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Horizon-Dependent Risk Aversion and the Timing and Pricing of Uncertainty

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Abstract

Inspired by experimental evidence, we amend the recursive utility model to let risk aversion decrease with the temporal horizon. Our pseudo-recursive preferences retain appealing features of the standard model and remain analytically tractable. In a macro-finance application, we formally derive the pricing of risk in our framework and confront two major challenges to the long-run risk asset pricing model: that it requires too strong a preference for early resolutions of uncertainty to explain the risk premia observed in the data; and that it cannot explain the evidence on term structures of expected returns, in particular the slope reversals documented during the 2007–09 financial crisis. Both puzzles are solved by introducing horizon-dependent risk aversion.

Key words: risk aversion, early resolution, term structure, volatility risk

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1 Introduction

We propose a model that relaxes the assumption, standard in the economics literature, that risk aversion is constant for all payoff horizons. We define pseudo-recursive preferences similar to Epstein-Zin (Epstein and Zin, 1989) but generalized to allow for horizon-dependent risk aversions. Our model remains tractable, and usual recursive techniques can be applied. As the experimental evidence indicates, we assume that agents are more risk averse for short horizons payoffs. We focus on a macro-finance application and derive equilibrium risk prices and individuals' attitudes towards the early or late resolutions of their future consumption shocks within the long-run risk framework.

We find that horizon-dependent risk aversion solves two important puzzles of the standard model. First, we show that our utility model can be calibrated to match the usual asset pricing moments without implying a strong preference for early resolutions of uncertainty. This result addresses the fundamental challenge of Epstein, Farhi, and Strzalecki (2014) that the standard calibration of long-run risk under Epstein-Zin preferences to explain risk prices in the data (e.g. Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2009) requires a very high timing premium which is difficult to reconcile with micro evidence and introspection. Second, we analyze our model's implications for the term structures of equity risk premia. These term structures are always upward sloping in the calibrations of the standard long-run risk model, a result challenged by the existing evidence (van Binsbergen, Brandt, and Koijen, 2012).[1] In contrast, we find that under horizon-dependent risk aversion, term structures are upward sloping on average, as in the data, but can turn sharply downward sloping when liquidity breaks down, such as in the recent financial crisis of 2007–2009.

Our first contribution is methodological: we introduce horizon-dependent risk aversion within the standard recursive utility model of Epstein and Zin (1989), which allows us to build on its success at explaining asset pricing moments when combined with long-run risk. We show that commonly used recursive techniques can be adapted to our setting of pseudo-recursive preferences, enabling us to derive closed-form solutions. Our baseline model can accommodate numerous extensions, be it on the valuation of risk (e.g. habit formation, disappointment aversion, loss aversion), or on the quantity of risk (e.g. rare disasters, production-based models). Further, under our preference model, *inter-temporal* decisions for deterministic payoffs are unchanged from the standard, time consistent, model; but *intra-temporal* allocations across risky assets are time inconsistent. We can therefore

[1]The term structures of risk premia observed in the data are also inconsistent with the habit-formation model of Campbell and Cochrane (1999) and the rare disaster models of Gabaix (2012) and Wachter (2013).

study the optimal decisions and pricing impact of horizon-dependent risk aversion in isolation from quasi-hyperbolic discounting, and in general from models of time inconsistent inter-temporal decisions.

We assume risk aversion decreases with the temporal horizon, as suggested by experimental evidence, and analyze the pricing implications of the resulting preference model. Combining recursive Epstein-Zin preferences with risk to the expected growth and volatility of consumption, the standard long-run risk model has had great success at matching asset pricing moments and at explaining their apparent "puzzles" (see Cochrane, 2016 for a review of the literature). It explains the high equity premium, matches various cross-sectional evidence, captures the macroeconomic announcement premium (excess returns around the Federal Reserve's regular monetary policy meetings), and results in time-varying risk premia that rationalize the volatility puzzle and return predictability. To do so, however, the standard long-run risk model also implies that the representative agent has a high timing premium, a measure of her preference for early versus late resolutions of uncertainty. Measured as the portion of her lifetime consumption the agent would be willing to forego in order to be told all her future consumption shocks in the next period rather than over time, the timing premium is 30% in the calibration of Bansal and Yaron (2004) and 80% in Bansal, Kiku, and Yaron (2009). Such a strong preference for early resolutions of uncertainty appears inconsistent with the evidence, for instance on investors' inattention to their wealth, and with commonsense considerations; raising doubts as to the validity of the whole model (Epstein, Farhi, and Strzalecki, 2014). As our second contribution, we show that horizon-dependent risk aversion can reconcile the two sets of evidence, the macroeconomic asset pricing data with microeconomic attitudes towards risk and information.

We analyze first how horizon-dependent risk aversion affects the willingness to pay for early resolutions of uncertainty. Specifically, we derive how two consumption streams with identical risk but different timing for information arrivals are valued: one where shocks are revealed gradually as they are realized over time, the other where all future shocks are revealed at the same early date; replicating for our preferences the analysis Epstein, Farhi, and Strzalecki (2014) do for Epstein-Zin preferences. Under horizon-dependent preferences, agents also value these consumption streams differently, even though the ex-ante distributions of risk are rigorously identical. Whether and how the two valuations differ depends on the wedge between risk aversions for short-horizon payoffs versus for long-horizon payoffs in addition to their values relative to the elasticity of inter-temporal substitution. A consumption stream with early resolution of uncertainty shifts the risk of all future shocks into a short-horizon risk, moving from a risk assessment using the lower risk aversion at long horizons to a risk assessment using the higher risk aversion at short horizons. This lowers the attractiveness of early resolution of uncertainty, compared to the standard framework with Epstein-Zin preferences. We formalize this intuition and prove that the timing premium is unambiguously lowered when risk aversion is decreasing in horizon.

We then apply our utility model and methodology to equilibrium asset pricing. We consider a representative agent who trades and clears the market every period, and, as such, cannot pre-commit to any specific strategy. Unable to commit to future behavior but aware of her dynamic inconsistency, in the spirit of Strotz (1955), the agent optimizes in the current period, fully anticipating re-optimization in future periods. Solving our model this way yields a canonical one-period pricing problem in which the Euler equation is satisfied, and the law of one price and no-arbitrage conditions hold. The stochastic discount factor of our pseudo-recursive model nests the standard case of the Epstein-Zin model, but with a new multiplicative term that loads on the wedge arising from the preferences' dynamic inconsistency between the $t+1$ continuation value used for optimization at a given time t and the actual valuation at $t+1$.

In a Lucas-tree endowment economy with long-run risk, we derive equilibrium prices and analyze how they are affected by the new term in the stochastic discount factor due to horizon-dependent risk aversions. We find that the pricing of shocks that impact consumption *levels* are unchanged from the standard model — reflecting that the dynamic inconsistency in our model does not concern inter-temporal decisions. In contrast, shocks to consumption *risk* (volatility) directly affect intra-temporal decisions, and their pricing changes under horizon-dependent risk aversion: the lower risk aversion at long horizons reduces the pricing of volatility shocks and this effect accumulates over time. Because of this dichotomy in the pricing impact of our model, lowering long-horizon risk aversions compared to the short-horizon increases the impact of consumption level shocks on the equity premium relative to volatility shocks — immediate and long run, with the effect increasing in the horizon. This allows us to discipline the calibration of our model: we use recent evidence on the macroeconomic announcement premium as a share of the total equity premium (Lucca and Moench, 2015; Ai and Bansal, 2018) to quantify the wedge between short and long-horizon risk aversions. We find that our model thus calibrated can match the usual asset pricing moments *and* a reasonable level of timing premia, lower than 10%. It therefore resolves all concerns regarding preferences for early or late resolution of uncertainty in the long-run risk framework, and reconciles the evidence from the macroeconomic and microeconomic literatures.

Finally we turn to the model's implications for how risk premia vary with the horizon of asset payoffs — the term structure of expected returns — and explore the impact a downward sloping term structure of risk aversions may have. The term structure of risk premia is at the center of an important debate in empirical finance: van Binsbergen, Brandt, and Koijen (2012), followed by numerous others, find evidence of downward sloping term structures for various excess returns, in contradiction with the calibrated long-run risk asset pricing model. How robust these results are and how important a puzzle they represent is not as yet fully established (see the literature review below). Nonetheless, they raise an important challenge to the standard model, which built its success on its ability to match asset pricing moments. A conceptually sound and empirically consistent model for the pricing of risk at different horizons is not only crucial for the valuation of new assets and for long-term versus short-term investment decisions, but also for fields in economics with clear policy implications such as climate change policy (Gollier, 2013; Giglio, Maggiori, Stroebel, and Weber, 2015). Understanding the term structure evidence is therefore key and our third contribution.

We derive the term structure of dividend risk excess returns under horizon-dependent risk aversion, assuming that agents trade every period. Under this "business as usual" setup, our calibrated model implies that term structures are upward sloping, though flatter than in the standard model with Epstein-Zin preferences; decreasing term structures of risk aversions therefore do *not* imply decreasing term structures of risk premia. This result explains the evidence that unconditional term structures of dividend risk premia are upward sloping but not why the conditional term structures sometimes become sharply downward sloping, as observed during the financial crisis of 2007–2009 (van Binsbergen, Hueskes, Koijen, and Vrugt, 2013; Bansal, Miller, Song, and Yaron, 2019). To account for the deep liquidity crunch over that period, we deviate from the representative agent assumption and let illiquidity lead some investors to adopt buy-and-hold strategies. While it is always counterfactually upward sloping in the standard model, a downward sloping term structure of risk premia can emerge in our calibrated horizon-dependent risk aversion model when enough investors opt for committed buy-and-hold strategies, in particular for assets with long horizons — a realistic assumption when liquidity breaks down.

In sum, the model of preferences we propose, where risk aversion differs for short-horizon and long-horizon payoffs, can address the early versus late resolution of uncertainty challenge of Epstein, Farhi, and Strzalecki (2014) and generate term structures of expected returns consistent with the slope reversal dynamics described in van Binsbergen, Hueskes, Koijen, and Vrugt (2013) and Bansal, Miller, Song, and Yaron (2019). We can solve these challenges to the long-run risk framework concerning the timing and pricing of uncertainty without compromising on the model's ability to match the usual asset pricing moments, and without departing from the methodology of the widely-used Epstein-Zin preferences.

After a review of the literature, we present our model of preferences in Section 2. We analyze the preference for early or late resolution of uncertainty in Section 3. In Section 4, we derive the risk pricing implications of our model. Section 5 presents its quantitative predictions, and shows that the calibrated model can reconcile the evidence on the equity and macroeconomic announcement premia with realistic levels for the timing premium. In Section 6, we analyze the term structure implications under the one-period trading assumption. Section 7 presents the pricing implications of our model under liquidity crunches. Section 8 concludes. All mathematical proofs are in the Appendix.

Related literature

This paper is the first to solve for equilibrium asset prices in an economy populated by agents with dynamically inconsistent risk aversions. Our methodology, which guarantees the no-arbitrage condition despite time inconsistency, follows Luttmer and Mariotti (2003), and our work complements theirs. They show that dynamically inconsistent preferences for inter-temporal trade-offs of the kind examined by Harris and Laibson (2001) have only limited implications for asset pricing, and little power to explain cross-sectional variations in asset returns. Given that cross-sectional asset pricing involves intra-period risk-return tradeoffs, it is indeed quite intuitive that inter-temporal dynamic inconsistency is not suitable to address puzzles related to risk premia.

Our model generalizes Epstein-Zin preferences by relaxing the dynamic consistency axiom of Kreps and Porteus (1978) to analyze the relationship between the timing and pricing of uncertainty. We choose the CRRA model for risk adjustments, standard to the macro-finance literature. In contrast, Routledge and Zin (2010), Bonomo et al. (2011) and Schreindorfer (2014) follow Gul (1991) and relax the independence axiom to analyze the asset pricing impact of disappointment aversion within a recursive framework. They find that their models generate endogenous predictability (Routledge and Zin, 2010); match various asset pricing moments (Bonomo et al., 2011); and price the cross-section of options better than the standard model (Schreindorfer, 2014). Similarly, Andries (2015) introduces loss aversion in recursive preferences à la Epstein and Zin (1989) and shows it helps match the security market line, while Dew-Becker (2012) uses a model of habit to obtain time varying risk premia. Our framework can also accommodate these various utility functions for the valuation of risk. Within the classical model of Epstein and Zin (1989), none of the above-mentioned preference models address the "excessive preference for early resolutions of uncertainty puzzle", pointed out by Epstein et al. (2014) or quantitatively match the term structures of risk prices — the two questions of interest in our analysis.

To capture various asset pricing moments, the long-run risk literature relies on the pricing of shocks to consumption growth and to consumption volatility. Hansen et al. (2008) directly measure consumption growth shocks in the data, and Bryzgalova and Julliard (2015) use cross-sections of returns to provide evidence consumption growth shocks are priced, which is consistent also with equity premia around the Federal Open Market Committee (FOMC) meetings (Lucca and Moench, 2015; Ai and Bansal, 2018). The importance of a volatility risk channel is supported by Campbell et al. (2016), who show that it is crucial for asset returns in a CAPM framework, and who relate this to other works on the relation between volatility risk and returns (Ang et al., 2006; Adrian and Rosenberg, 2008; Drechsler and Yaron, 2010; Bollerslev and Todorov, 2011; Menkhoff et al., 2012; Boguth and Kuehn, 2013). However, direct evidence in the data of time-varying uncertainty in the consumption process remains elusive.

Epstein et al. (2014) point out that calibrating the long-run risk model to match the observed market equity premium implies an extremely strong preference for early resolutions of uncertainty, as measured by a high "timing premium". Ai and Bansal (2018) argue that macro-announcement premia around the FOMC meetings in the data provide direct evidence of such preferences for early information. We show these two questions and the related evidence are not necessarily linked: our calibrated model can simultaneously match a high price of consumption growth risk and a low or even negative timing premium.

The puzzle of a downward sloping term structure of excess returns has emerged in the empirical literature, starting with van Binsbergen et al. (2012) who find expected excess returns for short-term dividends above those for long-term dividends (see also Boguth et al., 2012; van Binsbergen and Koijen, 2011; van Binsbergen et al., 2013). Van Binsbergen and Koijen (2016) document downward sloping Sharpe ratios of excess returns, across a variety of risky assets. Giglio et al. (2014) show a similar pattern exists for discount rates over much longer horizons using real estate data; Lustig et al. (2016) also find these patterns for currency carry trade risk premia. Weber (2016) sorts stocks by the duration of their cash flows and finds significantly higher returns for short-duration stocks. Dew-Becker et al. (2016) use data on variance swaps to show the volatility risk is priced mostly at very short horizons. Using different methodologies and standard index option data, Andries et al. (2016) also find a negative price of variance risk for maturities up to four months, and a strongly nonlinear downward sloping term structure (in absolute value).

These striking empirical findings have triggered numerous new theoretical works. Various models generate the desired implications — downward sloping term structures of risk premia — by making structural assumptions about the shocks affecting the economy and how they are priced. For example, Ai et al. (2015) derive term structure results in a production-based real business cycle model in which capital vintages face heterogeneous shocks to aggregate productivity. Other production-based models with implications for the term structure of equity risk are, e.g. Kogan and Papanikolaou (2010, 2014), and Gârleanu et al. (2012). Favilukis and Lin (2015), Belo et al. (2015), and Marfe (2015) offer wage rigidities to explain why risk levels and thus risk premia could be higher at short horizons. Croce et al. (2015) use informational frictions to generate a downward-sloping equity term structure. Backus et al. (2016) propose the inclusion of jumps to account for the discrepancy between short-horizon and long-horizon returns, while Nzesseu (2018) shows it is sufficient to add negative covariation between the consumption shocks and the volatility shocks to the long-run risk model of Bansal and Yaron (2004). Other models focus, as we do, on the risk prices rather than on the quantity of risk: for example, Andries (2015) and Curatola (2015) propose preferences with first-order risk aversion to explain the observed term structures; Khapko (2015) and Guo (2015) both study other dynamic extensions to Eisenbach and Schmalz (2016).[2]

All these papers explicitly focus on matching downward sloping term structures of risk prices. However, Bansal et al. (2019) cast doubt on the validity of the term structure puzzle. The authors document that expected excess returns of dividend strips are *upward* sloping on average, in line with most standard asset pricing models including the long-run risk framework. They postulate that the aforementioned empirical findings of downward sloping term structures are due to short time series of data in which the sharply inverted term structure of expected excess equity returns during the financial crisis of 2007–2009 is over-represented. They explain the slope reversals during crises periods by modeling regime shifts in the consumption and dividend risk processes. Andries et al. (2016) however find the term structure for the price of variance risk is downward sloping (in absolute value) in and out of crisis, inconsistent with a regime shift framework but not with our model of liquidity constraints.

None of the theoretical papers cited above simultaneously matches the evidence that term structures of risk premia can be sometimes upward sloping and sometimes down-

[2] They do so in a time-separable model, which confounds dynamically inconsistent risk preferences with dynamically inconsistent time preferences (hyperbolic discounting). That approach makes the two ingredients' relative contributions opaque. Further, the approach does not accommodate formal solutions, and thus formal interpretations.

ward sloping, and proposes solutions to the early versus late resolution of uncertainty puzzle; in contrast to our horizon-dependent risk aversion model.

2 Preferences with horizon-dependent risk aversion

Field and laboratory experiments document that risk-taking behavior is affected by how far in the future a risk occurs: subjects tend to be more averse to risks in the near future than to risks in the distant future. Early work by Jones and Johnson (1973) provides evidence for such horizon-dependent risk aversions from a simulated medical trial. More recent studies use the standard protocol of Holt and Laury (2002) to elicit risk aversion — Noussair and Wu (2006) in a within-subjects design and Coble and Lusk (2010) in an across-subjects design — and find risk aversion decreases as risk becomes more distant in time. The same pattern is documented by Sagristano, Trope, and Liberman (2002) and Baucells and Heukamp (2010) using binary choice among lotteries, as well as by Onculer (2000) and Abdellaoui, Diecidue, and Onculer (2011) using certainty equivalents.

Figure 1 provides an example of preferences with horizon-dependent risk aversion. Under this illustrative example, all subjects are asked to rank a lottery with payoff $x = 1$ for certain versus a lottery with payoff $x = 3$ with a 50% chance, and $x = 0$ otherwise. All subjects choose their rankings at time $t = 0$; however for some the lottery happens at time $t = 2$ (the "distant risk" case), and for some the lottery happens at time $t = 1$ (the "imminent risk" case).

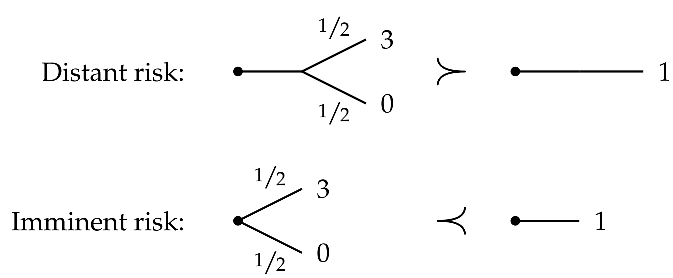

Figure 1: Preferences with horizon-dependent risk aversion.

The experimental evidence shows that subjects may prefer the certain lottery over the risky one when the risk is immediate but prefer the same risky lottery over the certain one when the risk is distant in the future. For a real life intuitive example, think of someone paying a considerable amount of money for a parachute jumping experience, and then refusing to actually jump once in the plane. This is the notion of *horizon-dependent risk aversion* as introduced by Eisenbach and Schmalz (2016) in a static, time separable, framework.

In the illustrative example above, one subgroup ranks lotteries with horizon $t = 1$ and the other subgroup ranks lotteries with horizon $t = 2$: within each subgroup the ranking is for lotteries that will happen at the same time. That the rankings change with the horizon reveals a dynamic inconsistency in *intra-temporal* choices, not in *inter-temporal* choices. In particular, the well documented hyperbolic discounting (e.g. Phelps and Pollak, 1968; Laibson, 1997) or other time inconsistencies concerning inter-temporal decisions do not influence, or cause, the evidence discussed above.[3]

2.1 Dynamic preference model

The experimental evidence that subjects are more risk averse for short-horizon than for long-horizon payoffs seems particularly relevant when considering the relationship between the timing and the pricing of risk — at the center of the recent challenges to the long-run risk framework. To explore the formal implications of horizon-dependent risk aversion in a dynamic framework, we introduce it in the recursive utility Epstein-Zin preferences, the standard model for long-run risk pricing. Epstein-Zin preferences are dynamically consistent (by definition). We generalize their model by relaxing the dynamic consistency axiom of Kreps and Porteus (1978).

To simplify the exposition, we present the model with only two levels of risk aversion γ and $\widetilde{\gamma}$: we assume that the agent treats immediate uncertainty with risk aversion γ, and all delayed uncertainty with risk aversion $\widetilde{\gamma}$, where $\gamma > \widetilde{\gamma} \geq 1$ in line with the experimental evidence. Our approach, with only two levels of risk aversion, is analogous to the way the β-δ framework (Phelps and Pollak, 1968; Laibson, 1997) is a special case of the general non-exponential discounting model of Strotz (1955). In Appendix A, we present the model for general sequences $\{\gamma_h\}_{h\geq 1}$ of risk aversion at horizon h. As long as risk aversions reach a constant level beyond a given horizon, closed form solutions similar to those derived in Sections 3, 4, 6 and 7 obtain.

At any time t, we denote by $E_t[\,\cdot\,] = E[\,\cdot \mid \mathcal{I}_t]$ the expectation conditional on $\mathcal{I}_t$, the information set at time t.

Definition 1 (Dynamic horizon-dependent risk aversion). *The agent's life-time utility in period t is given by*

$$V_t = \left((1-\beta)\, C_t^{1-\rho} + \beta E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \tag{1}$$

[3] Eisenbach and Schmalz (2016) also show that horizon-dependent risk aversion is conceptually orthogonal to time-varying risk aversion (Constantinides, 1990; Campbell and Cochrane, 1999) .

where the continuation value $\widetilde{V}_{t+1}$ satisfies the recursion

$$\widetilde{V}_{t+1} = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1} \left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}} \right]^{\frac{1-\rho}{1-\widetilde{\gamma}}} \right)^{\frac{1}{1-\rho}} . \tag{2}$$

As in Epstein-Zin preferences, the lifetime utility V_t depends on the deterministic current consumption C_t and on the certainty equivalent $E_t \left[\widetilde{V}_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$ of the continuation value $\widetilde{V}_{t+1}$, where the aggregation of the two periods occurs with constant elasticity of intertemporal substitution given by $1/\rho > 0$ under the subjective time discount $\beta > 0$. However, in contrast to Epstein-Zin preferences, the certainty equivalent of consumption starting at $t+1$ is calculated with relative risk aversion $\gamma > 0$, wherein the certainty equivalents of consumption starting at $t+2$ and beyond are calculated with relative risk aversion $\widetilde{\gamma} > 0$.

This is the concept of horizon-dependent risk aversion applied to the recursive valuation of certainty equivalents, as in Epstein-Zin preferences, but with risk aversion γ for imminent uncertainty and risk aversion $\widetilde{\gamma}$ for delayed uncertainty. Our model nests the Epstein-Zin model when $\gamma = \widetilde{\gamma}$, and, in turn, nests the standard time-separable model with constant relative risk aversion (CRRA) when $\gamma = \widetilde{\gamma} = \rho$. Any difference in the results we derive below under the preferences of Definition 1 to those obtained under the standard Epstein-Zin model thus hinges on $\widetilde{\gamma} \neq \gamma$.

The horizon-dependent valuation of risk implies a dynamic inconsistency, as the uncertain consumption stream starting at $t+1$ is evaluated as $\widetilde{V}_{t+1}$ by the agent's self at t and as V_{t+1} by the agent's self at $t+1$:

$$\widetilde{V}_{t+1} = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1} \left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}} \right]^{\frac{1-\rho}{1-\widetilde{\gamma}}} \right)^{\frac{1}{1-\rho}} \neq V_{t+1} \;\; = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1} \left[\widetilde{V}_{t+2}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}$$

Crucially, this disagreement between the agent's continuation value $\widetilde{V}_{t+1}$ at t and the agent's utility V_{t+1} at $t+1$ arises only for uncertain consumption streams. For any deterministic consumption stream the horizon dependence in Equation (1) becomes irrelevant and we have

$$\widetilde{V}_{t+1} = V_{t+1} = \left((1-\beta) \sum_{h \geq 0} \beta^h C_{t+1+h}^{1-\rho} \right)^{\frac{1}{1-\rho}} .$$

Our model implies dynamically inconsistent *risk* preferences while maintaining dynamically consistent *time* preferences, focusing strictly on the experimental evidence described above. The results we obtain in the analysis that follows can therefore be attributed to horizon-dependent risk aversion, orthogonal to extant models of time inconsistency, such as hyperbolic discounting.

2.2 Generalized preference model

In the preferences of Definition 1, we opted for CRRA risk adjustments. However, similarly to the Epstein-Zin model, our model of horizon-dependent risk aversion accommodates any preferences in the Chew-Dekel class of betweenness-respecting models (Dekel, 1986; Chew, 1989). The general model is defined as:

Definition 2 (**Generalized dynamic horizon-dependent risk aversion**). *The agent's utility in period t is given by*

$$V_t = \left((1-\beta)\, C_t^{1-\rho} + \beta \left(\mathcal{R}_t \left[\widetilde{V}_{t+1} \right] \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{3}$$

where the continuation value $\widetilde{V}_{t+1}$ satisfies the recursion

$$\widetilde{V}_{t+1} = \left((1-\beta)\, C_{t+1}^{1-\rho} + \beta \left(\widetilde{\mathcal{R}}_{t+1} \left[\widetilde{V}_{t+2} \right] \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \tag{4}$$

and $\mathcal{R}_t[\,\cdot\,]$ and $\widetilde{\mathcal{R}}_t[\,\cdot\,]$ are certainty-equivalent operators for utility functions $\mathcal{U}$ and $\widetilde{\mathcal{U}}$ in the Chew-Dekel class of betweenness-respecting models.

Examples of certainty equivalent operators other than CRRA (of Equations (1) and (2)) could be those of a CRRA habit model (Campbell and Cochrane, 1999) with risk aversions $\gamma > \widetilde{\gamma}$ or those of the disappointment aversion model (Gul, 1991) with first-order risk aversion coefficients $\theta > \widetilde{\theta}$. As mentioned in our review of the literature, introducing these "exotic" risk adjustments helps explain cross-sectional evidence (Routledge and Zin, 2010; Bonomo et al., 2011; Schreindorfer, 2014; Andries, 2015), orthogonal to the timing and pricing of risk we analyze in this paper and to our notion of horizon-dependent risk aversion. The cross-sectional results derived under the standard Epstein-Zin model would remain valid under the preferences of Definition 2.

At a deeper level, the preferences of Definition 2 allow for great flexibility: agents could have first-order risk aversion (disappointment aversion or loss aversion) for immediate risk but standard concave utility for longer horizons; they could have time-varying risk aversion for immediate risks only; the gap between their immediate and long-term risk aversions could vary with market conditions; etc. Our first contribution is conceptual: we propose a model of preferences that allows for the analysis of new and complex forms of dynamic inconsistencies within a simple framework.

2.3 Timing of risk and dynamic inconsistency

An agent with the time-inconsistent preferences of Definition 1 or Definition 2 can be either naive or sophisticated about the disagreement between her temporal selves; in addition, she may be able to commit to multi-period strategies or be compelled to reoptimize every period. The valuation of early versus late resolution of uncertainty, which we analyze first in Section 3, is by nature a static problem: its solutions are the same for naive and sophisticated investors, with or without commitment. But these modeling choices matter for dynamic outcomes, in particular the equilibrium asset prices we then derive.

In Sections 4, 5, 6 and 7, we follow the tradition of Strotz (1955), and assume the agent is fully rational and sophisticated when making choices in period t to maximize V_t. Self t realizes that her valuation of future consumption, given by $\widetilde{V}_{t+1}$, differs from the objective function V_{t+1} which self $t+1$ will maximize. The solution then corresponds to the subgame-perfect equilibrium in the sequential game played among the agent's different selves (see Appendix A.1). We assume no commitment in our general case, as appropriate for a representative agent who trades and clears the market at all times, and as such cannot precommit to a given strategy — similar to the framework of Luttmer and Mariotti (2003) for non-geometric discounting. However, in Section 7, we let the sophisticated agents commit to certain strategies when we explore the implications for illiquid assets and periods of liquidity crises in which one-period pricing breaks down.

Extending our results to an agent naive about her own dynamic inconsistencies is straightforward and does not present any conceptual challenge. We briefly discuss and derive formal results for this alternative approach in Appendix A.3.

3 Preference for early or late resolution of uncertainty

Our first question of interest is: how and to what extent do the horizon-dependent risk aversion preferences of Definition 1 affect agents' decisions regarding the timing of information arrivals? To analyze this issue, and determine whether agents have a preference for early or late resolutions of uncertainty, we strictly follow the set up of Epstein et al. (2014). Two types of consumption streams, subject to the exact same shocks over time, are evaluated at a given time t. In the first case, consumption shocks are revealed gradually, whenever they are realized: the shock affecting consumption at time $t+h$ is revealed at $t+h$, for all horizons $h \geq 1$. In the second case, all future consumption shocks are revealed in the next period, at time $t+1$, even when they affect consumption at a later period: the shock affecting consumption at $t+h$ is revealed at time $t+1$, for all $h \geq 1$.

Crucially, even when she receives the information about her future consumption shocks earlier, the agent cannot act on the information to change her future consumption stream. From the point of view of time t, when the agent evaluates the two consumption streams with or without early resolution of uncertainty, the distributions of future risks are therefore exactly the same in both cases; in the expected utility framework, she would assign them the exact same value. However, in the non time-separable models of Epstein and Zin (1989) and Definition 1, two consumption streams with ex ante identical risks, but different timing for the resolution of uncertainty, can have different values.

An agent with Epstein-Zin utility prefers early resolutions of uncertainty if and only if her risk aversion is greater than her inverse elasticity of inter-temporal substitution: $\gamma > \rho$.[4] How much she prefers early resolutions depends on the wedge $\gamma - \rho$ and on the magnitude of the uncertainty in the consumption shocks. As Epstein et al. (2014) point out, the parameters used in the long-run risk literature imply a strong preference for early resolution of uncertainty. For example, in the calibration of Bansal and Yaron (2004), the representative agent would be willing to forgo more than 30%, and in the more recent calibration of Bansal et al. (2009) more than 80%, of her consumption stream in exchange for all uncertainty to be resolved the next month instead of gradually over time.[5] We discuss below why giving up more than one third of one's wealth to learn early about future shocks – shocks that will happen no matter what – seems excessive: a "puzzle" according to Epstein et al. (2014). First, however, we demonstrate why horizon-dependent risk aversion in the preferences of Definition 1 can greatly influence the appetite for early versus late resolution of uncertainty.

Choosing a consumption stream with an early resolution (i.e., where all shocks are revealed at time $t+1$) rather than the same consumption stream with late resolutions (i.e., where shocks are revealed as they come over time) corresponds to shifting all future risk, short-term and long-term, to a next-period risk. Whether long-term risks are evaluated with the same risk aversion as immediate risks will thus matter for the relative values of the two theoretical consumption streams, and therefore for the preference for early or late resolutions of uncertainty. Importantly, assigning values to the two consumption streams above is a static problem: the agent evaluates the two (infinite) streams of consumption, with early or late resolution of uncertainty, exactly once. How her preferences change over time, whether she is naive or sophisticated about it, whether she can commit to specific

[4] To see why, note that in the case where all future shocks are revealed at $t+1$, the shocks to consumption from $t+2$ onward are evaluated with the inverse elasticity of inter-temporal substitution ρ since they are no longer uncertain; whereas, when shocks are revealed over time, variations in consumption from $t+2$ onward are still risky at $t+1$ and thus evaluated with risk aversion γ.

[5] See Figure 2 and Table 2 in Section 5.

future choices, are irrelevant to the relative values she assigns to the two consumption streams, i.e. to her preference for early or late information.

To formalize this argument, and derive how an agent with the horizon-dependent risk aversion preferences of Definition 1 assesses the early resolution of uncertainty, we replicate the formal analysis of Epstein et al. (2014). We assume, as they do, a unit elasticity of inter-temporal substitution, $\rho = 1$, and log-normal consumption growth with time varying drift, corresponding to long-run risk. Using lowercase letters to denote logs throughout, e.g. $c_t = \log C_t$, we let consumption follow the process

$$\begin{aligned} c_{t+1} - c_t &= \mu_c + \phi_c x_t + \alpha_c \sigma w_{c,t+1}, \\ x_{t+1} &= \nu_x x_t + \alpha_x \sigma w_{x,t+1}. \end{aligned} \tag{5}$$

For simplicity x_t, which represents time variations in the average consumption growth, is one-dimensional and the shocks $w_{c,t}$ and $w_{x,t}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal. The drift is stationary, i.e. ν_x is contracting. In the following sections, we add volatility shocks to the consumption process (5): the economic uncertainty becomes time varying, σ_t. We present the results in this section for the specific case $\sigma_t = \sigma$ as they are more readily interpretable and convey all the relevant intuitions.[6]

Denoting by V_t^* the agent's utility at t if all uncertainty (i.e., the entire sequence of shocks $\{w_{c,t+h}, w_{x,t+h}\}_{h \geq 1}$ in the consumption process (5)) is resolved at $t+1$, and by V_t the agent's utility if uncertainty is revealed over time, the timing premium is defined as

$$\mathrm{TP_t} = \frac{V_t^* - V_t}{V_t^*}.$$

This timing premium represents the fraction of utility, or equivalently the fraction of lifetime consumption, the agent is willing to forgo for an early rather than late resolution of uncertainty. As mentioned above, in the calibration with constant volatility of the classical Epstein-Zin model in Bansal and Yaron (2004), the timing premium is positive and quite high, at 30% (Epstein et al., 2014).

Proposition 1. *An agent with the horizon-dependent risk aversion preferences of Definition 1 with $\rho = 1$, facing the consumption process (5), has a constant timing premium*

$$\mathrm{TP} = 1 - \exp\left(\frac{1}{2}\left(1 - \left(\gamma - (1+\beta)\left(\gamma - \widetilde{\gamma}\right)\right)\right) \frac{\beta^2}{1-\beta^2}\, \alpha_v^2 \sigma^2\right), \tag{6}$$

[6] The interested reader can find the derivations for the case with time varying volatility in Appendix B.1.

where $\alpha_v^2 = \alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2 \alpha_x^2$.

To highlight the role played by horizon-dependent risk aversion, note that an agent with the standard Epstein-Zin preferences with risk aversion γ has a timing premium given by $\mathrm{TP} = 1 - \exp\left(\frac{1}{2}(1-\gamma)\frac{\beta^2}{1-\beta^2}\alpha_v^2\sigma^2\right)$, obtained by setting $\gamma = \widetilde{\gamma}$ in Equation (6). When $\gamma > \widetilde{\gamma}$, the timing premium is instead determined by:

$$\gamma - (1+\beta)(\gamma - \widetilde{\gamma}) < \gamma.$$

Corollary 1. *For an agent with horizon-dependent risk aversion, $\gamma > \widetilde{\gamma}$ unambiguously lowers the timing premium.*

To understand the intuition behind Corollary 1, observe that a consumption stream with an early resolution of uncertainty concentrates all the risk on the first period, over which the agent is the most risk averse, with immediate risk aversion γ. In contrast, a consumption stream with late resolutions of uncertainty has risk spread over multiple horizons, over some of which the agent is moderately risk averse since $\widetilde{\gamma} < \gamma$.[7]

Note that in following the analysis of Epstein et al. (2014) and assuming only two levels of risk aversion $\gamma, \widetilde{\gamma}$, we are implicitly mixing two comparisons: gradual resolution versus one-shot resolution and early resolution versus late resolution. In addition, we are placing the early resolution at time $t+1$, exactly in the period where the risk aversion changes from γ to $\widetilde{\gamma}$. However, we show in Appendix B.2 that the results of Proposition 1 and Corollaries 1 and 2 below are robust by (i) allowing for a general decreasing sequence of risk aversions $\{\gamma_h\}_{h=1}^{\infty}$ to show that the result is based on horizon-dependent risk aversion and not on a particular period and (ii) comparing resolution of all uncertainty at $t+1$ to resolution of all uncertainty at $t+2$ to show that the relevant comparison is between early and late resolution, not between gradual and one-shot resolution.

When would the timing premium turn negative, indicating a preference for *late* resolution? For an Epstein-Zin agent, this happens if and only if $\gamma < \rho$. In our model, with

[7] The same intuitive argument applies for other dynamic inconsistencies on inter-temporal rather than intra-temporal choices, our focus. In Appendix B.1, we derive the timing premium under hyperbolic discounting, whereby $\gamma = \widetilde{\gamma}$ but, at time t, the value V_t is derived with time discount parameter β, and the continuation value $\widetilde{V}_{t+1}$ is derived with time discount parameter $\widetilde{\beta} > \beta$. The preference for an early resolution of uncertainty still holds if and only if $\gamma > \rho$, but the magnitude of the timing premium is lower than if the time discount is $\widetilde{\beta}$ everywhere (and greater than if it is β everywhere). Introducing hyperbolic discounting has, however, a small quantitative effect: e.g. under the calibration of Bansal and Yaron (2004) with constant volatility, $\gamma = 10$, $\rho = 1$, and $\beta = 0.8, \widetilde{\beta} = 0.998$, the timing premium only goes from 27% (under $\beta = \widetilde{\beta} = 0.998$) to 22.5%.

$\rho = 1$ and the consumption process (5), the timing premium is negative if and only if

$$\gamma < 1 + (1+\beta)\left(\gamma - \widetilde{\gamma}\right). \tag{7}$$

When $\gamma > \widetilde{\gamma}$, we immediately obtain $1 + (1+\beta)(\gamma - \widetilde{\gamma}) > \rho = 1$, and the agent with horizon-dependent risk aversion can have a preference for late resolution, even when both risk aversions γ and $\widetilde{\gamma}$ are greater than the inverse elasticity of inter-temporal substitution — as long as the decline in risk aversion across horizons is sufficiently large. For example, suppose we set immediate risk aversion $\gamma = 10$ and β close to 1. Then the agent will prefer uncertainty to be resolved late rather than early according to the condition of Equation (7) as long as $\widetilde{\gamma} < 5.5$ which is substantially larger than $\rho = 1$.[8]

Corollary 2. *An agent with horizon-dependent risk aversion can prefer a late resolution of uncertainty even when all risk aversions exceed the inverse elasticity of inter-temporal substitution, i.e. when $\gamma > \widetilde{\gamma} > \rho$.*

The result of Corollary 2 is of particular interest because extant calibrations of the long-run risk model with Epstein-Zin preferences require γ greater than ρ by an order of magnitude to match equilibrium asset pricing moments — hence the high timing premia they imply. Under horizon-dependent risk aversion, the same calibration for γ and ρ no longer automatically implies such a strong preference for early resolutions of uncertainty. This is true even when the long-run risk aversion $\widetilde{\gamma}$ also remains above the inverse elasticity of inter-temporal substitution, in line with the micro evidence.

Ai and Bansal (2018) document a high macroeconomic announcement premium, as measured by the high share of the equity premium that realizes around pre-scheduled FOMC meetings (55% over the 1961–2014 period), and argue that this pricing of shocks to future consumption levels implies a strong preference for early resolution of uncertainty. Indeed, they show the link between the two is tight for time consistent recursive preferences. In the Epstein-Zin model in particular, the macroeconomic announcement premium, corresponding to the pricing of shocks to the consumption growth variable x_t in the process (5), is determined by $\gamma - \rho$ *exactly like* the timing premium. In contrast, Corollary 2 establishes that a high $\gamma - \rho$ need not imply a high or even a positive timing premium under the horizon-dependent risk averse preferences of Definition 1; whereas we show next in Section 4 that the macroeconomic announcement premium remains entirely determined by $\gamma - \rho$ in our model. Our framework decouples microeconomic interpretations regarding preferences for early or late information from the direct evidence in

[8] In the calibrated model of Section 5, in which we add time varying volatility to the consumption process (5), we obtain a preference for late resolution whenever $\widetilde{\gamma} < 4.42$.

macroeconomic data (macroeconomic announcement premia or asset pricing moments). We find in Section 5 that an equity premium and macroeconomic announcement premium consistent with the evidence can obtain under preference for early *or* late resolution (see Table 2).

But why is that progress? Why can't we simply acknowledge how well the long-run risk model fits the macroeconomic evidence and infer it implies the correct timing premia, as argued in Ai and Bansal (2018)? First, it is worth noting that the recursive utility Epstein-Zin model has little microeconomics or experimental foundation, contrary to other models of preferences commonly used in finance, e.g. prospect theory (Kahneman and Tversky, 1979), disappointment aversion (Gul, 1991), habit and dynamic inconsistency such as hyperbolic discounting (Laibson, 1997) or our model of preferences (Definition 1). The long-run risk model built its success solely on its ability to match macroeconomics evidence, meaning microeconomic inferences should be subject to deep scrutiny.

Second, we argue, in line with Epstein et al. (2014), that the magnitudes for the timing premia implied by calibrations of the long-run risk model with standard Epstein-Zin preferences are excessive. There is no direct evidence on the "correct" values of timing premia, by construction a purely theoretical question: we do not know how much an agent who cannot act to modify the consumption stream she will receive would pay to receive early information about it. But it seems somewhat unreasonable that she would be willing to forgo a large fraction of her wealth for earlier resolutions. Even more problematic for the timing premia obtained under the long-run risk calibration of Epstein-Zin preferences, the microeconomic evidence indicates many individuals behave as if they prefer to delay receiving information and avoid early resolutions, even in cases where information can be used to improve outcomes. In the health economics literature for instance, various examples of "information avoidance" are documented, whereby individuals prefer to not be told about their own test results, including concerning life-threatening diseases (e.g. Oster et al., 2013; Persoskie et al., 2014). Golman et al. (2016) provide an extensive survey of such behaviors. Closer to the theoretical framework we use to derive the timing premium, investors' inattention to their own wealth disputes the notion of a strong preference for early resolution of consumption risk; even more so because early information is instrumental in this case (inertia in portfolio allocations comes at a cost, e.g. Brunnermeier and Nagel, 2008; Calvet et al., 2009; Bilias et al., 2010; Andersen et al., 2015). These examples do not, per se, constitute a direct proof of a preference for late resolution of uncertainty, but they appear inconsistent with the high timing premium implied by the existing calibrations of the long-run risk model (usual citations).[9] Further, Karlsson et al., 2009; Alvarez et al., 2012; Sicherman et al., 2016 document that more risk averse investors are also more inattentive. This is inconsistent with the standard model: from Proposition 1 for the case $\gamma = \widetilde{\gamma}$ (Epstein-Zin preferences), the timing premium is strictly increasing in γ, corresponding to a stronger preference for early resolutions of risk, or less inattention for the more risk averse investors. In contrast, our model may be consistent with the evidence: more risk averse investors may also have more strongly horizon-dependent preferences (see Proposition 1 for the respective roles of γ and $\gamma - \widetilde{\gamma}$ in the timing premium).

Though circumstantial, the numerous examples above where agents prefer not to observe early information even when they can act on it make the magnitude of the timing premia under the standard long-run risk model appear unreasonable. A representative agent whose implied preferences appear contrary to commonsense considerations — here on early versus late resolution of uncertainty — raises doubts as to the legitimacy of the long-run risk model, despite its ability to match the macroeconomic evidence on equilibrium asset prices.[10]

In this section, we formally showed that introducing the notion of horizon-dependent risk aversions within recursive preferences (Definition 1) strictly lowers the timing premium, relative to the standard Epstein-Zin model. We further established that preferences for either early or late resolution of uncertainty are compatible with $\gamma > \widetilde{\gamma} > \rho$, i.e. with risk aversions (for any horizon of payoffs) greater than the inverse of the elasticity of intertemporal substitution (Corollary 2).[11] As we mentioned above, this result is key: $\gamma > \rho$ is crucial to the pricing of equilibrium asset pricing moments in both the standard Epstein-Zin model and in ours, as we establish next, when we analyze the equilibrium asset pricing implications of our model. This allows our model to match the macroeconomics evidence without imposing a strong preference for early resolutions of uncertainty, and thus to provide a reasonable answer, grounded in the experimental evidence, to the challenge posed by Epstein et al. (2014).

[9]Golman et al., 2016 discuss other theoretical rationalizations; Andries and Haddad, 2015 propose a model of information aversion that explains investors' inattention in the data.

[10]Aggregation theorems for Epstein and Zin (1989) preferences (Duffie and Lions, 1992) indicate that if most individuals have low or even negative timing premia, so would the marginal, representative, investor who sets prices.

[11]The results of Proposition 1, Corollary 1 and Corollary 2 are established for the two consumption streams — either all revealed in one period or all over time — strictly as in Epstein et al. (2014). However, the result that horizon-dependent risk aversion limits the appeal of early revelations remains valid for other structures of information arrivals. We discuss these extensions in Appendix B.2.

4 Asset prices

The decision to opt for an early resolution of uncertainty is by nature a multi-horizon problem, as the agent chooses how valuable it is to discover all her future risk at the next immediate period, rather than slowly over time. In this multi-horizon problem, introducing a wedge between the immediate risk aversion and the long-horizon risk aversion has a first-order impact on the agent's valuations — as we show in Proposition 1. In contrast, asset prices are set by agents who can, in general, reduce their risk allocation decisions to a repeated one-period problem. When nothing prevents agents from trading every period, prices at equilibrium must be such that the immediate consumption utility loss from investing a marginal amount of wealth today is strictly offset by the expected *next-period* utility gain when evaluating the investment's payoff. This fundamental difference between the static multi-horizon problem of preferences for early or late resolution of uncertainty versus the dynamic one-period problem of equilibrium asset prices is at the core of our analysis, and the reason why horizon-dependent risk aversion can address the puzzle pointed out in Epstein et al. (2014). When the conditions for the one-period set-up are satisfied, they naturally limit the impact horizon-dependent risk aversion can have on equilibrium asset prices without affecting its influence on the timing premium.

We derive the marginal pricing of risk in our model using a standard consumption-based asset pricing framework, in which a representative agent with the horizon-dependent preferences of Definition 1 sets equilibrium prices. All decisions are made in sequential one-period problems, where the no-arbitrage condition is automatically satisfied despite the agent's time inconsistent preferences (see Appendix A.1 for details). This one-period pricing framework is the classical approach.

Since prices are determined by the representative agent's perception of her next period utility and optimization problem, they crucially depend on whether (i) she is aware of her own dynamic inconsistencies and (ii) she can limit her future opportunity sets through commitment. As the representative agent, she cannot commit to refrain from trading every period: we make the stronger assumption that she cannot commit at all. We also assume she is fully sophisticated about the dynamic inconsistencies inherent in her preferences, which turns out to be the most interesting case in terms of asset pricing implications. We study the naive representative agent equilibrium outcomes in Appendix A.3, and find they differ strongly from those under sophistication.[12] As we discuss below in Sections 6 and 7, under standard market conditions the evidence in the data is consistent with a sophisticated representative agent, and not with a naive one.

[12] This contrasts with our results in Section 3, which remain the same for sophisticated and naive agents.

Assuming a sophisticated representative agent with the preferences of Definition 1, who trades and re-optimizes her utility every period and cannot commit to any specific strategy, the object of interest for asset pricing purposes is the stochastic discount factor (SDF). The SDF's derivation is based on the inter-temporal marginal rate of substitution

$$\Pi_{t,t+1} = \frac{dV_t/dW_{t+1}}{dV_t/dC_t},$$

which satisfies the Euler equation, whereby the equilibrium price at time t of a future payoff X_{t+1} is given by $P_t = E_t[\Pi_{t,t+1}X_{t+1}]$.

Proposition 2. *An agent with the horizon-dependent risk aversion preferences of Definition 1 has a one-period stochastic discount factor*

$$\Pi_{t,t+1} = \underbrace{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}}_{\text{(I)}} \times \underbrace{\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma}}_{\text{(II)}} \times \underbrace{\left(\frac{\widetilde{V}_{t+1}}{V_{t+1}}\right)^{1-\rho}}_{\text{(III)}}. \tag{8}$$

The SDF consists of three multiplicative parts. The first term (I) is standard, capturing the inter-temporal substitution between t and $t+1$, and is governed by the time discount factor β and the elasticity of inter-temporal substitution $1/\rho$.

The second term (II) captures the unexpected shocks realized in $t+1$ to consumption in the long-run, i.e. beyond $t+1$. It compares the ex-post realized $t+1$ utility $\widetilde{V}_{t+1}$ to its ex-ante certainty equivalent $E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$; both the comparison as well as the certainty equivalent are evaluated with immediate risk aversion γ. The same term obtains under standard Epstein-Zin preferences with the difference that, in our model, the $t+1$ utility of self t ($\widetilde{V}_{t+1}$) differs from that of self $t+1$ (V_{t+1}).

Finally, the third term (III) captures the dynamic inconsistency in our model by loading on the disagreement between selves t and $t+1$ when evaluating their $t+1$ utilities, given by the ratio $\widetilde{V}_{t+1}/V_{t+1}$.

To interpret what aggregate shocks the three terms (I), (II) and (III) in the stochastic discount factor of Equation (8) price, we analyze a standard Lucas-tree endowment economy with a log-normal consumption process where both the expected growth and uncertainty are time varying. This generalizes the consumption process (5) by adding stochastic volatility, in line with the long-run risk literature (e.g. Bansal and Yaron, 2004; Bansal et al., 2009):

$$
\begin{aligned}
c_{t+1} - c_t &= \mu_c + \phi_c x_t + \alpha_c \sigma_t w_{c,t+1} \\
x_{t+1} &= \nu_x x_t + \alpha_x \sigma_t w_{x,t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \nu_\sigma \left(\sigma_t^2 - \sigma^2 \right) + \alpha_\sigma w_{\sigma,t+1}
\end{aligned}
\tag{9}
$$

For simplicity, we assume that x_t is one dimensional and the three shocks $w_{c,t}$, $w_{x,t}$ and $w_{\sigma,t}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal.[13] Both ν_x and ν_σ are contracting.

Before deriving the pricing of the shocks $\{w_{c,t}, w_{x,t}, w_{\sigma,t}\}$ under horizon-dependent risk aversion, we briefly explain the role they play in the long-run risk model. This allows us to clarify the comparisons we draw later between ours and the classical framework.

The consumption process (9) accounts for time variations in expected consumption growth, through the state variable x_t, consistent with direct evidence in the data (Hansen et al., 2008). Cross-sectional asset pricing returns demonstrate further that shocks to x_t are priced in the data (Hansen et al., 2008; Bryzgalova and Julliard, 2015), capturing in particular the value premium from Fama and French (1993); while the analysis of the macro-announcement premium shows their pricing contributes to a large portion of the market equity premium (55% in Ai and Bansal, 2018, and 80% in Lucca and Moench, 2015, who study a shorter, more recent, time period). This set of evidence provides a foundation for combining expected growth risk in the consumption process (9) with recursive non time-separable preferences such as Epstein-Zin preferences: the long-run risk framework.

The time variations in the volatility σ_t, in consumption process (9) have a separate role to play: though not directly observable in the data, they are necessary to generate time-varying risk premia, and for the model to capture the volatility puzzle (Shiller, 1981).[14]

We now turn to how horizon-dependent risk aversion (Definition 1) affects the pricing of these consumption shocks. To derive closed-form solutions, we again focus on the case $\rho = 1$, a unit elasticity of inter-temporal substitution.[15] From Proposition 2, the variable of interest in our analysis is the ratio between the $t+1$ value of self t ($\widetilde{V}_{t+1}$) and that of self

[13] These assumptions can be generalized. We employ them here to make our results comparable to those of Bansal and Yaron (2004) and Bansal et al. (2009).

[14] Just like the standard Epstein-Zin model, our framework allows for the analysis of additional shocks in the consumption process (9), e.g. jumps. Drechsler and Yaron (2010) show such shocks help capture other features in the data, notably the pricing of variance swaps and their ability to predict market equity returns (Bollerslev et al., 2009).

[15] In Appendix C, we consider $\rho \neq 1$ and the approximation of a rate of time discount close to zero, $\beta \approx 1$. We show our main results remain valid as long as the elasticity of inter-temporal substitution is greater or equal to one ($1/\rho \geq 1$) — a constraint the standard long-run risk model must also satisfy to match asset pricing data.

$t+1$ (V_{t+1}). Taking logs, we obtain:

Lemma 1. *Under the Lucas-tree endowment process* (9) *and* $\rho = 1$,

$$\widetilde{v}_{t+1} - v_{t+1} = \frac{1}{2}\,\beta\,(\gamma - \widetilde{\gamma})\left(\alpha_c^2 + \phi_v^2\alpha_x^2 + \psi_v(\widetilde{\gamma})^2\,\alpha_\sigma^2\right)\sigma_{t+1}^2, \tag{10}$$

where ϕ_v *is independent of both* γ *and* $\widetilde{\gamma}$*, and* $\psi_v(\widetilde{\gamma}) < 0$ *is independent of* γ*:*

$$\phi_v = \frac{\beta\phi_c}{1-\beta\nu_x}, \tag{11}$$

$$\psi_v(\widetilde{\gamma}) = \frac{1}{2}\,\frac{\beta\,(1-\widetilde{\gamma})}{1-\beta\nu_\sigma}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right). \tag{12}$$

Equation (10) reflects that the $t+1$ value of self t ($\widetilde{v}_{t+1}$) and that of self $t+1$ (v_{t+1}) only differ in their $t+1$ valuation of uncertain consumption starting in $t+2$ onwards, which is governed by volatility σ_{t+1}. Self t evaluates this uncertainty with low risk aversion $\widetilde{\gamma}$ while self $t+1$ evaluates it with high risk aversion γ; implying that $\widetilde{v}_{t+1} - v_{t+1}$ is positive, and increasing in $\gamma - \widetilde{\gamma}$ and in the amount of uncertainty driven by volatility σ_{t+1}.

From terms (II) and (III) in Equation (8), horizon-dependent risk aversion affects only the pricing of shocks that correlate with variations in the ratio $\widetilde{V}_t/V_t$, therefore with variations in σ_t. From Lemma 1, we derive the central result:

Proposition 3. *Horizon-dependent risk aversion does not affect the equilibrium risk prices of shocks to consumption levels (immediate consumption shocks and shocks to consumption growth).*

If the agent faced consumption level shocks only, she could anticipate how her future self reoptimizes, and her time inconsistency would not cause additional uncertainty in her one-period decision making. Only unanticipated changes in her intra-temporal decisions, when the quantity of risk varies through time, interact with her dynamic inconsistency to modify risky assets' excess returns compared to the time consistent model. This result crucially hinges on the fact that, in our preference framework, only intra-temporal decisions are time inconsistent: inter-temporal decisions are unchanged from the standard model.

One important implication of Proposition 3 is that the macroeconomic announcement premium described and analyzed in Lucca and Moench (2015) and Ai and Bansal (2018) is the same under standard Epstein-Zin preferences and horizon-dependent risk aversion. Together with Corollary 1 — that horizon-dependent risk aversion preferences with $\widetilde{\gamma} < \gamma$ lower the timing premium compared to the standard Epstein and Zin (1989) model — Proposition 3 establishes that a high macroeconomic announcement premium need not coincide with a preference for early resolution of uncertainty. It shows that our model can qualitatively offer a solution to the timing premium puzzle derived by Epstein et al. (2014) in the constant volatility case.[16] Our analysis contrasts with Ai and Bansal (2018) who formally link high macroeconomic announcement premia with strong preferences for early resolutions of uncertainty.

Let us now turn to the pricing of all shocks, including shocks to volatility σ_t. From Lemma 1 we obtain:

Proposition 4. *Under the Lucas-tree endowment process* (9) *and* $\rho = 1$, *the stochastic discount factor satisfies*

$$\pi_{t,t+1} - E_t[\pi_{t,t+1}] = -\gamma\alpha_c\sigma_t w_{c,t+1} + (1-\gamma)\,\phi_v\alpha_x\sigma_t w_{x,t+1} \\ + (1-\gamma)\,\psi_v(\widetilde{\gamma})\,\alpha_\sigma w_{\sigma,t+1}. \tag{13}$$

The risk free rate is independent of $\widetilde{\gamma}$:

$$r_{f,t} = -\log\beta + \mu_c + \phi_c x_t + \left(\frac{1}{2} - \gamma\right)\alpha_c^2\sigma_t^2 \tag{14}$$

The pricing of the immediate consumption shocks, given by the term $\gamma\alpha_c\sigma_t w_{c,t+1}$ in Equation (13); the pricing of drift shocks, the term $(1-\gamma)\,\phi_v\alpha_x\sigma_t w_{x,t+1}$; as well as the risk-free rate in Equation (14); all depend only on the immediate risk aversion γ, and are unchanged from the standard long-run risk model.[17] In contrast, from Equations (12) and (13), we obtain the formal result:

Corollary 3. *For an agent with horizon-dependent risk aversion,* $\gamma > \widetilde{\gamma}$ *unambiguously lowers the pricing of volatility shocks:*

$$\frac{\psi_v(\widetilde{\gamma})}{\psi_v(\gamma)} = \frac{1-\widetilde{\gamma}}{1-\gamma} < 1. \tag{15}$$

Our model yields a negative price for volatility shocks: $(1-\gamma)\,\psi_v(\widetilde{\gamma})\,\alpha_\sigma w_{\sigma,t+1}$ in Equation (13). Assets with payoffs that covary with aggregate volatility provide valuable insurance, consistent with the existing long-run risk literature and the observed evidence from variance swaps and option straddles returns (see Dew-Becker et al., 2016, and Andries et al., 2016 for recent examples). The volatility shock prices in Equation (13) depend on both the immediate risk aversion γ, and on the longer-horizon one through $\psi_v(\widetilde{\gamma})$:

[16] That it can do so quantitatively under the consumption process (9) with time-varying volatility is in Section 5.

[17] When $\rho \neq 1$, the risk-free rate can depend on $\tilde{\gamma}$, though not risk prices for immediate consumption shocks and drift shocks – see Appendix C.

shocks to volatility make future intra-temporal decisions uncertain and, for this reason, how risky they are depends on horizon-dependent risk aversion. Due to the lower risk aversion $\widetilde{\gamma} < \gamma$, their implied long-run uncertainty does not "feel" as costly, which reduces the value of hedges against volatility shocks; the intuition behind Corollary 3.

Before turning to the quantitative analysis of our model, let us pause to interpret the qualitative implications of our results. First, as discussed above, the pricing of shocks to consumption levels, i.e. to C_{t+1}/C_t and to x_t, allows the standard long-run risk model to match the market equity premium, the macroeconomic announcement premium and the value premium. From Equations (11) and (13), the pricing of these shocks is exactly the same under horizon-dependent risk aversion: the preference model of Definition 1 retains the same ability to match these market premia.

Second, the shocks to consumption growth uncertainty in process (9) allow to obtain time-varying risk premia and explain the market volatility puzzle in the standard long-run risk model. How does the result of Corollary 3 affect the ability of our model, under horizon-dependent risk aversion, to do the same? As Equation (13) shows, time variations in risk premia arise from the pricing of both the immediate consumption shocks and the long-run consumption growth shocks — the terms $\gamma \alpha_c \sigma_t w_{c,t+1}$ and $(1-\gamma)\,\phi_v \alpha_x \sigma_t w_{x,t+1}$ — and *not* through the pricing of shocks to volatility. Variations in the pricing of risk thus remain unchanged by the introduction of horizon-dependent risk aversion with $\widetilde{\gamma} < \gamma$, providing the exact same rationalization of the volatility puzzle.

Third, in calibrations of the consumption process (9), the pricing of shocks to volatility under the Epstein-Zin model also contributes to the equity premia, sometimes to a large extent (e.g. Bansal et al., 2009). Introducing horizon-dependent risk aversion unambiguously reduces the magnitude of their impact. In the extreme case $\widetilde{\gamma} \approx 1$, the pricing of volatility shocks goes to zero (Corollary 3). To assess our model, we appeal to the evidence concerning the macroeconomic announcement premium and its share of the equity premium (80% in Lucca and Moench, 2015, 55% in Ai and Bansal, 2018). As driven exclusively by the pricing of shocks to consumption levels, the magnitude of this share provides direct evidence on the relative values of the risk prices of immediate and long-term consumption shocks versus volatility shocks; and, in turn, on how small the long-run risk aversion $\widetilde{\gamma}$ of the representative agent must be compared to her immediate risk aversion γ. This allows us to discipline the calibration of our model in the next section.

The results of Lemma 1 and Proposition 4 thus show that our framework with horizon-dependent risk aversion can qualitatively match the same asset pricing moments as the standard long-run risk framework. At the same time, Corollary 1 in Section 3 establishes $\widetilde{\gamma} < \gamma$ strictly lowers the timing premium. But how quickly does $\widetilde{\gamma} < \gamma$ affect the represen-

Table 1: Calibration.

Process	Parameters			
c_t	$\mu_c = 0.15\%$	$\phi_c = 1$	$\alpha_c = 1$	
x_t		$\nu_x = 0.975$	$\alpha_x = 0.0038$	
σ_t	$\sigma = 0.72\%$	$\nu_\sigma = 0.999$	$\alpha_\sigma = 0.00028\%$	
d_t	$\mu_d = 0.15\%$	$\phi_d = 2.5$	$\alpha_d = 5.96$	$\chi = 2.6$

tative agent's preference for early resolutions of uncertainty? Does the wedge between $\widetilde{\gamma}$ and γ that captures the share of the macroeconomic announcement premium in the market equity premium implies a "reasonable" level for the timing premium? We now turn to this quantitative assessment.

5 Model calibration

The consumption processes (9) is calibrated, Table 1, strictly as in Bansal et al. (2009).[18] This allows us to highlight how the horizon-dependent risk aversion preference model of Definition 1, rather than changes in the calibration for the endowment process, affects prices. In line with Bansal et al. (2009), we use $\beta = 0.9989$ for the monthly rate of time discount, and $\gamma = 10$ for the immediate risk aversion. The elasticity of inter-temporal substitution is 1 throughout (see Appendix C for $\rho \neq 1$ results).

We first study the quantitative implications of horizon-dependent risk aversion on the agent's willingness to pay for an early resolution of all consumption uncertainty. Figure 2 plots the timing premium for both horizon-dependent risk aversion and for standard Epstein-Zin preferences. As pointed out by Epstein et al. (2014), calibrating a standard Epstein-Zin representative agent to match asset pricing moments implies an extremely high willingness to pay for early resolution — more than 80% of the value of her expected consumption under the calibration of Bansal et al. (2009).[19] Depending on $\widetilde{\gamma}$, an agent with horizon-dependent risk aversion can have a significantly lower willingness to pay for an early resolution. In fact, for delayed risk aversion $\widetilde{\gamma} \leq 4.42$, the agent with our preference model prefers a late resolution of risk (negative timing premium). Figure 2 illustrates the first-order impact that horizon-dependent risk aversion has on the timing premium, and the potential for our model to solve the timing premium puzzle of Epstein et al. (2014).

[18]Table 1 also provides the calibration of the dividend growth processes (16) in Sections 6 and 7.

[19]In the calibration of Bansal and Yaron (2004) with stochastic volatility, but a lesser persistence in the volatility shocks, the timing premium is "just" 30%.

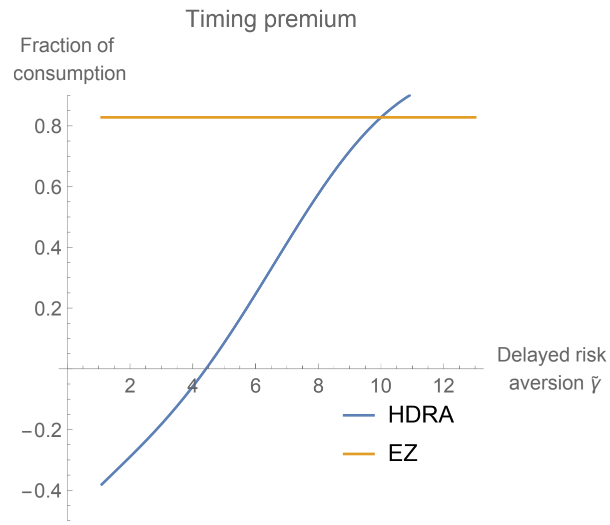

Figure 2: Effect of horizon-dependent risk aversion (HDRA) on willingness to pay for early resolution of uncertainty (timing premium), compared to Epstein-Zin preferences (EZ) with $\gamma = 10$.

The wedge between the immediate risk aversion γ and the long horizon risk aversion $\widetilde{\gamma}$, by reducing the pricing of shocks to volatility (see Corollary 3), also impacts the equity premium. And crucially, because $\gamma \neq \widetilde{\gamma}$ affects only the pricing of volatility shocks in the consumption process (9), but not that of shocks to both immediate and long-run consumption growth, increases in the wedge $\gamma - \widetilde{\gamma}$ lead monotonically to similar increases in the share of the equity premium that comes from the pricing of consumption level shocks, measured in the data by the macroeconomic announcement premium. We quantify this relation in Table 2. Under the calibration of the consumption process (9) in Bansal et al. (2009) (Table 1) and given the data estimates in Lucca and Moench (2015), we calibrate the horizon-dependent risk aversion model of Definition 1 with $\gamma = 10$ and $\widetilde{\gamma} = 5$.[20]

These levels of risk aversions, immediate and long-term, that allow the model to match the asset pricing moments, imply a timing premium of 8.5%, corresponding to the share of her lifetime consumption the representative investor would be willing to pay to observe all her future consumption shocks next period. The introspection as well as the available evidence we discuss in Section 3, circumstantial as it may be, indicate this is a considerably more reasonable value than the 30% obtained for the standard Epstein-Zin model in the calibration of Bansal and Yaron (2004), and of course the 80% timing premium under

[20]The calibration of the standard Epstein and Zin (1989) model in Bansal et al. (2009) matches the macroeconomic announcement premium in Ai and Bansal (2018) but underestimates it relative to Lucca and Moench (2015); ours matches Lucca and Moench (2015). Both calibrations generate equity premia consistent with the data.

Table 2: Equity and macro-announcement premia versus timing premium.

		Market returns	Macro-announcement share	Timing premium
Data		7.66%	55% – 80%	–
Calibration	$\widetilde{\gamma} \approx 1$	5.44%	97%	–39%
	$\widetilde{\gamma} = 2$	5.81%	92%	–29%
	$\widetilde{\gamma} = 3$	6.17%	87%	–18%
	$\widetilde{\gamma} = 4.42$	6.70%	81%	0
	$\widetilde{\gamma} = 5$	6.91%	77%	8.5%
	$\widetilde{\gamma} = 7$	7.64%	71%	38%
	$\widetilde{\gamma} = \gamma = 10$	8.75%	54%	83%

Annualized returns under $\gamma = 10$, $\rho = 1$ and the calibration of Table 1; Data is from Bansal et al. (2009), annual 1930–2008; and from Lucca and Moench (2015); Ai and Bansal (2018).

Bansal et al. (2009).

The results we derive in Table 2 demonstrate the success of the horizon-dependent risk aversion model of Definition 1: under our framework, calibrating asset pricing moments no longer precludes a reasonable preference for early resolutions of uncertainty.

6 Term structure implications

Several papers (van Binsbergen et al., 2012; van Binsbergen and Koijen, 2016; Giglio et al., 2014; Dew-Becker et al., 2016; Andries et al., 2016) provide empirical evidence of downward sloping term structures of expected excess returns for various types of risk; a puzzle for the long-run risk model. Bansal et al. (2019) on the other hand, find that the term structure for equity risk premia is upward sloping on average, but sharply downward sloping during the 2007–2009 financial crisis (see also van Binsbergen et al., 2013); sufficiently so to drive the downward sloping averages derived over short periods in van Binsbergen et al. (2012) and van Binsbergen and Koijen (2016). Finally, Gormsen (2016) further indicates that low price-dividend ratios correspond to more *upward* sloping term structures of expected excess returns. In the model of Section 4, such low price-dividend ratios are driven, in particular, by periods of high volatility.

The preference model of Definition 1 formalizes the notion there may be non-trivial term structures of risk aversions. Introducing the concept of risk aversions that decrease with the horizon of payoff uncertainty appears perfectly tailored to obtain a downward sloping term structure of expected returns. If an asset delivers payoffs at a long horizon, for which the investor has low risk aversion, it seems immediate that this asset should yield a lower price of risk than an asset with payoffs at a short horizon over which the investor is very risk averse. This simple narrative of a one-for-one relation between the term structure of risk aversions and that of risk prices does not take into account, however, that *all* assets are priced at next period horizon, no matter when payoffs are paid, if trading can occur every period, the one period pricing model. We nonetheless show below the term-structure of risk aversion does affect the term structures of risk prices in that framework.

To formally derive the implications of our model with respect to the evidence in van Binsbergen et al. (2012); van Binsbergen and Koijen (2016); Gormsen (2016); Bansal et al. (2019), we analyze the expected excess returns on dividend strip futures.[21] In line with the long-run risk literature, we assume that dividends have log-normal growth:

$$d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t w_{c,t+1} + \alpha_d \sigma_t w_{d,t+1}, \tag{16}$$

where the shocks $w_{d,t}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal to the consumption shocks $w_{c,t}$, $w_{x,t}$ and $w_{\sigma,t}$; ϕ_d captures the link between the mean consumption growth and the mean dividend growth; χ the correlation between immediate consumption and dividend shocks in the business cycle.[22]

Formally deriving the returns on dividend strip futures, we obtain the following:

Proposition 5. *Under the Lucas-tree endowment process (9) and the dividend process (16), the slopes of the term structures of dividend strips' expected returns and expected excess returns*

- *are flatter when $\gamma > \widetilde{\gamma}$, but of the same sign, than under the standard model $\gamma = \widetilde{\gamma}$;*
- *vary over time with volatility, without changing sign: they are steeper when σ_t is high.*

Proposition 5 establishes that horizon-dependent risk aversion affects the pricing of equity assets, in levels and term structures, when volatility is time varying. However it also makes clear horizon-dependent risk aversions and horizon-dependent excess returns need not be tightly linked under the standard fully dynamic one-period asset pricing framework. In the calibrations of the long-run risk model with Epstein-Zin preferences (Bansal and Yaron, 2004; Bansal et al., 2009), the term structure of expected returns of dividend strip futures is upward sloping. Proposition 5 states that relaxing the model, as we do,

[21] Under a dividend strip futures contract with horizon h at time t, the dividends paid on an index over the year $t + h$ will be exchanged at time $t + h$ against a fixed payment that is set at time t. The dividend strip futures were first introduced on the Eurostoxx 50 in 2008. A similar analysis on the term structure of risk-free zero-coupon bond yields can be found in Appendix B.5.

[22] Once again, these assumptions can be generalized, but they are those of Bansal and Yaron (2004) and Bansal et al. (2009).

Figure 3: Term structure of dividend strips expected returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.

to let the long-run risk aversion $\widetilde{\gamma}$ be lower than the immediate risk aversion γ does not change the sign of the slope. Figure 3 depicts the unconditional expected one-month returns of dividend strips (annualized) under horizon-dependent risk aversion with $\gamma = 10$ and $\widetilde{\gamma} = 5$, as per the calibration of Section 5, as well as under standard Epstein-Zin preferences with $\gamma = 10$.[23]

The term structures we obtain are consistent with the most recent evidence in Bansal et al. (2019): outside of the crisis years 2007–2009, these authors find increasing term structures over the first 7-year horizons for dividend strips' expected returns and Sharpe ratios, with just slightly lower levels on the whole index, suggesting a flattening or very slight decrease beyond a given horizon.[24] In addition, the second, time series, result of Proposition 5 whereby a higher σ_t leads to a steeper slope is supported by the empirical evidence in Gormsen (2016).

However, the one-period pricing framework implies upward sloping term structures at all points in time in our calibrated model: Proposition 5 holds for both conditional and unconditional term structures. This is inconsistent with Bansal et al. (2019), who find the term structure of dividend strip futures' expected returns was sharply downward sloping during the financial crisis (December 2007–June 2009) (see also van Binsbergen et al., 2013); and in direct contradiction with van Binsbergen et al. (2012) and van Binsbergen

[23]Figures for expected returns, expected excess returns and Sharpe ratios for different calibrations of $\widetilde{\gamma}$ are provided in Appendix D.

[24]Their term structure moments are obtained over the short 2005–2017 period, so we do not try to match their levels, just their overall shapes.

and Koijen (2016).

Outside the financial crisis of 2007–2009, the horizon-dependent risk aversion model matches the term structure of expected returns on equity, and its variations over time. We turn to term-structure behaviors during crisis periods next, and show that our preference model offers a rationalization of the slope reversal over 2007–2009 under the reasonable assumption that the financial crisis was accompanied by a drop in liquidity.

7 Asset pricing under liquidity constraints

Our analysis so far rests on the assumption of a one-period framework: the stochastic discount factor derived in Proposition 2 hinges on the agent's option to retrade every period. This standard modeling choice is appropriate for a representative agent who determines the equilibrium asset pricing moments we consider in Sections 4, 5 and 6. However, for dynamically inconsistent preferences such as ours, partial equilibrium outcomes under the one-period framework may depart from those under lower trading frequencies, so this modeling choice is not innocuous. We investigate how beak-downs in the one-period trading option affect equilibrium prices in this section, focusing on the term structure of expected returns.

Our choice to analyze term structures of returns under low trading frequencies is grounded in the evidence. The specific period in which the term structure of dividend strip excess returns behaves "unusually", as downward sloping, is the financial crisis of 2007-2009 (Bansal et al., 2019; van Binsbergen et al., 2013), which core feature is that it was accompanied by a dramatic drop in market liquidity (see e.g.Brunnermeier, 2009 and Muir, 2016).[25] We interpret illiquidity as a departure from the one-period trading paradigm. This can be exogenously imposed, e.g. through infrequent trading opportunities, or endogenously optimal, e.g. when buy-and-hold strategies help avoid rising trading costs. The literature on asset prices with liquidity risk points out the additional risk premium directly attributable to illiquidity (e.g. Acharya and Pedersen, 2005; Lee, 2011; Muir, 2016).[26] Our approach here is complementary since our focus is on the slope of the term structure of risk premia, not on its level.

Proposition 6. *Under the horizon-dependent risk aversion preferences of Definition 1 and* $\rho = 1$,

[25]We note also that one asset class, the medium-to-long horizon variance swaps and option straddles, documented to have downward sloping term structures of returns in *and* out of crisis (Dew-Becker et al., 2016; Andries et al., 2016) is characterized by its lack of liquidity.

[26]See also Duffie (2010) and Tirole (2011) for surveys of the literature on liquidity.

the stochastic discount factor for a buy-and-hold strategy with horizon h is given by[27]

$$\Pi_{t,t+h}^{\textit{buy-and-hold}} = \beta^h \left(\frac{C_{t+h}}{C_t}\right)^{-1} \times \underbrace{\frac{\widetilde{V}_{t+1}^{1-\gamma}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]}}_{\textit{high } \gamma} \times \underbrace{\frac{\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}\right]} \times \cdots \times \frac{\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}}{E_{t+h-1}\left[\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}\right]}}_{\textit{low } \widetilde{\gamma}}.$$

Compared to the one-period investor, with implicit risk aversion γ for future shocks at all horizons, the buy-and-hold agent who assumes no retrading at intermediate dates evaluates the shocks between $t+2$ and $t+h$ with lower risk aversion $\widetilde{\gamma}$ — suggesting a higher willingness to pay for risky assets and therefore lower expected returns than under frequent intermediate trading.

Proposition 6 concerns individuals' optimization problems. Because it implies investors have asset-specific discount factors, Proposition 6 implicitly makes several assumptions: that it is exogenously imposed or internally optimal for investors to choose buy-and-hold strategies when liquidity falls (e.g. because of higher transaction costs); and that they can commit to these strategies over time.[28] We do not, on the other hand, make any assumption about whether the law of one price and no-arbitrage pricing break down in times of liquidity crunches: equilibrium prices may still be set by one strictly positive stochastic discount factor (that may not resemble those of Proposition 6). A growing literature establishes that households' demand influences general equilibrium outcomes, in particular via intermediaries holdings (He and Krishnamurthy, 2013; Koijen and Yogo, 2015; Haddad and Muir, 2017), and therefore affects prices. We appeal to these results to derive the following:

Proposition 7. *Under consumption process* (9) *and dividend risk* (16), *buy-and-hold investors with the horizon-dependent risk aversion preferences of Definition* 1 *have a downward impact on the slope of the term structure of dividend strips' expected excess returns. The downward pressure on the term structure is greater when the economy is more volatile, i.e.* σ_t *is high.*

Proposition 7 derives from the intuition, explored and discussed in previous sections, that the less dynamic choices are, the more time inconsistencies in the agents' preferences can affect equilibrium outcomes. In particular, for buy-and-hold investors, the relation between horizon-dependent risk aversions and horizon-dependent risk prices becomes tighter. This result proposes an explanation for the evidence in Bansal et al. (2019) that the term structure of expected excess returns was sharply downward sloping during the

[27] The more general case with $\rho \neq 1$ is provided in Appendix A.2.

[28] We show in Appendix A.3 that naive agents in the one-period standard framework behave as the buy-and-hold investors in Proposition 6, making the naive investors assumption inconsistent with the observed upward sloping term structures in "normal" markets (see Proposition 7 below).

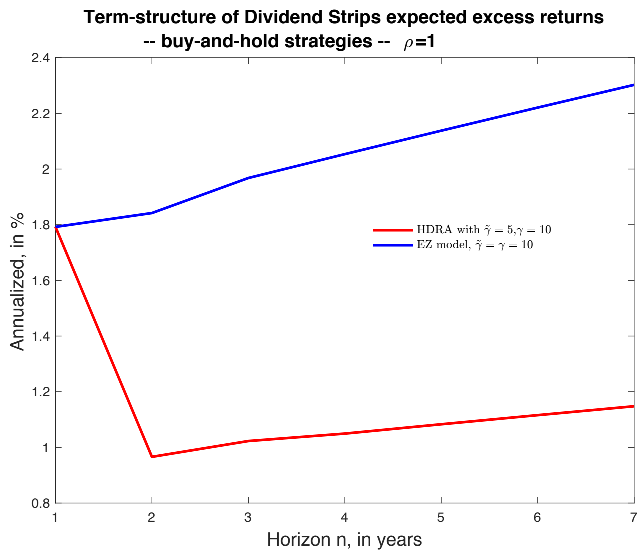

Figure 4: Term structure of dividend strips expected excess returns for buy-and-hold strategies under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1; case with σ_t two standard deviations above average.

financial crisis of 2007–2009. That prices over that period may have been driven, at least partly, by buy-and-hold investors, seems realistic (see our discussion above and e.g. Brunnermeier, 2009). Proposition 7 also sheds light on differences in term structures across markets, suggesting a more downward sloping term structure in markets with less liquidity and/or with longer trading horizons — in line with the empirical evidence in Dew-Becker et al. (2016); Andries et al. (2016) on medium-to-long horizon variance swaps and option straddles. Empirical work estimating risk prices in option markets segmented by asset maturities shows these prices correspond to downward sloping risk aversions consistent with our model (Andries et al., 2016; Lazarus, 2018). We note these assets did not display term-structure slope variations during the financial crisis, inconsistent with a simple consumption process regime shift narrative.

Figure 4 illustrates the conditional expected returns of dividend futures under the assumption new buy-and-hold investors enter the market every period and entirely determine equilibrium prices, in the calibration of Section 5. Our model successfully generates the sharply downward sloping term structure observed during the financial crisis of 2007–2009, whereas the standard Epstein-Zin model fails unambiguously.

As Bansal et al. (2019) point out, bid-ask spreads during the financial crisis were so large as to make estimates of the one-period returns very inaccurate. Our quantitative results in Figure 4 are therefore mostly illustrative. Moreover, they are derived in the corner case where buy-and-hold investors fully set prices, an extreme assumption. Proposition 7 nonetheless suggests that horizon-dependent risk aversion preferences provide a potential rationalization for the term structure slope reversal during the financial crisis of 2007 – 2009; one that does not require but could complement regime shift process adjustments in consumption and dividend growth over sharp economic downturns.

8 Conclusion

Calibrations of the long-run risk model (Bansal and Yaron, 2004; Bansal et al., 2009) are difficult to reconcile with the microeconomic foundations of the preferences they employ. Epstein et al. (2014) point out they imply a willingness to pay for earlier resolutions of uncertainty that defies both observed behaviors in the data and the introspection. We show that relaxing the restriction of Epstein and Zin (1989) that risk preferences be constant across horizons makes it possible to retain the desirable pricing properties of the long-run risk model, including the matching of the equity premium and of the macroeconomic announcement premium, and at the same time obtain reasonable implications for the timing of the resolution of uncertainty.

Established equilibrium asset pricing models have also been criticized because they make counterfactual predictions about the term structure of risk prices (e.g., van Binsbergen et al., 2012, 2013; van Binsbergen and Koijen, 2016; Bansal et al., 2019). We show horizon-dependent risk aversion preferences formally imply term structures of expected returns in line with the evidence: the unconditional upward slope but also its conditional time variations — higher slope under higher volatility and the slope reversal in liquidity crises and for illiquid markets.

We conclude that formalizing a model where risk aversion is higher at short-horizons than long-horizons, consistent with the experimental evidence, provides a useful new tool for asset pricing and macro-finance. We focused our attention on applications to finance but the tractability of this model makes it suitable to analyze features of other markets, such as housing, where agents are typically locked in over long horizons, or health decisions, where attitudes towards risk and time inconsistencies are key.

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Appendix (For online publication)

A Derivations under general sequence of risk aversions

Let $\{\gamma_h\}_{h\geq 1}$ be a decreasing sequence representing risk aversion at horizon h. In period t, the agent evaluates a consumption stream starting in period $t+h$ by

$$V_{t,t+h} = \left((1-\beta)\, C_{t+h}^{1-\rho} + \beta E_{t+h} \left[V_{t,t+h+1}^{1-\gamma_{h+1}} \right]^{\frac{1-\rho}{1-\gamma_{h+1}}} \right)^{\frac{1}{1-\rho}} \quad \text{for all} \quad h \geq 0. \tag{17}$$

The agent's utility in period t is given by setting $h=0$ in (17) which we denote by $V_t \equiv V_{t,t}$ for all t:

$$V_t = \left((1-\beta)\, C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}}$$

As in the Epstein-Zin model, utility V_t depends on deterministic current consumption C_t and a certainty equivalent $E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1}{1-\gamma_1}}$ of uncertain continuation values $V_{t,t+1}$, where the aggregation of the two periods occurs with constant elasticity of inter-temporal substitution given by $1/\rho$, regardless of the horizon h. However, in contrast to the Epstein-Zin model, the certainty equivalent of consumption starting at $t+1$ is calculated with relative risk aversion γ_1, wherein the certainty equivalent of consumption starting at $t+2$ is calculated with relative risk aversion γ_2, and so on. This is the concept of horizon-dependent risk aversion applied to the nested valuation of certainty equivalents, as in the Epstein-Zin model, but with relative risk aversion γ_h for the certainty equivalent formed at horizon h. Our model therefore nests the Epstein-Zin model if we set $\gamma_h = \gamma$ for all h, which, in turn, nests the standard time-separable model for $\gamma = \rho$.

An interesting question is the possibility to axiomatize the horizon-dependent risk aversion preferences we propose. Our dynamic model builds on the functional form of Epstein and Zin (1989) which captures non-time-separable preferences of the form axiomatized by Kreps and Porteus (1978). However, our generalization of the Epstein-Zin model explicitly violates Axiom 3.1 (temporal consistency) of Kreps and Porteus (1978) which is necessary for the recursive structure. In contrast to Epstein-Zin, the preference of our model captured by $V_t \equiv V_{t,t}$ is *not* recursive since $V_{t+1} \equiv V_{t+1,t+1}$ does not recur in the definition of V_t.

In order to derive the closed-form solution for $V_t \equiv V_{t,t}$, we assume that risk aversion is decreasing until some horizon H and constant thereafter, $\gamma_h > \gamma_{h+1}$ for $h < H$ and $\gamma_h = \widetilde{\gamma}$ for $h \geq H$. Starting with $V_{t,t+H}$, our model then corresponds to the standard Epstein-Zin recursion with risk aversion $\widetilde{\gamma}$ for which we can use the standard solution. Determining V_t then is just a matter of solving backwards.

A.1 Stochastic discount factor

We present the derivation of the stochastic discount factor with a general sequence of risk aversions $\{\gamma_h\}_{h\geq 1}$. The equations simplify to the ones in the main text by setting $\gamma_1 = \gamma$ and $\gamma_h = \widetilde{\gamma}$ for $h \geq 2$.

Proof of Proposition 2. This appendix derives the stochastic discount factor of our dynamic model using an approach similar to the one used by Luttmer and Mariotti (2003) for dynamic inconsistency due to non-geometric discounting. In every period t the agent chooses consumption C_t for the current period and state-contingent levels of wealth $\{W_{t+1,s}\}$ for the next period to maximize current utility V_t subject to a budget constraint *and* anticipating optimal choice C^*_{t+h} in all following periods ($h \geq 1$):

$$\max_{C_t, \{W_{t+1}\}} \left((1-\beta)\, C_t^{1-\rho} + \beta E_t \left[\left(V^*_{t,t+1}\right)^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}}$$

$$\text{s.t.} \quad \Pi_t C_t + E_t[\Pi_{t+1} W_{t+1}] \leq \Pi_t W_t$$

$$V^*_{t,t+h} = \left((1-\beta) \left(C^*_{t+h}\right)^{1-\rho} + \beta E_{t+h} \left[\left(V^*_{t,t+h+1}\right)^{1-\gamma_{h+1}} \right]^{\frac{1-\rho}{1-\gamma_{h+1}}} \right)^{\frac{1}{1-\rho}} \quad \text{for all} \quad h \geq 1.$$

Denoting by λ_t the Lagrange multiplier on the budget constraint for the period-t problem, the first order conditions are:[29]

- For C_t:

$$\left((1-\beta)\, C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} (1-\beta)\, C_t^{-\rho} = \lambda_t.$$

- For each $W_{t+1,s}$:

$$\frac{1}{1-\rho} \left((1-\beta)\, C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} \beta \frac{d}{dW_{t+1,s}} \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}}$$
$$= \Pr[t+1,s]\, \frac{\Pi_{t+1,s}}{\Pi_t} \lambda_t.$$

Combining the two, we get an initial equation for the SDF:

$$\frac{\Pi_{t+1,s}}{\Pi_t} = \beta \frac{\frac{1}{1-\rho} \frac{1}{\Pr[t+1,s]} \frac{d}{dW_{t+1,s}} E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}}}{1} \frac{1}{(1-\beta)\, C_t^{-\rho}}. \tag{18}$$

The agent in state s at $t+1$ maximizes

$$\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left(V^*_{t+1,s,t+2}\right)^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}}$$

and has the analogous first order condition for $C_{t+1,s}$:

$$\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} (1-\beta)\, C_{t+1,s}^{-\rho} = \lambda_{t+1,s}.$$

[29] For notational ease we drop the star from all Cs and Vs in the following optimality conditions but it should be kept in mind that all consumption values are the ones optimally chosen by the corresponding self.

The Lagrange multiplier $\lambda_{t+1,s}$ is equal to the marginal utility of an extra unit of wealth in state $t+1, s$:

$$\lambda_{t+1,s} = \frac{1}{1-\rho}\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}\left[V_{t+1,s,t+2}^{1-\gamma_1}\right]^{\frac{1-\rho}{1-\gamma_1}}\right)^{\frac{1}{1-\rho}-1}$$
$$\times \frac{d}{dW_{t+1,s}}\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}\left[V_{t+1,s,t+2}^{1-\gamma_1}\right]^{\frac{1-\rho}{1-\gamma_1}}\right).$$

Eliminating the Lagrange multiplier $\lambda_{t+1,s}$ and combining with the initial Equation (18) for the SDF, we get:

$$\frac{\Pi_{t+1,s}}{\Pi_t} = \beta \frac{\frac{1}{\Pr[t+1,s]}\frac{d}{dW_{t+1,s}} E_t\left[V_{t,t+1}^{1-\gamma_1}\right]^{\frac{1-\rho}{1-\gamma_1}}}{\frac{d}{dW_{t+1,s}}\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}\left[V_{t+1,s,t+2}^{1-\gamma_1}\right]^{\frac{1-\rho}{1-\gamma_1}}\right)}\left(\frac{C_{t+1,s}}{C_t}\right)^{-\rho}.$$

Expanding the V expressions, we can proceed with the differentiation in the numerator:

$$\begin{aligned}\frac{\Pi_{t+1,s}}{\Pi_t} = E_t&\left[\left((1-\beta)\, C_{t+1}^{1-\rho} + \beta E_{t+1}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)^{\frac{1-\gamma_1}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_1}-1} \\ &\times\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)^{\frac{1-\gamma_1}{1-\rho}-1} \\ &\quad\times \beta\frac{\frac{d}{dW_{t+1,s}}\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)}{\frac{d}{dW_{t+1,s}}\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\gamma_1}}\right)}\left(\frac{C_{t+1,s}}{C_t}\right)^{-\rho}.\end{aligned} \tag{19}$$

For Markov consumption $C = \phi W$, we can divide by $C_{t+1,s}$ and solve both differentiations:

- For the numerator:

$$\begin{aligned}&\frac{d}{dW_{t+1,s}}\left((1-\beta)\, C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}\left[\left((1-\beta)\, C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\gamma_3}}\right)^{\frac{1-\gamma_2}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_2}}\right) \\ &= \left((1-\beta)\, 1 + \beta E_{t+1,s}\left[\left((1-\beta)\left(\frac{C_{t+2}}{C_{t+1,s}}\right)^{1-\rho} + \beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\gamma_3}}\right)^{\frac{1-\gamma_2}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_2}}\right) \\ &\quad\times \phi_{t+1,s}^{1-\rho} W_{t+1,s}^{-\rho}.\end{aligned}$$

- For the denominator:

$$\frac{d}{dW_{t+1,s}}\left((1-\beta)\,C_{t+1,s}^{1-\rho}+\beta E_{t+1,s}\left[\left((1-\beta)\,C_{t+2}^{1-\rho}+\beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)^{\frac{1-\gamma_1}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_1}}\right)$$
$$=\left((1-\beta)\,1+\beta E_{t+1,s}\left[\left((1-\beta)\left(\frac{C_{t+2}}{C_{t+1,s}}\right)^{1-\rho}+\beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)^{\frac{1-\gamma_1}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_1}}\right)$$
$$\times\,\phi_{t+1,s}^{1-\rho}W_{t+1,s}^{-\rho}.$$

Substituting these into Equation (19) and canceling we get:

$$\frac{\Pi_{t+1,s}}{\Pi_t}=\frac{(1-\beta)\,C_{t+1,s}^{1-\rho}+\beta E_{t+1,s}\left[\left((1-\beta)\,C_{t+2}^{1-\rho}+\beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\gamma_3}}\right)^{\frac{1-\gamma_2}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_2}}}{(1-\beta)\,C_{t+1,s}^{1-\rho}+\beta E_{t+1,s}\left[\left((1-\beta)\,C_{t+2}^{1-\rho}+\beta E_{t+2}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)^{\frac{1-\gamma_1}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma_1}}}$$
$$\times\,\beta\left(\frac{C_{t+1,s}}{C_t}\right)^{-\rho}\left(\frac{(1-\beta)\,C_{t+1,s}^{1-\rho}+\beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\gamma_2}}}{E_t\left[\left((1-\beta)\,C_{t+1,s}^{1-\rho}+\beta E_{t+1}[\dots]^{\frac{1-\rho}{1-\gamma_2}}\right)^{\frac{1-\gamma_1}{1-\rho}}\right]}\right)^{\rho-\gamma_1}.$$

Simplifying and cleaning up notation, we arrive at

$$\Pi_{t,t+1}=\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{V_{t,t+1}}{E_t\left[V_{t,t+1}^{1-\gamma_1}\right]^{\frac{1}{1-\gamma_1}}}\right)^{\rho-\gamma_1}\left(\frac{V_{t,t+1}}{V_{t+1}}\right)^{1-\rho},$$

as stated in the text. □

A.2 Stochastic discount factor — illiquid markets

To derive the h-period ahead stochastic discount factor, we use the inter-temporal marginal rate of substitution

$$\Pi_{t,t+h}=\frac{dV_t/dW_{t+h}}{dV_t/dC_t}$$

where

$$\frac{dV_t}{dW_{t+h}}=\frac{dV_t}{dV_{t,t+h}}\times\frac{dV_{t,t+h}}{dW_{t+h}}$$
$$=\frac{dV_t}{dV_{t,t+1}}\times\prod_{\tau=1}^{h-1}\frac{dV_{t,t+\tau}}{dV_{t,t+\tau+1}}\times\frac{dV_{t,t+h}}{dW_{t+h}}.$$

Due to the homotheticity of our preferences, we can rely on the fact that both $V_{t,t+h}$ and V_{t+h} are homogeneous of degree one which implies that

$$\frac{dV_{t,t+h}/dW_{t+h}}{dV_{t+h}/dW_{t+h}} = \frac{V_{t,t+h}}{V_{t+h}}.$$

This allows us to derive the h-period SDF $\Pi_{t,t+h}$ as

$$\Pi_{t,t+h} = \beta^h \left(\frac{C_{t+h}}{C_t}\right)^{-\rho} \left(\frac{V_{t,t+h}}{V_{t+h}}\right)^{1-\rho} \prod_{\tau=1}^{h} \left(\frac{V_{t,t+\tau}}{E_{t+\tau-1}\left[V_{t,t+\tau}^{1-\gamma_\tau}\right]^{\frac{1}{1-\gamma_\tau}}}\right)^{\rho-\gamma_\tau}.$$

A.3 Naive investors

In our analysis so far, we assumed agents are self-aware about their own dynamic inconsistencies. If our agent is naive about it instead, she wrongly assumes she will optimize on $V_{t,t+h}$ instead of V_{t+h} for all $h \geq 1$. In particular, the envelope conditions at $t+1$ applies to $V_{t,t+1}$ in her one-period SDF, which becomes:

$$\Pi_{t,t+1}^{\text{naive}} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \left(\frac{V_{t,t+1}}{E_t\left[V_{t,t+1}^{1-\gamma_1}\right]^{\frac{1}{1-\gamma_1}}}\right)^{\rho-\gamma_1}$$

The following one-period SDFs for $h \geq 1$ are then given by:

$$\Pi_{t+h,t+h+1}^{\text{naive}} = \beta \left(\frac{C_{t+h+1}}{C_{t+h}}\right)^{-\rho} \left(\frac{V_{t,t+h+1}}{E_{t+h}\left[V_{t,t+h+1}^{1-\gamma_{h+1}}\right]^{\frac{1}{1-\gamma_{h+1}}}}\right)^{\rho-\gamma_{h+1}}$$

When $\rho = 1$, naive agents behave as the buy-and-hold investors in Proposition 6 :

$$\Pi_{t,t+1}^{\text{naive}} \times \cdots \times \Pi_{t+h-1,t+h}^{\text{naive}} \,|_{\rho=1} = \Pi_{t,t+h}^{\text{buy-and-hold}} \,|_{\rho=1}.$$

B Exact solutions for $\rho = 1$

This appendix presents the exact solutions derived for unit elasticity of inter-temporal substitution, $1/\rho = 1$, and log-normal uncertainty. Denoting logs by lowercase letters, our general model (17) becomes

$$v_t = (1-\beta)\, c_t + \beta \left(E_t[v_{t,t+1}] + \frac{1}{2}(1-\gamma_1)\, \mathrm{var}_t(v_{t,t+1})\right), \tag{20}$$

with the continuation value $v_{t,t+1}$ satisfying the recursion

$$v_{t,t+h} = (1-\beta)\, c_{t+h} + \beta \left(E_{t+1}[v_{t,t+h+1}] + \frac{1}{2}(1-\gamma_{h+1})\, \mathrm{var}_{t+1}(v_{t,t+h+1})\right).$$

B.1 Valuation of risk and temporal resolution

Proof of Proposition 1. Starting at horizon $t+1$, Equation (20) corresponds to the standard recursion

$$\widetilde{v}_{t+1} = (1-\beta)\, c_{t+1} + \frac{\beta}{1-\widetilde{\gamma}} \log(E_{t+1}[\exp\left((1-\widetilde{\gamma})\, \widetilde{v}_{t+2}\right)]).$$

If consumption follows process (5), guess and verify that the solution to the recursion satisfies

$$\widetilde{v}_t - c_t = \widetilde{\mu}_v + \widetilde{\phi}_v x_t.$$

Substituting in and matching coefficients yields

$$\widetilde{v}_t - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\frac{\beta\left(1-\widetilde{\gamma}\right)}{1-\beta}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2.$$

From the perspective of period t,

$$v_t = \left(1-\beta\right)c_t + \frac{\beta}{1-\gamma}\log(E_t[\exp\left(\left(1-\gamma\right)\widetilde{v}_{t+1}\right)])$$

and

$$v_t - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\frac{\beta}{1-\beta}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2\left(\left(1-\gamma\right)+\beta\left(\gamma-\widetilde{\gamma}\right)\right),$$

as stated in the text.

If all risk is resolved at $t+1$, log continuation utility $v^*_{t,t+1}$ is given by

$$\begin{aligned} v^*_{t+1} &= \left(1-\beta\right)c_{t+1} + \beta\Big(\left(1-\beta\right)c_{t+2} + \beta\left(\left(1-\beta\right)c_{t+3} + \cdots\right)\Big) \\ &= c_{t+1} + \sum_{h=1}^{\infty}\beta^h\left(c_{t+h+1} - c_{t+h}\right). \end{aligned}$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$\begin{aligned} E[v^*_{t+1}] &= c_t + \frac{1}{1-\beta}\mu + \frac{\phi_c}{1-\beta\nu_x}x_t, \\ \operatorname{var}(v^*_{t+1}) &= \frac{1}{1-\beta^2}\sigma^2\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right). \end{aligned}$$

Using these expressions, we can derive the early resolution utility at t as

$$v^*_t - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\frac{\beta\left(1-\gamma\right)}{1-\beta^2}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2.$$

Subtracting this from the utility v_t under gradual resolution, we arrive at a timing premium given by

$$\mathrm{TP} = 1 - \exp\left(\frac{1}{2}\frac{\beta^2\left(1-\gamma\right)}{1-\beta}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2\left(\frac{\gamma-\widetilde{\gamma}}{1-\gamma} + \frac{1}{1+\beta}\right)\right),$$

as stated in the text. □

Case with stochastic volatility: If consumption follows process (9) with stochastic volatility, guess and verify that the solution to the recursion for $\widetilde{v}_t$ satisfies

$$\widetilde{v}_t - c_t = \widetilde{\mu}_v + \phi_v x_t + \widetilde{\psi}_v\sigma_t^2$$

where

$$\widetilde{\mu}_v = \frac{\beta}{1-\beta}\left(\mu_c + \widetilde{\psi}_v \sigma^2 (1-\nu_\sigma) + \frac{1}{2}(1-\widetilde{\gamma})\widetilde{\psi}_v^2 \alpha_\sigma^2\right)$$
$$\phi_v = \frac{\beta\phi_c}{1-\beta\nu_x}$$
$$\widetilde{\psi}_v = \frac{1}{2}\frac{\beta(1-\widetilde{\gamma})}{1-\beta\nu_\sigma}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right).$$

We then obtain:

$$v_t - \widetilde{v}_t = -\frac{1}{2}\beta(\gamma-\widetilde{\gamma})\left[\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right]$$

$$v_t - c_t = \frac{\beta}{1-\beta}\left(\mu_c + \widetilde{\psi}_v\sigma^2(1-\nu_\sigma) + \frac{1}{2}\widetilde{\psi}_v^2\left((1-\gamma)+\beta(\gamma-\widetilde{\gamma})\right)\alpha_\sigma^2\right)$$
$$+ \phi_v x_t + \frac{\widetilde{\psi}_v}{1-\widetilde{\gamma}}\left((1-\gamma)+\beta\nu_\sigma(\gamma-\widetilde{\gamma})\right)\sigma_t^2$$

If all risk is resolved at $t+1$, log continuation utility $v_{t,t+1}^*$ is given by

$$v_{t+1}^* = (1-\beta)c_{t+1} + \beta\left((1-\beta)c_{t+2} + \beta\left((1-\beta)c_{t+3}+\cdots\right)\right)$$
$$= c_{t+1} + \sum_{h=1}^{\infty}\beta^h\left(c_{t+h+1}-c_{t+h}\right).$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$E_t[v_{t+1}^*] = c_t + \frac{1}{1-\beta}\mu + \frac{\phi_c}{1-\beta\nu_x}x_t,$$
$$\mathrm{var}_t(v_{t+1}^*) = \frac{1}{1-\beta^2\nu_\sigma}\left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2}\sigma^2(1-\nu_\sigma)\right)\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right).$$

Using these expressions, we can derive the early resolution utility at t as

$$v_t^* - c_t = \frac{\beta}{1-\beta}\mu_c + \frac{\beta\phi_c}{1-\beta\nu_x}x_t + \frac{1}{2}\frac{\beta(1-\gamma)}{1-\beta^2\nu_\sigma}\left(\alpha_c^2 + \left(\frac{\beta\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2}\sigma^2(1-\nu_\sigma)\right)$$

and

$$v_t - v_t^* = \frac{\beta}{1-\beta}\widetilde{\psi}_v\sigma^2(1-\nu_\sigma)\left(1 - \frac{1-\gamma}{1-\widetilde{\gamma}}\frac{1-\beta\nu_\sigma}{1-\beta^2\nu_\sigma}\frac{\beta}{1+\beta}\right)$$
$$+ \widetilde{\psi}_v\nu_\sigma\sigma_t^2\frac{\beta}{1-\widetilde{\gamma}}\left((1-\gamma)\frac{1-\beta}{1-\beta^2\nu_\sigma} + (\gamma-\widetilde{\gamma})\right)$$
$$+ \frac{1}{2}\beta\left[\frac{(1-\gamma)+\beta(\gamma-\widetilde{\gamma})}{1-\beta}\right]\widetilde{\psi}_v^2\alpha_\sigma^2$$

Time premium under hyperbolic discounting "β-δ" model Assume $\gamma = \widetilde{\gamma}$, but $\beta < \widetilde{\beta}$.

$$\widetilde{v}_t - c_t = \frac{\widetilde{\beta}}{1-\widetilde{\beta}}\mu_c + \frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}x_t + \frac{1}{2}\frac{\widetilde{\beta}(1-\gamma)}{1-\widetilde{\beta}}\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}\right)^2\alpha_x^2\right)\sigma^2$$

$$v_t - c_t = \frac{\beta}{1-\gamma} E_t \left[\exp\left(1-\gamma\right)\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right]$$

$$\widetilde{v}_t - c_t = \frac{\widetilde{\beta}}{1-\gamma} E_t \left[\exp\left(1-\gamma\right)\left(\widetilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t\right)\right]$$

$$\begin{aligned} v_t - c_t &= \frac{\beta}{\widetilde{\beta}}\left(\widetilde{v}_t - c_t\right) \\ &= \frac{\beta}{1-\widetilde{\beta}}\mu_c + \frac{\beta\phi_c}{1-\widetilde{\beta}\nu_x}x_t + \frac{1}{2}\frac{\beta\left(1-\gamma\right)}{1-\widetilde{\beta}}\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}\right)^2\alpha_x^2\right)\sigma^2 \end{aligned}$$

If all risk is resolved at $t+1$, log continuation utility $v^*_{t,t+1}$ is given by

$$\begin{aligned} v^*_{t+1} &= \left(1-\widetilde{\beta}\right)c_{t+1} + \widetilde{\beta}\left(\left(1-\beta\right)c_{t+2} + \widetilde{\beta}\left(\left(1-\beta\right)c_{t+3} + \cdots\right)\right) \\ &= c_{t+1} + \sum_{h=1}^{\infty}\widetilde{\beta}^h\left(c_{t+h+1} - c_{t+h}\right) \\ &= c_t + \sum_{h=0}^{\infty}\widetilde{\beta}^h\left(c_{t+h+1} - c_{t+h}\right). \end{aligned}$$

From the perspective of period t, this continuation utility is normally distributed with mean and variance given by

$$\begin{aligned} E_t[v^*_{t+1}] &= c_t + \frac{1}{1-\widetilde{\beta}}\mu_c + \frac{\phi_c}{1-\widetilde{\beta}\nu_x}x_t, \\ \mathrm{var}_t(v^*_{t+1}) &= \frac{1}{1-\widetilde{\beta}^2}\sigma^2\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}\right)^2\alpha_x^2\right). \end{aligned}$$

Using these expressions, we can derive the early resolution utility at t as

$$v^*_t - c_t = \frac{\beta}{1-\gamma}E_t\left[\exp\left(1-\gamma\right)\left(v^*_{t+1} - c_t\right)\right]$$

$$v^*_t - c_t = \frac{\beta}{1-\widetilde{\beta}}\mu_c + \frac{\beta\phi_c}{1-\widetilde{\beta}\nu_x}x_t + \frac{1}{2}\frac{\beta\left(1-\gamma\right)}{1-\widetilde{\beta}^2}\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\beta\nu_x}\right)^2\alpha_x^2\right)\sigma^2$$

and

$$v_t - v^*_t = \frac{1}{2}\frac{\beta\widetilde{\beta}\left(1-\gamma\right)}{1-\widetilde{\beta}^2}\left(\alpha_c^2 + \left(\frac{\widetilde{\beta}\phi_c}{1-\widetilde{\beta}\nu_x}\right)^2\alpha_x^2\right)\sigma^2$$

with $\beta < \widetilde{\beta}$, $\frac{\widetilde{\beta}^2}{1-\widetilde{\beta}^2} > \frac{\beta\widetilde{\beta}}{1-\widetilde{\beta}^2} > \frac{\beta^2}{1-\beta^2}$.

When $\gamma > \rho$, the timing premium under $\{\beta, \widetilde{\beta}\}$ is greater than under the β -only model and lower than under the $\widetilde{\beta}$ -only model.

B.2 Extension to other information arrival structures

General sequence of risk aversions and comparison $t+1$ vs. $t+2$. In the main text, we show that while an agent with Epstein-Zin preferences prefers early resolution iff $\gamma > \rho = 1$, our agent with horizon-dependent risk aversion can prefer late resolution, even if $\gamma > \tilde{\gamma} > 1$, as long as

$$\gamma - \widetilde{\gamma} > \frac{\gamma - 1}{1 + \beta},$$

i.e. as long as γ is sufficiently greater than $\tilde{\gamma}$. Suppose we have a sequence of risk aversions $\{\gamma_h\}_{h=1}^{\infty}$ that is decreasing to some horizon H and then constant at $\tilde{\gamma}$. For the comparison of gradual resolution vs. resolution at $t+1$, denoted by v_t^{t+1}, we have

$$v_t - v_t^{t+1} = \frac{1}{2}\left(1 - \left(\gamma_1 - (1+\beta)\sum_{h=1}^{\infty}\beta^{h-1}\left(\gamma_h - \gamma_{h+1}\right)\right)\right)\frac{\beta^2}{1-\beta^2}\,\alpha_v^2\sigma^2,$$

which has the same structure as in the timing premium for just two levels of risk aversion in equation (6). The agent prefers gradual later resolution if

$$\sum_{h=1}^{\infty}\beta^{h-1}\left(\gamma_h - \gamma_{h+1}\right) > \frac{\gamma_1 - 1}{1+\beta}$$

i.e. as long as the sequence $\{\gamma_h\}_{h=1}^{\infty}$ is sufficiently decreasing.

For the comparison of resolution at $t+1$ vs. resolution at $t+2$ we have

$$v_t^{t+1} - v_t^{t+2} = \frac{1}{2}\left(\beta\left(1-\gamma_1\right) - \left(1-\gamma_2\right)\right)\frac{\beta^2}{1-\beta^2}\,\alpha_v^2\sigma^2.$$

While an agent with Epstein-Zin preferences prefers early resolution iff $\gamma > 1$ since

$$\beta\left(1-\gamma\right) - \left(1-\gamma\right) = \left(1-\beta\right)\left(\gamma - 1\right),$$

our agent can prefer late resolution, as long as

$$\gamma_1 - \gamma_2 > \left(1-\beta\right)\left(\gamma_1 - 1\right)$$

i.e. as long as γ_1 is sufficiently greater than γ_2.

B.3 Stochastic discount factor

We now specialize to the case of two levels of risk aversion, setting $\gamma_1 = \gamma$ and $\gamma_h = \widetilde{\gamma}$ for $h \geq 2$.

Proof of Lemma 1. Under the stochastic process (9), we can guess and verify that the solution to the recursion for $\widetilde{v}_t$ satisfies

$$\widetilde{v}_t - c_t = \widetilde{\mu}_v + \phi_v x_t + \widetilde{\psi}_v \sigma_t^2$$

where we write $\widetilde{\psi}_v = \psi_v\left(\widetilde{\gamma}\right)$ throughout for simplification, and

$$\begin{aligned}
\widetilde{\mu}_v &= \frac{\beta}{1-\beta}\left(\mu_c + \widetilde{\psi}_v\sigma^2\left(1-\nu_\sigma\right) + \frac{1}{2}\left(1-\widetilde{\gamma}\right)\widetilde{\psi}_v^2\alpha_\sigma^2\right)\\
\phi_v &= \frac{\beta\phi_c}{1-\beta\nu_x}\\
\widetilde{\psi}_v &= \frac{1}{2}\frac{\beta\left(1-\widetilde{\gamma}\right)}{1-\beta\nu_\sigma}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right).
\end{aligned}$$

Substituting these into (20), we arrive at the solution for v_t:

$$v_t - \widetilde{v}_t = -\frac{1}{2}\beta\left(\gamma - \widetilde{\gamma}\right)\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right]$$

and

$$\begin{aligned} v_t - c_t = &\frac{\beta}{1-\beta}\left(\mu_c + \widetilde{\psi}_v\sigma^2\left(1-\nu_\sigma\right) + \frac{1}{2}\widetilde{\psi}_v^2\left(\left(1-\gamma\right) + \beta\left(\gamma - \widetilde{\gamma}\right)\right)\alpha_\sigma^2\right) \\ &+ \phi_v x_t + \frac{\widetilde{\psi}_v}{1-\widetilde{\gamma}}\left(\left(1-\gamma\right) + \beta\nu_\sigma\left(\gamma - \widetilde{\gamma}\right)\right)\sigma_t^2 \end{aligned}$$

□

Proof of Proposition 4. Using the results of Lemmas 1 and (20), the expression for the SDF follows from Equation (8):

$$\begin{aligned} \pi_{t,t+1} = &\overbrace{\log\beta - \mu_c - \phi_c x_t - \frac{1}{2}\left(1-\gamma\right)^2\left(\left[\alpha_c^2 + \phi_v^2\alpha_x^2\right]\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right)}^{\bar{\pi}_t} \\ &- \gamma\alpha_c\sigma_t w_{c,t+1} + \left(1-\gamma\right)\phi_v\alpha_x\sigma_t w_{x,t+1} \\ &+ \left(1-\gamma\right)\psi_v(\widetilde{\gamma})\,\alpha_\sigma w_{\sigma,t+1}, \end{aligned}$$

The risk-free rate is defined as $r_{f,t} = -\log E_t\left(\Pi_{t,t+1}\right)$ and simplifies to

$$r_{f,t} = -\log\beta + \mu_c + \phi_c x_t + \left(\frac{1}{2} - \gamma\right)\alpha_c^2\sigma_t^2$$

as stated in the text. □

B.4 Equity premium

To derive the equity premium, we log-linearize the returns on the dividend stream:

$$\begin{aligned} r_{m,t+1} &= \log\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \\ &= \Delta d_{t+1} + \log\left(1 + e^{z_{t+1}}\right) - z_t \\ &\approx k_0 + k_1 z_{t+1} - z_t + \Delta d_{t+1} \end{aligned}$$

where $z_t = p_t - d_t$ and $k_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$.

From

$$E_t\left(\Pi_{t,t+1}R_{m,t+1}\right) = 1$$

we obtain a recursion in z_t.

Guess: $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$

$$\log E_t\left(\exp\left(\left\{\begin{array}{l}\log\beta - \mu_c - \frac{1}{2}\left(1-\gamma\right)^2 \widetilde{\psi}_v^2 \alpha_\sigma^2 + k_0 + k_1 A_2 \sigma^2\left(1-\nu_\sigma\right) \\ -\phi_c x_t - A_1 x_t + \phi_d x_t + k_1 A_1 \nu_x x_t \\ -\frac{1}{2}\left(1-\gamma\right)^2\left[\alpha_c^2 + \phi_v^2 \alpha_x^2\right]\sigma_t^2 - A_2\sigma_t^2 + k_1 A_2 \nu_\sigma \sigma_t^2 \\ -\gamma\alpha_c\sigma_t W_{c,t+1} + \chi\alpha_c\sigma_t W_{t+1} + \alpha_d\sigma_t W_{t+1} + \\ \left(1-\gamma\right)\phi_v\alpha_x\sigma_t W_{x,t+1} + k_1 A_1 \alpha_x \sigma_t W_{x,t+1} \\ +\left(1-\gamma\right)\psi_v(\widetilde{\gamma})\,\alpha_\sigma W_{\sigma,t+1} + k_1 A_2 \alpha_\sigma W_{\sigma,t+1}\end{array}\right.\right)\right) = 0$$

and so:

$$\frac{\phi_d - \phi_c}{1 - k_1 \nu_x} = A_1$$

$$\begin{aligned} &-\frac{1}{2}\left(1-\gamma\right)^2\left[\alpha_c^2 + \phi_v^2\alpha_x^2\right] \\ +\frac{1}{2}\alpha_d^2 + \frac{1}{2}\left(\chi-\gamma\right)^2\alpha_c^2 + &\frac{1}{2}\left(k_1 A_1 + \left(1-\gamma\right)\phi_v\right)^2\alpha_x^2 = A_2\left(1 - k_1\nu_\sigma\right)\end{aligned}$$

Note A_1 and A_2 are both unaffected by $\widetilde{\gamma}$, and therefore identical to the standard model.

Since the equity premium is determined by the covariation between the returns $r_{m,t+1}$ and the stochastic discount factor $\pi_{t,t+1}$, and the loadings on the consumption level shocks are unchanged relative to the standard model for both the market returns and the SDF, the only term that is impacted is the cross-term for the loadings on the volatility shocks. The contribution of volatility shocks to the equity premium under horizon dependent risk aversion is simply the one under the standard model multiplied by $\frac{1-\widetilde{\gamma}}{1-\gamma}$ (see Corollary 3).

B.5 Term structure of returns

B.5.1 General claims

To make the problem as general as possible, we analyze horizon-dependent claims that are priced recursively as

$$Y_{t,h} = E_t\left[\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1}\right],$$

that is

$$y_{t,h} = E_t\left[\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}\right] + \frac{1}{2}\mathrm{var}_t\left(\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}\right),$$

where

$$\begin{aligned} g_{y,t+1} = {} & \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ & + \alpha_{y,c}\alpha_c\sigma_t w_{c,t+1} + \alpha_{y,x}\alpha_x\sigma_t w_{x,t+1} + \alpha_{y,\sigma}\alpha_\sigma\sigma_t w_{\sigma,t+1} + \alpha_{y,d}\alpha_d\sigma_t w_{d,t+1},\end{aligned}$$

and $Y_{t,0} = 1$.

Guess that

$$Y_{t,h} = \exp\left(\widetilde{\mu}_{y,h} + \phi_{y,h} x_t + \psi_{y,h}\sigma_t^2\right).$$

Suppose $h \geq 1$, then:

$$\log \widetilde{\Pi}_{t,t+1} G_{t,t+1} Y_{t+1,h-1} = \begin{cases} \log\beta - \mu_c - \phi_c x_t - \frac{1}{2}\left(1-\gamma\right)^2 \left[\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2 \alpha_\sigma^2\right] \\ +\mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ +\widetilde{\mu}_{y,h-1} + \phi_{y,h-1}\nu_x x_t + \psi_{y,h-1}\left(\sigma^2\left(1-\nu_\sigma\right) + \nu_\sigma \sigma_t^2\right) \\ + \left(-\gamma + \alpha_{y,c}\right)\alpha_c \sigma_t W_{t+1} + \left(\left(1-\gamma\right)\phi_v + \alpha_{y,x} + \phi_{y,n-1}\right)\alpha_x \sigma_t W_{t+1} \\ + \left(\left(1-\gamma\right)\widetilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,h-1}\right)\alpha_\sigma W_{t+1} \\ +\alpha_{y,d}\alpha_d \sigma_t W_{t+1} \end{cases}$$

Matching coefficients, we find the recursions, for $h \geq 1$:

- Terms in x_t:

$$\begin{aligned} & \phi_{y,h} = -\phi_c + \phi_y + \phi_{y,h-1}\nu_x \\ \Rightarrow \quad & \phi_{y,h} = \left(-\phi_c + \phi_y\right)\frac{1-\nu_x^h}{1-\nu_x} \end{aligned}$$

- Terms in σ_t^2:

$$\begin{aligned} \psi_{y,h} = & -\frac{1}{2}\left(1-\gamma\right)^2\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) + \psi_{y,h-1}\nu_\sigma + \psi_y \\ & + \frac{1}{2}\left(\left(-\gamma + \alpha_{y,c}\right)^2\alpha_c^2 + \left(\left(1-\gamma\right)\phi_v + \alpha_{y,x} + \phi_{y,h-1}\right)^2\alpha_x^2 + \alpha_{y,d}^2\alpha_d^2\right) \end{aligned}$$

  and thus the solution, for $h \geq 1$:

$$\begin{aligned} \psi_{y,h} = & \left[-\frac{1}{2}\left(1-\gamma\right)^2\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) + \psi_y + \frac{1}{2}\left(\left(-\gamma+\alpha_{y,c}\right)^2\alpha_c^2 + \alpha_{y,d}^2\alpha_d^2\right)\right]\frac{1-\nu_\sigma^h}{1-\nu_\sigma} \\ & + \frac{1}{2}\sum_{n=0}^{h-1}\nu_\sigma^n\left(\left(1-\gamma\right)\phi_v + \alpha_{y,x} + \phi_{y,n-1-h}\right)^2\alpha_x^2 \end{aligned}$$

- Constant:

$$\begin{aligned} \widetilde{\mu}_{y,h} - \widetilde{\mu}_{y,h-1} = & \log\beta - \mu_c + \mu_y + \sigma^2\left(1-\nu_\sigma\right)\psi_{y,h-1} \\ & + \frac{1}{2}\left(\left(\left(1-\gamma\right)\widetilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,h-1}\right)^2 - \left(1-\gamma\right)^2\widetilde{\psi}_v^2\right)\alpha_\sigma^2 \end{aligned}$$

and thus the solution, for $h \geq 1$:

$$\begin{aligned} \widetilde{\mu}_{y,h} = & h\left(\log\beta - \mu_c + \mu_y - \frac{1}{2}\left(1-\gamma\right)^2\widetilde{\psi}_v^2\alpha_\sigma^2\right) \\ & + \sum_{n=0}^{h-1}\left[\sigma^2\left(1-\nu_\sigma\right)\psi_{y,n} + \frac{1}{2}\left(\left(1-\gamma\right)\widetilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,n}\right)^2\alpha_\sigma^2\right] \end{aligned}$$

Note only the constant terms $\left\{\widetilde{\mu}_{y,h}\right\}$ are affected by the wedge between γ and $\widetilde{\gamma}$.

□

In line with the specification of van Binsbergen and Koijen (2016), we consider one-period holding returns for these claims of the form

$$
\begin{aligned}
1+R_{t+1,h}^{Y} &= \frac{G_{y,t+1}Y_{t+1,h-1}}{Y_{t,h}} = \frac{G_{y,t+1}Y_{t+1,h-1}}{E_t\left[\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right]} \\
&= R_{f,t}\frac{E_t[\Pi_{t,t+1}]\,G_{y,t+1}Y_{t+1,h-1}}{E_t\left[\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right]},
\end{aligned}
$$

with the risk-free rate

$$
R_{f,t} = \frac{1}{E_t[\Pi_{t,t+1}]}.
$$

The conditional Sharpe Ratio is

$$
\begin{aligned}
\mathrm{SR}_{t,h}^{Y} &= \frac{E_t\left[1+R_{t+1,h}^{Y}\right]-1}{\sqrt{\mathrm{var}_t\left(1+R_{t+1,h}^{Y}\right)}} \\
&= \frac{E_t\left(1+R_{t+1,h}^{Y}\right)-1}{\sqrt{E_t\left(\left(1+R_{t+1,h}^{Y}\right)^2\right)-\left(E_t\left(1+R_{t+1,h}^{Y}\right)\right)^2}} \\
&\approx \frac{r_{f,t}+\begin{cases}\left(\gamma\alpha_{y,c}\alpha_c^2-(1-\gamma)\,\phi_v\left(\alpha_{y,x}+\phi_{y,h-1}\right)\alpha_x^2\right)\sigma_t^2 \\ -(1-\gamma)\,\widetilde{\psi}_v\left(\alpha_{y,\sigma}+\psi_{y,h-1}\right)\alpha_\sigma^2\end{cases}}{\sqrt{\sigma_t^2\left(\alpha_{y,c}^2\alpha_c^2+\left(\alpha_{y,x}+\phi_{y,h-1}\right)^2\alpha_x^2+\alpha_{y,d}^2\alpha_d^2\right)+\left(\alpha_{y,\sigma}+\psi_{y,h-1}\right)^2\alpha_\sigma^2}}.
\end{aligned}
$$

In line with the specification of van Binsbergen and Koijen (2016), we also consider one-period holding returns for futures on these claims of the form

$$
\begin{aligned}
R_{t+1,h}^{F,Y}+1 &= \frac{1+R_{t+1,h}^{Y}}{1+R_{t+1,h}^{B}} = \frac{G_{y,t+1}Y_{t+1,h-1}}{Y_{t,h}}\frac{B_{t,h}}{B_{t+1,h-1}} \\
&= \frac{G_{y,t+1}Y_{t+1,h-1}}{E_t\left(\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right)}\frac{E_t\left(\Pi_{t,t+1}B_{t+1,h-1}\right)}{B_{t+1,h-1}},
\end{aligned}
$$

where $B_{t,h}$ is the price of \$1 at horizon h, i.e. the price of a Bond with horizon h.

Their conditional Sharpe Ratio is

$$
\begin{aligned}
\mathrm{SR}_{t,h}^{F,Y} &= \frac{E_t\left(1+R_{t+1,h}^{F,Y}\right)-1}{\sqrt{\operatorname{var}_t\left(1+R_{t+1,h}^{F,Y}\right)}} \\
&= \frac{E_t\left(1+R_{t+1,h}^{F,Y}\right)-1}{\sqrt{E_t\left(\left(1+R_{t+1,h}^{F,Y}\right)^2\right)-\left(E_t\left(1+R_{t+1,h}^{F,Y}\right)\right)^2}} \\
&\approx \frac{\begin{cases}\sigma_t^2\left(\gamma\alpha_{y,c}\alpha_c^2-\left(\alpha_{y,x}+\phi_{y,h-1}-\phi_{b,h-1}\right)\left((1-\gamma)\,\phi_v+\phi_{b,h-1}\right)\alpha_x^2\right) \\ -\left(\alpha_{y,\sigma}+\psi_{y,h-1}-\psi_{b,h-1}\right)\left((1-\gamma)\,\widetilde{\psi}_v+\psi_{b,h-1}\right)\alpha_\sigma^2\end{cases}}{\sqrt{\sigma_t^2\left(\alpha_{y,c}^2\alpha_c^2+\left(\alpha_{y,x}+\phi_{y,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\alpha_{y,d}^2\alpha_d^2\right)+\left(\alpha_{y,\sigma}+\psi_{y,h-1}-\psi_{b,h-1}\right)^2\alpha_\sigma^2}}.
\end{aligned}
$$

For the *unconditional* Sharpe ratio observe that the volatility process

$$\sigma_{t+1}^2-\sigma^2=\nu_\sigma\left(\sigma_t^2-\sigma^2\right)+\alpha_\sigma W_{t+1}$$

is stationary under the constraint $\nu_\sigma<1$ with normal distribution with mean σ^2 and variance $\Sigma_\sigma=\frac{\alpha_\sigma^2}{1-\nu_\sigma^2}$.
and therefore $E\left(\exp\left(a\sigma_t^2\right)\right)=\exp\left(a\sigma^2+\frac{1}{2}a^2\frac{\alpha_\sigma^2}{1-\nu_\sigma^2}\right)$.

B.5.2 Bonds

Bond prices Let the price at time t for \$1 in h periods be $B_{t,h}$ with $B_{t,0}=1$. For $h\geq 1$, we have

$$B_{t,h}=E_t[\Pi_{t,t+1}B_{t+1,h-1}]$$

This is the general problem from above with $g_{y,t+1}=0$ for all t and therefore

$$b_{t,h}=\widetilde{\mu}_{b,h}+\phi_{b,h}x_t+\psi_{b,h}\sigma_t^2,$$

with

$$\phi_{b,h}=-\phi_c\frac{1-\nu_x^h}{1-\nu_x}$$

$$
\begin{aligned}
\psi_{b,h}= &-\frac{1}{2}(1-\gamma)^2\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)+\psi_{b,h-1}\nu_\sigma \\
&+\frac{1}{2}\left(\gamma^2\alpha_c^2+\left((1-\gamma)\,\phi_v+\phi_{b,h-1}\right)^2\alpha_x^2\right)
\end{aligned}
$$

and

$$\psi_{b,1}=\left(\gamma-\frac{1}{2}\right)\alpha_c^2>0$$

and $\psi_{b,h}>0$ for all h, and $\psi_{b,h}$ increasing in h.

Further,

$$\widetilde{\mu}_{b,h} - \widetilde{\mu}_{b,h-1} = \log\beta - \mu_c + \sigma^2 (1-\nu_\sigma)\,\psi_{b,h-1} + \left((1-\gamma)\,\widetilde{\psi}_v \psi_{b,h-1} + \frac{1}{2}\psi_{b,h-1}^2 \right) \alpha_\sigma^2$$

increasing in h. But $\widetilde{\mu}_{b,h}$ can be decreasing if $\log\beta - \mu_c < 0$.

Bond returns The one-period returns are given by:

$$R^B_{t+1,h} = \frac{B_{t+1,h-1}}{B_{t,h}} - 1$$

and therefore

$$\log\left(R^B_{t+1,h} + 1\right) = -\log\beta + \mu_c - \left((1-\gamma)\,\widetilde{\psi}_v \psi_{b,h-1} + \frac{1}{2}\psi_{b,h-1}^2 \right)\alpha_\sigma^2 + \phi_c x_t + \left(\psi_{b,h-1}\nu_\sigma - \psi_{b,h}\right)\sigma_t^2 + \psi_{b,h-1}\alpha_\sigma W_{t+1} + \phi_{b,h-1}\alpha_x \sigma_t W_{t+1}$$

the term structure of expected returns is given by:

$$E_t\left(R^B_{t+1,h} + 1\right) \approx -\log\beta + \mu_c - (1-\gamma)\,\widetilde{\psi}_v \psi_{b,h-1}\alpha_\sigma^2 + \phi_c x_t - \left(\left(\gamma - \frac{1}{2}\right)\alpha_c^2 + (1-\gamma)\,\phi_v \phi_{b,h-1}\alpha_x^2 \right)\sigma_t^2$$

$$E_t\left(R^B_{t+1,h+1}\right) - E_t\left(R^B_{t+1,h}\right) \approx (\gamma - 1)\,\widetilde{\psi}_v\left(\psi_{b,h} - \psi_{b,h-1}\right)\alpha_\sigma^2 + (\gamma - 1)\,\phi_v\phi_c \frac{\nu_x^h - \nu_x^{h-1}}{1-\nu_x}\alpha_x^2\sigma_t^2 \leq 0.$$

The only impact of $\widetilde{\gamma}$ is through $\widetilde{\psi}_v$, and makes the slope less decreasing (but not increasing).

Risk-free rate The risk-free rate is given by

$$r_{f,t} = -\log B_{t,1}$$

i.e.

$$r_{f,t} = -\log\beta + \mu_c + \phi_c x_t - \left(\gamma - \frac{1}{2}\right)\alpha_c^2\sigma_t^2$$

B.5.3 Dividend strips

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$ with $P_{t,0} = D_t$. Then for $h \geq 1$:

$$\frac{P_{t,h}}{D_t} = E_t\left(\Pi_{t,t+1}\frac{D_{t+1}}{D_t}\frac{P_{t+1,h-1}}{D_{t+1}}\right),$$

which is the general problem from above with

$$g_{p,t+1} = d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi\alpha_c\sigma_t W_{t+1} + \alpha_d\sigma_t W_{t+1},$$

for all t and therefore

$$p_{t,h} - d_t = \widetilde{\mu}_{p,h} + \phi_{d,h}x_t + \psi_{d,h}\sigma_t^2,$$

with

$$\phi_{d,h} = \left(-\phi_c + \phi_d\right)\frac{1-\nu_x^h}{1-\nu_x}$$

$$\psi_{d,h} = -\frac{1}{2}(1-\gamma)^2\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) + \psi_{d,h-1}\nu_\sigma$$
$$+ \frac{1}{2}\left((-\gamma+\chi)^2\alpha_c^2 + ((1-\gamma)\phi_v + \phi_{d,h-1})^2\alpha_x^2 + \alpha_d^2\right)$$

$$\psi_{d,1} = \frac{1}{2}\alpha_d^2 + (\chi+1-2\gamma)(\chi-1)\frac{1}{2}\alpha_c^2$$

the sign depends on the parameters of the model.

$$\widetilde{\mu}_{d,h} - \widetilde{\mu}_{d,h-1} = \log\beta - \mu_c + \mu_d + \sigma^2(1-\nu_\sigma)\psi_{d,h-1} + \left((1-\gamma)\widetilde{\psi}_v\psi_{d,h-1} + \frac{1}{2}\psi_{d,h-1}^2\right)\alpha_\sigma^2$$

where the sign depends again on the parameters of the model.

For the dividend strips, the spot one-period returns are given by

$$R^P_{t+1,h} + 1 = \frac{P_{t+1,h-1}/D_{t+1}}{P_{t,h}/D_t}\frac{D_{t+1}}{D_t},$$

$$\log\left(R^P_{t+1,h}+1\right) = -\log\beta + \mu_c - \left((1-\gamma)\widetilde{\psi}_v\psi_{d,h-1} + \frac{1}{2}\psi_{d,h-1}^2\right)\alpha_\sigma^2$$
$$+ \phi_c x_t + (\psi_{d,h-1}\nu_\sigma - \psi_{d,h})\sigma_t^2$$
$$+ \psi_{d,h-1}\alpha_\sigma W_{t+1} + \phi_{d,h-1}\alpha_x\sigma_t W_{t+1} + \chi\alpha_c\sigma_t W_{t+1} + \alpha_d\sigma_t W_{t+1}$$

the conditional expected one-period returns are

$$E_t\left(R^P_{t+1,h}+1\right) \approx -\log\beta + \mu_c - (1-\gamma)\widetilde{\psi}_v\psi_{d,h-1}\alpha_\sigma^2 + \phi_c x_t$$
$$-\left(\left(\gamma(1-\chi) - \frac{1}{2}\right)\alpha_c^2 + (1-\gamma)\phi_v\phi_{d,h-1}\alpha_x^2\right)\sigma_t^2$$

$$E_t\left(R^P_{t+1,h+1}\right) - E_t\left(R^P_{t+1,h}\right) \approx \underbrace{(\gamma-1)\widetilde{\psi}_v}_{\leq 0}(\psi_{d,h}-\psi_{d,h-1})\alpha_\sigma^2 + \underbrace{(\gamma-1)\phi_v(\phi_c-\phi_d)\frac{\nu_x^h - \nu_x^{h-1}}{1-\nu_x}\alpha_x^2\sigma_t^2}_{\geq 0}$$

We need $(\psi_{d,h} - \psi_{d,h-1}) \geq 0$ to generate a downward sloping term structure, but that does not depend on the choice of $\widetilde{\gamma}$. If $(\psi_{d,h} - \psi_{d,h-1}) \leq 0$, then the returns are upward sloping, but less so in our model.

Note, that the returns are MORE upward sloping when σ_t is high...

□

The future one-period returns are given by:

$$R^{F,P}_{t+1,h} + 1 = \frac{1 + R^P_{t+1,h}}{1 + R^B_{t+1,h}}$$

$$\log\left(R^{F,P}_{t+1,h}+1\right) = -\left((1-\gamma)\,\widetilde{\psi}_v\left(\psi_{d,h-1}-\psi_{b,h-1}\right)+\frac{1}{2}\left(\psi^2_{d,h-1}-\psi^2_{b,h-1}\right)\right)\alpha^2_\sigma$$
$$+\left(\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\nu_\sigma-\left(\psi_{d,h}-\psi_{b,h}\right)\right)\sigma^2_t$$
$$+\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\alpha_\sigma W_{t+1}+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)\alpha_x\sigma_t W_{t+1}+\chi\alpha_c\sigma_t W_{t+1}+\alpha_d\sigma_t W_{t+1}$$

$$E_t\left(R^{F,P}_{t+1,h}+1\right) = -\left(\underbrace{\left((1-\gamma)\,\widetilde{\psi}_v+\psi_{b,h-1}\right)}_{\geq 0 \text{ and increasing}}\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\right)\alpha^2_\sigma$$
$$+\left(\gamma\chi\alpha^2_c+\underbrace{\left((\gamma-1)\,\phi_v-\phi_{b,h-1}\right)\left(\phi_{d,h-1}-\phi_{b,h-1}\right)}_{\geq 0 \text{ and increasing}}\alpha^2_x\right)\sigma^2_t$$

Note:

$$\psi_{d,h}-\psi_{b,h} = \left(\psi_{d,h-1}-\psi_{b,h-1}\right)\nu_\sigma$$
$$+\left(\chi\underbrace{\left(\frac{1}{2}\chi-\gamma\right)}_{\leq 0}\alpha^2_c+\underbrace{\left((1-\gamma)\,\phi_v+\frac{1}{2}\left(\phi_{d,h-1}+\phi_{b,h-1}\right)\right)}_{\leq 0 \text{ for } \gamma \text{ high enough}}\underbrace{\left(\phi_{d,h-1}-\phi_{b,h-1}\right)}_{\geq 0}\alpha^2_x+\underbrace{\frac{1}{2}\alpha^2_d}_{\geq 0}\right)$$

the sign depends on the parameters. But if it is positive increasing, $\widetilde{\gamma}$ reduces the downward impact of it on the term structure of expected returns. Only if it is negative and decreasing does our model help relative to the standard model, but then the slope is upward sloping....

Note, a higher σ_t means a MORE upward sloping term structure again

□

the Sharpe ratio term structure is given by:

$$SR^{F,P}_{t,n} \approx \frac{\begin{cases}\sigma^2_t\left(\gamma\chi\alpha^2_c-\left(\phi_{d,h-1}-\phi_{b,h-1}\right)\left((1-\gamma)\,\phi_v+\phi_{b,h-1}\right)\alpha^2_x\right)\\ -\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\left((1-\gamma)\,\widetilde{\psi}_v+\psi^1_{b,h-1}\right)\alpha^2_\sigma\end{cases}}{\sqrt{\sigma^2_t\left(\chi^2\alpha^2_c+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha^2_x+\alpha^2_d\right)+\left(\psi_{d,h-1}-\psi_{b,h-1}\right)^2\alpha^2_\sigma}}$$

If the expected returns term structure is upward sloping with $\psi_{d,h}-\psi_{b,h}\leq 0$ and decreasing, then $\widetilde{\gamma}$ can help make the sharpe ratio term structure downward sloping.

The unconditional Sharpe ratio term structure is:

$$\mathrm{SR}_h^{F,P} \approx \frac{\begin{cases} \sigma^2\left(\gamma\chi\alpha_c^2 - \left(\phi_{d,h-1}-\phi_{b,h-1}\right)\left(\left(1-\gamma\right)\phi_v+\phi_{b,h-1}\right)\alpha_x^2\right) \\ +\frac{1}{2}\frac{\alpha_\sigma^2}{1-\nu_\sigma^2}\left(\gamma\chi\alpha_c^2 - \left(\phi_{d,h-1}-\phi_{b,h-1}\right)\left(\left(1-\gamma\right)\phi_v+\phi_{b,h-1}\right)\alpha_x^2\right)^2 \\ -\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\left(\left(1-\gamma\right)\widetilde{\psi}_v+\psi_{b,h-1}^1\right)\alpha_\sigma^2 \end{cases}}{\sqrt{\begin{cases} \sigma^2\left(\chi^2\alpha_c^2+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\alpha_d^2\right) \\ +2\left(\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\nu_\sigma-\left(\psi_{d,h}-\psi_{b,h}\right)+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\chi^2\alpha_c^2+\alpha_d^2\right)^2\Sigma_\sigma \\ -\left(\left(\psi_{d,h-1}-\psi_{b,h-1}\right)\nu_\sigma-\left(\psi_{d,h}-\psi_{b,h}\right)+\frac{1}{2}\left(\left(\phi_{d,h-1}-\phi_{b,h-1}\right)^2\alpha_x^2+\chi^2\alpha_c^2+\alpha_d^2\right)\right)^2\Sigma_\sigma \\ +\left(\psi_{d,h-1}-\psi_{b,h-1}\right)^2\alpha_\sigma^2 \end{cases}}}.$$

B.6 Term structure of returns - Illiquid markets

We analyze horizon-dependent dividend claims when markets are illiquid and prices are set by buy-and-hold investors. From above, the SDF for a horizon h investor is (when $\rho = 1$):

$$\Pi_{t,t+h} = \beta^h\left(\frac{C_{t+h}}{C_t}\right)^{-1}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}$$

Consider a dividend with horizon h priced at time t under $\Pi_{t,t+h}$,

$$P_{t,h} = E_t[\Pi_{t,t+h}D_{t+h}]\,,$$

$$P_{t,h} = E_t\left[\beta^h\left(\frac{C_{t+h}}{C_t}\right)^{-1}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}D_{t+h}\right],$$

The price at time $t+1$ is under $\Pi_{t+1,t+1+(h-1)}$,

$$\frac{P_{t+1,h-1}}{D_{t+1}} = E_{t+1}\left[\Pi_{t+1,t+1+h-1}\frac{D_{t+h}}{D_t}\right],$$

$$P_{t+1,h-1} = E_{t+1}\left[\beta^{h-1}\left(\frac{C_{t+h}}{C_{t+1}}\right)^{-1}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+3}}{E_{t+2}\left[\widetilde{V}_{t+3}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}D_{t+h}\right],$$

The one-period return is given by:

$$R_{t+1,h}^{F,P} + 1 = \frac{\frac{P_{t+1,h-1}}{P_{t,h}}}{\frac{B_{t+1,h-1}}{B_{t,h}}}$$

so

$$E_t\left(R^P_{t+1,h}\right)=\frac{E_t\left[\beta^{h-1}\left(\frac{C_{t+h}}{C_{t+1}}\right)^{-1}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}^{1-\gamma}_{t+2}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+3}}{E_{t+2}\left[\widetilde{V}^{1-\widetilde{\gamma}}_{t+3}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}^{1-\widetilde{\gamma}}_{t+h}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\frac{D_{t+h}}{D_t}\right]}{E_t\left[\beta^{h}\left(\frac{C_{t+h}}{C_{t}}\right)^{-1}\left(\frac{\widetilde{V}_{t+1}}{E_{t}\left[\widetilde{V}^{1-\gamma}_{t+1}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}^{1-\widetilde{\gamma}}_{t+2}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}^{1-\widetilde{\gamma}}_{t+h}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\frac{D_{t+h}}{D_t}\right]}$$

To simplify notations, write:

$$\frac{\frac{D_{t+h}}{D_t}\left(\frac{C_{t+h}}{C_t}\right)^{-1}}{E_t\left(\frac{D_{t+h}}{D_t}\left(\frac{C_{t+h}}{C_t}\right)^{-1}\right)}=\exp\left(\sum_{j=1}^{h}\Delta_j W_{t+j}\right)$$

where

$$\Delta_j W_{t+j}=\sigma_{t+j-1}\left(\left(\phi_d-\phi_c\right)\frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x w_{x,t+j}+\alpha_d w_{d,t+j}+\left(\chi-1\right)\alpha_c w_{c,t+j}\right)$$

and

$$\left(\frac{\widetilde{V}_{t+j}}{E_{t+j-1}\left[\widetilde{V}^{1-\widetilde{\gamma}}_{t+j}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}=\exp\left(\left(1-\widetilde{\gamma}\right)\Sigma_j W_{t+j}-\frac{1}{2}\left|\left(1-\widetilde{\gamma}\right)\Sigma_j\right|^2\right)$$

(substitute $\widetilde{\gamma}$ with γ when necessary) where

$$\Sigma_j=\sigma_{t+j-1}\left(\phi_v\alpha_x w_{x,t+j}+\alpha_c w_{c,t+j}\right)+\widetilde{\psi}_v\alpha_\sigma w_{\sigma,t+j}$$

where W_{t+j} is the 4×1 vector of the independent iid shocks at time $t+j$, and $\Delta_{j,t+j-1},\Sigma_{j,t+j-1}$ is written Δ_j,Σ_j to simplify the formulas.

We obtain:

$$E_t\left(R_{t+1,h}\right)=\frac{E_t\left(\frac{C_{t+1}}{C_t}\right)\exp\left[\sum_{j=3}^{h}\left[\left(\Delta_j+\left(1-\widetilde{\gamma}\right)\Sigma_j\right)W_{t+j}-\frac{1}{2}\left|\left(1-\widetilde{\gamma}\right)\Sigma_j\right|^2\right]+\left(\Delta_2+\left(1-\gamma\right)\Sigma_2\right)W_{t+2}-\frac{1}{2}\left|\left(1-\gamma\right)\Sigma_2\right|^2+\Delta_1 W_{t+1}\right]}{\beta E_t\left[\exp\left[\sum_{j=2}^{h}\left[\left(\Delta_j+\left(1-\widetilde{\gamma}\right)\Sigma_j\right)W_{t+j}-\frac{1}{2}\left|\left(1-\widetilde{\gamma}\right)\Sigma_j\right|^2\right]+\left(\Delta_1+\left(1-\gamma\right)\Sigma_1\right)W_{t+1}-\frac{1}{2}\left|\left(1-\gamma\right)\Sigma_1\right|^2\right]\right]}$$

Because the shocks are iid, we obtain, when volatility is constant:

$$E_t\left(R^P_{t+1,h}\right)=\frac{\beta^{-1}E_t\left(\frac{C_{t+1}}{C_t}\right)\exp\left[\left(\Delta_2+\left(1-\gamma\right)\Sigma_2\right)W_{t+2}-\frac{1}{2}\left|\left(1-\gamma\right)\Sigma_2\right|^2+\Delta_1 W_{t+1}\right]}{E_t\left[\exp\left[\left(\Delta_2+\left(1-\widetilde{\gamma}\right)\Sigma_2\right)W_{t+2}-\frac{1}{2}\left|\left(1-\widetilde{\gamma}\right)\Sigma_2\right|^2+\left(\Delta_1+\left(1-\gamma\right)\Sigma_1\right)W_{t+1}-\frac{1}{2}\left|\left(1-\gamma\right)\Sigma_1\right|^2\right]\right]}$$

$$\log E_t\left(R^P_{t+1,h}\right)=-\log\beta+\mu_c+\phi_c x_t+\frac{1}{2}\alpha_c^2\sigma^2+cov\left(\Delta_1,\alpha_c\right)+\left(\widetilde{\gamma}-\gamma\right)cov\left(\Delta_2,\Sigma_2\right)-\left(1-\gamma\right)cov\left(\Delta_1,\Sigma_1\right)$$

$$\begin{aligned}\log E_t\left(R^P_{t+1,h}\right)=&-\log\beta+\mu_c+\phi_c x_t+\left(\chi-\frac{1}{2}\right)\alpha_c^2\sigma^2-\left(1-\gamma\right)\sigma^2\left[\phi_v\left(\phi_d-\phi_c\right)\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2+\left(\chi-1\right)\alpha_c^2\right]\\&+\underbrace{\left(\widetilde{\gamma}-\gamma\right)\sigma^2\left[\phi_v\left(\phi_d-\phi_c\right)\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2+\left(\chi-1\right)\alpha_c^2\right]}_{<0}\end{aligned}$$

Even when volatility is constant, HDRA impacts the term structure of expected returns when investors choose buy-and-hold strategies. The negative impact of HDRA increases with the horizon.

□

To obtain the returns on bonds, and the expected excess returns, replace ϕ_d, α_d and χ by 0 in the formula above:

$$\begin{aligned}\log E_t\left(R^B_{t+1,h}\right) &= -\log\beta+\mu_c+\phi_c x_t+-\frac{1}{2}\alpha_c^2\sigma^2+(1-\gamma)\,\sigma^2\left[\phi_v\phi_c\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2+\alpha_c^2\right]\\ &\quad-(\widetilde{\gamma}-\gamma)\,\sigma^2\left[\phi_v\phi_c\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2+\alpha_c^2\right]\end{aligned}$$

and

$$\begin{aligned}\log E_t\left(R^{P,F}_{t+1,h}\right) &= \gamma\chi\alpha_c^2\sigma^2-(1-\gamma)\,\sigma^2\left[\phi_v\phi_d\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2\right]\\ &\quad+\underbrace{(\widetilde{\gamma}-\gamma)\,\sigma^2\left[\phi_v\phi_d\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2+\chi\alpha_c^2\right]}_{<0}\end{aligned}$$

When volatility is time varying, we can rewrite,

$$\frac{E_t\left(\frac{C_{t+1}}{C_t}\right)\exp\left[\sum_{j=3}^{h}\left[\left(\Delta_j+(1-\widetilde{\gamma})\,\Sigma_j\right)W_{t+j}-\frac{1}{2}\left|(1-\widetilde{\gamma})\,\Sigma_j\right|^2\right]+(\Delta_2+(1-\gamma)\,\Sigma_2)\,W_{t+2}-\frac{1}{2}\left|(1-\gamma)\,\Sigma_2\right|^2+\Delta_1 W_{t+1}\right]}{\beta E_t\left[\exp\left[\sum_{j=2}^{h}\left[\left(\Delta_j+(1-\widetilde{\gamma})\,\Sigma_j\right)W_{t+j}-\frac{1}{2}\left|(1-\widetilde{\gamma})\,\Sigma_j\right|^2\right]+(\Delta_1+(1-\gamma)\,\Sigma_1)\,W_{t+1}-\frac{1}{2}\left|(1-\gamma)\,\Sigma_1\right|^2\right]\right]}=$$

$$\frac{\exp\left(-\log\beta+\mu_c+\phi_c x_t\right)E_t\exp\left[\sum_{j=3}^{h}\left[\widetilde{\Psi}_j\sigma^2_{t+j-1}\right]+(\Delta_2+(1-\gamma)\,\Sigma_2)\,W_{t+2}-\frac{1}{2}\left|(1-\gamma)\,\Sigma_2\right|^2+(\Delta_1 W_{t+1}+\alpha_c\sigma_t w_{c,t+1})\right]}{E_t\exp\left[\sum_{j=3}^{h}\left[\widetilde{\Psi}_j\sigma^2_{t+j-1}\right]+(\Delta_2+(1-\widetilde{\gamma})\,\Sigma_2)\,W_{t+2}-\frac{1}{2}\left|(1-\widetilde{\gamma})\,\Sigma_2\right|^2+(\Delta_1+(1-\gamma)\,\Sigma_1)\,W_{t+1}-\frac{1}{2}\left|(1-\gamma)\,\Sigma_1\right|^2\right]}$$

where

$$\widetilde{\Psi}_j=\frac{1}{2}\left(\left((\phi_d-\phi_c)\,\frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x\right)^2+\alpha_d^2+(\chi-1)^2\,\alpha_c^2\right)+(1-\widetilde{\gamma})\left[\phi_v\,(\phi_d-\phi_c)\,\frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x^2+(\chi-1)\,\alpha_c^2\right]$$

$$\widetilde{\Psi}_\infty=\frac{1}{2}\left(\left((\phi_d-\phi_c)\,\frac{\alpha_x}{1-\nu_x}\right)^2+\alpha_d^2+(\chi-1)^2\,\alpha_c^2\right)+(1-\widetilde{\gamma})\left[\phi_v\,(\phi_d-\phi_c)\,\frac{\alpha_x^2}{1-\nu_x}+(\chi-1)\,\alpha_c^2\right]$$

replace $\widetilde{\gamma}$ with γ to get Ψ_j

$$
\begin{aligned}
&\frac{E_t \exp\left[\sum_{j=3}^{h}\left[\widetilde{\Psi}_j\sigma^2_{t+j-1}\right] + \left(\Delta_2 + (1-\gamma)\Sigma_2\right)W_{t+2} - \frac{1}{2}\left|(1-\gamma)\Sigma_2\right|^2\right]}{E_t\left[\exp\left[\sum_{j=2}^{h}\left[\widetilde{\Psi}_j\sigma^2_{t+j-1}\right] + \left(\Delta_2 + (1-\widetilde{\gamma})\Sigma_2\right)W_{t+2} - \frac{1}{2}\left|(1-\widetilde{\gamma})\Sigma_2\right|^2 + (1-\gamma)\widetilde{\psi}_v\alpha_\sigma w_{\sigma,t+1} - \frac{1}{2}\left|(1-\gamma)\widetilde{\psi}_v\alpha_\sigma\right|^2\right]\right]} \\
&= \frac{E_t \exp\left[\alpha_\sigma\left(\nu_\sigma w_{\sigma,t+1} + w_{\sigma,t+2}\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + \left(\Delta_2 + (1-\gamma)\Sigma_2\right)W_{t+2} - \frac{1}{2}\left|(1-\gamma)\Sigma_2\right|^2\right]}{E_t \exp\left[\alpha_\sigma\left(\nu_\sigma w_{\sigma,t+1} + w_{\sigma,t+2}\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + \left(\Delta_2 + (1-\widetilde{\gamma})\Sigma_2\right)W_{t+2} - \frac{1}{2}\left|(1-\widetilde{\gamma})\Sigma_2\right|^2 + (1-\gamma)\widetilde{\psi}_v\alpha_\sigma w_{\sigma,t+1} - \frac{1}{2}\left|(1-\gamma)\widetilde{\psi}_v\alpha_\sigma\right|^2\right]} \\
&= \frac{E_t \exp\left[\alpha_\sigma\nu_\sigma w_{\sigma,t+1}\left(\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + \Psi_2\right) + \Psi_2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right) + \frac{1}{2}\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}\left(\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + 2(1-\gamma)\widetilde{\psi}_v\right)\alpha_\sigma^2\right]}{E_t \exp\left[\frac{1}{2}\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}\left(\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + 2(1-\widetilde{\gamma})\widetilde{\psi}_v\right)\alpha_\sigma^2 + \widetilde{\Psi}_2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right) + \left(\nu_\sigma\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + (1-\gamma)\widetilde{\psi}_v + \widetilde{\Psi}_2\right)\alpha_\sigma w_{\sigma,t+1} - \frac{1}{2}\left|(1-\gamma)\widetilde{\psi}_v\alpha_\sigma\right|^2\right]} \\
&= \frac{\exp\left[\alpha_\sigma^2\left[\frac{1}{2}(\Psi_2)^2 + \frac{1+\nu_\sigma^2}{2}\left(\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}\right)^2 + \left(\Psi_2\nu_\sigma + (1-\gamma)\widetilde{\psi}_v\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}\right] + \Psi_2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right)\right]}{\exp\left[\alpha_\sigma^2\left[\frac{1}{2}\left(\widetilde{\Psi}_2\right)^2 + \frac{1+\nu_\sigma^2}{2}\left(\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}\right)^2 + \left(\widetilde{\Psi}_2\nu_\sigma + \left[(1-\widetilde{\gamma}) + \nu_\sigma(1-\gamma)\right]\widetilde{\psi}_v\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + (1-\gamma)\widetilde{\psi}_v\widetilde{\Psi}_2\right] + \widetilde{\Psi}_2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right)\right]} \\
&= \frac{\exp\left[\alpha_\sigma^2\left[\frac{1}{2}(\Psi_2)^2 + \left(\Psi_2\nu_\sigma + (1-\gamma)\widetilde{\psi}_v\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}\right] + \Psi_2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right)\right]}{\exp\left[\alpha_\sigma^2\left[\frac{1}{2}\left(\widetilde{\Psi}_2\right)^2 + \left(\widetilde{\Psi}_2\nu_\sigma + \left[(1-\widetilde{\gamma}) + \nu_\sigma(1-\gamma)\right]\widetilde{\psi}_v\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} + (1-\gamma)\widetilde{\psi}_v\widetilde{\Psi}_2\right] + \widetilde{\Psi}_2\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right)\right]} \\
&= \exp\left[\frac{1}{2}\alpha_\sigma^2\left(\Psi_2^2 - \widetilde{\Psi}_2^2\right) + \left(\Psi_2 - \widetilde{\Psi}_2\right)\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2 + \alpha_\sigma^2\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2}\right) + (\widetilde{\gamma}-\gamma)\widetilde{\psi}_v\alpha_\sigma^2\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} - (1-\gamma)\widetilde{\psi}_v\alpha_\sigma^2\sum_{j=2}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\log E_t\left(R^P_{t+1,h}\right) = {} & -\log\beta + \mu_c + \phi_c x_t + \left(\chi - \frac{1}{2}\right)\alpha_c^2\sigma_t^2 - (1-\gamma)\sigma_t^2\left[\phi_v(\phi_d - \phi_c)\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2 + (\chi-1)\alpha_c^2\right] \\
& + \underbrace{(\widetilde{\gamma}-\gamma)\left[\phi_v(\phi_d - \phi_c)\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2 + (\chi-1)\alpha_c^2\right]}_{<0}\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right) \\
& + \alpha_\sigma^2\left[\frac{1}{2}\left(\Psi_2^2 - \widetilde{\Psi}_2^2\right) + \left(\Psi_2 - \widetilde{\Psi}_2\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2} + (\widetilde{\gamma}-\gamma)\widetilde{\psi}_v\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3} - (1-\gamma)\widetilde{\psi}_v\sum_{j=2}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\log E_t\left(R^P_{t+1,h}\right) = {} & -\log\beta + \mu_c + \phi_c x_t + \left(\chi - \frac{1}{2}\right)\alpha_c^2\sigma_t^2 - (1-\gamma)\sigma_t^2\left[\phi_v(\phi_d - \phi_c)\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2 + (\chi-1)\alpha_c^2\right] \\
& + \underbrace{(\widetilde{\gamma}-\gamma)\left[\phi_v(\phi_d - \phi_c)\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2 + (\chi-1)\alpha_c^2\right]}_{<0}\left(\sigma^2(1-\nu_\sigma) + \nu_\sigma\sigma_t^2\right) \\
& + \alpha_\sigma^2\underbrace{(\gamma-1)\widetilde{\psi}_v}_{<0}\underbrace{\sum_{j=2}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2}}_{<0 \text{ under } \gamma \text{ but } >0 \text{ for sufficiently low } \widetilde{\gamma}} \\
& + \alpha_\sigma^2\left[\frac{1}{2}\left(\Psi_2^2 - \widetilde{\Psi}_2^2\right) + \underbrace{\left(\Psi_2 - \widetilde{\Psi}_2\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2} + (\widetilde{\gamma}-\gamma)\widetilde{\psi}_v\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}}_{<0 \text{ for sufficiently low } \widetilde{\gamma}}\right]
\end{aligned}
$$

Note: we write $\Phi_k = \frac{\widetilde{\Psi}_{h+1-k}}{\nu_\sigma^k} \Longrightarrow \nu_\sigma^{h-2}\sum_{k=1}^{h-2}\Phi_k = \sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-3}$ in the matlab document

To obtain the returns on bonds, and their expected excess returns, replace ϕ_d, α_d and χ by 0 in the formula above:

$$\log E_t\left(R^B_{t+1,h}\right) = -\log\beta + \mu_c + \phi_c x_t - \frac{1}{2}\alpha_c^2\sigma_t^2 + (1-\gamma)\,\sigma_t^2\left[\phi_v\phi_c\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2 + \alpha_c^2\right]$$
$$\underbrace{-\left(\widetilde{\gamma}-\gamma\right)\left[\phi_v\phi_c\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2+\alpha_c^2\right]}_{<0}\left(\sigma^2\left(1-\nu_\sigma\right)+\nu_\sigma\sigma_t^2\right)$$
$$+\,\alpha_\sigma^2\underbrace{\left(\gamma-1\right)\widetilde{\psi}_v}_{<0}\qquad\underbrace{\sum_{j=2}^{h}\widetilde{\Psi}_{B,j}\nu_\sigma^{j-2}}_{<0 \text{ under } \gamma \text{ but } >0 \text{ for sufficiently low } \widetilde{\gamma}}$$
$$+\,\alpha_\sigma^2\left[\frac{1}{2}\left(\Psi_{B,2}^2-\widetilde{\Psi}_{B,2}^2\right)+\underbrace{\left(\Psi_{B,2}-\widetilde{\Psi}_{B,2}\right)\sum_{j=3}^{h}\widetilde{\Psi}_{B,,j}\nu_\sigma^{j-2}+\left(\widetilde{\gamma}-\gamma\right)\widetilde{\psi}_v\sum_{j=3}^{h}\widetilde{\Psi}_{B,j}\nu_\sigma^{j-3}}_{<0 \text{ for sufficiently low } \widetilde{\gamma}}\right]$$

where

$$\widetilde{\Psi}_{B,j} = \frac{1}{2}\left(\left(\phi_c\frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x\right)^2+\alpha_c^2\right)-\left(1-\widetilde{\gamma}\right)\left[\phi_v\phi_c\frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x^2+\alpha_c^2\right]$$

and

$$\widetilde{\Psi}_j-\widetilde{\Psi}_{B,j} = \frac{1}{2}\left(\phi_d\left(\phi_d-2\phi_c\right)\left(\frac{1-\nu_x^{h-j}}{1-\nu_x}\alpha_x\right)^2+\alpha_d^2+\chi\left(\chi-2\right)\alpha_c^2\right)+\left(1-\widetilde{\gamma}\right)\left[\phi_v\phi_c\frac{1-\nu_x^{h-2}}{1-\nu_x}\alpha_x^2+\alpha_c^2\right]$$

$$\log E_t\left(R^{P,F}_{t+1,h}\right) = \gamma\chi\alpha_c^2\sigma_t^2-\left(1-\gamma\right)\sigma_t^2\left[\phi_v\phi_d\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2\right]$$
$$\underbrace{+\left(\widetilde{\gamma}-\gamma\right)\left[\phi_v\phi_d\frac{1-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2+\chi\alpha_c^2\right]}_{<0}\left(\sigma^2\left(1-\nu_\sigma\right)+\nu_\sigma\sigma_t^2\right)$$
$$+\,\alpha_\sigma^2\underbrace{\left(\gamma-1\right)\widetilde{\psi}_v}_{<0}\qquad\underbrace{\sum_{j=2}^{h}\left(\widetilde{\Psi}_j-\widetilde{\Psi}_{B,j}\right)\nu_\sigma^{j-2}}_{<0 \text{ under } \gamma \text{ but } >0 \text{ for sufficiently low } \widetilde{\gamma}}$$
$$+\,\alpha_\sigma^2\left[\begin{cases}\frac{1}{2}\left(\Psi_2^2-\widetilde{\Psi}_2^2\right)+\left(\Psi_2-\widetilde{\Psi}_2\right)\sum_{j=3}^{h}\widetilde{\Psi}_j\nu_\sigma^{j-2}\\-\frac{1}{2}\left(\Psi_{B,2}^2-\widetilde{\Psi}_{B,2}^2\right)+\left(\Psi_{B,2}-\widetilde{\Psi}_{B,2}\right)\sum_{j=3}^{h}\widetilde{\Psi}_{B,j}\nu_\sigma^{j-2}\end{cases}+\left(\widetilde{\gamma}-\gamma\right)\widetilde{\psi}_v\sum_{j=3}^{h}\left(\widetilde{\Psi}_j-\widetilde{\Psi}_{B,j}\right)\nu_\sigma^{j-3}\right]$$

□

Using

$$r_{f,t} = -\log\beta+\mu_c+\phi_c x_t-\left(\gamma-\frac{1}{2}\right)\alpha_c^2\sigma_t^2$$

we have

$$E_t\left[\beta^h\left(\frac{C_{t+h}}{C_t}\right)^{-1}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\right]\propto$$

$$\exp\left[\left(\left(\tilde{\gamma}-\frac{1}{2}\right)\frac{1-\nu_\sigma^h}{1-\nu_\sigma}+(\gamma-\tilde{\gamma})\right)\alpha_c^2\sigma_t^2\right]\times$$

$$E_t\left[\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h-1}}{E_{t+h-2}\left[\widetilde{V}_{t+h-1}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\exp\left(-\phi_c\left(x_t+\ldots+x_{t+h-1}\right)\right)\right]$$

$$\begin{aligned}\exp\left(-\phi_c x_{t+h-1}\right)&=\exp\left(-\phi_c\left(\nu_x^2 x_{t+h-3}+\nu_x\alpha_x\sigma_{t+h-3}W_{t+h-2}+\alpha_x\sigma_{t+h-2}W_{t+h-1}\right)\right)\\&=\exp\left(-\phi_c\left(\nu_x^{h-1}x_t+\alpha_x\left(\nu_x^{h-2}\sigma_t W_{t+1}+\ldots+\sigma_{t+h-2}W_{t+h-1}\right)\right)\right)\end{aligned}$$

$$\exp\left(-\phi_c x_{t+h-2}\right)=\exp\left(-\phi_c\left(\nu_x^{h-2}x_t+\alpha_x\left(\nu_x^{h-3}\sigma_t W_{t+1}+\ldots+\sigma_{t+h-3}W_{t+h-2}\right)\right)\right)$$

$$\exp\left(-\phi_c\left(x_t+\ldots+x_{t+h-1}\right)\right)=$$
$$\exp\left(-\frac{\phi_c}{1-\nu_x}\left(\left(1-\nu_x^h\right)x_t+\alpha_x\left(\left(1-\nu_x^{h-1}\right)\sigma_t W_{t+1}+\ldots+\left(1-\nu_x\right)\sigma_{t+h-2}W_{t+h-1}\right)\right)\right)$$

$$E_t\left[\left(\frac{C_{t+h}}{C_t}\right)^{-1}\left(\frac{\widetilde{V}_{t+1}}{E_t\left[\widetilde{V}_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma}\left(\frac{\widetilde{V}_{t+2}}{E_{t+1}\left[\widetilde{V}_{t+2}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\cdots\left(\frac{\widetilde{V}_{t+h}}{E_{t+h-1}\left[\widetilde{V}_{t+h}^{1-\widetilde{\gamma}}\right]^{\frac{1}{1-\widetilde{\gamma}}}}\right)^{1-\widetilde{\gamma}}\right]\propto$$
$$\exp\left(\sum_{j=1}^{h-1}\left[\frac{1-\nu_x^j}{1-\nu_x}\left(\frac{1}{2}\phi_c\frac{1-\nu_x^j}{1-\nu_x}-(1-\tilde{\gamma})\phi_v\right)\right]\nu_\sigma^{h-1-j}+\frac{1-\nu_x^{h-1}}{1-\nu_x}\left(\left(\gamma-\tilde{\gamma}\right)\phi_v\right)\right)\phi_c\alpha_x^2\sigma_t^2$$

We painfully arrive at

$$\begin{aligned}\Sigma_{h,t}^B=&\left(\sum_{j=1}^{h-1}\left[\frac{1-\nu_x^j}{1-\nu_x}\left(\frac{1}{2}\phi_c\frac{1-\nu_x^j}{1-\nu_x}-(1-\tilde{\gamma})\phi_v\right)\right]\nu_\sigma^{h-1-j}+\frac{1-\nu_x^{h-1}}{1-\nu_x}\left(\left(\gamma-\tilde{\gamma}\right)\phi_v\right)\right)\phi_c\alpha_x^2\\&+\left(\left(\tilde{\gamma}-\frac{1}{2}\right)\frac{1-\nu_\sigma^h}{1-\nu_\sigma}+(\gamma-\tilde{\gamma})\right)\alpha_c^2\end{aligned}$$

□

C Approximation for $\beta \approx 1$

As in Appendix B, consider the simplified model with only two levels of risk aversion:

$$V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathcal{R}_{t,\gamma}\left(\widetilde{V}_{t+1}\right)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$
$$\widetilde{V}_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathcal{R}_{t,\widetilde{\gamma}}\left(\widetilde{V}_{t+1}\right)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where

$$\mathcal{R}_{t,\lambda}(X) = \left(E_t\left(X^{1-\lambda}\right)\right)^{\frac{1}{1-\lambda}}.$$

Also, as in Appendix B, take the evolutions:

$$c_{t+1} - c_t = \mu + \phi_c x_t + \alpha_c \sigma_t W_{t+1},$$
$$x_{t+1} = \nu_x x_t + \alpha_x \sigma_t W_{t+1},$$
$$\sigma_{t+1}^2 - \sigma^2 = \nu_\sigma\left(\sigma_t^2 - \sigma^2\right) + \alpha_\sigma W_{t+1},$$

and suppose the three shocks are independent. (We can relax this assumption.)

For β close to 1, we have:

$$\left(\frac{\widetilde{V}_t}{C_t}\right)^{1-\widetilde{\gamma}} \approx \beta^{\frac{1-\widetilde{\gamma}}{1-\rho}} E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\widetilde{\gamma}}\right].$$

This is an eigenfunction problem with eigenvalue $\beta^{-\frac{1-\widetilde{\gamma}}{1-\rho}}$ and eigenfunction $\left(\widetilde{V}/C\right)^{1-\widetilde{\gamma}}$ known up to a multiplier. Let's assume:

$$\widetilde{v}_t - c_t = \tilde{\mu}_v + \phi_v x_t + \tilde{\psi}_v \sigma_t^2.$$

Then we have:

- Terms in x_t (standard formula with $\beta = 1$):

$$\phi_v = \phi_c\left(I - \nu_x\right)^{-1}$$

- Terms in σ_t^2:

$$\tilde{\psi}_v = \frac{1}{2}\frac{1-\widetilde{\gamma}}{1-\nu_\sigma}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) < 0$$

- Constant terms:

$$\log\beta = -(1-\rho)\left(\mu + \tilde{\psi}_v\sigma^2(1-\nu_\sigma) + \frac{1}{2}(1-\tilde{\gamma})\tilde{\psi}_v^2\alpha_\sigma^2\right)$$

  we verify the solution for β is such that $\beta < 1$ and $\beta \approx 1$. We find that, as long as $\tilde{\gamma} \leq 5$, $\beta < 1 \Leftrightarrow \rho < 1$; and $\beta \approx 1$ is easily satisfied even for very low levels of ρ. e.g. in the calibration of Section (5), $1 > \beta \geq 0.9988$ for $\rho = 0.2$ and $\tilde{\gamma} \leq 5$.

For β close to 1, we have:

$$\frac{V_t}{\widetilde{V}_t} \approx \frac{\mathcal{R}_{t,\gamma}\left(\widetilde{V}_{t+1}\right)}{\mathcal{R}_{t,\widetilde{\gamma}}\left(\widetilde{V}_{t+1}\right)} = \frac{\left(E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\left(E_t\left[\left(\frac{\widetilde{V}_{t+1}}{C_{t+1}}\frac{C_{t+1}}{C_t}\right)^{1-\widetilde{\gamma}}\right]\right)^{\frac{1}{1-\widetilde{\gamma}}}},$$

and therefore:

$$v_t - \widetilde{v}_t = -\frac{1}{2}\left(\gamma - \widetilde{\gamma}\right)\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right],$$

The stochastic discount factor becomes:

$$\begin{aligned}\pi_{t,t+1} = \bar{\pi}_t - \gamma\alpha_c\sigma_t W_{t+1} + \left(\rho - \gamma\right)\phi_v\alpha_x\sigma_t W_{t+1} \\ + \left[\left(\rho - \gamma\right) + \left(1 - \rho\right)\left(\gamma - \widetilde{\gamma}\right)\frac{1 - \nu_\sigma}{1 - \widetilde{\gamma}}\right]\tilde{\psi}_v\alpha_\sigma W_{t+1},\end{aligned}$$

where

$$\begin{aligned}\bar{\pi}_t = \log\beta - \rho\mu_c - \rho\phi_c x_t - \left(\rho - \gamma\right)\frac{1}{2}\left(1 - \gamma\right)\left(\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2 + \widetilde{\psi}_v^2\alpha_\sigma^2\right]\right) \\ + \left(1 - \rho\right)\frac{1}{2}\left(\gamma - \widetilde{\gamma}\right)\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left(\nu_\sigma\sigma_t^2 + \sigma^2\left(1 - \nu_\sigma\right)\right) + \widetilde{\psi}_v^2\alpha_\sigma^2\right]\end{aligned}$$

$$\begin{aligned}\bar{\pi}_t = & -\mu_c - \rho\phi_c x_t - \left(1 - \rho\right)\frac{1}{2}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left(\frac{1 - \widetilde{\gamma}}{1 - \nu_\sigma} - \left(\gamma - \widetilde{\gamma}\right)\right)\sigma^2\left(1 - \nu_\sigma\right) \\ & - \frac{1}{2}\left(1 - \gamma\right)^2\tilde{\psi}_v^2\alpha_\sigma^2 \\ & - \frac{1}{2}\left(\left(\rho - \gamma\right)\left(1 - \gamma\right) - \left(1 - \rho\right)\left(\gamma - \widetilde{\gamma}\right)\nu_\sigma\right)\left(\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2\right]\right)\end{aligned}$$

The risk-free rate is defined as $r_{f,t} = -\log E_t\left(\Pi_{t,t+1}\right):$

$$\begin{aligned}r_{f,t} = & \ \mu_c + \rho\phi_c x_t + \left(1 - \rho\right)\frac{1}{2}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left(\frac{1 - \widetilde{\gamma}}{1 - \nu_\sigma} - \left(\gamma - \widetilde{\gamma}\right)\right)\sigma^2\left(1 - \nu_\sigma\right) \\ & + \frac{1}{2}\left[\left(1 - \gamma\right)^2 - \left[\left(\rho - \gamma\right) + \left(1 - \rho\right)\left(\gamma - \widetilde{\gamma}\right)\frac{1 - \nu_\sigma}{1 - \widetilde{\gamma}}\right]^2\right]\tilde{\psi}_v^2\alpha_\sigma^2 \\ & + \frac{1}{2}\left(\left(\rho - \gamma\right)\left(1 - \gamma\right) - \left(1 - \rho\right)\left(\gamma - \widetilde{\gamma}\right)\nu_\sigma\right)\left(\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2\right]\right) \\ & - \frac{1}{2}\left(\gamma^2\alpha_c^2\sigma_t^2 + \left(\rho - \gamma\right)^2\phi_v^2\alpha_x^2\sigma_t^2\right)\end{aligned}$$

Note the risk-free rate now depends on $\tilde{\gamma}$. □

C.1 Term structure of returns

C.1.1 General claims

To make the problem as general as possible, we analyze horizon-dependent claims that are priced recursively as

$$Y_{t,h} = E_t\left[\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right],$$

that is

$$y_{t,h} = E_t\left[\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}\right] + \frac{1}{2}\mathrm{var}_t\left(\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}\right),$$

where

$$g_{y,t+1} = \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ + \alpha_{y,c}\alpha_c\sigma_t w_{c,t+1} + \alpha_{y,x}\alpha_x\sigma_t w_{x,t+1} + \alpha_{y,\sigma}\alpha_\sigma\sigma_t w_{\sigma,t+1} + \alpha_{y,d}\alpha_d\sigma_t w_{d,t+1},$$

and $Y_{t,0} = 1$.

Guess that

$$Y_{t,h} = \exp\left(\widetilde{\mu}_{y,h} + \phi_{y,h}x_t + \tilde{\psi}_{y,h}\sigma_t^2\right).$$

Suppose $h \geq 1$, then:

$$\pi_{t,t+1} = \bar{\pi}_t - \gamma\alpha_c\sigma_t W_{t+1} + (\rho-\gamma)\,\phi_v\alpha_x\sigma_t W_{t+1} \\ + \left[(\rho-\gamma) + (1-\rho)\,(\gamma-\widetilde{\gamma})\,\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\tilde{\psi}_v\alpha_\sigma W_{t+1},$$

$$\log \widetilde{\Pi}_{t,t+1}G_{t,t+1}Y_{t+1,h-1} = \begin{cases} \bar{\pi}_t \\ +\mu_y + \phi_y x_t + \psi_y\sigma_t^2 \\ +\widetilde{\mu}_{y,h-1} + \phi_{y,h-1}\nu_x x_t + \psi_{y,h-1}\left(\sigma^2\left(1-\nu_\sigma\right) + \nu_\sigma\sigma_t^2\right) \\ +\left(-\gamma+\alpha_{y,c}\right)\alpha_c\sigma_t W_{t+1} + \left((\rho-\gamma)\,\phi_v + \alpha_{y,x} + \phi_{y,n-1}\right)\alpha_x\sigma_t W_{t+1} \\ +\left(\left[(\rho-\gamma) + (1-\rho)\,(\gamma-\widetilde{\gamma})\,\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,h-1}\right)\alpha_\sigma W_{t+1} \\ +\alpha_{y,d}\alpha_d\sigma_t W_{t+1} \end{cases}$$

where

$$\bar{\pi}_t = -\mu_c - \rho\phi_c x_t - (1-\rho)\,\frac{1}{2}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left(\frac{1-\widetilde{\gamma}}{1-\nu_\sigma} - (\gamma-\widetilde{\gamma})\right)\sigma^2\left(1-\nu_\sigma\right) \\ -\frac{1}{2}\,(1-\gamma)^2\,\tilde{\psi}_v^2\alpha_\sigma^2 \\ -\frac{1}{2}\left((\rho-\gamma)\,(1-\gamma) - (1-\rho)\,(\gamma-\widetilde{\gamma})\,\nu_\sigma\right)\left(\left[\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\sigma_t^2\right]\right)$$

Matching coefficients, we find the recursions, for $h \geq 1$:

- Terms in x_t:

$$\phi_{y,h} = -\rho\phi_c + \phi_y + \phi_{y,h-1}\nu_x \\ \Rightarrow \quad \phi_{y,h} = \left(-\rho\phi_c + \phi_y\right)\frac{1-\nu_x^h}{1-\nu_x}$$

- Terms in σ_t^2:

$$\tilde{\psi}_{y,h} = -\frac{1}{2}\left((\rho-\gamma)\,(1-\gamma) - (1-\rho)\,(\gamma-\widetilde{\gamma})\,\nu_\sigma\right)\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right) + \tilde{\psi}_{y,h-1}\nu_\sigma + \psi_y \\ + \frac{1}{2}\left(\left(-\gamma+\alpha_{y,c}\right)^2\alpha_c^2 + \left((\rho-\gamma)\,\phi_v + \alpha_{y,x} + \phi_{y,h-1}\right)^2\alpha_x^2 + \alpha_{y,d}^2\alpha_d^2\right)$$

- Constant:

$$\widetilde{\mu}_{y,h} - \widetilde{\mu}_{y,h-1} = -\mu_c - (1-\rho)\frac{1}{2}\left(\alpha_c^2 + \phi_v^2\alpha_x^2\right)\left(\frac{1-\widetilde{\gamma}}{1-\nu_\sigma} - (\gamma - \widetilde{\gamma})\right)\sigma^2(1-\nu_\sigma)$$
$$+\frac{1}{2}\left[\left(\left[(\rho-\gamma) + (1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v + \alpha_{y,\sigma} + \tilde{\psi}_{y,h-1}\right)^2 - (1-\gamma)^2\widetilde{\psi}_v^2\right]\alpha_\sigma^2$$
$$+\mu_y + \sigma^2(1-\nu_\sigma)\tilde{\psi}_{y,h-1}$$

Note only both the constant terms $\left\{\widetilde{\mu}_{y,h}\right\}$ and the loadings on the volatility shocks $\left\{\widetilde{\psi}_{y,h}\right\}$ are affected by the wedge between γ and $\widetilde{\gamma}$.

□

In line with the specification of van Binsbergen and Koijen (2016), we consider one-period holding returns for these claims of the form

$$1 + R^Y_{t+1,h} = \frac{G_{y,t+1}Y_{t+1,h-1}}{Y_{t,h}} = \frac{G_{y,t+1}Y_{t+1,h-1}}{E_t\left[\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right]}$$
$$= R_{f,t}\frac{E_t[\Pi_{t,t+1}]\,G_{y,t+1}Y_{t+1,h-1}}{E_t\left[\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right]},$$

with the risk-free rate

$$R_{f,t} = \frac{1}{E_t[\Pi_{t,t+1}]}.$$

In line with the specification of van Binsbergen and Koijen (2016), we also consider one-period holding returns for futures on these claims of the form

$$R^{F,Y}_{t+1,h} + 1 = \frac{1 + R^Y_{t+1,h}}{1 + R^B_{t+1,h}} = \frac{G_{y,t+1}Y_{t+1,h-1}}{Y_{t,h}}\frac{B_{t,h}}{B_{t+1,h-1}}$$
$$= \frac{G_{y,t+1}Y_{t+1,h-1}}{E_t\left(\Pi_{t,t+1}G_{y,t+1}Y_{t+1,h-1}\right)}\frac{E_t\left(\Pi_{t,t+1}B_{t+1,h-1}\right)}{B_{t+1,h-1}},$$

where $B_{t,h}$ is the price of \$1 at horizon h, i.e. the price of a Bond with horizon h.

Their conditional Sharpe Ratio is

$$\mathrm{SR}^{F,Y}_{t,h} = \frac{E_t\left(1 + R^{F,Y}_{t+1,h}\right) - 1}{\sqrt{\mathrm{var}_t\left(1 + R^{F,Y}_{t+1,h}\right)}}$$
$$= \frac{E_t\left(1 + R^{F,Y}_{t+1,h}\right) - 1}{\sqrt{E_t\left(\left(1 + R^{F,Y}_{t+1,h}\right)^2\right) - \left(E_t\left(1 + R^{F,Y}_{t+1,h}\right)\right)^2}}$$
$$\approx \frac{\begin{cases}\sigma_t^2\left(\gamma\alpha_{y,c}\alpha_c^2 - \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1}\right)\left((\rho-\gamma)\phi_v + \phi_{b,h-1}\right)\alpha_x^2\right) \\ -\left(\alpha_{y,\sigma} + \tilde{\psi}_{y,h-1} - \tilde{\psi}_{b,h-1}\right)\left(\left[(\rho-\gamma) + (1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v + \tilde{\psi}_{b,h-1}\right)\alpha_\sigma^2\end{cases}}{\sqrt{\sigma_t^2\left(\alpha_{y,c}^2\alpha_c^2 + \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1}\right)^2\alpha_x^2 + \alpha_{y,d}^2\alpha_d^2\right) + \left(\alpha_{y,\sigma} + \tilde{\psi}_{y,h-1} - \tilde{\psi}_{b,h-1}\right)^2\alpha_\sigma^2}}.$$

C.1.2 Bonds

Let the price at time t for \$1 in h periods be $B_{t,h}$ with $B_{t,0} = 1$. For $h \geq 1$, we have

$$B_{t,h} = E_t[\Pi_{t,t+1} B_{t+1,h-1}]$$

This is the general problem from above with $g_{y,t+1} = 0$ for all t and therefore

$$b_{t,h} = \widetilde{\mu}_{b,h} + \phi_{b,h} x_t + \tilde{\psi}_{b,h} \sigma_t^2,$$

with

$$\phi_{b,h} = -\rho \phi_c \frac{1 - \nu_x^h}{1 - \nu_x}$$

$$\begin{aligned}\tilde{\psi}_{b,h} = &-\frac{1}{2}\left((\rho - \gamma)(1 - \gamma) - (1 - \rho)(\gamma - \widetilde{\gamma})\nu_\sigma\right)\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right) + \tilde{\psi}_{b,h-1}\nu_\sigma \\ &+ \frac{1}{2}\left(\gamma^2 \alpha_c^2 + \left((\rho - \gamma)\phi_v + \phi_{b,h-1}\right)^2 \alpha_x^2\right)\end{aligned}$$

C.1.3 Dividend strips

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$ with $P_{t,0} = D_t$. Then for $h \geq 1$:

$$\frac{P_{t,h}}{D_t} = E_t\left(\Pi_{t,t+1} \frac{D_{t+1}}{D_t} \frac{P_{t+1,h-1}}{D_{t+1}}\right),$$

which is the general problem from above with

$$g_{p,t+1} = d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1},$$

for all t and therefore

$$p_{t,h} - d_t = \widetilde{\mu}_{p,h} + \phi_{d,h} x_t + \tilde{\psi}_{d,h} \sigma_t^2,$$

with

$$\phi_{d,h} = (-\rho \phi_c + \phi_d) \frac{1 - \nu_x^h}{1 - \nu_x}$$

$$\begin{aligned}\tilde{\psi}_{d,h} = &-\frac{1}{2}\left((\rho - \gamma)(1 - \gamma) - (1 - \rho)(\gamma - \widetilde{\gamma})\nu_\sigma\right)\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right) + \tilde{\psi}_{d,h-1}\nu_\sigma \\ &+ \frac{1}{2}\left((-\gamma + \chi)^2 \alpha_c^2 + \left((\rho - \gamma)\phi_v + \phi_{d,h-1}\right)^2 \alpha_x^2 + \alpha_d^2\right)\end{aligned}$$

$$\begin{aligned}\widetilde{\mu}_{y,h} - \widetilde{\mu}_{y,h-1} = &-\mu_c - (1 - \rho)\frac{1}{2}\left(\alpha_c^2 + \phi_v^2 \alpha_x^2\right)\left(\frac{1 - \widetilde{\gamma}}{1 - \nu_\sigma} - (\gamma - \widetilde{\gamma})\right)\sigma^2 (1 - \nu_\sigma) \\ &+ \frac{1}{2}\left[\left(\left[(\rho - \gamma) + (1 - \rho)(\gamma - \widetilde{\gamma})\frac{1 - \nu_\sigma}{1 - \widetilde{\gamma}}\right]\widetilde{\psi}_v + \tilde{\psi}_{d,h-1}\right)^2 - (1 - \gamma)^2 \widetilde{\psi}_v^2\right]\alpha_\sigma^2 \\ &+ \mu_d + \sigma^2 (1 - \nu_\sigma)\tilde{\psi}_{d,h-1}\end{aligned}$$

For the dividend strips, the spot one-period returns are given by

$$R^P_{t+1,h} + 1 = \frac{P_{t+1,h-1}/D_{t+1}}{P_{t,h}/D_t}\frac{D_{t+1}}{D_t},$$

$$\begin{aligned}
\log\left(R^P_{t+1,h}+1\right) &= \mu_c + (1-\rho)\frac{1}{2}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left(\frac{1-\widetilde{\gamma}}{1-\nu_\sigma} - (\gamma-\widetilde{\gamma})\right)\sigma^2(1-\nu_\sigma) \\
&- \frac{1}{2}\left[\left(\left[(\rho-\gamma)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v + \tilde{\psi}_{d,h-1}\right)^2 - (1-\gamma)^2\widetilde{\psi}_v^2\right]\alpha_\sigma^2 \\
&+ \rho\phi_c x_t + \left(\tilde{\psi}_{d,h-1}\nu_\sigma - \tilde{\psi}_{d,h}\right)\sigma_t^2 \\
&+ \tilde{\psi}_{d,h-1}\alpha_\sigma W_{t+1} + \phi_{d,h-1}\alpha_x\sigma_t W_{t+1} + \chi\alpha_c\sigma_t W_{t+1} + \alpha_d\sigma_t W_{t+1}
\end{aligned}$$

the conditional expected one-period returns are

$$\begin{aligned}
E_t\left(R^P_{t+1,h}+1\right) \approx \text{constant (in h)} &- \left[(\rho-\gamma)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v\tilde{\psi}_{d,h-1}\alpha_\sigma^2 \\
&+ \left(\tilde{\psi}_{d,h-1}\nu_\sigma - \tilde{\psi}_{d,h}\right)\sigma_t^2 + \frac{1}{2}\left(\phi_{d,h-1}^2\alpha_x^2\sigma_t^2\right)
\end{aligned}$$

$$\begin{aligned}
E_t\left(R^P_{t+1,h}+1\right) \approx \text{constant (in h)} &- \left[(\rho-\gamma)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v\tilde{\psi}_{d,h-1}\alpha_\sigma^2 \\
&- \left((\rho-\gamma)\phi_v\phi_{d,h-1}\right)\alpha_x^2\sigma_t^2
\end{aligned}$$

$$\begin{aligned}
E_t\left(R^P_{t+1,h}\right) - E_t\left(R^P_{t+1,h-1}\right) \approx &\underbrace{\left[(\gamma-\rho)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v}_{\leq 0}\left(\tilde{\psi}_{d,h} - \tilde{\psi}_{d,h-1}\right)\alpha_\sigma^2 \\
&+ \underbrace{(\gamma-\rho)\phi_v(\rho\phi_c-\phi_d)\frac{\nu_x^h-\nu_x^{h-1}}{1-\nu_x}\alpha_x^2\sigma_t^2}_{\geq 0}
\end{aligned}$$

We need $(\tilde{\psi}_{d,h} - \tilde{\psi}_{d,h-1}) \geq 0$ to generate a downward sloping term structure. If $(\tilde{\psi}_{d,h} - \tilde{\psi}_{d,h-1}) \leq 0$, then the returns are upward sloping, but less so in our model.

Note, that the returns are MORE upward sloping when σ_t is high...

The future one-period returns are given by:

$$R^{F,P}_{t+1,h} + 1 = \frac{1+R^P_{t+1,h}}{1+R^B_{t+1,h}}$$

$$\log\left(R^{P}_{t+1,h}+1\right) = \mu_c + (1-\rho)\frac{1}{2}\left(\alpha_c^2+\phi_v^2\alpha_x^2\right)\left(\frac{1-\widetilde{\gamma}}{1-\nu_\sigma}-(\gamma-\widetilde{\gamma})\right)\sigma^2(1-\nu_\sigma)$$
$$-\frac{1}{2}\left[\left(\left[(\rho-\gamma)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v+\tilde{\psi}_{d,h-1}\right)^2-(1-\gamma)^2\widetilde{\psi}_v^2\right]\alpha_\sigma^2$$
$$+\rho\phi_c x_t + \left(\tilde{\psi}_{d,h-1}\nu_\sigma-\tilde{\psi}_{d,h}\right)\sigma_t^2$$
$$+\tilde{\psi}_{d,h-1}\alpha_\sigma W_{t+1}+\phi_{d,h-1}\alpha_x\sigma_t W_{t+1}+\chi\alpha_c\sigma_t W_{t+1}+\alpha_d\sigma_t W_{t+1}$$

$$\log\left(R^{F,P}_{t+1,h}+1\right) = -\left(\left[(\rho-\gamma)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v\left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)+\frac{1}{2}\left(\tilde{\psi}^2_{d,h-1}-\tilde{\psi}^2_{b,h-1}\right)\right)\alpha_\sigma^2$$
$$+\left(\left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\nu_\sigma-\left(\tilde{\psi}_{d,h}-\tilde{\psi}_{b,h}\right)\right)\sigma_t^2$$
$$+\left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\alpha_\sigma W_{t+1}+\left(\phi_{d,h-1}-\phi_{b,h-1}\right)\alpha_x\sigma_t W_{t+1}+\chi\alpha_c\sigma_t W_{t+1}+\alpha_d\sigma_t W_{t+1}$$

$$E_t\left(R^{F,P}_{t+1,h}+1\right) = -\left(\underbrace{\left(\left[(\rho-\gamma)+(1-\rho)(\gamma-\widetilde{\gamma})\frac{1-\nu_\sigma}{1-\widetilde{\gamma}}\right]\widetilde{\psi}_v+\tilde{\psi}_{b,h-1}\right)}_{\geq 0\text{ and increasing}}\left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\right)\alpha_\sigma^2$$
$$+\left(\gamma\chi\alpha_c^2+\underbrace{\left(\left(\gamma-\rho\right)\phi_v-\phi_{b,h-1}\right)\left(\phi_{d,h-1}-\phi_{b,h-1}\right)}_{\geq 0\text{ and increasing}}\alpha_x^2\right)\sigma_t^2$$

Note:

$$\tilde{\psi}_{d,h}-\tilde{\psi}_{b,h} = \left(\tilde{\psi}_{d,h-1}-\tilde{\psi}_{b,h-1}\right)\nu_\sigma$$
$$+\left(\chi\underbrace{\left(\frac{1}{2}\chi-\gamma\right)}_{\leq 0}\alpha_c^2+\underbrace{\left((\rho-\gamma)\phi_v+\frac{1}{2}\left(\phi_{d,h-1}+\phi_{b,h-1}\right)\right)}_{\leq 0\text{ for }\gamma\text{ high enough}}\underbrace{\left(\phi_{d,h-1}-\phi_{b,h-1}\right)}_{\geq 0}\alpha_x^2+\underbrace{\frac{1}{2}\alpha_d^2}_{\geq 0}\right)$$

the sign depends on the parameters. But if it is positive increasing, $\widetilde{\gamma}$ reduces the downward impact of it on the term structure of expected returns. Only if it is negative and decreasing does our model help relative to the standard model, but then the slope is upward sloping....

Note, a higher σ_t means a MORE upward sloping term structure again. □

D Additional figures

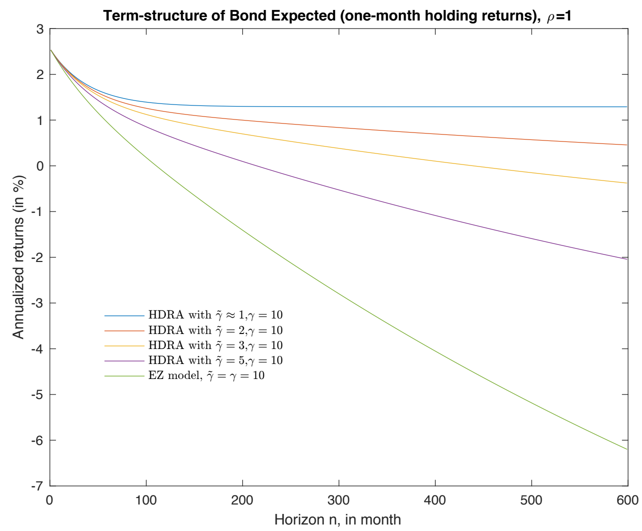

Figure 5: Term structure of bond returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1. Returns are conditional, with state variables set at their means: $x_t = 0$ and $\sigma_t = \sigma$.

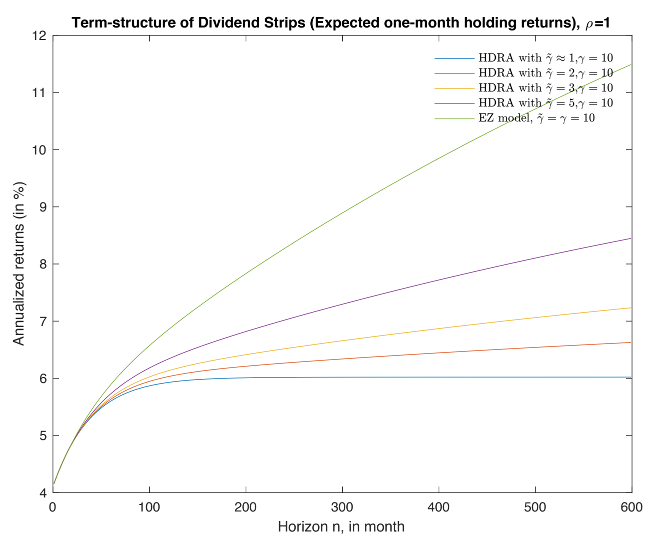

Figure 6: Term structure of dividend strip expected returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1. Returns are conditional, with state variables set at their means: $x_t = 0$ and $\sigma_t = \sigma$.

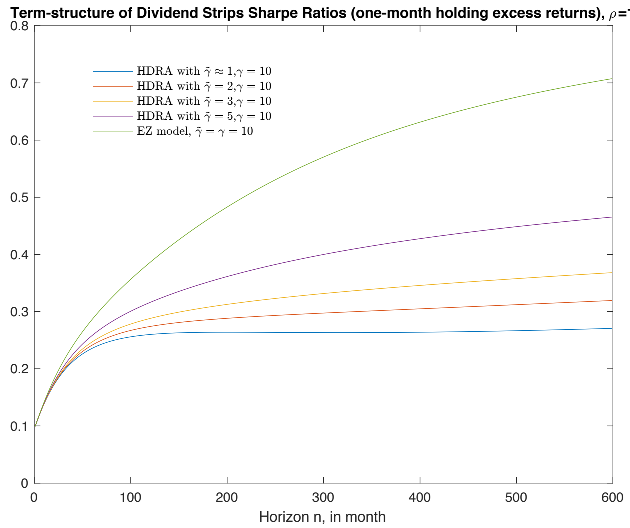

Figure 7: Term structure of dividend strip expected excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1. Returns are conditional, with state variables set at their means: $x_t = 0$ and $\sigma_t = \sigma$.

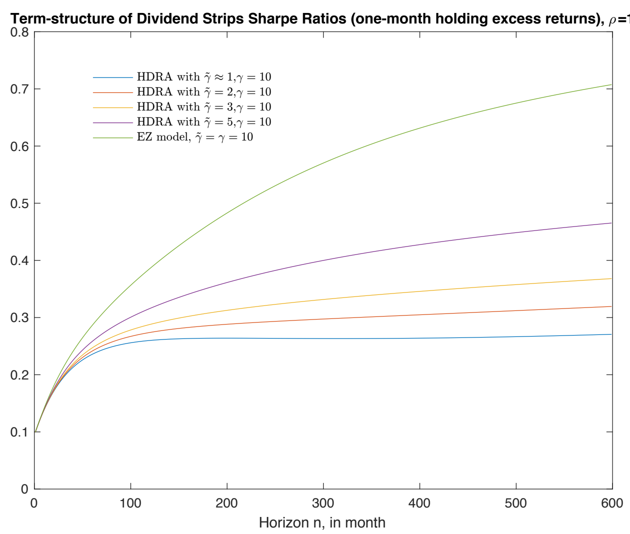

Figure 8: Term structure of dividend strip unconditional Sharpe ratios of excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.

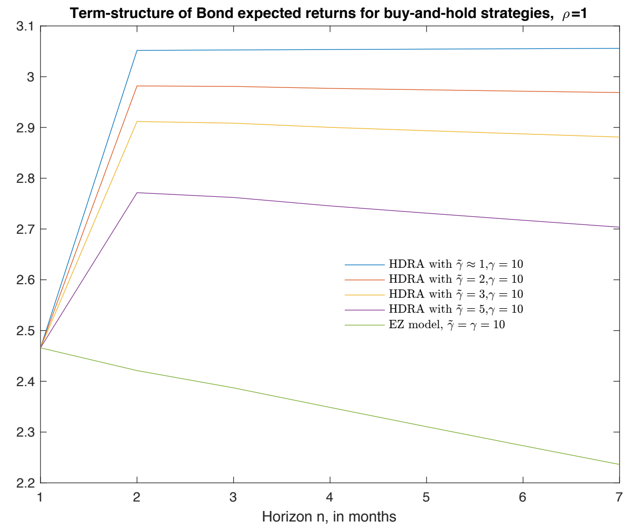

Figure 9: Term structure of of bond returns under illiquid buy-and-hold strategies, under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.

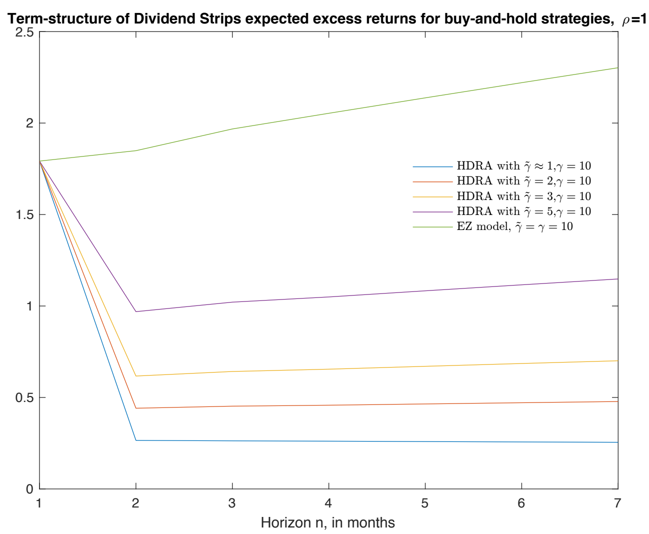

Figure 10: Term structure of dividend strip expected excess returns under illiquid buy-and-hold strategies, under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.