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# Coordinating Monetary and Macroprudential Policies

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**Abstract**

The financial crisis has prompted macroeconomists to think of new policy instruments that could help ensure financial stability. Policymakers are interested in understanding how these should be set in conjunction with monetary policy. We contribute to this debate by analyzing how monetary and macroprudential policy should be conducted to reduce the costs of macroeconomic fluctuations. We do so in a model in which such costs are driven by nominal rigidities and credit constraints. We find that, if faced with cost-push shocks, policy authorities should cooperate and commit to a given course of action. In a world in which monetary and macroprudential tools are set independently and under discretion, our findings suggest that assigning conservative mandates (á la Rogoff [1985]) and having one of the authorities act as a leader can mitigate coordination problems. At the same time, choosing monetary and macroprudential tools that work in a similar fashion can increase such problems.

Key words: monetary policy, macroprudential policy, commitment, discretion, policy coordination, borrowing constraints

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*"Let's suppose that we have a house price bubble and the Financial Policy Committee increases capital requirements [...]. The Monetary Policy Committee then lowers the policy rate to push inflation back to target. Can we be confident that the lowering of the policy rate accompanying the widening in lending margins does not keep the house price boom going?"*

SUSHIL B. WADHWANI
Former member of the Bank of England's Monetary Policy Committee
*Remarks at the London School of Economics, July 14, 2010*

# 1 Introduction

Recent developments in financial markets and the world economy have motivated macroeconomists to think of new policy instruments that could help ensure financial stability. Central banks and financial regulators have begun their search for the appropriate macroprudential tool. But apart from finding the right instrument, policymakers are also interested in understanding how such instruments should be set in conjunction with monetary policy. Some commentators have suggested that separate committees for monetary and macroprudential policies could lead to coordination problems. For example, Wadhwani (2010) warned of the risk of "push-me, pull-you" behavior between policy committees as shown by the quotation in the epigraph. The Committee on International Economic and Policy Reform called for the Tinbergen separation principle to be retired[1]. They stressed that rather than having one instrument devoted entirely to one objective, the macro-stabilization exercise must be viewed as a joint optimization problem where monetary and regulatory policies are used in concert in pursuit of monetary and macroprudential objectives.

To contribute to this debate, we study how monetary and macroprudential policy should be coordinated so as to minimise the social costs of macroeconomic fluctuations. To do so, we incorporate an endogenous macroprudential tool into a version of the model of Carlstrom, Fuerst, and Paustian (2010) in which firms borrow in advance to finance the wage bill. This framework encompasses the usual New Keynesian model with nominal rigidities and endogenous monetary policy. But it also incorporates credit frictions (along the lines of Kiyotaki and Moore (1997)) that give rise to borrowing

[1]See Eichengreen et al (2011)

constraints and a credit wedge that depends on the tightness of these constraints. In this model there is a role for macroprudential policy to minimize the costs associated with variations in such spreads.

We derive a utility-based welfare criterion that characterizes the underlying distortions in this economy. This quadratic loss function comprises the typical terms in the variability of inflation and the output gap that are common in standard sticky price models. In addition, it also features the variation in credit market conditions. Crucially, the credit distortion is given by the sum of the post-tax short-term nominal interest rate and a credit spread – or the *effective* interest rate. Movement in the nominal interest rate creates inefficiencies because it affects the borrowing costs of firms when financing part of their wage bill – as in Ravenna and Walsh (2005). At the same time, borrowing of firms is restricted by their net worth. While collateral constraints rule out default in equilibrium, the Lagrange multiplier associated with such constraints can be interpreted as the shadow cost of borrowing over and above the risk-free rate – and it is thus isomorphic to a credit spread.

Macroprudential policy is modeled as a cyclical tax on the borrowing of firms, and monetary policy sets the short-run interest rate. Our key focus is on the interactions between the monetary and macroprudential instruments. Such interactions arise naturally in our framework. For instance, the impact of higher credit spreads on aggregate conditions and welfare could be directly offset by lowering the nominal interest. But a particular path of the nominal interest rate that helps offset movements in the credit spread is in general not consistent with stabilizing the price level. Hence, there is a need for an additional policy instrument targeted directly at the credit friction.

Our analysis is split into two parts, one in which policymakers can commit to a given policy plan and another in which they act under discretion. In each setting, we compare the case in which the monetary and macroprudential authority cooperate with a non-cooperative Nash game between the two authorities.

We start by asking whether introducing an additional macroprudential instrument improves welfare. Although in a non-cooperative setting, more tools need not improve social welfare[2], under our baseline assignment of

[2]Our non-cooperative setup does not assume that both authorities have an identical objective function such as social welfare. This is motivated by a concern for accountability. In practice, different government authorities are often responsible for a more narrow set of objectives so that they can be held accountable when missing them. In line with this, we

mandates, the additional macroprudential tool does improve outcomes – no matter what shock drives macroeconomic fluctuations. This is good news as it shows that the benefits from the extra policy instrument are not outweighed by any potential coordination problems between the two policy authorities.

Nevertheless, our analysis shows that coordination problems (measured by the difference in social welfare between cooperative and non-cooperative outcomes) can be significant under discretion following cost-push shocks. So, in this environment, we study which institutional arrangements could be beneficial.

First, we study the case in which one authority is a "leader" when setting its instruments. This can be seen as a reduced-form way of characterizing an environment in which the two authorities may move at slightly different frequencies (e.g., quarterly for the leader, monthly for the follower). Leadership is equivalent to within-period commitment and so it clearly makes the leader weakly better off. But our results suggest that leadership also improves *overall* social welfare – especially in the case where the macroprudential authority has the advantage of setting its instrument first.

Second, we evaluate the case of conservative institutional mandates. It is well known from Rogoff (1985), among others, that outcomes under discretion can be improved by making the central bank more inflation-averse than is indicated by the social welfare function. We revisit this issue and find that, in our non-cooperative game between the two institutions, reducing the weight on the output gap for each authority can substantially improve welfare.

Our final analysis relates to the choice of macroprudential instruments. We find that a state-contingent loan-to-value ratio acts in a similar fashion to a time-varying tax on borrowings – and both are effective tools. Our results suggest, however, that a subsidy (or tax) on households' deposits should be treated with caution as a macroprudential instrument. In a non-cooperative setting, this tool leads to large welfare losses or even nonexistence of a equilibrium. Intuitively, this is because both deposit taxes and interest rates affect economic conditions in a similar fashion. As a result, when policymakers act independently, there is a costly tug-of-war between authorities with different objectives and similar instruments.

assume that each authority cares only about a subset of the distortions contributing to the overall loss function. This creates the possibility of coordination problems, and additional tools may exacerbate these problems.

## Related literature

There is a growing literature incorporating macroprudential policy in monetary models (see references in Beau, Clerc, and Mojon (2012) and Galati and Moessner (2010)). To the extent that this literature performs a welfare analysis, it typically assumes a coordinated setup with one institution setting all available policy tools. For example, Angeloni and Faia (2009) study a sticky price model with a fragile banking sector. Their analysis includes welfare-maximizing rules for capital ratios and the short term interest rate. The authors find that capital ratios should be countercyclical and that monetary policy should respond to leverage and asset prices. Similarly, Darracq Paris, Soerensen, and Palenzuela (2010) analyze a model with a capital-constrained banking sector, but their focus is mainly positive. In the normative part of the paper, they consider a cooperative setting and determine optimal rules for monetary policy and capital ratios that maximize an ad-hoc loss function.

Closer to our analysis is the paper by Collard, Dellas, Diba, and Loisel (2012), who study the jointly optimal plans for monetary and macroprudential policies in a model with limited liabilities and deposit insurance. They outline the conditions under which prudential policies prevent inefficient risk taking, while monetary policy deals with the business cycle fluctuations. In that way, they provide what might be seen as a natural delegation of objectives between policy institutions.

While the recent literature has advanced in providing a rationale for the use of macroprudential tools, we aim at contributing to the debate by formalizing the interactions between such tools and monetary policy. In this respect, our approach is related to that of Dixit and Lambertini (2003), who study the interactions between monetary and fiscal policy under commitment and under discretion.

Our work is also related to that of Angelini, Neri, and Panetta (2012), who also study the connections between monetary and macroprudential policies. A key contribution of theirs is an elaborate model of the banking sector. Nevertheless, this comes at the cost of policy objectives that are not derived from the microfoundations of the model – and thus are not clearly related to the underlying economic distortions.

Regarding the methodology, we use a linear-quadratic framework for optimal policy along the lines of Woodford (2003). This necessarily involves a financial constraint that is always binding. There is, nonetheless, a growing literature on optimal prudential policy that considers occasionally binding constraints (e.g., Jeanne and Korinek (2010), Bianchi (2011) and Otrok, Benigno, Chen, Rebucci, and Young (2012)). This literature highlights the conditions under which it is beneficial from a welfare perspective to reduce borrowing in non-crisis times in order to reduce the likelihood and severity of a crisis (induced by a constraint that becomes binding). In this sense, such policies are truly prudential in nature. Our model abstracts from this distinction between "normal times" and "crisis times" and focuses on the business cycle implications of always-binding constraints. For the purpose of studying policy coordination, the linear-quadratic framework is more tractable than the nonlinear setup considered in these papers, because the linear-quadratic approach allows for a simple characterization of welfare in terms of stabilization objectives that is amenable to delegation between different authorities.[3]

Finally, our computational framework builds upon the algorithm of Dennis (2007) and Soederlind (1999) to compute the Nash (and leadership) equilibrium for the game between the two policy authorities under discretion.

# 2 Model

The core of the model is a New Keynesian sticky price framework without capital accumulation. In addition, our model features an agency problem that captures the relationship between leverage and credit spreads as in Carlstrom and Fuerst (1997). In particular, we follow Carlstrom, Fuerst, and Paustian (2010) and assume that firms can only borrow a certain fraction of their net worth. In our setup, firms always want to borrow up to the maximum amount possible given the borrowing constraint, which implies a rate of return that exceeds the time preference rate of entrepreneurs. We also allow the policy authority to set a regulatory tool that affects the overall amount of borrowing of entrepreneurs. Effectively, in our framework the policy authority sets a tax on the amount firms borrow, so as to control the economy's overall level of leverage. The economy is populated by two types of agents: households and entrepreneurs. We describe the problem of each agent in turn.

[3]Delegation is not straightforward if the welfare criterion is not broken down into the separate distortions, but exists only in direct form as household utility.

## 2.1 Households

The typical household consumes the final good ($c_t$) and sells two types of labor input ($L_t$ and $u_t$) to the entrepreneurs at factor prices $w_t$ and $r_t$. Preferences are given by

$$U(c_t, L_t, u_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - B_1 \frac{L_t^{1+\theta}}{1+\theta} - B_2 \frac{u_t^{1+\theta}}{1+\theta}.$$

As in Carlstrom, Fuerst, and Paustian (2010), one can think of the above specification as having two distinct types of labor: $L_t$ is the labor input subject to credit constraints and $u_t$ is a conventional unconstrained labor input. Alternatively, the second labor input can be interpreted as "capital utilization", which has utility costs for households.

Both households and entrepreneurs can purchase shares at nominal price $Q_t^n$, which are claims to the dividends $D_t^n$ paid by the sticky price firms. Ownership of these shares shifts endogenously between households and entrepreneurs. The aggregate amount of shares is normalized to one, and $e_t$ denotes the fraction of shares owned by entrepreneurs.

Households enter period $t$ with cash holdings $M_t$. They receive their wage bill for each input $P_t(w_t L_t + r_t u_t)$ as well as the returns on their shares $(Q_t^n(1-e_{t-1}) + D_t^n(1-e_{t-1}))$ paid as cash at the start of the period. They use this cash to make deposits $A_t$ at the financial intermediary and buy new shares. Their remaining cash balances (given by $M_t + w_t L_t + r_t u_t + (Q_t^n + D_t^n)(1-e_{t-1}) - B_t - Q_t^n(1-e_t))$ are available to purchase consumption goods subject to a cash-in-advance constraint. That is,

$$P_t c_t \leq M_t + P_t w_t L_t + P_t r_t u_t + (Q_t^n + D_t^n)(1-e_{t-1}) - A_t - Q_t^n(1-e_t).$$

At the end of the period, households receive interest on their deposits ($R_t A_t$). We allow these to be taxed at a rate $\tau_{d,t}$. As a result, cash carried over to the next period is given by:[4]

$$M_{t+1} = M_t + P_t w_t L_t + P_t r_t u_t + (Q_t^n + D_t^n)(1-e_{t-1}) - A_t - Q_t^n(1-e_t) - P_t c_t + R_t A_t.$$

[4] In principle, households would own financial intermediaries and thus receive their profits. But these intermediaries are assumed to be solely a veil in a competitive industry, so they make zero profits.

In equilibrium, with a positive interest rate, the cash-in-advance constraint will always bind. The household budget constraint is

$$P_t c_t - P_t w_t L_t - P_t r_t u_t - (Q_t^n + D_t^n)(1 - e_{t-1}) + A_t + Q_t^n(1 - e_t) = R_{t-1} A_{t-1}.$$

One can write this budget constraint in real terms, and assume that $\Omega_w$ and $\Omega_r$ are subsidies to factor payments financed by the lump-sum tax , $T$, such that

$$c_t + Q_t(1 - e_t) + A_t = \Omega_w w_t L_t + \Omega_r r_t u_t + (Q_t + D_t)(1 - e_{t-1}) + \frac{R_{t-1}}{\pi_t} A_{t-1} + T_t.$$

Motivation for these subsidies is provided in a later section when we discuss the welfare-based loss function faced by the policymakers.

The first-order conditions for labor supply, stock purchases, and bond holdings are

$$B_1 \frac{L_t^\theta}{c_t^{-\sigma}} = \Omega_w w_t \tag{1}$$

$$B_2 \frac{u_t^\theta}{c_t^{-\sigma}} = \Omega_r r_t \tag{2}$$

$$c_t^{-\sigma} = \beta E_t \left( c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right) \tag{3}$$

$$c_t^{-\sigma} = \beta E_t \left( c_{t+1}^{-\sigma} \frac{Q_{t+1} + D_{t+1}}{Q_t} \right). \tag{4}$$

## 2.2 Entrepreneurs

There is a continuum of long-lived entrepreneurs with linear consumption preferences. Entrepreneurs hire labor from households and use these inputs in a constant-returns-to-scale production function to produce an intermediate good. But to pay the wage bill associated with the input $L_t$, entrepreneurs have to borrow from intermediaries at the rate $R_t$. So profits are given by

$$profits_t = p_t x_t - \tau_{b,t} R_t w_t L_t - r_t u_t,$$

where $p_t$ denotes the relative price of the intermediate good and $x_t = L_t^\alpha u_t^{1-\alpha}$ denotes its production. Here, $\tau_{b,t}$ is the macroprudential instrument that taxes or subsidizes the borrowing cost of the firm. The parameter $\alpha$ governs the importance of agency costs in the model. If $\alpha = 0$ the model collapses to the simple one-sector sticky price model without agency costs. When $\alpha = 1$, there is an agency cost but it causes no distortions in terms of the allocation of resources between factors of production.

Entrepreneurs face a constraint on their hiring of the labor input $L_t$. The constraint is a reduced-form approach to introducing a friction in borrowing for the wage bill $R_t w_t L_t$. We assume that this wage bill, taxed at a rate $\tau_{b,t}$, can be no larger than some function $g$ that captures the borrowing friction and is increasing in net worth and profits. We parameterize $g$ as a Cobb-Douglas function with parameter $b$ such that

$$\tau_{b,t} R_t w_t L_t \leq (nw_t)^b (p_t x_t - r_t u_t)^{1-b}. \tag{5}$$

Here, $nw_t \equiv e_{t-1}(Q_t + D_t) + n_t$ denotes the entrepreneur's after-tax net worth that is carried over from last period. The credit friction is motivated by a hold-up problem as in Hart and Moore (1994)). The Cobb-Douglas constraint over collateral and operating profit generalizes the Kiyotaki-Moore borrowing constraint such that the *linearized* model is isomorphic to a costly-state verification framework. We also include an exogenous shock to firms' net worth $n_t$.[5]

Let $\phi_t$ denote the Lagrange multiplier on the borrowing constraint. The first-order conditions for the choice of input factors are

$$\alpha p_t x_t = \tau_{b,t} R_t (1 + b\phi_t) w_t L_t \tag{6}$$
$$(1-\alpha) p_t x_t = r_t u_t. \tag{7}$$

Substituting these two FOC into the borrowing constraint, we obtain

$$\left(\frac{\alpha p_t x_t}{nw_t}\right)^b = (1 + b\phi_t). \tag{8}$$

Equation (6) shows that the Lagrange multiplier affects the choice of the constrained labor input like an interest rate on a loan financing the wage

[5]This net worth shock has no direct structural interpretation, but it has a long tradition in the analysis of business cycle models with credit constraints. It stands for a lump-sum redistribution of wealth between households and entrepreneurs. Such a redistribution would have no effect on aggregate outcomes in a model with frictionless financial markets, but it has an aggregate consequence in our model.

bill. In addition, from the equation above it is clear that the multiplier is an increasing function of leverage, defined as the ratio of firm's project value $p_t x_t$ to firm's own funds $nw_t$ Both of these characteristics are shared with the costly-state verification (CSV) framework. In that sense, our setup - once log-linearized - is isomorphic to CSV models such as those of Bernanke, Gertler, and Gilchrist (2000) or Carlstrom and Fuerst (1997). In particular, one can use this isomorphism to calibrate the parameter $b$.[6]

The budget constraint of the entrepreneur is given by

$$c_t^e + e_t Q_t = e_{t-1}(Q_t + D_t) + profit_t + n_t.$$

In equilibrium entrepreneurs postpone consumption because the return to internal funds exceeds their discount factor.[7] To limit self-financing in the long run, we assume that entrepreneurs die unexpectedly with probability $1-\gamma$. In case of death, their funds are transferred lump sum to households. This transfer assumption simplifies the welfare analysis substantially, because it is no longer needed to track welfare for both households and entrepreneurs. A similar assumption is effectively made in Gertler and Kiyotaki (2010), who assume that bankers and households belong to one big family, whose utility then serves as a welfare metric. In equilibrium, the budget constraint reduces to

$$e_t Q_t = \gamma \left[ e_{t-1}(Q_t + D_t) + profit_t + n_t \right],$$

which can be expressed equivalently as

$$e_t Q_t = \gamma \alpha p_t x_t \left[ \frac{b\phi}{1+b\phi_t} + \left( \frac{1}{1+b\phi_t} \right)^{1/b} \right]. \tag{9}$$

## 2.3 Final Good Production and Market Clearing

Monopolistically competitive firms indexed by $j$ produce final goods $y_{t,j}$ that are aggregated to an output bundle according to a CES aggregator

$$y_t = \left[ \int_0^1 y_{t,j}^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right]^{\frac{\epsilon_t}{\epsilon_t - 1}},$$

[6]This calibration can also be used to make sure the constraint is always binding in equilibrium.

[7]The fact that entrepreneurs never consume implies that they do not need to be included in the social planner's objective function.

where $\epsilon_t > 1$ is assumed to be a time-varying markup parameter. The final goods firms purchase the intermediate good from entrepreneurs at relative price $p_t$. The production function is given by $y_{j,t} = a_t x_{t,j}$, where productivity $a_t$ follows an exogenous AR(1) process in logs. Since the production function is linear, real marginal cost is given by

$$z_t = p_t / a_t.$$

There are Rotemberg (1982) quadratic costs of price adjustment, which enter the profit function of firm $j$ as $\frac{\phi}{2}[(p_{t,j} - p_{t-1,j})/p_{t-1,j}]^2 y_t$, with $\phi > 0$. These costs disappear from the linearized version of the social resource constraint if the steady-state inflation rate is zero. In a symmetric equilibrium, the Rotemberg price-setting problem gives rise to a Phillips curve relationship:

$$0 = (1 - \varepsilon_t) + \varepsilon_t z_t - \varphi\left(\pi_t - 1\right)\pi_t - \beta\frac{\lambda_{t+1}}{\lambda_t} E_t\left[\varphi\left(\pi_{t+1} - 1\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right]. \quad (10)$$

We calibrate the adjustment cost parameter $\varphi$ such that the slope of the log-linearized Phillips curve is identical to the one arising from Calvo pricing with an average duration of four quarters. Monopolistic competition implies that firms earn profits in equilibrium. These profits are paid out as dividends to shareholders of the sticky price firms. These dividends are given by

$$d_t = y_t(1 - z_t) - \frac{\varphi}{2}\left(\pi_t - 1\right)^2 y_t. \quad (11)$$

In a symmetric equilibrium where $y_{jt} = y_t \,\forall j$, aggregate output is given by

$$y_t = a_t L^\alpha u_t^{1-\alpha}.$$

The market-clearing condition for the final good is:

$$y_t = c_t + \frac{\varphi}{2}\left(\pi_t - 1\right)^2 y_t.$$

## 2.4 Policy

The model is closed by a description of policy behavior. In subsequent sections, we will consider different assumptions for how policymakers set their monetary and macroprudential instruments, $R_t$ and $\tau_{b,t}$. The main focus of our analysis will be on the case of optimal policy. But in doing so, we will analyze the case in which the monetary authority and the macroprudential authority work in a cooperative as well as a non-cooperative fashion. We will also consider the case in which policymakers can commit to a policy plan and the case in which they act under discretion.

## 2.5 Summary of Equilibrium Dynamics

In the appendix we present the full system of equilibrium conditions written in log-deviations from steady state. Such system can be reduced to the following equations:

$$\widehat{R}_t = \sigma E_t \Delta \widehat{y}^g_{t+1} + \sigma \frac{1+\theta}{\sigma+\theta} E_t \Delta \widehat{a}_{t+1} + E_t \widehat{\pi}_{t+1} \tag{12}$$

$$\widehat{\pi}_t = \lambda[(\sigma+\theta)\widehat{y}^g_t + \alpha(\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t) + \epsilon^{\pi}_t] + \beta E_t \widehat{\pi}_{t+1} \tag{13}$$

$$\begin{aligned} \beta \widehat{e}_t = &\widehat{e}_{t-1} - (1-\beta)\varepsilon(\sigma+\theta)\widehat{y}^g_t - \alpha(1-\beta)\varepsilon(\widehat{R}_t + \widehat{\tau}_{b,t}) \\ &+ [1 - \beta\Lambda' - (1-\beta)\varepsilon\alpha b)\widehat{\phi}_t + \widehat{n}_t \end{aligned} \tag{14}$$

$$\begin{aligned} (1-\Lambda')\widehat{\phi}_t = &(\theta+1)E_t \Delta \widehat{y}^g_{t+1} - (1-\alpha b)E_t \Delta \widehat{\phi}_{t+1} - E_t n_{t+1} \\ &+ \frac{(\theta+1)}{(\sigma+\theta)}[\alpha E_t(\Delta \widehat{R}_{t+1} + \Delta \widehat{\tau}_{b,t+1}) - (\sigma-1)E_t \Delta \widehat{a}_{t+1}] \end{aligned} \tag{15}$$

where $\widehat{y}^g_t = \widehat{y}_t - \widehat{y}^e_t$ and $\widehat{y}^e_t = \frac{1+\theta}{\sigma+\theta}\widehat{a}_t$.

Equation (12) in the system above represents the economy's Euler, or IS, equation. The Phillips curve equation (13) illustrates how both the cost channel and the credit channel introduce a form of endogenous markup into the model. That is, cost-push fluctuations arise from movements in the exogenous shock $\epsilon^{\pi}_t$ as well as fluctuations in the effective interest rate – given by $\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t$. Equation (13) also shows how the Lagrange multiplier on the credit constraint $\widehat{\phi}_t$ can be thought of as a credit spread affecting entrepreneurs' borrowing costs. Equation (14) is a backward-looking condition determining the evolution of firms' net worth. Finally, Equation (15) is the forward-looking condition (derived from the optimality condition for share purchases) for the evolution of the Lagrange multiplier or "credit spreads" $\widehat{\phi}_t$.

For the purpose of illustrating the main channels through which the credit distortions affect aggregate fluctuations, we consider the case in which policymakers follow simple rules. In particular, we assume that the nominal interest rate is adjusted according to a simple Taylor rule and the borrowing tax is kept fixed at its steady-state value:

$$\widehat{R}_t = \alpha_\pi \widehat{\pi}_t + \alpha_y \widehat{y}_t^g + \alpha_r \widehat{R}_{t-1} \tag{16}$$

$$\widehat{\tau}_{b,t} = \overline{\tau}_b. \tag{17}$$

The equilibrium for $\widehat{R}_t$, $\widehat{\pi}_t$, $\widehat{y}_t^g$, $\widehat{e}_t$, $\widehat{\phi}_t$, and $\widehat{\tau}_{b,t}$ is determined by equations (12)–(17) plus an exogenous stochastic process for the vector of forcing variables $\{\widehat{n}_t, \widehat{a}_t, \widehat{\epsilon}_t^m\}$. We assume that each of these variables follows a stationary AR(1) process with independent innovations.

The calibration of the model follows closely the one considered in Carlstrom, Fuerst, and Paustian (2010). The discount factor $\beta$ is set at 0.99. The elasticity of substitution between the differentiated goods $\epsilon$ is set to 10 (implying a monopolistic markup of around 11%). The Rotemberg price adjustment cost $\varphi$ is set to 173.08 (implying that prices stay fixed for an average of five quarters in an equivalent Calvo setting). The share of constrained labor $\alpha-$ or the share of intermediate good that are collateral constrained - is 0.5. Following Woodford (2003), we further assume an elasticity of labor supply consistent with $\theta = 0.47$. We consider different values for the elasticity of intertemporal substitution (EIS – or $\sigma^{-1}$). In our benchmark case we assume that risk aversion is 2 (so the elasticity of intertemporal substitution is $\sigma^{-1} = 0.5$). But, following Rotemberg and Woodford (1999), we also consider the case of $\sigma^{-1} = 6$. We assume that the total factor productivity follows an AR(1) process for autoregressive coefficient 0.95, whereas net worth and markup processes are assumed to be slightly less persistent with AR coefficients 0.9.

Figure 1 presents the evolution of inflation, output gap, nominal interest rates and credit spreads following an increase in productivity. The charts shows that, under our benchmark calibration, a positive productivity shock increases the value of the Lagrange multiplier – as higher credit demand leads to tighter borrowing conditions. The opposite holds when agents have a high elasticity of intertemporal substitution. In this case, the shock generates a large fall in interest rates, which in turn leads to a large increase in net worth

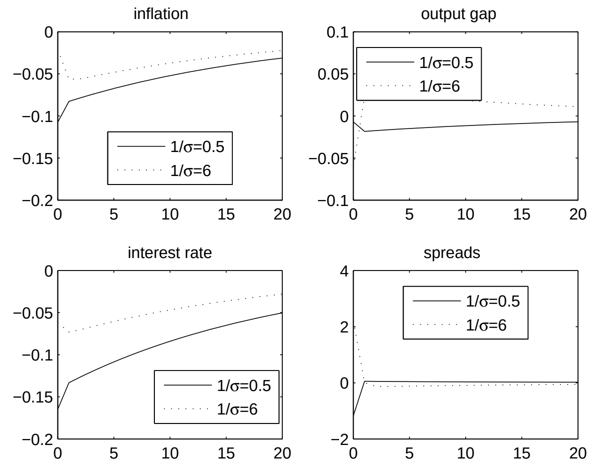


Figure 1: Impulse-response functions after a positive productivity shock.

– and, thus, credit supply.

As shown in Figure 2, a positive shock to net worth leads to looser credit conditions while the shock is inflationary. Markup shocks, on the other hand, lead credit to ease and inflation to fall (see Figure 3).

# 3 Economic Efficiency and Welfare

In our model, the presence of nominal rigidities, credit and cash constraints, and exogenous cost-push shocks generates inefficient economic dynamics. In this section we illustrate these inefficiencies in two ways. First, we represent the distortions in our model in terms of "wedges" between the efficient equilibrium and the competitive allocation in the nonlinear representation of our model. Second, we highlight the welfare costs associated with these distortions by deriving a quadratic micro-founded loss function.

## 3.1 Inefficient Wedges

Three equilibrium conditions are important to illustrate the wedge between the efficient allocation and the competitive equilibrium in our model. In the efficient equilibrium, real marginal cost is constant, while in our model

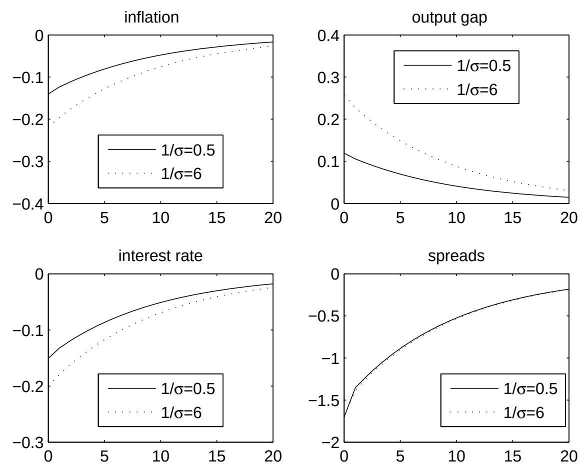


Figure 2: Impulse-response functions after a positive net worth shock.

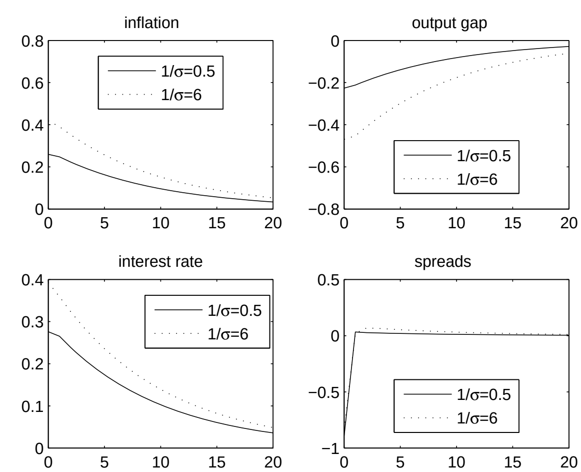


Figure 3: Impulse-response functions after a positive markup shock.

Equation (10) shows that movements in inflation and markup shocks create fluctuations in marginal costs. Second, the presence of cash in advance and credit constraint implies that "effective interest rates" affect the demand for the constrained labor $L_t$, this is represented by the term $\tau_{b,t}R_t(1+b\phi_t)$ in equation (6. Finally, movements in this rate will also affect the allocation of resources between the two labour inputs (seen by contrasting equations (6) and (7)).

That is, while the efficient allocation implies that the marginal rate of substitution equals the marginal rate of transformation for both inputs, i.e.,

$$MRS_{L,t} \equiv B_1 \frac{L_t^{\theta}}{c_t^{-\sigma}} = \alpha a_t L_t^{\alpha-1} u_t^{1-\alpha} \equiv MRT_{L,t}$$

$$MRS_{u,t} \equiv B_2 \frac{u_t^{\theta}}{c_t^{-\sigma}} = (1-\alpha) a_t L_t^{\alpha} u_t^{-\alpha} \equiv MRT_{u,t},$$

under the competitive equilibrium

$$\begin{aligned} MRS_{L,t} &= \frac{\Omega_w z_t}{\tau_{b,t} R_t (1+b\phi_t)} MRT_{L,t} \\ MRS_{u,t} &= \Omega_r z_t MRT_{u,t}, \end{aligned}$$

where

$$z_t = \varepsilon_t^{-1} \left\{ \varphi\left(\pi_t - 1\right)\pi_t + \beta \frac{\lambda_{t+1}}{\lambda_t} E_t \left[ \varphi\left(\pi_{t+1} - 1\right)\pi_{t+1} \frac{y_{t+1}}{y_t} \right] - (1-\varepsilon_t) \right\}.$$

It is possible to set $\Omega_w$ and $\Omega_r$ to ensure that the steady state is efficient. Nevertheless, our model is not dynamically efficient. The inefficiencies are driven by movements in the effective interest rates $\tau_{b,t}R_t(1+b\phi_t)$ and movements in marginal costs $z_t$, which in turn are driven by fluctuations in inflation $\pi_t$ and markup shocks $\varepsilon_t$.

## 3.2 Welfare Losses

If we assume, as suggested above, that the time-invariant subsidies $\Omega_w$ and $\Omega_r$ are set such that the welfare-relevant variables are efficient in the steady state,[8] we can derive the following quadratic welfare criterion:

$$\mathcal{L} = \frac{1}{2}\left[\frac{(\varepsilon-1)}{\lambda}\widehat{\pi}_t^2 + (\sigma+\theta)\left(\widehat{y}_t^g\right)^2 + \frac{\alpha(1-\alpha)}{1+\theta}(\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2\right]. \tag{18}$$

Such a quadratic welfare criterion is useful because it can be evaluated accurately up to second order by considering linearized equilibrium conditions.[9] Also, a quadratic loss function is a useful way of illustrating how economic distortions generate welfare losses in this economy. Apart from relative price misallocations, there are two other channels that lead to deviations from market efficiency: the so-called cost channel and the credit channel. Entrepreneurs in our model have to borrow in advance to pay the wage bill of labor input $L_t$. In fact, if we set $\alpha = 1$ and remove the borrowing constraint, we replicate the model and the loss function in Ravenna and Walsh (2005). But, in general, the amount entrepreneurs have to borrow is constrained by their net worth and profits. If entrepreneurs did not have to borrow in advance (but only within each period), our model and loss function would collapse to the one in Carlstrom, Fuerst, and Paustian (2010).

One could also write the social loss function (18), using equation ( 13), as

$$\mathcal{L} = \frac{1}{2}\left[\begin{array}{c} \frac{(\varepsilon-1)}{\lambda}\widehat{\pi}_t^2 + (\sigma+\theta)[\frac{1}{\lambda}(\widehat{\pi}_t - \beta E_t\widehat{\pi}_{t+1}) - \epsilon_t^{\pi} - \alpha(\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t)]^2 \\ +\frac{\alpha(1-\alpha)}{1+\theta}(\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2 \end{array}\right] \tag{19}$$

Consistent with the inefficient wedges shown in the previous section, Equation (19) illustrates how welfare losses come from movements in the credit spread, inflation, changes in policy instruments and markup shocks. It is

[8]We assume that these subsidies are only available to correct average distortions but cannot be used as state-contingent stabilization tools. This assumption is based on practical considerations, namely that such fiscal policy instruments are currently not likely to be available for stabilizing the distortions created by credit frictions and inflation. However, if such state-contingent tools were available to the fiscal authority, results similar to those obtained in Correia, Nicolini, and Teles (2008) would go through: namely, that certain distortions do not affect the set of implementable allocations if the right state-contingent instruments exist. A similar point has recently been made in the context of the zero lower bound by Correia, Farhi, Nicolini, and Teles (2011).

[9]See Woodford (2003) for details.

important to note that both the cost and the credit channel generate inefficiencies in the same margin: namely, they both affect firms' demand for labor $L_t$.

So, the first term in equation (18) or (19) is a result of relative price distortions and the second is a result of inefficient cost-push fluctuations. But when $\alpha \neq 1$ the model also features a "sectoral misallocation", which is represented by the last term in Equations (18) or (19). Such misallocation comes from the fact that only one factor of production is constrained and thus resources are inefficiently allocated between the two input factors.

# 4 Optimal Policy

Our focus is on the gains from coordinating macroprudential and monetary policies. To study these possible welfare gains, we compare cooperative and non-cooperative equilibria. Under cooperation, one institution sets both the monetary policy and the macroprudential tool in order to maximize the social welfare function given in (18). When studying non-cooperative equilibria, we need to specify separate objectives for each institution.

Such a delegation of mandates is necessarily somewhat subjective. There is no model-implied objective function for each institution as there is in the open economy monetary policy coordination literature with domestic and foreign welfare. Assigning to both institutions the common objective of social welfare maximization is likely to eliminate coordination issues. When the cooperative problem has a unique solution, the optimality conditions of such a problem are identical to the non-cooperative ones (see Blake and Kirsanova (2011), footnote 14). Clearly, if the optimal cooperative solution is unique and minimizes the loss function of the independent authorities, such authorities have no incentive to deviate from the cooperative solution. But, in practice, separate institutions are often made responsible for achieving narrower goals on the grounds of accountability.

One very natural way to assign objectives to each institution is to assume the monetary authority cares about the social welfare function except for the credit spread term, because the latter is taken care of by the macroprudential authority. Similarly, the macroprudential authority cares about social welfare except for the inflation term, which it assumes is taken care of by the monetary authority. Both care about the output gap term because it is affected by nominal rigidities and credit frictions. We will consider this as our benchmark delegation of objectives (labeled Non-coop I in the tables to follow). To summarize, our baseline delegation of objectives is given by

$$\mathcal{L}^{cb} = \frac{(\varepsilon - 1)}{\lambda}\widehat{\pi}_t^2 + (\sigma + \theta)\left(\widehat{y}_t^g\right)^2 \tag{20}$$

$$\mathcal{L}^{mp} = (\sigma + \theta)\left(\widehat{y}_t^g\right)^2 + \frac{\alpha(1-\alpha)}{1+\theta}(\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2. \tag{21}$$

This delegation has the desirable property that the sum of the mandates for both authorities covers all the economic distortions present in the social welfare function. Furthermore, each authority attaches the correct relative welfare weights to the two terms in its respective objective function. Therefore, the two authorities jointly cover all distortions and trade them off in the correct way. Nevertheless, there are non-internalized spillover effects between authorities that are at the heart of our analysis.

While this baseline delegation has clear appeal, there is a second natural way to delegate the mandates. It is plausible to think that policy authorities are assigned mandates to eliminate particular inefficiencies in the economy and ignore others. For example, the central bank could be assigned the mandate to eliminate inefficiencies generated by nominal rigidities and cost-push fluctuations, assuming that the macroprudential authority will deal with inefficiencies arising from credit frictions - and viceversa. So our alternative delegation is based on the following simple principle. We assume that the monetary authority has the mandate to stabilize those distortions that would arise in a world with a frictionless credit market, i.e., absent the borrowing constraint in (5). The macroprudential authority stabilizes those additional distortions that stem from the borrowing constraint. One could also think of this delegation as one in which the central bank maintains its traditional mandate (and model of the output gap) while, upon realization that credit frictions create inefficiencies (arguably driven by financial developments mentioned in the introduction), an additional authority is created with the sole mandate of minimizing fluctuations in the spread between borrowing rates and the risk-free interest rate.

If there were no borrowing constraints, the social loss function for our economy would be given by

$$\mathcal{L}^{cb} = \frac{1}{2}\left[\frac{(\varepsilon - 1)}{\lambda}\widehat{\pi}_t^2 + (\sigma + \theta)(\widehat{y}_t^{cb})^2 + \frac{\alpha(1-\alpha)}{1+\theta}\widehat{R}_t^{\,2}\right], \tag{22}$$

where $\widehat{y}_t^{cb}$ is the output gap that would prevail if there were no credit constraint. The term $\hat{R}_t^2$ appears in the loss function because the hiring of one input factor must be financed by borrowing in advance, but crucially without any collateral constraint. This is the so-called working capital channel as in in Walsh (2005). From the Phillips curve, movements in this output gap are given by changes in inflation, interest rates, and markup shocks:

$$\widehat{y}_t^{cb} = \frac{1}{\lambda}(\widehat{\pi}_t - \beta E_t \widehat{\pi}_{t+1}) - \epsilon_t^{\pi} - \alpha \widehat{R}_t. \tag{23}$$

Now consider a world with perfectly flexible prices and where borrowing would be intra-period. But labor will still be constrained by their net worth. The social welfare function would then be driven by changes in the credit spread and borrowing taxes:

$$\mathcal{L}^{mp} = \frac{1}{2}\left[(\widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2\right]. \tag{24}$$

This delegation of mandates is a plausible alternative. However, it does not have the property that adding up both objective functions covers all economic distortions. This is because terms involving the covariances of various frictions are ignored by both institutions. For instance, the social welfare function penalizes $(\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2$. But jointly, the two authorities care only about $\hat{R}^2$ and $(\widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2$, thus ignoring the covariance of the working capital channel distortion with the collateral constraint distortion. A similar argument can be made about the output gap term. One may therefore suspect that this second delegation scheme performs worse than the first.

Our analysis considers both discretion and commitment. Under discretion, we are looking for Markov feedback equilibria where decision rules are linear functions of current-period state variables, sometimes known as closed-loop feedback equilibria. The private sector is assumed to move after the monetary and macroprudential authorities, which in turn are moving simultaneously. For sensitivity, we also consider a within-period leadership structure, where one authority moves before the other within the period. Regarding the methodology, we use straightforward extensions of the algorithms in Dennis (2007) and Soederlind (1999) to compute the Nash equilibrium for the game between the two policy authorities under discretion.

When computing Nash equilibria under commitment, we follow common practice and look for open-loop equilibria (see, for example, Coenen, Lombardo, Smets, and Straub (2008)). Each authority chooses its plan at the beginning of time, taking as given the equilibrium *behavior* of the other authority.

## 4.1 Optimal Policy under Commitment

We now consider the case in which the monetary and the macroprudential authorities set policy under commitment – i.e., the authorities have the ability to deliver on past promises no matter what the current situation is today. When doing so, we compare the case in which the monetary and macroprudential authorities cooperate – or, equivalently, one policymaker sets both policy instruments – with the case of a non-cooperative Nash game between the two authorities.

### 4.1.1 Commitment: The Cooperative Setting

First we assume that the monetary and the macroprudential authorities follow the optimal policy in a cooperative manner. In this case, the optimal policy is derived by minimizing the loss function (18) subject to the constraints (12)-(15).

**Proposition 1** *When the two policy authorities cooperate, the first best can be achieved following productivity and net worth shocks.*
***Proof.*** *Absent markup shocks, the solution that implies* $\widehat{\pi}_t = 0$, $\widehat{y}_t = \frac{1+\theta}{\sigma+\theta}\widehat{a}_t$, $\widehat{R}_t = -\sigma\frac{1+\theta}{\sigma+\theta}(1-\rho)\widehat{a}_t$, *and* $\widehat{\tau}_{b,t} = -b\widehat{\phi}_t - \sigma\frac{1+\theta}{\sigma+\theta}(\rho-1)\widehat{a}_t$ *is consistent with the equilibrium conditions 12-15 and fully eliminates economic distortions – i.e., from the loss function* (18)*, we see that* $\mathcal{L} = 0$. ∎

Following a net worth shock, the policy prescription is trivial: Taxes should be set to offset credit spreads. As a result, not only interest rates but also the effective rate faced by entrepreneurs are kept constant (as $\widehat{R}_t = 0$ and $\widehat{\tau}_{b,t} = -b\widehat{\phi}_t$).

In the case of productivity shocks, interest rates move so as to avoid movements in inflation: e.g., higher productivity always warrants a policy cut (i.e. $\widehat{R}_t = -\sigma\frac{1+\theta}{\sigma+\theta}(1-\rho)\widehat{a}_t$). But the optimal response of the tax depends crucially on the value of the elasticity of intertemporal substitution, or $\sigma^{-1}$. In fact, one can write the reduced-form solution for the system under optimal policy in the case of productivity and net worth shocks as

$$\widehat{\phi}_t = \Phi_e \widehat{e}_{t-1} + \Phi_a \widehat{a}_t + \Phi_n \widehat{n}_t \tag{25}$$

$$\widehat{\tau}_t = -b\Phi_n(\widehat{e}_{t-1} + \widehat{n}_t) - \left( b\Phi_a - \frac{\sigma(1-\rho)(\theta+1)}{\sigma+\theta} \right) \widehat{a}_t, \tag{26}$$

where $\Phi_e$, $\Phi_a$, and $\Phi_n$ are defined in the appendix. Note that $\Phi_a > 0$ if $\sigma^{-1} > 1$. That is, as in the case of simple rules shown in Section 3, a productivity shock leads to higher credit spreads when the elasticity of intertemporal substitution is high. In this case, it is optimal to offset the suboptimal level of borrowing with lower taxes. On the other hand, when this elasticity is low, the optimal tax increases after the shock in order to offset the fall in spreads. Whether a productivity shock increases credit supply by more or less than credit demand is crucial in determining the correlation between the macroprudential and the monetary policy tool.

**Proposition 2** *In the case of markup shocks, optimal policy ensures efficiency if* $\alpha = 1$.
***Proof.*** *When there are only markup shocks and* $\alpha = 1$, *the solution that implies* $\widehat{\pi}_t = 0$, $\widehat{y}_t = 0$, $\widehat{R}_t = 0$,[10] *and* $\widehat{\tau}_{b,t} = -b\widehat{\phi}_t - \epsilon_t^{\pi}$ *is consistent with the equilibrium conditions* (12)-(15) *and fully eliminates economic distortions – i.e., from the loss function* (18), *we see that* $\mathcal{L} = 0$. ∎

Markup shocks affect entrepreneurs' costs in a fashion similar to the credit spreads. When there are no sectoral distortions ($\alpha = 1$), the optimal borrowing tax moves to generate a spread that fully offsets the markup shock. This way there are no cost-push fluctuations and interest rates can be kept constant.

When there are different factors of production (i.e., $\alpha \neq 1$), the strategy of moving taxes to offset cost-push fluctuations would generate misallocations of resources between such factors. The misallocation happens as the effective interest rate paid to hire one type of labor moves with the shock.

As shown in Figure 4, when $\alpha \neq 1$, the markup shock increases inflation and lowers the output gap. The optimal path of interest rates actually falls

[10] The equilibrium behavior of the instruments in Propositions 1 and 2 is not a statement about the rules that could implement the optimal allocation. It is well known that an interest rate rule reacting only to exogenous variables results in indeterminacy. We do not seek implementation, but instead characterize the movement of the instruments under optimal policy.

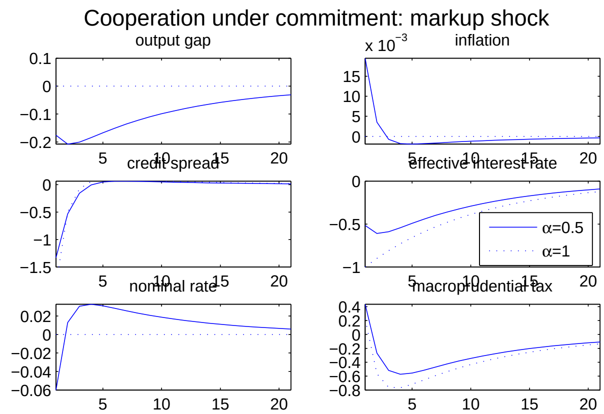


Figure 4: Response to markup shock under cooperation and commitment.

on impact to mitigate the effect of the shock on the output gap. But there is a credible commitment to higher interest rates in the future that helps lower inflation today. Borrowing taxes fall so as to reduce entrepreneurs' costs and counterbalance the higher markup. Note that, to contain the sectoral misallocation, the movement in effective interest rates is smaller when $\alpha \neq 1$.

#### 4.1.2 Commitment: The Non-cooperative Setting

We now consider the case in which the monetary and the macroprudential authorities set policy in a non-cooperative setting. In particular, under our benchmark delegation, the monetary authority sets interest rates in order to minimize its loss function – given by Equation (20) – and the macroprudential authority sets taxes to minimize its loss function (21). We also consider delegation of mandates in which the central bank's loss function is given by (22) and the macroprudential authority minimizes (24).

**Proposition 3** *Under our benchmark delegation of mandates, the first best can be achieved following productivity and net worth shocks.*
***Proof.*** *Absent markup shocks, the solution that implies* $\widehat{\pi}_t = 0$, $\widehat{y}_t = \frac{1+\theta}{\sigma+\theta}\widehat{a}_t$, $\widehat{R}_t = -\sigma\frac{1+\theta}{\sigma+\theta}(1-\rho)\widehat{a}_t$, *and* $\widehat{\tau}_{b,t} = -b\widehat{\phi}_t - \sigma\frac{1+\theta}{\sigma+\theta}(\rho-1)\widehat{a}_t$ *is consistent with the equilibrium conditions* (12)-(15)*; fully eliminates economic distortions – i.e., from the loss function 18 we see that* $\mathcal{L} = 0.$*; and also implies zero losses for*

*the different authorities – i.e., from the loss functions* (20) *and* (21) *we see that* $\mathcal{L}^{cb} = 0$ *and* $mathcal L^{mp} = 0$. ∎

**Proposition 4** *Under our alternative delegation of mandates, there are no gains from cooperation following net worth shocks. Following productivity shocks, the gains from cooperation also disappear as* $\rho \to 1$.

***Proof.*** *Given the macroprudential authority's loss function* (24)*, the non-cooperative solution implies* $\widehat{\tau}_{b,t} = -b\widehat{\phi_t}$. *This solution is identical to the one that would prevail in the cooperative setting (see Equation* (26)*) following net worth shocks and following productivity shocks when* $\rho \to 1$. ∎

Table 1 shows the welfare losses under the non-cooperative versus the cooperative setting when policymakers can commit. We also compare those with the case in which there is no macroprudential instrument. The results confirm that there are no gains from cooperation in the presence of net worth shocks as the macroprudential instrument can fully offset the shock, leaving no job for the monetary policy authority.

The table also confirms that the gains from cooperation following productivity shocks are zero for our baseline delegation (Non-coop I). In this case, the macroprudential instrument eliminates any cost-push fluctuations coming from movements in the effective interest rate while the central bank ensures price stability. Under our alternative delegation (Non-coop II) there are positive gains from cooperation following productivity shocks, but these are small. Because shocks are very persistent in our benchmark calibration, the model does not produce significant interest rate movements. So eliminating fluctuations in the credit spread (as dictated by the loss function (21)) mitigates most of the welfare losses coming from movements in effective borrowing rates.

Table 1: Welfare losses under commitment

| Shocks | No Macropru | Cooperative | Non-coop I | Non-coop II |
|---|---|---|---|---|
| Productivity | $0.03\sigma^2$ | 0 | 0 | $0.005\sigma^2$ |
| Net worth | $0.62\sigma^2$ | 0 | 0 | 0 |
| Markup | $0.93\sigma^2$ | $0.59\sigma^2$ | $0.80\sigma^2$ | $1.45\sigma^2$ |

For the case of markup shocks, the welfare losses in the non-cooperative setting given the baseline delegation (Non-coop I) are in between the losses arising under cooperation and from a setting without any macroprudential

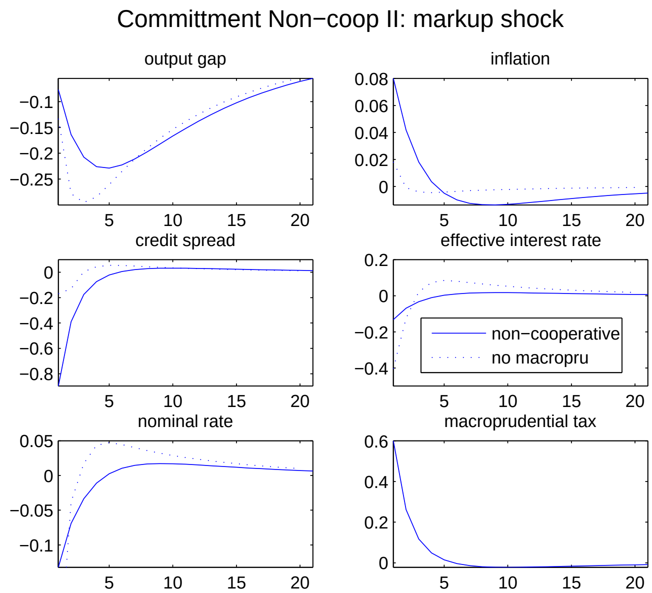


Figure 5: Impulse response to markup shock under commitment: No tool versus Non-coop II.

instrument. But this result is by no means general. Under the alternative delegation of objectives (Non-coop II), the non-cooperative setting delivers a worse outcome than the case in which there is no macroprudential instrument. As shown in Figure 5, when there is no macroprudential instrument, higher markups reduce demand for loans and lead to a fall in the credit spread. Lower spreads and lower effective interest rates then counterbalance the increase in costs generated by the shock, and the overall cost-push fluctuation is smaller (see Equation (13)). On the other hand, when the macroprudential authority acts by offsetting the fall in spread with higher taxes, effective interest rates are higher and markups have stronger effects on entrepreneurs' marginal costs. That is, the overall cost-push effect on inflation is higher in the non-cooperative setting than when there is an active macroprudential policy.

## 4.2 Optimal Policy under Discretion

We now consider the case in which the monetary and the macroprudential authorities set policy under discretion. In the discretionary setting, current policymakers perceive future policymakers to set their instrument according to an exogenously given Markov feedback rule. We then calculate the best response feedback rule of current policymakers and iterate until convergence reaches a fixed point. The two policymakers are assumed to move simultaneously and before the private sector (see Appendix D for details about the computational algorithm). It is well known that, in models with endogenous state variables, such discretionary equilibria need not be unique. To explore this, we initialized our algorithm from different starting points and found only a single equilibrium.

### 4.2.1 Discretion: The Cooperative Setting

**Proposition 5** *As in the case of commitment, when the two policy authorities cooperate, the first best can be achieved following productivity and net worth shocks.*

***Proof.*** *Since the equilibrium in which* $\widehat{\pi}_t = 0$, $\widehat{y}_t = \frac{1+\theta}{\sigma+\theta}\widehat{a}_t$, $\widehat{R}_t = -\sigma\frac{1+\theta}{\sigma+\theta}(1-\rho)\widehat{a}_t$, *and* $\widehat{\tau}_{b,t} = -b\widehat{\phi}_t - \sigma\frac{1+\theta}{\sigma+\theta}(\rho-1)\widehat{a}_t$ *is feasible and eliminates welfare losses in every period, this is also the solution for the discretionary equilibrium.* ■

Following markup shocks, also similar to the case of commitment, the discretionary equilibrium cannot mimic the first best, even when the policy authorities cooperate. Moreover, the discretionary equilibrium further suffers from the typical "inflationary bias" problem arising from policymaker's inability to deliver on past promises. This bias is present as long as there are cost-push fluctuations generating a trade-off between inflation and output. That is, if the public had anchored inflation expectations (i.e., they expect inflation to be at a given target), it would be beneficial for the monetary authority to set a policy that delivers higher inflation but larger output following a markup shock. Knowing this, agents will not expect inflation to be at a given target. And without anchored expectations, a policy of higher inflation will no longer deliver the beneficial increase in output. The inability of the monetary authority to commit to a policy that does not exploit the inflation and output trade-off leads to bigger welfare losses.

Table 2: Welfare losses under discretion

| shocks | No Macropru | Cooperative | Non-coop I | Non-coop II |
|---|---|---|---|---|
| Productivity | $0.06\sigma^2$ | 0 | 0 | $0.11\sigma^2$ |
| Net worth | $2.31\sigma^2$ | 0 | 0 | 0 |
| Markup | $3.46\sigma^2$ | $0.88\sigma^2$ | $2.11\sigma^2$ | $30.03\sigma^2$ |

#### 4.2.2 Discretion: The Non-cooperative Setting

We now consider the case in which the monetary and the macroprudential authorities set policy in a non-cooperative setting. We use the same delegation of loss functions as the one considered under commitment. As shown in Table 2, under our baseline delegation (Non-coop I), there are no gains from cooperation following net worth shocks and productivity shocks because the first best can be achieved even in the non-cooperative setting. As was the case under commitment, the welfare loss in the non-cooperative setting conditional on markup shocks is somewhere in between the cooperative outcome and the outcome in the absence of a macroprudential tool. The table also clearly shows that the gains from cooperation conditional on markup shocks are much larger in the discretionary case than in the commitment case.

We again find that our baseline delegation (Non-coop I) performs better than the alternative delegation scheme (Non-coop II). This can be traced back to two effects. First, the baseline delegation has some overlapping objectives between the authorities in that both care about the output gap. Such overlap can reduce coordination problems. Second, because the full effect of fluctuations in the effective interest rate does not enter the macroprudential loss function, the alternative delegation of mandates implies a non-cooperative outcome with large cost-push fluctuations.[11] Because (as discussed above) such fluctuations are particularly costly under discretion, introducing a macroprudential tool with this delegation of mandates (Non-coop II) reduces welfare – even following productivity shocks.

The results are even stronger in the case of markup shocks. Figure 6 shows the optimal path of inflation, output gap, and interest rates under our

[11] Effectively, as discussed earlier, the alternative delegation ignores the covariances between the various distortions in the mandate of either authority. Ignoring how the various distortions co-move is irrelevant when it is feasible to set each distortion to zero at all times, such as with net worth shocks.

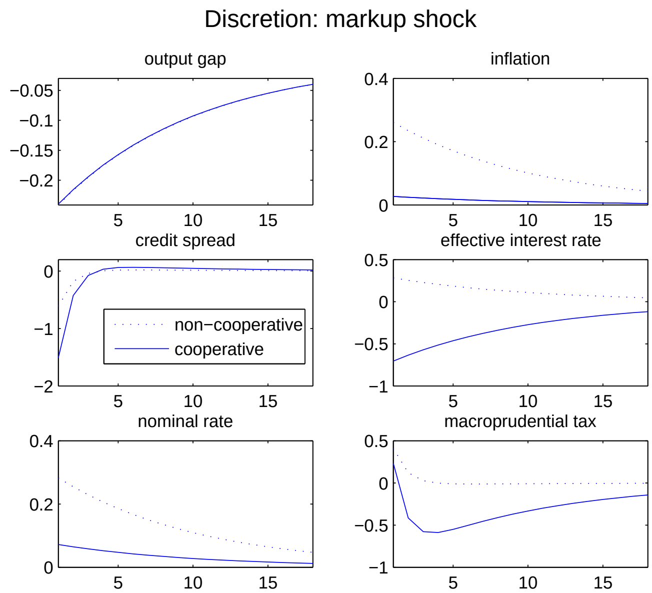


Figure 6: Impulse response to markup shock under discretion: Coop versus Non-coop II.

alternative delegation. Macroprudential policy moves taxes on borrowing to offset movements in credit spreads. This leads to an increase in effective borrowing rates – which increase inflation relative to the cooperative equilibrium. In the Nash equilibrium, the original shock is accompanied by a further increase in firms' borrowing costs. This worsens the trade-off between the output gap and inflation. Hence, when mandates are chosen inefficiently as in our alternative delegation (Non-coop II), there can be substantial coordination problems in a discretionary setup.

## 4.3 Leadership Setting

Would the coordination problem be mitigated if one authority could lead the decision-making process? Such within-period timing is natural in the context of macroprudential policy that is often seen to move at a lower frequency. While such different frequencies of decision making are difficult to model fully in our framework, one crude way to capture this setting is to let the macroprudential instrument be chosen first within the period, i.e. the macroprudential authority leads the monetary authority. We also investigate leadership by the monetary authority. Leadership can be thought of as within-period commitment by one player, which clearly makes the leader better off. However, it is in general not the case that a leadership setup improves social welfare vis-a-vis a simultaneous move of both players. The solution algorithm for the leadership game is described in Appendix D.

We consider the leadership setting only for the baseline delegation of mandates. In that game, welfare losses arise only for markup shocks and hence the leadership question is relevant only for this type of shock. The results are given in the following table.

Table 3: Welfare losses under leadership with discretion

| Shocks | Nash | Monetary Leads | Macropru Leads |
|---|---|---|---|
| Markup | $2.11\sigma^2$ | $1.60\sigma^2$ | $1.44\sigma^2$ |

Interestingly, either leadership structure improves welfare relative to the Nash game between the authorities. The smallest welfare loss occurs when the macroprudential authority moves first within the period. By inspection of the Euler equation, one can see that output is fully pinned down by the central bank's decision over the path of interest rates and expected inflation. But future inflation and future interest rates in our discretionary equilibrium are independent of current macroprudential policy. Because the macroprudential authority has no influence over the output gap, it uses its instrument exclusively to stabilize the credit market distortion. Such a one-sided use of the tool is in general not fully optimal whenever there are trade-offs.

Leadership by the central bank also improves welfare relative to the Nash equilibrium. In the latter, the central bank is shy when moving interest rates because it knows such movements have cost-push effects (given by the "cost channel"). When the central bank leads, it takes into account that the macroprudential authority will offset any movements in the effective interest rates. As a result, the central bank is more aggressive against inflation – and this reduces the discretionary bias.

## 4.4 Conservative Mandates

Would our results differ if mandates for policymakers were conservative in the style of Rogoff (1985)? It is well known that assigning the monetary authority

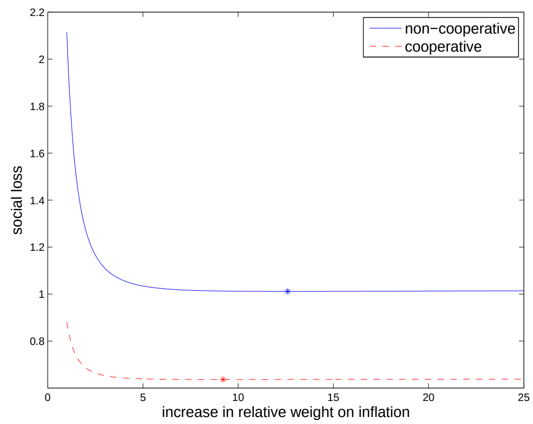


Figure 7: Social loss as a function of the degree of conservatism

a larger concern for inflation stabilization than what is indicated by the true social welfare function can improve welfare if the central bank acts under discretion. This case for a conservative central banker was first highlighted by Rogoff. The intuition is that this mitigates the discretionary bias that arises from the inability of the central bank to refrain from exploiting predetermined inflation expectations in order to reduce output volatility. But when the concern for inflation stabilization is large relative to the concern for output volatility, there is little desire by the discretionary central bank to reduce output gap volatility in the first place. In this sense, narrow mandates can partially substitute for the inability to commit.

What is not known from the existing literature is whether the case for a conservative central banker is stronger or weaker in Nash games between two authorities that act under discretion compared to the cooperative outcome under discretion. To study this question, we work with our baseline delegation of mandates, but increase the relative concern for non-output-gap variations by a scalar $\kappa$. The mandates in the Nash game now become

$$\mathcal{L}^{cb} = \kappa \frac{(\varepsilon - 1)}{\lambda} \widehat{\pi}_t^2 + (\sigma + \theta) \left(\widehat{y}_t^g\right)^2 \tag{27}$$

$$\mathcal{L}^{mp} = (\sigma + \theta) \left(\widehat{y}_t^g\right)^2 + \kappa \frac{\alpha(1-\alpha)}{1+\theta} (\widehat{R}_t + \widehat{\tau}_{b,t} + b\widehat{\phi}_t)^2. \tag{28}$$

Figure 7 plots the social loss conditional on markup shocks as a function of $\kappa$, which one may interpret as the degree of conservatism of the authorities. Clearly, the reduction in the welfare loss achievable by making the authorities more conservative is greater in the non-cooperative setup than in the cooperative setup. In line with this result, the degree of conservatism that minimizes social loss (indicated by the asterisk on each plot) is greater when the authorities do not cooperate than when they do. The reason for this is as follows. Increases in $\kappa$ have the usual benefit of reducing the inflationary bias under discretion. This is common to cooperation and non-cooperation. But in the non-cooperative game, increases in $\kappa$ have a second benefit in that they reduce coordination problems between authorities. When $\kappa$ tends to infinity, each authority has only one stabilization objective and one tool. In our model, this implies that each goal variable is perfectly stabilized no matter whether the authorities cooperate or not. Such a reduction in coordination problems comes at the cost of placing too small a weight on the output gap, but the output gap does not have a large weight in the social loss of New Keynesian models. Consequently, the optimal degree of conservatism is larger in the non-cooperative setting.

## 4.5 Alternative Macroprudential Tools

So far the macroprudential policy was modeled as a cyclical tax on borrowing of firms, and monetary policy sets the short-run interest rate. In this section, we consider two alternative tools. First is the case in which the macroprudential authority sets a loan-to-value ratio. In particular, we consider the case in which the collateral constraint of entrepreneurs is given by

$$R_t w_t L_t \leq (\delta_t n w_t)^b (p_t x_t - r_t u_t)^{1-b}.$$

In this case, $\delta_t$ is a regulatory tool that affects entrepreneurs' overall amount of borrowing. In a more elaborate model with an explicit banking sector, regulatory constraints on bank capital are likely to have similar affects as the leverage constraints that we impose on firms directly.[12] Such a tool, where $\delta_t > 1$, may also be interpreted as a form of credit guarantee scheme that allows firms to borrow more than what private-market conditions dictate. Alternatively, $\delta_t < 1$ implies that regulators are imposing restrictions on the amount of borrowing a firm can do.

[12]The macroprudential policy could also be implemented as a tax/subsidy in firms' profit.

Our results suggest that, although such an instrument does not directly affect firm's profits and does not enter the loss function (see Appendix B) – as in the case of $\tau_{b,t}$ – it would actually act in a similar fashion to the borrowing tax. In fact, regardless of whether policymakers cooperate and whether they can commit, the welfare performance is pretty much identical to the case of $\tau_{b,t}$ reported in Tables 1 and 2.

As an alternative to instruments that directly affect credit conditions, we also consider the case in which the macroprudential authority affects the incentive of households to save by subsidizing or taxing deposit rates. In particular, households are taxed at a rate $\tau_{d,t}$ so the budget constraint would be given by

$$c_t + Q_t e_t^h + A_t = \Omega_w w_t L_t + \Omega_r r_t u_t + (Q_t + D_t) e_{t-1}^h + \frac{\tau_{d,t-1} R_{t-1}}{\pi_t} A_{t-1} + T_t.$$

Such policy instruments would act in a similar fashion as the interest rate, and it would also affect agents' intertemporal decision:

$$c_t^{-\sigma} = \beta E_t \left( c_{t+1}^{-\sigma} \frac{R_t \tau_{d,t}}{\pi_{t+1}} \right), \tag{29}$$

But different from the interest rate, deposit taxes do not enter in the loss function, as they do not affect agents' borrowing constraints (see also Appendix B).

Our findings show that using such policy instruments would also lead to identical results when policymakers cooperate: The first best can be achieved following net worth and productivity shocks, and markup shocks lead to welfare losses similar to those reported in Tables 1 and 2.

But the results are quite different when policymakers act independently in a non-cooperative setup. Here, we find that this can cause extremely large welfare losses in response to productivity and net worth shocks or even absent a Nash equilibrium for markup shocks. We do not report these losses here, since volatilities are so large that the linear perturbation solution to our model can no longer be taken as a valid approximation. Intuitively, this is because both deposit taxes and interest rates are demand-side instruments. That is, both monetary and macroprudential tools affect the same margin. As a result, when policymakers act independently, there may be a tug-of-war between authorities with opposite objectives and similar instruments.

It is well known that the choice of instruments can matter tremendously in a non-cooperative game. In our model, large welfare losses or non-existence of equilibrium occurs when the central bank controls the nominal interest rate and the macroprudential authority controls the deposit tax/subsidy. In our framework, this is the wrong assignment. Firms' hiring decisions are distorted directly by the nominal interest rate and credit constraints, and aggregate demand depends on the deposit tax. Hence, a more natural assignment of objectives is to use the deposit tax as an aggregate demand management tool that stabilizes inflation and to use the nominal interest rate as a tool for correcting the firm's hiring decision in order to offset the distortion caused by credit constraints. This suggests a switching of tools (or equivalently objectives): The monetary authority controls the deposit tax and the macroprudential authority the nominal interest rates. For the case of our baseline delegation of objectives and this assignment of tools, the non-cooperative game always has an equilibrium and welfare losses are small as reported in the table below.

Table 4: Welfare losses under alternative assignment of instruments

| Shocks | Commitment | Discretion |
|---|---|---|
| Productivity | 0 | 0 |
| Net worth | 0 | 0 |
| Markup | $1.22\sigma^2$ | $0.91\sigma^2$ |

It may seem odd that the social loss under commitment is larger than the one under discretion. When all instruments are chosen jointly by one institution, commitment always results in smaller welfare losses than discretion. But note that the table above refers to a non-cooperative setting, where such results need not hold.

# 5 Conclusion

This paper tries to shed light on the potential implications of introducing a macroprudential tool. We focus on coordination issues that might arise when monetary policy sets an instrument that also affects such conditions but has a different objective. Our main conclusions are the following: 1) The introduction of an additional instrument targeted directly at credit market distortions can substantially improve welfare and coordination with monetary policy following cost-push shocks; 2) if the monetary and macroprudential authorities do not cooperate and act under discretion, assigning a conservative policy mandate for the different institutions would be valuable; 3) a leadership structure where the macroprudential authority moves first would also be beneficial; and 4) choosing a macroprudential tool that is too similar to that of the monetary authority can lead to costly coordination problems among policy-making institutions.

Our analysis considers the interaction between policy instruments in a classic business cycle framework. In practice, it may be the case that the macroprudential policy instrument operates at a lower frequency and addresses mainly medium-term imbalances that build up slowly over time rather than shorter-term fluctuations that monetary policy typically responds to. Modeling the interactions in such a setup is substantially more difficult. Similarly, one can conceive of macroprudential policy as guarding against disaster risk that occurs very infrequently. These are important questions that could be addressed in future research. Moreover, in order to keep the policy problem tractable, our framework does not feature an explicit banking sector. Introducing such financial intermediation explicitly could be another fruitful avenue for further research as it would help capture practical issues in the design of macroprudential instruments.

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# Appendix

# A Log-linear Model and Solution

We now present a system of equilibrium conditions written in log-deviations from steady state. We want to abstract from the average distortion being introduced by the credit friction and focus on the cyclical implications. We therefore assume that wage subsidies are in place such that the steady state is efficient ($MRS = MRT$) despite a borrowing constraint that is binding in the steady state.[13]

[13]We can think of our macroprudential instrument $\delta_t$ as a time-varying tax on firms' profits. If this tax rate is positive in the steady state, the wage subsidy needed to bring about an efficient steady state is affected.

$$
\begin{aligned}
\widehat{w}_t &= \sigma\widehat{y}_t + \theta\widehat{L}_t && \text{(L.1)}\\
\widehat{r}_t &= \sigma\widehat{y}_t + \theta\widehat{u}_t && \text{(L.2)}\\
\sigma(E_t\widehat{y}_{t+1} - \widehat{y}_t) &= \widehat{R}_t + \widehat{\tau}_{d,t} - E_t\widehat{\pi}_{t+1} && \text{(L.3)}\\
\sigma(E_t\widehat{y}_{t+1} - \widehat{y}_t) &= \beta E_t\widehat{q}_{t+1} - \widehat{q}_t + (1-\beta)E_t\widehat{d}_{t+1} && \text{(L.4)}\\
\widehat{\tau}_{b,t} + \widehat{R}_t + \widehat{w}_t + \widehat{L}_t &= b(\widehat{\delta}_t + \widehat{e}_{t-1} + \beta\widehat{q}_t + (1-\beta)\widehat{d}_t + \widehat{n}_t) && \text{(L.5)}\\
&\quad + (1-b)(\widehat{y}_t + \widehat{z}_t) && \\
\widehat{y}_t + \widehat{z}_t &= \widehat{\tau}_{b,t} + \widehat{R}_t + \widehat{w}_t + \widehat{L}_t + b\widehat{\phi}_t && \text{(L.6)}\\
\widehat{y}_t + \widehat{z}_t &= \widehat{r}_t + \widehat{u}_t && \text{(L.7)}\\
\widehat{e}_t + \widehat{q}_t &= \widehat{y}_t + \widehat{z}_t - \Lambda'\widehat{\phi}_t - \varpi\widehat{\delta}_t && \text{(L.8)}\\
\widehat{z}_t &= \widehat{p}_t - \widehat{a}_t && \text{(L.9)}\\
\widehat{\pi}_t &= \lambda\widehat{z}_t + \beta E_t\widehat{\pi}_{t+1} + \lambda\epsilon_t^{\pi} && \text{(L.10)}\\
\widehat{d}_t &= \widehat{y}_t - (\varepsilon - 1)\widehat{z}_t && \text{(L.11)}\\
\widehat{y}_t &= \widehat{a}_t + (1-\alpha)\widehat{u}_t + \alpha\widehat{L}_t && \text{(L.12)}\\
\widehat{\phi}_t &\equiv (\phi_t - \phi)/(1 + b\phi), && \text{(L.13)}
\end{aligned}
$$

where we define the following parameters

$$
\begin{aligned}
\Lambda' &= -\frac{F_\phi}{F}(1 + b\phi)\\
F &= \left[\frac{b\phi}{1+b\phi} + \delta^{-1}\left(\frac{1}{1+b\phi}\right)^{1/b}\right]\\
F_\phi &= \left[\frac{b}{1+b\phi} - \frac{b^2\phi}{(1+b\phi)^2} - \delta^{-1}\left(\frac{1}{1+b\phi}\right)^{\frac{1}{b}+1}\right]\\
\varpi &= -\frac{F_\delta}{F}\delta\\
F_\delta &= -\left[\delta^{-2}\left(\frac{1}{1+b\phi}\right)^{1/b}\right]
\end{aligned}
$$

Again, the system is closed with the specification of monetary and macroprudential policy.

# B Loss Function

A quadratic approximation to the welfare function around a steady state with zero inflation is given by

$$\frac{U(t) - U^*(t)}{U_c(ss)\, c_{ss}} \approx -\frac{1}{2}\left[\frac{(\varepsilon-1)}{\lambda}\widehat{\pi}_t^2 + (\sigma+\theta)(\hat{y}_t^g)^2 + \frac{\alpha(1-\alpha)}{1+\theta}(\hat{\tau}_{b,t} + \hat{R}_t + b\hat{\phi}_t)^2\right], \tag{14}$$

where $U^*(t)$ denotes welfare in a first best economy without agency or price adjustment costs.

**Proof.** The derivation follows closely that in Carlstrom, Fuerst, and Paustian (2010). We take a quadratic approximation to the utility function

$$\begin{aligned} U(t) - U(ss) \approx\, & U_c\tilde{c}_t + U_L\tilde{L}_t + U_u\hat{u}_t + \frac{1}{2}\left(U_{cc}\tilde{c}_t^2 + U_{LL}\tilde{L}_t^2 + U_{uu}\tilde{u}_t^2\right) \\ & + U_{cL}\tilde{c}_t\tilde{L}_t + U_{cu}\tilde{c}_t\tilde{u}_t + U_{Lu}\tilde{c}_t\tilde{L}_t. \end{aligned} \tag{15}$$

The resource constraint is given by

$$c_t = A_t L_t^{\alpha} u_t^{1-\alpha}\left[1 - \frac{\varphi(\pi_t-1)^2}{2}\right] \equiv A_t f(L_t, u_t)\left[1 - \frac{\varphi(\pi_t-1)^2}{2}\right]. \tag{16}$$

A quadratic approximation to this expression around the zero inflation steady state gives

$$\begin{aligned} \tilde{c}_t \approx\, & c_{ss}\tilde{A}_t + f_L(ss)\,\tilde{L}_t + f_u(ss)\,\tilde{u}_t \\ & + \frac{1}{2}\left(f_{LL}(ss)\,\tilde{L}_t^2 + f_{uu}(ss)\,\tilde{u}_t^2 - c_{ss}\varphi\widetilde{\pi}_t^2\right) \\ & + f_{LA}(ss)\,\tilde{L}_t\tilde{A}_t + f_{uA}(ss)\,\tilde{u}_t\tilde{A}_t + f_{Lu}(ss)\,\tilde{L}_t\tilde{u}_t. \end{aligned} \tag{17}$$

Since we have assumed separable utility, and that the steady-state is efficient, we have

$$\begin{aligned} U(t) \approx\, & U(ss) + \frac{1}{2}\left(U_{cc}\tilde{c}_t^2 + U_{LL}\tilde{L}_t^2 + U_{uu}\tilde{u}_t^2\right) \\ & + \frac{1}{2}U_c\left(f_{LL}(ss)\,\check{L}_t^2 + f_{uu}(ss)\,\tilde{u}_t^2 - c_{ss}\varphi\widetilde{\pi}_t^2 + f_{LA}(ss)\,\tilde{L}_t\tilde{A}_t\right. \\ & \left. + f_{uA}(ss)\,\tilde{u}_t\tilde{A}_t + f_{Lu}(ss)\,\tilde{L}_t\tilde{u}_t\right) + tip. \end{aligned} \tag{18}$$

Here, *tip* denotes terms independent of policy. The equilibrium choice of $u$ is given by

$$U_u(t) = U_c(t) f_u(t) P_t. \tag{19}$$

Log-linearizing this expression and imposing efficiency in the steady state, we have

$$\hat{u}_t = \frac{\alpha\,(1-\sigma)}{\alpha\,(1-\sigma)+(\sigma+\theta)}\hat{L}_t + \frac{1}{\alpha\,(1-\sigma)+(\sigma+\theta)}\hat{z}_t + \frac{(1-\sigma)}{\alpha\,(1-\sigma)+(\sigma+\theta)}\hat{a}_t. \tag{20}$$

We will use this expression to eliminate $u$ from the welfare function (18).

We now define the labor gap to be the gap between actual labor and efficient labor. Substituting this into (18), and simplifying and subtracting the same approximations in an efficient economy with flexible prices and no credit frictions (which is independent of policy), we have

$$\frac{U\,(t)-U^*\,(t)}{U_c\,(ss)\,c_{ss}} \approx -\frac{1}{2}\left[\varphi\widehat{\pi}_t^2 + \frac{\alpha\,(1+\theta)\,(\sigma+\theta)}{\theta+\alpha+\sigma(1-\alpha)}\left(\hat{L}_t^g\right)^2 + \frac{1-\alpha}{\theta+\alpha+\sigma(1-\alpha)}\hat{z}_t^2\right]. \tag{21}$$

Next, we will eliminate marginal cost and the labor gap from the loss function. We use the following expression for labor choice:

$$U_u(t)\,f_L(t) = U_L(t) f_U(t) \tau_{b,t} R_t (1+b\phi_t).$$

Expanding this up to the first order about an efficient steady state, we have:

$$b\hat{\phi}_t + \hat{R}_t + \hat{\tau}_{b,t} = (\theta+1)\hat{u}_t - (\theta+1)\hat{L}_t.$$

Using this expression, one can write the labor and output gaps as

$$\hat{Y}_t^g = \frac{1}{\sigma+\theta}\hat{Z}_t - \frac{\alpha}{\sigma+\theta}\left(\hat{\tau}_{b,t} + \hat{R}_t + b\hat{\phi}_t\right) \tag{22}$$

$$L_t^g = \frac{1}{\sigma+\theta}\hat{Z}_t - \frac{\alpha(1+\theta)+(1-\alpha)(\sigma+\theta)}{(\sigma+\theta)(1+\theta)}\left(\hat{\tau}_{b,t} + \hat{R}_t + b\hat{\phi}_t\right). \tag{23}$$

Substituting these into the loss function, we arrive at

$$\frac{U\,(t)-U^*\,(t)}{U_c\,(ss)\,c_{ss}} \approx -\frac{1}{2}\left[\frac{(\varepsilon-1)}{\lambda}\widehat{\pi}_t^2 + (\sigma+\theta)(\hat{y}_t^g)^2 + \frac{\alpha(1-\alpha)}{1+\theta}(\hat{R}_t + \hat{\tau}_{b,t} + b\hat{\phi}_t)^2\right]. \tag{24}$$

Finally, one can use $\lambda = (\epsilon - 1)/\varphi$ to rewrite the weight on inflation in terms of the slope of the Phillips curve. ■

# C The Reduced Form Solution: a Special Case

We now show the reduced form solution for the optimal cooperative policy under commitment when the economy is subject to net worth and productivity shocks. In this case one can achieve the first best, so imposing $\widehat{y}_t = \frac{1+\theta}{\sigma+\theta}\widehat{a}_t$ (or $\widehat{y}^g_{t+1} = 0$) and $\pi_t = 0$ in the system above would imply the following dynamics for the spread:

$$E_t\widehat{\phi}_{t+1} = C_\phi\widehat{\phi}_t + C_a\widehat{a}_t + C_n\widehat{n}_t, \tag{25}$$

where

$$\begin{aligned} C_\phi &= \frac{\alpha b(1-\sigma) + \Lambda'(\sigma+\theta)}{\alpha b(1-\sigma) + (\sigma+\theta)} \\ C_a &= \frac{(\sigma-1)(1+\theta)(1-\rho)}{\alpha b(1-\sigma) + (\sigma+\theta)} \\ C_n &= \frac{\rho_n(\sigma+\theta)}{\alpha b(1-\sigma) + (\sigma+\theta)}. \end{aligned}$$

Moreover, using the method of undetermined coefficient, one can derive the reduced-form solution for spreads as follows:

$$\widehat{\phi}_t = \Phi_e\widehat{e}_{t-1} + \Phi_a\widehat{a}_t + \Phi_n\widehat{n}_t,$$

where $\Phi_e = \frac{\alpha b\beta(1-\sigma)(1-\Lambda')}{(1-\beta\Lambda')(\sigma(1-\alpha b)+\theta+\alpha b)} - 1$ and $\Phi_a = \frac{\beta(1-\rho)(1-\sigma)(\theta+1)}{(1-\beta\rho)(\sigma(1-\alpha b)+\theta+\alpha b)}$ and $\Phi_n = -\frac{(1-\beta\Lambda')(\sigma+\theta)(1-\beta\rho)+\alpha b(1-\beta)(1-\sigma)}{(1-\beta\Lambda')(1-\beta\rho)(\sigma(1-\alpha b)+\theta+\alpha b)}$. So $\Phi_a > 0$ if $\sigma^{-1} > 1$, and this would also imply a negative value of $\Phi_n$.

The reduced-form solution for taxes is

$$\widehat{\tau}_t = T_e\widehat{e}_{t-1} + T_a\widehat{a}_t + T_n\widehat{n}_t$$

where $T_e = -b\Phi_n$ and $T_a = -b\Phi_a + \frac{\sigma(1-\rho)(\theta+1)}{\sigma+\theta}$ and $T_n = -b\Phi_n$. Or

$$\widehat{\tau}_{b,t} = -b\Phi_n\widehat{e}_{t-1} - \left(b\Phi_a - \frac{\sigma(1-\rho)(\theta+1)}{\sigma+\theta}\right)\widehat{a}_t - b\Phi_n\widehat{n}_t.$$

# D Algorithm for Computing Nash Equilibrium under Discretion

This algorithm is based on Dennis (2007), who shows how to compute optimal policy under discretion in a fairly general setup. This appendix covers the case of two policy institutions setting their respective instruments in a Nash game between each other. The private sector is a follower to both. Within-period leadership of one policymaker vis-a-vis the other institution is also considered.

## Environment

There are two policy institutions. Equilibrium is determined by a set of linear equations of the following form:

$$A_0 Y_t = A_1 Y_{t-1} + A_2 Y_{t+1}^e + A_3 X_t + \tilde{A}_3 \tilde{X}_t + A_4 X_{t+1}^e + \tilde{A}_4 \tilde{X}_{t+1}^e + A_5 v_t, \quad (26)$$

$Y_t$ are the $n_y$ non-policy variables, $X_t$ are the $n_1$ policy variables of player 1, and $\tilde{X}_t$ are the $n_2$ policy variables of player 2. The pure i.i.d. shocks are $v_t$ with covariance matrix $\Omega$. We require that there be $n_y$ non-policy equations and that $A_0$ is invertible.

The solution under discretion in a Markov-perfect equilibrium posits the following equilibrium decision rules:

$$Y_t = H_1 Y_{t-1} + H_2 v_t \quad (27)$$

$$X_t = F_1 Y_{t-1} + F_2 v_t \quad (28)$$

$$\tilde{X}_t = \tilde{F}_1 Y_{t-1} + \tilde{F}_2 v_t. \quad (29)$$

Substituting decision rules into the equilibrium conditions, we have

$$DY_t = A_1 Y_{t-1} + A_3 X_t + \tilde{A}_3 \tilde{X}_t + A_5 v_t$$

where: $D \equiv A_0 - A_2 H_1 - A_4 F_1 - \tilde{A}_4 \tilde{F}_1$

The players optimize the following loss function

$$\mathcal{L}^1 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ Y_t' W Y_t + X_t' Q X_t \right] \quad (30)$$

$$\mathcal{L}^2 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ Y_t' \tilde{W} Y_t + \tilde{X}_t' \tilde{Q} \tilde{X}_t \right]. \quad (31)$$

Here, $W$, $Q$,$\tilde{W}$,$\tilde{Q}$ are positive semi-definite matrices reflecting the preferences of policymakers 1 and 2, respectively.

Dennis shows that the loss can be expressed as

$$\mathcal{L}^1 = Y_t' P Y_t + X_t' Q X_t + \frac{\beta}{1-\beta} \mathrm{tr}\left[\left(F_2' Q F_2 + H_2' P H_2\right) \Omega\right] \tag{32}$$

$$\mathcal{L}^2 = Y_t' \tilde{P} Y_t + \tilde{X}_t' \tilde{Q} \tilde{X}_t + \frac{\beta}{1-\beta} \mathrm{tr}\left[\left(\tilde{F}_2' \tilde{Q} \tilde{F}_2 + H_2' \tilde{P} H_2\right) \Omega\right], \tag{33}$$

where $P$ and $\tilde{P}$ are the solutions to

$$P = W + \beta F_1' Q F_1 + \beta H_1' P H_1 \tag{34}$$

$$\tilde{P} = \tilde{W} + \beta \tilde{F}_1' \tilde{Q} \tilde{F}_1 + \beta H_1' \tilde{P} H_1. \tag{35}$$

Substituting for $Y_t$ into the loss functions, we arrive at

$$\begin{aligned}
\mathcal{L}^1 = & \left[(A_1 + \tilde{A}_3 \tilde{F}_1) Y_{t-1} + A_3 X_t + (A_5 + \tilde{A}_3 \tilde{F}_2) v_t\right]' (D^{-1})' P ... \\
& (D^{-1}) \left[(A_1 + \tilde{A}_3 \tilde{F}_1) Y_{t-1} + A_3 X_t + (A_5 + \tilde{A}_3 \tilde{F}_2) v_t\right] \\
& + X_t' Q X_t + \frac{\beta}{1-\beta} \mathrm{tr}\left[\left(F_2' Q F_2 + H_2' P H_2\right) \Omega\right] \qquad (36) \\
\mathcal{L}^2 = & \left[(A_1 + A_3 F_1) Y_{t-1} + \tilde{A}_3 \tilde{X}_t + (A_5 + A_3 F_2) v_t\right]' (D^{-1})' \tilde{P} ... \qquad (37) \\
& (D^{-1}) \left[(A_1 + A_3 F_1) Y_{t-1} + \tilde{A}_3 \tilde{X}_t + (A_5 + A_3 F_2) v_t\right] \\
& + \tilde{X}_t' \tilde{Q} \tilde{X}_t + \frac{\beta}{1-\beta} \mathrm{tr}\left[\left(\tilde{F}_2' \tilde{Q} \tilde{F}_2 + H_2' \tilde{P} H_2\right) \Omega\right]. \qquad (38)
\end{aligned}$$

Differentiation with respect to $X_t$ and $\tilde{X}_t$ results in the FONC

$$\begin{aligned}
0 = & A_3' (D')^{-1} P D^{-1} \left[(A_1 + \tilde{A}_3 \tilde{F}_1) Y_{t-1} + (A_5 + \tilde{A}_3 \tilde{F}_2) v_t\right] \\
& + \left(Q + A_3' (D')^{-1} P D^{-1} A_3\right) X_t \qquad (39) \\
0 = & \tilde{A}_3' (D')^{-1} \tilde{P} D^{-1} \left[(A_1 + A_3 F_1) Y_{t-1} + (A_5 + A_3 F_2) v_t\right] \\
& + \left(\tilde{Q} + \tilde{A}_3' (D')^{-1} \tilde{P} D^{-1} \tilde{A}_3\right) \tilde{X}_t. \qquad (40)
\end{aligned}$$

Solving these for $X_t$ and $\tilde{X}_t$ we obtain

$$X_t = F_1 Y_{t-1} + F_2 v_t$$

$$\tilde{X}_t = \tilde{F}_1 Y_{t-1} + \tilde{F}_2 v_t,$$

where

$$F_1 \equiv -\left[Q + A_3'(D')^{-1}PD^{-1}A_3\right]^{-1} A_3'(D')^{-1}PD^{-1}\left[A_1 + \tilde{A}_3\tilde{F}_1\right] \tag{41}$$

$$F_2 \equiv -\left[Q + A_3'(D')^{-1}PD^{-1}A_3\right]^{-1} A_3'(D')^{-1}PD^{-1}\left[A_5 + \tilde{A}_3\tilde{F}_2\right] \tag{42}$$

$$\tilde{F}_1 \equiv -\left[\tilde{Q} + \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\tilde{A}_3\right]^{-1} \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\left[A_1 + A_3F_1\right] \tag{43}$$

$$\tilde{F}_2 \equiv -\left[\tilde{Q} + \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\tilde{A}_3\right]^{-1} \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\left[A_5 + A_3F_2\right]. \tag{44}$$

Plugging these back into the system of model equations, we obtain

$$Y_t = H_1Y_{t-1} + H_2v_t,$$

where

$$H_1 \equiv D^{-1}\left(A_1 + A_3F_1 + \tilde{A}_3\tilde{F}_1\right) \tag{45}$$

$$H_2 \equiv D^{-1}\left(A_5 + A_3F_2 + \tilde{A}_3\tilde{F}_2\right). \tag{46}$$

**The algorithm**

1. Initialize $H_1, F_1, \tilde{F}_1$.
2. Solve for $P$ and $\tilde{P}$ using doubling algorithm.
3. Update $F_1, \tilde{F}_1, H_1$, according to (41), (43) and (45).
4. Iterate until convergence.
5. $F_2$ and $\tilde{F}_2$ can be computed from the system formed by (42) and (44) and then $H_2$ from (46).

# Leadership

Suppose that $X_t$ is set *within* the period before $\tilde{X}_t$. This is the so-called leadership equilibrium, which can be thought of as a within-period commitment by one policymaker. In this case, the instrument of the follower will depend on the instrument of the leader, as the latter is effectively pre-determined within the period. The leader can thus choose a value on the followers' best response function and exploit this ability optimally.

So we can write:

$$\tilde{X}_t = \tilde{F}_1 Y_{t-1} + G X_t + \tilde{F}_2 v_t. \tag{47}$$

Substituting decision rules into the equilibrium conditions, we have

$$D Y_t = A_1 Y_{t-1} + A_3 X_t + \tilde{A}_3 \tilde{X}_t + A_5 v_t,$$

where $D$ is now defined as

$$D \equiv A_0 - A_2 H_1 - A_4 F_1 - \tilde{A}_4 \tilde{F}_1. - \tilde{A}_4 G F_1. \tag{48}$$

The players still optimize the following loss function:

$$\mathcal{L}^1 = E_0 \sum_{t=0}^{\infty} \beta^t \left[Y_t' W Y_t + X_t' Q X_t\right] \tag{49}$$

$$\mathcal{L}^2 = E_0 \sum_{t=0}^{\infty} \beta^t \left[Y_t' \tilde{W} Y_t + \tilde{X}_t' \tilde{Q} \tilde{X}_t\right]. \tag{50}$$

The loss can be expressed as

$$\mathcal{L}^1 = Y_t' P Y_t + X_t' Q X_t + \frac{\beta}{1-\beta} \mathrm{tr}\left[\left(F_2' Q F_2 + H_2' P H_2\right)\Omega\right] \tag{51}$$

$$\mathcal{L}^2 = Y_t' \tilde{P} Y_t + \tilde{X}_t' \tilde{Q} \tilde{X}_t + \frac{\beta}{1-\beta} \mathrm{tr}\left[\left(\left(\tilde{F}_2 + G F_2\right)' \tilde{Q} \left(\tilde{F}_2 + G F_2\right) + H_2' \tilde{P} H_2\right)\Omega\right], \tag{52}$$

where $P$ and $\tilde{P}$ are the solutions to

$$P = W + \beta F_1' Q F_1 + \beta H_1' P H_1 \tag{53}$$

$$\tilde{P} = \tilde{W} + \beta \left(\tilde{F}_1 + G F_1\right)' \tilde{Q} \left(\tilde{F}_1 + G F_1\right) + \beta H_1' \tilde{P} H_1. \tag{54}$$

Substituting for $Y_t$ into the loss functions, we arrive at

$$\mathcal{L}^1 = \left[(A_1 + \tilde{A}_3\tilde{F}_1)Y_{t-1} + (A_3 + \tilde{A}_3 G)X_t + (A_5 + \tilde{A}_3\tilde{F}_2)v_t\right]' (D^{-1})'P...$$
$$(D^{-1}) \left[(A_1 + \tilde{A}_3\tilde{F}_1)Y_{t-1} + (A_3 + \tilde{A}_3 G)X_t + (A_5 + \tilde{A}_3\tilde{F}_2)v_t\right]$$
$$+ X_t'QX_t + \frac{\beta}{1-\beta}\mathrm{tr}\left[(F_2'QF_2 + H_2'PH_2)\,\Omega\right] \tag{55}$$
$$\mathcal{L}^2 = \left[A_1 Y_{t-1} + A_3 X_t + \tilde{A}_3\tilde{X}_t + A_5 v_t\right]' (D^{-1})'\tilde{P}... \tag{56}$$
$$(D^{-1}) \left[A_1 Y_{t-1} + A_3 X_t + \tilde{A}_3\tilde{X}_t + A_5 v_t\right]$$
$$+ \tilde{X}_t'\tilde{Q}\tilde{X}_t + \frac{\beta}{1-\beta}\mathrm{tr}\left[\left(\left(\tilde{F}_2 + GF_2\right)' \tilde{Q}\left(\tilde{F}_2 + GF_2\right) + H_2'\tilde{P}H_2\right)\Omega\right].$$

Differentiation with respect to $X_t$ and $\tilde{X}_t$ results in the FONC

$$0 = (A_3 + \tilde{A}_3 G)'(D')^{-1}PD^{-1}\left[(A_1 + \tilde{A}_3\tilde{F}_1)Y_{t-1} + (A_5 + \tilde{A}_3\tilde{F}_2)v_t\right]$$
$$+ \left(Q + (A_3 + \tilde{A}_3 G)'(D')^{-1}PD^{-1}(A_3 + \tilde{A}_3 G)\right)X_t \tag{57}$$
$$0 = \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\left[A_1 Y_{t-1} + A_3 X_t + A_5 v_t\right]$$
$$+ \left(\tilde{Q} + \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\tilde{A}_3\right)\tilde{X}_t. \tag{58}$$

Solving these for $X_t$ and $\tilde{X}_t$ we obtain

$$X_t = F_1 Y_{t-1} + F_2 v_t$$
$$\tilde{X}_t = \tilde{F}_1 Y_{t-1} + GX_t + \tilde{F}_2 v_t,$$

where

$$G \equiv -\left[\tilde{Q} + \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\tilde{A}_3\right]^{-1}\tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}A_3 \tag{59}$$
$$\tilde{F}_1 \equiv -\left[\tilde{Q} + \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\tilde{A}_3\right]^{-1}\tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}A_1 \tag{60}$$
$$\tilde{F}_2 \equiv -\left[\tilde{Q} + \tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}\tilde{A}_3\right]^{-1}\tilde{A}_3'(D')^{-1}\tilde{P}D^{-1}A_5 \tag{61}$$
$$F_1 \equiv -\left[Q + (A_3 + \tilde{A}_3 G)'(D')^{-1}PD^{-1}(A_3 + \tilde{A}_3 G)\right]^{-1}(A_3 + \tilde{A}_3 G)'(D')^{-1}PD^{-1}\left[A_1 + \tilde{A}_3\tilde{F}_1\right] \tag{62}$$
$$F_2 \equiv -\left[Q + (A_3 + \tilde{A}_3 G)'(D')^{-1}PD^{-1}(A_3 + \tilde{A}_3 G)\right]^{-1}(A_3 + \tilde{A}_3 G)'(D')^{-1}PD^{-1}\left[A_5 + \tilde{A}_3\tilde{F}_2\right]. \tag{63}$$

Plugging these back into the system of model equations, we obtain

$$Y_t = H_1 Y_{t-1} + H_2 v_t,$$

where

$$H_1 \equiv D^{-1}\left(A_1 + A_3 F_1 + \tilde{A}_3 \tilde{F}_1 + \tilde{A}_3 G F_1\right) \tag{64}$$

$$H_2 \equiv D^{-1}\left(A_5 + A_3 F_2 + \tilde{A}_3 \tilde{F}_2 + \tilde{A}_3 G F_2\right). \tag{65}$$

**The algorithm**

1. Initialize $H_1, F_1, \tilde{F}_1, G$.
2. Solve for $P$ and $\tilde{P}$ using doubling algorithm.
3. Update $G, \tilde{F}_1, F_1, H_1$, according to (62),(60),(59), and (64) in that order.
4. Iterate until convergence.
5. $F_2$ $\tilde{F}_2$, and $H_2$ can be computed from (63),(61), and (65).

## Alternative Algorithm

As an alternative to this algorithm, we can also compute the Nash equilibrium via an algorithm that does not explicitly take account of the two-policymaker setup.

## Environment

This algorithm is in the spirit of Soederlind (1999) but is more general. It does not require that the model can be written in state-space form. In practice, this means that the matrix $A_2$ below need not be invertible, whereas Soederlind requires this to be non-singular. In practice, $A_2$ is often singular.

Equilibrium is determined by a set of linear equations of the following form:

$$A_0 Y_t = A_1 Y_{t-1} + A_2 Y_{t+1}^e + A_3 X_t + A_4 X_{t+1} + A_5 v_t, \tag{66}$$

where $Y_t$ are the $n_y$ non-policy variables, and $X_t$ are the $n_1$ policy variables. The pure i.i.d. shocks are $v_t$ with covariance matrix $\Omega$. We require that there be $n_y$ non-policy equations and that $A_0$ is invertible.

The solution under discretion in a Markov-perfect equilibrium posits the following equilibrium decision rules:

$$\begin{aligned} Y_t &= H_1 Y_{t-1} + H_2 v_t && (67) \\ X_t &= F_1 Y_{t-1} + F_2 v_t. && (68) \end{aligned}$$

The policymakers' problem is:

$$\min_{X_t, Y_t} Y_t' W Y_t + X_t' Q X_t + \beta Y_t' V Y_t + \Lambda - 2\lambda_t \left\{ D Y_t - A_1 Y_{t-1} - A_3 X_t - A_5 v_t \right\},$$

where $W$, $Q$, are positive semi-definite matrices reflecting the preferences of the policymaker and $D \equiv (A_0 - A_2 H_1 - A_4 F_1)$.

Differentiation with respect to $Y_t$, $X_t$, and $\lambda_t$ results in the FONC

$$\begin{aligned} 0 &= (W + \beta V) Y_t - D' \lambda_t \\ 0 &= Q X_t + A_3' \lambda_t \\ Y_t &= D^{-1}(A_1 Y_{t-1} + A_3 X_t + A_5 v_t). \end{aligned}$$

Solve the first for $\lambda_t$

$$\lambda_t = (D')^{-1}(W + \beta V) Y_t.$$

Plugging this into the second FONC and then combining with (66) yields

$$\begin{aligned} 0 &= \Delta_1 X_t + \Delta_2 Y_{t-1} + \Delta_3 v_t \\ \text{where: } \Delta_1 &\equiv Q + A_3'(D')^{-1}(W + \beta V) D^{-1} A_3 \\ \Delta_2 &\equiv A_3'(D')^{-1}(W + \beta V) D^{-1} A_1 \\ \Delta_3 &\equiv A_3'(D')^{-1}(W + \beta V) D^{-1} A_5. \end{aligned}$$

If $\Delta_1$ is invertible, we can solve this for $X_t$ and obtain

$$\begin{aligned} X_t &= -\Delta_1^{-1} \left( \Delta_2 Y_{t-1} + \Delta_3 v_t \right) && (69) \\ X_t &= F_1 Y_{t-1} + F_2 v_t. && (70) \end{aligned}$$

Plugging this into the constraint, we obtain

$$Y_t = D^{-1} \left[ \left( A_1 + A_3 F_1 \right) Y_{t-1} + \left( A_5 + A_3 F_2 \right) v_t \right].$$

Hence, we have an updating rule of the form

$$F_1 \equiv -\Delta_1^{-1}\Delta_2 \tag{71}$$
$$F_2 \equiv -\Delta_1^{-1}\Delta_3 \tag{72}$$
$$H_1 \equiv D^{-1}(A_1 + A_3F_1) \tag{73}$$
$$H_2 \equiv D^{-1}(A_5 + A_3F_2). \tag{74}$$

Given these decision rules, the current-period value function is given by

$$V_t = Y_t'WY_t + X_t'QX_t + \beta Y_t'VY_t + \Lambda.$$

Plugging in the assumed laws of motion, we obtain

$$\begin{aligned} V_{t-1} = &Y_{t-1}'(H_1'WH_1 + F_1'QF_1 + \beta H_1'VH_1)Y_{t-1} \\ &+ E_{t-1}[v_t'(H_2'WH_2 + F_2'WF_2 + \beta H_2'VH_2)v_t]. \end{aligned} \tag{75}$$

Thus, we have updating rules for the value function

$$V \equiv H_1'WH_1 + F_1'QF_1 + \beta H_1'VH_1 \tag{76}$$
$$\Lambda \equiv E_{t-1}[v_t'(H_2'WH_2 + F_2'WF_2 + \beta H_2'VH_2)v_t]. \tag{77}$$

**The Algorithm**

1. Initialize $H_1, F_1, V$.
2. Update $F_1, H_1, V$ according to (71),(73), and (76).
3. Iterate until convergence.
4. Upon convergence, $F_2$, $H_2$, $\Lambda$ can be computed from (72), (74), and (77).

To adapt this to a game between two policymakers, we can take the reaction function of policymaker 1 as given and subsume it in the vector $Y_t$. Conditional on this reaction function, we solve for the optimal discretionary response of policymaker 2 as outlined in the above algorithm. We start the algorithm again, but take the obtained reaction function of policymaker 2 as given and compute the optimal response of policymaker 1. We then iterate until convergence by passing the reaction functions.