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Abstract

This paper proposes a theory of the fiscal foundations of inflation based on imperfect knowledge and learning. The theory is similar in spirit to, but distinct from, unpleasant monetarist arithmetic and the fiscal theory of the price level. Because the assumption of imperfect knowledge breaks Ricardian equivalence, details of fiscal policy, such as the average scale and composition of the public debt, matter for inflation. As a result, fiscal policy constrains the efficacy of monetary policy. Heavily indebted economies with debt maturity structures observed in many countries require aggressive monetary policy to anchor inflation expectations. The model predicts that the Great Moderation period would not have been so moderate had fiscal policy been characterized by a scale and composition of public debt now witnessed in some advanced economies in the aftermath of the 2007-09 global recession.

Key words: debt management policy, maturity structure, monetary policy, expectations stabilization, Great Moderation

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1 Introduction

In the aftermath of the 2007-2009 global recession many countries have experienced a sharp increase in their public debt-to-GDP ratios as a result of expansionary fiscal policy (**figure 1**, left panel). An important theoretical and practical issue concerns the consequences of these fiscal developments for future macroeconomic stability, in particular for inflation. This paper proposes a theory of the inflation consequences of fiscal policy based on imperfect knowledge and learning. Permitting beliefs to depart either temporarily or permanently from those consistent with rational expectations equilibrium leads to departures from Ricardian equivalence, creating a link between the path of government debt, taxes and inflation.[1] For economies with a high level of government debt of average duration commonly observed in many countries, this link is sufficiently strong to hinder a central bank's pursuit of price stability. The theory is then used to evaluate the role of fiscal policy during the Great Moderation in the US. To date the literature has focused on "good luck" versus "good monetary policy", with little discussion of the contributing role of fiscal policy to economic stability.[2] The results reveal that if the US had debt levels now witnessed in many advanced countries, then the Great Moderation would not have been so moderate — macroeconomic volatility would have been similar to earlier decades.

These findings stand in stark contrast with the conventional view of stabilization policy which emerged during the years of the Great Moderation — see Clarida, Gali, and Gertler (1999). According to this view fiscal policy satisfies a strong irrelevance property. Monetary policy provides the nominal anchor by responding aggressively to inflation, while fiscal policy maintains the value of the public debt. In the language of Leeper (1991) monetary policy is active, fiscal policy is passive, and the equilibrium is Ricardian. Changes in the size and maturity composition of nominal government liabilities have no impact on inflation. This result, however, depends strongly on the assumption of rational expectations and, in particular, a complete understanding of the current and future policy regime at any point in time. Given the profound uncertainty surrounding recent monetary and fiscal frameworks in many countries, and a constantly changing economic environment, this benchmark can only be viewed

[1] See Evans, Honkapohja, and Mitra (2012), Eusepi and Preston (2012) and Woodford (2012) for relevant discussion.

[2] See, for example, Stock and Watson (2002).

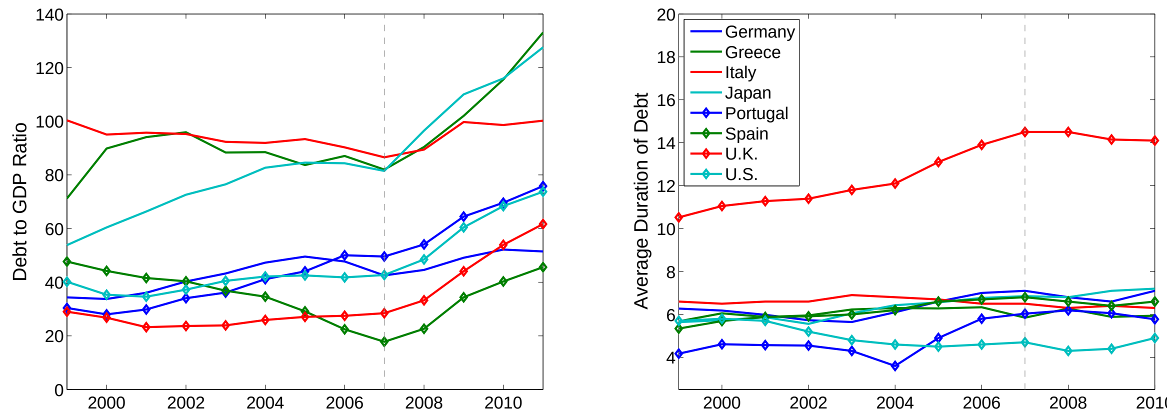

Figure 1: **Size and maturity composition of debt**. The figure shows the evolution of debt-to-GDP ratios and average maturity of debt for a selected group of countries. The debt-to-GDP time series is measured as net financial liabilities as a percentage of nominal GDP; the average maturity of debt is measured as the average term to maturity of total outstanding government debt. The data source is the OECD database.

as a very stringent assumption.[3]

Consider a flexible-price endowment economy with long-term government debt. Monetary and fiscal policy are specified by simple rules. The monetary rule prescribes that the short-term nominal rate responds more than proportionally to inflation, while the fiscal rule adjusts lump-sum taxes more than proportionally to changes in government debt. Under rational expectations this policy framework induces a Ricardian equilibrium. Fiscal policy has no monetary consequences. Now suppose agents have imperfect knowledge, modeled as uncertainty about the long-term equilibrium level of inflation and taxes. Interpret this as either fundamental uncertainty about the policy regime, or imperfect credibility about policy objectives. Following Marcet and Sargent (1989) and Evans and Honkapohja (2001), to learn about the long-run objectives of policy, agents employ a simple linear econometric model with an unobserved drift, estimated each period as new data become available. Estimates of average inflation and taxes are updated in response to past forecast errors. This is an intuitive model of expectations formation supported by empirical evidence.[4] This kind of belief

[3] See Davig and Leeper (2006) and Bianchi (2010).

[4] See Adam, Marcet, and Nicolini (2012), Adam, Beutel, and Marcet (2013), Eusepi and Preston (2011), Milani (2007) and Slobodyan and Wouters (2012).

structure is an example of "end-point uncertainty" — see, for example, Kozicki and Tinsley (2001) for an asset pricing application.

In this simple imperfect-knowledge economy analytic results for stability — defined as the set of policies which ensure agents correctly learn the long-run objectives of policy — reveal that elevated debt levels and moderate-maturity structures, between 2 and 7 years, and therefore similar to those displayed by many countries (figure 1, right panel), are destabilizing. To anchor inflation expectations monetary policy must respond aggressively to changes in inflation, over and above adjustments in the stance of policy prescribed by the Taylor principle. Conversely, both low and high average duration debt are desirable as they promote stability even in heavily indebted economies. The results contrast markedly with a rational expectations analysis of the model: the Taylor principle is sufficient for expectations stability regardless of the properties of issued debt.

How and why do the scale and composition of debt matter for inflation stabilization? Two aspects of the model are decisive: the wealth effects arising from shifts in expected taxes and the self-referential nature of the agents' learning process. Consumption demand depends both on intertemporal substitution of consumption and on wealth effects attached to holdings of the public debt. Under rational expectations the wealth effects are zero at all times: fiscal consequences of monetary policy are fully offset by anticipated changes in the present value of taxes. The transmission channel of monetary policy operates only through the intertemporal substitution of consumption.[5] Under learning, the estimated present discounted value of taxes does not necessarily offset changes in debt holdings, as agents are uncertain about their long-run tax burden. The resulting mismatch between government debt holdings and the estimated present discounted value of taxes generates wealth effects on consumption demand.[6] The strength of non-Ricardian effects on consumption and their implications for macroeconomic stability depend on the relative strength of substitution and wealth components of consumption demand. Imperfect knowledge re-weights the relative importance of wealth and substitution effects compared to a rational expectations analysis of the model. This is the

[5] See Clarida, Gali, and Gertler (1999) and Woodford (2003).

[6] In this way the theory is close related but distinct from the fiscal theory of the price level — see Leeper (1991), Sims (1994), Woodford (1996) and Cochrane (2001). The fiscal theory of the price level asserts that when monetary policy is passive and fiscal policy is active rationally anticipated shifts in the present value of tax obligations generate wealth effects on aggregate demand. The theory proposed here does not rely on this alternative configuration of policy. Wealth effects instead arise from a misspecified model of tax obligations.

first key model property.

While the result that learning can induce temporary deviations from Ricardian equivalence is unsurprising, the question is: under what conditions does it have significant implications for monetary policy? The answer depends on the second key property of the model. The model is "self-referential" in the sense of Marcet and Sargent (1989): beliefs affect the data-generating process, which in turn affect beliefs. Changes in expected monetary policy, reflecting changes in inflation expectations, shift the price of long-term government bonds and, as a result, the path of government debt accumulation and taxes. The resulting changes in taxes and tax expectations lead to wealth effects on consumption demand which feed back into inflation dynamics and monetary policy expectations, closing the loop. As monetary and fiscal expectations errors are propagated through the economy, they become partially self-fulfilling, opening the door to instability.

The degree of self-referentiality depends on the interaction of monetary and fiscal policy, and in particular, the scale and composition of the public debt. High average levels of public debt increase the relative weight of wealth effects on consumption discussed above. In turn, the sensitivity of debt accumulation to policy expectations depends non-monotonically on the average duration of government debt. With one-period debt the bond price does not depend on expectations — tax dynamics are independent of expectations and not self-referential. However, as the average duration rises, bond price becomes increasingly sensitive to monetary policy expectations. At the same time the fraction of outstanding debt rolled over in any period diminishes. These two countervailing effects — the first destabilizing and the second stabilizing — deliver the non-monotonicity. In the limit of consol bonds debt is never rolled over so the price effects vanish — tax dynamics are again independent of expectations and not self-referential.

High levels of intermediate-duration government debt make the economy highly self-referential: even small deviations from rational beliefs have long-lasting effects on the equilibrium dynamics. Conversely, in economies where government debt has low or very large average duration, or where the debt-to-GDP ratio is small, the Taylor principle is restored. This finding underscores an important general principle: models with limited self-referentiality are well approximated by a rational expectations analysis.

The theoretical results suggest fiscal policy is potentially important to our understanding of US monetary history, and in particular, important to explaining the Great Moderation. An interesting feature of the data over this period is the relative stability of the US economy, coupled with the gradual decline in long-term inflation expectations. This adjustment, which started with the Volcker disinflation and spans the 1990s, can be interpreted as market participants' gradually learning about a new monetary policy regime with low average inflation. A large literature has attributed the relative stability of this period either to good monetary policy, as described by a policy rule satisfying the Taylor principle, or to 'good luck', a period of relatively low volatility of shocks. These analyses, however, do not investigate the role of key aspects of fiscal policy, like the size and composition of government debt. To explore this possibility, the model is extended to include nominal rigidities, endogenous labor supply and distortionary taxes. The model is used to recover economic disturbances and shown to capture well movements in various measures of inflation expectations. Counterfactual simulations making different assumptions about the size and composition of government debt, and leaving monetary policy and the estimated disturbances unchanged, demonstrate that the Great Moderation period, 1984-2007, would have been less moderate had fiscal policy been characterized by high-debt levels and short-maturity structures.

Improved monetary policy or declining volatility of economic disturbances did not alone deliver the Great Moderation. It also required judicious debt-management policy in terms of having a low level of government debt. Taking as given the average maturity structure of US government debt, had the government debt-to-GDP ratio been above 150% the US economy would have experienced volatility in inflation and detrended output not much lower than over the period 1955-1983. Moreover, long-maturity structures of debt, in excess of 15 years, would have maintained inflation stability, even if the US economy had very high levels of debt. This suggests that countries where the average maturity of debt is tilted toward very long maturities can, *ceteris paribus*, afford to have higher debt-to-GDP ratios without creating macroeconomic volatility. As shown on **figure 1**, the only country with such a long maturity of debt in our sample is the United Kingdom.

The findings of this analysis have clear predictions for the near-term evolution of the US and many other economies which face severe fiscal imbalances. To support aggregate demand, these economies have shifted to high levels of public indebtedness and a shortened maturity structure due to large-scale asset purchase programs. The above results indicate that further deterioration in fiscal conditions could lead to macroeconomic volatility, as central banks' ability to stabilize inflation would be severely impaired.

2 An Endowment Economy

This section presents a simple flexible-price endowment economy with long-term nominal bonds. This permits analytical characterization of the interactions between monetary and fiscal policy, and specifically the constraints imposed by the scale and composition of the public debt on the choice of monetary policy rule.

The pivotal modeling departure from standard analyses is the assumption that agents have incomplete knowledge about the economic environment: they form expectations using data from the economic system in which they operate. Learning is introduced following the anticipated utility approach as described by Kreps (1998) and Sargent (1999). The analysis follows Marcet and Sargent (1989) and Preston (2005), solving for optimal decisions conditional on current beliefs.

2.1 Households

The economy is populated by a continuum of households seeking to maximize future expected discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \frac{C_T(i)^{1-\sigma}}{1-\sigma} \tag{1}$$

where $\sigma > 0$, $0 < \beta < 1$ and $C_t(i)$ denotes household$-i$ consumption in period t. The operator $\hat{E}_t^i$ denotes the beliefs at time t held by each household i, described below. Households have access to two types of nominal assets supplied by the government: one-period debt, B_t^s, with price P_t^s; and a more general portfolio of debt, B_t^m, with price P_t^m. Following Woodford (1998, 2001), the latter asset has payment structure $\rho^{T-(t+1)}$ for $T > t$ and $0 \leq \rho \leq 1$. The value of such an instrument issued in period t in any future period $t+j$ is $P_{t+j}^{m-j} = \rho^j P_{t+j}^m$. The asset can be interpreted as a portfolio of infinitely many bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$. Varying the parameter ρ varies the average maturity of the asset.[7] For example, when $\rho = 0$ the portfolio comprises one-period debt; and when $\rho = 1$ the portfolio comprises console bonds.

[7] An elegant feature of this structure is that it permits discussion of debt maturity with the addition of single state variable.

Define P_t as the price level at period t. Letting $b_t^s(i) \equiv B_t^s(i)/P_t$ and $b_t^m(i) \equiv B_t^m(i)/P_t$, household i's real wealth is defined by $\mathbb{W}_t(i) \equiv P_t^s b_t^s(i) + P_t^m b_t^m(i)$. The budget constraint is given by

$$\mathbb{W}_t(i) \leq R_t^m \pi_t^{-1} \mathbb{W}_{t-1}(i) + \left(R_{t-1}^s - R_t^m\right) P_{t-1}^s b_{t-1}^s(i) + y_t(i) - \tau_t(i) - C_t(i) \tag{2}$$

where $R_t^m = \left(1 + \rho P_t^m\right)/P_{t-1}^m$ and $R_{t-1}^s = 1/P_{t-1}^s$ denote realized returns from holding each asset, with the latter implicitly defining the period nominal interest rate, the instrument of central bank policy. Each period agents receive a stochastic endowment, $y_t(i)$, assumed for simplicity to be an i.i.d. random variable, and pay lump-sum taxes $\tau_t(i)$. In addition agents face a no-Ponzi constraint of the form

$$\lim_{T\to\infty} \hat{E}_t^i \left(\prod_{s=0}^{T-t} R_{t+s}^m \pi_{t+s}^{-1}\right)^{-1} \mathbb{W}_T(i) \geq 0 \tag{3}$$

where $\pi_t = P_t/P_{t-1}$.

To summarize households choose sequences $\{C_T(i), \mathbb{W}_T(i), b_T^s(i)\}_{T=t}^{\infty}$ to maximize utility, (1), subject to (2) and (3), given initial wealth $\mathbb{W}_{t-1}(i)$ and their beliefs regarding the evolution of the endowment, taxes and asset returns. Conditional on beliefs, optimality requires (2) and (3) hold with equality and satisfaction of

$$C_t^{-\sigma}(i) = \hat{E}_t^i \left[R_t^s \frac{C_{t+1}^{-\sigma}(i)}{\pi_{t+1}}\right] \tag{4}$$

$$C_t^{-\sigma}(i) = \hat{E}_t^i \left[\frac{1 + \rho P_{t+1}^m}{P_t^m} \frac{C_{t+1}^{-\sigma}(i)}{\pi_{t+1}}\right] \tag{5}$$

the Euler equations corresponding to the two assets.

2.2 Monetary and fiscal policy

The central bank implements monetary policy according to the family of interest-rate rules

$$R_t^s = \bar{R}^s \pi_t^{\phi_\pi} \tag{6}$$

where $\phi_\pi \geq 0$ and $\bar{R}^s$ the steady-state gross interest rate. The steady-state inflation rate is assumed to be zero. The fiscal authority finances exogenously determined government purchases, G_t, assumed here to be zero in each period, by issuing public debt and levying taxes. One-period debt, B_t^s, is in zero supply, while $B_t^m > 0$ in all periods t. Imposing the restriction that one-period debt is in zero supply, the real flow budget constraint of the government is given by

$$P_t^m b_t^m = \pi_t^{-1} b_{t-1}^m \left(1 + \rho P_t^m\right) - \tau_t. \tag{7}$$

Tax policy is determined by a rule of the form

$$\tau_t = \bar{\tau} \left(\frac{b_{t-1}^m}{\bar{b}^m} \right)^{\phi_b}, \tag{8}$$

where $\phi_b \geq 0$ and $\bar{b}^m$ is the steady-state level of debt, prescribing a tax response to changes in the real amount (at face value) of debt issued.[8]

2.3 Market clearing and equilibrium

The analysis considers a symmetric equilibrium in which all households are identical, though they do not know this to be true. Given that households have identical initial asset holdings, preferences, endowment, taxes and beliefs, and face common constraints, they make identical state-contingent decisions. Equilibrium requires all goods and asset markets to clear. The former requires the aggregate restriction

$$\int_0^1 C_t(i)\, di = C_t = \int_0^1 y_t(i)\, di \tag{9}$$

where C_t denotes aggregate consumption demand. The latter requires

$$\int_0^1 B_t^s(i)\, di = 0 \text{ and } \int_0^1 B_t^m(i)\, di = B_t^m \tag{10}$$

with $B_{-1}^s(i) = 0$ and $B_{-1}^m(i) > 0$ for all households $i \in [0, 1]$. Equilibrium is then a sequence of prices $\{P_t, P_t^m, R_t^s\}$ and allocations $\{C_t(i), B_t^m(i), \tau_t\}$ satisfying individual optimality and market clearing conditions, given $y_t(i)$ for $i \in [0, 1]$.

The policy regime. Focus is given to a policy regime where monetary policy is 'active', satisfying the Taylor principle $\phi_\pi > 1$, and fiscal policy is 'passive', $1 < \phi_b < (1 + \beta)/(1 - \beta)$, implying that in equilibrium government intertemporal solvency is guaranteed under all

[8]The results do not change if taxes respond instead to the value of nominal debt. We also assume that each agent faces the same tax burden in equilibrium. Generalizing to permit heterogeniety in tax obligations, where these obligations remain in fixed proportion, deliver identical results.

circumstances — see Leeper (1991). Under the assumption of rational expectations, this policy regime implies a locally unique bounded equilibrium in which the evolution of nominal liabilities have no monetary consequences; the size and duration of public debt do not affect inflation.

Globally, the policy regime displays multiple equilibria under rational expectations. These equilibria, including deflationary traps and explosive equilibria, are well understood and have been discussed extensively in the New Keynesian literature.[9] Our analysis is restricted to the neighborhood of the locally unique equilibrium under rational expectations with zero inflation. Introducing incomplete knowledge and learning is shown to dramatically change the properties of this 'good' equilibrium. Studying the global properties of the model is left to future research.

3 Aggregate Dynamics

Subsequent analysis employs a log-linear approximation in the neighborhood of the non-stochastic steady state of zero inflation. For any variable k_t denote $\hat{k}_t = \ln\left(k_t/\bar{k}\right)$ the log deviation from steady state with the exception of the net short-term interest rate $\hat{\imath}_t = \ln\left(\frac{R_t^s}{R^s}\right)$.

The optimal consumption decision rule for each household i is a joint implication of the Euler equations, the flow budget constraint and transversality:

$$\begin{aligned}\hat{C}_t(i) = {} & (1-\beta)\,\hat{y}_t(i) - \sigma^{-1}\beta\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + \\ & \delta\left(\hat{b}_{t-1}^m(i) - \hat{\pi}_t + \rho\beta\hat{P}_t^m + \beta\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T - \hat{\pi}_{T+1} - (\beta^{-1}-1)\hat{\tau}_t(i)\right)\right)\end{aligned} \tag{11}$$

where σ^{-1} denotes the consumption intertemporal elasticity of substitution and $\delta = (\beta^{-1} - 1)P^m b^m/\bar{Y}$ measures the steady-state debt-to-income ratio.

Optimal consumption decisions require forecasts of nominal interest rates, inflation and taxes into the indefinite future. The top line in (11) describes the evolution of consumption in absence of wealth effects from holding debt. This captures the standard transmission mechanism of monetary policy in the model under rational expectations, under the Ricardian policy regime that we study. The bottom line in (11), referred to as the 'non-Ricardian'

[9]See for example Benhabib, Schmitt-Grohe, and Uribe (2001), Woodford (2003) and Cochrane (2011).

component of consumption demand, measures the wealth effects from holding government debt net of taxes. It comprises three components. The first is the real value of debt holdings, the second measures the present value of real returns from holding debt (purchased in the current period) and the third component denotes the present value of taxes. Under rational expectations, in equilibrium these terms sum precisely to zero. Under imperfect knowledge, incorrect forecasts of returns and taxes imply the public debt is perceived as net wealth. The key to model dynamics is the relative strength of the 'standard' and 'non-Ricardian' components, referred to loosely as "substitution" and "income" effects. These in turn are determined by the relative magnitude of the intertemporal elasticity of consumption, σ^{-1}, and the debt-to-income ratio, δ.

From the households' first-order conditions for asset holdings, a log-linear approximation to the no-arbitrage restriction yields the familiar expectations hypothesis of the yield curve. The price of the bond portfolio is

$$\hat{P}_t^m = -\hat{E}_t^i \sum_{T=t}^{\infty} (\beta\rho)^{T-t} \hat{\imath}_T. \tag{12}$$

The multiple-maturity debt portfolio is priced as the expected present discounted value of all future one-period interest rates, where the discount factor is given by $\beta\rho$. The average duration of the portfolio is given by $(1-\beta\rho)^{-1}$. Relation (12) is consistent with the existence of a unique equilibrium bond price because agents have the same beliefs: $\hat{E}_t = \hat{E}_t^i$ for every $i \in [0,1]$. This paper abstracts from asset pricing issues arising from financial market participants having heterogeneous non-nested information sets, consistent with our information assumptions. For simplicity it is assumed that each agent supposes they are the marginal trader in all future periods when determining desired asset allocations. Equilibrium affirms this supposition as all agents are identical. This resolves, albeit by essentially pushing the issue aside, the difficulty of having an aggregate quantity on the left-hand-side determined by a quantity on the right-hand-side that depends on an individual's beliefs.

Aggregating (11) over the continuum $i \in [0,1]$ and imposing goods market clearing, $C_t(i) = C_t = y_t$, provides

$$\begin{aligned} \sigma\hat{C}_t \;=\; & \sigma(1-\beta)\hat{y}_t - \beta\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} (\hat{\imath}_T - \hat{\pi}_{T+1}) + \\ & \psi\left(\hat{b}_{t-1}^m - \hat{\pi}_t + \rho\beta\hat{P}_t^m + \beta\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left(\hat{\imath}_T - \hat{\pi}_{T+1} - (\beta^{-1}-1)\hat{\tau}_t\right)\right) \end{aligned} \tag{13}$$

where the parameter $\psi \equiv \delta/\sigma^{-1}$ measures the relative strength of substitution and wealth effects, which will play a central role in model dynamics under imperfect knowledge. Because aggregate consumption is determined by the exogenous endowment, this relation determines the current real interest rate. In turn, given the monetary and fiscal rules, the endowment process and agents' expectations, this equation determines inflation.

Combining a log-linear approximation to the government budget constraint, (7), the tax rule, (8) and the bond price, (12), taxes evolve according to

$$\begin{aligned}\hat{\tau}_{t+1} &= \phi_b \hat{b}_t^m \\ &= \lambda_b \hat{\tau}_t - \phi_b \left(\beta^{-1} - (1-\rho)\,\phi_\pi\right) \hat{\pi}_t + \phi_b (1-\rho)\,\rho\beta \hat{E}_t \sum_{T=t}^{\infty} (\rho\beta)^{T-t} \hat{\imath}_{T+1}\end{aligned} \quad (14)$$

where the parameter $\lambda_b = \beta^{-1}(1-(1-\beta)\,\phi_b)$ satisfies $|\lambda_b| < 1$. In equilibrium the evolution of taxes depends on expectations about the future path of monetary policy. The degree to which policy expectations affect the evolution of taxes, equivalently debt, depends nonlinearly on the parameter ρ, which indexes the average duration of debt. For very low and very long-debt maturities these effects are small, and indeed vanish in the case of one-period debt, $\rho = 0$, and console bonds, $\rho = 1$. At low levels of duration, the bond price only reflects changes in the short-term interest rate. At very high levels of duration changes in policy expectation are fully reflected in the price of debt, with little effect on debt issuance and taxes. In contrast, for intermediate values of duration, changes in policy expectations are reflected both in the price of debt and debt issuance, which in turn drives the evolution of taxes. These observations are central to the mechanism by which imperfect knowledge engenders fiscal foundations of inflation.

Substituting the bond price, (12), and the monetary policy rule, (6), into the consumption and tax equations, permits the equilibrium dynamics of the model — equations (13) and (14) — to be compactly represented as the two-dimensional system

$$z_t = A_1 z_{t-1} + \sum_{j=1}^{2} A_{j+1} \hat{E}_t \sum_{T=t}^{\infty} \left(\beta\rho^{j-1}\right)^{T-t} z_{T+1} + A_4 \hat{y}_t. \quad (15)$$

where $z_t = \begin{pmatrix} \hat{\pi}_t & \hat{\tau}_{t+1} \end{pmatrix}'$ and the matrices A_1, A_2 and A_3 depend on composites of the parameters ψ, ρ, ϕ_b and ϕ_π.

This representation of equilibrium dynamics assumes agents understand the form and details of the monetary policy rule. They make interest-rate forecasts that are consistent with rule (6). This assumption is made for analytical convenience and has no implications for the stability results discussed below. Also, in this simple model the only stochastic disturbance is the endowment. Exogenous policy, preference and government spending shocks could also be introduced without loss of generality. In the following section we focus on the effects of a shift in expected inflation and taxes, independently of the source of disturbance. These assumptions are all relaxed in section 5 which develops an empirical New Keynesian model.

4 Information, Learning and Stability

4.1 Beliefs

Specifying beliefs completes the model. Households have incomplete knowledge about the true structure of the economy. They observe only their own objectives, constraints and realizations of aggregate variables as well as prices that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: even though their decision problems are identical, they do not know this to be true. The fact that agents have no knowledge of other agents' preferences and beliefs and have imperfect knowledge about the prevailing policy regime implies that they do not know the equilibrium evolution of inflation and taxes.

Rational Expectations equilibrium. To anchor ideas, the model has a unique bounded rational expectations equilibrium of the form

$$\hat{\pi}_t = -\frac{\sigma}{\phi_\pi}\hat{y}_t \tag{16}$$

and

$$\hat{\tau}_{t+1} = \lambda_b \hat{\tau}_t + \phi_b \left(\phi_\pi^{-1}\beta^{-1} - (1-\rho) \right) \sigma \hat{y}_t. \tag{17}$$

Inflation is a linear function of the endowment process and independent of fiscal variables. The equilibrium is Ricardian — debt has no monetary consequences.

Learning about long-term drifts. To ensure agents can learn the underlying rational expectations equilibrium, we assume they employ a simple linear econometric model: a Vector Auto Regression (VAR) in the variables $\begin{pmatrix} \hat{\pi}_t & \hat{\tau}_t \end{pmatrix}'$. A VAR with one lag nests the minimum-state-variable stationary rational expectations solution. The forecasting model is

$$z_t = \omega + \Phi z_{t-1} + e_t \tag{18}$$

where $z_t = \begin{pmatrix} \hat{\pi}_t & \hat{\tau}_t \end{pmatrix}'$ and e_t is the noise term. The model coefficients (ω, Φ) are updated over time, as additional data become available.

The minimum-state-variable rational expectations solution is defined by the coefficients

$$\omega^* = \mathbf{0}_{2\times 1};\ \Phi^* = \begin{bmatrix} 0 & 0 \\ 0 & \lambda_b \end{bmatrix}.$$

While the rational expectations solution does not contain an intercept, it has a natural interpretation under learning of capturing incomplete knowledge about the long-term evolution of inflation and taxes. Specifically, long-horizon expectations are tied to agents' perceptions about long-run policy targets for inflation and taxes. In order to keep the analysis as simple as possible, subsequent analysis gives focus to the evolution of the intercept terms ω. Agents possess knowledge of the rational expectations equilibrium estimates of the slope coefficients $\Phi = \Phi^*$. Assumptions of this kind have been used extensively in earlier literature studying various aspects of policy uncertainty and credibility. For example, Kozicki and Tinsley (2001) explore asymmetric information about long-run inflation objectives as an explanation for failures of the expectations hypothesis of the term structure. Davig (2004) studies the implications of regime switches in the average level of public debt for the economics of distortionary income taxation. More generally, any econometric filtering problem — and, hence, all models of real-time learning — partitions new information into transitory and permanent components for optimal forecasting. Assuming knowledge of the dynamic components of the model gives emphasis to innovations in the permanent component of the filtering problem.

In any event, proceeding in this fashion is without loss of generality. Beliefs of this kind represent a first-order accurate log-linear approximation to a richer class of beliefs in which agents must learn the rational expectations values, Φ^*. As remaining model equations are evaluated to the first-order, the dynamics resulting from learning these values are second order and therefore negligible under the maintained assumption that disturbances are sufficiently small. Permitting agents to learn these coefficients leaves the results unchanged.[10]

Expectations and recursive estimation. In period t agents form expectations using the forecasting model based on data available up to period $t-1$. Denoting period-t beliefs

[10] The appendix derives the first-order approximation to beliefs. Note also, in the authors experience, it is always the dynamics of constant coefficients that impose the most stringent requirements for learnability of rational expectations equilibrium. While a theorem is not available, numerical results confirm this for the current model.

$\hat{\omega}_{t-1} = \begin{pmatrix} \hat{\omega}^{\pi}_{t-1} & \hat{\omega}^{\tau}_{t-1} \end{pmatrix}$, agents use model (18) to evaluate expectations for inflation and taxes in (15) to provide[11]

$$\hat{E}_t \sum_{T=t+1}^{\infty} \beta^{T-t} \hat{\tau}_{T+1} = \frac{1}{(1-\beta)(1-\beta\lambda_b)} \hat{\omega}^{\tau}_{t-1} + \frac{\lambda_b}{1-\beta\lambda_b} \hat{\tau}_{t+1} \tag{19}$$

$$\hat{E}_t \sum_{T=t}^{\infty} \left(\beta\rho^{j-1}\right)^{T-t} \hat{\pi}_{T+1} = \frac{1}{1-\beta\rho^{j-1}} \hat{\omega}^{\pi}_{t-1}, \; j = 1, 2.$$

Agents use the following recursive algorithm to update their time$-t$ estimates, $\hat{\omega}_t$, of ω

$$\hat{\omega}_t = \hat{\omega}_{t-1} + g_t t^{-1} \epsilon_t \tag{20}$$

and

$$\epsilon_t = z_t - (\hat{\omega}_{t-1} + \Phi^* z_{t-1})$$

is the prediction error. Different assumptions about the variable g_t deliver different gains in the filtering problem. When $g_t = 1$ the updating rule (20) is recursive least squares. When $g_t = \bar{g}t$ the recursive updating is given by a constant-gain algorithm, implying that past observations are discounted more heavily. An observation n periods old receives a weight of $(1-\bar{g})^n$. A constant $\bar{g}$ insures against potential shifts in the structure of the economy (i.e. a policy regime shift). The analysis employs both gain assumptions for reasons explicated in the next section.

4.2 Self-Referential dynamics and stability

The data-generating process implicitly defines a mapping between agents' beliefs, $(\hat{\omega}_{t-1}, \Phi^*)$, and the actual coefficients describing observed dynamics. Substituting (19) in (15), the true data-generating process consists of the equations for inflation and taxes

$$\hat{\pi}_t = -T_{\pi\pi}(\psi, \phi_\pi, \rho)\, \hat{\omega}^{\pi}_{t-1} - T_{\pi\tau}(\psi, \phi_\pi)\, \hat{\omega}^{\tau}_{t-1} - \frac{\sigma}{\phi_\pi} \hat{y}_t \tag{21}$$

$$\hat{\tau}_{t+1} = \phi_b T_{\tau\pi}(\rho)\, \phi_\pi \hat{\omega}^{\pi}_{t-1} + \lambda_b \hat{\tau}_t - \phi_b \left(\beta^{-1} - (1-\rho)\, \phi_\pi\right) \hat{\pi}_t, \tag{22}$$

[11] To avoid a difficult simultaneity problem, agents use previous-period estimates when forming current forecasts. This is standard in the leaning literature. Beliefs are a state variable.

where

$$T_{\pi\pi}(\psi, \phi_\pi, \rho) = \frac{(1-\psi)\left(\beta - \phi_\pi^{-1}\right)}{1-\beta} + \frac{\beta\psi\rho^2}{1-\beta\rho} + \psi T_{\tau\pi}(\rho);$$

$$T_{\pi\tau}(\psi, \phi_\pi) = \frac{\phi_\pi^{-1}\psi\beta}{\phi_b(1-\beta)}; \quad T_{\tau\pi}(\rho) = \frac{(1-\rho)\rho\beta}{1-\rho\beta}.$$

Equations (21) and (22), which together with the updating rule (20) describe the equilibrium evolution of the economy, clarify the self-referential dynamics of inflation and debt. Agents' beliefs affect the actual evolution of inflation and taxes. These data are in turn used to update beliefs. As a result, the true data-generating process displays a time-varying drift, captured by the $T(\cdot)$ coefficients. Two key coefficients measure feedback from beliefs to the actual evolution of taxes and inflation. The coefficient $T_{\pi\tau}(\psi, \phi_\pi)$ in (21) captures wealth effects on consumption demand and inflation stemming from changes in agents' beliefs about long-run average taxes, $\hat{\omega}_{t-1}^{\tau}$. That tax beliefs affect the dynamics of inflation formally establishes the departure from Ricardian equivalence. The coefficient $T_{\tau\pi}(\rho)$ in (22) measures the sensitivity of taxes to shifts in beliefs about long-run average inflation, $\hat{\omega}_{t-1}^{\pi}$. The size of this coefficient depends non-monotonically on the average duration of government debt — recall the discussion of equation (14). These two terms depend on monetary and fiscal policy parameters. The dynamic behavior of the economy can be expressed compactly as

$$z_t = T(\psi, \rho, \phi_\pi)\hat{\omega}_{t-1} + \Phi^* z_{t-1} + \hat{u}_t$$

where

$$T(\psi, \rho, \phi_\pi) = \begin{bmatrix} -T_{\pi\pi}(\psi, \phi_\pi, \rho) & -T_{\pi\tau}(\psi, \phi_\pi) \\ \kappa\phi_b \cdot T_{\pi\pi}(\psi, \phi_\pi, \rho) + T_{\tau\pi}(\rho)\phi_b\phi_\pi & \kappa\phi_b \cdot T_{\pi\tau}(\psi, \phi_\pi) \end{bmatrix},$$

which defines $\kappa = \left(\beta^{-1} - (1-\rho)\phi_\pi\right)$ and $\hat{u}_t = \begin{pmatrix} -1 & \phi_b\kappa \end{pmatrix}' \sigma\phi_\pi^{-1}\hat{y}_t$. Comparing these dynamics with those under rational expectations — (16) and (17) — the only deviation from rational expectations is the drift term $T(\psi, \rho, \phi_\pi)\hat{\omega}_{t-1}$.

We can now characterize under what conditions the size and composition of government debt affect the dynamics of inflation. Following Marcet and Sargent (1989) and Evans and Honkapohja (2001), the limiting behavior of agents' beliefs can be described by an ordinary differential equation, reflecting the mapping between the agents' perceived drift $\hat{\omega}_t$ in (18) and the actual drift as described in (21) and (22). The learning literature refers to the implied dynamics as the 'mean dynamics'. In compact terms, the ODE is

$$\begin{bmatrix} \dot{\omega}^{\pi} \\ \dot{\omega}^{\tau} \end{bmatrix} = \underbrace{T(\psi, \rho, \phi_{\pi}) \begin{bmatrix} \omega^{\pi} \\ \omega^{\tau} \end{bmatrix}}_{\text{Actual drift}} - \underbrace{\begin{bmatrix} \omega^{\pi} \\ \omega^{\tau} \end{bmatrix}}_{\text{Perceived drift}} . \tag{23}$$

The fixed point of (23) is the rational expectations equilibrium $\omega^* = 0$. The self-referential behavior of the economy depends on the interaction between the agents' perceived drift and the realized drift. This in turn depends on the properties of the matrix $T(\cdot)$.

Two kinds of stability result can be established. If the fixed point of the ODE is stable, implying all eigenvalues have negative real parts, then: 1) for decreasing gain algorithms, $g_t = 1$, as $t \to \infty$ beliefs $\hat{\omega}_t$ converge point-wise to rational expectations equilibrium ω^*. Such convergence is called expectational stability; and 2) for constant-gain algorithms, $g_t = \bar{g}t$, and $\bar{g}$ sufficiently small, $\hat{\omega}_t$ converges to a limiting distribution centered on ω^* — see Evans and Honkapohja 2001. The first stability result is exploited to understand the interactions of monetary and fiscal policy, and the constraints placed by long-term debt on monetary control. The second stability result, premised on the first, is then exploited to explore model dynamics and the empirical relevance of our theory. Worth underscoring is that conditions derived for expectational stability apply to a broad range of adaptive learning algorithms, of which least-squares learning is but one example. In this sense the results are quite general. And while a constant-gain algorithm is only one of many possible ways of modeling expectations formation, the mechanism rests on the intuitively appealing assumption that beliefs are revised in response to past forecast errors. The choice of a constant-gain algorithm reflects its long-standing use in the learning literature; that it is a practical procedure to guard against underlying structural change and, therefore, a convenient and elegant way to capture imperfect knowledge about policy; and evidence that such forecasting procedures explain properties of macroeconomic and survey forecast data — see Adam, Marcet, and Nicolini (2012), Adam, Beutel, and Marcet (2013), Eusepi and Preston (2011), Milani (2007) and Slobodyan and Wouters (2012).

A special case: taxes are not self-referential. Before stating a general stability result, consider two special cases of the model. First, the ratio of the debt-to-income ratio and the intertemporal elasticity of consumption, ψ, was earlier asserted to be fundamental to stability as its magnitude governs the relative importance of income and substitution effects on aggregate consumption demand. For wealth effects that are relatively small compared to substitution effects — so $\psi \to 0$ and therefore $T_{\pi\tau}(\psi, \phi_\pi) \to 0$ — the economy displays the same mean dynamics as in the standard account of monetary policy. The evolution of inflation expectations is de-coupled from taxes and tax expectations, removing an important source of self-referentiality from the model. It is straightforward to see that (23) implies

$$\dot{\omega}^\pi = -\frac{1-\phi_\pi^{-1}}{1-\beta}\omega^\pi. \tag{24}$$

Convergence to rational expectations equilibrium occurs provided the Taylor principle is satisfied.

Second, the average duration of government debt, indexed by ρ, measures the impact of interest-rate expectations on bond issuance and taxes. Suppose that $\psi > 0$ and $\rho = 0$ or 1. Looking a the evolution of taxes in (14), for these extreme parameter values debt issuance and taxes are independent of inflation expectations. This is captured by the fact that $T_{\tau\pi}(0) = T_{\tau\pi}(1) = 0$ in (22). As a result expected inflation affects taxes only through realized inflation. From (23), beliefs about taxes and inflation move proportionally, with

$$\begin{aligned}\dot{\omega}^\tau &= \kappa\phi_b\left[T_{\pi\pi}(\psi, \phi_\pi, \rho)\,\omega^\pi + T_{\pi\tau}(\psi, \phi_\pi)\,\omega^\tau\right] \\ &= -\kappa\phi_b\dot{\omega}^\pi.\end{aligned}$$

As in the case where $\psi = 0$, only inflation exhibits self-referential behavior: simple algebra establishes the evolution of inflation expectations conforms to (24).[12]

The general case. With a positive value for $\psi = \delta\sigma$ and $\rho \in (0,1)$ both taxes and inflation exhibit self-referential behavior. Shifts in expected inflation affect debt issuance and tax expectations which, in turn, impact consumption demand and inflation. Such economies exhibit stronger self-referential behavior. **Figure 2** summarizes the different channels through which self-referential behavior of inflation and taxes arises. The feedback from beliefs to taxes and inflation will be stronger when: 1) debt issuance and taxes are most sensitive to inflation expectations; and 2) debt exerts larger wealth effects on consumption demand. The next

[12]Notice that $T(\psi, \rho, \phi_\pi)$ has linearly dependent rows.

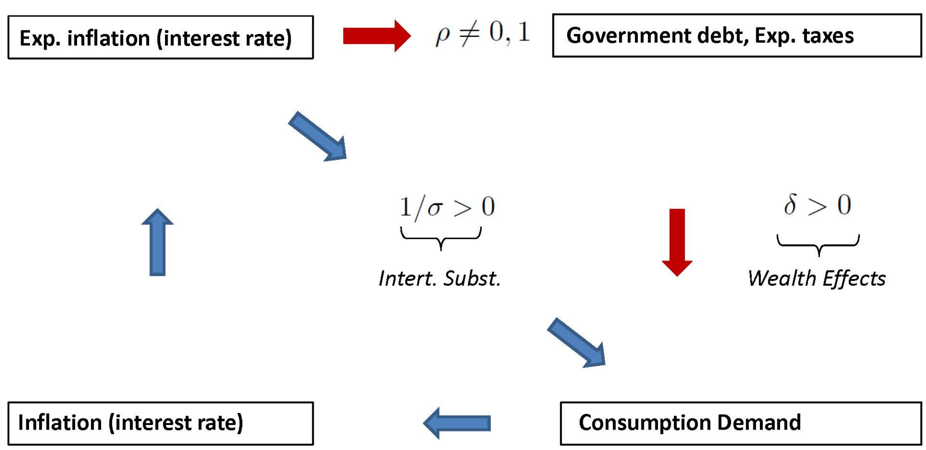

Figure 2: **Non-Ricardian effects and self-referentiality**. The figure shows the self-referential behavior of taxes and inflation. The blue arrows show the standard trasmission mechanism through which monetary policy expectations affect inflation and inflation expectations. This mechanism, which operates via intertemporal substitution of consumption, is stabilizing. The red arrows indicate a second transmission mechanism generated by the interaction between monetary policy expectations, government debt and expected taxes. This mechanism, which is destabilizing, operates though wealth effect on consumption demand.

section provides intuition and the precise economic mechanism driving results. The following proposition summarizes the results.

Proposition 1 *Consider the policy regime defined by:* $\phi_\pi > 1$; $1 < \phi_b < (1+\beta)/(1-\beta)$.

– *Under rational expectations, neither* ψ *nor* ρ *affect inflation.*

– *Under learning fiscal policy will generally constrain monetary policy. Convergence to rational expectations occurs if and only if*

$$\phi_\pi > \max\left(1, \frac{1}{(2-\beta) - \psi \cdot \beta T_{\tau\pi}(\rho)}\right).$$

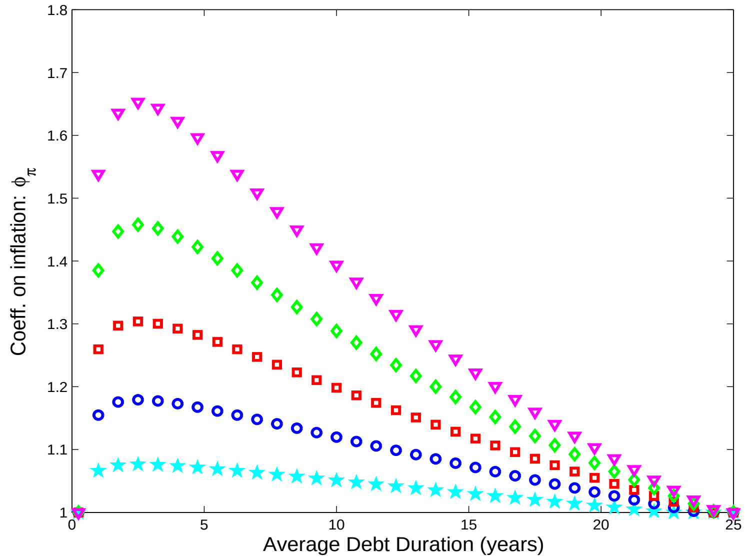

Figure 3: **Stability frontiers: the role of wealth effects and debt duration**. The figure shows stability frontiers for different parameter configurations. The frontiers on top correspond to relatively strong wealth effects. For each frontier, the area (below) above denote un-stable equilibria.

The set of parameters (ψ, ρ, ϕ_π) consistent with stability is described in **figure 3**. The figure plots the "stability frontiers" of the model, highlighting the interaction between monetary policy and the average duration of debt. The discount factor β is set to 0.99. Regions above each contour delineate policy configurations consistent with the stability of the rational expectations equilibrium. Each frontier corresponds to different values of ψ which measures the size of debt scaled by the intertemporal elasticity of substitution for consumption. The values considered are: 0.1, 0.2, 0.3, 0.4 and 0.5.[13] For a given average maturity of debt, higher average levels of indebtedness require more aggressive monetary policy. For a given scale of public debt, variation in the average maturity of public debt engenders non-monotonic constraints on monetary policy. Fiscal regimes with average-debt durations between 2 to 7 years are conducive to instability.

Worth noting is a special property of one-period debt — tax dynamics are not self-

[13]We do not here give emphasis to the precise numerical values underlying this ratio — that is left to the empircal model of section 5 which gives economic content to these values. However, assuming log utility gives immediate magnitudes to the implied average debt levels.

referential as the price of debt depends only on contemporaneous inflation. The monetary policy rule is critical to this conclusion. For policy rules that respond to inflation expectations the stability frontiers are monotonic for a given level of debt, as equilibrium taxes depend on expectations even with one-period debt. Long average durations of debt are conducive to stability. The appendix contains further discussion.

Stability results in many models of learning typically do not depend on opportunities to substitute intertemporally. This is because most earlier literature fails to solve for optimal decisions conditional on beliefs. An implication is that agents need not forecast policy variables to make current decisions — see Eusepi and Preston (2012) for a discussion. There is no uncertainty about the prevailing policy regime. The point that stability regions depend on the interaction of policy parameters with opportunities to substitute intertemporally appears nonetheless quite general. Davig and Leeper (2007), for example, find the same phenomenon in a rational expectations model with only monetary policy, but where policy regime is subject to recurring change. Even with a simple Taylor rule that responds only to inflation, parameters that describe the intertemporal elasticity of substitution and the degree of price stickiness matter for determinacy regions. This is not true in a model with a single regime. Important then is the existence of fundamental uncertainty about the policy regime.[14]

4.3 Anchoring inflation expectations: the role of fiscal policy

The previous section underscores the limiting behavior of the economy and how fiscal policy can dramatically alter the requirements for macroeconomic stability under imperfect knowledge. To reinforce these ideas and give further insight, a numerical example now explores the dynamic response of the economy to a small increase in expected inflation, holding all the other variables at the rational expectations equilibrium.[15] In this experiment assume $\beta = .99$ and a constant endowment process $y_t = \bar{y}$ for all t. The coefficients of the policy rules are set to $\phi_\pi = 1.5$ and $\phi_b = 1.2$, respectively, which ensures the above expectational stability results apply. The average duration of debt is determined by $\rho = .93$, which implies an average maturity of just over three years. We consider an economy with zero debt and an economy with a debt-to-output ratio of 200% in annual terms. The preference parameter $\sigma = 4.5$ is

[14] The Frisch elasticity of labor supply is also important. See the appendix for a discussion in the context of the empirical model developed in the sequel.

[15] Similar conclusions would be reached if instead we considered a rise in expected taxes.

chosen to imply a low intertemporal elasticity of substitution for consumption, but it remains sufficiently large that the learning process converges. Finally, we assume that agents' learning rule (20) has a forgetting factor $g_t = \bar{g}t$ where $\bar{g} = 0.025$, the same value chosen for the empirical model developed later.[16]

Figure 4, top-left panel, shows the evolution of inflation in response to a small increase in $\hat{\omega}_0^\pi$, holding, at impact, all other variables at steady-state values. In particular, we assume the initial conditions $\hat{\omega}_0^\pi = 0.01$ and $\hat{\omega}_0^\tau = 0$. Given these initial conditions, the model's dynamic response is described by equations (21), (22) and (20). In both the economy with zero debt (red-dashed line) and high debt (blue-solid line), inflation and the bond price drop in response to higher inflation expectations; agents expect a more-than-proportional increase in future interest rates because monetary policy satisfies the Taylor principle. Subsequently, inflation is brought back to equilibrium by lower short-term real rates. The response of inflation in the high-debt economy is different in three ways. First, the impact response is smaller; second, the response is more persistent; and third, inflation overshoots its steady-state value.

To gain intuition on the role of the size and composition of debt, the bottom panels of **figure 4** display the evolution of each component of consumption demand in (11) for the high-debt economy. In the zero-debt economy non-Ricardian effects are absent, and the transmission of monetary policy operates only through intertemporal substitution of consumption, in accordance with the standard new-Keynesian view. Inspecting the bottom-left panel, in the high-debt economy the non-Ricardian component (red dashed line) counteracts the expansionary effects of lower real interest rates measured in the Ricardian component (blue solid line), preventing fast convergence.

To explain the drop in the non-Ricardian component, the bottom-right panel of **figure 4** shows its three subcomponents. Taxes and tax expectations are predetermined. On impact, the higher present discounted value of the short-term real interest income (black dashed-dotted line) and the decline of the price level generates positive wealth effects, which explains the milder initial fall in inflation relative to the zero-debt economy. Subsequently the non-Ricardian term turns negative, preventing consumption demand and inflation from adjusting to their steady-state values as fast as in the economy without debt. This is explained by both the decline in expected interest rates triggered by the lower short-term rate (in accordance

[16]Because there are no shocks in this economy, the estimated drift converges point-wise to the the rational expectation equilibrium.

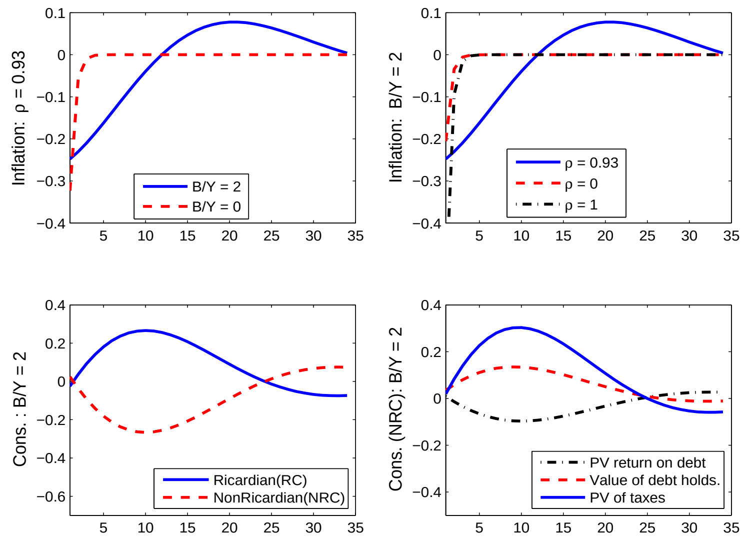

Figure 4: **Response to an increase in inflation expectations**. The top-left panel shows the response of inflation in two economies with zero and high government debt respectively. The lower panels display the response of the different components of consumption demand in the economy with high debt. The top-right panel shows the response of inflation in the high-debt economy under alternative assumed duration of debt.

with the Taylor principle), and the fact that the present discounted value of expected taxes (blue solid line) exceeds the increase in the real value of debt (red dashed line). Debt and taxes gradually increase for two reasons. First, the initial decline in the price of the long-term bond leads to an increase in debt issuance. Second, the subsequent gradual increase in the bond price, which mirrors the decline in expected interest rates, is more than off-set by the low inflation rate. This leads to further increases in debt and taxes, with households attributing part of this increase as permanent. Because actual policy behavior is unchanged, the perceived increase in the level of long-run taxes, net of the increase in debt holdings, produces a negative wealth effect. This explains the persistent response of inflation in the high-debt economy.

Finally, the persistently low level of inflation implies a low level of the short-term rate, allowing aggregate demand to overshoot. At the same time, the negative wealth effects from net debt holdings ease and eventually become stimulative, as both inflation and the price of the bond continue rising. This restarts the cycle. The overshooting of inflation and output, reminiscent of Sims' (2011b) "stepping on a rake" phenomenon, emphasizes a critical difference in the theories of imperfect knowledge and the fiscal theory of the price level. The fiscal foundations of inflation induced by imperfect knowledge do not rely on passive monetary and active fiscal policy.

As explained above, the key to this propagation mechanism lies in the self-referential nature of the model. The top-right panel in **figure 4** further clarifies the stability results of **proposition 1**. The figure shows that for $\rho = 0$ and $\rho = 1$ the initial increase in inflation expectations do not have lasting effects on inflation. In fact, except for the first couple of periods, the inflation response resembles that observed in the zero-debt economy. This is because inflation expectations do not feed back on the evolution of taxes; learning is fast in this case.

5 The Great Moderation: did fiscal policy help?

The theory indicates that under imperfect knowledge details of fiscal policy can matter significantly for the control of inflation expectations. That more heavily indebted economies constrain monetary policy certainly resonates with public pronouncements of policy makers. However, the analytical insights of the previous section are asymptotic in nature, posing the hypothetical question of whether, given enough data, agents' beliefs would converge to the rational expectations equilibrium. A natural question is what properties are induced by learning dynamics outside of rational expectations equilibrium and are they quantitatively important? Is it the case that high-debt and moderate-maturity economies induce macroeconomic volatility — even when the policy regime is consistent with the long-run stability of expectations? This section explores these issues in the context of the Great Moderation period using an estimated version of the model.

Why focus on the Great Moderation? Two main reasons. First, an interesting feature of US data over the period 1984Q1-2007Q2 is the relative stability of the US economy, coupled with the gradual decline in long-term inflation expectations that commenced with the Volcker disinflation — see, for example, Stock and Watson (2002). This adjustment, which spans the 1990s, can be interpreted as market participants gradually learning about a new monetary policy regime with low average inflation. Second, policy during this period has been characterized as a regime where monetary policy was active and fiscal policy was passive, the policy mix considered in this paper. Under the previous policy regime, associated with the high inflation of the seventies, monetary policy was passive (a violation of the Taylor principle).

Much recent research has sought to understand whether it is changes in the conduct of monetary policy or changes in the volatility of economic disturbances — often referred to as good policy versus good luck — that best account for the Great Moderation.[17] A notable feature of these analyses is the absence of fiscal variables. A specific hypothesis of interest is to what extent was the Great Moderation the result of good fiscal policy? Given that in the model considered here equilibrium is jointly determined by choices of monetary and fiscal policy, surely fiscal policy itself is a candidate explanation of the Great Moderation. While the subsequent analysis does not attempt a thorough investigation of the contribution of changes in the volatility of exogenous disturbances, changes in monetary policy and changes in fiscal policy, it demonstrates the Great Moderation is not a necessary implication of better monetary policy — it also required good fiscal policy in a sense to be made precise.

5.1 A New Keynesian model

To provide a numerically plausible account of the data, the endowment economy is now generalized in several dimensions. These include incorporation of monopolistic competition, nominal rigidities and endogenous labor supply. A more general class of monetary and fiscal policy are also permitted. The model is New Keynesian, similar in spirit to Clarida, Gali, and Gertler (1999) and Woodford (2003), used in many recent studies of monetary policy, extended to include multiple-maturity debt. The appendix provides model details and discusses implications for expectations stability and macroeconomic dynamics. For current purposes it suffices to note that the basic mechanisms revealed by the endowment economy analysis continue to hold in this environment. Imperfect knowledge is the critical assumption that engenders fiscal foundations of inflation. Other model complications are included on the grounds of realism, comparability to earlier work and enhancement of the basic mechanism induced by imperfect knowledge. The latter features are briefly noted as they arise.

There is a continuum of monopolistically competitive firms. Each differentiated consump-

[17] Important contributions include, inter alia, Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Sims and Zha (2006), Primiceri (2005), Justiniano and Primiceri (2008) and Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2010).

tion good is produced according to the linear production function

$$Y_t(j) = A_t H_t(j) \tag{25}$$

where A_t denotes an exogenous aggregate stationary technology process. Each firm faces a demand curve $Y_t(j) = (P_t(j)/P_t)^{-\theta} Y_t$, where Y_t denotes aggregate output, and solves a Rotemberg-style price-setting problem, taking wages, the aggregate price level and technology as given.[18] A price $p_t(j)$ is chosen to maximize the expected discounted value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} Q_{t,T}^F \Gamma_T(j)$$

where

$$\Gamma_T(j) = p_T(j)^{1-\theta} P_T^{\theta} Y_T - p^{-\theta} P_T^{\theta} Y_T W_T / A_T - \chi \left(p_T(j) / p_{T-1}(j) - 1 \right)^2 \tag{26}$$

denotes period T profits, W_t denotes the hourly wage, and $\chi > 0$ scales the quadratic cost of price adjustment. Given market incompleteness, it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income

$$Q_{t,T}^F = \beta^{T-t} P_t Y_T / (P_T Y_t)$$

for $T \geq t$.

Households maximize the following intertemporal utility

$$\frac{1}{1-\sigma} \hat{E}_t^i \sum_{T=t}^{\infty} \xi_T \beta^{T-t} C_T^{1-\sigma}(i) \cdot \left(1 - \frac{\psi}{1+\gamma} H_T^{1+\gamma}(i) \right)^{1-\sigma}$$

where $\sigma > 1, \gamma > 0$ and $C_T(i)$ is a standard Dixit-Stiglitz aggregator, $H_T(i)$ is the amount of labor supplied to the production of goods, and ξ_t is an exogenous shock to the discount factor. The degree of intertemporal substitution in leisure affects the relative importance of wealth effects from holding the public debt. The greater the preparedness to substitute leisure intertemporally, the smaller are wealth effects. Complementarities between consumption and leisure therefore mitigate the negative wealth effects on labor supply, leading to larger expenditure effects for given scale and composition of public debt. The household's flow budget constraint is now

$$\mathbb{W}_t(i) \leq R_t^m \pi_t^{-1} \mathbb{W}_{t-1}(i) + \left(R_{t-1}^s - R_t^m \right) P_{t-1}^s b_{t-1}^s(i) + (1-\tau_t^w) w_t H_t(i) + \Gamma_t - \tau_t^{LS} - C_t(i) \tag{27}$$

[18] Because we consider a first-order approximation to equilibrium dynamics, this is equivalent to assuming Calvo pricing.

where $w_t = W_t/P_t$, and τ_t^w, τ_t^{LS} denote labor income and lump-sum taxes. Aside from realism, distortionary taxation, through its effects on firms' marginal cost structures, makes inflation dynamics more self-referential. This strengthens the fiscal effects on inflation. Subsequent results in no way depend on encountering a fiscal limit determined by a Laffer curve, never a relevant constraint on monetary policy.

The central bank implements monetary policy according to the family of interest-rate rules

$$R_t^s = R_t^* \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \tag{28}$$

where $\phi_\pi, \phi_y \geq 0$; and $Y_t = C_t + G_t$ is aggregate output with a steady-state $\bar{Y}$. Interest-rate policy responds to deviations of inflation and output from steady-state levels.[19] The term $R_t^* = (1 + \bar{\imath})e^{m_t}$ captures exogenous shifts in the intercept, where $\bar{\imath}$ is the steady-state level of the net interest rate and m_t is an exogenous stochastic process to be defined below. The steady-state inflation rate is assumed to be zero. The flow budget constraint of the government is given by

$$P_t^m B_t^m = B_{t-1}^m \left(1 + \rho P_t^m\right) - P_t S_t. \tag{29}$$

where the real structural surplus is

$$S_t = T_t/P_t - G_t. \tag{30}$$

The government has access to both lump-sum taxes, τ_t^{LS}, and labor income taxes, τ_t^w, which generates total tax revenue

$$T_t/P_t = \tau_t^{LS} + \tau_t^w w_t H_t$$

where W_t denotes hourly wages and H_t total hours worked. Tax policy is determined by tax rules of the form

$$\tau_t^{LS} = \bar{\tau}^{LS} \left(\frac{l_t}{\bar{l}} \right)^{\phi_{\tau_l}} \quad \text{and} \quad \tau_t^w = \bar{\tau}^w \left(\frac{l_t}{\bar{l}} \right)^{\phi_{\tau_l^w}}, \tag{31}$$

where $l_t = B_{t-1}^m \left(1 + \rho P_t^m\right)/P_{t-1}$ is a measure of real government liabilities in period t. The policy parameters satisfy $\phi_{\tau_l}, \phi_{\tau_l^w} \geq 0$. Such rules are consistent with empirical work by Davig and Leeper (2006).

[19] The analysis eschews the study of optimal policy to give emphasis to the interaction of monetary policy with various dimensions of fiscal policy. See Eusepi, Giannoni, and Preston (2012) for an analysis of optimal policy in the context of this model.

5.2 Learning

As in the simple model, agents use a linear econometric model nesting the stationary rational expectations equilibrium. Here it takes the form

$$\mathbb{Z}_t = \Omega + \Phi_b \hat{b}^m_{t-1} + \Phi_{\mathbb{S}} \mathbb{S}_{t-1} + \mathbf{e}_t \tag{32}$$

where the vector $\mathbb{Z}_t = \left(\hat{\imath}_t, \pi_t, \hat{w}_t, \hat{\Gamma}_t, \hat{\tau}^{LS}_t, \hat{\tau}^w_t, \hat{b}^m_t\right)'$ includes all endogenous variables beyond the control of individual agents, and $\mathbb{S}_t = \left(\hat{A}_t, \hat{\xi}_t, \hat{G}_t, \hat{m}_t\right)'$ is the vector of exogenous disturbances and $\mathbf{e}_t$ denotes a vector of *i.i.d.* errors. Government debt $\hat{b}^m_t$ is the only endogenous state variable of the model. In contrast to the endowment economy, agents do not know the monetary policy rule. The structural relationships between interest rates and inflation on the one hand, and taxes and debt on the other hand, are two of the many rational expectations equilibrium restrictions about which agents must learn.

In log deviations the exogenous processes evolve according to the first-order vector autoregression

$$\mathbb{S}_t = F\mathbb{S}_{t-1} + Q\epsilon_t \tag{33}$$

where the variance-covariance matrix of the innovations ϵ_t is the identity matrix; Q is a lower triangular matrix; and F has all eigenvalues in the unit circle. As customary in this literature, the law of motion (33) is known to the agents. Being free of self-referential dynamics ensures coefficients are learnable with probability one — standard econometric asymptotics apply.

Agents update only their estimates of the intercept $\hat{\Omega}_t$ using the updating rule (20). The remaining coefficients take their rational expectations values so that $\Phi_b = \Phi^*_b$ and $\Phi_{\mathbb{S}} = \Phi^*_{\mathbb{S}}$. Because of distortionary taxation, the column vector Φ^*_b has all non-zero elements. Provided agents' estimates of Ω_t are sufficiently close to their values under rational expectations, subjective beliefs of this kind represent a first-order approximation of a richer forecasting model in which all coefficients are updated. The appendix shows that under the specific formulation of beliefs adopted here, the updating of non-intercept coefficients have only second-order effects on model dynamics.[20]

[20] This formulation is commonly used in the adaptive learning literature — see Evans and Honkapohja (2001).

5.3 Estimation

The model is confronted with data by use of set of parametric assumptions about the model's structural parameters and estimation of the exogenous disturbances. Parametric choices reflect conventional values in the relevant micro and macro literatures and are intended to give a minimally realistic account of the data. The model is parameterized at a quarterly frequency. The households' discount factor is $\beta = 0.99$ and their Frisch elasticity of labor supply is set to 0.6, in line with micro-evidence summarized by, inter alia, Hall (2009). The elasticity of intertemporal substitution of consumption is $\sigma^{-1} = 1/4$. This is consistent with maintained values in the large literature on medium- to large-scale stochastic general equilibrium models — see, for example, Coenen et al. (2012). Moreover, it does not appear to be inconsistent with US data, conditional on the simple model analyzed here.[21] Turning to firms, nominal rigidities are determined by setting the cost parameter χ to be consistent with empirical measures of price stickiness. The chosen value implies a price average duration of a about five quarters.[22] The elasticity of demand across differentiated goods is $\theta = 6$. These values are in line with an extensive New Keynesian empirical literature.

We fix the constant gain in the updating rule (20) to $\bar{g} = 0.025$, implying that 25 year-old observations receive a weight of less than 0.1.[23] Subsequent results show this parameter choice is consistent with the behavior of long-term expectations during the US Great Moderation.

In the baseline parameterization the response coefficients to government debt liabilities are set to $\phi_l = 1.3$ (lump-sum taxation) and $\phi_l^w = 0.09$ (labor tax rate). These parameter configurations are chosen to be consistent with a passive fiscal regime in the sense of Leeper (1991). Labor taxes are not very responsive to changes in government liabilities. This assumption limits the role of distortionary taxes in providing a link between government debt and inflation.[24] The empirical analysis focuses on this link as emerging from imperfect information and learning. As a result, the steady-state labor tax rate, $\bar{\tau}^w$, is 15%, lower than in the US data. Consistent with US data over the sample considered, the steady-state debt-to-output ratio is 40%, in annual terms, and the average maturity of debt is 5.4 years. In subsequent

[21] In the estimation exercise discussed below, the model with a low elasticity provides a better fit as measured by the likelihood.

[22] In terms of the Calvo model, which is isomorphic to our specification, the probability that a firm does *not* reset the price in any given period is set to eighty percent.

[23] The weight is calculated as $(1 - 0.025)^{100} \simeq 0.07$.

[24] The chosen parameter is in line with the estimate of Traum and Yang (2011).

analysis, counterfactual experiments vary the steady-state debt-to-output ratio. Steady-state taxes are always adjusted to ensure intertemporal solvency of the government accounts. The ratio of government spending to output is 0.22, in line with post-war US data, implying a consumption-to-output ratio is 0.78. The monetary rule takes values consistent with Taylor (1993), setting $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$.

Given the calibrated parameters, the parameters of the shock process $\mathbb{S}_t$ are estimated using Maximum Likelihood. We use data for GDP growth, three-month Treasury-Bill rate, GDP deflator inflation and debt-to-GDP ratio. The data for GDP and GDP deflator come from the National Accounts, while the value of federal government debt comes from the Federal Reserve Bank of Dallas. The sample used for estimation is 1984Q1-2007Q2.[25] The estimation is performed using demeaned variables. The specification of the exogenous processes in (33), is similar to the wedges specification of Chari, Kehoe and McGrattan (2007). For the model to be identified we impose a restriction on the F matrix, namely that the lagged correlation between the government spending, $\hat{G}_t$, and preference, $\hat{\xi}_t$, disturbance is zero — this guarantees that the likelihood is locally sharp. There is no attempt to identify specific shocks. Subsequent results only rely on the estimated variance-covariance matrix. The linear state-space model is defined in the appendix.

The results of the estimation are summarized in **Table 1** in the appendix, which includes the parameter estimates for F and Q together with the 90% confidence intervals computed 1000 bootstrapped replications.

Figure 5 suggests that the model does a reasonable job capturing salient features of de-trended output and various measures of inflation expectations during the Great Moderation. Model-implied predictions for these series are generated using the estimated latent states inferred from the Kalman smoother. The black solid lines show the model predictions using the point estimates of the parameters and the shaded area corresponds to the 95^{th} percent confidence regions.[26] The red lines correspond to the US data. For de-trended output we use the output gap measure from the Congressional Budget Office (CBO). Measures of inflation expectations correspond to the GDP deflator. The one- and four-quarters-ahead forecasts are from the Survey of Professional Forecasters, while the five-to-ten year inflation forecast, avail-

[25] We use data starting from 1982Q2 as a training sample.

[26] To capture the elevated level of inflation expectations prior to the Volker disinflation we initialize the state of the economy in 1980Q3. This is earlier than the sample chosen for the estimation of the exogenous processes, which was selected to capture the great moderation.

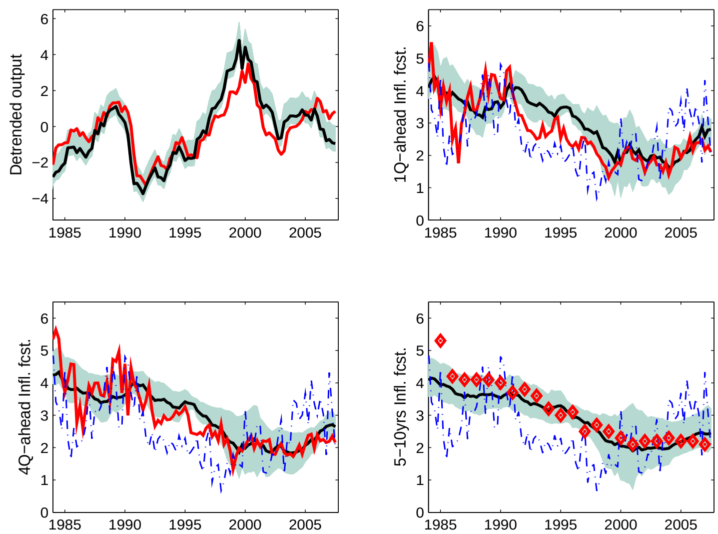

Figure 5: **Detrended output and inflation expectations**. The solid black line denotes the model-implied path for the four variables under the point estimates. The light-shaded area denotes the 95^{th} percent bands obtained from 1000 bootstrapped replications. The red solid lines denotes actual data. For detrended output we use the CBO estimate of the output gap. One- and four-quarters- ahead GDP-deflator forecasts are from SPF survey while the five-to-ten- years forecast is from the Blue Chip survey. Finally, the dashed blue line in the bottom-right box is GDP deflator.

able at a biannual frequency, is constructed using the Blue Chip Economic Indicators Survey. None of these series are used in the estimation. The blue line shows the US GDP deflator which is plotted to allow a comparison with the adjustment in inflation expectations. The model captures quite well the general decline and key turning points in inflation expectations at different forecasting horizons, in particular the long-term forecast. Similarly, the output gap is quite well explained, though with some discrepancies, notably the late 1990s and the recent crisis period.

5.4 Counterfactuals

An advantage of estimating a structural model is the ability to conduct counterfactual experiments. Model predictions under alternative configurations of policy can be determined, assuming the economy is subject to the same sequences of disturbances identified in estima-

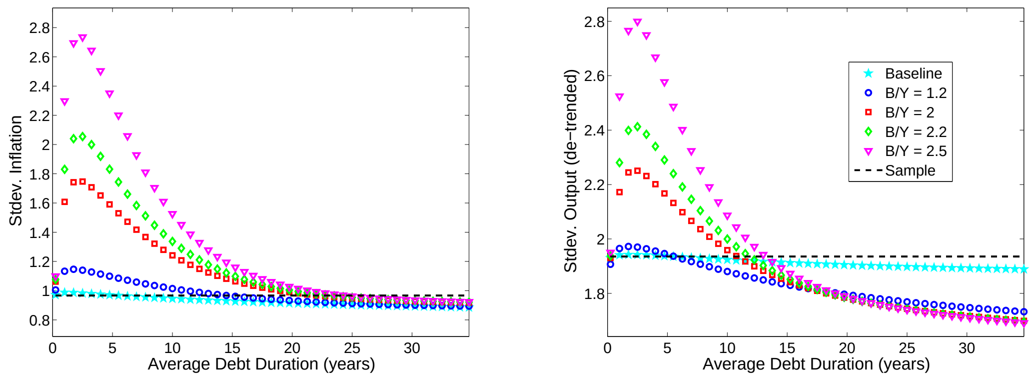

Figure 6: **Inflation and output volatility**. The figure shows the change in the standard deviation of inflation and de-trended output over the sample, in counterfactuals where the average maturity of debt and the debt-to-output ratio vary. In all experiments the realized shocks are the same and correspond to the Kalman smoother estimates under the baseline calibration. The black dotted line shows the standard deviation of output inflation in sample.

tion. This permits evaluating whether monetary control would have been as precise had the fiscal environment been different to that experienced over the sample period under consideration.

5.4.1 Alternative fiscal scenarios

Figure 6 illustrates the results of the main counterfactual exercise showing the evolution of inflation (left panel) and de-trended output (right panel) under alternative fiscal policy configurations but the same shock history as estimated in the baseline model. The other parameters in the model, including the monetary policy rule, are unchanged. We compute the standard deviation of inflation and de-trended output over the period 1984Q1-2007Q2. Each point on the plotted curves represents the result attached to a specific fiscal policy configuration. In this and all subsequent counterfactuals initial beliefs, together with the other state variables, are the same as under the baseline calibration.

The counterfactual exercise yields two main conclusions. First, consistent with the stability results discussed above, for a given level of debt-to-output ratio the volatility of inflation peaks for values of debt duration between 2-to-5 years. Moreover, higher values of the debt-to-output ratio boost inflation and output volatility. Notice that for the chosen fiscal and monetary policy configurations the underlying rational expectations equilibrium is stable under learning. However, a fiscal regime that promotes high levels of debt at relatively short maturity would have implied more volatile output and inflation during the time period of the Great Moderation. To offer some perspective, the standard deviation of GDP deflator and output gap, as measured by the CBO, in the years 1955Q1-1983Q4 were 2.9% and 3.2% respectively. They are not much higher than the values in **figure 6**, corresponding to elevated levels of debt-to-GDP ratios.

Second, fiscal regimes with long-term debt appear to have a stabilizing effect on the economy. Regardless of the steady-state levels of debt, if government debt had an average duration above 15 years, then both de-trended output and inflation would have been less volatile. Among the countries described in **figure 1,** only the United Kingdom, with an average maturity of debt of about 14 years, comes close to satisfying this condition. The result accords with the results of section 4.2, underscoring the fact that long-maturity debt mitigates economic volatility through lower sensitivity of debt issuance to inflation expectations.

To offer further insight, **figure 7** shows the counterfactual paths of inflation, three-month Treasury-bill, de-trended output and long-term inflation expectations for two specific fiscal policy configurations. The solid green line corresponds to the baseline calibration for the US; the dashed blue line denotes a high debt-to-output ratio, 200% in annualized terms, and an average maturity of 5.4 years, corresponding to the baseline calibration for the US; the dashed-dotted red line labels a policy regime with the same high debt-to-output ratio but with an average duration of debt of 30 years. Under the high debt-to-output ratio and baseline average maturity, inflation is significantly higher in the first part of the sample and it undershoots relative to history in the late 1990s, as long-term inflation expectations adjust rapidly towards 2% and undershoots. Inflation dynamics under the high-debt regime accords with the intuition provided in section 4. Interestingly, a fiscal regime that instead has a high average maturity predicts an evolution of inflation and other variables close to the historical pattern. In fact, the dashed red line hugs fairly closely the solid green line, with the exception of the beginning of the sample where inflation in the high-debt regime is significantly lower and the early 2000s, where inflation is somewhat above.

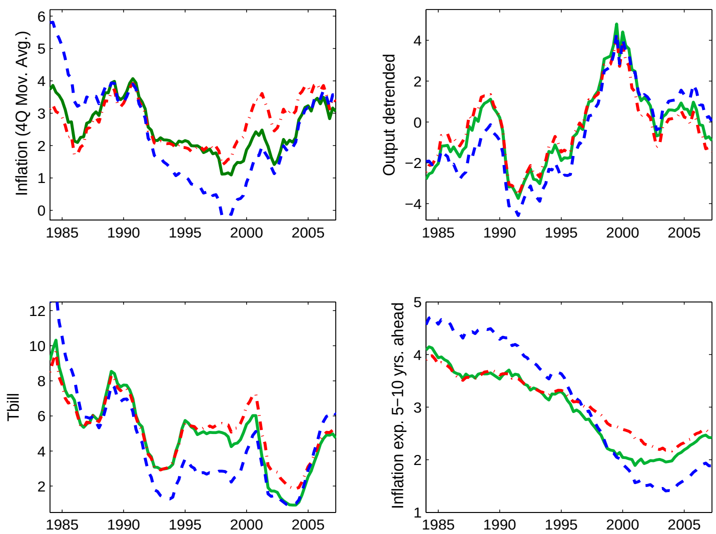

Figure 7: **Counterfactual simulations**. The figure shows counterfactual simulations with different fiscal policy configurations. The solid green line corresponds to the baseline calibration, the dashed blue line corresponds to a debt-to-output ratio of 200 percent, with an average maturity of debt corresponding to the baseline specification. Finally, the dashed-dotted red line shows an economy with debt-to-output ratio of 200 percent but an average maturity of debt of about 30 years.

5.4.2 Responding to output

Figure 8 shows counterfactual simulations for the short-term interest rate and inflation under different monetary and fiscal configurations, emphasizing the role played by output responses of monetary policy. The solid green line corresponds to the US data; the dashed blue line corresponds to a monetary policy rule which is more aggressive towards inflation ($\phi_\pi = 2$) but retains the same response to de-trended output ($\phi_y = 0.5/4$). To evaluate the effects of a more aggressive monetary rule, compare the left panel, describing the baseline fiscal configuration to the right panel, where the government debt-to-output ratio is 200% and the average debt duration is 3.5 years.[27] Despite the sizable differences in the fiscal regimes, a more aggressive response to inflation keeps inflation in check; the difference between the blue lines in the left and right panels are not very large.

[27] This is roughly the average duration of government bonds in the US over the period 1975-1984.

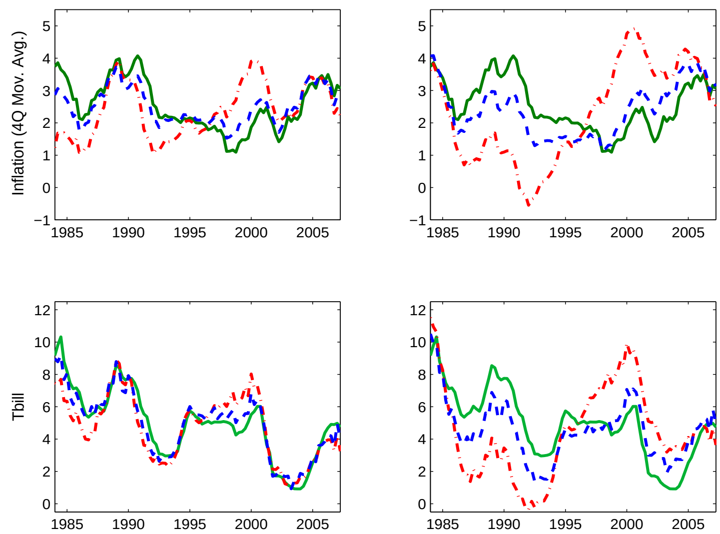

Figure 8: **Monetary policy rule**. The figure shows counterfactual simulations for T-bill and inflation under different monetary and fiscal policy configurations. The solid green line corresponds to the US data; the dashed blue line corresponds to a monetary policy rule with a response coefficient of 2 on inflation and 0.5/4 on de-trended output; the red dashed-dotted line represents a monetary policy rule with a response coefficient of 2 on inflation and 0.1/4 on de-trended output. Finally, the panels on the left correspond to the baseline fiscal configuration (debt-to-output ratio of 40 percent and average maturity of debt of 5.4 years). The panels on the right correspond to a debt-to-output ratio of 200 percent and average maturity of debt of 3.5 years.

The dashed-dotted red line tells a very different story; here the response to inflation remains strong ($\phi_\pi = 2$) but the response to detrended output is greatly diminished ($\phi_y = 0.1/4$). The difference between left and right panels is substantial. In a fiscal regime with high government debt of short duration, the central bank fails to control inflation. The US economy during the Great Moderation would have experienced deflation in the early 1990s and substantial inflation in 2000. Looking at the short-term interest rate, the counterfactual simulation indicates that in the high-debt regime the zero lower bound would have been violated over the period 1991-1993, while interest rates would have reached double-digits in the early 2000. An aggressive response to inflation *per se* would not suffice to control inflation in an economy with high debt of low duration.

5.4.3 Inflation expectations and economic volatility

Earlier results indicate that higher debt and lower average maturities than actually observed would have rendered the Great Moderation less moderate. An important feature of the data over this sample is the gradual decline in inflation expectations. It remains then to understand the role played by inflation expectations inherited by Volcker at the onset of the Great Moderation period.

A final experiment shows that most of the volatility in inflation under different fiscal policy regimes is due to the adjustment in inflation expectations over the sample. To see this, simulate the model using the estimated parameter values for the shocks and consider two scenarios. In the first, use as initial conditions for each simulation the state vector estimated from the US data in 1980Q3 — for reasons enumerated in footnote 26. In the second, simulate the model at its stationary distribution. That is, impose as a starting condition the steady state of the model and discard the first 200 periods before computing model statistics.

The results of this experiment are shown in **figure 9**. Each statistic in the four panels, corresponding to a particular fiscal policy configuration, is obtained by averaging 1000 replications of identical length samples. The top-left panel reports the 'conditional' simulations, displaying counterfactual histories that are comparable to those documented in **figure 6**, which uses the historical shocks.

However, the 'unconditional' simulations in the top-right panel reveal much less inflation volatility under alternative fiscal regimes. This shows that as inflation expectations converge to the new low-inflation regime, and remain anchored, alternative fiscal policy configurations do not have, *on average*, large effects on observed inflation volatility. This conclusion depends on inflation expectations remaining stable over time. A sequence of shocks, or any structural change, leading to a sudden shift in long-term expectations would lead to greater volatility in economies with high levels of debt of moderate average duration.

To close this section, it is worth underscoring that the findings of this analysis have clear predictions for the near-term evolution of the US and many other economies affected by the 2007-2009 global recession. The crisis has witnessed a high degree of uncertainty about the economic environment and host of new policy initiatives, many unfamiliar to agents. Focusing on the US, there is a great deal of uncertainty about the future course of monetary policy, specifically regarding the exit strategy from the zero lower bound and the unwinding of the Fed

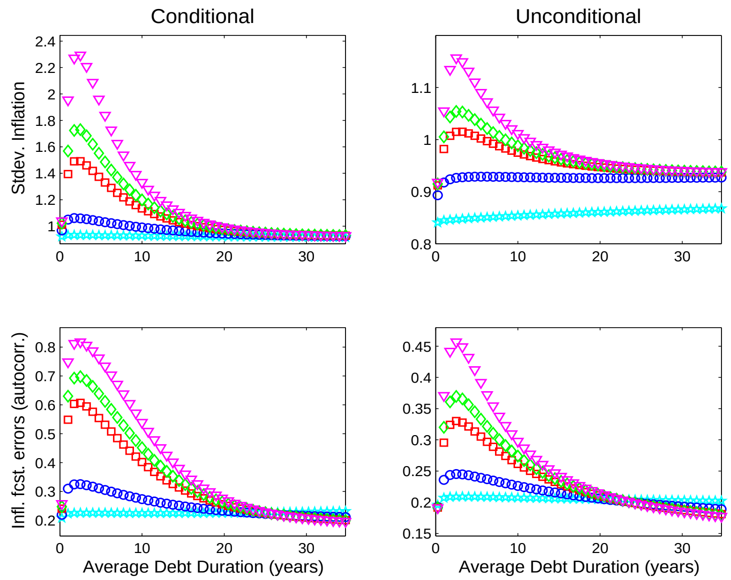

Figure 9: **Simulations**. The figure shows model simulations with different fiscal policy configurations. The column labeled 'Conditional' shows results of simulating the model with initial conditions corresponding to the estimated state of the economy using US data. The label 'Unconditional' show the standard deviation of inflation and autocorrelation in forecast errors evaluated at the model's unconditional distribution. This is obtained simulating the model with initial conditions corresponding to the rational expectations equilibrium and discarding the first 200 observations. The figure at the top show the standard deviation of inflation for the same policy experiments as in Figure 2. It is obtained by averaging 1000 replications for each policy configuration. The figure at the bottom displays the coefficient on lagged forecast errors of one-quarter-ahead inflation forecasts, where the regression equation is the same as in the text. The same regression on survey data yields a coefficient of 0.59 for the sample 1984Q1-2007Q2 (with a t-stat of 6.7) and a coefficient of .60 for the sample 1982Q3-2007Q2 (with a t-stat of 7.2).

balance-sheet. Moreover, the stance of fiscal policy has altered in response to the recession, with substantial increases in the level of the public debt. There is little hope that current imbalances will be remedied quickly, with substantial risk that they could worsen at least in the short-to-medium term. At the same time, the economy has shifted to a shortened maturity structure, due to large-scale asset purchase programs and inflation expectations could be viewed to be, or at least are at some risk of being, unusually low. These observations suggest initial conditions less propitious than observed at the commencement of the great moderation period. Drifting inflation expectations together with deteriorating fiscal conditions may limit the efficacy of monetary policy.

5.4.4 Some limitations of the analysis

Throughout this paper the constant gain $\bar{g}$ in the learning rule is taken as an invariant parameter. In a more realistic model, the gain would adjust to changes in the economic environment and, in particular, to shifts in monetary and fiscal policy. To gauge what forecasting errors agents would make under different regimes, we study the pattern of error autocorrelation in inflation forecasts. The lower panels of **figure 9** provide information on the autocorrelation structure of inflation forecast errors across fiscal regimes under both the unconditional and conditional scenario. Here we consider the one-quarter-ahead forecasts. In each simulation we run the simple regression:

$$fe_t^\pi = \beta_0 + \beta_1 fe_{t-1}^\pi + e_t$$

where fe_t^π denotes the one-period-ahead forecast error. For each fiscal policy configuration, the bottom panels of **figure 9** show the mean estimate of β_1 over 1000 simulations. As one would expect the forecast errors exhibit positive autocorrelation. The pattern of autocorrelation is more pronounced in the conditional simulations. The correlation increases the higher the debt-to-output ratio and the lower the average maturity of debt. However, looking at survey forecasts for the GDP deflator during the Great Moderation, we find substantial autocorrelation in forecast errors. The same regression on survey data yields a coefficient of $\beta_1 = 0.59$ for the sample 1984Q1-2007Q2, with a t-statistic of 6.7. The model implications are therefore plausible despite the assumption of a fixed gain coefficient.

Finally, in this simple model the size of debt required for a substantial impact on economic volatility is quite large, higher than currently observed in most countries. This likely reflects both the simplicity of the model used and the specific experiment that we consider. Regarding the latter, recall that we focus only on the adjustment of long-term expectations; that is, the dynamics of the intercept in agents' perceived law of motion. We assume that agents have perfect knowledge about the short-term dynamics of the economy. This includes the coefficients of the monetary and fiscal policy rules together with their implications for the economic variables. It is, however, realistic to assume that fiscal and monetary policy rules change over time, and that agents would need to update their beliefs not only about their model's intercept but also about all other coefficients. Davig and Leeper (2006), Bianchi

(2012) and Bianchi and Ilut (2012), among others, find evidence of monetary and fiscal regime switches in the post-war US years, and in particular before and after Volcker. It is reasonable to expect that a version of this model embedding these structural changes would generate more macroeconomic volatility for a given size of government debt. The study of such a model is left for further research.

6 Discussion

The paper has built a fiscal theory of inflation based on imperfect knowledge. It provides insights relevant for the interpretation of US monetary history, and gives predictions about macroeconomic adjustment in the current monetary and fiscal environment. The approach is now related to various other literatures that argue the importance of debt to a proper understanding of inflation dynamics. Indeed, the paper can be viewed as building on these literatures by proposing a new theory of the fiscal determinants of inflation.

Policy design and adaptive learning. These results build on a now large literature on learning dynamics and inflation control. Bullard and Mitra (2002) and Evans and Honkapohja (2003) consider the stability properties of interest-rate rules in a New Keynesian model in which one-period-ahead expectations matter and there is no public debt. Preston (2005) and Preston (2006) explore similar questions in a model of optimal decision making. In models with one-period debt Evans and Honkapohja (2007) and Eusepi and Preston (2012) explore the interactions of fiscal and monetary policy, characterizing learning analogues to the seminal insights of Leeper's (1991) rational expectations analysis. A specific implication is the standard account of monetary policy, with active monetary policy and passive fiscal policy, is shown to be always stable under learning, regardless of the size of debt if interest rates are adjusted in response to current inflation. Eusepi and Preston (2012) shows that when monetary policy rules respond to inflation expectations instability can occur in more heavily indebted economies. The present paper advances these contributions, demonstrating that the maturity structure itself is a critical determinant of inflation control in models of imperfect knowledge. Instability under learning arises even in the benchmark case where the central bank responds to current information. Moreover, earlier contributions only characterized stability regions attached to particular policy rules. The analysis here takes a much more significant step, establishing the empirical relevance of learning dynamics for our understanding of monetary and fiscal interactions, and US monetary history. Finally, Eusepi, Giannoni, and Preston (2012) apply the model of this paper to the question of optimal policy design when agent's beliefs violate the expectations hypothesis of the term structure.

Unpleasant Monetarist Arithmetic and the Fiscal Theory of the Price Level. Sargent and Wallace (1981) demonstrated that under certain circumstances fiscal policy could render monetary policy impotent. A dominant fiscal authority was envisaged that independently set its budgets, including the entire future sequence of structural surpluses. When deficits cannot be financed by debt issuance, the monetary authority must provide the requisite revenue by printing money. Inflation control is subordinated by demands for seigniorage.

The fiscal theory of the price level — see Leeper (1991), Sims (1994), Woodford (1996) and Cochrane (2001)— asserts a distinct mechanism by which debt determines inflation. In contrast to the unpleasant monetarist arithmetic, the connection between debt and inflation is not determined causally by printing money — though money balances might adjust because of equilibrium considerations. Rather, the theory contends that certain choices of fiscal policy can render future structural surpluses insufficiently responsive to outstanding debt. The only way intertemporal solvency of government accounts can be restored is through adjustments in the price level to ensure consistency between the real value of current outstanding debt and the real present discounted value of structural surpluses. Here fiscal policy determines inflation, while monetary policy maintains the value of the public debt. This theory predicts that debt has monetary consequences. These theories advance our understanding of the inflationary consequences of fiscal imbalances. They have been invoked by Sims (2011b) and Bianchi and Ilut (2012) to explain the surge in inflation in the 1970s, when monetary policy has been characterized as passive, and employed by Davig, Leeper, and Walker (2011) to generate predictions about the potential inflationary pressures from growing unfunded liabilities attached to various entitlement programs.

Regime Uncertainty. The property that learning induces dynamics that out-of-rational-expectations equilibrium depend on outstanding debt has much in common with regime switching models of policy. Starting with Davig and Leeper (2006) there has been a concerted effort to understand the consequence of shifts in policy regime for macroeconomic dynamics. The central idea is that while there are periods in which policy is conducted according to conventional wisdom, with monetary policy providing a nominal anchor, there may also be periods in which fiscal policy determines the price level, with monetary policy stabilizing the level of the public debt. To the extent that there is non-zero probability weight on this second regime, debt will have monetary consequences, even during periods when policy is conducted according to the first regime. In some innovative work Bianchi (2010) and Bianchi and Ilut (2012) exploit these insights to understand how postwar inflation data depend upon agents' beliefs about the likelihood of different policy regimes. Davig, Leeper, and Walker (2011) study the consequences of high levels of the public debt for current inflation and transfer/entitlements reform.

More closely related to our paper is Sims (2011a). In contrast to our analysis, Sims proposes that agents make model consistent forecasts except for inflation. Conditional expectations of inflation are assumed to depend on debt. This is a reduced-form description of beliefs that would arise in a formal model of policy regime change discussed above. Like our paper, it does not require explicit characterization of alternative regimes. Unlike our paper, it is somewhat less general, restricting the possible influence of alternative regimes to inflation expectations alone. Nonetheless, Sims demonstrates, consistent with the analysis of Eusepi and Preston (2012) and this paper, that tighter monetary policy can lead to bursts of future inflation in the medium term — even when monetary and fiscal policy have conventional assignments. Sims (2011a) refers to this as "stepping on a rake" — see also Sims (2011b).

7 Conclusions

Using a theory of debt management policy based on imperfect knowledge, this paper provides fiscal foundations of inflation. The existence of imperfect knowledge implies that holdings of the public debt are perceived as net wealth, giving scope for the scale and composition of debt to be relevant to inflation dynamics. It is shown that both the scale and composition of debt place constraints on monetary control. High debt and moderate maturity economies require more aggressive monetary policy to deliver expectations stability.

An estimated version of the model reveals that the Great Moderation was not a necessary implication of better monetary policy — it depended crucially on the choice of fiscal policy. Counterfactual experiments reveal higher and more moderate maturity debt structures would have delivered greater macroeconomic volatility over the great moderation period. Furthermore, the extent of moderation would have been greater had the US economy issued much longer debt.

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8 Appendix (not for publication)

8.1 Proof of the Proposition

For the Jacobian to have eigenvalues with negative real parts requires its trace to be negative and determinant to be positive. The Jacobian is:

$$\begin{bmatrix} -\left(\frac{(1-\psi)\left(\beta-\phi_\pi^{-1}\right)}{1-\beta}+\frac{\beta\psi\rho^2}{1-\beta\rho}+\psi T_{\tau\pi}+1\right) & -\frac{\phi_\pi^{-1}\psi\beta}{\phi_b(1-\beta)} \\ \left(\beta^{-1}-(1-\rho)\,\phi_\pi\right)\phi_b\left(\frac{(1-\psi)\left(\beta-\phi_\pi^{-1}\right)}{1-\beta}+\frac{\beta\psi\rho^2}{1-\beta\rho}+\psi T_{\tau\pi}\right)+T_{\tau\pi}\phi_b\phi_\pi & \left(\beta^{-1}-(1-\rho)\,\phi_\pi\right)\phi_b\frac{\phi_\pi^{-1}\psi\beta}{\phi_b(1-\beta)}-1 \end{bmatrix}$$

which implies the trace is

$$-\frac{1}{\phi_\pi(\beta-1)(\beta\rho-1)}\begin{pmatrix} 2\phi_\pi+\beta\rho-\beta\phi_\pi+\beta^2\rho\phi_\pi+\psi\phi_\pi T_{\tau\pi}(\rho)-2\beta\rho\phi_\pi \\ +\beta\psi\rho^2\phi_\pi-\beta\psi\phi_\pi T_{\tau\pi}(\rho)-\beta\psi\rho\phi_\pi+\beta^2\psi\rho\phi_\pi T_{\tau\pi}(\rho)-\beta\psi\rho\phi_\pi T_{\tau\pi}(\rho)-1 \end{pmatrix}$$

and the determinant

$$\frac{1}{\phi_\pi(\beta-1)(\beta\rho-1)}\left(\phi_\pi+\beta\rho+\psi\phi_\pi T_{\tau\pi}(\rho)-\beta\rho\phi_\pi+\beta\psi\rho^2\phi_\pi-\beta\psi\rho\phi_\pi-\beta\psi\rho\phi_\pi T_{\tau\pi}(\rho)-1\right).$$

For the trace to be negative requires

$$\begin{aligned} 0 \;<\; & 2\phi_\pi+\beta\rho-\beta\phi_\pi+\beta^2\rho\phi_\pi+\psi\phi_\pi T_{\tau\pi}(\rho)-2\beta\rho\phi_\pi \\ & +\beta\psi\rho^2\phi_\pi-\beta\psi\phi_\pi T_{\tau\pi}(\rho)-\beta\psi\rho\phi_\pi+\beta^2\psi\rho\phi_\pi T_{\tau\pi}(\rho)-\beta\psi\rho\phi_\pi T_{\tau\pi}(\rho)-1 \end{aligned}$$

which requires

$$\phi_\pi\left(2-\beta+\beta^2\rho+\psi T_{\tau\pi}(\rho)-2\beta\rho+\beta\psi\rho^2-\beta\psi T_{\tau\pi}(\rho)-\beta\psi\rho+\beta^2\psi\rho T_{\tau\pi}(\rho)-\beta\psi\rho T_{\tau\pi}(\rho)\right)>1-\beta\rho.$$

Now consider the term on the LHS

$$\begin{aligned} LHS &= \phi_\pi \left(2-\beta+\beta^2\rho+\psi T_{\tau\pi}(\rho)-2\beta\rho+\beta\psi\rho^2-\beta\psi T_{\tau\pi}(\rho)-\beta\psi\rho+\beta^2\psi\rho T_{\tau\pi}(\rho)-\beta\psi\rho T_{\tau\pi}(\rho)\right) \\ &= \phi_\pi \left(2-\beta+\beta^2\rho-2\beta\rho-\beta\psi\rho+\beta\psi\rho^2+\psi T_{\tau\pi}(\rho)\left(1-\beta+\beta^2\rho-\beta\rho\right)\right) \\ &= \phi_\pi \left((1-\beta\rho)\left[(2-\beta)-\psi T_{\tau\pi}(\rho)\right]+\psi T_{\tau\pi}(\rho)(1-\beta)(1-\beta\rho)\right). \end{aligned}$$

It follows that

$$\phi_\pi \left[(1-\beta\rho)\left[(2-\beta)-\psi T_{\tau\pi}(\rho)\right]+\psi T_{\pi\tau}(\rho)(1-\beta)(1-\beta\rho)\right] > 1-\beta\rho$$

which is equivalent to

$$\begin{aligned} 1 &< \phi_\pi \left(\left[(2-\beta)-\psi T_{\tau\pi}(\rho)\right]+\psi T_{\tau\pi}(\rho)(1-\beta)\right) \\ &= \phi_\pi \left(\left[(2-\beta)-\psi T_{\tau\pi}(\rho)\right]+\psi T_{\tau\pi}(\rho)(1-\beta)\right) \\ &= \phi_\pi \left[(2-\beta)-\psi\beta T_{\tau\pi}(\rho)\right]. \end{aligned}$$

The determinant needs to be positive which requires

$$\phi_\pi+\psi\phi_\pi T_{\tau\pi}(\rho)-\beta\rho\phi_\pi+\beta\psi\rho^2\phi_\pi-\beta\psi\rho\phi_\pi-\beta\psi\rho\phi_\pi T_{\tau\pi}(\rho) > 1-\beta\rho.$$

What about the LHS? Consider the term

$$\begin{aligned} LHS &= \phi_\pi+\psi\phi_\pi T_{\tau\pi}(\rho)-\beta\rho\phi_\pi+\beta\psi\rho^2\phi_\pi-\beta\psi\rho\phi_\pi-\beta\psi\rho\phi_\pi T_{\tau\pi}(\rho) \\ &= \phi_\pi \left(1+\psi T_{\tau\pi}(\rho)-\beta\rho+\beta\psi\rho^2-\beta\psi\rho-\beta\psi\rho T_{\tau\pi}(\rho)\right) \\ &= \phi_\pi \left(1-\beta\rho+\beta\psi\rho(\rho-1)+\psi(1-\rho)\rho\beta\right) \\ &= \phi_\pi (1-\beta\rho). \end{aligned}$$

Therefore the determinant is positive whenever

$$\phi_\pi > 1$$

which is the Taylor Principle.

8.2 Model

This section reports model equations in log-linear form.

Households. The first-order conditions for bond holdings yield two Euler equations

$$-\hat{\imath}_t = \hat{E}_t^i\left[\hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1}(i) - \hat{\lambda}_t(i) - \hat{\pi}_{t+1}\right] \tag{34}$$

$$\hat{P}_t^m = \hat{E}_t^i\left[\left(\hat{\xi}_{t+1} - \hat{\xi}_t + \hat{\lambda}_{t+1}(i) - \hat{\lambda}_t(i) - \hat{\pi}_{t+1}\right) + \rho\beta\hat{P}_{t+1}^m\right] \tag{35}$$

where $\hat{\lambda}_t$ denotes the Lagrangian multiplier associated with the flow budget constraint, which, expressed in terms of the marginal utility of consumption, is

$$-\hat{C}_t(i) + \Theta\hat{H}_t(i) = \sigma^{-1}\hat{\lambda}_t(i), \tag{36}$$

where

$$\Theta = \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1}\frac{(1-\bar{\tau}^w)(\theta-1)}{\theta}.$$

Combining (34) and (35) yields the no-arbitrage condition (12). Combining the first-order condition for hours and (36) gives the constant-consumption labor supply equation

$$(\gamma+\Theta)\,\hat{H}_t(i) = \hat{w}_t - \frac{\bar{\tau}^w}{(1-\bar{\tau}^w)}\hat{\tau}_t^w - \hat{C}_t(i). \tag{37}$$

Finally, the Frisch elasticity of labor supply for KPR preferences is

$$\left[\gamma + \frac{2\sigma-1}{\sigma}\Theta\right]\hat{H}_t = \hat{w}_t + \sigma^{-1}\hat{\lambda}_t$$

which implies the parameter restriction: $\gamma + \frac{2\sigma-1}{\sigma}\Theta \geq 0$. The household intertemporal budget constraint to a first-order approximation is

$$\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\hat{C}_T(i) = \bar{b}_Y\left[\beta^{-1}\left(\hat{b}_{t-1}^m(i) - \hat{\pi}_t + \rho\beta\hat{P}_t^m\right) + \hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right] +$$

$$+\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left[\frac{(1-\bar{\tau}^w)(\theta-1)}{\theta}\left(\hat{H}_t(i) + \hat{w}_T - \frac{\bar{\tau}^w}{(1-\bar{\tau}^w)}\hat{\tau}_T^w\right) + \theta^{-1}\hat{\Gamma}_T - \frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_T^{LS}\right]$$

where

$$\bar{b}_Y = \frac{\bar{P}^m\bar{b}^m}{\bar{Y}} = \frac{\beta}{1-\beta}\frac{\bar{S}}{\bar{Y}},$$

and where the arbitrage condition is assumed to hold in all future periods.[28] Using the Euler equation (34) and the marginal utility of consumption (36), recursive backwards substitution

[28] That is:

$$\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{P}_T^m - \rho\beta\hat{P}_{T+1}^m\right) = -\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\hat{\imath}_T.$$

and taking expectations at time t gives

$$\hat{E}_t^i\left[\hat{C}_T(i)+\Theta\hat{H}_T(i)\right]=\sigma^{-1}\hat{E}_t^i\left(\hat{\xi}_T-\hat{\xi}_t\right)+\sigma^{-1}\hat{E}_t^i\sum_{s=t}^{T-1}\left(\hat{\imath}_s-\hat{\pi}_{s+1}\right).$$

Substituting back into the intertemporal budget constraint, combined with the constant-consumption labor supply (37) gives the consumption decision rule

$$\begin{aligned}\hat{C}_t &= (1-\sigma^{-1})\Theta\hat{H}_t+\sigma^{-1}\hat{\xi}_t-\beta\left[\sigma^{-1}-\bar{s}_C^{-1}\frac{\bar{S}}{\bar{Y}}\right]\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left(\hat{\imath}_T-\hat{\pi}_{T+1}\right)\\ &\quad+\bar{s}_C^{-1}\frac{\bar{S}}{\bar{Y}}\left[\hat{b}_{t-1}^m-\hat{\pi}_t+\rho\beta\hat{P}_t^m\right]\\ &\quad+\frac{(1-\beta)}{\bar{s}_C}\hat{E}_t^i\sum_{T=t}^{\infty}\beta^{T-t}\left[\sigma^{-1}\hat{\xi}_T+\theta^{-1}\hat{\Gamma}_T+\psi_w\left(\hat{w}_T-\frac{\bar{\tau}^w}{1-\bar{\tau}^w}\hat{\tau}_T^w\right)-\frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_T^{LS}\right],\end{aligned} \tag{38}$$

where

$$\begin{aligned}\bar{s}_C^{-1}(&= \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1}\frac{1+(1-\sigma^{-1})\frac{\Theta}{\gamma+\Theta}}{1+\frac{\Theta}{\gamma+\Theta}}\\ \psi_w &= \left(\frac{\bar{C}}{\bar{Y}}\right)\left[\Theta+\left(\frac{\sigma^{-1}}{1+(1-\sigma^{-1})\frac{\Theta}{\gamma+\Theta}}\right)\frac{\Theta}{\gamma+\Theta}\right].\end{aligned}$$

Firms. The first-order condition for the optimal price decision of firms, to a log-linear approximation, satisfies

$$\hat{p}_t(i)=\alpha\hat{p}_{t-1}(j)+\psi_w\alpha\hat{E}_t^j\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\hat{w}_T-\hat{A}_T+\hat{P}_T\right]$$

where $\hat{p}_t(j)=\log\left(p_t(j)/P_t\right)$ and where $\psi_w\equiv(\theta-1)\bar{Y}/\chi=(1-\alpha\beta)(1-\alpha)\alpha^{-1}>0$, and α satisfies the restrictions $0<\alpha<1$. Aggregating price decisions over the continuum of firms gives a generalized Phillips curve

$$\hat{\pi}_t=\psi_\pi\left(\hat{w}_t-A_t\right)+\hat{E}_t\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[\psi_\pi\alpha\beta\left(\hat{w}_{T+1}-A_{T+1}\right)+(1-\alpha)\beta\hat{\pi}_{T+1}\right]. \tag{39}$$

Firm profits and the production function are

$$\hat{\Gamma}_t=\hat{Y}_t-(\theta-1)\left(\hat{w}_t-\hat{A}_t\right) \tag{40}$$

and

$$\hat{H}_t = \hat{Y}_t - \hat{A}_t. \tag{41}$$

Finally, equilibrium in the goods markets yields the aggregate resource constraint

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t. \tag{42}$$

Monetary and fiscal policy. The nominal interest-rate rule satisfies the approximation

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t + \hat{m}_t \tag{43}$$

The activities of the fiscal authority are summarized by a log-linear approximation to (29), the definition of liabilities, (31), the definition of the structural surplus, and tax rules to give:

$$\hat{b}_t^m = \beta^{-1}\left(\hat{b}_{t-1}^m - \hat{\pi}_t\right) + (\rho - 1)\hat{P}_t^m - \left(\beta^{-1} - 1\right)\hat{s}_t \tag{44}$$

$$\hat{l}_t = \hat{b}_{t-1}^m + \beta\rho\hat{P}_t^m \tag{45}$$

$$\frac{\bar{s}}{\bar{Y}}\hat{s}_t = \frac{\bar{\tau}^{LS}}{\bar{Y}}\hat{\tau}_t^{LS} + \bar{\tau}^w\left(1 - \theta^{-1}\right)\left(\hat{\tau}_t^w + \hat{w}_t + \hat{H}_t\right) - \frac{\bar{G}}{\bar{Y}}\hat{G}_t \tag{46}$$

$$\hat{\tau}_t^{LS} = \phi_{\tau_l^{LS}}\hat{l}_t \tag{47}$$

$$\hat{\tau}_t^w = \phi_{\tau_l^w}\hat{l}_t. \tag{48}$$

Equilibrium. The *symmetric* equilibrium, for given expectations and exogenous processes S_t, is defined by the 13 equations (38), (39), (12), (37), (40), (41), (42), (43), (44)-(48) in the endogenous variables $(\hat{C}_t, \hat{H}_t, \hat{Y}_t, \hat{b}_t^m, \hat{l}_t, \hat{s}_t, \hat{\tau}_t^{LS}, \hat{\tau}_t^w, \hat{\imath}_t, \hat{\pi}_t, \hat{P}_t^m, \hat{w}_t, \hat{\Gamma}_t)$.

8.3 Actual Law of Motion

The equilibrium defined above can be reduced to a system of seven equations in the variables $\mathbb{Z}_t = \left(\hat{\imath}_t, \pi_t, \hat{w}_t, \hat{\Gamma}_t, \hat{\tau}_t^{LS}, \hat{\tau}_t^w, \hat{b}_t^m\right)'$. First, use the constant-consumption labor supply (37) and profits (40), coupled with the production function (41) and the resource constraint (42), to express consumption, hours and profits in terms of $\hat{w}_t$ and exogenous shocks. This yields

$$\hat{C}_t = \hat{C}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right) = \left[(\gamma+\Theta)\frac{\bar{C}}{\bar{Y}}+1\right]^{-1}\left(\hat{w}_t + (\gamma+\Theta)\hat{A}_t - (\gamma+\Theta)\frac{\bar{G}}{\bar{Y}}\hat{G}_t\right) \quad (49)$$

$$\hat{H}_t = \hat{H}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right) = (\gamma+\Theta)^{-1}\left[\hat{w}_t - \hat{C}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right)\right] \quad (50)$$

$$\hat{\Gamma}_t = \hat{H}_t(\hat{w}_t, \hat{A}_t, \hat{G}_t) + \hat{A}_t - (\theta-1)\left(\hat{w}_t - \hat{A}_t\right). \quad (51)$$

Notice that the model implies a negative relation between wages and profits, holding shocks and taxes constant.[29] Second, combine the flow government budget constraint (44) with (46) and the bond pricing equation (12) to get

$$\hat{b}_t^m = \hat{b}_t^m(\hat{b}_{t-1}^m, \hat{\tau}_t^{LS}, \hat{\tau}_t^w \hat{\imath}_t, \hat{\pi}_t, \hat{w}_t, \hat{A}_t, \hat{G}_t). \quad (52)$$

Finally, the monetary policy rule (43) can be expressed as

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y\left(\hat{H}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right) + \hat{A}_t\right) + \hat{m}_t. \quad (53)$$

The reduced model is then described by the consumption decision rule (*38*), after substituting for $\hat{C}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right)$ and $\hat{H}_t\left(\hat{w}_t, \hat{A}_t, \hat{G}_t\right)$; the Phillips curve (39); profits (51); the government flow budget constraint (52); and the policy rules (47), (48) and (53). The model can be written compactly as

$$\begin{bmatrix} \mathbb{Z}_t \\ \mathbb{S}_t \end{bmatrix} = \sum_{s=1}^{3} A_s\left(\hat{E}_t \sum_{T=t}^{\infty} \gamma_s^{T-t} \begin{bmatrix} \mathbb{Z}_{T+1} \\ \mathbb{S}_{T+1} \end{bmatrix}\right) + B\begin{bmatrix} \mathbb{Z}_{t-1} \\ \mathbb{S}_{t-1} \end{bmatrix} + C\epsilon_t \quad (54)$$

where $A_1, A_2, A_3, B, C,$ are matrices defining the equations for the 7 endogenous variables, $\mathbb{Z}_t$, and 4 exogenous variables, $\mathbb{S}_t$. The parameters $\gamma_1 = \beta$, $\gamma_2 = \alpha\beta$ and $\gamma_3 = \rho\beta$ denote the discount factors in the consumption, inflation and bond-price equations. Given the agents' PLM (18), forecasts can be computed as

$$\hat{E}_t\begin{bmatrix} \mathbb{Z}_{T+1} \\ \mathbb{S}_{T+1} \end{bmatrix} = (I_{11}-\Omega)^{-1}\left(I_{11}-\Omega^{T-t+1}\right)\begin{bmatrix} \Omega_0 \\ 0_{4\times 1} \end{bmatrix} + \Omega^{T-t+1}\begin{bmatrix} \mathbb{Z}_t \\ \mathbb{S}_t \end{bmatrix}$$

[29] Solving for (51) we get, for constant shocks and taxes,

$$\hat{\Gamma}_t = \psi_\Gamma \hat{w}_t,$$

where $\psi_\Gamma = \left[\left(\frac{\bar{Y}}{\bar{C}}\left(1+\frac{\bar{C}}{\bar{Y}}\Theta\right)+\gamma\right)^{-1} - (\theta-1)\right] < 0.$

where

$$\Omega = \begin{bmatrix} \Omega_{\mathbb{Z}} & \Omega_{\mathbb{S}} \\ 0_{7\times7} & F \end{bmatrix},$$

and

$$\hat{E}_t \sum_{T=t}^{\infty} \gamma_s^{T-t} \begin{bmatrix} \mathbb{Z}_{T+1} \\ \mathbb{S}_{T+1} \end{bmatrix} = \Psi_0^s\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \begin{bmatrix} \Omega_0 \\ 0_{4\times1} \end{bmatrix} + \Psi_1^s\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) \begin{bmatrix} \mathbb{Z}_t \\ \mathbb{S}_t \end{bmatrix} \tag{55}$$

where

$$\Psi_0^s\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) = \left(I_{11} - \Omega\right)^{-1}\left[\left(1-\gamma_s\right)^{-1} I_{11} - \Omega\left(I_{11} - \gamma_s\Omega\right)^{-1}\right]$$

$$\Psi_1^s\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right) = \Omega\left(I_{11} - \gamma_s\Omega\right)^{-1}.$$

Inserting the forecasts (55) in (54) we get the true data-generating process

$$\mathbb{Z}_t = T_0\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right)\cdot\Omega_0 + T_{\mathbb{Z}}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right)\mathbb{Z}_{t-1} + T_{\mathbb{S}}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right)\mathbb{S}_{t-1} + T_{\epsilon}\left(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}}\right)\epsilon_t.$$

8.4 Model with real-time learning

8.4.1 Approximation

The models of Section 4 and 5 employ a simplified belief structure in which agents learn only about average outcomes. Such beliefs can be shown to be a linear approximation of a more general belief structure in which agents update all coefficients, where the approximation is taken in the neighborhood of the mean dynamics of the beliefs and the rational expectations steady state. To see this suppose beliefs are given by

$$\mathbb{Z}_t = \Omega_0 + \Omega_{\mathbb{Z}}\mathbb{Z}_{t-1} + \Omega_{\mathbb{S}}\mathbb{S}_{t-1} + \mathbf{e}_t$$

where the vector $\mathbb{Z}_t = \left(\hat{\imath}_t, \pi_t, \hat{w}_t, \hat{\Gamma}_t, \hat{\tau}_t^{LS}, \hat{\tau}_t^{w}, \hat{b}_t^{m}\right)'$ includes all endogenous variables beyond the control of individual agents, and $\mathbb{S}_t = \left(\hat{A}_t, \hat{\xi}_t, \hat{G}_t, \hat{m}_t\right)'$ is the vector of exogenous shocks and $\mathbf{e}_t$ denotes a vector of *i.i.d.* errors. Given estimates of this model

$$\left(\tilde{\Omega}_{0,t-1}, \tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right)$$

the true data generating process is

$$\begin{aligned} \mathbb{Z}_t \;=\; & T_0\left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right)\cdot\tilde{\Omega}_{0,t-1} + T_{\mathbb{Z}}\left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right)\mathbb{Z}_{t-1} \\ & + T_{\mathbb{S}}\left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right)\mathbb{S}_{t-1} + T_{\epsilon}\left(\tilde{\Omega}_{\mathbb{Z},t-1}, \tilde{\Omega}_{\mathbb{S},t-1}\right)\epsilon_t. \end{aligned}$$

Taking a first-order linear approximation provides

$$\mathbb{Z}_t = T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)\tilde{\Omega}_{0,t-1} + \bar{\Omega}_{\mathbb{Z}}\mathbb{Z}_{t-1} + \bar{\Omega}_{\mathbb{S}}\mathbb{S}_{t-1} + T_\epsilon\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)\epsilon_t + \mathbb{O}(\,\|\epsilon_t\|^2\,) \tag{56}$$

where $\mathbb{O}(\,\|\epsilon_t\|^2\,)$ captures all terms of order $\|\epsilon_t\|^2$ or smaller if $\tilde{\Omega}_{0,t-1}$ is first order and $\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)$ denote ration expectations values of $(\Omega_{\mathbb{Z}}, \Omega_{\mathbb{S}})$.

Under what conditions is $\tilde{\Omega}_{0,t-1}$ first order? The learning algorithm is

$$\begin{aligned} \Xi_t &= \Xi_{t-1} + gR_t^{-1}z_{t-1}z_{t-1}'\left[T(\Xi_{t-1}) - \Xi_{t-1}\right] \\ R_t &= R_{t-1} + g\left[z_{t-1}z_{t-1}' - R_{t-1}\right] \end{aligned} \tag{57}$$

where we define

$$\Xi_t' = \left(\tilde{\Omega}_{0,t}, \tilde{\Omega}_{\mathbb{Z},t}, \tilde{\Omega}_{\mathbb{S},t}\right) \text{ and } z_t' = (1, \mathbb{Z}_t', \mathbb{S}_t')$$

and assume a constant gain g rather than a decreasing gain as in the stability results of Section 4. As shown in Evans and Honkapohja (2001), for sufficiently small g and large t, the mean dynamics of the algorithm have the property

$$\lim_{t\to\infty} E\left[z_{t-1}(\Xi)\, z_{t-1}'(\Xi)\right] = M(\Xi),$$

where E denotes the unconditional expectation taken with respect to the invariant distribution for the process $\mathbb{S}_t$, for a fixed value of Ξ. Since $z_t(\Xi)$ is asymptotically stationary for Ξ close to $\bar{\Xi}$, the limit $M(\Xi)$ is finite. Moreover, Evans and Honkapohja (2001) and Sargent and Williams (2005) show that R_t converges locally to $M(\Xi)$ so that in the mean dynamics we have $R_t \to R = M(\Xi)$ and therefore

$$R^{-1}M(\Xi) = I.$$

Approximate (57) in the neighborhood of the mean dynamics so that

$$\hat{\Xi}_t = \hat{\Xi}_{t-1} + g\left[DT(\bar{\Xi}) - I\right]\hat{\Xi}_{t-1} + \mathbb{O}(\,\|\epsilon_t\|^2\,)$$

where $DT(\bar{\Xi})$ is the Jacobian of the $T-$map and $\hat{\Xi}_t = \Xi_t - \bar{\Xi}$. The latter expression can be written

$$\begin{aligned} \tilde{\Omega}_{0,t} &= \tilde{\Omega}_{0,t-1} + g\left[T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)\tilde{\Omega}_{0,t-1} - \tilde{\Omega}_{0,t-1}\right] + \mathbb{O}(\,\|\epsilon_t\|^2\,) \\ \hat{\Omega}_{\mathbb{Z},t} &= \hat{\Omega}_{\mathbb{Z},t-1} + g\left[DT_{\mathbb{Z}}(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}})\hat{\Omega}_{\mathbb{Z},t-1} - \hat{\Omega}_{\mathbb{Z},t-1}\right] + \mathbb{O}(\,\|\epsilon_t\|^2\,) \\ \hat{\Omega}_{\mathbb{S},t} &= \hat{\Omega}_{\mathbb{S},t-1} + g\left[DT_{\mathbb{S}}(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}})\hat{\Omega}_{\mathbb{S},t-1} - \hat{\Omega}_{\mathbb{S},t-1}\right] + \mathbb{O}(\,\|\epsilon_t\|^2\,) \end{aligned}$$

where $\hat{\Omega}_{i,t} = \tilde{\Omega}_{i,t} - \bar{\Omega}_{i,t}$ for $i = \mathbb{Z}, \mathbb{S}$. The eigenvalues of $DT_i(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}) - I$, $i = \mathbb{Z}, \mathbb{S}$ determine whether there is is convergence in the mean dynamics (they are the stability conditions). Rewriting these expressions in terms of each individual variable delivers the expressions assumed in the paper. The dynamics of $\left(\tilde{\Omega}_{0,t}, \hat{\Omega}_{\mathbb{Z},t}, \hat{\Omega}_{\mathbb{S},t}\right)$ are all first-order and converge to the rational expectations equilibrium so long as the conditions for expectations stability are satisfied. Furthermore, it is immediate the dynamics of $\left(\hat{\Omega}_{\mathbb{Z},t}, \hat{\Omega}_{\mathbb{S},t}\right)$ are contribute variation of order $\mathbb{O}(\, \|\epsilon_t\|^2\,)$ to the true data-generating process (56).

8.5 State-space model

The model with real-time learning implies the simple linear ALM

$$\begin{aligned} \mathbb{Z}_t &= T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right)\tilde{\Omega}_{0,t-1} + \bar{\omega}_b \hat{b}^m_{t-1} + \bar{\Omega}_{\mathbb{S}}\mathbb{S}_{t-1} + \bar{\omega}_\epsilon \epsilon_t \\ \tilde{\Omega}_{0,t} &= \tilde{\Omega}_{0,t-1} + g \cdot \left[\left(T_0\left(\bar{\Omega}_{\mathbb{Z}}, \bar{\Omega}_{\mathbb{S}}\right) - I_7\right)\tilde{\Omega}_{0,t-1} + \bar{\omega}_\epsilon \epsilon_t\right]. \end{aligned}$$

To estimate the model we augment the state-space we the two following variables: log-output changes and government debt-to-out ratio in deviation from its steady state level,

$$\begin{aligned} \Delta \ln Y_t &= \hat{Y}_t - \hat{Y}_{t-1} \\ \frac{P^m_t b^m_t}{Y_t} - \frac{P^m \bar{b}^m}{\bar{Y}} &= \left(\frac{P^m \bar{b}^m}{\bar{Y}}\right)^{-1}\left(\hat{P}^m_t + \hat{b}^m_t - \hat{Y}_t\right). \end{aligned}$$

The state-space model then takes the standard form

$$\Xi_t = \mathbb{F}_\Xi(\eta)\Xi_{t-1} + \mathbb{F}_{\mathbf{w}}(\eta)\mathbf{w}_t$$

where Ξ_t is the appropriately augmented state vector, η denotes the model's structural parameters and

$$E\mathbf{w}_t\mathbf{w}_t' = \mathbf{\Sigma}_w.$$

The observation equation is then

$$\begin{bmatrix} \Delta \ln GDP_t - \overline{\Delta \ln GDP} \\ \Delta \ln DEFL_t - \overline{\Delta \ln DEFL} \\ TBill_t - \overline{TBill} \\ B_t/GDP_t - \overline{B/GDP} \end{bmatrix} = H(\eta)\,\Xi_t.$$

The parameters of the exogenous processes defined by F and Q are estimated using Maximum Likelihood.

8.6 Expectations stability results in the empirical model

The following section demonstrates that the empirical model inherits the stability properties derived for the endowment economy. It also draws out some of the motivation for the inclusion of several features in the empirical model.

Expectations Stability and dynamics. Analytical results are not feasible for the empirical model. We therefore employ a numerical analysis, using the parameterization described in section 5. These results make clear that the various features included in the richer model do not much alter earlier insights. **Figure 10** plots stability frontiers for the empirical model, analogous to **Figure 3** for the endowment economy. Regions above each contour delineate policy configurations consistent with expectational stability. Both the scale and composition of the public debt constrain the design of monetary policy in almost identical fashion to the endowment economy. For a given average maturity of debt, higher average levels of indebtedness require more aggressive monetary policy. For a given scale of public debt, variation in the average maturity of public debt engenders non-monotonic constraints on monetary policy. Fiscal regimes with average debt durations between 2 and 7 years are conducive to expectational instability. Interestingly, most countries in **figure 1** display average debt maturities within this range, with the notable exception of the UK. In the case of a debt-to-GDP ratio of 250 percent, and an average maturity of 2 years, the coefficient on inflation in the policy rule must be greater than 1.9 to deliver stability.

Figure 11 plots dynamics of several model variables in different economies in response to a shock to inflation expectations. The top panels show the response of output and inflation for economies with zero (red dashed line) and high (blue solid line) debt-to-output ratios respectively. Learning induces stable dynamics. In the zero-debt economy higher inflation expectations raise the expected path of the real interest rate, lowering consumption, real wages and, via the Phillips curve (39), inflation.[30] [31] As a result, inflation expectations converge to their steady state independently of the scale and composition of debt. Notice that the short-term nominal interest rate falls below its steady state in response to lower output and

[30] Equivalently, one could consider an 'high substitution' economy with separable preferences where $\bar{s}_C^{-1}(1) \approx 0$.

[31] Wages are proportional to consumption and therefore follow the same dynamic path. In consquence, higher expected wages stimulate consumption mainly through the intertemporal substitution of leisure. Because wages and profits are negatively related in this model, the positive income effects of higher expected wages are partially offset the expected decline in profits.

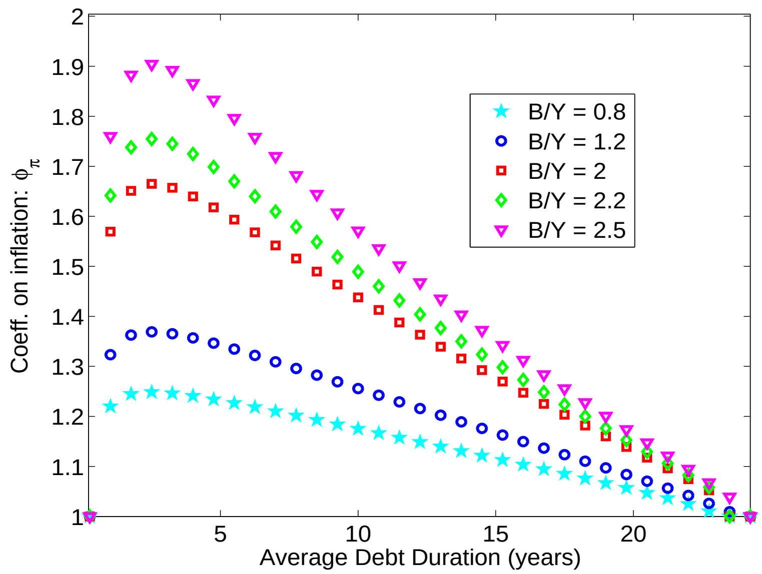

Figure 10: **Stability frontiers in the New Keynesian model**. The figure shows stability frontiers for different parameter configurations. For each frontier, the area (below) above denote (un)stable equilibria. The figure displays stability regions for alternative values of the average debt maturity and debt-to-output ratio in the baseline model with a Taylor rule that responds to inflation only.

inflation. This is the standard argument for following simple monetary policy rules which satisfy the Taylor principle.

With non-zero debt, fiscal policy affects the response of consumption to an increase in inflation expectations through the analogous non-Ricardian term identified in the endowment economy. Debt affects the response of output and inflation in three ways. First, the impact response for inflation is smaller; second, the response is more persistent; and third, output and inflation overshoot their steady state values.

To see the contributing role of fiscal policy, the bottom panels of **figure 11** display the different components of aggregate consumption. The bottom-left panel shows the evolution of the 'Ricardian' (solid blue) and 'non-Ricardian' (dashed red) components of aggregate consumption; the bottom-right panel displays the three sub-components of the 'non-Ricardian' term analogous to **figure 4** for the endowment economy. The higher present discounted value of real interest income (black dashed-dotted line) on impact generates positive wealth

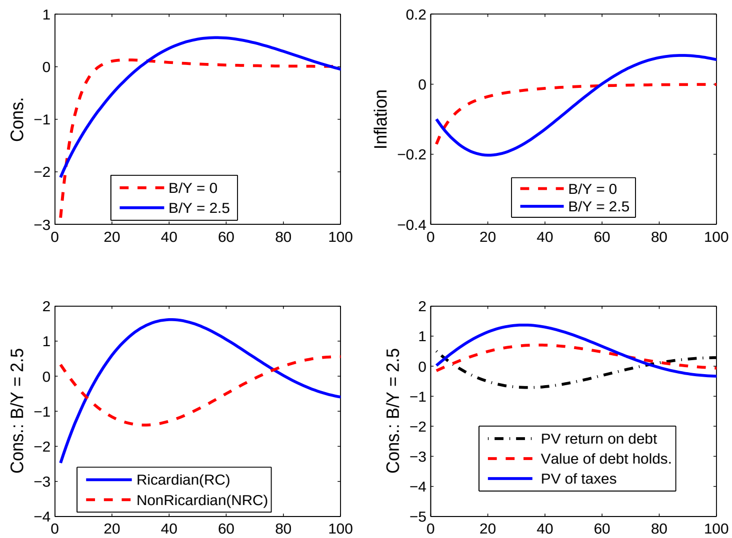

Figure 11: **Response to an increase in inflation expectations in the NK model**. The upper panels show the response of consumption and inflation in two economies with zero and high government debt respectively. The lower panels display the response of the different components of consumption in the economy with high debt.

effects, which explain the milder initial fall of output and inflation in the high-debt economy. Subsequently the 'non-Ricardian' term turns negative, preventing consumption and inflation from adjusting to their steady state values as fast as in the economy without debt. This is explained by both the decline in interest income, as the short-term rate falls, and the fact that the present discounted value of expected taxes (blue solid line) exceeds the increase in the real value of debt (red dashed line).

Finally, the persistently low level of output and inflation imply a low level of the short-term rate, allowing aggregate demand to overshoot. At the same time, the negative wealth effects from net debt holdings ease and eventually become stimulative, restarting the cycle. Comparing the impulse response functions for the empirical and endowment economies makes clear that the important mechanism is imperfect knowledge.

Non-separable preferences. Non-separable preferences permit a more general treatment of wealth effects on consumption demand. The following proposition underscores that high substitution economies are economies in which wealth effects are necessarily small. The more prepared are agents to substitute leisure and consumption intertemporally, the smaller will be the wealth effects from holding the public debt. For example, in the case of separable preferences and an infinite Frisch elasticity of labor supply the wealth effects are zero. Similarly, following directly from proposition 1, risk neutral agents with infinite intertemporal elasticity of consumption substitution, will also imply smaller wealth effects from the public debt. The inclusion of non-separabilities serves to mute to a small degree the effects of endogenous labor supply on the scale of wealth effects attached to a given scale and composition of public debt.

Proposition 2 *For given $\bar{C}/\bar{Y}$, and debt-to-GDP ratio, δ, the scale of wealth effects are indexed by*

$$\bar{s}_C^{-1} = \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1} \frac{1 + (1-\sigma^{-1})\frac{\Theta}{\gamma+\Theta}}{1+\frac{\Theta}{\gamma+\Theta}}.$$

The following properties are immediate:

$$\lim_{\sigma\to\infty} \bar{s}_C^{-1} = \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1}$$

$$\lim_{\gamma\to\infty} \bar{s}_C^{-1} = \left(\frac{\bar{C}}{\bar{Y}}\right)^{-1}$$

and

$$\lim_{\gamma\to-\Theta} \bar{s}_C^{-1}|_{\sigma=1} = 0.$$

The first two properties show the scale of wealth effects are maximized in two limiting cases: when labor supply is fixed, corresponding to a constant-consumption elasticity of labor supply equal to zero; and when consumption elasticity of intertemporal substitution is equal to zero. Non-separable preferences, by increasing the marginal utility of consumption with hours worked, mute the negative income effects on labor supply. The third result further underscores the importance of intertemporal substitution of leisure. In the case of separable preferences over consumption and leisure, and a constant-consumption elasticity, $(\gamma+\Theta)^{-1}$, that is infinite, the wealth effects are zero, and the path of consumption is determined by intertemporal substitution of consumption and labor; consumption depends only on the paths of the real interest rate and the real wage.[32] [33] Conversely, wealth effects, and therefore the

[32] In the model with $\sigma = 1$ the Frish and constant-consumption elasticities are the same.

[33] Note that

$$\lim_{\gamma\to-\Theta} \frac{\psi_w}{\bar{s}_C(1,1)} = 1.$$

evolution of government debt holdings net of expected taxes, will be more important when agents have limited incentives to substitute intertemporally.

Distortionary taxation. Like non-separable preferences, inclusion of distortionary taxation, in addition to being a realistic feature of modern economies, serves to enhance the fiscal constraints on monetary policy identified in the endowment economy. By making a firm's marginal costs structure depend on the evolution of debt, prices themselves will directly depend on debt issuance. This renders the model more "self-referential" enhancing the wealth effects for a given scale and composition of the public debt.

Figure 12 plots stability regions in monetary policy and average-debt-maturity space. Three contours are shown indexed by increasing levels of average debt. The non-monotonicity identified in the endowment economy is again evident. In contrast to those earlier results, at longer average debt maturities, the Taylor principle is not restored for high levels of the steady state debt-to-GDP ratio. This arises because the scale parameter $\bar{s}_C^{-1}$ — which regulates the expenditures effects from shifting evaluations of the public debt — depends on the steady-state level of income taxation $\bar{\tau}^w$. As average indebtedness increases so does the steady-state income tax burden to support intertemporal solvency. This is not the case under lump-sum taxation.

While this model feature makes comparison across different levels of indebtedness somewhat opaque, it is obvious that the basic tenor of results remain unchanged, despite having a conventional value for the inverse elasticity of intertemporal substitution. Several other comments are worth making about this extension. First, inclusion of distortionary taxation gives an alternative rationale for departures from Ricardian equivalence and conditional expectations depending debt. Absent this departures form Ricardian equivalence arise solely because of imperfect knowledge about the policy regime. Second, an objection might be that it is well understood that distortionary taxation alters the determinacy conditions of this model under simple rules — see, for example, Benhabib and Eusepi (2005). It is straightforward to show that for maintained parameter values, satisfaction of the Taylor principle is sufficient of determinacy of rational expectations equilibrium. Third, alternative mechanisms which are equally realistic extensions of the model, will deliver similar reductions in the elasticity of substitution. For example, Benhabib and Eusepi (2005) show that essentially the same mechanism for indeterminacy arises with the inclusion of capital as in the case of distortionary

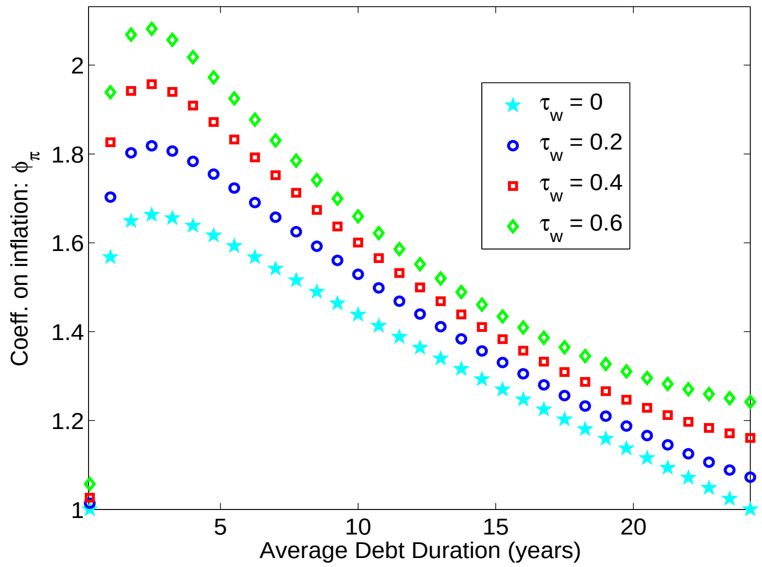

Figure 12: **E-stability frontiers with distortionary taxation**. The figure shows stability frontiers corresponding to different debt-response coefficients in the fiscal rule with distortionary taxation. To emphasize the role of distortionary taxation steady state labor tax rate is assumed to be 25 percent instead of 15 percent in the baseline calibration. Finally, the assumed debt-to-output ratio is 200 percent (in annual terms).

taxation. Fourth, the identified expenditure effects do not arise due to a fiscal limit operating through a Laffer curve.

Details of the policy rule. it is often argued that monetary policy ought to be specified in terms of a reaction function in which nominal interest rates respond to expectations of next-period inflation rather than realizations of current-period inflation. Indeed, there is a variety of empirical evidence supporting central bank reaction functions of this kind — see, for example, Clarida, Gali, and Gertler (1998) and Clarida, Gali, and Gertler (2000). The learning literature has also argued in favor of such rules — see Bullard and Mitra (2002) and Evans and Honkapohja (2003). To this end, consider a rule of the form

$$\hat{\imath}_t = \phi_\pi \hat{E}_t \hat{\pi}_{t+1}. \tag{58}$$

It is assumed that in implementing this interest-rate rule, the central bank responds to observed private-sector inflation expectations. An alternative, but equivalent assumption, is that the central bank has the same forecasting model of inflation as households and firms.

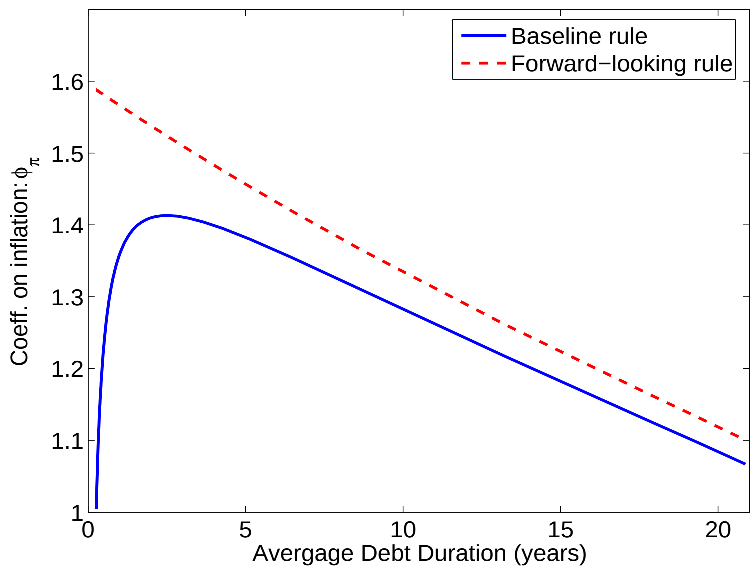

Figure 13: **E-stability frontiers with forward-looking policy rules**. The figure shows stability frontiers corresponding to the baseline policy rule (solid blue line) and the forward-looking policy rule (dashed red line) . The assumed debt-to-output ratio is 150 percent (in annual terms).

One additional assumption is required for interesting results: households understand that monetary policy is determined according to equation (58). Absent this assumption, rules of this kind engender considerable instability. This was first noted by Preston (2006) in a model almost identical to that proposed here, assuming an economy with no debt, no government purchases and no taxation.[34] Eusepi and Preston (2010, 2012) develop that analysis further and interpret the assumption of knowledge of the monetary policy rule as central bank communication. As details of the monetary policy strategy are known, households can make policy-consistent forecasts. Eusepi and Preston (2010) show that this assists stability as aggregate demand management through interest-rate policy is more effective. Agents knowing the rule ensures that projections of nominal interest rates satisfy the Taylor principle. This leads to the appropriate restraint of aggregate demand. Without such knowledge, demand management fails because households project very flat profiles for the real interest rate in response to various disturbances.

[34]That analysis also failed to account for the endogeneity of labor supply.

Figure 13 plots stability regions for the contemporaneous inflation and inflation expectation-based rules assuming a debt-to-output ratio of 150%. The other parameters are the same as in **figure 10**. A notable implication arising from expectations-based instrument rules is that the effect of increasing average maturity of debt is monotonic. As before, the intuition relies on the dynamics of real debt. Under the expectations-based rule the low of motion for government debt becomes

$$\hat{b}_t^m = \beta^{-1}\hat{b}_{t-1}^m - \beta^{-1}\hat{\pi}_t + (1-\rho)\,\phi_\pi \hat{E}_t\hat{\pi}_{t+1} +$$

$$+ (1-\rho)\,\rho\beta\hat{E}_t \sum_{T=t}^{\infty} (\rho\beta)^{T-t}\,\phi_\pi\hat{\pi}_{T+2} - \left(\beta^{-1}-1\right)\hat{s}_t.$$

Now even for small values of ρ real debt depends on expected inflation, which is the source of instability at short maturities. The special case of $\rho = 0$ is discussed in Eusepi and Preston (2012), where it is shown, consistently with **figure 13**, that the Taylor principle is not sufficient for stability. The region of instability is largest in the case of a debt portfolio comprised only of one-period instruments. As the maturity structure increases, monetary policy can be less aggressive from the perspective of expectations stabilization. This is consistent with the discussion in the previous section. To the extent that central banks will always in practice need to respond to a forecast of inflation, these results suggest that longer-maturity debt is more desirable on the ground of protecting against expectations-driven instability from learning dynamics.[35]

[35] A more realistic timing assumption might be to assume the nominal interest-rate policy is determined as a function of $\hat{E}_{t-1}\hat{\pi}_t$. The findings of Eusepi and Preston (2012) suggest similar results would be expected to obtain.

*Table I. Parameters of the VAR Stochastic Process[a]

Coefficient Matrix F on Lagged States				Coefficient Matrix Q, where $V = QQ'$			
Quarterly Data, 1984Q1-2007Q2							
.6659 (0.4137, 0.8201)	−0.0983 (−0.1345, −0.0925)	0.0332 (0.0222, 0.0465)	−4.6485 (−5.6262, −3.4063)	0.0195 (0.0178, 0.0219)	0	0	0
−0.4901 (−0.7378, −0.3096)	0.8262 (0.7495, 0.8447)	0 (−)	−1.8173 (−2.7657, −0.6181)	−0.0055 (−0.0148, 0.0073)	0.0417 (0.0314, 0.0514)	0	0
−0.2871 (−0.9856, 0.1540)	0 (−)	1.0764 (1.0519, 1.1062)	−9.1406 (−11.8105, −5.4230)	0.0122 (−0.0248, 0.0408)	0.027 (0.0021, 0.0025)	0.1121 (0.0691, 0.1539)	0
−0.050 (−0.0776, −0.0254)	−0.0175 (−0.0227, −0.0156)	0.0052 (0.0036, 0.0080)	0.2328 (0.0854, 0.3980)	0.0022 (−0.0019, 0.0560)	0.0007 (0.0003, 0.0011)	0.0008 (0.0004, 0.0012)	0.0011 (0.0009, 0.0013)

[a] The model is estimated using Maximum Likelihood. To ensure stationarity, we add to the likelihood function a penalty term proportional to $\max(|\lambda_{\max}| - 0.99)^2$, where $\lambda_{\max}$ is the maximum eigenvalue of F. Numbers in parentheses are 90% confidence intervals for a bootstrapped distribution with 1000 replications.To ensure that the variance-covariance matrix V is positive semidefinite, we estimate Q rather than $V = QQ'$.