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Marianne Andries | Thomas M. Eisenbach | Martin C. Schmalz

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Abstract

Inspired by experimental evidence, we amend the recursive utility model to let risk aversion decrease with the temporal horizon. Our pseudo-recursive preferences remain tractable and retain appealing features of the long-run risk framework, notably its success at explaining asset pricing moments. In addition, our model addresses two challenges to the standard model. Calibrating the agents' preferences to explain the equity premium no longer implies an extreme preference for early resolutions of uncertainty. Horizon-dependent risk aversion helps resolve key puzzles in finance on the valuation of assets across maturities and captures the term structure of equity risk premia and its dynamics.

Key words: risk aversion, early resolution, term structure, volatility risk

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Eisenbach: Federal Reserve Bank of New York (email: thomas.eisenbach@ny.frb.org). Andries: University of Southern California (email: andries@usc.edu). Schmalz: University of Oxford, CEPR, and ECGI (email: martin.schmalz@sbs.ox.ac.uk). This paper was previously circulated under the title "Asset Pricing with Horizon-Dependent Risk Aversion." For helpful comments and discussions, the authors thank Daniel Andrei, Jaroslav Borovička, Markus Brunnermeier, John Campbell, Ing-Haw Cheng, Max Croce, Ian Dew-Becker, Bob Dittmar, Ralph Koijen, Ali Lazrak, Anh Le, Seokwoo Lee (discussant), Erik Loualiche, Matteo Maggiori, Thomas Mariotti, Stefan Nagel, Dimitris Papanikolaou, Nikolai Roussanov (discussant), Martin Schneider (discussant), David Schreindorfer, Avanidhar Subrahmanyam, and Adrien Verdelhan, as well as conference participants at AFA, SED, Econometric Society, EEA, NBER Asset Pricing Meeting, Adam Smith, and WFA. They also thank Wentao Li for his work as a research assistant. Schmalz is grateful for generous financial support through an NTT Fellowship from the Mitsui Life Financial Center. Marianne Andries acknowledges funding from the French National Research Agency (ANR) under the Investments for the Future (Investissements d'Avenir) program, grant ANR-17-EURE-0010.

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1 Introduction

We propose a model that relaxes the assumption, standard in the economics and finance literature, that an agent's risk aversion is the same for all payoff horizons. We define pseudo-recursive preferences similar to Epstein-Zin (Epstein and Zin, 1989) but generalized to allow for horizon-dependent risk aversion. We assume that agents are more risk averse at short horizons than at long horizons, as the experimental evidence indicates. Within a standard long-run risk economy in the tradition of Bansal and Yaron (2004), we find that our model retains the Epstein-Zin model's ability to match standard asset pricing moments. In addition, we show that allowing for horizon-dependent risk aversion resolves two important puzzles in finance. First, our model can be calibrated to match the equity premium without implying an extreme timing premium (the share of wealth an agent would be willing to pay for resolving all future consumption uncertainty early), a fundamental challenge in the long-run risk framework (Epstein et al., 2014). Second, our model can explain several important stylized facts in finance on the valuation of long-term risks, in particular that risk premia have upward or downward sloping term-structures depending on market conditions, consistent with the evidence in the data and contrary to standard asset pricing models.

Our first contribution is methodological: we introduce horizon-dependent risk aversion within the standard recursive utility model of Epstein and Zin (1989), which allows us to build on its success at explaining asset pricing moments when combined with the long-run risk in consumption growth of Bansal and Yaron (2004). The usual recursive techniques, adapted to our setting of pseudo-recursive preferences, enable us to derive closed-form solutions. Our baseline model can accommodate numerous extensions, be it on the valuation of risk (habit formation, disappointment aversion, loss aversion, etc.), or on the quantity of risk (rare disasters, production-based models, etc.). Further, under our preference model, *inter-temporal* decisions for deterministic payoffs are unchanged from the standard model and remain time consistent; but *intra-temporal* allocations across risky assets are dynamically time inconsistent. We can therefore study the effects of horizon-dependent risk aversion on optimal decisions and equilibrium pricing in isolation of any effects of time inconsistent inter-temporal preferences, such as quasi-hyperbolic discounting.

The second contribution of our paper is to address the two key challenges to the long-run risk asset pricing model mentioned above, the timing premium and the term-structure of risk premia, and formally show that our preference model can reconcile the usual asset pricing moments with attitudes towards information and risk valuations at different maturities. The standard long-run risk model (e.g. Bansal and Yaron, 2004; Bansal et al., 2012) has had great success at matching asset pricing moments and at explaining their apparent "puzzles" by combining recursive Epstein-Zin preferences with risk to the expected growth and volatility of consumption (see Cochrane, 2017, for a review of the literature). It explains the high equity premium, results in time-varying risk premia that rationalize the volatility puzzle and return predictability, and matches various cross-sectional evidence.

However, to match these moments, the standard model also implies that the representative agent has a high to extreme timing premium, a measure of her preferences for early versus late resolution of uncertainty (Epstein et al., 2014). In the calibration of Bansal et al. (2012), in a set-up where she cannot act on information she receives, the representative agent would nonetheless be willing to pay 85% of her wealth to resolve all her future consumption shocks early. Such a strong preference for an early resolution of uncertainty appears inconsistent with evidence, e.g. on investors' inattention to their wealth, and with commonsense considerations, thus raising doubts as to the validity of the standard model.

In addition, the long-run risk at the heart of the standard model implies that agents face greater aggregate shocks at longer horizons and therefore require a greater compensation, or risk premia, to invest in long-term assets. The data, however, provides a range of counter-examples where investors appear to demand low expected excess returns at long horizons: e.g. the low risk premia of buy-and-hold assets such as private equity and housing (Moskowitz and Vissing-Jørgensen, 2002; Giglio et al., 2015; Chambers et al., 2021); the low demand for long-term insurance and for options with medium and long maturities (Garleanu et al., 2008; Akaichi et al., 2020); and the evidence of downward sloping term-structures of excess equity returns first documented in van Binsbergen et al. (2012) (see the literature review below).

To address these challenges to the long-run risk model, we first analyze how horizon-dependent risk aversion affects the timing premium — the willingness to pay for early resolutions of uncertainty. We formally derive how two consumption streams with identical risk but different timing for information arrivals are valued: one where shocks are revealed gradually as they are realized over time, the other where all future shocks are revealed next period. Even though the ex-ante distributions of risk are rigorously identical, agents value these consumption streams differently under Epstein-Zin preferences. They have a strong preference to receive information early when the ratio of their risk aversion to their inverse of the elasticity of intertemporal substitution is high; a high value for this ratio is also a necessary condition for the model to capture the equity premium in the data, hence the critique of Epstein et al. (2014). Under horizon-dependent risk aversion, however, a consumption stream with early resolution of uncertainty shifts the risk of all future shocks into a short-horizon risk, moving from a risk assessment using the low long-term risk aversion to a risk assessment using the high short-term risk aversion; this lowers the attractiveness of early resolution of uncertainty compared to the standard framework. We formalize this intuition and show that the timing premium is unambiguously lower when the risk aversion decreases with the horizon. We quantify this result and show that our calibration of the model can simultaneously match the equity premium and generate a low timing premium.

We then apply our model to equilibrium asset pricing. We first consider a representative agent who trades and clears the market every period. Unable to commit to future behavior but aware of her dynamic inconsistency, the agent optimizes in the current period, fully anticipating reoptimization in future periods (in the spirit of Strotz, 1955). Solving our model this way yields a canonical one-period pricing problem in which the Euler equation is satisfied, and the law of one price and no-arbitrage conditions hold. We consider a Lucas-tree endowment economy with long-run risk, using the same consumption and dividend risk processes as in the standard model (Bansal and Yaron, 2004; Bansal et al., 2012), where shocks to consumption levels (immediate consumption shocks and shocks to consumption growth) and shocks to consumption risk (volatility shocks) are priced. Our formal results show that the pricing of shocks to consumption levels does not change under horizon-dependent risk aversion — reflecting that the dynamic inconsistency in our model does not concern inter-temporal decisions. In contrast, shocks to consumption risk (volatility) directly concern *intra*-temporal decisions, and their pricing changes under horizon-dependent risk aversion: the lower risk aversion at long horizons reduces the pricing of volatility shocks and this effect accumulates over time.

Our model can still be calibrated to capture the usual asset pricing moments, in particular the equity premium and the return predictability puzzles. However, the lower pricing of consumption volatility shocks relative to consumption growth shocks has two important implications. First, it means that a greater share of the equity premium is compensation for consumption growth risk. This provides an explanation of the high average expected returns on macroeconomic announcement days (Lucca and Moench, 2015; Ai and Bansal, 2018) as periods when news about future economic growth is revealed. In contrast, the standard model, where a greater share of the equity premium is compensation for consumption volatility risk, requires that macroeconomic announcements mostly provide news about future economic volatility to justify the evidence in the data; this is a more constraining interpretation of the information provided on those days.

Second, because the horizon-dependent risk aversion model requires lower compensation for consumption volatility risk, the greater exposure of longer-horizon assets to volatility risk does not increase their expected returns as much as in the standard model. We therefore derive a flatter, though still upward sloping, average term-structure of equity risk premia, that can lead to a downward sloping average term-structure of equity Sharpe ratios at long-horizons, both consistent with the evidence (see Giglio et al. (2023) and the literature review). The slope of equity premia becomes more upward sloping in periods of increased consumption volatility, as the uncertainty of longer-horizon assets increases relative to shorter-horizon assets. Since these periods also correspond to lower market price levels, we obtain a negative correlation between the slope of the term-structure of dividend strip returns and the market price-dividend ratio, a dynamic result consistent with the evidence in Gormsen (2021).

The data, however, also shows periods of downward sloping term-structures of equity premia (see van Binsbergen et al. (2012) and the literature review below), inconsistent with the formal asset pricing results we derive for a representative agent with horizon-dependent risk aversion who trades every period. This may appear counter-intuitive: if risk aversion decreases with the horizon, should risk compensations not also decrease with the horizon, with a one-for-one relation between the term-structure of risk aversions and the term-structure of risk premia? This intuition turns out to be incorrect: because the representative agent trades every period, she prices *all* financial assets at the next period horizon no matter when payoffs occur. Given this simple observation on the mechanism of our model, we extend our analysis to consider the implications of horizon-dependent risk aversion when agents no longer assume they will trade every period, in line with the evidence in the data on the low frequency of investors' trading (e.g. Alvarez et al., 2012; Sicherman et al., 2016). Despite maintaining the long-run risk assumption, i.e. longer horizon assets are exposed to greater consumption risks, we find that our model can generate downward sloping term-structures of risk premia for buy-and-hold strategies. We believe the differential pricing implications of our model for liquid one-period risks versus long-term locked-in investments identifies a key channel to explain why the term-structure of risk premia has a negative slope for some assets or for some periods, and not for others; an important puzzle in finance.

Finally, we take our analytical results to the data to confirm their quantitative relevance. We find that the horizon-dependent risk aversion model can be calibrated to match the evidence in Giglio et al. (2023) on the slowly upward sloping average term structure of equity premia and the first upward then downward sloping Sharpe ratios in "normal times," i.e. when NBER recessions are excluded from the data sample, where we assume the standard one-period pricing framework applies; as well as the downward sloping term structures in Giglio et al. (2023) during NBER recessions, interpreted as periods of illiquid buy-and-hold pricing. These quantitative results, which improve markedly on the standard long-run risk model, can be achieved while also matching the equity premium and the macroeconomic announcement premium, and while implying a reasonable timing premium close to zero.

In sum, the model of preferences we propose, where risk aversion differs for short-horizon and long-horizon uncertainty, can address the early versus late resolution of uncertainty critique and explain several important stylized facts in finance on the valuation of long-term risks. We can solve these challenges concerning the timing and pricing of uncertainty without compromising on the model's ability to match the usual asset pricing moments, and without departing from the methodology of the widely-used Epstein-Zin preferences and long-run risk framework.

After a review of the literature, we present our utility model in Section 2. We analyze the preference for early or late resolution of uncertainty in Section 3. In Section 4, we derive the risk pricing implications of horizon-dependent risk aversion. Section 5 relates our results to the evidence in the data and proposes a calibration of our model. Section 6 concludes. All mathematical proofs are in the appendix.

Related literature

This paper is the first to solve for equilibrium asset prices in an economy populated by agents with dynamically inconsistent risk aversions. Our methodology, which guarantees the no-arbitrage condition despite time inconsistency, follows Luttmer and Mariotti (2003), and our work complements theirs. They show that dynamically inconsistent preferences for inter-temporal trade-offs of the kind examined by Harris and Laibson (2001) have only limited implications for asset pricing, and little power to explain cross-sectional variations in asset returns. Given that cross-sectional asset pricing involves intra-period risk-return tradeoffs, it is indeed quite intuitive that intra-temporal dynamic inconsistency, such as horizon-dependent risk aversion, rather than inter-temporal dynamic inconsistency can address puzzles related to risk premia.[1]

Our model generalizes Epstein-Zin preferences by relaxing the dynamic consistency axiom of Kreps and Porteus (1978) to analyze the relationship between the timing and pricing of uncertainty. We choose the CRRA model for risk adjustments, standard to the macro-finance literature. In contrast, Routledge and Zin (2010), Bonomo et al. (2011) and

[1] Eisenbach and Schmalz (2016) show that dynamic inconsistency for intra-temporal risk trade-offs is orthogonal to dynamic inconsistency for inter-temporal consumption trade-offs due to non-geometric discounting as in Strotz (1955), Phelps and Pollak (1968), and Laibson (1997). Heimer et al. (2023) document dynamic inconsistency in path-dependent risk taking where agents' observed behavior displays the classic disposition effect although their planned behavior calls for the opposite.

Schreindorfer (2014) follow Gul (1991) and relax the independence axiom to analyze the asset pricing impact of disappointment aversion within a recursive framework. They find that their models generate endogenous predictability (Routledge and Zin, 2010); match various asset pricing moments (Bonomo et al., 2011); and price the cross-section of options better than the standard model (Schreindorfer, 2014). Andries (2021) introduces loss aversion in recursive preferences à la Epstein and Zin (1989) and shows it helps match the security market line, while Dew-Becker (2014) uses a model of habit to obtain time varying risk premia. Our framework can also accommodate these non-standard utility functions for the valuation of risk. Within the classical model of Epstein and Zin (1989), none of the above-mentioned preference models address the excessive preference for early resolution of uncertainty puzzle pointed out by Epstein et al. (2014) or explain the term structure of equity risk premia — the two questions of interest in our analysis.

Starting with van Binsbergen et al. (2012), several papers provide empirical evidence of downward sloping term structures of expected excess returns for various types of risk (e.g. Lustig et al., 2019; van Binsbergen, 2016; Giglio et al., 2015; Dew-Becker et al., 2017; Andries et al., 2023). These empirical findings, which cannot be explained by the standard asset pricing models (Zviadadze, 2021), gave rise to numerous new works explicitly focused on systematically deriving downward sloping term structures of risk prices (e.g. Kogan and Papanikolaou, 2010; Gârleanu et al., 2012; Belo et al., 2015; Croce et al., 2015; Marfe, 2015; Favilukis and Lin, 2016; Marfe, 2017; Ai et al., 2018; Backus et al., 2018; Tinang, 2019). However, recent empirical work on the dynamics of the term-structure of equity risk premia show its slope varies over time. Gormsen (2021) documents that the index price-dividend ratio correlates negatively with the slope of the term structure of dividend strip futures expected returns, corresponding to, on average, a more upward sloping term-strucure in "bad" times/ low index price (relative to dividends). On the other hand, Bansal et al. (2021), using data from 2004 to 2017, document that expected excess returns of dividend risk are upward sloping on average, but became sharply downward sloping during the financial crisis of 2007–2009. Similarly, Chabi-Yo and Loudis (2020) show that the short-end term structure is upward-sloping in expansions and downward-sloping in recessions (see also Golez and Jackwerth, 2020). Finally, Giglio et al. (2023), using a principal components method to extend the sample period to 1973-2020, find a slightly positive slope outside of NBER crises and a negative one during NBER crises. Other asset classes on the other hand display downward sloping term-structures in and out of crisis, e.g. Giglio et al. (2015) for housing and Andries et al. (2023) for the price of variance risk.

The horizon-dependent risk aversion model captures the average, slowly upward sloping term-structure of equity premia, as well as their Sharpe ratios, and reconciles the dynamics results in Gormsen (2021) with seemingly contradictory evidence (e.g. Chabi-Yo and Loudis, 2020; Golez and Jackwerth, 2020; Bansal et al., 2021; Giglio et al., 2023): during crisis periods, if liquidity breaks down (e.g. Pedersen, 2009; Brunnermeier, 2009; Nagel, 2012) and investors resort to buy-and-hold strategies, we obtain smaller or even negative slopes in the term structures of expected returns, as well as downward sloping term-structures of forward equity yields. Additional supporting evidence that liquid versus illiquid assets are priced differently in the term-structure, as implied by our horizon-dependent risk aversion model, can be found in Weber (2018), who shows that higher cashflow durations of equity shares have a downward influence on expected returns only within short-sale constrained stocks.

Other explanations for the dynamics of the term structure of equity returns have been proposed. Ai et al. (2018) introduce a production-based general equilibrium framework to model the dynamics of the discount factor and cash flows. Gonçalves (2021) show that short-duration dividend strips have a higher reinvestment risk and a lower market risk, which generates variations in the term structure depending on market conditions. In contrast to our approach, these papers rely on more complex macroeconomic shocks than the standard model to explain the term-structure results. Finally, Cassella et al. (2023) document that investors have biased expectations, with greater optimism for long-horizon assets; and that time variations in the horizon bias correlate with the term-structure dynamics. Their empirical findings, on investors' expectations, complement our approach, on investors' preferences.

2 Horizon-dependent risk aversion

Experiments document that risk attitudes are affected by how far in the future a risk occurs (resolution and payoff) and that subjects tend to be more risk averse at short horizons than at long horizons. We focus on three papers in particular, that use modern techniques of experimental economics such as real monetary payoffs and that explicitly study or allow us to infer how risk attitudes are affected by the horizon at which a lottery is resolved and paid out: Noussair and Wu (2006), Baucells and Heukamp (2010), and Abdellaoui, Diecidue, and Onculer (2011).[2] Across the three papers, a consistent picture of the quantitative importance of decreasing horizon-dependent risk aversion emerges. While all three possibilities of horizon-dependent risk aversion are present among subjects (decreasing,

[2] For related evidence, see Jones and Johnson (1973); Rachlin and Siegel (1994); Shelley (1994); Keren and Roelofsma (1995); Ahlbrecht and Weber (1997); Öncüler (2000); Sagristano et al. (2002); Coble and Lusk (2010).

constant and increasing), the share of subjects with decreasing risk aversion is between 40% and 60%, which significantly outweighs the share with increasing risk aversion and is at least comparable to the share with constant risk aversion. When averaging across subjects, risk aversion is decreasing in horizon in the vast majority of comparisons and the decrease tends to be statistically significant. We discuss the three papers in detail in Appendix A.1.

Based on this evidence, Eisenbach and Schmalz (2016) propose the theoretical concept of *horizon-dependent risk aversion* and explore implications across several domains using a two-period setup with time-separable expected utility: in period 2, the agent evaluates uncertain consumption as $E[v(c_2)]$ while in period 1, she evaluates uncertain consumption as $E[v(c_1) + \delta u(c_2)]$, where δ is a time discount and v has higher risk aversion than u. In this framework, horizon-dependent risk aversion can explain evidence on consumer demand for short-horizon insurance and for commitment devices to take risk.

To explore the dynamic asset pricing implications of horizon-dependent risk aversion, we depart from the essentially static, time separable preferences of Eisenbach and Schmalz (2016) and model instead a generalization of the Epstein-Zin recursive preferences that relaxes the dynamic consistency axiom of Kreps and Porteus (1978). This approach yields a tractable "pseudo-recursive" model that captures horizon-dependent risk aversion and enables us to study its effects in a standard asset pricing framework following the tradition of Epstein and Zin (1989), Bansal and Yaron (2004), and Hansen et al. (2008). Our pseudo-recursive preferences also allow us to introduce a dynamic inconsistency on *intra-temporal* risk decisions while maintaining the dynamic consistency on *inter-temporal* allocation decisions. In particular, the well documented hyperbolic discounting (e.g. Phelps and Pollak, 1968; Laibson, 1997) or other time inconsistencies concerning inter-temporal decisions do not influence, or cause, the results we derive.[3]

2.1 Dynamic preference model

To simplify the exposition, we present a model with only two levels for the coefficient of relative risk aversion, γ and $\widetilde{\gamma}$. We assume that the agent treats immediate uncertainty with risk aversion γ, and all delayed uncertainty with risk aversion $\widetilde{\gamma}$, with $\widetilde{\gamma}$ lower than γ in line with the experimental evidence. In Appendix A.2, we extend our model to general sequences $\{\gamma_h\}_{h \geq 1}$ of risk aversion at horizon h. As long as risk aversion reaches a constant level beyond a given horizon, closed form solutions similar to those derived in the main

[3]Eisenbach and Schmalz (2016) show that horizon-dependent risk aversion is also conceptually orthogonal to time-varying risk aversion and the habit models of Constantinides (1990); Campbell and Cochrane (1999).

text obtain.[4]

At any time t, we denote by $E_t[\,\cdot\,] = E[\,\cdot \mid \mathcal{I}_t]$ the expectation conditional on $\mathcal{I}_t$, the information set at time t.

Definition 1 (**Dynamic horizon-dependent risk aversion**). *In period t, the agent evaluates the uncertain consumption stream $\{C_\tau\}_{\tau \geq t}$ as*

$$V_t = \left((1-\beta)\, C_t^{1-\rho} + \beta E_t \big[\widetilde{V}_{t+1}^{1-\gamma} \big]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{1}$$

where the continuation value $\widetilde{V}_{t+1}$ satisfies the recursion

$$\widetilde{V}_{t+1} = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1} \big[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}} \big]^{\frac{1-\rho}{1-\widetilde{\gamma}}} \right)^{\frac{1}{1-\rho}}, \tag{2}$$

and the preference parameters satisfy $\beta \in (0,1)$ and $\rho, \gamma, \widetilde{\gamma} > 0$.[5]

The lifetime utility V_t depends on the deterministic current consumption C_t and on the certainty equivalent $E_t\big[\widetilde{V}_{t+1}^{1-\gamma}\big]^{\frac{1}{1-\gamma}}$ of the uncertain continuation value $\widetilde{V}_{t+1}$, where the aggregation of the two periods occurs with constant elasticity of intertemporal substitution given by $1/\rho$ under the subjective time discount β. However, the certainty equivalent of consumption starting at $t+1$ is calculated with relative risk aversion γ, whereas the certainty equivalents of consumption starting at $t+2$ and beyond are calculated with relative risk aversion $\widetilde{\gamma}$. Our model nests the Epstein-Zin model when $\gamma = \widetilde{\gamma}$, and, in turn, nests the standard time-separable model when $\gamma = \widetilde{\gamma} = \rho$. Any difference in the results we derive below under the preferences of Definition 1 to those obtained under the standard Epstein-Zin model thus hinges on $\widetilde{\gamma} \neq \gamma$.

The horizon-dependent risk aversion implies a dynamic inconsistency, as the uncertain consumption stream starting at $t+1$ is evaluated as $\widetilde{V}_{t+1}$ by the agent's self at t and as V_{t+1} by the agent's self at $t+1$:

$$\begin{aligned} \widetilde{V}_{t+1} &= \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1} \big[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}} \big]^{\frac{1-\rho}{1-\widetilde{\gamma}}} \right)^{\frac{1}{1-\rho}} \\ \neq V_{t+1} &= \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1} \big[\widetilde{V}_{t+2}^{1-\gamma} \big]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}. \end{aligned}$$

[4]Our approach with only two levels of risk aversion is analogous to the β-δ framework (Phelps and Pollak, 1968; Laibson, 1997) as a special case of the general non-exponential discounting model of Strotz (1955).

[5]The cases $\rho = 1$, $\gamma = 1$ or $\widetilde{\gamma} = 1$ correspond to log-utility equivalents.

However, the disagreement between the agent's continuation value $\widetilde{V}_{t+1}$ at t and the agent's utility V_{t+1} at $t+1$ arises only for uncertain consumption streams. For any deterministic consumption stream the horizon dependence in Equation (1) becomes irrelevant and we have

$$\widetilde{V}_{t+1} = V_{t+1} = \left((1-\beta) \textstyle\sum_{h \geq 0} \beta^h C_{t+1+h}^{1-\rho} \right)^{\frac{1}{1-\rho}}.$$

Our model therefore implies dynamically inconsistent *risk* preferences while maintaining dynamically consistent *time* preferences, and is orthogonal to existing models of time inconsistency, such as hyperbolic discounting.

2.2 Generalized preference model

In the preferences of Definition 1, we opt for risk adjustments with constant relative risk aversion (CRRA). However, similarly to the Epstein-Zin model, our model of horizon-dependent risk aversion accommodates any preferences in the Chew-Dekel class of betweenness-respecting models (Dekel, 1986; Chew, 1989). The general model is defined as:

Definition 2 (**Generalized dynamic horizon-dependent risk aversion**)**.** *In period t, the agent evaluates the uncertain consumption stream $\{C_\tau\}_{\tau \geq t}$ as*

$$V_t = \left((1-\beta)\, C_t^{1-\rho} + \beta \left(\mathcal{R}_t \big[\widetilde{V}_{t+1} \big] \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{3}$$

where the continuation value $\widetilde{V}_{t+1}$ satisfies the recursion

$$\widetilde{V}_{t+1} = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta \left(\widetilde{\mathcal{R}}_{t+1} \big[\widetilde{V}_{t+2} \big] \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{4}$$

and $\mathcal{R}_t[\,\cdot\,]$ and $\widetilde{\mathcal{R}}_t[\,\cdot\,]$ are certainty-equivalent operators for utility functions $\mathcal{U}$ and $\widetilde{\mathcal{U}}$ in the Chew-Dekel class of betweenness-respecting models.

Examples of certainty equivalent operators other than CRRA could be those of a Campbell and Cochrane (1999) habit model with risk aversions $\gamma > \widetilde{\gamma}$ or those of a Gul (1991) disappointment aversion model with first-order risk aversion coefficients $\theta > \widetilde{\theta}$. As mentioned in our review of the literature, introducing these "exotic" risk adjustments helps explain cross-sectional evidence (Routledge and Zin, 2010; Bonomo et al., 2011; Schreindorfer, 2014; Andries, 2021), orthogonal to the timing and pricing of risk we analyze in this paper and to our notion of horizon-dependent risk aversion. The cross-sectional results derived under the standard Epstein-Zin model would remain valid under the preferences of Definition 2.

At a deeper level, the preferences of Definition 2 allow for great flexibility: Agents could have first-order risk aversion (disappointment aversion or loss aversion) for immediate risk but standard concave utility for longer horizons; they could have time-varying risk aversion for immediate risks only; the gap between their immediate and long-term risk aversions could vary with market conditions; etc. Definition 2 proposes a model of preferences that allows for the analysis of new and complex forms of dynamic inconsistencies within a simple framework.

2.3 Timing of risk and dynamic inconsistency

An agent with the time-inconsistent preferences of Definition 1 or Definition 2 can be either naive or sophisticated about the disagreement between her temporal selves; in addition, she may be able to commit to multi-period strategies or be compelled to re-optimize every period. The valuation of early versus late resolution of uncertainty, derived in Section 3, is by nature a static problem. Its solutions are the same for naive and sophisticated investors, with or without commitment. However, these modeling choices matter for dynamic outcomes, and the asset prices we derive in Section 4.

We follow the tradition of Strotz (1955), and assume that the agent is fully rational and sophisticated when making choices in period t to maximize V_t. Self t realizes that her valuation of future consumption, given by $\widetilde{V}_{t+1}$, differs from the objective function V_{t+1} which self $t+1$ will maximize. The solution then corresponds to the subgame-perfect equilibrium in the sequential game played among the agent's different selves.

We assume no commitment in the one-period framework in Section 4.1 and 4.2, as appropriate for a representative agent who trades and clears the market at all times, and as such cannot precommit to a given strategy — analogous to the approach of Luttmer and Mariotti (2003) for non-geometric discounting. However, we also analyze the implications of letting sophisticated agents commit to buy-and-hold strategies in Section 4.3, e.g. for illiquid assets or periods of liquidity crises in which one-period pricing may break down.[6]

[6]Extending our results to an agent naive about her own dynamic inconsistencies is straightforward and does not present any conceptual challenge. We briefly discuss and derive formal results for this alternative approach in Appendix A.3.

3 Preference for early or late resolution of uncertainty

To what extent do the horizon-dependent risk aversion preferences of Definition 1 affect agents' decisions regarding the timing of information arrivals? To analyze this issue, and determine whether agents have a preference for early or late resolutions of uncertainty, we strictly follow the set up of Epstein et al. (2014). Two consumption streams, subject to the exact same shocks over time, are evaluated at a given time t. In the first case, consumption shocks are revealed gradually when they are realized: the shock affecting consumption at time $t+h$ is revealed at $t+h$, for all horizons $h \geq 1$. In the second case, all future consumption shocks are revealed in the next period, at time $t+1$, even when they affect consumption at a later period: the shock affecting consumption at $t+h$ is revealed at time $t+1$, for all $h \geq 1$.

Crucially, even when she receives early information about her future consumption shocks, the agent cannot act on this information to change her future consumption stream. From the point of view of time t, the distributions of future risks are therefore the same with or without early resolution of uncertainty. In the expected utility framework, the agent would assign the same value to the two consumption streams at time t. However, in the non time-separable models of Epstein and Zin (1989) and our Definition 1, two consumption streams with ex ante identical risks, but different timing for the resolution of uncertainty, can have different values, as we derive below.

Assigning values to the two consumption streams above is a static problem: the agent evaluates the two streams of consumption exactly once. How her preferences change over time, whether she is naive or sophisticated about it or whether she can commit to specific future choices are irrelevant to the relative values she assigns to the two consumption streams, i.e. to her preference for early or late resolution of uncertainty.

3.1 Timing premium

As in Epstein et al. (2014), we present and discuss our formal results for the timing premium under the corner case of a unit elasticity of inter-temporal substitution ($\rho = 1$), and given the long-run risk consumption dynamics

$$\begin{aligned} c_{t+1} - c_t &= \mu_c + \phi_c x_t + \alpha_c \sigma w_{c,t+1}, \\ x_{t+1} &= \nu_x x_t + \alpha_x \sigma w_{x,t+1}, \end{aligned} \tag{5}$$

where the time-varying consumption growth x_t is one dimensional, $|\nu_x| < 1$, and the shocks $w_{c,t}$ and $w_{x,t}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal to each other. Throughout lower-case letters c, x and w denote logs, e.g. $c_t = \log C_t$.[7]

Denote by V_t^* the agent's utility at t if the sequence of shocks $\{w_{c,t+h}, w_{x,t+h}\}_{h\geq 1}$ in consumption process (5) is revealed at $t+1$, and by V_t the agent's utility if uncertainty is resolved over time. The timing premium is defined as

$$\mathrm{TP_t} = 1 - \frac{V_t}{V_t^*}.$$

This represents the fraction of utility, or equivalently the fraction of lifetime consumption, the agent is willing to forgo for an early rather than late resolution of uncertainty.[8]

Proposition 1 (**Timing premium**). *An agent with the horizon-dependent risk aversion preferences of Definition 1 with $\rho = 1$, facing the consumption process (5) has timing premium*

$$\mathrm{TP} = 1 - \exp\left(\frac{1}{2}\left(1 - \left(\gamma - (1+\beta)\left(\gamma - \widetilde{\gamma}\right)\right)\right)\frac{\beta^2}{1-\beta^2}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2 \alpha_x^2\right)\sigma^2\right). \quad (6)$$

Proof. See Appendix B.1. □

Corollary 1. *For an agent with horizon-dependent risk aversion, $\gamma > \widetilde{\gamma}$ unambiguously lowers the timing premium.*

Proof. See Appendix B.2. □

Choosing a consumption stream with an early resolution at date $t+1$ rather than the same consumption stream with late resolutions at dates $\{t+h\}_{h\geq 1}$ corresponds to shifting all future risk, short-term and long-term, to a single next-period risk. It thus matters for the relative values of the two theoretical consumption streams, i.e. for the preference for early or late resolution of uncertainty, that our model of horizon-dependent risk aversion evaluates long-term risks with a different risk aversion than short-term risks. Shifting risks

[7] In Sections 4 and 5, we let the consumption volatility σ vary over time and consider $\rho \neq 1$. Formal derivations of the timing premium for the case with time varying volatility and $\rho = 1$ are in Appendix B.1; the methodology under which we derive numerical estimates of the timing premium when $\rho \neq 1$ is in Appendix D.2.

[8] In following the analysis of Epstein et al. (2014) and assuming only two levels of risk aversion γ and $\widetilde{\gamma}$, we are implicitly mixing two comparisons: gradual resolution versus one-shot resolution and early resolution versus late resolution. In addition, we are placing the early resolution at time $t+1$, exactly in the period where the risk aversion changes from γ to $\widetilde{\gamma}$. We show in Appendix B.5 that the results of Proposition 1 and Corollaries 1 and 2 are robust to (i) allowing for a general decreasing sequence of risk aversions $\{\gamma_h\}_{h=1}^{\infty}$ to show that the result is based on horizon-dependent risk aversion and not on a particular period and (ii) comparing resolution of all uncertainty at $t+1$ to resolution of all uncertainty at $t+2$ to show that the relevant comparison is between early and late resolution, not between gradual and one-shot resolution.

to an earlier horizon is subject to greater risk discounts and thus "costly" when $\gamma > \widetilde{\gamma}$, which lowers the value of early resolution of uncertainty relative to the standard model.[9]

3.2 Preference for early or late resolution of uncertainty?

An agent with the standard Epstein-Zin preferences prefers early resolution of uncertainty if and only if $\gamma > \rho$.[10] This well known result is re-obtained in Proposition 1: when setting $\gamma = \widetilde{\gamma}$, the timing premium of Equation (6) is positive if and only if $\gamma > 1 = \rho$. However, in the horizon-dependent risk aversion model, we obtain a higher threshold: the timing premium is positive if and only if $\gamma > 1 + (1+\beta)(\gamma - \widetilde{\gamma}) > 1 = \rho$. The agent may therefore have a preference for late resolution, even when both risk aversions γ and $\widetilde{\gamma}$ are greater than $\rho = 1$, as long as the decline in risk aversions across horizons is sufficiently large. For example, suppose we set immediate risk aversion $\gamma = 10$ and β close to 1. Then the inequality above shows that the agent will prefer early resolution of uncertainty only if $\widetilde{\gamma} > 5.5$, which is substantially larger than $1 = \rho$.

Corollary 2. *An agent with horizon-dependent risk aversion can prefer late resolution of uncertainty even when all coefficients of relative risk aversion exceed the inverse elasticity of inter-temporal substitution, i.e. even when $\gamma > \widetilde{\gamma} > \rho$.*

Proof. See Appendix B.3. □

The result of Corollary 2 is of particular interest because existing calibrations of the long-run risk model with Epstein-Zin preferences require γ greater than ρ by an order of magnitude to match equilibrium asset pricing moments — hence the high timing premia they imply, and the critique of Epstein et al. (2014). Under horizon-dependent risk aversion, $\gamma \gg \rho$ no longer automatically implies such a strong preference for early resolution of uncertainty. This is true even when the long-run risk aversion also remains above the inverse elasticity of inter-temporal substitution, i.e. when $\widetilde{\gamma} > \rho$. Figure 1 plots the timing premium for both horizon-dependent risk aversion and for standard Epstein-Zin

[9] In principle, the same argument could be made for other dynamically inconsistent models such as hyperbolic discounting. In Appendix B.4, we derive the timing premium under hyperbolic discounting where $\widetilde{\gamma} = \gamma$ but the value V_t uses time discount parameter β while the continuation value $\widetilde{V}_{t+1}$ uses $\widetilde{\beta} > \beta$. We show that the preference for early resolution of uncertainty still holds if and only if $\gamma > \rho$ (as in the standard model), and that the hyperbolic discounting has only a small quantitative effect. Under the calibration of Bansal and Yaron (2004) with constant volatility, $\gamma = 10$ and $\rho = 1$, the timing premium only goes from 27% under $\beta = \widetilde{\beta} = 0.998$ to 22.5% under $\beta = 0.8, \widetilde{\beta} = 0.998$.

[10] To see why, note that in the case where all future shocks are revealed at $t+1$, the shocks to consumption from $t+2$ onward are evaluated with the inverse elasticity of inter-temporal substitution ρ since they are no longer uncertain; whereas, when shocks are revealed over time, variations in consumption from $t+2$ onward are still risky at $t+1$ and thus evaluated with risk aversion γ.

preferences, under the consumption calibration of Bansal et al. (2012) with time-varying volatility. It illustrates the first-order impact that horizon-dependent risk aversion has on the timing premium, and the potential for our model to address the critique of Epstein et al. (2014).

We believe this result is key for several reasons. First, it is worth noting that the recursive utility model has little microeconomic or experimental foundation, contrary to other models of preferences commonly used in finance, e.g. habit (Campbell and Cochrane, 1999) or prospect theory (Barberis et al., 2001). Since the long-run risk model built its success solely on its ability to match macroeconomics evidence, microeconomic inferences should be subject to scrutiny.

Second, we argue, in line with Epstein et al. (2014), that the magnitudes for the timing premia implied by calibrations of the long-run risk model with standard Epstein-Zin preferences are excessive (30% in the calibration of Bansal and Yaron (2004), 85% under Bansal et al. (2012)). There is no direct evidence on the "correct" values of timing premia, by construction a purely theoretical question. However, the microeconomic evidence indicates many individuals behave as if they prefer to delay receiving information (see e.g. Oster et al. (2013) and Oster et al. (2013) in the health economics literature; and Golman et al. (2017) for an extensive survey of such behaviors). Closer to our theoretical framework, investors' inattention to their own wealth disputes the notion of a strong preference for early resolution of consumption risk; even more so because the resulting inertia in their portfolio allocations is costly to them (Brunnermeier and Nagel, 2008; Calvet et al., 2009; Bilias et al., 2010; Andersen et al., 2020). Finally, more risk averse investors are also found to be more inattentive (Karlsson et al., 2009; Alvarez et al., 2012; Sicherman et al., 2016).[11] This is inconsistent with the standard model: from Proposition 1 for the case $\gamma = \widetilde{\gamma}$ (Epstein-Zin preferences), the timing premium is strictly increasing in γ, corresponding to a stronger preference for early resolution of uncertainty, or less inattention for the more risk averse investors. In contrast, our model is consistent with the evidence if more risk averse investors also have more strongly horizon-dependent risk aversion (see Proposition 1 for the respective roles of γ and $\gamma - \widetilde{\gamma}$ in the timing premium).

Though circumstantial, the numerous examples above make the magnitude of the timing premia under the standard long-run risk model appear unreasonable. A representative agent whose implied preferences appear contrary to commonsense considerations raises doubts as to the legitimacy of the long-run risk model, despite its ability to match the

[11] Andries and Haddad (2020) propose a model of information aversion that explains investors' inattention in the data. Beshears et al. (2017) show that information aggregation increases risk taking only with sufficiently large payoffs at sufficiently short horizons. This is consistent with information aggregation effects being quantitatively relevant when risk aversion is sufficiently high (large payoffs and short horizon).

macroeconomic evidence on equilibrium asset prices.[12]

4 Asset prices

In Section 3, we analyze how an agent with horizon-dependent risk aversion would choose between two information structures, one with and one without an early resolution of uncertainty, while keeping fixed the consumption stream she receives. In this section, instead, we keep fixed the information structure (shocks are revealed as they occur over time) and let the agent consider investment choices that generate different future consumption streams. While the timing premium is a static problem, investment choices where financial assets can be traded over time before their payoffs realize are by nature repeated dynamic optimizations.

We consider an agent who trades off the utility cost of purchasing an asset at time t from her current wealth W_t in exchange for the expected utility payoff from the resulting changes to her future wealth W_{t+T}, where T is the time interval after which she re-optimizes and may choose to sell the asset. To derive the equilibrium risk prices determined by a representative agent with horizon-dependent risk aversion, we assume in Sections 4.1 and 4.2 that she re-optimizes every period ($T = 1$). However, in Section 4.3 we also analyze how an agent who commits, or is constrained, to trade less frequently would price risky assets ($T > 1$). As we show below, the time inconsistency in our agent's risk taking behavior results in crucial differences between the $T = 1$ and the $T > 1$ frameworks.

4.1 One-period risk pricing

We derive the marginal pricing of risk in a standard Lucas-tree endowment economy, in which a representative agent with the horizon-dependent preferences of Definition 1 sets equilibrium prices. All decisions are made in sequential one-period problems, where the no-arbitrage condition is automatically satisfied despite the agent's time inconsistent preferences, similar to Luttmer and Mariotti (2003). As discussed in Section 2, we assume the agent is sophisticated, i.e. aware of her own time inconsistency.

Proposition 2 (**Stochastic discount factor**)**.** *A sophisticated agent with the horizon-dependent*

[12] Aggregation theorems for Epstein and Zin (1989) preferences indicate that if most individuals have low or even negative timing premia, so would the marginal, representative, investor who sets prices (Duffie and Lions, 1992).

risk aversion preferences of Definition 1 has a one-period stochastic discount factor (SDF)

$$\Pi_{t,t+1} = \underbrace{\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}}_{\text{(I)}} \times \underbrace{\left(\frac{\widetilde{V}_{t+1}}{E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}}_{\text{(II)}} \times \underbrace{\left(\frac{\widetilde{V}_{t+1}}{V_{t+1}} \right)^{1-\rho}}_{\text{(III)}}. \tag{7}$$

Proof. See Appendix C.1. □

The SDF consists of three multiplicative parts. The first term (I) is standard, capturing the substitution between consumption at time t and at time $t+1$, and is governed by the time discount factor β and the elasticity of inter-temporal substitution $1/\rho$. The second term (II) captures the shocks to the future consumption stream that are realized at $t+1$, comparing the ex-post realized utility $\widetilde{V}_{t+1}$ to its ex-ante certainty equivalent $E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$. The SDF at time t evaluates these $t+1$ shocks with immediate risk aversion γ. An analogous term obtains under standard Epstein-Zin preferences with the difference that, in our model, the $t+1$ utility of self t ($\widetilde{V}_{t+1}$) differs from that of self $t+1$ (V_{t+1}). Finally, the third term (III) captures the dynamic inconsistency in our model by loading on the disagreement between selves t and $t+1$ when evaluating their $t+1$ utilities, given by the ratio $\widetilde{V}_{t+1}/V_{t+1}$. Horizon-dependent risk aversion affects the pricing of shocks through terms (II) and (III) in the stochastic discount factor of Equation (7), i.e. via the difference between $\widetilde{V}_{t+1}$ andV_{t+1}; only term (I) is unchanged from the standard long-run risk framework.

To model the macro-economic shocks that determine asset prices, we assume a log-normal endowment consumption process in line with the long-run risk literature where both the expected growth and uncertainty are time varying (e.g. Bansal and Yaron, 2004; Bansal et al., 2012):

$$\begin{aligned} c_{t+1} - c_t &= \mu_c + \phi_c x_t + \alpha_c \sigma_t w_{c,t+1} \\ x_{t+1} &= \nu_x x_t + \alpha_x \sigma_t w_{x,t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + \nu_\sigma \left(\sigma_t^2 - \sigma^2 \right) + \alpha_\sigma w_{\sigma,t+1} \end{aligned} \tag{8}$$

Both x_t and σ_t are one dimensional, $|\nu_x| < 1$ and $|\nu_\sigma| < 1$, and the three shocks $w_{c,t+1}$, $w_{x,t+1}$ and $w_{\sigma,t+1}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal to each other.[13]

Before deriving the pricing of the shocks $\{w_{c,t}, w_{x,t}, w_{\sigma,t}\}$ under horizon-dependent

[13] These assumptions can be generalized. We employ them here to make our results comparable to those of Bansal and Yaron (2004) and Bansal et al. (2012).

risk aversion, we briefly explain the role each shock plays in the long-run risk model. This allows us to clarify the comparisons we draw later between the standard model and ours.

The consumption process (8) accounts for time variation in expected consumption growth through the state variable x_t and its shocks $\{w_{x,t}\}$, in addition to the immediate consumption shocks captured by $\{w_{c,t}\}$, i.e. each period's deviations from the trend. Both sets of shocks are consistent with direct evidence in consumption data (Hansen et al., 2008). In addition, empirical analyses of cross-sectional asset returns demonstrate that the shocks to x_t are priced (Hansen et al., 2008; Bryzgalova and Julliard, 2021) and help capture, in particular, the value premium of Fama and French (1993). These results are the bedrock of the long-run risk framework: they provide the foundation for combining expected growth shocks in the consumption process (8) with preferences that can price them: the recursive non time-separable utility model of Epstein-Zin in the standard approach.

Consumption process (8) also accounts for time variation in the volatility σ_t via the shocks $\{w_{\sigma,t}\}$. In contrast to the consumption level and growth shocks $\{w_{c,t}\}$ and $\{w_{x,t}\}$, such volatility shocks are not easily observable in the data. They are, however, necessary to generate time-varying risk premia and for the model to capture the volatility puzzle (Shiller, 1981).[14]

We now derive how horizon-dependent risk aversion affects the pricing of these consumption shocks. As in the standard Epstein-Zin model, exact closed-form solutions only obtain in the case $\rho = 1$.[15] To analyze the impact of variation in the elasticity of intertemporal substitution $1/\rho$ on asset prices, necessary for comparisons to usual calibrations of the standard long-run risk model such as Bansal and Yaron (2004) and Bansal et al. (2012), we derive approximate solutions to the value function of Definition 1 assuming $\beta \approx 1$, i.e. for a rate of time discount close to zero; a reasonable approximation at the monthly frequency.[16] This approximation method determines the value function of Definition 1 up to a constant, and allows us to analyze equity premia but not risk-free assets: our formal results below are restricted to the analysis of excess returns. Given that the dynamic inconsistency in our preference model concerns only intra-temporal choices across risky assets, and not intertemporal decisions, we believe this is a valid approach: our results on equity premia should capture most of the impact of horizon-dependent risk aver-

[14]Just like the standard Epstein-Zin model, our framework allows for the analysis of additional shocks in the consumption process (8), e.g. jumps. Drechsler and Yaron (2010) show such shocks help capture other features in the data, notably the pricing of variance swaps and their ability to predict market equity returns (Bollerslev et al., 2009).

[15]The closed-form solutions for $\rho = 1$ are derived in Appendix C.5.

[16]E.g. Bansal et al. (2012) use $\beta = 0.9989$. The description of the $\beta \approx 1$ methodology is in Appendix C.2.

sions on asset prices. In fact, in the case $\rho = 1$ for which we derive closed-form solutions, the risk-free rate is strictly unchanged by $\widetilde{\gamma} \neq \gamma$ (see Appendix C.5); which validates our $\beta \approx 1$ methodology and the exclusive analysis of equity premia.

The results we derive remain valid as long as $1/\rho \geq 1, \gamma \geq 1$, and $\widetilde{\gamma} \geq 1$, an assumption we maintain throughout Section 4 — a constraint the standard long-run risk model must also satisfy to match asset pricing data.

From Proposition 2, the variable of interest in our analysis is the ratio $\widetilde{V}_{t+1}/V_{t+1}$, i.e. the wedge between the $t+1$ value of self t and of self $t+1$. Taking logs, we obtain:

Lemma 1. *Under the Lucas-tree endowment process* (8) *and the preferences of Definition 1 with* $\beta \approx 1$,

$$\widetilde{v}_{t+1} - v_{t+1} = \frac{1}{2}\left(\gamma - \widetilde{\gamma}\right)\left(\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right)\sigma_{t+1}^2 + \psi_v(\widetilde{\gamma})^2\, \alpha_\sigma^2\right) > 0, \tag{9}$$

where ϕ_v is independent of both γ and $\widetilde{\gamma}$, and $\psi_v(\widetilde{\gamma})$ is independent of γ:

$$\phi_v = \frac{\phi_c}{1 - \nu_x} > 0, \tag{10}$$

$$\psi_v(\widetilde{\gamma}) = -\frac{1}{2}\,\frac{\widetilde{\gamma} - 1}{1 - \nu_\sigma}\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right) < 0. \tag{11}$$

Proof. See Appendix C.2. □

The wedge in Equation (9) reflects that the $t+1$ value of self t and of self $t+1$ only differ in their valuation of uncertain consumption starting from $t+2$ onward, which is governed by volatility σ_{t+1}. Self t evaluates this uncertainty with low risk aversion $\widetilde{\gamma}$ while self $t+1$ evaluates it with high risk aversion γ. The wedge $\widetilde{v}_{t+1} - v_{t+1}$ is therefore positive, and increasing in $\gamma - \widetilde{\gamma}$ and in the amount of uncertainty driven by volatility σ_{t+1}.

From terms (II) and (III) in the stochastic discount factor (7), horizon-dependent risk aversion affects only the pricing of shocks that correlate with variations in the ratio $\widetilde{V}_{t+1}/V_{t+1}$, and therefore with variations in σ_{t+1}. From Lemma 1, we derive the result:

Proposition 3. *Under the Lucas-tree endowment process* (8)*, horizon-dependent risk aversion does not affect the equilibrium risk prices of shocks to consumption levels* (*immediate consumption shocks and consumption growth shocks*).

Proof. See Appendix C.3. □

If the agent faced only consumption level shocks, she could anticipate how her future self re-optimizes, and her time inconsistency would not cause additional uncertainty in her one-period decision making. Only unanticipated changes in her intra-temporal decisions, when the quantity of risk varies through time, interact with her time inconsistency to modify asset prices compared to the time consistent model.

Let us now turn to the pricing of all shocks, including shocks to volatility σ_t. From Lemma 1 we obtain:

Proposition 4. *Under the Lucas-tree endowment process* (8) *and the preferences of Definition 1 with* $\beta \approx 1$*, the stochastic discount factor satisfies*

$$\begin{aligned}\pi_{t,t+1} - E_t[\pi_{t,t+1}] = &-\gamma\alpha_c\sigma_t w_{c,t+1} - (\gamma-\rho)\,\phi_v\alpha_x\sigma_t w_{x,t+1} \\ &+ \left[(\gamma-\rho) + (1-\rho)\,(\gamma-\widetilde{\gamma})\,\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\left|\psi_v(\widetilde{\gamma})\right|\alpha_\sigma w_{\sigma,t+1}. \qquad (12)\end{aligned}$$

Proof. See Appendix C.3. □

In line with Proposition 3, we find that the pricing of the immediate consumption shocks, given by the term $\gamma\alpha_c\sigma_t w_{c,t+1}$, and the pricing of the consumption growth shocks, given by the term $(\gamma-\rho)\,\phi_v\alpha_x\sigma_t w_{x,t+1}$, depend only on the immediate risk aversion γ and are unchanged from the standard long-run risk model. In contrast, the pricing of shocks to consumption volatility is affected by horizon-dependent risk aversion and Equations (11) and (12) imply the following result.

Corollary 3. *Under the Lucas-tree endowment process* (8) *and the preferences of Definition 1 with* $\beta \approx 1$*, the volatility shocks have lower risk prices* (*in absolute magnitude*) *than in the standard model with* $\gamma = \widetilde{\gamma}$*:*

$$\left[(\gamma-\rho) + (1-\rho)\,(\gamma-\widetilde{\gamma})\,\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\left|\psi_v(\widetilde{\gamma})\right| < (\gamma-\rho)\left|\psi_v(\gamma)\right|. \qquad (13)$$

Proof. See Appendix C.4. □

Our model yields a negative price for volatility shocks: assets with payoffs that covary with aggregate volatility provide valuable insurance, consistent with the existing long-run risk literature and the observed evidence from variance swaps and option straddles returns (see e.g. Christoffersen et al., 2013). However, shocks to volatility make future intra-temporal decisions uncertain and, for this reason, how risky they are depends on horizon-dependent risk aversions. Due to the lower long-run risk aversion $\widetilde{\gamma} < \gamma$, their implied long-run uncertainty does not "feel" as costly, which reduces the value of hedges against volatility shocks.

The pricing of shocks to consumption levels, i.e. to C_{t+1}/C_t and to x_t, allows the standard long-run risk model to match the market equity premium and the value premium, even when assuming volatility is constant in process (8) (see e.g. Bansal and Yaron, 2004; Hansen et al., 2008). From Proposition 3, the pricing of these shocks is exactly the same under horizon-dependent risk aversion.

The pricing of shocks to consumption volatility in process (8) allows the standard long-run risk model to obtain time-varying risk premia which, in turn, explain the market volatility puzzle and the predictability of the index price-dividend ratio. Proposition 4 shows that time variation in risk premia arises, both in the standard model and in ours, from the pricing of the consumption level shocks and *not* from the pricing of consumption volatility shocks. Time-varying risk premia resulting from time-varying volatility thus remain unchanged by the introduction of horizon-dependent risk aversion.

In sum, the preference model of Definition 1 with horizon-dependent risk aversion retains the ability of the standard model to match the market equity premium and the value premium, and to explain the volatility puzzle and the predictability puzzle. We can therefore focus our analysis on how the lower pricing of volatility risk under horizon-dependent risk aversion affects the relative consumption risk contributions to the equity premium (macro-announcement premium in Section 4.2) and the term-structure of equity risk premia (in Section 4.3).

4.2 Market returns

In line with Bansal and Yaron (2004); Bansal et al. (2012), we model the dividend growth process as

$$d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t w_{c,t+1} + \alpha_d \sigma_t w_{d,t+1}, \tag{14}$$

where $d_t = \log D_t$, and the shocks $w_{d,t}$ are i.i.d. $\mathcal{N}(0, 1)$ and orthogonal to the consumption shocks $w_{c,t}$, $w_{x,t}$ and $w_{\sigma,t}$ of process (8). The coefficient ϕ_d captures the link between the mean consumption growth and the mean dividend growth, and χ the correlation between immediate consumption and dividend shocks in the business cycle.

As in Section 4.1, we assume a representative agent with the preferences of Definition 1 trades and clears the market every period, and thus cannot commit to any pre-determined strategy.

4.2.1 Equity premium

The equity premium is defined as $\mathrm{EP}_t = \log E_t R_{m,t+1} - \log R_{f,t}, \forall t$, where $\{R_{m,t}\}$ are the market returns and $\{R_{f,t}\}$ the time-varying risk-free rate. We follow the standard log-linearization method to formally derive the equity premium, i.e. a first-order approximation around the average log price dividend ratio $\overline{p-d}$.

Proposition 5 (**Equity Premium**). *Under the Lucas-tree endowment process* (8)*, the dividend process* (14)*, and the preferences of Definition 1 with $\beta \approx 1$, the equity premium at time t is*

$$\begin{aligned}\mathrm{EP}_t = {} & \gamma\chi\alpha_c^2\sigma_t^2 + (\gamma-\rho)\,\phi_v\lambda_{m,x}\alpha_x^2\sigma_t^2 \\ & + \left[(\gamma-\rho) + (1-\rho)\,(\gamma-\widetilde{\gamma})\,\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\left|\psi_v(\widetilde{\gamma})\right|\lambda_{m,\sigma}\,(\gamma,\widetilde{\gamma})\,\alpha_\sigma^2. \end{aligned} \tag{15}$$

$\lambda_{m,x}$ is the market's loading on $\{w_{x,t}\}$ and $-\lambda_{m,\sigma}(\gamma,\widetilde{\gamma})$ its loading on $\{w_{\sigma,t}\}$:

$$\begin{cases}\lambda_{m,x} = \frac{\kappa}{1-\kappa\nu_x}\,(\phi_d - \rho\phi_c) \\ \lambda_{m,\sigma}(\gamma,\widetilde{\gamma}) = \frac{1}{2}\frac{\kappa}{1-\kappa\nu_\sigma} \times \begin{cases}\left((\gamma-1)\,(\gamma-\rho) - (1-\rho)\,(\gamma-\widetilde{\gamma})\,\nu_\sigma\right)\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) \\ -\left((\gamma-\chi)^2\,\alpha_c^2 + ((\gamma-\rho)\,\phi_v - \lambda_{m,x})^2\,\alpha_x^2 + \alpha_d^2\right)\end{cases}\end{cases},$$

where $\kappa = \frac{\exp\left(\overline{p-d}\right)}{1+\exp\left(\overline{p-d}\right)}$, such that $\lambda_{m,\sigma}(\gamma,\widetilde{\gamma}) < \lambda_{m,\sigma}(\gamma,\gamma)$ when $\gamma > \widetilde{\gamma}$.

Proof. See Appendix C.6. □

The first term in Equation (15), $\gamma\chi\alpha_c^2\sigma_t^2$, is the share of the equity premium coming from the market returns' loading χ on the immediate consumption shocks $\{w_{c,t}\}$; the second term, $(\gamma-\rho)\,\phi_v\lambda_{m,x}\alpha_x^2\sigma_t^2$, is the share coming from the market returns' loading $\lambda_{m,x}$ on the consumption growth shocks $\{w_{x,t}\}$. Propositions 4 and 5 show that both the model-implied covariations of market returns with these consumption level shocks and the pricing of these shocks are unchanged by horizon-dependent risk aversion.

The third term in Equation (15), $\left[(\gamma-\rho) + (1-\rho)\,(\gamma-\widetilde{\gamma})\,\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\left|\psi_v(\widetilde{\gamma})\right|\lambda_{m,\sigma}\,(\gamma,\widetilde{\gamma})\,\alpha_\sigma^2$, is the share of the equity premium coming from the market returns' loading $-\lambda_{m,\sigma}\,(\gamma,\widetilde{\gamma})$ on the consumption volatility shocks $\{w_{\sigma,t}\}$. Corollary 3 shows that horizon-dependent risk aversion with $\gamma > \widetilde{\gamma}$ generates a lower price of volatility risk (in absolute magnitude). Proposition 5 shows further that, under horizon-dependent risk aversion, market returns covary less (in absolute terms) with volatility shocks: $\lambda_{m,\sigma}\,(\gamma,\widetilde{\gamma}) < \lambda_{m,\sigma}\,(\gamma,\gamma)$. Combined, these two results imply that horizon-dependent risk aversion unambiguously reduces the contribution of consumption volatility shocks to the equity premium.

4.2.2 Macroeconomic announcement premium

The contribution of each of the shocks in process (8) to the equity premium has direct implications on how to interpret the evidence on the macroeconomic announcement premium. The literature documents large returns on equities around pre-scheduled macroeconomic news releases. Savor and Wilson (2013) consider three kinds of announcements (inflation, labor market, and Federal Open Market Committee (FOMC) meetings) from 1958 to 2009 and find that the average excess returns on U.S. equities on the announcement days account for over 60% of the annual equity premium in their sample. Lucca and Moench (2015) find excess returns of about 50% on and around FOMC announcements in the 1980 to 2011 period. Ai and Bansal (2018) extend their analysis to five kinds of announcements (those of Savor and Wilson (2013) plus releases of GDP data and of the ISM manufacturing report), covering about 30 days per year in their sample (1961 to 2014), and find the average excess returns on macroeconomic announcement days account for 55% of the equity premium.

Most macroeconnomic announcements (e.g. inflation, FOMC meetings) are not direct signals about the consumption shocks as we model them in process (8). The announcements however, such as monetary policy, may be influenced by expectations about growth and uncertainty, and, as such, are informative about the risks we consider in our framework. E.g. Cieslak and Pang (2021) study the channels through which the Fed affects asset prices (bonds and stocks), and find risk premium shocks generate nearly 70% of the average FOMC-day increase in stock returns, while monetary easing shocks account for only 25%. Accordingly, Ai and Bansal (2018) formalize the macroeconomic announcement premium as reflecting all information about the next-period continuation value that is not immediately revealed in consumption, corresponding to the shocks $\{w_{x,t}\}$ and $\{w_{\sigma,t}\}$ in our framework. Ai et al. (2021) instead propose a production economy model in which the macroeconomic announcement premium captures information on productivity growth shocks, similar to the consumption growth shocks $\{w_{x,t}\}$ in process (8).[17] We take an agnostic approach and use estimates of the macroeconomic announcement premium to provide bounds on the relative contributions of the immediate consumption shocks, of the consumption growth shocks and of the volatility shocks. We then use these bounds to discipline the calibration of our model in Section 5.

If we assume that macroeconomic announcements reveal only information about con-

[17] Ai et al. (2021) do not model variations in consumption volatility and the comparison to our economy is not exact. Wachter and Zhu (2022) propose a model where macroeconomic announcements reduce the uncertainty of disaster risks, orthogonal to the shocks we consider.

sumption growth, then Proposition 5 implies that the calibration of our model must satisfy

$$\frac{\mathrm{EP} - [\mathrm{EP}]_{\alpha_x=0}}{\mathrm{EP}} \approx \frac{(\gamma - \rho)\, \phi_v \lambda_{m,x} \alpha_x^2 \sigma^2}{E[\mathrm{EP}_t]} \geq \mathrm{MAP}, \tag{16}$$

where MAP is the macro-announcement premium, $\mathrm{EP} = \log ER_{m,t+1} - \log ER_{f,t}$ is the unconditional equity premium, and $[\mathrm{EP}]_{\alpha_x=0}$ is the unconditional equity premium in an economy with no consumption growth risk.[18] The inequality in (16) captures that information about the consumption growth shocks $\{w_{x,t}\}$ may also arrive on days other than those used to calculate the macroeconomic announcement premium, so the contribution to the equity premium of the pricing of these shocks must exceed the share of 55% estimated in Ai et al. (2018).

If, instead, we assume that macroeconomic announcements also reveal information about future consumption volatility, then we obtain the less constraining bound

$$\begin{aligned} &\frac{\mathrm{EP} - [\mathrm{EP}]_{\alpha_x=0,\alpha_\sigma=0}}{\mathrm{EP}} \\ &\approx \frac{(\gamma - \rho)\, \phi_v \lambda_{m,x} \alpha_x^2 \sigma^2 + \left[(\gamma - \rho) + (1 - \rho)\,(\gamma - \widetilde{\gamma})\, \frac{1 - \nu_\sigma}{\widetilde{\gamma} - 1}\right] \left|\psi_v(\widetilde{\gamma})\right| \lambda_{m,\sigma}\,(\gamma, \widetilde{\gamma})\, \alpha_\sigma^2}{E\,[\mathrm{EP}_t]} \\ &\geq \mathrm{MAP}, \end{aligned} \tag{17}$$

where $[\mathrm{EP}]_{\alpha_x=0,\alpha_\sigma=0}$ is the unconditional equity premium in an economy with no consumption growth or volatility risk.[18]

Satisfying inequality (17) is relatively easy in calibrations of the standard long-run risk model where consumption volatility shocks command a high risk premium. However, by reducing the relative contribution of the consumption growth shocks, such calibrations automatically make satisfying inequality (16) more difficult. They require that macroeconomic announcements be interpreted as providing information about consumption volatility rather than consumption growth, to explain the evidence in the data.

From Equations (16) and (17), satisfying the macroeconomic announcement premium bounds in the standard model with $\widetilde{\gamma} = \gamma$ depends entirely on the wedge $\gamma - \rho$, which also determines the timing premium under the preferences of Epstein-Zin (as shown in Section 3). In response to the critique of Epstein et al. (2014), Ai and Bansal (2018) therefore argue that the high macroeconomic announcement premium in the data can be viewed as evidence of a strong preference for early resolutions of uncertainty. Allowing for horizon-

[18] The formal solutions for EP and $[\mathrm{EP}]_{\alpha_x=0}$ and $[\mathrm{EP}]_{\alpha_x=0,\alpha_\sigma=0}$ are provided in Appendix C.6. The relation in Equations (16) and (17) is approximate because $E[\mathrm{EP}_t]$ does not account for the Jensen terms in the unconditional equity premium EP.

dependent risk aversion with $\gamma > \widetilde{\gamma}$ helps decouple microeconomic interpretations regarding the preference for early or late resolution from the evidence on the macroeconomic announcement premium by strictly lowering the timing premium (Corollary 1) while increasing the contribution of the consumption growth shocks to the equity premium. As we show in our calibration of Section 5, matching the equity premium while satisfying the macroeconomic announcement premium bounds no longer implies a strong preference for early resolution of uncertainty.

4.3 Term-structure of equity returns

We turn to the implications of horizon-dependent risk aversion for the term-structure of equity risk pricing, both its average slope and its time variation. This analysis is crucial to determine how our model differs from, and may improve upon, the standard long-run risk framework in capturing asset prices in the data.

In line with the specification of van Binsbergen and Koijen (2017) and Gormsen (2021), we consider one-period holding returns for dividend strip futures with maturity h, realized at $t+1$ as

$$R^F_{t+1,h} = \frac{P_{t+1,h-1}/P_{t,h}}{B_{t+1,h-1}/B_{t,h}}, \forall h \geq 1,$$

where, at any time t, $P_{t,h}$ is the price of an asset that pays the dividend D_{t+h} in $h \geq 1$ periods, $B_{t,h}$ is the price of a risk-free bond that pays \$1 in $h \geq 1$ periods; and $P_{t,0} = D_t$, $B_{t,0} = \$1$.

4.3.1 Liquid markets, one period pricing

We first assume that prices are set by a representative agent who trades and clears the market every period, as in Sections 4.1 and 4.2.

Proposition 6 (**One-period pricing of dividend strip futures**). *Assuming one-period pricing, the expected returns and Sharpe ratios of dividend strip futures under the Lucas-tree endowment process (8), the dividend process (14), and the preferences of Definition 1 with $\beta \approx 1$, are determined, for any time t and maturity $h \geq 1$ as*

$$\log E_t\left[R^F_{t+1,h}\right] = \phi_h \sigma_t^2 + \left(a_h - (\gamma - \widetilde{\gamma})\, b_h\right) \psi_h \alpha_\sigma^2, \tag{18}$$

$$\mathrm{SR}_t\left(R^F_{t+1,h}\right) = \frac{1 - 1\Big/E_t\left[R^F_{t+1,h}\right]}{\sqrt{\exp\left(\Phi_h \sigma_t^2 + \psi_h^2 \alpha_\sigma^2\right) - 1}}, \tag{19}$$

where

$$\begin{cases} \phi_h = \gamma\chi\alpha_c^2 + \frac{\left(\gamma - \rho\nu_x^{h-1}\right)\left(1-\nu_x^{h-1}\right)}{(1-\nu_x)^2}\phi_c\phi_d\alpha_x^2 > 0 \\ \Phi_h = \chi^2\alpha_c^2 + \left(\frac{1-\nu_x^{h-1}}{1-\nu_x}\right)^2\phi_d^2\alpha_x^2 + \alpha_d^2 \\ a_h > a_h - \left(\gamma - \widetilde{\gamma}\right) b_h > 0 \end{cases} \tag{20}$$

and coefficients $\{\psi_h, a_h, b_h\}$ vary with the maturity h and the preference parameters $\{\gamma, \rho\}$ but not with $\widetilde{\gamma}$ (see Appendix C.7 for the closed-from solutions for $\{\psi_h, a_h, b_h\}$).

Proof. See Appendix C.7. □

In the term-structure of equity risk pricing of Equation (18), the first term $\phi_h\sigma_t^2$ represents the pricing of the dividend strip futures' loadings on the immediate consumption shocks $\alpha_c\sigma_t w_{c,t}$ and on the consumption growth shocks $\alpha_x\sigma_t w_{x,t}$. From Equation (20), ϕ_h is unambiguously positive and increasing such that $\phi_h\sigma_t^2$ in Equation (18) contributes towards an upward sloping term-structure of equity risk premia. This is consistent with the notion of long-run risk: with riskier long-horizon assets equity risk premia are increasing in the standard model. As in our previous results, the pricing of the consumption level shocks is unaffected by horizon-dependent risk aversion so the long-run risk "push" towards an upward sloping term-structure of equity premia remains.

The second term $\left(a_h - \left(\gamma - \widetilde{\gamma}\right) b_h\right)\psi_h\alpha_\sigma^2$ in Equation (18) corresponds to the pricing of the dividend futures' loading on the volatility shocks $\alpha_\sigma w_{\sigma,t}$. In contrast to the first term $\phi_h\sigma_t^2$, and similar to our previous results, horizon-dependent risk aversion with $\gamma > \widetilde{\gamma}$ unambiguously reduces the pricing of volatility shocks (in absolute value) compared to the standard long-run risk model: from $a_h > a_h - \left(\gamma - \widetilde{\gamma}\right) b_h > 0$ in Equation (20), we obtain $\left(a_h - \left(\gamma - \widetilde{\gamma}\right) b_h\right) |\psi_h|\,\alpha_\sigma^2 < a_h\,|\psi_h|\,\alpha_\sigma^2$ for all h. Calibrations of the consumption process (8), dividend process (14), and preference parameters $\{\gamma, \rho\}$ under which ψ_h is positive and increasing thus imply that horizon-dependent risk aversion with $\gamma > \widetilde{\gamma}$ generates a *flatter* term-structure of equity risk prices compared to the standard long-run risk model, albeit still an increasing one. As we show in Section 5, this is what we obtain in the calibration of Bansal et al. (2012).

In the Sharpe ratios of Equation (19), the standard deviation of dividend strip futures' returns at a given horizon h (the denominator in Equation (19)) is unaffected by the wedge $\gamma > \widetilde{\gamma}$. Given Φ_h positive and increasing (see Equation (20)), as is ψ_h in usual calibration of the long-run risk model (e.g. Bansal et al. (2012)), dividend strip future returns are increasingly volatile at longer horizons. Combined with the flatter slope for the term-structure of expected returns, horizon-dependent risk aversion unambiguously results in a lower slope, and potentially a downward sloping term-structure of equity Sharpe ratios. This is what we obtain at medium to long-horizon for some calibrations of $\gamma > \widetilde{\gamma}$ in Section 5.

Proposition 7 (**Term-structure dynamics**). *Under the conditions of Proposition 6, the slope of the term-structure of dividend strip futures' expected returns varies with the index price-dividend ratio according to*

$$\operatorname{cov}\left(\log E_t\left[R^F_{t+1,h}\right], \log \frac{P_t}{D_t}\right) = -\phi_h \frac{\lambda_{m,\sigma}(\gamma,\widetilde{\gamma})}{\kappa} \frac{\alpha_\sigma^2}{1-\nu_\sigma^2}, \quad \forall h \geq 1, \tag{21}$$

where $\lambda_{m,\sigma}(\gamma,\widetilde{\gamma})\,/\kappa$ is defined in Proposition 5 and ϕ_h in Proposition 6.

Proof. See Appendix C.8. □

From Equation (18), the only source of time-series variations in the expected returns of dividend strip futures at horizon h comes from their loading ϕ_h on the time varying consumption volatility σ_t^2, where ϕ_h is positive and increasing in h. Since the market index dynamics also load on the consumption volatility σ_t^2, via the term $\lambda_{m,\sigma}(\gamma,\widetilde{\gamma})$ in Proposition 5, Equation (21) obtains. Periods of high consumption volatility correspond to a lower index value relative to dividends, but to a steeper slope for the term-structure of expected dividend strip returns, resulting in a negative relation in Equation (21).

This result is consistent with the empirical evidence in Gormsen (2021), who shows the term-structure is counter-cyclical: more upward sloping in "bad" times, when the index price level is relatively low.[19]

4.3.2 Illiquid markets, buy-and-hold pricing

The results of Propositions 6 and 7 report equilibrium asset prices under the representative agent assumption and the one-period trading paradigm, analogous to our analysis of index returns in Proposition 5. However, the assets via which we derive the term-structure of equity risk premia — dividend strip futures with increasing maturities $h \geq 1$ — are traded in markets considerably less liquid and transparent than the index returns (Bansal et al., 2021). In general, it is sub-optimal to trade every period when facing transaction

[19]Because $\lambda_{m,\sigma}(\gamma,\widetilde{\gamma}) < \lambda_{m,\sigma}(\gamma,\gamma)$, the covariation in Equation (21) is lower, in absolute value, than under the standard model. Gormsen (2021), however, analyzes the regression of future returns on the current dividend-price ratio, corresponding in our model to $\log E_t\left[R^F_{t+1,h}\right]$ regressed on $\log \frac{D_t}{P_t}$, for which the β loadings are derived in Appendix C.8, and their magnitude when $\gamma > \widetilde{\gamma}$ relative to $\gamma = \widetilde{\gamma}$ is ambiguous. Figure 6 displays the term-structures of $\{\beta_h\}$ for the calibrations of $\{\gamma,\widetilde{\gamma}\}$ we consider in Section 5. Though the model generates lower covariations with the index dividend-price ratio than the point estimates in Gormsen (2021), they remain within their 95% confidence intervals.

costs and/or information costs (see e.g. Alvarez et al., 2012; Abel et al., 2013; Andries and Haddad, 2020), a theoretical result that is consistent with the low frequencies of investor trading in the data (e.g. once a year in Sicherman et al., 2016).

Infrequent trading, whether for illiquid assets or during periods of high overall market illiquidity, may affect pricing under horizon-dependent risk aversion due to the preferences' time inconsistency.[20] Taking Propositions 6 and 7 directly to the data, without allowing for potential deviations from the one-period framework, would then be incorrect under certain market conditions. To address this issue, we consider the case of an investor who, when valuing an asset with payoff at maturity h, assumes she will hold on to it until maturity, thus avoiding all transaction and/or information costs:

Proposition 8 (**Term-structure of equity returns in illiquid markets**). *Assuming prices are set by buy-and-hold strategies, the slope of the term-structure of dividend strip futures' expected returns under the Lucas-tree endowment process* (8)*, the dividend process* (14)*, and the preferences of Definition 1 with* $\beta \approx 1$*, calculated at* $\sigma_t = \sigma$ *satisfies*

$$\begin{aligned}\lim_{h\to\infty}&\left(\log E_t\left[R^F_{t+1,h}\right] - \log E_t\left[R^F_{t+1,1}\right]\right)\\ &= -\left(\gamma - \widetilde{\gamma}\right)\chi\alpha_c^2\sigma^2 + \widetilde{\gamma}\phi_c\phi_d\left(\frac{\alpha_x}{1-\nu_x}\right)^2\sigma^2 + \Psi(\widetilde{\gamma})\left(\frac{\alpha_\sigma}{1-\nu_\sigma}\right)^2, \qquad (22)\end{aligned}$$

where

$$\Psi(\widetilde{\gamma}) = \frac{1}{8}\left[\left(\widetilde{\gamma}^2\alpha_c^2 + \widetilde{\gamma}^2\phi_c^2\left(\frac{\alpha_x}{1-\nu_x}\right)^2\right)^2 - \left(\left(\widetilde{\gamma}-\chi\right)^2\alpha_c^2 + \left(\widetilde{\gamma}\phi_c - \phi_d\right)^2\left(\frac{\alpha_x}{1-\nu_x}\right)^2\right)^2\right].$$

Proof. See Appendix C.9. □

Equation (22) determines the limit slope of the term-structure of equity risk premia: the difference between the infinite-horizon and the next-horizon dividend strip futures' expected returns.

The second and third terms in Equation (22), $\widetilde{\gamma}\phi_c\phi_d\left(\frac{\alpha_x}{1-\nu_x}\right)^2\sigma^2$ and $\Psi(\widetilde{\gamma})\left(\frac{\alpha_\sigma}{1-\nu_\sigma}\right)^2$, correspond to the limit slope under the standard long-run risk model, calculated with long-term risk aversion $\widetilde{\gamma}$. Since the long-term assets are valued under buy-and-hold strategies in Proposition 8, they are priced via their corresponding horizon-dependent risk aversion

[20] The effects are in addition to the risk premium directly attributable to liquidity constraints (e.g. Acharya and Pedersen, 2005; Lee, 2011; Muir, 2017). See also Duffie (2010) and Tirole (2011) for surveys of the literature on liquidity. Our approach here is complementary since our focus is on how illiquidity, in the form of low trading frequencies, affects the term-structure of risk premia in the presence of time inconsistent preferences.

$\widetilde{\gamma}$. As in the standard model, these two terms contribute to higher expected returns in the long-run, i.e. an upward sloping price of equity risk, albeit less so if $\widetilde{\gamma}$ is low.

The first term in Equation (22), $-\left(\gamma-\widetilde{\gamma}\right)\chi\alpha_c^2\sigma^2$, captures the impact of the horizon-dependent risk aversion model: $\gamma>\widetilde{\gamma}$ unambiguously pushes the pricing of next-horizon equity risk up relative to the long-horizon, so the term-structure may become downward sloping.[21]

Proposition 8 therefore formalizes the intuition that time inconsistency in investors' risk aversion affects asset valuations more if choices are less dynamic. For buy-and-hold investors, the relation between horizon-dependent risk aversion and horizon-dependent risk premia becomes tight: since long-horizon asset are priced via the low long-horizon risk aversion, they can require overall lower risk compensation than short-horizon assets despite their higher outright long-run risk.

We acknowledge that the partial equilibrium results of Proposition 8 are not directly comparable, without a fully-fledged model for the market clearing conditions of illiquid assets, to the general equilibrium risk pricing of Propositions 5, 6 and 7 derived for a representative agent in the one-period framework. However, we appeal to the growing literature that establishes that households' demand influences general equilibrium outcomes and therefore affects prices, in particular via intermediaries holdings (He and Krishnamurthy, 2013; Koijen and Yogo, 2019; Haddad and Muir, 2021), to take the limit result of Equation (22) to the data in Section 5; and verify that horizon-dependent risk aversion interacting with time-varying market liquidity induces dynamics in the term-structure of risk pricing consistent with the observed evidence.

5 Data and model calibration

We assess how well the horizon-dependent risk aversion model fits the data and its potential to explain the documented puzzles on the slope and dynamics of the term-structure of equity risk premia; while implying a reasonable timing premium.

To highlight how the horizon-dependent risk aversion model of Definition 1, rather than changes in the calibration for the endowment process, affects prices, we calibrate the consumption processes (8) and the dividend growth processes (14) strictly as in Bansal et al. (2012) (see Table 1). Bansal et al. (2012) document that their calibration matches the evidence on consumption and dividend growth dynamics.

Given the calibration of Table 1, and the approximation $\beta\approx 1$ under which the formal

[21] The limit result of Equation (22) for the case $\sigma_t\neq\sigma$ is provided in Appendix C.9. Whether $\sigma_t>\sigma$ induces a steeper downward sloping term-structure depends on the parameters of the model.

results of Section 4 are derived, our remaining choice variables to calibrate asset prices are the preference parameters $\{\rho, \gamma, \widetilde{\gamma}\}$.

To determine the impact of the elasticity of intertemporal substitution EIS $= 1/\rho$ on asset prices, we consider $\rho = 1/2$ and $\rho = 1/1.5$, standard in the asset pricing literature.[22] In our choice of the long-term risk aversion $\widetilde{\gamma}$, we consider the case $\widetilde{\gamma} = 2$, a level of risk aversion common in the microeconomic literature; and $\widetilde{\gamma} = 1$, corresponding to the strongest departure of our horizon-dependent risk aversion model from the standard framework. For each $\{\rho, \widetilde{\gamma}\}$ pair, we choose the short-term risk aversion γ so as to match the equity premia (dividend strip futures returns) in the data.

The calibration results are in Table 2. Column (1) provides the estimates in the data for the moments we consider (discussed below). Column (2) provides the results under the standard long-run risk framework in the calibration of Bansal et al. (2012). Columns (3) and (4) provide the results of our horizon-dependent risk aversion model for the case $\rho = 1/2$, and $\widetilde{\gamma} = 2$ (Column (3)) and $\widetilde{\gamma} = 1$ (Column (4)). Finally, Columns (5) and (6) provide the equivalent results for $\rho = 1/1.5$.

As we discuss in more detail below, the horizon-dependent risk aversion model with $\rho = 1/2$ fits the data quite well, both for the case $\widetilde{\gamma} = 2, \gamma = 11$ (Column(3)) and the case $\widetilde{\gamma} = 1, \gamma = 13$ (Column(4)). To illustrate these results, we display the term-structure of the dividend strip future expected returns (expected equity premia) in Figure 2, the term-structure of the dividend strip future returns Sharpe ratios in Figure 3, and the term-structure of expected forward equity yields during recessions in Figure 4; for the calibrations of Table 2. The calibration of the standard long-run risk model is also displayed, in a dotted black line, alongside the results of the horizon-dependent risk aversion model. The calibrations of Columns (3) and (4), which best capture the data, are in bold blue and red lines.

Figures 2, 3 and 4 make readily apparent how horizon-dependent risk aversion improves on the standard long-run risk model at capturing the evidence, both in normal times and in recessions.

5.1 Data and interpretation of individual results

We discuss below the data sources we use and the interpretation of our results, starting with the term-structure results, our core contribution in the matching of asset pricing data:

[22] As we show in Table 2, lower (higher) EIS (ρ) parameters would fit the data more poorly.

5.1.1 Term structure of equity premia

Normal times/ liquid market (Table 2 and Figures 2 and 3). The conditional expected returns of dividend strip futures with maturities $h \geq 1$ are derived in Proposition 6 for a representative agent who trades and clears the market every period, i.e. the standard approach. We derive the corresponding unconditional returns in Appendix C.7, and obtain the annual dividend strip counterparts.[23] We compare them to the average returns and Sharpe ratios of dividend strip futures of maturity 1 year to 15 years in Giglio et al. (2023), where, in line with their notion of "normal times", we remove from their 1973–2020 sample all NBER recession months.[23] As seen in Figures 2 and 3, as well as in Table 2, the shape of the slowly upward sloping equity risk premia (Figure 2) and of the first upward then downward sloping Sharpe ratios (Figure 3) are very well captured by the horizon-dependent risk aversion model. E.g. comparing Column (3) to the data in Column (1), the calibration $\widetilde{\gamma} = 2, \gamma = 11, \rho = 1/2$ matches the evidence, not only in levels: $\log E\left[R_2^F\right] = 3.4\%$ versus 3.7%, $\mathrm{SR}\left[R_2^F\right] = 0.18$ versus 0.19; but also, crucially, in the slopes: $\log E\left[R_{15}^F\right] - \log E\left[R_5^F\right] = 2.6\%$ versus 2.4%, $\mathrm{SR}\left[R_{15}^F\right] - \mathrm{SR}\left[R_5^F\right] = 0.07$ versus 0.04. In contrast, the standard model of Column (2) generates excessively steep slopes: $\log E\left[R_{15}^F\right] - \log E\left[R_5^F\right] = 8.7\%$ versus 2.4% in the data; $\mathrm{SR}\left[R_{15}^F\right] - \mathrm{SR}\left[R_5^F\right] = 0.31$ versus 0.04 in the data.

Recessions/ Illiquid market (Table 2 and Figure 4). In Section 4.3, we consider how prices are set by an investor who uses buy-and-hold strategies, and formally derive the limit slope of the term-structure as the expected return of the infinite-horizon versus the next-period dividend strip futures in Proposition 8. Evidence in the data indicates the slope is negative during recessions (van Binsbergen and Koijen, 2017; Bansal et al., 2021). In Appendix C.9, we formally derive the corresponding expected forward equity yields, defined as $\mathrm{FEY}_{t,h} = -\frac{1}{h}\log\left(\frac{P_{t,h}}{D_t} \times \frac{1}{B_{t,h}}\right)$ where $P_{t,h}$ is the price of the dividend strip and $B_{t,h}$ the price of a risk-free bond at horizon h, under buy-and-hold strategies. This allows us to obtain precise numerical solutions that we then take directly to the dividend strip equity yield data in Giglio et al. (2023).[24] We restrict their 1973–2020 sample to NBER recessions, as proxy-periods of liquidity break-downs, in line with the evidence in Pedersen (2009), Brunnermeier (2009), and Bansal et al. (2021) for the Great Financial Crisis.[23] As seen in Figure 4 and Table 2, the horizon-dependent risk aversion model captures the negative

[23]See Appendix D.1 for details.

[24]We analyze forward equity yields for the *Recessions/ Illiquid market*, and not equity premia, because we have too few data points on which to estimate annual returns: since 1973 only two recessions lasted more than 12 months.

slope of the term-structure of forward equity yields and of equity premia. Our calibration cannot fully match the magnitude of the downward slope in the data: the front-end of the forward-yield curve is too low relative to the data and the back-end too high. E.g. comparing the calibration $\widetilde{\gamma} = 2, \gamma = 11, \rho = 1/2$ (Column (3)) to the evidence in Giglio et al. (2023) (Column (1)), we obtain, for the levels: $E[\text{FEY}_2] = -2.0\%$ versus -0.8%; and for the slopes: $E[\text{FEY}_5] - E[\text{FEY}_2] = -0.3\%$ versus -2.9%, and $E[\text{FEY}_{15}] - E[\text{FEY}_5] = -0.2\%$ versus -1.4%.[25] On the other hand, the standard long-run risk model is grossly off, with an upward sloping term-structure of forward equity yields, and an unrealistically high infinite horizon expected return; the standard model of Column (2) does not just generate the wrong magnitudes, but the wrong signs: $E[\text{FEY}_5] - E[\text{FEY}_2] = +1.2\%$ versus -2.9% in the data, and $E[\text{FEY}_{15}] - E[\text{FEY}_5] = +4.2\%$ versus -1.4% in the data.

Other data estimates. As discussed in the introduction, analyzing the slope of the term structure of equity risk premia has been the focus of active research in the finance literature since van Binsbergen et al. (2012), and empirical estimates vary across papers and sample periods. To be consistent in the analysis of the term-structure results described above, we chose to systematically compare them to estimates from the same database of dividend strip (proxies) in Giglio et al. (2023), who provide, to our knowledge, the longest sample (1973–2020). The main result we exploit above, that the slope of the term-structure of risk premia is slowly upward sloping in "normal times" and downward sloping during NBER recessions, is confirmed in the proprietary data on dividend strip futures at maturity 1 year to 7 years in Gormsen (2021), which also displays a steeply downward sloping term-structure during the Great Financial Crisis; as well as in Bansal et al. (2021).[26]

5.1.2 Timing premium

As discussed in Section 3, there is no clear estimate in the data to which to compare the timing premium we formally derived in Proposition 1. However, the micro evidence on attitudes towards information, as well as commonsense considerations, suggest that the timing premium should be close to zero. From Figure 1, horizon-dependent risk aversion can generate a wide range of timing premia, and the larger the wedge between γ

[25] Considering other calibrations of the EIS would not improve on this result: ρ barely affects the term-structure of forward equity premia (see Table 2). For this reason, only the calibrations with $\rho = 1/2$ are illustrated in Figure 4. We posit that extending the preference model of Definition 1 to a decreasing sequence of risk aversions, as in Appendix A.2, could improve on the result we obtain for the $\{\gamma, \widetilde{\gamma}\}$ framework, though this is left for future research.

[26] We are grateful to Niels Gormsen for providing us with the average term-structure results for the December 2007 to June 2009 period, over which the 7-year horizon strip return is 23 percentage points below that of the 1-year horizon strip.

and $\widetilde{\gamma}$ the lower the timing premium, including preferences for late resolution of uncertainty. This is what we obtain in Table 2, with a timing premium as low as -42% for $\widetilde{\gamma} = 1, \gamma = 14, \rho = 1/1.5$ (Column (6)), an unrealistic preference against early information.[27] At the other extreme, in line with Epstein et al. (2014) and the results of Section 3, we find that the standard long-run risk model generates an unrealistically high timing premium of 85% (Column (2)). In contrast, the horizon-dependent risk aversion model under the calibration $\widetilde{\gamma} = 2, \gamma = 11, \rho = 1/2$ of Column (3) generates a realistic timing premium of 1%, corresponding to a slight preference for early resolution. Given that $\widetilde{\gamma} = 2, \gamma = 11, \rho = 1/2$ also captures the asset pricing evidence, it offers a clear resolution to the puzzle of Epstein et al. (2014).

5.1.3 Equity premium

The formal solution for the conditional market equity premium is in Proposition 5. To take it to the data, we derive its unconditional counterpart in Appendix C.6. We compare it to the estimate, obtained for the 1930–2008 period, in Bansal et al. (2012), and to their calibrated equity premium in Column (2). Our calibrated equity premium is close to Bansal et al. (2012) (between 5.4% and 5.7% versus 6.6%) and their data estimate (6.8%). This is achieved while keeping the immediate risk aversion γ as low as possible: $\gamma = 11$ in our preferred calibration with $\widetilde{\gamma} = 2$ and $\rho = 1/2$, versus $\gamma = 10$ in the standard model.

5.1.4 Macro announcement premium

In Section 4.2, we describe how the evidence on the macro-announcement premium provides lower bounds on the relative contributions of the different consumption shocks to the equity premium. The two bounds — the contribution of the consumption growth shocks for the first bound, and the contribution of both the consumption growth shocks and the consumption volatility shocks for the second bound — are formally derived in Appendix C.6; and compared to the macro-announcement premium estimate in the data of Ai et al. (2018).[28] In all the calibrations of horizon-dependent risk aversion in Table 2, 80% or more of the macro-announcement premium can be explained via the signals on the consumption growth shocks $\{w_{x,t}\}$ provided on announcement days. In contrast, in the standard long-run risk model in the calibration of Bansal et al. (2012), more than 40% of the macro-announcement premium is explained via the pricing of the consumption volatility shocks $\{w_{\sigma,t}\}$.

[27] The results for $\rho \neq 1$ in Table 2 are numerically derived, under the procedure described in Appendix D.2.

[28] As discussed in Section 4.2, other estimates of the macro-announcement premium in the literature yield similar results.

5.2 Discussion

The calibration of Table 2 and Figures 2, 3, and 4 show that the horizon-dependent risk aversion model can match the evidence in the data and is a clear improvement on the standard long-run risk model: First, the slowly upward sloping average term-structure of equity premia matches the evidence in Giglio et al. (2023), as do the upward then downward sloping Sharpe ratios; second, the dynamics of the slope of equity premia matches the evidence during recessions, where the term-structure of expected forward equity yields becomes downward sloping (Giglio et al., 2023), as well as in "normal times", where equity premia grow at a steeper slope when the index price is low (see Proposition 7 and the evidence in Gormsen, 2021); third, the implied timing premium is close to zero (in the calibration $\widetilde{\gamma} = 2, \gamma = 11, \rho = 1/2$); and fourth, this obtains within a calibration that can match the equity premium and satisfy the macro-announcement premium bounds without having to rely heavily on consumption volatility risks to be revealed on announcement days.

6 Conclusion

Relaxing the restriction of Epstein and Zin (1989) that risk preferences be constant across horizons makes it possible to retain the tractability and desirable pricing properties of the long-run asset pricing model, while addressing key challenges to the standard framework.

Matching the equity premium, and the macroeconomic announcement premium, can be achieved without implying a willingness to pay for earlier resolutions of uncertainty that defies both observed behaviors in the data and the introspection. In addition horizon-dependent risk aversion preferences provide a rationalization why the term-structure of risk premia is upward sloping for some assets or over some periods, and downward sloping for others, as observed in the data.

Under the standard one-period trading pricing paradigm, *all* assets, no matter their maturity, are priced at the next-period horizon, so the (low) long-term risk aversion has a limited impact on risk premia. As the risk exposures of equity assets increase with the horizon, so does the term-structure of their expected returns. However, for assets with high trading costs and/or low liquidity, investors may prefer to consider buy-and-hold strategies, in which case the dynamic inconsistency in their preferences has a greater "bite": when longer maturity assets are evaluated with the corresponding longer horizon risk aversion, the term-structure of risk premia can become downward sloping.

This feature of our model allows us to explain, both qualitatively and quantitatively, important puzzles in empirical finance on the term-structure of equity risk premia and its negative slope during NBER recessions. Additional implications include the abnormally low returns in private equity and housing investments, or the very low trading volumes for medium to long-term options and insurance.

We conclude that formalizing a model where risk aversion is higher at short-horizons than long-horizons, consistent with the experimental evidence, provides a useful new tool for asset pricing and macro-finance. We focused our attention on applications to finance but the tractability of this model makes it suitable to analyze features of other markets, such as health decisions, where attitudes towards risk and time inconsistencies are key.

References

Abdellaoui, M., E. Diecidue, and A. Onculer (2011). Risk preferences at different time periods: An experimental investigation. *Management Science 57*(5), 975–987.

Abel, A. B., J. C. Eberly, and S. Panageas (2013). Optimal inattention to the stock market with information costs and transactions costs. *Econometrica 81*(4), 1455–1481.

Acharya, V. V. and L. H. Pedersen (2005). Asset pricing with liquidity risk. *Journal of Financial Economics 77*(2), 375–410.

Ahlbrecht, M. and M. Weber (1997). An empirical study on intertemporal decision making under risk. *Management Science 43*(6), 813–826.

Ai, H. and R. Bansal (2018). Risk preferences and the macroeconomic announcement premium. *Econometrica 86*(4), 1383–1430.

Ai, H., R. Bansal, J. Im, and C. Ying (2021). Macroeconomic announcement premium with production. Working Paper.

Ai, H., M. M. Croce, A. M. Diercks, and K. Li (2018). News shocks and the production-based term structure of equity returns. *Review of Financial Studies 31*(7), 2423–2467.

Akaichi, F., J. Costa-Font, and R. Frank (2020). Uninsured by choice? A choice experiment on long term care insurance. *Journal of Economic Behavior & Organization 173*, 422–434.

Alvarez, F., L. Guiso, and F. Lippi (2012). Durable consumption and asset management with transaction and observation costs. *American Economic Review 102*(5), 2272–2300.

Andersen, S., J. Y. Campbell, K. M. Nielsen, and T. Ramadorai (2020). Sources of inaction in household finance: Evidence from the danish mortgage market. *American Economic Review 110*(10), 3184–3230.

Andries, M. (2021). Risk-pricing under gain-loss asymmetry. Working Paper.

Andries, M., T. M. Eisenbach, M. C. Schmalz, and R. J. Kahn (2023). The term structure of the price of variance risk. Working Paper.

Andries, M. and V. Haddad (2020). Information aversion. *Journal of Political Economy 128*(5), 1901–1939.

Backus, D., N. Boyarchenko, and M. Chernov (2018). Term structures of asset prices and returns. *Journal of Financial Economics 129*(1), 1–23.

Bansal, R., D. Kiku, and A. Yaron (2012). An empirical evaluation of the long-run risks model for asset prices. *Critical Finance Review 1*, 183–221.

Bansal, R., S. Miller, D. Song, and A. Yaron (2021). The term structure of equity risk premia. *Journal of Financial Economics 142*(3), 1209–1228.

Bansal, R. and A. Yaron (2004). Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance 59*(4), 1481–1509.

Barberis, N., M. Huang, and T. Santos (2001). Prospect Theory and Asset Prices. *Quarterly Journal of Economics 116*(1), 1–53.

Baucells, M. and F. Heukamp (2010). Common ratio using delay. *Theory and Decision 68*(1-2), 149–158.

Belo, F., P. Collin-Dufresne, and R. S. Goldstein (2015). Dividend dynamics and the term structure of dividend strips. *Journal of Finance 70*(3), 1115–1160.

Beshears, J., J. J. Choi, D. Laibson, and B. C. Madrian (2017). Does Aggregated Returns Disclosure Increase Portfolio Risk Taking? *Review of Financial Studies 30*(6), 1971–2005.

Bilias, Y., D. Georgarakos, and M. Haliassos (2010). Portfolio inertia and stock market fluctuations. *Journal of Money, Credit and Banking 42*(4), 715–742.

van Binsbergen, J. H. (2016). Good-specific habit formation and the cross-section of expected returns. *Journal of Finance 71*(4), 1699–1732.

van Binsbergen, J. H., M. Brandt, and R. Koijen (2012). On the timing and pricing of dividends. *American Economic Review 102*(4), 1596–1618.

van Binsbergen, J. H., W. Hueskes, R. Koijen, and E. Vrugt (2013). Equity yields. *Journal of Financial Economics 110*(3), 503–519.

van Binsbergen, J. H. and R. S. Koijen (2017). The term structure of returns: Facts and theory. *Journal of Financial Economics 124*(1), 1–21.

Bollerslev, T., G. Tauchen, and H. Zhou (2009). Expected stock returns and variance risk premia. *Review of Financial Studies 22*(11), 4463–4492.

Bonomo, M., R. Garcia, N. Meddahi, and R. Tédongap (2011). Generalized disappointment aversion, long-run volatility risk, and asset prices. *Review of Financial Studies 24*(1), 82–122.

Brunnermeier, M. K. (2009). Deciphering the liquidity and credit crunch 2007–2008. *Journal of Economic perspectives 23*(1), 77–100.

Brunnermeier, M. K. and S. Nagel (2008). Do wealth fluctuations generate time-varying risk aversion? Micro-evidence on individuals. *American Economic Review 98*(3), 713–736.

Bryzgalova, S. and C. Julliard (2021). Consumption. Working Paper.

Calvet, L. E., J. Y. Campbell, and P. Sodini (2009). Fight or flight? Portfolio rebalancing by individual investors. *Quarterly Journal of Economics 124*(1), 301.

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*(2), 205–251.

Cassella, S., B. Golez, H. Gulen, and P. Kelly (2023). Horizon bias and the term structure of equity returns. *Review of Financial Studies 36*(3), 1253–1288.

Chabi-Yo, F. and J. Loudis (2020). The conditional expected market return. *Journal of Financial Economics 137*(3), 752–786.

Chambers, D., C. Spaenjers, and E. Steiner (2021, 03). The rate of return on real estate: Long-run micro-level evidence. *Review of Financial Studies 34*(8), 3572–3607.

Chew, S. H. (1989). Axiomatic utility theories with the betweenness property. *Annals of operations Research 19*(1), 273–298.

Christoffersen, P., S. Heston, and K. Jacobs (2013). Capturing option anomalies with a variance-dependent pricing kernel. *Review of Financial Studies 26*(8), 1963–2006.

Cieslak, A. and H. Pang (2021). Common shocks in stocks and bonds. *Journal of Financial Economics 142*(2), 880–904.

Coble, K. and J. Lusk (2010). At the nexus of risk and time preferences: An experimental investigation. *Journal of Risk and Uncertainty 41*, 67–79.

Cochrane, J. H. (2017). Macro-finance. *Review of Finance 21*(3), 945–985.

Constantinides, G. M. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy 98*(3), 519–543.

Croce, M. M., M. Lettau, and S. C. Ludvigson (2015). Investor information, long-run risk, and the term structure of equity. *Review of Financial Studies 28*(3), 706–742.

Dekel, E. (1986). An axiomatic characterization of preferences under uncertainty: Weakening the independence axiom. *Journal of Economic theory 40*(2), 304–318.

Dew-Becker, I. (2014). A model of time-varying risk premia with habits and production. Working Paper.

Dew-Becker, I., S. Giglio, A. Le, and M. Rodriguez (2017). The price of variance risk. *Journal of Financial Economics 123*(2), 225–250.

Drechsler, I. and A. Yaron (2010). What's vol got to do with it. *Review of Financial Studies 24*(1), 1–45.

Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance 65*(4), 1237–1267.

Duffie, D. and P.-L. Lions (1992). Pde solutions of stochastic differential utility. *Journal of Mathematical Economics 21*(6), 577–606.

Eisenbach, T. M. and M. C. Schmalz (2016). Anxiety in the face of risk. *Journal of Financial Economics 121*(2), 414–426.

Epstein, L. G., E. Farhi, and T. Strzalecki (2014). How much would you pay to resolve long-run risk? *American Economic Review 104*(9), 2680–2697.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–969.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Favilukis, J. and X. Lin (2016). Wage rigidity: A quantitative solution to several asset pricing puzzles. *Review of Financial Studies 29*(1), 148–192.

Gârleanu, N., L. Kogan, and S. Panageas (2012). Displacement risk and asset returns. *Journal of Financial Economics 105*(3), 491–510.

Garleanu, N., L. H. Pedersen, and A. M. Poteshman (2008). Demand-based option pricing. *Review of Financial Studies 22*(10), 4259–4299.

Giglio, S., B. T. Kelly, and S. Kozak (2023). Equity term structures without dividend strips data. *Journal of Finance* (forthcoming).

Giglio, S., M. Maggiori, and J. Stroebel (2015). Very long-run discount rates. *Quarterly Journal of Economics 130*(1), 1–53.

Golez, B. and J. C. Jackwerth (2020). Quarter-of-a-century of dividend pricing. Working Paper.

Golman, R., D. Hagmann, and G. Loewenstein (2017). Information avoidance. *Journal of Economic Literature 55*(1), 96–135.

Gonçalves, A. S. (2021). Reinvestment risk and the equity term structure. *Journal of Finance 76*(5), 2153–2197.

Gormsen, N. J. (2021). Time variation of the equity term structure. *Journal of Finance 76*(4), 1959–1999.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica 59*(3), 667–686.

Gürkaynak, R. S., B. Sack, and J. H. Wright (2007). The US Treasury yield curve: 1961 to the present. *Journal of Monetary Economics 54*(8), 2291–2304.

Haddad, V. and T. Muir (2021). Do intermediaries matter for aggregate asset prices? *Journal of Finance 76*(6), 2719–2761.

Hansen, L. P., J. C. Heaton, and N. Li (2008). Consumption strikes back? Measuring long-run risk. *Journal of Political economy 116*(2), 260–302.

Harris, C. and D. Laibson (2001). Dynamic choices of hyperbolic consumers. *Econometrica 69*(4), 935–957.

He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. *American Economic Review 103*(2), 732–70.

Heimer, R., Z. Iliewa, A. Imas, and M. Weber (2023). Dynamic inconsistency in risky choice: Evidence from the lab and field. Working paper.

Holt, C. A. and S. K. Laury (2002). Risk aversion and incentive effects. *American Economic Review 92*(5), 1644–1655.

Jones, E. E. and C. A. Johnson (1973). Delay of consequences and the riskiness of decisions. *Journal of Personality 41*(4), 613–637.

Karlsson, N., G. Loewenstein, and D. Seppi (2009). The ostrich effect: Selective attention to information. *Journal of Risk and Uncertainty 38*, 95–115.

Keren, G. and P. Roelofsma (1995). Immediacy and certainty in intertemporal choice. *Organizational Behavior and Human Decision Processes 63*(3), 287–297.

Kogan, L. and D. Papanikolaou (2010). Growth opportunities and technology shocks. *American Economic Review 100*(2), 532–536.

Koijen, R. S. and M. Yogo (2019). A demand system approach to asset pricing. *Journal of Political Economy 127*(4), 1475–1515.

Kreps, D. M. and E. L. Porteus (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica 46*(1), 185–200.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. *Quarterly Journal of Economics 112*(2), 443–477.

Lee, K.-H. (2011). The world price of liquidity risk. *Journal of Financial Economics 99*(1), 136 – 161.

Lucca, D. O. and E. Moench (2015). The pre-FOMC announcement drift. *Journal of Finance 70*(1), 329–371.

Lustig, H., A. Stathopoulos, and A. Verdelhan (2019). The term structure of currency carry trade risk premia. *American Economic Review 109*(12), 4142–77.

Luttmer, E. G. J. and T. Mariotti (2003). Subjective discounting in an exchange economy. *Journal of Political Economy 111*(5), 959–989.

Marfe, R. (2015). Labor rigidity and the dynamics of the value premium. In *Paris December 2015 Finance Meeting EUROFIDAI-AFFI*.

Marfe, R. (2017). Income insurance and the equilibrium term structure of equity. *Journal of Finance 72*(5), 2073–2130.

Moskowitz, T. J. and A. Vissing-Jørgensen (2002). The returns to entrepreneurial investment: A private equity premium puzzle? *American Economic Review 92*(4), 745–778.

Muir, T. (2017). Financial crises and risk premia. *Quarterly Journal of Economics 132*(2), 765–809.

Nagel, S. (2012). Evaporating liquidity. *Review of Financial Studies 25*(7), 2005–2039.

Noussair, C. and P. Wu (2006). Risk tolerance in the present and the future: An experimental study. *Managerial and Decision Economics 27*(6), 401–412.

Öncüler, A. (2000). Intertemporal choice under uncertainty: A behavioral perspective. Working Paper.

Oster, E., I. Shoulson, and E. R. Dorsey (2013). Optimal expectations and limited medical testing: Evidence from huntington disease. *American Economic Review 103*(2), 804–830.

Pedersen, L. H. (2009). When everyone runs for the exit. *International Journal of Central Banking 5*(4), 177–199.

Phelps, E. S. and R. A. Pollak (1968). On second-best national saving and game-equilibrium growth. *Review of Economic Studies 35*(2), 185–199.

Rachlin, H. and E. Siegel (1994). Temporal patterning in probabilistic choice. *Organizational Behavior and Human Decision Processes 59*(2), 161–176.

Routledge, B. R. and S. E. Zin (2010). Generalized disappointment aversion and asset prices. *Journal of Finance 65*(4), 1303–1332.

Sagristano, M. D., Y. Trope, and N. Liberman (2002). Time-dependent gambling: Odds now, money later. *Journal of Experimental Psychology: General 131*(3), 364–376.

Savor, P. and M. Wilson (2013). How much do investors care about macroeconomic risk? Evidence from scheduled economic announcements. *Journal of Financial and Quantitative Analysis 48*(2), 343–375.

Schreindorfer, D. (2014). Tails, fears, and equilibrium option prices. Working Paper.

Shelley, M. K. (1994). Gain/loss asymmetry in risky intertemporal choice. *Organizational Behavior and Human Decision Processes 59*(1), 124–159.

Shiller, R. J. (1981). The use of volatility measures in assessing market efficiency. *Journal of Finance 36*(2), 291–304.

Sicherman, N., G. Loewenstein, D. J. Seppi, and S. P. Utkus (2016). Financial attention. *Review of Financial Studies 29*(4), 863–897.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies 23*(3), 165–180.

Tinang, J. (2019). Macro uncertainty and the term structure of risk premium. Working Paper.

Tirole, J. (2011). Illiquidity and all its friends. *Journal of Economic Literature 49*(2), 287–325.

Wachter, J. A. and Y. Zhu (2022). A model of two days: Discrete news and asset prices. *Review of Financial Studies 35*(5), 2246–2307.

Weber, M. (2018). Cash flow duration and the term structure of equity returns. *Journal of Financial Economics 128*(3), 486–503.

Zviadadze, I. (2021). Term structure of risk in expected returns. *Review of Financial Studies 34*(12), 6032–6086.

Tables and Figures

Table 1: Calibration parameters

Process	Parameters			
c_t	$\mu_c = 0.0015$	$\phi_c = 1$	$\alpha_c = 1$	
x_t		$\nu_x = 0.975$	$\alpha_x = 0.038$	
σ_t	$\sigma = 0.0072$	$\nu_\sigma = 0.999$	$\alpha_\sigma = 0.0000028$	
d_t	$\mu_d = 0.0015$	$\phi_d = 2.5$	$\chi = 2.6$	$\alpha_d = 5.96$

Table 2: Data and model calibration

	Data	Standard LRR	Horizon-dependent risk aversion			
			$\rho = 1/2$		$\rho = 1/1.5$	
			$\widetilde{\gamma} = 2$	$\widetilde{\gamma} = 1$	$\widetilde{\gamma} = 2$	$\widetilde{\gamma} = 1$
Moment	Estimate	Bansal et al. (2012)	$\gamma = 11$	$\gamma = 13$	$\gamma = 12$	$\gamma = 14$
	(1)	(2)	(3)	(4)	(5)	(6)
Term Stucture of Equity Premia						
Normal times/ liquid market:						
$\log E\left[R_2^F\right]$	3.7%	3.1%	3.4%	3.7%	3.3%	4.1%
$\log E\left[R_5^F\right] - \log E\left[R_2^F\right]$	2.1%	3.3%	2.3%	2.5%	2.5%	2.6%
$\log E\left[R_{15}^F\right] - \log E\left[R_5^F\right]$	**2.4%**	**8.7%**	**2.6%**	1.9%	3.0%	2.1%
$\mathrm{SR}\left[R_2^F\right]$	0.19	0.17	0.18	0.20	0.18	0.22
$\mathrm{SR}\left[R_5^F\right] - \mathrm{SR}\left[R_2^F\right]$	**0.09**	**0.15**	**0.09**	0.10	0.10	0.10
$\mathrm{SR}\left[R_{15}^F\right] - \mathrm{SR}\left[R_5^F\right]$	**0.04**	**0.31**	**0.07**	0.03	0.08	0.03
Recessions/ Illiquid market:						
$\log E\left[R_\infty^F\right] - \log E\left[R_1^F\right]$	$\leq$ **0**	**90.2%**	**-1.0%**	-1.9%	-1.1%	-2.0%
$E[\mathrm{FEY}_2]$	-0.8%	-0.5%	-2.0%	-2.1%	-2.0%	-2.1%
$E[\mathrm{FEY}_5] - E[\mathrm{FEY}_2]$	**- 2.9%**	**1.2%**	**- 0.3%**	-0.5%	-0.4%	-0.5%
$E[\mathrm{FEY}_{15}] - E[\mathrm{FEY}_5]$	**- 1.4%**	**4.2%**	**- 0.2%**	-0.3%	-0.2%	-0.3%
Timing Premium	***NA*, $\approx$ 0**	**85%**	**1%**	-14%	-22%	-42%
Equity Premium						
$\log E[R_{m,t+1}] - \log E\left[R_{f,t}\right]$	6.8%	6.6%	5.6%	5.4%	6.0%	5.5%
Macro-Announcement Premium						
Bound on $\{w_{x,t}\}$ shocks	**55%**	**32%**	**49%**	61%	45%	58%
Bound on $\{w_{x,t}\}$ and $\{w_{\sigma,t}\}$ shocks	55%	75%	68%	61%	68%	58%

Data is from Bansal et al. (2012) (1930 to 2008 sample) for the equity premium, from Ai et al. (2018) (1961 to 2014 sample) for the macro-announcement premium, and Giglio et al. (2023) for the term-structure data using dividend strip proxies (1973 to 2020 sample excluding NBER recessions for the *Normal times/ liquid market* estimates and restricted to NBER recessions for the *Recessions/ Illiquid market* estimates). *Standard LRR* stands for the standard "long-run risk" model (Bansal et al., 2012). All returns are annual or annualized. The *Normal times/ liquid market* returns $\log E\left[R_h^F\right]$ and Sharpe ratios $\mathrm{SR}\left[R_h^F\right]$ are those of dividend strip futures at the $h \geq 2$ year horizon under one-period trading. The *Recessions/ Illiquid market* returns $\log E\left[R_\infty^F\right] - \log E\left[R_1^F\right]$ is derived in Proposition 8 under buy-and-hold trading. The *Recessions/ Illiquid market* forward equity yields $E[\mathrm{FEY}_h]$ are the average equity yield of the dividend strips futures at the $h \geq 1$ horizon under buy-and-hold trading. The *Timing Premium* is described in Section 3. We derive its value for $\rho \neq 1$ according to the algorithm described in Appendix D.2, under $\beta = 0.9989$. The *Equity Premium* and *Macro-announcement Premium*, including the description of the *Bound on* $\{w_{x,t}\}$ *shocks* and *Bound on* $\{w_{x,t}\}$ *and* $\{w_{\sigma,t}\}$ *shocks*, are in Section 4.2. See Appendix D.1 for additional details. Estimates where the calibration of horizon-dependent risk aversion most clearly improves on the standard model are in bold.

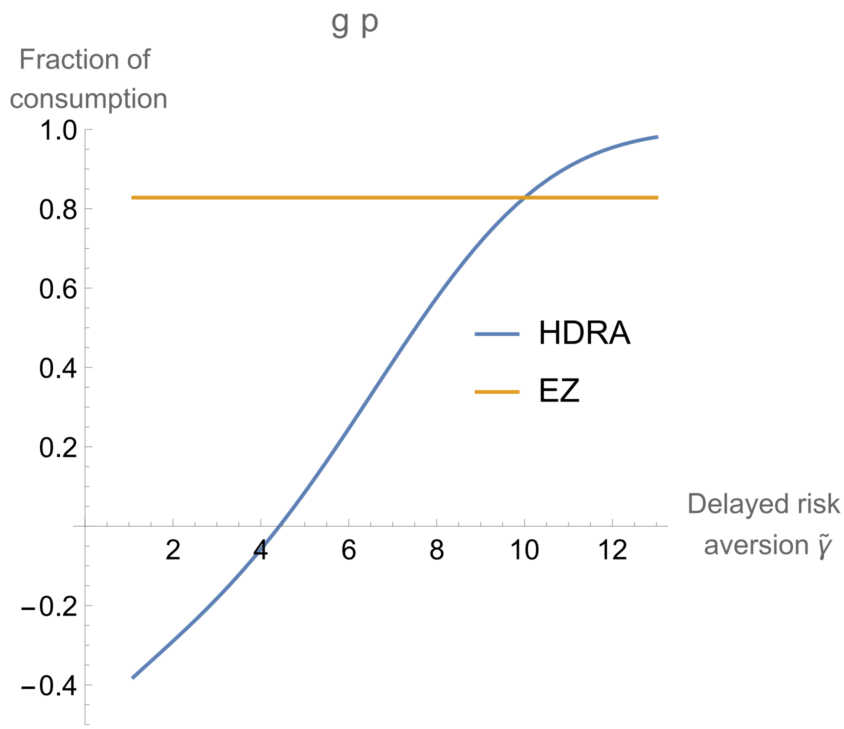

Figure 1: Timing premium

Effect of horizon-dependent risk aversion (HDRA) on willingness to pay for early resolution of uncertainty (timing premium), compared to Epstein-Zin preferences (EZ) with $\gamma = 10$. Calibration of Table 1, with $\rho = 1$ and $\beta = 0.9989$.

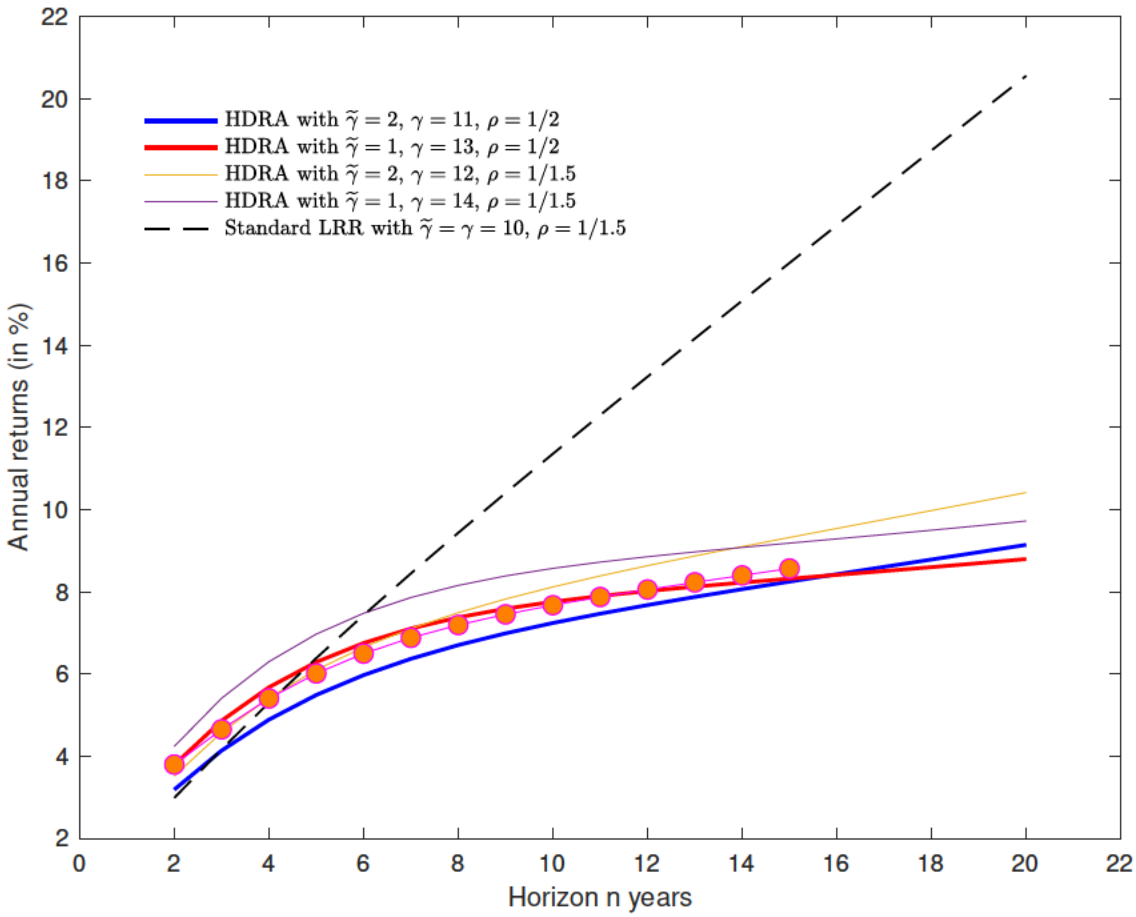

Figure 2: Term structure of dividend strip future returns

Expected annual returns of dividend strip futures at horizon $h \geq 2$ under one-period trading. *HDRA* stands for "Horizon-dependent risk aversion", *Standard LRR* stands for the standard "Long-run risk" model ($\tilde{\gamma} = \gamma = 10$ and $\rho = 1/1.5$ are the parameters of Bansal et al. (2012)). The calibration of both models is from Table 1. Data is from Giglio et al. (2023) using dividend strip proxies (1973-2020 sample excluding NBER recessions). See Appendix D.1 for details.

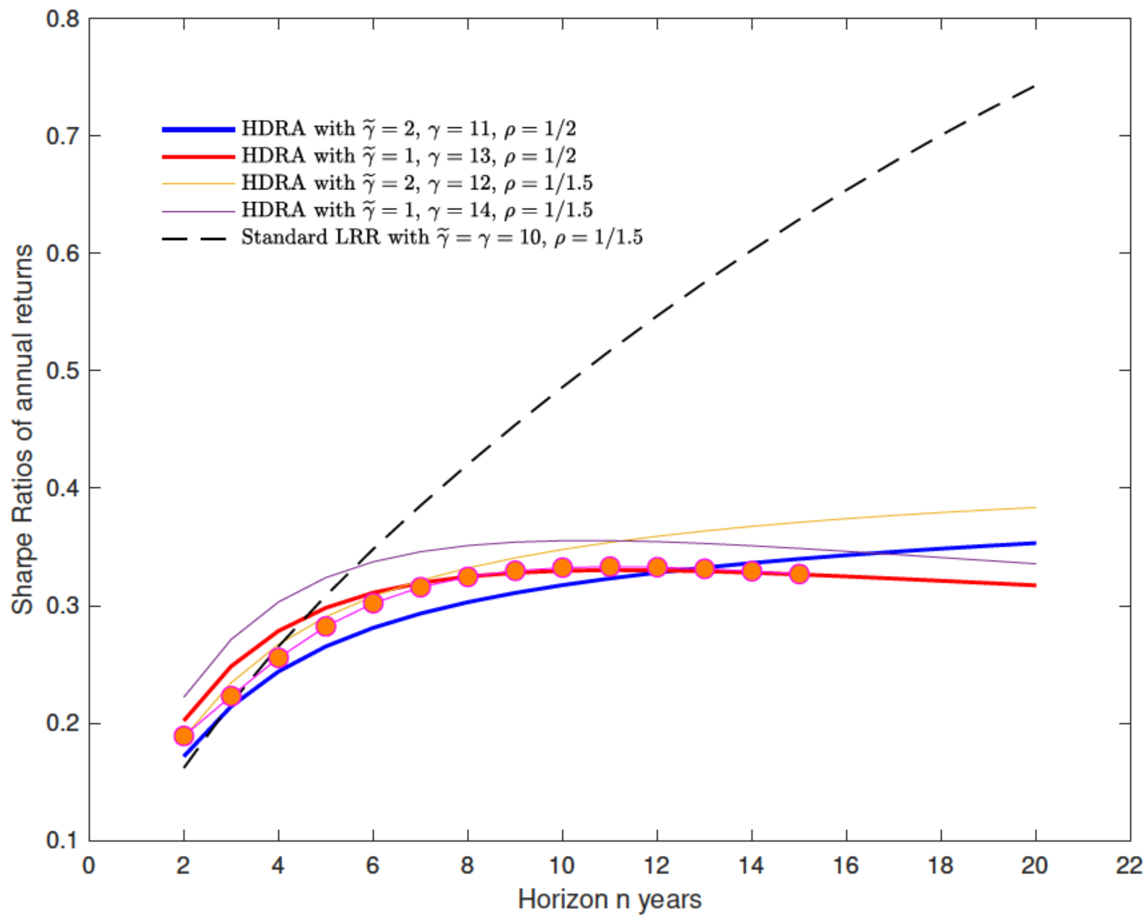

Figure 3: Dividend strip future Sharpe ratios

Dividend strip futures Sharpe ratios at horizon $h \geq 2$ under one-period trading. *HDRA* stands for "Horizon-dependent risk aversion", *Standard LRR* stands for the standard "Long-run risk" model ($\widetilde{\gamma} = \gamma = 10$ and $\rho = 1/1.5$ are the parameters of Bansal et al. (2012)). The calibration of both models is from Table 1. Data is from Giglio et al. (2023) using dividend strip proxies (1973-2020 sample excluding NBER recessions). See Appendix D.1 for details.

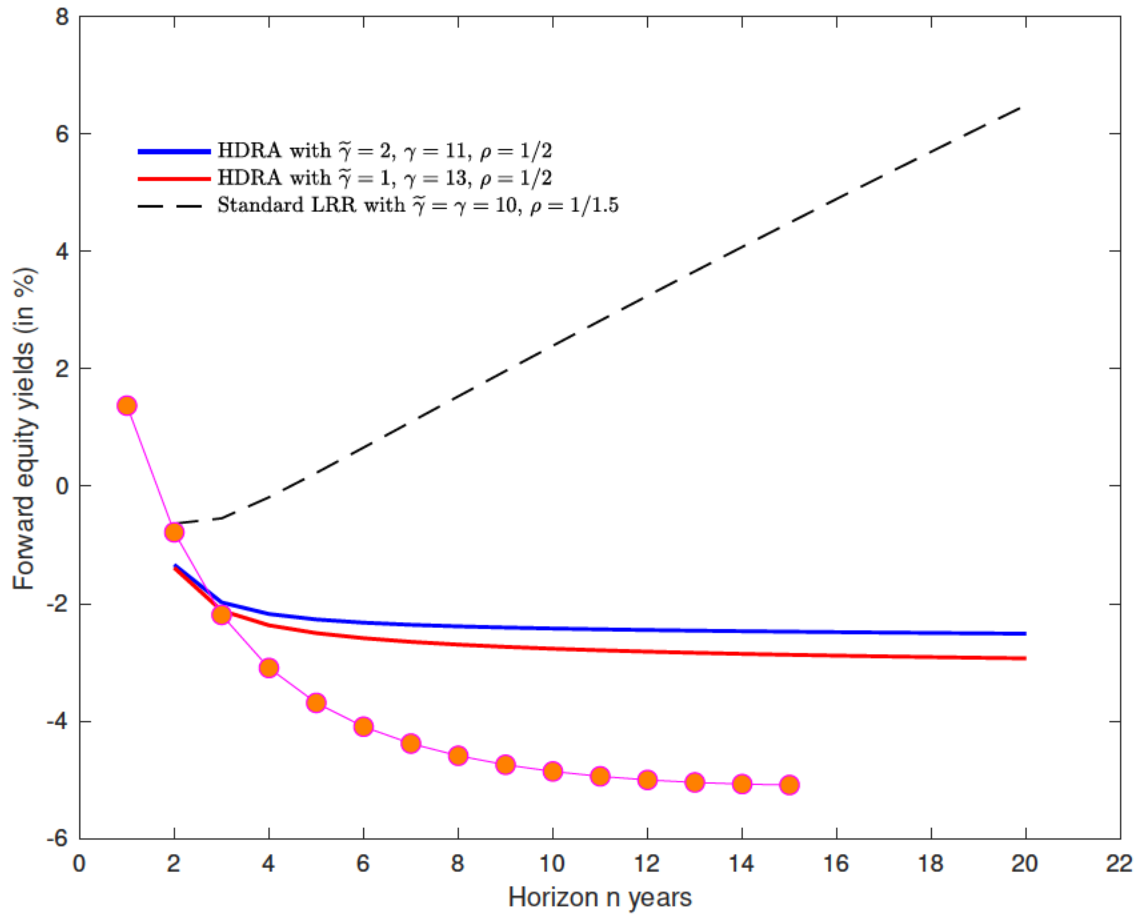

Figure 4: Term structure of forward equity yields: Recessions

Dividend strips futures average equity yield at horizon $h \geq 2$ under buy-and-hold trading. *HDRA* stands for "Horizon-dependent risk aversion", *Standard LRR* stands for the standard "Long-run risk" model ($\widetilde{\gamma} = \gamma = 10$ and $\rho = 1/1.5$ are the parameters of Bansal et al. (2012)). The calibration of both models is from Table 1. Data is from Giglio et al. (2023) using dividend strip proxies (1973-2020 sample restricted to NBER recessions). See Appendix D.1 for details.

Appendix

A Appendix to Section 2

A.1 Experimental evidence for horizon-dependent risk aversion

In this appendix, we discuss in detail the experimental evidence for risk aversion decreasing with horizon. The experimental literature on risk attitudes is large, spans the fields of psychology and economics and predominantly investigates deviations from the standard expected utility model. We therefore focus on three papers that use modern techniques of experimental economics such as real monetary payoffs and that explicitly study or allow us to infer how risk attitudes are affected by the horizon at which a lottery is resolved and paid out: Noussair and Wu (2006), Baucells and Heukamp (2010), and Abdellaoui, Diecidue, and Onculer (2011).

Noussair and Wu (2006) employ the protocol of Holt and Laury (2002), a widely used method in experimental economics, to gauge risk aversion. Subjects are presented with a list of choices between two binary lotteries in each row. The lottery in the first column always has two intermediate prizes, e.g., \$10.00 and \$8.00, whereas the lottery in the second column always has a high and a low prize, e.g., \$19.25 and \$0.50. Going down the list, only the respective probabilities of the two prizes change, varying, e.g. from $(0.1, 0.9)$ to $(1, 0)$. As probability mass shifts from the second prize to the first prize of both lotteries, the second, riskier lottery becomes increasingly attractive compared to the first, safer lottery. Subjects are asked to pick one of the two lotteries in each row and typically switch from the safer lottery to the riskier lottery at some row. A subject who switches at a later row exhibits greater risk aversion than a subject who switches at an earlier row.

Noussair and Wu (2006) use this protocol with real payoffs on a sample of undergraduate students from economics courses at Emory University. In the main treatment (sessions 1–6), each subject makes choices for resolution and payout to occur immediately and also for resolution and payout to occur three months later.[29] Pooling the six sessions, the sample consists of 44 subjects, five of which make choices inconsistent with expected utility maximization (switching back and forth) and are excluded from the analysis. Of the remaining 39 subjects, 15 subjects (38.5%) exhibit risk aversion decreasing in horizon, 21 subjects (53.8%) exhibit constant risk aversion and three subjects (7.7%) exhibit increasing risk aversion (their Table 2). Decreasing risk aversion is therefore five times more prevalent in the sample than increasing risk aversion.

[29]The experiment includes one additional session that does not include immediate resolution and one that increase the payoffs of the lotteries resolved in the future.

Figure 5 provides a simplified example of the experiment of Noussair and Wu (2006), and illustrates the notion of *horizon-dependent risk aversion* (HDRA). In Figure 5, subjects are asked to rank a lottery with payoff $x = 1$ for certain versus a lottery with payoff $x = 3$ with a 50% chance, and $x = 0$ otherwise. All subjects choose their rankings at time $t = 0$; however for some the lottery happens at time $t = 2$ (the "distant risk" case), and for some the lottery happens at time $t = 1$ (the "imminent risk" case).

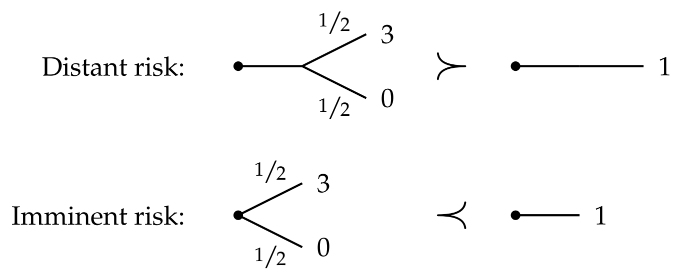

Figure 5: Preferences with horizon-dependent risk aversion.

To gauge statistical significance, we show 95% confidence intervals for the three proportions of subjects in Table 3 and find that the confidence intervals for the proportions with increasing and decreasing risk aversion do not overlap.[30] Noussair and Wu (2006) report that a paired t-test rejects the hypothesis that decreasing and increasing risk aversion are equally likely at the 1% level.

Table 3: Proportions of subjects with different forms of HDRA (Horizon-dependent risk aversion). The table reports proportions of subjects with decreasing, constant and increasing HDRA and corresponding Wald 95% confidence intervals for the horizon comparisons in the papers listed at the top (NW is Noussair and Wu (2006); ADO is Abdellaoui, Diecidue, and Onculer (2011)). For NW, the last confidence interval is clipped at the lower bound.

	NW	ADO		
	0–3 months	0–6 months	0–12 months	6–12 months
Decreasing HDRA	0.385	0.569	0.627	0.465
	$[0.232, 0.537]$	$[0.525, 0.612]$	$[0.584, 0.669]$	$[0.422, 0.509]$
Constant HDRA	0.538	0.250	0.187	0.373
	$[0.382, 0.695]$	$[0.213, 0.290]$	$[0.154, 0.223]$	$[0.331, 0.416]$
Increasing HDRA	0.077	0.181	0.187	0.162
	$[0, 0.161]$	$[0.149, 0.217]$	$[0.154, 0.223]$	$[0.131, 0.196]$
Num. obs.	39	52×10	52×10	52×10

[30] Noussair and Wu (2006) include the five subjects with inconsistent choices when they report 95% confidence interval of the proportion with decreasing and increasing risk aversion as $[0.242, 0.520]$ and $[0.006, 0.176]$, respectively, on their page 406. Their footnote 11 reports the confidence intervals excluding the inconsistent subjects.

Baucells and Heukamp (2010) study a sample of 221 MBA and EMBA students at IESE Business School who choose between binary lotteries that pay either a positive amount or zero (real payoffs). Similar to the protocol of Holt and Laury (2002), the first lottery is safer (higher probability of a lower payoff or deterministic) while the second lottery is riskier (lower probability of a higher payoff) such that the share of subjects choosing the safer lottery is a proxy for average risk aversion across subjects. In seven groups of choices, the lotteries are kept constant within each group and only the horizon of the resolution and payoff of both lotteries differs. Two of the groups cover horizons 0, 3, 6 and 12 months (tasks 1–4 and 8–11 in their Table 5); two cover 0, 3 and 6 months (tasks 5–7 and 12–14); two cover 0, 1 and 3 months (tasks 15–17 and 18–20); and one covers 0 and 3 months (tasks 21–22).[31] Within each of the seven groups, we can compare horizons $h > 0$ to horizon $h = 0$, yielding 15 comparisons. Subjects exhibit decreasing average risk aversion for 12 out of the 15 comparisons (more subjects choosing the safe alternative at horizon $h > 0$ than at horizon $h = 0$). Baucells and Heukamp (2010) do not report results of paired t-tests; using the data reported in their Table 5 (sample size and share of subjects choosing each lottery), we can only conduct two-sample t-tests. In the plausible case that risk aversion is positively correlated across horizon at the subject level, the two-sample t-statistic is biased downward compared to the paired t-statistic so we are biased against finding a significant difference. Nevertheless, we find that five of those 12 cases of decreasing risk aversion are significant at conventional levels against the alternative of increasing risk aversion (three at the 10% level, one at 5% and one at 1%). In contrast, subjects exhibit increasing risk aversion in only three out of the 15 comparisons with the difference never significant at conventional levels. Baucells and Heukamp (2010) do not report results that would allow us to calculate the proportions of subjects with decreasing, constant and increasing risk aversion.

Abdellaoui, Diecidue, and Onculer (2011) who gauge risk attitudes on a sample of 52 undergraduates from economics courses at Bogazici University. The experiment considers 10 different binary lotteries varying in their prizes and probabilities (real payoffs). For each subject, the experiment elicits the certainty equivalents for the 10 lotteries at each of three horizons (0, 6 and 12 months), where the certainty equivalent is paid at the same time as the lottery would be.[32] Their Table 2 reports means and standard deviations for the certainty equivalents of the 10 lotteries at the three horizons; their Table 3 reports t-statistics for paired tests against the hypothesis of equal certainty equivalents at horizons

[31] The experiment includes three additional tasks which do not allow comparison of the same lotteries across different horizons.

[32] The experiment includes a fourth session with uncertain horizon.

0 vs. 6 months and 0 vs. 12 months (also 6 vs. 12). Analogous to our discussion of Baucells and Heukamp (2010) above, we have 20 comparisons of horizons $h > 0$ to horizon $h = 0$. Subjects exhibit strictly decreasing average risk aversion for 18 out of the 20 comparisons (the mean certainty equivalent is higher at horizon $h > 0$ than at horizon $h = 0$). Now using the reported t-statistics for the proper paired test, 14 of the 18 differences are significant at conventional levels against the alternative of equal risk aversion (two at the 10% level, two at 5% and ten at 1%). In contrast, subjects exhibit increasing risk aversion in only two out of the 20 comparisons with the difference never significant at conventional levels. Abdellaoui, Diecidue, and Onculer (2011) also report that one-way ANOVA tests detect significantly decreasing risk aversion at the 5% level for seven of the 10 lotteries. For risk premia ($EV/CE - 1$), their Table 2 implies that the average risk premium across the 10 lotteries at horizon 0, 6 and 12 months is 35.7%, 28.4% and 25.6%, respectively, while the median risk premium is 43.8%, 32.6% and 25.9%. Risk premia are therefore 40% to 70% higher for the present than for the 12 month horizon.

Further, their Table 3 reports, for each lottery, the number of subjects with decreasing and with increasing risk aversion which allows us to calculate the proportion of subjects with decreasing, constant and increasing risk aversion with corresponding 95% confidence intervals. Pooling across lotteries, our Table 3 shows that the share of subjects with decreasing risk aversion is between 46.5% and 56.9% across the three horizon comparisons, the share with constant risk aversion is between 18.7% and 37.3% and the share with increasing risk aversion between 16.2% and 18.1%; the 95% confidence intervals for the share with decreasing risk aversion never overlap with the confidence intervals for the other two shares. Including the results from Noussair and Wu (2006) shown in the first column of Table 3, the share of subjects that display decreasing risk aversion is higher at longer horizons which is consistent with the effect being stronger and thus easier to detect at longer horizons.

In sum, a consistent picture of the quantitative importance of decreasing HDRA emerges across the three papers by Noussair and Wu (2006), Baucells and Heukamp (2010), and Abdellaoui, Diecidue, and Onculer (2011). While all three possibilities of HDRA are present among subjects (decreasing, constant and increasing), the share of subjects with decreasing risk aversion is between 40% and 60%, significantly outweighs the share with increasing risk aversion and is at least comparable to the share with constant risk aversion. Second, when averaging across subjects, risk aversion is decreasing in horizon in the vast majority of comparisons, with the decrease both statistically and economically significant.

A.2 General sequence of risk aversions

Let $\{\gamma_h\}_{h\geq 1}$ be a sequence representing risk aversion at horizon h. In period t, the agent evaluates a consumption stream starting in period $t+h$ by

$$V_{t,h} = \left((1-\beta)\, C_{t+h}^{1-\rho} + \beta E_{t+h}\left[V_{t,h+1}^{1-\gamma_{h+1}} \right]^{\frac{1-\rho}{1-\gamma_{h+1}}} \right)^{\frac{1}{1-\rho}} \quad \text{for all} \quad h \geq 0.$$

The agent's utility in period t, corresponding to $h=0$, is denoted $V_t \equiv V_{t,0}$ for all t.

Assuming the risk aversion sequence becomes a constant at some horizon H, i.e. $\gamma_h = \widetilde{\gamma}$ for $h \geq H$, then $V_{t,H}$, corresponds to the standard Epstein-Zin recursion with risk aversion $\widetilde{\gamma}$, for which we can use the standard solution. Solving backwards from $V_{t,H}$ to $V_{t,H-1}$, then $V_{t,H-2}$ etc. determines V_t. See Appendix B.5 for additional details.

A.3 Naive investors

If our agent is naive about her own dynamic inconsistencies, she wrongly assumes she will optimize on $\widetilde{V}_{t+h}$ instead of V_{t+h} for all $h \geq 1$. The envelope conditions at $t+1$ thus applies to $\widetilde{V}_{t+1}$ in her one-period SDF, which becomes:

$$\Pi_{t,t+1}^{\text{naive}} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}$$

The following one-period SDFs for $h \geq 1$ are then given by:

$$\Pi_{t+h,t+h+1}^{\text{naive}} = \beta \left(\frac{C_{t+h+1}}{C_{t+h}} \right)^{-\rho} \left(\frac{\widetilde{V}_{t+h}}{E_{t+h}\left[\widetilde{V}_{t+h+1}^{1-\widetilde{\gamma}} \right]^{\frac{1}{1-\widetilde{\gamma}}}} \right)^{\rho-\widetilde{\gamma}}$$

When $\rho = 1$, naive agents behave as the buy-and-hold investors of Proposition 8:

$$\Pi_{t,t+1}^{\text{naive}} \times \cdots \times \Pi_{t+h-1,t+h}^{\text{naive}} \,|_{\rho=1} = \Pi_{t,t+h}^{\text{buy-and-hold}} \,|_{\rho=1} .$$

B Appendix to Section 3

Section 3 discusses the timing premium for the case with constant volatility of consumption, i.e. process (5), but we derive all proofs for the more general case with time varying volatility, using consumption process (8).

The results presented in Proposition 1, Corollary 1, and Corollary 2, correspond to the case $\sigma_t = \sigma$. Figure 1 reports the timing premium under consumption process (8) and the calibration of Bansal et al. (2012).

B.1 Proof of Proposition 1

Under $\rho = 1$, and denoting logs by lowercase letters, our general model (1), (2) becomes

$$v_t = (1-\beta)\, c_t + \frac{\beta}{1-\gamma} \log(E_t[\exp\left((1-\gamma)\, \widetilde{v}_{t+1}\right)]) \,,$$

and

$$\widetilde{v}_{t+1} = (1-\beta)\, c_{t+1} + \frac{\beta}{1-\widetilde{\gamma}} \log(E_{t+1}[\exp\left((1-\widetilde{\gamma})\, \widetilde{v}_{t+2}\right)]) \,.$$

The continuation value $\widetilde{V}_{t+1}$ is determined by a standard Epstein-Zin recursion, for which closed-form solutions obtain under the lognormal stochastic process (8). Following the usual methodology, we guess a solution of the form

$$\widetilde{v}_t - c_t = \widetilde{\mu}_v + \phi_v x_t + \widetilde{\psi}_v \sigma_t^2,$$

where we write $\widetilde{\psi}_v = \psi_v\left(\widetilde{\gamma}\right)$ throughout the appendix to simplify the notation.

Then, using

$$\widetilde{v}_t = (1-\beta)\, c_t + \frac{\beta}{1-\widetilde{\gamma}} \log(E_t[\exp\left((1-\widetilde{\gamma})\, \widetilde{v}_{t+1}\right)]) \,, \text{ i.e.}$$
$$\widetilde{v}_t - c_t = \frac{\beta}{1-\widetilde{\gamma}} \log(E_t[\exp\left((1-\widetilde{\gamma})\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right)]) \,,$$

it is straightforward to verify that the linear functional form is the solution to the recursion for $\widetilde{v}_t - c_t$, with:

$$\begin{aligned}
\phi_v &= \frac{\beta\phi_c}{1-\beta\nu_x} \\
\widetilde{\psi}_v &= \frac{1}{2}\frac{\beta\left(1-\widetilde{\gamma}\right)}{1-\beta\nu_\sigma}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) \\
\widetilde{\mu}_v &= \frac{\beta}{1-\beta}\left(\mu_c + \widetilde{\psi}_v\sigma^2\left(1-\nu_\sigma\right) + \frac{1}{2}\left(1-\widetilde{\gamma}\right)\widetilde{\psi}_v^2\alpha_\sigma^2\right).
\end{aligned}$$

From

$$v_t = (1-\beta)\, c_t + \frac{\beta}{1-\gamma} \log(E_t[\exp\left((1-\gamma)\, \widetilde{v}_{t+1}\right)])\,, \text{ i.e.}$$
$$v_t - c_t = \frac{\beta}{1-\gamma} \log(E_t[\exp\left((1-\gamma)\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right)])\,,$$

we obtain

$$\widetilde{v}_t - v_t = \frac{\beta}{1-\widetilde{\gamma}} \log(E_t[\exp\left((1-\widetilde{\gamma})\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right)])$$
$$- \frac{\beta}{1-\gamma} \log(E_t[\exp\left((1-\gamma)\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right)])\,.$$

The constant terms in the expectations cancel out; the volatility terms remain with coefficient $\frac{1}{2}\beta(1-\widetilde{\gamma}) - \frac{1}{2}\beta(1-\gamma)$.

We arrive at the solution

$$\widetilde{v}_t - v_t = \frac{1}{2}\beta\left(\gamma - \widetilde{\gamma}\right)\left[\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2 \alpha_\sigma^2\right],$$

which, using the solution for $\widetilde{\psi}_v$, we can rewrite as

$$v_t - \widetilde{v}_t = -\frac{1}{2}\beta\left(\gamma - \widetilde{\gamma}\right)\widetilde{\psi}_v^2 \alpha_\sigma^2 - \left(\gamma - \widetilde{\gamma}\right)\frac{1-\beta\nu_\sigma}{1-\widetilde{\gamma}}\widetilde{\psi}_v \sigma_t^2.$$

Combining this with our solution for $\widetilde{v}_t - c_t$, we obtain

$$v_t - c_t = \frac{\beta}{1-\beta}\left(\mu_c + \widetilde{\psi}_v \sigma^2\left(1-\nu_\sigma\right) + \frac{1}{2}\widetilde{\psi}_v^2\left(\left(1-\gamma\right) + \beta\left(\gamma - \widetilde{\gamma}\right)\right)\alpha_\sigma^2\right)$$
$$+ \phi_v x_t + \frac{\widetilde{\psi}_v}{1-\widetilde{\gamma}}\left(\left(1-\gamma\right) + \beta\nu_\sigma\left(\gamma - \widetilde{\gamma}\right)\right)\sigma_t^2,$$

the solution to the value function at time t if consumption follows risk process (8).

If all risk is resolved at $t+1$, the log continuation utility $v_{t,t+1}^*$ is given by

$$v_{t,t+1}^* = (1-\beta)\, c_{t+1} + \beta\left(\left(1-\beta\right) c_{t+2} + \beta\left(\left(1-\beta\right) c_{t+3} + \cdots\right)\right)$$
$$= c_{t+1} + \sum_{h=1}^{\infty} \beta^h \left(c_{t+h+1} - c_{t+h}\right).$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$
\begin{aligned}
E_t[v^*_{t,t+1}] &= c_t + \frac{1}{1-\beta}\,\mu_c + \frac{\phi_c}{1-\beta\nu_x}x_t,\\
\mathrm{var}_t(v^*_{t,t+1}) &= \frac{1}{1-\beta^2\nu_\sigma}\left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2}\sigma^2\left(1-\nu_\sigma\right)\right)\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right).
\end{aligned}
$$

Using these expressions, and the relation

$$
\begin{aligned}
v_t^* &= (1-\beta)\,c_t + \frac{\beta}{1-\gamma}\log\big(E_t\big[\exp\left(\left(1-\gamma\right)v^*_{t,t+1}\right)\big]\big)\\
&= (1-\beta)\,c_t + \beta\left(E_t\big[v^*_{t,t+1}\big] + \frac{1}{2}\left(1-\gamma\right)\mathrm{var}_t\big(v^*_{t,t+1}\big)\right),
\end{aligned}
$$

we obtain the early resolution utility at t as

$$
\begin{aligned}
v_t^* - c_t &= \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\frac{\beta\left(1-\gamma\right)}{1-\beta^2\nu_\sigma}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2}\sigma^2\left(1-\nu_\sigma\right)\right)\\
&= \frac{\beta}{1-\beta}\mu_c + \phi_v x_t + \frac{\left(1-\gamma\right)\left(1-\beta\nu_\sigma\right)}{\left(1-\widetilde{\gamma}\right)\left(1-\beta^2\nu_\sigma\right)}\widetilde{\psi}_v\left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2}\sigma^2\left(1-\nu_\sigma\right)\right).
\end{aligned}
$$

From the definition of the timing premium,

$$
\log\left(1-\mathrm{TP}_t\right) = v_t - v_t^*,
$$

and from the calculations above, we derive

$$
\begin{aligned}
v_t - v_t^* = &\frac{\beta}{1-\beta}\widetilde{\psi}_v\sigma^2\left(1-\nu_\sigma\right)\left(1 - \frac{1-\gamma}{1-\widetilde{\gamma}}\frac{1-\beta\nu_\sigma}{1-\beta^2\nu_\sigma}\frac{\beta}{1+\beta}\right)\\
&+ \widetilde{\psi}_v\nu_\sigma\sigma_t^2\frac{\beta}{1-\widetilde{\gamma}}\left(\left(1-\gamma\right)\frac{1-\beta}{1-\beta^2\nu_\sigma} + \left(\gamma-\widetilde{\gamma}\right)\right)\\
&+ \frac{1}{2}\frac{\beta}{1-\beta}\left(\left(1-\gamma\right) + \beta\left(\gamma-\widetilde{\gamma}\right)\right)\widetilde{\psi}_v^2\alpha_\sigma^2.
\end{aligned}
$$

In the case with constant volatility, $\sigma_t = \sigma$, $\alpha_\sigma = 0$, $\nu_\sigma = 1$, we obtain

$$
\begin{aligned}
v_t - v_t^* &= \widetilde{\psi}_v\sigma^2\frac{\beta}{1-\widetilde{\gamma}}\left(\left(1-\gamma\right)\frac{1-\beta}{1-\beta^2} + \left(\gamma-\widetilde{\gamma}\right)\right)\\
&= \frac{1}{2}\frac{\beta^2}{1-\beta}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2\left(\left(1-\gamma\right)\frac{1-\beta}{1-\beta^2} + \left(\gamma-\widetilde{\gamma}\right)\right).
\end{aligned}
$$

We thus obtain the timing premium

$$\log\left(1-\mathrm{TP}_t\right)=\frac{1}{2}\frac{\beta^2}{1-\beta^2}\left(\alpha_c^2+\left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2\left((1-\gamma)+\ (1+\beta)\left(\gamma-\widetilde{\gamma}\right)\right),$$

or, as stated in the proposition,

$$\mathrm{TP}=1-\exp\left(\frac{1}{2}\left(1-\left(\gamma-(1+\beta)\left(\gamma-\widetilde{\gamma}\right)\right)\right)\frac{\beta^2}{1-\beta^2}\left(\alpha_c^2+\left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2\right).$$

□

B.2 Proof of Corollary 1

From Proposition 1,

$$\log\left(1-\mathrm{TP}_t\right)=\underbrace{\frac{1}{2}\frac{\beta^2}{1-\beta^2}\left(\alpha_c^2+\left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2}_{\geq 0}\left((1-\gamma)+(1+\beta)\underbrace{\left(\gamma-\widetilde{\gamma}\right)}_{\geq 0\text{ iff }\gamma\geq\widetilde{\gamma}}\right),$$

so $\gamma\geq\widetilde{\gamma}$ unambiguously increases $\log\left(1-\mathrm{TP}_t\right)$, i.e. unambiguously lowers TP_t.

□

B.3 Proof of Corollary 2

From Proposition 1,

$$\log\left(1-\mathrm{TP}_t\right)=\underbrace{\frac{1}{2}\frac{\beta^2}{1-\beta^2}\left(\alpha_c^2+\left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2}_{\geq 0}\left((1-\gamma)+\ (1+\beta)\left(\gamma-\widetilde{\gamma}\right)\right),$$

so

$$\begin{aligned}\mathrm{TP}_t\leq 0\quad&\Leftrightarrow\quad(1-\gamma)+(1+\beta)\left(\gamma-\widetilde{\gamma}\right)\geq 0\\&\Leftrightarrow\quad\widetilde{\gamma}\leq\frac{1+\beta\gamma}{1+\beta}.\end{aligned}$$

Let $\gamma>\rho=1$. Then $\gamma>\frac{1+\beta\gamma}{1+\beta}>1$, and any $\widetilde{\gamma}$ such that $\gamma>\frac{1+\beta\gamma}{1+\beta}>\widetilde{\gamma}>1=\rho$ implies a preference for a late resolution of uncertainty.

$\square$

B.4 Timing premium under hyperbolic discounting "β-δ model"

We now assume risk aversion constant across horizons $\gamma = \widetilde{\gamma}$, but instead two different discount factors $\beta < \widetilde{\beta}$. The preference model becomes

$$V_t = \left((1-\beta) C_t^{1-\rho} + \beta E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}},$$

and

$$\widetilde{V}_t = \left(\left(1-\widetilde{\beta}\right) C_t^{1-\rho} + \widetilde{\beta} E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}.$$

Assuming $\rho = 1$, and focusing on the simpler case $\sigma_t = \sigma$ of process (5), we can use the recursions

$$\begin{aligned}
\widetilde{v}_t - c_t &= \frac{\widetilde{\beta}}{1-\gamma} E_t \left[\exp\left(1-\gamma\right)\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right] \\
v_t - c_t &= \frac{\beta}{1-\gamma} E_t \left[\exp\left(1-\gamma\right)\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right],
\end{aligned}$$

to derive the solutions

$$\begin{aligned}
\widetilde{v}_t - c_t &= \frac{\widetilde{\beta}}{1-\widetilde{\beta}}\mu_c + \frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x} x_t + \frac{1}{2}\frac{\widetilde{\beta}\left(1-\gamma\right)}{1-\widetilde{\beta}} \left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x} \right)^2 \alpha_x^2 \right) \sigma^2 \\
v_t - c_t &= \frac{\beta}{1-\widetilde{\beta}}\mu_c + \frac{\beta\phi_c}{1-\widetilde{\beta}\nu_x} x_t + \frac{1}{2}\frac{\beta\left(1-\gamma\right)}{1-\widetilde{\beta}} \left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x} \right)^2 \alpha_x^2 \right) \sigma^2.
\end{aligned}$$

If all risk is resolved at $t+1$, the log continuation utility $v_{t,t+1}^*$ is given by

$$\begin{aligned}
v_{t+1}^* &= \left(1-\widetilde{\beta}\right) c_{t+1} + \widetilde{\beta} \left(\left(1-\widetilde{\beta}\right) c_{t+2} + \widetilde{\beta} \left(\left(1-\widetilde{\beta}\right) c_{t+3} + \cdots \right) \right) \\
&= c_t + \sum_{h=0}^{\infty} \widetilde{\beta}^h \left(c_{t+h+1} - c_{t+h}\right).
\end{aligned}$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$E_t[v^*_{t+1}] = c_t + \frac{1}{1-\widetilde{\beta}}\mu_c + \frac{\phi_c}{1-\widetilde{\beta}\nu_x}x_t,$$
$$\mathrm{var}_t(v^*_{t+1}) = \frac{1}{1-\widetilde{\beta}^2}\sigma^2\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}\right)^2\alpha_x^2\right).$$

Using these expressions, we can derive the early resolution utility at t as

$$\begin{aligned} v^*_t &= c_t + \frac{\beta}{1-\gamma}E_t\left[\exp\left(1-\gamma\right)\left(v^*_{t+1}-c_t\right)\right] \\ &= c_t + \frac{\beta}{1-\widetilde{\beta}}\mu_c + \frac{\beta\phi_c}{1-\widetilde{\beta}\nu_x}x_t + \frac{1}{2}\frac{\beta\left(1-\gamma\right)}{1-\widetilde{\beta}^2}\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2, \end{aligned}$$

so we have

$$v_t - v^*_t = \frac{1}{2}\frac{\beta\widetilde{\beta}\left(1-\gamma\right)}{1-\widetilde{\beta}^2}\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}\right)^2\alpha_x^2\right)\sigma^2.$$

In the case of hyperbolic discounting where $\beta < \widetilde{\beta}$, we have

$$\frac{\widetilde{\beta}^2}{1-\widetilde{\beta}^2} > \frac{\beta\widetilde{\beta}}{1-\widetilde{\beta}^2} > \frac{\beta^2}{1-\beta^2}.$$

Given $\log\left(1-\mathrm{TP}_t\right) = v_t - v^*_t$, and assuming $\gamma > 1 = \rho$, the timing premium under the β-$\widetilde{\beta}$ hyperbolic model is greater than under the β-only model and lower than under the $\widetilde{\beta}$-only model. Under the β-$\widetilde{\beta}$ hyperbolic model, agents have a preference for early resolutions of uncertainty if and only if $\gamma > 1 = \rho$.

B.5 Extension to other information arrival structures

General sequence of risk aversions

Suppose we have a sequence of risk aversions $\{\gamma_h\}_{h=1}^{\infty}$ that is decreasing until horizon H and constant afterwards: $\gamma_h = \widetilde{\gamma}, \forall h \geq H$. We consider the case $\rho = 1$ and $\sigma_t = \sigma$, i.e. process (5), as in Proposition 1. Following the same methodology as in Appendix B.1, we start from the standard recursion

$$v_{t,H-1} = (1-\beta)\, c_{t+1} + \frac{\beta}{1-\gamma_H} \log(E_{t+1}[\exp\left((1-\gamma_H)\, v_{t+1,H-1}\right)]) \,,$$

with solution

$$v_{t,H-1} - c_t = \mu_{v,H} + \phi_v x_t,$$

where

$$\begin{aligned}
\phi_v &= \frac{\beta \phi_c}{1-\beta \nu_x} \\
\mu_{v,H} &= \frac{\beta}{1-\beta} \left(\mu_c + \frac{1}{2}\left(1-\gamma_H\right)\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right)\right).
\end{aligned}$$

Then, from

$$v_{t,H-2} = (1-\beta)\, c_t + \frac{\beta}{1-\gamma_{H-1}} \log(E_t[\exp\left((1-\gamma_{H-1})\, v_{t+1,H-1}\right)]) \,,$$

we obtain

$$v_{t,H-2} - c_t = \mu_{v,H-1} + \phi_v x_t,$$

where

$$\mu_{v,H-1} = \beta \left(\mu_c + \frac{1}{2}\left(1-\gamma_{H-1}\right)\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right) + \mu_{v,H}\right).$$

Similarly, we obtain:

$$v_{t,H-3} - c_t = \mu_{v,H-2} + \phi_v x_t,$$

where

$$\mu_{v,H-2} = \beta \left(\mu_c + \frac{1}{2}\left(1-\gamma_{H-2}\right)\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right) + \mu_{v,H-1}\right).$$

Iterating down to $v_{t,0} = v_t$, we obtain:

$$v_t - c_t = \mu_{v,1} + \phi_v x_t,$$

where

$$\mu_{v,1} = \beta\mu_c\left(1+\beta+\cdots+\beta^{H-2}\right)+\beta^{H-1}\mu_{v,H}$$
$$+\frac{1}{2}\beta\left[(1-\gamma_1)+\beta(1-\gamma_2)+\cdots+\beta^{H-2}(1-\gamma_{H-1})\right]\left(\alpha_c^2+\phi_v^2\alpha_x^2\right).$$

Rearranging the terms in $1-\gamma_h$ as $1-\gamma_1+\gamma_1-\gamma_2+\cdots$, and using the recursive solution for $\mu_{v,H}$ above, we obtain

$$\mu_{v,1} = \frac{\beta}{1-\beta}\mu_c+\frac{1}{2}\frac{\beta}{1-\beta}\left[(1-\gamma_1)+\sum_{h=1}^{H-1}\beta^h(\gamma_h-\gamma_{h+1})\right]\left(\alpha_c^2+\phi_v^2\alpha_x^2\right).$$

The value under early resolution of uncertainty is unchanged from the $\gamma,\widetilde{\gamma}$ framework:

$$v_t^- c_t = \frac{\beta}{1-\beta}\mu_c+\phi_v x_t+\frac{1}{2}\frac{\beta}{1-\beta^2}(1-\gamma_1)\left(\alpha_c^2+\phi_v^2\alpha_x^2\right).$$

Combining these results, we obtain

$$v_t - v_t^* = \frac{1}{2}\frac{\beta}{1-\beta}\left[(1-\gamma_1)\frac{\beta}{1+\beta}+\sum_{h=1}^{H-1}\beta^h(\gamma_h-\gamma_{h+1})\right]\left(\alpha_c^2+\phi_v^2\alpha_x^2\right).$$

The agent therefore prefers later resolutions of uncertainty if and only if

$$\textstyle\sum_{h=1}^{H-1}\beta^{h-1}(\gamma_h-\gamma_{h+1}) > \dfrac{\gamma_1-1}{1+\beta},$$

i.e. as long as the sequence $\{\gamma_h\}_{h=1}^H$ is sufficiently decreasing.

Resolution of uncertainty at $t+2$ versus at $t+1$

As seen in Appendix B.1, if all risk is resolved at $t+1$, the log continuation utility $v_{t,t+1}^{*,1}$ is given by

$$v_{t,t+1}^{*,1} = c_{t+1}+\sum_{h=1}^{\infty}\beta^h(c_{t+h+1}-c_{t+h}),$$

such that

$$E_t[v^{*,1}_{t,t+1}] = c_t + \frac{1}{1-\beta}\,\mu_c + \frac{\phi_c}{1-\beta\nu_x}x_t,$$
$$\text{var}_t(v^{*,1}_{t,t+1}) = \frac{\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2}{1-\beta^2}.$$

From

$$v^{*,1}_t = c_t + \frac{\beta}{1-\gamma}\log\Big(E_t\Big[\exp\left((1-\gamma)\left(v^{*,1}_{t,t+1} - c_t\right)\right)\Big]\Big),$$

we obtain the early resolution utility at t given by

$$v^{*,1}_t = c_t + \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\frac{\beta\,(1-\gamma)}{1-\beta^2}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right).$$

If all risk is resolved at $t+2$ instead, we can apply the same methodology as above to derive $v^{*,2}_{t,t+1}$:

$$v^{*,2}_{t,t+1} = c_{t+1} + \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_{t+1} + \frac{1}{2}\frac{\beta\,(1-\widetilde{\gamma})}{1-\beta^2}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)$$

To which we apply

$$v^{*,2}_t = c_t + \frac{\beta}{1-\gamma}\log\Big(E_t\Big[\exp\left((1-\gamma)\left(v^{*,2}_{t,t+1} - c_{t+1} + c_{t+1} - c_t\right)\right)\Big]\Big),$$

to obtain

$$v^{*,2}_t = c_t + \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\beta\left(\frac{\beta\,(1-\widetilde{\gamma})}{1-\beta^2} + 1 - \gamma\right)\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right).$$

The difference in utility between resolution at $t+2$ and at $t+1$ is therefore given by

$$v^{*,2}_t - v^{*,1}_t = \frac{1}{2}\frac{\beta^2}{1-\beta^2}\left((1-\widetilde{\gamma}) - \beta\,(1-\gamma)\right)\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right).$$

An agent with standard Epstein-Zin preferences, i.e. $\gamma = \widetilde{\gamma}$, prefers the later resolution at $t+2$ to the earlier resolution at $t+1$, i.e. $v^{*,1}_t > v^{*,2}_t$, if and only if $\gamma > 1$. In contrast, an agent with horizon dependent risk aversion and $\gamma > 1$ prefers the later resolution at $t+2$

to the earlier resolution at $t+1$ if and only if $\gamma - \widetilde{\gamma} > (1-\beta)(\gamma - 1)$.

C Appendix to Section 4

C.1 Proof of Proposition 2

To derive the one-period ahead stochastic discount factor, we use the inter-temporal marginal rate of substitution

$$\Pi_{t,t+1} = \frac{dV_t / dW_{t+1}}{dV_t / dC_t}.$$

We have:

$$\frac{dV_t}{dW_{t+1}} = \frac{dV_t}{d\widetilde{V}_{t+1}} \times \frac{d\widetilde{V}_{t+1}}{dW_{t+1}}$$

Due to the homotheticity of our preferences, we can rely on the fact that both $\widetilde{V}_{t+1}$ and V_{t+1} are homogeneous of degree one which implies that

$$\frac{d\widetilde{V}_{t+1}/dW_{t+1}}{dV_{t+1}/dW_{t+1}} = \frac{\widetilde{V}_{t+1}}{V_{t+1}}.$$

Further, the envelope condition guarantees:

$$\frac{dV_t}{dW_t} = \frac{dV_t}{dC_t}.$$

This allows us to derive:

$$\Pi_{t,t+1} = \frac{dV_t / d\widetilde{V}_{t+1} \times dV_{t+1}/dC_{t+1} \times \widetilde{V}_{t+1}/V_{t+1}}{dV_t / dC_t}.$$

Given

$$V_t = \left((1-\beta) C_t^{1-\rho} + \beta E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}},$$

we have:

$$\frac{dV_t}{dC_t} = \left((1-\beta)\,C_t^{1-\rho} + \beta E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}-1}(1-\beta)\,C_t^{-\rho}$$
$$= V_t^{\rho}\,(1-\beta)\,C_t^{-\rho},$$

and

$$\frac{dV_t}{d\widetilde{V}_{t+1}} = \left((1-\beta)\,C_t^{1-\rho} + \beta E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}}\right)^{\frac{1}{1-\rho}-1}\beta\widetilde{V}_{t+1}^{-\gamma}E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1-\rho}{1-\gamma}-1}$$
$$= V_t^{\rho}\beta\widetilde{V}_{t+1}^{-\gamma}E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{\gamma-\rho}{1-\gamma}}.$$

Combining these, we obtain:

$$\Pi_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\left(\frac{\widetilde{V}_{t+1}}{V_{t+1}}\right)^{1-\rho}.$$

□

C.2 Proof of Lemma 1

In the preferences of Definition 1, the recursion in $\widetilde{V}$ can be rewritten as:

$$\left(\frac{\widetilde{V}_t}{C_t}\right)^{1-\widetilde{\gamma}} = \left[(1-\beta)^{1-\rho} + \beta E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\widetilde{\gamma}}\right]^{\frac{1-\rho}{1-\widetilde{\gamma}}}\right]^{\frac{1-\widetilde{\gamma}}{1-\rho}},$$

so, for $\beta = 1-\epsilon$, with $\epsilon << 1$, we obtain the approximation:

$$\left(\frac{\widetilde{V}_t}{C_t}\right)^{1-\widetilde{\gamma}} \approx \beta^{\frac{1-\widetilde{\gamma}}{1-\rho}}E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\widetilde{\gamma}}\right].$$

This is an eigenfunction problem with eigenvalue $\beta^{-\frac{1-\widetilde{\gamma}}{1-\rho}}$ and eigenfunction $\left(\widetilde{V}/C\right)^{1-\widetilde{\gamma}}$, known up to a multiplier.

As usual, we assume and verify:

$$\widetilde{v}_t - c_t \approx \tilde{\mu}_v + \phi_v x_t + \tilde{\psi}_v \sigma_t^2,$$

where we obtain from the recursion above

$$\begin{aligned}
\phi_v &= \frac{\phi_c}{1-\nu_x} \\
\tilde{\psi}_v &= \frac{1}{2}\frac{1-\widetilde{\gamma}}{1-\nu_\sigma}\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right) < 0.
\end{aligned}$$

Using the approximation $\beta \approx 1$ throughout, we derive:

$$\frac{V_t}{\widetilde{V}_t} \approx = \frac{\left(E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\left(E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\widetilde{\gamma}}\right]\right)^{\frac{1}{1-\widetilde{\gamma}}}},$$

and therefore:

$$v_t - \widetilde{v}_t \approx -\frac{1}{2}\left(\gamma - \widetilde{\gamma}\right)\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2 \alpha_\sigma^2\right].$$

□

C.3 Proof of Proposition 3 and Proposition 4

From Proposition 2:

$$\Pi_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}}{E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]}\right)^{\frac{\rho-\gamma}{1-\gamma}}\left(\frac{\widetilde{V}_{t+1}}{V_{t+1}}\right)^{1-\rho}.$$

Using the solution for $\widetilde{v}_t - c_t$ and for $\widetilde{v}_t - v_t$ derived in Appendix C.2, denoting logs by lowercase letters, and defining $\bar{\pi}_t = E_t\left[\pi_{t,t+1}\right]$, the expression for the SDF becomes:

$$\begin{aligned}
\pi_{t,t+1} = \bar{\pi}_t &- \gamma\alpha_c\sigma_t w_{c,t+1} + (\rho-\gamma)\,\phi_v\alpha_x\sigma_t w_{x,t+1} \\
&+ \left[(\rho-\gamma) + (1-\rho)\left(\gamma-\widetilde{\gamma}\right)\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\tilde{\psi}_v\alpha_\sigma w_{\sigma,t+1}.
\end{aligned}$$

□

C.4 Proof of Corollary 3

From $\psi_v(\widetilde{\gamma}) < 0$ (see Appendix C.2), and given $\gamma > \widetilde{\gamma} > 1 > \rho$, we immediately obtain:

$$\left[(\rho-\gamma) + (1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\tilde{\psi}_v > 0,$$

corresponding to a negative price of risk for volatility shocks.

From Appendix C.2, $\tilde{\psi}_v = \psi_v(\widetilde{\gamma}) = \frac{1-\widetilde{\gamma}}{1-\gamma}\psi_v(\gamma) = \left(1 + \frac{\gamma-\widetilde{\gamma}}{1-\gamma}\right)\psi_v(\gamma)$, so we can rewrite the pricing of volatility shocks under horizon dependent risk aversion as:

$$\left[(\rho-\gamma) + (1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\tilde{\psi}_v = (\rho-\gamma)\psi_v + \underbrace{\left[(1-\gamma) - \nu_\sigma(1-\rho)\right]\frac{\gamma-\widetilde{\gamma}}{1-\gamma}\psi_v}_{\leq 0}.$$

$\square$

C.5 Exact Solutions, $\rho = 1$

Using Proposition 2 for the case $\rho = 1$, we obtain:

$$\begin{aligned}\Pi_{t,t+1} &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{\widetilde{V}_{t+1}^{1-\gamma}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]} \\ &= \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}}{E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]}.\end{aligned}$$

Using the solution $\widetilde{v}_t - c_t = \widetilde{\mu}_v + \phi_v x_t + \widetilde{\psi}_v\sigma_t^2$ and $\widetilde{v}_t - v_t = \frac{1}{2}\beta(\gamma-\widetilde{\gamma})\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right]$ derived in Appendix B when $\rho = 1$, the expression for the SDF becomes:

$$\begin{aligned}\pi_{t,t+1} = &\overbrace{\log\beta - \mu_c - \phi_c x_t - \frac{1}{2}(1-\gamma)^2\left(\left[\alpha_c^2 + \phi_v^2\alpha_x^2\right]\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right)}^{\bar{\pi}_t = E_t[\pi_{t,t+1}]} \\ &- \gamma\alpha_c\sigma_t w_{c,t+1} + (1-\gamma)\phi_v\alpha_x\sigma_t w_{x,t+1} \\ &+ (1-\gamma)\widetilde{\psi}_v\alpha_\sigma w_{\sigma,t+1}.\end{aligned}$$

The log risk-free rate is given by $r_{f,t} = -\log E_t(\Pi_{t,t+1})$:

$$r_{f,t} = -\bar{\pi}_t - \frac{1}{2}\left(\gamma^2\alpha_c^2\sigma_t^2 + (1-\gamma)^2\phi_v^2\alpha_x^2\sigma_t^2 + (1-\gamma)^2\widetilde{\psi}_v^2\alpha_\sigma^2\right),$$

which simplifies to

$$r_{f,t} = -\log\beta + \mu_c + \phi_c x_t + \left(\frac{1}{2} - \gamma\right)\alpha_c^2\sigma_t^2.$$

C.6 Proof of Proposition 5

To derive the equity premium, we log-linearize the market returns $\{r_{m,t}\}$ as a first-order approximation around the average log price-dividend ratio $\overline{z} = \overline{p-d}$:

$$\begin{aligned} r_{m,t+1} &= \log\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \\ &= \Delta d_{t+1} + \log\left(1 + e^{z_{t+1}}\right) - z_t \\ &\approx \kappa_0 + \kappa z_{t+1} - z_t + \Delta d_{t+1} \end{aligned}$$

where $z_t = p_t - d_t$ and $\kappa = \frac{e^{\overline{z}}}{1+e^{\overline{z}}} < 1$, $\kappa_0 = \log\left(1 + e^{\overline{z}}\right) - \overline{z}\kappa$, and $\overline{z} = \overline{p-d}$ is taken from the data.

From

$$E_t\left(\Pi_{t,t+1} R_{m,t+1}\right) = 1$$

we obtain a recursion in z_t:

$$\log E_t\left(\exp\left[\pi_{t,t+1} + \kappa_0 + \kappa z_{t+1} - z_t + \Delta d_{t+1}\right]\right) = 0.$$

From Appendix C.2 and C.3:

$$\begin{aligned} \pi_{t,t+1} = \bar{\pi}_t &- \gamma\alpha_c\sigma_t w_{c,t+1} + (\rho - \gamma)\,\phi_v\alpha_x\sigma_t w_{x,t+1} \\ &+ \left[(\rho - \gamma) + (1-\rho)\,(\gamma - \widetilde{\gamma})\,\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\tilde{\psi}_v\alpha_\sigma w_{\sigma,t+1}. \end{aligned}$$

where,

$$\bar{\pi}_t = \bar{\pi} - \rho\phi_c x_t - \frac{1}{2}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left[(\rho-\gamma)\,(1-\gamma) - (1-\rho)\,(\gamma - \widetilde{\gamma})\,\nu_\sigma\right]\sigma_t^2.$$

Guess: $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$, and verify:

$$\log E_t \left(\exp \left(\left\{ \begin{array}{l} \bar{\pi} + \kappa_0 + (\kappa - 1) A_0 + \kappa A_2 \sigma^2 (1 - \nu_\sigma) \\ -\rho \phi_c x_t + (\kappa \nu_x - 1) A_1 x_t + \phi_d x_t \\ -\frac{1}{2} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \left[(\rho - \gamma)(1 - \gamma) - (1 - \rho)(\gamma - \widetilde{\gamma}) \nu_\sigma \right] \sigma_t^2 + (\kappa \nu_\sigma - 1) A_2 \sigma_t^2 \\ +(-\gamma + \chi) \alpha_c \sigma_t w_{c,t+1} \\ +((\rho - \gamma) \phi_v + \kappa A_1) \alpha_x \sigma_t w_{x,t+1} \\ + \left(\left[(\rho - \gamma) + (1 - \rho)(\gamma - \widetilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \widetilde{\gamma}} \right] \tilde{\psi}_v + \kappa A_2 \right) \alpha_\sigma w_{\sigma,t+1} \\ +\alpha_d \sigma_t w_{d,t+1} \end{array} \right. \right) \right) = 0.$$

We obtain:

$$A_1 = \frac{\phi_d - \rho \phi_c}{1 - \kappa \nu_x},$$

$$\begin{aligned} A_2 (1 - \kappa \nu_\sigma) = & -\frac{1}{2} \left[(\rho - \gamma)(1 - \gamma) - (1 - \rho)(\gamma - \widetilde{\gamma}) \nu_\sigma \right] \left[\alpha_c^2 + \phi_v^2 \alpha_x^2 \right] \\ & + \frac{1}{2} (\chi - \gamma)^2 \alpha_c^2 + \frac{1}{2} (\kappa A_1 + (\rho - \gamma) \phi_v)^2 \alpha_x^2 + \frac{1}{2} \alpha_d^2. \end{aligned}$$

Given these results, we can rewrite:

$$r_{m,t+1} = \overline{r_{m,t}} + \chi \alpha_c \sigma_t w_{c,t+1} + \kappa A_1 \alpha_x \sigma_t w_{x,t+1} + \kappa A_2(\gamma, \widetilde{\gamma}) \alpha_\sigma w_{\sigma,t+1} + \alpha_d \sigma_t w_{d,t+1},$$

where $A_2(\gamma, \widetilde{\gamma}) = A_2(\gamma) + \frac{1}{2(1 - \kappa \nu_\sigma)} \left[(1 - \rho)(\gamma - \widetilde{\gamma}) \nu_\sigma \right] \left[\alpha_c^2 + \phi_v^2 \alpha_x^2 \right] \geq A_2(\gamma)$, strictly so when $\gamma > \widetilde{\gamma}$: the market returns covary *less* negatively with the consumption volatility shocks.

Given $E_t \left(\Pi_{t,t+1} R_{m,t+1} \right) = 1$, we obtain:

$$\begin{aligned} \log E_t R_{m,t+1} - \log R_{f,t} = & \gamma \chi \alpha_c^2 \sigma_t^2 + (\gamma - \rho) \phi_v \kappa A_1 \alpha_x^2 \sigma_t^2 \\ & + \left[(\gamma - \rho) + (1 - \rho)(\gamma - \widetilde{\gamma}) \frac{1 - \nu_\sigma}{\widetilde{\gamma} - 1} \right] \tilde{\psi}_v \kappa A_2(\gamma, \widetilde{\gamma}) \alpha_\sigma^2. \end{aligned}$$

□

To derive the unconditional equity premium, we note that the volatility process is stationary under the constraint $\nu_\sigma < 1$, such that $\sigma_t^2 \sim \mathcal{N} \left(\sigma^2, \Sigma_\sigma^2 \right)$, where $\Sigma_\sigma^2 = \frac{\alpha_\sigma^2}{1 - \nu_\sigma^2}$.

Since the shocks to x_t and to σ_t^2 between $t-1$ and t are independent and $E(x_t) = 0$, we have

$$\begin{aligned}\operatorname{cov}\left(x_t,\sigma_t^2\right) &= EE_{t-1}\left(x_t\sigma_t^2\right)\\ &= \nu_x\nu_\sigma E\left(x_{t-1}\sigma_{t-1}^2\right),\end{aligned}$$

i.e. $\operatorname{cov}\left(x_t,\sigma_t^2\right)=0$.

We note:

$$\begin{aligned}\log R_{f,t} &= -\log E_t\Pi_{t,t+1}\\ &= \rho\phi_c x_t + \frac{1}{2}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left[(\rho-\gamma)(1-\gamma)-(1-\rho)(\gamma-\widetilde{\gamma})\nu_\sigma\right]\sigma_t^2\\ &- \frac{1}{2}\left(\gamma^2\alpha_c^2+(\rho-\gamma)^2\phi_v^2\alpha_x^2\right)\sigma_t^2+\overline{r_f}\\ &= \rho\phi_c x_t+\overline{r_{f,\sigma}}\sigma_t^2+\overline{r_f},\end{aligned}$$

where $\overline{r_f}$ contains all the constant terms, and $\overline{r_{f,\sigma}}$ contains all the terms in σ_t^2, in $\log R_{f,t}$.

From:

$$\begin{aligned}\log E_tR_{m,t+1}-\log R_{f,t} &= \gamma\chi\alpha_c^2\sigma_t^2+(\gamma-\rho)\phi_v\kappa A_1\alpha_x^2\sigma_t^2\\ &+\left[(\gamma-\rho)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\tilde{\psi}_v\kappa A_2(\gamma,\widetilde{\gamma})\alpha_\sigma^2,\end{aligned}$$

we obtain:

$$\begin{aligned}\log ER_{m,t+1}-\log ER_{f,t} &= \gamma\chi\alpha_c^2\sigma^2+(\gamma-\rho)\phi_v\kappa A_1\alpha_x^2\sigma^2\\ &+\left[(\gamma-\rho)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\tilde{\psi}_v\kappa A_2(\gamma,\widetilde{\gamma})\alpha_\sigma^2\\ &+\frac{1}{2}\left[\gamma\chi\alpha_c^2+(\gamma-\rho)\phi_v\kappa A_1\alpha_x^2\right]^2\Sigma_\sigma^2\\ &+\left[\gamma\chi\alpha_c^2+(\gamma-\rho)\phi_v\kappa A_1\alpha_x^2\right]\overline{r_{f,\sigma}}\Sigma_\sigma^2,\end{aligned}$$

where $\overline{r_{f,\sigma}}=\frac{1}{2}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left[(\rho-\gamma)(1-\gamma)-(1-\rho)(\gamma-\widetilde{\gamma})\nu_\sigma\right]-\frac{1}{2}\left(\gamma^2\alpha_c^2+(\rho-\gamma)^2\phi_v^2\alpha_x^2\right)$.

Finally, the unconditional macro-announcement bounds are determined as the contribution to the unconditional equity premium of all terms coming from the $\{w_{x,t}\}$ shocks; or all the terms coming from the $\{w_{x,t},w_{\sigma,t}\}$ shocks:

$$\operatorname{MAP}\left(\{w_{x,t}\}\right)=\frac{\left[\log ER_{m,t+1}-\log ER_{f,t}\right]-\left[\log ER_{m,t+1}-\log ER_{f,t}\right]_{\alpha_x=0}}{\log ER_{m,t+1}-\log ER_{f,t}}$$

$$\text{MAP}\left(\{w_{x,t}, w_{\sigma,t}\}\right) = \frac{\left[\log ER_{m,t+1} - \log ER_{f,t}\right] - \left[\log ER_{m,t+1} - \log ER_{f,t}\right]_{\alpha_x=0,\alpha_\sigma=0}}{\log ER_{m,t+1} - \log ER_{f,t}}.$$

C.7 Proof of Proposition 6

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$, and $P_{t,0} = D_t$. Then for $h \geq 1$:

$$\frac{P_{t,h}}{D_t} = E_t\left(\Pi_{t,t+1}\frac{D_{t+1}}{D_t}\frac{P_{t+1,h-1}}{D_{t+1}}\right).$$

We guess and verify

$$\frac{P_{t,h}}{D_t} = \exp\left(\widetilde{\mu}_{d,h} + \phi_{d,h}x_t + \tilde{\psi}_{d,h}\sigma_t^2\right),$$

where $\widetilde{\mu}_{d,0} = \phi_{d,0} = \tilde{\psi}_{d,0} = 0$.

The recursion above becomes, for $h \geq 1$:

$$E_t\left(\exp\left(\begin{cases}\bar{\pi} + \mu_d + \widetilde{\mu}_{d,h-1} + \tilde{\psi}_{d,h-1}(1-\nu_\sigma)\sigma^2 \\ +\left(-\rho\phi_c + \phi_d + \phi_{d,h-1}\nu_x\right)x_t \\ +\left(-\frac{1}{2}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left[(\rho-\gamma)(1-\gamma) - (1-\rho)(\gamma-\widetilde{\gamma})\nu_\sigma\right] + \tilde{\psi}_{d,h-1}\nu_\sigma\right)\sigma_t^2 \\ +(-\gamma+\chi)\alpha_c\sigma_t w_{c,t+1} \\ +((\rho-\gamma)\phi_v + \phi_{d,h-1})\alpha_x\sigma_t w_{x,t+1} \\ +\left(\left[(\rho-\gamma) + (1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\tilde{\psi}_v + \tilde{\psi}_{d,h-1}\right)\alpha_\sigma w_{\sigma,t+1} \\ +\alpha_d\sigma_t w_{d,t+1}\end{cases}\right)\right)$$

$$= \exp\left(\widetilde{\mu}_{d,h} + \phi_{d,h}x_t + \tilde{\psi}_{d,h}\sigma_t^2\right).$$

Matching coefficients, we find the recursions, for $h \geq 1$:

- Terms in x_t:

$$\begin{aligned} \phi_{d,h} &= -\rho\phi_c + \phi_d + \phi_{d,h-1}\nu_x \\ \Rightarrow \quad \phi_{d,h} &= \left(-\rho\phi_c + \phi_d\right)\frac{1-\nu_x^h}{1-\nu_x} \end{aligned}$$

- Terms in σ_t^2:

$$\begin{aligned}\tilde{\psi}_{d,h} &= -\frac{1}{2}\left(\left(\rho-\gamma\right)\left(1-\gamma\right)-\left(1-\rho\right)\left(\gamma-\widetilde{\gamma}\right)\nu_\sigma\right)\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)+\tilde{\psi}_{d,h-1}\nu_\sigma \\ &+\frac{1}{2}\left(\left(-\gamma+\chi\right)^2\alpha_c^2+\left(\left(\rho-\gamma\right)\phi_v+\phi_{d,h-1}\right)^2\alpha_x^2+\alpha_d^2\right)\end{aligned}$$

- Constant:

$$\begin{aligned}\widetilde{\mu}_{d,h}-\widetilde{\mu}_{d,h-1} &= \bar{\pi}+\mu_d+\tilde{\psi}_{d,h-1}\left(1-\nu_\sigma\right)\sigma^2 \\ &+\frac{1}{2}\left(\left[\left(\rho-\gamma\right)+\left(1-\rho\right)\left(\gamma-\widetilde{\gamma}\right)\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v+\tilde{\psi}_{d,h-1}\right)^2\alpha_\sigma^2.\end{aligned}$$

We replicate the method above to the case of bond assets, i.e. to constant \$1 payoffs, such that:

$$B_{t,h}=\exp\left(\widetilde{\mu}_{b,h}+\phi_{b,h}x_t+\tilde{\psi}_{b,h}\sigma_t^2\right),$$

where $B_{t,h}$ is the price of \$1 at horizon h, and obtain:

- Terms in x_t:

$$\phi_{b,h}=-\rho\phi_c\frac{1-\nu_x^h}{1-\nu_x}.$$

- Terms in σ_t^2:

$$\begin{aligned}\tilde{\psi}_{b,h} &= -\frac{1}{2}\left(\left(\rho-\gamma\right)\left(1-\gamma\right)-\left(1-\rho\right)\left(\gamma-\widetilde{\gamma}\right)\nu_\sigma\right)\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)+\tilde{\psi}_{b,h-1}\nu_\sigma \\ &+\frac{1}{2}\left(\gamma^2\alpha_c^2+\left(\left(\rho-\gamma\right)\phi_v+\phi_{b,h-1}\right)^2\alpha_x^2\right).\end{aligned}$$

- Constant:

$$\begin{aligned}\widetilde{\mu}_{b,h}-\widetilde{\mu}_{b,h-1} &= \bar{\pi}+\tilde{\psi}_{b,h-1}\left(1-\nu_\sigma\right)\sigma^2 \\ &+\frac{1}{2}\left(\left[\left(\rho-\gamma\right)+\left(1-\rho\right)\left(\gamma-\widetilde{\gamma}\right)\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v+\tilde{\psi}_{b,h-1}\right)^2\alpha_\sigma^2.\end{aligned}$$

The returns for dividend strips futures are:

$$R^F_{t+1,h} = \frac{P_{t+1,h-1}/D_{t+1}}{P_{t,h}/D_t}\frac{D_{t+1}}{D_t}\frac{B_{t,h}}{B_{t+1,h-1}}$$
$$= \frac{\Pi_{t,t+1}\frac{D_{t+1}}{D_t}\frac{P_{t+1,h-1}}{D_{t+1}}}{E_t\left(\Pi_{t,t+1}\frac{D_{t+1}}{D_t}\frac{P_{t+1,h-1}}{D_{t+1}}\right)}\frac{E_t\left(\Pi_{t,t+1}B_{t+1,h-1}\right)}{\Pi_{t,t+1}B_{t+1,h-1}}.$$

We obtain:

$$\begin{aligned}\log\left(R^F_{t+1,h}\right) &= \chi\alpha_c\sigma_t w_{c,t+1} + \left(\phi_{d,h-1}-\phi_{b,h-1}\right)\alpha_x\sigma_t w_{x,t+1} + \left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\alpha_\sigma w_{\sigma,t+1} + \alpha_d\sigma_t w_{d,t+1}\\ &\quad - \left(\left[(\rho-\gamma)+(1-\rho)\left(\gamma-\widetilde{\gamma}\right)\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v\left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)+\frac{1}{2}\left(\tilde{\psi}^2_{d,h-1}-\tilde{\psi}^2_{b,h-1}\right)\right)\alpha^2_\sigma\\ &\quad - \frac{1}{2}\left(\left(-2\gamma\chi+\chi^2\right)\alpha^2_c+\left(2\left(\rho-\gamma\right)\phi_v\left(\phi_{d,h-1}-\phi_{b,h-1}\right)+\left(\phi^2_{d,h-1}-\phi^2_{b,h-1}\right)\right)\alpha^2_x+\alpha^2_d\right)\sigma^2_t,\end{aligned}$$

so

$$\begin{aligned}\log E_t\left(R^F_{t+1,h}\right) &= \left(\left[(\gamma-\rho)+(1-\rho)\left(\gamma-\widetilde{\gamma}\right)\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\widetilde{\psi}_v-\tilde{\psi}_{b,h-1}\right)\left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\alpha^2_\sigma\\ &\quad + \left(\gamma\chi\alpha^2_c+\left(\left(\gamma-\rho\right)\phi_v-\phi_{b,h-1}\right)\left(\phi_{d,h-1}-\phi_{b,h-1}\right)\alpha^2_x\right)\sigma^2_t.\end{aligned}$$

We note:

First

$$\left(\left(\gamma-\rho\right)\phi_v-\phi_{b,h-1}\right)\left(\phi_{d,h-1}-\phi_{b,h-1}\right) = \left(\gamma-\rho\nu_x^{h-1}\right)\frac{\phi_c}{1-\nu_x}\left(1-\nu_x^{h-1}\right)\frac{\phi_d}{1-\nu_x},$$

strictly positive and increasing in h.

Second,

$$\begin{aligned}\tilde{\psi}_{d,h}-\tilde{\psi}_{b,h} &= \left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\nu_\sigma\\ &\quad + \underbrace{\chi\left(\frac{1}{2}\chi-\gamma\right)\alpha^2_c+\left(\left(\rho-\gamma\right)\phi_v+\frac{1}{2}\left(\phi_{d,h-1}+\phi_{b,h-1}\right)\right)\left(\phi_{d,h-1}-\phi_{b,h-1}\right)\alpha^2_x+\frac{1}{2}\alpha^2_d}_{\text{does not depend on }\widetilde{\gamma}},\end{aligned}$$

so $\tilde{\psi}_{d,h}-\tilde{\psi}_{b,h}=\psi_{d,h}-\psi_{b,h}, \forall h$.

Third,

$$\tilde{\psi}_{b,h} = \tilde{\psi}_{b,h-1}\nu_\sigma + \underbrace{\Psi_{h-1}}_{\text{does not depend on } \widetilde{\gamma}} + \frac{1}{2}(1-\rho)(\gamma-\widetilde{\gamma})\nu_\sigma\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right),$$

where $\Psi_{h-1} = -\frac{1}{2}(\rho-\gamma)(1-\gamma)\left(\alpha_c^2+\phi_v^2\alpha_x^2\right) + \frac{1}{2}\left(\gamma^2\alpha_c^2 + \left((\rho-\gamma)\phi_v + \phi_{b,h-1}\right)^2\alpha_x^2\right) > 0$, when $\gamma > \widetilde{\gamma} > 1 \geq \rho$. So:

$$\tilde{\psi}_{b,h} = \psi_{b,h} + \frac{1}{2}(1-\rho)(\gamma-\widetilde{\gamma})\nu_\sigma\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\frac{1-\nu_\sigma^h}{1-\nu_\sigma},$$

where $\psi_{b,h} \geq 0, \forall h$. We obtain:

$$\begin{aligned}\left[(\gamma-\rho) + (1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{\widetilde{\gamma}-1}\right]\widetilde{\psi}_v - \tilde{\psi}_{b,h-1} &= \left((\gamma-\rho)\psi_v - \psi_{b,h-1}\right) \\ &\quad + \frac{1}{2}\frac{\gamma-\widetilde{\gamma}}{1-\nu_\sigma}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left((\gamma-1)+(1-\rho)\nu_\sigma^h\right),\end{aligned}$$

where the second term (positive) cannot offset the first term (negative) when $\gamma > \widetilde{\gamma} > 1 \geq \rho$.

Taking these results together, we obtain:

$$\begin{aligned}&\log E_t\left(R^F_{t+1,h}\right) = \\ &\underbrace{\left((\gamma-\rho)|\psi_v| + \psi_{b,h-1} - \underbrace{\frac{1}{2}\frac{\gamma-\widetilde{\gamma}}{1-\nu_\sigma}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left[(\gamma-1)+(1-\rho)\nu_\sigma^h\right]}_{\geq 0}\right)}_{\geq 0} \times \left(\psi_{b,h-1} - \psi_{d,h-1}\right)\alpha_\sigma^2 \\ &+ \left(\gamma\chi\alpha_c^2 + \underbrace{\left(\gamma - \rho\nu_x^{h-1}\right)\frac{\phi_c}{1-\nu_x}\left(1-\nu_x^{h-1}\right)\frac{\phi_d}{1-\nu_x}}_{\geq 0 \text{ and increasing}}\alpha_x^2\right)\sigma_t^2.\end{aligned}$$

To derive the unconditional expected returns, observe that the volatility process is stationary under the constraint $\nu_\sigma < 1$ such that: $\sigma_t^2 \sim \mathcal{N}\left(\sigma^2, \Sigma_\sigma^2\right)$, where $\Sigma_\sigma^2 = \frac{\alpha_\sigma^2}{1-\nu_\sigma^2}$, so

$$\begin{aligned}\log E\left(R^F_{t+1,h}\right) &= E\log E_t\left(R^F_{t+1,h}\right) \\ &\quad + \frac{1}{2}\left(\gamma\chi\alpha_c^2 + \left(\gamma-\rho\nu_x^{h-1}\right)\frac{\phi_c}{1-\nu_x}\left(1-\nu_x^{h-1}\right)\frac{\phi_d}{1-\nu_x}\alpha_x^2\right)^2\Sigma_\sigma^2.\end{aligned}$$

Turning to the Sharpe Ratio:

$$\begin{aligned}
\mathrm{SR}^F_{t,h} &= \frac{E_t\left(R^F_{t+1,h}-1\right)}{\sqrt{\mathrm{var}_t\left(R^F_{t+1,h}\right)}} \\
&= \frac{1-1/E_t\left(R^F_{t+1,h}\right)}{\sqrt{\exp^{\sigma_t^2\left(\chi^2\alpha_c^2+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\alpha_d^2\right)+\left(\psi_{d,h-1}-\psi_{b,h-1}\right)^2\alpha_\sigma^2}-1}},
\end{aligned}$$

where the denominator is unchanged from the standard model.

Finally the unconditional Sharpe ratio is:

$$\begin{aligned}
\mathrm{SR}^F_{h} &= \frac{E\left(R^F_{t+1,h}-1\right)}{\sqrt{\mathrm{var}\left(R^F_{t+1,h}\right)}} \\
&= \frac{1-1/E\left(R^F_{t+1,h}\right)}{\sqrt{\exp\left(\begin{cases}\sigma^2\left(\chi^2\alpha_c^2+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\alpha_d^2\right)+\left(\psi_{d,h-1}-\psi_{b,h-1}\right)^2\alpha_\sigma^2 \\ +\frac{7}{4}\left(\chi^2\alpha_c^2+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\alpha_d^2\right)^2\Sigma_\sigma^2\end{cases}\right)-1}},
\end{aligned}$$

where the denominator is unchanged from the standard model.

We define:

$$\begin{aligned}
\phi_h &= \gamma\chi\alpha_c^2+\left(\gamma-\rho\nu_x^{h-1}\right)\frac{\phi_c}{1-\nu_x}\left(1-\nu_x^{h-1}\right)\frac{\phi_d}{1-\nu_x}\alpha_x^2, \\
\psi_h &= \psi_{b,h-1}-\psi_{d,h-1}, \\
\Psi_h &= \chi^2\alpha_c^2+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\alpha_d^2, \\
a_h &= (\gamma-\rho)\left|\psi_v\right|+\psi_{b,h-1}, \\
b_h &= \frac{1}{2}\frac{1}{1-\nu_\sigma}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left[(\gamma-1)+(1-\rho)\,\nu_\sigma^h\right],
\end{aligned}$$

to derive Equation (18) and Equation (19). □

C.8 Proof of Proposition 7

From Section C.6, the log price-dividend ratio at time is determined by $z_t = A_0 + A_1 x_t + A_2\sigma_t^2$, and, in the notations of Proposition 5, $\lambda_{m,x} = \kappa A_1$ and $\lambda_{m,\sigma}(\gamma,\widetilde{\gamma}) = -\kappa A_2$, where $\kappa = \frac{e^z}{1+e^z}$.

From Equation (18), the only time variations in $\log E_t\left(R^F_{t+1,h}\right)$ come from the term $\phi_h\sigma_t^2$.

The volatility process is stationary under the constraint $\nu_\sigma < 1$, such that $\sigma_t^2 \sim \mathcal{N}\left(\sigma^2, \Sigma_\sigma^2\right)$, where $\Sigma_\sigma^2 = \frac{\alpha_\sigma^2}{1-\nu_\sigma^2}$.

Since the shocks to x_t and to σ_t^2 between $t-1$ and t are independent and $E\left(x_t\right) = 0$, we have

$$\begin{aligned}
\operatorname{cov}\left(x_t, \sigma_t^2\right) &= EE_{t-1}\left(x_t\sigma_t^2\right) \\
&= \nu_x\nu_\sigma E\left(x_{t-1}\sigma_{t-1}^2\right),
\end{aligned}$$

i.e. $\operatorname{cov}\left(x_t, \sigma_t^2\right) = 0$.

In addition,

$$\begin{aligned}
\operatorname{cov}\left(x_t, x_t\right) &= EE_{t-1}\left(x_t^2\right) \\
&= \nu_x^2 E\left(x_{t-1}^2\right) + \alpha_x^2 E\left(\sigma_{t-1}^2\right),
\end{aligned}$$

so $\operatorname{var}\left(x_t\right) = \frac{\alpha_x^2\sigma^2}{1-\nu_x^2}$.

We obtain:

$$\operatorname{cov}\left(p_t - d_t, \log E_t\left(R^F_{t+1,h}\right)\right) = -\phi_h \frac{\lambda_{m,\sigma}\left(\gamma, \widetilde{\gamma}\right)}{\kappa}\frac{\alpha_\sigma^2}{1-\nu_\sigma^2},$$

and,

$$\operatorname{var}\left(p_t - d_t\right) = \left(\frac{\lambda_{m,x}}{\kappa}\right)^2 \frac{\alpha_x^2\sigma^2}{1-\nu_x^2} + \left(\frac{\lambda_{m,\sigma}\left(\gamma, \widetilde{\gamma}\right)}{\kappa}\right)^2 \frac{\alpha_\sigma^2}{1-\nu_\sigma^2}.$$

The regression of $\log E_t\left[R^F_{t+1,h}\right]$ on $\log \frac{P_t}{D_t}$ yields

$$\beta_h = \frac{-\phi_h \frac{\lambda_{m,\sigma}(\gamma,\widetilde{\gamma})}{\kappa}\frac{\alpha_\sigma^2}{1-\nu_\sigma^2}}{\left(\frac{\lambda_{m,x}}{\kappa}\right)^2 \frac{\alpha_x^2\sigma^2}{1-\nu_x^2} + \left(\frac{\lambda_{m,\sigma}(\gamma,\widetilde{\gamma})}{\kappa}\right)^2 \frac{\alpha_\sigma^2}{1-\nu_\sigma^2}}.$$

□

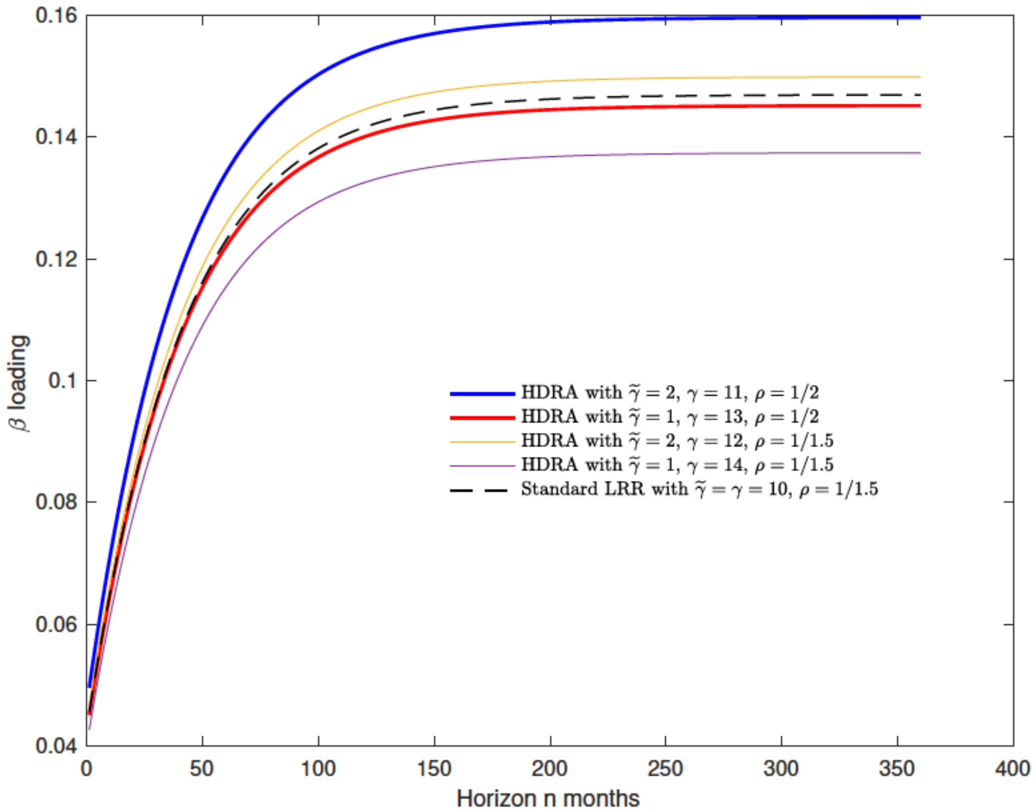

Figure 6: Dividend strip future expected returns loading on Index dividend-price ratio

β-loadings of the regressions of dividend strip futures expected returns at horizon $h \geq 1$ $\log E_t\left[R_h^F\right]$ on the index dividend/price ratio $d_t - p_t$; under one-period trading. *HDRA* stands for "Horizon-dependent risk aversion", *Standard LRR* stands for the standard "Long-run risk" model ($\widetilde{\gamma} = \gamma = 10$ and $\rho = 1/1.5$ are the parameters of Bansal et al. (2012)). The calibration of both models is from Table 1. All returns are monthly, annualized.

C.9 Proof of Proposition 8

To derive the term-structure of dividend strip futures expected returns under buy-and-hold strategies, we start by deriving the h-period ahead stochastic discount factor, using the inter-temporal marginal rate of substitution

$$\Pi_{t,t+h} = \frac{dV_t/dW_{t+h}}{dV_t/dC_t},$$

where

$$\frac{dV_t}{dW_{t+h}} = \frac{dV_t}{d\widetilde{V}_{t+1}} \times \prod_{\tau=1}^{h-1} \frac{d\widetilde{V}_{t+\tau}}{d\widetilde{V}_{t+\tau+1}} \times \frac{d\widetilde{V}_{t+h}}{dW_{t+h}}.$$

Due to the homotheticity of our preferences, we can rely on the fact that both $\widetilde{V}_{t+h}$ and V_{t+h} are homogeneous of degree one which implies

$$\frac{d\widetilde{V}_{t+h}/dW_{t+h}}{dV_{t+h}/dW_{t+h}} = \frac{\widetilde{V}_{t+h}}{V_{t+h}},$$

where the envelope condition guarantees:

$$\frac{dV_{t+h}}{dW_{t+h}} = \frac{dV_{t+h}}{dC_{t+h}}.$$

From the preferences of Definition 1:

$$\frac{dV_t}{dC_t} = V_t^{\rho}\left(1-\beta\right)C_t^{-\rho},$$

$$\frac{dV_t}{d\widetilde{V}_{t+1}} = V_t^{\rho}\beta\widetilde{V}_{t+1}^{-\gamma}E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{\gamma-\rho}{1-\gamma}},$$

and

$$\frac{d\widetilde{V}_t}{d\widetilde{V}_{t+1}} = \widetilde{V}_t^{\rho}\beta\widetilde{V}_{t+1}^{-\widetilde{\gamma}}E_t\left[\widetilde{V}_{t+1}^{1-\widetilde{\gamma}}\right]^{\frac{\widetilde{\gamma}-\rho}{1-\widetilde{\gamma}}}.$$

Combining these results allows us to derive the h-period SDF $\Pi_{t,t+h}$ as

$$\Pi_{t,t+h} = \beta^h\left(\frac{C_{t+h}}{C_t}\right)^{-\rho}\left(\frac{\widetilde{V}_{t+h}}{V_{t+h}}\right)^{1-\rho}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}\prod_{\tau=2}^{h}\left(\frac{\widetilde{V}_{t+\tau}}{E_{t+\tau-1}\left[\widetilde{V}_{t+\tau}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{\rho-\widetilde{\gamma}}.$$

Consider a dividend strip and bond at horizon h priced at time t under $\Pi_{t,t+h}$,

$$\begin{aligned} P_{t,h}/D_t &= E_t[\Pi_{t,t+h}D_{t+h}/D_t], \\ B_{t,h} &= E_t[\Pi_{t,t+h}], \end{aligned}$$

then, at time $t+1$, priced under $\Pi_{t+1,t+1+h-1}$

$$\begin{aligned} P_{t+1,h-1}/D_{t+1} &= E_{t+1}[\Pi_{t+1,t+1+h-1}D_{t+h}/D_{t+1}], \\ B_{t+1,h-1} &= E_{t+1}[\Pi_{t+1,t+1+h-1}]. \end{aligned}$$

The one-period return of the dividend strip future at horizon h is:

$$R^F_{t+1,h} = \frac{\left[\left(P_{t+1,h-1}/D_{t+1}\right) / \left(P_{t,h}/D_t\right)\right] \times \left(D_{t+1}/D_t\right)}{B_{t+1,h-1}/B_{t,h}}.$$

We note:

$$d_{t+h} - d_t = h\mu_d + \frac{1-\nu_x^{h-1}}{1-\nu_x}\phi_d x_t + \sum_{j=1}^{h} \sigma_{t+j-1} \left(\chi\alpha_c w_{c,t+j} + \phi_d \frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x w_{x,t+j} + \alpha_d w_{d,t+j} \right),$$

$$c_{t+h} - c_t = h\mu_c + \frac{1-\nu_x^{h-1}}{1-\nu_x}\phi_c x_t + \sum_{j=1}^{h} \sigma_{t+j-1} \left(\alpha_c w_{c,t+j} + \phi_c \frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x w_{x,t+j} \right),$$

$$\widetilde{v}_{t+h} - v_{t+h} = \frac{1}{2}\left(\gamma - \widetilde{\gamma}\right) \left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) \left(\sigma^2 + \nu_\sigma^h \left(\sigma_t^2 - \sigma^2\right) + \alpha_\sigma \sum_{j=1}^{h} \nu_\sigma^{h-j} w_{\sigma,t+j} \right) + \widetilde{\psi}_v^2 \alpha_\sigma^2 \right],$$

and

$$\begin{aligned}
\log \left(\frac{\widetilde{V}_{t+\tau}^{1-\widetilde{\gamma}}}{E_{t+\tau-1}\left[\widetilde{V}_{t+\tau}^{1-\widetilde{\gamma}}\right]} \right)^{\frac{\rho-\widetilde{\gamma}}{1-\widetilde{\gamma}}} &= \left(\rho - \widetilde{\gamma}\right) \left(\sigma_{t+\tau-1} \left(\alpha_c w_{c,t+\tau} + \phi_v \alpha_x w_{x,t+\tau} \right) + \widetilde{\psi}_v \alpha_\sigma w_{\sigma,t+\tau} \right) \\
&\quad - \frac{1}{2}\left(\rho - \widetilde{\gamma}\right)\left(1 - \widetilde{\gamma}\right) \left(\sigma_{t+\tau-1}^2 \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) + \widetilde{\psi}_v^2 \alpha_\sigma^2 \right), \\
\log \left(\frac{\widetilde{V}_{t+1}^{1-\gamma}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]} \right)^{\frac{\rho-\gamma}{1-\gamma}} &= \left(\rho - \gamma\right) \left(\sigma_t \left(\alpha_c w_{c,t+1} + \phi_v \alpha_x w_{x,t+1} \right) + \widetilde{\psi}_v \alpha_\sigma w_{\sigma,t+1} \right) \\
&\quad - \frac{1}{2}\left(\rho - \gamma\right)\left(1 - \gamma\right) \left(\sigma_t^2 \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) + \widetilde{\psi}_v^2 \alpha_\sigma^2 \right).
\end{aligned}$$

Write $\alpha_v^2 = \alpha_c^2 + \phi_v^2 \alpha_x^2$, and

$$d_{t+h} - d_t = \overline{d_{t+h} - d_t} + \sum_{j=1}^{h} \sigma_{t+j-1} \Delta_{j,h} W_{t+j},$$

$$\pi_{t,t+h} = \overline{\pi_{t,t+h}} + \sum_{j=1}^{h} \sigma_{t+j-1} Q_{j,h} W_{t+j} + \sum_{j=1}^{h} q_{j,h} w_{\sigma,t+j} + \pi \sum_{j=2}^{h} \sigma_{t+j-1}^2,$$

where

$$\overline{d_{t+h} - d_t} = h\mu_d + \frac{1-\nu_x^{h-1}}{1-\nu_x}\phi_d x_t,$$

and

$$
\begin{aligned}
\Delta_{j,h} W_{t+j} &= \chi \alpha_c w_{c,t+j} + \phi_d \frac{1-\nu_x^{h-j}}{1-\nu_x} \alpha_x w_{x,t+j} + \alpha_d w_{d,t+j} \\
Q_{j,h} W_{t+j} &= -\widetilde{\gamma} \alpha_c w_{c,t+\tau} + \left(\rho \nu_x^{h-j} - \widetilde{\gamma} \right) \phi_v \alpha_x w_{x,t+\tau}, \forall j \geq 2 \\
Q_{1,h} W_{t+1} &= -\gamma \alpha_c w_{c,t+1} + \left(\rho \nu_x^{h-1} - \gamma \right) \phi_v \alpha_x w_{x,t+1} \\
q_{j,h} &= \alpha_\sigma \left[\left(\rho - \widetilde{\gamma} \right) \widetilde{\psi}_v + \frac{1}{2} \alpha_v^2 \left(1 - \rho \right) \left(\gamma - \widetilde{\gamma} \right) \nu_\sigma^{h-j} \right], \forall j \geq 2 \\
q_{1,h} &= \alpha_\sigma \left[\left(\rho - \gamma \right) \widetilde{\psi}_v + \frac{1}{2} \alpha_v^2 \left(1 - \rho \right) \left(\gamma - \widetilde{\gamma} \right) \nu_\sigma^{h-1} \right] \\
\pi &= -\frac{1}{2} \left(\rho - \widetilde{\gamma} \right) \left(1 - \widetilde{\gamma} \right) \alpha_v^2.
\end{aligned}
$$

So:

$$
\log \frac{P_{t,h}}{D_t} = \overline{d_{t+h} - d_t} + \overline{\pi_{t,t+h}} + \log E_t \exp \left[\sum_{j=1}^{h} \sigma_{t+j-1} \left(\Delta_{j,h} + Q_{j,h} \right) W_{t+j} + \sum_{j=1}^{h} q_{j,h} w_{\sigma,t+j} + \pi \sum_{j=2}^{h} \sigma_{t+j-1}^2 \right].
$$

Relatively straightforward calculations yield:

$$
\begin{aligned}
& E_t \exp \left[\sum_{j=1}^{h} \sigma_{t+j-1} \left(\Delta_{j,h} + Q_{j,h} \right) W_{t+j} + \sum_{j=1}^{h} q_{j,h} w_{\sigma,t+j} + \pi \sum_{j=2}^{h} \sigma_{t+j-1}^2 \right] \\
& = \exp \begin{cases} \sum_{j=1}^{h} \left(\frac{1}{2} \left| \Delta_{j,h} + Q_{j,h} \right|^2 + \pi \right) \left(\nu_\sigma^{j-1} \sigma_t^2 + \sigma^2 \left(1 - \nu_\sigma^{j-1} \right) \right) \\ + \frac{1}{2} q_{h,h}^2 + \frac{1}{2} \sum_{j=1}^{h-1} \left(q_{j,h} + \alpha_\sigma \sum_{k=j+1}^{h} \nu_\sigma^{k-(j+1)} \left(\frac{1}{2} \left| \Delta_{k,h} + Q_{k,h} \right|^2 + \pi \right) \right)^2 \end{cases}
\end{aligned}
$$

We obtain

$$
\begin{aligned}
& \log \frac{P_{t,h}}{D_t} - \log B_{t,h} = h \mu_d + \frac{1-\nu_x^{h-1}}{1-\nu_x} \phi_d x_t + \frac{1}{2} \sum_{j=1}^{h} \left(\left| \Delta_{j,h} + Q_{j,h} \right|^2 - \left| Q_{j,h} \right|^2 \right) \left(\sigma^2 \left(1 - \nu_\sigma^{j-1} \right) + \nu_\sigma^{j-1} \sigma_t^2 \right) \\
& + \frac{1}{2} \sum_{j=1}^{h-1} \left(q_{j,h} + \alpha_\sigma \sum_{k=j+1}^{h} \nu_\sigma^{k-(j+1)} \left(\frac{1}{2} \left| \Delta_{k,h} + Q_{k,h} \right|^2 + \pi \right) \right)^2 \\
& - \frac{1}{2} \sum_{j=1}^{h-1} \left(q_{j,h} + \alpha_\sigma \sum_{k=j+1}^{h} \nu_\sigma^{k-(j+1)} \left(\frac{1}{2} \left| Q_{k,h} \right|^2 + \pi \right) \right)^2 .
\end{aligned}
$$

Using

$$
\begin{aligned}
\Delta_{j+1,h} &= \Delta_{j,h-1}, \forall j \geq 1 \\
Q_{j+1,h} &= Q_{j,h-1}, \forall j \geq 2 \\
q_{j+1,h} &= q_{j,h-1}, \forall j \geq 2 \\
q_{2,h} &= q_{1,h-1} + \alpha_\sigma \left(\gamma - \widetilde{\gamma}\right) \widetilde{\psi}_v,
\end{aligned}
$$

we derive:

$$
\begin{aligned}
&\log E_t R^F_{t+1,h} = \\
&\frac{1}{2}\left(\chi^2\alpha_c^2 + \left(\frac{1-\nu_x^{h-2}}{1-\nu_x}\right)^2 \phi_d^2\alpha_x^2 + \alpha_d^2\right)\sigma_t^2 \\
&-\frac{1}{2}\left(\left(|Q_{1,h-1}|^2 - |\Delta_{1,h-1} + Q_{1,h-1}|^2\right) - \left(|Q_{2,h}|^2 - |\Delta_{2,h} + Q_{2,h}|^2\right)\right)\left(\sigma^2\left(1-\nu_\sigma\right) + \nu_\sigma\sigma_t^2\right) \\
&+\frac{1}{2}\left(|Q_{1,h}|^2 - |\Delta_{1,h} + Q_{1,h}|^2\right)\sigma_t^2 \\
&+\frac{1}{4}\left(\sum_{j=3}^{h}\left(\left|Q_{j,h}\right|^2 - \left|\Delta_{j,h} + Q_{j,h}\right|^2\right)\nu_\sigma^{j-2} + \left(|Q_{1,h-1}|^2 - |\Delta_{1,h-1} + Q_{1,h-1}|^2\right)\right)^2 \alpha_\sigma^2 \\
&+\frac{1}{2}\alpha_\sigma^2\left(\gamma - \widetilde{\gamma}\right)\widetilde{\psi}_v \sum_{j=3}^{h}\nu_\sigma^{j-3}\left(\left|Q_{j,h}\right|^2 - \left|\Delta_{j,h} + Q_{j,h}\right|^2\right) \\
&+\frac{1}{2}\left[\left(q_{1,h} + \alpha_\sigma\sum_{k=2}^{h}\nu_\sigma^{k-2}\left(\frac{1}{2}\left|Q_{k,h}\right|^2 + \pi\right)\right)^2 - \left(q_{1,h} + \alpha_\sigma\sum_{k=2}^{h}\nu_\sigma^{k-2}\left(\frac{1}{2}\left|\Delta_{k,h} + Q_{k,h}\right|^2 + \pi\right)\right)^2\right]
\end{aligned}
$$

Re-arranging, and using:

$$
\left|Q_{j,h}\right|^2 - \left|\Delta_{j,h} + Q_{j,h}\right|^2 = \left(\left|Q_{j,h}\right|^2 - \left|\Delta_{j,h} + Q_{j,h}\right|^2\right)_\gamma - 2\left(\gamma - \widetilde{\gamma}\right)\left(\chi\alpha_c^2 + \phi_d\frac{1-\nu_x^{h-j}}{1-\nu_x}\phi_v\alpha_x^2\right)
$$

and

$$
\left(|\Delta_{1,h} + Q_{1,h}|^2 - |Q_{1,h}|^2\right) - \left(|\Delta_{1,1} + Q_{1,1}|^2 - |Q_{1,1}|^2\right) = \phi_d\frac{1-\nu_x^{h-1}}{1-\nu_x}\left(\phi_d\frac{1-\nu_x^{h-1}}{1-\nu_x} + 2\left(\rho\nu_x^{h-1} - \gamma\right)\phi_v\right)
$$

we obtain:

$$
\begin{aligned}
&\log E_t R^F_{t+1,1} - \log E_t R^F_{t+1,h} = \\
&-\phi_d \frac{1-\nu_x^{h-1}}{1-\nu_x}\left(\gamma - \rho\nu_x^{h-1}\right)\phi_v \alpha_x^2 \sigma_t^2 + \frac{1}{2}\nu_x^{h-2}\left(\frac{1-\nu_x^{h-2}}{1-\nu_x} + \frac{1-\nu_x^{h-1}}{1-\nu_x}\right)\phi_d^2\alpha_x^2\sigma_t^2 \\
&+ (\gamma - \widetilde{\gamma})\begin{cases}\left(\chi\alpha_c^2 + \phi_d \frac{1-\nu_x^{h-2}}{1-\nu_x}\phi_v\alpha_x^2\right)\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right) \\ +\frac{1}{2}\alpha_\sigma^2\left|\widetilde{\psi}_v\right|\sum_{j=3}^h \nu_\sigma^{j-3}\left(\left|Q_{j,h}\right|^2 - \left|\Delta_{j,h} + Q_{j,h}\right|^2\right)\end{cases} \\
&- \frac{1}{2}\left[\left(q_{1,h} + \alpha_\sigma \sum_{k=2}^h \nu_\sigma^{k-2}\left(\frac{1}{2}\left|Q_{k,h}\right|^2 + \pi\right)\right)^2 - \left(q_{1,h} + \alpha_\sigma \sum_{k=2}^h \nu_\sigma^{k-2}\left(\frac{1}{2}\left|\Delta_{k,h} + Q_{k,h}\right|^2 + \pi\right)\right)^2\right].
\end{aligned}
$$

Using

$$
\begin{aligned}
\left(q_{1,h} + \alpha_\sigma \frac{\pi}{1-\nu_\sigma}\right) &\underset{h\to+\infty}{\longrightarrow} \alpha_\sigma\left|\widetilde{\psi}_v\right|(\gamma - \widetilde{\gamma}) \\
\sum_{j=2}^h \nu_\sigma^{j-2}\nu_x^{h-j} &\underset{h\to+\infty}{\longrightarrow} 0
\end{aligned}
$$

We obtain

$$
\begin{aligned}
&\log E_t R^F_{t+1,1} - \log E_t R^F_{t+1,h} \\
&\underset{h\to+\infty}{\longrightarrow} (\gamma - \widetilde{\gamma})\left[\chi\alpha_c^2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right) + \frac{\phi_d\phi_v}{1-\nu_x}\alpha_x^2\left(\sigma^2 - \sigma_t^2\right)(1-\nu_\sigma)\right] \\
&- \widetilde{\gamma}\frac{\phi_d\phi_v}{1-\nu_x}\alpha_x^2\sigma_t^2 - \frac{1}{8}\left(\frac{\alpha_\sigma}{1-\nu_\sigma}\right)^2 \underbrace{\left[\widetilde{\gamma}^4\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)^2 - \left((\widetilde{\gamma} - \chi)^2\alpha_c^2 + \left(\widetilde{\gamma}\phi_v - \frac{\phi_d}{1-\nu_x}\right)^2\alpha_x^2\right)^2\right]}_{>0}
\end{aligned}
$$

Calculated at $\sigma_t = \sigma$:

$$
\begin{aligned}
&\log E_t R^F_{t+1,1} - \log E_t R^F_{t+1,h} \\
&\underset{h\to+\infty}{\longrightarrow} \left((\gamma - \widetilde{\gamma})\chi\alpha_c^2 - \widetilde{\gamma}\frac{\phi_d\phi_v}{1-\nu_x}\alpha_x^2\right)\sigma^2 \\
&- \frac{1}{8}\left(\frac{\alpha_\sigma}{1-\nu_\sigma}\right)^2 \underbrace{\left[\widetilde{\gamma}^4\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)^2 - \left((\widetilde{\gamma} - \chi)^2\alpha_c^2 + \left(\widetilde{\gamma}\phi_v - \frac{\phi_d}{1-\nu_x}\right)^2\alpha_x^2\right)^2\right]}_{>0}.
\end{aligned}
$$

□

D Appendix to Section 5

D.1 Term-structure: estimates and model solutions

Our formal results are derived for monthly dividend strip, i.e. at time t the dividend strip with maturity h is a claim to the monthly dividend at time $t+h$. To convert these to annual dividend strips prices, we use the closed-form solutions for the monthly strips prices for any $\{x_t, \sigma_t, d_t\}$, in Appendix C.7 and Appendix C.9, to infer each annual dividend strip price as the sum of the 12 corresponding monthly strips, divided by 12 to convert the monthly dividend D_t into an annual equivalent. From the initial conditions $\{C_t = 1, x_t = 0, \sigma_t = \sigma, D_t = 1\}$, we simulate 1,000 paths of 1,000 months for $\{c_t, x_t, \sigma_t, d_t\}$ to obtain time series of dividend strips prices.

We observe that both the $P_{t,h}$ dividend strips and the $B_{t,h}$ bonds are known up to the multiplier constant $\exp(\bar{\pi})$ (see Appendix C.7), but since we only analyze ratios $P_{t,h}/B_{t,h}$, we set $\bar{\pi} = 0$ without loss of generality.

For our "Normal times/ liquid market" calibration, we obtain the annual returns on the dividend strip futures following the formula:

$$R^F_{t+1,h} = \frac{P_{t+1,h-1}/P_{t,h}}{B_{t+1,h-1}/B_{t,h}}, \forall h \geq 2,$$

at the annual frequency: $t+1$ is $t+$ 12 months, h is the horizon in years.

Since the dividend strips at horizon less than 1 year are not defined, we obtain a monthly times series of annual returns for horizon $h \geq 2$. We use the simulated sample averages and standard deviations to measure the model's equity premia $\log E\left[R^F_h\right]$ and Sharpe ratios $\mathrm{SR}\left[R^F_h\right]$, $\forall h \geq 2$.

To estimate their values in the data, we use the equity yield proxy data provided in Giglio et al. (2023)'s replication package corresponding to $P_{t,h}/D_t$ for $h \geq 1$ at the monthly frequency. To obtain the bond prices, we follow the methodology of Giglio et al. (2023), and use the estimates from Gürkaynak et al. (2007). Finally, to obtain the monthly dividends, we proceed as in van Binsbergen et al. (2013), using the difference between returns with and without dividends times the lagged value of index, which allows us to obtain $P_{t,h}$ from $P_{t,h}/D_t$. We then apply the same returns formula as above to calculate the annual returns at the monthly frequency, for $h \geq 2$.

We exclude from the data the annual returns realized during NBER recessions, to derive the estimates in "Normal times".

For our "Recessions/ Illiquid market" calibration, we analyze the average forward equity yields directly, where the forward equity yield of the dividend strip of horizon h is given by:

$$\text{FEY}_{t,h} = -\frac{1}{h} \log \left(\frac{P_{t,h}}{D_t} \times \frac{1}{B_{t,h}} \right).$$

The closed-form solutions of Appendix C.9 allow us to obtain the expected forward equity yield $E[\text{FEY}_h]$ for $h \geq 2$ for annual dividend strips under buy-and-hold, without resorting to our simulated data. To obtain the corresponding data estimates, we use the equity yield proxy data provided in Giglio et al. (2023)'s replication package as well as bond estimates from Gürkaynak et al. (2007), where we restrict the 1973-2020 sample to the NBER recession months.

D.2 Timing Premium when $\rho \neq 1$

When $\rho \neq 1$, we cannot derive closed-form solutions for the timing premium. We solve numerically instead, using the following method:

1. Starting from $x_0 = 0$ and $\sigma_0 = \sigma$, simulate $T = 100$ periods of $\{x_t, \sigma_t\}$ from process (8).
2. To derive the value with slow release of information:
   (a) For each t, make 1000 random draws of possible next-period $\{x_{t+1}, \sigma_{t+1}\}$.
   (b) define $\widetilde{\text{vc}}_t = \widetilde{v}_t - c_t, \forall t$. For each random draw, derive $\widetilde{\text{vc}}_{t+1}(0)$, the exact solution to the value function of Equation (2) under $\rho = 1$, as a function of $\{x_t, \sigma_t\}$ (see Appendix B.1)
   (c) Adapting Equation (2) to

   $$\widetilde{\text{vc}}_t(1) = \frac{1}{1-\rho} \log \left((1-\beta) + \beta \left(E_t \left[\exp\left[(1-\widetilde{\gamma}) \left(\widetilde{\text{vc}}_{t+1}(0) + c_{t+1} - c_t \right) \right] \right) \right)^{\frac{1-\rho}{1-\widetilde{\gamma}}} \right),$$

   obtain $\widetilde{\text{vc}}_t(1)$ from $\widetilde{\text{vc}}_t(0)$, as a function of $\{x_t, \sigma_t\}$.
   (d) Repeat the method to obtain $\widetilde{\text{vc}}_t(2)$ from $\widetilde{\text{vc}}_t(1)$, then $\widetilde{\text{vc}}_t(3)$ from $\widetilde{\text{vc}}_t(2)$ etc, all functions of $\{x_t, \sigma_t\}$. Stop after $N = 100$. We construct a 40x40 grid for $\{x_t, \sigma_t\}$, and interpolate for pairs $\{x_{t+1}, \sigma_{t+1}\}$ not on the grid points.
   (e) For each t, average across the random draws to get the expectation of the next-period $t+1$ value using $\widetilde{\text{vc}}_t(N)$; then calculate the first-period t value using Equation (1) adapted to value-to-consumption ratios.

3. To derive the value with early release of information:

    (a) for each t, simulate 1000 random paths of 10,000 years each.

    (b) Average across the random draws to get the expectation of the next-period $t+1$ value; then calculate the first-period t value using Equation (1) adapted to value-to-consumption ratios.

4. Calculate the timing premium for each t

5. Take the average over T periods of the timing premium.