Fire-Sale Spillovers and Systemic Risk

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Abstract

We reveal and track over time the factors making the financial system vulnerable to fire sales by constructing an index of aggregate vulnerability. The index starts increasing in 2004, before any other major systemic risk measure, more than doubling by 2008. The fire-sale-specific factors of deleveraging speed and concentration of illiquid assets account for the majority of this increase. Individual banks’ contributions to aggregate vulnerability are an excellent five-year-ahead predictor of SRISK, one of the most prominent systemic risk measures. Had our estimates been available at the time, they would have been a useful early indicator of when and where vulnerabilities were building up.

Key words: systemic risk, fire-sale externalities, leverage, linkage, concentration, bank holding company

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1 Introduction

Fire sale spillovers have long been recognized as a potentially important source of contagion in financial markets and therefore a systemic risk concern. The mechanisms, systemic implications, and welfare costs of fire sales have been abundantly studied in the theoretical literature. In contrast, the empirical literature is understandably thinner, as it is difficult to conclusively identify fire sales. A few notable papers have documented the existence and severity of fire sale spillovers for particular episodes or asset classes by exploiting one-time changes in the environment or specific institutional peculiarities that allow for credible identification strategies. From an aggregate welfare perspective, however, we are ostensibly more concerned with fire sales that affect a large portion of the financial sector and many different markets simultaneously, especially in states of high marginal utilities — crises being the paradigmatic example. Clean identification in such turbulent times is quixotic at best. Even if possible, it would be too late to do much about them in terms of welfare save for costly liquidity or other interventions.

A more promising complementary goal is to understand the ex ante vulnerability of the system to fire sales, especially to those with aggregate consequences. In addition to circumventing the issue of identification, if detection of vulnerability can be done far enough in advance, then it may be possible for the affected parties and policymakers to intervene before the fire sales materialize. Detecting ex-ante vulnerabilities comes with its own set of challenges. What are the factors that make the financial system vulnerable to fire sales? Can we track them over time? Is it possible to predict not only when, but where in the financial sector vulnerabilities lurk?

In this paper, we address these questions by constructing an index of aggregate vulnerability to fire sales of large bank holding companies, the core of the U.S. financial system. The index decomposes additively into each bank’s “systemicness” (its contribution to a fire sale) as well as multiplicatively into aggregate versus cross-sectional factors that drive fire-sale vulnerability. We find that the aggregate vulnerability index (AV, for short) starts increasing already in 2004, before any other major systemic risk measure. It then rises steadily, more than doubling before the crisis. After the crisis, AV decreases equally dramatically, ending in late 2016 at roughly 20 percent of its initial level in 2000. This indicates that the the U.S. banking system has materially reduced its vulnerability to fire

1 Acharya et al. (2009); Brunnermeier (2009); Caballero (2010); Duffie (2010); Shleifer and Vishny (2011); Hanson et al. (2011); Ellul et al. (2014).

2 Shleifer and Vishny (1992); Allen and Gale (1994); Mitchell et al. (2007); Acharya et al. (2009); Brunnermeier and Pedersen (2009); Gromb and Vayanos (2010); Diamond and Rajan (2011).

3 Coval and Stafford (2007); Mitchell et al. (2007); Ellul et al. (2011); Merrill et al. (2012); Feldhütter (2012); Mitchell and Pulvino (2012).
We identify the fire-sale specific factors of deleveraging speed and concentration of illiquid assets which jointly account for 50% of the growth of AV and 40% of its variance between the beginning of our sample and 2008q3, when AV peaks. Using dynamic panel regressions and real-time data to minimize look-ahead bias, we show that individual banks’ contributions to AV — which we call “systemicness” — are an excellent five-year-ahead predictors of SRISK, one of the most prominent and widely used measures of firm-specific systemic risk and itself a validated predictor of negative crisis outcomes in the cross-section (Brownlees and Engle, 2016). Even after controlling for contemporaneous SRISK, ΔCoVaR, and other risk measures, an increase in systemicness of 1 percent is associated with an increase in SRISK of 2.7 percent five years later with a 1 percent level of statistical significance. Had they been available at the time, our measures would therefore have been a useful early indicator of when and where vulnerabilities were building up.

Our analysis builds on the cross-sectional “vulnerable banks” framework of Greenwood, Landier, and Thesmar (2015), adapting it to a panel analysis to track and dissect vulnerabilities over time as well as across banks. The framework takes as given banks’ leverage, asset holdings, asset liquidation behavior, and price impact of liquidating assets. It then considers a hypothetical large negative shock that leads to an increase in leverage. Banks respond by selling assets and paying off debt to at least partially retrace the increase in leverage. These asset fire sales have a price impact that depends on the liquidity of the assets and the amount sold. Any bank that happens to hold assets similar to those that were fire-sold, even if not initially shocked, will see the value of these asset holdings decline, a fire-sale spillover. AV is the sum of all of these spillover losses — as opposed to the initial direct losses — as a share of the total equity capital in the system.

We offer three contributions relative to the framework of Greenwood et al. (2015) that are instrumental in deriving and interpreting our empirical results. First, we separate the role of aggregate versus cross-sectional drivers of fire-sale vulnerability by decomposing AV into aggregate factors and a cross-sectional measure we call “illiquidity concentration.” While it is well known that size and leverage — two of the aggregate factors — are relevant to systemic risk for various reasons, we isolate in illiquidity concentration a factor specific to fire-sale spillovers. Its magnitude and therefore the vulnerability of the system to fire sales depends on the cross-sectional distributions of bank size, leverage, and holdings of illiquid assets.

Second, in Greenwood et al. (2015), a bank’s pre-shock leverage is assumed to be its post-shock target leverage, and the bank is assumed to fully adjust back to its target leverage following a shock. While this is innocuous for the single cross-section of banks that
they consider (because changing the target or the degree of partial adjustment amounts to a simple rescaling), it is a strong assumption for a dynamic application with as long a sample period as ours. Maintaining the assumption would (i) implicitly interpret observed variation over time in a bank’s leverage as variation in the bank’s leverage target and (ii) rule out variation over time in the degree of adjustment toward target following a shock. We generalize this part of the framework and assume that, in response to shocks, a bank partially adjusts leverage toward a latent target, with time variation both in the target and in the adjustment speed. Importantly, we are able to integrate the partial adjustment seamlessly into the AV framework of Greenwood et al. (2015), providing a new dynamic factor in the decomposition: the adjustment speed to target leverage. Spillover losses and therefore vulnerability to fire sales are increasing in the adjustment speed, i.e. how quickly banks delever after a shock.

Third, we provide a theoretical foundation to the reduced form AV framework. We develop a rational expectations model of optimal dynamic capital structure with liquidity costs that generates the bank behavior assumed in calculating AV. Having such a model addresses the Lucas critique, offers a concrete set of assumptions that justify the original AV framework of Greenwood et al. (2015), and precisely explains how those assumptions need to be modified to obtain the dynamic panel version of AV with partial leverage adjustment that we construct. In the model, a bank dynamically chooses a portfolio of assets, dividend distributions, and equity capital issuance in order to maximize shareholder value. If equity issuance is not allowed and assets have no liquidation costs, the bank behaves as in the original AV framework of Greenwood et al. (2015), maintaining a fixed leverage target. When equity capital issuance is subject to a fixed cost and asset liquidations have a positive and heterogeneous price impact, the bank instead behaves as in the new AV version we construct, partially adjusting to a leverage target with a time-varying speed of adjustment. Neither regulatory capital nor liquidity constraints are needed in the model to generate leverage targets. Nevertheless, adding them preserves the mapping between the model and AV, changing only the value of the leverage target and the adjustment speed towards it.

We apply the AV framework to a quarterly panel of 200 large U.S. bank holding companies (BHCs) from 2000 to 2016. We focus on BHCs for several reasons: they are a large fraction of the entire U.S. financial sector, including not only commercial banks but also large broker-dealers and other financial institutions; they are a good window into the broader shadow banking system; detailed regulatory data on their balance sheets is pub-

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4 Throughout the paper we also refer to BHCs as “banks” for simplicity.
5Bord et al. (2012); Cetorelli et al. (2012); Adrian et al. (2015); Glode and Opp (2016).
licitly available; and they were forced to fire-sell assets in the face of deteriorating equity capital during the financial crisis.\(^6\)

Looking at the drivers of overall vulnerability, we find that each of the AV factors contributes differently to the total and that the contributions change over time. Size and leverage — known factors of systemic risk — show the expected trends, increasing in the pre-crisis period and decreasing towards the end of the sample. The two new factors that we identify as specific to fire-sale spillovers — adjustment speed and illiquidity concentration — also play important roles in the evolution of AV and in the cross-section of bank systemicness.

Leverage adjustment speed shows a slow decreasing trend until 2006, before increasing by roughly 70 percent and causing AV to spike in late 2008. This is notable since, in our estimation, we control for any adjustments via equity issuance. The increase in estimated adjustment speed during the crisis therefore captures greater delevering through balance sheet contraction, consistent with fire sales. Illiquidity concentration, the measure capturing vulnerabilities stemming from the cross-sectional distribution of assets and their liquidities across banks with different size and leverage, has a positive trend starting in late 2002, giving an even earlier signal of systemic risk buildup than the overall AV index. Until early 2007, illiquidity concentration increases by roughly 25 percent.

We confirm the importance of the cross-sectional illiquidity concentration with two further exercises. First, we decompose the variance of AV into the contributions of the variances (and covariances) of the constituent factors; illiquidity concentration, after size, is the second greatest contributor to variation in AV pre-crisis and it is the most stable factor in terms of its contribution across pre-crisis/crisis/post-crisis subsamples. Second, we compare actual AV to a hypothetical AV for a counterfactual banking system with the same aggregate portfolio and leverage held by homogeneous banks; over the majority of our sample, AV is roughly 20 percent higher due to the heterogeneity of banks in the data.

In addition to being the first measure of systemic risk specific to fire-sale spillovers, AV has other unique features that complement and improve upon other existing systemic risk measures. First, AV is constructed from the bottom up using detailed balance sheet information of individual asset classes at each bank. In contrast, the predominant strategy in the literature relies on market prices or macroeconomic aggregates to build top-down indicators. The more than 30 measures considered by Bisias et al. (2012) and Mitra et al. (2011) all use market prices or macroeconomic aggregates as key inputs. The three measures that also use balance sheet information rely only on book equity, total assets and

\(^6\)Bernanke (2009).
total liabilities; none use holdings disaggregated by asset class.\footnote{Acharya et al. (2012) and Billio et al. (2012) use a combination of asset prices and book equity for each institution examined while Fender and McGuire (2010) use consolidated balance sheet information for European banks aggregated geographically by country. Although not among the surveyed articles, Pierret (2014) uses a combination of market prices and a less granular subset of short-term assets and liabilities to study the solvency-liquidity nexus of banks rather than systemic risk due to fire sales.} Although there are many advantages to using market prices, one important disadvantage is that volatilities and risk premia are usually compressed just prior to a crisis, pushing models based on market prices towards low values of systemic risk despite the underlying buildup in vulnerability. In contrast, AV signals increased systemic risk ahead of all other measures of which we are aware.

Finally, our measure is — and already has been — immediately useful for policymakers and regulators. The designation of systemically important financial institutions (SIFIs) has become an active if controversial area in post-crisis regulation. The Dodd-Frank Act requires, among other standards, that a financial firm be designated a SIFI when it “holds assets that, if liquidated quickly, would cause a fall in asset prices and thereby [...] cause significant losses [...] for other firms with similar holdings,” a description that almost exactly matches the exercise in this paper.\footnote{Final rule and interpretive guidance to Section 113 of the Dodd-Frank Wall Street Reform and Consumer Protection Act.} An earlier version of our measure was used in the designation of AIG, Metlife, and other companies as systemically important by the Financial Stability Oversight Council (FSOC), and in the evaluation and dismissal of Fidelity and other asset manager’s cases.\footnote{United States Department of the Treasury (2012); Financial Stability Oversight Council (2015); Financial Stability Board (2016); U.S. House of Representatives (2016).} It has also been adapted to other countries and markets.\footnote{Levy-Carciente et al. (2015); Zhou et al. (2016); Fricke and Fricke (2017); McKeown et al. (2017); Ellul et al. (2018).}

Bank stress testing has become another standard tool, yet current implementations mainly consider initial individual losses at large financial institutions, and all but ignore the second-round losses that can create systemic risk.\footnote{Current stress tests do consider macroeconomic shocks that could exogenously embed the second-round shocks (Tarullo, 2017). However, they are assumed rather than derived. Greenlaw et al. (2012) argue that in their current form, stress tests are more micro- than macro-prudential. Cont and Schaanning (2017) use a modified version of the Greenwood et al. (2015) framework to make the case for macroprudential stress testing. Aymanns et al. (2017) argue for the inclusion of fire-sale effects like the ones we propose in macroprudential stress tests.} Our analysis can be interpreted as a stylized macro-prudential stress test in which the regulator provides a scenario (the initial exogenous shocks to assets) and the framework computes spillover losses for the system as a whole. Even though the framework is equally easy to implement for any combination of shocks (any scenario), we calculate the time series of AV by applying the same
shock every quarter, allowing us to understand in a consistent way if changes in the system from one quarter to the next have affected the vulnerability of the system to fire sales.

Additionally, our framework can easily produce counterfactuals that can be used to evaluate past policies or proposals for future reform. For example, Duarte and Eisenbach (2014) evaluate how vulnerable the system would have been without the Troubled Asset Relief Program (TARP) and how close the actual implementation of the program was to the framework’s optimal policy.

The rest of the paper is structured as follows. In Section 2, we present the framework used to calculate fire-sale spillovers. In Section 3, we develop and solve a model of an optimizing bank that produces the bank behavior assumed in the framework of Section 2. In Section 4, we describe the estimation of leverage targets and adjustment speed. In Section 5 we present and discuss the results on fire-sale spillovers. In Section 7 we document the predictive power of the measures.

2 Framework

To calculate potential spillovers from fire sales, we build on the “vulnerable banks” framework of Greenwood et al. (2015). The framework assumes a simple fire-sale scenario where after an exogenous shock to assets banks suffer direct losses and sell assets to delever. Aggregate fire sales have price impact which represents another endogenous “shock” to assets, causing the indirect fire-sale spillovers that are the focus of the analysis. We emphasize where we deviate materially from Greenwood et al. (2015).

Banks are indexed by $i = 1, \ldots, N$ and assets (or asset classes) are indexed by $k = 1, \ldots, K$. In period $t$, bank $i$ has total assets $a_{it}$ with portfolio weight $m_{ikt}$ on asset $k$ such that $\sum_k m_{ikt} = 1$. On the liabilities side, bank $i$ has debt $d_{it}$ and equity capital $e_{it}$, resulting in leverage $b_{it} = d_{it}/e_{it}$. We let $a_t = \sum_i a_{it}$ denote the total assets of the system, $e_t = \sum_i e_{it}$ system equity capital, $d_t = \sum_i d_{it}$ system debt, and $b_t = d_t/e_t$ system leverage. Other than differentiating between debt and equity, we are making no further assumptions on banks’ liabilities.

We start with an initial vector of given shocks $(f_{1t}, \ldots, f_{Kt})^\top > 0$ across asset classes.\footnote{We are interested in negative shocks leading to sales and losses; for notational simplicity, we denote these quantities as positive numbers.} The shock leads to direct losses for bank $i$ given by $a_{it} \sum_k m_{ikt} f_{kt}$, which increase the bank’s leverage.\footnote{Banks hold some cash which should not receive a shock; to simplify the analysis, we subtract all cash holdings from both assets and debt, focusing on net leverage. We assume that all remaining assets are marked to market. We show in Section 6.1 that not marking to market loans does not significantly affect the}
leads to a “passive” leverage given by

\[ b_{it}^p = \frac{d_{it}}{e_{it} - a_{it} \sum_k m_{ikt} f_{kt}}, \]  

(1)

Greenwood et al. (2015) assume that following the shock, banks return to their initial (pre-shock) leverage \( b_{it} \), motivated by the evidence of Adrian and Shin (2010, 2011). This is a strong assumption, however, especially for a dynamic application with a long sample period like ours: (i) it requires all observed variation in a bank’s leverage to be interpreted as variation in the bank’s leverage target and (ii) it rules out variation in the adjustment speed over time. We therefore generalize this part of the framework and assume that banks’ active leverage adjustment in response to shocks is given by a standard partial adjustment model

\[ b_{it+} = \lambda_t b_{it} + (1 - \lambda_t) b_{it}^p, \]  

(2)

where the leverage \( b_{it+} \) after shock and active adjustment is a convex combination of the passive leverage \( b_{it}^p \) and a leverage target \( b_{it}^* \) with \( \lambda_t \) representing the adjustment speed towards the target.\(^{14}\) This behavior is consistent with the optimizing model in Section 3.\(^{15}\)

Since our interest is in fire-sale spillovers, we restrict attention to leverage adjustments that involve selling assets and reducing debt but not raising equity. However, we do not assume that raising equity to reduce leverage is not possible, nor that all leverage adjustment is done through fire-sales. The model in Section 3 shows that depending on the size of the shock, to reduce leverage, banks can find it optimal to just sell assets or to issue equity and sell assets at the same time. In either case, the mapping between the model and AV is preserved. In Section 4, when we empirically estimate leverage targets, we count all equity issuance (including government injections) as part of passive leverage adjustment. Last, even though issuing equity is allowed in the framework, we are interested in systemic risk, which is usually accompanied by distress in capital markets and weak macroeconomic conditions. In such a scenario, raising equity can be limited, difficult or undesirable for economic, signaling or other reasons (Shleifer and Vishny, 1992).

Using the definition of passive leverage in equation (1) and with partial adjustment vulnerability index. For further discussion, see Allen and Carletti (2008); Sapra (2008); Ellul et al. (2014).

\(^{14}\)In order to obtain a robust estimate, we assume that the adjustment speed \( \lambda_t \) varies over time but not across banks. See Section 4 for details.

\(^{15}\)Greenwood et al. (2015) discuss a similar partial adjustment model in their Appendix B.2. Section 4 discusses in detail how we estimate \( b_{it}^* \) and \( \lambda_t \).
according to (2), the amount of cash \( x_{it} \) that needs to be raised to pay down debt satisfies

\[
\frac{d_{it} - x_{it}}{e_{it} - a_{it} \sum_k m_{ikt} f_{kt}} = \lambda_t b^*_t + (1 - \lambda_t) \frac{d_{it}}{e_{it} - a_{it} \sum_k m_{ikt} f_{kt}}.
\]

Solving for \( x_{it} \) we arrive at a simple expression made up of two parts:

\[
x_{it}^\text{total} = \lambda_t \frac{b_{it} - b^*_t}{b_{it} + 1} a_{it} + \lambda_t b^*_t a_{it} \sum_k m_{ikt} f_{kt}
\]

The first part, \( x_{it}^\text{base} \), is a baseline adjustment towards target that occurs even in the absence of any shock. If leverage is above target, \( b_{it} > b^*_t \), there are asset sales, \( x_{it}^\text{base} > 0 \), and vice versa for purchases. The second part, \( x_{it}^\text{shock} \), is an adjustment in response to the shock — the delevering we are interested in. Empirically, the shock part will dominate the baseline adjustment for appropriate stress scenarios. For example, with our data, a shock of 1% to all assets, \( f_{kt} = 0.01 \), results in a baseline part that is two orders of magnitude smaller than the shock part.\(^16\) For our analysis, we therefore focus on the shock part and denote it simply by \( x_{it} \).\(^17\)

The cash \( x_{it} \) is raised by selling assets. These asset sales have to be distributed in some way across the assets held by the bank. We denote by \( \tilde{m}_{ikt} \), the amount of each asset that the bank sells as a share of total sales, i.e., \( x_{ikt} = \tilde{m}_{ikt} x_{it} \). We assume for our benchmark that banks sell in proportion to their existing portfolio weights, \( \tilde{m}_{ikt} = m_{ikt} \) (as in Greenwood et al., 2015), to be agnostic about the relative importance of several opposing forces that could lead to more sales of relatively liquid or illiquid assets.\(^18\) We discuss in Section

\(^{16}\)We have \( \sum_i \sum_t \frac{b_{it} - b^*_t}{b_{it} + 1} = -0.005 \) and \( \sum_t \sum_i b^*_t \times 0.01 = 0.113.\)

\(^{17}\)Note that the shock part is analogous to the expression in Greenwood et al. (2015), \( x_{it}^\text{GLT} = b_{it} a_{it} \sum_k m_{ikt} f_{kt} \), where banks are simply assumed to return to their pre-shock leverage. The difference in our framework is that the adjustment is partial and to a target, \( \lambda_t \times b^*_t \), instead of full and to pre-shock leverage, \( 1 \times b_{it-1} \). For sufficiently large shocks, some banks may be selling all of their assets. We take this into account in our empirical implementation by using \( x_{it} = \min \{ a_{it}, \lambda_t b^*_t a_{it} \sum_k m_{ikt} f_{kt} \} \).

\(^{18}\)There is vast theoretical literature highlighting different dynamic trade-offs for portfolio liquidations. Liquid assets have an option value if markets can be even more illiquid in the future (Ang et al., 2014). The volatility and speed of mean-reversion of asset prices also play a role in shaping incentives of a risk averse manager to dispose of them (Almgren and Chriss, 1999, 2001; Gârleanu and Pedersen, 2016). Independent of the elasticity of prices with respect to the amount sold, whether price impacts are transitory or permanent can alter the timing of liquidation of assets with different degrees of liquidity (Almgren and Chriss, 1999, 2001; Brown et al., 2010). The dynamic considerations for asset liquidations interact strongly with the decision of what assets to sell first. Selling the most liquid assets first has the important advantage of minimizing the price impact of fire sales, which may reduce total losses. Illiquid assets could be optimally sold first if their price is expected to decrease more in the future irrespective of their liquidity, or precisely because of their liquidity, as more illiquid assets there are being fire sold will have a lower future price (Gârleanu and Pedersen, 2013). The elasticity of demand for assets outside of the banking sector can also be a key determi-
6.2 several alternatives for the liquidation strategy \( \bar{m}_{ikt} \) and how they affect the results. Summing the sales of asset \( k \) across banks then implies aggregate sales of asset \( k \) given by

\[
y_{kt} = \sum i m_{ikt} x_{ikt} = \sum i m_{ikt} \lambda_i b^* t a_i t \sum k' m_{ik't} f_{k't}.
\]  

(3)

Next, we assume that the asset sales have price impact that is linear in the volume sold. This is the predominant assumption in the empirical literature and seems to fit the patterns of the data well.\(^{19}\) In addition, we do not consider cross-asset price impacts, e.g. selling agency MBS has no direct impact on the price of corporate bonds. The asset classes we consider are sufficiently different that the first-order effects should be consistent with no cross-asset price impacts.\(^{20}\) Specifically, the price impact of asset \( k \) is proportional to its illiquidity \( \ell_k \) and inversely proportional to the wealth \( w_t \) of potential buyers of fire-sold assets (Shleifer and Vishny, 1992). Aggregate sales of \( y_{kt} \) dollars of asset \( k \) therefore have price impact \( (\ell_k / w_t) y_{kt} \).\(^{21}\) Combining this with the expression for aggregate sales in (3),

\(^{19}\)Almost all empirical papers that identify fire sells cited in footnote 3 have linear pricing. In the theoretical literature, the first-round price impact is almost always proportional to the amount sold, sometimes with multipliers arising only in subsequent liquidation rounds (Kyle, 1985; Glosten and Harris, 1988; Bertsimas and Lo, 1998; Obizhaeva and Wang, 2013). Quadratic and non-linear costs have also been used and estimated (Heaton and Lucas, 1996; Hasbrouck and Seppi, 2001; Almgren, 2003; Gârleanu and Pedersen, 2013; Kyle and Obizhaeva, 2016). On the other hand, over many days — which is the relevant horizon for AV — the non-linearities can smooth out and make price impacts are much closer to linear (Bouchaud, 2010).

\(^{20}\)In any case, the off-diagonal entries are difficult to estimate, but likely to be positive which would only exacerbate fire sale spillovers. Greenwood (2005) shows that in a model of limited arbitrage, similar assets in integrated markets can produce positive off-diagonal values. He uses Japanese stocks as an example.

\(^{21}\)This intuition follows directly from Shleifer and Vishny (1992). In their model, fire sold assets cannot be purchased by peers because they are also in distress (as in AV) and are instead purchased by “second best users” who have a lower valuation of the asset. The wealth of buyers is a key determinant of the price at which assets can be sold. In the original framework of Greenwood et al. (2015), the illiquidity is measured in units of percentage points of price change per dollar amount sold which is standard in the empirical literature (Amihud, 2002). However, as noted in Acharya and Pedersen (2005), this is inappropriate when working with longer periods where the relevant markets grow over time (see also Comerton-Forde et al., 2010; Hameed et al., 2010). We therefore decompose the illiquidity of Greenwood et al. (2015) as \( \ell_{kt}^{CLT} = \ell_k / w_t \) where \( \ell_k \) is a stationary measure of illiquidity for asset \( k \) expressed in percentage points of price change per dollar amount sold relative to dollar wealth available to purchase.
the fire-sale price impact for asset $k$ is given by

$$
\hat{f}_{kt} = \ell_k \frac{w_t}{a_t} \sum_i m_{ikt} \lambda_i b_{it}^* a_{it} \sum_k' m_{ik't} f_{k't}.
$$

Finally, the price impact of the fire-sale cause spillover losses to all banks holding the assets that were fire-sold, which we can calculate analogously to the very first step above as $a_{it} \sum_k m_{ikt} \hat{f}_{kt}$. Summing spillovers over all banks, we arrive at the total spillover losses $L_t$ suffered by the banking system which, can also be written in matrix form

$$
L_t = \sum_i a_{it}' \sum_k m_{i'kt} \ell_k \frac{w_t}{a_t} \sum_i m_{ikt} \lambda_i b_{it}^* a_{it} \sum_k' m_{ik't} f_{k't}
$$

where $1^\top$ is a row vector of ones, $F_t = (f_{1t}, \ldots, f_{Kt})^\top$ is the vector of shocks, $M_t$ the $N \times K$ matrix of portfolio weights, $A_t$ and $B_t^*$ are $N \times N$ diagonal matrices of, respectively, total assets and leverage targets, and $L$ is a $K \times K$ diagonal matrix of price impacts. It is important to note that $L_t$ captures only the indirect losses due to spillovers. It therefore does not include the direct losses due to the initial shock, given by $\sum_i a_{it} \sum_k m_{ikt} f_{k't}$. This makes our analysis different but complementary to the typical microprudential stress-test analysis that focuses on the direct losses for a given shock.

In principle, this first round of price declines in equation (4) can be thought of as a new (now endogenous) negative shock $\hat{f}_t$ that would induce banks to restart the delevering sequence. The process could then be repeated, potentially until convergence.²² In Section 6.3, we verify that our main results are virtually unchanged in a multi-round setup and study the variation in convergence speed over time.

We want to distinguish between the effects stemming from aggregate characteristics of the banking system and effects that arise due to the distribution of assets across banks. To do so, we denote by $a_{it} = a_{it} / a_t$ bank $i$’s assets as a share of system assets and by $b_{it}^* = b_{it}^* / \bar{b}_t^*$ bank $i$’s leverage target relative to the average leverage target $\bar{b}_t^* = \frac{1}{N} \sum_i b_{it}^*$. For the portfolio weights we denote by $m_{kt} = \sum_i m_{ikt} a_{it} / a_t$ the system portfolio weight for asset $k$ and by $\mu_{ikt} = m_{ikt} / m_{kt}$ bank $i$’s portfolio weight for asset $k$ relative to the system portfolio weight. The expression for total spillover losses $L_t$ in (5) can then be rearranged

²²Tepper and Borowiecki (2014) and Capponi and Larsson (2015) develop systemic risk measures based on how close the banking system is to being explosive due to high leverage and asset concentration. Braouezec and Wagalath (2017) study the fixed-point like equilibrium in a one-asset version of Greenwood et al. (2015).
as

\[ \mathcal{L}_t = \frac{a_t^2 b_t^*}{w_t} \sum_k \left[ m_{kt}^2 \ell_k \sum_i \left( \mu_{ikt} \beta_{iit}^* \alpha_{it} \sum_{k'} m_{ik't'} f_{k't'} \right) \right]. \]

The only variables not readily available to calculate \( \mathcal{L}_t \) are the price impacts \( \{\ell_k\}_{k=1}^K \) which are notoriously hard to estimate.\(^{24}\) We therefore normalize \( \mathcal{L}_t \) to 100 at the beginning of our sample period and treat it as an index, focusing on its changes over time rather than its level. Further, we choose the same shock across all assets, \( f_{kt} = f_t \) for all \( k \), to calculate an overall vulnerability of the system to spillovers while being agnostic about where a particular fire-sale episode may originate. In this case \( \sum_k m_{ikt} f_t = f_t \) so the exogenous shock \( f_t \) affects \( \mathcal{L}_t \) linearly. Since we are interested in studying changes in vulnerability over time, we need the shock to be constant, \( f_t = f \) for all \( t \) so that estimates are directly comparable. The magnitude of \( f \) then has no effect on the evolution of an index either so we drop it from the expressions below.

Based on the total spillover losses \( \mathcal{L}_t \) we define the following three measures of systemic risk.

**Aggregate vulnerability:** The fraction of system equity capital lost due to spillovers, \( \mathcal{L}_t/e_t \), captures the “aggregate vulnerability” (AV) of the system to fire-sale spillovers. It can be decomposed into four factors:\(^{25}\)

\[
\text{AV}_t = \frac{a_t}{w_t} \times (b_t + 1) \frac{b_t^*}{b_t} \times \lambda_t \times \sum_k \left[ m_{kt}^2 \ell_k \sum_i \left( \mu_{ikt} \beta_{iit}^* \alpha_{it} \right) \right]
\]

(6)

The first factor is the size of the system relative to the wealth of outside buyers; if the banking system grows faster than outside wealth then aggregate liquidity is lower and fire sales are more severe. The second factor combines two measures of leverage: aggregate leverage \( b_t + 1 = \sum_i a_{it}/\sum_i e_{it} \) since spillover losses relative to system equity are increasing in system leverage; and the average leverage target \( \bar{b}_t^* = \frac{1}{N} \sum_i b_{it}^* \) which captures how asset

\(^{23}\)Note that the sum over \( i' \) drops out since \( \sum_{i'} \alpha_{i'it} \mu_{i'kt} = 1 \).

\(^{24}\)Ellul et al. (2011) find a median price impact of 7.5 basis points per $10 billion for corporate bonds, with several basis points of variation depending on bond quality and other factors. Other empirical studies of the price impact of fire sales are Coval and Stafford (2007) for individual stocks, Jotikasthira et al. (2012) for emerging market stock indices and Merrill et al. (2012) for non-agency residential MBS. They find price impact estimates that are significantly higher than those for corporate bonds. However, the assets they study do not specifically fit our asset classes as well as corporate bonds (equities in tri-party repo are predominantly large caps and American or European indices).

\(^{25}\)Note that our decomposition differs slightly from the one in Greenwood et al. (2015). We compare the two versions below, after introducing the measures for individual banks and assets.
sales are increasing in the average leverage target. We combine them into a single term since the two measures and their effects on AV are not fundamentally different. The third factor is the adjustment speed from the partial adjustment model (2) since spillovers are larger if banks adjust more quickly towards target leverage.

The fourth factor, “illiquidity concentration,” captures how the cross-sectional distribution of assets, size and leverage across heterogeneous banks affects fire-sale vulnerability. Heterogeneity increases vulnerability if the cross-sectional correlations of $m_{kt}, \ell_k, \mu_{ikt}, \beta^*_it,$ and $\alpha_{it}$ are positive: the effect of asset class $k$ is large if it is (i) widely held with a high aggregate share $m_{kt},$ (ii) illiquid with a high $\ell_k,$ and (iii) concentrated in banks that are relatively levered and relatively large. If all banks were the same, equal to a representative bank with $\alpha_{it} = 1/N, \beta^*_it = 1, and \mu_{ikt} = 1$ for all $i$ and $k$, then illiquidity concentration collapses to $\sum_k m_{kt}^2 \ell_k.$ We study in Section 5.3 how important heterogeneity is quantitatively and find that it increases vulnerability by roughly 20 percent over most of our sample.

**Systemicness of bank $i$:** We define the systemicness of bank $i$ as the contribution to aggregate vulnerability of bank $i$, obtained by dropping the summation over $i$ in the expression for aggregate vulnerability (6). It can also be interpreted as the aggregate vulnerability resulting from a shock only to bank $i$. Highlighting the terms that are specific to bank $i$ we have:

$$SB_{it} = \frac{a_t}{\omega_t} (b_t + 1) \bar{b}_i \lambda_t \times \alpha_{it} \times \bar{\beta}^*_it \times \sum_k \left[ m_{kt}^2 \ell_k \mu_{ikt} \right]$$

(7)

The first term contains only aggregate factors so it does not vary across banks. The next factors are specific to bank $i$ and imply high systemicness if the bank (i) is large with a high $\alpha_{it}$, (ii) has a high leverage target $\beta^*_it,$ and (iii) has high “illiquidity linkage” by holding large and illiquid asset classes.

**Systemicness of asset $k$:** Similar to the measure for individual banks, we define the systemicness of asset $k$ as the contribution of asset $k$ to AV, equivalently obtained either by dropping the summation over $k$ in the expression for aggregate vulnerability (6), or as the aggregate vulnerability for a shock only to asset $k$ (with $f_{kt} = 0$ for $k' \neq k$). Highlighting

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26 Asset sales depend on both actual leverage and target leverage $b^*_it$. Under the partial adjustment framework (2), a bank’s actual leverage $b_{it}$ itself depends on its target leverage $b^*_it,$ at least on average. Once we ignore the empirically negligible adjustment towards target in the absence of shocks, sales depend only on $b^*_it$ (directly and indirectly through $b_{it}$).
the terms that are specific to asset class $k$ we have:

$$
\text{SA}_{kt} = \frac{a_t}{w_t} (b_t + 1) \tilde{B}_t^* \lambda_t \times m_{kt} \times \sum_{k'} \left[ m_{k't}^2 \ell' \sum_i (\mu_{ikt} \beta_{it}^* \alpha_{it}^t \mu_{ikt}) \right].
$$

(8)

Again, the first factors are aggregate and do not vary across assets. The rest of the factors show that a specific asset class $k$ is systemic if it is large in aggregate and if it is held by systemic banks.

**Vulnerability of bank $i$:** Instead of summing the spillover losses across all banks as in equation (5) and taking the ratio to total equity capital, we can consider the spillover losses suffered by an individual bank relative to its individual equity capital. This vulnerability of bank $i$ is given by

$$
\text{VB}_{it} = \frac{a_t}{w_t} \tilde{B}_t^* \lambda_t \times (b_{it} + 1) \times \sum_{k'} \left[ \mu_{ikt} m_{ikt}^2 \ell \sum_{i'} (\mu_{i'kt} \beta_{it}^* \alpha_{i't}^t) \right].
$$

(9)

Bank $i$ is more vulnerable if it is more levered or if it holds assets that are large, illiquid, and held by large and levered banks.

**Comparison to GLT:** Greenwood et al. (2015) use an additive decomposition of AV into the sum of individual banks’ systemicness,\(^{27}\)

$$
\text{AV}_t = \sum_i \text{SB}_{it} = \sum_i \gamma_{it}^{\text{GLT}} \lambda_t b_{it}^* a_{it}^{-1},
$$

(10)

where $\gamma_{it}^{\text{GLT}}$ is the Greenwood et al. (2015) connectedness of bank $i$:

$$
\gamma_{it}^{\text{GLT}} = \sum_{k} \left( \sum_{i'} a_{i't} m_{i'kt} \right) \frac{\ell'_k}{w_t} m_{ikt}
$$

Substituting $\gamma_{it}^{\text{GLT}}$ into (10) and separating aggregate terms from cross-sectional terms, we arrive at our multiplicative decomposition of AV already shown in equation (6),

$$
\text{AV}_t = \frac{a_t}{w_t} \times (b_t + 1) \tilde{B}_t^* \lambda_t \times \sum_i \gamma_{it} \beta_{it}^* \alpha_{it}^t,
$$

where

\(^{27}\) We’re showing how the Greenwood et al. (2015) decomposition applies to our measure of AV, i.e. with partial adjustment and leaving out the shock $r_{it} = \sum_k m_{ikt} f_{kt}$ which we set constant for all $i$ and $t$. 

13
where $\gamma_{it}$ is our “illiquidity linkage” for bank $i$ (from equation 7) which differs from connectedness in Greenwood et al. (2015) by a factor $(a_t / w_t)^{-1}$.\footnote{Note that $\sum_{i} \alpha_{iklt} \mu_{ikt} = 1$.}

$$\gamma_{it} = \sum_{k} n_{kt}^2 \ell_{k} \mu_{ikt} = \left( \frac{a_t}{w_t} \right)^{-1} \gamma_{GLT}$$

We choose the multiplicative decomposition for three reasons: (i) it separates aggregate determinants of fire-sale vulnerability from cross-sectional determinants (illiquidity concentration); (ii) it lends itself more readily to our focus on changes in AV over time since it allows us to track the evolution of each multiplicative factor; and (iii) it separates the fire-sale specific factors of adjustment speed and illiquidity concentration from the size and leverage factors, which are known to affect systemic risk for various reasons.

### 3 A model of optimal dynamic capital structure with liquidity costs

We develop a model for a single bank that maximizes shareholder value over time by optimally choosing its portfolio of assets, dividend distributions, and equity capital issuance. Optimal bank behavior will imply that leverage satisfies the partial adjustment model in equation (2), and that the bank liquidates assets in response to large negative shocks. The model also generates a liquidation rule $\tilde{m}_{ik}$ for which the amount of asset $k$ that is liquidated is increasing in the initial portfolio share $m_{ik}$.

Time is continuous, with $t \in [0, \infty)$. Uncertainty is described by a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ under which a Brownian motion $W_t$ and a Poisson process $N_t$ are defined. The Poisson process has constant jump intensity $\eta > 0$ and its increments $dN_t$ are independent of the Brownian motion and can take the values 0 or 1.

The bank’s investment opportunities consist of a riskless bond with price $B_t$ and a risky asset with price $S_t$ that follow the exogenous processes

$$dB_t = rB_t dt \quad (11)$$

$$dS_t = S_{t-} (\mu dt + \sigma dW_t - f dN_t) \quad (12)$$

where $r$, $\mu$, $\sigma$ are positive constants, $f \in (0, 1)$, and the subscript $t-$ in $S_{t-}$ denotes the limit of $S_{\tau}$ as $\tau$ approaches $t$ from below. When a jump $dN_t = 1$ occurs, the risky asset loses a fraction $f$ of its value. These infrequent but large exogenous downward jumps in
the risky asset capture a non-diversifiable aggregate “crisis” or “disaster” risk of the kind we postulate in AV. We first solve the model with a single risky asset and then discuss the introduction of a second risky asset.

There are three frictions. First, the risky asset is not perfectly liquid, so we refer to it as the illiquid asset. Selling one dollar of the illiquid asset results in only $1 - \ell$ dollars for the bank, where $\ell \in (0, 1)$. We interpret this proportional cost as the price impact of selling from the point of view of an individual bank acting as a price taker. Without loss of generality, we assume that buying the illiquid asset can be done without price impact. A price impact that is linear in volume is common among some of the seminal papers in the literature. We model the price impact as temporary as in Sadka (2006); Almgren and Chriss (1999, 2001); Gârleanu and Pedersen (2013); Brown et al. (2010), who also include a second, permanent price impact component. We abstract from the permanent component because it is unrelated to the price pressure mechanism related to fire-sales.

The second friction is that issuing equity has a fixed cost. At any time $t$, if the bank wants to inject $I$ units of cash into the firm, it must raise $I + \kappa$ units of cash, where $\kappa > 0$ is the fixed cost of issuing equity. Given the fixed cost structure, it is immediate that equity will not be issued continuously, but only at discrete intervals. The third friction is that the bank cannot buy back equity. This assumption greatly simplifies the analysis and is justified empirically by the insignificant role that equity buybacks play in the data sample we use.

Let $X_t$ and $Y_t$ be the amounts of money invested in the bond and the illiquid asset, respectively. The amount $X_t$ invested in the bond can be thought of as a cash account with all cash invested into perfectly liquid and riskless bonds. For banks, we expect leverage to be positive (borrowing can be done by shorting the bond, $X_t < 0$). In this case, the balance

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29 We think of the price impact perceived by an individual bank as proportional to the asset’s illiquidity in aggregate.

30 The version with buying and selling costs is a renormalization of the price and $\ell$.

31 See footnote 19 for a discussion. Quadratic costs can be accommodated in our model but would lead to non-linear boundaries between (buy, sell, etc.) regions; instead of thresholds, boundaries would become curves, increasing the dimensionality of the free-boundary problem.

32 The permanent price impact depends on the cumulative amount of the asset that is liquidated and is independent of the rate at which the asset is traded reflecting, for example, the value of information revelation.

33 Fixed costs of equity issuance are prevalent in the literature (Ross et al., 2000; Franks and Sanzhar, 2006; Décamps et al., 2011; Milne and Whalley, 2001).

34 In practice, banks use equity buybacks in lieu of cash dividends but not to meaningfully adjust their capital structure (Hirtle, 2004, 2016).
where \( a_t = Y_t \) are total book assets, \( d_t = -X_t \) is (short-term/instantaneous) debt and \( e_t = a_t - d_t = X_t + Y_t \) is book equity capital. Leverage, consistent with the AV framework, is \( b_t = d_t / e_t \). We define the time of bankruptcy \( \tau_B \) as the first time that the bank’s liquidated value of assets, \( X_t + (1 - \ell) Y_t \), is negative.\(^{35}\) When bankruptcy occurs, shareholders walk away empty-handed, debt holders receive whatever cash is left after liquidation, and the bank shuts down. In equilibrium, the bank decides its optimal bankruptcy boundary and can therefore default on its debt strategically. However, it turns out that default is never optimal, consistent with the assumption that the bank can borrow at the riskless rate.

The bank’s optimization problem is

\[
\tilde{V}(t, x, y) = \max_{\{I_t, \xi_{1t}, \xi_{2t}, C_t\}_{t \geq t}} \mathbb{E}_t \left[ \int_t^{\tau_B} e^{-\rho(s-t)} \frac{C_s^{1-\gamma}}{1 - \gamma} ds - \sum_{i: t \leq \tau_i \leq \tau_B} e^{-\rho(\tau_i - t)} (dI_{\tau_i} + \kappa) \right]
\]

s.t.

\[
\begin{align*}
\frac{dX_t}{dt} &= (rX_t - C_t) dt + dI_t - d\xi_{1t} + (1 - \ell) d\xi_{2t}, \\
\frac{dY_t}{dt} &= Y_{t-} (\mu dt + \sigma dW_t - f dN_t) + d\xi_{1t} - d\xi_{2t}, \\
X_{t-} &= x \quad \text{and} \quad Y_{t-} = y,
\end{align*}
\]

where \( \tilde{V} \) is the value function; \( x \) and \( y \) are the initial holdings of the bond and the illiquid asset; \( C_t \) is the amount of dividends distributed at time \( t \); \( I_t \) is a non-negative, non-decreasing, cadlag\(^{36}\) process of cumulative equity issued up to time \( t \); \( \tau_i \) with \( i = 1, 2, ... \) are the times when equity is issued, i.e., when \( dI_t > 0 \); \( d\xi_{1t} \) and \( d\xi_{2t} \) are non-negative, non-decreasing, right-continuous processes of cumulative purchases and sales, respectively, of the illiquid asset up to time \( t \); \( \rho > 0 \) is the constant discount rate of shareholders; and \( 1/\gamma > 0 \) is the constant elasticity of intertemporal substitution. The choice variables \( C_t, I_t, \xi_{1t}, \xi_{2t} \) must be \( \mathcal{F}_t \)-adapted for all \( t \) and the times of equity issuance \( \tau_i \) must be \( \mathcal{F}_t \)-stopping times, so that decisions cannot be made using future information. Since the problem is stationary and the discounting exponential, \( \tilde{V}(t, x, y) = e^{-\rho t} V(x, y) \) for some time-independent function \( V \).

\(^{35}\)Defining bankruptcy as the first time that book value \( X_t + Y_t \) is negative does not materially change the conclusions. In Figure 3a, the boundary between the sell and equity issuance regions would rotate counterclockwise and the boundary between the equity issuance and bankruptcy region would change slightly.

\(^{36}\)Right-continuous with left limits.
The objective function in equation (13) states that shareholders have preferences over dividend distributions and equity issuance. We assume that shareholders have a finite elasticity of intertemporal substitution with respect to dividends to capture their preference for smooth dividends. In contrast, the value function is linear in equity injections, producing lumpy issuance (if issuance were a continuous process like dividends, the fixed costs would lead to immediate bankruptcy). The linearity also captures the “deep pockets” of the pool of investors or of the bank’s own intermediaries with investors.

Equations (14) and (15) give the evolution of the amount of money invested in the bond and the illiquid asset, respectively. Since the bank is levered, the amount invested in the bond is negative and therefore (14) also describes the evolution of (the negative of) debt. If at time $t$ the bank paid no dividends, issued no equity, and neither bought nor sold the illiquid asset, then equations (14) and (15) would have dynamics identical to those of the exogenous prices in equations (11) and (12). When the bank pays dividends $C_t$, issues equity, or buys or sells the illiquid asset, the cash receipts and outlays flow through equation (14). The last term in equation (14) shows that selling one unit of the illiquid asset only yields $1 - \ell$ units of cash, which is immediately invested in bonds (if $X_t < 0$, the cash is used to pay down debt).

We define the bank’s leverage target $b_t^*$ to be the value of leverage for which (i) the bank finds it optimal to make no active adjustments in its capital structure or portfolio of assets, $d\xi_{1t} = d\xi_{2t} = dI_t = 0$, while (ii) leverage is expected to remain constant, $E_t[db_t] = 0$, and (iii) dividends $C_t$ are paid according to the optimal policy rule. We define passive leverage $b_t^p$ as the leverage the bank would have if, starting at time $t$, it made no adjustments to its capital structure, portfolio or dividend levels, $d\xi_{1t} = d\xi_{2t} = dI_t = dC_t = 0$, even if any of these decisions are suboptimal. The adjustment speed towards target $\lambda_t$ is the unique number that makes equation (2) hold given $b_t^*$, $b_t^p$ and the realized leverage $b_t$. Although $\lambda_t$ is defined as the forcing variable that makes equation (2) hold, the equation does not hold trivially because a leverage target may not exist. For example, pecking-order theories of capital structure, models with managerial empire-building and models in which debt is supply-driven generally fail to produce a leverage target. Another recent and concrete example is the “leverage ratchet” model in Admati et al. (2018), where leverage is history-dependent and downward rigid.

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37Hirtle (2016) shows that dividends remained smooth even during the last financial crisis.

38We have defined $b_t^*$ and $b_t^p$ as functions of bank actions over the next instant $dt$, but the definitions readily generalize to any horizon $t + s$ by asserting that the same requirements that hold over the next $dt$ also hold between $t$ and $t + s$. Since fire-sales in the model occur instantaneously after the jump shock, knowing $b_t^*$ and $b_t^p$ at that instant is all we need to make the connection to AV. Nevertheless, $b_t^*$ and $b_t^p$ are both well defined and constant as $s \to \infty$. 

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17
To understand the model, we first examine simpler subcases by adding or removing frictions.

**Merton portfolio.** The standard Merton portfolio problem has costless liquidation \((\ell = 0)\) but no external financing, so new equity cannot be issued \((\kappa \to \infty\) and \(I_t = 0\) for all \(t)\).\(^{39}\) The value function is

\[
V_M(e) = \begin{cases} 
K_M e^{1-\gamma}, & \text{if } e > 0 \\
0, & \text{otherwise}
\end{cases}
\]

where \(K_M\) is a constant (the subscript \(M\) is for Merton). Because there are no liquidity costs, debt and the illiquid asset can be converted into each other one-for-one, so the value function can be written as a function of just equity capital \(e_t\) (instead of the two state variables \(X_t\) and \(Y_t\)). The optimal strategy is to keep leverage constant at some level \(b_M\).

Figure 1a shows a Merton line in an \(x\)-\(y\) plane, where the \(x\) and \(y\) coordinates measure the bank’s bond and holdings of the illiquid asset, respectively. Half-lines from the origin — like the Merton line — have constant leverage, so we identify each half-line from the origin with a particular value of \(b_t\). Because banks are levered, we expect them to be in the north-west quadrant where debt is positive, i.e. the bond position is negative and the illiquid asset position is positive. As the line rotates counter-clockwise, leverage increases.

\(^{39}\) Although in the context of portfolio optimization the Merton problem is usually considered the frictionless case, in the present context of banking it corresponds to an extreme form of capital market frictions.
After a shock moves the bank away from $b_M$, the bank must rebalance instantaneously to maintain a constant leverage. Below the $b_M$ line, the bank sells the illiquid asset to return to $b_M$; above it, it buys. The bankruptcy region is the set of points below the line $x + y = 0$, where the liquidated value of equity — which in this case is equal to the book value of equity because $\ell = 0$ — is negative. The optimal strategy ensures that leverage is low enough so that bankruptcy never happens.

Figure 1b depicts a bank that, starting at $(X_t, Y_t)$, suffers a jump $dN_t = 1$. The bank’s illiquid asset holdings decline by $f$ percent, moving the bank to $(X^p_t, Y^p_t)$. Without any rebalancing, leverage would increase to $b^p_t > b_M$. To restore leverage to $b_M$, the bank must sell

$$d\xi_{2t} = Y^p_t - Y_t = b_M Y_{t-} - f = b_M a_{t-} f$$

of the illiquid asset and use the proceeds to pay down debt, ending up at $(X_t, Y_t)$. Since selling one unit of the illiquid asset yields one unit of the bond, the adjustment occurs along a line with slope $-1$, parallel to the boundary between the sell and bankruptcy regions. The amount sold is increasing in the initial position $Y_{t-}$, the size of the jump $f$, and initial leverage $b_{t-} = b_M$.

A leverage target exists and is $b^* = b_M$. Since actual leverage $b_t$ is always equal to $b_M$ (adjustments are instantaneous), the partial adjustment equation has $\lambda_t = 1$ and $b_t = b^*_t$ for all $t$. Passive leverage at time $t$ is the leverage $b^p_t = -X^p_t / (X^p_t + Y^p_t)$ that would result if the bank did not make any adjustments to go back to target after the shock.\footnote{The leverage target is constant because the opportunity set is constant; if $r, \mu, \sigma$ or $f$ were not constant, $b^*_{t+}$ would be time-varying, as in our empirical estimation of $b^*_{t+}$.} Last, equating the marginal value of dividends to the marginal cost of issuing equity gives $C^{-\gamma}_t = V^M_{\epsilon e}$, which determines the level of dividends (we use subscripts to denote partial derivatives, so $V^M_{\epsilon e}$ is the partial derivative of $V^M$ with respect to $e$).

This Merton portfolio case corresponds exactly to the behavior in Greenwood et al. (2015), where the bank returns to target leverage immediately after the shock solely by liquidating assets, and the target leverage is equal to pre-shock leverage.

**Merton portfolio with illiquidity.** Introducing illiquidity into the Merton portfolio problem ($\ell > 0$) but retaining the no-issuance constraint ($I_t = 0$) produces the optimal
strategy in Figure 2a.\textsuperscript{41} The bankruptcy region is the set of \((x, y)\) for which the bank’s liquidation value is negative, located below the line \(x + (1 - \ell)y = 0\). The other three regions are the buy, dividend distribution, and sell regions, all of which are cones from the origin. Because of the liquidation costs \(\ell\), continuous rebalancing would lead to certain bankruptcy, so it is no longer optimal to keep leverage constant. Instead, leverage is kept within the interval \([b_{\text{buy}}, b_{\text{sell}}]\), where \(b_{\text{sell}} > b_{\text{buy}}\) are constants determined by the parameters of the model and the optimal behavior of the bank. When \(b_t \in (b_{\text{buy}}, b_{\text{sell}})\), the bank does not adjust its portfolio \((d\xi_{1t} = d\xi_{2t} = 0)\) and pays dividends according to \(C_t^{-\gamma} = V_x(X_t, Y_t)\).\textsuperscript{42} For \(b_t > b_{\text{sell}}\), the bank delevers by instantaneously selling the illiquid asset until \(b_t = b_{\text{sell}}\). For \(b_t < b_{\text{buy}}\), it brings leverage back to \(b_{\text{buy}}\) by purchasing the asset. If, starting at \(b_t \in (b_{\text{buy}}, b_{\text{sell}})\), the bank hits the boundaries \(b_{\text{buy}}\) or \(b_{\text{sell}}\) at a later time, it buys or sells a small amount of the asset to reflect the process back to the dividend distribution region.\textsuperscript{43}

Figure 2b shows how delevering is achieved after a jump puts the bank in the selling region. The bank is initially at \((X_{t-}, Y_{t-})\) in the dividend distribution region. After the jump,

\textsuperscript{41}In all cases, we solve the model numerically using the algorithm in Muthuraman et al. (2008). Oksendal and Sulem (2002) solve this Merton problem with illiquidity quasi-explicitly when there is no leverage and Oksendal and Sulem (2005) provide general conditions for the existence and uniqueness of solutions for problems with combined stochastic control and impulse control of jump diffusions, like the one in our model.

\textsuperscript{42}In this region, \(Y_t\) is an uncontrolled geometric Brownian motion with jumps and \(X_t\) drifts to the left with speed \(|rX_t - C_t|\).

\textsuperscript{43}In technical terms, the bank issues an amount of equity equal to the “local time” of the reflected process.
but before any actions are taken, the bank is at \((X_t^p, Y_t^p)\). The transition from \((X_t^p, Y_t^p)\) to \((X_t, Y_t)\) occurs by selling the illiquid asset and paying down debt along a line with slope \((\ell - 1)^{-1}\), parallel to the boundary between the sell and bankruptcy regions, since selling one unit of the illiquid asset yields only \(1 - \ell\) units of the bond. The amount liquidated is

\[
Y_t^p - Y_t = \frac{1}{1 - \ell (1 + b_{sell})} \left( 1 - \frac{b_{sell} + 1}{b_t + 1} + b_{sell} f \right) Y_t. \tag{17}
\]

Equation (17) shows that the amount sold is increasing in the initial amount of illiquid asset on the bank’s balance sheet, \(Y_{t-}\), initial leverage, \(b_{t-}\), and the jump size, \(f\), but decreasing in the leverage boundary, \(b_{sell}\), if initial leverage is high enough \((b_{t-} > (1 - \ell)(1 - f))\). Conditional on the initial point \((X_t^p, Y_t^p)\) staying in the sell region and the amount sold being decreasing in \(b_{sell}\), changes in \(\ell\) have two opposing effects.\(^{44}\) After a jump, the initial leverage \(b_{t-}\) and the leverage boundary \(b_{sell}\) fully determine the amount of debt that must be paid down by the bank to reduce leverage from \(b_{p\ t}\) to \(b_{sell}\). The amount of illiquid asset that must be sold to raise this amount is increasing in \(\ell\) since the bank must sell \(Y_t^p / (1 - \ell)\) of the illiquid asset to receive \(Y_t^p\) dollars. On the other hand, to reduce price impact over time, optimality makes the bank choose a wider no-trade interval \([b_{buy}, b_{sell}]\) when \(\ell\) is larger, so \(b_{sell}\) is increasing in \(\ell\). Given initial leverage \(b_{t-}\), returning to a higher leverage requires a smaller reduction in debt and consequently a smaller fire sale. It turns out that the second effect dominates if \(b_{t-}\) is large enough. In this case, the amount liquidated is decreasing in \(\ell\).

The leverage target in the model must be inside the dividend distribution region, since only there is it optimal to choose \(d\xi_{1t} = d\xi_{2t} = 0\). Just as in the benchmark Merton portfolio case, the leverage target is constant,\(^{45}\) but could be time varying in a more general model in which the bank has a hedging demand inside the dividend distribution region. Passive leverage, \(b_{p\ t}^*\), is the leverage that would be realized if the bank did not adjust its

\[^{44}\text{This is the case we consider in AV. It is possible that a change in }\ell\text{ moves the boundaries of the sell region so that }\left(X_t^p, Y_t^p\right)\text{ is no longer there. We return to this case below when we discuss regulation.}\]

\[^{45}\text{The second condition that }b^*_t\text{ needs to satisfy is that leverage is expected to be constant, which happens if and only if}\]

\[
0 = E_t \left[ d \left( \frac{X_t}{Y_t} \right) \right]
\]

Using that the value function is homogeneous of degree \(1 - \gamma\), the above condition can be written

\[
0 = (r + J\lambda - \mu) \frac{X_t}{Y_t} - W' \left( \frac{X_t}{Y_t} \right)^{-\frac{1}{\gamma}} + \left( \sigma^2 - \frac{\lambda f^2}{f - 1} \right) \frac{X_t}{Y_t} \tag{18}
\]

where the function \(W\) is defined by \(W(z) = V(z, 1)\). Because \(W\) is increasing and concave, there is a unique value of the ratio \(X_t/Y_t\) that satisfies equation (18), which determines the leverage target \(b^*\).

21
portfolio after the jump. Since after the jump but before adjusting the bank is at \((X_t^p, Y_t^p)\), it follows that \(b_t^p = -x^p / (x^p + y^p)\). Because the bank did sell some of the illiquid asset, realized leverage is not \(b_t^p\) but \(b_t = b_{sell}\). After the jump, we have that \(b_t^p < b_t < b_t^*\). The adjustment speed, \(\lambda_t\), captures how much delevering was already done and how much more delevering the bank needs to undertake to return to target after time \(t\).

After \(t\), whether the adjustment to target entails selling more of the illiquid asset depends on the realization of shocks. In fact, the expected path for the bank is to reach its target leverage without selling (or buying) the illiquid asset. Delevering after \(t\) is expected to occur gradually through asset price appreciation: The fire sale effectively ends at time \(t\), consistent with our implementation of AV in which only active leverage adjustments produce fire-sales.

Because the Merton portfolio case with illiquidity just analyzed satisfies the partial adjustment behavior in equation (2) and has a proportional price impact \(\ell\), it generates equation 4 used in the construction of AV for the one asset case (except for the adjustment of liquidity to outside wealth given by \(1/w_t\)). In turn, equations 4 and (2) together determine the adjustment speed \(\lambda_t\) after a jump shock, and show that it depends on the bank’s current leverage, balance sheet characteristics and parameters of the exogenous price processes. In Section 4, when we estimate adjustment speeds empirically, we use variables that reflect this insight (and also add aggregate variables, which affect not only banks’ balance sheets but also asset prices). The liquidation rule \(\tilde{m}_{kt}\) is trivial in this case, since there is only one asset.

**Illiquidity and equity issuance.** We now return to our full model with illiquidity and costly equity issuance. Figure 3a displays the optimal strategy of the bank. Compared to Figure 2, the only new region is the equity issuance region. The equity issuance optimization decision is

\[
0 = \max_{dI \geq 0} \{ V(x + dI, y) - V(x, y) - (dI + \kappa) \} \tag{19}
\]

The benefit of issuing an amount \(dI\) of equity is that it is used to pay down debt, increasing the value by \(V(x + dI, y) - V(x, y)\). Because the objective function is linear in \(dI\), the costs of issuing \(dI\) in equity are \(dI\) plus the fixed cost, \(\kappa\). Equation (19) simply says that the bank should issue equity for as long as benefits exceed costs. The FOC for an interior solution \((dI > 0)\) to the problem is

\[
V_x(x + dI, y) = 1
\]

which equalizes the marginal cost of equity of 1 to its marginal benefit of \(V_x(x + dI, y)\).

Introducing equity issuance has two main implications regarding the behavior of the
bank after a negative jump in the price of the illiquid asset. Consult Figure 3a. If the jump is large enough to put the bank in the equity issuance region, the bank finds it optimal to issue some equity and liquidate some of the illiquid asset at the same time. In this case, fire sales are mitigated compared to the case in which the bank returns to $b_{\text{sell}}$ solely by selling the illiquid asset. Second, the option of recapitalizing, by providing insurance against negative outcomes, increases the leverage threshold $b_{\text{sell}}$ at which the bank starts selling the illiquid asset.

**Two illiquid assets.** A straightforward extension of the model is to consider two illiquid assets with different degrees of illiquidity. Having two illiquid assets is helpful because it informs our decision in picking a liquidation rule $\tilde{m}_{kt}$ in our multi-asset AV framework in a more direct way. The two assets have prices $S_{1t}$ and $S_{2t}$ determined by

\[
\begin{align*}
   dS_{1t} &= S_{1t-} (\mu_1 dt + \sigma_1 dW_{1t} - f_1 dN_t) \\
   dS_{2t} &= S_{2t-} (\mu_2 dt + \sigma_2 dW_{2t} - f_2 dN_t)
\end{align*}
\]

where $W_{1t}$ and $W_{2t}$ are Brownian motions that are allowed to be correlated. We assume that $\ell_1 > \ell_2$, so asset 1 is more illiquid.

If both assets are traded frictionlessly and equity issuance is not allowed (the standard Merton case), then the amount liquidated of each asset after a jump shock $dN_t = 1$ is

\[
\begin{align*}
   Y_{1t}^p - Y_{1t} &= b_M a_t - m_{1t} f_1 \\
   Y_{2t}^p - Y_{2t} &= b_M a_t - m_{2t} f_2
\end{align*}
\]

where, as in the AV framework, $m_{kt}$ denotes the portfolio weight of asset $k$. Note that if $f_1 = f_2$, this case corresponds to the selling rule $\tilde{m}_{kt} = m_{kt}$ in which assets are sold proportionally to their initial portfolio weights, and where banks adjust fully to their pre-shock leverage, exactly as in Greenwood et al. (2015).

Now we reintroduce liquidation costs ($\ell_1, \ell_2 > 0$) and equity issuance with fixed cost $\kappa$. Figure 3b shows the optimal behavior of the bank as a function of the holdings $y_1$ and $y_2$ of the illiquid assets for a constant level of debt $X_t = \bar{x}$. The dividend distribution region is still a cone.\(^{46}\) Figure 3b shows one of its cross-sections, again holding $X_t$ constant at $\bar{x}$. In the figure, the dividend distribution region is wider along the $y_1$ axis because the first asset is more illiquid — a wider region implies selling it less often economizing on liquidation costs. For this figure, we have assumed that $W_{1t}$ and $W_{2t}$ are uncorrelated. However,

\(^{46}\)More precisely, it is the conic hull defined by four vectors.
because both assets are exposed to $dN_t$, they are still positively correlated, making the dividend distribution region a parallelogram instead of a rectangle.

When a jump occurs, $Y_{1t}$ and $Y_{2t}$ both decrease. If $(1-f_1)Y_{1t}, (1-f_2)Y_{2t})$ is outside the dividend distribution region, the optimal response of the bank is — after perhaps issuing some equity — to sell enough of both assets to return to the boundary of the dividend distribution region — in this case, the south-west vertex of a parallelogram on a plane different from the one shown in the figure (as the bank liquidates assets and pays down debt, the value of $x_t$ increases). As was the case in the one-asset case, the boundaries of the dividend distribution region — together with the leverage and portfolio shares at the boundary — are determined by parameters and are not time-varying.

Appendix B shows that the liquidation rule is

$$\tilde{m}_{1t} = m_{2t-} + \Theta_1$$
$$\approx m_{1t-} + (3f - 4) (m_{1t-} - m_{1,\text{sell}}) \quad (20)$$

$$\tilde{m}_{2t} = m_{2t-} + \Theta_2$$
$$\approx m_{2t-} + (3f - 4) (m_{2t-} - m_{2,\text{sell}}) \quad (21)$$

where $m_{1,\text{sell}}$ and $m_{2,\text{sell}}$ are the portfolio shares at the boundary of the dividend distribution region that the bank has at time $t$ after selling assets, and $\Theta_1, \Theta_2$ are functions of initial leverage, $b_{1-}$, initial portfolio positions, $m_{1t-}, m_{2t-}$, and parameters of the model. If $\Theta_1 = \Theta_2 = 0$, the liquidation rule in the model would be identical to the proportional
rule assumed in AV. Equations 20 and 21 give linear approximations for $\Theta_1$ and $\Theta_2$ to investigate the factors driving the first-order differences between the liquidation rules in the model and in AV. Equations 20 and 21 imply that if the bank was closer to the selling boundary before the shock, then the model’s selling rule will be more similar to AV. Everything else equal, this will happen, for example, if pre-shock leverage is higher, if liquidation costs are smaller, or if interest rates are higher.

**Regulation.** Regulatory constraints can be incorporated into the model without any significant effort or changes in the results. Liquidity, leverage and risk-based capital constraints can all be written as

$$e_t \geq b_R (w_1 m_1 a_t + w_2 m_2 a_t) \quad (22)$$

For example, if equation (22) is interpreted as a risk-based capital constraint, it states that equity capital must be larger than a fraction $b_R$ of risk-weighted assets, where $b_R$ and the risk weights $w_k$ are picked by the regulator. If $b_R < b_{sell}$, then the constraint is binding. The optimal behavior of the bank is the same as without the constraint but replacing the endogenously determined boundary of the sell region $b_{sell}$ by the exogenously imposed $b_R$. For the one asset case in Figures 1a and 2a, the presence of the binding constraint pivots the boundary between the sell and dividend distribution regions around the origin in a clockwise direction, with corresponding changes in the other regions. The dividend distribution region shrinks, making the bank act as if the illiquid assets were effectively more liquid. Paradoxically, for the same initial conditions, if a fire sale occurs, it will be larger when the regulatory constraint is binding since the bank must pay down more debt to return to the lower leverage required by the constraint. On the other hand, some states that previously led to fire sales no longer do so: points that are in the dividend distribution region without the constraint but in the sell or equity issuance regions when the constraint is binding can no longer trigger a fire sale after a jump. For the two asset case in Figure 3b, the constraint forces the bank to adjust whenever it is below a line with negative slope. Depending on the current debt and asset positions of the bank and the parameters picked by the regulator, the constraint can change the boundary of some or all of the regions. Just as in the one asset case, a binding constraint implies that some states are no longer capable of triggering fire sales, but when a fire sale does occur, it is larger.
4 Estimation of leverage targets and adjustment speed

We use quarterly data from financial firms that file regulatory form FR Y-9C with the Federal Reserve. Form FR Y-9C provides consolidated balance sheet information for bank holding companies, savings and loans associations and securities holding companies. For convenience, we refer to all of them as banks. The information in the form is publicly available and is generally used by regulators to assess and monitor the condition of the financial sector. Banks with total assets over $150 million before 2006q1, over $500 million between 2006q1 and 2014q4, and over $1 billion thereafter are required to file. We include in our sample large banks (any bank that is ever in the top 100 by total assets in a quarter) because they have the most complete and uniform data and account for almost all assets (92% on average). Our measure of equity is tier 1 capital which becomes available in the data in 1996q1. Our sample therefore runs from 1996q1 to 2016q4. We subtract equity from total assets to obtain our measure of debt. To simplify the analysis, we then subtract all cash holdings from both assets and debt. Our measure of leverage is the ratio of debt to equity. We cap leverage at 30 whenever it exceeds this threshold.

As explanatory variables for our estimation of bank leverage targeting, we use bank characteristics commonly used in the empirical literature on capital structure of non-financial firms (e.g. Titman and Wessels, 1988; Rajan and Zingales, 1995; Frank and Goyal, 2009) that have been shown to apply equally to banks’ leverage (e.g. Berger et al., 2008; Gropp and Heider, 2010). We do not include time fixed effects that would absorb variation we are interested in capturing but use a set of aggregate variables instead (Korteweg and Strebulaev, 2015). Table 1 provides a list of the variables with descriptions and summary statistics.

In Table 2, we first confirm the validity of the explanatory variables by running panel regressions of leverage on lagged explanatory variables (Titman and Wessels, 1988; Rajan and Zingales, 1995; Frank and Goyal, 2009):

\[ b_{it} = \delta_x^T x_{it-1} + \delta_z^T z_{t-1} + \nu_i + \eta_t + \epsilon_{it}. \]

We include, in addition to bank fixed effects \( \nu_i \), either date fixed effects \( \eta_t \) (column 1) or lagged aggregate variables \( z_{t-1} \) (column 2). We can see that almost all variables are significant and have the expected signs: leverage is higher for looser capital requirements and lower for banks subject to CCAR stress tests. Banks that are more profitable and riskier
Table 1: Variables used in leverage target regressions. Sources: FR Y-9C, CRSP, Compustat, FRED.

<table>
<thead>
<tr>
<th>Bank characteristics</th>
<th>Mean</th>
<th>Std.</th>
<th>p5</th>
<th>p95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage</td>
<td>11.22</td>
<td>3.63</td>
<td>6.63</td>
<td>16.28</td>
</tr>
<tr>
<td>Regulatory max</td>
<td>32.82</td>
<td>14.30</td>
<td>19.65</td>
<td>47.86</td>
</tr>
<tr>
<td>CCAR</td>
<td>0.06</td>
<td>0.23</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Size</td>
<td>23.57</td>
<td>1.54</td>
<td>21.48</td>
<td>26.56</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Risk</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Loan share</td>
<td>0.64</td>
<td>0.17</td>
<td>0.27</td>
<td>0.82</td>
</tr>
<tr>
<td>Retail deposits</td>
<td>0.51</td>
<td>0.19</td>
<td>0.11</td>
<td>0.77</td>
</tr>
<tr>
<td>Public</td>
<td>0.79</td>
<td>0.41</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Public × MTB</td>
<td>5.84</td>
<td>3.05</td>
<td>0.00</td>
<td>8.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Aggregate variables</th>
<th>Mean</th>
<th>Std.</th>
<th>p5</th>
<th>p95</th>
</tr>
</thead>
<tbody>
<tr>
<td>dGDP</td>
<td>0.02</td>
<td>0.02</td>
<td>-0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>Term spread</td>
<td>1.19</td>
<td>0.93</td>
<td>-0.11</td>
<td>2.59</td>
</tr>
<tr>
<td>Recession</td>
<td>0.09</td>
<td>0.29</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\(a\) The tier-1 capital requirement is on the ratio of tier-1 capital (our measure of equity) to risk-weighted assets. We convert it to a maximum requirement on debt over equity as \((\rho_{e/d})^{-1} = (\rho_{c/rwa} rwa_{it}/d_{it})^{-1}\).
Table 2: Regressions of leverage on bank characteristics and aggregate variables. Columns (1) and (2) are panel regressions with fixed effects, Column (3) is a dynamic panel regression with fixed effects.

<table>
<thead>
<tr>
<th></th>
<th>(1) (leverage (static))</th>
<th>(2) (leverage (static))</th>
<th>(3) (leverage (dynamic))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulatory max</td>
<td>0.0172 (0.0154)</td>
<td>0.0366* (0.0219)</td>
<td>0.00131 (0.00187)</td>
</tr>
<tr>
<td>CCAR</td>
<td>-1.186*** (0.378)</td>
<td>-1.848*** (0.334)</td>
<td>-0.0851** (0.0697)</td>
</tr>
<tr>
<td>Size</td>
<td>1.122*** (0.413)</td>
<td>0.412 (0.253)</td>
<td>-0.0744 (3.041)</td>
</tr>
<tr>
<td>Profitability</td>
<td>-49.99** (22.21)</td>
<td>-27.76*** (10.29)</td>
<td>-0.0744 (2.097)</td>
</tr>
<tr>
<td>Risk</td>
<td>-25.88** (11.01)</td>
<td>-2.127 (2.080)</td>
<td>-0.0744 (3.022)</td>
</tr>
<tr>
<td>Loan share</td>
<td>-2.061 (2.007)</td>
<td>-1.981** (1.005)</td>
<td>-0.244 (0.182)</td>
</tr>
<tr>
<td>Retail deposits</td>
<td>-6.497** (2.796)</td>
<td>-8.439*** (2.207)</td>
<td>-2.640*** (0.421)</td>
</tr>
<tr>
<td>Public</td>
<td>0.648* (0.352)</td>
<td>0.943*** (0.258)</td>
<td>0.341*** (0.0551)</td>
</tr>
<tr>
<td>Public x MTB</td>
<td>2.732* (1.550)</td>
<td>0.531 (0.496)</td>
<td>-0.0506 (0.0543)</td>
</tr>
<tr>
<td>dGDP</td>
<td>-0.323*** (0.007)</td>
<td>-0.0297** (0.0145)</td>
<td>0.876*** (0.0194)</td>
</tr>
<tr>
<td>Term spread</td>
<td>0.883*** (0.168)</td>
<td>-0.0506 (0.0543)</td>
<td>0.876*** (0.0194)</td>
</tr>
<tr>
<td>Passive leverage</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank, date FEs</td>
<td>Yes, Yes</td>
<td>Yes, No</td>
<td>Yes, No</td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0.628</td>
<td>0.608</td>
<td>0.818</td>
</tr>
<tr>
<td>Observations</td>
<td>11306</td>
<td>11306</td>
<td>11305</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p < .1, ** p < .05, *** p < .01

have lower leverage.\textsuperscript{48} “Traditional” banks with high loan shares and deposit funding have lower leverage; publicly traded banks have lower leverage on average but increasing in their market-to-book ratio. In terms of the aggregate variables, leverage increases after high GDP growth and a low term spread but also, mechanically due to losses, in recessions.

Turning to our estimation of dynamic leverage adjustments, we use a standard dynamic capital structure model with partial adjustment toward a leverage target (Flannery and Rangan, 2006; Lemmon et al., 2008):

\[ b_{it} = \lambda b_{it}^* + (1 - \lambda) b_{it}^p + \epsilon_{it}, \]  

(23)

where \( b_{it}^* \) is bank \( i \)'s leverage target for period \( t \), \( b_{it}^p \) is the “passive” leverage that would result for period \( t \) without any active adjustment, and \( \lambda \) is the speed of adjustment to target.\textsuperscript{49} The leverage target \( b_{it}^* \) is modeled as a function of lagged bank characteristics,\textsuperscript{48} The coefficient on size is affected by the lack of time fixed effects in columns 2 and 3 due to the trend in assets.

\textsuperscript{49}As is standard in the literature, we assume a constant adjustment speed across banks. As detailed below, we then run rolling regressions to allow for changes in \( \lambda \) over time. There is a trade-off in capturing variation in \( \lambda \) across banks or across time. Our approach favors variation across time that is key for our multiplicative decomposition of AV.
aggregate variables, as well as a bank fixed effect:

\[ b_{it}^* = \delta^T x_{it-1} + \delta^T z_{t-1} + v_i. \]  

(24)

To distinguish between active and passive leverage adjustments, we use “passive” leverage instead of using lagged leverage as the baseline for active leverage adjustments (Faulkender et al., 2012). The starting point for passive leverage \( b_{it}^p \) is lagged leverage \( b_{it-1} = d_{it-1}/e_{it-1} \). Then we first increase the bank’s equity by net income, i.e. the profits or losses made in period \( t \). Second, since banks aim to pay smooth dividends, we subtract average dividends paid over the previous eight quarters.\(^{50}\) Third, we add the net issuance of equity in period \( t \); this is important since, as mentioned in Section 2, we are interested in delevering through asset sales, i.e. excluding new issuance of equity.\(^{51}\) Our measure of passive leverage is therefore given by\(^{52}\)

\[ b_{it}^p = \frac{d_{it-1}}{e_{it-1} + \text{net}_{\text{inc}_{it}} - \text{ave}_{\text{div}_{it-1}} + \text{net}_{\text{iss}_{it}}}. \]

Substituting into equation (23), our regression takes the form

\[ b_{it} = \lambda \left( \delta^T x_{it-1} + \delta^T z_{t-1} + v_i \right) + (1 - \lambda) b_{it}^p + \varepsilon_{it}. \]  

(25)

For illustration, column 3 of Table 2 shows the results from estimating the dynamic adjustment model (25) as a dynamic panel regression on the whole sample with bank fixed effects.\(^{53}\) We can recover an estimated adjustment speed \( \hat{\lambda} \) from the coefficient on \( b_{it}^p \) and

\(^{50}\) In addition to cash dividends, we include stock repurchases (gross purchases of treasury stocks) which are commonly used by banks instead of cash dividends (Hirtle, 2004, 2016).

\(^{51}\) The only time there is non-negligible equity issuance in our sample period is during the crisis and the majority of that is the TARP recapitalization. If we were to treat this as an active adjustment, it would increase our estimate of the adjustment speed and therefore fire-sale vulnerability. However, this was arguably the opposite of the leverage adjustment we are interested in: through the issuance of equity and in a coordinated fashion rather than through disorderly asset sales.

\(^{52}\) We also apply a constant correction to all \( b_{it}^p \) to ensure that the averages of passive leverage and estimated target leverage equal the overall average actual leverage within each estimation window, \( \Sigma_i \Sigma_t b_{it}^p = \Sigma_i \Sigma_t b_{it}^* = \Sigma_i \Sigma_t b_{it} \).

\(^{53}\) In dynamic panel models there is a risk of bias due to the correlation between the fixed effects and the lagged dependent variable (e.g. Baltagi, 2008). However, Flannery and Hankins (2013) show that in samples representative of corporate finance data such as ours, the direction and magnitude of the bias is not obvious and standard cross sectional regressions with firm fixed effects perform well compared to more sophisticated techniques. Our estimate of adjustment speed is also consistent with the one obtained by Berger et al. (2008) using system GMM (Blundell and Bond, 1998) on annual data; using our cross sectional regression with bank fixed effects on annual date and their sample period we estimate the same (annual) adjustment speed of 0.60.
an estimated leverage target $\hat{b}_{it}^*$ from the fitted values $\hat{b}_{it}$ as

$$\hat{b}_{it}^* = \frac{1}{\lambda} \left( \hat{b}_{it} - (1 - \lambda) b_{it}^p \right).$$

To minimize any look-ahead bias in our analysis of vulnerabilities and to allow for the possibility of changes in the coefficients over time we estimate equation (25) on rolling 16-quarter windows. From the regression on window $(t-15, \ldots, t)$ we then take the adjustment speed and the leverage targets for period $t$. Figure 4 shows the resulting time series of estimated adjustment speed and average estimated leverage target, compared to average actual leverage (left panel), as well as the estimated targets for the largest banks (right panel). We see that estimated leverage targets are generally quite close to actual leverage, albeit somewhat smoother. A notable exception is the period 2007–2008 when actual leverage increases due to banks suffering losses while the leverage target stays flat. The estimated adjustment speed $\hat{\lambda}$ is 0.3 on average with a spike during the crisis and a decline since then.\(^{54}\) The spike is quite notable given that we have adjusted the passive leverage starting point by all equity issuance (including TARP). The higher estimate of $\hat{\lambda}$ during the crisis is therefore not due to the unusual equity issuance to recapitalize but

\(^{54}\)Note that our data is at quarterly frequency, leading to a lower estimate of adjustment speed than in the literature using annual data. If we run our estimation on annual data, we obtain adjustment speeds varying between 0.45 and 0.65, consistent with the literature (Berger et al., 2008; Gropp and Heider, 2010).
Table 3: Summary statistics for balance sheet data. Source: FR Y-9C.

|                        | System | p5   | Med. | Mean | p95  | $\ell_k$
|------------------------|--------|------|------|------|------|---------
| Assets ($ billions)    | 10,592.5 | 6.4  | 17.2 | 105.9| 485.4|         
| Leverage               | 12.6   | 6.7  | 10.8 | 11.3 | 16.9 |         
| Portfolio shares (percent): |       |      |      |      |      |         
| Residential real estate loans | 15.3  | 0.2  | 16.4 | 16.8 | 36.3 | 12.0    
| C & I loans            | 10.8   | 0.4  | 12.1 | 13.2 | 27.2 | 15.0    
| Repo & fed funds loans | 9.9    | 0.0  | 0.3  | 2.7  | 14.9 | 2.0     
| Agency MBS             | 8.8    | 0.8  | 10.7 | 12.3 | 29.9 | 3.0     
| Consumer loans         | 8.7    | 0.1  | 4.3  | 7.0  | 18.1 | 15.0    
| Commercial real estate loans | 7.6   | 0.2  | 17.8 | 19.4 | 43.6 | 15.0    
| ABS & other debt securities | 6.8   | 0.0  | 0.7  | 2.7  | 11.8 | 7.0     
| U.S. Treasuries        | 2.2    | 0.0  | 0.2  | 1.4  | 6.6  | 1.0     
| Equities & other securities | 1.9   | 0.0  | 0.2  | 0.8  | 2.6  | 11.0    
| Non-agency MBS         | 1.8    | 0.0  | 0.3  | 1.6  | 7.5  | 13.0    
| Agency securities      | 1.7    | 0.0  | 1.6  | 3.7  | 14.5 | 3.0     
| Lease financings       | 1.5    | 0.0  | 0.6  | 1.5  | 6.1  | 15.0    
| Municipal securities   | 1.2    | 0.0  | 1.1  | 2.0  | 7.6  | 12.0    
| Other real estate loans| 1.0    | 0.0  | 0.2  | 1.2  | 3.6  | 15.0    
| Residual loans         | 4.6    | 0.0  | 2.2  | 3.7  | 10.9 | 15.0    
| Residual securities    | 4.3    | 0.0  | 0.0  | 0.8  | 4.1  | 20.0    
| Residual assets        | 11.8   | 3.2  | 8.0  | 9.4  | 20.0 | 20.0    

Note: System statistics are time-series means. All other statistics are over the entire panel.

rather to delevering through balance sheet contraction.

5 Calculation of fire-sale spillovers

The matrices of total assets $A_t$, portfolio weights $M_t$, and leverage $B_t$ come directly from the FR Y-9C balance sheet data described in Section 4. We group assets into the seventeen categories listed in Table 3 to construct the matrix of portfolio weights $M_t$. This is the finest subdivision we can construct while reasonably maintaining the assumption of no cross-asset price impacts of fire sales. Banks’ leverages are collected in the diagonal matrix $B_t$. To measure the wealth $w_t$ of potential buyers of fire-sold assets, we use the value of total financial sector assets minus the assets of the banks in our sample. There are no readily available estimates for the liquidity of most assets of banks. Greenwood et al. (2015) therefore assume the same price impact for all assets, $\ell_k = \ell$ for all $k$. Instead, we introduce heterogeneity in the liquidity of asset classes by using the information contained

55 Appendix D contains the mapping between these asset classes and entries in form FR Y-9C.
56 The Financial Accounts series for total assets of the financial sector is Z1/FL792000095.Q.
in the Net Stable Funding Ratio (NSFR) of the Basel III regulatory framework.\footnote{We use the NSFR instead of the Liquidity Coverage Ratio (LCR) since it distinguishes asset classes more finely.} The NSFR involves applying haircuts to different asset classes to account for differences in liquidity over a horizon of one year. The last column of Table 3 shows the values of price impact $\ell_k$, normalized to the value for Treasuries (since the absolute value does not affect the AV which is normalized to 100 at the beginning of the sample).\footnote{Appendix E shows in detail how we impute liquidity values for different assets using the NSFR guidelines.} To calculate AV, we include the top 100 banks every quarter that have leverage target estimates from Section 4. Our motivation is that many banks either appear, disappear or re-appear in different subperiods of our sample, e.g. due to mergers and acquisitions, bankruptcies and the conversion of non-bank financial institutions into bank holding companies and vice-versa. Restricting analysis to a balanced panel of institutions that are present for the entire sample period may therefore miss important trends. Our approach balances (i) maintaining consistency in the sample across time with (ii) accurately accounting for the state of the system at every point in time. We show robustness in Appendix A.4.

\section*{5.1 Results}

Figure 5 shows the evolution of AV as well as its comprising factors from equation (6), which we normalize to 100 at the beginning of the sample. AV shows no clear trend between 2000 and the end of 2003. Starting in early 2004, it increases steadily until the finan-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{aggregate_vulnerability.png}
\caption{Aggregate vulnerability index and decomposition into factors based on equation (6), all normalized to 100 at the beginning of the sample.}
\end{figure}
cial crisis, peaking in 2008q3 at over two times the initial level. If available at the time, our measure would have been useful as an early indicator of vulnerabilities building up; we explore this issue further with predictive regressions in Section 7. The measure decreases sharply over the course of 2009 and returns to its initial level in mid 2010. In the post-crisis period, the measure declines further before stabilizing in 2015 at around 20 percent of its initial level.

Studying the four comprising factors of AV in the right panel of Figure 5, we see that each factor contributes differently to the total and that the contributions change over time. Relative size of the banking system compared to the rest of the financial sector as well as leverage show expected trends, increasing in the pre-crisis period and decreasing towards the end of the sample. Of note are the sharp declines in leverage in late 2008 mostly due to bank recapitalizations (TARP, see Duarte and Eisenbach, 2014) as well as the increase in relative size in early 2009 due to the addition to the sample of firms such as Morgan Stanley and Goldman Sachs which became bank holding companies.\(^{59}\)

Size and leverage are known factors of systemic risk. It is therefore crucial for us that the two factors that are specific to fire-sale spillovers — adjustment speed and illiquidity concentration — also play important roles in the evolution of AV. We see that the adjustment speed shows a decreasing trend until 2006, before increasing by roughly 70 percent and causing AV to spike in late 2008. This is notable since, in the estimation in Section 4, we have controlled for any adjustments via equity issuance so the estimated increase in adjustment speed during the crisis reveals greater leverage adjustment explicitly, not through raising equity.

Turning to illiquidity concentration, the measure capturing concentration of more illiquid assets among relatively large and levered banks, we see a positive trend starting already in late 2002, before any of the other factors except size show any increase; between late 2002 and early 2007, illiquidity concentration increases by roughly 25 percent. In the post-crisis period it remains quite stable before starting a downward trend in 2013. Jointly, the two fire-sale specific factors of adjustment speed and illiquidity concentration account for over half of the increase in AV, whether measured from the beginning of the sample or from when AV starts increasing in early 2004.

Figure 6 reports the most systemic banks and assets, showing the evolution of the measures \(SB_{it}\) and \(SA_{kt}\) from equations (7) and (8), respectively. Note both systemicness measures individually sum up to AV. Since we normalize AV to 100 at the beginning of the sample for Figure 5, we normalize the systemicness measures so they sum to 100 at the

\(^{59}\) Appendix A.4 shows our measure is robust to using a balanced panel, so that firms that enter and exit the sample do not drive our results.
Among banks, Citigroup is the most systemic for the majority of the sample, with Bank of America and JPMorgan Chase following closely behind. Despite their overall systemicness measures being highly correlated, there are clearly differences in the patterns due to differences in the evolution of the bank specific factors in decomposition (7).

Among assets, residential real estate loans stand out as the most systemic and with the fastest growth in the run-up to the crisis. This is not just because they comprise a large fraction of total assets (Table 3) but also because they are held in large amounts by the most systemic banks. They are also a key determinant of the illiquidity concentration factor of AV: Since 2002 and until 2007, a large proportion of banks increased their portfolio share of residential real estate loans, making balance sheets across the system more similar. The next most systemic asset, C & I loans, are as systemic as residential real estate loans until 2001 when the bifurcation in their aggregate portfolio shares occurs.

5.2 Variance decomposition

To further disentangle the contribution of the individual factors of AV in equation (6), we can decompose the variance of log AV into the variances and covariances of the log factors $X_n$ according to

$$\text{var}(\log AV) = \text{var}(\sum_n \log X_n) = \sum_n \text{var}(\log X_n) + \sum_n \sum_{m \neq n} \text{cov}(\log X_n, \log X_m). \quad (26)$$
We then sum the contributions of each log factor, i.e. its variance and all covariances and express the total relative to the variance of log AV according to

\[
\text{contribution of factor } X_n \equiv \frac{\text{var}(\log X_n) + \sum_{m \neq n} \text{cov}(\log X_n, \log X_m)}{\text{var}(\sum_m \log X_m)}.
\]  

(27)

Figure 7 shows the results of this variance decomposition across three sub-periods of our sample: before the crisis (before 2007q1), around the crisis (2007q1–2009q4), and after the crisis (after 2009q4). We see that the contribution of relative system size is large pre-crisis but much smaller post-crisis and even negative during the crisis (due to negative covariances with other factors). In contrast, the contribution of leverage is greatest during the crisis and only half as large in the pre and post period. Variation in adjustment speed contributes very little to variation in AV before the crisis but turns into the main contributor during (together with leverage) and after the crisis. Finally, illiquidity concentration stands out as the second largest contributor to variation in AV pre-crisis and the most stable contributor over the whole sample. Jointly, adjustment speed and illiquidity concentration account for 40 percent of the variance of AV between the beginning of our sample and 2008q3, when AV peaks.

5.3 Effect of heterogeneity on AV

An important element of the AV framework is the non-neutrality with respect to the distribution of a given “aggregate balance sheet” across different institutions, as captured by the illiquidity concentration factor in the decomposition (6). We can study this effect of bank heterogeneity by constructing a counterfactual system where banks are homogeneous and comparing the resulting fire-sale vulnerability. To construct such a counterfactual measure we assume that all banks are equally sized, have the same leverage target, and hold...
the same asset portfolio, effectively creating a representative bank. This requires setting \( \alpha_{it} = 1/N, \beta_{it}^* = 1, \) and \( \mu_{ikt} = 1 \) for all \( i, k \) in the expressions for aggregate vulnerability in (6):

\[
AV_{t}^{\text{hom}} = \frac{a_t}{w_t} (b_t + 1) \beta_t^* \lambda_t \sum_k \left( m_{ikt}^2 \ell_k \right).
\]

Taking the ratio of actual AV to the hypothetical homogeneous AV, the first three factors (which depend on aggregate variables only) cancel and we are left with a ratio of the respective illiquidity concentration factors:

\[
\frac{AV_t}{AV_t^{\text{hom}}} = \frac{\sum_k \left[ m_{ikt}^2 \ell_k \sum_i (\mu_{ikt} \beta_{it}^* \alpha_{it}) \right]}{\sum_k \left( m_{ikt}^2 \ell_k \right)}.
\]

Figure 8 shows the evolution of the ratio over time, highlighting that the effect of heterogeneity on AV can be large and variable over the sample. From the beginning of the sample until 2003, heterogeneity has little effect on AV. From 2005 to 2014, AV is roughly 20 percent higher due to heterogeneity. Then the effect declines and eventually reverses with AV about 20 percent lower due to bank heterogeneity at the end of the sample in 2016.

6 Robustness

We present three sets of robustness checks: (i) not marking to market loan losses, (ii) alternative rules for liquidating assets, and (iii) conducting multiple rounds of fire sales.
Several additional robustness checks are in Appendix A.

### 6.1 Not marking to market

Banks don’t mark-to-market every asset on their balance sheet. A portion of their balance sheet can be “held-to-maturity,” allowing interim unrealized losses to go unrecognized. In such cases, when confronted with a negative shock, banks may not recognize the full extent of the economic losses on their balance sheets. While the economic pressure to sell assets is still present, a more benign accounting-based leverage may relax the need to fire-sell assets, at least in the short run. We therefore consider the extreme case in which banks simply do not mark down any loans, residual securities, or residual assets (and mark-to-market the rest of their balance sheet). Figure 9 displays the results, comparing the benchmark AV to the version without marking-to-market. To illustrate the difference in magnitude, we have normalized the latter index by the relative size of the raw AV values at the beginning of the sample. When banks do not mark down illiquid assets at all, AV is cut by roughly two thirds but in terms of the behavior of the index over time, the results hardly change.

### 6.2 Alternative liquidation rules

Selling the most liquid assets first has the important advantage of minimizing the price impact of fire sales which reduces total losses. In addition, some illiquid assets may simply be impossible to sell. However, there are several good reasons for selling illiquid assets
first. If banks expect that markets will become more illiquid in the future, the liquidity premium should be smaller today than tomorrow, creating an incentive to hold on to liquidity until it is more valuable. Regulatory requirements on risk-weighted assets create an incentive to sell assets with high risk-weights first, which tend to be more illiquid. At the same time, however, assets with high risk weights tend to be more illiquid. This creates a tension between selling assets that have high risk-weights but are less liquid – which eases the capital requirement but imposes higher liquidation costs – or selling assets that have low risk-weights but are more liquid.

For robustness we calculate AV under three alternative liquidation rules: (i) sell liquid assets first, (ii) sell liquid assets last, (iii) sell assets proportional to liquidity, and (iv) minimize price impact subject to a risk-based capital requirement.

**Sell liquid assets first.** We first assume a simple “waterfall” strategy, whereby banks sell assets in decreasing order of liquidity until they achieve their desired leverage. The results are in Figure 10 (left panel), where, for level comparison to the benchmark index, we have normalized the alternative AV indeces by the relative size of the raw AV values at the beginning of the sample, as we did before. When selling liquid assets first, the level

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60 For example, in the summer of 2008, Lehman Brothers sold some of its less liquid assets including commercial MBS, commercial mortgage inventory, leveraged loans and LBO-related debt while keeping a relatively constant liquidity buffer (Valukas, 2010).

61 Scholes (2000); Acharya and Pedersen (2005); Brown et al. (2009); Krishnamurthy (2010).

62 Cifuentes et al. (2005); Hameed et al. (2010); Merrill et al. (2012). Post-crisis regulations such as Basel III’s liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR) create further pressure to dispose of illiquid assets first.
of AV is cut to roughly one fifth but in terms of the behavior of the index over time, the results don’t materially change. The reduction in levels comes from the fact that assets with higher NSFR liquidity not only have lower price impact but tend to be held in a concentrated fashion by large and levered banks with large security portfolios.

**Sell liquid assets last.** This strategy is the opposite of the previous one so assets are now sold in *increasing* order of liquidity. As shown in Figure 10, this assumption roughly doubles the level of AV but in terms of the behavior of the index over time, the results again hardly change.

**Sell proportional to liquidity.** We now assume that banks sell assets inversely proportional to price impact. Figure 10 (right panel) shows that that under the assumption of selling proportional to asset liquidity, the level of AV is about one third lower but in terms of the index over time, the results hardly change.

**Risk weights and liquidity.** We now assume that banks minimize the price impact of their fire-sales subject to a risk-based capital requirement; the details of the analysis are in Appendix A.1. Figure 10 (right panel) shows that AV under the resulting trade-off between risk weights and liquidity is considerably smaller than the benchmark, but not as small as under the simple liquid first strategy (left panel). However, as before, the profile of vulnerability over time retains its shape.

### 6.3 Multiple rounds

We now study how AV changes when we iterate the one-shot fire-sale mechanism that we used in our main specification. We think of the spillover losses that arise due to the initial exogenous shock as a new endogenous “shock” $\tilde{f}$ that triggers a second round of fire-sales, given in equation (4). The spillover losses of this new round serve as a shock for the next round, and so on. The multi-round AV is the sum of spillover losses in all rounds as a fraction of initial system equity.

We need to account for fire-sold assets leaving the system in the current round before we can proceed to the next. Total assets inside the system thus decrease in each round of fire-sales. Once we explicitly allow assets at the beginning of the round, $A_1$, and assets

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63If an asset is sold but does not leave the banking system, it means that another bank bought it. In this case, one would expect little price impact, i.e. not a fire sale. See also footnote 21.
at the end of the round, $A_2$, to differ, the first-round fire-sale spillovers are:

$$AV_1 = \frac{1}{e^{\lambda w}} 1^\top A_2 MLM^\top B^* A_1 MF_1.$$  

(28)

The assumption that all fire-sold assets exit the system implies that $A_2$ is given by the following relation.$^{64}$

$$A_2 1 = A_1 1 - \lambda B^* A_1 MF_1,$$

Using $A_2$ as initial assets for the second round and the first-round fire-sale effects $F_2 = \frac{\lambda}{w} LM^\top B^* A_1 MF_1$ as the new shock, we find second-round spillover losses:

$$AV_2 = AV_1 + \frac{1}{e^{\lambda w}} 1^\top A_3 MLM^\top B^* A_2 MF_2.$$

We can iterate this process indefinitely with resulting fire-sale spillovers given by $AV_\infty = AV_1 + AV_2 + AV_3 + \ldots$. Figure 11 shows how multiple rounds of fire-sales affect AV (normalized by the relative size of the raw AV values at the beginning of the sample). We see that convergence is achieved fairly quickly and that the shape of AV is preserved. Interestingly, the fraction of converged AV\_\text{\infty} not accounted for by the first round AV\_1 is not constant over time (Figure 11, right panel). There is a positive relationship between the absolute effect of multiple rounds and the level of AV. Among the factors of AV, the contribution of additional rounds is highly correlated with leverage (0.78), adjustment speed

$^{64}$We multiply the diagonal matrices $A_1$ and $A_2$ by a vector of ones to make them conformable with the vector $B^* A_1 MF$ of dollar amounts that each bank must sell to return to target leverage. Since banks sell assets proportionally to their holdings, their portfolio shares $M$ remain unchanged across rounds.
AV shows a clear increasing trend pre-crisis, especially between 2004q1 and 2007q1. In contrast, none of the other measures signal increased risk until 2007, and most do not overcome their 2003q1 levels until 2007q3. Identifying this steady build-up of systemic risk seems crucial to be able to respond to it — identifying trends and implementing policy actions can take time.

Of course, given that there is just one crisis in our sample, any time-series analysis relies on what is effectively a single identifying observation. To overcome this limitation, we exploit the panel data underlying the construction of AV. We show that bank systemicness $SB_{it}$ from equation (7) is an excellent predictor of $SRISK_{it}$, one of the most prominent measures.

We thank Stefano Giglio, Bryan Kelly and Seth Pruitt for generously sharing with us their data on systemic risk measures. See Appendix C for the sources of the different systemic risk measures. To aid interpretation, we adjust the sign of the systemic risk measures so that a higher value always denotes higher systemic risk. The only two measures we do not plot are $\Delta Absorption(1)$ and $\Delta Absorption(2)$ because they are too volatile compared to the other measures, but they nevertheless clearly exhibit no increasing trend pre-crisis. We also plot the TED spread in logs to bring it to a scale similar to the other measures (its 2008q4 value is more than 2,000 percent larger than its 2003q1 value).
and widely used measures of firm-specific systemic risk and itself a validated predictor of negative crisis outcomes in the cross-section (Brownlees and Engle, 2016). Using the quarterly data from 1996q1 to 2016q4, we run the dynamic panel regression

\[
\text{SRISK}_{i,t+k} = \beta \text{SB}_{i,t} + \delta \text{SRISK}_{i,t} + \gamma \text{controls}_{i,t} + \nu_i + \eta_t + \epsilon_{it}
\]  

(29)

where \(\nu_i\) are bank-specific fixed effects, \(\eta_t\) are time fixed effects, \(\epsilon_{it}\) is an error term assumed to be uncorrelated to the regressors, and \(\text{controls}_{i,t}\) is a vector of the following control variables: \(\Delta\text{CoVaR}, \text{MES} \) (marginal expected shortfall), conditional CAPM beta, stock returns, volatility of stock returns, physical probability of default over the next year, conditional value-at-risk at the 95 level, maturity mismatch and number of subsidiaries.66 \(\Delta\text{CoVaR} \) (Adrian and Brunnermeier, 2016) and \(\text{MES} \) (Acharya et al., 2012) are two successful measures of firm-specific systemic risk that we include to highlight that our measure contains new information.67 The rest of the controls are meant to capture bank characteristics that could in principle affect the systemicness of each bank but are broadly unrelated to the specific fire-sale mechanism we consider.68 Because regression (29) contains a lag of the dependent variable as a regressor, we use the system GMM estimator (Arellano and Bond, 1995; Blundell and Bond, 1998).69

The coefficient of interest is \(\beta\). If \(\beta\) is positive and significant, then systemicness \(\text{SB}\) is a good predictor of \(\text{SRISK}\). The magnitude of \(\beta\) is also of interest. Since \(\text{SB}\) and \(\text{SRISK}\) are

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66SRISK, MES (marginal expected shortfall) and CAPM betas are from the Volatility Laboratory (https://vlab.stern.nyu.edu/). Stock returns are from CRSP. Volatility of stock returns are the sum of squared daily returns over the corresponding horizon. One-year ahead physical probabilities of default are from Moody’s Analytics (formerly KMV). The maturity mismatch, conditional value-at-risk and \(\Delta\text{CoVaR} \) are from Adrian and Brunnermeier (2016) (Publicly available at https://www.aeaweb.org/articles?id=10.1257/aer.20120555). The number of subsidiaries is from Cetorelli and Goldberg (2016). Troubled Asset Relief Program (TARP) capital injections are from the U.S. Treasury (Publicly available at https://www.treasury.gov/initiatives/financial-stability/TARP-Programs/Pages/default.aspx).

67It is important to point out that \(\text{SRISK}, \Delta\text{CoVaR} \) and \(\text{MES} \) can be computed for a much broader set of firms than just banks, for a longer span of time, and at higher frequency, so any comparison must take this into account.

68The conditional CAPM beta, stock returns and the volatility of stock returns are included to capture the overall riskiness of banks; the probability of default and value-at-risk measure the tail risk; maturity mismatch correlates with funding risks; and the number of subsidiaries proxies for the complexity of each bank (Cetorelli and Goldberg, 2016).

69We use time fixed effects as exogenous instruments and all lags for the rest of the regressors, which we consider endogenous, as instruments but “collapsing” the instrument matrix as in Calderon et al. (2002); Beck and Levine (2004); Carkovic and Levine (2005) to keep the number of instruments small. The high p-values for the Arellano and Bond (1991) test for auto-correlated residuals that are reported in Table 4 mean that the null hypothesis that instruments are valid cannot be rejected at high confidence levels. Appendix G shows that the results in Table 4 are robust to similar specifications, including a standard fixed-effects regression (instead of GMM instrumented with lags) that could be more appropriate given that our sample has 160 quarters, perhaps not falling in the “small T” category.
both measured in units of dollars divided by equity capital, the magnitude of $\beta$ can be easily interpreted.

Column 1 in Table 4 shows the results when the prediction horizon is 5 years ($k = 20$ quarters). The coefficient of interest is significant at the 10 percent level and equal to $\beta = 2.13$. This implies that an increase in systemicness SB of 1 percent at time $t$ is associated with an increase in SRISK of 2.13 percent 5 years later. Columns 2 through 5 repeat the same regression for the prediction horizons $k = 4, 3, 2, 1$ years. For these horizons, $\beta$ is significant at the 1 percent level and its magnitude is larger, peaking at 4.36 for $k = 4$.

Columns 6 through 10 show the results of running the same regression as equation (29) but replacing $SB_{it}$ by its constituent factors from equation (7) that vary across banks, $\log(size_{it}), \log(leverage_{it}), \log(illiquidity \ linkage_{it})$, and their interactions, $\log(size_{it}) \times \log(leverage_{it}), \log(size_{it}) \times \log(illiquidity \ linkage_{it})$ and $\log(leverage_{it}) \times \log(illiquidity \ linkage_{it})$. We use logs to adapt the multiplicative decomposition of equation (7) to the additive form of the regression. We include the interaction terms to allow for the possibility that the covariances between factors, and not just their levels, drive predictability. The main goal of these regressions is to further understand what economic forces drive the good predictability properties of our bank-specific systemicness measure. A secondary goal is to assess the usefulness of the decomposition in equation (7) and the overall measure SB. For example, if only one factor — say size — were the single source of predictability, a case could be made that one should dispense with SB in favor of that simpler factor. The conclusion from Columns 6 through 10 of Table 4, however, is that all factors play a role, albeit at different horizons and with different magnitudes. For 5, 4 and 3 year ahead predictions, leverage shows up as significant at the 1 percent level and with a large coefficient. For shorter horizons, the t-statistic on leverage and the magnitude of its coefficient are both noticeably smaller. Illiquidity linkage shows a different pattern, with very low statistical significance for 5 and 4 year horizons, but large coefficients at the 3, 2 and 1 year horizons that are significant at the 1 percent level. Some of the coefficients on the interaction terms are also economically and statistically significant.

As a last exercise, we use our measure of individual bank vulnerability, $VB_{it}$ from equation (9), to predict capital injections conducted during the crisis under the Troubled Asset Relief Program (TARP). We view this exercise as an important complement to the predictive panel regressions discussed above. First, it tests a different dimension of the fire-sale framework — whether more vulnerable banks do indeed have worse outcomes, as opposed to what banks contribute the most to systemic risk, which do not necessarily need to

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70 We use HAC standard errors clustered at the bank level and adjusted for small samples using the Windmeijer (2005) correction.
Table 4: Regressions of SRISK on lagged SRISK, bank systemicness or its factors (equation 29).

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<td></td>
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<tr>
<td>SRISK_{it}</td>
<td>-0.12</td>
<td>-0.037</td>
<td>0.21***</td>
<td>0.33***</td>
<td>0.61***</td>
<td>-0.0081</td>
<td>0.071</td>
<td>0.29***</td>
<td>0.39***</td>
<td>0.65***</td>
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<tr>
<td></td>
<td>(-1.43)</td>
<td>(-0.96)</td>
<td>(11.50)</td>
<td>(7.83)</td>
<td>(14.27)</td>
<td>(-0.10)</td>
<td>(1.02)</td>
<td>(6.32)</td>
<td>(8.11)</td>
<td>(15.35)</td>
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<tr>
<td>∆CoVaR_{it}</td>
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<td>-2.80</td>
<td>0.36</td>
<td>-0.32</td>
<td>0.33</td>
<td>-6.69</td>
<td>-3.48</td>
<td>-0.26</td>
<td>-1.07</td>
<td>0.40</td>
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<td></td>
<td>(-1.05)</td>
<td>(-1.28)</td>
<td>(0.20)</td>
<td>(-0.19)</td>
<td>(0.23)</td>
<td>(-1.15)</td>
<td>(-1.27)</td>
<td>(-0.14)</td>
<td>(-0.51)</td>
<td>(0.32)</td>
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<tr>
<td>AR(2) p-value</td>
<td>0.71</td>
<td>0.74</td>
<td>0.56</td>
<td>0.35</td>
<td>0.95</td>
<td>0.89</td>
<td>0.78</td>
<td>0.28</td>
<td>0.69</td>
<td>0.47</td>
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<tr>
<td>Num Obs</td>
<td>1472</td>
<td>1698</td>
<td>1928</td>
<td>2163</td>
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<td>1455</td>
<td>1681</td>
<td>1910</td>
<td>2144</td>
<td>2288</td>
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</tbody>
</table>

* t statistics in parentheses
* * p < .1, ** p < .05, *** p < .01
have poor outcomes themselves. Second, in contrast to fire-sale externalities and systemic risk, the negative outcomes that should be associated with vulnerability, if not directly observable, have a much closer empirical proxy, providing a more direct empirical test of the framework (in this case, TARP is a proxy for equity needs during the crisis). Third, although narrowing attention to a single cross section of banks during the crisis foregoes the dynamic information contained in the panel, it allows us to show that our framework is relevant not solely on average but also when it matters most.

We use the assumptions and specification in Brownlees and Engle (2016). For a fixed time $\tau$, we run the cross-sectional regression

$$
\log C_{i}^* = \alpha_\tau + \beta_\tau \log(\text{VB}_{i\tau}) + \gamma_\tau \text{controls}_{i\tau} + \epsilon_{i\tau}
$$

(30)

where $\log C_{i}^*$ are the capital needs of bank $i$ during the crisis, $\log(\text{VB}_{i\tau})$ is log individual bank vulnerability for bank $i$ at the fixed time $\tau$, $\text{controls}_{i\tau}$ is a vector of explanatory variables and $\epsilon_{i\tau}$ is a Gaussian error term assumed to be uncorrelated with the regressors. We run two specifications. In the first one, there are no controls. In the second one, the controls are $\text{SRISK}_{i\tau}$, $\Delta\text{CoVaR}_{i\tau}$, $\text{MES}_{i\tau}$, volatility of stock returns, log assets, and equity capital fall between 2007q2 and 2008q2 as a share of assets. Brownlees and Engle (2016) further assume that TARP injections are carried out only if the amount injected is positive, leading the econometrician to observe the censored variable $\log C_{i} = \max\{\log C_{i}^*, 0\}$. Under these conditions, equation (30) is a Tobit regression that can be estimated consistently by maximum likelihood.

Columns 1 and 2 of Table 5 show the estimated coefficients for $\tau = 2004q4$. Both with and without controls, our measure of individual bank vulnerability has a coefficient $\hat{\beta}_{2004q4}$ that is statistically significant at the 10 and 5 percent levels, respectively. In addition, the magnitude of the coefficient is similar in the two specifications and economically large: a one percent increase in bank vulnerability in 2004q4 is associated with either an 11.74 or a 14.16 percent increase in TARP injections depending on whether controls are included. Columns 3 through 6 repeat the exercise for $\tau = 2005q4$ and 2006q4. Without controls, the coefficient on vulnerability is now insignificant. On the other hand, when controls are included, $\hat{\beta}_{\tau}$ is significant at the 1 percent level and still economically large.\footnote{Appendix G shows that vulnerability also predicts the probability of receiving TARP injections by running a Probit regression (with a TARP indicator as dependent variable).}

\footnote{Unlike Brownlees and Engle (2016), we do not include industry fixed effects because our sample consists only of banks. See Brownlees and Engle (2016) for a more detailed discussion of the assumptions and interpretation of equation (30).}
<table>
<thead>
<tr>
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<th>$\tau = 2006q4$</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<td><strong>Ind. Vulnerability</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$_{i\tau}$</td>
<td>11.7***</td>
<td>14.2*</td>
<td>9.33</td>
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<td></td>
<td>(2.11)</td>
<td>(1.82)</td>
<td>(1.48)</td>
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<tr>
<td><strong>SRISK</strong></td>
<td>-0.77</td>
<td>-1.83*</td>
<td>-1.90</td>
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<td>$_{i\tau}$</td>
<td>(-0.56)</td>
<td>(-1.87)</td>
<td>(-1.18)</td>
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<tr>
<td></td>
<td>(0.72)</td>
<td>(-0.75)</td>
<td>(-0.29)</td>
</tr>
<tr>
<td><strong>$\Delta$CoVaR</strong></td>
<td>48.4</td>
<td>-33.4</td>
<td>-21.2</td>
</tr>
<tr>
<td>$_{i\tau}$</td>
<td>(0.72)</td>
<td>(-0.75)</td>
<td>(-0.29)</td>
</tr>
<tr>
<td><strong>Equity Fall</strong></td>
<td>215.5</td>
<td>-35.9</td>
<td>268.2</td>
</tr>
<tr>
<td>$i_{(07q2-08q2)}$</td>
<td>(0.75)</td>
<td>(-0.15)</td>
<td>(0.92)</td>
</tr>
<tr>
<td><strong>Stock Vol</strong></td>
<td>0.25</td>
<td>-4.47***</td>
<td>1.55</td>
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<tr>
<td>$i_{\tau}$</td>
<td>(0.16)</td>
<td>(-2.81)</td>
<td>(1.18)</td>
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<tr>
<td><strong>Assets</strong></td>
<td>0.14</td>
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<td>-3.06</td>
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<tr>
<td>$i_{(log)}$</td>
<td>(0.04)</td>
<td>(-2.15)</td>
<td>(-0.86)</td>
</tr>
<tr>
<td><strong>MES</strong></td>
<td>5.45</td>
<td>27.4***</td>
<td>17.1*</td>
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<tr>
<td>$i_{\tau}$</td>
<td>(0.59)</td>
<td>(3.50)</td>
<td>(1.72)</td>
</tr>
<tr>
<td><strong>Num Obs</strong></td>
<td>100</td>
<td>38</td>
<td>100</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

$^*$ $p < .1$, $^{**} p < .05$, $^{***} p < .01$
8 Conclusion

In this paper, we study the factors making the financial system vulnerable to fire sales. We construct an index of aggregate vulnerability to fire sales of large bank holding companies that decomposes additively into each bank’s “systemicness” as well as multiplicatively into aggregate versus cross-sectional factors that drive fire-sale vulnerability.

We use this framework to track vulnerability and its drivers over time. Our index starts increasing in 2004, before any other major systemic risk measure, more than doubling by 2008. We identify the fire-sale specific factors of delevering speed and concentration of illiquid assets as accounting for the majority of this increase. After the crisis, the index decreases equally dramatically, ending in late 2016 at roughly 20 percent of its initial level in 2000. This indicates that the the U.S. banking system has materially reduced its vulnerability to fire sales.

We show that it is possible to predict not only when, but where in the financial sector vulnerabilities lurk. Individual banks’ contributions to aggregate vulnerability are an excellent five-year-ahead predictor of SRISK, one of the most prominent and widely used measures of firm-specific systemic risk and itself a validated predictor of negative crisis outcomes in the cross-section. Had they been available at the time, our measures would have been a useful early indicator of when and where vulnerabilities were building up.
References


Duarte, F. and T. M. Eisenbach (2014, April 15). On fire-sale externalities, TARP was close to optimal. Liberty Street Economics Blog.


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Financial Stability Oversight Council (2015). Explanation of the basis of the Financial Stability Oversight Council’s final determination that material financial distress at MetLife could pose a threat to U.S. financial stability and that MetLife should be supervised by the Board of Governors of the Federal Reserve System and be subject to prudential standards. Available at U.S. Chamber of Litigation http://www.chamberlitigation.com/sites/default/files/cases/files/2015/Final%20Designation%20%5B%20paginated%20from%20RJA%20vols%204%20and%205%5D%20-%20%20MetLife%20v.%20FSOC%20%20-DDC%20.pdf (2018/14/05).


Appendix

A Robustness checks

A.1 Details on risk-based capital requirements

We capture the trade-off between risk weights and price impact in a simple model. Bank \( i \)'s equity capital must exceed a fixed percentage of its risk-weighted assets

\[
e_{it} \geq \kappa a_{it}^\omega = \kappa \sum_{k=1}^{K} \omega_k m_{ikt} a_{it},
\]

where \( \kappa \in (0, 1) \) is a fixed number picked by the regulator and \( \omega_k \geq 0 \) is the risk weight of asset \( k \).

We maintain the partial adjustment model for leverage and the implied cash amount \( x_{it} \) but assume that the bank wants to minimize the total price discount it suffers when selling assets but still satisfy the capital requirement (equation (31)) and the budget constraint (that it has to raise \( x_{it} \)).\(^{73}\) The price impact for bank \( i \) of selling a share \( \rho_{ikt} \in [0, 1] \) of its holdings in asset \( k \) is \( \ell_k \rho_{ikt} m_{ikt} a_{it} \) basis points, since \( \rho_{ikt} m_{ikt} a_{it} \) is the dollar amount sold (before any price impact) and \( \ell_k \) is the liquidity of the asset in units of basis points per dollar. Hence, the loss to the bank due to the price impact is \( \ell_k (\rho_{ikt} m_{ikt} a_{it})^2 \) dollars. The amount of asset \( k \) remaining on the balance sheet is \( (1 - \rho_{ikt}) m_{ikt} a_{it} \) dollars. The bank’s optimization problem is then

\[
\min_{\rho_{ikt}} \sum_{k=1}^{K} \ell_k (\rho_{ikt} m_{ikt} a_{it})^2
\]

\[
\text{s.t. } e_{it} \geq \kappa \sum_{k=1}^{K} \omega_k (1 - \rho_{ikt}) m_{ikt} a_{it},
\]

\[
x_{it} = \sum_{k=1}^{K} \rho_{ikt} m_{ikt} a_{it} - \sum_{k=1}^{K} \ell_k (\rho_{ikt} m_{ikt} a_{it})^2
\]

\[
0 \leq \rho_{ikt} \leq 1
\]

We calibrate risk weights \( \omega_k \) by using the “standardized approach” of capital requirements in Basel III.\(^{74}\) For the tightness of the risk-based capital requirement we pick \( \kappa = 0.06, \)

\(^{73}\) We allow neither short-selling nor purchases of assets — only sales of assets on the balance sheet are permitted.

\(^{74}\) Appendix F shows the details. Most large banks use the “advanced approach” instead of the “standardized approach”, which usually produces lower overall risk-weights. We use the standardized approach.
which means banks must hold at least six percent of risk-weighted assets in equity. This number corresponds to the minimum Tier 1 capital requirement from Basel III.

A.2 Shocks to equity capital

Instead of considering a shock to the value of assets, we now consider a shock that exogenously reduces the equity capital of banks. Conceptually, an equity shock is an appealing way to model idiosyncratic financial distress at a particular firm or set of firms, while asset shocks seem a better way to model market-wide distress, or disruptions in specific asset classes. Modeling capital losses large enough to put firms close to insolvency could be useful when trying to evaluate whether firms should be designated as systemically important financial institutions (SIFIs).\textsuperscript{75}

We calibrate the equity shock to have the same average initial direct losses as our benchmark of a one percent uniform asset shock.\textsuperscript{76} To do so, we first compute the size \( g_{it} \) of the equity shock needed so that each bank \( i \) has the same direct losses in each time period \( t \) as when hit by a one percent asset shock:

\[
g_{it} = \frac{0.01 \times a_{it}}{e_{it}}.
\]

Then, we take the average of \( g_{it} \) across all banks \( i \) and all time periods \( t \) to arrive at a uniform equity shock \( g \). The linearity of the framework is still preserved, so shocking each bank’s equity capital separately and then adding the resulting fire-sale spillovers is equivalent to shocking the equity capital of all banks simultaneously.

Figure 13 shows that, for the most part, equity shocks produce lower AV than asset shocks. This is due to the fact that less levered banks also tend to be smaller and have

\textsuperscript{75}For example, the Dodd-Frank act requires, among other standards, that a firm be designated as a SIFI if, whenever it experiences “material financial distress or failure”, it “holds assets that, if liquidated quickly, would cause a fall in asset prices and thereby significantly disrupt trading or funding in key markets or cause significant losses or funding problems for other firms with similar holdings.” (Final rule and interpretive guidance to Section 113 of the Dodd-Frank Wall Street Reform and Consumer Protection Act.) Our framework with equity shocks embodies the spirit of this so-called “asset liquidation channel” quite well if we interpret material financial distress as a severe depletion of equity capital. Note that the law starts with the presumption of material finance distress or failure and does not require reasons or probabilities for that event. Modeling equity shocks as exogenous is therefore very much in accordance with the law.

\textsuperscript{76}While for each single bank there is a one-to-one correspondence between asset shocks and equity shocks, it is not possible to construct a uniform system-wide equity shock (with the same shock magnitude for all banks) that exactly reproduces the outcome of a uniform system-wide asset shock. This is because leverage is not constant across firms. For a given asset shock, a more levered firm experiences higher initial capital losses than a less levered firm. Hence, a uniform shock to equity capital with the same initial aggregate losses causes larger capital declines in less levered firms.
lower illiquidity linkage, therefore amplifying and transmitting less externalities.

### A.3 Liquidity across assets and time

For BHCs, the benchmark specification has liquidity of asset classes based on the Net Stable Funding Ratio (NSFR) of the Basel III regulatory framework. Greenwood et al. (2015) instead assume the same price impact for all assets, $\ell_k = \ell$ for all $k$ and base the estimate on the liquidity of corporate bonds. Figure 14 (left panel) shows that AV is lower under this assumption (since most bank assets are less liquid than corporate bonds).

In our benchmark, we adjust liquidity across time by the wealth $w_t$ of potential buyers of fire-sold assets. We proxy for $w_t$ using total financial sector assets net of BHCs, respectively. We now explore two alternatives. First, we can assume that the entire economy has the capacity to absorb assets when they are fire-sold, instead of just the financial sector. For this first scenario, we replace $w_t$ by nominal GDP in all periods $t$. Second, we can assume that aggregate liquidity is constant across time periods. The price impact, expressed in units of basis points per dollar sold, is therefore independent of the total size of financial markets or the economy. This is an extreme case and implies that wealth of potential buyers remains constant, even in nominal terms.\footnote{This choice could make AV non-stationary, as the total assets of the banks we consider are presumably co-integrated with total assets in the financial system or the economy.} For this scenario, we set $w_t = \text{const.}$ in all periods $t$. Figure 14 (right panel) shows the implications of the two scenarios.\footnote{By construction, AV under all three scenarios is the same in 2011q3, respectively, the quarter we use to normalize absolute asset liquidity, to be consistent with Greenwood et al. (2015).} There is a
notable difference only for the constant liquidity scenario and only in the pre-crisis period, where the growth in AV is faster than in the benchmark. This is due to the fact that over this period, financial sector assets grew significantly faster than GDP. When taking the entire economy as a reference for potential buyers of fire-sold assets, the potential spillovers therefore grow faster as financial sector growth outpaces the rest of the economy.

A.4 Analysis with balanced panels of institutions

In our main analysis, we include the top 100 banks every quarter that have leverage target estimates from Section 4 which may be different sets in each period. Figure 15 displays AV when we only keep banks that have been present throughout the entire sample. Because some large, levered and linked institutions are dropped from the sample, aggregate vulnerability decreases. The qualitative behavior of the measure remains the same, with the curve essentially shifting downwards for all time periods and the run up to the crisis becoming more pronounced.

B Derivation of liquidation rule with two assets

The dividend distribution region is a convex cone from the origin defined by four vectors (the region inside the convex combination of the four vectors defines the cone). After a jump shock $dN_t = 1$ that puts the bank outside the dividend distribution region, we know that the bank will adjust so as to return to the edge of the dividend distribution region cone.
that is closest to the line defined by $y_1 = y_2 = 0$ (as explained in Section 3, see Figure 3). Let $\varphi$ denote this edge; it can be parametrized as

$$
\varphi = \{ (x, y_1, y_2) \text{ s.t. } (x, y_1, y_2) = t (x_{\text{sell}}, y_{1,\text{sell}}, y_{2,\text{sell}}) \text{ and } t \geq 0 \},
$$

where $(x_{\text{sell}}, y_{1,\text{sell}}, y_{2,\text{sell}})$ is any non-zero point on the edge we are interested in. Note that $(x_{\text{sell}}, y_{1,\text{sell}}, y_{2,\text{sell}})$ is determined solely by the parameters of the model. Given a liquidation value of the bank $W$, the set of points that have the same liquidation value is the iso-wealth plane

$$
W = x + (1 - \ell_1) y_1 + (1 - \ell_2) y_2.
$$

Given a value $b$ for leverage, the set of points that have the same leverage is the iso-leverage plane

$$
0 = (b + 1) x + by_1 + by_2.
$$

Consider a bank with initial balance sheet position $(x_0, y_{10}, y_{20})$, where $x_0$ is the amount invested in bonds, and $y_{10}, y_{20}$ are the amounts invested in the two illiquid assets. After a jump shock $dN_t = 1$, without any adjustments, the bank would be at $(x^p, y^p_{1}, y^p_{2}) = (x_0, (1 - f) y_{10}, (1 - f) y_{20})$ with wealth

$$
W^p = x^p + (1 - \ell_1) y^p_{1} + (1 - \ell_2) y^p_{2}
$$

$$
= x_0 + (1 - \ell_1) (1 - f) y_{10} + (1 - \ell_2) (1 - f) y_{20}.
$$

Assume that $(x_0, y_{10}, y_{20})$ is in the dividend distribution region and that $(x^p, y^p_{1}, y^p_{2})$ is in
the sell region. The bank adjusts by selling the illiquid assets until it is at a point \((x_1, y_{11}, y_{21})\) on the edge \(\varphi\):

\[
(x_1, y_{11}, y_{21}) = t_1 (x_{\text{sell}}, y_{1,\text{sell}}, y_{2,\text{sell}})
\] (35)

for some \(t_1\). Let \(W_1\) be the liquidation value of the bank at \((x_1, y_{11}, y_{21})\). Since selling assets does not change the liquidation value of the bank, we must have that

\[
W^p = W_1.
\] (36)

Geometrically, \((x_1, y_{11}, y_{21})\) is the point that is on the iso-wealth plane defined by \(W^p\) and also on the line \(t_1 (x_{\text{sell}}, y_{1,\text{sell}}, y_{2,\text{sell}})\). The only point that satisfies equations (35) and (36) is parametrized by

\[
t_1 = \frac{W^p}{x_{\text{sell}} + (1 - \ell_1) y_{1,\text{sell}} + (1 - \ell_2) y_{2,\text{sell}}}
= \frac{x_0 + (1 - \ell_1) (1 - f) y_{10} + (1 - \ell_2) (1 - f) y_{20}}{x_{\text{sell}} + (1 - \ell_1) y_{1,\text{sell}} + (1 - \ell_2) y_{2,\text{sell}}}.
\]

The amount of each of the illiquid assets that the bank sells to get from \((x^p, y^p_1, y^p_2)\) to \((x_1, y_{11}, y_{21})\) is

\[
y^p_1 - y_{11} = (1 - f) y_{10} - y_{11}
= (1 - f) y_{10} - t_1 y_{1,\text{sell}}
\]

\[
y^p_2 - y_{21} = (1 - f) y_{20} - y_{21}
= (1 - f) y_{20} - t_1 y_{2,\text{sell}}
\]

while the total sold is the sum

\[
(y^p_1 - y_{11}) + (y^p_2 - y_{21}) = (1 - f) (y_{10} + y_{20}) - t_1 (y_{1,\text{sell}} + y_{2,\text{sell}}).
\]

Out of the total sold, the share coming from the first asset is

\[
\frac{y^p_1 - y_{11}}{(y^p_1 - y_{11}) + (y^p_2 - y_{21})} = \frac{(1 - f) y_{10} - t_1 y_{1,\text{sell}}}{(1 - f) (y_{10} + y_{20}) - t_1 (y_{1,\text{sell}} + y_{2,\text{sell}})}.
\] (37)
Substituting \( t_1 \) into equation (37) and using the following relations

\[
\begin{align*}
    m_{it} &= \frac{y_{it}}{a_t} \\
    b_t &= \frac{d_t}{e_t} \\
    e_t &= x_t + y_{1t} + y_{2t} \\
    a_t &= y_{1t} + y_{2t} \\
    d_t &= -x_t,
\end{align*}
\]

where \( i = 0, 1, p, B \) and \( t = 1, 2 \), we get

\[
\frac{y^p_1 - y_{11}}{(y^p_1 - y_{11}) + (y^p_2 - y_{21})} = m_{10} + \Theta_1,
\]

(38)

with

\[
\begin{align*}
    \Theta_1 &\equiv \frac{Y}{Y + 1 - f} (m_{1B} - m_{10}) \\
    Y &\equiv (Y_1 - Y_2) m_{10} + Y_2 + Y_3 \\
    Y_1 &\equiv \frac{(\ell_1 - 1) (f - 1)}{\Psi} \\
    Y_2 &\equiv \frac{(\ell_2 - 1) (f - 1)}{\Psi} \\
    Y_3 &\equiv \frac{b_0}{b_0 + 1} \\
    \Psi &\equiv (\ell_1 - 1) m_{1B} + (\ell_2 - 1) m_{2B} - \frac{b_1}{b_1 + 1}.
\end{align*}
\]

Linearizing equation (38) with respect to \((f, \ell_1, \ell_2, b_0, b_1)\) around \((1, 0, 0, 1, 1)\) gives

\[
\frac{y^p_1 - y_{11}}{(y^p_1 - y_{11}) + (y^p_2 - y_{21})} \approx m_{10} + (3f - 4) (m_{10} - m_{1B}).
\]

The analogous expression holds for the second illiquid asset.

If after the \( dN_t = 1 \) jump the bank is in the equity issuance region, then the same calculations apply but interpreting \((x^p, y^p_1, y^p_2)\) as the point the bank is at after it has finished issuing equity.
### Systemic risk measures

#### Table 6: Systemic risk measures used in Section 7.

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<tr>
<th>Measures</th>
<th>Sources</th>
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<tbody>
<tr>
<td>GZ</td>
<td>Gilchrist and Zakrajsek (2012)</td>
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<tr>
<td>Absorption, ∆Absorption</td>
<td>Kritzman et al. (2011)</td>
</tr>
<tr>
<td>CoVaR, ∆CoVaR</td>
<td>Adrian and Brunnermeier (2016)</td>
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<tr>
<td>MES (APPR), SysRisk</td>
<td>Acharya et al. (2017)</td>
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<tr>
<td>MES (SRISK)</td>
<td>Brownlees and Engle (2016)</td>
</tr>
<tr>
<td>Intl. Spillover</td>
<td>Diebold and Yilmaz (2009)</td>
</tr>
<tr>
<td>Turbulence</td>
<td>Kritzman and Li (2010)</td>
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Table 6 lists the sources for the various systemic risk measures we use. See Giglio et al. (2016) for details.
### Mapping between asset classes and form FR Y-9C

<table>
<thead>
<tr>
<th>Category</th>
<th>Codes on FR Y-9C</th>
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<td><strong>Total assets</strong></td>
<td>Entire sample bhck2170</td>
</tr>
<tr>
<td><strong>Equity</strong></td>
<td>Starting 2014q1 bhck8274 or bhca8274</td>
</tr>
<tr>
<td></td>
<td>Up to 2013q4 bhck8274</td>
</tr>
<tr>
<td><strong>Cash</strong></td>
<td>Entire sample bhck0081 + bhck0395 + bhck0397</td>
</tr>
<tr>
<td><strong>U.S. Treasuries</strong></td>
<td>Starting 2008q1 bhck0211 + bhck1287 + bhcm3531</td>
</tr>
<tr>
<td></td>
<td>Up to 2007q4 bhck0211 + bhck1287 + bhck3531</td>
</tr>
<tr>
<td><strong>Agency securities</strong></td>
<td>Starting 2008q1 bhck1289 + bhck1294 + bhck1293 + bhck1298 + bhcm3532</td>
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<tr>
<td></td>
<td>Up to 2007q4 bhck1289 + bhck1294 + bhck1293 + bhck1298 + bhck3532</td>
</tr>
<tr>
<td><strong>Municipal securities</strong></td>
<td>Starting 2008q1 bhck8496 + bhck8499 + bhcm3533</td>
</tr>
<tr>
<td></td>
<td>2001q1 to 2007q4 bhck8496 + bhck8499 + bhck3533</td>
</tr>
<tr>
<td></td>
<td>Up to 2000q4 bhck8531 + bhck8535 + bhck8534 + bhck8538</td>
</tr>
<tr>
<td><strong>Agency MBS</strong></td>
<td>Starting 2011q1 bhckg300 + bhckg304 + bhckg312 + bhckg316 + bhckk142 + bhckk150 + bhckg303 + bhckg307 + bhckg315 + bhckg319 + bhckk145 + bhckk153 + bhckg379 + bhckg380 + bhckk197</td>
</tr>
<tr>
<td></td>
<td>2009q2 to 2010q4 bhckg300 + bhckg304 + bhckg312 + bhckg316 + bhckg303 + bhckg307 + bhckg315 + bhckg319 + bhckg379 + bhckg380 + (bhckg324 + bhckg328 + bhckg327 + bhckg331 + bhckg382)/2</td>
</tr>
<tr>
<td></td>
<td>2008q1 to 2009q1 bhck1698 + bhck1703 + bhck1714 + bhck1718 + bhck1702 + bhck1707 + bhck1717 + bhck1732 + bhcm3534 + bhcm3535</td>
</tr>
<tr>
<td></td>
<td>Up to 2007q4 bhck1698 + bhck1703 + bhck1714 + bhck1718 + bhck1702 + bhck1707 + bhck1717 + bhck1732 + bhck3534 + bhck3535</td>
</tr>
<tr>
<td><strong>Non-agency MBS</strong></td>
<td>Starting 2011q1 bhckg308 + bhckg320 + bhckk146 + bhckk154 + bhckg311 + bhckg323 + bhckk149 + bhckk157 + bhckg381 + bhckk198</td>
</tr>
<tr>
<td></td>
<td>2009q2 to 2010q4 bhckg308 + bhckg320 + bhckg311 + bhckg323 + bhckg381 + (bhckg324 + bhckg328 + bhckg327 + bhckg331 + bhckg382)/2</td>
</tr>
<tr>
<td></td>
<td>2008q1 to 2009q1 bhck1709 + bhck1733 + bhck1713 + bhck1736 + bhcm3536</td>
</tr>
<tr>
<td></td>
<td>Up to 2007q4 bhck1709 + bhck1733 + bhck1713 + bhck1736 + bhck3536</td>
</tr>
<tr>
<td>Category</td>
<td>Codes on FR Y-9C</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>ABS &amp; other debt securities</td>
<td>Starting 2009q2: bhckc026 + bhckg336 + bhckg340 + bhckg344 + bhck1737 + bhck1742 + bhck1746 + bhckc027 + bhckg339 + bhckg343 + bhckg347 + bhck1741 + bhck1746 + bhckg383 + bhckg384 + bhckg385 + bhckg386</td>
</tr>
<tr>
<td></td>
<td>2008q1 to 2009q1: bhckc026 + bhckg336 + bhckg340 + bhckg344 + bhck1737 + bhck1742 + bhck1746 + bhckc027 + bhckg339 + bhckg343 + bhckg347 + bhck1741 + bhck1746 + bhcm3537</td>
</tr>
<tr>
<td></td>
<td>2006q1 to 2007q4: bhckc026 + bhckg336 + bhckg340 + bhckg344 + bhck1737 + bhck1742 + bhck1746 + bhckc027 + bhckg339 + bhckg343 + bhckg347 + bhck1741 + bhck1746 + bhck3537</td>
</tr>
<tr>
<td></td>
<td>2001q1 to 2005q4: bhckb838 + bhckb842 + bhckb846 + bhckb850 + bhckb854 + bhckb858 + bhck1737 + bhck1742 + bhckb841 + bhckb845 + bhckb849 + bhckb853 + bhckb857 + bhckb861 + bhck1741 + bhck1746 + bhck3537</td>
</tr>
<tr>
<td></td>
<td>Up to 2000q4: bhck1754 + bhck1773 - (bhck0211 + bhck1287 + bhck3531 + bhck1289 + bhck1294 + bhck1293 + bhck1298 + bhck3532 + bhck8531 + bhck8535 + bhck8534 + bhck8538 + bhck1698 + bhck1703 + bhck1714 + bhck1718 + bhck1702 + bhck1707 + bhck1717 + bhck1732 + bhck1709 + bhck1733 + bhck1713 + bhck1736 + bhck8544 + bhck8550) + bhck3537</td>
</tr>
<tr>
<td>Equities &amp; other securities</td>
<td>Starting 2001q1: bhcka511 + bhcm3541</td>
</tr>
<tr>
<td></td>
<td>Up to 2000q4: bhck8544 + bhck8550</td>
</tr>
<tr>
<td>Residual securities</td>
<td>Entire sample: bhck1754 + bhck1773 + bhck3545 – all securities above</td>
</tr>
<tr>
<td>Repo and fed funds loans</td>
<td>Starting 2002q1: bhdm987 + bhckb989</td>
</tr>
<tr>
<td></td>
<td>1997q1 to 2001q4: bhck1350</td>
</tr>
<tr>
<td></td>
<td>Up to 1996q4: bhck0276 + bhck0277</td>
</tr>
<tr>
<td>Residential real estate loans</td>
<td>Entire sample: bhdm1797 + bhdm5367 + bhdm5368 + bhdmf606 + bhdmf607 + bhdmf611</td>
</tr>
<tr>
<td>Commercial real estate loans</td>
<td>Starting 2007q1: bhckf158 + bhckf159 + bhdm1460 + bhckf160 + bhckf161 + bhdmf604 + bhdmf612 + bhdmf613</td>
</tr>
<tr>
<td></td>
<td>Up to 2006q4: bhdm1415 + bhdm1460 + bhdm1480 + bhdmf604 + bhdmf612 + bhdmf613</td>
</tr>
<tr>
<td>Other real estate loans</td>
<td>Starting 2007q1: bhck1410 – (bhdm1797 + bhdm5367 + bhdm5368 + bhckf158 + bhckf159 + bhdm1460 + bhckf160 + bhckf161) + bhckf610 – (bhdmf606 + bhdmf607 + bhdmf611 + bhdmf604 + bhdmf612 + bhdmf613)</td>
</tr>
<tr>
<td>Category</td>
<td>Codes on FR Y-9C</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>
| C & I loans       | Up to 2006q4  \(bhck1410 - (bhdm1797 + bhdm5367 + bhdm5368 + bhdm1415 + bhdm1460 + bhdm1480) + bhckf610 - (bhdmf606 + bhdmf607 + bhdmf611 + bhdmf604 + bhdmf612 + bhdmf613)\)  
                  | Entire sample  \(bhck1763 + bhck1764 + bhckf614\)                                                                                                                                                              |
| Consumer loans    | Up to 2000q4  \(bhckb538 + bhckb539 + bhckk137 + bhckk207 + bhckf615 + bhckf616 + bhckk199 + bhckk210\)                                                                                                    |
                  | Starting 2011q1 \(bhckb538 + bhckb539 + bhckk137 + bhckk207 + bhckf615 + bhckf616 + bhckk199 + bhckk210\)                                                                                                    |
                  | 2001q1 to 2010q4 \(bhckb538 + bhckb539 + bhckk2011 + bhckf615 + bhckf616 + bhckf617\)                                                                                                                   |
| Lease financings  | Up to 2000q4  \(bhck2008 + bhck2011\)                                                                                                                                                                          |
                  | Starting 2007q1 \(bhckf162 + bhckf163\)                                                                                                                                                                        |
                  | Up to 2006q4  \(bhck2182 + bhck2183\)                                                                                                                                                                         |
| Residual loans    | Entire sample \(bhckf618 - all loans above\)                                                                                                                                                                    |
| Residual assets   | Entire sample \(bhck2170 - all assets above\)                                                                                                                                                                    |

Note: We combine all categories under trading assets with the corresponding categories under securities and loans. We use amortized cost for all securities reported as held-to-maturity and fair value for all securities reported as available-for-sale. We use loans and trading assets on a consolidated basis where available. From 2009q2 to 2010q4 commercial MBS are not broken out into agency MBS and non-agency MBS; we allocate them 50:50. Up to 2000q4 municipal securities include small amounts of MBS which are also included in agency MBS and non-agency MBS; we replace negative values of ABS and other debt securities with 0. In the calculation of total assets, loans are adjusted by unearned income but the loan breakdown is unadjusted; we replace negative values of residual loans with 0.

## E NSFR weights

We use price impacts \(\ell_k\) based on the weights laid out under the Net Stable Funding Ratio (NSFR). The NSFR is fully described in “Basel III: The Net Stable Funding Ratio,” issued in October 2014 by the Basel Committee on Banking Supervision. When necessary, we refer to risk-weights assigned in “International Convergence of Capital Measurement and Capital Standards,” issued in June 2006 by the Basel Committee on Banking Supervision. Of course, some level of judgment is used in assigning these weights, as our asset classes do not align perfectly with those described by the documentation on the NSFR. In addition, some of our asset categories contain assets with heterogeneous liquidity weights, whose relative magni-
attitudes are not possible to determine using Y-9C data. Nevertheless, we believe the weights are broadly representative and are sufficiently reasonable to illustrate the effect of heterogeneous liquidity.

We determine liquidity weights based on the NSFR as follows:

<table>
<thead>
<tr>
<th>Asset class</th>
<th>NSFR haircut</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Treasuries</td>
<td>5%</td>
<td>See section II.B, paragraph 37.</td>
</tr>
<tr>
<td>Repo &amp; fed funds loans</td>
<td>10%</td>
<td>We take the collateral underlying reverse repurchase agreements as the relevant assets in determining liquidity weights (see section II.B, paragraph 32 for details). The collateral for most repos is U.S. Treasuries (5% liquidity weight), followed by agency MBS (15% liquidity weight).</td>
</tr>
<tr>
<td>Agency MBS</td>
<td>15%</td>
<td>See section II.B, paragraph 39.</td>
</tr>
<tr>
<td>Agency securities</td>
<td>15%</td>
<td>Identical treatment to agency MBS.</td>
</tr>
<tr>
<td>ABS &amp; other debt securities</td>
<td>35%</td>
<td>A heterogeneous group of asset types with liquidity weights ranging from 5% to 100%. We judge that portfolio weights are slanted towards more liquid assets and thus assign a liquidity weight of 35%. See section II.B, paragraphs 37–42.</td>
</tr>
<tr>
<td>Equities &amp; other securities</td>
<td>55%</td>
<td>Non-financial, exchange-traded common equity shares receive a liquidity weight of 50%, while all other equity received a liquidity weight of 100%. See section II.B, paragraphs 40 and 43.</td>
</tr>
<tr>
<td>Municipal securities</td>
<td>60%</td>
<td>NSFR Liquidity weights depend on the duration of the residual maturity as well as the assigned risk weight according to “International Convergence of Capital Measurement and Capital Standards.” Weights range from 50% to 65%. See section II.B, paragraphs 40–41.</td>
</tr>
<tr>
<td>Residential real estate loans</td>
<td>60%</td>
<td>NSFR liquidity weights for residential real estate loans depend on the residual maturity of the loan as well as the assigned risk weight according to “International Convergence of Capital Measurement and Capital Standards.” Weights range from 50% to 65%. See section II.B, paragraphs 40–41.</td>
</tr>
<tr>
<td>Non-agency MBS</td>
<td>65%</td>
<td>NSFR Liquidity weights depend on the duration of the residual maturity as well as the assigned risk weight according to “International Convergence of Capital Measurement and Capital Standards.” Weights range from 50% to 85%. See section II.B, paragraphs 40–42.</td>
</tr>
<tr>
<td>Asset class</td>
<td>NSFR haircut</td>
<td>Notes</td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>C &amp; I loans</td>
<td>75%</td>
<td>NSFR liquidity weights for commercial real estate loans depend on the residual maturity of the loan as well as the assigned risk weight according to “International Convergence of Capital Measurement and Capital Standards.” Weights range from 50% to 85%. See section II.B, paragraphs 40–42.</td>
</tr>
<tr>
<td>Commercial real estate loans</td>
<td>75%</td>
<td>Identical treatment as C &amp; I loans</td>
</tr>
<tr>
<td>Consumer loans</td>
<td>75%</td>
<td>Identical treatment as C &amp; I loans</td>
</tr>
<tr>
<td>Lease financings</td>
<td>75%</td>
<td>Identical treatment as C &amp; I loans</td>
</tr>
<tr>
<td>Other real estate loans</td>
<td>75%</td>
<td>Identical treatment as C &amp; I loans</td>
</tr>
<tr>
<td>Residual loans</td>
<td>75%</td>
<td>Identical treatment as C &amp; I loans</td>
</tr>
<tr>
<td>Residual assets</td>
<td>100%</td>
<td>See section II.B, paragraph 43.</td>
</tr>
<tr>
<td>Residual securities</td>
<td>100%</td>
<td>See section II.B, paragraph 43.</td>
</tr>
</tbody>
</table>

### F Basel III capital risk weights

We base our risk weights on the “International Convergence of Capital Measurement and Capital Standards,” issued in June 2006 by the Basel Committee on Banking Supervision. When the Basel standards are very different from the U.S. implementation, or too general, we use the Federal Register, Vol. 77, No. 169, August 30, 2012, Part III and the Federal Register, Vol. 78, No. 198, October 11, 2013. When possible, we use the standardized approach. Of course, there is substantial judgment in assigning risk-weights and the advanced approaches could lead to very different risk weights.\(^{79}\) In addition, some of our asset categories contain assets with heterogeneous risk-weights, whose relative magnitudes are not possible to determine using Y-9C data. Nevertheless, we believe the weights are broadly representative and are sufficiently reasonable to illustrate the effect of capital requirements. We determine the weights as follows:

<table>
<thead>
<tr>
<th>Asset class</th>
<th>Risk weight</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>0%</td>
<td>Has no credit risk.</td>
</tr>
<tr>
<td>U.S. Treasuries</td>
<td>0%</td>
<td>The U.S. has an ECA risk score of 0 to 1, thus receives zero risk weight on its sovereign debt. See Annex 11, Section I.A, paragraph 2 of Basel Committee, 2006.</td>
</tr>
<tr>
<td>Repo &amp; fed funds loans</td>
<td>0%</td>
<td>By virtue of Part II, Section 2.D, paragraphs 170 and 171, and since virtually all the collateral in our data are U.S. Treasuries and Agency MBS, we assign a risk-weight of zero.</td>
</tr>
<tr>
<td>Agency MBS</td>
<td>20%</td>
<td>Treated as claims on banks and securities firms according to Annex 11, Section I.B, paragraph 7. Based on Annex 11, Section I.C, paragraph 8, we assign a 20% risk weight.</td>
</tr>
<tr>
<td>Agency securities</td>
<td>20%</td>
<td>Identical treatment as agency MBS.</td>
</tr>
<tr>
<td>ABS &amp; other debt securities</td>
<td>100%</td>
<td>Other assets with 100% risk weight (Annex 11, Section I.J, paragraph 23).</td>
</tr>
<tr>
<td>Equities &amp; other securities</td>
<td>100%</td>
<td>Other assets with 100% risk weight (Annex 11, Section I.J, paragraph 23).</td>
</tr>
<tr>
<td>Municipal securities</td>
<td>10%</td>
<td>Treated the same as agency securities according to Annex 11, Section I.B, paragraph 7 and thus generically receive a risk weight of 20%. However, the characteristics detailed in footnote 260 are satisfied by a large number of municipal securities, which should then receive a 0% risk weight.</td>
</tr>
<tr>
<td>Residential real estate loans</td>
<td>65%</td>
<td>Annex 11, Section I.F, paragraph 15 proposes 35%. The risk weight could be higher if local regulator deems appropriate (Annex 11, Section I.F, paragraph 16). In the U.S., the implementation of the standardized approach has significantly higher risk-weights, ranging from 50% to 100% depending on the characteristics of the loan (Federal Register, Vol. 78, No. 198, October 11, 2013).</td>
</tr>
<tr>
<td>Non-agency MBS</td>
<td>35%</td>
<td>See Annex 11, Section I.F, paragraph 15.</td>
</tr>
<tr>
<td>C &amp; I loans</td>
<td>100%</td>
<td>A heterogeneous group of asset types with risk weights ranging from 75% to 150%. See Annex 11, Section I.D-I.I.</td>
</tr>
<tr>
<td>Commercial real estate loans</td>
<td>100%</td>
<td>See Annex 11, Section I.G, paragraph 17.</td>
</tr>
<tr>
<td>Consumer loans</td>
<td>75%</td>
<td>See Annex 11, Section I.E, paragraphs 12-13. Risk-weight of 75% assumes orientation, product and granularity criteria are met, could be higher if not met. In the U.S., consumer loans get 100% risk-weight under the standardized approach (Federal Register, Vol. 77, No. 169, August 30, 2012, Part III).</td>
</tr>
<tr>
<td>Lease financings</td>
<td>100%</td>
<td>Other assets with 100% risk weight (Annex 11, Section I.J, paragraph 23).</td>
</tr>
<tr>
<td>Other real estate loans</td>
<td>100%</td>
<td>Mostly collateralized by farmland, treated as commercial real estate loans.</td>
</tr>
<tr>
<td>Residual loans</td>
<td>100%</td>
<td>Other assets with 100% risk weight (Annex 11, Section I.J, paragraph 23).</td>
</tr>
<tr>
<td>Asset class</td>
<td>Risk weight</td>
<td>Notes</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>Residual assets</td>
<td>100%</td>
<td>Other assets with 100% risk weight (Annex 11, Section I.J, paragraph 23).</td>
</tr>
<tr>
<td>Residual securities</td>
<td>100%</td>
<td>Other assets with 100% risk weight (Annex 11, Section I.J, paragraph 23).</td>
</tr>
</tbody>
</table>
### Table 10: Probit regression $1_{\text{TARP}_\tau} = \alpha_\tau + \beta_\tau \log(\text{VB}_\tau) + \gamma_\tau \text{controls}_\tau + \varepsilon_\tau$

<table>
<thead>
<tr>
<th></th>
<th>$\tau = 2004q4$</th>
<th>$\tau = 2005q4$</th>
<th>$\tau = 2006q4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ind. Vulnerability$_\tau$</td>
<td>0.56$^*$</td>
<td>1.46$^*$</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(1.94)</td>
<td>(1.86)</td>
<td>(1.38)</td>
</tr>
<tr>
<td>SRISK$_\tau$</td>
<td>-0.18</td>
<td>-0.57$^{***}$</td>
<td>-0.29</td>
</tr>
<tr>
<td></td>
<td>(-0.88)</td>
<td>(-2.83)</td>
<td></td>
</tr>
<tr>
<td>$\Delta$CoVaR$_\tau$</td>
<td>6.59</td>
<td>-0.52</td>
<td>-2.23</td>
</tr>
<tr>
<td></td>
<td>(1.03)</td>
<td>(-0.09)</td>
<td></td>
</tr>
<tr>
<td>Equity Fall$_i$ (07q2-08q2)</td>
<td>27.4</td>
<td>7.58</td>
<td>43.7</td>
</tr>
<tr>
<td></td>
<td>(0.88)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>Stock Vol$_\tau$</td>
<td>0.052</td>
<td>-0.86$^{***}$</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(-2.71)</td>
<td></td>
</tr>
<tr>
<td>Assets$_\tau$ (log)</td>
<td>-0.12</td>
<td>-1.21$^{***}$</td>
<td>-0.46</td>
</tr>
<tr>
<td></td>
<td>(-0.35)</td>
<td>(-2.71)</td>
<td></td>
</tr>
<tr>
<td>MES$_\tau$</td>
<td>0.50</td>
<td>5.55$^{***}$</td>
<td>2.09$^*$</td>
</tr>
<tr>
<td></td>
<td>(0.59)</td>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>Constant$_i$</td>
<td>1.15$^*$</td>
<td>4.89</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>(1.65)</td>
<td>(0.57)</td>
<td>(1.21)</td>
</tr>
<tr>
<td>Num Obs</td>
<td>100</td>
<td>38</td>
<td>100</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses

$^*$ $p < .1$, $^**$ $p < .05$, $^{***}$ $p < .01$
Table 11: \( \text{SRISK}_{i,t+k} = \beta \text{SB}_{it} + \delta \text{SRISK}_{it} + \gamma \text{controls}_{it} + \nu_t + \eta_i + \epsilon_{it} \)

<table>
<thead>
<tr>
<th></th>
<th>Pooled OLS</th>
<th>Fixed Effects</th>
<th>Dynamic Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systemicness(_{it})</td>
<td>1.99(^{***}) (8.69)</td>
<td>5.34(^{***}) (4.75)</td>
<td>2.75(^{***}) (3.32)</td>
</tr>
<tr>
<td>( \text{SRISK}_{it} )</td>
<td>0.36(^{***}) (7.14)</td>
<td>0.16(^{***}) (2.80)</td>
<td>0.12(^{***}) (6.50)</td>
</tr>
<tr>
<td>( \Delta \text{CoVaR}_{it} )</td>
<td>-0.20 (-0.40)</td>
<td>0.023 (0.02)</td>
<td>0.36 (0.20)</td>
</tr>
<tr>
<td>( \text{MES}_{it} )</td>
<td>0.013 (0.07)</td>
<td>1.23 (0.99)</td>
<td>0.95 (-0.95)</td>
</tr>
<tr>
<td>Stock Ret(_{it}) (log)</td>
<td>-0.00046 (-0.83)</td>
<td>0.00028 (0.41)</td>
<td>0.00023 (0.41)</td>
</tr>
<tr>
<td>Stock Vol(_{it})</td>
<td>0.012(^*) (1.94)</td>
<td>-0.0094 (-1.11)</td>
<td>-0.0074 (-1.05)</td>
</tr>
<tr>
<td>CAPM beta(_{it})</td>
<td>-0.81 (-1.58)</td>
<td>-4.62 (-0.99)</td>
<td>4.99 (-1.33)</td>
</tr>
<tr>
<td>Prob. of Default(_{it})</td>
<td>6.76(^*) (1.90)</td>
<td>30.1(^{**}) (2.54)</td>
<td>21.4(^{**}) (2.07)</td>
</tr>
<tr>
<td>95% VaR(_{it})</td>
<td>-0.61(^{***}) (-2.75)</td>
<td>-1.18 (-1.22)</td>
<td>-1.78 (-1.67)</td>
</tr>
<tr>
<td>Maturity Mismatch(_{it})</td>
<td>-0.23(^*) (-1.79)</td>
<td>-3.14 (-1.13)</td>
<td>-2.71 (-1.23)</td>
</tr>
<tr>
<td># of subs(_{it}) ( \times 10^{-3} )</td>
<td>-0.020 (-0.13)</td>
<td>-0.13 (-0.21)</td>
<td>-0.093 (-0.21)</td>
</tr>
<tr>
<td>L.SRK(_{it})</td>
<td>0.049(^{***}) (2.85)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Adj. R\(^2\) | 0.18 | 0.35 | 0.36 | 0.29 | 0.37 | 0.32 | 0.40 | 0.41 | 0.43 |
| AR(2) p-value | 0.71 | 0.56 | 0.80 |
| FE (Bank,Time) | N, N | N, N | N, N | Y, N | Y, Y | Y, N | Y, Y | Y, Y | Y, Y |

\(^{i}\) statistics in parentheses

\(^{*}\) \(p < .1\), \(^{**}\) \(p < .05\), \(^{***}\) \(p < .01\)
Table 12: \( SRISK_{it+k} = \beta SBR_{it} + \delta SRISK_{it} + \gamma \text{controls}_{it} + \nu_i + \eta_t + \epsilon_{it} \)

<table>
<thead>
<tr>
<th></th>
<th>Pooled OLS</th>
<th>Fixed Effects</th>
<th>Dynamic Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rel. Assets (_{it}) (log)</td>
<td>0.50***</td>
<td>-0.72</td>
<td>-0.52</td>
</tr>
<tr>
<td></td>
<td>(4.48)</td>
<td>(-0.66)</td>
<td>(-1.46)</td>
</tr>
<tr>
<td></td>
<td>0.25**</td>
<td>0.83</td>
<td>-1.47***</td>
</tr>
<tr>
<td></td>
<td>(2.38)</td>
<td>(0.66)</td>
<td>(-2.78)</td>
</tr>
<tr>
<td></td>
<td>-0.043</td>
<td>-0.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.29)</td>
<td>(-1.03)</td>
<td></td>
</tr>
<tr>
<td>Rel. Leverage (_{it}) (log)</td>
<td>8.68***</td>
<td>17.4</td>
<td>9.26</td>
</tr>
<tr>
<td></td>
<td>(6.98)</td>
<td>(1.38)</td>
<td>(1.62)</td>
</tr>
<tr>
<td></td>
<td>7.36***</td>
<td>13.6</td>
<td>3.85</td>
</tr>
<tr>
<td></td>
<td>(5.12)</td>
<td>(1.26)</td>
<td>(0.91)</td>
</tr>
<tr>
<td></td>
<td>6.14***</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.02)</td>
<td>(1.53)</td>
<td></td>
</tr>
<tr>
<td>Illiquidity Link (_{it}) (log)</td>
<td>-6.02***</td>
<td>14.0**</td>
<td>10.6*</td>
</tr>
<tr>
<td></td>
<td>(-4.60)</td>
<td>(2.22)</td>
<td>(1.78)</td>
</tr>
<tr>
<td></td>
<td>-1.89</td>
<td>9.29</td>
<td>17.8**</td>
</tr>
<tr>
<td></td>
<td>(-1.51)</td>
<td>(1.52)</td>
<td>(2.64)</td>
</tr>
<tr>
<td></td>
<td>-0.32</td>
<td>15.0**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.21)</td>
<td>(2.05)</td>
<td></td>
</tr>
<tr>
<td>Assets (<em>{it}) \times Lev (</em>{it}) (log)</td>
<td>1.30***</td>
<td>2.72</td>
<td>1.62*</td>
</tr>
<tr>
<td></td>
<td>(6.28)</td>
<td>(1.35)</td>
<td>(1.78)</td>
</tr>
<tr>
<td></td>
<td>1.16***</td>
<td>2.17</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>(5.04)</td>
<td>(1.25)</td>
<td>(1.03)</td>
</tr>
<tr>
<td></td>
<td>1.02***</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.10)</td>
<td>(1.25)</td>
<td></td>
</tr>
<tr>
<td>Assets (<em>{it}) \times Illiq (</em>{it}) (log)</td>
<td>-1.07***</td>
<td>2.42**</td>
<td>1.74*</td>
</tr>
<tr>
<td></td>
<td>(-4.54)</td>
<td>(2.26)</td>
<td>(1.82)</td>
</tr>
<tr>
<td></td>
<td>-0.35</td>
<td>1.92*</td>
<td>3.21**</td>
</tr>
<tr>
<td></td>
<td>(-1.55)</td>
<td>(1.78)</td>
<td>(2.59)</td>
</tr>
<tr>
<td></td>
<td>-0.66</td>
<td>2.56**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.24)</td>
<td>(2.08)</td>
<td></td>
</tr>
<tr>
<td>Lev (<em>{it}) \times Illiq (</em>{it}) (log)</td>
<td>0.64</td>
<td>-2.10</td>
<td>3.39*</td>
</tr>
<tr>
<td></td>
<td>(1.14)</td>
<td>(-0.86)</td>
<td>(1.72)</td>
</tr>
<tr>
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<td>1.07**</td>
<td>-0.60</td>
<td>3.96</td>
</tr>
<tr>
<td></td>
<td>(2.48)</td>
<td>(-0.25)</td>
<td>(1.61)</td>
</tr>
<tr>
<td></td>
<td>1.23***</td>
<td>-0.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.58)</td>
<td>(-0.35)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SRISK (_{it})</td>
<td>0.39***</td>
<td>0.30***</td>
</tr>
<tr>
<td></td>
<td>(7.79)</td>
<td>(12.45)</td>
<td>(5.64)</td>
</tr>
<tr>
<td></td>
<td>0.36***</td>
<td>0.27***</td>
<td>(6.32)</td>
</tr>
<tr>
<td></td>
<td>(6.43)</td>
<td>(10.52)</td>
<td>(11.40)</td>
</tr>
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<td></td>
<td>( \Delta \text{CoVaR}_{it})</td>
<td>-0.47</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(-0.87)</td>
<td>(0.23)</td>
<td>(0.48)</td>
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<tr>
<td></td>
<td>MES (_{it})</td>
<td>-0.18</td>
<td>1.97**</td>
</tr>
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<td>(-0.92)</td>
<td>(2.03)</td>
<td>(1.49)</td>
</tr>
<tr>
<td></td>
<td>Stock Ret (_{it}) (log)</td>
<td>-0.00091</td>
<td>0.00023</td>
</tr>
<tr>
<td></td>
<td>(-1.58)</td>
<td>(0.33)</td>
<td>(0.59)</td>
</tr>
<tr>
<td></td>
<td>Stock Vol (_{it})</td>
<td>0.018**</td>
<td>0.0068</td>
</tr>
<tr>
<td></td>
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<td>(0.0045)</td>
<td>(0.0046)</td>
</tr>
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<td></td>
<td>(2.43)</td>
<td>(0.70)</td>
<td>(0.70)</td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>CAPM beta&lt;sub&gt;it&lt;/sub&gt;</td>
<td>0.029</td>
<td>-5.78</td>
<td>-4.52</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(-1.57)</td>
<td>(-1.41)</td>
</tr>
<tr>
<td>Prob. of Default&lt;sub&gt;it&lt;/sub&gt;</td>
<td>9.61**</td>
<td>18.0</td>
<td>13.7</td>
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<td>(2.35)</td>
<td>(1.36)</td>
<td>(1.37)</td>
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<td>95% VaR&lt;sub&gt;it&lt;/sub&gt;</td>
<td>-0.25</td>
<td>-1.69*</td>
<td>-1.85</td>
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<td>(-1.12)</td>
<td>(-1.74)</td>
<td>(-1.56)</td>
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<tr>
<td>Maturity Mismatch&lt;sub&gt;it&lt;/sub&gt;</td>
<td>0.44</td>
<td>-3.78</td>
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<td></td>
<td>(1.40)</td>
<td>(-1.17)</td>
<td>(-1.08)</td>
</tr>
<tr>
<td># of subs&lt;sub&gt;it&lt;/sub&gt; ×10&lt;sup&gt;-3&lt;/sup&gt;</td>
<td>0.78***</td>
<td>1.02</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>(4.65)</td>
<td>(1.25)</td>
<td>(0.75)</td>
</tr>
<tr>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.10</td>
<td>0.28</td>
<td>0.30</td>
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<tr>
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<td>0.68</td>
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<tr>
<td>AR(2) p-value</td>
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</tbody>
</table>

* t statistics in parentheses
* * p < .1, ** p < .05, *** p < .01