DETERMINANTS OF CURRENCY RISK PREMIUMS

Abstract

This paper presents a theoretical model of exchange-rate determination intended to address the forward premium puzzle. It also explains the empirical observation that risk premiums depend on interest differentials. The model's closed-form solution indicates that currency risk premiums depend on two factors: interest differentials and the current deviation of the exchange rate from its long-run equilibrium. If speculators have an alternative to exchange-rate speculation, then there is no presumption that uncovered interest parity holds even approximately in long-run equilibrium. The model is consistent with existing evidence suggesting that forward premiums are negatively related to rationally expected future exchange rate changes. New empirical evidence is provided in support of the model. (JEL F31, F47)

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One of the most intriguing empirical results in the area of international money and finance is the phenomenon of forward discount bias. Under the familiar conditions of uncovered interest parity and rational expectations, the forward premium (that is, the difference between the forward exchange rate and the current spot rate) should be an unbiased predictor of future exchange-rate changes. Existing evidence shows, however, that the actual change in a spot exchange rate is poorly predicted by the forward premium. In fact, the implied prediction is seriously biased and often has the wrong sign (Froot and Thaler 1990, Engel 1996, Lewis 1995).

Another implication of the joint hypothesis of uncovered interest parity and rational expectations is that rationally expected excess returns, often referred to as risk premiums, should be identically zero. As a corollary, the empirically identified forward bias implies the existence of a non-zero risk premium. Though the existence of forward bias was identified almost two decades ago, empirical research has not been successful at reconciling the behavior of risk premiums with existing theoretical models. When confronted with actual data on risk premiums, the models generally have extremely low explanatory power, and theoretically important variables are frequently statistically insignificant.

The relationship between risk premiums and interest differentials is a particularly puzzling aspect of this mystery. Many empirical studies have concluded that risk premiums are strongly related to interest differentials. This is true for studies in which the rational expectations assumption justifies the use of realized excess returns as a proxy for risk premiums (Froot and Thaler 1990, Engel 1996), as well as for studies in which risk premiums are measured using survey data (Frankel and Chinn 1991). However, existing theoretical models almost universally fail to provide an explanation for the strong observed relationship between risk premiums and interest differentials.

This paper presents a theoretical model of exchange-rate determination in which interest differentials are among the primary determinants of risk premiums. The multiperiod model incorporates two types of agents: rational speculators, whose currency demand depends on expected exchange-rate movements; and nonspeculative agents, whose demand depends on current exchange-rate levels. The model's simple, closed-form solution indicates that currency risk premiums should depend on two factors: interest differentials and the current deviation of the exchange rate from its long-run equilibrium. Supporting empirical evidence is provided.

One strength of the model presented here is its focus on exchange-rate dynamics at relatively short horizons, corresponding to those typically examined in empirical studies of risk premiums, which vary from a week to a year. This focus is embodied in the modeling of the rational speculative agents, who are intended to represent foreign exchange market participants such as currency traders and hedge fund managers, whose trading horizons generally fall well short of a year. This focus is also embodied in the assumption that equilibrium in the foreign exchange market is determined by the requirement that flow demand equal flow supply. The importance of flow demand and supply for exchange-rate equilibrium has been established empirically in recent studies of short-term exchange rate dynamics (Lyons 1995; Goodhart and Payne 1996; Evans 1997). Much consistent evidence has also been found for U.S. equities markets (Shleifer 1986, Holthausen *et al.*

1990). In our model, the utility-maximizing speculators must be satisfied with their portfolio allocations each period, consistent with portfolio balance models, but exchange rates are determined according to the condition of flow equilibrium.

Alternative models of the foreign exchange market that connect currency risk premiums and interest differentials have been put forward by Hagiwara and Hierce (1998), Mark and Wu (1999), and Obstfeld and Rogoff (1998). The conclusions of these papers are likely complementary to those presented here, since the models used in these papers are most relevant to relatively long time horizons. This is true of the model in Hagiwara and Herce (1998) because it relies on portfolio balance equilibrium to determine exchange rates. Models with this equilibrium condition have not been very successful empirically in explaining or forecasting short-term exchange-rates. The Obstfeld and Rogoff (1998) model is probably most relevant for longer horizons because it relies on purchasing power parity to establish exchange rates. Purchasing power parity seems to provide a reasonable first approximation to exchange rates over long periods, but it generally does not provide good guidance to short-term exchange rate dynamics.

Mark and Wu (1999), whose model does seem appropriate for short-term exchange-rate analysis, also provide a potential source of forward bias that is complementary to that presented here. In their model, the relationship between risk premiums and interest differentials depends critically on the presence of noise traders, speculators whose expectations of exchange-rate changes depend more heavily on interest differentials than is consistent with full rationality. Exchange-rate dynamics in our model do not depend on departures from rationality.

Section I of the paper discusses current knowledge about foreign exchange risk premiums. Section II introduces our model of exchange-rate determination with rational speculators. Section III discusses the behavior of risk premiums in the model. Uncovered Interest Parity would hold only if speculation were infinite, which is not consistent with long-run equilibrium. If speculation were infinite, expected speculator utility would be at a global minimum and some speculators would leave the market. Section IV modifies the model to achieve greater realism. Section V provides evidence consistent with the model, and Section VI concludes.

I. FOREIGN EXCHANGE RISK PREMIUMS

I.A. The Puzzling Behavior of Realized Risk Premiums

A foreign exchange risk premium represents the market's anticipated excess return to holding foreign currency relative to holding domestic currency:

$$rp_t = {}_ts^e_{t+k} - s_t + r^*_t - r_t \tag{1}$$

Here, s_t represents the (log) spot exchange rate at time t, measured as domestic currency units per foreign currency unit; r^*_t and r_t represent foreign and domestic interest rates, respectively; the superscript "e" denotes an anticipated future value; the prescript "t" indicates that the anticipations are formed at time t.

Risk premiums should be constant at zero under the strictest version of the familiar Uncovered Interest Parity condition, which states that speculative activity should drive the forward rate to equal the expected future spot exchange rate: $f_{t,t+k} = {}_ts^e_{t+k}$. Here, $f_{t,t+k}$ represents the (log) forward rate at time t for contracts maturing at time $t+k$, and covered interest parity has been invoked. Under rational expectations, this condition is further restricted: $f_{t,t+k} = E_t s^e_{t+k}$, where E_t indicates expectations formed rationally using information available as of time t.

The joint hypothesis that Uncovered Interest Parity holds and expectations are formed rationally has been widely tested and rejected. The standard test begins with the insight that, under Uncovered Interest Parity alone, the estimated value of A should be zero and the estimated value of B should be unity in the following regression:

$$ {}_t s^e_{t+k} - s_t = A + B d_t + \varphi_{t+k} $$

where $d_t = f_{t,t+k} - s_t$ represents the forward premium on foreign currency or, equivalently given covered interest parity, d_t represents the interest differential, $d_t = r_t - r^*_{t.}$ The final term, φ_t, represents a mean-zero random disturbance.

Under rational expectations, the realized and expected future spot exchange rates differ by a mean zero, unpredictable noise term. In this case, one can estimate a modified version of equation (2) in which the expected exchange rate change is replaced by the actual change:

$$ s_{t+k} - s_t = \alpha + \beta d_t + \kappa_{t+k} \ . \tag{2} $$

Here, κ_t represents a mean zero random disturbance that can be interpreted as the sum of φ_t, above, and the period-t expectational error. Under the null, the estimated values of α and β should be zero and unity, respectively.

Froot and Thaler (1990) and Engel (1996) discuss evidence that the estimates of β are often negative. In fact, estimates of β are typically closer to minus one than to plus one, unless, as shown by Flood and Taylor (1996), the interest differentials are very large.

This puzzling evidence is frequently referred to as "forward discount bias" and, as discussed by Isard (1995, p. 142), "economists do not yet fully understand" it.

We reproduce the standard evidence in Tables 1A and 1B, which examine risk premiums at the 3-, 6-, and 12-month horizons for five currencies relative to the U.S. dollar: the Deutschemark, the Japanese yen, the U.K. pound, the Swiss franc, and the Canadian dollar. The monthly data cover January, 1970 through August, 1998. The risk premiums are calculated as the actual exchange-rate change (log difference) minus the maturity-adjusted, continuous-time interest rate differential. Euromarket rates are used for the relevant interest rates, since these are most closely comparable across countries. To ensure that the samples were non-overlapping, we use only January, April, July, and October data for the 3-month regressions. For the six-month regressions we use only January and July data. For the annual regressions we use only January data.[1]

As shown in Table 1A, when equation (2) is estimated using OLS the 15 estimated values of ψ_s are consistently negative and greater than unity in absolute value. With two exceptions, all the estimates are significant at the 5 percent level. Frankel and Chinn (1991) note that OLS estimates can likely be improved by using the Seemingly Unrelated Regressions (SUR) technique, which takes into account the likely existence of nonzero correlation in the error terms across countries. We provide such results in Table 1B. The overall conclusion that interest differentials are significantly and negatively related to risk premiums is sustained.

[1] All variables were taken as month averages because this was the only option for the price data used in later regressions.

From the empirical failure of the joint hypothesis of Uncovered Interest Parity and rational expectations we can infer that there are predictable, nonzero excess returns to foreign exchange speculation. Empirical evidence has provided two other important insights into the behavior of these excess returns. First, they vary substantially over time. In fact, as shown by Fama (1984) and Hodrick and Srivastava (1986), if one assumes rational expectations, one must conclude from the empirical evidence that risk premiums are more variable than expected exchange rate changes. Second, the excess returns are strongly related to interest rate differentials.

Below, we develop a rational expectations model consistent with these findings. In particular, the model generates endogenous, variable risk premiums strongly related to interest rate differentials across countries.

I.B. Survey Evidence

One potential source of predictable excess returns in the foreign exchange market would be imperfect rationality of exchange rate expectations. Some evidence exists, provided by Froot and Frankel (1989) and others, supporting the hypothesis that such expectations are not fully rational. Using survey data of exchange rate expectations, they estimate the standard expression for Uncovered Interest Parity, equation (2). Consistent with Uncovered Interest Parity, and inconsistent with rational expectations, they find that B is usually close to one. In this case the risk premium is captured by the variable A. They find that A differs significantly from zero, and that it varies substantially across time. Thus Froot and Frankel's evidence suggests that risk premiums are large and variable even if one abandons the hypothesis of rational expectations. Further evidence, provided by

Frankel and Chinn (1991) and Marston (1997) shows that risk premiums are strongly related to interest differentials.

McCallum (1994) voices concerns about these results that no doubt reflect others' reactions as well. He writes (pp. 122-123): "Many researchers will be reluctant to accept the Froot-Frankel argument. ... Even if one is willing to contemplate the abandonment of expectational rationality, he/she may be highly dubious regarding this particular form of irrationality, especially since no psychologically appealing rationale is provided by its proponents."

I.C. Time-Varying Risk Premiums

If irrationality is not the source of the predictable, time-varying risk premiums, then we should be able to understand them through economic theory. Given Fama's conclusion that risk premiums vary substantially if expectations are indeed formed rationally, it is reassuring to note that standard equilibrium asset-pricing models imply that the risk premium should vary across time. In the Capital Asset Pricing Model, for example, the risk premium should depend on the risk-free rate prevailing at time t, the return on an appropriate market portfolio from period t through t+1 expected at time t, and the conditional covariance between returns to speculating on the spot market and returns to the market portfolio divided by the variance of the returns to the market portfolio (Hodrick 1987).

As noted by Froot and Thaler, if these are the only determinants of risk premiums then it is difficult to explain their behavior during certain periods. In the early 1980s, for example, when U.S. interest rates were relatively high, this risk premium interpretation would require that “dollar-denominated assets were perceived to be much riskier than assets denominated in other currencies.” However, this was “exactly the opposite of the ‘safe haven’ hypothesis which was frequently offered at that time as an explanation for the dollar’s strength.” Thus, they conclude that it “is hard to see how one could rely on the risk-premium interpretation alone to explain the dollar of the 1980’s.”

Lewis (1995) presents a related model that can address this criticism by broadening the potential determinants of risk premiums. In her model, the risk premium depends on risk aversion, portfolio holdings of domestic and foreign assets at home and abroad, the conditional variance of the exchange rate, and the covariances between exchange rates and domestic and foreign inflation. In discussing a similar framework, Engel (1996) comments that the variance of prices is generally dwarfed by the variance of the exchange rate itself. In this case, as Lewis (1995, pp. 1926-7) notes, “the sign of the risk premium would depend on the difference between … domestic holdings of foreign bonds and foreign holdings of domestic bonds. When domestic residents are net creditors … then the overall effect on the risk premium is to compensate domestic investors for net holdings of foreign deposits.” The model presented below builds on this insight.

Lewis (1995) calculates risk premiums for the Deutschmark, Yen and Pound against the dollar and finds not only substantial variations but also relatively frequent switches in sign. She then asks why this model does not seem to explain the foreign exchange risk premium, and gives two reasons. First, based on estimates by Engel and Rodrigues (1989), the conditional variance of the exchange rate varies far less than would be needed to explain variations in the risk premium. Second, the risk premium appears to switch sign much more frequently than countries switch between overall creditor and debtor status. Her interpretation (p. 1928) is "[T]he infrequent shifts between net debtor and creditor positions and the lack of variability in conditional variances suggest this model cannot explain the changes in sign in predictable returns either."

There may be an important mismatch between the time horizons of the risk premiums typically examined in empirical work and the time horizons of much of the international capital movements associated with overall national net asset positions. Short-horizon risk premiums are probably of greatest concern to those with correspondingly short trading horizons, such as currency traders, hedge fund managers, and the managers of mutual fund portfolios (who are typically re-evaluated at quarterly intervals). Other international capital flows, such as those associated with syndicated bank loans and direct investment, may not be significantly affected by short-term risk premiums. In this case, the low frequency of shifts in countries' overall net creditor and debtor positions may not be relevant for short-term risk premiums.

II. EXCHANGE-RATE DETERMINATION WITH RATIONAL SPECULATORS

In this section we present a model of the foreign exchange market in which short-term currency risk premiums are determined primarily by agents operating with short horizons. In the following sections, we show that the model's endogenous, time-varying risk premiums depend on interest differentials and that they are unlikely to be competed away. The model imposes flow equilibrium in the foreign exchange market, in common with Driskill and McCafferty (1980), among others. It shares with many models the property that exchange rates ultimately converge to their long-run equilibrium.

II.A. The Model

The rational, utility-maximizing speculators in this model are agents who exploit expected short-run exchange-rate changes to make profits. These agents typically devote all their working time to this activity, operating from dealing rooms of large banks or other financial institutions. Their ranks would include not only foreign exchange dealers but also managers of internationally invested institutional investment funds, such as mutual funds, hedge funds, and pension funds. They are extensively trained and they closely monitor market developments in an effort to assess the likely future direction of exchange rates. Since their performance is evaluated at least once per year, and more frequently in many cases, their speculative horizons will be correspondingly short.

To earn profits a speculator takes a position of size b_t, measured in units of foreign currency. The profits on this position will be proportional to the change in (the log of) the exchange rate, $s_{t+1} - s_t$, minus the short-term interest differential across countries:

$$\pi_{t+1} = b_t \, [s_{t+1} - s_t - d_t]. \tag{3}$$

In common with many microstructure models, we assume that speculators choose positions to maximize the following welfare function

$$W_t = E_t(\pi_{t+1}) - (\theta/2) Var_t(\pi_{t+1}) \tag{4}$$

where θ is a measure of risk aversion. $E_t(\pi_{t+1})$ denotes the expected level and $Var_t(\pi_{t+1})$ the conditional variance of a speculator's profits with information as of time t. This is equivalent to maximizing the expected value of a constant absolute risk aversion utility function when the exchange rate has a conditional normal distribution.

The speculators' optimal bet will be proportional to expected profits and inversely proportional to risk aversion and risk itself:

$$b_t = q\, [E_t(s_{t+1}) - s_t - d_t] = q\, rp_t \tag{5}$$

where

$$q = 1/\theta\, Var(s_{t+1})\ . \tag{6}$$

$Var(s_{t+1})$ is the expected variance of the exchange rate conditional on information at time t. As shown by Carlson and Osler (1999), if exogenous influences on the conditional variance of the exchange rate are constant then $Var(s_{t+1})$ is also constant. This is assumed here. When the expected return on foreign assets exceeds that on domestic assets, and the risk premium is positive, speculators take a long position in foreign currency. Conversely, speculators will take a short position when the risk premium is negative.

In equilibrium, the exchange rate adjusts so that total net demand for foreign currency equals zero. Net demand from speculators will correspond to the *change* in their aggregate desired foreign currency position. If there are N speculators, this can be written:

$$B_t - B_{t-1} = N\,(b_t - b_{t-1}) = Q\,(rp_t - rp_{t-1})\ , \tag{7}$$

where the symbol $Q \equiv Nq$ will be useful later.

Net foreign currency demand from nonspeculative agents is assumed to depend on the level of the exchange rate:

$$FX_t = C_t - S\, s_t\ , \qquad S > 0. \tag{8}$$

The assumption that S is positive, which will be maintained hereafter, corresponds to the assumption that net foreign exchange demand from nonspeculative agents satisfies the Marshall-Lerner-Robinson condition familiar from international trade theory.

We take "nonspeculative" currency demand to include all traditional current account activities, including trade in goods and services, transfers of investment income, and both public and private unilateral transfers. We also take it to include foreign direct investment, because empirical evidence suggests that exchange rate levels are a major influence on direct investment (Ray 1989, Froot and Stein 1991, Blonigen 1997). If C_t is random, this portion of total demand could also incorporate liquidity trading and some forms of noise trading.

In theory, the agents that engage in these activities could also engage in short-term speculation. In reality, they choose to specialize, recognizing that they lack the necessary expertise, information, and time to speculate with much success. The maximization problems they solve in their areas of specialization we leave unspecified, choosing to focus on a relatively abstract interpretation of their behavior.

With this definition of nonspeculative currency demand in mind, the term C_t can be taken as summarizing the influence of all factors other than the exchange rate that might affect this net demand, including goods and services prices, real incomes, barriers to trade, and political factors. For now we assume that interest rates do not affect nonspeculative demand, an admittedly unrealistic assumption that is relaxed later.

Setting net foreign currency demand from speculators equal to net demand from nonspeculative agents, the equilibrium condition determining the exchange rate becomes:

$$FX_t + B_t - B_{t-1} = 0. \tag{9}$$

Note that this expression embodies the assumption that exchange rates adjust to achieve flow equilibrium in the foreign exchange market. As noted in the introduction, reliance on a flow equilibrium condition is consistent with mounting evidence of the effect of flow demand and supply on both exchange rates and U.S. stocks. Investors in this model are always in utility-maximizing stock equilibrium, in contrast to ad hoc capital flows in the Mundell-Fleming model.

II.B. Equilibrium

Assuming rational expectations, the exchange-rate equilibrium condition becomes:

$$E_t s_{t+1} - (1 + S/Q)s_t - E_{t-1}s_t + s_{t-1} = - C_t/Q + \Delta_t \tag{10}$$

where $\Delta_t \equiv d_t - d_{t-1}$ represents the change in the interest differential.[2] The bubble-free solution, derived in the appendix, is:

$$s_t = \lambda\, s_{t-1} + (1-\lambda)\sum_{j=0}^{\infty} l^j (E_t C_{t+j} - l E_{t-1} C_{t+j})\,/\,S$$

$$- \frac{l}{1-l}\sum_{j=0}^{\infty} l^j (E_t \Delta_{t+j} - l E_{t-1} \Delta_{t+j}) \tag{11}$$

where λ is the smaller root of the associated characteristic equation: $\lambda^2 - (2+S/Q)\lambda + 1 = 0$. Note that $0 < \lambda < 1$, $\lim_{Q\to 0} \lambda = 0$, $\partial l/\partial Q > 0$, and $\lim_{Q/S\to\infty} \lambda = 1$.

Equation (11) states that the current exchange rate depends on its own lagged value, on expected future values of C_t, representing the primary determinants of nonspeculative demand, and on expected future values of Δ_t, the change in the interest differential. To derive more substantive results we must be more specific about the behavior of C_t and Δ_t, which are the system's two sources of randomness.

With regard to C_t, suppose this component of nonspeculative currency demand is subject to i.i.d. mean-zero shocks denoted by ε_t: $C_t = \overline{C} + \varepsilon_t$.[3] In this case,

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[2] This expression is derived by substituting from (5) and (7) into (9) and collecting terms.

$$E_t C_{t+j} = \overline{C} \text{ for j = 1,2,3,...} \tag{12}$$

Define $\bar{s} \equiv \overline{C}$/S as the exchange-rate's equilibrium level in the absence of speculators, established by setting net nonspeculative demand, FX_t, equal to zero in equation (8). As we will show later, this will also be the exchange-rate's long-run level in the presence of speculators.

With regard to interest rate differentials, we assume these are mean-reverting, consistent with evidence provided by McCallum (1994) and others. As in Mark and Wu (1999), we also assume that interest differentials are exogenous.[4] Specifically, we assume:

$$d_t = \rho\, d_{t-1} + \eta_t. \tag{13}$$

where $0 < \rho < 1$ and η_t represents a mean zero, i.i.d. shock.

With these assumptions regarding the behavior of C_t and Δ_t (or equivalently, C_t and d_t), the solution for the exchange rate becomes (see the appendix for details):

$$s_{t+1} = \bar{s} + \lambda(s_t - \bar{s}) + (1\text{-}\lambda)\, \varepsilon_{t+1} - \frac{l}{1-rl}\, \eta_{t+1} + \frac{l\,(1-r)}{1-rl}\, d_t\,. \tag{14}$$

The first term on the right-hand side of this expression shows that one important exchange rate determinant is its equilibrium value in the absence of speculators, $\bar{s}$. The second term on the right shows that, in the absence of other influences, the exchange rate will converge to $\bar{s}$ monotonically, eliminating the fraction 1-λ of any discrepancy between $\bar{s}$ and s_t each period. Since the remaining three exchange rate determinants (ε_{t+1}, η_{t+1}, and

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[3] All the conclusions of the paper are unchanged if we assume, instead, that C_t is subject to permanent as well as transitory influences. The appendix derives the results for this more general case.
[4] The assumption that interest rates are strictly exogenous is not critical to the results, which are unchanged so long as interest rates are subject to at least one influence exogenous to the rest of the model, such as national monetary policies.

d_t) all have a central tendency of zero, the exchange rate in this model will tend in the long run towards $\bar{s} = \bar{C}/S$ even in the presence of speculators.

The fourth term on the right-hand side of (14) shows that the exchange rate is directly influenced by any change in the interest rate differential: not surprisingly, a rise in foreign interest rates (a negative η_{t+1}) has the immediate effect of appreciating the foreign currency. The current shock to nonspeculative demand also influences the current exchange rate, as does the previous period's interest differential. To understand why the coefficient on the latter is positive, keep in mind that, with mean reversion, a high current interest-rate differential means declining differentials over the future. This, in turn, implies that speculators will be planning concurrent decreases in their holdings of foreign exchange. The effect is stronger when mean reversion occurs more rapidly (when ρ is smaller).

The introduction of speculators transforms exchange-rate determination. In the absence of speculation the exchange rate would always satisfy $s_t = C_t/S$. Interest differentials would have no effect on exchange rates whatsoever. In the presence of speculators, both the level and the change in interest differentials affect current exchange rates. In the absence of speculators, any nonzero value for the shock to non-speculative demand, ε_t, would be immediately and fully reflected in the current exchange rate and would have no impact thereafter. By contrast, when speculators are present, the exchange rate's immediate response to an ε-shock is reduced substantially, and ε-shocks affect all future exchange rates.

The influence of speculators can be summarized by the variable λ. Since λ is monotonically related to $Q = Nq = N/\theta Var(s)$, which in turn can be viewed as a measure of average speculative activity, we can take λ as a measure of average speculative activity so long as other exogenous variables, such as risk aversion and the statistical distributions of the shocks, remain constant. This implies that increasing the activity of speculators reduces the initial effect of an ε-shock and lengthens the exchange-rate's convergence towards its long-run equilibrium. Increasing their activity also intensifies the exchange rate's response to the change in interest differentials and to past differentials. (See Carlson and Osler 1999 for further elaboration of these points.)

III. RISK PREMIUMS IN LONG-RUN EQUILIBRIUM

This section examines the behavior of risk premiums in the model. In the first subsection we show that the risk premium is determined endogenously, that it varies across time, and that it is determined in part by interest differentials, all of which is consistent with existing empirical results. In the second subsection, we show that risk premiums are unlikely to be driven to zero by competition among market participants.

III.A. The Risk Premium

To begin, note that the expected change in the exchange rate at time t can be derived from equation (14) as:

$$E_t s_{t+1} - s_t = (1-\lambda)\,(\bar{s} - s_t) + \beta\, d_t \tag{15}$$

where $\beta = \dfrac{l\ (1-r)}{1-r\,l} < 1$. The risk premium then takes the form:

$$rp_t = (1-\lambda)(\overline{s} - s_t) + (\beta-1)\, d_t \tag{16}$$

So long as λ falls short of unity, the model predicts that the risk premium, measured here as the expected excess return to foreign currency, varies over time and is determined by two factors: the gap between current and long-run exchange rates (which we will call the "exchange-rate gap," for convenience), and the interest differential.

The reason that risk premiums are determined by the gap between current and long-run exchange rates is best understood through a simple example. Suppose that interest differentials are fixed at zero, and that the exchange rate is below its long-run value. This induces speculators to take a long position in foreign currency. The exchange-rate risk associated with that position is compensated by the expected appreciation of the foreign currency. The expected compensation, the risk premium, is equal to $(1-\lambda)$ times the current exchange-rate gap.

The importance of portfolio allocations for risk also explains the relationship between risk premiums and interest differentials. Once again, a scenario will help clarify the intuition involved. Suppose dollar interest rates rise relative to interest rates on assets in other currencies, as they did in the early 1980s. The attempt to buy dollar assets and sell foreign assets bids up the value of the dollar. As foreign speculators obtain more dollar assets, their positions as denominated in their own currencies are subject to greater potential variation in value. If they are risk averse, these speculators limit their exposure even though they could earn an excess return by increasing still more their dollar holdings. The expected excess return (risk premium) on dollars compensates speculators for their increased exposures to dollar-denominated assets.

This analysis shows that riskiness, as measured by the risk premium in this model, does not arise exclusively from exchange-rate volatility; it also depends on the size of speculative positions, consistent with the Lewis (1995) specification discussed above, and with standard portfolio balance models (Branson 1977). This interpretation of the dollar's behavior during the early 1980s provides a reconciliation of the "safe haven" hypothesis cited by Froot and Thaler (1990) with the risk premium hypothesis for the failure of Uncovered Interest Parity. The causation runs from interest differentials to speculative positions to risk premiums, rather than the other way. (See Goodhart (1988) and Carlson (1998) for further discussion of this view.)

The analysis is not undermined by the fact that overall, national net creditor and debtor positions vary little over time compared with the variability of risk premiums, cited by Lewis (1995). This is because risk premiums in this model are determined by the net foreign exchange exposure of a subset of capital account agents, specifically those with very short speculative horizons. These agents' views about the relative attractiveness of different currencies are known to swing widely over short horizons. In consequence, these agents' net creditor position could likewise change dramatically over relatively short periods.

III.B. Long-Run Equilibrium

If speculation were extremely active (and λ close to unity), risk premiums would be approximately constant at zero, and the model would conform approximately to Uncovered Interest Parity. However, foreign exchange speculation is naturally limited by competition from other markets, so λ will almost certainly fall short of unity in long-run equilibrium. We model this point explicitly below.

The number of speculators, N, should depend on whether foreign exchange speculation appears to be a better business than other speculative activities. We use unconditional expected welfare, $E(W_t)$, as a measure of the desirability of being in this market. This measure will be compared with W^*, an exogenous parameter representing expected welfare from being in other markets.[5] If $E(W_t)$ is greater than W^*, then additional speculators have an incentive to enter the foreign exchange market, and vice versa.

We choose to model the participation choice with respect to unconditional expected welfare to enhance the correspondence of our model with the reality of foreign exchange markets, where short-term trading is dominated by interbank traders. Those who manage dealing rooms decide whether to have dealers in this market, and how many. These managers hire and train the dealers and usually keep them for a matter of years. For these reasons, decisions about the extent of participation in foreign exchange markets are made on a low frequency basis, while a single period in the model corresponds to medium- or high-frequency exchange rate dynamics.

As shown in the appendix, unconditional expected welfare can be expressed as follows:

$$E(W_t) = \frac{(1-l)^3}{2q(1+l)}\ [\ \frac{\mathrm{var}(e)}{\mathrm{var}(n)} + \frac{(1+rl)}{(1-rl)^3(1-r^2)}\ \frac{\mathrm{var}(h)}{\mathrm{var}(n)}\ ]. \qquad (17)$$

5 This is a construct that has also been used by Osler (1998).

Thus, unconditional expected welfare depends on λ, ρ, $Var(\varepsilon)$, and $Var(\eta)$. Through λ, which is endogenous, welfare also depends on the number of speculators, N, and their risk aversion, θ.

The implications of (17) for the long-run level of speculative activity are not immediately obvious because the relationship between unconditional welfare and λ, or equivalently between unconditional welfare and the amount of speculative activity, is not generally monotonic. However, the first differential of (17) with respect to λ indicates that there will always be some λ^* above which unconditional expected welfare declines monotonically with further increases in speculative activity. Further, as evident in equation (17), unconditional expected welfare becomes arbitrarily small for large values of λ. Thus, in long-run equilibrium, when $E(W_t) = W^*$, there is an amount of speculation beyond which there is no incentive for additional speculators to enter the foreign exchange market. Thus long-run equilibrium speculation is finite, and equilibrium values of λ are bounded away from unity. In short, in this model there is no presumption that Uncovered Interest Parity will hold even approximately.

IV. AN EXTENSION OF THE MODEL

In this section we modify the model to allow nonspeculative activity to depend on interest rate levels. Import demand seems likely to be negatively affected by higher domestic interest rates and export demand negatively affected by higher foreign interest rates, with corresponding effects on currency demands. In this case, risk premiums still depend on interest differentials, but the direction of their effect can be either positive or negative.

More specifically, we modify the expression for net nonspeculative foreign currency demand to the following:

$$FX_t = C_t - S\, s_t - I\, d_t \tag{18}$$

where I represents the sensitivity of these flows to interest-rate differentials. When foreign interest rates rise, for example, foreign importers presumably reduce their demand for domestic goods, and net demand for foreign currency rises.

As shown in the appendix, the solution for the exchange rate now takes the form:

$$s_{t+1} = \bar{s} + \lambda(s_t - \bar{s}) + \frac{l\,(1-r)-(I/S)r(1-l)^2}{1-rl}\, d_t$$

$$+ (1-\lambda)\,\varepsilon_{t+1} - \frac{l+(1-l)(I/S)}{1-rl}\,\eta_{t+1}\,. \tag{19}$$

The foreign exchange risk premium is now:

$$rp_t = (1-\lambda)(\bar{s} - s_t) + \left[\frac{l\,(1-r)-(I/S)r(1-l)^2}{1-rl} - 1\right] d_t \tag{20}$$

If $\frac{I}{S} > \frac{l\,(1-r)}{r(1-l)^2}$, the coefficient on the interest differential d_t is negative in equation (19) and less than minus one in equation (20). In this case, the foreign currency will tend to appreciate when the foreign interest rate exceeds the domestic rate, and vice versa (other things equal). To satisfy this condition, the interest sensitivity of net demand must be high relative to the exchange-rate sensitivity of net demand.

As discussed in the introduction, there are three alternative explanations in the literature for the relationship between forward premiums and rationally expected exchange rate changes. Among these, two are potentially consistent with the negative relationship found empirically. Obstfeld and Rogoff (1998) suggests that countries with high interest rates may be countries with high inflation and relatively volatile monetary policy. If money is a good hedge, in real terms, to consumption shocks, then the currency of the country with more volatile monetary policy (and higher interest rates) will carry a positive risk premium. Mark and Wu (1999) suggest that forward bias can occur if noise traders are in sufficient supply and their speculative positions depend directly on interest differentials.

V. EMPIRICAL EVIDENCE

The risk premiums in the model presented above should be determined by interest differentials and by the gap between current and long-run exchange rates, as shown in equation (20). To estimate this it would be appropriate to run the following regression:

$$rp_t = \alpha_{\mathrm{m}}(\bar{s} - s_t) + \psi_{\mathrm{m}} d_t + \upsilon_t \qquad (21)$$

where the subscript m refers to the model developed here and rp_t, for purposes of estimation, will be the actual excess return. This expression is closely related to the standard risk premium regression model,

$$rp_t = \alpha_s + \psi_s d_t + \xi_t \tag{22}$$

where the subscript s stands for the standard regression equation. As discussed earlier and shown in Tables 1A-B, evidence based on this equation suggests that risk premiums and interest differentials are often negatively related, with coefficients less than minus one.

Though it might seem that these results are consistent with the extended version of our model, in which nonspeculative currency demand is interest sensitive, the model presented here suggests that equation (22) is mis-specified. In particular, the model indicates that the exchange-rate gap variable is missing from equation (22), and, in consequence, the estimated values for ψ_s could be biased estimates of the true ψ_m.

To develop an empirical risk premium model consistent with the theoretical model presented above, it is necessary to develop a measure of the gap between current and long-run equilibrium exchange rates. For this we turn to simple purchasing power parity, since this is possibly the only standard model of long-run exchange-rates with any empirical reliability. Using data from January, 1970 through August, 1998, we regress the log of the exchange rate level between currency A and currency B on log producer price indexes for both countries. All the price coefficients have the expected sign and are statistically significant. Residuals from these regressions serve as exchange-rate gaps in the risk premium regressions.

Regressions of equation (21), reported in Table 2A, have adjusted R^2s which average around 0.15, fairly high values by the standards of excess returns data at this frequency.[6] Further, the estimated coefficients are fully consistent with the model, which implies that α_m should be positive but less than one and that ψ_m could be either positive or negative.

All 15 of the coefficients on interest differentials, ψ_m, are negative, and all but two are significant at the 5-percent level. Further, all of them fall below minus one, indicating that the relationship between forward premiums are negatively related to rationally expected exchange-rate changes, consistent with the more standard risk premium regressions presented in Table 1.

The coefficients on the exchange-rate gap, α_m, which are crudely analogous to simple Error Correction Model (ECM) coefficients for the purchasing power parity regression, all have the theoretically expected, positive sign. The magnitudes of these coefficients are also reasonable, since they suggest that between 10 and 40 percent of a given year's exchange-rate gap is closed during the following year. The fact that these coefficients rise monotonically with maturity suggests that a simple ECM-type adjustment mechanism is a reasonable characterization of the actual process by which exchange rates adjust to inflation differentials.

Though most of the exchange-rate gap coefficients in Table 2A are not statistically significant, this is not surprising given the shortness of the sample, and does not necessarily undermine the theory. The available evidence for purchasing power parity supports the hypothesis, on balance, but strong statistical support is generally limited to samples that are very large, either across time or across countries. In support of this, consider the results presented in Table 2B. Here, the purchasing power parity regressions

[6] A constant was included in each regression.

and the risk premium regressions were all accomplished using SUR, which effectively increases the sample size. With this estimation technique we find that 12 of the 15 exchange-rate gap coefficients are statistically significant.

The use of SUR does not change the regressions' qualitative implications: the coefficients on exchange-rate gaps remain positive, have reasonable magnitudes, and increase with the maturity horizon of the risk premium. Further, almost all the coefficients on interest-rate differentials remain statistically significant, and all of them fall below minus one. Overall, these statistical results support the model's implication that risk premiums vary endogenously, and that they are strongly related to interest differentials and exchange-rate gaps.

VI. CONCLUSION

This paper presents a theoretical exchange rate model with rational speculators in which the risk premium varies endogenously. Risk premiums are determined, in particular, by interest differentials and by the gap between current and long-run equilibrium exchange rates. These two determinants affect speculators' positions, which in turn affect the implied relative riskiness of the marginal purchases of domestic and foreign assets. The paper also provides empirical evidence supporting the model.

Though exchange rates in this model would be consistent with Uncovered Interest Parity if speculation were infinite, infinite speculation is not consistent with long-run equilibrium. The expected utility of being a speculator becomes arbitrarily small when there is sufficient speculative activity in the market. Since speculators can always shift to other markets, in the long run there will be a finite equilibrium number of foreign-exchange speculators. In short, there is no *a priori* presumption that Uncovered Interest Parity holds even approximately in long-run foreign-exchange market equilibrium.

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Table 1A: Econometric Estimates of the Standard Risk Premium Equation. All equations estimated with OLS.

The table shows econometric estimates of the following equation:

$$rp_t = \alpha_s + \psi_s d_t + \xi_t$$

where rp_t is the risk premium, d_t is the interest differential, and ξ_t is a random disturbance. Interest rates were calculated using euromarket interest rates of the appropriate maturity. Data span the period January, 1970-August, 1998.

Country	3-Month	6-Month	12-Month
Japan			
Interest differential	-4.401***	-3.764***	-3.958***
Adj R-Squared	0.191	0.232	0.394
Germany			
Interest differential	-1.547**	-1.626*	-1.343
Adj R-Squared	0.029	0.040	0.039
UK			
Interest differential	-2.166***	-2.384***	-2.512**
Adj R-Squared	0.072	0.102	0.154
Canada			
Interest differential	-2.090***	-2.161***	-2.011**
Adj R-Squared	0.137	0.212	0.216
Switzerland			
Interest differential	-2.234***	-2.108***	-1.907**
Adj R-Squared	0.083	0.100	0.130

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 1B. Econometric Estimates of the Standard Risk Premium Equation. All equations estimated simultaneously using SUR.

The table shows econometric estimates of the following equation:

$$rp_t = \alpha_s + \psi_s d_t + \xi_t$$

where rp_t is the risk premium, d_t is the interest differential, and ξ_t is a random disturbance. Interest rates were calculated using euromarket interest rates of the appropriate maturity. Data span the period January, 1970-August, 1998.

Country	**3-Month**	**6-Month**	**12-Month**
Japan			
Interest differential	-4.700***	-4.118***	-3.812***
Adj R-Squared	0.191	0.230	0.394
Germany			
Interest differential	-1.892***	-1.368**	-1.272*
Adj R-Squared	0.024	0.035	0.006
UK			
Interest differential	-3.038***	-2.779***	-2.890***
Adj R-Squared	0.100	0.135	0.181
Canada			
Interest differential	-2.507***	-2.481***	-2.339***
Adj R-Squared	0.131	0.207	0.208
Switzerland			
Interest differential	-2.115***	-1.445***	-1.396**
Adj R-Squared	0.066	0.063	0.084

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 2A: Econometric Estimates of the Model With Long-Run Exchange Rates Determined by PPP. All regressions using OLS.

The table shows econometric estimates of the following equation:

$$rp_t = \alpha_m(\bar{s} - s_t) + \psi_m d_t + \upsilon_t$$

where rp_t is the risk premium, ($\bar{s} - s_t$) represents the gap between the long-run equilibrium exchange rate ($\bar{s}$) and the current exchange rate, d_t is the interest differential, and ν_t is a random disturbance. Long-run equilibrium exchange rates are fitted values from the following regressions:

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p^*_t + \nu_t,$$

where p_t and p^*_t represent (log) domestic and foreign price levels, respectively, and ν_t represents a random disturbance. Interest rates were calculated using euromarket interest rates of the appropriate maturity. Data span January, 1970-August, 1998.

Country	**3-Month**	**6-Month**	**12-Month**
Japan			
XRGap	0.034	0.139	0.193
Interest differential	-4.439***	-3.859***	-3.843***
Adj R-Squared	0.185	0.240	0.383
Germany			
XRGap	0.044	0.143	0.215
Interest differential	-1.543**	-1.685*	-1.383
Adj R-Squared	0.030	0.062	0.036
UK			
XRGap	0.081**	0.215**	0.371**
Interest differential	-2.093***	-2.413***	-2.278**
Adj R-Squared	0.099	0.184	0.278
Canada			
XRGap	0.029	0.050	0.197
Interest differential	-2.278***	-2.342***	-2.412**
Adj R-Squared	0.130	0.197	0.199
Switzerland			
XRGap	0.040	0.114	0.226
Interest differential	-2.293***	-2.239***	-2.012**
Adj R-Squared	0.084	0.113	0.147

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 2b. Econometric Estimates of the Model With Long-Run Exchange Rates Determined by PPP. All regressions using SUR.

The table shows econometric estimates of the following equation:

$$rp_t = \alpha_m(\bar{s} - s_t) + \psi_m d_t + \upsilon_t$$

where rp_t is the risk premium, $(\bar{s} - s_t)$ represents the gap between the long-run equilibrium exchange rate ($\bar{s}$) and the current exchange rate, d_t is the interest differential, and ν_t is a random disturbance. Long-run equilibrium exchange rates are fitted values from the following regressions:

$$s_t = \beta_0 + \beta_1 p_t + \beta_2 p^*_t + \nu_t,$$

where p_t and p^*_t represent (log) domestic and foreign price levels, respectively, and ν_t represents a random disturbance. Interest rates were calculated using euromarket interest rates of the appropriate maturity. Data span January, 1970-August, 1998.

Country	**3-Month**	**6-Month**	**12-Month**
Japan			
XRGap	0.123***	0.301***	0.455***
Interest differential	-4.989***	-4.432***	-3.820***
Adj R-Squared	0.163	0.208	0.353
Germany			
XRGap	0.121***	0.236***	0.527***
Interest differential	-2.290***	-1.889***	-1.714**
Adj R-Squared	-0.004	0.042	-0.032
UK			
XRGap	0.076**	0.178**	0.363***
Interest differential	-3.175***	-3.013***	-3.094***
Adj R-Squared	0.129	0.211	0.270
Canada			
XRGap	0.021	0.026	0.315
Interest differential	-2.643***	-2.519***	-3.056***
Adj R-Squared	0.124	0.192	0.180
Switzerland			
XRGap	0.109***	0.219***	0.480***
Interest differential	-2.512***	-1.939***	-1.830***
Adj R-Squared	0.023	0.011	-0.006

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Appendix

<u>Derivation of the solution for the model</u>

Our model implies the following difference equation:

(A.1) $E_t s_{t+1} - (1 + S/Q)s_t - E_{t-1}s_t + s_{t-1} = - X_t$

with $X_t = [C_t - Q(d_t - d_{t-1})]/Q$

To find the solution, first take expectations of (A.1) as of time t-1, and denote by F the forward operator which increases the date on s but not the date on the expectations operator E and by $L = F^{-1}$ the lag operator that decreases the date on s but does not change the date of the expectations operator. Then collect terms:

$$[F^2 - (2 + S/Q)F + 1]\, L\, E_{t-1}s_t = - E_{t-1}X_t$$

By factorization:

(A.2) $(F - \lambda)(F - \frac{1}{l})\, L\, E_{t-1}s_t = - E_{t-1}X_t$

where λ is the smaller root of the characteristic equation: $\lambda^2 - (2+S/Q)\lambda + 1 = 0$.

Multiply (A.2) through by $-\lambda/(1-\lambda F)$ and expand to get:

(A.3) $$E_{t-1}s_t = l s_{t-1} + l \sum_{j=0}^{\infty} l^j E_{t-1}X_{t+j} + K l^{-t}$$

where K is an arbitrary constant. With the assumption of no explosive bubbles, K = 0. When (A.3), with K = 0, is used to substitute in (A.1) for $E_{t-1}s_t$ and, with a suitable change in the time index, for $E_t s_{t+1}$, the resulting expression after collecting terms is:

(A.4) $$(1 - l + \frac{S}{Q})s_t = (1 - l)s_{t-1} + X_t + \sum_{j=0}^{\infty} l^{j+1} E_t X_{t+1+j} - l \sum_{j=0}^{\infty} l^j E_{t-1}X_{t+j}$$

From the factorization, the sum of the roots can be written $l + \frac{1}{l} = 2 + \frac{S}{Q}$ and so:

(A.5) $$1 - l + \frac{S}{Q} = \frac{(1 - l)}{l}$$

and

(A.6) $$\frac{l}{(1-l)} = (1-l)\frac{Q}{S}$$

From (A.5) and (A.6)

(A.7) $$\frac{1}{(1-l+\frac{S}{Q})} = (1-l)\frac{Q}{S}$$

Multiply both sides of (A.4) by (A.7), and note that $(1\text{-}\lambda)^2Q/S = \lambda$, to get

$$s_t = l s_{t-1} + (1-l)\frac{Q}{S}\sum_{j=0}^{\infty} l^j [E_t X_{t+j} - l E_{t-1} X_{t+j}]$$

or substituting for $X_t = [C_t - Q\,\Delta d_t\,]/Q$, and noting again that $Q/S = \lambda/(1\text{-}\lambda)^2$,

(A.8) $$s_t = \lambda\, s_{t-1} + (1\text{-}\lambda)\sum_{j=0}^{\infty} l^j (E_t C_{t+j} - l E_{t-1} C_{t+j})/S$$

$$- \frac{l}{1-l}\sum_{j=0}^{\infty} l^j (E_t \Delta d_{t+j} - l E_{t-1}\Delta d_{t+j})$$

Assume that C_t is subject to both permanent and transitory shocks, as postulated by Muth (1960). In that case $E_t\, C_{t+j} = E_t\, C_{t+1}$ for j = 2,3,… Define $\bar{s}_t = E_t\, C_{t+1}/S$ and let $\varepsilon_t = [C_t - E_tC_{t+1}]/S$ be the perceived transitory shock to net liquidity demand. The summation involving C terms can then be written

(A.9) $$(1\text{-}\lambda)\sum_{j=0}^{\infty} l^j (E_t C_{t+j} - l E_{t-1} C_{t+j})/S = [(1\text{-}\lambda)\, C_t + \lambda\, E_tC_{t+1} - \lambda\, E_{t-1}C_t]/S$$

$$= \bar{s}_t - \lambda \bar{s}_{t-1} + (1\text{-}\lambda)\,\varepsilon_t$$

Since we have assumed that $d_t = \rho d_{t-1} + \eta_t$, note that

$$d_t - d_{t-1} = (\rho-1)d_{t-1} + \eta_t\,,$$

$$E_{t-1}\,(d_t - d_{t-1}) = (\rho-1)d_{t-1},$$

$$E_t\,(d_{t+j} - d_{t-1+j}) = (\rho-1)\rho^{\,j-1}\, d_t = (\rho-1)(\rho^{\,j}\, d_{t-1} + \rho^{\,j-1}\,\eta_t)$$

$$E_{t-1}\,(d_{t+j} - d_{t-1+j}) = (\rho-1)\rho^{\,j}\, d_{t-1}$$

With these substitutions and using (A9) in (A.8), the solution becomes:

(A.10) $$s_t - \bar{s}_t = \lambda(s_{t-1} - \bar{s}_{t-1}) + (1\text{-}\lambda)\,\varepsilon_t - \frac{l}{1-r\,l}\,\eta_t + \frac{l\,(1-r)}{1-r\,l}\,d_{t-1}.$$

Expected Welfare

Conditional expected welfare is given as:

(A.11) $W_t = E_t(\pi_{t+1}) - (\theta/2)Var_t(\pi_{t+1})$

The profitability of position b_t is:

(A.12) $\pi_{t+1} = b_t\,[s_{t+1} - s_t - d_t]$.

The position itself is given by

(A.13) $b_t = q\ rp_t$ where $q = \dfrac{1}{q\mathrm{Var}(s)}$

The risk premium rp_t is defined by

(A.14) $rp_t = E_t s_{t+1} - s_t - d_t$ with $d_t = r_t - r^*_t$.

The unanticipated change in the exchange rate takes the form:

(A.15) $\nu_{t+1} = (1\text{-}\lambda)\ \varepsilon_{t+1} - \dfrac{l}{1 - r\,l}\ \eta_{t+1}$

Therefore

(A.16) $s_{t+1} = E_t s_{t+1} + \nu_{t+1}$

and

(A.17) $\mathrm{Var}(s) = (1\text{-}\lambda)^2\ \mathrm{var}(\varepsilon) + \dfrac{l^2}{(1 - r\,l\,)^2}\ \mathrm{var}(\eta) = \mathrm{var}(\nu)$

The payoff to a rational speculative position, after substituting (A.13), (A.14) and (A.16) into (A.12) is:

(A.18) $\pi_{t+1} = q\ rp_t\ [rp_t + \nu_{t+1}]$

From (A.18), we have

(A.19) $E_t(\pi_{t+1}) = q\ (rp_t)^2$

(A.20) $Var_t(\pi_{t+1}) = q^2\ (rp_t)^2\ \mathrm{Var}(s)$

Substituting these into (A.11) and simplifying yields

(A.21) $$W_t = \frac{(rp_t)^2}{2qVar(s)}$$

In the model, since $E_t \bar{s}_{t+1} = \bar{s}_t$, the risk premium is given by:

(A.22) $$rp_t = (1-\lambda)(\bar{s}_t - s_t) + (\beta - 1)\, d_t \qquad \text{where } \beta = \frac{l\ (1-r)}{1-r\,l}$$

Substitute this into (A.21) and take the unconditional expected value:

(A.23) $$E(W_t) = \frac{(1-l\,)^2 E\{(\bar{s}_t - s_t) - [1/(1-l\,r\,)]d_t\}^2}{2qVar(s)}$$

The numerator in (A.23) can be written

(A.24) $$(1-\lambda)^2 \left\{ E(s_t - \bar{s}_t)^2 + \frac{2}{(1-r\,l\,)} Ed_t(s_t - \bar{s}_t) + \frac{1}{(1-r\,l\,)^2} E\, d_t^2 \right\}$$

with

(A.25) $$d_t = \rho\, d_{t-1} + \eta_t \qquad 0 < \rho < 1.$$

(A.26) $$s_t - \bar{s}_t = \lambda(s_{t-1} - \bar{s}_{t-1}) + \nu_t + \frac{l\ (1-r)}{1-r\,l}\, d_{t-1}$$

To evaluate $E\, d_t^2$, note from (A.25) that d_t can be written in moving average form:

$$d_t = \eta_t + \rho\eta_{t-1} + \rho^2\eta_{t-2} + \ldots$$

Therefore, assuming independent, mean-zero η's:

(A.27) $$E\, d_t^2 = var(d) = \frac{1}{1-r^2}\ var(\eta)$$

For the first term in (A.24), the moving average representation for $(s_t - \bar{s}_t)$ can be written (with $b = \frac{l\ (1-r)}{1-r\,l}$):

$$s_t - \bar{s}_t = \nu_t + \lambda\, \nu_{t-1} + \lambda^2\, \nu_{t-2} + \ldots + bd_{t-1} + \lambda bd_{t-2} + \lambda^2\, bd_{t-3} + \lambda^3\, bd_{t-4} \quad \ldots$$

Therefore:

$$
\begin{aligned}
E(s_t - \bar{s}_t)^2 = {} & (1 + \lambda^2 + \lambda^4 \ldots)\mathrm{var}(\nu) \\
& + b^2\, E[d_{t-1}^{\;2} + \lambda^2 d_{t-2}^{\;3} + \lambda^4 d_{t-3}^{\;2} \ \ldots \\
& \qquad + 2\,\lambda\, d_{t-1}d_{t-2} + 2\,\lambda^2\, d_{t-2}d_{t-3} + 2\lambda^5\, d_{t-3}d_{t-4} + \ldots \\
& \qquad + 2\,\lambda^2\, d_{t-1}d_{t-3} + 2\,\lambda^4\, d_{t-2}d_{t-4} + \ \ldots\ ] \\
& + 2b\, E[\lambda \nu_{t-1}d_{t-1} + \lambda^2\, \nu_{t-2}d_{t-1} + \lambda^3\, \nu_{t-3}d_{t-1} + \ldots \\
& \qquad + \lambda^3\, \nu_{t-2}d_{t-2} + \lambda^4\, \nu_{t-3}d_{t-2} + \ldots\ ]
\end{aligned}
$$

$$
= \frac{1}{1-l^2}\{\mathrm{var}(\nu) + b^2\, [1 + \frac{2rl}{1-rl}]\mathrm{var}(d) - 2b\frac{l}{1-rl}\,\frac{l}{1-rl}\,\mathrm{var}(\eta)\,\}
$$

$$
= \frac{1}{1-l^2}\{(1-\lambda)^2\,\mathrm{var}(\varepsilon) + \frac{l^2}{(1-rl)^2}\ \mathrm{var}(\eta)
$$

$$
+ [\frac{l^2\ (1-r)^2}{(1-rl)^3}\,\frac{1+rl}{1-r^2} - \frac{2l^3(1-r)}{(1-rl)^3}]\ \mathrm{var}(\eta)\ \}
$$

$$
= \frac{1-l}{1+l}\ \mathrm{var}(\varepsilon) +
$$

$$
\frac{l^2\ [(1-rl)(1-r^2)+(1-r)^2(1+rl)-2l(1-r)(1-r^2)]}{(1-l^2)(1-rl)^3(1-r^2)}\ \mathrm{var}(\eta)
$$

$$
= \frac{1-l}{1+l}\ \mathrm{var}(\varepsilon) +
$$

$$
\frac{l^2\ (1-r)[(1-rl)(1+r)+(1-r)(1+rl)-2l(1-r^2)]}{(1-l^2)(1-rl)^3(1-r^2)}\ \mathrm{var}(\eta)
$$

$$
= \frac{1-l}{1+l}\ \mathrm{var}(\varepsilon) + \frac{l^2\ (1-r)2(1-l)}{(1-l^2)(1-rl)^3(1-r^2)}\ \mathrm{var}(\eta)
$$

(A.28) $$E(s_t - \bar{s}_t)^2 = \frac{1-l}{1+l}\ \mathrm{var}(\varepsilon) + \frac{2l^2}{(1+l)(1-rl)^3(1+r)}\ \mathrm{var}(\eta)$$

The foregoing used $E\, d_t d_{t-j} = \rho^j\, \mathrm{var}(d)$ and $E\, \nu_{t-j}\, d_t = -\frac{l}{1-rl}\,\rho^j\, \mathrm{var}(\eta)$.

For the middle term in (A.24)

$$E\,(s_t - \bar{s}_t)d_t = E(\nu_t + \lambda\, \nu_{t-1} + \lambda^2\, \nu_{t-2} \ldots bd_{t-1} + \lambda bd_{t-2} + \lambda^2\, bd_{t-3}\ \ \ldots)\, d_t$$

$$
= -\frac{l}{1-rl}(1+\rho\lambda + (\rho\lambda)^2 + \ldots)\mathrm{var}(\eta) + b\rho(1 + \rho\lambda + (\rho\lambda)^2 + \ldots)\mathrm{var}(d)
$$

$$
= \frac{-l(1-r^2) + rl\ (1-r)}{(1-rl)^2}\mathrm{var}(d) = \frac{-l(1-r\ )}{(1-rl)^2}\ \mathrm{var}(d)
$$

(A.29) $\qquad E\,(s_t - \bar{s}_t)d_t = \dfrac{-\,l}{(1-r\,l)^2(1+r)}\ \mathrm{var}(\eta)$

Note the negative unconditional covariance between the interest rate differential and deviations in the exchange rate from its long-run level. Higher domestic interest rates tend to be associated with an appreciated currency.

Putting (A.27), (A.28) and (A.29) into (A.24) and the result into (A.23) yields:

$$2\theta\ E(W_t) = \frac{(1-l)^3}{1+l}\ \frac{\mathrm{var}(e)}{\mathrm{var}(n)}$$

$$+\ \frac{(1-l)^2}{(1-r\,l)^2}\left[\frac{2l^2}{(1+l)(1-r\,l)(1+r)} - \frac{2l}{(1-r\,l)(1+r)} + \frac{1}{1-r^2}\right]\frac{\mathrm{var}(h)}{\mathrm{var}(n)}$$

$$= \frac{(1-l)^3}{1+l}\ \frac{\mathrm{var}(e)}{\mathrm{var}(n)}$$

$$+(1-\lambda)^2\left[\frac{2l^2\ (1-r) - 2l(1+l)(1-r) + (1+l)(1-r\,l)}{(1+l)(1-r\,l)^3(1-r^2)}\right]\frac{\mathrm{var}(h)}{\mathrm{var}(n)}$$

Therefore

(A.30) $\qquad E(W_t) = \dfrac{(1-l)^3}{2q(1+l)}\left[\dfrac{\mathrm{var}(e)}{\mathrm{var}(n)} + \dfrac{(1+r\,l)}{(1-r\,l)^3(1-r^2)}\ \dfrac{\mathrm{var}(h)}{\mathrm{var}(n)}\right]$

<u>Interest Rates and Non-Speculating Traders</u>

To include non-speculating traders who respond to interest-rate differences, write the net demand as:

$$L_t = C_t + S\ s_t - I\ d_t.$$

In this case X_t has the term $-(I/Q)d_t$ and the right side of (A.9) has the added expression:

$$-(1-l)\frac{Q}{S}\frac{I}{Q}\sum_{j=1}^{\infty} l^j[E_t d_{t+j} - lE_{t-1}d_{t+j}]$$

$$= \quad (1-l)\frac{I}{S}{}_{j\ \ 1}\ l^j[r^j\ \ {}_t - l\,r^{j\ \ 1}d_t\ \ ]$$

$$= -(1-l)\frac{I}{S}\sum_{j=1}^{\infty} l^j[r^{j+1}d_{t-1} + r^j h_t - l\,r^{j+1}d_{t-1}]$$

$$= -(1-l)\frac{I}{S}\left[\frac{r(1-l)}{1-r\,l}d_{t-1} + \frac{1}{1-r\,l}h_t\right]$$

Add this to the right side of (A.10) to get the equation (19) in the text.