kind of announcements made by the FOMC, we have focused here on the effect of announcements about changes in the nominal interest rate at various horizons. Our proposed solution to the forward guidance puzzle involves more discounting than in the standard consumption Euler equation (similarly to McKay et al. (2015)), but also more discounting of future economic conditions for price-setting decisions, thereby mitigating the concerns raised by Carlstrom et al. (2012b), Chung et al. (2014), and Kiley (2014) about

Figure 9: Effect of an Announced Nominal Interest Rate Cut 4 Periods Ahead in the SWBY Model for Different Values of p



the New Keynesian Phillips curve. To understand the role of the endogenous response of inflation, we report in Figures A3–A5 of the Appendix the effects of announcements about changes in the *real* short-term interest rate at various horizons. Such experiments are also more closely related to those performed by McKay et al. (2015). The results are overall qualitatively similar to those reported above with the responses being more muted with positive death probability p than when $p = 0$. However, the magnitudes of the responses are smaller when we consider announcements about real interest rate changes than is the case when the announcements pertain to nominal interest rate changes. This difference is due to the fact that the inflation response tends to amplify the effects of an announcement about the nominal rate on the real rate: the announcement of a drop in the nominal rate stimulates the economy, pushes inflation up, hence lowers the real rate by more than the nominal rate. Given that the responses are smaller, the attenuation provided by positive death probabilities, while still considerable, is also smaller.

Figure 10: Effect of an Announced Nominal Interest Rate Cut 8 Periods Ahead in the SWBY Model for Different Values of p



5 Conclusion

Forward guidance has become an essential tool for monetary policy in many industrial economies confronted with the effective lower bound on interest rates. Yet, little is know about the quantitative effects of forward guidance announcements on the economy. In this paper, we document the impact of forward guidance announcements on a broad cross section of financial markets data and on forecasts of key economic variables reported in the panel of Blue Chip forecasts. We find that these impacts have been very heterogeneous and depend in very subtle ways on the type forward guidance announcements. We show that once we control for these other elements, forward guidance had, on average, positive and meaningful effects on output and inflation.

The paper analyzes the effect of forward guidance announcements in the context of a fairly standard medium scale DSGE model, and shows that this model tends to grossly overestimate the impact of forward guidance on the macroeconomy, a phenomenon we call the "forward guidance puzzle." We argue that the model's excess response to announcements about future policy changes is likely due to the lack of discounting of future economic outcomes. We propose a tentative resolution to this puzzle based on the fact that life is finite. We show that incorporating a perpetual youth structure into a DSGE model leads to an aggregate economy that discounts the future more heavily. This implies that announcements of policy changes far in the future generate significantly smaller effects on current aggregate variables than is the case in models with infinitely lived agents.

While the paper has focused on effects of forward guidance announcements, its message is more general. Our discussion illustrates that conventional DSGE models tend to generate excessively strong responses of key macroeconomic variables to news about conditions far in the future. We have argued that incorporating a Blanchard-Yaari perpetual youth structure in medium-scale DSGE models is desirable, as it is feasible, tractable, and likely to generate more realistic model dynamics.

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A Appendix

A.1 Additional Empirical Results

A.1.1 Description of Policy Decompositions in 2.2

In order to assign numerical values to the FOMC language, we used the followings rules. In the future, we hope to use better machine learning techniques to improve on this albeit rough codification:

- If the statement said that inflation was expected to be "at or below target", we coded it as having neutral inflation language. Otherwise, the statement was said to have "concerned inflation language."
- If the statement used the words "moderate" or "continued to recover" to describe output growth, then the statement was coded as having neutral GDP language. If the words "slowed," "contracted" or "paused" were used, it received a 1 for "Bad GDP language" and conversely, if the words "picked up," "strengthened' or "rebounded" were used, then it received a 1 for "Good GDP language."
- If calendar-based forward guidance was newly announced on that date, it received a 1 for "Forward Guidance". Note that this does not consider the forward guidance that was announced in December 2008 along with the movement of interest rates downward to the zero lower bound. Additionally, it does not consider forward guidance changes that were implicit in the unemployment rate target in 2013 and 2014.
- If the statement contained the first announcement of an asset purchase program, the date received a 1 for "QE announcement."
- If the statement contained an announcement that it was going to continue its previously announced asset purchases, it received a 1 for "QE continuation." Note that we did not separately code the announcements for the tapering of QE. This choice would be something to check the robustness of in future specifications.

- If the statement announced that it would continue "reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction," it also received a 1 for the "maturity" dummy.

A.1.2 Additional Tables and Figures

Table A1: Forward Guidance FOMC Statements Language – Economic Conditions

August 2011: Information received ... indicates that economic growth so far this year has been considerably slower than the Committee had expected. ... Inflation picked up earlier in the year, ... More recently, inflation has moderated as prices of energy and some commodities have declined from their earlier peaks. The Committee now expects a somewhat slower pace of recovery over coming quarters than it did at the time of the previous meeting ... Moreover, downside risks to the economic outlook have increased...
January 2012: Information received ... suggests that the economy has been expanding moderately, notwithstanding some slowing in global growth ...The Committee expects economic growth over coming quarters to be modest ...The Committee also anticipates that over coming quarters, inflation will run at levels at or below those consistent with the Committee's dual mandate.
September 2012: Information received ... suggests that economic activity has continued to expand at a moderate pace in recent months. ... Inflation has been subdued, although the prices of some key commodities have increased recently.

Table A2: Forward Guidance FOMC Statements Language – Policy

August 2011:

...The Committee currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. ...

January 2012:

...the Committee expects to maintain a highly accommodative stance for monetary policy. In particular, the Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. ... The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September.

September 2012:

The Committee is concerned that, without further policy accommodation, economic growth might not be strong enough to generate sustained improvement in labor market conditions. ... the Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month... The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities ... These actions ... will increase the ... holdings of longer-term securities by about $85 billion each month through the end of the year, should put downward pressure on longer-term interest rates, ... and help to make broader financial conditions more accommodative ... If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability ...

To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the economic recovery strengthens. In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.

Table A3: List of Economic News and Asset Prices Regressors in Equation (1)

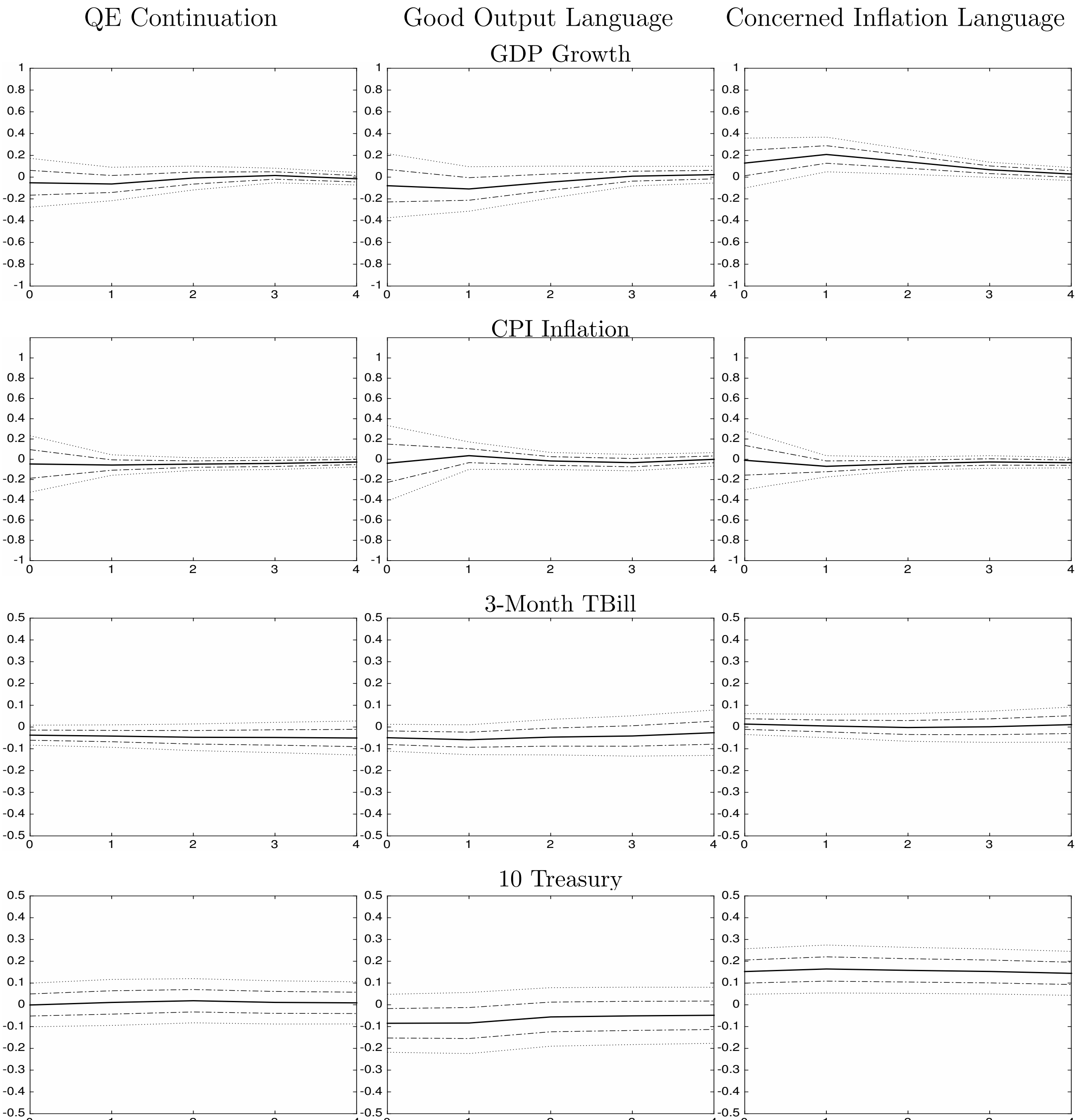
Name	Description	Unit
Economic News (ΔM)		
GDP	real GDP QoQ: Advanced, Preliminary, Final	% change
NAPMPMI	ISM Manufacturing PMI SA	level
NFP	US Employees on Nonfarm Payrolls Total MoM Net Change SA	level
USURTOT	U-3 US Unemployment Rate	
	Total in Labor Force Seasonally Adjusted	level
RSTAMOM	Adjusted Retail & Food Services Sales SA Total Monthly % Change	% change
CPI	US CPI Urban Consumers MoM SA	% change
CPTI	US Capacity Utilization % of Total Capacity SA	level
INJCJC	US Initial Jobless Claims SA	level
CONCCONF	Conference Board Consumer Confidence SA 1985=100	level
LEI	Conference Board US Leading Index MoM	% change
NHSLTOT	US New One Family Houses Sold Annual Total SAAR	level
Asset Prices (ΔAP)		
BBOX	Barclays Swaption Volatility	log change
MBS30	Average of Mortgage Backed Securities	
	(GNMA, FMNMA, GOLD), 30 Year Constant Maturity	bp difference
SP500	S&P 500 Index	log change
FXEDUS	EURUSD Spot Exchange Rate - Price of 1 EUR in USD	log change
DGS10	US Treasury Yield Curve Rate	
	T Note Constant Maturity 10 Year	bp difference
TIPS10	S&P 10 Year US TIPS Index Average Yield	bp difference

Table A4: Robustnes Checks for Figure 2

	GDP	CPI	3-Month TBill	10 Treasury
Grouped Dummies				
August 2011	-0.426	0.170	0.023	-0.350
Baseline s.e.	(0.40)	(0.52)	(0.09)	(0.19)
Heteroskedastic s.e.	(0.12)	(0.16)	(0.02)	(0.04)
Factor s.e.	(0.36)	(0.44)	(0.07)	(0.175)
January 2012	0.182	0.1662	0.025	-0.035
Baseline s.e.	(0.34)	(0.45)	(0.08)	(0.17)
Heteroskedastic s.e.	(0.10)	(0.14)	(0.02)	(0.04)
Factor s.e.	(0.31)	(0.40)	(0.06)	(0.15)
September 2012	1.013	0.857	0.1651	0.296
Baseline s.e.	(0.39)	(0.51)	(0.09)	(0.19)
Heteroskedastic s.e.	(0.11)	(0.15)	(0.02)	(0.042)
Factor s.e.	(0.34)	(0.45)	(0.07)	(0.17)
Not-Excluding 2-Day Window of Asset Prices				
August 2011	-0.581	0.116	0.026	-0.120
	(0.40)	(0.52)	(0.06)	(0.14)
January 2012	0.185	0.264	-0.018	0.050
	(0.36)	(0.47)	(0.06)	(0.13)
September 2012	0.773	0.864	0.024	0.062
	(0.45)	(0.60)	(0.08)	(0.17)

Notes: The coefficient reported in the table is the $\beta^{e}_{(k,1)}$ coefficient from Equation 1 for GDP, inflation, 3-Month TBill and the 10-year TBill.

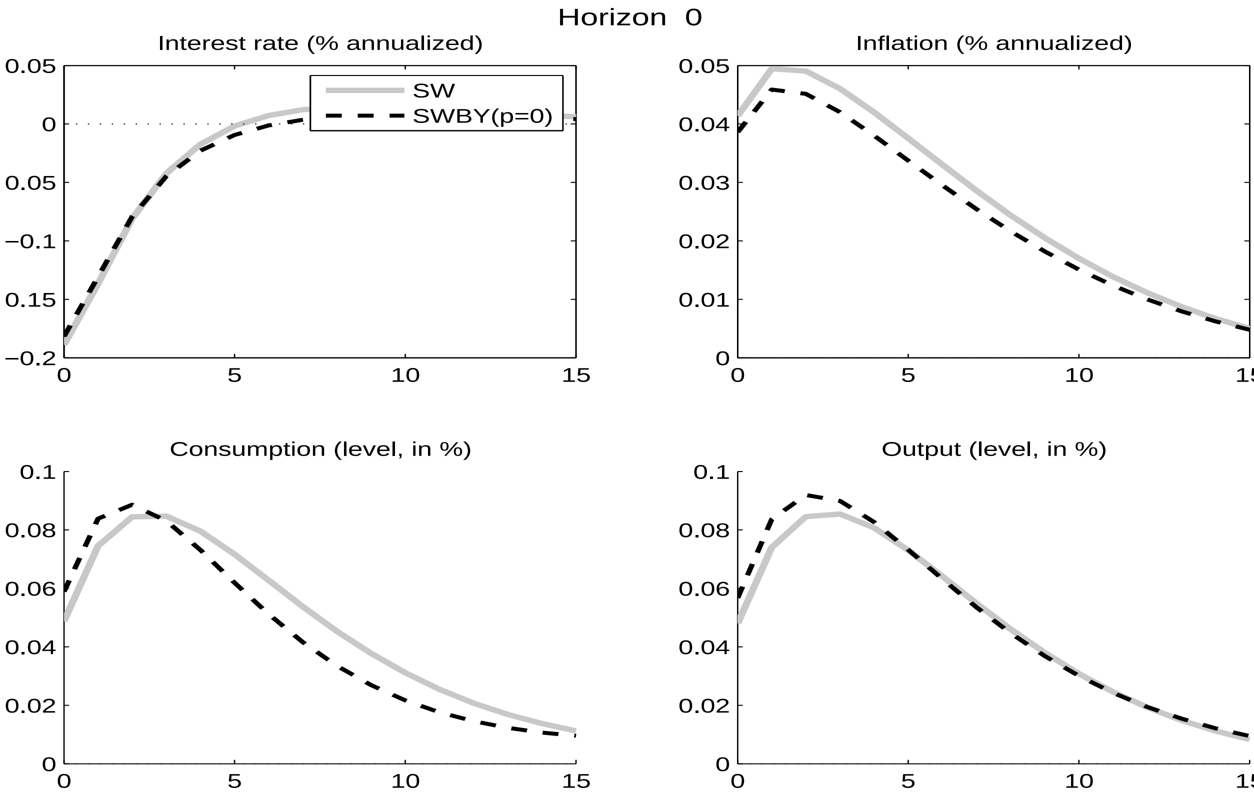
Figure A1: Decomposing FOMC Statements: The Effect of QE Continuation, Good Output Language and Concerned Inflation Language



Notes: The panel in Figure **??** shows the estimates of $\beta(k,h)^e$ for three different elements of the FOMC statement – forward guidance announcements, QE announcements and bad GDP language – and four different variables, GDP growth, CPI inflation, the 3-month TBill, and the 10-year Treasury rate. Variables and events correspond to rows and columns in the panel, respectively, while the horizon h is in the horizontal axis of each plot.For each triplet (e,k,h) we report the OLS estimate of $\beta(k,h)^e$ (solid black) and the 68 and 90 percent bands (dash-and-dotted and dotted lines, respectively) computed using heteroskedasticity-robust standard errors. The sample for each regression is $t = 2008.06, .., 2015.02$.

A.1.3 Impulse Responses to Unanticipated and Anticipated Shocks to Interest Rates

Figure A2: Effect of a Contemporaneous Monetary Policy Shock: SW vs SWBY($p = 0$)



Notes: The figure compares the impulse response functions to anticipated monetary shocks at horizons in the model of Smets and Wouters (2007) (gray solid lines) with those of the SWBY model in which the death probability p is set to 0 (black dashed lines). Both models differ in their specification of the household utility function.

Figure A3: Effect of a Contemporaneous Real Interest Rate Cut in the SWBY Model for Different Values of p

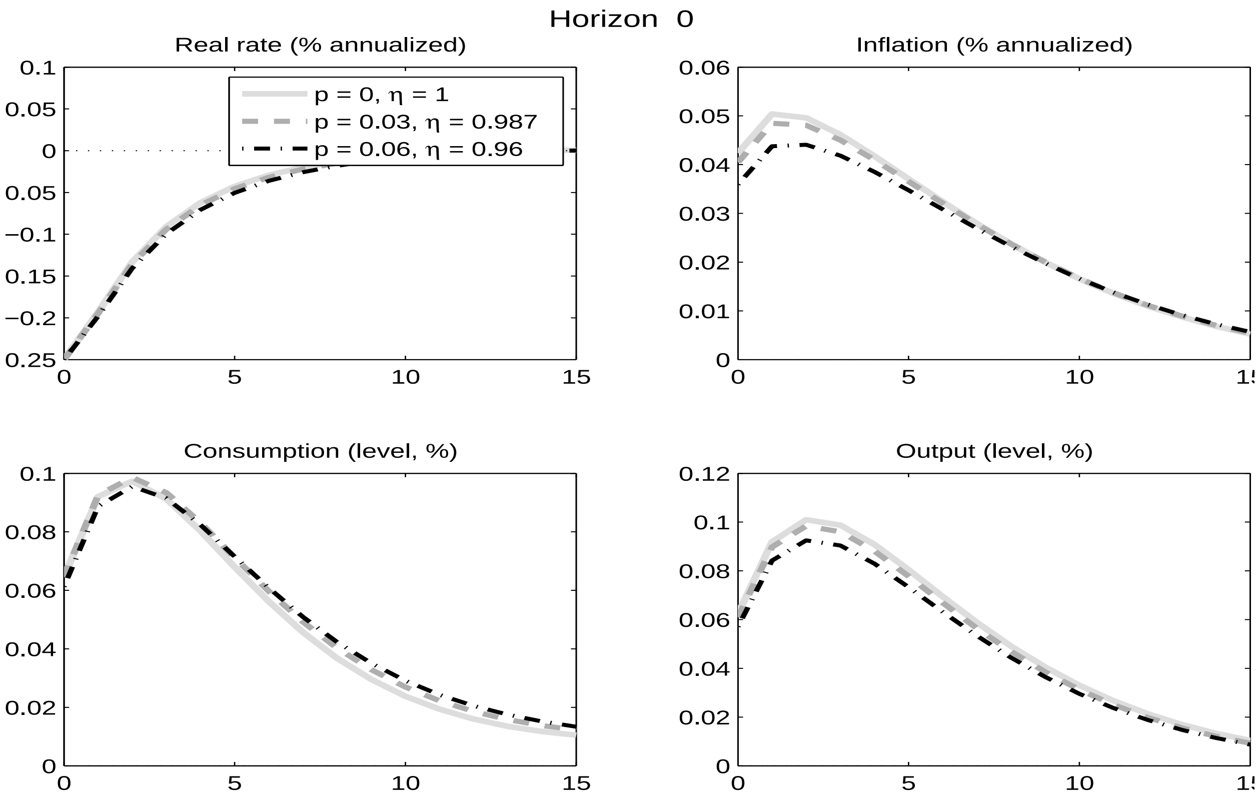


Figure A4: Effect of an Announced Real Interest Rate Cut 4 Periods Ahead in the SWBY Model for Different Values of p

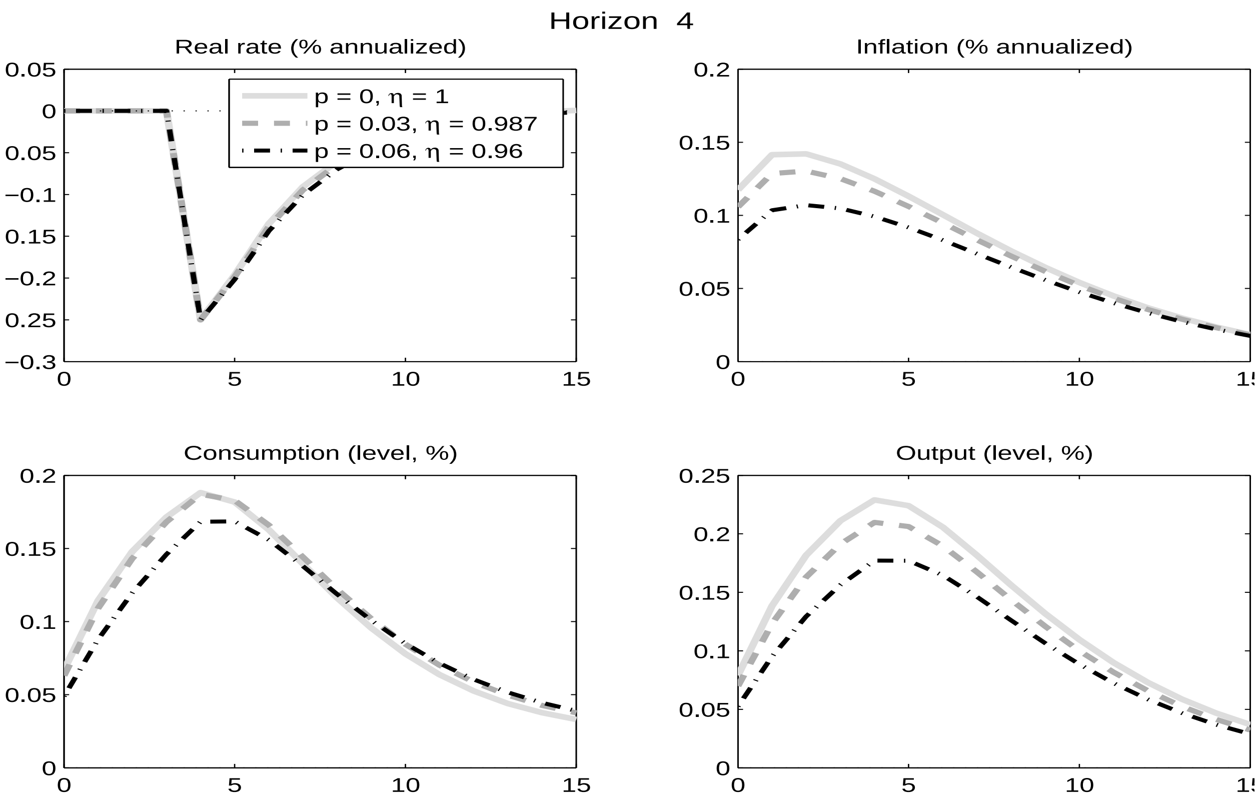
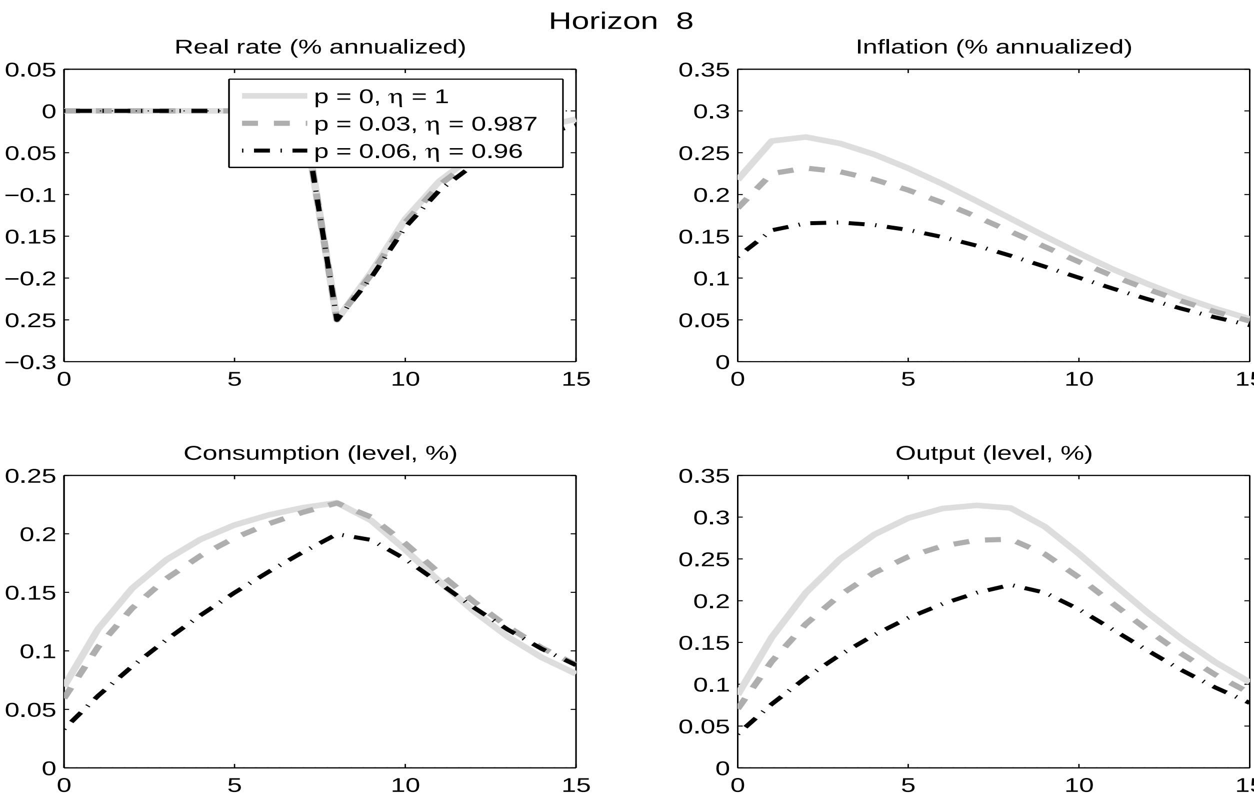


Figure A5: Effect of an Announced Real Interest Rate Cut 8 Periods Ahead in the SWBY Model for Different Values of p



A.2 Model Description

In this Appendix, we summarize the log-linear equations that characterize the FRBNY DSGE model used in Section 3. The microeconomic foundations of the model are described in Del Negro et al. (2013). Because the model has a source of non-stationarity in the process for technology Z_t, to solve the model we first rewrite its equilibrium conditions in terms of stationary variables, and then solve for the non-stochastic steady state of the transformed model. Finally we take a log-linear approximation of the transformed model around its steady state. This approximation generates a set of log-linear equations, which we solve to obtain the model's state-space representation, using the method of Sims (2002). We then use the state-space representation in the estimation procedure.

Below we list the log-linear equations of the model. We follow the usual convention of denoting log-deviations from steady state with hatted variables: for any stationary variable x_t, $\widehat{x}_t \equiv \log(x_t/x_*)$, where x_* denotes its steady state value. The steady state itself is a function of the model's parameters. Equations describing the mapping between parameters and steady state variables are available upon request.

The *Consumption Euler Equation* that characterizes the optimal allocation of consumption over time is given by

$$\hat{\xi}_t = \hat{R}_t + \mathbb{E}_t[\hat{\xi}_{t+1}] - \mathbb{E}_t[\hat{z}_{t+1}] - \mathbb{E}_t[\hat{\pi}_{t+1}], \tag{63}$$

where $\hat{R}_t$ is the gross nominal interest rate on government bonds, and $\hat{\xi}_t$ is the marginal utility of consumption.

The *Marginal Utility of Consumption* ξ_t evolves according to

$$(1 - \tilde{h}\beta)(1 - \tilde{h})\hat{\xi}_t = -\,(1 + \beta\tilde{h}^2)\hat{c}_t + \tilde{h}\hat{c}_{t-1} - \tilde{h}\hat{z}_t + \beta\tilde{h}\mathbb{E}_t[\hat{c}_{t+1}] + \beta\tilde{h}\mathbb{E}_t[\hat{z}_{t+1}], \tag{64}$$

where $\hat{c}_t$ is consumption, e^{γ} is the steady-state (gross) growth rate of the economy, h captures habit persistence in consumption, and $\tilde{h} \equiv he^{-\gamma}$.

The *Capital Stock* follows

$$\hat{\bar{k}}_t = -(1 - \frac{i_*}{k_*})\hat{z}_t + (1 - \frac{i_*}{k_*})\hat{\bar{k}}_{t-1} + \frac{i_*}{k_*}\hat{\mu}_t + \frac{i_*}{k_*}\hat{\imath}_t, \tag{65}$$

where $\hat{\bar{k}}_t$ is installed capital, $\hat{z}_t$ is the growth rate of productivity, i_* and k_* are steady state investment and the level of capital, respectively, and $\hat{\mu}_t$ is the exogenous process that affects the efficiency by which a foregone unit of consumption contributes to capital utilization.

The *Effective Capital* $\hat{k}_t$ is in turn given by

$$\hat{k}_t = \hat{u}_t - \hat{z}_t + \hat{\bar{k}}_{t-1}, \tag{66}$$

where $\hat{u}_t$ is the level of capital utilization.

Capital Utilization is given by

$$r_*^k \hat{r}_t^k = a''(u) \;\; \hat{u}_t, \tag{67}$$

where r_*^k is the steady state rental rate of capital and the function $a(u)$ captures the utilization cost.

The *Optimal Investment* decision satisfies the Euler equation

$$\hat{i}_t = \frac{1}{1+\beta} I\!\!E_t[\hat{\imath}_{t-1} - \hat{z}_t] + \frac{\beta}{1+\beta} I\!\!E_t[\hat{\imath}_{t+1} + \hat{z}_{t+1}] + \frac{1}{(1+\beta)\phi'' e^{2\gamma}} \hat{q}_t^k + \frac{1}{(1+\beta)\phi'' e^{2\gamma}} \hat{\mu}_t, \tag{68}$$

where $\hat{i}_t$ is investment, $\phi(.)$ is the cost of adjusting capital, with ϕ' and $\phi'' > 0$, and $\hat{q}_t^k$ is the price of capital.

The *Realized Return on Capital* is given by:

$$\hat{\tilde{R}}_t^k - \hat{\pi}_t = \frac{r_*^k}{r_*^k + (1-\delta)} \hat{r}_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} \hat{q}_t^k - \hat{q}_{t-1}^k, \tag{69}$$

where δ is the rate of capital depreciation, π_t is the inflation rate, whose evolution is described below, $\hat{r}_t^k$ is the capital rental rate and $\hat{q}_t^k$ is the price of capital.

The *Expected Excess Return on Capital* (or ‘spread’)

$$I\!\!E_t \left[\hat{\tilde{R}}_{t+1}^k - \hat{R}_t \right] = \zeta_{sp,b} \left(\hat{q}_t^k + \hat{\bar{k}}_t - \hat{n}_t \right) + \tilde{\sigma}_{\omega,t} \tag{70}$$

can be expressed as a function of the entrepreneurs’ leverage (i.e., the ratio of the value of capital to nominal net worth) and exogenous fluctuations in the volatility of the entrepreneurs’ idiosyncratic productivity, $\tilde{\sigma}_{\omega,t} \equiv \zeta_{sp,\sigma_\omega} \hat{\sigma}_{\omega,t}$. The parameter $\zeta_{sp,b}$ is the elasticity of the spread with respect to leverage, and ζ_{sp,σ_ω} is the elasticity of the spread with respect to the volatility of the spread shock.

The *Entrepreneurs’ Net Worth,* $\hat{n}_t$, evolves according to

$$\hat{n}_t = \zeta_{n,\tilde{R}^k} \left(\hat{\tilde{R}}_t^k - \hat{\pi}_t \right) - \zeta_{n,R} \left(\hat{R}_{t-1} - \hat{\pi}_t \right) + \zeta_{n,qK} \left(\hat{q}_{t-1}^k + \hat{\bar{k}}_{t-1} \right) + \zeta_{n,n} \hat{n}_{t-1} - \gamma^e \frac{v_*}{n_*} \hat{z}_t - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}, \tag{71}$$

where $\zeta_{n,\tilde{R}^k}$, $\zeta_{n,R}$, ζ_{n,q^K}, $\zeta_{n,n}$, and ζ_{n,σ_ω} are the elasticities of net worth to the return on capital, the nominal interest rate, the cost of capital, net worth itself and the volatility σ_ω, respectively, and γ^e is the fraction of entrepreneurs who survive each period.

The evolution of the *Aggregate Nominal Wage* is then given by

$$\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \frac{1-\zeta_w}{\zeta_w}\hat{\tilde{w}}_t, \tag{72}$$

where ζ_w is the fraction of workers who cannot adjust their wages in a given period and $\hat{\tilde{w}}_t$ is the optimal wage chosen by workers that can freely set it, or optimal reset wage.

The *Optimal Reset Wage* follows

$$\begin{aligned}(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\tilde{w}}_t \;+\; (1+\zeta_w\beta\nu_l(\frac{1+\lambda_w}{\lambda_w}))\hat{w}_t =& \\ \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\mathbb{E}_t[\hat{\tilde{w}}_{t+1}+\hat{w}_{t+1}] \;+\; \hat{\varphi}_t \;+\; (1-\zeta_w\beta)(\nu_l\hat{L}_t-\hat{\xi}_t)& \\ +\; \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\mathbb{E}_t[\hat{\pi}_{t+1}+\hat{z}_{t+1}],&\end{aligned}$$

where $\hat{\varphi}_t$ is a stochastic preference shifter affecting the marginal utility of leisure and λ_w is the parameter that determines the elasticity of substitution between differentiated labor services.

The optimal price-setting decision yields a *Phillips Curve* equation

$$\hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p}\hat{mc}_t + \frac{1}{\zeta_p}\tilde{\lambda}_{f,t}, \tag{73}$$

where $\hat{\pi}_t$ is inflation, $\hat{mc}_t$ is nominal marginal cost, β is the discount factor, and ζ_p is the Calvo parameter, representing the fraction of firms that cannot adjust their prices each period. $\tilde{\lambda_{f,t}}$ is the following re-parametrization of the cost-push shock $\lambda_{f,t}$: $\tilde{\lambda_{f,t}} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)]\lambda_{f,t}$, where λ_f is the steady state value of the markup shock.

The *Marginal Cost (or labor share)* satisfies

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}^k_t, \tag{74}$$

where α is the output elasticity to capital and $\hat{r}^k_t$ is the capital rental rate.

The *Production Function* is given by

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)\hat{L}_t, \tag{75}$$

where the *Capital-Labor Ratio* satisfies

$$\hat{k}_t = \hat{w}_t - \hat{r}^k_t + \hat{L}_t. \tag{76}$$

The *Resource Constraint* is

$$\hat{y}_t = \hat{g}_t + \frac{c_*}{c_* + i_*}\hat{c}_t + \frac{i_*}{c_* + i_*}\hat{\imath}_t + \frac{r^k_* k_*}{c_* + i_*}\hat{u}_t, \tag{77}$$

where $\hat{y}_t$ is output and $\hat{g}_t$ is government spending.

Finally, the *Policy Rule* is

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left(\psi_\pi \sum_{j=0}^{3} \hat{\pi}_{t-j} + \psi_y \sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})\right) + \epsilon^R_t + \sum_{k=1}^{K} \epsilon^R_{k,t-k}, \tag{78}$$

where $\sum_{j=0}^{3} \hat{\pi}_{t-j}$ is 4-quarter inflation expressed in deviation from the Central Bank's objective π_* (which corresponds to steady state inflation), $\sum_{j=0}^{3} (\hat{y}_{t-j} - \hat{y}_{t-j-1} + \hat{z}_{t-j})$ is the 4-quarter growth rate of real GDP expressed in deviation from steady state growth, ϵ^R_t is the standard contemporaneous policy shock, and the terms $\epsilon^R_{k,t-k}$ are anticipated policy shocks, known to agents at time $t-k$.

A.3 The Exogenous Processes

The exogenous processes $\widehat{z}_t, \widehat{\varphi}_t, \tilde{\lambda}_{f,t}, \hat{\mu}_t, \tilde{\sigma}_{\omega,t}$ and $\hat{g}_t$ are assumed to follow AR(1) processes with autocorrelation parameters denoted by $\rho_z, \rho_\varphi, \rho_{\lambda_f}, \rho_\mu, \rho_{\sigma_\omega}$, and ρ_g, respectively. The innovations to these processes are structural shocks driving the model dynamics. They are assumed to be normally distributed with mean zero and a standard deviation denoted by $\sigma_z, \sigma_\varphi, \sigma_{\lambda_f}, \sigma_\mu, \sigma_{\sigma_\omega}$, and σ_g, respectively. The remaining structural shocks are the monetary policy shocks, both unanticipated, ϵ^R_t, and anticipated, $\epsilon^R_{k,t-k}$, all assumed *i.i.d.*

A.4 Measurement Equations

Real output growth (%, annualized)	$400(\ln Y_t - \ln Y_{t-1})$	$=$	$400(\widehat{y}_t - \widehat{y}_{t-1} + z_t)$
Hours (%)	$100 \ln L_t$	$=$	$100(L_t + \ln L^{adj})$
Labor Share (%)	$100 \ln LS_t$	$=$	$100(\widehat{L}_t + \widehat{w}_t - \widehat{y}_t + \ln LS_*)$
Inflation (%,annualized)	π_t^{Core}	$=$	$400(\widehat{\pi}_t + \ln \pi_*)$
Interest Rate (%,annualized)	FFR_t	$=$	$400(\widehat{R}_t + \ln R_*)$,
Spread (%,annualized)	SP_t	$=$	$400(\mathbb{E}_t \left[\widehat{\tilde{R}}^k_{t+1} - \hat{R}_t\right] + SP_*)$,

where the parameter L^{adj} captures the units of measured hours.

A.5 Prior and Posterior

Parameters	Prior Mean	Prior Stdd	Post Mean	90% Lower Band	90% Upper Band
Policy Parameters					
ψ_1	2.000	0.250	2.016	1.782	2.257
ψ_2	0.200	0.100	0.273	0.175	0.369
ρ_r	0.500	0.200	0.762	0.721	0.804
σ_r	0.200	4.000	0.152	0.132	0.170
Nominal Rigidities Parameters					
ζ_p	0.750	0.100	0.879	0.852	0.908
ζ_w	0.750	0.100	0.904	0.872	0.938
Other "Endogenous Propagation and Steady State" Parameters					
α	0.330	0.020	0.350	0.345	0.355
a''	0.200	0.100	0.294	0.129	0.444
h	0.700	0.050	0.704	0.636	0.770
ϕ''	4.000	1.500	3.121	2.247	3.994
ν_l	2.000	0.750	1.273	0.465	2.017
r^*	1.500	1.000	0.288	0.038	0.525
π^*	2.000	0.250	2.382	2.115	2.655
γ	2.750	0.500	1.687	1.307	2.063
g^*	0.300	0.100	0.195	0.090	0.300
ζ_{sp}	0.050	0.020	0.070	0.041	0.100
spr_*	2.000	0.500	1.163	0.750	1.556
ρs and σs					
ρ_z	0.400	0.250	0.487	0.369	0.605
ρ_ϕ	0.750	0.150	0.284	0.165	0.397
ρ_{λ_f}	0.750	0.150	0.470	0.364	0.572
ρ_μ	0.750	0.150	0.991	0.982	1.000
ρ_g	0.750	0.150	0.927	0.847	0.999
ρ_{σ_ω}	0.750	0.150	0.965	0.938	0.995
σ_z	0.300	4.000	0.788	0.695	0.877
σ_ϕ	3.000	4.000	29.403	12.878	45.636
σ_{λ_f}	0.200	4.000	0.087	0.072	0.101
σ_μ	0.750	4.000	0.358	0.271	0.450
σ_g	0.500	4.000	0.224	0.180	0.266
σ_{σ_ω}	0.050	4.000	0.085	0.076	0.095

Note: The following parameters are fixed: $\delta = 0.025$, $\nu_m = 2$, $\lambda_w = 0.3$, $\chi \; = 0.1$, $\lambda_f = 0.15$, $F(\omega) = 0.15$, $\gamma_* = 0.99$. L^{adj} has a prior mean of 253.500, with standard deviation at 5.

A.6 Anticipated Shocks to Condition on Interest-Rate Path

The following algorithm determines the time $T+1$ monetary policy shocks as a function of the desired interest rate sequence $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$ to generate predictions conditional on an announced interest rate path. The announced interest rate path will be attained in expectation. We denote by ϵ_t the vector that collects the innovations of the unanticipated shocks (both policy and non policy shocks), and by $\epsilon^R_{1:K,t}$ the vector of anticipated policy shocks.

Algorithm 1. Drawing Counterfactual Forecasts via Anticipated Shocks.[48]

1. Use the Kalman filter to compute the mean $s_{T|T}$ of the distribution $p(s_T|\theta, Y_{1:T})$.

2. Consider the following system of equations, omitting the θ argument of the system matrices:

$$\begin{aligned}
\bar{R}_{T+1} &= \Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T + \Psi_{R,2}\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\ \prime}_{1:K,T+1}]' \\
\bar{R}_{T+2} &= \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^2 s_T + \Psi_{R,2}\Phi_1\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\ \prime}_{1:K,T+1}]' \\
&\vdots \\
\bar{R}_{T+\bar{H}} &= \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T + \Psi_{R,2}(\Phi_1)^{\bar{H}-1}\Phi_\epsilon[\bar{\epsilon}^R_{T+1}, 0, \ldots, 0, \bar{\epsilon}^{R\ \prime}_{1:K,T+1}]'
\end{aligned} \tag{79}$$

   This linear system of $\bar{H}$ equations with $\bar{H}$ unknowns can be solved for for the vector of policy shocks $\bar{\epsilon}^R = [\bar{\epsilon}^R_{T+1}, \bar{\epsilon}^{R\prime}_{1:K,T+1}]'$. Specifically, rewrite the system (79) as

$$b = M_{\bar{H}}\bar{\epsilon}^R, \tag{80}$$

   where

$$\begin{aligned}
b &= [\bar{R}_{T+1}, ..., \bar{R}_{T+\bar{H}}]' - [\Psi_{R,1} + \Psi_{R,2}\Phi_1 s_T, ..., \Psi_{R,1} + \Psi_{R,2}(\Phi_1)^{\bar{H}} s_T]', \\
M_{\bar{H}} &= [\Psi_{R,2}, \Psi_{R,2}\Phi_1, ..., \Psi_{R,2}(\Phi_1)^{\bar{H}-1}]\Phi_{\epsilon,R},
\end{aligned} \tag{81}$$

   and $\Phi_{\epsilon,R}$ collects the columns of the matrix Φ_ϵ corresponding to the vector of policy shocks $\bar{\epsilon}^R$. The solution of (79) is then

$$\bar{\epsilon}^R = M^{-1}_{\bar{H}} b. \tag{82}$$

[48] The algorithm in Del Negro and Schorfheide (2013) describes how to draw from the entire counterfactual predictive distribution, conditional on draws of θ from the posterior density. Here we focus on the mode of the posterior density for θ.

3. Starting from $s_{T|T}$, iterate the state transition equation (4) forward to obtain a sequence $s_{T+1:T+H|T}$:

$$s_{t|T} = \Phi_1(\theta^{(j)})s_{t-1|T} + \Phi_\epsilon(\theta^{(j)})[\epsilon_t^R, 0, \ldots, 0, \epsilon_{1:K,t}^{R\,\prime}]', \quad t = T+1, \ldots, T+H,$$

where (i) $\epsilon_{T+1}^R = \bar{\epsilon}_{T+1}^R$ and $\epsilon_t^R = 0$ for $t = T+2, \ldots, T+\bar{H}$; (ii) $\epsilon_{1:K,T+1}^R = \bar{\epsilon}_{1:K,T+1}^R$ and $\epsilon_{1:K,t}^R = 0$ for $t = T+2, \ldots, T+\bar{H}$ (that is, in both cases use solved-for values in period $T+1$ and zeros thereafter).

4. Use the measurement equation (5) to compute $y_{T+1:T+H}$ based on $s_{T+1:T+H|T}$. □

A.7 Steady State of the SWBY Model

This Appendix characterizes the steady state of the SWBY model described in Section 4.2. Steady-state values are denoted by $*$-subscripts. As discussed in the main text, all nonstationary variables of the model are detrended by $Z_t = e^{\gamma + \frac{1}{1-\alpha}\tilde{z}_t}$.

Households

The detrended versions of expressions (26), (28), and (29) are respectively:

$$(1+\nu)c_t - \nu h e^{-z_t^*} c_{t-1} + I\!E_t\left[f_{t+1}s_{t+1}\right] = s_t + w_t^h - \mathtt{t}_t, \tag{83}$$

$$c_t - he^{-z_t^*}c_{t-1} = \frac{p}{1-p}\frac{(1-\beta(1-p))}{\beta(1+\nu)} I\!E_t\left[f_{t+1}s_{t+1}\right] + I\!E_t\left[\beta^{-1}f_{t+1}(c_{t+1} - he^{-z_{t+1}^*}c_t)\right], \tag{84}$$

$$w_t^h(1-L_t) = \nu(c_t - he^{-z_t^*}c_{t-1}), \tag{85}$$

At steady state, defining $\tilde{h} \equiv he^{-\gamma}$, these expressions reduce to

$$(1+\nu)c_* - \nu\tilde{h}c_* + f_*s_* = s_* + w_*^h - \mathtt{t}_*, \tag{86}$$

$$c_* = \frac{p}{1-p}\frac{(1-\beta(1-p))}{\beta(1+\nu)(1-\tilde{h})} f_*s_* + \beta^{-1}f_*c_*, \tag{87}$$

$$w_*(1-L_*) = \nu(1-\tilde{h})c_*. \tag{88}$$

Mutual Funds

The detrended version of equations (31) and (32) become:

$$k_t = u_t e^{-z_t^*}\bar{k}_{t-1}, \tag{89}$$

$$\bar{k}_t = (1-\delta)e^{-z_t^*}\bar{k}_{t-1} + \mu_t\left(1 - \phi(\frac{i_t}{i_{t-1}}e^{z_t^*})\right)i_t. \tag{90}$$

They deliver the steady-state relationships:

$$k_* = e^{-\gamma}\bar{k}_*, \tag{91}$$

$$i_* = \mu\left(1-(1-\delta)e^{-\gamma}\right)\bar{k}_*, \tag{92}$$

under the assumption that $\phi(e^{\gamma}) = 0$.

The detrended version of (35) becomes

$$1 = R_t b_t \mathbb{E}_t[f_{t+1}e^{-z^*_{t+1}}\pi^{-1}_{t+1}], \tag{93}$$

which implies at steady state:

$$R_* = f_*^{-1}\pi_* e^{\gamma}. \tag{94}$$

Equations (36), (37), and (38) become:

$$\begin{aligned} & q^k_t \mu_t \left(1 - \phi(\frac{i_t}{i_{t-1}}e^{z^*_t}) - \phi'(\frac{i_t}{i_{t-1}}e^{z^*_t})\frac{i_t}{i_{t-1}}e^{z^*_t}\right) \\ & + \mathbb{E}_t[e^{-z^*_{t+1}}f_{t+1}q^k_{t+1}\mu_{t+1}\phi'(\frac{i_{t+1}}{i_t}e^{z^*_{t+1}})(\frac{i_{t+1}}{i_t}e^{z^*_{t+1}})^2] = 1, \end{aligned} \tag{95}$$

$$q^k_t = \mathbb{E}_t\left[e^{-z^*_{t+1}}f_{t+1}\left(r^k_{t+1}u_{t+1} - a(u_{t+1}) + q^k_{t+1}(1-\delta)\right)\right], \tag{96}$$

$$r^k_t = a'(u_t). \tag{97}$$

Under the assumptions that $\phi'(e^{\gamma}) = 0$, $u_* = 1$ and $a(u_*) = 0$, the above equations imply at steady state

$$q^k_* = 1, \tag{98}$$

$$r^k_* = f_*^{-1}e^{\gamma} - (1-\delta), \tag{99}$$

$$r^k_* = a'(u_*), \tag{100}$$

where (98) implies $q^k_* = 1$ (note the $a(.)$ function can be normalized so to make $a'(1)$ be whatever the steady state r^k_* is).

Government

The detrended version of expression (39) is:

$$y_*\tilde{g}_t e^{-\frac{1}{1-\alpha}\tilde{z}_t} + \frac{b^g_{t-1}e^{-z^*_{t-1}}}{\pi_t} = \mathtt{t}_t + \frac{b^g_t}{b_t R_t}, \tag{101}$$

It follows that steady state taxes are:

$$\frac{\mathtt{t}_*}{y_*} = \tilde{g}_* + \left(1 - \frac{e^{z_*^*}}{r_*}\right) \frac{b_*^g}{y_*} \frac{e^{-z_*^*}}{\pi_*}. \tag{102}$$

Resource constraints

If the technology process is stationary, the resource constraint becomes:

$$y_* \tilde{g}_t e^{-\frac{1}{1-\alpha}\tilde{z}_t} + c_t + i_t + a(u_t)\bar{e}^{-z_t^*} k_{t-1} = y_t, \tag{103}$$

otherwise it becomes:

$$y_* g_t + c_t + i_t + a(u_t)\bar{e}^{-z_t^*} k_{t-1} = y_t. \tag{104}$$

Detrended output is also given as a function of inputs by:

$$y_t = k_t^\alpha L_t^{1-\alpha} - \Phi e^{-\frac{1}{1-\alpha}\tilde{z}_t}. \tag{105}$$

At steady state we have:

$$\frac{1}{1-g_*}(c_* + i_*) = y_*, \tag{106}$$

and

$$y_* = k_*^\alpha L_*^{1-\alpha} - \Phi. \tag{107}$$

A.7.1 Implied Steady-State Relationships

Fix L_* to some value (ν will rationalize that value). Define the real rate as

$$r_* = \frac{R_*}{\pi_*}, \tag{108}$$

and call $\tilde{\beta}$ the steady-state value of the stochastic discount factor (that is, $\tilde{\beta} = f_*$) Equation (94) defines the relationship between $\tilde{\beta}$ and the (gross) real rate r_*:

$$r_* = \tilde{\beta}^{-1} e^\gamma. \tag{109}$$

From (99), we have

$$r_*^k = r_* - (1-\delta). \tag{110}$$

The optimal pricing decision of intermediate goods produces implies:

$$w_* = \left(\frac{1}{1+\lambda_f}\alpha^\alpha (1-\alpha)^{(1-\alpha)} r_*^{k\ -\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{111}$$

and the cost minimization of intermediate firms implies:

$$k_* = \frac{\alpha}{1-\alpha}\frac{w_*}{r_*^k}L_*. \tag{112}$$

From (91) and (92):

$$\bar{k}_* = e^{\gamma}k_*, \tag{113}$$

$$i_* = \left(1-(1-\delta)e^{-\gamma}\right)\bar{k}_*. \tag{114}$$

From (107):

$$y_* = k_*^{\alpha}L_*^{1-\alpha} - \Phi. \tag{115}$$

SW use the reparameterization $\Phi_p = \frac{y_* + \Phi}{y_*}$, implying that steady state output is given by:

$$y_* = \frac{k_*^{\alpha}L_*^{1-\alpha}}{\Phi_p}. \tag{116}$$

From (106):

$$c_* = (1-g_*)y_* - i_*, \tag{117}$$

The ν that rationalizes the value of L_* is given by (118)

$$\nu = \frac{w_*}{c_*}\frac{(1-L_*)}{(1-\tilde{h})}. \tag{118}$$

Solving equation (87) for β as a function of $\tilde{\beta}$ and the wealth to consumption ratio $\frac{s_*}{c_*}$, we obtain

$$\beta = \left(1 + \frac{p}{(1+\nu)(1-\tilde{h})}\frac{s_*}{c_*}\tilde{\beta}\right)^{-1}\left(1 + \frac{1}{(1-p)}\frac{p}{(1+\nu)(1-\tilde{h})}\frac{s_*}{c_*}\right)\tilde{\beta}. \tag{119}$$

Expression (86) determines the wealth to consumption ratio given $\tilde{\beta}$ and steady-state taxes

$$\frac{s_*}{c_*} = (1-\tilde{\beta})^{-1}\left(1 - \frac{w_*}{c_*}L_* + \frac{\mathtt{t}_*}{c_*}\right), \tag{120}$$

where the steady-state taxes are given by (102):

$$\frac{\mathtt{t}_*}{y_*} = \tilde{g}_* + \left(1 - \frac{e^{\gamma}}{r_*}\right)\frac{b_*^g}{y_*}\frac{e^{-\gamma}}{\pi_*}. \tag{121}$$