Floor Systems and the Friedman Rule:
The Fiscal Arithmetic of
Open Market Operations

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FRBNY
Staff
REPORTS

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Federal Reserve Bank of New York Staff Reports, no. 754
December 2015
JEL classification: E42, E52, E58

Abstract

In a floor system of monetary policy implementation, the central bank remunerates bank reserves at or near the market rate of interest. Some observers have expressed concern that operating such a system will have adverse fiscal consequences for the public sector and may even require the government to subsidize the central bank. We show that this is not the case. Using the monetary general equilibrium model of Berentsen et al. (2014), we show how a central bank that supplies reserves through open market operations can always generate non-negative net income, even when using a floor system to implement the Friedman rule.

Key words: monetary policy implementation, central bank operations, interest on reserves

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1   Introduction

Prior to the financial crisis of 2007-2009, many central banks implemented monetary policy using a set of procedures collectively known as a *corridor (or channel) system.* In such a system, the central bank offers to lend to commercial banks at an interest rate above its target for the market rate and remunerates banks' deposits at the central bank (called *reserves*) at an interest rate below the target. These two administered rates form a "corridor" around the target rate, which is often set at the midpoint. The central bank supplies a limited quantity of reserves and banks meet their liquidity needs by actively borrowing/lending these scarce reserves in the interbank market. The central bank then uses small, frequent adjustments to the supply of reserves to steer the market interest rate to its target.

In recent years, however, some central banks have adopted new implementation frameworks as large-scale asset purchases and/or increased lending activity have created a dramatic increase in the quantity of reserves held by commercial banks. In the U.S., for example, reserve balances increased from \$11 billion in July 2007 to a peak of \$2.8 trillion in 2014-2015.[1] When the supply of reserves is so large, it is no longer possible for a central bank to steer market interest rates using the small adjustments to reserve supply that characterize a corridor system. Instead, there has been a shift toward operating a *floor system* in which market rates are controlled primarily by adjusting the interest rate the central bank pays on reserves (the "floor" of the corridor) with no meaningful change in the quantity of these balances.

Once the stance of monetary policy begins to tighten, central banks will be faced with a choice between continuing to use a floor system to implement monetary policy and returning to a corridor system. While there has been some discussion of this choice in policy circles,[2] there is still relatively little formal economic analysis of the issue and the merits and potential side effects of each approach are not fully understood. In the U.S., the Federal Open Market Committee (FOMC) has indicated that it will conduct monetary policy using a type of floor system in the years ahead, but its choice of framework over a longer time horizon remains open.[3]

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[1] Data from Federal Reserve Statistical Release H.4.1, available at http://www.federalreserve.gov/releases/h41/. See Keister and McAndrews (2009) for a discussion of the relationship between the policy response to the financial crisis and the quantity of bank reserves. See Ennis and Wolman (2015) for an analysis of how these reserves are distributed across banks.

[2] See, for example, Bernhardsen and Kloster (2010) and Gagnon and Sack (2014).

[3] See FOMC (2014) for additional details about the Committee's planned approach to normalizing monetary policy. FOMC (2015) states that the Federal Reserve will "soon begin an extended effort to evaluate potential long-run monetary policy implementation frameworks."

Several recent papers have investigated the potential fiscal consequences of implementing monetary policy using a floor system with a large supply of bank reserves. Many of these studies have focused on transitional issues associated with rising interest rates. When interest rates increase, a central bank that holds long-maturity bonds will experience capital losses. If, in addition, the central bank pays interest on reserves at or near the short-term market interest rate, it may experience negative cash flow in some periods. Carpenter et al. (2015), Christensen et al. (2015) and Greenlaw et al. (2013) provide quantitative analyses of the possible evolution of the Federal Reserve's balance sheet and income under different scenarios for interest rates.[4] It is important to recognize, however, that these papers do not examine the consequences of operating a floor system *per se*. Rather, they study the implications of relying on a floor system to raise interest rates when the central bank holds a particular, large portfolio of assets that were accumulated through unconventional policy measures.

Some observers express a separate concern about the *steady-state* fiscal implications of using a floor system. If a central bank pays interest on reserves at or near its target for the market interest rate, how are these payments financed? Plosser (2010) worries that "the interest [paid on reserves] is financed by government revenues" and "may come at the cost of central bank independence." Berentsen et al. (2014) study a general equilibrium model in which bank reserves facilitate exchange and show that "the optimal floor system requires either transfers from the fiscal authority to the central bank or a reduction in seigniorage payments from the central bank to the government." They label this result the "unpleasant fiscal arithmetic of a floor system" and echo Plosser's concern than these fiscal implications will jeopardize a central bank's independence. Both sets of authors conclude that a central bank is better off using a corridor system to implement monetary policy.

We argue that concerns about the fiscal implications of a floor system are misplaced. We study the model in Berentsen et al. (2014) with a small, but important, change: we assume the central bank changes the supply of reserves through open market operations using short-term government bonds, rather than through transfers. This assumption has an immediate implication for the balance sheet of the central bank: for every dollar of reserves in circulation, the central bank owns a bond that was purchased when that dollar was created. The central bank

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[4]See also Fleming et al. (2013). Bassetto and Messer (2013), Del Negro and Sims (2015) and Hall and Reis (2015) study central bank solvency more generally and its implications for institutional design.

earns interest on these bonds at the market rate, which depends in equilibrium on the supply of reserves and on the central bank's two administered rates.[5] We show that when the central bank operates this way, it will never require transfers from the fiscal authority. Regardless of the supply of reserves and whether it operates a corridor or floor system, the central bank's equilibrium asset revenue will always be sufficient to cover its interest expense.

There is a long tradition in monetary economics of focusing primarily on the finances of the consolidated public sector, which includes both the fiscal authority and the central bank. At this consolidated level, our approach is identical to that in Berentsen et al. (2014). The consolidated approach has proved useful for studying a wide range of issues because it allows one to clearly state the economically meaningful constraints while abstracting from institutional details. However, additional structure is needed to discuss the division of assets and revenue between the fiscal authority and the central bank. In their baseline analysis, Berentsen et al. (2014) assume the central bank owns no assets. Implicitly, this approach implies that the reserves held by banks were created by a transfer from the public sector to private agents. Under our accounting, the debt created by this transfer would be an obligation of the fiscal authority, which would issue bonds to pay for the expenditure, and not of the central bank. Berentsen et al. (2014) later allow for the possibility that the central bank owns some assets and show that if these assets are large enough, no transfers from the fiscal authority are required. However, their approach provides no guidance on how the quantity of assets owned by the central bank is determined or how likely the problem of negative revenue is to arise. Our approach of explicitly accounting for open market operations pins down the central bank's assets and revenue in a way that matches accounting standards used in practice. Depending on the details of its policy, the central bank's equilibrium net income may be positive or zero, but we show that it will not be negative.

The optimal monetary policy in the environment we study here is to set the interest rate on reserves equal to the equilibrium interest rate on an illiquid one-period bond. This policy is a form of the Friedman rule and ensures that banks face no opportunity cost of holding reserves. We show that the central bank's net income in a stationary equilibrium under this policy is zero in all periods. This result reflects the well-known fact that a central bank typically generates

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[5]This point was emphasized by Goodfriend (2002) in an early discussion of the possible benefits of operating a floor system.

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no seigniorage revenue when following the Friedman rule. Notice, however, that the policy is perfectly feasible for the central bank, since the interest it must pay on its outstanding reserves is covered by the interest it earns on the assets it purchased when those reserves were created. Moreover, we show that this policy has no adverse fiscal consequences for the public sector as a whole. The level of taxes the government must collect from agents under the optimal policy is exactly the same as would be required in an environment in which credit markets work perfectly and there is no need for money or a central bank.

In the next section, we present the model and describe our assumptions about the operation of the central bank in detail. In Section 3, we present our results on the fiscal arithmetic of open market operations in general and under the optimal monetary policy. We examine two extensions in Section 4. In the first, we suppose the central bank is required to generate strictly positive net income for some exogenous reason, and we allow it to set reserve requirements. We show that the optimal policy in this modified environment is again a floor system, but with required reserves remunerated at a lower interest rate than excess reserves. In the second extension, we allow the central bank to hold an arbitrary portfolio of assets and provide sufficient conditions for its net income to be positive. Overall, our analysis suggests that the choice between a corridor and a floor system should be made based on the ability of each approach to implement monetary policy effectively and efficiently, rather than any concern about fiscal implications.

2   The model

We begin by reviewing the model of Berentsen et al. (2014; hereafter BMW), using their notation except where indicated.[6] Time is discrete and indexed by $t = 0, 1, 2, ...$ The economy is inhabited by two types of infinitely-lived agents, labeled 'buyers' and 'sellers,' each with unit measure and a discount factor $\beta < 1$. There is also a government with the ability to levy taxes, make transfers, and issue one-period nominal discount bonds, as well as a central bank that can create money.

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[6]The model builds on the basic framework of Lagos and Wright (2005) as adapted to study monetary policy implementation by Berentsen and Monnet (2008). Alternative approaches to modeling the general equilibrium effects of paying interest on reserves can be found in Bianchi and Bigio (2014), Ennis (2014), and Williamson (2015), among others.
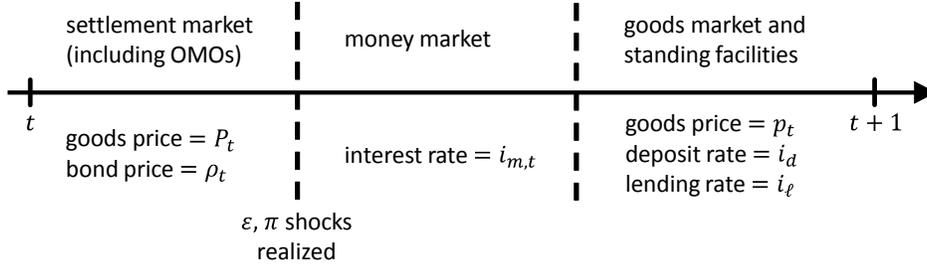
Figure 1: Timing of markets and notation

2.1 Markets, preferences and timing

In each period, three perfectly-competitive markets open sequentially as depicted in Figure 1.

The settlement market. In the first market, agents can trade money, bonds and a consumption good, and all claims from the previous period are settled. Let P_t denote the price level in this market and let ρ_t denote the price of a one-period nominal discount bond. Buyers and sellers can both consume and produce goods in this market and have linear utility: consuming x units of goods yields x units of utility, while producing h units of goods entails disutility $-h$. Buyers produce in this market to acquire the funds needed to pay off any loans taken in the previous period and to build a portfolio of assets (money and bonds) that will be carried through the current period. The linearity of preferences implies that all buyers will exit the settlement market holding the same portfolio. Sellers use this market to consume the proceeds of assets acquired over the course of the previous period. Sellers are assumed to exit the settlement market with zero assets; given the linearity in preferences, this assumption is without any loss of generality. Both the government and the central bank also participate in this market; their activities are described below.

The money market. After the settlement market has closed, a second market opens in which a subset of buyers are able to borrow or lend money against collateral, in the form of government bonds, at an interest rate $i_{m,t}$. At the beginning of this market, a buyer's preferences over consumption in the upcoming goods market is revealed. Specifically, a buyer of type ε will receive utility $\varepsilon \ln(q)$ from consuming q units in the goods market. Buyers' types are drawn from a continuous distribution $F(\varepsilon)$ with support $[0, 1]$ and are independent across individuals

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and over time.[7] The idiosyncratic nature of this shock creates a non-degenerate distribution of liquidity needs; buyers with high realizations of ε will desire to hold larger money balances than buyers with low realizations. However, only a randomly-chosen fraction π of buyers have access to this market. A buyer who has access to the money market is called *active* and a buyer without access is called *non-active*. All financial contracts have a one-period maturity; loans taken in the money market must be repaid in the next period's settlement market.

The goods market. The fundamental motive for trade in the model comes from the third market, where money can be used to purchase goods at price p_t. Only sellers can produce in this market and they have linear disutility: producing q_s units of the good brings utility $-q_s$. As described above, a buyer of type ε receives utility $\varepsilon \ln(q)$ from consuming q units of this good. Households are anonymous to each other and cannot commit to honor intertemporal promises. Bonds are illiquid and cannot be used as media of exchange in the goods market. Only money, being a tangible asset that is fully recognizable, can be used as a medium of exchange.

Standing facilities. While the goods market is open, buyers have the option of borrowing money from the central bank's *lending facility*; such loans must be repaid in the settlement market in the following period at interest rate i_ℓ. Agents can also deposit money at the central bank's *deposit facility* and earn the interest rate i_d. These two interest rates are policy choices of the central bank and are assumed to be constant over time. The deposit facility remains open until the end of the period, which allows sellers to deposit the proceeds from their goods-market sales. If the interest rate i_d is strictly positive, agents will choose to deposit all of their money holdings with the central bank.[8] If the money market interest rate $i_{m,t}$ lies strictly between the central bank's lending rate i_ℓ and deposit rate i_d, active buyers will strictly prefer to adjust their money holdings in the money market rather than at the central bank's facilities. In this case, only non-active buyers would choose to use the central bank's lending facility and only non-active buyers and sellers would use the deposit facility.

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[7]There is a small difference from BMW here: they assume the distribution of ε is unbounded from above, which simplifies the analysis in some dimensions but creates technical problems when trying to exactly implement the optimal policy.

[8]For this reason, money in this model is best interpreted as representing bank reserves rather than currency in circulation.

2.2 The government

The government buys goods, levies lump-sum taxes and transfers, and issues one-period nominal bonds; all of this activity takes place in the settlement market. Let G_t denote the government's nominal spending in period t and let T_t denote its nominal tax revenue, so that $\Delta_t \equiv G_t - T_t$ is the government's primary deficit. In the initial period, the government purchases $\Psi > 0$ goods in the settlement market.[9] We think of these resources as being used to provide a public good at $t = 0$ (e.g., infrastructure, national defense, etc.), but none of the results below depend on this interpretation; the government could instead simply make lump-sum transfers to households. The government levies no taxes at $t = 0$, so its initial-period deficit is

$$\Delta_0 = P_0 \Psi. \tag{1}$$

Following BMW, we assume that government spending is zero in all subsequent periods and, hence, the government has a primary surplus equal to its tax revenue, or

$$\Delta_t = -T_t \quad \text{for } t \geq 1.$$

The public finance aspect of the model concerns how the initial deficit is financed over time.

In the initial period settlement market, the government issues bonds to finance its deficit. Letting B_t denote the face value of bonds issued in period t, its budget constraint in the initial period is

$$\rho_0 B_0 = \Delta_0. \tag{2}$$

In each subsequent period, the government must pay off the maturing bonds and receives the operating surplus of the central bank, S_t, which may be positive or negative. The government's budget constraint in these periods can be written as

$$\rho_t B_t + S_t = B_{t-1} + \Delta_t. \tag{3}$$

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[9]Our notation here deviates slightly from that in BMW: we use Δ rather than $\mathcal{D}$ to avoid confusion with the notation for deposits in the next subsection and we use $t + 1$ subscripts rather than "+" superscripts to denote next-period variables. We also introduce explicit notation for the government's purchase of goods in the settlement market; the importance of the parameter ψ is discussed in Section 3.4.

2.3 The central bank

BMW do not explicitly model the central bank's balance sheet and their baseline assumption is that the central bank holds no assets. This assumption plays an important role in their "unpleasant fiscal arithmetic" result, as we show below. In contrast, we explicitly model an institution that operates as a central *bank*, using the double-entry accounting rules required of any bank to link its assets and liabilities. We assume the central bank changes the money supply by purchasing or selling government bonds in the settlement market, which matches standard practice among many central banks.

Assumption 1 *The central bank issues/retires money through open market operations; that is, by purchasing or selling one-period government bonds in the settlement market.*

The central bank begins each period with zero equity. In the settlement market of each period $t \geq 0$, it buys a quantity B_t^C of newly-issued government bonds, paying with $\rho_t B_t^C$ units of newly-issued money. This money is a liability of the central bank and, therefore, its balance sheet as the end of the settlement market is given by

$$
\begin{array}{c|c}
\text{Assets} & \text{Liabilities} \\
\hline
\text{Bonds} \quad \rho_t B_t^C & \text{Money} \quad M_t
\end{array}
\tag{4}
$$

Importantly, a central bank that begins with zero equity will still have zero equity at the close of the settlement market since every dollar of money created was used to purchase a dollar's worth of assets.

During the goods market, the central bank operates its standing facilities. Let L_t denote the amount lent to buyers through the lending facility and let D_t denote the quantity of deposits accepted. Loans are made using newly-issued money and thus increase the stock of money in circulation. When agents deposit funds with the central bank, this money is removed from circulation and the central bank has a new liability in the form of interest-bearing deposits. The central bank's balance sheet at the end of the goods market is thus given by

$$
\begin{array}{cc|cc}
\text{Assets} & & \text{Liabilities} & \\
\hline
\text{Bonds} & \rho_t B_t^C & \text{Money} & M_t + L_t - D_t \\
\text{Loans} & L_t & \text{Deposits} & D_t
\end{array}
\tag{5}
$$

As mentioned above, agents will choose to deposit all of their money holdings with the central bank at the end of the goods market if i_d is strictly positive. In this case, D_t will equal $M_t + L_t$ and the quantity of money that remains in circulation between periods is zero.

At the beginning of period $t + 1$, the bonds owned by the central bank mature and the loans it made are repaid with interest. At the same time, the central bank must pay interest to agents who used the deposit facility. Its balance sheet when the settlement market opens is given by

Assets		Liabilities		
Bonds	B_t^C	Money	$M_t + L_t - D_t$	(6)
Loans	$(1 + i_\ell) L_t$	Deposits	$(1 + i_d) D_t$	
		Surplus	S_{t+1}	

where S_{t+1} denotes the central bank's operating surplus for the period. This term is defined to equate the central bank's assets and liabilities, that is,

$$S_{t+1} \equiv B_t^C + (1 + i_\ell) L_t - (M_t + L_t - D_t) - (1 + i_d) D_t.$$

Define i_t to be the nominal interest rate on government bonds issued in period t, so that $(1 + i_t) \equiv \rho_t^{-1}$ holds. Then, using the relationship $M_t = \rho_t B_t^C$ from the initial balance sheet (4), the expression above can be simplified to

$$S_{t+1} = i_t M_t + i_\ell L_t - i_d D_t. \tag{7}$$

The central bank's surplus thus has two main components: the revenue it earns on its bond portfolio (which is equal in size to the money supply) and the net revenue from its two standing facilities. In principle, this surplus could be positive, zero, or negative. We assume the resources are remitted to the government if it is positive. If S_{t+1} is negative, the government transfers resources to the central bank to erase the deficit. Either way, S_{t+1} enters the government's budget constraint in period $t + 1$ as shown in equation (3). This rule leaves the central bank with exactly zero equity and, after it purchases bonds in the period $t + 1$ settlement market, its balance sheet will once again be of the form in (4).

Our primary interest is in determining the conditions under which the central bank requires transfers from the government to fund its operations. BMW argue that such transfers are generally required when the central bank operates a floor system and refer to this result as *unpleasant fiscal arithmetic*. We follow their terminology.

Definition 1 *Unpleasant fiscal arithmetic* occurs when the central bank's surplus is negative in some period: $S_t < 0$.

Bear in mind that our definition of the central bank's surplus is different from that used in BMW, reflecting the difference in accounting from our Assumption 1. Comparing our equation (7) with equation (1) in BMW shows that our accounting allows the central bank to earn interest on its asset portfolio, which is equal in size to the monetary base. BMW, in contrast, assume the central bank holds no assets in their benchmark case and only earns revenue from its lending facility and new money creation.

2.4   The consolidated public sector

Substituting the expression for the central bank's surplus in equation (7) into the government's budget constraint (3) yields the budget constraint of the consolidated public sector

$$\rho_t B_t + i_{t-1} M_{t-1} + i_\ell L_{t-1} = B_{t-1} + i_d D_{t-1} + \Delta_t. \tag{8}$$

This constraint shows that total public revenue in period t, including new bond issuance and seigniorage revenue, must cover payments to existing bond holders and depositors in the central bank as well as the government's primary deficit. Define

$$B_t^P \equiv B_t - B_t^C$$

to be the quantity of bonds held by private agents. Then we can rearrange terms to write equation (8) as

$$\Delta_t = \rho_t \left(B_t^P + B_t^C \right) - \left(B_{t-1}^P + B_{t-1}^C \right) + i_{t-1} M_{t-1} + i_\ell L_{t-1} - i_d D_{t-1}.$$

Using the central bank's balance sheet identity in (4) for periods t and $t-1$, this becomes

$$\Delta_t = M_t - M_{t-1} + i_\ell L_{t-1} - i_d D_{t-1} + \rho_t B_t^P - B_{t-1}^P,$$

which is identical to equation (3) in BMW. The fact that the public sector's consolidated budget constraint is the same here as in BMW confirms that we are studying the same underlying model. Assumption 1 simply imposes a clear and realistic accounting of how resources are divided between the government and the central bank.

3   Results

In this section, we study the equilibrium properties of the central bank's surplus, focusing throughout on equilibria in which money has value in all periods. We follow BMW in assuming that the government chooses a constant gross growth rate $\eta > \beta$ for the supply of nominal bonds and that the central bank chooses this same growth rate for the nominal money supply.

3.1   Corridors and floors

The remaining policy choices for the central bank in this model are the interest rates at its two standing facilities, i_ℓ and i_d. These two interest rates create bounds for the equilibrium market interest rates, as established in the following proposition.

Proposition 1 *In any monetary equilibrium, $i_{m,t} \in [i_d, i_\ell]$ and $i_t \in [i_d, i_\ell]$ hold for all $t \geq 0$.*

The first part of the proposition is established in BMW (see their equation (19)) and can be understood by thinking about a buyer's incentive to borrow/lend in the money market. If $i_{m,t} < i_d$ held in some period, buyers with money holdings above their desired goods-market expenditure would prefer to wait and deposit these funds with the central bank rather than lend them in the money market. This shift would reduce the supply of funds in the money market and cause the interest rate to rise until $i_{m,t} \geq i_d$ holds. If $i_{m,t} > i_\ell$ held in some period, buyers would want to lend all of their money holdings in the money market and borrow whatever funds they use in the goods market from the central bank. In order for the money market to clear, the interest rate must fall until $i_{m,t} \leq i_\ell$ holds.

The second part of the proposition follows from buyers' portfolio choices in the settlement market. If $i_t < i_d$ held in some period, buyers would choose to exit the period-t settlement market holding only money and no bonds because money would both offer a higher return and be useful for goods-market transactions. Given that the supply of bonds is positive, their price ρ_t must fall until $i_t \geq i_d$ holds. Similarly, if $i_t > i_\ell$ held in some period, buyers would choose a portfolio consisting of only bonds and would borrow whatever money they use in the goods market from the central bank. In order for the settlement market to clear with a positive value of money, the price ρ_t of bonds must rise until $i_t \leq i_\ell$ holds.

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Note that an immediate consequence of this result is that the central bank must set $i_d \leq i_\ell$ in order for a monetary equilibrium to exist. In addition, the central bank must set $i_d \leq \frac{\eta}{\beta} - 1$; otherwise the demand to hold money and deposit at the central bank would be unbounded. We assume both of these conditions hold throughout.

The literature on monetary policy implementation refers to the interval $[i_d, i_\ell]$ as the central bank's interest rate *corridor*. A central bank is said to operate a *corridor system* if it sets a target for the money market interest rate that is in the interior of this corridor, often at the midpoint. Under a *floor system*, the central bank aims to act so that the market interest rate is equal to the rate i_d at its deposit facility, which is the "floor" of this corridor.

Definition 2 *The central bank operates a <u>corridor system</u> in period t if $i_d < i_{m,t} < i_\ell$. It operates a <u>floor system</u> if $i_d = i_{m,t} \leq i_\ell$.*

Notice that this definition refers to a property of an equilibrium, as it depends on the equilibrium interest rate in the money market.[10]

3.2 Equilibrium fiscal arithmetic

Combining Proposition 1 with Assumption 1 yields our central result: the central bank's surplus defined in equation (7) is always non-negative in equilibrium. In other words, unpleasant fiscal arithmetic never arises when the central bank operates according to Assumption 1.

Proposition 2 *Under Assumption 1, $S_{t+1} \geq 0$ holds for all $t \geq 0$ in any monetary equilibrium.*

Proof: Using $i_d \leq i_t \leq i_\ell$ from Proposition 1 in equation (7) yields

$$S_{t+1} \geq i_d \left(M_t + L_t - D_t \right) \geq 0.$$

The second inequality reflects the fact that the amount of money deposited with the central bank (D_t) cannot exceed the total supply of money created through open market operations (M_t) plus central bank lending (L_t). ∎

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[10]BMW define a floor system somewhat differently, requiring that the interest rates at the central bank's two facilities be equal, $i_d = i_\ell$. Given that i_ℓ is a ceiling for the market interest rate, it is easy to see that $i_d = i_{m,t}$ must hold under their definition as well. The differences between these two definitions are immaterial for our results, but our definition is formulated to more closely reflect the terminology typically used by central banks. See, for example, Bowman et al. (2010), Bernhardsen and Kloster (2010), and FOMC (2011).

Proposition 2 shows how a central bank that creates money through open market operations using one-period government bonds will always be self-financing. Notice that this result places no restriction on the path of market interest rates, nor does it rely on particular features of the BMW model; it follows from simply combining the no-arbitrage relationships in Proposition 1 with the accounting relationships in (4) - (6).

The equilibrium size of the operating surplus will depend on the parameters of the model as well as the specific policies followed by the central bank. In the next subsection, we study the equilibrium behavior of S_t under the optimal policy arrangement.

3.3 Fiscal arithmetic under the optimal policy

Focusing on stationary equilibria, BMW show that the optimal policy in their environment is a type of Friedman rule. In particular, the central bank should set the interest rate at its deposit facility equal to the nominal interest rate on an illiquid government bond. By doing so, it will remove the opportunity cost of holding money and, in equilibrium, buyers will choose to be satiated in real money balances.

The same result holds in our setting with one additional caveat. Under Assumption 1, the supply of money can be no larger than the outstanding government debt. We assume this debt is sufficiently large that the central bank can meet the demand for money that arises under the optimal policy using open market operations. In particular, since the debt is created by government purchases in period 0, we assume these purchases are sufficiently large.

Assumption 2 $\psi \geq 1$.

At the Friedman rule, each buyer will want to hold at least one unit of real money balances, which allows her to consume the first-best quantity in the goods market even if she is excluded from the money market and receives the highest realization of the preference shock, $\varepsilon = 1$. Assumption 2 guarantees that the central bank can buy enough government bonds so that the real per-capita money supply in the settlement market is at least 1 in every period. If this condition did not hold, running a floor system would require the central bank to buy some private assets in addition to government bonds; we return to this issue in the discussion in Section 3.4. With this assumption in hand, the following result follows immediately from Proposition 2 in BMW.

Proposition 3 *(Berentsen et al., 2014) The optimal policy sets $1 + i_d = \frac{\eta}{\beta}$. This policy implements the first-best allocation and corresponds to a floor system as $i_t = i_{m,t} = i_d$ holds in every period.*

The interest rate on the central bank's lending facility can be set to any value $i_\ell \geq i_d$ in this policy. Since buyers never borrow from the central bank in this equilibrium, the actual choice of rate is immaterial. Our next result characterizes the fiscal arithmetic of this equilibrium, showing that the central bank's surplus S_t is exactly zero in all periods.

Proposition 4 *Under Assumptions 1 and 2, if the central bank follows the optimal policy of setting $1 + i_d = \frac{\eta}{\beta}$, then $S_{t+1} = 0$ holds for all $t \geq 0$.*

The proof of this result is straightforward using the expression for surplus in (7). Under the optimal policy, L_t is zero and deposits at the central bank, D_t, are equal to the money supply, M_t. Since $i_t = i_d$ holds in all periods under the optimal policy, equation (7) immediately implies $S_{t+1} = 0$ in all periods. The intuition is also clear. We assumed that the central bank has zero equity capital, so its assets are equal in value to its liabilities in (4) and (5). Under the optimal policy, the yield on the central bank's assets and liabilities is the same; this is the logic of the Friedman rule. Hence, the central bank has exactly zero surplus in each period.

3.4 Discussion

Propositions 2 and 4 show that a central bank can implement a range of policies – including the Friedman rule – without requiring transfers from the government. In this subsection, we discuss the relationship between these results and those in BMW in more detail, focusing on why our conclusions differ so starkly. We also discuss other factors that can affect a central bank's net income in practice.

Hidden transfers? BMW discuss the possibility that the central bank could own government bonds and use the interest revenue from these bonds to finance the interest paid at its deposit facility. They conclude that:

> Endowing the central bank with government bonds is simply a way to hide transfer payments from the government to the central bank. In this case, the government has to levy taxes to finance interest payments on the government bonds, which it then hands over to the central bank. The central bank, then uses these funds to pay interest on reserves. (p. 535)

To evaluate these statements, it is useful to look back at the initial period of the model. In this period, the government purchases goods in the settlement market, as shown in equation (1). Under our accounting rules, the government issues bonds to finance this purchase, as shown in equation (2), and the central bank purchases some of these bonds to create money, as shown in (4). In particular, the central bank is not "endowed" with bonds; it *purchases* bonds in the settlement market as part of the money-creation process. In BMW, in contrast, only part of the government's period-0 purchase of goods is financed by issuing bonds; the remainder is financed by the newly issued money. In other words, their approach implicitly assumes that the central bank creates money by purchasing goods in period 0 and transfers these goods to the government (or directly to households); this transfer leaves the central bank with negative equity. They then label any use of tax revenue to finance the central bank's part of the period-0 goods purchase as a "transfer" from the government to the central bank.

In our view, this approach conflates fiscal and monetary policy. Our Assumption 1 draws a clear line between these two types of policies by stating that the central bank does not buy goods or make transfers to households. Instead, the central bank only buys/sells government bonds and all fiscal policy is conducted by the government. The debt created by fiscal policy in period 0 is then an obligation of the government and not of the central bank. Over time, the government budget constraint requires that the interest expense associated with this initial debt be paid from a combination of taxes and seigniorage revenue, as shown in equation (3). Proposition 3 shows that seigniorage revenue under the optimal policy is zero, so that the government's purchases are entirely financed by direct taxes.[11] In this sense, there is a clear separation between the roles of the central bank and the government under the optimal policy: the central bank provides an efficient means of payment while the government raises taxes to pay for its expenditures.

Central bank independence. BMW's focus on the central bank's surplus S_t is motivated by political economy considerations. In particular, they worry – correctly, in our view – that transfers from the government to the central bank may come with strings attached and thereby compromise central bank independence. We interpret their results as saying (i) if the central bank remits all of its revenue to the government each period, then it will have negative equity,

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[11]This result is not surprising given that the government has access to lump-sum taxes in this model. It is worth bearing in mind, however, that versions of the Friedman rule have been shown to be optimal in a range of models without lump-sum taxes as well; see, for example, Chari et al. (1996).

and (ii) a central bank with negative equity must either rely on transfers from the government or follow a suboptimal monetary policy. Our results show that this problem does not arise when the central bank operates through open market operations and only remits its operating surplus (or net income) to the government each period. These results may help explain why the monetary authority in a country is typically organized as a *bank*, with assets that match its liabilities, rather than as an agency that simply issues money and holds no assets. Operating as a (central) bank makes the monetary authority independent in the sense that it can implement any monetary policy – including the Friedman rule – without requiring transfers from the government.

Fiscal policy and resource costs. Our treatment of fiscal policy also differs from that in BMW in a more subtle way: we consider the public sector's purchases of goods in the initial period, ψ, to be a fixed parameter of the economy, independent of monetary considerations. One can imagine, for example, that there is a fixed-scale public works project (infrastructure, national defense, etc.) that must be undertaken in period 0. As a result, the debt needed for the central bank to follow the optimal monetary policy using open market operations already exists. In BMW, in contrast, the government's purchases in the initial period are endogenous and depend on the central bank's policy choice. In particular, when the central bank operates a floor system, the demand for real money balances is very large.[12] When the central bank issues these large money balances in period 0, BMW implicitly assume the public sector *purchases more goods* in this period. These larger initial purchases create a larger real interest burden for the government in subsequent periods. This fact leads BMW to conclude that "running a floor system is costly to the taxpayer." (p.535) Our analysis shows that the resource costs identified by BMW come not from the central bank's choice of an operational framework *per se*, but from the paper's assumptions that link public expenditure to that choice. When public expenditure in the initial period is held fixed, as we assume here, operating a floor system entails no real resource cost.

A useful comparison. To emphasize this last point, consider an alternative environment that is identical to the one we study here except that buyers can make purchases in the goods market on credit. There is no need for money or a central bank in this alternative environment; instead,

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[12] Given BMW's assumption that the distribution of the preference shock ε is unbounded from above, these purchases would actually be infinite at the optimal policy. For this reason, the proofs of their results focus on the limiting case where policy approaches the Friedman rule.

buyers credibly commit to pay for goods-market purchases in the following-period settlement market. The government still issues bonds to purchase ψ goods in the initial period and collects taxes to pay the interest on this debt each period. It is easy to see that the equilibrium real interest rate in this non-monetary model must equal β^{-1} in all periods. Using this rate, the real tax revenue the government must collect in each period to fund its initial purchase can be shown to be

$$\tau \equiv \frac{1 - \beta}{\beta} \psi. \tag{9}$$

This value of τ represents the real steady-state fiscal burden associated with the government's purchases in the initial period. Returning to our monetary model, it is straightforward to show that, under the optimal policy in Proposition 3, the real taxes collected by the government in the stationary monetary equilibrium are exactly equal to τ in (9). In other words, introducing a central bank that operates a floor system has no effect on the government's fiscal policy or on the real tax burden. This comparison makes clear that operating a floor system does not create any cost for the taxpayer.

Foregone seigniorage. Operating a floor system in this model does imply that the central bank forgoes the possibility of raising seigniorage revenue. In a corridor system, in contrast, the central bank is able to generate a positive surplus both because some banks borrow from its lending facility and because it pays a below-market interest rate at its deposit facility. This surplus is remitted to the government and thereby decreases the tax revenue the government must directly collect. Notice, however, that the total tax burden on private agents is unchanged. While taxes levied directly by the government are lower, agents are implicitly taxed on their money holdings and when they borrow from the central bank. The budget constraint of the consolidated public sector (8) ensures that total tax revenue is the same under both policy regimes. Moreover, Proposition 3 establishes that agents are strictly worse off under the corridor system because of the distortions it creates in trade. In other words, taking everything into account, a corridor system is actually *more* costly to the taxpayer than a floor system.

Scarce government debt. The fact that the central bank can implement the optimal policy through open market operations relies on Assumption 2 to ensure that the existing government debt is large enough. Given the current levels of public debt in the U.S., Europe and elsewhere,

this assumption appears to be the empirically relevant one. However, there are cases in which it may be violated. In Australia, for example, the value of all bonds issued by the central government was less than 5% of GDP as recently as 2007.[13] In such situations, the central bank may face difficult decisions about what type(s) of assets to purchase under either a corridor or a floor system. Operating a floor system may exacerbate this problem because it requires the central bank to create somewhat larger reserve balances. In the model we study here, this problem has an easy solution because there is no cost to having the government issue more debt; agents are indifferent about receiving a larger transfer in the initial period and paying correspondingly larger taxes in subsequent periods. In other environments, however, creating additional public debt could be costly. It is conceivable that if the assets available for the central bank to purchase are sufficiently unattractive, the optimal policy could be a corridor system that aims to minimize the size of the central bank's balance sheet. Understanding the precise conditions under which such a result would arise is an interesting topic for future research. We would emphasize, however, that this type of concern is very different from the notion of unpleasant fiscal arithmetic in Definition 1.

Central bank surplus in practice. Central banks have several sources of revenue, as well as some expenses, that are absent in the model studied here. The most important of these items is the net income associated with currency. Historically, the liabilities of many central banks have consisted primarily of currency in circulation, which does not earn interest. At the end of 2006, for example, Federal Reserve notes accounted for 93% of the Federal Reserve's total liabilities. In practice, the income associated with currency can be quite large. We can get a rough idea of its size by looking at the Federal Reserve's balance sheet and income at the end of 2006. The total value of Federal Reserve notes outstanding at that time was $783 billion. The Fed earned an average interest rate of approximately 4.5% on the government securities it held that year, which implies that its net income associated with these outstanding notes was approximately $35 billion. For comparison, the Federal Reserve's revenue from lending to depository institutions in 2006 was $12 million, less than one-tenth of one percent of its total revenue.[14] The income associated with currency in circulation thus provides a sizeable cushion

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[13]Source: Mid-Year Economic and Fiscal Outlook 2014-15, Commonwealth of Australia, p. 274; http://www.budget.gov.au/2014-15/content/myefo/download/MYEFO_2014-15.pdf

[14]These numbers are drawn from the 94th Annual Report of the Board of Governors of the Federal Reserve System, 2007; http://www.federalreserve.gov/boarddocs/RptCongress/annual07/pdf/AR07.pdf

that supports a central bank's financial position and independence. Importantly, this income is largely invariant to the central bank's choice of framework for implementing monetary policy; it will be present regardless of whether the central bank operates a corridor or a floor system.

4 Extensions

In this section, we present two extensions that clarify the conditions under which our results obtain. We first show how a floor system may still be optimal even if the central bank is required to generate a positive surplus in each period. We then study the general conditions on a central bank's asset holdings under which it is self-financing.

4.1 A surplus requirement

Suppose that, for some reason outside the model, the central bank must generate a strictly positive surplus in each period. Would this requirement change the optimal way of implementing monetary policy? To answer this question, we now require the central bank to satisfy

$$S_t \geq \bar{S}_t > 0 \text{ for all } t.$$

We allow the surplus requirement to change over time, but assume that its real value remains uniformly bounded,

$$\frac{\bar{S}_t}{P_t} \leq \sigma \text{ for all } t, \tag{10}$$

where σ is a positive constant. What is the optimal way of implementing monetary policy when the central bank faces this new constraint?

BMW show that when the central bank must generate income, the constrained efficient allocation in their environment is implemented by a corridor system; see their Proposition 3. Intuitively, since the central bank owns no assets and the growth rate of the money supply is fixed, the only way for it to generate additional revenue is to have some buyers borrow at the lending facility. By lowering the interest rate paid on reserves, the central bank makes holding reserves costly and thereby encourages borrowers to accumulate smaller real cash balances in the settlement market. Buyers whose realized value of the preference shock ε is large enough will then borrow from the central bank to increase their consumption; the repayment of these loans generates the desired revenue for the central bank.

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This result relies critically on the assumption that the central has no other source(s) of revenue. To demonstrate this fact, we modify the model by introducing an additional policy tool: required reserves. In particular, we allow the central bank to require that each buyer hold a minimum of C_t units of money on deposit at the central bank in period t. The central bank remunerates funds held to meet this requirement at interest rate i_c, which may be different from the deposit-facility rate i_d. If a buyer does not have enough money on hand at the end of the goods market to meet this requirement, she must borrow from the central bank's lending facility to ensure the requirement is satisfied. As long as $i_d > 0$, buyers will place any money balances above the required amount in the central bank's deposit facility at the end of the period; we now use D_t to denote these excess deposits (also called *excess reserves*).

To see how introducing required reserves changes the central bank's surplus, note that its balance sheet when the settlement market opens is now given by

	Assets		Liabilities	
Bonds	B_t^C	Money	$M_t + L_t - C_t - D_t$	
Loans	$(1+i_\ell)\,L_t$	Required reserves	$(1+i_c)\,C_t$	(11)
		Excess deposits	$(1+i_d)\,D_t$	
		Surplus	S_{t+1}	

As before, the surplus term is defined to equate the central bank's assets and liabilities,

$$S_{t+1} \equiv B_t^C + (1+i_\ell)\,L_t - (M_t + L_t - C_t - D_t) - (1+i_c)\,C_t - (1+i_d)\,D_t.$$

Using the relationship $M_t = \rho_t B_t^C$ from the initial balance sheet (4), this expression now simplifies to

$$S_{t+1} = i_t M_t + i_\ell L_t - i_c C_t - i_d D_t. \tag{12}$$

Together, (11) and (12) show how the interest costs associated with the central bank's liabilities now depend on a combination of the interest rates i_c and i_d.

Suppose the central bank follows the optimal policy from Section 3 by setting $1 + i_d = \eta/\beta$. It is straightforward to show that this policy again corresponds to a floor system, with $i_t = i_{m,t} = i_d$ for all t. Because there is no opportunity cost of holding excess deposits, buyers will again choose to exit the settlement market satiated in real money balances, which allows them to purchase the first-best level of consumption in the goods market regardless of their realized preference

shock without borrowing from the central bank. Using this fact (i.e., $L_t = 0$) and that all money balances will either be used to meet reserve requirements or held as excess deposits (i.e., $M_t = C_t + D_t$), the expression for central bank surplus in (12) simplifies to[15]

$$S_{t+1} = (i_d - i_c)C_t.$$

If the interest rate on required reserves is set below the interest rate at the deposit facility, the central bank's surplus will be positive and proportional to the size of the reserve requirement. This surplus corresponds to the net interest margin the central bank earns on assets that are funded by the money held to meet reserve requirements. Suppose, for example, that i_c is set to zero. Using $1 + i_d = \eta/\beta$, we can write the expression for equilibrium surplus as

$$S_{t+1} = \left(\frac{\eta - \beta}{\beta}\right) C_t.$$

The central bank can then meet its surplus requirement in period $t + 1$ by setting

$$C_t = \left(\frac{\beta}{\eta - \beta}\right) \bar{S}_{t+1}. \tag{13}$$

By setting reserve requirements according to (13), the central bank can generate the required level of surplus without distorting agents' spending/consumption decisions. This policy still corresponds to the Friedman rule in the sense that there is no opportunity cost of holding additional money balances at the margin. It remains to be verified that the policy can, in fact, be implemented through open market operations as required by Assumption 1. Each buyer will want to hold sufficient real money balances to satisfy the reserve requirement *and* be able to purchase the first-best quantity in the goods market even if she is excluded from the money market and receives the highest realization of the preference shock, $\varepsilon = 1$. This plan requires that the real value of the money supply in period t be at least

$$\begin{aligned}
\frac{M_t}{P_t} &\geq 1 + \left(\frac{\beta}{\eta - \beta}\right) \frac{\bar{S}_{t+1}}{P_t} \\
&= 1 + \left(\frac{\eta\beta}{\eta - \beta}\right) \frac{\bar{S}_{t+1}}{P_{t+1}},
\end{aligned}$$

where the second line uses the equilibrium inflation rate $P_{t+1}/P_t = \eta$. The assumption in (10)

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[15] The variable C_t here could also be interpreted as representing currency in circulation rather than required reserves, in which case the interest rate i_c would be zero. This equation could then be used to calculcate the net income associated with currency as discussed in the previous section.

places a uniform upper bound on this expression. It follows that the policy is indeed feasible for the central bank as long as we adjust our assumption about size of the government's purchases in period 0.

Assumption 2b $\psi \geq 1 + \bar{\sigma}\frac{\eta\beta}{\eta-\beta}$.

This modified assumption ensures that there is enough government debt in the settlement market for the central bank to meet the demand for real money balances that arises under this policy using open market operations.[16]

It is straightforward to show that the stationary equilibrium under this policy generates *exactly* the same allocation of resources as before for any value of the interest rate i_c. When required reserves are added to the model, the central bank creates more money by purchasing additional government bonds. Private agents hold more money and correspondingly fewer bonds. If i_c is set lower than the market interest rate, the central bank earns positive surplus and remits these resources to the government each period. As a result, the government collects lower direct taxes from agents, and these lower taxes exactly offset the costs associated with meeting the reserve requirement. In effect, remunerating required reserves at a below-market interest rate acts as a lump-sum tax on buyers in this model. In equilibrium, the public sector collects exactly the same amount of revenue in each period; increasing required reserves simply implies that more of this revenue accrues to the central bank and less is collected directly by the fiscal authority. We summarize this discussion in the following result.

Proposition 5 *For any bounded revenue requirement* $\left\{\overline{S}_t\right\}$, *the central bank can implement the first-best allocation under Assumptions 1 and 2b by setting* $1 + i_d = \frac{\eta}{\beta}$ *and* $i_c = 0$.

We should emphasize that we are not advocating that central banks use required reserves as a way of generating income. In practice, imposing reserve requirements and remunerating required reserves at a below-market rate may distort banks' decisions in ways that are not captured by the model here. Instead, our goal is to show that a floor system may still be optimal even if the central bank is required to generate strictly positive surplus each period, depending on the other policy tools at its disposal.

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[16] If the central bank can pay a negative interest rate on required reserves, this assumption can be weakened to require only $\psi \geq 1 + \beta\bar{\sigma}$.

4.2 Arbitrary asset holdings

BMW also consider a version of the model in which the central bank holds some asset other than government bonds, which is assumed to earn an exogenous rate of return r. Introducing a new asset complicates the analysis in general equilibrium, since the return on other assets in the economy may need to adjust, so BMW consider a partial equilibrium setting. Despite its limitations, this exercise is interesting because it allows us to think about some features of actual central bank balance sheets that are absent in the general equilibrium analysis above. For example, the returns on a central bank's assets and liabilities are typically not equal, even under a floor system. Reserves – which are risk free, perfectly liquid and have the shortest maturity – typically have a lower yield than the types of assets a central bank holds. In this subsection, we apply our accounting to this modified version of the model and ask under what conditions the central bank's surplus will be negative. To keep the analysis close to that in BMW, we return to the setting with no reserve requirements.

Suppose the central bank starts with K units of the other asset and M units of money outstanding. To simplify matters, we assume the money supply is constant over time and there are no open market operations. The central bank's equity $E \equiv K - M$ is held constant over time and may be either positive or negative. Its balance sheet at the end of the settlement market is

Assets		Liabilities	
Other asset	K	Money	M
		Equity	E

(14)

During the goods market, the central bank operates its standing facilities as described above. Its balance sheet at the opening of the settlement market in the next period is then

Assets		Liabilities	
Other asset	$(1+r)K$	Money	$M + L_t - D_t$
Loans	$(1+i_\ell)L_t$	Deposits	$(1+i_d)D_t$
		Equity	E
		Surplus	S_{t+1}

The surplus term S_{t+1} is defined to equate the central bank's assets and liabilities, holding equity

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E constant across periods,

$$S_{t+1} = (1+r)\,K + (1+i_\ell)\,L_t - (M + L_t - D_t) - (1+i_d)\,D_t - E.$$

Using the balance sheet identity from (14), this expression can be simplified to

$$S_{t+1} = rK + i_\ell L_t - i_d D_t.$$

Notice the similarity between this simplified expression and (7). The evolution of the central bank's balance sheet – and the surplus it generates – is the same as before; the only difference is the source of its asset revenue. If the central bank operates a floor system as discussed above, buyers do not use the central bank's lending facility and $L_t = 0$. Using the assumption in this section that $\eta = 1$ and the fact that all money is deposited at the end of the goods market (i.e., $D_t = M$), we can further reduce the expression for the central bank's surplus to

$$S_t = rK - i_d M.$$

The following result follows immediately from this expression.

Proposition 6 *Suppose the central bank holds an asset with arbitrary return r. If (i) $E \geq 0$ and (ii) $r \geq i_d$, then $S_t \geq 0$ for all t.*

The first condition simply requires that the value of the assets owned by the central bank be at least as large as the outstanding money supply $(K \geq M)$. The second condition requires that the rate of return the central bank earns on its assets is at least as large as the rate it pays at is deposit facility. If, for example, the central bank holds longer-term government bonds, this condition reduces to the requirement that the yield curve not be downward sloping. These conditions are not necessary, of course. A central bank with negative equity may nevertheless generate positive surplus if the per-unit return on its assets is sufficiently large compared to the interest it pays on reserves. Similarly, a central bank that earns less per unit of assets than the interest it pays on reserves could nonetheless have positive surplus if its equity is large enough.

BMW provide a condition under which the central bank's surplus is nonnegative with arbitrary asset holdings; see their Proposition 4. However, this condition is difficult to interpret, leading them to conclude that "having a significant holding of real assets may still not be sufficient to avoid having to receive fiscal transfers under the optimal floor system." (p. 536)

Interpreting the model and its results is easier under our approach. In particular, Proposition 6 shows that unpleasant fiscal arithmetic only arises if the central bank has negative equity and/or earns a below-market rate of return on its assets.

5   Concluding remarks

We have shown how a central bank that supplies money through open-market operations using one-period government bonds is always self-financing. In particular, it can use a floor system to implement the optimal monetary policy without requiring transfers from the government in any period. Moreover, we have shown that this policy has no adverse fiscal consequences for the consolidated public sector or for taxpayers. These results suggest that the choice between a corridor and a floor system should be made based on effectiveness of each approach in implementing monetary policy, rather than any concern about fiscal implications. They may also help explain why countries tend to organize their monetary authority as a bank, that is, a corporation with assets that match liabilities, rather than as an agency that simply issues money. This form of organization can cleanly disentangle the direct fiscal effects of monetary policy from the fiscal policies of the consolidated government sector and, in doing so, may help preserve the independence of monetary policy.

We have assumed throughout our analysis that the central bank buys short-term government bonds in order to separate the effects of the central bank's choice of operational framework from the effects of its choice of asset holdings. In practice, central banks often hold longer-term government bonds and sometimes hold private assets. While holding other assets will tend to raise a central bank's net interest revenue on average, it also exposes the central bank to interest rate and/or credit risk. When it is exposed to such risks, a central bank may have negative income in some periods and states of nature. For example, a central bank holding primarily long-term government bonds may have negative income if interest rates rise rapidly and it pays interest on reserves. Importantly, these episodes can occur under either a corridor or a floor system, although the larger balance sheet associated with a floor system may expose the central bank to larger risks.

The point we want to emphasize in this paper, however, is that the question of what assets the central bank purchases is logically separate from the choice of how it implements monetary

policy. Our analysis above has shown how a central bank can use a floor system without taking any interest rate or credit risk and can thereby insure that its net income is never negative. In practice, a central bank may choose to hold other assets for a variety of reasons. In comparing the floor and corridor systems, however, it can always choose to hold the *additional* assets associated with a floor system in short-term government bonds. In doing so, the central bank can realize the efficiencies associated with this implementation of the Friedman rule without changing the level of interest rate and/or credit risk it faces.

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