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Abstract

Inspired by experimental evidence, we amend the recursive utility model to let risk aversion decrease with the temporal horizon. Our pseudo-recursive preferences remain tractable and retain appealing features of the long-run risk framework, notably its success at explaining asset pricing moments. Calibrating the agents' preferences to explain the market returns observed in the data no longer implies an extreme preference for early resolutions of uncertainty and captures key puzzles in finance on the valuation and demand for risk at long maturities.

Key words: risk aversion, early resolution, term structure, volatility risk

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1 Introduction

We propose a model that relaxes the assumption, standard in the economics literature, that risk aversion is constant for all payoff horizons. We define pseudo-recursive preferences similar to Epstein-Zin (Epstein and Zin, 1989) but generalized to allow for horizon-dependent risk aversions. As the experimental evidence indicates, we assume agents are more risk averse for short horizons payoffs. Our model remains tractable, and usual recursive techniques can be applied. We find horizon-dependent risk aversion solves important puzzles in finance. First, the model can be calibrated to match the usual asset pricing moments without implying an extreme preference for early resolutions of uncertainty, difficult to reconcile with micro evidence and introspection and a fundamental challenge to the standard framework as pointed out in Epstein et al. (2014). Second, horizon-dependent risk aversion can reconcile the high market returns observed in the data with a strong appetite for long-term risk holdings. This explains, among others, the low risk premia of buy-and-hold assets such as private equity and housing documented in e.g. Moskowitz and Vissing-Jørgensen (2002); Giglio et al. (2014); Chambers et al. (2019); the downward sloping term-structures of excess equity returns during the recent financial crisis of 2007–2009 in van Binsbergen et al. (2012); Bansal et al. (2019); and the low demand for long-term insurance and for options with medium and long maturities, a puzzle discussed in e.g. Garleanu et al. (2008); Akaichi et al. (2019).

Our first contribution is methodological: we introduce horizon-dependent risk aversion within the standard recursive utility model of Epstein and Zin (1989), which allows us to build on its success at explaining asset pricing moments when combined with long-run risk. We assume risk aversions decrease with the temporal horizon, as suggested by experimental evidence, and analyze the pricing implications of our preference model. We show that commonly used recursive techniques can be adapted to our setting of pseudo-recursive preferences, enabling us to derive closed-form solutions. Our baseline model can accommodate numerous extensions, be it on the valuation of risk (habit formation, disappointment aversion, loss aversion, etc.), or on the quantity of risk (rare disasters, production-based models, etc.). Further, under our preference model, *inter-temporal* decisions for deterministic payoffs are unchanged from the standard, time consistent, model; but *intra-temporal* allocations across risky assets are dynamically time inconsistent. We can

therefore study the optimal decisions and pricing impact of horizon-dependent risk aversion in isolation from quasi-hyperbolic discounting, and in general from models of time inconsistent inter-temporal decisions.

Combining recursive Epstein-Zin preferences with risk to the expected growth and volatility of consumption, the standard long-run risk model has had great success at matching asset pricing moments and at explaining their apparent "puzzles" (see [Cochrane \(2016\)](#) for a review of the literature). It explains the high equity premium, matches various cross-sectional evidence, captures the macroeconomic announcement premium (excess returns around the Federal Reserve's regular monetary policy meetings), and results in time-varying risk premia that rationalize the volatility puzzle and return predictability. To do so, however, the model also implies that the representative agent has a high to extreme timing premium, a measure of her preferences for early versus late resolutions of uncertainty: 30% in the calibration of [Bansal and Yaron \(2004\)](#) and 80% in [Bansal, Kiku, and Yaron \(2009\)](#). This corresponds to the portion of her lifetime consumption the agent would be willing to forego in order to be told all her future consumption shocks in the next period rather than over time. Such a strong preference for an early resolution of uncertainty appears inconsistent with the evidence, for instance on investors' inattention to their wealth, and with commonsense considerations; raising doubts as to the validity of the whole model ([Epstein, Farhi, and Strzalecki, 2014](#)). Further, agents are exposed to larger aggregate risks the longer the horizon of asset payoffs, making it hard for the standard model to explain several observed features of the data: the low risk premia of illiquid long-term buy-and-hold assets such as private equity and housing; the examples of downward sloping term-structures of risk premia; the low demand for long-term insurance and for options with medium or long maturities.

The core contribution of our paper is to formally show horizon-dependent risk aversion can reconcile these sets of evidence, the macroeconomic asset pricing data with microeconomic attitudes towards information, risk valuations and risk appetites. To apply our utility model and methodology to equilibrium asset pricing, we consider a representative agent who trades and clears the market every period, and, as such, cannot pre-commit to any specific strategy: unable to commit to future behavior but aware of her dynamic inconsistency, in the spirit of [Strotz \(1955\)](#), the agent optimizes in the current period, fully anticipating reoptimization in future periods. Solving our model this way

yields a canonical one-period pricing problem in which the Euler equation is satisfied, and the law of one price and no-arbitrage conditions hold. In a Lucas-tree endowment economy with long-run risk, we find that the pricing of shocks that impact consumption *levels* is unchanged from the standard model — reflecting that the dynamic inconsistency in our model does not concern inter-temporal decisions. In contrast, shocks to consumption *risk* (volatility) directly affect intra-temporal decisions, and their pricing changes under horizon-dependent risk aversion: the lower risk aversion at long horizons reduces the pricing of volatility shocks and this effect accumulates over time. The model can be calibrated to capture the usual asset pricing moments, but because of this dichotomy in the pricing impact of our model, lowering long-horizon risk aversions compared to the short-horizon increases the impact of consumption level shocks on the equity premium relative to volatility shocks. This allows us to discipline the calibration of our model: we use recent evidence on the macroeconomic announcement premium as a share of the total equity premium (Lucca and Moench, 2015; Ai and Bansal, 2018) to quantify the wedge between short and long-horizon risk aversions. We obtain a long-term risk aversion roughly half that of the the immediate risk aversion, consistent with the estimates in the experimental literature (Onculer, 2000). In contrast, calibrating the same level of risk aversion for immediate and long-term risks, the standard model, overestimates the contribution of volatility risk prices to the equity premium.

We then analyze how horizon-dependent risk aversion affects the timing premium — the willingness to pay for early resolutions of uncertainty. Specifically, we formally derive how two consumption streams with identical risk but different timing for information arrivals are valued: one where shocks are revealed gradually as they are realized over time, the other where all future shocks are revealed at the same early date. Agents value these consumption streams differently, even though the ex-ante distributions of risk are rigorously identical. Whether and how the two valuations differ depends on the wedge between risk aversions for short-horizon payoffs versus for long-horizon payoffs as well as on their values relative to the elasticity of inter-temporal substitution. A consumption stream with early resolution of uncertainty shifts the risk of all future shocks into a short-horizon risk, moving from a risk assessment using the lower risk aversion at long horizons to a risk assessment using the higher risk aversion at short horizons. This lowers the attractiveness of early resolution of uncertainty, compared to the standard framework with Epstein-Zin

preferences. We formalize this intuition and prove the timing premium is unambiguously lowered when risk aversion is decreasing in horizon. Our model calibrated to match the usual asset pricing moments and the macro-announcement premium results in a reasonable level of timing premia, lower than 10%.

Finally we turn to the model's implications for the valuation and demand for risk at different horizons of asset payoffs. If agents trade every period, our calibrated model implies the compensation, or excess returns, they require for taking risk increases with the temporal horizon of payoffs: decreasing term structures of risk aversions do *not* imply decreasing term structures of risk premia. On the other hand, investors who adopt buy-and-hold strategies have greater risk appetites for assets with longer payoff horizon. The differential pricing implications of our model for liquid one-period risks and long-term locked-in investments can explain important puzzles in finance, in particular the low risk premia for private equity and housing, the sharply downward sloping term-structures of equity returns during the financial crisis of 2007–2009 and the low demand for long-term insurance and hedging options at all but immediate maturities.

In sum, the model of preferences we propose, where risk aversions differ for short-horizon and long-horizon payoffs, can address the early versus late resolution of uncertainty critique and explain several important puzzles in finance on the valuation and appetite for long-term risks. We can solve these challenges to the long-run risk framework concerning the timing and pricing of uncertainty without compromising on the model's ability to match the usual asset pricing moments, and without departing from the methodology of the widely-used Epstein-Zin preferences.

After a review of the literature, we present our model of preferences in Section 2. In Section 3, we derive the risk pricing implications of our model and its calibration. We analyze the preference for early or late resolution of uncertainty in Section 4. In Section 5, we analyze the valuation and demand for risk at different horizons; and how it relates to the evidence in the data. Section 6 concludes. All mathematical proofs are in the Appendix.

Related literature

This paper is the first to solve for equilibrium asset prices in an economy populated by agents with dynamically inconsistent risk aversions. Our methodology, which guarantees

the no-arbitrage condition despite time inconsistency, follows [Luttmer and Mariotti \(2003\)](#), and our work complements theirs. They show that dynamically inconsistent preferences for inter-temporal trade-offs of the kind examined by [Harris and Laibson \(2001\)](#) have only limited implications for asset pricing, and little power to explain cross-sectional variations in asset returns. Given that cross-sectional asset pricing involves intra-period risk-return tradeoffs, it is indeed quite intuitive that inter-temporal dynamic inconsistency is not suitable to address puzzles related to risk premia.

Our model generalizes Epstein-Zin preferences by relaxing the dynamic consistency axiom of [Kreps and Porteus \(1978\)](#) to analyze the relationship between the timing and pricing of uncertainty. We choose the CRRA model for risk adjustments, standard to the macro-finance literature. In contrast, [Routledge and Zin \(2010\)](#), [Bonomo et al. \(2011\)](#) and [Schreindorfer \(2014\)](#) follow [Gul \(1991\)](#) and relax the independence axiom to analyze the asset pricing impact of disappointment aversion within a recursive framework. They find that their models generate endogenous predictability ([Routledge and Zin, 2010](#)); match various asset pricing moments ([Bonomo et al., 2011](#)); and price the cross-section of options better than the standard model ([Schreindorfer, 2014](#)). Similarly, [Andries \(2015\)](#) introduces loss aversion in recursive preferences à la [Epstein and Zin \(1989\)](#) and shows it helps match the security market line, while [Dew-Becker \(2012\)](#) uses a model of habit to obtain time varying risk premia. Our framework can also accommodate these non-standard utility functions for the valuation of risk. Within the classical model of [Epstein and Zin \(1989\)](#), none of the above-mentioned preference models address the "excessive preference for early resolutions of uncertainty puzzle", pointed out by [Epstein et al. \(2014\)](#) or explain the pricing and demand for risk at long-horizons — the two questions of interest in our analysis.

To capture various asset pricing moments, the long-run risk literature relies on the pricing of shocks to consumption growth and to consumption volatility. [Hansen et al. \(2008\)](#) directly measure consumption growth shocks in the data, and [Bryzgalova and Julliard \(2015\)](#) use cross-sections of returns to provide evidence consumption growth shocks are priced, which is consistent also with equity premia around the Federal Open Market Committee (FOMC) meetings ([Lucca and Moench, 2015](#); [Ai and Bansal, 2018](#)). The importance of a volatility risk channel is supported by [Campbell et al. \(2016\)](#), who show that it is crucial for asset returns in a CAPM framework, and who relate this to other works on the

relation between volatility risk and returns (Ang et al., 2006; Adrian and Rosenberg, 2008; Drechsler and Yaron, 2010; Bollerslev and Todorov, 2011; Menkhoff et al., 2012; Boguth and Kuehn, 2013). However, direct evidence in the data of time-varying uncertainty in the consumption process remains elusive.

The literature concerning the preferences for early resolutions of uncertainty in the long-run risk framework, and the related evidence is further discussed in Section 4; that concerning the demand and valuation of risk at different payoff horizons is at the core of Section 5.3, and presented in details there.

2 Preferences with horizon-dependent risk aversion

Field and laboratory experiments document that risk-taking behavior is affected by how far in the future a risk occurs: subjects tend to be more averse to risks in the near future than to risks in the distant future. Early work by Jones and Johnson (1973) provides evidence for such horizon-dependent risk aversions from a simulated medical trial. More recent studies use the standard protocol of Holt and Laury (2002) to elicit risk aversion — Noussair and Wu (2006) in a within-subjects design and Coble and Lusk (2010) in an across-subjects design — and find risk aversion decreases as risk becomes more distant in time. The same pattern is documented by Sagristano, Trope, and Liberman (2002) and Baucells and Heukamp (2010) using binary choice among lotteries, as well as by Onculer (2000) and Abdellaoui, Diecidue, and Onculer (2011) using certainty equivalents. Onculer (2000) thus quantifies the premium for risk at different horizons and shows it is twice higher for immediate payoffs than for delayed lotteries.¹

Figure 1 provides an example of preferences with horizon-dependent risk aversion. Under this illustrative example, all subjects are asked to rank a lottery with payoff $x = 1$ for certain versus a lottery with payoff $x = 3$ with a 50% chance, and $x = 0$ otherwise. All subjects choose their rankings at time $t = 0$; however for some the lottery happens at time $t = 2$ (the "distant risk" case), and for some the lottery happens at time $t = 1$ (the "imminent risk" case).

The experimental evidence shows that subjects may prefer the certain lottery over the

¹The premium for risk is measured as the difference between the expected payoff of a lottery and the value, or certainty equivalent, subjects assign to it.

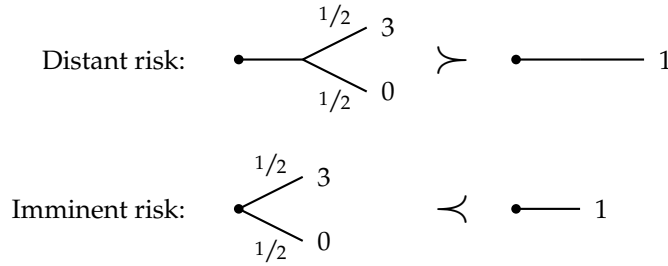


Figure 1: Preferences with horizon-dependent risk aversion.

risky one when the risk is immediate but prefer the same risky lottery over the certain one when the risk is distant in the future. For a real life intuitive example, think of someone paying a considerable amount of money for a parachute jumping experience, and then refusing to actually jump once in the plane. This is the notion of *horizon-dependent risk aversion* as introduced by Eisenbach and Schmalz (2016) in a static, time separable, framework.

In the illustrative example above, one subgroup ranks lotteries with horizon $t = 1$ and the other subgroup ranks lotteries with horizon $t = 2$: within each subgroup the ranking is for lotteries that will happen at the same time. That the rankings change with the horizon reveals a dynamic inconsistency in *intra-temporal* choices, not in *inter-temporal* choices. In particular, the well documented hyperbolic discounting (e.g. Phelps and Pollak, 1968; Laibson, 1997) or other time inconsistencies concerning inter-temporal decisions do not influence, or cause, the evidence discussed above.²

2.1 Dynamic preference model

To explore the formal implications of horizon-dependent risk aversion in a dynamic framework, we introduce it in the recursive utility Epstein-Zin preferences, the standard model for long-run risk pricing. Epstein-Zin preferences are dynamically consistent (by definition). We generalize their model by relaxing the dynamic consistency axiom of Kreps and Porteus (1978). To simplify the exposition, we present the model with only two levels of risk aversion γ and $\tilde{\gamma}$: we assume that the agent treats immediate uncertainty with risk aversion γ , and all delayed uncertainty with risk aversion $\tilde{\gamma}$, where $\gamma > \tilde{\gamma} \geq 1$ in line with

²Eisenbach and Schmalz (2016) also show that horizon-dependent risk aversion is conceptually orthogonal to time-varying risk aversion (Constantinides, 1990; Campbell and Cochrane, 1999).

the experimental evidence.³

At any time t , we denote by $E_t[\cdot] = E[\cdot | \mathcal{I}_t]$ the expectation conditional on \mathcal{I}_t , the information set at time t .

Definition 1 (Dynamic horizon-dependent risk aversion). *The agent's life-time utility in period t of a consumption stream $\{C_\tau\}_{\tau \geq t}$ is given by*

$$V_t = \left((1 - \beta) C_t^{1-\rho} + \beta E_t[\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad (1)$$

where the continuation value \tilde{V}_{t+1} satisfies the recursion

$$\tilde{V}_{t+1} = \left((1 - \beta) C_{t+1}^{1-\rho} + \beta E_{t+1}[\tilde{V}_{t+2}^{1-\tilde{\gamma}}]^{\frac{1-\rho}{1-\tilde{\gamma}}} \right)^{\frac{1}{1-\rho}}. \quad (2)$$

The lifetime utility V_t depends on the deterministic current consumption C_t and on the certainty equivalent $E_t[\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}$ of the continuation value \tilde{V}_{t+1} , where the aggregation of the two periods occurs with constant elasticity of intertemporal substitution given by $1/\rho > 0$ under the subjective time discount $\beta > 0$. However, the certainty equivalent of consumption starting at $t + 1$ is calculated with relative risk aversion $\gamma > 0$, wherein the certainty equivalents of consumption starting at $t + 2$ and beyond are calculated with relative risk aversion $\tilde{\gamma} > 0$. Adapted to the preferences of Definition 1, the experimental evidence in [Onculer \(2000\)](#) is consistent with $\tilde{\gamma} \approx \frac{1}{2}\gamma$.⁴

This is the concept of horizon-dependent risk aversion applied to the recursive valuation of certainty equivalents, as in Epstein-Zin preferences, but with risk aversion γ for imminent uncertainty and risk aversion $\tilde{\gamma}$ for delayed uncertainty. Our model nests the Epstein-Zin model when $\gamma = \tilde{\gamma}$, and, in turn, nests the standard time-separable model with constant relative risk aversion (CRRA) when $\gamma = \tilde{\gamma} = \rho$. Any difference in the results we derive below under the preferences of Definition 1 to those obtained under the

³Our approach, with only two levels of risk aversion, is analogous to the β - δ framework ([Phelps and Pollak, 1968](#); [Laibson, 1997](#)) as a special case of the general non-exponential discounting model of [Strotz \(1955\)](#). In Appendix A, we present the model for general sequences $\{\gamma_h\}_{h \geq 1}$ of risk aversion at horizon h . As long as risk aversions reach a constant level beyond a given horizon, closed form solutions similar to those derived in Sections 3, 4, 5 and 5.3 obtain.

⁴In Section 3.2, we find a calibration of the model with $\tilde{\gamma} \approx \frac{1}{2}\gamma$ allows to match the asset pricing evidence.

standard Epstein-Zin model thus hinges on $\tilde{\gamma} \neq \gamma$.

The horizon-dependent valuation of risk implies a dynamic inconsistency, as the uncertain consumption stream starting at $t + 1$ is evaluated as \tilde{V}_{t+1} by the agent's self at t and as V_{t+1} by the agent's self at $t + 1$:

$$\tilde{V}_{t+1} = \left((1 - \beta) C_{t+1}^{1-\rho} + \beta E_{t+1} [\tilde{V}_{t+2}^{1-\tilde{\gamma}}]^{\frac{1-\rho}{1-\tilde{\gamma}}} \right)^{\frac{1}{1-\rho}} \neq V_{t+1} = \left((1 - \beta) C_{t+1}^{1-\rho} + \beta E_{t+1} [\tilde{V}_{t+2}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}$$

Crucially, this disagreement between the agent's continuation value \tilde{V}_{t+1} at t and the agent's utility V_{t+1} at $t + 1$ arises only for uncertain consumption streams. For any deterministic consumption stream the horizon dependence in Equation (1) becomes irrelevant and we have

$$\tilde{V}_{t+1} = V_{t+1} = \left((1 - \beta) \sum_{h \geq 0} \beta^h C_{t+1+h}^{1-\rho} \right)^{\frac{1}{1-\rho}}.$$

Our model implies dynamically inconsistent *risk* preferences while maintaining dynamically consistent *time* preferences, focusing strictly on the experimental evidence described above. The results we obtain in the analysis that follows can therefore be attributed to horizon-dependent risk aversion, orthogonal to extant models of time inconsistency, such as hyperbolic discounting.

2.2 Generalized preference model

In the preferences of Definition 1, we opted for CRRA risk adjustments. However, similarly to the Epstein-Zin model, our model of horizon-dependent risk aversion accommodates any preferences in the Chew-Dekel class of betweenness-respecting models (Dekel, 1986; Chew, 1989). The general model is defined as:

Definition 2 (Generalized dynamic horizon-dependent risk aversion). *The agent's utility in period t is given by*

$$V_t = \left((1 - \beta) C_t^{1-\rho} + \beta \left(\mathcal{R}_t [\tilde{V}_{t+1}] \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \quad (3)$$

where the continuation value \tilde{V}_{t+1} satisfies the recursion

$$\tilde{V}_{t+1} = \left((1 - \beta) C_{t+1}^{1-\rho} + \beta \left(\tilde{\mathcal{R}}_{t+1}[\tilde{V}_{t+2}] \right)^{1-\rho} \right)^{\frac{1}{1-\rho}}, \quad (4)$$

and $\mathcal{R}_t[\cdot]$ and $\tilde{\mathcal{R}}_t[\cdot]$ are certainty-equivalent operators for utility functions \mathcal{U} and $\tilde{\mathcal{U}}$ in the Chew-Dekel class of betweenness-respecting models.

Examples of certainty equivalent operators other than CRRA (of Equations (1) and (2)) could be those of a CRRA habit model (Campbell and Cochrane, 1999) with risk aversions $\gamma > \tilde{\gamma}$ or those of the disappointment aversion model (Gul, 1991) with first-order risk aversion coefficients $\theta > \tilde{\theta}$. As mentioned in our review of the literature, introducing these "exotic" risk adjustments helps explain cross-sectional evidence (Routledge and Zin, 2010; Bonomo et al., 2011; Schreindorfer, 2014; Andries, 2015), orthogonal to the timing and pricing of risk we analyze in this paper and to our notion of horizon-dependent risk aversion. The cross-sectional results derived under the standard Epstein-Zin model would remain valid under the preferences of Definition 2.

At a deeper level, the preferences of Definition 2 allow for great flexibility: agents could have first-order risk aversion (disappointment aversion or loss aversion) for immediate risk but standard concave utility for longer horizons; they could have time-varying risk aversion for immediate risks only; the gap between their immediate and long-term risk aversions could vary with market conditions; etc. Our first contribution is conceptual: we propose a model of preferences that allows for the analysis of new and complex forms of dynamic inconsistencies within a simple framework.

2.3 Timing of risk and dynamic inconsistency

An agent with the time-inconsistent preferences of Definition 1 or Definition 2 can be either naive or sophisticated about the disagreement between her temporal selves; in addition, she may be able to commit to multi-period strategies or be compelled to reoptimize every period. These modeling choices matter for dynamic outcomes, and the asset prices we derive. In contrast, the valuation of early versus late resolution of uncertainty is by nature a static problem: its solutions are the same for naive and sophisticated investors, with or without commitment.

We follow the tradition of [Strotz \(1955\)](#), and assume the agent is fully rational and sophisticated when making choices in period t to maximize V_t . Self t realizes that her valuation of future consumption, given by \tilde{V}_{t+1} , differs from the objective function V_{t+1} which self $t + 1$ will maximize. The solution then corresponds to the subgame-perfect equilibrium in the sequential game played among the agent's different selves (see [Appendix A.1](#)).

We assume no commitment in [Section 3](#), as appropriate for a representative agent who trades and clears the market at all times, and as such cannot precommit to a given strategy — similar to the framework of [Luttmer and Mariotti \(2003\)](#) for non-geometric discounting. However, in [Section 5](#), we analyze the implications of letting the sophisticated agents commit to buy-and-hold strategies, e.g. for illiquid assets and periods of liquidity crises in which one-period pricing breaks down.

Extending our results to an agent naive about her own dynamic inconsistencies is straightforward and does not present any conceptual challenge. We briefly discuss and derive formal results for this alternative approach in [Appendix A.3](#).

3 Asset prices

We derive the marginal pricing of risk in a standard Lucas-tree endowment economy, in which a representative agent with the horizon-dependent preferences of [Definition 1](#) sets equilibrium prices. All decisions are made in sequential one-period problems, where the no-arbitrage condition is automatically satisfied despite the agent's time inconsistent preferences (see [Appendix A.1](#) for details). This one-period pricing framework is the classical approach.

Assuming a sophisticated representative agent with the preferences of [Definition 1](#), who trades and re-optimizes her utility every period and cannot commit to any specific strategy, the object of interest for asset pricing purposes is the stochastic discount factor (SDF). The SDF's derivation is based on the inter-temporal marginal rate of substitution

$$\Pi_{t,t+1} = \frac{dV_t/dW_{t+1}}{dV_t/dC_t},$$

which satisfies the Euler equation, whereby the equilibrium price at time t of a future payoff X_{t+1} is given by $P_t = E_t[\Pi_{t,t+1}X_{t+1}]$.

Proposition 1. *An agent with the horizon-dependent risk aversion preferences of Definition 1 has a one-period stochastic discount factor*

$$\Pi_{t,t+1} = \underbrace{\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}}_{(I)} \times \underbrace{\left(\frac{\tilde{V}_{t+1}}{E_t[\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\rho-\gamma}}_{(II)} \times \underbrace{\left(\frac{\tilde{V}_{t+1}}{V_{t+1}} \right)^{1-\rho}}_{(III)}. \quad (5)$$

The SDF consists of three multiplicative parts. The first term (I) is standard, capturing the inter-temporal substitution between t and $t + 1$, and is governed by the time discount factor β and the elasticity of inter-temporal substitution $1/\rho$.

The second term (II) captures the unexpected shocks realized in $t + 1$ to consumption in the long-run, i.e. beyond $t + 1$. It compares the ex-post realized $t + 1$ utility \tilde{V}_{t+1} to its ex-ante certainty equivalent $E_t[\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}$; both the comparison as well as the certainty equivalent are evaluated with immediate risk aversion γ . The same term obtains under standard Epstein-Zin preferences with the difference that, in our model, the $t + 1$ utility of self t (\tilde{V}_{t+1}) differs from that of self $t + 1$ (V_{t+1}).

Finally, the third term (III) captures the dynamic inconsistency in our model by loading on the disagreement between selves t and $t + 1$ when evaluating their $t + 1$ utilities, given by the ratio \tilde{V}_{t+1}/V_{t+1} .

3.1 Risk model, risk prices

To interpret what aggregate shocks the three terms (I), (II) and (III) in the stochastic discount factor of Equation (5) price, we assume a log-normal endowment consumption process where both the expected growth and uncertainty are time varying, in line with the long-run risk literature (e.g. [Bansal and Yaron, 2004](#); [Bansal et al., 2009](#)):

$$\begin{aligned} c_{t+1} - c_t &= \mu_c + \phi_c x_t + \alpha_c \sigma_t \omega_{c,t+1} \\ x_{t+1} &= \nu_x x_t + \alpha_x \sigma_t \omega_{x,t+1} \\ \sigma_{t+1}^2 &= \sigma^2 + \nu_\sigma \left(\sigma_t^2 - \sigma^2 \right) + \alpha_\sigma \omega_{\sigma,t+1} \end{aligned} \quad (6)$$

For simplicity, we assume that x_t is one dimensional and the three shocks $w_{c,t}$, $w_{x,t}$ and $w_{\sigma,t}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal.⁵ Both v_x and v_σ are contracting. Throughout, $c_t = \log C_t$.

Before deriving the pricing of the shocks $\{w_{c,t}, w_{x,t}, w_{\sigma,t}\}$ under horizon-dependent risk aversion, we briefly explain the role they play in the long-run risk model. This allows us to clarify the comparisons we draw later between ours and the classical framework.

The consumption process (6) accounts for time variations in expected consumption growth, through the state variable x_t , consistent with direct evidence in the data (Hansen et al., 2008). Cross-sectional asset pricing returns demonstrate further that shocks to x_t are priced in the data (Hansen et al., 2008; Bryzgalova and Julliard, 2015), capturing in particular the value premium from Fama and French (1993); while the analysis of the macro-announcement premium shows their pricing contributes to a large portion of the market equity premium (55% in Ai and Bansal, 2018, and 80% in Lucca and Moench, 2015, who study a shorter, more recent, time period). This set of evidence provides a foundation for combining expected growth risk in the consumption process (6) with recursive non time-separable preferences such as Epstein-Zin preferences: the long-run risk framework.

The time variations in the volatility σ_t , in consumption process (6) have a separate role to play: though not directly observable in the data, they are necessary to generate time-varying risk premia, and for the model to capture the volatility puzzle (Shiller, 1981).⁶

We now turn to how horizon-dependent risk aversion (Definition 1) affects the pricing of these consumption shocks. To derive closed-form solutions, we focus on the case $\rho = 1$, a unit elasticity of inter-temporal substitution.⁷ From Proposition 1, the variable of interest in our analysis is the ratio between the $t + 1$ value of self t (\tilde{V}_{t+1}) and that of self $t + 1$ (V_{t+1}). Taking logs, we obtain:

⁵These assumptions can be generalized. We employ them here to make our results comparable to those of Bansal and Yaron (2004) and Bansal et al. (2009).

⁶Just like the standard Epstein-Zin model, our framework allows for the analysis of additional shocks in the consumption process (6), e.g. jumps. Drechsler and Yaron (2010) show such shocks help capture other features in the data, notably the pricing of variance swaps and their ability to predict market equity returns (Bollerslev et al., 2009).

⁷In Appendix C, we consider $\rho \neq 1$ and the approximation of a rate of time discount close to zero, $\beta \approx 1$. We show our main results remain valid as long as the elasticity of inter-temporal substitution is greater or equal to one ($1/\rho \geq 1$) — a constraint the standard long-run risk model must also satisfy to match asset pricing data.

Lemma 1. *Under the Lucas-tree endowment process (6) and $\rho = 1$,*

$$\tilde{v}_{t+1} - v_{t+1} = \frac{1}{2} \beta (\gamma - \tilde{\gamma}) \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 + \psi_v(\tilde{\gamma})^2 \alpha_\sigma^2 \right) \sigma_{t+1}^2, \quad (7)$$

where ϕ_v is independent of both γ and $\tilde{\gamma}$, and $\psi_v(\tilde{\gamma}) < 0$ is independent of γ :

$$\phi_v = \frac{\beta \phi_c}{1 - \beta v_x}, \quad (8)$$

$$\psi_v(\tilde{\gamma}) = \frac{1}{2} \frac{\beta (1 - \tilde{\gamma})}{1 - \beta v_\sigma} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right). \quad (9)$$

Equation (7) reflects that the $t + 1$ value of self t (\tilde{v}_{t+1}) and that of self $t + 1$ (v_{t+1}) only differ in their $t + 1$ valuation of uncertain consumption starting in $t + 2$ onwards, which is governed by volatility σ_{t+1} . Self t evaluates this uncertainty with low risk aversion $\tilde{\gamma}$ while self $t + 1$ evaluates it with high risk aversion γ ; implying that $\tilde{v}_{t+1} - v_{t+1}$ is positive, and increasing in $\gamma - \tilde{\gamma}$ and in the amount of uncertainty driven by volatility σ_{t+1} .

From terms (II) and (III) in Equation (5), horizon-dependent risk aversion affects only the pricing of shocks that correlate with variations in the ratio \tilde{V}_t / V_t , therefore with variations in σ_t . From Lemma 1, we derive the central result:

Proposition 2. *Horizon-dependent risk aversion does not affect the equilibrium risk prices of shocks to consumption levels (immediate consumption shocks and shocks to consumption growth).*

If the agent faced consumption level shocks only, she could anticipate how her future self reoptimizes, and her time inconsistency would not cause additional uncertainty in her one-period decision making. Only unanticipated changes in her intra-temporal decisions, when the quantity of risk varies through time, interact with her dynamic inconsistency to modify risky assets' excess returns compared to the time consistent model. This result crucially hinges on the fact that, in our preference framework, only intra-temporal decisions are time inconsistent: inter-temporal decisions are unchanged from the standard model. One important implication of Proposition 2 is that the macroeconomic announcement premium described and analyzed in Lucca and Moench (2015) and Ai and Bansal (2018) is the same under standard Epstein-Zin preferences and horizon-dependent risk aversion.

Let us now turn to the pricing of all shocks, including shocks to volatility σ_t . From Lemma 1 we obtain:

Proposition 3. *Under the Lucas-tree endowment process (6) and $\rho = 1$, the stochastic discount factor satisfies*

$$\begin{aligned} \pi_{t,t+1} - E_t[\pi_{t,t+1}] &= -\gamma \alpha_c \sigma_t w_{c,t+1} + (1 - \gamma) \phi_v \alpha_x \sigma_t w_{x,t+1} \\ &\quad + (1 - \gamma) \psi_v(\tilde{\gamma}) \alpha_\sigma w_{\sigma,t+1}. \end{aligned} \quad (10)$$

The risk free rate is independent of $\tilde{\gamma}$:

$$r_{f,t} = -\log \beta + \mu_c + \phi_c x_t + \left(\frac{1}{2} - \gamma \right) \alpha_c^2 \sigma_t^2 \quad (11)$$

The pricing of the immediate consumption shocks, given by the term $\gamma \alpha_c \sigma_t w_{c,t+1}$; the pricing of drift shocks, the term $(1 - \gamma) \phi_v \alpha_x \sigma_t w_{x,t+1}$; as well as the risk-free rate, in Equations (10) and (11); all depend only on the immediate risk aversion γ , and are unchanged from the standard long-run risk model.⁸ In contrast, from Equations (9) and (10), we obtain the formal result:

Corollary 1. *For an agent with horizon-dependent risk aversion, $\gamma > \tilde{\gamma}$ unambiguously lowers the pricing of volatility shocks:*

$$\frac{\psi_v(\tilde{\gamma})}{\psi_v(\gamma)} = \frac{1 - \tilde{\gamma}}{1 - \gamma} < 1. \quad (12)$$

Our model yields a negative price for volatility shocks: $(1 - \gamma) \psi_v(\tilde{\gamma}) \alpha_\sigma w_{\sigma,t+1}$ in Equation (10). Assets with payoffs that covary with aggregate volatility provide valuable insurance, consistent with the existing long-run risk literature and the observed evidence from variance swaps and option straddles returns (see Dew-Becker et al., 2016, and Andries et al., 2016 for recent examples). However, shocks to volatility make future intra-temporal decisions uncertain and, for this reason, how risky they are depends on horizon-dependent risk aversion. Due to the lower risk aversion $\tilde{\gamma} < \gamma$, their implied long-run uncertainty does not "feel" as costly, which reduces the value of hedges against volatility shocks; the intuition behind Corollary 1.

Before turning to the quantitative analysis of our model, let us pause to interpret the

⁸When $\rho \neq 1$, the risk-free rate can depend on $\tilde{\gamma}$, though not the risk prices for immediate consumption shocks and drift shocks – see Appendix C.

qualitative implications of our results. First, as discussed above, the pricing of shocks to consumption levels, i.e. to C_{t+1}/C_t and to x_t , allows the standard long-run risk model to match the market equity premium, the macroeconomic announcement premium and the value premium. From Equations (8) and (10), the pricing of these shocks is exactly the same under horizon-dependent risk aversion: the preference model of Definition 1 retains the same ability to match these market premia.

Second, the shocks to consumption growth uncertainty in process (6) allow to obtain time-varying risk premia and explain the market volatility puzzle in the standard long-run risk model. As Equation (10) shows, time variations in risk premia arise from the pricing of both the immediate consumption shocks and the long-run consumption growth shocks — the terms $\gamma\alpha_c\sigma_t w_{c,t+1}$ and $(1 - \gamma)\phi_v\alpha_x\sigma_t w_{x,t+1}$ — and *not* through the pricing of shocks to volatility. Variations in the pricing of risk thus remain unchanged by the introduction of horizon-dependent risk aversion with $\tilde{\gamma} < \gamma$, providing the exact same rationalization of the volatility puzzle.

Third, in calibrations of the consumption process (6), the pricing of shocks to volatility under the Epstein-Zin model also contributes to the equity premia, sometimes to a large extent (e.g. [Bansal et al., 2009](#)). Introducing horizon-dependent risk aversion unambiguously reduces the magnitude of their impact. In the extreme case $\tilde{\gamma} \approx 1$, the pricing of volatility shocks goes to zero (Corollary 1). To assess our model, we appeal to the evidence concerning the macroeconomic announcement premium and its share of the equity premium (80% in [Lucca and Moench, 2015](#), 55% in [Ai and Bansal, 2018](#)). As driven exclusively by the pricing of shocks to consumption levels, the magnitude of this share provides direct evidence on the relative values of the risk prices of immediate and long-term consumption shocks versus volatility shocks; and, in turn, on how small the long-run risk aversion $\tilde{\gamma}$ of the representative agent must be compared to her immediate risk aversion γ . This allows us to discipline the calibration of our model.

3.2 Model calibration

The consumption processes (6) is calibrated, Table 1, strictly as in [Bansal et al. \(2009\)](#).⁹ This allows us to highlight how the horizon-dependent risk aversion preference model

⁹Table 1 also provides the calibration of the dividend growth processes (15) in Sections 5 and 5.3.

Table 1: Calibration.

Process	Parameters			
c_t	$\mu_c = 0.15\%$	$\phi_c = 1$	$\alpha_c = 1$	
x_t		$\nu_x = 0.975$	$\alpha_x = 0.0038$	
σ_t	$\sigma = 0.72\%$	$\nu_\sigma = 0.999$	$\alpha_\sigma = 0.00028\%$	
d_t	$\mu_d = 0.15\%$	$\phi_d = 2.5$	$\alpha_d = 5.96$	$\chi = 2.6$

of Definition 1, rather than changes in the calibration for the endowment process, affects prices. In line with [Bansal et al. \(2009\)](#), we use $\beta = 0.9989$ for the monthly rate of time discount. The elasticity of inter-temporal substitution is $\frac{1}{\rho} = 1$ throughout (see Appendix C for $\rho \neq 1$ results).

The wedge between the immediate risk aversion γ and the long horizon risk aversion $\tilde{\gamma}$, by reducing the pricing of shocks to volatility (see Corollary 1), impacts the equity premium. And because $\gamma \neq \tilde{\gamma}$ affects only the pricing of volatility shocks in the consumption process (6), but not that of shocks to both immediate and long-run consumption growth, increases in the wedge $\gamma - \tilde{\gamma}$ lead monotonically to similar increases in the share of the equity premium that comes from the pricing of consumption level shocks, measured in the data by the macroeconomic announcement premium. We quantify this relation in Table 2. Under the calibration of the consumption process (6) in [Bansal et al. \(2009\)](#) (Table 1) and given the data estimates in [Lucca and Moench \(2015\)](#); [Ai and Bansal \(2018\)](#), we calibrate the horizon-dependent risk aversion model of Definition 1 with $\gamma = 11$ and $\tilde{\gamma} = 5.3$.¹⁰ The immediate risk aversion $\gamma = 11$, slightly higher than under [Bansal et al. \(2009\)](#), compensates for the lower pricing of volatility shocks when $\tilde{\gamma} < \gamma$ in order to capture the equity premium. Crucially, the calibration of the wedge between the immediate and long-term risk aversions under which the macro-announcement premium contribution to the equity premium obtains as in the data matches exactly the experimental evidence in [Onculer \(2000\)](#): $\tilde{\gamma} \approx \frac{1}{2}\gamma$.

¹⁰The calibration of the standard [Epstein and Zin \(1989\)](#) model in [Bansal et al. \(2009\)](#), with $\gamma = \tilde{\gamma} = 10$ matches the macroeconomic announcement premium in [Ai and Bansal \(2018\)](#) but underestimates it relative to [Lucca and Moench \(2015\)](#); ours falls between the two estimates in [Ai and Bansal \(2018\)](#) and [Lucca and Moench \(2015\)](#).

Table 2: Equity and macro-announcement premia.

	Market returns	Macro-announcement share
Data	7.66%	55% – 80%
Calibration $\tilde{\gamma} \approx 1, \gamma = 11$	5.90%	96%
$\tilde{\gamma} = 2, \gamma = 11$	6.31%	89%
$\tilde{\gamma} = 4, \gamma = 11$	7.13%	77%
$\tilde{\gamma} = 4.65, \gamma = 11$	7.40%	73%
$\tilde{\gamma} = 5.3, \gamma = 11$	7.66%	70%
$\tilde{\gamma} = 7, \gamma = 11$	7.95%	64%
$\tilde{\gamma} = 8, \gamma = 11$	8.76%	60%
Bansal et al. (2009): $\tilde{\gamma} = \gamma = 10$	8.75%	54%

Annualized returns under $\rho = 1$ and the calibration of Table 1;

Data is from Bansal et al. (2009), annual 1930–2008; and from Lucca and Moench (2015); Ai and Bansal (2018).

4 Preference for early or late resolution of uncertainty

To what extent do the horizon-dependent risk aversion preferences of Definition 1 affect agents' decisions regarding the timing of information arrivals? To analyze this issue, and determine whether agents have a preference for early or late resolutions of uncertainty, we strictly follow the set up of Epstein et al. (2014). Two types of consumption streams, subject to the exact same shocks over time, are evaluated at a given time t . In the first case, consumption shocks are revealed gradually, whenever they are realized: the shock affecting consumption at time $t + h$ is revealed at $t + h$, for all horizons $h \geq 1$. In the second case, all future consumption shocks are revealed in the next period, at time $t + 1$, even when they affect consumption at a later period: the shock affecting consumption at $t + h$ is revealed at time $t + 1$, for all $h \geq 1$.

Crucially, even when she receives the information about her future consumption shocks earlier, the agent cannot act on the information to change her future consumption stream. From the point of view of time t , when the agent evaluates the two consumption streams with or without early resolution of uncertainty, the distributions of future risks are therefore exactly the same in both cases; in the expected utility framework, she would assign them the exact same value. However, in the non time-separable models of Epstein and Zin

(1989) and Definition 1, two consumption streams with ex ante identical risks, but different timing for the resolution of uncertainty, can have different values.

An agent with Epstein-Zin utility prefers early resolutions of uncertainty if and only if her risk aversion is greater than her inverse elasticity of inter-temporal substitution: $\gamma > \rho$.¹¹ How much she prefers early resolutions depends on the wedge $\gamma - \rho$ and on the magnitude of the uncertainty in the consumption shocks. However, choosing a consumption stream with an early resolution (i.e., where all shocks are revealed at time $t + 1$) rather than the same consumption stream with late resolutions (i.e., where shocks are revealed as they come over time) corresponds to shifting all future risk, short-term and long-term, to a next-period risk. Whether long-term risks are evaluated with the same risk aversion as immediate risks will thus matter for the relative values of the two theoretical consumption streams, and therefore for the preference for early or late resolutions of uncertainty.¹²

Importantly, assigning values to the two consumption streams above is a static problem: the agent evaluates the two (infinite) streams of consumption, with early or late resolution of uncertainty, exactly once. How her preferences change over time, whether she is naive or sophisticated about it, whether she can commit to specific future choices, are irrelevant to the relative values she assigns to the two consumption streams, i.e. to her preference for early or late information.

4.1 Timing premium

Denoting by V_t^* the agent's utility at t if all uncertainty (i.e., the entire sequence of shocks $\{w_{t+h}\}_{h \geq 1}$ in the consumption process (6)) is resolved at $t + 1$, and by V_t the agent's utility

¹¹To see why, note that in the case where all future shocks are revealed at $t + 1$, the shocks to consumption from $t + 2$ onward are evaluated with the inverse elasticity of inter-temporal substitution ρ since they are no longer uncertain; whereas, when shocks are revealed over time, variations in consumption from $t + 2$ onward are still risky at $t + 1$ and thus evaluated with risk aversion γ .

¹²In Appendix B.1, we derive the timing premium under hyperbolic discounting, whereby $\gamma = \tilde{\gamma}$ but, at time t , the value V_t is derived with time discount parameter β , and the continuation value \tilde{V}_{t+1} is derived with time discount parameter $\tilde{\beta} > \beta$. The preference for an early resolution of uncertainty still holds if and only if $\gamma > \rho$, but the magnitude of the timing premium is lower than if the time discount is $\tilde{\beta}$ everywhere (and greater than if it is β everywhere). Introducing hyperbolic discounting has, however, a small quantitative effect: e.g. under the calibration of Bansal and Yaron (2004) with constant volatility, $\gamma = 10$, $\rho = 1$, and $\beta = 0.8$, $\tilde{\beta} = 0.998$, the timing premium only goes from 27% (under $\beta = \tilde{\beta} = 0.998$) to 22.5%.

if uncertainty is revealed over time, the timing premium is defined as

$$\text{TP}_t = \frac{V_t^* - V_t}{V_t^*}.$$

This timing premium represents the fraction of utility, or equivalently the fraction of life-time consumption, the agent is willing to forgo for an early rather than late resolution of uncertainty. As before, we assume a unit elasticity of inter-temporal substitution. The formal derivations are presented for the case $\sigma_t = \sigma$ in consumption process (6), where they are more readily interpretable and convey all the relevant intuitions. The interested reader can find the derivations for the case with time varying volatility in Appendix B.1.

Proposition 4. *An agent with the horizon-dependent risk aversion preferences of Definition 1 with $\rho = 1$, facing the consumption process (6) with $\sigma_t = \sigma$, has a constant timing premium*

$$\text{TP} = 1 - \exp\left(\frac{1}{2} \left(1 - (\gamma - (1 + \beta)(\gamma - \tilde{\gamma}))\right) \frac{\beta^2}{1 - \beta^2} \alpha_v^2 \sigma^2\right), \quad (13)$$

where $\alpha_v^2 = \alpha_c^2 + \left(\frac{\beta\phi_c}{1 - \beta v_x}\right)^2 \alpha_x^2$.

To highlight the role played by horizon-dependent risk aversion, note that an agent with the standard Epstein-Zin preferences with risk aversion γ has a timing premium given by $\text{TP} = 1 - \exp\left(\frac{1}{2} (1 - \gamma) \frac{\beta^2}{1 - \beta^2} \alpha_v^2 \sigma^2\right)$, obtained by setting $\gamma = \tilde{\gamma}$ in Equation (13). When $\gamma > \tilde{\gamma}$, the timing premium is instead determined by:

$$\gamma - (1 + \beta)(\gamma - \tilde{\gamma}) < \gamma.$$

Corollary 2. *For an agent with horizon-dependent risk aversion, $\gamma > \tilde{\gamma}$ unambiguously lowers the timing premium.*

When would the timing premium turn negative, indicating a preference for *late* resolution? For an Epstein-Zin agent, this happens if and only if $\gamma < \rho$. In our model, with $\rho = 1$ and the consumption process (6) with $\sigma_t = \sigma$, the timing premium is negative if and only if

$$\gamma < 1 + (1 + \beta)(\gamma - \tilde{\gamma}). \quad (14)$$

When $\gamma > \tilde{\gamma}$, we immediately obtain $1 + (1 + \beta)(\gamma - \tilde{\gamma}) > \rho = 1$, and the agent with horizon-dependent risk aversion can have a preference for late resolution, even when both risk aversions γ and $\tilde{\gamma}$ are greater than the inverse elasticity of inter-temporal substitution — as long as the decline in risk aversion across horizons is sufficiently large. For example, suppose we set immediate risk aversion $\gamma = 10$ and β close to 1. Then the agent will prefer uncertainty to be resolved late rather than early according to the condition of Equation (14) as long as $\tilde{\gamma} < 5.5$ which is substantially larger than $\rho = 1$.¹³

Corollary 3. *An agent with horizon-dependent risk aversion can prefer a late resolution of uncertainty even when all risk aversions exceed the inverse elasticity of inter-temporal substitution, i.e. when $\gamma > \tilde{\gamma} > \rho$.*

The result of Corollary 3 is of particular interest because extant calibrations of the long-run risk model with Epstein-Zin preferences require γ greater than ρ by an order of magnitude to match equilibrium asset pricing moments — hence the high timing premia they imply. Under horizon-dependent risk aversion, the same calibration for γ and ρ no longer automatically implies such a strong preference for early resolutions of uncertainty. This is true even when the long-run risk aversion $\tilde{\gamma}$ also remains above the inverse elasticity of inter-temporal substitution, in line with the micro evidence.¹⁴

Ai and Bansal (2018) document a high macroeconomic announcement premium, as measured by the high share of the equity premium that realizes around pre-scheduled FOMC meetings (55% over the 1961–2014 period), and argue that this pricing of shocks to future consumption levels implies a strong preference for early resolution of uncertainty. Indeed, the link between the two is tight for time consistent recursive preferences: both are determined by the wedge $\gamma - \rho$ (see Proposition 1). In contrast, Corollary 3 establishes that a high $\gamma - \rho$ need not imply a high or even a positive timing premium under the horizon-

¹³In the calibrated model of Section 3.2, with time varying volatility in the consumption process (6), we obtain a preference for late resolution whenever $\tilde{\gamma} < 4.42$ when $\gamma = 10$.

¹⁴In following the analysis of Epstein et al. (2014) and assuming only two levels of risk aversion $\gamma, \tilde{\gamma}$, we are implicitly mixing two comparisons: gradual resolution versus one-shot resolution and early resolution versus late resolution. In addition, we are placing the early resolution at time $t + 1$, exactly in the period where the risk aversion changes from γ to $\tilde{\gamma}$. However, we show in Appendix B.2 that the results of Proposition 4 and Corollaries 2 and 3 below are robust by (i) allowing for a general decreasing sequence of risk aversions $\{\gamma_h\}_{h=1}^{\infty}$ to show that the result is based on horizon-dependent risk aversion and not on a particular period and (ii) comparing resolution of all uncertainty at $t + 1$ to resolution of all uncertainty at $t + 2$ to show that the relevant comparison is between early and late resolution, not between gradual and one-shot resolution.

dependent risk averse preferences of Definition 1; whereas we showed in Section 3 that the macroeconomic announcement premium is unchanged by our model. Our framework decouples microeconomic interpretations regarding preferences for early or late information from the direct evidence in macroeconomic data (macroeconomic announcement premia or asset pricing moments). We show in below, in Section 4.2 that an equity premium and macroeconomic announcement premium consistent with the evidence no longer implies a strong preference for early resolutions of uncertainty (see Table 2).

We believe this result is key for several reasons. First, it is worth noting that the recursive utility Epstein-Zin model has little microeconomics or experimental foundation, contrary to other models of preferences commonly used in finance, e.g. prospect theory (Kahneman and Tversky, 1979), disappointment aversion (Gul, 1991), habit and dynamic inconsistency such as hyperbolic discounting (Laibson, 1997) or our model of preferences (Definition 1). The long-run risk model built its success solely on its ability to match macroeconomics evidence, meaning microeconomic inferences should be subject to deep scrutiny.

Second, we argue, in line with Epstein et al. (2014), that the magnitudes for the timing premia implied by calibrations of the long-run risk model with standard Epstein-Zin preferences are excessive. There is no direct evidence on the "correct" values of timing premia, by construction a purely theoretical question: we do not know how much an agent who cannot act to modify the consumption stream she will receive would pay to receive early information about it. But it seems somewhat unreasonable that she would be willing to forgo a large fraction of her wealth for earlier resolutions. Even more problematic for the timing premia obtained under the long-run risk calibration of Epstein-Zin preferences, the microeconomic evidence indicates many individuals behave as if they prefer to delay receiving information and avoid early resolutions, even in cases where information can be used to improve outcomes. In the health economics literature for instance, various examples of "information avoidance" are documented, whereby individuals prefer to not be told about their own test results, including concerning life-threatening diseases (e.g. Oster et al., 2013; Persoskie et al., 2014). Golman et al. (2016) provide an extensive survey of such behaviors. Closer to the theoretical framework we use to derive the timing premium, investors' inattention to their own wealth disputes the notion of a strong preference for early resolution of consumption risk; even more so because early information is instrumental in this case (inertia in portfolio allocations comes at a cost, e.g. Brunnermeier and

Nagel, 2008; Calvet et al., 2009; Biliias et al., 2010; Andersen et al., 2015). These examples do not, per se, constitute a direct proof of a preference for late resolution of uncertainty, but they appear inconsistent with the high timing premium implied by the existing calibrations of the long-run risk model (usual citations).¹⁵ Further, Karlsson et al., 2009; Alvarez et al., 2012; Sicherman et al., 2016 document that more risk averse investors are also more inattentive. This is inconsistent with the standard model: from Proposition 4 for the case $\gamma = \tilde{\gamma}$ (Epstein-Zin preferences), the timing premium is strictly increasing in γ , corresponding to a stronger preference for early resolutions of risk, or less inattention for the more risk averse investors. In contrast, our model may be consistent with the evidence: more risk averse investors may also have more strongly horizon-dependent preferences (see Proposition 4 for the respective roles of γ and $\gamma - \tilde{\gamma}$ in the timing premium).

Though circumstantial, the numerous examples above where agents prefer not to observe early information even when they can act on it make the magnitude of the timing premia under the standard long-run risk model appear unreasonable. A representative agent whose implied preferences appear contrary to commonsense considerations — here on early versus late resolution of uncertainty — raises doubts as to the legitimacy of the long-run risk model, despite its ability to match the macroeconomic evidence on equilibrium asset prices.¹⁶

4.2 Timing premium in the calibration of Section 3.2

Figure 2 plots the timing premium for both horizon-dependent risk aversion and for standard Epstein-Zin preferences, when the immediate risk aversion is $\gamma = 11$, as in Section 3.2. It illustrates the first-order impact that horizon-dependent risk aversion has on the timing premium, and the potential for our model to address the critique of Epstein et al. (2014). Depending on $\tilde{\gamma}$, an agent with horizon-dependent risk aversion can have a significantly lower willingness to pay for an early resolution than in the standard model. In fact, for delayed risk aversion $\tilde{\gamma} \leq 4.65$, the agent prefers a late resolution of risk (negative

¹⁵Golman et al., 2016 discuss other theoretical rationalizations; Andries and Haddad, 2018 propose a model of information aversion that explains investors' inattention in the data.

¹⁶Aggregation theorems for Epstein and Zin (1989) preferences (Duffie and Lions, 1992) indicate that if most individuals have low or even negative timing premia, so would the marginal, representative, investor who sets prices.

timing premium).

Delayed risk aversion $\tilde{\gamma} = 5.3$ combined with immediate risk aversion $\gamma = 11$, which matches both the equity and the macro-announcement premia (Section 3.2), imply a timing premium of 10%, corresponding to the share of her lifetime consumption the representative investor would be willing to pay to observe all her future consumption shocks next period. The introspection as well as the available evidence we discuss above, circumstantial as it may be, indicate this is a considerably more reasonable value than the 30% obtained for the standard Epstein-Zin model in the calibration of [Bansal and Yaron \(2004\)](#), and of course the 80% timing premium under [Bansal et al. \(2009\)](#).

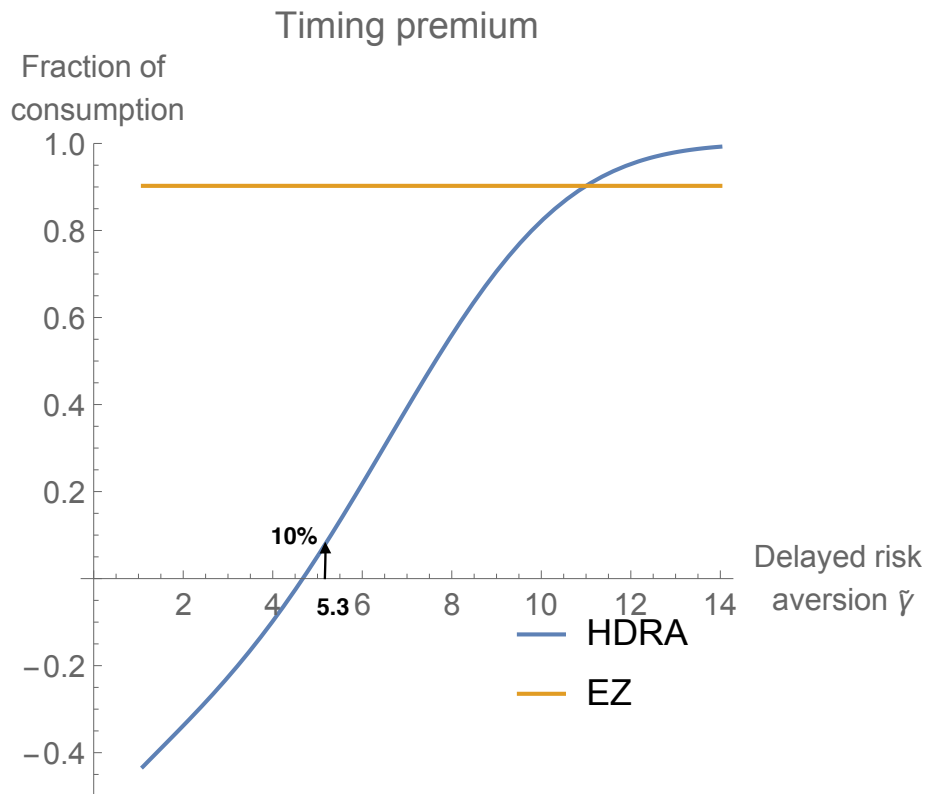


Figure 2: Effect of horizon-dependent risk aversion (HDRA) on willingness to pay for early resolution of uncertainty (timing premium), compared to Epstein-Zin preferences (EZ) with $\gamma = 11$.

5 Long-term valuations, long-term demand

Many puzzles remain on the valuation and demand for long-horizon assets. The long-run risk model with standard Epstein-Zin preferences calibrated to match asset pricing moments (e.g. [Bansal and Yaron, 2004](#); [Hansen et al., 2008](#); [Bansal et al., 2009](#)) implies agents face greater aggregate shocks at longer horizons, and therefore require a greater compensation, or risk premia, to invest in long-term assets. But counter-examples to this simple rule abound, whereby investors appear to demand low expected excess returns at long horizons and to be willing to hold considerable risks, as we review in details in [Section 5.3](#).

The model of [Definition 1](#) formalizes the experimental evidence that agents have lower risk aversions for long-horizon payoffs than for immediate risks. Under these preferences, it seems immediate that a financial asset expected to deliver payoffs far in the future should be evaluated with a lower price of risk than one with payoffs at a short horizon. This simple narrative yields potentially opposite implications than the standard model, i.e. lower risk premia for long-term assets; and a potential explanation for the evidence in the data. It does not take into account, however, that investors may not assess risky assets within buy-and-hold strategies, under which the horizon of payoffs may matter. In the one period pricing model for instance, where trading occurs every period, *all* financial assets are priced at the next period horizon, no matter when payoffs are to be paid, so the one-for-one relation between risk aversions and risk prices need not obtain.

To derive the implications of our model with respect to the valuation and demand for assets with different horizons, we focus on equity risk. In line with the long-run risk literature, we assume that dividends have log-normal growth:

$$d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t w_{c,t+1} + \alpha_d \sigma_t w_{d,t+1}, \quad (15)$$

where the shocks $w_{d,t}$ are i.i.d. $\mathcal{N}(0,1)$ and orthogonal to the consumption shocks $w_{c,t}$, $w_{x,t}$ and $w_{\sigma,t}$ of process (6); ϕ_d captures the link between the mean consumption growth and the mean dividend growth; χ the correlation between immediate consumption and dividend shocks in the business cycle.¹⁷

¹⁷Once again, these assumptions can be generalized, but they are those of [Bansal and Yaron \(2004\)](#) and [Bansal et al. \(2009\)](#).

5.1 Long-term risk premia

In Section 3, we derived equilibrium asset prices under the representative agent assumption in a one-period trading paradigm. However, individual investors trade a much lower frequencies. Recent direct evidence on trading activities in their retirement accounts indicates investors re-adjust their portfolios once a year on average (Sicherman et al., 2016). Why investors trade so rarely may be exogenously imposed, e.g. through infrequent trading opportunities, or endogenously optimal, e.g. when buy-and-hold strategies help avoid rising information costs (Alvarez et al., 2012; Andries and Haddad, 2018), or the high trading costs of illiquid assets. The literature on asset prices with liquidity risk points out the additional risk premium directly attributable to illiquidity (e.g. Acharya and Pedersen, 2005; Lee, 2011; Muir, 2016).¹⁸ Our approach here is complementary since our focus is on how illiquidity, in the form of low trading frequencies, affect the slope of risk premia in their variations with the risk horizons.

Proposition 5. *Under the horizon-dependent risk aversion preferences of Definition 1 and $\rho = 1$, the stochastic discount factor for an investment strategy at horizon $h > 1$ is given by¹⁹*

$$\Pi_{t,t+h}^{buy-and-hold} = \beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1} \times \underbrace{\frac{\tilde{V}_{t+1}^{1-\gamma}}{E_t[\tilde{V}_{t+1}^{1-\gamma}]}}_{high \ \gamma} \times \underbrace{\frac{\tilde{V}_{t+2}^{1-\tilde{\gamma}}}{E_{t+1}[\tilde{V}_{t+2}^{1-\tilde{\gamma}}]} \times \dots \times \frac{\tilde{V}_{t+h}^{1-\tilde{\gamma}}}{E_{t+h-1}[\tilde{V}_{t+h}^{1-\tilde{\gamma}}]}}_{low \ \tilde{\gamma}}.$$

Compared to the one-period investor, with implicit risk aversion γ for future shocks at all horizons, an agent who assumes no re trading at the intermediate dates between t and $t+h$ evaluates the shocks between $t+2$ and $t+h$ with lower risk aversion $\tilde{\gamma}$ — suggesting a higher willingness to pay for risky assets and therefore lower expected returns than under frequent intermediate trading.

Proposition 6. *Under the Lucas-tree endowment process (6) and the dividend process (15),*

1. *Under high frequency (one-period) strategies,*

¹⁸See also Duffie (2010) and Tirole (2011) for surveys of the literature on liquidity.

¹⁹The more general case with $\rho \neq 1$ is provided in Appendix A.2.

- the difference between long-term versus short-term risk compensations investors require is lower when $\gamma > \tilde{\gamma}$, but of the same sign, than under the standard model $\gamma = \tilde{\gamma}$;²⁰
- the difference between long-term versus short-term risk compensations investors require is greater when σ_t is high.

2. Under investment strategies at horizon $h > 1$,

- the long-term risk compensations investors require decreases with $h > 1$ when $\tilde{\gamma} < \gamma$; they can become lower than for short-term risks.
- the downward pressure from $h > 1$ and $\tilde{\gamma} < \gamma$ is greater when σ_t is high.

Proposition 6 derives from the intuition, explored and discussed in previous sections, that the less dynamic choices are, the more time inconsistencies in the agents' preferences can affect equilibrium outcomes. In particular, for buy-and-hold investors, the relation between horizon-dependent risk aversions and horizon-dependent risk prices becomes tighter. Because their appetite for risk is greater at longer horizons of payoffs, illiquidity has an ambiguous impact on prices: it makes asset holdings riskier because less capable to compensate income shocks — the traditional channel

Propositions 5 and 6 contain partial equilibrium results: they derive the valuations of risk for an investor who chooses a fixed strategy for the next h periods and commits to it.²¹ Nonetheless, a growing literature establishes that households' demand influences general equilibrium outcomes, in particular via intermediaries holdings (He and Krishnamurthy, 2013; Koijen and Yogo, 2015; Haddad and Muir, 2017), and therefore affects prices. We appeal to these results to derive the following testable implications:

Corollary 4. *Under the Lucas-tree endowment process (6) and the dividend process (15), and the calibration of Table 1:*

1. *Equity expected returns are higher at long horizons for assets with no trading costs.*

²⁰Similarly, we derive a flatter slope, but of same sign for the term-structure of zero-coupon bond yields in Appendix B.5. Taking into account inflation risk, as in e.g. Bansal and Shaliastovich (2013), allows to obtain upward sloping bond yields and match the evidence.

²¹We show in Appendix A.3 that naive agents in the one-period standard framework behave as buy-and-hold investors: an asset with payoff horizon h is evaluated at frequency h under Proposition 5.

2. *The higher the trading costs, the lower the expected returns at long horizons relative to short horizons.*
3. *Differences in the pricing of long and short horizon assets are greater the higher the consumption volatility.*

We assess in Section 5.3 how well the testable predictions of Corollary 4 fare in the data; and how much the horizon-dependent risk aversion model helps understand puzzling empirical evidence on the pricing of long-term assets.

5.2 Long-term risk taking

We turn to the analysis of agents' optimal risk taking, as a function of the horizon of their portfolios' rebalancing. An investor with the horizon-dependent risk aversion preferences of Definition 1 and horizon $h \geq 1$ optimizes at time t the value V_t of her wealth W_t by choosing her consumption plan $\{C_t, C_{t+1}, \dots, C_{t+h-1}\}$ and her investment portfolio, under the constraint her risk share position remains constant over h periods. She has access to a risky asset with dividend process (15), on which she invests a share θ_t of her savings. The rest of her portfolio accrues at the risk-free rate. Her optimization problem at time t for horizon $h \geq 1$ is given by:

$$\begin{aligned}
V_t(W_t) &= \max_{\{C_t, \dots, C_{t+h-1}, \theta_t\}} \left[(1 - \beta) C_t^{1-\rho} + \beta \mathcal{R}_t \left(\tilde{V}_{t+1}(W_{t+1}) \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \\
&\quad \text{s.t.} \\
W_{t+\tau} &= \theta_t (W_t - C_{t \rightarrow t+h}) R_{t,t+\tau}, \forall \tau \in \{1, h\}, \\
C_{t \rightarrow t+h} &= \sum_{\tau=0}^{h-1} e^{-\tau r_f} C_{t+\tau}.
\end{aligned} \tag{16}$$

To simplify the analysis, we analyze the risk dividend process (15) in the special case $\sigma_t = \sigma$.²²

Proposition 7. *Under the horizon-dependent risk aversion preferences of Definition 1 and $\rho = 1$, an investor with horizon $h \geq 1$ optimizing Problem (16) under dividend process (15) in the special*

²²The case with time varying volatility is analyzed in Appendix B.7

case $\sigma_t = \sigma$ invests, on average, a share $\theta = \theta_M + \theta_H$ of her savings into the risky asset, such that the myopic demand is:

$$\theta_M = \frac{1}{1 + \frac{(\tilde{\gamma} - \gamma)}{\gamma} \left(1 - \frac{\sigma_{1,h}^2}{\sigma_h^2}\right)} \frac{E(R_{m,h} - R_{f,h})}{\gamma \sigma_h^2} + o(1 - \beta), \quad (17)$$

and the hedging demand is:

$$\theta_H = \frac{1}{1 + \frac{(\tilde{\gamma} - \gamma)}{\gamma} \left(1 - \frac{\sigma_{1,h}^2}{\sigma_h^2}\right)} \frac{\text{cov}\left(\frac{\tilde{V}_{t+1}}{W_{t+1}}, R_{m,t+1}\right)}{\gamma \sigma_h^2} \times \begin{cases} (1 - \gamma) & \text{if } h = 1 \\ (1 - \tilde{\gamma}) & \text{if } h > 1 \end{cases} + o(1 - \beta). \quad (18)$$

$E(R_{m,h} - R_{f,h})$ and σ_h^2 are the expected excess returns and the variance of the total risky returns between t and $t + h$, with $\sigma_{1,h}^2$ the contribution of the $t + 1$ shocks.²³

The solutions of Proposition 7 simplify greatly under an i.i.d dividend process, in which case the hedging demand disappears:

$$\theta = \frac{1}{1 + \frac{(\tilde{\gamma} - \gamma) h - 1}{\gamma h}} \frac{E(R_m - R_f)}{\gamma \sigma_1^2} + o(1 - \beta),$$

in which case it is transparently clear the risk share increases with the horizon h if and only if $\tilde{\gamma} < \gamma$. This increase occurs rapidly, e.g. the demand for risk at the two-month horizon in the calibration of Section 3.2 with $\tilde{\gamma} = 5.3$ and $\gamma = 11$ is more than 1/3 greater than at the one month horizon.

When the dividend process of the risky asset has predictable components, the variance of the cumulated risk returns $\sigma_{m,h}^2$ increases, but less than linearly, with the horizon h . From

²³The solutions of Equations (17) and (18) are exact for $h = 1$; for $h > 1$, the exact solution for θ is:

$$\theta = \frac{\frac{E(R_{m,h} - R_{f,h})}{\sigma_{m,h}^2} + (1 - \tilde{\gamma}) \frac{\text{cov}\left(\frac{\tilde{V}_{t+1}}{W_{t+1}}, R_{m,t+1}\right)}{\sigma_{m,h}^2}}{\left(\beta\gamma + (1 - \beta) \left(\beta + (\gamma - 1) \frac{\sigma_m^2}{\sigma_{m,h}^2}\right)\right) + (\tilde{\gamma} - \gamma) \left(1 - \frac{\sigma_{1,h}^2}{\sigma_{m,h}^2} + \frac{\sigma_m^2}{\sigma_{m,h}^2} \left(\frac{1 - \beta}{\beta}\right)\right)}.$$

Equations (17) and (18), the myopic demand increases and the term $\frac{\text{cov}\left(\frac{\tilde{V}_{t+1}}{\tilde{W}_{t+1}}, R_{m,t+1}\right)}{\gamma\sigma_{m,h}^2}$ in the hedging demand decreases in absolute value with h , even when $\tilde{\gamma} = \gamma$. Both variations are however greatly amplified by $\tilde{\gamma} < \gamma$. In the calibration of Section 3.2 with $\tilde{\gamma} = 5.3$ and $\gamma = 11$, the hedging demand for the two-period horizon is 40% smaller, in absolute value, than under the standard model, and the myopic demand 35% greater. Since $\text{cov}\left(\frac{\tilde{V}_{t+1}}{\tilde{W}_{t+1}}, R_{m,t+1}\right) > 0$ in the solution to optimization problem (16), these variations in the myopic and the hedging demand both contribute to a greater risk share θ as the horizon h increases.

Corollary 5. *The demand for risk in optimization problem (16) with dividend process (15) is increasing in the investment horizon h :*

- *faster when $\tilde{\gamma} < \gamma$ if dividends have predictable components,*
- *only when $\tilde{\gamma} < \gamma$ if dividends have i.i.d growth,*
- *in the calibration of Section 3.2, the demand for risk is 35% greater at the two-period than at the one-period horizon.*

5.3 Related empirical evidence

We turn to how well the implications derived in Corollaries 4 and 5 match the empirical evidence; and argue the horizon-dependent risk aversion model provides a reasonable answer to various important puzzles in finance, without sacrificing the ability to match the usual asset pricing moments (as seen in Section 3).

Illiquid long-term assets. Illiquidity has been extensively analyzed as an additional source of risk to the investors, resulting in higher returns compensations (e.g. Acharya and Pedersen, 2005). Corollary 4 however proposes another viewpoint whereby illiquidity, by making trades less frequent and the assessment horizons longer, serves as a commitment device to evaluate long-term payoffs with lower long-term risk aversion. This second channel via which illiquidity decreases the pricing of risk, specific to horizon-dependent risk aversion, can rationalize the low risk premia we observe in the data for several illiquid

long-term assets, a puzzle otherwise. The abnormally low excess returns in private equity investments (e.g. Moskowitz and Vissing-Jørgensen, 2002) and in real estate holdings (Giglio et al., 2014; Chambers et al., 2019) are two extensively documented and analyzed such examples in the literature.

Term-structures of expected returns. Starting with van Binsbergen et al. (2012), several recent papers (van Binsbergen et al., 2012; Lustig et al., 2016; van Binsbergen and Koijen, 2016; Giglio et al., 2014; Dew-Becker et al., 2016; Andries et al., 2016) provide empirical evidence of downward sloping term structures of expected excess returns for various types of risk; a puzzle for the long-run risk model. These striking empirical findings have started a vigorous debate and triggered numerous new theoretical works (Kogan and Papanikolaou, 2010, 2014; Ai et al., 2015; Gârleanu et al., 2012; Favilukis and Lin, 2015; Croce et al., 2015; Andries, 2015; Curatola, 2015; Backus et al., 2016; Marfe, 2015; Nzesseu, 2018), explicitly focused on systematically deriving downward sloping term structures of risk prices. However, Bansal et al. (2019), but also van Binsbergen et al. (2013); van Binsbergen and Koijen (2016), document that expected excess returns of dividend risk are *upward* sloping on average, but became sharply downward sloping during the financial crisis of 2007–2009. Gormsen (2016) further indicates that low price-dividend ratios driven, in particular, by periods of high volatility, correspond to more *upward* sloping term structures of dividend expected excess returns. Other asset classes on the other hand display downward sloping term-structures in and out of crisis, e.g. Andries et al. (2016) for the price of variance risk and Giglio et al. (2014) for housing.

Horizon-dependent risk aversion and the results of Corollary 4 allow to shed light on these various conflicting results: they suggest differences in trading costs across markets and across different time periods can greatly influence the slopes of the term-structures of expected returns with smaller or even negative slopes under lower liquidity. They explain why expected excess returns on dividend risk are increasing in the horizon of payoffs in normal one-period pricing conditions, even more so when volatility is high as pointed out in Gormsen (2016) (Points 1. and 3. in Corollary 4); but turned downward sloping during the recent financial crisis when liquidity broke down and trading costs, i.e. bid-ask spreads, shot up dramatically (see e.g. Pedersen, 2009; Brunnermeier, 2009; Bansal et al., 2019), as van Binsbergen et al. (2013); van Binsbergen and Koijen (2016); Bansal et al.

(2019) document (Point 2. in Corollary 4). Additional supporting evidence for our horizon-dependent risk aversion model can be found in [Weber \(2016\)](#) who shows that higher cash-flow durations, i.e. longer payoff horizons, of equity shares have a downward influence on expected returns *only* within short-sale constrained stocks, in line with the results of Corollary 4.

Risk positions and investment horizons Abstracting from labor income and life-cycle considerations, which are not modeled in our analysis, our horizon-dependent risk aversion model results in a simple rule of thumb for risk taking decisions, Corollary 5: the longer the rebalancing horizon, the more investors should be willing to take risks.

In line with this result, [Shum and Faig \(2006\)](#) find, using data from the 1995 Survey of Consumer Finances, that those who are saving for the long-term (retirement) allocate more of their financial portfolios to equities, than those saving for the short-term, e.g. to buy or renovate a home. To assess the validity of Corollary 5 further, we compare the risk-taking evidence of mutual funds and hedge funds investors. The evidence shows hedge funds, which are characterized by their considerably more binding constraints, have more volatile and higher risk than mutual funds ([Ackermann et al., 1999](#)), and even more so for hedge funds with greater lockup constraints ([Aragon, 2007](#)). Similarly, mutual funds with higher exit costs tend to be more actively traded and to hold more illiquid assets ([Pastor et al., 2017](#)), resulting in riskier returns ([Kacperczyk et al., 2005](#)). Both lockup conditions and exit costs provide valid proxies to assess the trading frequencies of the different portfolios, with greater constraints corresponding to longer investment horizons. The evidence above thus provides further support for the results derived under horizon-dependent risk aversion.

Insurance and hedging at different horizons The counterpart to the greater risk-taking in financial portfolios at longer horizons, and another direct result from Corollary 5, is that the demand for insurance should decrease with the risk horizon. In addition, investors assign lower values to buy-and-hold long-term term hedging assets under Corollary 4. As the supply side for such assets (insurances or banks) assesses their risk at high frequencies (e.g. daily P&L running), the horizon-dependent risk aversion model predicts trading volumes sharply decreasing with the horizon of insurable or hedgeable risks.

The evidence in the data strongly supports the results of Corollaries 4 and 5. In [Akaichi](#)

et al. (2019), the authors find the willingness to pay for lifetime insurance policies falls below the existing market rates, explaining why they are largely not being sold anymore (American Association for Long-Term Care Insurance, 2015). The authors show further the additional premium individuals in their survey would be willing to pay for an additional year of coverage decreases fast with the horizon; they are below their actuarial fair values beyond the three-year horizon. Further evidence is found in option markets. Garleanu et al. (2008) formalize why end-users hedging demands impact option prices, whereby a higher demand can sustain higher prices and thus higher risk-aversions in option suppliers (e.g. banks) and greater volumes. Under this channel, horizon-dependent risk aversion predicts low volumes and low prices in options, at all but short maturities. This is verified by Dew-Becker et al. (2016); Andries et al. (2016) who find close to zero assigned value for medium and long-term variance risk insurance. It is also reflected by the trading volumes in option markets. Garleanu et al. (2008) reports the average non-market-maker net demand for put and call equity option contracts is 10 times higher for options up to six-month maturities than for the 6-months to one-year maturities, and 15 times higher than for options at any higher than one year maturities. Both sets of results, important puzzles in empirical finance, are well explained by our horizon-dependent risk aversion model; including why the decrease in volumes and hedging costs decrease so fast (Corollary (5)).

6 Conclusion

Calibrations of the long-run risk model (Bansal and Yaron, 2004; Bansal et al., 2009) are difficult to reconcile with the microeconomic foundations of the preferences they employ. Epstein et al. (2014) point out they imply a willingness to pay for earlier resolutions of uncertainty that defies both observed behaviors in the data and introspection. We show that relaxing the restriction of Epstein and Zin (1989) that risk preferences be constant across horizons makes it possible to retain the desirable pricing properties of the long-run risk model, including the matching of the equity premium and of the macroeconomic announcement premium, and at the same time obtain reasonable implications for the timing of the resolution of uncertainty.

We show further horizon-dependent risk aversion preferences formally imply assets

with high trading costs and/or low liquidity can have relatively low risk premia at long-horizon; and that investors may be willing to take more risk over longer lock-in periods — illiquidity provides a form of commitment device to accept more risk, with lower risk compensation. This feature of our model allows to explain several features of the data and important puzzles in empirical finance, such as the abnormally low returns in private equity and housing investments, the downward sloping term-structure of equity risk during the financial crisis of 2007–2009, and the very low trading volumes for medium to long-term options and insurance.

We conclude that formalizing a model where risk aversion is higher at short-horizons than long-horizons, consistent with the experimental evidence, provides a useful new tool for asset pricing and macro-finance. We focused our attention on applications to finance but the tractability of this model makes it suitable to analyze features of other markets, such as health decisions, where attitudes towards risk and time inconsistencies are key.

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Appendix

A Derivations under general sequence of risk aversions

Let $\{\gamma_h\}_{h \geq 1}$ be a decreasing sequence representing risk aversion at horizon h . In period t , the agent evaluates a consumption stream starting in period $t + h$ by

$$V_{t,t+h} = \left((1 - \beta) C_{t+h}^{1-\rho} + \beta E_{t+h} \left[V_{t,t+h+1}^{1-\gamma_{h+1}} \right]^{\frac{1-\rho}{1-\gamma_{h+1}}} \right)^{\frac{1}{1-\rho}} \quad \text{for all } h \geq 0. \quad (19)$$

The agent's utility in period t is given by setting $h = 0$ in (19) which we denote by $V_t \equiv V_{t,t}$ for all t :

$$V_t = \left((1 - \beta) C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}}$$

As in the Epstein-Zin model, utility V_t depends on deterministic current consumption C_t and a certainty equivalent $E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1}{1-\gamma_1}}$ of uncertain continuation values $V_{t,t+1}$, where the aggregation of the two periods occurs with constant elasticity of inter-temporal substitution given by $1/\rho$, regardless of the horizon h . However, in contrast to the Epstein-Zin model, the certainty equivalent of consumption starting at $t + 1$ is calculated with relative risk aversion γ_1 , wherein the certainty equivalent of consumption starting at $t + 2$ is calculated with relative risk aversion γ_2 , and so on. Our model therefore nests the Epstein-Zin model if we set $\gamma_h = \gamma$ for all h , which, in turn, nests the standard time-separable model for $\gamma = \rho$.

In order to derive the closed-form solution for $V_t \equiv V_{t,t}$, we assume that risk aversion is decreasing until some horizon H and constant thereafter, $\gamma_h > \gamma_{h+1}$ for $h < H$ and $\gamma_h = \tilde{\gamma}$ for $h \geq H$. Starting with $V_{t,t+H}$, our model then corresponds to the standard Epstein-Zin recursion with risk aversion $\tilde{\gamma}$ for which we can use the standard solution. Determining V_t then is just a matter of solving backwards.

A.1 Stochastic discount factor

We present the derivation of the stochastic discount factor with a general sequence of risk aversions $\{\gamma_h\}_{h \geq 1}$. The equations simplify to the ones in the main text by setting $\gamma_1 = \gamma$ and $\gamma_h = \tilde{\gamma}$ for $h \geq 2$.

Proof of Proposition 1. This appendix derives the stochastic discount factor of our dynamic model using an approach similar to the one used by [Luttmer and Mariotti \(2003\)](#) for dynamic inconsistency due to non-geometric discounting. In every period t the agent chooses consumption C_t for the current period and state-contingent levels of wealth $\{W_{t+1,s}\}$ for the next period to maximize current utility V_t subject to a budget constraint *and* anticipating optimal choice C_{t+h}^* in all following periods ($h \geq 1$):

$$\begin{aligned} \max_{C_t, \{W_{t+1}\}} & \left((1 - \beta) C_t^{1-\rho} + \beta E_t \left[(V_{t,t+1}^*)^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}} \\ \text{s.t.} & \quad \Pi_t C_t + E_t[\Pi_{t+1} W_{t+1}] \leq \Pi_t W_t \\ & \quad V_{t,t+h}^* = \left((1 - \beta) (C_{t+h}^*)^{1-\rho} + \beta E_{t+h} \left[(V_{t,t+h+1}^*)^{1-\gamma_{h+1}} \right]^{\frac{1-\rho}{1-\gamma_{h+1}}} \right)^{\frac{1}{1-\rho}} \quad \text{for all } h \geq 1. \end{aligned}$$

Denoting by λ_t the Lagrange multiplier on the budget constraint for the period- t problem, the first order conditions are:²⁴

- For C_t :

$$\left((1 - \beta) C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho} - 1} (1 - \beta) C_t^{-\rho} = \lambda_t.$$

²⁴For notational ease we drop the star from all Cs and Vs in the following optimality conditions but it should be kept in mind that all consumption values are the ones optimally chosen by the corresponding self.

- For each $W_{t+1,s}$:

$$\frac{1}{1-\rho} \left((1-\beta) C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} \beta \frac{d}{dW_{t+1,s}} \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} = \Pr[t+1,s] \frac{\Pi_{t+1,s}}{\Pi_t} \lambda_t.$$

Combining the two, we get an initial equation for the SDF:

$$\frac{\Pi_{t+1,s}}{\Pi_t} = \beta \frac{\frac{1}{1-\rho} \frac{1}{\Pr[t+1,s]} \frac{d}{dW_{t+1,s}} E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}}}{1} \frac{1}{(1-\beta) C_t^{-\rho}}. \quad (20)$$

The agent in state s at $t+1$ maximizes

$$\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[(V_{t+1,s,t+2}^*)^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}}$$

and has the analogous first order condition for $C_{t+1,s}$:

$$\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} (1-\beta) C_{t+1,s}^{-\rho} = \lambda_{t+1,s}.$$

The Lagrange multiplier $\lambda_{t+1,s}$ is equal to the marginal utility of an extra unit of wealth in state $t+1,s$:

$$\lambda_{t+1,s} = \frac{1}{1-\rho} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} \times \frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right).$$

Eliminating the Lagrange multiplier $\lambda_{t+1,s}$ and combining with the initial Equation (23) for the SDF, we get:

$$\frac{\Pi_{t+1,s}}{\Pi_t} = \beta \frac{\frac{1}{\Pr[t+1,s]} \frac{d}{dW_{t+1,s}} E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}}}{\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)} \left(\frac{C_{t+1,s}}{C_t} \right)^{-\rho}.$$

Expanding the V expressions, we can proceed with the differentiation in the numerator:

$$\begin{aligned} \frac{\Pi_{t+1,s}}{\Pi_t} &= E_t \left[\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} [\dots]^{\frac{1-\rho}{1-\gamma_2}} \right)^{\frac{1-\gamma_1}{1-\rho}} \right]^{\frac{1-\rho}{1-\gamma_1}-1} \\ &\quad \times \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} [\dots]^{\frac{1-\rho}{1-\gamma_2}} \right)^{\frac{1-\gamma_1}{1-\rho}-1} \\ &\quad \times \beta \frac{\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} [\dots]^{\frac{1-\rho}{1-\gamma_2}} \right)}{\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} [\dots]^{\frac{1-\rho}{1-\gamma_1}} \right)} \left(\frac{C_{t+1,s}}{C_t} \right)^{-\rho}. \end{aligned} \quad (21)$$

For Markov consumption $C = \phi W$, we can divide by $C_{t+1,s}$ and solve both differentiations:

- For the numerator:

$$\begin{aligned} &\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2} [\dots]^{\frac{1-\rho}{1-\gamma_3}} \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\rho}{1-\gamma_1}} \right) \\ &= \left((1-\beta) 1 + \beta E_{t+1,s} \left[\left((1-\beta) \left(\frac{C_{t+2}}{C_{t+1,s}} \right)^{1-\rho} + \beta E_{t+2} [\dots]^{\frac{1-\rho}{1-\gamma_3}} \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\rho}{1-\gamma_1}} \right) \\ &\quad \times \phi_{t+1,s}^{1-\rho} W_{t+1,s}^{-\rho}. \end{aligned}$$

- For the denominator:

$$\begin{aligned}
& \frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\gamma_1}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma_1}} \\
&= \left((1-\beta) 1 + \beta E_{t+1,s} \left[\left((1-\beta) \left(\frac{C_{t+2}}{C_{t+1,s}} \right)^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\gamma_1}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma_1}} \\
&\quad \times \phi_{t+1,s}^{1-\rho} W_{t+1,s}^{-\rho}.
\end{aligned}$$

Substituting these into Equation (24) and canceling we get:

$$\begin{aligned}
\frac{\Pi_{t+1,s}}{\Pi_t} &= \frac{(1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_3}} \right]^{\frac{1-\gamma_2}{1-\rho}}}{(1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\gamma_1}{1-\rho}}} \\
&\quad \times \beta \left(\frac{C_{t+1,s}}{C_t} \right)^{-\rho} \left(\frac{(1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\gamma_2}}}{E_t \left[\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1}[\dots]^{\frac{1-\rho}{1-\gamma_2}} \right)^{\frac{1-\gamma_1}{1-\rho}} \right]} \right)^{\rho-\gamma_1}.
\end{aligned}$$

Simplifying and cleaning up notation, we arrive at

$$\Pi_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t,t+1}}{E_t [V_{t,t+1}^{1-\gamma_1}]^{\frac{1}{1-\gamma_1}}} \right)^{\rho-\gamma_1} \left(\frac{V_{t,t+1}}{V_{t+1}} \right)^{1-\rho},$$

as stated in the text. □

A.2 Stochastic discount factor — horizon $h > 1$

Proof of Proposition 6 . To derive the h -period ahead stochastic discount factor, we use the inter-temporal marginal rate of substitution

$$\Pi_{t,t+h} = \frac{dV_t/dW_{t+h}}{dV_t/dC_t}$$

where

$$\begin{aligned} \frac{dV_t}{dW_{t+h}} &= \frac{dV_t}{dV_{t,t+h}} \times \frac{dV_{t,t+h}}{dW_{t+h}} \\ &= \frac{dV_t}{dV_{t,t+1}} \times \prod_{\tau=1}^{h-1} \frac{dV_{t,t+\tau}}{dV_{t,t+\tau+1}} \times \frac{dV_{t,t+h}}{dW_{t+h}}. \end{aligned}$$

Due to the homotheticity of our preferences, we can rely on the fact that both $V_{t,t+h}$ and V_{t+h} are homogeneous of degree one which implies that

$$\frac{dV_{t,t+h}/dW_{t+h}}{dV_{t+h}/dW_{t+h}} = \frac{V_{t,t+h}}{V_{t+h}}.$$

This allows us to derive the h -period SDF $\Pi_{t,t+h}$ as

$$\Pi_{t,t+h} = \beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-\rho} \left(\frac{V_{t,t+h}}{V_{t+h}} \right)^{1-\rho} \prod_{\tau=1}^h \left(\frac{V_{t,t+\tau}}{E_{t+\tau-1} \left[V_{t,t+\tau}^{1-\gamma_\tau} \right]^{\frac{1}{1-\gamma_\tau}}} \right)^{\rho-\gamma_\tau}.$$

A.3 Naive investors

In our analysis so far, we assumed agents are self-aware about their own dynamic inconsistencies. If our agent is naive about it instead, she wrongly assumes she will optimize on $V_{t,t+h}$ instead of V_{t+h} for all $h \geq 1$. In particular, the envelope conditions at $t + 1$ applies to $V_{t,t+1}$ in her one-period SDF, which becomes:

$$\Pi_{t,t+1}^{\text{naive}} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t,t+1}}{E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1}{1-\gamma_1}}} \right)^{\rho-\gamma_1}$$

The following one-period SDFs for $h \geq 1$ are then given by:

$$\Pi_{t+h,t+h+1}^{\text{naive}} = \beta \left(\frac{C_{t+h+1}}{C_{t+h}} \right)^{-\rho} \left(\frac{V_{t,t+h+1}}{E_{t+h} [V_{t,t+h+1}^{1-\gamma_{h+1}}]^{\frac{1}{1-\gamma_{h+1}}}} \right)^{\rho-\gamma_{h+1}}$$

When $\rho = 1$, naive agents behave as the buy-and-hold investors in Proposition 5 :

$$\Pi_{t,t+1}^{\text{naive}} \times \dots \times \Pi_{t+h-1,t+h}^{\text{naive}} \Big|_{\rho=1} = \Pi_{t,t+h}^{\text{buy-and-hold}} \Big|_{\rho=1} .$$

B Exact solutions for $\rho = 1$

This appendix presents the exact solutions derived for unit elasticity of inter-temporal substitution, $1/\rho = 1$, and log-normal uncertainty. Denoting logs by lowercase letters, our general model (19) becomes

$$v_t = (1 - \beta) c_t + \beta \left(E_t[v_{t,t+1}] + \frac{1}{2} (1 - \gamma_1) \text{var}_t(v_{t,t+1}) \right), \quad (22)$$

with the continuation value $v_{t,t+1}$ satisfying the recursion

$$v_{t,t+h} = (1 - \beta) c_{t+h} + \beta \left(E_{t+1}[v_{t,t+h+1}] + \frac{1}{2} (1 - \gamma_{h+1}) \text{var}_{t+1}(v_{t,t+h+1}) \right).$$

B.1 Valuation of risk and temporal resolution

Proof of Proposition 4. Starting at horizon $t + 1$, Equation (22) corresponds to the standard recursion

$$\tilde{v}_{t+1} = (1 - \beta) c_{t+1} + \frac{\beta}{1 - \tilde{\gamma}} \log(E_{t+1}[\exp((1 - \tilde{\gamma}) \tilde{v}_{t+2})]).$$

If consumption follows process (6) with $\sigma_t = \sigma$, guess and verify that the solution to the recursion satisfies

$$\tilde{v}_t - c_t = \tilde{\mu}_v + \tilde{\phi}_v x_t.$$

Substituting in and matching coefficients yields

$$\tilde{v}_t - c_t = \frac{\beta}{1-\beta} \mu_c + \frac{\beta \phi_c}{1-\beta v_x} x_t + \frac{1}{2} \frac{\beta(1-\tilde{\gamma})}{1-\beta} \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1-\beta v_x} \right)^2 \alpha_x^2 \right) \sigma^2.$$

From the perspective of period t ,

$$v_t = (1-\beta) c_t + \frac{\beta}{1-\gamma} \log(E_t[\exp((1-\gamma) \tilde{v}_{t+1})])$$

and

$$v_t - c_t = \frac{\beta}{1-\beta} \mu_c + \frac{\beta \phi_c}{1-\beta v_x} x_t + \frac{1}{2} \frac{\beta}{1-\beta} \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1-\beta v_x} \right)^2 \alpha_x^2 \right) \sigma^2 ((1-\gamma) + \beta(\gamma - \tilde{\gamma})),$$

as stated in the text.

If all risk is resolved at $t+1$, log continuation utility $v_{t,t+1}^*$ is given by

$$\begin{aligned} v_{t,t+1}^* &= (1-\beta) c_{t+1} + \beta \left((1-\beta) c_{t+2} + \beta \left((1-\beta) c_{t+3} + \dots \right) \right) \\ &= c_{t+1} + \sum_{h=1}^{\infty} \beta^h (c_{t+h+1} - c_{t+h}). \end{aligned}$$

From the perspective of period t , this continuation utility is normally distributed with mean and variance given by

$$\begin{aligned} E[v_{t,t+1}^*] &= c_t + \frac{1}{1-\beta} \mu + \frac{\phi_c}{1-\beta v_x} x_t, \\ \text{var}(v_{t,t+1}^*) &= \frac{1}{1-\beta^2} \sigma^2 \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1-\beta v_x} \right)^2 \alpha_x^2 \right). \end{aligned}$$

Using these expressions, we can derive the early resolution utility at t as

$$v_t^* - c_t = \frac{\beta}{1-\beta} \mu_c + \frac{\beta \phi_c}{1-\beta v_x} x_t + \frac{1}{2} \frac{\beta(1-\gamma)}{1-\beta^2} \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1-\beta v_x} \right)^2 \alpha_x^2 \right) \sigma^2.$$

Subtracting this from the utility v_t under gradual resolution, we arrive at a timing pre-

mium given by

$$\text{TP} = 1 - \exp\left(\frac{1}{2} \frac{\beta^2 (1 - \gamma)}{1 - \beta} \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1 - \beta v_x}\right)^2 \alpha_x^2\right) \sigma^2 \left(\frac{\gamma - \tilde{\gamma}}{1 - \gamma} + \frac{1}{1 + \beta}\right)\right),$$

as stated in the text. □

Case with stochastic volatility: If consumption follows process (6) with stochastic volatility, guess and verify that the solution to the recursion for \tilde{v}_t satisfies

$$\tilde{v}_t - c_t = \tilde{\mu}_v + \phi_v x_t + \tilde{\psi}_v \sigma_t^2$$

where

$$\begin{aligned} \tilde{\mu}_v &= \frac{\beta}{1 - \beta} \left(\mu_c + \tilde{\psi}_v \sigma^2 (1 - v_\sigma) + \frac{1}{2} (1 - \tilde{\gamma}) \tilde{\psi}_v^2 \alpha_\sigma^2 \right) \\ \phi_v &= \frac{\beta \phi_c}{1 - \beta v_x} \\ \tilde{\psi}_v &= \frac{1}{2} \frac{\beta (1 - \tilde{\gamma})}{1 - \beta v_\sigma} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right). \end{aligned}$$

We then obtain:

$$\begin{aligned} v_t - \tilde{v}_t &= -\frac{1}{2} \beta (\gamma - \tilde{\gamma}) \left[\left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \sigma_t^2 + \tilde{\psi}_v^2 \alpha_\sigma^2 \right] \\ v_t - c_t &= \frac{\beta}{1 - \beta} \left(\mu_c + \tilde{\psi}_v \sigma^2 (1 - v_\sigma) + \frac{1}{2} \tilde{\psi}_v^2 \left((1 - \gamma) + \beta (\gamma - \tilde{\gamma}) \right) \alpha_\sigma^2 \right) \\ &\quad + \phi_v x_t + \frac{\tilde{\psi}_v}{1 - \tilde{\gamma}} \left((1 - \gamma) + \beta v_\sigma (\gamma - \tilde{\gamma}) \right) \sigma_t^2 \end{aligned}$$

If all risk is resolved at $t + 1$, log continuation utility $v_{t,t+1}^*$ is given by

$$\begin{aligned} v_{t+1}^* &= (1 - \beta) c_{t+1} + \beta \left((1 - \beta) c_{t+2} + \beta \left((1 - \beta) c_{t+3} + \dots \right) \right) \\ &= c_{t+1} + \sum_{h=1}^{\infty} \beta^h (c_{t+h+1} - c_{t+h}). \end{aligned}$$

From the perspective of period t , this continuation utility is normally distributed with mean and variance given by

$$E_t[v_{t+1}^*] = c_t + \frac{1}{1-\beta} \mu + \frac{\phi_c}{1-\beta v_x} x_t,$$

$$\text{var}_t(v_{t+1}^*) = \frac{1}{1-\beta^2 v_\sigma} \left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2} \sigma^2 (1-v_\sigma) \right) \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1-\beta v_x} \right)^2 \alpha_x^2 \right).$$

Using these expressions, we can derive the early resolution utility at t as

$$v_t^* - c_t = \frac{\beta}{1-\beta} \mu_c + \frac{\beta \phi_c}{1-\beta v_x} x_t + \frac{1}{2} \frac{\beta (1-\gamma)}{1-\beta^2 v_\sigma} \left(\alpha_c^2 + \left(\frac{\beta \phi_c}{1-\beta v_x} \right)^2 \alpha_x^2 \right) \left(\sigma_t^2 + \frac{\beta^2}{1-\beta^2} \sigma^2 (1-v_\sigma) \right)$$

and

$$v_t - v_t^* = \frac{\beta}{1-\beta} \tilde{\psi}_v \sigma^2 (1-v_\sigma) \left(1 - \frac{1-\gamma}{1-\tilde{\gamma}} \frac{1-\beta v_\sigma}{1-\beta^2 v_\sigma} \frac{\beta}{1+\beta} \right)$$

$$+ \tilde{\psi}_v v_\sigma \sigma_t^2 \frac{\beta}{1-\tilde{\gamma}} \left((1-\gamma) \frac{1-\beta}{1-\beta^2 v_\sigma} + (\gamma - \tilde{\gamma}) \right)$$

$$+ \frac{1}{2} \beta \left[\frac{(1-\gamma) + \beta(\gamma - \tilde{\gamma})}{1-\beta} \right] \tilde{\psi}_v^2 \alpha_\sigma^2$$

Time premium under hyperbolic discounting " β - δ " model Assume $\gamma = \tilde{\gamma}$, but $\beta < \tilde{\beta}$.

$$\tilde{v}_t - c_t = \frac{\tilde{\beta}}{1-\tilde{\beta}} \mu_c + \frac{\tilde{\beta} \phi_c}{1-\tilde{\beta} v_x} x_t + \frac{1}{2} \frac{\tilde{\beta} (1-\gamma)}{1-\tilde{\beta}} \left(\alpha_c^2 + \left(\frac{\tilde{\beta} \phi_c}{1-\tilde{\beta} v_x} \right)^2 \alpha_x^2 \right) \sigma^2$$

$$v_t - c_t = \frac{\beta}{1-\gamma} E_t [\exp(1-\gamma) (\tilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t)]$$

$$\tilde{v}_t - c_t = \frac{\tilde{\beta}}{1-\gamma} E_t [\exp(1-\gamma) (\tilde{v}_{t+1} - c_{t+1} + c_{t+1} - c_t)]$$

$$\begin{aligned}
v_t - c_t &= \frac{\beta}{\tilde{\beta}} (\tilde{v}_t - c_t) \\
&= \frac{\beta}{1 - \tilde{\beta}} \mu_c + \frac{\beta \phi_c}{1 - \tilde{\beta} v_x} x_t + \frac{1}{2} \frac{\beta (1 - \gamma)}{1 - \tilde{\beta}} \left(\alpha_c^2 + \left(\frac{\tilde{\beta} \phi_c}{1 - \tilde{\beta} v_x} \right)^2 \alpha_x^2 \right) \sigma^2
\end{aligned}$$

If all risk is resolved at $t + 1$, log continuation utility $v_{t,t+1}^*$ is given by

$$\begin{aligned}
v_{t+1}^* &= (1 - \tilde{\beta}) c_{t+1} + \tilde{\beta} \left((1 - \beta) c_{t+2} + \tilde{\beta} \left((1 - \beta) c_{t+3} + \dots \right) \right) \\
&= c_{t+1} + \sum_{h=1}^{\infty} \tilde{\beta}^h (c_{t+h+1} - c_{t+h}) \\
&= c_t + \sum_{h=0}^{\infty} \tilde{\beta}^h (c_{t+h+1} - c_{t+h}).
\end{aligned}$$

From the perspective of period t , this continuation utility is normally distributed with mean and variance given by

$$\begin{aligned}
E_t[v_{t+1}^*] &= c_t + \frac{1}{1 - \tilde{\beta}} \mu_c + \frac{\phi_c}{1 - \tilde{\beta} v_x} x_t, \\
\text{var}_t(v_{t+1}^*) &= \frac{1}{1 - \tilde{\beta}^2} \sigma^2 \left(\alpha_c^2 + \left(\frac{\tilde{\beta} \phi_c}{1 - \tilde{\beta} v_x} \right)^2 \alpha_x^2 \right).
\end{aligned}$$

Using these expressions, we can derive the early resolution utility at t as

$$\begin{aligned}
v_t^* - c_t &= \frac{\beta}{1 - \gamma} E_t \left[\exp(1 - \gamma) (v_{t+1}^* - c_t) \right] \\
v_t^* - c_t &= \frac{\beta}{1 - \tilde{\beta}} \mu_c + \frac{\beta \phi_c}{1 - \tilde{\beta} v_x} x_t + \frac{1}{2} \frac{\beta (1 - \gamma)}{1 - \tilde{\beta}^2} \left(\alpha_c^2 + \left(\frac{\tilde{\beta} \phi_c}{1 - \tilde{\beta} v_x} \right)^2 \alpha_x^2 \right) \sigma^2
\end{aligned}$$

and

$$v_t - v_t^* = \frac{1}{2} \frac{\beta \tilde{\beta} (1 - \gamma)}{1 - \tilde{\beta}^2} \left(\alpha_c^2 + \left(\frac{\tilde{\beta} \phi_c}{1 - \tilde{\beta} v_x} \right)^2 \alpha_x^2 \right) \sigma^2$$

with $\beta < \tilde{\beta}$, $\frac{\tilde{\beta}^2}{1 - \tilde{\beta}^2} > \frac{\beta \tilde{\beta}}{1 - \beta^2} > \frac{\beta^2}{1 - \beta^2}$.

When $\gamma > \rho$, the timing premium under $\{\beta, \tilde{\beta}\}$ is greater than under the β -only model and lower than under the $\tilde{\beta}$ -only model.

B.2 Extension to other information arrival structures

General sequence of risk aversions and comparison $t + 1$ vs. $t + 2$. In the main text, we show that while an agent with Epstein-Zin preferences prefers early resolution iff $\gamma > \rho = 1$, our agent with horizon-dependent risk aversion can prefer late resolution, even if $\gamma > \tilde{\gamma} > 1$, as long as

$$\gamma - \tilde{\gamma} > \frac{\gamma - 1}{1 + \beta},$$

i.e. as long as γ is sufficiently greater than $\tilde{\gamma}$. Suppose we have a sequence of risk aversions $\{\gamma_h\}_{h=1}^{\infty}$ that is decreasing to some horizon H and then constant at $\tilde{\gamma}$. For the comparison of gradual resolution vs. resolution at $t + 1$, denoted by v_t^{t+1} , we have

$$v_t - v_t^{t+1} = \frac{1}{2} \left(1 - \left(\gamma_1 - (1 + \beta) \sum_{h=1}^{\infty} \beta^{h-1} (\gamma_h - \gamma_{h+1}) \right) \right) \frac{\beta^2}{1 - \beta^2} \alpha_v^2 \sigma^2,$$

which has the same structure as in the timing premium for just two levels of risk aversion in equation (13). The agent prefers gradual later resolution if

$$\sum_{h=1}^{\infty} \beta^{h-1} (\gamma_h - \gamma_{h+1}) > \frac{\gamma_1 - 1}{1 + \beta}$$

i.e. as long as the sequence $\{\gamma_h\}_{h=1}^{\infty}$ is sufficiently decreasing.

For the comparison of resolution at $t + 1$ vs. resolution at $t + 2$ we have

$$v_t^{t+1} - v_t^{t+2} = \frac{1}{2} \left(\beta(1 - \gamma_1) - (1 - \gamma_2) \right) \frac{\beta^2}{1 - \beta^2} \alpha_v^2 \sigma^2.$$

While an agent with Epstein-Zin preferences prefers early resolution iff $\gamma > 1$ since

$$\beta(1 - \gamma) - (1 - \gamma) = (1 - \beta)(\gamma - 1),$$

our agent can prefer late resolution, as long as

$$\gamma_1 - \gamma_2 > (1 - \beta)(\gamma_1 - 1)$$

i.e. as long as γ_1 is sufficiently greater than γ_2 .

B.3 Stochastic discount factor

We now specialize to the case of two levels of risk aversion, setting $\gamma_1 = \gamma$ and $\gamma_h = \tilde{\gamma}$ for $h \geq 2$.

Proof of Lemma 1. Under the stochastic process (6), we can guess and verify that the solution to the recursion for \tilde{v}_t satisfies

$$\tilde{v}_t - c_t = \tilde{\mu}_v + \phi_v x_t + \tilde{\psi}_v \sigma_t^2$$

where we write $\tilde{\psi}_v = \psi_v(\tilde{\gamma})$ throughout for simplification, and

$$\begin{aligned} \tilde{\mu}_v &= \frac{\beta}{1 - \beta} \left(\mu_c + \tilde{\psi}_v \sigma^2 (1 - v_\sigma) + \frac{1}{2} (1 - \tilde{\gamma}) \tilde{\psi}_v^2 \alpha_\sigma^2 \right) \\ \phi_v &= \frac{\beta \phi_c}{1 - \beta v_x} \\ \tilde{\psi}_v &= \frac{1}{2} \frac{\beta (1 - \tilde{\gamma})}{1 - \beta v_\sigma} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right). \end{aligned}$$

Substituting these into (22), we arrive at the solution for v_t :

$$v_t - \tilde{v}_t = -\frac{1}{2}\beta(\gamma - \tilde{\gamma}) \left[(\alpha_c^2 + \phi_v^2 \alpha_x^2) \sigma_t^2 + \tilde{\psi}_v^2 \alpha_\sigma^2 \right]$$

and

$$\begin{aligned} v_t - c_t &= \frac{\beta}{1 - \beta} \left(\mu_c + \tilde{\psi}_v \sigma^2 (1 - \nu_\sigma) + \frac{1}{2} \tilde{\psi}_v^2 ((1 - \gamma) + \beta(\gamma - \tilde{\gamma})) \alpha_\sigma^2 \right) \\ &\quad + \phi_v x_t + \frac{\tilde{\psi}_v}{1 - \tilde{\gamma}} ((1 - \gamma) + \beta \nu_\sigma (\gamma - \tilde{\gamma})) \sigma_t^2 \end{aligned}$$

□

Proof of Proposition 3. Using the results of Lemmas 1 and (22), the expression for the SDF follows from Equation (5):

$$\begin{aligned} \pi_{t,t+1} &= \overbrace{\log \beta - \mu_c - \phi_c x_t - \frac{1}{2} (1 - \gamma)^2 \left([\alpha_c^2 + \phi_v^2 \alpha_x^2] \sigma_t^2 + \tilde{\psi}_v^2 \alpha_\sigma^2 \right)}^{\tilde{\pi}_t} \\ &\quad - \gamma \alpha_c \sigma_t w_{c,t+1} + (1 - \gamma) \phi_v \alpha_x \sigma_t w_{x,t+1} \\ &\quad + (1 - \gamma) \psi_v(\tilde{\gamma}) \alpha_\sigma w_{\sigma,t+1}, \end{aligned}$$

The risk-free rate is defined as $r_{f,t} = -\log E_t(\Pi_{t,t+1})$ and simplifies to

$$r_{f,t} = -\log \beta + \mu_c + \phi_c x_t + \left(\frac{1}{2} - \gamma \right) \alpha_c^2 \sigma_t^2$$

as stated in the text.

□

B.4 Equity premium

To derive the equity premium, we log-linearize the returns on the dividend stream:

$$\begin{aligned}
r_{m,t+1} &= \log \left(\frac{P_{t+1} + D_{t+1}}{P_t} \right) \\
&= \Delta d_{t+1} + \log (1 + e^{z_{t+1}}) - z_t \\
&\approx k_0 + k_1 z_{t+1} - z_t + \Delta d_{t+1}
\end{aligned}$$

where $z_t = p_t - d_t$ and $k_1 = \frac{e^z}{1+e^z}$.

From

$$E_t (\Pi_{t,t+1} R_{m,t+1}) = 1$$

we obtain a recursion in z_t .

Guess: $z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$

$$\log E_t \left(\exp \left(\begin{pmatrix} \log \beta - \mu_c - \frac{1}{2} (1 - \gamma)^2 \tilde{\psi}_v^2 \alpha_\sigma^2 + k_0 + k_1 A_2 \sigma^2 (1 - \nu_\sigma) \\ -\phi_c x_t - A_1 x_t + \phi_d x_t + k_1 A_1 \nu_x x_t \\ -\frac{1}{2} (1 - \gamma)^2 [\alpha_c^2 + \phi_v^2 \alpha_x^2] \sigma_t^2 - A_2 \sigma_t^2 + k_1 A_2 \nu_\sigma \sigma_t^2 \\ -\gamma \alpha_c \sigma_t W_{c,t+1} + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1} + \\ (1 - \gamma) \phi_v \alpha_x \sigma_t W_{x,t+1} + k_1 A_1 \alpha_x \sigma_t W_{x,t+1} \\ + (1 - \gamma) \psi_v(\tilde{\gamma}) \alpha_\sigma W_{\sigma,t+1} + k_1 A_2 \alpha_\sigma W_{\sigma,t+1} \end{pmatrix} \right) \right) = 0$$

and so:

$$\begin{aligned}
\frac{\phi_d - \phi_c}{1 - k_1 \nu_x} &= A_1 \\
-\frac{1}{2} (1 - \gamma)^2 [\alpha_c^2 + \phi_v^2 \alpha_x^2] \\
+\frac{1}{2} \alpha_d^2 + \frac{1}{2} (\chi - \gamma)^2 \alpha_c^2 + \frac{1}{2} (k_1 A_1 + (1 - \gamma) \phi_v)^2 \alpha_x^2 &= A_2 (1 - k_1 \nu_\sigma)
\end{aligned}$$

Note A_1 and A_2 are both unaffected by $\tilde{\gamma}$, and therefore identical to the standard model.

Since the equity premium is determined by the covariation between the returns $r_{m,t+1}$ and the stochastic discount factor $\pi_{t,t+1}$, and the loadings on the consumption level shocks are unchanged relative to the standard model for both the market returns and the SDF, the only term that is impacted is the cross-term for the loadings on the volatility shocks. The contribution of volatility shocks to the equity premium under horizon dependent risk aversion is simply the one under the standard model multiplied by $\frac{1-\tilde{\gamma}}{1-\gamma}$ (see Corollary 1).

B.5 Term structure of returns

B.5.1 General claims

To make the problem as general as possible, we analyze horizon-dependent claims that are priced recursively as

$$Y_{t,h} = E_t [\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1}],$$

that is

$$y_{t,h} = E_t [\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}] + \frac{1}{2} \text{var}_t (\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1}),$$

where

$$g_{y,t+1} = \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ + \alpha_{y,c} \alpha_c \sigma_t \omega_{c,t+1} + \alpha_{y,x} \alpha_x \sigma_t \omega_{x,t+1} + \alpha_{y,\sigma} \alpha_\sigma \sigma_t \omega_{\sigma,t+1} + \alpha_{y,d} \alpha_d \sigma_t \omega_{d,t+1},$$

and $Y_{t,0} = 1$.

Guess that

$$Y_{t,h} = \exp(\tilde{\mu}_{y,h} + \phi_{y,h} x_t + \psi_{y,h} \sigma_t^2).$$

Suppose $h \geq 1$, then:

$$\log \tilde{\Pi}_{t,t+1} G_{t,t+1} Y_{t+1,h-1} = \begin{cases} \log \beta - \mu_c - \phi_c x_t - \frac{1}{2} (1 - \gamma)^2 [(\alpha_c^2 + \phi_v^2 \alpha_x^2) \sigma_t^2 + \tilde{\psi}_v^2 \alpha_\sigma^2] \\ + \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ + \tilde{\mu}_{y,h-1} + \phi_{y,h-1} \nu_x x_t + \psi_{y,h-1} (\sigma^2 (1 - \nu_\sigma) + \nu_\sigma \sigma_t^2) \\ + (-\gamma + \alpha_{y,c}) \alpha_c \sigma_t W_{t+1} + ((1 - \gamma) \phi_v + \alpha_{y,x} + \phi_{y,n-1}) \alpha_x \sigma_t W_{t+1} \\ + ((1 - \gamma) \tilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,h-1}) \alpha_\sigma W_{t+1} \\ + \alpha_{y,d} \alpha_d \sigma_t W_{t+1} \end{cases}$$

Matching coefficients, we find the recursions, for $h \geq 1$:

- Terms in x_t :

$$\begin{aligned} \phi_{y,h} &= -\phi_c + \phi_y + \phi_{y,h-1} \nu_x \\ \Rightarrow \phi_{y,h} &= (-\phi_c + \phi_y) \frac{1 - \nu_x^h}{1 - \nu_x} \end{aligned}$$

- Terms in σ_t^2 :

$$\begin{aligned} \psi_{y,h} &= -\frac{1}{2} (1 - \gamma)^2 (\alpha_c^2 + \phi_v^2 \alpha_x^2) + \psi_{y,h-1} \nu_\sigma + \psi_y \\ &\quad + \frac{1}{2} \left((-\gamma + \alpha_{y,c})^2 \alpha_c^2 + ((1 - \gamma) \phi_v + \alpha_{y,x} + \phi_{y,h-1})^2 \alpha_x^2 + \alpha_{y,d}^2 \alpha_d^2 \right) \end{aligned}$$

and thus the solution, for $h \geq 1$:

$$\begin{aligned} \psi_{y,h} &= \left[-\frac{1}{2} (1 - \gamma)^2 (\alpha_c^2 + \phi_v^2 \alpha_x^2) + \psi_y + \frac{1}{2} \left((-\gamma + \alpha_{y,c})^2 \alpha_c^2 + \alpha_{y,d}^2 \alpha_d^2 \right) \right] \frac{1 - \nu_\sigma^h}{1 - \nu_\sigma} \\ &\quad + \frac{1}{2} \sum_{n=0}^{h-1} \nu_\sigma^n \left((1 - \gamma) \phi_v + \alpha_{y,x} + \phi_{y,n-1-h} \right)^2 \alpha_x^2 \end{aligned}$$

- Constant:

$$\begin{aligned}\tilde{\mu}_{y,h} - \tilde{\mu}_{y,h-1} &= \log \beta - \mu_c + \mu_y + \sigma^2 (1 - \nu_\sigma) \psi_{y,h-1} \\ &\quad + \frac{1}{2} \left(((1 - \gamma) \tilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,h-1})^2 - (1 - \gamma)^2 \tilde{\psi}_v^2 \right) \alpha_\sigma^2\end{aligned}$$

and thus the solution, for $h \geq 1$:

$$\begin{aligned}\tilde{\mu}_{y,h} &= h \left(\log \beta - \mu_c + \mu_y - \frac{1}{2} (1 - \gamma)^2 \tilde{\psi}_v^2 \alpha_\sigma^2 \right) \\ &\quad + \sum_{n=0}^{h-1} \left[\sigma^2 (1 - \nu_\sigma) \psi_{y,n} + \frac{1}{2} \left((1 - \gamma) \tilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,n} \right)^2 \alpha_\sigma^2 \right]\end{aligned}$$

Note only the constant terms $\{\tilde{\mu}_{y,h}\}$ are affected by the wedge between γ and $\tilde{\gamma}$. □

In line with the specification of [van Binsbergen and Kojen \(2016\)](#), we consider one-period holding returns for these claims of the form

$$\begin{aligned}1 + R_{t+1,h}^Y &= \frac{G_{y,t+1} Y_{t+1,h-1}}{Y_{t,h}} = \frac{G_{y,t+1} Y_{t+1,h-1}}{E_t[\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1}]} \\ &= R_{f,t} \frac{E_t[\Pi_{t,t+1}] G_{y,t+1} Y_{t+1,h-1}}{E_t[\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1}]},\end{aligned}$$

with the risk-free rate

$$R_{f,t} = \frac{1}{E_t[\Pi_{t,t+1}]}.$$

The conditional Sharpe Ratio is

$$\begin{aligned}
\text{SR}_{t,h}^Y &= \frac{E_t \left[1 + R_{t+1,h}^Y \right] - 1}{\sqrt{\text{var}_t \left(1 + R_{t+1,h}^Y \right)}} \\
&= \frac{E_t \left(1 + R_{t+1,h}^Y \right) - 1}{\sqrt{E_t \left(\left(1 + R_{t+1,h}^Y \right)^2 \right) - \left(E_t \left(1 + R_{t+1,h}^Y \right) \right)^2}} \\
&\approx \frac{r_{f,t} + \begin{cases} (\gamma \alpha_{y,c} \alpha_c^2 - (1 - \gamma) \phi_v (\alpha_{y,x} + \phi_{y,h-1}) \alpha_x^2) \sigma_t^2 \\ - (1 - \gamma) \tilde{\psi}_v (\alpha_{y,\sigma} + \psi_{y,h-1}) \alpha_\sigma^2 \end{cases}}{\sqrt{\sigma_t^2 \left(\alpha_{y,c}^2 \alpha_c^2 + (\alpha_{y,x} + \phi_{y,h-1})^2 \alpha_x^2 + \alpha_{y,d}^2 \alpha_d^2 \right) + (\alpha_{y,\sigma} + \psi_{y,h-1})^2 \alpha_\sigma^2}}.
\end{aligned}$$

In line with the specification of [van Binsbergen and Koijen \(2016\)](#), we also consider one-period holding returns for futures on these claims of the form

$$\begin{aligned}
R_{t+1,h}^{F,Y} + 1 &= \frac{1 + R_{t+1,h}^Y}{1 + R_{t+1,h}^B} = \frac{G_{y,t+1} Y_{t+1,h-1}}{Y_{t,h}} \frac{B_{t,h}}{B_{t+1,h-1}} \\
&= \frac{G_{y,t+1} Y_{t+1,h-1}}{E_t (\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1})} \frac{E_t (\Pi_{t,t+1} B_{t+1,h-1})}{B_{t+1,h-1}},
\end{aligned}$$

where $B_{t,h}$ is the price of \$1 at horizon h , i.e. the price of a Bond with horizon h .

Their conditional Sharpe Ratio is

$$\begin{aligned}
\text{SR}_{t,h}^{F,Y} &= \frac{E_t \left(1 + R_{t+1,h}^{F,Y} \right) - 1}{\sqrt{\text{var}_t \left(1 + R_{t+1,h}^{F,Y} \right)}} \\
&= \frac{E_t \left(1 + R_{t+1,h}^{F,Y} \right) - 1}{\sqrt{E_t \left(\left(1 + R_{t+1,h}^{F,Y} \right)^2 \right) - \left(E_t \left(1 + R_{t+1,h}^{F,Y} \right) \right)^2}} \\
&\approx \frac{\begin{cases} \sigma_t^2 \left(\gamma \alpha_{y,c} \alpha_c^2 - \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1} \right) \left((1-\gamma) \phi_v + \phi_{b,h-1} \right) \alpha_x^2 \right) \\ - \left(\alpha_{y,\sigma} + \psi_{y,h-1} - \psi_{b,h-1} \right) \left((1-\gamma) \tilde{\psi}_v + \psi_{b,h-1} \right) \alpha_\sigma^2 \end{cases}}{\sqrt{\sigma_t^2 \left(\alpha_{y,c}^2 \alpha_c^2 + \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1} \right)^2 \alpha_x^2 + \alpha_{y,d}^2 \alpha_d^2 \right) + \left(\alpha_{y,\sigma} + \psi_{y,h-1} - \psi_{b,h-1} \right)^2 \alpha_\sigma^2}}.
\end{aligned}$$

For the *unconditional* Sharpe ratio observe that the volatility process

$$\sigma_{t+1}^2 - \sigma^2 = \nu_\sigma \left(\sigma_t^2 - \sigma^2 \right) + \alpha_\sigma W_{t+1}$$

is stationary under the constraint $\nu_\sigma < 1$ with normal distribution with mean σ^2 and variance $\Sigma_\sigma = \frac{\alpha_\sigma^2}{1-\nu_\sigma^2}$.

and therefore $E \left(\exp \left(a \sigma_t^2 \right) \right) = \exp \left(a \sigma^2 + \frac{1}{2} a^2 \frac{\alpha_\sigma^2}{1-\nu_\sigma^2} \right)$.

B.5.2 Bonds

Bond prices Let the price at time t for \$1 in h periods be $B_{t,h}$ with $B_{t,0} = 1$. For $h \geq 1$, we have

$$B_{t,h} = E_t[\Pi_{t,t+1} B_{t+1,h-1}]$$

This is the general problem from above with $g_{y,t+1} = 0$ for all t and therefore

$$b_{t,h} = \tilde{\mu}_{b,h} + \phi_{b,h} x_t + \psi_{b,h} \sigma_t^2,$$

with

$$\phi_{b,h} = -\phi_c \frac{1 - \nu_x^h}{1 - \nu_x}$$

$$\begin{aligned} \psi_{b,h} &= -\frac{1}{2} (1 - \gamma)^2 (\alpha_c^2 + \phi_v^2 \alpha_x^2) + \psi_{b,h-1} \nu_\sigma \\ &\quad + \frac{1}{2} (\gamma^2 \alpha_c^2 + ((1 - \gamma) \phi_v + \phi_{b,h-1})^2 \alpha_x^2) \end{aligned}$$

and

$$\psi_{b,1} = \left(\gamma - \frac{1}{2} \right) \alpha_c^2 > 0$$

and $\psi_{b,h} > 0$ for all h , and $\psi_{b,h}$ increasing in h .

Further,

$$\tilde{\mu}_{b,h} - \tilde{\mu}_{b,h-1} = \log \beta - \mu_c + \sigma^2 (1 - \nu_\sigma) \psi_{b,h-1} + \left((1 - \gamma) \tilde{\psi}_v \psi_{b,h-1} + \frac{1}{2} \psi_{b,h-1}^2 \right) \alpha_\sigma^2$$

increasing in h . But $\tilde{\mu}_{b,h}$ can be decreasing if $\log \beta - \mu_c < 0$.

Bond returns The one-period returns are given by:

$$R_{t+1,h}^B = \frac{B_{t+1,h-1}}{B_{t,h}} - 1$$

and therefore

$$\begin{aligned} \log \left(R_{t+1,h}^B + 1 \right) &= -\log \beta + \mu_c - \left((1 - \gamma) \tilde{\psi}_v \psi_{b,h-1} + \frac{1}{2} \psi_{b,h-1}^2 \right) \alpha_\sigma^2 + \phi_c x_t + (\psi_{b,h-1} \nu_\sigma - \psi_{b,h}) \sigma_t^2 \\ &\quad + \psi_{b,h-1} \alpha_\sigma W_{t+1} + \phi_{b,h-1} \alpha_x \sigma_t W_{t+1} \end{aligned}$$

the term structure of expected returns is given by:

$$E_t \left(R_{t+1,h}^B + 1 \right) \approx -\log \beta + \mu_c - (1 - \gamma) \tilde{\psi}_v \psi_{b,h-1} \alpha_\sigma^2 + \phi_c x_t - \left(\left(\gamma - \frac{1}{2} \right) \alpha_c^2 + (1 - \gamma) \phi_v \phi_{b,h-1} \alpha_x^2 \right) \sigma_t^2$$

$$E_t \left(R_{t+1,h+1}^B \right) - E_t \left(R_{t+1,h}^B \right) \approx (\gamma - 1) \tilde{\psi}_v (\psi_{b,h} - \psi_{b,h-1}) \alpha_\sigma^2 + (\gamma - 1) \phi_v \phi_c \frac{\nu_x^h - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 \sigma_t^2 \leq 0.$$

The only impact of $\tilde{\gamma}$ is through $\tilde{\psi}_v$, and makes the slope less decreasing (but not increasing).

Risk-free rate The risk-free rate is given by

$$r_{f,t} = -\log B_{t,1}$$

i.e.

$$r_{f,t} = -\log \beta + \mu_c + \phi_c x_t - \left(\gamma - \frac{1}{2} \right) \alpha_c^2 \sigma_t^2$$

B.5.3 Dividend strips

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$ with $P_{t,0} = D_t$. Then for $h \geq 1$:

$$\frac{P_{t,h}}{D_t} = E_t \left(\Pi_{t,t+1} \frac{D_{t+1}}{D_t} \frac{P_{t+1,h-1}}{D_{t+1}} \right),$$

which is the general problem from above with

$$g_{p,t+1} = d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1},$$

for all t and therefore

$$p_{t,h} - d_t = \tilde{\mu}_{p,h} + \phi_{d,h} x_t + \psi_{d,h} \sigma_t^2,$$

with

$$\phi_{d,h} = (-\phi_c + \phi_d) \frac{1 - \nu_x^h}{1 - \nu_x}$$

$$\begin{aligned} \psi_{d,h} &= -\frac{1}{2} (1 - \gamma)^2 (\alpha_c^2 + \phi_v^2 \alpha_x^2) + \psi_{d,h-1} \nu_\sigma \\ &+ \frac{1}{2} \left((-\gamma + \chi)^2 \alpha_c^2 + ((1 - \gamma) \phi_v + \phi_{d,h-1})^2 \alpha_x^2 + \alpha_d^2 \right) \end{aligned}$$

$$\psi_{d,1} = \frac{1}{2} \alpha_d^2 + (\chi + 1 - 2\gamma) (\chi - 1) \frac{1}{2} \alpha_c^2$$

the sign depends on the parameters of the model.

$$\tilde{\mu}_{d,h} - \tilde{\mu}_{d,h-1} = \log \beta - \mu_c + \mu_d + \sigma^2 (1 - \nu_\sigma) \psi_{d,h-1} + \left((1 - \gamma) \tilde{\psi}_v \psi_{d,h-1} + \frac{1}{2} \psi_{d,h-1}^2 \right) \alpha_\sigma^2$$

where the sign depends again on the parameters of the model.

For the dividend strips, the spot one-period returns are given by

$$R_{t+1,h}^P + 1 = \frac{P_{t+1,h-1}/D_{t+1}}{P_{t,h}/D_t} \frac{D_{t+1}}{D_t},$$

$$\begin{aligned} \log \left(R_{t+1,h}^P + 1 \right) &= -\log \beta + \mu_c - \left((1 - \gamma) \tilde{\psi}_v \psi_{d,h-1} + \frac{1}{2} \psi_{d,h-1}^2 \right) \alpha_\sigma^2 \\ &+ \phi_c x_t + (\psi_{d,h-1} \nu_\sigma - \psi_{d,h}) \sigma_t^2 \\ &+ \psi_{d,h-1} \alpha_\sigma W_{t+1} + \phi_{d,h-1} \alpha_x \sigma_t W_{t+1} + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1} \end{aligned}$$

the conditional expected one-period returns are

$$E_t \left(R_{t+1,h}^P + 1 \right) \approx -\log \beta + \mu_c - (1 - \gamma) \tilde{\psi}_v \psi_{d,h-1} \alpha_\sigma^2 + \phi_c x_t \\ - \left(\left(\gamma (1 - \chi) - \frac{1}{2} \right) \alpha_c^2 + (1 - \gamma) \phi_v \phi_{d,h-1} \alpha_x^2 \right) \sigma_t^2$$

$$E_t \left(R_{t+1,h+1}^P \right) - E_t \left(R_{t+1,h}^P \right) \approx \underbrace{(\gamma - 1) \tilde{\psi}_v (\psi_{d,h} - \psi_{d,h-1}) \alpha_\sigma^2}_{\leq 0} + \underbrace{(\gamma - 1) \phi_v (\phi_c - \phi_d) \frac{\nu_x^h - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 \sigma_t^2}_{\geq 0}$$

We need $(\psi_{d,h} - \psi_{d,h-1}) \geq 0$ to generate a downward sloping term structure, but that does not depend on the choice of $\tilde{\gamma}$. If $(\psi_{d,h} - \psi_{d,h-1}) \leq 0$, then the returns are upward sloping, but less so in our model.

Note, that the returns are MORE upward sloping when σ_t is high...

□

The future one-period returns are given by:

$$R_{t+1,h}^{F,P} + 1 = \frac{1 + R_{t+1,h}^P}{1 + R_{t+1,h}^B}$$

$$\log \left(R_{t+1,h}^{F,P} + 1 \right) = - \left((1 - \gamma) \tilde{\psi}_v (\psi_{d,h-1} - \psi_{b,h-1}) + \frac{1}{2} (\psi_{d,h-1}^2 - \psi_{b,h-1}^2) \right) \alpha_\sigma^2 \\ + ((\psi_{d,h-1} - \psi_{b,h-1}) \nu_\sigma - (\psi_{d,h} - \psi_{b,h})) \sigma_t^2 \\ + (\psi_{d,h-1} - \psi_{b,h-1}) \alpha_\sigma W_{t+1} + (\phi_{d,h-1} - \phi_{b,h-1}) \alpha_x \sigma_t W_{t+1} + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1}$$

$$E_t \left(R_{t+1,h}^{F,P} + 1 \right) = - \left(\underbrace{((1-\gamma)\tilde{\psi}_v + \psi_{b,h-1})}_{\geq 0 \text{ and increasing}} (\psi_{d,h-1} - \psi_{b,h-1}) \right) \alpha_\sigma^2$$

$$+ \left(\gamma\chi\alpha_c^2 + \underbrace{((\gamma-1)\phi_v - \phi_{b,h-1})}_{\geq 0 \text{ and increasing}} (\phi_{d,h-1} - \phi_{b,h-1}) \alpha_x^2 \right) \sigma_t^2$$

Note:

$$\psi_{d,h} - \psi_{b,h} = (\psi_{d,h-1} - \psi_{b,h-1}) \nu_\sigma$$

$$+ \left(\underbrace{\chi \left(\frac{1}{2}\chi - \gamma \right)}_{\leq 0} \alpha_c^2 + \underbrace{\left((1-\gamma)\phi_v + \frac{1}{2}(\phi_{d,h-1} + \phi_{b,h-1}) \right)}_{\leq 0 \text{ for } \gamma \text{ high enough}} \underbrace{(\phi_{d,h-1} - \phi_{b,h-1})}_{\geq 0} \alpha_x^2 + \underbrace{\frac{1}{2}\alpha_d^2}_{\geq 0} \right)$$

the sign depends on the parameters. But if it is positive increasing, $\tilde{\gamma}$ reduces the downward impact of it on the term structure of expected returns. Only if it is negative and decreasing does our model help relative to the standard model, but then the slope is upward sloping....

Note, a higher σ_t means a MORE upward sloping term structure again

□

the Sharpe ratio term structure is given by:

$$SR_{t,n}^{F,P} \approx \frac{\begin{cases} \sigma_t^2 (\gamma\chi\alpha_c^2 - (\phi_{d,h-1} - \phi_{b,h-1}) ((1-\gamma)\phi_v + \phi_{b,h-1}) \alpha_x^2) \\ - (\psi_{d,h-1} - \psi_{b,h-1}) \left((1-\gamma)\tilde{\psi}_v + \psi_{b,h-1}^1 \right) \alpha_\sigma^2 \end{cases}}{\sqrt{\sigma_t^2 \left(\chi^2\alpha_c^2 + (\phi_{d,h-1} - \phi_{b,h-1})^2 \alpha_x^2 + \alpha_d^2 \right) + (\psi_{d,h-1} - \psi_{b,h-1})^2 \alpha_\sigma^2}}$$

If the expected returns term structure is upward sloping with $\psi_{d,h} - \psi_{b,h} \leq 0$ and decreasing, then $\tilde{\gamma}$ can help make the sharpe ratio term structure downward sloping.

The unconditional Sharpe ratio term structure is:

$$\text{SR}_h^{F,P} \approx \frac{\begin{cases} \sigma^2 (\gamma \chi \alpha_c^2 - (\phi_{d,h-1} - \phi_{b,h-1}) ((1 - \gamma) \phi_v + \phi_{b,h-1}) \alpha_x^2) \\ + \frac{1}{2} \frac{\alpha_\sigma^2}{1 - \nu_\sigma^2} (\gamma \chi \alpha_c^2 - (\phi_{d,h-1} - \phi_{b,h-1}) ((1 - \gamma) \phi_v + \phi_{b,h-1}) \alpha_x^2)^2 \\ - (\psi_{d,h-1} - \psi_{b,h-1}) \left((1 - \gamma) \tilde{\psi}_v + \psi_{b,h-1}^1 \right) \alpha_\sigma^2 \end{cases}}{\sqrt{\begin{cases} \sigma^2 (\chi^2 \alpha_c^2 + (\phi_{d,h-1} - \phi_{b,h-1})^2 \alpha_x^2 + \alpha_d^2) \\ + 2 \left((\psi_{d,h-1} - \psi_{b,h-1}) \nu_\sigma - (\psi_{d,h} - \psi_{b,h}) + (\phi_{d,h-1} - \phi_{b,h-1})^2 \alpha_x^2 + \chi^2 \alpha_c^2 + \alpha_d^2 \right)^2 \Sigma_\sigma \\ - \left((\psi_{d,h-1} - \psi_{b,h-1}) \nu_\sigma - (\psi_{d,h} - \psi_{b,h}) + \frac{1}{2} \left((\phi_{d,h-1} - \phi_{b,h-1})^2 \alpha_x^2 + \chi^2 \alpha_c^2 + \alpha_d^2 \right) \right)^2 \Sigma_\sigma \\ + (\psi_{d,h-1} - \psi_{b,h-1})^2 \alpha_\sigma^2 \end{cases}}}$$

B.6 Term structure of returns - Illiquid markets

We analyze horizon-dependent dividend claims when markets are illiquid and prices are set by buy-and-hold investors. From above, the SDF for a horizon h investor is (when $\rho = 1$):

$$\Pi_{t,t+h} = \beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1} \left(\frac{\tilde{V}_{t+1}}{E_t [\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \cdots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1} [\tilde{V}_{t+h}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}}$$

Consider a dividend with horizon h priced at time t under $\Pi_{t,t+h}$,

$$P_{t,h} = E_t [\Pi_{t,t+h} D_{t+h}],$$

$$P_{t,h} = E_t \left[\beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1} \left(\frac{\tilde{V}_{t+1}}{E_t [\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \cdots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1} [\tilde{V}_{t+h}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} D_{t+h} \right],$$

The price at time $t + 1$ is under $\Pi_{t+1,t+1+(h-1)}$,

$$\frac{P_{t+1,h-1}}{D_{t+1}} = E_{t+1} \left[\Pi_{t+1,t+1+(h-1)} \frac{D_{t+h}}{D_t} \right],$$

$$P_{t+1,h-1} = E_{t+1} \left[\beta^{h-1} \left(\frac{C_{t+h}}{C_{t+1}} \right)^{-1} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma} \left(\frac{\tilde{V}_{t+3}}{E_{t+2} [\tilde{V}_{t+3}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} \cdots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1} [\tilde{V}_{t+h}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} D_{t+h} \right],$$

The one-period return is given by:

$$R_{t+1,h}^{F,P} + 1 = \frac{\frac{P_{t+1,h-1}}{P_{t,h}}}{\frac{B_{t+1,h-1}}{B_{t,h}}}$$

so

$$E_t \left(R_{t+1,h}^P \right) = \frac{E_t \left[\beta^{h-1} \left(\frac{C_{t+h}}{C_{t+1}} \right)^{-1} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma} \left(\frac{\tilde{V}_{t+3}}{E_{t+2} [\tilde{V}_{t+3}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} \cdots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1} [\tilde{V}_{t+h}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} \frac{D_{t+h}}{D_t} \right]}{E_t \left[\beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1} \left(\frac{\tilde{V}_{t+1}}{E_t [\tilde{V}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\gamma} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} \cdots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1} [\tilde{V}_{t+h}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} \frac{D_{t+h}}{D_t} \right]}$$

To simplify notations, write:

$$\frac{\frac{D_{t+h}}{D_t} \left(\frac{C_{t+h}}{C_t} \right)^{-1}}{E_t \left(\frac{D_{t+h}}{D_t} \left(\frac{C_{t+h}}{C_t} \right)^{-1} \right)} = \exp \left(\sum_{j=1}^h \Delta_j W_{t+j} \right)$$

where

$$\Delta_j W_{t+j} = \sigma_{t+j-1} \left((\phi_d - \phi_c) \frac{1 - \nu_x^{h-j}}{1 - \nu_x} \alpha_x w_{x,t+j} + \alpha_d w_{d,t+j} + (\chi - 1) \alpha_c w_{c,t+j} \right)$$

and

$$\left(\frac{\tilde{V}_{t+j}}{E_{t+j-1} [\tilde{V}_{t+j}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{1-\tilde{\gamma}} = \exp \left((1 - \tilde{\gamma}) \Sigma_j W_{t+j} - \frac{1}{2} |(1 - \tilde{\gamma}) \Sigma_j|^2 \right)$$

(substitute $\tilde{\gamma}$ with γ when necessary) where

$$\Sigma_j = \sigma_{t+j-1} (\phi_v \alpha_x w_{x,t+j} + \alpha_c w_{c,t+j}) + \tilde{\psi}_v \alpha_\sigma w_{\sigma,t+j}$$

where W_{t+j} is the 4×1 vector of the independent iid shocks at time $t+j$, and $\Delta_{j,t+j-1}, \Sigma_{j,t+j-1}$ is written Δ_j, Σ_j to simplify the formulas.

We obtain:

$$E_t(R_{t+1,h}) = \frac{E_t\left(\frac{C_{t+1}}{C_t}\right) \exp\left[\sum_{j=3}^h \left[(\Delta_j + (1-\tilde{\gamma})\Sigma_j) W_{t+j} - \frac{1}{2} |(1-\tilde{\gamma})\Sigma_j|^2\right] + (\Delta_2 + (1-\gamma)\Sigma_2) W_{t+2} - \frac{1}{2} |(1-\gamma)\Sigma_2|^2 + \Delta_1 W_{t+1}\right]}{\beta E_t\left[\exp\left[\sum_{j=2}^h \left[(\Delta_j + (1-\tilde{\gamma})\Sigma_j) W_{t+j} - \frac{1}{2} |(1-\tilde{\gamma})\Sigma_j|^2\right] + (\Delta_1 + (1-\gamma)\Sigma_1) W_{t+1} - \frac{1}{2} |(1-\gamma)\Sigma_1|^2\right]\right]}$$

Because the shocks are iid, we obtain, when volatility is constant:

$$E_t\left(R_{t+1,h}^P\right) = \frac{\beta^{-1} E_t\left(\frac{C_{t+1}}{C_t}\right) \exp\left[(\Delta_2 + (1-\gamma)\Sigma_2) W_{t+2} - \frac{1}{2} |(1-\gamma)\Sigma_2|^2 + \Delta_1 W_{t+1}\right]}{E_t\left[\exp\left[(\Delta_2 + (1-\tilde{\gamma})\Sigma_2) W_{t+2} - \frac{1}{2} |(1-\tilde{\gamma})\Sigma_2|^2 + (\Delta_1 + (1-\gamma)\Sigma_1) W_{t+1} - \frac{1}{2} |(1-\gamma)\Sigma_1|^2\right]\right]}$$

$$\log E_t\left(R_{t+1,h}^P\right) = -\log \beta + \mu_c + \phi_c x_t + \frac{1}{2} \alpha_c^2 \sigma^2 + \text{cov}(\Delta_1, \alpha_c) + (\tilde{\gamma} - \gamma) \text{cov}(\Delta_2, \Sigma_2) - (1-\gamma) \text{cov}(\Delta_1, \Sigma_1)$$

$$\begin{aligned} \log E_t\left(R_{t+1,h}^P\right) &= -\log \beta + \mu_c + \phi_c x_t + \left(\chi - \frac{1}{2}\right) \alpha_c^2 \sigma^2 - (1-\gamma) \sigma^2 \left[\phi_v (\phi_d - \phi_c) \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 + (\chi - 1) \alpha_c^2\right] \\ &\quad + \underbrace{(\tilde{\gamma} - \gamma) \sigma^2 \left[\phi_v (\phi_d - \phi_c) \frac{1 - \nu_x^{h-2}}{1 - \nu_x} \alpha_x^2 + (\chi - 1) \alpha_c^2\right]}_{<0} \end{aligned}$$

Even when volatility is constant, HDRA impacts the term structure of expected returns when investors choose buy-and-hold strategies. The negative impact of HDRA increases with the horizon. □

To obtain the returns on bonds, and the expected excess returns, replace ϕ_d, α_d and χ by 0 in the formula above:

$$\begin{aligned} \log E_t\left(R_{t+1,h}^B\right) &= -\log \beta + \mu_c + \phi_c x_t + -\frac{1}{2} \alpha_c^2 \sigma^2 + (1-\gamma) \sigma^2 \left[\phi_v \phi_c \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 + \alpha_c^2\right] \\ &\quad - (\tilde{\gamma} - \gamma) \sigma^2 \left[\phi_v \phi_c \frac{1 - \nu_x^{h-2}}{1 - \nu_x} \alpha_x^2 + \alpha_c^2\right] \end{aligned}$$

and

$$\begin{aligned} \log E_t \left(R_{t+1,h}^{P,F} \right) &= \gamma \chi \alpha_c^2 \sigma^2 - (1 - \gamma) \sigma^2 \left[\phi_v \phi_d \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 \right] \\ &\quad + \underbrace{(\tilde{\gamma} - \gamma) \sigma^2 \left[\phi_v \phi_d \frac{1 - \nu_x^{h-2}}{1 - \nu_x} \alpha_x^2 + \chi \alpha_c^2 \right]}_{<0} \end{aligned}$$

When volatility is time varying, we can rewrite,

$$\begin{aligned} &\frac{E_t \left(\frac{C_{t+1}}{C_t} \right) \exp \left[\sum_{j=3}^h \left[(\Delta_j + (1 - \tilde{\gamma}) \Sigma_j) W_{t+j} - \frac{1}{2} |(1 - \tilde{\gamma}) \Sigma_j|^2 \right] + (\Delta_2 + (1 - \gamma) \Sigma_2) W_{t+2} - \frac{1}{2} |(1 - \gamma) \Sigma_2|^2 + \Delta_1 W_{t+1} \right]}{\beta E_t \left[\exp \left[\sum_{j=2}^h \left[(\Delta_j + (1 - \tilde{\gamma}) \Sigma_j) W_{t+j} - \frac{1}{2} |(1 - \tilde{\gamma}) \Sigma_j|^2 \right] + (\Delta_1 + (1 - \gamma) \Sigma_1) W_{t+1} - \frac{1}{2} |(1 - \gamma) \Sigma_1|^2 \right] \right]} = \\ &\frac{\exp \left(-\log \beta + \mu_c + \phi_c x_t \right) E_t \exp \left[\sum_{j=3}^h \left[\tilde{\Psi}_j \sigma_{t+j-1}^2 \right] + (\Delta_2 + (1 - \gamma) \Sigma_2) W_{t+2} - \frac{1}{2} |(1 - \gamma) \Sigma_2|^2 + (\Delta_1 W_{t+1} + \alpha_c \sigma_t w_{c,t+1}) \right]}{E_t \exp \left[\sum_{j=3}^h \left[\tilde{\Psi}_j \sigma_{t+j-1}^2 \right] + (\Delta_2 + (1 - \tilde{\gamma}) \Sigma_2) W_{t+2} - \frac{1}{2} |(1 - \tilde{\gamma}) \Sigma_2|^2 + (\Delta_1 + (1 - \gamma) \Sigma_1) W_{t+1} - \frac{1}{2} |(1 - \gamma) \Sigma_1|^2 \right]} \end{aligned}$$

where

$$\begin{aligned} \tilde{\Psi}_j &= \frac{1}{2} \left(\left((\phi_d - \phi_c) \frac{1 - \nu_x^{h-j}}{1 - \nu_x} \alpha_x \right)^2 + \alpha_d^2 + (\chi - 1)^2 \alpha_c^2 \right) + (1 - \tilde{\gamma}) \left[\phi_v (\phi_d - \phi_c) \frac{1 - \nu_x^{h-j}}{1 - \nu_x} \alpha_x^2 + (\chi - 1) \alpha_c^2 \right] \\ \tilde{\Psi}_\infty &= \frac{1}{2} \left(\left((\phi_d - \phi_c) \frac{\alpha_x}{1 - \nu_x} \right)^2 + \alpha_d^2 + (\chi - 1)^2 \alpha_c^2 \right) + (1 - \tilde{\gamma}) \left[\phi_v (\phi_d - \phi_c) \frac{\alpha_x^2}{1 - \nu_x} + (\chi - 1) \alpha_c^2 \right] \end{aligned}$$

replace $\tilde{\gamma}$ with γ to get Ψ_j

$$\begin{aligned}
& \frac{E_t \exp \left[\sum_{j=3}^h \left[\tilde{\Psi}_j \sigma_{t+j-1}^2 \right] + (\Delta_2 + (1-\gamma) \Sigma_2) W_{t+2} - \frac{1}{2} |(1-\gamma) \Sigma_2|^2 \right]}{E_t \left[\exp \left[\sum_{j=2}^h \left[\tilde{\Psi}_j \sigma_{t+j-1}^2 \right] + (\Delta_2 + (1-\tilde{\gamma}) \Sigma_2) W_{t+2} - \frac{1}{2} |(1-\tilde{\gamma}) \Sigma_2|^2 + (1-\gamma) \tilde{\psi}_v \alpha_\sigma w_{\sigma,t+1} - \frac{1}{2} |(1-\gamma) \tilde{\psi}_v \alpha_\sigma|^2 \right] \right]} = \\
& \frac{E_t \exp \left[\alpha_\sigma (v_\sigma w_{\sigma,t+1} + w_{\sigma,t+2}) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + (\Delta_2 + (1-\gamma) \Sigma_2) W_{t+2} - \frac{1}{2} |(1-\gamma) \Sigma_2|^2 \right]}{E_t \exp \left[\alpha_\sigma (v_\sigma w_{\sigma,t+1} + w_{\sigma,t+2}) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + (\Delta_2 + (1-\tilde{\gamma}) \Sigma_2) W_{t+2} - \frac{1}{2} |(1-\tilde{\gamma}) \Sigma_2|^2 + (1-\gamma) \tilde{\psi}_v \alpha_\sigma w_{\sigma,t+1} - \frac{1}{2} |(1-\gamma) \tilde{\psi}_v \alpha_\sigma|^2 \right]} = \\
& \frac{E_t \exp \left[\alpha_\sigma v_\sigma w_{\sigma,t+1} \left(\sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + \Psi_2 \right) + \Psi_2 (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) + \frac{1}{2} \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} \left(\sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + 2(1-\gamma) \tilde{\psi}_v \right) \alpha_\sigma^2 \right]}{E_t \exp \left[\frac{1}{2} \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} \left(\sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + 2(1-\tilde{\gamma}) \tilde{\psi}_v \right) \alpha_\sigma^2 + \tilde{\Psi}_2 (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) + (v_\sigma \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + (1-\gamma) \tilde{\psi}_v + \tilde{\Psi}_2) \alpha_\sigma w_{\sigma,t+1} - \frac{1}{2} |(1-\gamma) \tilde{\psi}_v \alpha_\sigma|^2 \right]} = \\
& \frac{\exp \left[\alpha_\sigma^2 \left[\frac{1}{2} (\Psi_2)^2 + \frac{1+v_\sigma^2}{2} \left(\sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} \right)^2 + (\Psi_2 v_\sigma + (1-\gamma) \tilde{\psi}_v) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} \right] + \Psi_2 (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) \right]}{\exp \left[\alpha_\sigma^2 \left[\frac{1}{2} (\tilde{\Psi}_2)^2 + \frac{1+v_\sigma^2}{2} \left(\sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} \right)^2 + (\tilde{\Psi}_2 v_\sigma + [(1-\tilde{\gamma}) + v_\sigma (1-\gamma)] \tilde{\psi}_v) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + (1-\gamma) \tilde{\psi}_v \tilde{\Psi}_2 \right] + \tilde{\Psi}_2 (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) \right]} = \\
& \frac{\exp \left[\alpha_\sigma^2 \left[\frac{1}{2} (\Psi_2)^2 + (\Psi_2 v_\sigma + (1-\gamma) \tilde{\psi}_v) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} \right] + \Psi_2 (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) \right]}{\exp \left[\alpha_\sigma^2 \left[\frac{1}{2} (\tilde{\Psi}_2)^2 + (\tilde{\Psi}_2 v_\sigma + [(1-\tilde{\gamma}) + v_\sigma (1-\gamma)] \tilde{\psi}_v) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} + (1-\gamma) \tilde{\psi}_v \tilde{\Psi}_2 \right] + \tilde{\Psi}_2 (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) \right]} = \\
& \exp \left[\frac{1}{2} \alpha_\sigma^2 (\Psi_2^2 - \tilde{\Psi}_2^2) + (\Psi_2 - \tilde{\Psi}_2) \left(\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2 + \alpha_\sigma^2 \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-2} \right) + (\tilde{\gamma} - \gamma) \tilde{\psi}_v \alpha_\sigma^2 \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} - (1-\gamma) \tilde{\psi}_v \alpha_\sigma^2 \sum_{j=2}^h \tilde{\Psi}_j v_\sigma^{j-2} \right]
\end{aligned}$$

$$\begin{aligned}
\log E_t \left(R_{t+1,h}^P \right) &= -\log \beta + \mu_c + \phi_c x_t + \left(\chi - \frac{1}{2} \right) \alpha_c^2 \sigma_t^2 - (1-\gamma) \sigma_t^2 \left[\phi_v (\phi_d - \phi_c) \frac{1-v_x^{h-1}}{1-v_x} \alpha_x^2 + (\chi-1) \alpha_c^2 \right] \\
&+ \underbrace{(\tilde{\gamma} - \gamma) \left[\phi_v (\phi_d - \phi_c) \frac{1-v_x^{h-2}}{1-v_x} \alpha_x^2 + (\chi-1) \alpha_c^2 \right]}_{<0} (\sigma^2 (1-v_\sigma) + v_\sigma \sigma_t^2) \\
&+ \alpha_\sigma^2 \left[\frac{1}{2} (\Psi_2^2 - \tilde{\Psi}_2^2) + (\Psi_2 - \tilde{\Psi}_2) \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-2} + (\tilde{\gamma} - \gamma) \tilde{\psi}_v \sum_{j=3}^h \tilde{\Psi}_j v_\sigma^{j-3} - (1-\gamma) \tilde{\psi}_v \sum_{j=2}^h \tilde{\Psi}_j v_\sigma^{j-2} \right]
\end{aligned}$$

$$\begin{aligned}
\log E_t \left(R_{t+1,h}^P \right) &= -\log \beta + \mu_c + \phi_c x_t + \left(\chi - \frac{1}{2} \right) \alpha_c^2 \sigma_t^2 - (1 - \gamma) \sigma_t^2 \left[\phi_v (\phi_d - \phi_c) \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 + (\chi - 1) \alpha_c^2 \right] \\
&\quad + \underbrace{(\tilde{\gamma} - \gamma) \left[\phi_v (\phi_d - \phi_c) \frac{1 - \nu_x^{h-2}}{1 - \nu_x} \alpha_x^2 + (\chi - 1) \alpha_c^2 \right]}_{<0} \left(\sigma^2 (1 - \nu_\sigma) + \nu_\sigma \sigma_t^2 \right) \\
&\quad + \alpha_\sigma^2 \underbrace{(\gamma - 1) \tilde{\psi}_v}_{<0} \underbrace{\sum_{j=2}^h \tilde{\Psi}_j \nu_\sigma^{j-2}}_{<0 \text{ under } \gamma \text{ but } >0 \text{ for sufficiently low } \tilde{\gamma}} \\
&\quad + \alpha_\sigma^2 \left[\frac{1}{2} \left(\Psi_2^2 - \tilde{\Psi}_2^2 \right) + \underbrace{\left(\Psi_2 - \tilde{\Psi}_2 \right) \sum_{j=3}^h \tilde{\Psi}_j \nu_\sigma^{j-2} + (\tilde{\gamma} - \gamma) \tilde{\psi}_v \sum_{j=3}^h \tilde{\Psi}_j \nu_\sigma^{j-3}}_{<0 \text{ for sufficiently low } \tilde{\gamma}} \right]
\end{aligned}$$

Note: we write $\Phi_k = \frac{\tilde{\Psi}_{h+1-k}}{\nu_\sigma^k} \implies \nu_\sigma^{h-2} \sum_{k=1}^{h-2} \Phi_k = \sum_{j=3}^h \tilde{\Psi}_j \nu_\sigma^{j-3}$ in the matlab document

To obtain the returns on bonds, and their expected excess returns, replace ϕ_d , α_d and χ by 0 in the formula above:

$$\begin{aligned}
\log E_t \left(R_{t+1,h}^B \right) &= -\log \beta + \mu_c + \phi_c x_t - \frac{1}{2} \alpha_c^2 \sigma_t^2 + (1 - \gamma) \sigma_t^2 \left[\phi_v \phi_c \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 + \alpha_c^2 \right] \\
&\quad - \underbrace{(\tilde{\gamma} - \gamma) \left[\phi_v \phi_c \frac{1 - \nu_x^{h-2}}{1 - \nu_x} \alpha_x^2 + \alpha_c^2 \right]}_{<0} \left(\sigma^2 (1 - \nu_\sigma) + \nu_\sigma \sigma_t^2 \right) \\
&\quad + \alpha_\sigma^2 \underbrace{(\gamma - 1) \tilde{\psi}_v}_{<0} \underbrace{\sum_{j=2}^h \tilde{\Psi}_{B,j} \nu_\sigma^{j-2}}_{<0 \text{ under } \gamma \text{ but } >0 \text{ for sufficiently low } \tilde{\gamma}} \\
&\quad + \alpha_\sigma^2 \left[\frac{1}{2} \left(\Psi_{B,2}^2 - \tilde{\Psi}_{B,2}^2 \right) + \underbrace{\left(\Psi_{B,2} - \tilde{\Psi}_{B,2} \right) \sum_{j=3}^h \tilde{\Psi}_{B,j} \nu_\sigma^{j-2} + (\tilde{\gamma} - \gamma) \tilde{\psi}_v \sum_{j=3}^h \tilde{\Psi}_{B,j} \nu_\sigma^{j-3}}_{<0 \text{ for sufficiently low } \tilde{\gamma}} \right]
\end{aligned}$$

where

$$\tilde{\Psi}_{B,j} = \frac{1}{2} \left(\left(\phi_c \frac{1 - \nu_x^{h-j}}{1 - \nu_x} \alpha_x \right)^2 + \alpha_c^2 \right) - (1 - \tilde{\gamma}) \left[\phi_v \phi_c \frac{1 - \nu_x^{h-j}}{1 - \nu_x} \alpha_x^2 + \alpha_c^2 \right]$$

and

$$\tilde{\Psi}_j - \tilde{\Psi}_{B,j} = \frac{1}{2} \left(\phi_d (\phi_d - 2\phi_c) \left(\frac{1 - \nu_x^{h-j}}{1 - \nu_x} \alpha_x \right)^2 + \alpha_d^2 + \chi (\chi - 2) \alpha_c^2 \right) + (1 - \tilde{\gamma}) \left[\phi_v \phi_c \frac{1 - \nu_x^{h-2}}{1 - \nu_x} \alpha_x^2 + \alpha_c^2 \right]$$

$$\begin{aligned} \log E_t \left(R_{t+1,h}^{D,F} \right) &= \gamma \chi \alpha_c^2 \sigma_t^2 - (1 - \gamma) \sigma_t^2 \left[\phi_v \phi_d \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 \right] \\ &\quad + \underbrace{(\tilde{\gamma} - \gamma) \left[\phi_v \phi_d \frac{1 - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 + \chi \alpha_c^2 \right]}_{<0} (\sigma^2 (1 - \nu_\sigma) + \nu_\sigma \sigma_t^2) \\ &\quad + \alpha_\sigma^2 \underbrace{(\gamma - 1) \tilde{\psi}_v}_{<0} \underbrace{\sum_{j=2}^h (\tilde{\Psi}_j - \tilde{\Psi}_{B,j}) \nu_\sigma^{j-2}}_{<0 \text{ under } \gamma \text{ but } >0 \text{ for sufficiently low } \tilde{\gamma}} \\ &\quad + \alpha_\sigma^2 \left[\left\{ \begin{aligned} &\frac{1}{2} (\Psi_2^2 - \tilde{\Psi}_2^2) + (\Psi_2 - \tilde{\Psi}_2) \sum_{j=3}^h \tilde{\Psi}_j \nu_\sigma^{j-2} \\ &-\frac{1}{2} (\Psi_{B,2}^2 - \tilde{\Psi}_{B,2}^2) + (\Psi_{B,2} - \tilde{\Psi}_{B,2}) \sum_{j=3}^h \tilde{\Psi}_{B,j} \nu_\sigma^{j-2} \end{aligned} \right\} + (\tilde{\gamma} - \gamma) \tilde{\psi}_v \sum_{j=3}^h (\tilde{\Psi}_j - \tilde{\Psi}_{B,j}) \nu_\sigma^{j-3} \right] \end{aligned}$$

□

Using

$$r_{f,t} = -\log \beta + \mu_c + \phi_c x_t - \left(\gamma - \frac{1}{2} \right) \alpha_c^2 \sigma_t^2$$

we have

$$\begin{aligned} E_t \left[\beta^h \left(\frac{C_{t+h}}{C_t} \right)^{-1} \left(\frac{\tilde{V}_{t+1}}{E_t [\tilde{V}_{t+1}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \dots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1} [\tilde{V}_{t+h}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \right] &\propto \\ \exp \left[\left(\left(\tilde{\gamma} - \frac{1}{2} \right) \frac{1 - \nu_\sigma^h}{1 - \nu_\sigma} + (\gamma - \tilde{\gamma}) \right) \alpha_c^2 \sigma_t^2 \right] &\times \\ E_t \left[\left(\frac{\tilde{V}_{t+1}}{E_t [\tilde{V}_{t+1}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \left(\frac{\tilde{V}_{t+2}}{E_{t+1} [\tilde{V}_{t+2}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \dots \left(\frac{\tilde{V}_{t+h-1}}{E_{t+h-2} [\tilde{V}_{t+h-1}^{1-\tilde{\gamma}}]^{\frac{1}{1-\tilde{\gamma}}}} \right)^{1-\tilde{\gamma}} \exp(-\phi_c (x_t + \dots + x_{t+h-1})) \right] & \end{aligned}$$

$$\begin{aligned}\exp(-\phi_c x_{t+h-1}) &= \exp\left(-\phi_c \left(\nu_x^2 x_{t+h-3} + \nu_x \alpha_x \sigma_{t+h-3} W_{t+h-2} + \alpha_x \sigma_{t+h-2} W_{t+h-1}\right)\right) \\ &= \exp\left(-\phi_c \left(\nu_x^{h-1} x_t + \alpha_x \left(\nu_x^{h-2} \sigma_t W_{t+1} + \dots + \sigma_{t+h-2} W_{t+h-1}\right)\right)\right)\end{aligned}$$

$$\exp(-\phi_c x_{t+h-2}) = \exp\left(-\phi_c \left(\nu_x^{h-2} x_t + \alpha_x \left(\nu_x^{h-3} \sigma_t W_{t+1} + \dots + \sigma_{t+h-3} W_{t+h-2}\right)\right)\right)$$

$$\begin{aligned}\exp(-\phi_c (x_t + \dots + x_{t+h-1})) &= \\ \exp\left(-\frac{\phi_c}{1-\nu_x} \left(\left(1-\nu_x^h\right) x_t + \alpha_x \left(\left(1-\nu_x^{h-1}\right) \sigma_t W_{t+1} + \dots + \left(1-\nu_x\right) \sigma_{t+h-2} W_{t+h-1}\right)\right)\right)\end{aligned}$$

$$\begin{aligned}E_t \left[\left(\frac{C_{t+h}}{C_t}\right)^{-1} \left(\frac{\tilde{V}_{t+1}}{E_t[\tilde{V}_{t+1}]^{\frac{1}{1-\gamma}}}\right)^{1-\gamma} \left(\frac{\tilde{V}_{t+2}}{E_{t+1}[\tilde{V}_{t+2}]^{\frac{1}{1-\tilde{\gamma}}}}\right)^{1-\tilde{\gamma}} \dots \left(\frac{\tilde{V}_{t+h}}{E_{t+h-1}[\tilde{V}_{t+h}]^{\frac{1}{1-\tilde{\gamma}}}}\right)^{1-\tilde{\gamma}} \right] &\propto \\ \exp\left(\sum_{j=1}^{h-1} \left[\frac{1-\nu_x^j}{1-\nu_x} \left(\frac{1}{2}\phi_c \frac{1-\nu_x^j}{1-\nu_x} - (1-\tilde{\gamma})\phi_v\right)\right] \nu_\sigma^{h-1-j} + \frac{1-\nu_x^{h-1}}{1-\nu_x} ((\gamma-\tilde{\gamma})\phi_v)\right) &\phi_c \alpha_x^2 \sigma_t^2\end{aligned}$$

We painfully arrive at

$$\begin{aligned}\Sigma_{h,t}^B &= \left(\sum_{j=1}^{h-1} \left[\frac{1-\nu_x^j}{1-\nu_x} \left(\frac{1}{2}\phi_c \frac{1-\nu_x^j}{1-\nu_x} - (1-\tilde{\gamma})\phi_v\right)\right] \nu_\sigma^{h-1-j} + \frac{1-\nu_x^{h-1}}{1-\nu_x} ((\gamma-\tilde{\gamma})\phi_v)\right) \phi_c \alpha_x^2 \\ &+ \left(\left(\tilde{\gamma} - \frac{1}{2}\right) \frac{1-\nu_\sigma^h}{1-\nu_\sigma} + (\gamma-\tilde{\gamma})\right) \alpha_c^2\end{aligned}$$

□

B.7 Demand for risk

Wealth:

- start with V_t homogenous of degree one in $\{C_t, V_{t,t+1}\}$:

$$\begin{aligned} V_t &= \frac{dV_t}{dC_t} C_t + E_t \left(\frac{dV_t}{dV_{t,t+1}} V_{t,t+1} \right) \\ &= \frac{dV_t}{dC_t} \left[C_t + E_t \left(\frac{\frac{dV_t}{dV_{t,t+1}} \frac{dV_{t,t+1}}{dW_{t+1}} V_{t,t+1}}{\frac{dV_t}{dC_t} \frac{dV_{t,t+1}}{dW_{t+1}}} \right) \right] \end{aligned}$$

and

$$\frac{V_t}{\frac{dV_t}{dC_t}} = C_t + E_t \left(\Pi_{t,t+1} \frac{V_{t,t+1}}{\frac{dV_{t,t+1}}{dW_{t+1}}} \right)$$

- remember from above:

$$\frac{V_{t,t+1}}{\frac{dV_{t,t+1}}{dW_{t+1}}} = \frac{V_{t+1}}{\frac{dV_{t+1}}{dW_{t+1}}}$$

so we end up with

$$W_t = \frac{V_t}{\frac{dV_t}{dC_t}}$$

and

$$\frac{W_t}{C_t} = \frac{1}{1-\beta} \left(\frac{V_t}{C_t} \right)^{1-\rho}$$

which is the standard formula

- what if the next optimization is at $h > 1$?

$$\begin{aligned}
V_t &= \frac{dV_t}{dC_t} C_t + E_t \left(\frac{dV_t}{dV_{t,t+1}} V_{t,t+1} \right) \\
&= \frac{dV_t}{dC_t} \left[C_t + E_t \left(\frac{\frac{dV_t}{dV_{t,t+1}} \frac{dV_{t,t+1}}{dW_{t+1}} V_{t,t+1}}{\frac{dV_t}{dC_t} \frac{dV_{t,t+1}}{dW_{t+1}}} \right) \right] \\
&= \frac{dV_t}{dC_t} \left[C_t + E_t \left(\frac{\frac{dV_t}{dV_{t,t+1}} \frac{dV_{t,t+1}}{dW_{t+1}}}{\frac{dV_t}{dC_t} \frac{dV_{t,t+1}}{dW_{t+1}}} \left(C_{t+1} + \frac{dV_{t,t+1}}{dV_{t+1,t+2}} V_{t+1,t+2} \right) \right) \right] \\
&= \frac{dV_t}{dC_t} \left[C_t + E_t \left(\Pi_{t,t+1} C_{t+1} + \Pi_{t,t+2} \frac{dV_{t+2}}{dC_{t+2}} \right) \right]
\end{aligned}$$

we still obtain

$$W_t = \frac{V_t}{\frac{dV_t}{dC_t}}$$

- when $\rho = 1$, $C_t = \alpha W_t$

Market returns:

- long-linearization of market returns:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{D_{t+1}}{D_t} \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t}$$

$$r_{t+1} = \Delta d_{t+1} + k_1 z_{t+1} - z_t + \kappa_0$$

where $z_t = \log \frac{P_t}{D_t}$; from earlier calculations, $z_t = A_0 + A_1 x_t$, so

$$r_{m,t+1} = a_{m,0} + a_{m,1} x_t + \chi \alpha_c \sigma_t w_{c,t+1} + \chi_2 \alpha_x \sigma_t w_{x,t+1} + \alpha_d \sigma_t w_{d,t+1}$$

$$d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t w_{c,t+1} + \alpha_d \sigma_t w_{d,t+1}$$

- to simplify, we assume $\sigma_t = \sigma$

$$\begin{aligned} r_{m,t+1} &= \mu_m + (k_1 v_x - 1) A_1 x_t + \chi \alpha_c \sigma w_{c,t+1} + k_1 A_1 \alpha_x w_{x,t+1} + \alpha_d w_{d,t+1} \\ &= \mu_m + (\phi_c - \phi_d) x_t + \sigma_m w_{m,t+1} \\ &= \mu_{m,t} + \sigma_m w_{m,t+1} \end{aligned}$$

and $k_1 = \frac{e^z}{1+e^z}$, $A_1 = \frac{\phi_d - \phi_c}{1 - k_1 v_x}$

- portfolio investment θ :

$$\begin{aligned} e^{r_{p,t+1}} &= R_f + \theta_t (R_m - R_f) \\ &= R_f (1 + \theta_t (e^{r_m - r_f} - 1)) \end{aligned}$$

if log-normal:

$$E(e^{r_{p,t+1}}) = \mu_p + \frac{1}{2} \sigma_p^2 = e^{r_f} \left(1 + \theta_t \left(e^{\mu_m - r_f + \frac{1}{2} \sigma_m^2} - 1 \right) \right)$$

first-order approximation:

$$r_{p,t+1} - r_f = \theta_t (r_{m,t+1} - r_f) + \frac{1}{2} \theta_t (1 - \theta_t) \sigma_m^2$$

s.t. $E(r_{p,t+1}) = r_f + \theta_t (\mu_{m,t} - r_f) + \frac{1}{2} \theta_t \sigma_m^2$

One period optimization:

$$V_t = \max_{\{C_t, \theta_t\}} C_t^{1-\beta} \mathcal{R}_t (\tilde{V}_{t+1})^\beta$$

s.t.

$$W_{t+1} = (W_t - C_t) e^{r_{p,t+1}}$$

By homogeneity in the value function, we have $V_t = V(x_t) W_t$, $\tilde{V}_t = \tilde{V}(x_t) W_t$ and $C_t = \alpha W_t$, so the optimization becomes:

$$V(x_t) = \max_{\{\alpha, \theta_t\}} \alpha^{1-\beta} (1-\alpha)^\beta \mathcal{R}_t(\tilde{V}(x_{t+1}) e^{r_{p,t+1}})^\beta$$

so θ_t is chosen to maximize:

$$\log \mathcal{R}_t(\tilde{V}(x_{t+1}) e^{r_{p,t+1}}) = \frac{1}{1-\gamma} \log E \exp \left[(1-\gamma) \left(\tilde{v}(x_{t+1}) + \theta_t (\mu_{m,t} - r_{f,t} + \sigma_m w_{m,t+1}) + \frac{1}{2} \theta_t (1-\theta_t) \sigma_m^2 \right) \right]$$

- If $cov(w_{m,t+1}, w_{x,t+1}) = 0$, we get back to the standard myopic demand:

$$\max \left(\theta_t (\mu_{m,t} - r_{f,t}) + \frac{1}{2} \theta_t (1-\theta_t) \sigma_m^2 + \frac{1}{2} \theta_t^2 (1-\gamma) \sigma_m^2 \right)$$

so:

$$(\mu_{m,t} - r_{f,t}) + \frac{1}{2} \sigma_m^2 - \gamma \theta_t \sigma_m^2 = 0$$

i.e.

$$\theta_t = \frac{E(R_m - R_f)}{\gamma \sigma_m^2}$$

- When $cov(w_{m,t+1}, w_{x,t+1}) \neq 0$, we get:

$$\max \left(\theta_t (\mu_{m,t} - r_{f,t}) + \frac{1}{2} \theta_t (1-\theta_t) \sigma_m^2 + \frac{1}{2} (1-\gamma) \left(\theta_t^2 \sigma_m^2 + 2\theta_t \sigma_m cov(\tilde{v}(x_{t+1}), w_{m,t+1}) \right) \right)$$

so:

$$(\mu_{m,t} - r_{f,t}) + \frac{1}{2} \sigma_m^2 - \gamma \theta_t \sigma_m^2 + (1-\gamma) (\sigma_m cov(\tilde{v}(x_{t+1}), w_{m,t+1})) = 0$$

i.e.

$$\theta_t = \frac{E(R_m - R_f)}{\gamma \sigma_m^2} + \frac{(1-\gamma) cov(\tilde{v}(x_{t+1}), w_{m,t+1})}{\gamma \sigma_m}$$

which combines the standard myopic demand with the hedging demand due to the predictability of consumption growth shocks.

Two period optimization:

$$V_t = \max_{\{C_t, C_{t+1}\theta_t\}} C_t^{1-\beta} \mathcal{R}_t \left(C_{t+1}^{1-\beta} \tilde{\mathcal{R}}_{t+1} (\tilde{V}_{t+2})^\beta \right)^\beta$$

s.t.

$$W_{t+2} = (W_t - C_{t \rightarrow t+2}) e^{r_{p,t+2}}$$

As above, we have $V_t = V(x_t) W_t$, $\tilde{V}_t = \tilde{V}(x_t) W_t$ and $C_t = \alpha_1 W_t$, $C_{t+1} = \alpha_2 W_t R_f$ so the optimization becomes:

$$V(x_t) = \max_{\{\alpha_1, \alpha_2, \theta_t\}} \left(\alpha_1^{1-\beta} (\alpha_2 R_f)^{\beta(1-\beta)} (1 - (\alpha_1 + \alpha_2))^{\beta^2} \right) \mathcal{R}_t \left(\tilde{\mathcal{R}}_{t+1} (\tilde{V}(x_{t+2}) e^{r_{p,t+2}})^\beta \right)^\beta$$

We have:

$$r_{p,t+2} - r_{f,t,2} = \theta_t (r_{m,t+2} - r_{f,t,2}) + \frac{1}{2} \theta_t (1 - \theta_t) \sigma_{m,2}^2$$

$$\begin{aligned} r_{m,t+2} &= r_{m,t+1} + r_{m,t+1,t+2} \\ &= 2\mu_m + (\phi_c - \phi_d) (1 + \nu_x) x_t + (\phi_c - \phi_d) \alpha_x w_{x,t+1} + \sigma_m w_{m,t+1} + \sigma_m w_{m,t+2} \\ &= \mu_{m,t,2} + ((\phi_c - \phi_d) \alpha_x w_{x,t+1} + \sigma_m w_{m,t+1}) + \sigma_m w_{m,t+2} \end{aligned}$$

so getting rid of the terms that do not affect the optimization,

$$\begin{aligned} \log \tilde{\mathcal{R}}_{t+1} (\tilde{V}(x_{t+2}) e^{r_{p,t+2}}) &= \phi_v \nu_x x_{t+1} + \theta_t (\mu_{m,t+2} - r_{f,t,2} + ((\phi_c - \phi_d) \alpha_x w_{x,t+1} + \sigma_m w_{m,t+1})) \\ &\quad + \frac{1}{2} \theta_t (1 - \theta_t) \sigma_{m,2}^2 \\ &\quad + \frac{1}{2} (1 - \tilde{\gamma}) \left(\theta_t^2 \sigma_m^2 + 2\theta_t \sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1}) \right) \end{aligned}$$

and therefore

$$\log \mathcal{R}_t \left(\tilde{\mathcal{R}}_{t+1} \left(\tilde{V}(x_{t+2}) e^{r_{p,t+2}} \right)^\beta \right) = \beta \left\{ \begin{aligned} & \left(\phi_\sigma v_x^2 x_t + \theta_t (\mu_{m,t+2} - r_{f,t,2}) + \frac{1}{2} \theta_t (1 - \theta_t) \sigma_{m,2}^2 \right) \\ & + \frac{1}{2} (1 - \tilde{\gamma}) (\theta_t^2 \sigma_m^2 + 2\theta_t \sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1})) \\ & + \frac{1}{2} \beta^2 (1 - \gamma) \theta_t^2 (\sigma_{m,2}^2 - \sigma_m^2) \end{aligned} \right.$$

so θ_t is chosen to maximize:

$$\max \left\{ \begin{aligned} & \theta_t (\mu_{m,t+2} - r_{f,t,2}) + \frac{1}{2} \theta_t (1 - \theta_t) \sigma_{m,2}^2 \\ & + \frac{1}{2} (1 - \tilde{\gamma}) (\theta_t^2 \sigma_m^2 + 2\theta_t \sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1})) \\ & + \frac{1}{2} \beta^2 (1 - \gamma) \theta_t^2 (\sigma_{m,2}^2 - \sigma_m^2) \end{aligned} \right.$$

At horizon $h \geq 2$, we expand to get:

$$\left\{ \begin{aligned} & \mu_{m,t+h} - r_{f,t,h} + \frac{1}{2} \sigma_{m,h}^2 + (1 - \tilde{\gamma}) \sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1}) \\ & + \theta_t \left(\sigma_m^2 (1 - \beta) (1 - \gamma) - \beta (\gamma + (1 - \beta)) \sigma_{m,h}^2 \right) \\ & + \theta_t (\gamma - \tilde{\gamma}) \beta \left(\sigma_{m,h}^2 - \sigma_{1,h}^2 + \sigma_m^2 \left(\frac{1-\beta}{\beta} \right) \right) \end{aligned} \right. = 0$$

where $\sigma_{1,h}^2$ is the variance of the w_{t+1} shocks in $r_{m,t+h}$.

when $\beta \approx 1$:

$$\left\{ \begin{aligned} & \mu_{m,t+h} - r_{f,t,h} + \frac{1}{2} \sigma_{m,h}^2 + (1 - \tilde{\gamma}) \sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1}) \\ & + \theta_t \left(-\gamma \sigma_{m,h}^2 \right) \\ & + \theta_t (\gamma - \tilde{\gamma}) \left(\sigma_{m,h}^2 - \sigma_{1,h}^2 \right) \end{aligned} \right. = 0$$

and

$$\theta_t = \frac{\frac{E(R_{m,h} - R_f)}{\gamma \sigma_{m,h}^2}}{1 + \frac{(\tilde{\gamma} - \gamma)}{\gamma} \left(1 - \frac{\sigma_{1,h}^2}{\sigma_{m,h}^2} \right)} + \frac{(1 - \tilde{\gamma}) \frac{\sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1})}{\gamma \sigma_{m,h}^2}}{1 + \frac{(\tilde{\gamma} - \gamma)}{\gamma} \left(1 - \frac{\sigma_{1,h}^2}{\sigma_{m,h}^2} \right)}$$

with no predictability, we would get:

$$\theta_t = \frac{1}{1 + \frac{(\tilde{\gamma} - \gamma)h - 1}{\gamma}} \frac{E(R_m - R_f)}{\gamma \sigma_m^2}$$

with β :

$$\begin{cases} \mu_{m,t+h} - r_{f,t,h} + \frac{1}{2}\sigma_{m,h}^2 + (1 - \tilde{\gamma}) \sigma_m \text{cov}(\tilde{v}(x_{t+1}), w_{m,t+1}) \\ -\theta_t \sigma_{m,h}^2 \left(\beta \gamma + (1 - \beta) \left(\beta + (\gamma - 1) \frac{\sigma_m^2}{\sigma_{m,h}^2} \right) \right) \\ +\theta_t (\gamma - \tilde{\gamma}) \beta \left(\sigma_{m,h}^2 - \sigma_{1,h}^2 + \sigma_m^2 \left(\frac{1 - \beta}{\beta} \right) \right) \end{cases} = 0$$

and

$$\theta = \frac{\frac{E(R_{m,h} - R_f)}{\sigma_{m,h}^2} + (1 - \tilde{\gamma}) \frac{\text{cov}\left(\frac{\tilde{V}_{t+1}}{W_{t+1}}, R_{m,t+1}\right)}{\sigma_{m,h}^2}}{\left(\beta \gamma + (1 - \beta) \left(\beta + (\gamma - 1) \frac{\sigma_m^2}{\sigma_{m,h}^2} \right) \right) + (\tilde{\gamma} - \gamma) \left(1 - \frac{\sigma_{1,h}^2}{\sigma_{m,h}^2} + \frac{\sigma_m^2}{\sigma_{m,h}^2} \left(\frac{1 - \beta}{\beta} \right) \right)}$$

Both the myopic and the hedging demands are increased by $\tilde{\gamma} > \gamma$ when the horizon of optimization is $h > 1$, as long as $\text{cov}(w_{x,t+1}, w_{m,t+1}) \geq 0$.

C Approximation for $\beta \approx 1$

As in Appendix B, consider the simplified model with only two levels of risk aversion:

$$\begin{aligned} V_t &= \left[(1 - \beta) C_t^{1-\rho} + \beta \left(\mathcal{R}_{t,\gamma} \left(\tilde{V}_{t+1} \right) \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \\ \tilde{V}_t &= \left[(1 - \beta) C_t^{1-\rho} + \beta \left(\mathcal{R}_{t,\tilde{\gamma}} \left(\tilde{V}_{t+1} \right) \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \end{aligned}$$

where

$$\mathcal{R}_{t,\lambda}(X) = \left(E_t \left(X^{1-\lambda} \right) \right)^{\frac{1}{1-\lambda}}.$$

Also, as in Appendix B, take the evolutions:

$$\begin{aligned}c_{t+1} - c_t &= \mu + \phi_c x_t + \alpha_c \sigma_t W_{t+1}, \\x_{t+1} &= \nu_x x_t + \alpha_x \sigma_t W_{t+1}, \\\sigma_{t+1}^2 - \sigma^2 &= \nu_\sigma (\sigma_t^2 - \sigma^2) + \alpha_\sigma W_{t+1},\end{aligned}$$

and suppose the three shocks are independent. (We can relax this assumption.)

For β close to 1, we have:

$$\left(\frac{\tilde{V}_t}{C_t}\right)^{1-\tilde{\gamma}} \approx \beta^{\frac{1-\tilde{\gamma}}{1-\rho}} E_t \left[\left(\frac{\tilde{V}_{t+1} C_{t+1}}{C_{t+1} C_t}\right)^{1-\tilde{\gamma}} \right].$$

This is an eigenfunction problem with eigenvalue $\beta^{-\frac{1-\tilde{\gamma}}{1-\rho}}$ and eigenfunction $(\tilde{V}/C)^{1-\tilde{\gamma}}$ known up to a multiplier. Let's assume:

$$\tilde{v}_t - c_t = \tilde{\mu}_v + \phi_v x_t + \tilde{\psi}_v \sigma_t^2.$$

Then we have:

- Terms in x_t (standard formula with $\beta = 1$):

$$\phi_v = \phi_c (I - \nu_x)^{-1}$$

- Terms in σ_t^2 :

$$\tilde{\psi}_v = \frac{1}{2} \frac{1-\tilde{\gamma}}{1-\nu_\sigma} (\alpha_c^2 + \phi_v^2 \alpha_x^2) < 0$$

- Constant terms:

$$\log \beta = -(1-\rho) \left(\mu + \tilde{\psi}_v \sigma^2 (1-\nu_\sigma) + \frac{1}{2} (1-\tilde{\gamma}) \tilde{\psi}_v^2 \alpha_\sigma^2 \right)$$

we verify the solution for β is such that $\beta < 1$ and $\beta \approx 1$. We find that, as long as $\tilde{\gamma} \leq 5$, $\beta < 1 \Leftrightarrow \rho < 1$; and $\beta \approx 1$ is easily satisfied even for very low levels of ρ . e.g. in the calibration of Section (3.2), $1 > \beta \geq 0.9988$ for $\rho = 0.2$ and $\tilde{\gamma} \leq 5$.

For β close to 1, we have:

$$\frac{V_t}{\tilde{V}_t} \approx \frac{\mathcal{R}_{t,\gamma}(\tilde{V}_{t+1})}{\mathcal{R}_{t,\tilde{\gamma}}(\tilde{V}_{t+1})} = \frac{\left(E_t \left[\left(\frac{\tilde{V}_{t+1} C_{t+1}}{C_t} \right)^{1-\gamma} \right]\right)^{\frac{1}{1-\gamma}}}{\left(E_t \left[\left(\frac{\tilde{V}_{t+1} C_{t+1}}{C_t} \right)^{1-\tilde{\gamma}} \right]\right)^{\frac{1}{1-\tilde{\gamma}}}},$$

and therefore:

$$v_t - \tilde{v}_t = -\frac{1}{2}(\gamma - \tilde{\gamma}) \left[(\alpha_c^2 + \phi_v^2 \alpha_x^2) \sigma_t^2 + \tilde{\psi}_v^2 \alpha_\sigma^2 \right],$$

The stochastic discount factor becomes:

$$\begin{aligned} \pi_{t,t+1} &= \bar{\pi}_t - \gamma \alpha_c \sigma_t W_{t+1} + (\rho - \gamma) \phi_v \alpha_x \sigma_t W_{t+1} \\ &\quad + \left[(\rho - \gamma) + (1 - \rho)(\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v \alpha_\sigma W_{t+1}, \end{aligned}$$

where

$$\begin{aligned} \bar{\pi}_t &= \log \beta - \rho \mu_c - \rho \phi_c x_t - (\rho - \gamma) \frac{1}{2} (1 - \gamma) \left(\left[(\alpha_c^2 + \phi_v^2 \alpha_x^2) \sigma_t^2 + \tilde{\psi}_v^2 \alpha_\sigma^2 \right] \right) \\ &\quad + (1 - \rho) \frac{1}{2} (\gamma - \tilde{\gamma}) \left[(\alpha_c^2 + \phi_v^2 \alpha_x^2) (\nu_\sigma \sigma_t^2 + \sigma^2 (1 - \nu_\sigma)) + \tilde{\psi}_v^2 \alpha_\sigma^2 \right] \end{aligned}$$

$$\begin{aligned} \bar{\pi}_t &= -\mu_c - \rho \phi_c x_t - (1 - \rho) \frac{1}{2} (\alpha_c^2 + \phi_v^2 \alpha_x^2) \left(\frac{1 - \tilde{\gamma}}{1 - \nu_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1 - \nu_\sigma) \\ &\quad - \frac{1}{2} (1 - \gamma)^2 \tilde{\psi}_v^2 \alpha_\sigma^2 \\ &\quad - \frac{1}{2} ((\rho - \gamma)(1 - \gamma) - (1 - \rho)(\gamma - \tilde{\gamma}) \nu_\sigma) \left(\left[(\alpha_c^2 + \phi_v^2 \alpha_x^2) \sigma_t^2 \right] \right) \end{aligned}$$

The risk-free rate is defined as $r_{f,t} = -\log E_t(\Pi_{t,t+1})$:

$$\begin{aligned}
r_{f,t} &= \mu_c + \rho\phi_c x_t + (1-\rho) \frac{1}{2} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \left(\frac{1-\tilde{\gamma}}{1-\nu_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1-\nu_\sigma) \\
&+ \frac{1}{2} \left[(1-\gamma)^2 - \left[(\rho-\gamma) + (1-\rho)(\gamma-\tilde{\gamma}) \frac{1-\nu_\sigma}{1-\tilde{\gamma}} \right]^2 \right] \tilde{\psi}_v^2 \alpha_\sigma^2 \\
&+ \frac{1}{2} \left((\rho-\gamma)(1-\gamma) - (1-\rho)(\gamma-\tilde{\gamma})\nu_\sigma \right) \left(\left[\left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \sigma_t^2 \right] \right) \\
&- \frac{1}{2} \left(\gamma^2 \alpha_c^2 \sigma_t^2 + (\rho-\gamma)^2 \phi_v^2 \alpha_x^2 \sigma_t^2 \right)
\end{aligned}$$

Note the risk-free rate now depends on $\tilde{\gamma}$. □

C.1 Term structure of returns

C.1.1 General claims

To make the problem as general as possible, we analyze horizon-dependent claims that are priced recursively as

$$Y_{t,h} = E_t \left[\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1} \right],$$

that is

$$y_{t,h} = E_t \left[\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1} \right] + \frac{1}{2} \text{var}_t \left(\pi_{t,t+1} + g_{y,t+1} + y_{t+1,h-1} \right),$$

where

$$\begin{aligned}
g_{y,t+1} &= \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\
&+ \alpha_{y,c} \alpha_c \sigma_t \omega_{c,t+1} + \alpha_{y,x} \alpha_x \sigma_t \omega_{x,t+1} + \alpha_{y,\sigma} \alpha_\sigma \sigma_t \omega_{\sigma,t+1} + \alpha_{y,d} \alpha_d \sigma_t \omega_{d,t+1},
\end{aligned}$$

and $Y_{t,0} = 1$.

Guess that

$$Y_{t,h} = \exp \left(\tilde{\mu}_{y,h} + \phi_{y,h} x_t + \tilde{\psi}_{y,h} \sigma_t^2 \right).$$

Suppose $h \geq 1$, then:

$$\begin{aligned} \pi_{t,t+1} &= \bar{\pi}_t - \gamma \alpha_c \sigma_t W_{t+1} + (\rho - \gamma) \phi_v \alpha_x \sigma_t W_{t+1} \\ &\quad + \left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v \alpha_\sigma W_{t+1}, \\ \log \tilde{\Pi}_{t,t+1} G_{t,t+1} Y_{t+1,h-1} &= \begin{cases} \bar{\pi}_t \\ + \mu_y + \phi_y x_t + \psi_y \sigma_t^2 \\ + \tilde{\mu}_{y,h-1} + \phi_{y,h-1} \nu_x x_t + \psi_{y,h-1} (\sigma^2 (1 - \nu_\sigma) + \nu_\sigma \sigma_t^2) \\ + (-\gamma + \alpha_{y,c}) \alpha_c \sigma_t W_{t+1} + ((\rho - \gamma) \phi_v + \alpha_{y,x} + \phi_{y,h-1}) \alpha_x \sigma_t W_{t+1} \\ + \left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \alpha_{y,\sigma} + \psi_{y,h-1} \right) \alpha_\sigma W_{t+1} \\ + \alpha_{y,d} \alpha_d \sigma_t W_{t+1} \end{cases} \end{aligned}$$

where

$$\begin{aligned} \bar{\pi}_t &= -\mu_c - \rho \phi_c x_t - (1 - \rho) \frac{1}{2} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \left(\frac{1 - \tilde{\gamma}}{1 - \nu_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1 - \nu_\sigma) \\ &\quad - \frac{1}{2} (1 - \gamma)^2 \tilde{\psi}_v^2 \alpha_\sigma^2 \\ &\quad - \frac{1}{2} \left((\rho - \gamma) (1 - \gamma) - (1 - \rho) (\gamma - \tilde{\gamma}) \nu_\sigma \right) \left(\left[\left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \sigma_t^2 \right] \right) \end{aligned}$$

Matching coefficients, we find the recursions, for $h \geq 1$:

- Terms in x_t :

$$\begin{aligned} \phi_{y,h} &= -\rho \phi_c + \phi_y + \phi_{y,h-1} \nu_x \\ \Rightarrow \phi_{y,h} &= (-\rho \phi_c + \phi_y) \frac{1 - \nu_x^h}{1 - \nu_x} \end{aligned}$$

- Terms in σ_t^2 :

$$\begin{aligned} \tilde{\psi}_{y,h} &= -\frac{1}{2} \left((\rho - \gamma) (1 - \gamma) - (1 - \rho) (\gamma - \tilde{\gamma}) \nu_\sigma \right) \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) + \tilde{\psi}_{y,h-1} \nu_\sigma + \psi_y \\ &\quad + \frac{1}{2} \left((-\gamma + \alpha_{y,c})^2 \alpha_c^2 + ((\rho - \gamma) \phi_v + \alpha_{y,x} + \phi_{y,h-1})^2 \alpha_x^2 + \alpha_{y,d}^2 \alpha_d^2 \right) \end{aligned}$$

- Constant:

$$\begin{aligned}\tilde{\mu}_{y,h} - \tilde{\mu}_{y,h-1} &= -\mu_c - (1 - \rho) \frac{1}{2} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \left(\frac{1 - \tilde{\gamma}}{1 - \nu_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1 - \nu_\sigma) \\ &\quad + \frac{1}{2} \left[\left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \alpha_{y,\sigma} + \tilde{\psi}_{y,h-1} \right)^2 - (1 - \gamma)^2 \tilde{\psi}_v^2 \right] \alpha_\sigma^2 \\ &\quad + \mu_y + \sigma^2 (1 - \nu_\sigma) \tilde{\psi}_{y,h-1}\end{aligned}$$

Note only both the constant terms $\{\tilde{\mu}_{y,h}\}$ and the loadings on the volatility shocks $\{\tilde{\psi}_{y,h}\}$ are affected by the wedge between γ and $\tilde{\gamma}$. □

In line with the specification of [van Binsbergen and Kojien \(2016\)](#), we consider one-period holding returns for these claims of the form

$$\begin{aligned}1 + R_{t+1,h}^Y &= \frac{G_{y,t+1} Y_{t+1,h-1}}{Y_{t,h}} = \frac{G_{y,t+1} Y_{t+1,h-1}}{E_t[\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1}]} \\ &= R_{f,t} \frac{E_t[\Pi_{t,t+1}] G_{y,t+1} Y_{t+1,h-1}}{E_t[\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1}]},\end{aligned}$$

with the risk-free rate

$$R_{f,t} = \frac{1}{E_t[\Pi_{t,t+1}]}.$$

In line with the specification of [van Binsbergen and Kojien \(2016\)](#), we also consider one-period holding returns for futures on these claims of the form

$$\begin{aligned}R_{t+1,h}^{F,Y} + 1 &= \frac{1 + R_{t+1,h}^Y}{1 + R_{t+1,h}^B} = \frac{G_{y,t+1} Y_{t+1,h-1}}{Y_{t,h}} \frac{B_{t,h}}{B_{t+1,h-1}} \\ &= \frac{G_{y,t+1} Y_{t+1,h-1}}{E_t(\Pi_{t,t+1} G_{y,t+1} Y_{t+1,h-1})} \frac{E_t(\Pi_{t,t+1} B_{t+1,h-1})}{B_{t+1,h-1}},\end{aligned}$$

where $B_{t,h}$ is the price of \$1 at horizon h , i.e. the price of a Bond with horizon h .

Their conditional Sharpe Ratio is

$$\begin{aligned}
\text{SR}_{t,h}^{F,Y} &= \frac{E_t \left(1 + R_{t+1,h}^{F,Y} \right) - 1}{\sqrt{\text{var}_t \left(1 + R_{t+1,h}^{F,Y} \right)}} \\
&= \frac{E_t \left(1 + R_{t+1,h}^{F,Y} \right) - 1}{\sqrt{E_t \left(\left(1 + R_{t+1,h}^{F,Y} \right)^2 \right) - \left(E_t \left(1 + R_{t+1,h}^{F,Y} \right) \right)^2}} \\
&\approx \frac{\left\{ \sigma_t^2 \left(\gamma \alpha_{y,c} \alpha_c^2 - \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1} \right) \left((\rho - \gamma) \phi_v + \phi_{b,h-1} \right) \alpha_x^2 \right) \right.}{\sqrt{\sigma_t^2 \left(\alpha_{y,c}^2 \alpha_c^2 + \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1} \right)^2 \alpha_x^2 + \alpha_{y,d}^2 \alpha_d^2 \right) + \left(\alpha_{y,\sigma} + \tilde{\psi}_{y,h-1} - \tilde{\psi}_{b,h-1} \right)^2 \alpha_\sigma^2}} \\
&\quad \left. - \left(\alpha_{y,\sigma} + \tilde{\psi}_{y,h-1} - \tilde{\psi}_{b,h-1} \right) \left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \tilde{\psi}_{b,h-1} \right) \alpha_\sigma^2 \right\}}{\sqrt{\sigma_t^2 \left(\alpha_{y,c}^2 \alpha_c^2 + \left(\alpha_{y,x} + \phi_{y,h-1} - \phi_{b,h-1} \right)^2 \alpha_x^2 + \alpha_{y,d}^2 \alpha_d^2 \right) + \left(\alpha_{y,\sigma} + \tilde{\psi}_{y,h-1} - \tilde{\psi}_{b,h-1} \right)^2 \alpha_\sigma^2}}.
\end{aligned}$$

C.1.2 Bonds

Let the price at time t for \$1 in h periods be $B_{t,h}$ with $B_{t,0} = 1$. For $h \geq 1$, we have

$$B_{t,h} = E_t[\Pi_{t,t+1} B_{t+1,h-1}]$$

This is the general problem from above with $g_{y,t+1} = 0$ for all t and therefore

$$b_{t,h} = \tilde{\mu}_{b,h} + \phi_{b,h} x_t + \tilde{\psi}_{b,h} \sigma_t^2,$$

with

$$\phi_{b,h} = -\rho \phi_c \frac{1 - \nu_x^h}{1 - \nu_x}$$

$$\begin{aligned}
\tilde{\psi}_{b,h} &= -\frac{1}{2} \left((\rho - \gamma) (1 - \gamma) - (1 - \rho) (\gamma - \tilde{\gamma}) \nu_\sigma \right) \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) + \tilde{\psi}_{b,h-1} \nu_\sigma \\
&\quad + \frac{1}{2} \left(\gamma^2 \alpha_c^2 + \left((\rho - \gamma) \phi_v + \phi_{b,h-1} \right)^2 \alpha_x^2 \right)
\end{aligned}$$

C.1.3 Dividend strips

Let the price at time t for the full dividend D_{t+h} in h periods be $P_{t,h}$ with $P_{t,0} = D_t$. Then for $h \geq 1$:

$$\frac{P_{t,h}}{D_t} = E_t \left(\Pi_{t,t+1} \frac{D_{t+1}}{D_t} \frac{P_{t+1,h-1}}{D_{t+1}} \right),$$

which is the general problem from above with

$$g_{p,t+1} = d_{t+1} - d_t = \mu_d + \phi_d x_t + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1},$$

for all t and therefore

$$p_{t,h} - d_t = \tilde{\mu}_{p,h} + \phi_{d,h} x_t + \tilde{\psi}_{d,h} \sigma_t^2,$$

with

$$\phi_{d,h} = (-\rho \phi_c + \phi_d) \frac{1 - v_x^h}{1 - v_x}$$

$$\begin{aligned} \tilde{\psi}_{d,h} = & -\frac{1}{2} ((\rho - \gamma) (1 - \gamma) - (1 - \rho) (\gamma - \tilde{\gamma}) v_\sigma) (\alpha_c^2 + \phi_v^2 \alpha_x^2) + \tilde{\psi}_{d,h-1} v_\sigma \\ & + \frac{1}{2} \left((-\gamma + \chi)^2 \alpha_c^2 + ((\rho - \gamma) \phi_v + \phi_{d,h-1})^2 \alpha_x^2 + \alpha_d^2 \right) \end{aligned}$$

$$\begin{aligned} \tilde{\mu}_{y,h} - \tilde{\mu}_{y,h-1} = & -\mu_c - (1 - \rho) \frac{1}{2} (\alpha_c^2 + \phi_v^2 \alpha_x^2) \left(\frac{1 - \tilde{\gamma}}{1 - v_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1 - v_\sigma) \\ & + \frac{1}{2} \left[\left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - v_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \tilde{\psi}_{d,h-1} \right)^2 - (1 - \gamma)^2 \tilde{\psi}_v^2 \right] \alpha_\sigma^2 \\ & + \mu_d + \sigma^2 (1 - v_\sigma) \tilde{\psi}_{d,h-1} \end{aligned}$$

For the dividend strips, the spot one-period returns are given by

$$R_{t+1,h}^P + 1 = \frac{P_{t+1,h-1}/D_{t+1}}{P_{t,h}/D_t} \frac{D_{t+1}}{D_t},$$

$$\begin{aligned}
\log \left(R_{t+1,h}^P + 1 \right) &= \mu_c + (1 - \rho) \frac{1}{2} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \left(\frac{1 - \tilde{\gamma}}{1 - \nu_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1 - \nu_\sigma) \\
&\quad - \frac{1}{2} \left[\left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \tilde{\psi}_{d,h-1} \right)^2 - (1 - \gamma)^2 \tilde{\psi}_v^2 \right] \alpha_\sigma^2 \\
&\quad + \rho \phi_c x_t + (\tilde{\psi}_{d,h-1} \nu_\sigma - \tilde{\psi}_{d,h}) \sigma_t^2 \\
&\quad + \tilde{\psi}_{d,h-1} \alpha_\sigma W_{t+1} + \phi_{d,h-1} \alpha_x \sigma_t W_{t+1} + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1}
\end{aligned}$$

the conditional expected one-period returns are

$$\begin{aligned}
E_t \left(R_{t+1,h}^P + 1 \right) &\approx \text{constant (in h)} - \left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v \tilde{\psi}_{d,h-1} \alpha_\sigma^2 \\
&\quad + (\tilde{\psi}_{d,h-1} \nu_\sigma - \tilde{\psi}_{d,h}) \sigma_t^2 + \frac{1}{2} \left(\phi_{d,h-1}^2 \alpha_x^2 \sigma_t^2 \right)
\end{aligned}$$

$$\begin{aligned}
E_t \left(R_{t+1,h}^P + 1 \right) &\approx \text{constant (in h)} - \left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v \tilde{\psi}_{d,h-1} \alpha_\sigma^2 \\
&\quad - ((\rho - \gamma) \phi_v \phi_{d,h-1}) \alpha_x^2 \sigma_t^2
\end{aligned}$$

$$\begin{aligned}
E_t \left(R_{t+1,h}^P \right) - E_t \left(R_{t+1,h-1}^P \right) &\approx \underbrace{\left[(\gamma - \rho) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v (\tilde{\psi}_{d,h} - \tilde{\psi}_{d,h-1}) \alpha_\sigma^2}_{\leq 0} \\
&\quad + \underbrace{(\gamma - \rho) \phi_v (\rho \phi_c - \phi_d) \frac{\nu_x^h - \nu_x^{h-1}}{1 - \nu_x} \alpha_x^2 \sigma_t^2}_{\geq 0}
\end{aligned}$$

We need $(\tilde{\psi}_{d,h} - \tilde{\psi}_{d,h-1}) \geq 0$ to generate a downward sloping term structure. If $(\tilde{\psi}_{d,h} - \tilde{\psi}_{d,h-1}) \leq 0$, then the returns are upward sloping, but less so in our model.

Note, that the returns are MORE upward sloping when σ_t is high...

The future one-period returns are given by:

$$R_{t+1,h}^{F,P} + 1 = \frac{1 + R_{t+1,h}^P}{1 + R_{t+1,h}^B}$$

$$\begin{aligned} \log \left(R_{t+1,h}^P + 1 \right) &= \mu_c + (1 - \rho) \frac{1}{2} \left(\alpha_c^2 + \phi_v^2 \alpha_x^2 \right) \left(\frac{1 - \tilde{\gamma}}{1 - \nu_\sigma} - (\gamma - \tilde{\gamma}) \right) \sigma^2 (1 - \nu_\sigma) \\ &\quad - \frac{1}{2} \left[\left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \tilde{\psi}_{d,h-1} \right)^2 - (1 - \gamma)^2 \tilde{\psi}_v^2 \right] \alpha_\sigma^2 \\ &\quad + \rho \phi_c x_t + (\tilde{\psi}_{d,h-1} \nu_\sigma - \tilde{\psi}_{d,h}) \sigma_t^2 \\ &\quad + \tilde{\psi}_{d,h-1} \alpha_\sigma W_{t+1} + \phi_{d,h-1} \alpha_x \sigma_t W_{t+1} + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1} \end{aligned}$$

$$\begin{aligned} \log \left(R_{t+1,h}^{F,P} + 1 \right) &= - \left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v (\tilde{\psi}_{d,h-1} - \tilde{\psi}_{b,h-1}) + \frac{1}{2} (\tilde{\psi}_{d,h-1}^2 - \tilde{\psi}_{b,h-1}^2) \right) \\ &\quad + ((\tilde{\psi}_{d,h-1} - \tilde{\psi}_{b,h-1}) \nu_\sigma - (\tilde{\psi}_{d,h} - \tilde{\psi}_{b,h})) \sigma_t^2 \\ &\quad + (\tilde{\psi}_{d,h-1} - \tilde{\psi}_{b,h-1}) \alpha_\sigma W_{t+1} + (\phi_{d,h-1} - \phi_{b,h-1}) \alpha_x \sigma_t W_{t+1} + \chi \alpha_c \sigma_t W_{t+1} + \alpha_d \sigma_t W_{t+1} \end{aligned}$$

$$\begin{aligned} E_t \left(R_{t+1,h}^{F,P} + 1 \right) &= - \left(\underbrace{\left(\left[(\rho - \gamma) + (1 - \rho) (\gamma - \tilde{\gamma}) \frac{1 - \nu_\sigma}{1 - \tilde{\gamma}} \right] \tilde{\psi}_v + \tilde{\psi}_{b,h-1} \right)}_{\geq 0 \text{ and increasing}} (\tilde{\psi}_{d,h-1} - \tilde{\psi}_{b,h-1}) \right) \alpha_\sigma^2 \\ &\quad + \left(\gamma \chi \alpha_c^2 + \underbrace{((\gamma - \rho) \phi_v - \phi_{b,h-1}) (\phi_{d,h-1} - \phi_{b,h-1})}_{\geq 0 \text{ and increasing}} \alpha_x^2 \right) \sigma_t^2 \end{aligned}$$

Note:

$$\tilde{\psi}_{d,h} - \tilde{\psi}_{b,h} = (\tilde{\psi}_{d,h-1} - \tilde{\psi}_{b,h-1}) \nu_\sigma + \left(\underbrace{\chi \left(\frac{1}{2} \chi - \gamma \right)}_{\leq 0} \alpha_c^2 + \underbrace{\left((\rho - \gamma) \phi_v + \frac{1}{2} (\phi_{d,h-1} + \phi_{b,h-1}) \right)}_{\leq 0 \text{ for } \gamma \text{ high enough}} \underbrace{(\phi_{d,h-1} - \phi_{b,h-1})}_{\geq 0} \alpha_x^2 + \underbrace{\frac{1}{2} \alpha_d^2}_{\geq 0} \right)$$

the sign depends on the parameters. But if it is positive increasing, $\tilde{\gamma}$ reduces the downward impact of it on the term structure of expected returns. Only if it is negative and decreasing does our model help relative to the standard model, but then the slope is upward sloping....

Note, a higher σ_t means a MORE upward sloping term structure again. □

Internet Appendix

IA.1 Alternative derivation of stochastic discount factor

This appendix derives the stochastic discount factor of our dynamic model using an approach similar to the one used by [Luttmer and Mariotti \(2003\)](#) for dynamic inconsistency due to non-geometric discounting. In every period t the agent chooses consumption C_t for the current period and state-contingent levels of wealth $\{W_{t+1,s}\}$ for the next period to maximize current utility V_t subject to a budget constraint *and* anticipating optimal choice C_{t+h}^* in all following periods ($h \geq 1$):

$$\begin{aligned} \max_{C_t, \{W_{t+1,s}\}} & \left((1 - \beta) C_t^{1-\rho} + \beta E_t \left[(V_{t,t+1}^*)^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}} \\ \text{s.t.} & \quad \Pi_t C_t + E_t[\Pi_{t+1} W_{t+1}] \leq \Pi_t W_t \\ & \quad V_{t,t+h}^* = \left((1 - \beta) (C_{t+h}^*)^{1-\rho} + \beta E_{t+h} \left[(V_{t,t+h+1}^*)^{1-\gamma_{h+1}} \right]^{\frac{1-\rho}{1-\gamma_{h+1}}} \right)^{\frac{1}{1-\rho}} \quad \text{for all } h \geq 1. \end{aligned}$$

Denoting by λ_t the Lagrange multiplier on the budget constraint for the period- t problem, the first order conditions are:²⁵

- For C_t :

$$\left((1 - \beta) C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} (1 - \beta) C_t^{-\rho} = \lambda_t.$$

- For each $W_{t+1,s}$:

$$\begin{aligned} \frac{1}{1-\rho} \left((1 - \beta) C_t^{1-\rho} + \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} \beta \frac{d}{dW_{t+1,s}} \beta E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \\ = \Pr[t+1, s] \frac{\Pi_{t+1,s}}{\Pi_t} \lambda_t. \end{aligned}$$

²⁵For notational ease we drop the star from all Cs and Vs in the following optimality conditions but it should be kept in mind that all consumption values are the ones optimally chosen by the corresponding self.

Combining the two, we get an initial equation for the SDF:

$$\frac{\Pi_{t+1,s}}{\Pi_t} = \beta \frac{\frac{1}{1-\rho} \frac{1}{\Pr[t+1,s]} \frac{d}{dW_{t+1,s}} E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}}}{1} \frac{1}{(1-\beta) C_t^{-\rho}}. \quad (23)$$

The agent in state s at $t + 1$ maximizes

$$\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[(V_{t+1,s,t+2}^*)^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}}$$

and has the analogous first order condition for $C_{t+1,s}$:

$$\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} (1-\beta) C_{t+1,s}^{-\rho} = \lambda_{t+1,s}.$$

The Lagrange multiplier $\lambda_{t+1,s}$ is equal to the marginal utility of an extra unit of wealth in state $t + 1, s$:

$$\begin{aligned} \lambda_{t+1,s} &= \frac{1}{1-\rho} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)^{\frac{1}{1-\rho}-1} \\ &\quad \times \frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right). \end{aligned}$$

Eliminating the Lagrange multiplier $\lambda_{t+1,s}$ and combining with the initial Equation (23) for the SDF, we get:

$$\frac{\Pi_{t+1,s}}{\Pi_t} = \beta \frac{\frac{1}{\Pr[t+1,s]} \frac{d}{dW_{t+1,s}} E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}}}{\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[V_{t+1,s,t+2}^{1-\gamma_1} \right]^{\frac{1-\rho}{1-\gamma_1}} \right)} \left(\frac{C_{t+1,s}}{C_t} \right)^{-\rho}.$$

Expanding the V expressions, we can proceed with the differentiation in the numerator:

$$\begin{aligned}
\frac{\Pi_{t+1,s}}{\Pi_t} &= E_t \left[\left((1-\beta) C_{t+1}^{1-\rho} + \beta E_{t+1}[\dots]^{1-\rho} \right)^{\frac{1-\gamma_1}{1-\rho}} \right]^{\frac{1-\rho}{1-\gamma_1}-1} \\
&\quad \times \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{1-\rho} \right)^{\frac{1-\gamma_1}{1-\rho}-1} \\
&\quad \times \beta \frac{\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{1-\rho} \right)}{\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1}^{1-\rho} + \beta E_{t+1}[\dots]^{1-\rho} \right)} \left(\frac{C_{t+1,s}}{C_t} \right)^{-\rho}. \quad (24)
\end{aligned}$$

For Markov consumption $C = \phi W$, we can divide by $C_{t+1,s}$ and solve both differentiations:

- For the numerator:

$$\begin{aligned}
&\frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots]^{1-\rho} \right)^{\frac{1-\gamma_2}{1-\rho}} \right]^{\frac{1-\rho}{1-\gamma_2}} \right) \\
&= \left((1-\beta) 1 + \beta E_{t+1,s} \left[\left((1-\beta) \left(\frac{C_{t+2}}{C_{t+1,s}} \right)^{1-\rho} + \beta E_{t+2}[\dots]^{1-\rho} \right)^{\frac{1-\gamma_2}{1-\rho}} \right]^{\frac{1-\rho}{1-\gamma_2}} \right) \\
&\quad \times \phi_{t+1,s}^{1-\rho} W_{t+1,s}^{-\rho}.
\end{aligned}$$

- For the denominator:

$$\begin{aligned}
& \frac{d}{dW_{t+1,s}} \left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\gamma_1}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma_1}} \\
&= \left((1-\beta) 1 + \beta E_{t+1,s} \left[\left((1-\beta) \left(\frac{C_{t+2}}{C_{t+1,s}} \right)^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\gamma_1}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma_1}} \\
&\quad \times \phi_{t+1,s}^{1-\rho} W_{t+1,s}^{-\rho}.
\end{aligned}$$

Substituting these into Equation (24) and canceling we get:

$$\begin{aligned}
\frac{\Pi_{t+1,s}}{\Pi_t} &= \frac{(1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_3}} \right]^{\frac{1-\gamma_2}{1-\rho}}}{(1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s} \left[\left((1-\beta) C_{t+2}^{1-\rho} + \beta E_{t+2}[\dots] \right)^{\frac{1-\rho}{1-\gamma_2}} \right]^{\frac{1-\gamma_1}{1-\rho}}} \\
&\quad \times \beta \left(\frac{C_{t+1,s}}{C_t} \right)^{-\rho} \left(\frac{(1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1,s}[\dots]^{\frac{1-\rho}{1-\gamma_2}}}{E_t \left[\left((1-\beta) C_{t+1,s}^{1-\rho} + \beta E_{t+1}[\dots]^{\frac{1-\rho}{1-\gamma_2}} \right)^{\frac{1-\gamma_1}{1-\rho}} \right]} \right)^{\rho-\gamma_1}.
\end{aligned}$$

Simplifying and cleaning up notation, we arrive at

$$\Pi_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t,t+1}}{E_t \left[V_{t,t+1}^{1-\gamma_1} \right]^{\frac{1}{1-\gamma_1}}} \right)^{\rho-\gamma_1} \left(\frac{V_{t,t+1}}{V_{t+1}} \right)^{1-\rho},$$

as stated in the text. □

IA.2 Additional figures

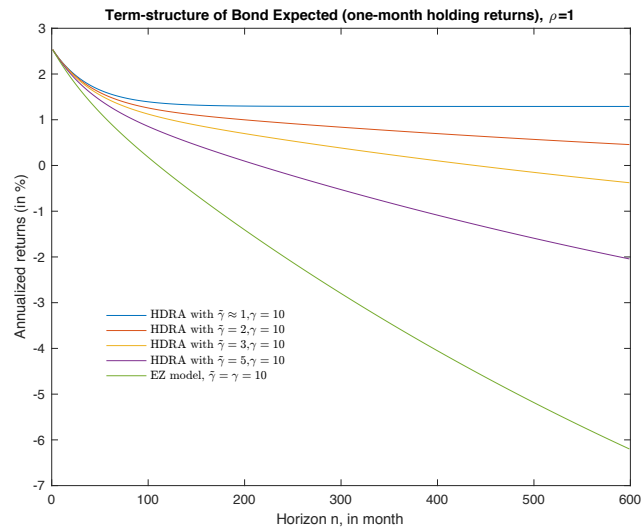


Figure IA.1: Term structure of bond returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1. Returns are conditional, with state variables set at their means: $x_t = 0$ and $\sigma_t = \sigma$.

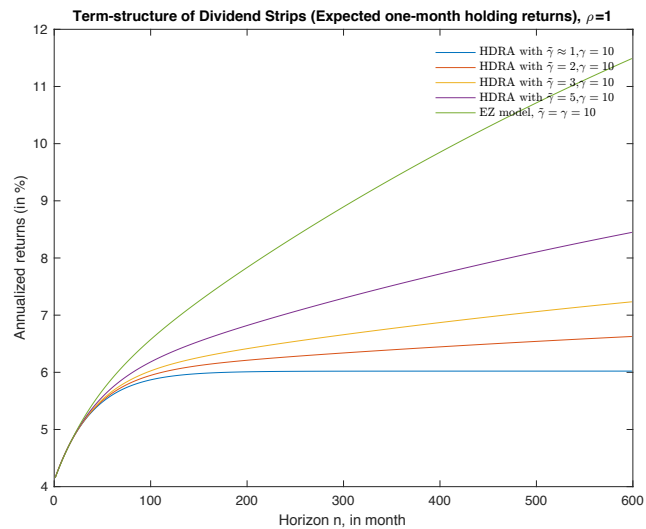


Figure IA.2: Term structure of dividend strip expected returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1. Returns are conditional, with state variables set at their means: $x_t = 0$ and $\sigma_t = \sigma$.

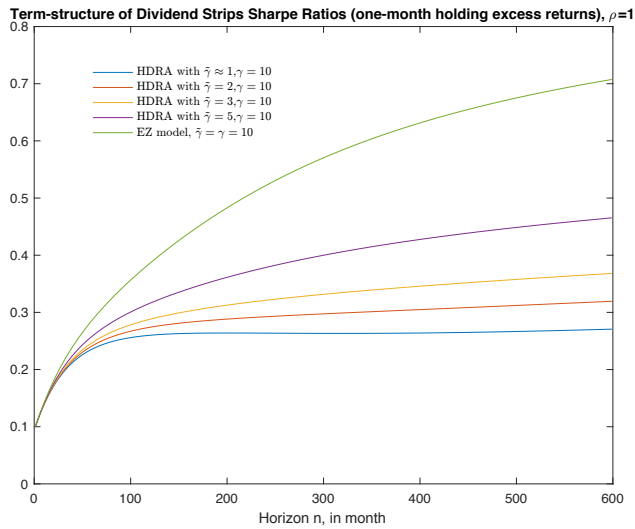


Figure IA.3: Term structure of dividend strip unconditional Sharpe ratios of excess returns under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.

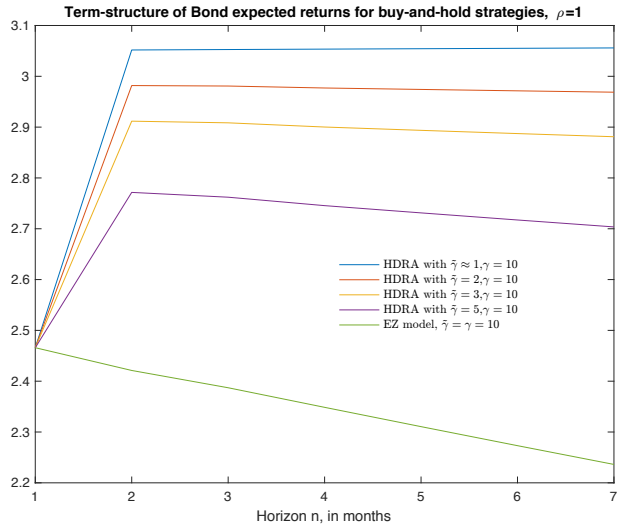


Figure IA.4: Term structure of of bond returns under illiquid buy-and-hold strategies, under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.

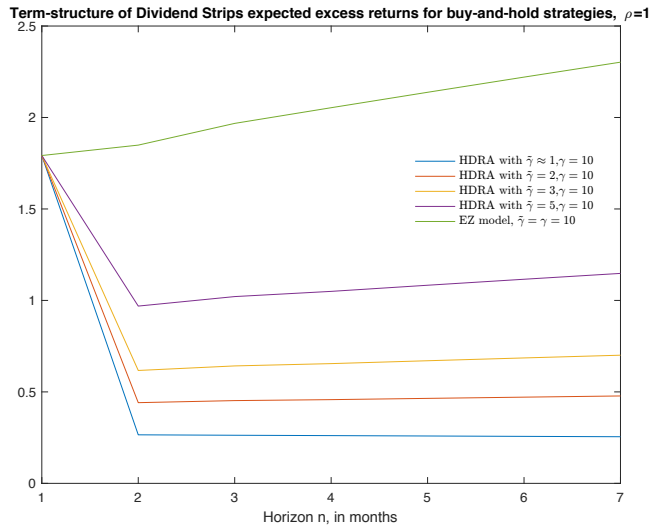


Figure IA.5: Term structure of dividend strip expected excess returns under illiquid buy-and-hold strategies, under horizon-dependent risk aversion (HDRA) and Epstein-Zin (EZ), with the calibration of Table 1.