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Abstract

This paper quantitatively evaluates the hypothesis that the housing bust in 2007 decreased geographical reallocation and increased the dispersion and level of unemployment during the Great Recession. We construct an equilibrium model of multiple locations with frictional housing and labor markets. When house prices fall, the amount of home equity declines, making it harder for homeowners to afford the down payment on a new house after moving. Consequently, the decline in house prices reduces migration and causes unemployment to rise differently in different locations. The model accounts for 90 percent of the increase in geographical dispersion of unemployment and the entire decline in net migration. However, despite large effects on migration and geographical dispersion of unemployment, the effect on aggregate unemployment is moderate: Our findings suggest that, absent the housing bust, aggregate unemployment would have been 0.5 percentage point lower.

Key words: geographical reallocation, housing bust, migration, unemployment

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1 Introduction

The unemployment rate in the United States increased from 5 percent in January 2007 to 10.1 percent in October 2009, as the economy experienced its deepest downturn in the postwar era. Equally important, but less well known and understood, is the fact that unemployment rates varied widely across locations. For example, the difference between the 90th and the 10th percentiles in the unemployment rate distribution more than doubled.[1] During the same period, following a sharp decline in house prices, the net migration rate declined by 50 percent to an all-time low.[2] In this paper, we develop a novel general equilibrium model of multiple locations with frictional housing and labor markets and use it to argue that the decline in the geographical reallocation of labor, triggered by the housing bust, caused the rise in unemployment dispersion across locations. We then use the model to measure the effect of the housing bust on aggregate unemployment during the Great Recession.

How and why does a decline in house prices affect geographical reallocation and the labor market? In this paper, we focus on a financial friction: the down payment requirement for purchasing a home. When house prices fall, the amount of home equity declines, making it harder for homeowners to afford the down payment on a new house after a move. To the extent that households care about owning a house, the decline in house prices affects their migration decisions. Some households that would normally move out of low-productivity regions may stay and look for jobs in distressed labor markets, further increasing local unemployment in those regions. Thus, the decline in geographical reallocation may cause unemployment to rise differently in different places. Furthermore, by increasing the fraction of population in low-productivity locations, the housing bust may cause aggregate unemployment to rise more.

Our economy consists of a finite number of locations populated by workers and firms. Each location has local labor and housing markets that are subject to search frictions and exogenous productivity shocks. Workers reside in different locations and may choose to move for a combination of idiosyncratic reasons and conditions in the labor and housing markets. Once in a location, workers may decide to purchase a home or remain as renters. To finance housing purchases, they can take on a mortgage after making a down payment from their savings. The difference between the two simulations quantifies the effect of the housing bust on migration and local and aggregate unemployment.

We then proceed to structurally estimating a two-location version of our model using the simulated method of moments. Specifically, we target national gross and net migration rates, homeownership rates,

[1]Throughout the paper, a location refers to a Metropolitan Statistical Area (MSA) in the United States. The average of the 90–10 differential across MSAs over the period 2000–2007 is 3 percent, whereas the average over the period 2008-2010 is 7 percent.

[2]Data on population flows are obtained from the IRS. For further details about the data used in this paper and the definitions, please see Appendix A.

median leverage, average time for selling a house, and aggregate statistics related to the labor market before the Great Recession. The calibrated model is quantitatively consistent with a range of other facts that are not explicitly targeted in the estimation. These include the correlation between leverage and mobility, the cyclicality of migration and unemployment dispersion, and the negative correlation between local unemployment and net flows.

We then use the model to study the role of the housing bust in the geographical dispersion of unemployment during the Great Recession. To that extent, we first group the Metropolitan Statistical Areas (MSAs) in the United States into two categories according to the decline in house prices during the housing bust. The decline in labor productivity is larger for the group with the larger housing bust. We isolate the impact of the housing bust by feeding into the model the observed declines in labor productivities for the two locations with and without the associated changes in house prices.

Our quantitative exercise shows that the model captures well the decline in migration rates during the Great Recession. We document that the net migration rate decreased from 0.8 percent in 2006 to 0.3 percent in 2009. In the model, we find that the housing bust and the recession resulted in a decline in net migration from a prerecession average of 0.8 percent to 0.2 percent. Moreover, the model predicts that, absent the housing bust, migration rates would have *increased*.[3] That increase occurs because the productivity shock is heterogeneous across locations, which raises the incentives to migrate out of the low-productivity location. The decline in house prices counteracts this force by decreasing the home equity of homeowners and making the down payment constraint a relevant friction. As households value owning over renting, many unemployed workers who would have otherwise migrated to the better location decide to stay. Quantitatively, the latter effect dominates the former and results in a decline in geographical reallocation.

The decline in migration due to "house-lock" has implications for the dispersion of unemployment across the two locations. In particular, by reducing the flow of unemployed workers out of the high-unemployment location, it drives a wedge between the two locations. Quantitatively, we find that the combination of house price and labor productivity shocks is able to generate almost 90 percent of the difference in unemployment rates between the two locations during the Great Recession.

It has been suggested that the decline in geographical mobility due to locked-in homeowners might be responsible for the sluggish performance of the labor market during the Great Recession.[4] The model

[3] Kaplan and Schulhofer-Wohl (2012) have shown that most of the decline in gross migration during the Great Recession is a consequence of a secular trend, implying a small cyclical component. Our findings suggest that the decline in migration observed during this period constitutes only a small portion of the effect of the housing bust.

[4] e.g. Sam Roberts reports in a *New York Times* article on April 23, 2009, "Experts said the lack of mobility was of concern on two fronts. It suggests that Americans were unable or unwilling to follow any job opportunities that may have existed around the country, as they have in the past. And the lack of movement itself, they said, could have an impact on the economy, reducing the economic activity generated by moves. ("Slump Creates Lack of Mobility for Americans.", http://www.nytimes.com/2009/04/23/us/23census.html)". Also, see Kocherlakota (2010).

developed in this paper enables us to quantify the aggregate impacts of a house price decline through its effects on geographical reallocation. Our counterfactual experiment suggests that the unemployment rate in the United States would have been 0.5 percentage points lower throughout the recession and during the recovery had it not been for the decline in house prices. Equivalently, the housing bust explains about 10 percent of the increase in unemployment during the Great Recession. It is worth mentioning that the estimated aggregate impact is smaller than one might expect, given the large drop in net migration and the sharp rise in the dispersion of unemployment. However, this finding can be easily reconciled by noting that the housing bust has *opposite* effects on the two locations: the low-productivity location has higher unemployment than in a recession without a housing bust because of the locked-in unemployed homeowners, whereas the other location has lower unemployment, thanks to the lack of the inflow of unemployed coming from the low-productivity location. Thus, it is not clear ex ante whether one should expect the reduced geographical reallocation to cause higher or lower level of aggregate unemployment. Our quantitative exercise suggests that while this mechanism results in a rise in aggregate unemployment, it is quantitatively small.

This paper is related to several strands of the literature. There is a large literature studying how regions react to adverse labor market shocks. In a seminal work, Blanchard and Katz (1992) have documented that the effect of an adverse shock on local unemployment is persistent in the short run and mean-reverting in the long run but that the effect on employment is permanent. They conclude that population flows are an important adjustment mechanism. Our paper builds on the premise that geographical reallocation is important for local economies and considers the effects of potential frictions for mobility on local and aggregate unemployment.

A growing literature studies the effects of housing equity, house-selling behavior, and mobility. Using data from the Boston condominium market in the 1990s, Genesove and Mayer (1997) document a positive relation between leverage and the posting price of houses. They find that a homeowner with a 100 percent leverage posts a 4 percent higher price compared to an otherwise similar homeowner with only 80 percent leverage, and it takes about 15 percent longer for a highly leveraged homeowner to sell the housing unit. Consistent with their findings, our model successfully captures the relationship between leverage and the time to sell. Using mortgage data from New Jersey, Chan (2001) finds that declining house prices significantly reduce the mobility rates of homeowners, in particular for those with a high loan-to-value ratio. More recently, using the American Housing Survey, Schulhofer-Wohl (2011) finds no relationship between leverage and residential mobility, whereas Ferreira et al. (2010, 2012) find that homeowners with negative equity move 30 percent less compared to homeowners with positive equity.

On the theoretical front, our model builds on the island framework of Lucas and Prescott (1974) and Alvarez and Shimer (2011). Head and Lloyd-Ellis (2012) is one of several recent papers studying the interactions among the housing market, migration, and the labor market. They study a model of multiple locations with search frictions in housing and labor markets and show that the illiquidity of housing can generate differences in unemployment rates and homeownership rates. Our paper is different from theirs, as we focus on the role of mortgage leverage in explaining population flows during the Great Recession, whereas they analyze a stationary environment with no assets. This approach requires deviation from a steady-state analysis by incorporating location-specific productivity shocks and allowing for asset accumulation.

A number of recent papers study the role of housing in labor reallocation. Nenov (2012) builds a multiregional economy with a fixed supply of housing and uses it to study the effects of lower mobility on labor market outcomes. In his model, the migration rate is determined by the exogenous fraction of immobile households. Our paper models the determinants of the decline in mobility and is thus able to isolate the relevance of the frictions coming from the housing market. Davis et al. (2010) build a model with a continuum of locations and use it to study the role of moving costs and inelastic housing supply on shaping the character and extent of labor reallocation in the United States.

Recently, several papers have studied the effect of the house-lock on aggregate unemployment. Sterk (2011) uses a business cycle model with a down payment requirement. In his model, a constant fraction of job offers requires households to move and buy a new house. Falling house prices make the housing transaction undesirable, reducing mobility and resulting in higher aggregate unemployment. Unlike in Sterk (2011), we explicitly model local labor markets. This modeling allows us to endogenize the importance of migration for aggregate unemployment in an environment with heteregeneous labor productivity shocks. Our findings suggest that, absent housing market related frictions, migration would have increased in response to local productivity differences. This increase implies that geographical reallocation is more important for the labor market during the Great Recession. Valletta (2012) investigates the differences in unemployment durations between homeowners and renters across geographic areas differentiated by the severity of the decline in home prices and concludes that the effect of the house-lock on unemployment has been small. Sahin et al. (2012) develop a measure of mismatch and quantify the extent of unemployment caused by mismatch across industries and locations during the Great Recession. They find that mismatch can account for 0.6 to 1.7 percentage points of the aggregate unemployment rate during the Great Recession but that most of the increase in structural unemployment is sectoral.

Finally, this paper is part of a recent literature that employs directed search models of the labor market to study economies with heterogeneity and aggregate shocks (e.g., Menzio and Shi (2010b,a, 2011); Schaal (2012); Kaas and Kircher (2011)). In our model, both the housing and the labor market is modeled with directed search. The use of directed search enables us to compute a *block-recursive* equilibrium of the model; that is, a particular recursive equilibrium, in which the endogenous distributions generated within the model are not part of the state space. Along this dimension, most closely related to us is Hedlund (2012), who develops a directed search model of the housing market and studies the implications of search frictions on house price dynamics.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 provides the details of our estimation and the model's fit along the targeted and untargeted dimensions of the data. Section 4 uses the model as a measurement tool to quantify the effect of the housing bust on the labor market. Finally, Section 5 concludes.

2 A Model of Labor and Housing Markets and Geographical Reallocation

In this section, we present a model of geographical reallocation. To illustrate the relationship between households' leverage and migration decision, we adopt a directed search model of the housing market. Facing a trade-off between the posting price and the time it takes to sell a house, households decide the posting price for their houses. As a by-product of directed search, we are able show that our model admits a block-recursive equilibrium: that is, a particular recursive equilibrium, in which the endogenous distributions generated within the model are not part of the state space.

2.1 Agents and Markets

The economy consists of a finite number of locations indexed by $i \in I$. There is a continuum of households of measure 1 located in different locations. There is a continuum of firms, housing market intermediaries (real estate managers and leasing companies), and construction companies with a positive measure. Time is discrete and continues forever: $t = 0, 1, 2, \cdots$.

Households: Households are ex ante identical and have a periodical utility function given by $u(c, l, h, \chi_i)$,

$$u : \mathbb{R}_+ \times \{l_0, l_1\} \times \{h_0, h_1\} \times [\underline{\chi}, \overline{\chi}] \to \mathbb{R},$$

defined over consumption (c), leisure (l), housing status (h), and preference for their current residence (χ_i). Leisure takes a value of l_0 or l_1 where $l_0 < l_1$, denoting whether the household is employed or unemployed, respectively. Similarly, housing status takes values h_0 or h_1 with $h_0 < h_1$, denoting whether the household is a renter or a homeowner, respectively. Households decide where to live and work, whether to purchase a house or live in a rented one and how much to save and consume. Each household maximizes the expected sum of periodical utilities discounted at the discount rate $\beta \in (0,1)$.

Firms and the Labor Market: Each firm operates a constant returns-to-scale technology that, if matched with a worker, turns one unit of labor into z_i units of consumption. The labor productivity z_i is the same for all firms in a given location but can be different across locations. Labor productivity in a location follows a Markov process and takes values in $Z = \{z_1, z_2, \cdots, z_N\}$, according to the transition matrix Υ_Z. At the beginning of each period, the state of the economy, ψ, is given by

$$\psi = (\{z_i\}_{i\in I}, \{n_i\}_{i\in I}, \{\Gamma_i\}_{i\in I}).$$

The first element of ψ denotes labor productivities at each location; n_i is the fraction of population in location i, and $\Gamma_i : \mathbb{R} \times \{h_0, h_1\} \times \mathbb{R}_+ \times \{l_0, l_1\} \times [\underline{\chi}, \overline{\chi}]^I \to [0,1]$ is a function denoting the measure of households in location i over assets, housing tenure, wages, employment status, and location preference.

Households and firms meet and produce output in a frictional labor market. The labor market is organized along a continuum of submarkets that differ in the wage contract that is offered. More specifically, we allow only for fixed-wage contracts. When a firm meets a worker in submarket w, the firm offers the worker an employment contract that pays the worker a wage of w every period until the match ends exogenously with probability δ. Firms and workers fully commit to this contract. Consequently, each submarket can be indexed by the wage offered w.

Search in the labor market is directed: firms choose the wage to offer to potential workers and the number of vacancies to post. Each vacancy requires the payment of a posting cost, k. Similarly, households decide in which submarket to look for jobs. We denote by $\theta_l^i(w;\psi)$ the market tightness of submarket w—the ratio of the number of vacancies created by firms in submarket w to the number of workers that are looking for jobs in the same submarket in location i. Once in a submarket, workers find jobs with probability $\pi_l[\theta_l^i(w;\psi)]$, and firms find workers with probability $q_l[\theta_l^i(w;\psi)] = \pi_l[\theta_l^i(w;\psi)]/\theta_l^i(w;\psi)$.

Housing Market Intermediaries and the Structure of the Housing Market: There are three types of companies in the housing market: construction companies, real estate managers (REMs), and leasing companies. Construction companies operate a constant returns-to-scale technology that turns μ_i units of the consumption good into one unit of housing in location i. Newly constructed houses can be sold to households or to leasing companies to be used as rental units. μ_i is assumed to be constant over time. We model the housing bust as an unexpected decrease in construction costs.

Housing transactions are facilitated by REMs in that they buy houses from sellers and sell them to buyers. In both of these markets, households and REMs meet in a frictional housing market. Similar to the labor market, the housing market consists of a continuum of submarkets. Each submarket is characterized by the transaction price of the house, p.

Search is directed in the housing market. Renters that would like to buy a house decide on the price at which they are willing to buy and look for a house in that submarket. There is a down payment requirement to purchasing a house: households are allowed to buy a house at price p only if their assets suffice to cover α fraction of the house price; i.e., $a \geq \alpha p$. REMs with a house decide on the selling price and post a vacancy accordingly. We define the market tightness of submarket p, $\theta_b^i(p;\psi)$, as the ratio of vacancies posted by REMs at price p to the number of households that are looking for a house at this price. Once in a submarket, households meet an REM and buys a house with probability $\pi_b[\theta_b^i(p;\psi)]$. The probability for a REM of meeting a household is given by $q_b[\theta_b^i(p;\psi)] = \pi_b[\theta_b^i(p;\psi)]/\theta_b^i(p;\psi)$. The directed nature of the search ensures that upon meeting a REM, a household is willing to buy the house at price p.

On the other side of the housing market, homeowners that would like to sell their house choose a selling price p. REMs decide the price to buy and look for sellers that are willing to sell at that price. The market tightness on this side of the housing market is denoted by $\theta_s^i(p;\psi)$. The probability of a trade for a seller is given by $\pi_s[\theta_s^i(p;\psi)]$, and the probability of a trade for the REM is given by $q_s[\theta_s^i(p;\psi)] = \pi_s[\theta_s^i(p;\psi)]/\theta_s^i(p;\psi)$.

Leasing companies buy houses from construction companies and turn them into rental houses at no cost. They then rent them out to households for one period to obtain a rent of $\rho^i(\psi)$. The market for rental units is perfectly competitive. Finally, rental units depreciate: a rental unit disappears every period with probability γ .

Financial Markets: Financial markets are incomplete. Households can save and borrow using a risk-free bond. The risk-free bond yields a constant interest rate r. When borrowing, homeowners and renters face (exogenously) different borrowing limits $\underline{a}_1$ and $\underline{a}_0$, where $\underline{a}_1$ is the borrowing limit of homeowners and $\underline{a}_0$ is the borrowing limit of renters. The borrowing limit is tighter for renters, $\underline{a}_0 > \underline{a}_1$.

Renters may use a mortgage to buy a house. As mentioned previously, households can purchase a house at price p, as long as their assets are larger than αp. The portion of the house price that is not paid at the time of purchase is borrowed at interest rate r. Households can then roll over their debt by paying the interest only or lower their balance by paying more every period. The details of the mortgage arrangement will be further explained in Section 2.3.

2.2 Timing of Events

The introduction of assets into a search model with aggregate shocks increases the dimensionality of workers' and firms' problems. In principle, the aggregate state includes the labor productivity shocks across locations, $\{z_i\}_{i\in I}$, and the population distribution, as well as the distribution of employment, wages, assets, preference shocks, and housing status within each location. The latter is critical because one must keep track of an infinite dimensional object in the state space, rendering the dynamics of the model computationally intractable. Fortunately, the structure of the model gives rise to a block-recursive equilibrium, an equilibrium in which firms' and workers' problems are independent of the distribution. We present the model in its general form and allow the distribution to be part of the state space. We then discuss, in the next section, what conditions give rise to this property.

Each period is divided into five stages: job separations, housing market transactions, migration, search in the labor market, and production. During the separation stage, employed households exogenously move into unemployment with probability δ.

After shocks are realized, housing markets open. If a homeowner wants to sell his house, he chooses the price to sell and looks for a buyer. If the homeowner successfully sells his unit, he becomes a renter for one period but may enter the housing market in the next period. If a renter wants to purchase a house, he chooses at what price to look for a house and visits the corresponding submarket to find a seller.

Upon completing housing transactions, households decide whether to remain in their current location or to move to another place. Unemployed renters and unemployed homeowners that sold their houses within this period decide whether and where to move.

Following the migration stage, labor markets open. Firms post vacancies in different submarkets, and unemployed households choose in which submarket to look for a job. Job search is local: only residents of location i are allowed to apply for vacancies posted in location i.

During the production stage, an unemployed household collects b units of the consumption good as unemployment benefits. Employed households in location i produce z_i units of output and are paid their wage w. Households then decide on their consumption and savings, and renters pay out the location-specific rent $\rho^i(\psi)$.

2.3 The Problem of the Household

In this section, we present the Bellman equations that govern the decision problems of households. The value functions are measured at the beginning of the consumption-savings stage—the last stage in a period. We use auxiliary value functions to denote the value functions at the job search stage and define them when necessary. We first consider the problem of an unemployed renter.

2.3.1 The Consumption-Savings Problem

We start by describing the problem at the consumption-savings stage. The search problem of buyers and sellers in the housing market is also explained in this section. We then turn to the search problem in the labor market.

Unemployed Renters: Equation (1) presents the problem of an unemployed renter at the consumption-savings stage

$$\begin{aligned} U^i(a, h_0, \chi; \psi) = & \max_{c,a'} u(c, l_1, h_0, \chi_i) \\ & + \beta \mathbb{E} \left\{ \max_{p,\, a' \geq \alpha p} \pi_b(p; \psi') D^i(a' - p, h_1, \chi'; \psi') + [1 - \pi_b(p; \psi')] \max_{j \in I} \left\{ D^j(a', h_0, \chi'; \psi') \right\} \right\} \\ & a + b = c + \rho^i(\psi) + \frac{a'}{1+r} \\ & a' \geq a_0. \end{aligned} \tag{1}$$

Here, $U^i(a, h_0, \chi; \psi)$ is the value of being unemployed in location i to a renter (h_0) with assets a and an $I \times 1$ vector of preference for locations $\chi \equiv \{\chi_i\}_{i \in I}$. The household chooses current consumption (c) and savings (a'), subject to the budget constraint that we present and discuss below, and obtains an instantaneous utility of $u(c, l_1, h_0, \chi_i)$ and goes to the next period. At the beginning of the next period, housing markets open. Since he is a renter, he has the option of purchasing a house. He chooses the purchasing price p, subject to the down payment constraint $a' \geq \alpha p$. For that price, his expected payoff is

$$\pi_b(p; \psi') D^i(a' - p, h_1, \chi'; \psi') + [1 - \pi_b(p; \psi')] \max_{j \in I} \left\{ D^j(a', h_0, \chi'; \psi') \right\} \tag{2}$$

The first term reflects the fact that he finds a seller with probability $\pi_b(p; \psi')$ and obtains the ownership of the house upon paying the transaction price. In that case, his assets are given by $a' - p$. He enters the migration stage as a homeowner and is not allowed to move. After the migration stage comes the job search stage. He looks for a job in his current location i. This delivers an expected payoff of $D^i(a' - p, h_1, \chi'; \psi')$. The value function D denotes the value of searching for a job in location i and will be defined below. The second term in equation (2) reflects the fact that the household does not find a seller with complementary probability. In that case, he is still a renter and can choose to move.

We now turn to the budget constraint facing the unemployed renter. His resources for the period are given by his assets and the unemployment benefit he collects. He uses these to finance current consumption c, rent $\rho^i(\psi)$, and savings. He can borrow and save at interest rate r, subject to the borrowing constraint that $a' \geq a_0$.

Unemployed Homeowners: Equation (3) shows the problem of an unemployed homeowner:

$$\begin{aligned} U^i(a, h_1, \chi; \psi) = &\max_{c,a'} u(c, l_1, h_1, \chi_i) \\ &+ \beta \mathbb{E}\left\{ \max_p \pi_s(p; \psi') \max_{j \in I} \left\{ D^j(a' + p, h_0, \chi'; \psi') \right\} + [1 - \pi_s(p; \psi')] D^i(a', h_1, \chi'; \psi') \right\} \\ &a + b = c + \frac{a'}{1 + r} \\ &a' \geq a_1. \end{aligned} \tag{3}$$

Here, $U^i(a, h_1, \chi; \psi)$ is the value of being unemployed in location i to a homeowner with assets a and preference for locations χ. The homeowner chooses current consumption (c) and savings (a'), subject to the budget constraint, and obtains an instantaneous utility of $u(c, l_1, h_1, \chi_i)$ and goes to the next period. At the beginning of the next period, housing markets open. The unemployed homeowner has the option of selling the house. He decides the selling price p that then determines the probability of finding a buyer. For that price, the expected payoff is given by:

$$\max_p \pi_s(p; \psi') \max_{j \in I} \{D^j(a' + p, h_0, \chi'; \psi')\} + [1 - \pi_s(p; \psi')] D^i(a', h_1, \chi'; \psi'). \tag{4}$$

The first term reflects the fact that the household transfers the ownership of the house to a realtor with probability $\pi_s(p; \psi')$ and receives the payment p. In that case, the household's assets are given by $a' + p$. Consequently, the household enters the migration stage as a renter and decides whether to move to another location or remain in the current location. Similar to the renter's problem, $\max_{j \in I} D^j(a' + p, h_0, \chi'; \psi')$ measures the value of looking for a job on another location. The second term in equation (4) reflects the fact that the household does not find a buyer with complementary probability. In that case, as a homeowner, the household is not allowed to migrate. As a consequence, he searches for an employer in the current location i and obtains a value of $D^i(a', h_1, \chi'; \psi')$.

Equation (4) highlights the option value of migration. By selling the house, the homeowner obtains this option value. Clearly, the more the household wants to migrate, the sooner he would like to sell the house. As a result, differences between labor markets and a homeowner's preference for his current location affect the price-posting decision.

Employed Renters: We now turn to the problem of employed households. We start by describing the decision problem of an employed renter. Equation 5 shows the Bellman equation for an employed renter in location i:

$$
\begin{aligned}
& W^i\left(w, a, h_0, \chi ; \psi\right) \qquad (5)\\
= & \max_{c, a'} u(c, l_0, h_0, \chi_i) + \beta \mathbb{E}\Bigg\{ (1-\delta)\left(\max_{a' \geq \alpha p} \pi_b(p;\psi') W^i(a'-p, h_1, \chi'; \psi') + [1-\pi_b(p;\psi')] W^i(a', h_0, \chi'; \psi') \right) \\
& \qquad + \delta\left(\max_{a' \geq \alpha p} \pi_b(p;\psi') D^i(a'-p, h_1, \chi'; \psi') + [1-\pi_b(p;\psi')] \max_{j \in I} \left\{ D^j(a'-p, h_1, \chi'; \psi') \right\} \right) \Bigg\} \\
& \qquad a + w = c + \rho^i(\psi) + \frac{a'}{1+r} \\
& \qquad a' \geq a_0 .
\end{aligned}
$$

Here, $W^i\left(w, a, h_0, \chi ; \psi\right)$ is the value of being employed at wage w in location i to a renter (h_0) with assets a and preference for locations χ. The household chooses current consumption (c) and savings (a'), subject to the budget constraint, and obtains an instantaneous utility of $u(c, l_0, h_0, \chi_i)$ and goes to the next period. At the beginning of the next period, job destruction shock δ and productivity shocks are realized. The second line in equation (5) captures the event that the employed renter keeps his job. The household has the option of buying a house in the housing market. The renter decides the buying price p (subject to the down payment constraint) that then determines the probability of finding a seller. For that price, the expected payoff is given by:

$$
\pi_b(p;\psi') W^i(a'-p, h_1, \chi'; \psi') + [1-\pi_b(p;\psi')] W^i(a', h_0, \chi'; \psi'). \qquad (6)
$$

Here, the first term measures the payoff associated with buying the house: upon finding a seller, the buyer pays the house price and becomes an employed renter. With complementary probability $1-\pi_b(p;\psi')$, he does not find a seller and remains an employed renter. Being employed, he is not allowed to migrate or look for another job and skips these two stages to obtain a value of $W^i(a'-p, h_0, \chi'; \psi')$.

The last two lines in equation (6) capture the event that the renter loses his job and becomes unemployed. For this household, the rest of the problem looks very similar to that of an unemployed renter: the renter decides whether to purchase a house or not. In the case of a successful purchase, he searches for a job locally. In the other case, he decides whether to move or not and then looks for a job. The budget constraint facing an employed renter is very similar to the one facing an unemployed renter, the difference being the labor income w instead of the unemployment benefits b.

Employed Homeowners: Equation (7) shows the Bellman equation for an employed owner in location i:

$$
\begin{aligned}
&W^i(w, a, h_1, \chi; \psi) &(7)\\
&= \max_{c,a'} u(c, l_0, h_1, \chi_i) + \beta\mathbb{E}\Bigg\{ (1-\delta)\Big(\max \pi_s(p; \psi')W^i(a'+p, h_0, \chi'; \psi') + [1-\pi_s(p;\psi')]W^i(a', h_1, \chi'; \psi')\Big) \\
&\qquad + \delta\Big(\max_p \pi_s(p;\psi') \max_{j\in I}\{D^j(a'+p, h_0, \chi'; \psi')\} + [1-\pi_s(p;\psi')]D^i(a', h_1, \chi'; \psi')\Big)\Bigg\} \\
&\qquad a + w = c + \frac{a'}{1+r} \\
&\qquad a' \geq a_1
\end{aligned}
$$

Here, $W^i(w, a, h_1, \chi; \psi)$ is the value of being employed at wage w in location i to a homeowner with assets a and preference for locations χ. The household chooses current consumption (c) and savings (a'), subject to the budget constraint, and obtains an instantaneous utility of $u(c, l_0, h_1, \chi_i)$ and goes to the next period. At the beginning of the next period, job destruction and productivity shocks are realized. The second line in equation (7) captures the event that the homeowner keeps his job. He has the option of selling the house in the housing market. Selling delivers a payoff of $W^i(a'+p, h_0, \chi'; \psi')$ and not selling delivers a payoff of $W^i(a', h_1, \chi'; \psi')$. If the homeowner becomes unemployed, by setting a selling price p, he may try to sell the house and get the option value of migration ($\max_{j\in I}\{D^j(a'+p, h_0, \chi'; \psi')$) or not sell it and get a payoff of $D^i(a', h_1, \chi'; \psi')$.

2.3.2 The Job Search Problem

So far, we have described the problem of employed and unemployed homeowners and renters at the consumption and savings stage as well as in the housing market. Here, we describe the search problem of households in the labor market. Recall that, by assumption, only unemployed households are allowed to search for jobs and that job search is local: households in a given location can apply only for vacancies in the same location.

Compared to a random-matching technology, where there is a single market tightness in the labor market $\theta(\psi)$, there is a continuum of wages and corresponding market tightnesses in this model, due to the directed nature of search. Households decide in which submarket (at what wage) to look for jobs. Submarkets are indexed by the fixed wage w, and the market tightness in this submarket is given by $\theta_l^i(w; \psi)$. Correspondingly, $\pi_l\left[\theta_l^i(w; \psi)\right]$ denotes the probability of a worker's finding a job as a function of the applied wage w.

The value of searching in the local labor market for a household with assets a and housing status h is given by

$$D^i\left(a,h,\chi;\psi\right)=\max_w \pi_l\left[\theta_l^i\left(w;\psi\right)\right]W^i\left(w,a,h,\chi;\psi\right)+\left(1-\pi_l\left[\theta_l^i\left(w;\psi\right)\right]\right)U^i(a,h,\chi;\psi). \tag{8}$$

Policy Functions: We now introduce the notation for optimal policy rules, since they will be used in the definition of a recursive equilibrium. The optimal rule for the savings decision of employed and unemployed households is denoted by g_W^i and g_U^i, respectively. The optimal house-buying price is denoted by p_b^i, and the optimal house-selling price is denoted by p_s^i. The optimal migration decision is denoted by m^i. Finally, we denote the optimal solution to the job search problem in equation (8) by w^i.

2.4 The Problem of the Firm

We now turn to firms in the labor market. Firms post vacancies to hire workers. Each vacancy lasts for one period. Recall that the job search is directed, so that when a firm decides to post a vacancy, it also decides in which submarket to post it. Our contract space allows only for fixed-wage contracts; therefore, vacancies are indexed by the offered wages w. The value to the firm of being matched with a worker and paying wage w in location $i \in I$ can be written as:

$$J^i\left(w;\psi\right)=z_i-w+\frac{1-\delta}{1+r}\mathbb{E}J^i\left(w;\psi'\right). \tag{9}$$

Posting a vacancy requires the payment of cost k. The value of creating a vacancy in location k with wage w is given by

$$V^i\left(w;\psi\right)=-k+q_l\left[\theta_l^i\left(w;\psi\right)\right]J^i\left(w;\psi\right), \tag{10}$$

where q_l denotes the probability of finding a worker at wage w and is a function of the labor market tightness $\theta_l^i\left(w;\psi\right)$. When the value of creating one vacancy at wage w is strictly positive, the firm finds it optimal to create infinite vacancies. When it is strictly negative, no vacancies are created in submarket w. When the value is zero, then the firm's profit is independent of the number of vacancies it creates in submarket w.

We assume free entry of firms. Therefore, in any submarket visited by a positive measure of workers, the following must hold:

$$k \geq q_l\left[\theta_l^i\left(w;\psi\right)\right]J^i\left(w;\psi\right), \tag{11}$$

with complementary slackness. That is, equation (11) must hold with equality if $\theta_l^i\left(w;\psi\right)>0$. When we focus on block-recursive equilibrium, we will focus on equilibria that have a positive number of entrants every period.

2.5 The Problem of Housing Market Intermediaries

Our work borrows from the insights of Menzio and Shi (2010a) and extends the notion of *block recursivity* to the housing market. In what follows, we will describe the structure of the housing market that gives rise to the existence of such an equilibrium. As we will see in the next section, this equilibrium requires a combination of directed search and free-entry conditions in every market. The introduction of the housing market intermediaries makes the existence possible. We have three types of firms in the housing market: real estate managers (REM), leasing companies, and construction companies.

REMs with a vacant house try to sell it to buyers. They get a payoff of p when they succeed in selling a house at price p but get no flow payoff from having vacant houses. Therefore, the value of holding a vacant house in location $i \in I$ to a real estate manager is

$$R^i(\psi) = \max_p q_b \left[\theta_b^i\,(p;\psi)\right] p + \left(1 - q_b \left[\theta_b^i\,(p;\psi)\right]\right) \frac{1}{1+r} \mathbb{E} R^i\,(\psi')\,, \tag{12}$$

where $\theta_b^i\,(p;\psi)$ is the market tightness for the housing submarket with price p. The subscript b indicates that this is the side of the housing market where households are buyers. Equation (12) holds that REMs choose the price p at which they are willing to sell the house and are successful in doing so with probability $q_b\left[\theta_b^i\,(p;\psi)\right]$. They cannot find a buyer with complementary probability and the house remains for one period.

We now turn to the other side of the housing market. In order to buy houses from sellers in the housing market, REMs post vacancies by paying a cost κ. As in the other markets, the search is directed so that when REMs decide to post vacancies, they also decide the price at which they are willing to buy. There is full commitment to the posted price, so that whenever a REM meets a homeowner, the housing unit is transferred to the REM at price p. The value of posting a vacancy for a REM in location $i \in I$ at price p is given by

$$Q^i\,(p;\psi) = -\kappa + q_s \left[\theta_s^i\,(p;\psi)\right] \left[R^i(\psi) - p\right]. \tag{13}$$

We assume free entry of REMs. Therefore, a free-entry condition similar to (11) holds for all the submarkets in the selling market that are visited by a positive measure of homeowners. This is given by

$$\kappa \geq q_s \left[\theta_s^i\,(p;\psi)\right] \left[R^i(\psi) - p\right], \tag{14}$$

with equation (14) holding with equality whenever $\theta_s^i\,(p;\psi) > 0$.

Leasing companies operate in a competitive rental market. The rental contract is for one period. At the end of every period, a constant fraction γ of rental houses depreciates. Depreciation is discrete: that is, these rental houses are completely destructed. Leasing companies get the rental rate, $\rho^i(\psi)$, as a flow payoff, until the unit disappears. Thus, the value of holding a rental house to leasing companies is given by

$$L^i(\psi) = \rho^i(\psi) + \frac{1-\gamma}{1+r}\mathbb{E}L^i(\psi'). \tag{15}$$

Depreciation is important for ensuring that in every period new houses are built, which in turn is important to making the free-entry conditions hold with equality. We elaborate on this issue when we discuss the existence of a block-recursive equilibrium.

Finally, we turn to construction companies. Construction companies can build a new house immediately at cost μ_i. They then have the choice of becoming REMs and trying to sell these units to renters or of becoming leasing companies and renting out the house. As long as the value of holding a house to a REM exceeds the cost of constructing a new one, there will be new construction. This setup introduces two additional free-entry conditions:

$$\mu_i \geq R^i(\psi) \tag{16}$$

$$\mu_i \geq L^i(\psi) \tag{17}$$

2.6 Equilibrium

We now define a recursive equilibrium for this economy. We denote the set of housing service types as $\mathcal{H} = \{0, 1\}$ and the set of locations as $I = \{1, 2, ...\bar{I}\}$. The set of local productivity shocks is given by $Z = [\underline{z}, \overline{z}]$. Define $\mathcal{W} = [b, z_N]$ to be the set of possible wages and $\mathcal{A} = [\underline{a}_1, \bar{a}]$ to be the set of possible assets.[5] Let Ξ denote the set of preference shocks. Finally, let Ψ denote the possible realizations of the aggregate state.

Definition 1. A *recursive equilibrium* comprises

- a set of value functions for households: $\Big\{W^i : \mathcal{W} \times \mathcal{A} \times \mathcal{H} \times \Xi \times \Psi \to \mathbb{R}$, $U^i : \mathcal{A} \times \mathcal{H} \times \Xi \times \Psi \to \mathbb{R}$, $D^i : \mathcal{A} \times \mathcal{H} \times \Xi \times \Psi \to \mathbb{R}\Big\}_{i \in I}$,
- a set of policy functions for households: $\Big\{g^i_W : \mathcal{W} \times \mathcal{A} \times H \times \Xi \times \Psi \to \mathbb{R}$, $g^i_U : \mathcal{A} \times H \times \Xi \times \Psi \to \mathbb{R}$, $p^i_b : \mathcal{W} \times \mathcal{A} \times H \times \Xi \times \Psi \to \mathbb{R}_+$, $p^i_s : \mathcal{W} \times \mathcal{A} \times H \times \Xi \times \Psi \to \mathbb{R}_+$, $m^i : \mathcal{A} \times \Xi \times \Psi \to \mathbb{R}_+$, and $w^i : \mathcal{A} \times H \times \Xi \times \Psi \to \mathbb{R}_+\Big\}_{i \in I}$,

[5] It is easy to prove that there are endogenous bounds on the set of possible wages that will be offered and thus on the set of assets that will be realized in equilibrium. The assumption of bounded sets is, in that sense, not an assumption but a result.

- value functions for firms: $\left\{J^i : \mathcal{W} \times \Psi \to \mathbb{R}\right\}_{i \in I}$
- value functions for intermediaries in the housing market: $\left\{R^i : \Psi \to \mathbb{R},\ L^i : \Psi \to \mathbb{R}\right\}_{i \in I}$
- market tightness functions in the labor market, $\left\{\theta_l^i : \mathcal{W} \times \Psi \to \mathbb{R}_+\right\}_{i \in I}$,
- market tightness functions in the housing market $\left\{\theta_s^i : \mathbb{R}_+ \times \Psi \to \mathbb{R}_+,\, \theta_b^i : \mathbb{R}_+ \times \Psi \to \mathbb{R}_+\right\}_{i \in I}$,
- a transition probability function for the aggregate state of the economy $\Phi : \Psi \times \Psi \to [0, 1]$; such that

1. **Households maximize:** Given the market tightness functions, the value functions solve (1), (3), (5), (7), and (8), and $\{g_W^i\}_{i \in I}, \{g_U^i\}_{i \in I}, \{p_b^i\}_{i \in I}, \{p_s^i\}_{i \in I}, \{m^i\}_{i \in I}$ and $\{w^i\}_{i \in I}$ are the associated policy functions.
2. **Firms and housing market intermediaries maximize:** $\{J^i\}_{i \in I}$ solves 9, and $\{R^i\}_{i \in I}$ and $\{L^i\}_{i \in I}$ satisfy (12) and (15), respectively.
3. **Free entry of firms:** Given the value function of firms $\{J^i\}_{i \in I}$, the market tightness function $\{\theta_l^i\}_{i \in I}$ satisfies (11).
4. **Free entry of real estate managers:** Given the value function of housing intermediaries, (14) holds.
5. **Free entry of construction companies:** (16) and (17) are satisfied.
6. **Law of motion for the aggregate state space:** Φ is derived from the policy functions of households and the transition function of the productivity shocks.

2.7 Existence of a Block-Recursive Equilibrium

Solving a recursive equilibrium outside of the steady state requires solving functional equations in which the functions depend on the entire distribution of workers across locations, assets, employment states, housing tenure, and preference shocks. This solution requires keeping track of an infinite-dimensional object. In general, this feature makes the problem difficult to solve, even numerically. In the presence of search frictions, this class of models becomes even more complex. To address this difficulty, we utilize the notion of *block recursivity*. We now define this property and show the existence of an equilibrium with that property. We then elaborate on the usefulness of this result and discuss the structure of the market that makes it possible.

Definition 2. A *block-recursive equilibrium (BRE)* is a recursive equilibrium such that the value functions, policy functions, and market tightness functions depend on the aggregate state of the economy ψ, only through the stochastic shocks $\{z_i\}_{i \in I}$, and not through any endogenous distributions generated within the economy ($\{\Gamma_i\}_{i \in I}$, or $\{n_i\}_{i \in I}$); and free-entry conditions (11), (14), (16), and (17) are satisfied with equality.

Proposition 1. *There exists a block-recursive equilibrium.*

Proof. Proof. See appendix B. ☐

Let us now elaborate on the usefulness of proposition 1. In general, there is no easy way to compute an equilibrium of this model because of the high dimensionality of the state space. A commonly used approach in the literature is to approximate the distribution with several moments and make a conjecture about a law of motion for them. One can iterate on this conjecture to make it consistent with the policy rule of the households.[6] Note that this procedure already adds a large number of (continuous) state variables: we need to add at least homeownership and unemployment rates, fraction of population across locations, mean assets, and mean wages. Typically, having a good description of the evolution of aggregate variables requires second-order moments. This approach renders our model impossible to compute.[7]

Proposition 1 asserts that there exists a block-recursive equilibrium. Moreover its proof reveals that it is possible to compute the market tightness functions in the housing and labor markets *without* solving the household's decision problem. With those, it is straightforward to solve the decision problem of the households. This makes it possible to solve the model in two steps: First, solve for the market tightness functions using the free-entry conditions. Second, solve the household's problem taking as given the market tightness functions. Further details regarding the computation of the model, see Appendix C.

It is important to note that the endogenous distribution of households matters for the evolution of the economy: migration decisions, job search decisions, and house-buying and -selling decisions all depend on individual characteristics. Therefore, the response of the aggregate variables (for example unemployment rate, homeownership rate, etc) to shocks will depend crucially on the distribution of households (across assets, wages, etc) at the time the shock hits the economy. Block-recursive equilibrium is an equilibrium in which prices do not depend on endogenous distributions generated in the economy, but the evolution of important endogenous variables do.

As stressed in Menzio and Shi (2010a), the directed nature of the search technology is important for Proposition 1. The reason is the following: if search is random (but there is still price posting), then a firm needs to forecast what type of worker will apply and show up. The necessity arises because the type of worker affects the probability that the job will be accepted. To compute expectations appropriately, the

[6]More precisely, one solves a household's decision problem, given this law of motion, to obtain optimal policy rules. By simulating data from the model with these policy rules, one can obtain the implied law of motion for aggregate variables and compare it to the conjectured law of motion. The conjecture is revised until the procedure converges.

[7]Solving only the household problem, taking as given the market tightness functions, takes on average 15 hours on a cluster with 20 cores.

firm needs to know the entire distribution of households. A similar problem arises in a housing market with random search. Real estate managers in the housing market would need to forecast what type of buyer will show up, as this determines the willingness to pay for the house. This, again, requires knowledge of the entire distribution.

Free entry of firms is also important as it pins down the relationship between the offered wage and the probability of finding a worker—hence the corresponding tightness of the submarket. The introduction of housing market intermediaries and construction companies gives rise to three free-entry conditions, as we have shown above. Free entry is critical for the existence of block-recursive equilibrium.[8]

3 Calibration

We now turn to the calibration of the model. We calibrate the model to match a number of targets related to the labor and housing markets, mobility patterns, and wealth distribution before the housing bust in 2007. Before addressing the Great Recession, we evaluate the model's performance along a number of untargeted dimensions such as business cycle statistics, cyclicality of migration rates, and correlation between net flow rates and local unemployment rates. We then use the model to study the Great Recession.

3.1 Functional Forms

Let us introduce functional forms for the utility function and the matching probabilities. The utility function takes the following form,

$$u\left(c,l,h,\chi\right)=\frac{\left(c+\lambda\left(1-l\right)+\phi_{h}\right)^{1-\sigma}}{1-\sigma}+\chi,$$

where ϕ_h is the consumption services from housing type h and λ is the value of home production and leisure. If $l=1$, the household is currently employed and $\lambda\left(1-l\right)=0$. If $l=0$, the household is currently unemployed and gets a flow consumption of λ from home production. Note that because home production is not tradable, it directly enters the utility function as a perfect substitute for the consumption good.

Following Menzio and Shi (2011) and Schaal (2012), we pick the contact rate functions with constant elasticity of substitution

$$p\left(\theta\right)=\theta\left(1+\theta^{\gamma}\right)^{-1/\gamma},\ \ q\left(\theta\right)=\left(1+\theta\right)^{-1/\gamma}$$

for both the labor and housing markets.[9] γ_l, γ_b, and γ_s denote the matching function parameters for the

[8]An alternative structure to ours could be to have directed search in the housing market but have households trade among themselves. That is, sellers post houses, and buyers look for houses. Although this is perhaps a more realistic setup, one needs free-entry conditions to pin down market tightnesses in the housing market.

[9]Apart from providing a good fit to the data, a constant-returns-to-scale matching function is needed for the existence of a block-recursive equilibrium.

labor market and for the buying and selling sides of the house-buying market, respectively. We assume that $\gamma_b = \gamma_s$.

3.2 Stochastic Process for Labor Productivity

We need to calibrate the stochastic processes for labor productivity, that is, to calibrate Z and its transition function Υ_Z. We measure labor productivity as output per worker and estimate the following specification to obtain the persistence and variance of local labor productivity shocks at annual frequency:

$$\log z_{i,t} = \alpha + \rho \log z_{i,t-1} + \epsilon_{i,t}.$$

Since our data are annual, we convert point estimates to monthly numbers. Results are reported in Table I. We discretize the process for local labor productivity using the Rouwenhorst method with four grid points.[10]

3.3 Calibration Strategy

Calibration proceeds in two steps. In the first step, we exogenously calibrate parameters that have direct counterparts in the data or can be taken from previous studies because they are not model dependent. The second step follows a simulated method of moments.

Parameters Calibrated a Priori: A period in the model corresponds to a month. We set the monthly interest rate r to match an annual interest rate of 3 percent $\left(r = (1+3\%)^{1/12} - 1 \approx 0.25\%\right)$. The risk-aversion coefficient in the utility function σ is set to 2. The down payment requirement for buying a home is set to 10 percent. This requirement is lower than the typical 20 percent used in most of the literature on housing but is consistent with the financial developments in the housing market before the housing bust in 2007. Average replacement rate in the unemployment insurance system is around 40 percent. Consistent with this, b is set to 0.4. The monthly job destruction rate δ is set to 3.4 percent, as reported in Shimer (2005).[11] Table II summarizes the parameters of the model. The top panel presents parameters that are calibrated outside the model, and the bottom panel presents those that are calibrated within the model.

Parameters Calibrated with the Simulated Method of Moments: Parameters in the bottom panel of Table II are estimated by the simulated method of moments. The parameters are chosen to minimize the

[10] A more common alternative in the literature is the Tauchen method. A drawback of this method is that it requires many grid points to approximate well a highly persistent process. Computational concerns limit our choice of the number of grid points. The Rouwenhorst method, on the other hand, performs a better job with fewer grid points. Using simulated data, we verify that four grid points suffice to provide a good fit.

[11] This is constructed in Shimer (2005) using data on employment, short-term unemployment, and the hiring rate. The number reported is the average monthly separation rate over the period 1951 to 2003.

distance between the model-generated statistics and the targets in the data. The distance is defined as the percentage deviation from the target and uses the identity matrix as the weighting matrix. We now explain the targeted moments in the data in detail.

We start by describing moments related to the housing market. We target a homeownership rate of 69 percent, and an average time to sell of 3.5 months.[12] Vacancies posted by homeowners to sell a house last for one period in the model. We define the model counterpart of time to sell as the inverse of selling probability $1/\pi_s$. To calibrate the elasticity of the matching function, we need a moment that relates the posted price to the time to sell. To that end, we target the findings of Genesove and Mayer (1997), which show that homeowners with 100 percent leverage post prices about 4 percent higher than homeowners with 80 percent leverage. They report that the corresponding time to sell is 15 percent lower for the highly leveraged homeowners.

We now elaborate further on this part of the calibration. There are two parameters on search in the housing market that need to be calibrated: vacancy posting cost κ of REMs and the "elasticity" parameters in the matching function $\gamma_s = \gamma_b$. The average time to sell is intimately linked to κ, as changes in this parameter shift the entire market tightness functions (and thus the relationship between selling price and the probability of trade). The elasticity parameter, however, governs how the probability of trade is affected by a change in selling price. The decision problem of the household provides a mapping between asset position and the optimal selling price. By taking the composition of this decision rule with the market tightness function that maps the price to the time to sell, we can construct the model counterpart of the relationship between leverage and time to sell. This moment depends tightly on the elasticity parameter and is therefore used to calibrate it.

How can the model generate a negative relationship between assets and time to sell? There are two forces in the model. Households with lower assets have a higher propensity to move for job-related reasons. This is because their marginal utility of consumption is higher compared to households with more liquid wealth. Households with fewer assets (and thus more leverage), however, find it harder to afford a new house after moving and end up renting for many periods because of the financial friction in the model—the down payment constraint. Because households obtain a higher utility from owning (also a calibrated parameter), highly leveraged households have a motive to post higher selling prices (and sell slower) than households with lower leverage. The relationship between leverage and time to sell is a result of this trade-off. It turns out that the model does generate the right relationship quantitatively.

This strategy is analogous to the standard one in the search literature that is used to calibrate the

[12] This is the homeownership rate in the United States right before the onset of the housing bust. See `http://www.census.gov/hhes/www/housing/hvs/charts/files/fig05.pdf`. Average time to sell is taken from the National Association of Realtors website. Different sources report different numbers that range between 2.5 and 5.5 months at times with "good" housing markets.

vacancy posting cost and matching function parameter. For example, Shimer (2005) uses average job finding probability and the correlation between the job finding rate and market tightness. The former is (mostly) informative about the average job finding rate, and the latter is informative about the correlation. The average time to sell is analogous to the job finding probability, and the elasticity of time to sell with respect to leverage is analogous to the elasticity of the job finding probability with respect to market tightness.

Finally, we target the ratio of average house prices to average monthly earnings in the model. To determine the empirical value of this moment, we compute the ratio of average house price to average monthly wages for each year over between 2001 and 2005. We then average this time series to obtain an average of 48.

Turning to the labor market dimension of the model, we target an average job finding rate of 45 percent and a correlation at the quarterly frequency between the (log) job finding rate and the market tightness of 0.94. Both targets are the same as in Shimer (2005). In the model presented above, there are multiple submarkets in the labor market at any time and thus multiple job finding rates at any point in time. Unlike in Shimer (2005), the elasticity cannot be calibrated before solving the model: we need to solve and simulate the model to obtain the average job finding probabilities across different submarkets, weighted by the number of workers that apply there.[13] The resulting series from the model is monthly. We obtain quarterly series by taking the average over three months. The quarterly time series for the (log) job finding probabilities and (log) market tightnesses are then filtered with an Hodrick–Prescott filter using a scaling parameter of 1600. The model counterpart of the correlation is computed using the detrended series.

The model also has predictions about mobility rates. We target two mobility-related moments: average gross mobility and average net mobility in the United States. Gross mobility is defined as the average of population inflow and outflow rates: that is, $gross\,mobility = \sum_t \sum_i |inflow_{i,t} + outflow_{i,t}| / 2NT$, where N is the number of MSAs for which we have data on population flows and T is the number of years our data span. Net mobility is defined as the average of absolute values of population net flows: that is, it is given by $net\,mobility = \sum_t \sum_i |inflow_{i,t} - outflow_{i,t}| / NT$. Empirical values for gross and net migration are computed from the IRS data. We use data for the period 2004–2007 to exclude the recession period. We find an average gross migration rate of 4.3 percent and an average net migration rate of 0.8 percent.

The key parameters that help us match the net and gross migration rates are the persistence and variance of preference shocks. In the model, net migration occurs because of differences in labor productivity across the two locations: households tend to relocate themselves to places with better labor productivity because it is easier to find jobs and also wages are higher. Unemployed households would like to look for a job in the location with higher labor productivity. Absent any other motive, the model-generated net mobility is the same as gross mobility. Preference shocks make people move for non-labor-market-related reasons and

[13] Alternatively, one can use the change in the unemployment rate in the model to infer average job finding probability.

make them move in both directions at the same time. That is, we observe that populations that lose workers also attract new workers, thus breaking the relationship between net migration and gross migration. If the persistence of preference shocks gets larger, households do not respond to local labor market differences as much, and the net migration rate decreases. Yet, gross mobility is large because households move whenever their preferences so dictate. This intuitive discussion suggests that the persistence of preference shocks helps us match the difference between gross and net migration rates. Increasing the variance of preference shocks, however, increases the gross migration rate as households are hit by larger preference shocks. By choosing the persistence and variance of preference shocks, we can calibrate the model to match the gross and net migration rates exactly.

We target a median leverage of 67 percent, which is computed from the 2004 wave of the Survey of Consumer Finances. Leverage in the model is computed as the ratio of household debt to the average house selling-price in a household location. Table II shows the resulting parameter values, and Table III summarizes the targets and the fit of the model with respect to the targeted moments.

3.4 The Model's Fit on Nontargeted Moments

Before using the model to address the Great Recession, we evaluate the model's performance along a number of untargeted dimensions of the data. Our model has predictions on how much the population of a location changes following a local labor market shock. We therefore compare three quantitative predictions of our model to the data: standard deviation of gross and net flows, the cyclicality of migration, and the correlation between local labor market conditions and local population flows.

3.4.1 Volatility of Migration

Table IV reports the volatility of aggregate gross and net migration rates computed from the model and from the data. While the volatility of migration rates is higher in the model than in the data, we find that, as in the data, gross migration is more volatile than net migration. Because only the unemployed people make mobility decisions in our model, migration rates tend to move together with local and aggregate unemployment.[14] As we discussed before, migration is a result of the trade-off between idiosyncratic tastes and differences in local productivity. Idiosyncratic shocks in the calibrated model are quite persistent and prevent many people from moving in response to differences across the two labor markets. Consequently, the response of net migration to differences in productivity across locations is dampened. However, gross

[14] In our calibrated model, unemployment is as volatile as in the data. Therefore, it is not surprising, given the structure of the model and the volatility of unemployment, that the volatility of migration rates in the model are higher than those in the data.

migration is greatly affected by aggregate unemployment. As unemployment goes up, more people move to follow their idiosyncratic taste for location. As a result, gross migration is more volatile than net migration.

3.4.2 Local Labor Market Conditions and Migration

Table V shows the regression coefficient of outflow rates on relative productivity. Relative productivity is defined as the deviation of local productivity from aggregate productivity $(\log(z_t) - \log(z_{i,t}))$ in a year, and is defined such that a positive value indicates that the productivity in the location is lower than aggregate productivity in the economy. The model is able to replicate the negative relationship between outflows and relative productivity, suggesting that in the benchmark model without housing-market-related frictions, households move out of low-productivity locations.

3.4.3 Cyclicality of Migration

Finally, we analyze the cyclicality of gross migration in our estimated model. In particular, as we will show in the next section, our model predicts a rise in migration rates during the Great Recession (absent frictions arising in the housing market). In what follows, we will show that this does not happen in a typical recession in our model and is a consequence of heterogeneous productivity shocks specific to the Great Recession. In fact, the model delivers, consistent with the data, a procyclical gross migration rate.

Table VI reports the regression coefficient of the log of gross migration rate on log unemployment. We use the numbers reported in Davis et al. (2010) on MSA-level gross migration rates. We regress the log of this variable on aggregate unemployment and report the coefficient. As Table VI shows, our model is able to generate a procyclical gross migration rate.

There are two main forces in the model that affect the cyclicality of gross migration. On the one hand, there is a composition effect: in our model, only the unemployed workers are allowed to migrate. Since in a recession there are more unemployed households, migration tends to increase during recessions. On the other hand, households care less about preference shocks during a recession because of the functional form of the utility function. Preference shocks enter the utility function in a separable fashion, implying that, for a given level of preference shock, the marginal benefit of migration is constant over time. However, moving is costly and entails a wealth loss because it involves selling the house, and selling takes place in a frictional housing market. Hence, there is a trade-off between moving for preference-related reasons and avoiding the loss. Since the marginal utility of consumption is higher in a recession, the migration rate tends to be procyclical.[15] It turns out that the income effect dominates the composition effect, resulting in a procyclical

[15] This argument is analogous to the ones used in the health literature to explain the rise in health expenditures over time and the differences in health expenditures between high- and low-income households. For example, see Hall and Jones (2007).

gross migration rate that is consistent with the data.

4 Housing Bust and the Great Recession

Using the calibrated model, we now quantify the effect of the housing bust on the dispersion of unemployment rates across MSAs during the recent recession. To study the effects of the housing bust through our two-location framework, we need to group the MSAs in the United States into two categories. We choose the groups based on the size of the housing bust: location A contains all the MSAs in our dataset where house prices declined by less than 35 percent, and location B contains the remaining MSAs.[16] Out of the 341 MSAs for which we have data on house prices, 250 are assigned to location A. According to our categorization, house prices declined by 19.6 percent in location A and 46.8 percent in location B. We engineer this decline in the model as the consequence of a one-time, unanticipated, and permanent decline in housing construction costs $\{\mu_i\}_{i=1,2}$.[17]

To isolate the effect of the housing bust on geographical reallocation during the Great Recession, we run the following two experiments. In the first simulation, which we label the factual simulation, we feed into the model the exact labor productivities observed in the data and house price shocks backed out through the model.[18] In our data, labor productivity declined by 1 percent in MSAs in location A and by 5 in MSAs assigned to location B. In the second simulation, which we call the counterfactual simulation, we feed into the model the same realizations of labor productivity for locations A and B. The parameters for housing construction costs do not change in this simulation, and, as a result, house prices decline by a much smaller amount.

4.1 Migration Rates

We start reporting the decline in migration rates. It is important that the decline in migration is consistent with the decline in the data, because according to our hypothesis, the decline in migration is the driving force behind the increase in unemployment dispersion. The factual simulation shows that our model generates a sizable decline in both gross and net migration rates, consistent with the data. Note that the decline in migration rates is not targeted at any point of estimation. Table VII summarizes the migration rates generated in the factual simulation. The model predicts a decline to 0.2 percent in net migration from a

[16] The size of the housing bust is defined as the percentage decline in house prices from the peak to the trough.

[17] Note that transaction prices in the housing market are endogenous in our model. To make the model consistent with the data, we choose the decline in construction costs in the two locations such that the decline in house-sales prices in the model exactly matches the decline observed in the data.

[18] Households' expectations are derived from the stochastic process of labor productivity. In other words, households expect these shocks to recover according to the $AR(1)$ coefficient.

prerecession level of 0.8 percent. The empirical counterpart of this, obtained through IRS data, shows a decline from 0.8 percent to 0.3 percent. For gross flows, the model predicts a decline from 4.3 percent to 3.3 percent, compared to a decline in the gross migration rate from 4.3 percent to 3.8 percent.

To isolate the effect of the housing bust on migration, we use our model to see what would have happened to migration rates in the absence of the housing bust. The last column of Table VIII shows net migration rates predicted by the counterfactual experiment. Interestingly, the model predicts a *rise* in net migration. Without the housing bust, responding to the asymmetric decline in labor productivity, more households would have migrated. As a result, the model generates an increase in net migration from 0.8 percent to 1.1 percent. However, in the factual simulation, the decline in house prices constrains the mobility of homeowners and results in a decline in net migration from 0.8 percent to 0.2 percent. The comparision of the two simulations indicates that the decline in migration caused by the housing bust is 0.8 percent, larger than the observed decline in the data. The decline in migration observed during the Great Recession is only 62.5 percent of the entire decline caused by the housing bust.

Figure I depicts the model's prediction for the gross migration rate. The model generates a fall by around one percentage point from its prerecession level during the Great Recession (shown in the dashed-dotted line). The counterfactual simulation (shown in the solid line) indicates that, absent the housing bust, gross migration would have increased to 5.3 percent. Similar to the net migration rate, the model predicts a rise in the gross migration rate in the counterfactual analysis without the housing bust. We conclude that the observed decline in migration (both gross and net) in the data constitutes only half the decline caused by the housing bust.

To understand why geographical reallocation declines as a result of the house price decline, we now investigate the policy rules of homeowners for migration. Figure II shows the optimal policy rule for migration of a homeowner with 80 percent leverage that currently resides in location B. The x-axis is the productivity at location A, and the y-axis is the preference of the household for B, its current residence. The policy rule illustrates the trade-offs between the two factors governing the decision to migrate: preference shocks and differences in local labor market conditions. Fixing the preference in the current location, a higher labor productivity in A makes households in B more likely to move out from their current residence. As the counterfactual simulation results indicate, in the absence of additional frictions coming from the housing market, the asymmetric decline in labor productivity during the Great Recession increases the benefit of migration, and thus workers in the relatively more distressed labor market move out.

Similarly, for a given level of labor productivity in A, only households below a cutoff preference enter the housing market to sell their houses. Those that successfully sell then move to location A. The top line shows the cutoff preference before the housing bust, while the bottom line shows the one after the housing bust. As the figure shows, the cutoff preference shifts down, suggesting that many households that would have moved decide to stay (thereby creating the region in the figure labeled "locked-in households"). The quantitative analysis suggests that the latter effect dominates the former and leads to a decline in geographical reallocation.

4.2 Geographical Dispersion of Unemployment Rates

This section studies the implications of the "house-lock" on local unemployment rates. The left panel of Figure III shows local unemployment rates in the simulation with house price declines and labor productivity shocks, and the right panel shows local unemployment rates in the counterfactual simulation (labor productivity shocks only). In both panels, we plot the deviation of unemployment from the level before the recession. The model predicts a rise in local unemployment by around 1.5 percentage points in A and 4 percentage points in B. In the data, the rise is around 4.5 and 7 percentage points for locations A and B, respectively. The right panel in Figure III highlights the role of the housing bust in local unemployment. Without the housing bust, the model predicts a rise in local unemployment of 2 and 2.5 percentage points for locations A and B, respectively. Thus, the housing bust increases the unemployment rate further in B (large housing bust region) while decreasing it in A (small housing bust region).

As a consequence of the house-lock, many unemployed homeowners that would normally be looking for jobs in the other location now look for jobs in B. This causes local unemployment in B to rise more in response to a labor productivity decline. At the same time, it causes unemployment in A to rise *less*, since the households that would be unemployed and looking for jobs in A are now still in B. This mechanism results in an increase in the dispersion of unemployment across these two locations: A faces a lower decline in labor productivity, and the housing bust *decreases* the effect of the labor productivity shock on local unemployment, whereas B faces a higher decline in labor productivity, and the housing bust *amplifies* the effect on local unemployment.

Figure IV plots the evolution of the difference of local unemployment rates between these two locations. The dashed line shows the data. In the solid line, we plot the difference in the simulation with both types of shocks, whereas in the dashed-dotted line we show the difference in the counterfactual simulation. We conclude that the housing bust substantially increases the model's ability to capture the rise in the dispersion of unemployment rates across MSAs.

4.3 Aggregate Unemployment Rate

Our model allows us to study the effects of the housing bust on aggregate unemployment. Results in the previous sections suggest that the housing bust increases the fraction of unemployed workers that look for jobs in the low-productivity location. Therefore, one would expect the housing bust to further increase aggregate unemployment. Figure V shows the evolution of the aggregate unemployment rate for the two simulations. The asymmetric decline in labor productivity, accompanied by an asymmetric decline in house prices, results in an increase in unemployment of around 2.5 percentage points (shown in the solid line). The blue line reveals that the increase in the aggregate unemployment rate would have been around 2 percentage points had there been no housing bust. We conclude that the effect of the housing bust on aggregate unemployment rate is 0.5 percentage points. This may seem in contrast with the large effects on gross and net migration rates as well as on the dispersion of unemployment. This inconsistency can be easily reconciled by noting that the housing bust has opposite effects on the two locations. Location A is having a relatively lower unemployment rate compared to a recession without a housing bust because of the decline of the inflow of unemployed households from location B. Location B, however, has a higher unemployment than in a counterfactual recession because of the locked-in unemployed homeowners. Thus, it is not clear ex ante in which direction the aggregate unemployment rate would respond. Our quantitative exercise suggests that despite large effects on migration and local unemployment rates, this mechanism can explain a small fraction of aggregate unemployment.

5 Conclusions

We have developed a computationally tractable equilibrium model of multiple locations with local housing and labor markets and used it to study the effects of the housing bust on local and aggregate labor markets during the Great Recession. Our analysis suggests that the housing bust is responsible for the decline of migration and the increase in the dispersion of unemployment across regions. A reduction in house prices reduces the home equity for households and causes the down payment constraint to bind for more households. It is because households prefer owning over renting that the decline in house prices distorts their migration decisions. Consequently, unemployment in the low-productivity region responds strongly to the decline in productivity, whereas the rise in the relatively better location is lower, due to a reduction in the inflow of unemployed workers. The opposite effects drive up the dispersion in unemployment but result in a smaller rise in aggregate unemployment. Despite a large decline in geographical reallocation and the resulting rise in unemployment dispersion, we found that the housing bust accounts for 0.5 percentage points of the rise in aggregate unemployment.

The model presented in this paper provides a quantitative framework for future research on housing and labor markets. Housing markets may affect various aspects of local labor markets, including local wages, local unemployment rates, inflow and outflow of workers, and the time it takes for the region to recover from adverse shocks. Our model has the essential ingredients to evaluate how policies that affect homeownership and housing debt influence local and aggregate labor market outcomes.

Several European countries are characterized by large and persistent unemployment differences across regions. They also typically have more rigid housing markets. Is there a link between the housing and rental market structure and unemployment dispersion? The model developed in this paper can be modified to study the implications of these differences for the labor markets in those countries. We defer this work to ongoing and future research.

6 Tables

Table I
Labor Productivity Process

Parameter	ρ	σ_ϵ
Value	0.98	0.01

Note: Results of the estimation of an $AR(1)$ process on the log of local labor productivity (output per worker).

Table II

Model Parameters

Parameter	Value	Description
Precalibrated		
α	10%	down payment requirement
r	0.25%	monthly interest rate
b	0.4	unemployment benefits
ζ_0	0	consumption flow from renting
δ	3.4%	job destruction probability
Within-the-model		
ζ_1	0.20	consumption flow from owning
β	0.984	discount rate
λ	0.51	home production
k	0.75	vacancy posting cost for firms
γ_l	1.80	labor market elasticity
κ	0.12	vacancy posting cost for REMs
γ_b, γ_s	0.80	housing market matching functions
ρ_ψ	0.991	persistence of the preference shock
σ_ψ	0.002	standard deviation of preference shocks
μ	48	housing construction cost
γ	0.05	depreciation rate of rental houses

Note: Table II reports the calibrated parameter values of the model. The upper panel reports parameters calibrated a priori. The lower panel reports the parameters calibrated within the model.

Table III

Matching the Calibration Targets

Moment	Data	Model
Homeownership rate	69%	69%
Average time to sell (in months)	3.5	3.5
Genesove and Mayer (1997)	15%	15%
Job finding probability	0.45	0.44
Elasticity of job finding probability	0.95	0.94
Volatility of job finding rate/labor productivity	5.5	5.5
Median leverage	67%	67%
House price/monthly wage	48	48
Gross mobility	4.3%	4.3%
Net mobility	0.8%	0.8%

Note: Table III reports calibration targets and their values from the model. See the discussion in the text for detailed information on the targets.

Table IV
Volatility of Gross and Net Flows

Standard Deviation	Model	Data
Gross migration	0.35	0.22
Net migration	0.20	0.12

Table V
Local Productivity and Population Flows

$outflow_{i,t}$	Model	Data
Relative productivity	-0.011	-0.0047
$\text{Log}(z_t) - \log(z_{i,t})$		(0.0019)

Table VI
Cyclicality of Gross Flows

$\log grossflow_t$	Model	Data
Log unemployment	-0.009	-0.011
$\text{Log}(u_t)$		(0.004)

Table VII
Migration: Data vs. Model

	Data		Model	
Year	Gross migration (%)	Net migration (%)	Gross migration (%)	Net migration (%)
2006–2007	4.3	0.8	4.3	0.8
2007–2008	4.1	0.5	3.6	0.3
2008–2009	3.8	0.3	3.3	0.2

Note: Table VII compares the gross and net migration rates in the data and the model.

Table VIII
Net Migration: Data vs. Model

Year	Data (%)	Model (%)	Counterfactual (%)
2006–2007	0.8	0.8	0.8
2007–2008	0.5	0.3	1.6
2008–2009	0.3	0.2	1.1

Note: Table VIII compares the net migration rates in the data and the model. The last column shows the net migration rate predicted by the model in the absence of a housing bust.

7 Figures

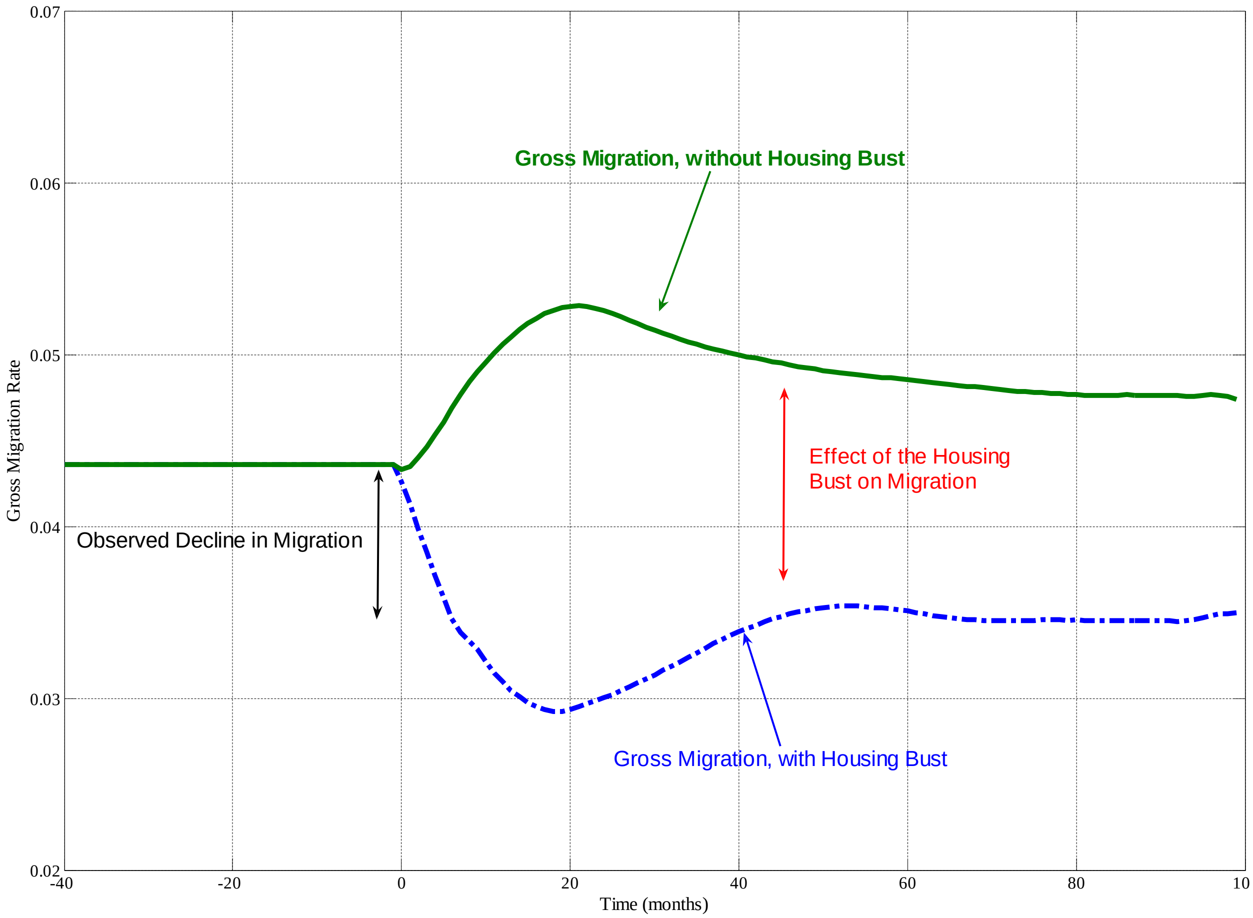

FIGURE I
GROSS MIGRATION AND THE HOUSING BUST

Note: Figure shows the gross migration rate predicted by the model. The blue line shows the decline in migration as a result of the housing bust and decline in labor productivity. The green line shows the counterfactual migration rate that would have obtained absent the housing bust. The model predicts a rise in migration. The difference between the two lines is the true effect of the housing bust on migration.

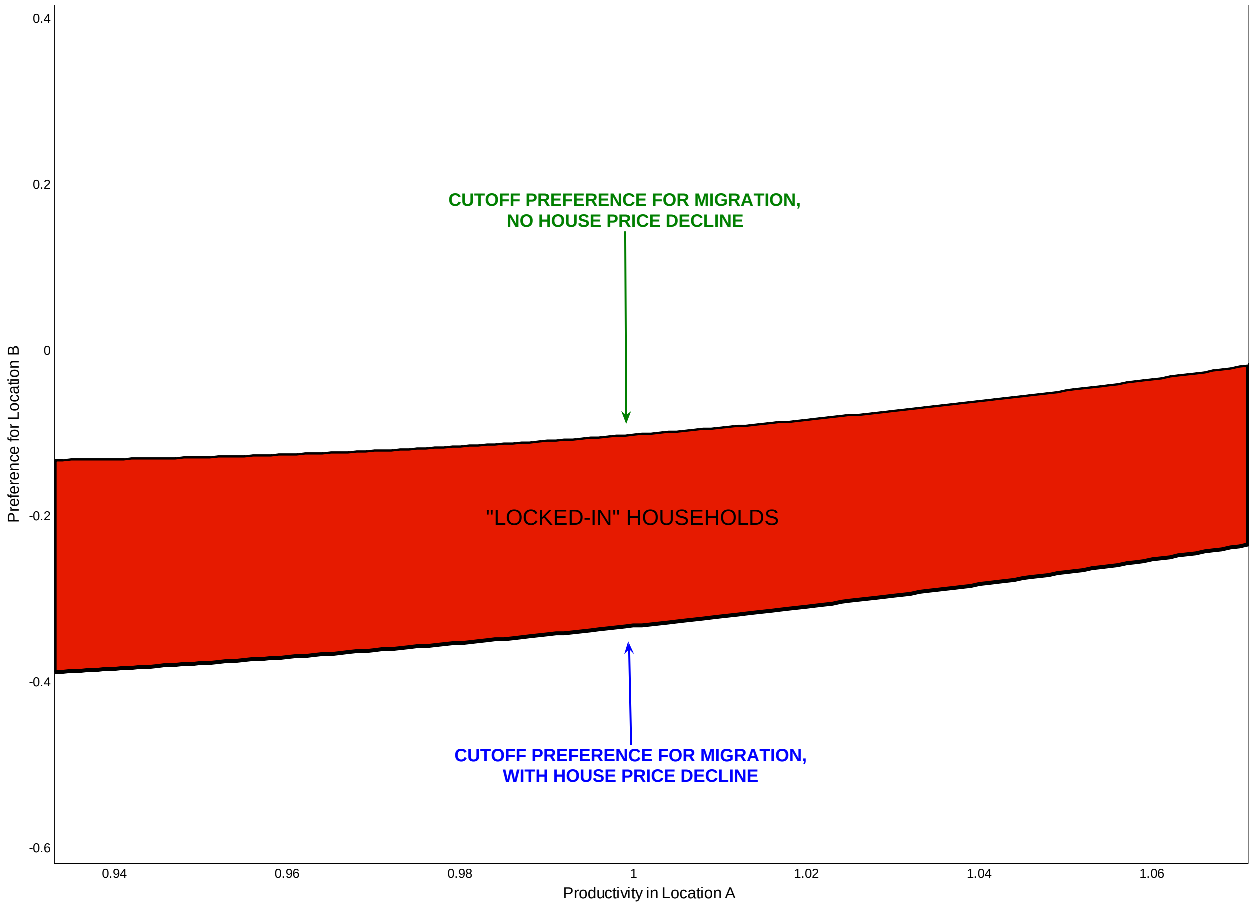

Figure II
Cutoff Rule for Migration

Note: Figure shows the migration decision of a household with 80% leverage living in location B. X-axis is the labor productivity in location A, and the y-axis is the preference of the household for the current residence, location B. The migration decision is characterized by a cutoff rule that is increasing in the productivity of the other location. The cutoff shifts down when house prices in MSA B fall. The shaded area designates the set of households that do not move due to the housing bust. We label them as locked-in households.

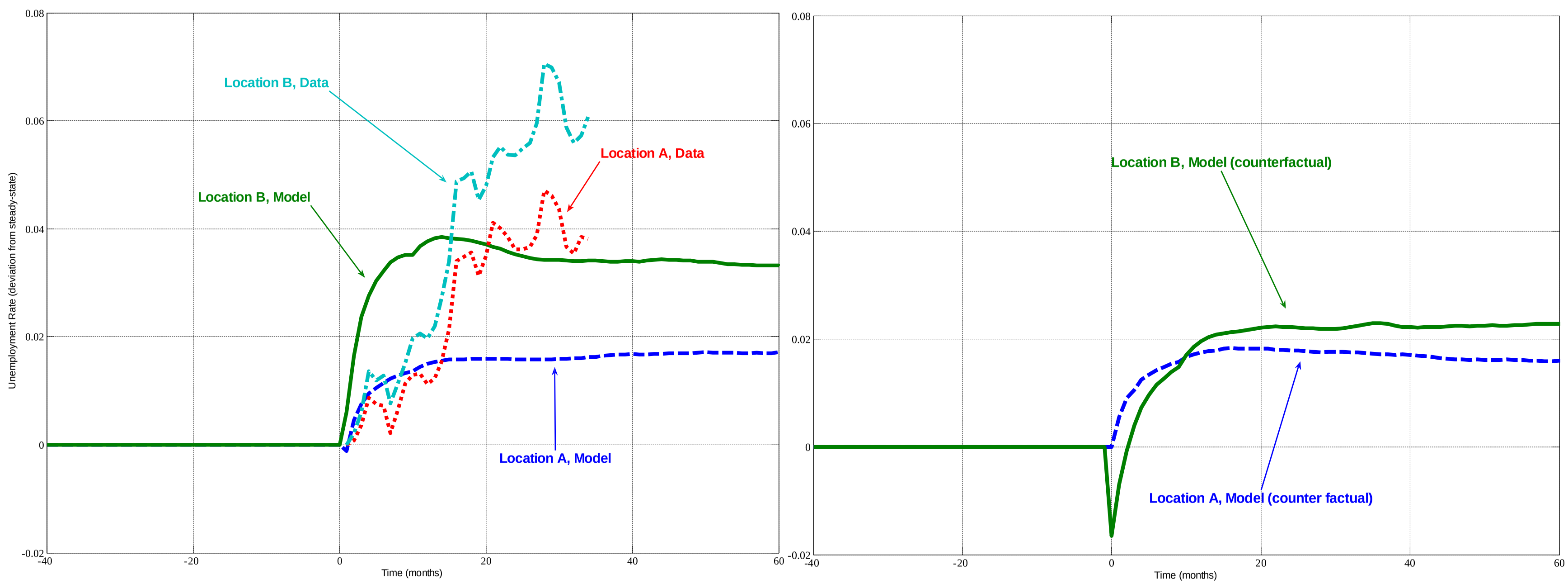

FIGURE III
LOCAL UNEMPLOYMENT AND THE HOUSING BUST

Note: Figure shows the unemployment rate for location A and location B. We plot the deviation from the unemployment rate before the recession. The left panel shows the simulation with a house price shock as well as a labor productivity shock. The right panel plots the case where the model is hit by a labor productivity shock only.

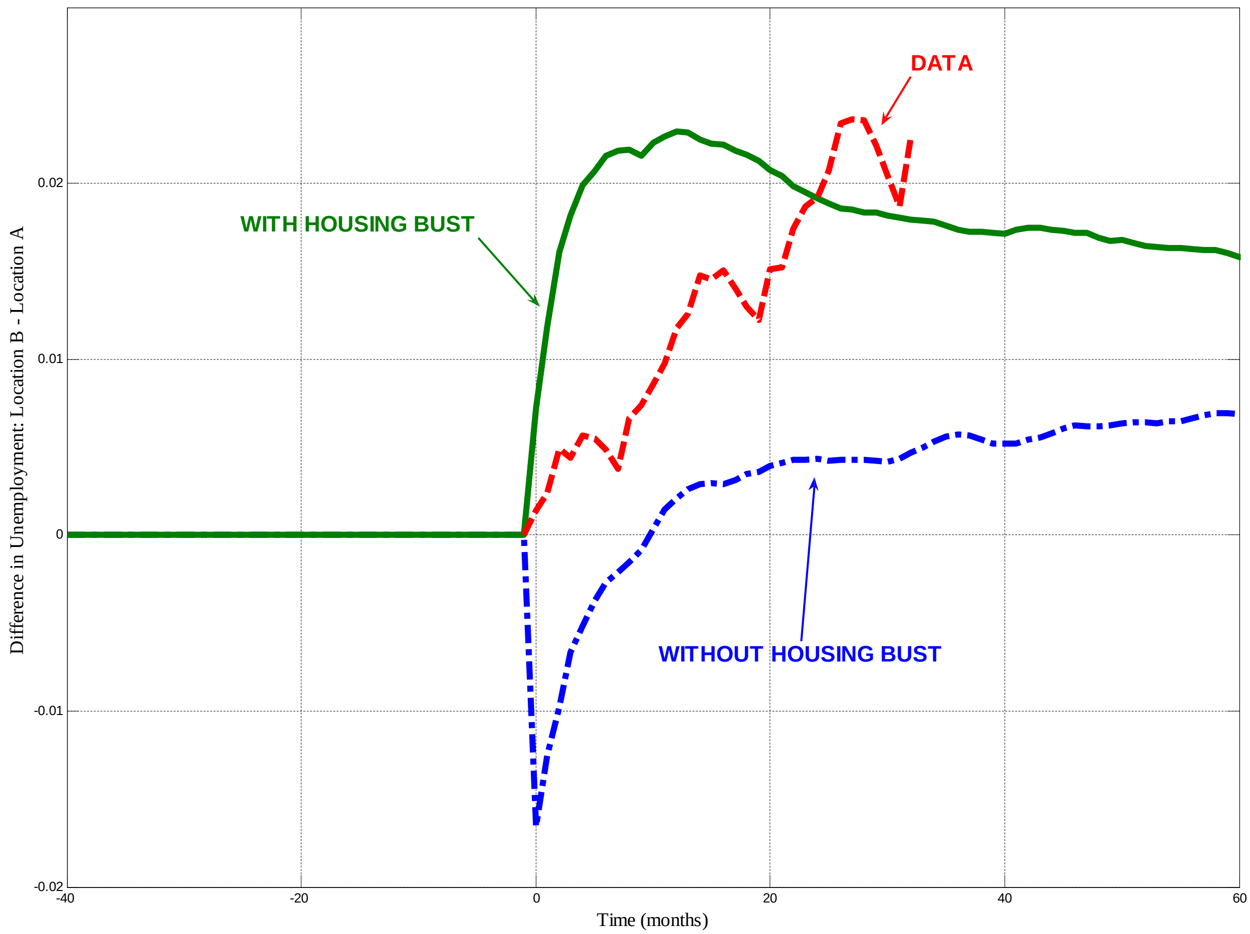

FIGURE IV
DISPERSION OF UNEMPLOYMENT AND THE HOUSING BUST

Note: Figure compares the prediction of the model for the difference in unemployment rates between location A and location B with the data. The solid line shows the difference that arises as a result of labor productivity and house price shock. The dashed-dotted line shows the effect of the decline in labor productivity without a housing bust. The dashed line is data.

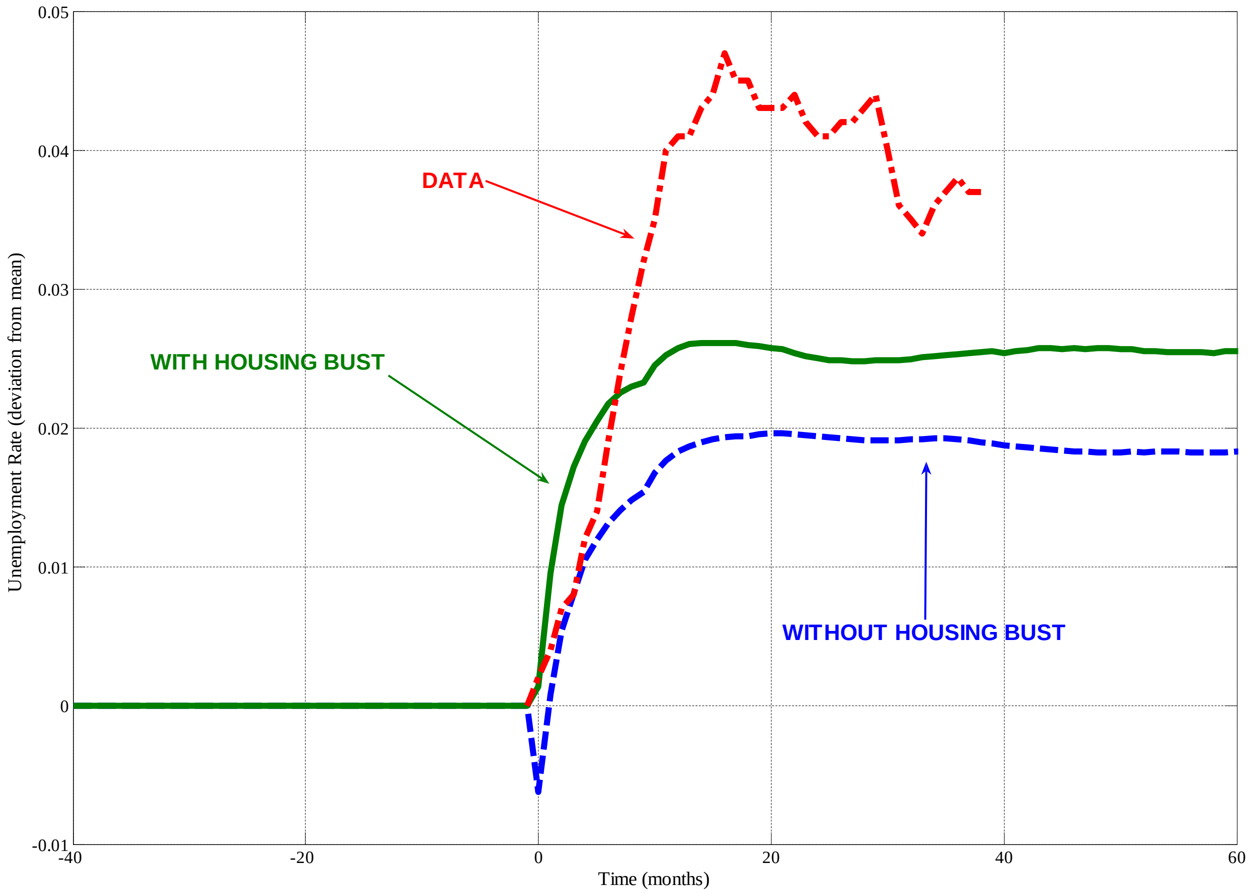

Figure V
Aggregate Unemployment and the Housing Bust

Note: In the solid line, we show the rise in aggregate unemployment as a response to house price and labor productivity shocks. The dashed line shows the rise with a labor productivity shock but without a housing bust. Finally, the dashed-dotted line shows the data.

A Data Appendix

Annual output data at the MSA level are available at the Bureau of Economic Analysis (BEA) website for the period 2001–2009.[19] We use real GDP by metropolitan area in millions of chained 2005 dollars (all industry total). Employment and unemployment data are taken from the Bureau of Labor Statistics (BLS). These variables are also available at monthly frequency. We construct a measure of local labor productivity for each MSA as the ratio of output to employment. Population data are taken from the Regional Economic Accounts of the Bureau of Economic Analysis (table CA1-3). These are available at http://www.bea.gov/regional/reis. Population numbers reported are midyear estimates. Quarterly data on house prices are obtained from the Federal Housing Finance Agency. We use all-transaction indexes (estimated using sales price and appraisal data). Annual estimates are computed as the average of quarterly observations.

Using county-county migration data based on tax return records of the Internal Revenue Service (IRS), we construct data on MSA-level population gross inflows, gross outflows, and net flows. These data are available from the IRS website for the period 2004–2009. For each year, IRS reports population inflows and outflows for all counties. These files report the origin and the destination counties and the number of migrants in two units: "returns" and "personal exemptions." We follow Davis et al. (2010) and use the exemptions data, as these data approximate the migrant population as opposed to the number of households as in returns data.[20] Gross inflows into an MSA are computed as the sum of all inflows into any county in that MSA from any other county in other MSAs. Gross outflows are computed analogously. Inflow and outflow rates are computed as the ratio of flow to the population in that year. Finally, the net flow rate is defined as the difference between the gross inflow rate and the gross outflow rate.

B Existence of a Block-Recursive Equilibrium

To prove proposition 1, we proceed in two steps. We first show that the functional equations and the corresponding free-entry conditions for firms and housing market intermediaries admit a solution, where the dependance of the value functions, market tightness functions, and the rental rate on the aggregate state is through exogenous shocks only. More formally, we show that there exists a set of market tightness functions $\{\theta_l^i(w;\psi), \theta_s^i(p;\psi), \theta_b^i(p;\psi)\}_{i\in I}$, rental rate $\{\rho^i(\psi)\}_{i\in I}$, and value functions of firms and housing market intermediaries $\{J^i, L^i, R^i\}_{i\in I}$, which depend on ψ only through exogenous shocks $(z_i, \mu_i)_{i\in I}$, and not through any endogenous distribution $\{\Gamma_i\}_{i\in I}$ or $\{n_i\}_{i\in I}$. That is, we can reduce the state space ψ into exogenous shocks, $(z_i, \mu_i)_{i\in I}$. In the second stage, we collapse the problem of households into one big

[19] http://bea.gov/regional/gdpmetro/

[20] For a more detailed description of the IRS data, see Davis et al. (2010).

functional equation and show that it is a contraction. We also show that the functional equation maps the set of functions that do not depend on the endogenous distribution into the same set, provided that the market tightness functions and the rental rate are independent from the endogenous distribution as well. This shows that there is a solution to the problem of households that, together with the value functions, market tightness functions, and rental rates of the first step, constitutes a block-recursive equilibrium of the economy.

B.1 Market Tightness Functions

- Define $\mathcal{J}(\mathcal{W} \times Z)$ as the set of continuous and bounded functions J such that $J : \mathcal{W} \times Z \to \mathbb{R}$ and denote T_J as an operator associated with (9). It is easy to verify that T_J maps $\mathcal{J}$ into $\mathcal{J}$. Applying Blackwell's sufficiency conditions, we can show that the operator $T_J : \mathcal{J} \to \mathcal{J}$ is a contraction. Denote the fixed point of T_J as $J^* \in \mathcal{J}$.[21]

- Substituting J^* into the free-entry condition for the labor market, (11), we get the labor market tightness function $\theta_l^*(w;\psi)$ as only a function of wage, and labor productivity shock z:

$$\theta_l^*(w;z) = \begin{cases} q_l^{-1}\left(\frac{k}{J^*(w;z)}\right) & \text{if } w \in \mathcal{W}(z) \\ 0 & o/w \end{cases}.$$

- Similarly, we define $\mathcal{R}(\mathcal{P}_b \times M)$ as the set of continuous and bounded functions mapping $\mathcal{P}_b \times M$ to $\mathbb{R}$, and $\mathcal{L}(M)$ as the set of continuous and bounded functions from M to $\mathbb{R}$. It is easy to show that the operator associated with (12) maps functions from $\mathcal{R}(\mathcal{P}_b \times M)$ into $\mathcal{R}(\mathcal{P}_b \times M)$ if $\theta_b(\cdot)$ depends on ψ only through μ. Similarly, one can show that the operator associated with (15) maps functions from $\mathcal{L}(M)$ into $\mathcal{L}(M)$ if $\psi_i(\cdot)$ depends on ψ only through μ. The standard contraction mapping argument can be applied to establish the existence of fixed points, $R^*$and L^*, of operators (12) and (15), respectively.

- Using the free-entry conditions for the housing markets, (14) and (16), and plugging them into the operators (12) and (15), respectively, one can solve for the market tightness functions $\theta_s^*(p;\mu)$ and $\theta_b^*(p;\mu)$:

$$\theta_s^*(p;\mu) = \begin{cases} \pi_s^{-1}\left(\frac{\kappa}{R^*(p;\mu)}\right) & \text{if } p \in \mathcal{P}_s(\mu) \\ 0 & o/w \end{cases}$$

[21] The assumption of full commitment to a constant wage contract of workers guarantees that $\mathcal{J}(\mathcal{W} \times Z)$ is a space of bounded and continuous functions.

$$\theta_b^*(p;\mu) = \begin{cases} \pi_b^{-1}\left(\frac{\mu-(1+r)^{-1}\mathbb{E}_{\psi'|\psi}[\mu'|\mu]}{p-(1+r)^{-1}\mathbb{E}_{\psi'|\psi}[\mu'|\mu]}\right) & \text{if } p \in \mathcal{P}_b(\mu) \\ 0 & o/w \end{cases}.$$

- Using the free-entry condition (17) in the operator (15), we get the rental rate of the economy $\rho(\psi)$ as a function of μ.

$$\rho^*(\mu) = \mu - \frac{1-\gamma}{1+r}\mathbb{E}_{\psi'|\psi}[\mu' \mid \mu].$$

B.2 Households' Value Function

- First, we reformulate the value functions of households as one function $V : I \times E \times \mathcal{W} \times A \times H \times \Xi^I \times \Psi \to \mathbb{R}$ such that

$$\begin{aligned} V(i, e=1, w, a, h=1, \chi; \psi) =& W^i(w, a, h_1, \chi; \psi) \\ V(i, e=1, w, a, h=0, \chi; \psi) =& W^i(w, a, h_0, \chi; \psi) \\ V(i, e=0, :, a, h=1, \chi; \psi) =& U^i(a, h_1, \chi; \psi) \\ V(i, e=0, :, a, h=0, \chi; \psi) =& U^i(a, h_0, \chi; \psi). \end{aligned}$$

- Using the above value function, V, we can define the labor market surplus function as the following:

$$\tilde{\Delta}(i, a, h; \psi) \quad = \quad \sum_{k=1}^{\bar{I}} \mathbf{1}_{\{\mathbf{k=i}\}} \left[\max_{w \in \mathcal{W}(z_i)} \pi_l\left(\theta_l^*(w; z_i)\right) \{V(i, 1, w, a, h, \chi; \psi) - V(i, 0, w, a, h, \chi; \psi)\} \right]$$

- In similar manner, we define the option value of migration as

$$\tilde{M}(i, a, h, \chi\psi) \quad = \quad (1-h)[\max_{j \in I} \tilde{\Delta}(j, a, h, \chi; \psi)].$$

- We define a set of functions $\mathcal{V} : I \times E \times \mathcal{W} \times A \times H \times \Xi^I \times Z^I \times M^I \to \mathbb{R}$ and T_V such that

$$
\begin{aligned}
& (T_V V)\,(i, e, w, a, h, \chi; \{z_i, \mu_i\}_{i \in I}) \\
= \quad & \sum_{k=1}^{\bar{I}} \mathbf{1_{k=i}} \times \Bigg\{ (1-e)\,(1-h) \left\{ \max_{a' \geq a_0} u(c, l_1, h_0, \chi_i) + \beta \mathbb{E}_{\psi'|\psi} \Bigg[\tilde{M}(i, a', 0, \chi; \psi') \right. \\
& \qquad + \max_{p \in \mathcal{P}_b(\mu_1'), a' \geq \alpha p} \pi_b^*(p; \mu_i') \times \Big\{ V(i, 0, w, a'-p, 1, \chi; \psi') + \tilde{\Delta}(i, a'-p, 1, \chi; \psi') - \tilde{M}(i, a', 0, \chi; \psi') \Big\} \Bigg] \Bigg\} \\
& + (1-e)\,h \Bigg\{ \max_{a' \geq a_1} u(c, l_0, h_1, \chi_i) + \beta \mathbb{E}_{\psi'|\psi} \Bigg[\tilde{\Delta}(i, a', 1, \chi; \psi') \\
& \qquad\qquad + \max_{p \in \mathcal{P}_s(\mu_i')} \pi_s^*(p; \mu_i') \times \Big\{ \tilde{M}(i, a'+p, 0, \chi; \psi') - \tilde{\Delta}(i, a', h, \chi; \psi') \Big\} \Bigg] \Bigg\} \\
& + e\,(1-h) \Bigg\{ \max_{a' \geq a_0} u(c, l_0, h_0, \chi_i) + \beta \mathbb{E}_{\psi'|\psi} \Bigg[(1-\delta) \Big\{ V(i, 1, w, a', 0, \chi; \psi') \\
& \qquad + \max_{p \in \mathcal{P}_b(\mu_i'), a' \geq \alpha p} \pi_b^*(p; \mu_i') \times \{ V(i, 1, w, a'-p, 1, \chi; \psi') - V(i, 1, w, a', 0, \chi; \psi') \} \Big\} \\
& \qquad + \delta \Big\{ \tilde{M}(i, a', 0, \chi; \psi') + \max_{p \in \mathcal{P}_b(\hat{\mu}_1), a' \geq p} \pi_b^*(p; \mu_i') \times \{ \tilde{\Delta} \left(i, a'-p, 1, \chi; \hat{\psi} \right) \\
& \qquad\qquad + V(i, 1, w, a'-p, 1, \chi; \hat{\psi}) - \tilde{M}(i, a', 0, \chi; \hat{\psi}) \} \Big\} \Bigg] \Bigg\} \\
& + eh \Bigg\{ \max_{a' \geq a_1} u(c, l_0, h_1, \chi_i) + \beta \mathbb{E}_{\psi'|\psi} \Bigg[(1-\delta) \Big\{ V(i, 1, w, a', 1, \chi; \psi') \\
& \qquad + \max_{p \in \mathcal{P}_s(\mu_i')} \pi_s^*(p; \mu_i') \times \{ V(i, 1, w, a'+p, 0, \chi; \psi') - V(i, 1, w, a', 1, \chi; \psi') \} \Big\} \\
& \qquad + \delta \Big\{ \tilde{\Delta}(i, a', 1, \chi; \psi') + \max_{p \in \mathcal{P}_s(\mu_i')} \pi_s^*(p; \mu_i') \times \{ \tilde{M}(i, a'+p, 0, \chi; \psi') - \tilde{\Delta}(i, a', 1, \chi; \psi') \} \Big\} \Bigg] \Bigg\} \Bigg\},
\end{aligned}
$$

where

$$a + ew + (1-e)b = c + (1-h)\mathbf{1_{\{k=i\}}} \rho_i^*(\mu_i) + \frac{a'}{1+r}.$$

- We can show that the operator T_V maps a function from $\mathcal{V}$ into $\mathcal{V}$, where $\mathcal{V}$ is the set of continuous and bounded functions on the appropriate domain, assuming a bounded and continuous utility function. From the definition of $\tilde{\Delta}(\cdot)$ and $\tilde{M}(\cdot)$, it is clear that if $V \in \mathcal{V}$, then $\tilde{\Delta}\,(i, a, h, \{\chi_i'\}_{i \in I}; \psi) = \tilde{\Delta}\,(i, a, h, \{\chi_i'\}_{i \in I}; z_i, \mu_i)$ and $\tilde{M}(i, a, h, \{\chi_i'\}_{i \in I}; \psi) = \tilde{M}(i, a, h, \{\chi_i'\}_{i \in I}; z_i, \mu_i)$. Substituting $\tilde{\Delta}\,(i, a, h, \{\chi_i'\}_{i \in I}; z_i, \mu_i)$ and $\tilde{M}(i, a, h, \{\chi_i'\}_{i \in I}; z_i, \mu_i)$ into the definition of T_V, we get $T_V : \mathcal{V} \to \mathcal{V}$.

- $\mathcal{V}$ is a complete metric space. Verifying Blackwell's sufficiency conditions, one can easily show that $T_V : \mathcal{V} \to \mathcal{V}$ is a contraction. Therefore, there exists a fixed point, $V^* \in \mathcal{V}$. This is a solution to the household's problem, and depends on the aggregate state only through $\{z_i, \mu_i\}_{i \in I}$.

C Computational Appendix—Not For Publication

In this section, we describe the details of our computational procedure. We employ a nested fixed point algorithm to estimate the model and match model generated moments to their empirical counterparts. In Section 2, we have shown that our model admits a block recursive equilibrium and the value functions, policy functions, and market tightness functions depend on the aggregate state of the economy ψ, only through the exogenous stochastic shocks. This property ensures that we can solve for the equilibrium price schedules in the labor and housing markets without solving the problems of the households. The algorithm to solve for a block recursive equilibrium of the model consists of two stages that we explain below. The state space is discretized.

C.1 Supply Side Problems

In the first stage, we solve the supply side problems to derive the market tightness functions. Recall that the value function of a firm matched with a worker in location i is given by:

$$J^i(w;\psi) = z_i - w + \frac{1-w}{1+r}\mathbb{E}J^i(w,\psi').$$

We start with a guess for J^i such that J^i depends on ψ only through the exogenous labor productivity shocks. The functional equation above is a contraction and has a unique fixed point. By construction, the fixed point is independent of the endogenous distribution of households across locations. Using the free-entry condition in the labor market presented below, we derive the corresponding labor market tightnesses in all the active submarkets:

$$\kappa = q_l\left[\theta_l^i(w;\psi)\right]J^i(w;\psi) \quad \text{for all } w \text{ with } \theta_l^i(w;\psi) > 0.$$

Similarly, we use the value functions of the intermediaries in the housing market together with the free-entry conditions to determine the rental rates and the price-probability schedules. Free entry in the rental market implies $\mu_i = L^i(\psi)$. Substituting this in the functional equation for L^i, we obtain the following equation, which can be easily solved for $\rho^i(\psi)$:

$$\mu_i = \rho^i(\psi) + \frac{1-\gamma}{1+r}\mathbb{E}_\mu\mu_i'.$$

The free-entry condition in the market in which REMs are house buyers is given by:

$$k = q_s \left[\theta_s^i \left(p; \psi\right)\right] \left[R^i \left(\psi\right) - p\right] \quad \text{for all } p \ \text{ with } \ \theta_s^i \left(p; \psi\right) > 0.$$

Using the free entry condition that $\mu_i = R^i \left(\psi\right)$, we obtain a closed-form solution for θ_s^i. In every active submarket ($\theta_s^i > 0$), the following relation holds:

$$\theta_s^i \left(p; \psi\right) = q_s^{-1} \left(\frac{k}{\mu_i - p}\right).$$

Lastly, recall that the value function of an REM with a house is given by:

$$R^i \left(\psi\right) = \max_p q_b \left[\theta_b^i \left(p; \psi\right)\right] p + \frac{1 - q_b \left[\theta_b^i \left(p; \psi\right)\right]}{1 + r} \mathbb{E} R^i \left(\psi'\right).$$

Combining the free-entry condition, $\mu_i = R^i \left(\psi\right)$, and the fact that each submarket with $\theta_b^i \left(p; \psi\right) > 0$ should deliver the same value to REMs, we obtain the following expression

$$\mu_i = q_b \left[\theta_b^i \left(p; \psi\right)\right] p + \frac{1 - q_b \left[\theta_b^i \left(p; \psi\right)\right]}{1 + r} \mathbb{E} \mu_i' \ \text{ for all } p \ \text{ with } \ \theta_b^i \left(p; \psi\right) > 0,$$

and we can solve for $\theta_b^i \left(p; \psi\right)$.

C.2 Household's Problem

The second stage solves the various problems — consumption-saving, job search, migration decision, and housing transaction — that a household faces at various stages within a period. We use standard value function iteration techniques to solve these problems. In solving every problem, the relevant choice set is discretized. We start with a guess for the value functions at the consumption-savings stage and solve the problems at the job search, migration and housing search stages. Using the policy rules obtained, we update our guess for the value functions at the consumption-savings stage according to the relevant Bellman equations. We repeat the procedure until the value functions converge to the fixed point of the Bellman equation.

C.3 Overview of the Algorithm

Our procedure can be summarized as follows:

1. **Loop 1**—Guess a vector of the structural parameters Θ.

(a) Compute the rental market prices $\rho^{i}(\psi)$ and the price-probability schedules in the housing market $\theta_{b}^{i}(p;\psi)$ and $\theta_{s}^{i}(p;\psi)$.

(b) **Loop 2**—Make an initial guess for the value functions of the household in the consumption-savings stage U_{0}^{i} and W_{0}^{i}.

   i. Solve the job search problem for unemployed households and obtain D^{i}.

   ii. Solve the migration problem to obtain the decision rule for migration.

   iii. Solve the housing market problem for homeowners and renters to obtain the price posting behavior of participants.

   iv. Using the policy rules for job search, migration and house selling and buying behaviors, obtain U_{1}^{i} and W_{1}^{i} according to the relevant Bellman equations.

   v. If for each location i, $\| W_{1}^{i} - W_{0}^{i} \| < \epsilon_{V}$ and $\| U_{1}^{i} - U_{0}^{i} \| < \epsilon_{V}$, end Loop 2, otherwise set $U_{0}^{i} = U_{1}^{i}$ and $V_{0}^{i} = V_{1}^{i}$ and go to i.

2. Simulate the economy and check that all the free-entry conditions hold.

3. Obtain long-run averages of model generated moments $\mathcal{M}^{MODEL}$.

4. If the moments satisfy $\sum \left(\frac{\mathcal{M}_{i}^{MODEL} - \mathcal{M}_{i}^{DATA}}{\mathcal{M}_{i}^{DATA}} \right)^{2}$, end Loop 1. Otherwise, return to 1.

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