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Abstract

This paper explores the role of capital flows and exchange rate dynamics in shaping the global economy's adjustment in a liquidity trap. Using a multi-country model with nominal rigidities, we shed light on the global adjustment since the Great Recession, a period when many advanced economies were pushed to the zero bound on interest rates. We establish three main results. First, when the North hits the zero bound, downstream capital flows alleviate the recession by reallocating demand to the South and switching expenditure toward North goods. Second, a free capital flow regime falls short of supporting efficient demand and expenditure reallocations and induces too little downstream (upstream) flows during (after) the liquidity trap. And third, when it comes to capital flow management, individual countries' incentives to manage their terms of trade conflict with aggregate demand stabilization and global efficiency. This underscores the importance of international policy coordination in liquidity trap episodes.

Key words: capital flows, international spillovers, liquidity traps, uncovered interest parity, capital flow management, policy coordination, optimal monetary policy

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1 Introduction

Following the 2007-2008 financial crisis, the world economy experienced a recession that originated in the United States before spreading to other countries. Central banks responded by engaging in expansionary monetary policy, and interest rates were slashed most dramatically in some major advanced economies, where they hit the zero bound. The policy response was more subdued in emerging economies, which were generally less affected by the financial crisis. The resulting interest rate differential between advanced and emerging economies, however, was associated with a surge in capital inflows and a currency appreciation pressure in the latter. Fearing an erosion of external competitiveness, policy makers in some emerging market countries (most notably India and Brazil) adopted measures to slow down capital inflows. Meanwhile, the aggressive response of advanced economies' monetary authorities generated a heated debate about international spillovers and the need for policy coordination at the zero lower bound.[1] A key theme of this debate concerned the desirability of capital flows and associated terms of trade movements. What role do capital flows play in the global macroeconomic adjustment when the world economy is subject to large asymmetric shocks? Should free capital flows be expected to fulfill this role efficiently? Is the zero bound on interest rates critical in this regard? Our goal is to make progress on these issues.

To this end, we use a stylized non-linear multi-country version of the open economy New Keyensian model of Gali and Monacelli (2005). We assume flexible exchange rates, divide the world economy into two blocks (North and South), and model a liquidity trap as the consequence of a large, unanticipated, negative demand shock.[2] Similar to Werning (2012)'s closed economy analysis, we adopt a deterministic continuous time formulation, which affords us analytical tractability.

We start by analyzing the optimal monetary policy response of individual countries to negative demand shocks in the presence of an explicit zero lower bound (ZLB) constraint on the interest rate. Large enough shocks make the ZLB bind, and consistent with earlier work on closed economies, we find that it is optimal to prolong the period at which the interest rate is kept at zero (see Eggertsson and Woodford 2003 and Werning 2012). Interestingly, openness adds both stabilizing

[1] This debate is commonly associated with statements by Reserve Bank of India's President Raghuram Rajan and Brazil's Finance Minister Guido Mantega. Rajan has repeatedly asked advanced economies to be mindful of international spillovers emanating from their policy decisions and asked for more international policy coordination (see, for instance, Rajan 2010 and Rajan 2014). Mantega is reported to have described the climate as that of an "international currency war" (see Financial Times of September 27th, 2010).

[2] A liquidity trap is defined as a situation where negative interest rates are needed to obtain the socially optimal allocation in a part of the world economy. We define the concept more precisely in Section 3.

and destabilizing forces into the picture. On the one hand, openness reduces the size of the interest rate adjustment required to stabilize domestic demand (thus offering an extra cushion toward the ZLB) in the face of home shocks. On the other hand, it makes domestic conditions vulnerable to foreign conditions (thus creating possibilities of a binding ZLB as a result of adverse shocks abroad). These forces, we argue, crucially depend on the ease with which capital can flow across countries. Under free capital flows, an adverse demand shock in one region can pull the entire world into a liquidity trap. Conversely, even with high trade integration, closed capital accounts shield the rest of the world at the expense of exacerbating the liquidity trap experience for the affected region.

We then analyze the role played by capital flows and exchange rate movements in facilitating the global economy's adjustment in a liquidity trap. Despite our model being able to generate liquidity traps that engulf the entire globe, we focus on a situation in which only a region of the world economy (i.e. the North) experiences one. We restrict our attention to this scenario because we think that it best captures the North-South divide during the Great Recession, when a large share of advanced economies hit the ZLB, but virtually no emerging economies did. We characterize the world equilibrium under the assumption that all countries conduct monetary policy optimally, and show how the degree of capital mobility critically influences the smoothness of the global macroeconomic adjustment.

Under free capital flows, the North's temporary desire to save during the liquidity trap is accommodated by an accumulation of claims vis-à-vis the South, who temporarily enjoy cheaper consumption. Capital, thus, flows downstream during the liquidity trap. These flows are accompanied by an exchange rate adjustment on account of the persistent interest rate differentials between the South and the North. North (South) currencies depreciate (appreciate) on impact, and then continuously appreciate (depreciate) during the time spent by the North at the ZLB. The terms of trade path induced by these exchange rate movements help alleviate the North recession by reallocating expenditure toward North goods in the initial stage of the liquidity trap, at the time when demand is most deficient. A comparison of the global dynamics under free capital flows and closed capital accounts reveals that the stabilizing effect of openness onto the North's adjustment has to do more with inter-temporal trade than with intra-temporal trade. Under closed capital accounts, despite intra-temporal trade, the North is unable to save by running a current account surplus, and the South does not experience a consumption boom. South currencies depreciate rather than appreciate on impact, which fails to promote expenditure switching in favor of North goods. As a result, the North experiences a more severe recession-boom cycle and optimally exits the ZLB later than under free capital flows. Greater capital mobility thus allows for more significant terms of trade adjustments and reduces the severity of the demand driven recession in the North.

Having established the positive result that capital flows promote a smoother adjustment in countries experiencing a liquidity trap, we ask whether a regime of free capital flows *efficiently* fulfills this stabilizing role. To this end, we formulate a planning problem in which a global planner chooses a path of taxes or subsidies on downstream capital flows to maximize world welfare, subject to monetary policy being set optimally. The taxes/subsidies allow the planner to restrict or encourage capital flows relative to the free capital flow case, so that this planning problem enables us to analyze whether/when capital flows too much or too little from a global efficiency standpoint. We find that while away from the zero bound, free capital flows are constrained efficient, they are constrained *inefficient* when a region of the world economy faces a binding ZLB. The inefficiency of decentralized international borrowing/savings decisions can be traced back to an aggregate demand externality resulting from the combination of output being demand determined and monetary policy being constrained by the zero bound in some countries. Agents do not internalize that their savings decisions have effects on both inter- and intra-temporal prices. In conjunction with nominal rigidities, these decisions affect the level of economic activity, both at home and abroad. Away from the ZLB, monetary authorities are able to address the externality. However, at the ZLB their ability to do so is limited. In such a situation, capital flow management can be used to complement monetary policy.

Perhaps surprisingly, the efficient capital flow regime entails larger downstream (upstream) capital flows during (after) the liquidity trap than the free capital flows benchmark. A binding ZLB translates into an overly appreciated North exchange rate in the early stage of the liquidity trap. Capital flow taxes allow exchange rate dynamics to decouple from interest rate dynamics, and thereby relax the ZLB constraint in the North without inflicting much harm to the South. In particular, they can implement a steeper (flatter) exchange rate path during (after) the liquidity trap. This extra tilting of the terms of trade path shifts expenditure toward (away from) North goods precisely when the demand for these goods are low (high). It also shifts expenditure away from and toward South goods, but these effects are offset by South monetary policy, which is not constrained by the ZLB. In analogy to the manner that delaying ZLB exit in a closed economy allows borrowing monetary policy room from the future, globally efficient capital flow management can be interpreted as enabling a transfer of monetary policy room across regions.

At first glance, our finding that capital does not flow sufficiently in a liquidity trap seemingly stand in sharp contrast to a recent literature on capital flow management that argues that free capital flows might instead be excessively volatile (see our literature review below). This literature, however, studies capital flow management from the perspective of individual capital flow recipient countries, whereas we take a global efficiency standpoint. To show that this distinction is crucial, we also consider an alternative setting where countries manage capital flows non-cooperatively. In this case, we show that the incentives of individual countries to manage capital flows respond to a desire to manage dynamic terms of trade.[3] In particular, from the point of view of South countries, it is individually optimal to slow down rather than speed up capital flows during the liquidity trap. In contrast, North countries experiencing a binding ZLB have an incentive to subsidize capital outflows to stabilize domestic aggregate demand. Consequently, capital flow management measures that are optimal from an inflow recipient perspective conflict with macroeconomic stabilization in countries with deficient aggregate demand during a liquidity trap. A Nash equilibrium where only South countries manage their capital account features a positive tax on downstream flows, which slows down (but neither shut down nor reverse) capital flows. One where all countries manage their capital account features a form of currency war, with subsidies to outflows by the North and taxes on inflows by the South nearly neutralizing each other. Our analysis thus points to the adverse effects of uncoordinated capital controls, in particular in liquidity trap episodes, and highlights the importance of global policy coordination in this area.

The rest of the paper is organized as follows. We conclude the introduction with a review of related literature. We then describe the model in Section 2. Section 3 highlights the role of capital flows at the zero bound, Section 4 analyzes capital flow efficiency, and Section 5 studies non-cooperative capital flow management. Section 6 concludes.

Related literature The paper relates to a large literature on optimal policy at the ZLB that developed following the seminal work of Krugman (1998) and Eggertsson and Woodford (2003). Our continuous time formulation of the optimal monetary policy problem is most closely related to Werning (2012)'s work in the closed economy context. Consistent with these papers, we find that optimal monetary policy calls for an extension of the time spent at the ZLB past the liquidity trap episode. Our work relates in particular to the literature on optimal monetary policy at the ZLB in the open economy. Svensson (2001, 2003, 2004) argues that a foolproof way of escaping a liquidity trap in a small open economy is by devaluing the currency, and temporarily adopting a peg and a price-level target. In the context of a two country model, Jeanne (2009) argues that a temporary increase in both countries' inflation targets may restore the first-best. In a similar framework,

[3]This motive arises inevery open economy model where countries have some degree of market power over a good they trade. The motive for managing the capital account applies to capital exporters and importers alike, and prevails independently from zero lower bound considerations.

Fujiwara et al. (2013) show that the optimal rate of inflation in one country is affected by whether or not the other country is in a liquidity trap. Haberis and Lipinska (2012) find that more stimulatory foreign policy worsens the home policymakers' trade-off between stabilizing inflation and the output gap when home and foreign goods are close substitutes. Cook and Devereux (2013) find that in a liquidity trap, the terms-of-trade may respond perversely and make it optimal for a partner country to raise interest rate even though its natural rate is below zero. In a closely related paper, Devereux and Yetman (2014) conclude that if monetary policy is set cooperatively then the use of capital controls at the ZLB is always welfare reducing. Overall, our contribution to the literature on optimal policy at the ZLB is twofold. First, we provide an analytical characterization of optimal monetary policy in the open economy, notably through a comparison of the optimal ZLB exit time across capital flow regimes. Second, we consider capital flow taxes/subsidies as an additional tool to overcome the limitations of monetary policy at the ZLB, and solve for optimal cooperative and non-cooperative capital flow management regimes in the world economy.[4]

The paper also connects to a large literature on capital flows and the regulation thereof in emerging markets. Calvo et al. (1993, 1996) argue that capital flows cycles to emerging market throughout the 1980s and 1990s were mainly associated with external factors, such as U.S. interest rates. This pattern seems to have prevailed again during the Great Recession. Our modeling of a capital flow cycle being the outcome of a negative demand shock in advanced economies is consistent with this association. Several recent papers have developed arguments in favor of capital account interventions based on imperfections in financial markets (e.g. Caballero and Krishnamurthy 2001, Korinek 2007, 2010, Jeanne and Korinek 2010, Bianchi 2011).[5] Others have shown that imperfections on goods markets may also provide a rationale for the optimal use of capital controls. De Paoli and Lipinska (2012) and Costinot et al. (2014) emphasize the role of market power and dynamic terms-of-trade management. Farhi and Werning (2012a, 2014) and Schmidth-Grohe and Uribe (forthcoming) stress the role of nominal rigidities. All these papers study optimal capital flow management from the perspective of individual countries. In contrast, we stress the benefits of capital flow taxes/subsidies in promoting efficiency at the level of the world economy. In questioning the efficiency property of a free capital flow regime, our approach is closer in spirit to that of Heathcote and Perri (2014) and Brunnermeier and Sannikov (2015). In models featuring financial market imperfections, they show that for some (extreme)

[4] Korinek (2014) (section 5.2) also analyzes the use of capital flow taxes at the ZLB in a small open economy that may fall into a liquidity trap as a result of a drop in the exogenous world interest rate.

[5] Gabaix and Maggiori (Forthcoming) also show that in the presence of financial frictions, capital controls can increase the potency of currency market interventions as a tool to combat exchange rate movements generated by financial turmoil.

parameterizations, shutting down capital accounts globally can be Pareto improving. Instead, we work with a model featuring imperfections on goods markets and question capital flow efficiency by solving for an entire time path of optimal capital flow taxes/subsidies along the transitional dynamics of a liquidity trap.

The inefficiency of free capital flows in our model results from aggregate demand externalities, a notion first discussed in Blanchard and Kiyotaki (1987) in the context of agents' price and wage setting decisions in economies where output depends on aggregate demand and is not at its first-best level. Our paper is closer to recent work on aggregate demand externalities by Farhi and Werning (2012a,b, 2013), Korinek and Simsek (2014) and Schmidth-Grohe and Uribe (forthcoming) that instead emphasizes the role of agents' debt choices. Farhi and Werning (2013) develop a general theory of aggregate demand externalities in economies with nominal rigidities and constraints on monetary policy, of which Farhi and Werning (2012a,b), Korinek and Simsek (2014) and our paper can be seen as applications, pertaining to, respectively, Mundell's trilemma, fiscal unions, macro-prudential policy ahead of a liquidity trap, and stimulative capital flow management during a global liquidity trap. In that regard, a distinguishing feature of our work is to emphasize the beneficial aspect of distorting static and dynamic relative prices in ways that shift expenditure toward (away from) goods that are relatively more (less) under-provided, independently from cross-sectional heterogeneity in consumers' marginal propensity to consume.

In parallel and independent work, Caballero et al. (2015) analyze issues similar to the ones we study in this paper. Like us, they find that liquidity traps can get transmitted across countries through capital flows. However, while they focus on a scenario where the entire world is engulfed in a liquidity trap, our analysis is mostly concerned with a case where a liquidity trap only affects a large region of the world economy. Further, they focus on a situation where a global liquidity trap is permanent and results in *secular stagnation*, while we analyze the short-run adjustment in a temporary liquidity trap. In light of the recent rate hike in the US, we feel that our model might be better positioned to understand global adjustment in the current scenario. Despite these differences, the two papers share common ideas. Caballero et al. (2015) argue that when the entire world is in a liquidity trap, countries privately have an incentive to use policy to manipulate exchange rates so as to gain a competitive advantage. This scenario is similar to our analysis of currency wars in Section 5.

2 Model

The world economy consists of a unit mass of countries, separated into two blocks. North economies consists of the countries for which $k \in [0, x]$ and South economies consists of the countries for which $k \in (x, 1]$.[6] Following a large body of literature, we adopt a parametrization featuring unitary inter- and intra-temporal elasticities of substitution. As is well known, this parametrization, popularized by Cole and Obstfeld (1991), results in economies being insular with respect to foreign monetary policy. As a result, it enables us to streamline cross-border spillovers arising from demand shocks and capital flow management policies.[7] We elaborate on these issues at the end of the Section.

2.1 Households

In each country k (we will refer to country k as the 'home' country for ease of exposition), there is a representative household with preferences represented by the utility functional

$$\int_0^\infty e^{-\int_0^t (\rho + \zeta_{k,h}) dh} \left[\log \mathbb{C}_{k,t} - \frac{(N_{k,t})^{1+\phi}}{1+\phi} \right] dt, \tag{1}$$

where $\mathbb{C}_{k,t}$ is consumption, $N_{k,t}$ is labor supply, ϕ is the inverse Frisch elasticity of labor supply, ρ is the (long run) discount rate and $\zeta_{k,t}$ is a time-varying and country specific preference shifter. We will refer to a negative realization of ζ_k as a *negative demand shock*, as such a shock lowers the demand for current consumption relative to future consumption (and hence increases the desire to save). The consumption index $\mathbb{C}_{k,t}$ is defined as

$$\mathbb{C}_{k,t} \equiv (1-\alpha)^{1-\alpha} \alpha^{\alpha} \left(C_{k,t}^H \right)^{1-\alpha} \left(C_{k,t}^F \right)^{\alpha} \tag{2}$$

where C_k^H denotes an index of domestically produced varieties, C_k^F is an index of imported goods and α is a home bias parameter representing the degree of openness. Letting $l \in [0, 1]$ index varieties, we define $C_k^H \equiv \left[\int_0^1 C_k^H(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$, where $C_k^H(l)$ denotes country k's consumption of variety l produced domestically, and $\epsilon > 1$ is the elasticity of substitution between varieties produced

[6] We think of the North block as representing *demand deficient* economies, and of the South as standing for the rest of the world.

[7] The Cole-Obstfeld parametrization also has the advantage of decisively improving the tractability of the non-linear model and has been used extensively in the open economy literature. It is known that under this parametrization, the model would have a log-linear structure absent discount rate shocks and capital flow taxes. With these features, the model does not have an exact log-linear structure but it remains analytically tractable.

within a given country. Similarly, we define $C_k^F \equiv \exp\left(\int_0^1 \log C_k^j dj\right)$ and $C_k^j \equiv \left[\int_0^1 C_k^j(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$, where C_k^j (resp. $C_k^j(l)$) denotes country k's consumption of the final good (resp. variety l) produced in country j.

The household's budget constraint is given by

$$\begin{aligned} \dot{a}_{k,t} &= i_{k,t}a_{k,t} + W_{k,t}N_{k,t} + T_{k,t} - \int_0^1 P_{k,t}^k(l)C_k^H(l)dl - \int_0^1 \int_0^1 P_k^j(l)C_k^j(l)dldj \\ &\quad + \int_0^1 \left[i_{j,t} - i_{k,t} + \tau_{k,t} - \tau_{j,t} + \frac{\dot{\mathcal{E}}_{k,t}^j}{\mathcal{E}_{k,t}^j}\right] \mathcal{E}_{k,t}^j D_{k,t}^j dj \end{aligned} \tag{3}$$

where $a_{k,t} \equiv \int_0^1 \mathcal{E}_{k,t}^j D_{k,t}^j dj$ are net assets expressed in country k's own currency, $\mathcal{E}_{k,t}^j$ is the nominal exchange rate between country j and k, $D_{k,t+1}^j$ are the bonds issued by country j and held by country k at time t, $W_{k,t}$ is the nominal wage and $T_{k,t}$ denotes lump-sum transfers including the payout of domestic firms. We explicitly allow for taxes and subsidies on capital flows. $\tau_{k,t}$ is a tax on capital inflows (or a subsidy on capital outflows) in country k, and similarly $\tau_{j,t}$ is a tax on capital inflows (or a subsidy on capital outflows) in country j. The proceeds of these taxes are rebated lump sum to the households of country k and j, respectively.

The lump-sum rebate $T_{k,t}$ is given in equilibrium by

$$T_{k,t} = -\tau_{k,t} \int_0^1 \mathcal{E}_{k,t}^j D_{k,t}^j dj + \Pi_{k,t}, \tag{4}$$

where $\Pi_{k,t}$ is firm profits.

2.2 Firms

Technology A firm in each economy k produces a differentiated good $l \in [0,1]$ with a linear technology: $Y_{k,t}(l) = AN_{k,t}(l)$. For simplicity, we assume that labor productivity A is constant and identical across countries.

Price setting We assume that the price of each variety is fully rigid, and normalize this price to 1. An implication of this assumption is that the producer price index (PPI) of a country in its own currency is fixed at 1.

The assumption of fully rigid prices can be regarded as an extreme one, but it has the virtue of significantly improving the analytical tractability of the model.[8] This assumption rules out

[8]It also allows for an a simple exact non-linear solution of the path of all relevant variables during and after a

PPI inflation or deflation, so our analysis abstracts from the distortion caused by price dispersion typically present in sticky price environments à la Calvo and instead solely focuses labor wedge distortions. At the same time, our rigid prices assumption does not eliminate the deflation-recession feedback loop that is a key characteristic of liquidity trap episodes, since the relevant measure for that mechanism is CPI inflation rather than PPI inflation, and CPI inflation does respond to nominal exchange rate fluctuations.

2.3 Terms of Trade, Exchange Rates and UIP

Expenditure minimization leads to the home country's consumer price index (CPI) definition

$$\mathbb{P}_k \equiv (P_k^H)^{1-\alpha}(P_k^F)^{\alpha}, \tag{5}$$

where $P_k^H \equiv \left[\int_0^1 P_k^H(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ is the home country's PPI, $P_k^F \equiv \exp\left(\int_0^1 \ln P_k^j dj\right)$ is the price index of imported goods , and $P_k^j \equiv \left[\int_0^1 P_k^j(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ is country j's PPI, $P_k^H(l)$ (resp. $P_k^j(l)$) denotes the price of variety l produced in the home country (resp. in country j).[9] A k subscript indicates a price or price index expressed in country k's currency.

$\mathcal{E}_k^j$ is the nominal exchange rate between country k and country j.[10] The law of one price (LOP) implies $P_k^j(l) = \mathcal{E}_k^j P_j^j(l)$. At the level of country j's final good, it implies $P_k^j = \mathcal{E}_k^j P_j^j$. Therefore, the price index of imported goods satisfies $P_k^F = \exp\left[\int_0^1 \ln\left(\mathcal{E}_k^j P_j^j\right) dj\right] = \mathcal{E}_k P^*$, for a world price index $P^* \equiv \exp\left(\int_0^1 \ln P_j^j dj\right)$ and a home country's effective nominal exchange $\mathcal{E}_k \equiv \exp\left(\int_0^1 \ln \mathcal{E}_k^j dj\right)$.

Using the above definitions, the household's budget constraint (3) can be expressed as

$$\dot{a}_{k,t} = i_{k,t}a_{k,t} + W_{k,t}N_{k,t} + T_{k,t} - \mathbb{P}_{k,t}\mathbb{C}_{k,t} + \int_0^1 \left[i_{j,t} - i_{k,t} + \tau_{k,t} - \tau_{j,t} + \frac{\dot{\mathcal{E}}_{k,t}^j}{\mathcal{E}_{k,t}^j}\right] \mathcal{E}_{k,t}^j D_{k,t}^j dj \tag{6}$$

The bilateral terms of trade between country k and country j are defined as the relative price of country j's good in terms of country k's good, $\mathcal{S}_k^j \equiv \frac{P_k^j}{P_k^k}$. The effective terms of trade of country

liquidity trap scenario, for all the policy regimes we consider. Such a non-linear solution is attractive, given the focus on large shocks and the well known unit-root property of the net foreign asset position of countries in deterministic open-economy settings.

[9] Note that $P_k^H \equiv P_k^k$.

[10] An increase in $\mathcal{E}_{k,t}^j$ is a depreciation of country k's currency and an appreciation of country j's currency ($\mathcal{E}_{k,t}^j = 1/\mathcal{E}_{j,t}^k$).

k are defined as $\mathcal{S}_k \equiv \frac{P_k^F}{P_k^k} = \exp\left(\int_0^1 \ln \mathcal{S}_k^j dj\right)$. The bilateral real exchange rate between country k and country j is further defined as the ratio of the two countries' CPIs $\mathcal{Q}_k^j \equiv \frac{\mathcal{E}_k^j \mathbb{P}_j}{\mathbb{P}_k}$, and the effective real exchange rate of country k is defined as $\mathcal{Q}_k \equiv \frac{P_k^F}{\mathbb{P}_k} = \frac{\mathcal{E}_k P^*}{\mathbb{P}_k}$.

2.4 Equilibrium conditions with symmetric North and South blocks

We now present equilibrium conditions from the perspective of a home country k in a case of symmetric North and South blocks. Equilibrium conditions comprise a set of bilateral Backus-Smith conditions, a goods market clearing condition, three equations relating bilateral and effective terms of trade and real exchange rates, a labor market clearing condition, a domestic bond Euler equation, a set of UIP conditions, and the country's resource constraint.

The Backus-Smith condition between country k and a representative North country is given by

$$\mathbb{C}_{k,t} = \Theta_{k,t}^n \mathbb{C}_{n,t} \mathcal{Q}_k^n \tag{7}$$

where $\Theta_{k,t}^n$ is a relative weight of country k with respect to the North country, whose path satisfies[11]

$$\Theta_{k,t}^n \equiv \Theta_{k,0}^n \exp\left[\int_0^t \left(\tau_{k,h} - \tau_{n,h} - \zeta_{k,h} + \zeta_{n,h}\right) dh\right]. \tag{8}$$

The relative weight $\Theta_{k,t}^n$ summarizes the dynamics of the distribution of wealth between country k and a representative North country, and its growth rate is proportional to the gaps between relative instantaneous discount rates and capital inflow taxes across countries.[12,13]

The market clearing condition for output of country k, defined as $Y_{k,t} \equiv \left[\int_0^1 Y_{k,t}(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$ is given by

$$Y_{k,t} = (1-\alpha)\left(\frac{\mathcal{Q}_{k,t}}{\mathcal{S}_{k,t}}\right)^{-1} \mathbb{C}_{k,t} + \alpha x \left(\mathcal{S}_{n,t}\mathcal{S}_{k,t}^n\right) \mathcal{Q}_{n,t}^{-1}\mathbb{C}_{n,t} + \alpha(1-x)\left(\mathcal{S}_{s,t}\mathcal{S}_{k,t}^s\right) \mathcal{Q}_{s,t}^{-1}\mathbb{C}_{s,t}, \tag{9}$$

[11] This weight is sometimes referred to as a Pareto weight, as it corresponds to the relative weight set on country k in a hypothetical planning problem.

[12] For equal capital flow taxes, if the home country is less patient than a North country at a given instant (i.e., $\zeta_{k,t} > \zeta_{n,t}$), its relative weight $\Theta_{k,t}^n$ declines. Similarly, for equal discount rates, if the home country has a higher capital inflow tax than a North country at a given instant (i.e., $\tau_{k,t} > \tau_{n,t}$), its relative weight $\Theta_{k,t}^n$ rises. In the first case, the home country is accumulating assets vis-à-vis a North country. In the second case, it is running down assets vis-à-vis a North country.

[13] A similar bilateral condition holds between country k and a South country, but this condition can alternatively be derived from (7) and the Backus-Smith condition between a North country and a South country.

where the three terms making up demand for the country k good represent domestic demand, foreign demand from North countries, and foreign demand from South countries, respectively.

The effective and bilateral terms of trade are related through $\mathcal{S}_{k,t} = \left(\mathcal{S}^n_{k,t}\right)^x \left(\mathcal{S}^s_{k,t}\right)^{1-x}$, the bilateral real exchange rate is related to the effective and bilateral terms of trade through $\mathcal{Q}^j_{k,t} = \left(\mathcal{S}^j_{k,t}\right)^{1-\alpha}$ for $j = \{n, s\}$, and the effective real exchange rates is related to the effective terms of trade through $\mathcal{Q}_k = \mathcal{S}_k^{1-\alpha}$. The labor market clearing condition is given by $N_{k,t} = \frac{Y_{k,t}}{A}$, and the Euler equation for the domestic bond by

$$\frac{\dot{\mathbb{C}}_{k,t}}{\mathbb{C}_{k,t}} = i_{k,t} - \pi_{k,t} - (\rho + \zeta_{k,t}), \tag{10}$$

where $\pi_k \equiv \frac{\dot{\mathbb{P}}_k}{\mathbb{P}_k} = \pi^H_k + \frac{\dot{\mathcal{S}}_k}{\mathcal{S}_k} - \frac{\dot{\mathcal{Q}}_k}{\mathcal{Q}_k}$ is CPI inflation. With fully rigid prices, producer prices are fixed at their at their $t = 0$ values in own currency terms, and as a result PPI inflation is always zero: $\pi^H_{k,t} = 0$.[14]

The interest parity condition between the home bond and a North country bond is given by[15]

$$i_{k,t} - \tau_{k,t} = i_{n,t} - \tau_{n,t} + \frac{\dot{\mathcal{E}}^n_k}{\mathcal{E}^n_k}. \tag{11}$$

Finally, country k's budget constraint is

$$\dot{B}_{k,t} = (\rho + \zeta_{n,t} - \tau_{n,t})\, B_{k,t} + \mathbb{C}^{-1}_{n,t} \left(\mathcal{Q}^n_{k,t}\right)^{-1} \left[(\mathcal{S}_{k,t})^{-\alpha} Y_{k,t} - \mathbb{C}_{k,t}\right] \tag{12}$$

where $B_{k,t} \equiv \frac{\mathbb{C}^{-1}_{n,t} a_{k,t}}{\mathcal{E}^n_{k,t}\mathbb{P}_{n,t}} = \frac{\mathbb{C}^{-1}_{n,t} \int_0^1 \mathcal{E}^j_{k,t} D^j_{k,t} dj}{\mathcal{E}^n_{k,t}\mathbb{P}_{n,t}}$ is a country's net foreign assets at t measured in terms of a North country's CPI $\mathbb{P}_{n,t}$ and normalized by a North country's marginal utility of consumption $\mathbb{C}^{-1}_{n,t}$. Imposing a No-Ponzi game condition, this budget constraint can be written in present value form as

$$B_{k,0} = -\int_0^\infty e^{-\int_0^t [\rho + \zeta_{n,s} - \tau_{n,s}]ds} \mathbb{C}^{-1}_{n,t} \left(\mathcal{Q}^n_{k,t}\right)^{-1} \left[(\mathcal{S}_{k,t})^{-\alpha} Y_{k,t} - \mathbb{C}_{k,t}\right] dt \tag{13}$$

Given the Cole-Obstfeld parametrization, the relative weight $\Theta^n_{k,t}$ in the Backus-Smith condition (7) has the interpretation of a ratio of expenditures: $\Theta^n_{k,t} = (\mathbb{P}_{k,t}\mathbb{C}_{k,t})/(\mathcal{E}^j_{k,t}\mathbb{P}_{j,t}\mathbb{C}_{n,t})$.[16] The goods market clearing condition can be written as to

$$Y_{k,t} = \left[(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}\right] \left(\mathcal{S}^s_{n,t}\right)^{\alpha(1-x)} \mathcal{S}^n_{k,t}\mathbb{C}_{n,t}, \tag{14}$$

[14] The price of all varieties produced in country k in their own currency is normalized to 1 at $t = 0$.

[15] A similar bilateral condition holds between country k and a South country, but this condition can alternatively be derived from (11) and the interest parity condition between a North country and a South country.

[16] Note that $\left(\mathcal{S}^n_{k,t}\right)^{1-\alpha} = \mathcal{Q}^n_{k,t} = \mathcal{E}^j_{k,t}\mathbb{P}_{j,t}/\mathbb{P}_{k,t}$.

and the home country's lifetime budget constraint as

$$B_{k,0} = \alpha \int_0^\infty e^{-\int_0^t (\rho+\zeta_{n,s}-\tau_{n,s})ds} \left[\Theta^n_{k,t} - x - (1-x)\Theta^n_{s,t}\right] dt. \tag{15}$$

(7), $\mathcal{Q}^j_{k,t} = \left(\mathcal{S}^j_{k,t}\right)^{1-\alpha}$ and (14), together with their counterparts for representative North and South countries, can be combined in a way that greatly simplifies the structure of the optimal policy problems we consider in the next sections. This is summarized in the following lemma.

Lemma 1 (Implementability constraints)**.** *An implementable allocation for a country k satisfies the consumption-output relationship*

$$\mathbb{C}_{k,t} = \Theta^n_{k,t} \left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right)^{1-\alpha} \left(\frac{Y_{n,t}}{\Lambda_{n,t}}\right)^{\alpha x} \left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{\alpha(1-x)}, \tag{16}$$

and the dynamic IS equation

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho + \zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha (1-x)\, \Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}}, \tag{17}$$

for $\Lambda_{k,t} \equiv (1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}$*, and* $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$.

Proof. See Appendix A.1. □

Lemma 1 describes constraints on implementable allocations. (16) implies that home consumption is a geometric average of appropriately normalized home and foreign output levels (adjusted by the expenditure ratio $\Theta^n_{k,t}$), while (17) is a non-linear dynamic New-Keynesian IS curve that relates output growth to the nominal interest rate, the discount rate and the growth of relative expenditure ratios.[17]

The dynamic IS curve (17) is one of the model's key equations and contains important information about the international spillovers at work in the model. Crucially, it reveals that domestic output is independent of foreign monetary policy. A foreign monetary expansion stimulates foreign consumption (through a standard inter-temporal substitution channel) and therefore stimulates demand for the domestic good. At the same time, by generating a domestic currency appreciation, it switches expenditure (by domestic and foreign consumers alike) away from the domestic good.

[17] (16) is obtained by combining the Backus-Smith conditions (7) for countries k and s with the the equation relating the bilateral real exchange rate with the bilateral terms of trade $\mathcal{Q}^j_{k,t} = \left(\mathcal{S}^j_{k,t}\right)^{1-\alpha}$, and the market clearing conditions (14) for countries k, s and n. (17) is obtained by differentiating country k's market clearing condition (14) and substituting the consumption Euler equations (10) for countries k, s and n. See Appendix A.1 for details.

As first noted by Corsetti and Pesenti (2001) in the context of a related model, under the joint assumption of unitary intra- and inter-temporal elasticity of substitution, these two effects exactly cancel out. Hence, under the retained parametrization, the model does not feature spillovers from foreign monetary policy onto domestic output. As is evident from (17), it does, however, feature spillovers from foreign demand shocks and capital flow taxes onto domestic output through their effects on the growth of expenditure ratios. In particular, positive (negative) foreign demand shocks are expansionary (contractionary) through their inter-temporal substitution effects on foreign consumption. Similarly, domestic capital flow taxes (subsidies) or foreign capital flow subsidies (taxes) are expansionary (contractionary) through their expenditure switching effects on domestic output.[18]

Also, notice that in the limit of *extreme home bias*, $\alpha \to 0$, the IS equation collapses to $\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - \rho - \zeta_{k,t}$ which also corresponds to the IS curve if country k was a closed economy. We will refer to this case as the *closed economy limit.* As is apparent, country k is insulated from the rest of the world in this case.

3 Capital flows and the zero bound

In a world with integrated financial markets, differences in nominal interest rates across countries are associated with exchange rate dynamics that may reallocate expenditures towards the relatively cheaper goods, both over production locations and over time. The presence of nominal rigidities in turn implies that such reallocations of expenditures impact the level of economic activity. In this section, we describe how monetary policy optimally adjusts to demand shocks originating at home or abroad, and how the induced interest rate differentials lead to global expenditure reallocation.

3.1 Optimal monetary policy in country k

A benevolent monetary authority in country k sets interest rates to maximize the utility of a domestic representative household. Using Lemma 1, the optimal policy problem can be compactly written as:

$$\max \int_0^\infty e^{-\int_0^t (\rho+\zeta_{k,h})dh} \left[(1-\alpha)\ln Y_{k,t} - \frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi} \right] dt \qquad (18)$$

[18]Domestic capital flow taxes (subsidies) or foreign capital flow subsidies (taxes) require a expected appreciation (depreciation) of the domestic currency, which in turn is associated with negative (positive) demand growth for the domestic good.

subject to:

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho + \zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{19}$$

$$i_{k,t} \geq 0. \tag{20}$$

with $\Lambda_{k,t} \equiv (1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}$, and $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$. (19) is the dynamic IS equation for country k output, (20) is the ZLB constraint, and $Y_{k,0}$ is free. This is an optimal control problem with control $i_{k,t}$ and state $Y_{k,t}$. The monetary authority's optimal plan is characterized by a two dimensional system of differential equations in the state variable $Y_{k,t}$ and its co-state $\mu_{k,t}$, consisting of (19) and

$$\dot{\mu}_{k,t} = -\frac{e^{-\int_0^t (\rho + \zeta_{k,t}) dh}}{Y_{k,t}} \left\{ (1-\alpha) - \left(\frac{Y_{k,t}}{A} \right)^{1+\phi} \right\} - \mu_{k,t} \frac{\dot{Y}_{k,t}}{Y_{k,t}}, \tag{21}$$

with $\mu_{k,t} i_{k,t} = 0$ and $\mu_{k,0} = 0$. $\mu_{k,t}$ is often referred to as the value of commitment. Proposition 1 characterizes optimal policy in country k in the absence of the ZLB.

Proposition 1 (Optimal monetary policy without ZLB)**.** *In the absence of a zero-bound on interest rates, the monetary authority stabilizes domestic output perfectly, achieving* $Y_{k,t} = A(1-\alpha)^{\frac{1}{1+\phi}} \equiv \overline{Y}$, *by setting an initial exchange rate of* $\mathcal{E}^n_{k,0} = \left(\overline{Y}/\Lambda_{k,0}\right)\left(Y_{n,0}/\Lambda_{n,0}\right)^{-1}$ *and an interest rate path given by*[19]

$$\mathcal{I}_k = (\rho + \zeta_{k,t}) + \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} + \frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{22}$$

Proof. See text below. □

In the absence of the ZLB (or for a small enough shocks), equation (21) indicates that the monetary authority aims to perfectly stabilize output at $Y_{k,t} = \overline{Y}$. By lowering the interest rate in response to a negative (domestic or foreign) demand shock, it stimulates demand for the domestic good through a standard inter-temporal substitution channel and an intra-temporal expenditure switching channel. The inter-temporal substitution channel concerns domestic agents only, while the expenditure switching channel, characteristic of open-economy settings, applies to home and foreign agents alike.

[19] A complete description of the monetary authority's actions requires the specification of an entire exchange rate path, or alternatively, of an initial exchange rate level and a path for the domestic interest rate. The optimal exchange rate path is given by $\mathcal{E}^n_{k,t} = \left(A(1-\alpha)^{\frac{1}{1+\phi}}/\Lambda_{k,t}\right)\left(Y_{n,t}/\Lambda_{n,t}\right)^{-1}$.

We refer to the optimal monetary policy outlined in Proposition 1 as the *unconstrained policy*. A large enough negative demand shock (either at home or abroad) can push the interest rate associated with this policy below 0, leading to a violation of the ZLB constraint. We refer to such a situation as a *liquidity trap* in country k. Proposition 2 describes optimal policy in such a situation.

Proposition 2 (Optimal monetary policy at ZLB)**.** *Suppose that the interest rate policy prescribed in Proposition 1 violates the ZLB constraint for $t \in [0, T)$ but not for $t \geq T$. Then the ZLB binds, with $i_{k,t} = 0$ for $t \in [0, \widehat{T}_k)$ and $i_{k,t} = \mathcal{I}_k$ for $t \geq \widehat{T}_k$. The ZLB exit time $\widehat{T}_k > T$ and the output path satisfy*

$$0 = \int_0^{\widehat{T}_k} e^{-\int_0^t (\rho + \zeta_{k,h}) dh} \left[1 - \left(\frac{Y_{k,t}}{\overline{Y}} \right)^{1+\phi} \right] dt, \tag{23}$$

and for $Y_{k,0}$ implicitly defined by (19), (23) and $Y_{k,\widehat{T}_k} = \overline{Y}$, the initial exchange rate is $\mathcal{E}^n_{k,0} = (Y_{k,0}/\Lambda_{k,0}) (Y_{n,0}/\Lambda_{n,0})^{-1}$.

Proof. See Appendix A.2. □

Hence, if the unconstrained policy violates the ZLB for some period of time, it is optimal to keep the interest rate at zero for longer. The commitment to do so, often referred to as "forward guidance," generates a demand boom after the liquidity trap, whose purpose is to alleviate the initial output contraction. Under optimal policy, an economy with a binding ZLB thus goes through a recession-boom cycle in output. Output growth is positive during the liquidity trap – from 0 to T – and negative between the end of the trap and the ZLB exit time – from T to $\widehat{T}_k$. Furthermore, the ZLB exit time is optimally chosen so as to minimize average deviations from the unconstrained output level $\overline{Y}$.

Our characterization of optimal policy at the ZLB is reminiscent of earlier results in the closed economy literature.[20] But in the open economy, monetary policy also operates through an expenditure switching channel, whose precise workings constitutes the main focus of our paper. By lowering interest rates, a monetary authority can create an interest rate differential between itself and other economies. This differential induces a depreciation of the home currency, increasing the competitiveness of its exports. The resulting global expenditure switching compensates for lower domestic demand and potentially alleviates the demand driven recession at home. This adjustment of the terms of trade or exchange rates is strongly linked to capital flows.

[20] See for example, Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2006) and Werning (2012). In particular, Werning (2012) obtains an analogous characterization of the optimal ZLB exit time under the scenario of rigid prices in a linearized closed economy model.

Is openness an unambiguous blessing? On the one hand, openness reduce a country's exposure to home demand shocks. But on the other hand, it increases its exposure to foreign demand shocks. We formally define the exposure of country k to a home demand shock as $\chi_{kk} \equiv \frac{\partial \mathcal{I}_k}{\partial \zeta_{k,t}}$, and its exposure to a foreign demand shock as $\chi_{kn} \equiv \frac{\partial \mathcal{I}_k}{\partial \zeta_{n,t}}$ (or $\chi_{ks} \equiv \frac{\partial \mathcal{I}_k}{\partial \zeta_{s,t}}$). These are natural measures of exposure, as they represent the aggressiveness with which the monetary authority needs to adjust the interest rate to stabilize demand in response to these shocks.

Proposition 3 (Openness and the incidence of demand shocks)**.** *The exposure to home shocks decreases with openness (i.e.* $\frac{\partial \chi_{kk}}{\partial \alpha} < 0$*), but the exposure to foreign shocks increases with openness (i.e.* $\frac{\partial \chi_{kn}}{\partial \alpha}, \frac{\partial \chi_{ks}}{\partial \alpha} > 0$*).*

Proof. See text below. □

The result follows directly from differentiating the exposure measures with respect to α. On the one hand, an economy hit by a domestic demand shock of a given size is less likely to experience a liquidity trap if it is open than if it is closed. On the other hand, openness creates possibilities that the economy may experience liquidity traps as a result of foreign shocks.[21] Thus, the reduced vulnerability to domestic shocks comes at the expense of an increased vulnerability to foreign shocks. The transmission of shocks is closely tied to capital flows: an economy hit by a negative demand shock exports savings into foreign economies, thereby contributing to appreciate their currencies and divert demand away from their locally produced goods. The more integrated economies are, the stronger this channel, as out next result illustrates.

Proposition 4 (Globally spreading liquidity traps)**.** *Under free capital flows, in the limit of no home bias (*$\alpha \to 1$*), liquidity traps are synchronized across all countries globally.*

Proof. See text below. □

The result follows directly from the fact that under free capital flows and $\alpha \to 1$, the unconstrained interest rate in (22) is equalized across countries and equal to $\mathcal{I}_k = \rho + \frac{x\zeta_{n,t} + (1-x)\Theta^n_{s,t}\zeta_{s,t}}{x + (1-x)\Theta^n_{s,t}}$. The more integrated the world economy, the easier it is for demand shocks to get transmitted accross countries. A direct implication of this result is that under free capital flows and no home

[21] For instance, under free capital flows, a home demand shock in a North economy k leads to an unconstrained interest rate of $\mathcal{I}_k = \rho + [(1-\alpha)\,\Theta^n_{k,t}/\Lambda_{k,t}]\zeta_{k,t}$ (with $0 < (1-\alpha)\,\Theta^n_{k,t}/\Lambda_{k,t} < 1$), which compares with unconstrained interest rate of $\mathcal{I}_k = \rho + \zeta_{k,t}$ in the closed economy. On the other hand, a foreign (North) demand shock leads to an unconstrained interest rate of $\mathcal{I}_k = \rho + (\alpha x/\Lambda_{k,t})\zeta_{n,t}$ (with $0 < \alpha x/\Lambda_{k,t} < 1$) in the open economy, which compares with an unconstrained interest rate of $\mathcal{I}_k = \rho$ in the closed economy.

bias, any demand shock that pushes a region to the ZLB necessarily also drags the entire world to the ZLB.[22]

To gain insights into the role played by expenditure switching in the global macroeconomic adjustment taking place in a liquidity trap, we put additional structure on the exogenous variables and take a world equilibrium perspective. More precisely, we consider a demand shock path that only push the North into a liquidity trap, and study the unique Nash equilibrium of game in which each country's monetary authority's solves (18) subject to (19) and (20). Our focus on such a scenario is motivated by the global economic environment of the Great Recession, during which several key advanced economies, but not emerging markets, were pushed to the ZLB. The analysis of the Nash equilibrium in the next section shows that global adjustment crucially depends on the prevailing capital flow regime.

3.2 Nash equilibrium of monetary policy game

Our interest is in analyzing a liquidity trap episode that primarily affects a region of the world economy. To this end, we put some additional structure onto the model's exogenous driving forces. Following standard practice in the literature, we generate a liquidity trap via an unanticipated temporary large *negative demand shock*.

Assumption 1. *At $t = 0$, agents learn about the path of demand shocks for $t \geq 0$. This path is given by $\zeta_{s,t} = 0\ \forall t \geq 0$ and*

$$\zeta_{n,t} = \begin{cases} -\bar{\zeta} & \text{for } t \in [0, T), \qquad \text{with } \bar{\zeta} > 0 \\ 0 & \text{for } t \geq T \end{cases}$$

The negative demand shock originates in the North, and prevails from 0 to T. Next, we bound the size of this demand shock to ensure that it is large enough to make the North experience a liquidity trap, yet small enough not to make the South experience one.[23,24]

[22] This is consistent with the findings of Cook and Devereux (2013), who point out that under the assumption of no home bias and unit elasticities, natural interest rate are equalized across countries. Caballero et al. (2015) also obtain a similar result.

[23] This scenario seems to be the most relevant one to describe the characteristics of the Great Recession, during which short term interest rates reached (or neared) the ZLB in several key advanced economies, but not in emerging markets.

[24] In the limiting case with extreme home bias ($\alpha \to 0$), this condition trivially reduces to $\rho < \bar{\zeta}$, which is analogous to the closed economy condition under which the natural rate becomes negative. But in general, the condition depends on the degree of openness α and on the mass of countries experiencing the demand shock x. In

Assumption 2. *The size of the demand shock satisfies:*

$$\rho + \frac{\alpha(1-x)\rho}{[1-\alpha(1-x)]\left(\rho-\bar{\zeta}\right)}\left[\rho - \bar{\zeta}e^{-(\rho-\bar{\zeta})T}\right] < \bar{\zeta} < \rho + \frac{(1-\alpha x)\rho}{\alpha x\left(\rho-\bar{\zeta}\right)}e^{-\bar{\zeta}T}\left[\rho - \bar{\zeta}e^{-(\rho-\bar{\zeta})T}\right].$$

This structure enables us to characterize the unique Nash equilibrium of the monetary policy game and construct a narrative of the global adjustment following a demand shock which drives the North, but not the South, to the ZLB. Unless indicated otherwise, Assumptions 1 and 2 are maintained throughout the rest of the paper.

Proposition 5. *Suppose that capital flow taxes are small (in absolute value). Then in the Nash equilibrium of the monetary policy game, the ZLB binds in the North but not in the South.*

Proof. See Appendix A.3. □

Under the maintained assumptions on the size of the demand shock, the ZLB prevents monetary policy from fully stabilizing aggregate demand in the North, but not in the South. This results for the North in a real interest rate that is "too high" (as in a closed economy) and in an exchange rate that is "too appreciated."

Integrating the dynamic IS equation from $t \geq 0$ to $\widehat{T}_n$ yields an expression for the ratio of North output to its unconstrained level:

$$\frac{Y_{n,t}}{Y_{n,\widehat{T}_n}} = \frac{\Lambda^n_{s,t}}{\Lambda^n_{s,\widehat{T}_n}} e^{\int_t^{\widehat{T}_n}\left(\rho+\zeta_{n,h}\right)dh} \tag{24}$$

Substituting (24) into (23) (specialized for a North economy) yields a single non-linear equation in the North's optimal ZLB exit time $\widehat{T}_n$:[25]

$$0 = \int_0^{\widehat{T}_n} e^{-\int_0^t(\rho+\zeta_{n,t})dh}\left[1 - \left(\frac{\Lambda^n_{s,t}}{\Lambda^n_{s,\widehat{T}_n}}\right)^{1+\phi} e^{(1+\phi)\int_t^{\widehat{T}_n}\left(\rho+\zeta_{n,h}\right)dh}\right]dt. \tag{25}$$

particular, a small enough α ensure that Assumption 2 is satisfied if $\rho < \bar{\zeta}$. Thus, as long as the North block is not too large, a large enough shock in the North need not push the South to the ZLB. More generally, the parameter set satisfying the condition of Assumption 2 is non-empty if and only if the following condition holds:

$$T\bar{\zeta} < \ln\left[1 + \frac{1-\alpha}{\alpha(1-x)\alpha x}\right].$$

Loosely speaking, this condition requires that for a given duration of the liquidity trap T the shock $\bar{\zeta}$ is not too large, or equivalently that for a given shock $\bar{\zeta}$, the duration of the trap T is not too long.

[25] A sufficient condition for (25) to have a solution larger than T is that North output growth is positive from 0 to T, which itself rules out situations where taxes on upstream flows or subsidies on downstream flows are very large. If a solution to (25) larger than T exists, then it is unique.

The ratio $\Lambda^n_{s,t}/\Lambda^n_{s,\widehat{T}_n}$ in (25) depends on the ease with which capital can flow between countries and thereby promote (or hamper) global adjustment when the North gets pushed to the ZLB. To illustrate this point, we characterize in detail the global adjustment associated with two stylized capital flow regimes: one with free capital flows, and another one with closed capital accounts.

Free capital flows The free capital flow regime corresponds to a case where capital flow taxes are zero at all times, i.e. $\tau_{s,t} = \tau_{n,t} = 0 \ \forall t$. Capital flows downstream during the liquidity trap only, i.e. $\dot{\Theta}^n_{s,t}/\Theta^n_{s,t} < 0$ for $0 \leq t < T$, and $\dot{\Theta}^n_{s,t}/\Theta^n_{s,t} = 0$ for $t \geq T$. There is both intra-temporal and inter-temporal trade. North economies reduce their nominal rate to zero and commit to keeping it there till after the trap has ended. South economies also lower interest rates, but not all the way down to zero, and do so only for the duration of the trap. A positive interest rate differential between the South and the North thus accompanies downstream capital flows during the liquidity trap, and persists for a short period after it. During this time, interest parity requires a continuous depreciation (appreciation) of South (North) currencies, i.e. $\dot{\mathcal{E}}^n_{s,t}/\mathcal{E}^n_{s,t} = i_{s,t} - i_{n,t} > 0$, which typically leads to an appreciation (depreciation) on impact. These exchange rate movements induce a terms of trade path that promotes expenditure switching. This expenditure switching has an intra-temporal dimension (North vs. South goods), and inter-temporal dimension (current vs. future goods). Both work to reallocate demand toward North goods in the initial part of the liquidity trap, precisely when demand for these goods is most depressed. Meanwhile, downstream capital flows allow a global reallocation of demand: during the trap, North consumption is depressed (i.e. is tilting up), but South consumption booms (i.e. is tilting down).[26] Finally, following the trap, the South runs a permanent trade surplus to cover interest payments on the accumulated foreign debt. The solid lines in Figure 1 graphically displays the global adjustment to the demand shock in the free capital flow regime, confirming the above described narrative.[27] The stabilizing role of capital flows is best illustrated by comparing the free capital flow outcome

[26]To see this, observe that during the liquidity trap, from the Euler equations we have $\dot{\mathbb{C}}_{n,t}/\mathbb{C}_{n,t} = \alpha(1-x)i_{s,t} - (\rho - \bar{\zeta}) > 0$ and $\dot{\mathbb{C}}_{s,t}/\mathbb{C}_{s,t} = (1-\alpha x)i_{s,t} - \rho < 0$.

[27]The parametrization used to generate the figure relies on standard values from the literature. Following Gali and Monacelli (2005), we set the discount rate to $\rho = 0.04$, the openness parameter set to $\alpha = 0.4$, and the inverse Frisch elasticity of labor supply to $\phi = 3$. For parameters pertaining to our liquidity trap scenario, we follow Werning (2012). The duration of the trap is set to $T = 2$ years, and the size of the demand shock is set to $\bar{\zeta} = 0.08$. In a closed economy benchmark, such a shock size would result in natural real interest rate of -4% for the duration of the liquidity trap. Finally, we set the relative size of the North block, for which there is no natural counterparts in standard models, to $x = 0.4$, aiming at generating a share of advanced economies in world GDP in line with recent numbers (alternative plausible values for that parameter deliver identical qualitative results). These parameters (easily) satisfy Assumption 2. Unless noted otherwise, they are used for all our following figures.

to a closed capital account scenario, to which we now turn.

Closed capital accounts The closed capital account scenario corresponds to a capital flow tax wedge $\tau_{s,t} - \tau_{n,t} = -\zeta_{n,t}$ that exactly shuts down capital flows, i.e. results in $\dot{\Theta}^n_{s,t}/\Theta^n_{s,t} = 0$ for all $t \geq 0$. Unlike the free capital flow case which features both inter- and intra-temporal trade, the closed capital account scenario only features intra-temporal trade. The dynamic IS equation for an economy k (19) reduces to $\dot{Y}_{k,t}/Y_{k,t} = i_{k,t} - (\rho + \zeta_{k,t})$, which coincides with the closed economy limit. Consequently, the optimal monetary policy takes the same form as in the closed economy. South interest rate do not move ($i_{s,t} = \rho\ \forall t$), while in the North, the ZLB binds and exit is delayed to $\widehat{T}_n^{\text{closed}} > T$. From 0 to T, North output grows at rate $\dot{Y}_{n,t}/Y_{n,t} = \bar{\zeta} - \rho > 0$, while from T to $\widehat{T}_n^{\text{closed}}$, it grows at rate $\dot{Y}_{n,t}/Y_{n,t} = -\rho < 0$, exactly as in the closed economy. In stark contrast with the free capital flow case, interest parity requires a continuous appreciation (depreciation) of South (North) currencies during the liquidity trap: $\dot{\mathcal{E}}^n_{s,t}/\mathcal{E}^n_{s,t} = \rho - \bar{\zeta} < 0$. The terms of trade thus move "the wrong way" from the perspective of aggregate demand stabilization in the North. Furthermore, the lack of capital flows prevents global demand reallocation: during the trap, consumption in both the North and the South is depressed (i.e. tilting up).[28] Finally, since the relative expenditure ratio $\Theta^n_{s,t}$ is constant, $\Lambda^n_{s,\widehat{T}_n}/\Lambda^n_{s,t} = 1$ in equations (24) and (25), and the exit time characterization coincides with that of Werning (2012). Our next result contrasts this exit time with the one prevailing under free capital flows.

Proposition 6. *The North exits the ZLB earlier under free capital flows than under closed capital accounts (or equivalently, than in a closed economy benchmark):* $\widehat{T}_n^{free} < \widehat{T}_n^{closed}$.

Proof. See Appendix A.4. □

This result is yet another materialization of the stabilizing effects of capital flows at the ZLB: not only do free capital flows yield a smoother output path for North economies, they also allow for a faster adjustment process. The dashed line in Figure 1 represents the variables' responses to the demand shock in the closed capital account regime, confirming the above narrative.

To sum up, the comparison of the dynamics under the two regimes sheds light on the stabilizing effects of capital flows when only one region experiences a liquidity trap. Downstream flows reallocate demand globally by inducing the South to experience a consumption boom at the precise time when demand is deficient in the North. Meanwhile, exchange rate movements associated with

[28] To see this, observe that during the liquidity trap, from the Euler equations we have $\dot{\mathbb{C}}_{n,t}/\mathbb{C}_{n,t} = -[1 - \alpha(1 - x)](\rho - \bar{\zeta}) > 0$ and $\dot{\mathbb{C}}_{s,t}/\mathbb{C}_{s,t} = -\alpha x(\rho - \bar{\zeta}) > 0$.

these flows foster a reallocation of expenditure toward North goods at the moment when their provision is the most depressed.

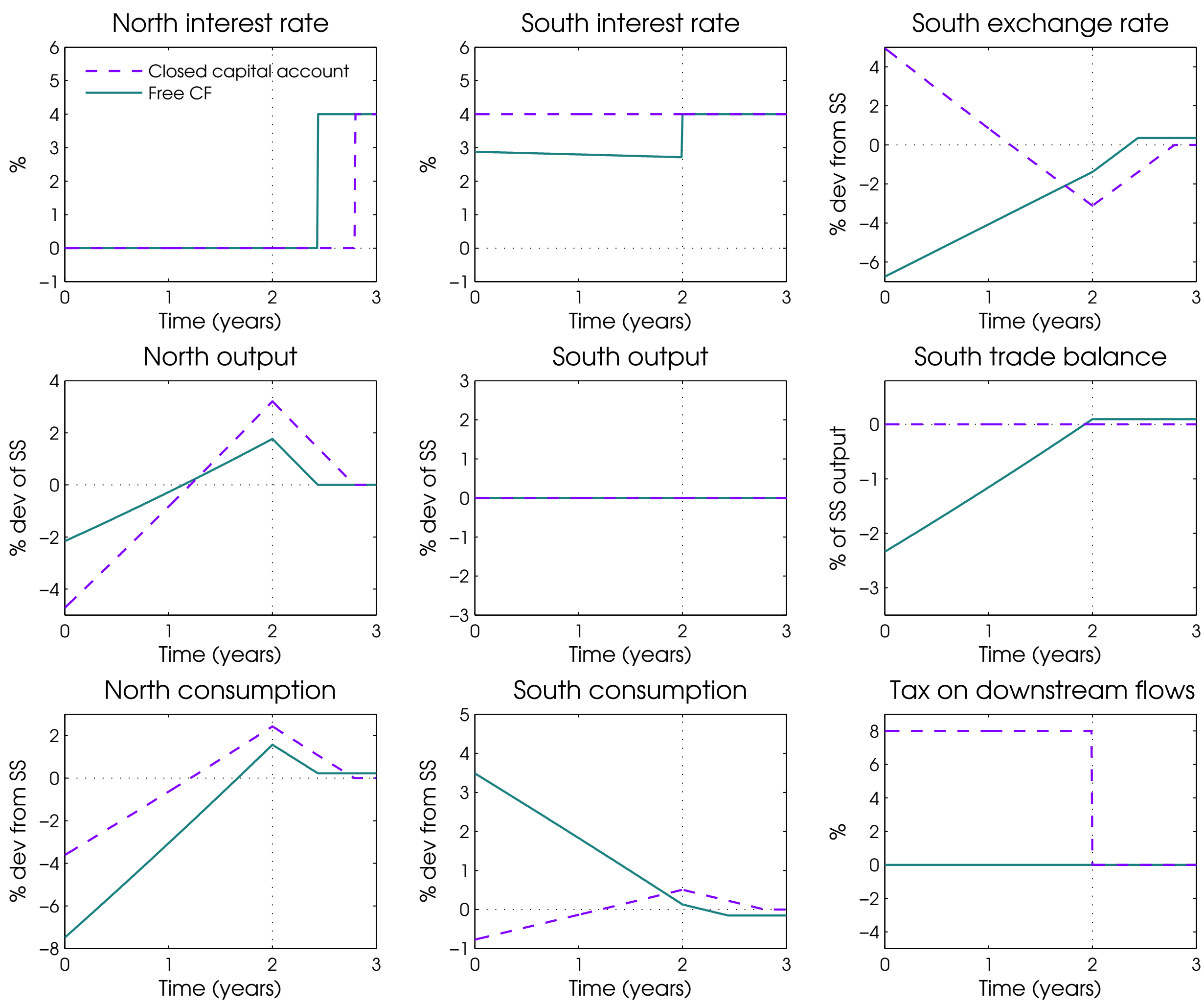

Figure 1: Variable paths under optimal monetary policy in all countries: free capital flows (solid) vs. closed capital accounts (dashed).

4 Efficient capital flows

The above analysis emphasized the stabilizing role played by capital flows in a scenario where only a region of the world economy faces a binding ZLB. It did not address, however, the question of the efficiency properties of a free capital flow regime. In this section, we tackle this issue by asking whether managed capital flows can raise welfare in some countries without reducing it in others.

We frame this efficiency question by considering a Ramsey planning problem. The objective of our utilitarian global planner is to maximize global welfare while ensuring that each country gets at least the same level of welfare as they enjoyed in the free capital flows case. We endow the planner with two instruments: (i) a transfer $\mathcal{T}$ from North to South countries at date 0, and (ii) a path for taxes (or subsidies) on downstream capital flows $\tau_{s,t}$.[29,30] The planner's choices are restricted by two sets of implementability constraints. First, she must respect all equilibrium conditions characterizing private agents' optimal decisions. Second, she must observe constraints representing optimal monetary policy making by individual countries. The planning problem can be written as:

$$\max_{\{i_{s,t},i_{n,t},\tau_{s,t},\mathcal{T}\}} \int_0^\infty e^{-\rho t}\left\{e^{-\int_0^h \zeta_{n,h}dh}\left[\log \mathbb{C}_{n,t} - \frac{1}{1+\phi}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right] + \Xi\left[\log \mathbb{C}_{s,t} - \frac{1}{1+\phi}\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right]\right\} dt$$

subject to:

$$V_k^{\text{free}} \leq \int_0^\infty e^{-\int_0^t(\rho+\zeta_{k,h})dh}\left\{\log \mathbb{C}_{k,t} - \frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right\} dt \tag{26}$$

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho+\zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}}\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{k,t}}\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{27}$$

[29] We focus on symmetric equilibria where the planner treats countries symmetrically within the North and South blocks.

[30] The relevant effective tax rate for cross-border savings decisions is the tax differential $\tau_{s,t} - \tau_{n,t}$. As a result, in the presence of transfers, the assumption that $\tau_{n,t} = 0$ is without loss of generality.

$$\mathbb{C}_{k,t} = \Theta^n_{k,t}\left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right)^{1-\alpha}\left(\frac{Y_{n,t}}{\Lambda_{n,t}}\right)^{\alpha x}\left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{\alpha(1-x)} \tag{28}$$

$$\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \tau_{s,t} + \zeta_{n,t} \tag{29}$$

$$\dot{\mu}_{k,t} = -\frac{e^{-\int_0^t(\rho+\zeta_{k,t})dh}}{Y_{k,t}}\left\{(1-\alpha) - \left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right\} - \mu_{k,t}\frac{\dot{Y}_{k,t}}{Y_{k,t}} \tag{30}$$

$$i_{k,t}\mu_{k,t} = 0\ ,\ i_{k,t} \geq 0 \tag{31}$$

$$B_{n,0} - \mathcal{T} = \alpha(1-x)\int_0^\infty e^{-\int_0^t[\rho+\zeta_{n,h}]dh}\left[1-\Theta^n_{s,t}\right]dt \tag{32}$$

with $\Lambda_{k,t} \equiv (1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}$ and $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$ for $k \in \{n, s\}$. Ξ is an exogenous relative weight assigned by the global planner to South countries. (26) is the constraint indicating that the planner must deliver to all countries at least the same level of welfare as under free capital flows (defined as V_k^{free}). (27) and (28) are dynamic IS equations and equilibrium consumption equations, respectively.[31] (29) is the law of motion for the relative expenditure ratio of South to North countries. (30) and (31) are the conditions representing optimal monetary policy responses by individual countries. Finally, (32) is a North country's inter-temporal budget constraint.[32]

The advantage of setting up the problem in this way is that we can evaluate the efficiency of a regime of free capital flows by asking a very simple question: *Is the optimal choice of the planner characterized by $\tau_{s,t} = 0$, $\forall t$?* If so, then the planner cannot achieve Pareto improvements by distorting international borrowing decisions, and we conclude that free capital flows are constrained Pareto efficient. However, if the planner does choose a non-zero tax path, then it means that the free capital flow regime is constrained Pareto inefficient. We now show that the Pareto efficiency of a free capital flow regime depends crucially on whether the North is experiencing a liquidity trap.

Proposition 7 (Constrained inefficiency of free capital flows in a liquidity trap)**.** *In and around the liquidity trap, the free capital flow regime is constrained inefficient.*

Proof. See Appendix A.5. □

[31] See Lemma 1 for details.

[32] $\mathcal{T}$ is the date 0 amount taxed away from a North country. As a result, $\frac{x}{1-x}\mathcal{T}$ is the amount transfered to a South country. Given the consistency condition for initial net foreign assets, $B_{s,0} = -\frac{xB_{n,0}}{1-x}$, a South country's inter-temporal budget constraint can be derived from (32).

This result indicates that a regime of active capital flow management Pareto dominates the free capital flows benchmark when one region of the world economy is at the ZLB. The proof is by contradiction and relies on the fact that a zero tax path does not satisfy the planner's optimality conditions. This negative result raises the question of how the efficient capital flow regime looks like. Does it entail more or less flows than the free capital flows benchmark? How does this regime reduces the distortions caused by the ZLB? Our next result provides an analytical characterization of the efficient capital flows regime.

Proposition 8 (Characterization of efficient capital flows)**.** *For small enough degrees of openness α, the optimal capital flow tax path satisfies*

$$\begin{aligned} &\tau_{s,t} < 0 \quad \text{for } 0 \leq t < T \\ &\tau_{s,t} > 0 \quad \text{for } T \leq t < \widehat{T}_n \\ &\tau_{s,t} = 0 \quad \text{for } t \geq \widehat{T}_n \end{aligned}$$

Furthermore, in the limit of extreme home bias ($\alpha \to 0$), it is given by $\tau_{s,t} = (1+\phi)(\rho + \zeta_{n,t})$ for $0 \leq t < \widehat{T}_n$ and $\tau_{s,t} = 0$ for $t \geq \widehat{T}_n$.

Proof. See Appendix A.6. □

The efficient capital flow regime thus features a subsidy to downstream flows (or tax on upstream flows) during the liquidity trap, followed by a tax on downstream flows (or subsidy to upstream flows) between the end of the trap and the ZLB exit time in the North.[33] Thus, from the point of view of the planner, in the free capital flow regime capital does not flow enough from the North to the South during the liquidity trap, and then capital fails to flow in the opposite direction between the end of the trap and the North's ZLB exit time.[34] Loosely speaking, the policy intervention achieves a superior output stabilization in the North than in the free capital flows benchmark, without destabilizing output in the South. The superior output stabilization is explained by a flattening of the output path during the time that the North spends at the ZLB. The mechanics can be gleaned from the North's dynamic IS equation during this period:

$$\frac{\dot{Y}_{n,t}}{Y_{n,t}} = -\zeta_{n,t} + \frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{n,t}} \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}.$$

[33] Note that the analytical characterization of the efficient regime is valid for small enough degree of openness as measured by α. For larger values of α, we cannot unambiguously sign the path of controls. However, we can confirm numerically that this characterization also holds for larger values of α in line with the ones commonly used in the literature (see Figure 2).

[34] Recall that in the free capital flow regime, capital only flows (downstream) during the liquidity trap. In particular, capital flows halt at the end of the trap.

Figure 2 illustrates the properties of the efficient capital flow regime by plotting the paths of key macro variables alongside their free capital flows counterparts. The figure helps build a narrative of the mechanics of the optimal policy intervention. The solid line represents the free capital flow regime, while the dashed line represents the efficient capital flow regime.

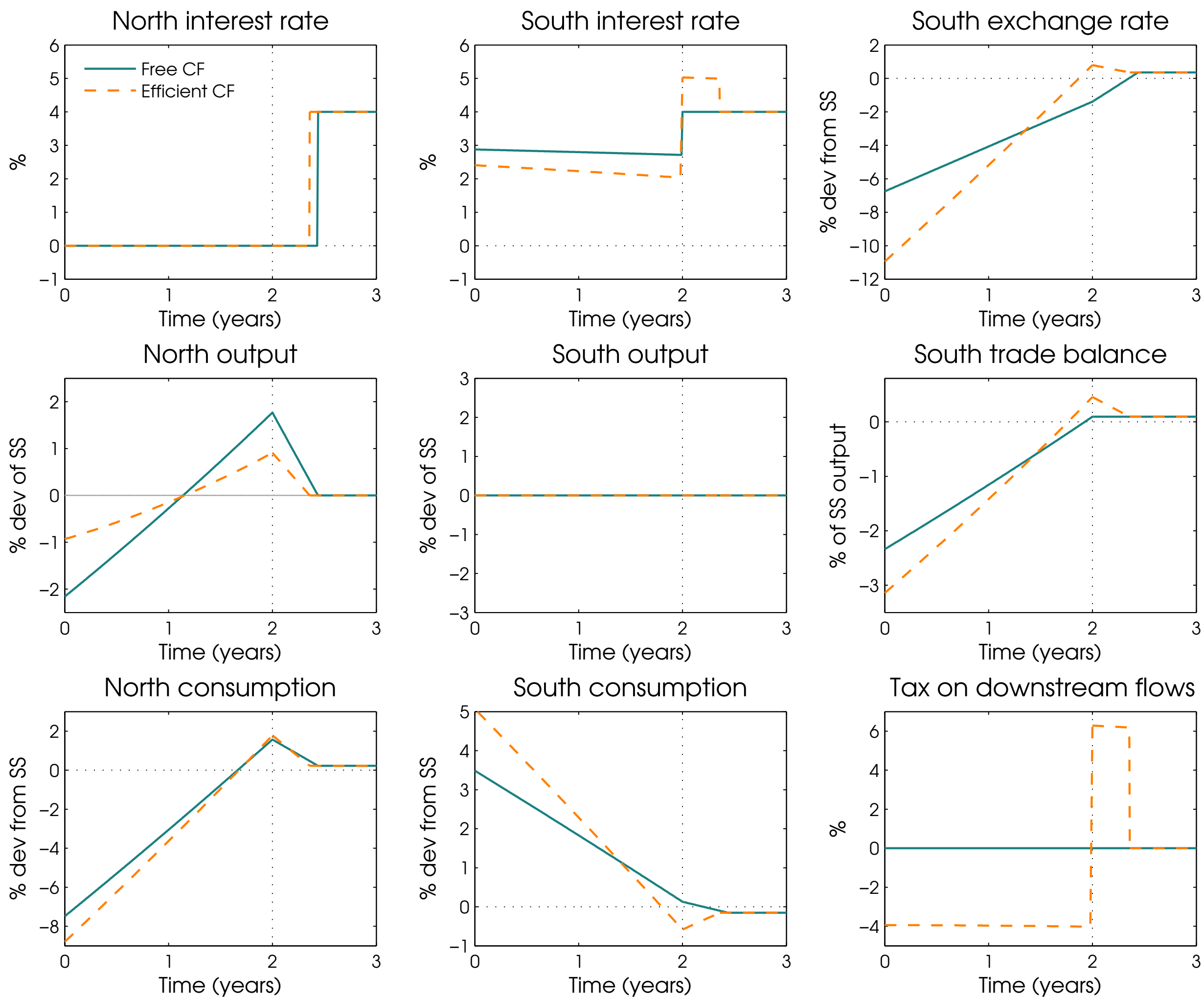

Figure 2: Variable paths under free capital flows (solid) vs. efficient capital flows (dashed).

The efficient regime features larger downstream flows during the trap, and positive upstream flows (rather than no flows) after the trap. These larger downstream flows during the trap induce a larger expected appreciation (depreciation) of the North (South) currencies, while the larger upstream flows (from the end of the trap till renormalization of policy in the North) induce an expected depreciation (appreciation) of the North (South) currencies. This later part is in contrast to the free capital flow scenario which featured a appreciation (depreciation) of the North (South) currency between the end of the trap and the North's ZLB exit time. The steeper exchange rate path during the trap translates into a steeper terms of trade path, and shifts global expenditure (i.e., originating from both the North and the South) toward North goods in the initial stage of the trap, and away from them in the late stage of it. Consequently, both the initial recession and the ensuing boom in the North are less pronounced than under free capital flows.

It is worth noting that the optimal capital flow tax path influences demand for South goods: it is contractionary during the liquidity trap, and expansionary after the trap. But since the monetary authority in the South is not constrained by the ZLB, it has the potency to adjust and offset these effects, by being more expansionary during the trap and more contractionary after the trap than in the free capital flow case. In terms of inter-temporal consumption allocations, the subsidy to downstream flows during the trap lowers the price of current consumption in the South, and generates an even larger temporary consumption boom there than under free capital flows. The South thus runs a larger temporary trade deficit, and the North a larger temporary trade surplus, than in the free capital flow benchmark.

Aggregate demand externalities and the ZLB Farhi and Werning (2013) argue that *aggregate demand externalities* generated by nominal rigidities and constraints on monetary policy generically render inter-temporal decisions constrained inefficient. In our model, constraints on monetary policy take two forms: (i) the non-cooperative nature of monetary policy setting, and (ii) the ZLB. One might therefore wonder whether it is the ZLB as such, or the non-cooperativeness assumption that render borrowing and savings decisions constrained inefficient. The following result establishes that the ZLB, rather than the non-cooperativeness, is key in that regard.

Proposition 9 (Constrained efficiency of free capital flows away from ZLB)**.** *In the absence of the ZLB (or for small enough demand shocks), the free capital flow regime is constrained efficient.*

Proof. See Appendix A.5. □

The result emphasizes that when non-cooperativeness is the only constraint on monetary policy, there is no role for capital flow management from an efficiency perspective. This sharp result is due to the knife-edge nature of the Cole-Obstfeld parametrization we adopt. It implies that absent the ZLB, monetary authorities target a given, constant, level of output of $A(1-\alpha)^{\frac{1}{1+\phi}}$ no matter what. This output targeting prevents aggregate demand externalities from operating, and denies any beneficial role to capital flow management policies.

In our setup, since monetary policy decisions do not entail cross-border externalities, the efficient regime of capital flow management is not motivated by inward-looking monetary policy decisions.[35] Instead, it is motivated by externalities associated with private agents' financial decisions interacting with constraints on monetary policy. Private agents take prices as given and do not internalize how their increased desire to save affects the economy as a whole. With nominal rigidities, the resulting fall in demand pushes the economy into a recession. The monetary authority, which is not a price taker, instead internalizes and attempts to nullify these effects by affecting prices. In the absence of the ZLB, it can lower rates enough to induce sufficient inter-temporal substitution and expenditure switching to eliminate any contraction in output and correct this aggregate demand externality. At the zero bound, however, the monetary authority is unable to lower rates sufficiently to do so. In this scenario, capital flow management can induce an additional stimulus to demand and curtail the severity of the boom-bust cycle. Nevertheless, capital flow taxes are an imperfect substitute and cannot fully eliminate the fall in output without hurting the other economies.

The efficiency gains of the capital account intervention can also be cast in terms of labor wedge stabilization. The labor wedge, defined for the good of a generic country $k \in \{n, s\}$ as

$$\omega_{k,t} \equiv 1 - \left(\mathcal{S}^n_{k,t}\right)^{\alpha} \left(\mathcal{S}^n_{s,t}\right)^{-\alpha(1-x)} \frac{\mathbb{C}_{k,t} N^{\phi}_{k,t}}{A} \tag{33}$$

is an indicator of the degree of under- or over-provision of this good relative to the first best allocation.[36] Figure 3 represents the paths of the labor wedge for the North and South goods under the free (solid) and efficient (dashed) capital flow regimes.[37] The figure shows that the efficient capital account intervention achieves a noticeably smoother labor wedge for the North

[35] Although the Nash equilibrium of the monetary policy game does not achieve the first best allocation that a unconstrained global social planner would chose, international savings decisions are nonetheless constrained efficient. To further stress the point that the non-cooperative nature of monetary policy is not the reason behind the inefficiency of free capital flows, in Appendix B, we show that the inefficiency follows a similar patter to that under non-cooperative monetary policy. We present a similar analysis where monetary policy in all countries is chosen by the global planner rather than by domestic monetary authorities. We refer to this case as the cooperative monetary policy case. Furthermore, in the limit of complete home bias $\alpha \to 0$, we show that the optimal path of taxes/subsidies are identical under cooperative and non-cooperative monetary policy.

[36] The labor wedge is zero at the first best, positive when the good is under-provided, and negative when it is over-provided.

[37] The fact that the labor wedge paths are not centered around zero is due to the assumption of non-cooperative monetary policy, under which countries limit their output as a result of being monopolistically competitive suppliers

goods, at the expense of a mildly less smooth labor wedge for the South goods. Optimal policy thus achieves a labor wedge stabilization across time, as well as across regions. [38]

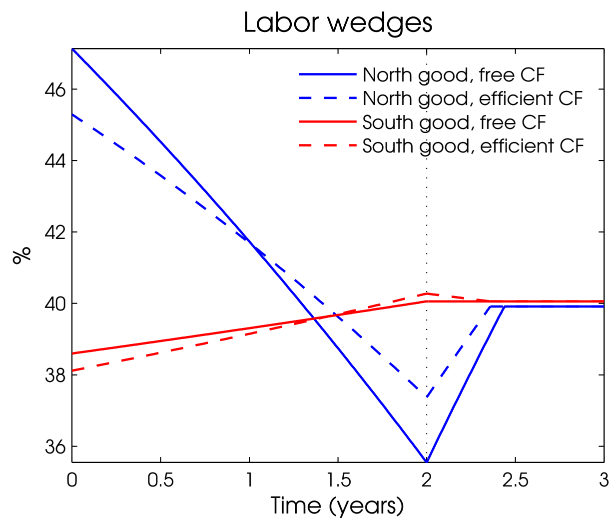

Figure 3: Labor wedges in free capital flows vs. efficient capital flow regimes

Are gift transfers alone desirable? In the absence of capital flow taxes, can the planner use the transfer alone to compensate for the constraints on monetary policy in the North? Since a transfer of resources from the North to the South renders the North poorer, it could potentially reduce the excess desire to save and improve outcomes. Our next result addresses this issue.

Lemma 2 (Undesirability of gift transfers)**.** *Suppose the global planner has no access to capital flow taxes. Then the optimal date 0 transfer is* $\mathcal{T} = 0$.

Proof. See Appendix A.5. □

of the good they export. By using capital flow taxes, our global planner can affect the inter-temporal allocation of consumption, and therefore output, across time, but is impotent when it comes to changing the overall level of economic activity.

[38]The idea underlying the optimal policy intervention is thus similar to that of Farhi and Werning (2012b), who emphasize the benefits of fiscal transfers as a way to correct aggregate demand externalities in a monetary union. In their framework, wealth transfers toward agents with the highest propensity to consumption on relatively more under-provided goods is desirable, because rigid relative prices force adjustment to happen through quantity variations. In contrast, in our model, optimal transfers take wealth away from agents with the highest propensity to consume on relatively more depressed goods, because they induces relative price adjustments allowing expenditure switching (by all agents worldwide) in favor of the goods whose provision is relatively more depressed.

If the planner is restricted to not using capital flow taxes, then she optimally chooses to make no use of the date 0 transfer. This indicates that the transfer is purely compensating and does not constitute a separate instrument to deal with aggregate demand externalities. In order to reduce the incentive to save, the planner needs to make the North poorer relative to the future. However, a transfer only at time 0 makes the North poorer on whole and not relative to the future. Instead, subsidies to downstream capital flows induce a change in relative prices which despite the assumed home bias in preferences makes all consumers tilt their expenditure in favor of North goods.

5 Capital flow management and currency wars

The previous section established that in a liquidity trap, larger capital flows implemented through capital account interventions could make all countries better off. Would this favorable outcome be achieved in a decentralized setting? We address this question by studying the Nash equilibrium of a game in which domestic policymakers choose both monetary and capital flow management policy non-cooperatively. We consider two scenarios: one where only South countries manage their capital account, and one where all countries do so. Section 5.1 characterizes the optimal choices of an individual country, while Sections and discusses Nash equilibrium outcomes.

5.1 Optimal policy in country k

Using Lemma 1, the problem of country k's policy authority can be written as maximizing

$$\int_0^\infty e^{-\int_0^t (\rho+\zeta_{k,s})ds} \left[\ln\left(\Theta^n_{k,t}\right) + (1-\alpha)\ln\left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi} \right] dt \tag{34}$$

subject to

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho+\zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}}\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{k,t}}\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{35}$$

$$\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} = \tau_{k,t} - \zeta_{k,t} - (\tau_{n,t} - \zeta_{n,t}), \tag{36}$$

$$i_{k,t} \geq 0, \tag{37}$$

$$B_{k,0} = \alpha \int_0^\infty e^{-\int_0^t (\rho+\zeta_{n,s}-\tau_{n,s})ds} \left[\Theta^n_{k,t} - x - (1-x)\Theta^n_{s,t}\right] dt, \tag{38}$$

with $\Lambda_{k,t} \equiv (1-\alpha)\Theta^n_{k,t} + \alpha x + \alpha(1-x)\Theta^n_{s,t}$, and $\frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} = \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}$. (35) is the dynamic IS equation, (36) is the law of motion for the expenditure ratio $\Theta^n_{k,t}$, (37) is the ZLB constraint and

(38) is the country's budget constraint. This problem is similar to the problem of the monetary authority in Section 3, except that the policy authority now has an additional instrument: it is able to influence the path of $\Theta^n_{k,t}$ by taxing or subsidizing capital inflows. We have the following result.

Proposition 10 (Individually optimal capital flow taxes)**.** *Country k's optimal tax on capital inflows balances a dynamic terms of trade manipulation motive with an aggregate demand stimulation motive, and satisfies:*

$$\tau_{k,t} = \underbrace{\Omega^1_{k,t}\left[(1-x)\,\Theta^n_{s,t}\left(\zeta_{k,t}+\tau_{s,t}\right)+x\left(\zeta_{k,t}-\zeta_{n,t}+\tau_{n,t}\right)\right]}_{\textit{dynamic terms of trade management}} + \underbrace{\Omega^2_{k,t}\frac{\dot{Y}_{k,t}}{Y_{k,t}}}_{\textit{aggregate demand management}} \tag{39}$$

for $\Omega^1_{k,t}, \Omega^2_{k,t} > 0$.

Proof. See Appendix A.7. □

The first term in (39) reflects incentives to manage dynamic terms of trade and is present regardless of the ZLB. The second term reflects an aggregate demand management motive and is present only when the ZLB binds country k. We now in turn discuss these individual aspects.

Corollary 1 (Optimal taming of capital flow cycles)**.** *The optimal capital flow tax is increasing in the home demand shock and decreasing in foreign demand shocks.*

Regardless of the zero bound, faced with a positive home demand shock or negative foreign demand shocks, a country experiences capital inflows and a temporary appreciation of its currency. As a result, it suffers from a temporarily lower foreign demand for its good. It is then optimal to tax inflows in order to curtail the temporary appreciation and inter-temporally smooth foreign demand for the country's domestic good. This is the dynamic terms of trade manipulation motive of capital flow management identified by Costinot et al. (2014), and also present in Farhi and Werning (2014). Thus, regardless of the zero bound, countries tend to optimally tame capital flow cycles caused by demand shocks.[39] Our next result concerns how countries react to foreign capital flow taxes.

Corollary 2 (Strategic complementarities in capital flow taxes)**.** *The optimal capital flow tax is increasing in foreign capital inflow taxes.*

[39] Note that this mechanism does not hinge on nominal rigidities, but on the monopolistically competitive structure of international goods markets.

This result follows naturally from the observation that foreign capital flow taxes have the same reduced form effects on a country's capital flows and exchange rate/ terms of trade profile as foreign demand shocks. In particular, a positive foreign tax (by North and South alike) acts as a negative foreign demand shock, increasing capital inflows and appreciating the home currency. It is thus optimal for the home country to respond to a positive foreign capital flow tax with a positive tax on inflows. Capital flow taxes are thus strategic complements among countries.

The motives discussed so far for managing the capital account are present regardless of the ZLB. We now turn to a motive stressed by Proposition 10 that is specific to ZLB episodes.

Corollary 3 (Aggregate demand stabilizing capital flow taxes at the ZLB)**.** *A country for which the ZLB binds combats the ZLB by setting a higher tax on inflows when its domestic output is rising, and a lower tax on inflows when its domestic output is falling.*

This result refers to the second term in (39). It reflects the fact that a country where the ZLB binds finds it optimal to adjust its tax on capital inflows to smooth domestic aggregate demand. Given the output profile of a country with a binding ZLB constraint outlined in Section 3, the tax on capital inflows (subsidy on outflows) should be higher during the liquidity trap (i.e., when $\dot{Y}_{k,t}/Y_{k,t} > 0$), and lower following the trap during the time where the monetary policy authority delays exit from the ZLB (i.e., when $\dot{Y}_{k,t}/Y_{k,t} < 0$).

In conclusion, absent (or away from) the ZLB, all countries use capital account management to tame the capital flow cycle caused by asymmetric demand shocks.[40] With the ZLB, however, North countries, in an effort to compensate for the impotency of monetary policy, additionally use capital flow taxes to smooth aggregate demand for their home good. This extra incentive creates a potential conflict between the objectives of North and South economies. The North may try to subsidize capital outflows to combat its liquidity trap, while South countries fight back by taxing capital inflows to avoid an excessive appreciation of their currency. This conflict has a flavor of currency wars.

5.2 Capital controls by the South

In the latest global capital flow cycle that constitutes the motivation for our paper, only emerging market countries engaged in capital flow management policies. To analyze such a scenario, we now consider the Nash equilibrium outcome of a game where only South countries are able to

[40]This is consistent with the common perception that countries may find it optimal to limit capital mobility.

optimally set capital flow taxes.[41] We have the following result.

Proposition 11 (South taxes capital inflows)**.** *In the symmetric Nash equilibrium of the capital flow management game between South countries only, the capital flow tax imposed by each South country satisfies*

$$\tau_{s,t} = -\Upsilon_{s,t}\zeta_{n,t}, \tag{40}$$

for some $0 < \Upsilon_{s,t} < 1$*. In other words, capital flow management by South countries slows down, but neither shuts down nor reverses capital flows during the liquidity trap.*

Proof. See Appendix A.8. □

Given that for each individual South country, (39) indicates that it is optimal to set a positive tax on inflows in repsonse to (i) a negative demand shock in the North, *and* (ii) in response to foreign capital flow taxes, in equilibrium South countries act to slow down capital flows. Each individual South country tries to curtail its own terms of trade improvement by limiting capital inflows. In doing so, it deflects capital flows toward other South countries, which are inclined to act in the same way. Following from this iterative process, the unique symmetric Nash equilibrium of the game features a positive tax on capital inflows by all South economies.

Figure 4 shows the paths of key macro variables in this Nash equilibrium and contrasts them with their counterparts in the free capital flow and efficient capital flow regimes. Downstream flows are smaller and the exchange rate path is flatter in the Nash case than under free capital flows. As a result, the North recession is deeper and the ZLB is more delayed. Thus, uncoordinated capital account interventions by the South limit the stabilizing forces of downstream flows during the liquidity trap, and hamper the macroeconomic adjustment in the North.

5.3 An all-out currency war!

When all countries manage their capital account optimally, all the forces laid out in Section 5.1 are simultaneously at play. This is a situation in which a currency war breaks out. North countries might want to subsidize capital outflows to stabilize their domestic aggregate demand, and in response South countries retaliate by taxing inflows. In such a Nash equilibrium, even though we can analytically characterize the optimal taxes set by the North and South, we are unable to sign them. We therefore illustrate this currency war Nash equilibrium numerically.

[41] This corresponds to a case where all countries maximize (34) subject to (35)-(38), but North countries in addition face the constraint that $\tau_{k,t} = 0 \ \forall t$.

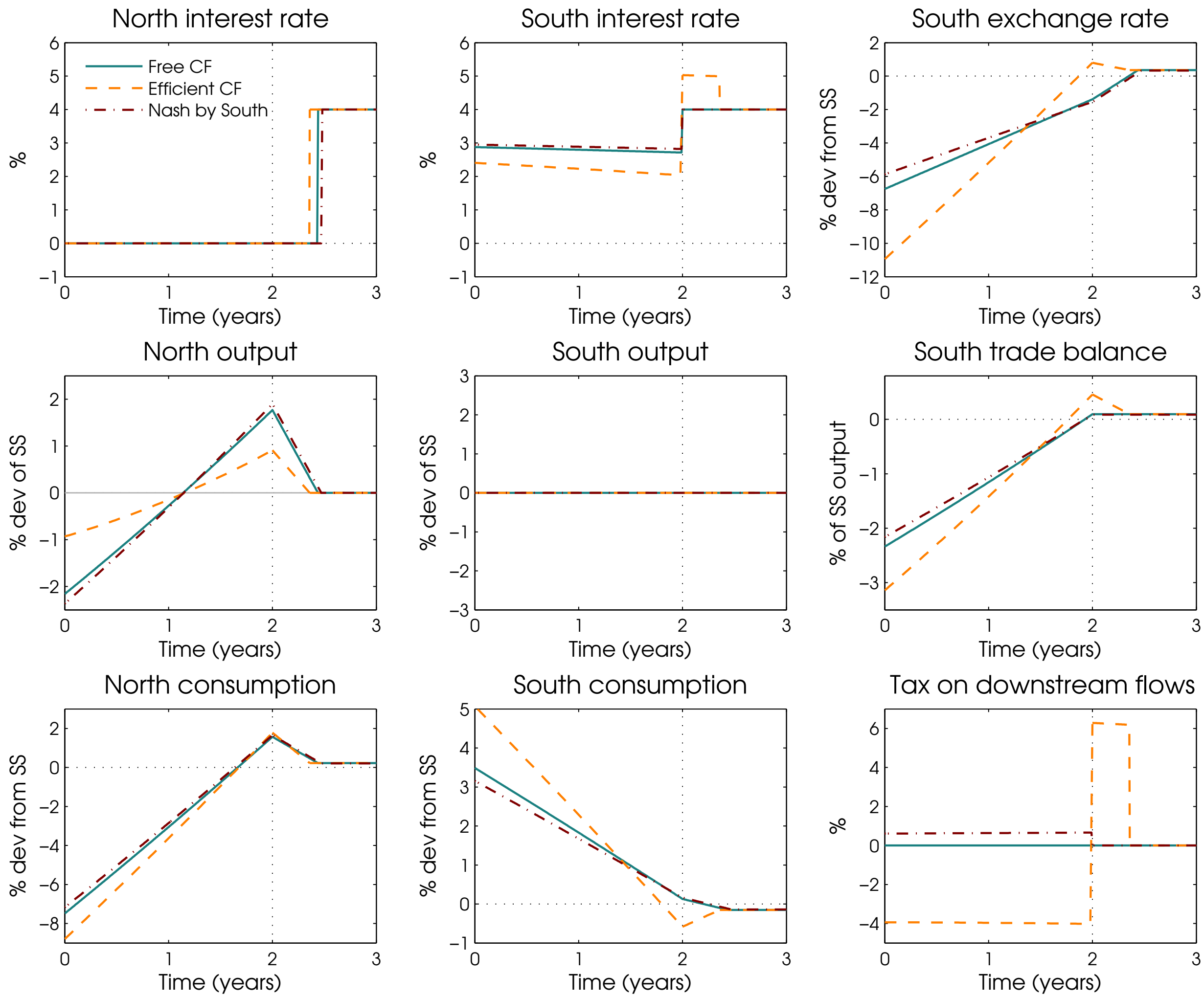

Figure 4: Variable paths under alternative regimes: free capital flows (solid) vs. efficient capital flows (dashed) vs. uncoordinated CF management by South countries (dot-dashed).

Figure 5 represents the tax wedge on downstream flows $\tau_{s,t} - \tau_{n,t}$, when all parameters but the inverse Frisch elasticity parameter are set to the previously specified values. This tax wedge is informative because it summarizes the net effects of the capital account interventions of North and South countries, and its sign determines in which direction capital flows differ from the free capital flows benchmark. We display results for alternative values of the inverse Frisch elasticity parameter ϕ, because it is the parameter with respect to which the tax wedge is most sensitive.

We experiment with values ranging from $\phi = 1.35$ (a common value in the Real Business Cycle literature) to $\phi = 3$ (a value often used in the New Keynesian literature, see e.g. Gali and Monacelli 2005). The left panel shows the entire tax wedge path for $\phi = 1.35$ (blue) and $\phi = 3$ (red). The middle panel shows the average tax wedge during the liquidity trap (i.e., between 0 and T) for an inverse Frisch elasticity ranging from $\phi = 1.35$ to $\phi = 3$. The right panel shows the average tax wedge after the liquidity trap while the North is still at the ZLB (i.e., between T and $\widehat{T}_n$) for the same range of ϕ. In all three panels, solid lines represent the tax wedges prevailing in a Nash equilibrium, while dashed lines represent the tax wedge path obtained in the efficient capital flow regime of Section 4.

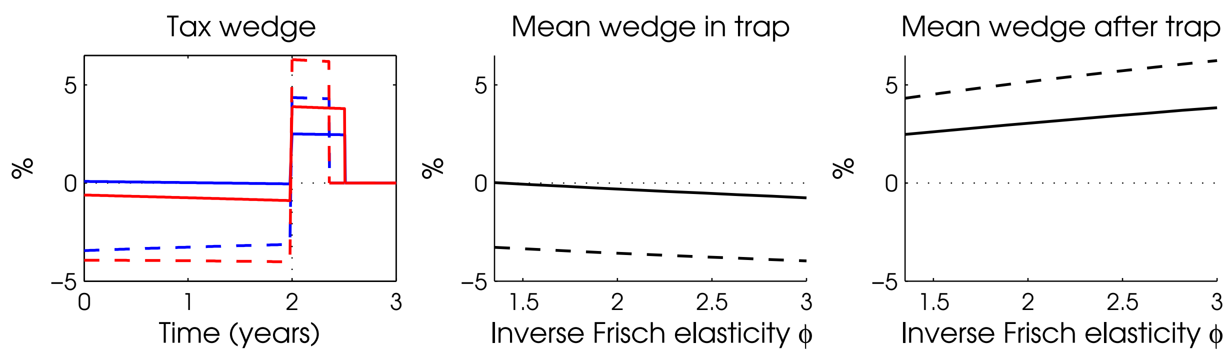

Figure 5: Tax wedge on downstream flows ($\tau_{s,t} - \tau_{n,t}$): efficient regime (dashed) vs. Nash regime (solid). Left panel shows paths for $\phi = 1.35$ (blue) and $\phi = 3$ (red). Middle panel shows average over liquidity trap (0 to T). Right panel shows average after liquidity trap (T to $\widehat{T}_n$).

The figure outlines that the currency war results in countries neutralizing each others' capital account interventions. North countries balance the terms of trade benefits of taxing capital outflows, with the aggregate demand stabilization benefits of subsidizing these.[42] South countries, on the other hand, fight against capital inflows brought about by the North demand shock as well as by other countries' taxes on inflows (or subsidies to outflows) by taxing these inflows. As a result, both during and after the liquidity trap, the tax wedge falls short of its value in the efficient regime, and capital does not flow enough to promote an efficient global adjustment.

[42] The benefits from stabilizing output at the ZLB is increasing in the inverse Frisch elasticity of labor supply ϕ. This can be seen from the fact that a larger ϕ implies a larger labor wedge from equation (33). Hence, for larger values of ϕ, the North puts a higher weight on aggregate demand stabilization and subsidizes capital outflows more aggressively.

6 Conclusion

We argue that when a large region of the world economy experiences a liquidity trap, global capital flows allow for a reallocation of demand and expenditures, and are therefore stabilizing. Due to aggregate demand externalities operating at the zero lower bound, free capital flows are nonetheless constrained inefficient and result in too small reallocations. Global efficiency requires larger flows during and after the liquidity trap, to compensate for the inability of monetary policy to stimulate aggregate demand in the region where the zero bound on interest rates is binding. Despite pointing to inefficient capital flows in a liquidity trap, our analysis does not support the management of capital flows by individual countries. To the contrary, it suggests that the terms of trade management objectives underlying such policies may interfere with aggregate demand stabilization, and thus hamper, rather than promote, a smooth global macroeconomic adjustment. Consequently, the analysis underscores the importance of international policy coordination.

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A Appendix

A.1 Proof of Lemma 1

We start by deriving the consumption expression (16). Substituting $\mathcal{Q}^n_{k,t} = \left(\mathcal{S}^n_{k,t}\right)^{1-\alpha}$ and (14) into (7) to eliminate $\mathcal{Q}^n_{k,t}$ and $\mathcal{S}^n_{k,t}$, we get

$$\mathbb{C}_{k,t} = \Theta^n_{k,t}\left(Y_{k,t}\right)^{1-\alpha}\left(\mathbb{C}_{n,t}\right)^{\alpha}\left[(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}\right]^{-(1-\alpha)}\left(\mathcal{S}^s_{n,t}\right)^{-\alpha(1-x)(1-\alpha)} \tag{A.1}$$

Specializing this expression for representative North and South countries, we have

$$\mathbb{C}_{n,t} = Y_{n,t}\left[1-\alpha\,(1-x) + \alpha\,(1-x)\,\Theta^n_{s,t}\right]^{-1}\left(\mathcal{S}^s_{n,t}\right)^{-\alpha(1-x)} \tag{A.2}$$

$$\mathbb{C}_{s,t} = \Theta^n_{s,t}\left(Y_{s,t}\right)^{1-\alpha}\left(\mathbb{C}_{n,t}\right)^{\alpha}\left[(1-\alpha x)\,\Theta^n_{s,t} + \alpha x\right]^{-(1-\alpha)}\left(\mathcal{S}^s_{n,t}\right)^{-\alpha(1-x)(1-\alpha)} \tag{A.3}$$

Furthermore, specializing the Backus-Smith condition for a representative South country, we have

$$\mathbb{C}_{s,t} = \Theta^n_{s,t}\mathbb{C}_{n,t}\left(\mathcal{S}^n_{s,t}\right)^{1-\alpha} \tag{A.4}$$

(A.1), (A.2), (A.3) and (A.4) are a log-linear system in $\mathbb{C}_{k,t}$, $\mathbb{C}_{n,t}$, $\mathbb{C}_{s,t}$ and $\mathcal{S}^n_{s,t}$ whose solution for $\mathbb{C}_{k,t}$ is given by (16).

Next, we derive the dynamic IS equation (17). Taking logs of the goods market clearing condition (14) and differentiating with respect to time, we get

$$\begin{aligned}\frac{\dot{Y}_{k,t}}{Y_{k,t}} = {} & \left[\frac{(1-\alpha)\,\Theta^n_{k,t}}{(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}} - \frac{1}{1-\alpha}\right]\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} + \frac{1}{1-\alpha}\frac{\dot{\mathbb{C}}_{k,t}}{\mathbb{C}_{k,t}} \\ & +\alpha\,(1-x)\left[\frac{\Theta^n_{s,t}}{(1-\alpha)\,\Theta^n_{k,t} + \alpha x + \alpha\,(1-x)\,\Theta^n_{s,t}} + \frac{1}{1-\alpha}\right]\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} - \frac{\alpha\,(1-x)}{1-\alpha}\frac{\dot{\mathbb{C}}_{s,t}}{\mathbb{C}_{s,t}} - \frac{\alpha x}{1-\alpha}\frac{\dot{\mathbb{C}}_{n,t}}{\mathbb{C}_{n,t}}\end{aligned} \tag{A.5}$$

Under the maintained rigid prices assumption, CPI inflation is given by

$$\pi_{k,t} = \alpha\frac{\dot{\mathcal{S}}_{k,t}}{\mathcal{S}_{k,t}} = \alpha\left[\frac{\dot{\mathcal{E}}^n_{k,t}}{\mathcal{E}^n_{k,t}} + (1-x)\frac{\dot{\mathcal{E}}^s_{n,t}}{\mathcal{E}^s_{n,t}}\right].$$

Substituting out the exchange rate depreciation terms above using the UIP conditions (11) specialized for country k and for a representative South country, we can write the Euler equation (10) as

$$\frac{\dot{\mathbb{C}}_{k,t}}{\mathbb{C}_{k,t}} = (1-\alpha)i_{k,t} + \alpha\left[x\left(i_{n,t} + \tau_{k,t} - \tau_{n,t}\right) + (1-x)\left(i_{s,t} + \tau_{k,t} - \tau_{s,t}\right)\right] - \left(\rho + \zeta_{k,t}\right)$$

Specializing this Euler equation for representative North and South countries, and substituting into the output growth expression (A.5), we obtain the dynamic IS equation (17).

A.2 Proof of Proposition 2

Equation (21) can be rewritten as

$$\frac{d\left(\mu_{k,t}Y_{k,t}\right)}{dt} = -e^{-\rho t+\int_0^t \zeta_{k,h}dh}\left[(1-\alpha)-\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right] \tag{A.6}$$

Defining $\widehat{T}_k$ as the time at which the ZLB stops binding in country k, and integrating both sides of (A.6) from 0 to $\widehat{T}_k$ yields

$$\mu_{k,\widehat{T}_k}Y_{k,\widehat{T}_k} - \mu_{k,0}Y_{k,0} = -\int_0^{\widehat{T}_k} e^{-\rho t+\int_0^t \zeta_{k,h}dh}\left[(1-\alpha)-\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]dt$$

Since $\mu_{k,0}$ is free and the ZLB (by construction) does not bind anymore at $\widehat{T}_k$, we have $\mu_{k,0} = \mu_{k,\widehat{T}_k} = 0$ and therefore

$$0 = \int_0^{\widehat{T}_k} e^{-\int_0^t(\rho+\zeta_{k,t})dh}\left[1-\left(\frac{Y_{k,t}}{\overline{Y}}\right)^{1+\phi}\right]dt \tag{A.7}$$

with $\overline{Y} \equiv A\left(1-\alpha\right)^{\frac{1}{1+\phi}}$.

The result that $\widehat{T}_k > T$ is easily established. First, $\widehat{T}_k < T$ can be ruled out, because it would require contradicting (for some $t < T$) the premise that the interest rate policy prescribed in Proposition 1 violates the ZLB for $[0, T)$. Second, $\widehat{T}_k = T$ can be ruled out using the observation that if the interest rate policy prescribed in Proposition 1 violates the ZLB for $[0, T)$, then a binding ZLB requires $\frac{\dot{Y}_{k,t}}{Y_{k,t}} > 0$ for $[0, T)$, which implies a strictly positive integral on the right hand side of (A.7).

A.3 Proof of Proposition 5

We proceed by proving the result for zero capital flow taxes, and then arguing that it must hold as well for small taxes due to the continuity of the interest rate policy function $\mathcal{I}_k$ in $\tau_{k,t}$, $\tau_{n,t}$ and $\tau_{s,t}$. In a symmetric equilibrium with zero capital flow taxes, the interest rate policy function specified in Proposition 1, for $0 \le t < T$, reduce to

$$\mathcal{I}_n = \rho - \frac{1-\alpha(1-x)}{1-\alpha(1-x)+\alpha\left(1-x\right)\Theta^n_{s,t}}\bar{\zeta}, \tag{A.8}$$

$$\mathcal{I}_s = \rho - \frac{\alpha x}{\left(1-\alpha x\right)\Theta^n_{s,t}+\alpha x}\bar{\zeta}. \tag{A.9}$$

for a North and for a South country, respectively. Now, noting that under the retained assumption of symmetric initial wealth positions $\Theta^n_{s,0} = \frac{\rho}{\rho-\bar{\zeta}} - \frac{\bar{\zeta}}{\rho-\bar{\zeta}}e^{-\left(\rho-\bar{\zeta}\right)T}$ and $\Theta^n_{s,T} = \Theta^n_{s,0}e^{-\bar{\zeta}T}$, the condition of Assumption 2 can be written as

$$\rho + \frac{\alpha\left(1-x\right)\rho}{\left[1-\alpha(1-x)\right]}\Theta^n_{s,0} < \bar{\zeta} < \rho + \frac{\left(1-\alpha x\right)\rho}{\alpha x}\Theta^n_{s,T}.$$

or

$$\frac{1-\alpha(1-x)+\alpha\left(1-x\right)\Theta^{n}_{s,0}}{1-\alpha(1-x)}\rho<\bar{\zeta}<\frac{\alpha x+\left(1-\alpha x\right)\Theta^{n}_{s,T}}{\alpha x}\rho. \tag{A.10}$$

Given that $\Theta^{n}_{s,T}<\Theta^{n}_{s,t}<\Theta^{n}_{s,0}$ for $0<t<T$, the left inequality in (A.10), together with (A.8), implies that $\mathcal{I}_n<0$ for $0\leq t<T$. Similarly, the right inequality in (A.10), together with (A.9), implies that $\mathcal{I}_s>0$ for $0\leq t<T$. For $t\geq T$, we have $\mathcal{I}_n=\mathcal{I}_s=\rho>0$. As a result, the unconstrained optimal policy prescribed in Proposition 1 is feasible for South countries, but not for North countries. These instead follow the constrained policy prescribed in Proposition 2.

A.4 Proof of Proposition 6

Defining the functions

$$\begin{aligned} f_1\left(z\right) &\equiv \int_0^T e^{-\left(\rho-\bar{\zeta}\right)t}\left\{1-\left(\frac{\Lambda^{n}_{s,t}}{\Lambda^{n}_{s,z}}\right)^{1+\phi}e^{(1+\phi)\left[\rho(z-t)-\bar{\zeta}(T-t)\right]}\right\}dt,\\ f_2\left(z\right) &\equiv -e^{\bar{\zeta}T}\int_T^z e^{-\rho t}\left\{1-\left(\frac{\Lambda^{n}_{s,t}}{\Lambda^{n}_{s,z}}\right)^{1+\phi}e^{(1+\phi)\rho(z-t)}\right\}dt, \end{aligned}$$

equation (25) can be written as

$$f_1(\widehat{T}_n)=f_2(\widehat{T}_n) \tag{A.11}$$

The functions satisfy $f_1'\left(z\right)<0$ and $f_2'\left(z\right)>0$, with $f_1\left(T\right)>0$, $f_2(T)=0$, $\lim_{z\to\infty}f_1\left(z\right)=-\infty$, and $\lim_{z\to\infty}f_2\left(z\right)=+\infty$.[43] (A.11) therefore has a unique solution $\widehat{T}_n>T$.

Now observe that under free capital flows, $\Lambda^{n}_{s,t}>\Lambda^{n}_{s,T}$ for $t<T$ and $\Lambda^{n}_{s,t}=\Lambda^{n}_{s,T}$ for $t\geq T$, while under closed capital accounts, $\Lambda^{n}_{s,t}=\Lambda^{n}_{s,T}=1$ for all $t\geq 0$. As a result, $f_1^{\text{free}}\left(z\right)<f_1^{\text{closed}}\left(z\right)$ and $f_2^{\text{free}}\left(z\right)=f_2^{\text{closed}}\left(z\right)$ for $z>T$. It must thus be that $\widehat{T}^{\text{free}}_n<\widehat{T}^{\text{closed}}_n$.

A.5 Proof of Proposition 7

The problem of the global planner in Section 4 is replicated here for convenience:

$$\max_{\{i_{n,t},i_{s,t},\tau_{s,t},\mathcal{T}\}}\int_0^\infty e^{-\rho t}\left\{xe^{\bar{\zeta}\min[t,T]}\mathbb{W}_{n,t}+\left(1-x\right)\Xi\mathbb{W}_{n,t}\right\}dt$$

[43] A sufficient condition for $f_1\left(T\right)>0$ is that $\frac{\dot{Y}_{n,t}}{Y_{n,t}}>0$ for $t\in[0,T)$. This follows from the fact that $\mathcal{I}_n(\cdot)<0$ for $t\in[0,T)$ and is guaranteed to hold under both free capital flows and closed capital accounts.

subject to:

$$V_n^{\text{free}} \leq \int_0^\infty e^{-\rho t+\bar{\zeta}\min[t,T]}\mathbb{W}_{n,t}dt \tag{A.12}$$

$$V_s^{\text{free}} \leq \int_0^\infty e^{-\rho t}\mathbb{W}_{s,t}dt \tag{A.13}$$

$$\mathbb{W}_{n,t} \equiv \log\left(\underbrace{\left(\frac{Y_{nt}}{\Lambda_{n,t}}\right)^{1-\alpha(1-x)}\left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{\alpha(1-x)}}_{\mathbb{C}_{n,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}$$

$$\mathbb{W}_{s,t} \equiv \log\left(\underbrace{\Theta_{s,t}^n\left(\frac{Y_{n,t}}{\Lambda_{n,t}}\right)^{\alpha x}\left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{1-\alpha x}}_{\mathbb{C}_{s,t}}\right) - \frac{1}{1+\phi}\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}$$

$$\frac{\dot{\Theta}_{s,t}^n}{\Theta_{s,t}^n} = \tau_{s,t} + \zeta_{n,t}$$

$$\frac{\dot{Y}_{n,t}}{Y_{n,t}} = i_{n,t} - (\rho + \zeta_{n,t}) + \frac{\alpha(1-x)\Theta_{s,t}^n}{\Lambda_{n,t}}\frac{\dot{\Theta}_{s,t}^n}{\Theta_{s,t}^n}$$

$$\frac{\dot{Y}_{s,t}}{Y_{s,t}} = i_{s,t} - \rho - \frac{\alpha x}{\Lambda_{s,t}}\frac{\dot{\Theta}_{s,t}^n}{\Theta_{s,t}^n}$$

$$B_{n,0} - \frac{1-x}{x}\mathcal{T} = \alpha(1-x)\int_0^\infty e^{-\rho t+\bar{\zeta}\min[t,T]}\left[1-\Theta_{s,t}^n\right]dt \tag{A.14}$$

$$\dot{\mu}_{n,t} = -\frac{e^{-\rho t+\bar{\zeta}\min[t,T]}}{Y_{n,t}}\left\{1-\alpha-\left[\frac{Y_{n,t}}{A}\right]^{1+\phi}\right\} - \mu_{n,t}\frac{\dot{Y}_{n,t}}{Y_{n,t}} \tag{A.15}$$

$$\dot{\mu}_{s,t} = -\frac{e^{-\rho t}}{Y_{s,t}}\left\{1-\alpha-\left[\frac{Y_{s,t}}{A}\right]^{1+\phi}\right\} - \mu_{s,t}\frac{\dot{Y}_{s,t}}{Y_{s,t}} \tag{A.16}$$

$$\mu_{n,t}i_{n,t} = 0 \tag{A.17}$$

$$\mu_{s,t}i_{s,t} = 0 \tag{A.18}$$

for $\Lambda_{n,t} \equiv 1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n$ and $\Lambda_{s,t} \equiv (1-\alpha x)\Theta_{s,t}^n + \alpha x$. Let $\Delta_n \geq 0$ and $\Delta_s \geq 0$ be the multipliers on the inequality constraints (A.12) and (A.13), respectively. The Hamiltonian associated with this problem is given by[44]

$$
\begin{aligned}
\mathcal{H} \quad = \quad & x\left(1+\Delta_n\right) e^{-\rho t+\bar{\zeta}\min[t,T]}\mathbb{W}_{n,t}+(1-x)\left(\Xi+\Delta_s\right) e^{-\rho t}\mathbb{W}_{s,t}+\mu_t\Theta^n_{s,t}\left(\tau_{s,t}+\zeta_{n,t}\right)\\
&+\lambda_{n,t}Y_{n,t}\left\{i_{n,t}-\left(\rho+\zeta_{n,t}\right)+\frac{\alpha\left(1-x\right)\Theta^n_{s,t}}{1-\alpha\left(1-x\right)+\alpha\left(1-x\right)\Theta^n_{s,t}}\left(\tau_{s,t}+\zeta_{n,t}\right)\right\}\\
&+\lambda_{s,t}Y_{s,t}\left\{i_{s,t}-\rho-\frac{\alpha x}{\left(1-\alpha x\right)\Theta^n_{s,t}+\alpha x}\left(\tau_{s,t}+\zeta_{n,t}\right)\right\}\\
&+\varphi_{n,t}\left[-\frac{e^{-\rho t+\zeta_{n,t}}}{Y_{n,t}}\left\{(1-\alpha)-\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right\}-\mu_{n,t}\left\{i_{n,t}-\left(\rho+\zeta_{n,t}\right)+\frac{\alpha\left(1-x\right)\Theta^n_{s,t}}{1-\alpha\left(1-x\right)+\alpha\left(1-x\right)\Theta^n_{s,t}}\left(\tau_{s,t}+\zeta_{n,t}\right)\right\}\right]\\
&+\varphi_{s,t}\left[-\frac{e^{-\rho t}}{Y_{s,t}}\left\{(1-\alpha)-\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right\}-\mu_{s,t}\left\{i_{s,t}-\rho-\frac{\alpha x}{\left(1-\alpha x\right)\Theta^n_{s,t}+\alpha x}\left(\tau_{s,t}+\zeta_{n,t}\right)\right\}\right]\\
&+\kappa_{n,t}\mu_{n,t}i_{n,t}+\kappa_{s,t}\mu_{s,t}i_{s,t}
\end{aligned}
$$

The state variables are Y_n, Y_s, μ_k, μ_s and Θ^n_s, and we define $\lambda_n, \lambda_s, \varphi_n, \varphi_s$ and μ as the respective co-states. κ_n, κ_s are the multiplier on the equality constraints (A.17) and (A.18). The optimality conditions are given by:

$$
\frac{\partial\mathcal{H}}{\partial i_{n,t}} = \lambda_{n,t}Y_{n,t}+\kappa_{n,t}\mu_{n,t}\leq 0, \qquad \lambda_{n,t}i_{n,t}=0 \tag{A.19}
$$

$$
\frac{\partial\mathcal{H}}{\partial i_{n,t}} = \lambda_{s,t}Y_{s,t}+\kappa_{s,t}\mu_{s,t}\leq, 0 \qquad \lambda_{s,t}i_{s,t}=0 \tag{A.20}
$$

$$
\frac{\partial\mathcal{H}}{\partial \tau_{s,t}} = \mu_t\Theta^n_{s,t}+\left(\lambda_{n,t}Y_{n,t}-\varphi_{n,t}\mu_{n,t}\right)\frac{\alpha\left(1-x\right)\Theta^n_{s,t}}{\Lambda_{n,t}}-\left(\lambda_{s,t}Y_{s,t}-\varphi_{s,t}\mu_{s,t}\right)\frac{\alpha x}{\Lambda_{s,t}}=0 \tag{A.21}
$$

$$
\begin{aligned}
-\dot{\mu}_t = \; & e^{-\rho t+\bar{\zeta}\min[t,T]}\left(1+\Delta_n\right)\left(x\frac{\partial\mathbb{W}_{n,t}}{\partial\Theta^n_{s,t}}+(1-x)\,\Xi^n_{s,t}\frac{\partial\mathbb{W}_{s,t}}{\partial\Theta^n_{s,t}}\right)+\mu_t\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}\\
&+\left(\lambda_{n,t}Y_{n,t}-\varphi_{n,t}\mu_{n,t}\right)\frac{\left[1-\alpha\left(1-x\right)\right]\alpha\left(1-x\right)}{\left(\Lambda_{n,t}\right)^2}\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}\\
&+\left(\lambda_{s,t}Y_{s,t}-\varphi_{s,t}\mu_{s,t}\right)\frac{\left(1-\alpha x\right)\alpha x}{\left(\Lambda_{s,t}\right)^2}\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}
\end{aligned} \tag{A.22}
$$

[44]The budget constraint (A.14) is omitted for convenience, since the first order condition for $\mathcal{T}$ will require that the multiplier on that constraint be zero.

$$-\dot{\lambda}_{n,t} = e^{-\rho t+\bar{\zeta}\min[t,T]}\left(1+\Delta_n\right)\left(x\frac{\partial \mathbb{W}_{n,t}}{\partial Y_{n,t}}+(1-x)\,\Xi^n_{s,t}\frac{\partial \mathbb{W}_{s,t}}{\partial Y_{n,t}}\right)+\lambda_{n,t}\frac{\dot{Y}_{n,t}}{Y_{n,t,}}$$
$$+\frac{\varphi_{n,t}}{Y_{n,t}}\frac{e^{-\rho t+\bar{\zeta}\min[t,T]}}{Y_{n,t}}\left[(1-\alpha)+\phi\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right] \quad (A.23)$$
$$-\dot{\lambda}_{s,t} = e^{-\rho t+\bar{\zeta}\min[t,T]}\left(1+\Delta_n\right)\left(x\frac{\partial \mathbb{W}_{n,t}}{\partial Y_{s,t}}+(1-x)\,\Xi^n_{s,t}\frac{\partial \mathbb{W}_{s,t}}{\partial Y_{s,t}}\right)+\lambda_{s,t}\frac{\dot{Y}_{s,t}}{Y_{s,t}}$$
$$+\frac{\varphi_{s,t}}{Y_{s,t}}\frac{e^{-\rho t}}{Y_{s,t}}\left[(1-\alpha)+\phi\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right] \quad (A.24)$$
$$-\dot{\varphi}_{n,t} = -\varphi_{n,t}\frac{\dot{Y}_{n,t}}{Y_{n,t}}+\kappa_{n,t}i_{n,t} \quad (A.25)$$
$$-\dot{\varphi}_{s,t} = -\varphi_{s,t}\frac{\dot{Y}_{s,t}}{Y_{s,t}}+\kappa_{s,t}i_{s,t} \quad (A.26)$$

where $\Xi^n_{s,t}\equiv e^{-\bar{\zeta}\min[t,T]}\frac{\Xi+\Delta_s}{1+\Delta_n}$, and

$$\frac{\partial \mathbb{W}_{n,t}}{\partial Y_{n,t}}=\left[1-\alpha\,(1-x)-\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right]\frac{1}{Y_{n,t}},\quad \frac{\partial \mathbb{W}_{n,t}}{\partial Y_{s,t}}=\alpha\,(1-x)\,\frac{1}{Y_{s,t}},\quad \frac{\partial \mathbb{W}_{n,t}}{\partial \Theta^n_{s,t}}=-\frac{\alpha\,(1-x)\,\Phi_t}{\Lambda_{n,t}\Lambda_{s,t}}, \quad (A.27)$$
$$\frac{\partial \mathbb{W}_{s,t}}{\partial Y_{n,t}}=\alpha x\frac{1}{Y_{n,t}},\quad \frac{\partial \mathbb{W}_{s,t}}{\partial Y_{s,t}}=\left[1-\alpha x-\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right]\frac{1}{Y_{s,t}},\quad \frac{\partial \mathbb{W}_{s,t}}{\partial \Theta^n_{s,t}}=\frac{\alpha x\Phi_t}{\Theta^n_{s,t}\Lambda_{n,t}\Lambda_{s,t}} \quad (A.28)$$

with $\Phi_t\equiv 1-\alpha\,(1-x)+(1-\alpha x)\,\Theta^n_{s,t}$.

A.5.1 Constrained efficiency of free capital flows away from the ZLB

Absent the ZLB, we have $\mu_{n,t}=\mu_{s,t}=0$ for all t. Then (A.15)-(A.16) require that that $Y_{n,t}=Y_{s,t}=A(1-\alpha)^{\frac{1}{1+\phi}}$ for all t and (A.19)-(A.20) imply that $\lambda_{n,t}=\lambda_{s,t}=0$ for all t. (A.21) then requires that $\mu_t=0$ for all t. Using this information in (A.22), we get $\Theta^n_{s,t}=\Xi^n_{s,t}$. Differentiating this equation with respect to time yields $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}=\zeta_{n,t}$, which implies an optimal choice of $\tau_{s,t}=0$ for all t. Thus, in the absence of the ZLB, free capital flows are constrained efficient.

A.5.2 Constrained inefficiency of free capital flows at the ZLB

In the sequel, we focus on a case where the ZLB binds only in the North, and later on verify that this is indeed the relevant scenario. We define as $\widehat{T}_n>T$ the time at which the North exits the ZLB. Then, for $t<\widehat{T}_n$, (A.25) implies that $\frac{\varphi_{n,t}}{Y_{n,t}}=\frac{\varphi_{n,\widehat{T}_n}}{Y_{n,\widehat{T}_n}}=\delta_n$. Further, for $t\geq\widehat{T}_n$, we must have $\mu_{n,t}=\lambda_{n,t}=0$ and $Y_{n,t}=A\,(1-\alpha)^{\frac{1}{1+\phi}}$. (A.23) then implies

$$\delta_n=-\frac{\alpha x}{(1-\alpha)\,(1+\phi)}\,(1+\Delta_n)\,[x+(1-x)\,\Xi_T]\,.$$

Differentiating (A.21) with respect to time yields

$$\begin{aligned}\dot{\mu}_t\Theta^n_{s,t}+\mu_t\dot{\Theta}^n_{s,t} &= -\left(\dot{\lambda}_{n,t}Y_{n,t}+\lambda_{n,t}\dot{Y}_{n,t}-\dot{\varphi}_{n,t}\mu_{n,t}-\varphi_{n,t}\dot{\mu}_{n,t}\right)\frac{\alpha(1-x)\Theta^n_{s,t}}{\Lambda_{n,t}}\\ &\quad -\left(\lambda_{n,t}Y_{n,t}-\varphi_{n,t}\mu_{n,t}\right)\alpha(1-x)\left[1-\alpha(1-x)\right]\frac{\dot{\Theta}^n_{s,t}}{(\Lambda_{n,t})^2}\end{aligned} \tag{A.29}$$

Using (A.22), (A.23), (A.25) and (A.15) to substitute into (A.29) leads to

$$-\Psi\alpha x\left\{[1-\alpha(1-x)]+\alpha(1-x)\Xi_t\right\}\frac{\Theta^n_{s,t}}{\Lambda_{n,t}}+\frac{\Theta^n_{s,t}}{\Lambda_{n,t}}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}-\Psi\alpha x\frac{\Phi_t}{\Lambda_{n,t}\Lambda_{s,t}}\left[\Xi_t-\Theta^n_{s,t}\right]=0 \tag{A.30}$$

for

$$\Psi\equiv\frac{1-\alpha}{\left[1-\alpha(1-x)+\alpha(1-x)\Xi^n_{s,T}\right]\alpha x}.$$

We now prove that under the retained Assumption 2, which guarantees that the ZLB binds in the North under free capital flows, the free capital flow regime is constrained inefficient. We prove the result by showing that a zero capital flow tax path is not a solution to the planning problem. We start by establishing the following intermediate result.

Lemma 3. *Conditional on a zero capital flow tax path $\tau_{s,t}=0\ \forall t\geq 0$, the optimal transfer is $\mathcal{T}=0$.*

Proof. We proceed in two steps. First, we show that the optimal transfer is zero for one particular welfare weight. Then, we argue that the optimal transfer must also be zero for arbitrary welfare weights. The relevant planning problem is the one described above, with the differences that $\tau_{s,t}$ is not a control variable, and as a result, (A.21) drops out of the set of optimality conditions, and $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}=\zeta_{n,t}$.

For $t\geq T$, since $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}=\zeta_{n,t}=0$ (A.22) implies

$$\frac{d\left(\mu_t\Theta^n_{s,t}\right)}{dt}=-e^{-\rho t+\bar{\zeta}T}\left(1+\Delta_n\right)\frac{\Phi_T}{\Lambda_{n,T}\Lambda_{s,T}}\alpha x(1-x)\left\{\Xi^n_{s,T}-\Theta^n_{s,T}\right\}$$

Integrating from T to ∞, and imposing the terminal condition $\lim_{t\to\infty}\mu_t\Theta^n_{s,t}=0$, we get

$$0=\mu_T\Theta^n_{s,T}-\frac{\Phi_T}{\Lambda_{n,T}\Lambda_{s,T}}\alpha x(1-x)e^{(\bar{\zeta}-\rho)T}\left(1+\Delta_n\right)\left\{\Xi^n_{s,T}-\Theta^n_{s,T}\right\}\frac{1}{\rho}$$

Since the state variable $\Theta^n_{s,t}$ is free, we have $\mu_T=0$ and therefore $\Theta^n_{s,T}=\Xi^n_{s,T}$. It follows that $\Theta^n_{s,0}=\Xi^n_{s,0}$.

Now, fix the welfare weight to the symmetric value $\Xi=(\bar{\zeta}e^{(\bar{\zeta}-\rho)T}-\rho)/(\bar{\zeta}-\rho)$, and consider the relaxed problem where the constraints requiring that all countries are at least as well off as in the free capital flow scenario (A.12) and (A.13) are dropped. In this case, $\Delta_n=\Delta_s=0$ and the solution to the planner's problem features $\Theta^n_{s,0}=\Xi^n_{s,0}=\Xi=(\bar{\zeta}e^{(\bar{\zeta}-\rho)T}-\rho)/(\bar{\zeta}-\rho)$, and therefore entails a zero transfer. Since this optimal plan trivially satisfies the constraints (A.12) and (A.13), it is also the solution to the more constrained version of the problem including these two extra constraints.

To prove that a zero transfer is also optimal for arbitrary welfare weights, we proceed by contradiction. Suppose that a zero transfer is not optimal. This requires that there is a non-zero transfer that makes either the North or the South (or both) better off without making neither of the two worse off. But if this were the case, then for the symmetric welfare weight $\Xi = (\bar{\zeta} e^{(\bar{\zeta}-\rho)T} - \rho)/(\bar{\zeta} - \rho)$, the optimal transfer cannot be zero either. This is a contradiction with the result above. It follows that the optimal transfer is zero for arbitrary welfare weights. $\square$

We now proceed to show by contradiction that a zero capital flow tax path is not a solution to the planning problem. Suppose it was the case. Then, by the principle of optimality, Lemma 3 implies that an optimal transfer of $\mathcal{T} = 0$. It then follows that $\Theta^n_{s,t} = \Xi^n_{s,t} = e^{-\bar{\zeta} \min[t,T]}(\bar{\zeta} e^{(\bar{\zeta}-\rho)T} - \rho)/(\bar{\zeta} - \rho)$. The allocations thus coincide with those of the free capital flow regime, and Proposition 5 established that in this case the ZLB binds in the North but not in the South. (A.30) then becomes

$$\Lambda_{n,t} = \Lambda_{n,T} \frac{1}{1-\alpha} \left(\frac{Y_{n,t}}{A} \right)^{1+\phi}.$$

Differentiating this expression with respect to time yields

$$\frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{n,t}} \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = (1+\phi) \frac{\dot{Y}_{n,t}}{Y_{n,t}}.$$

For $0 \le t < T$, we have $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = -\bar{\zeta} < 0$ but $\frac{\dot{Y}_{n,t}}{Y_{n,t}} > 0$, while for $T \le t < \widehat{T}_n$, we have $\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = 0$ but $\frac{\dot{Y}_{n,t}}{Y_{n,t}} < 0$. In either case, we have a contradiction.

A.6 Proof of Proposition 8

Using the labor wedge definition in (33), which amount for North and South countries to $\omega_{n,t} = 1 - \frac{\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}}{\Lambda_{n,t}}$ and $\omega_{s,t} = 1 - \frac{(1-\alpha)\Theta^n_{s,t}}{\Lambda_{s,t}}$, we can rewrite (A.30) as

$$\Theta^n_{s,t} = \Psi \frac{\Xi^n_{s,t} - \left[\alpha x + (1-\alpha x)\Xi^n_{s,t}\right] \frac{1-\alpha x}{1-\alpha} (1-\omega_{s,t})}{(1-\omega_{n,t})}. \tag{A.31}$$

Differentiating this equation with respect to time, we obtain

$$\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \frac{\zeta_{n,t}\Xi^n_{s,t} - \frac{1-\alpha x}{1-\alpha} \left\{(1-\alpha x)\zeta_{n,t}\Xi^n_{s,t}(1-\omega_{s,t}) - \left[\alpha x + (1-\alpha x)\Xi^n_{s,t}\right]\dot{\omega}_{s,t}\right\}}{\Xi^n_{s,t} - \left[\alpha x + (1-\alpha x)\Xi^n_{s,t}\right] \frac{1-\alpha x}{1-\alpha}(1-\omega_{s,t})} + \frac{\dot{\omega}_{n,t}}{1-\omega_{n,t}}$$

and therefore

$$\tau_{s,t} = \frac{\alpha x (1-\omega_{s,t})\zeta_{n,t} + \left[\alpha x + (1-\alpha x)\Xi^n_{s,t}\right]\dot{\omega}_{s,t}}{\frac{1-\alpha}{1-\alpha x}\Xi^n_{s,t} - \left[\alpha x + (1-\alpha x)\Xi^n_{s,t}\right](1-\omega_{s,t})} + \frac{\dot{\omega}_{n,t}}{1-\omega_{n,t}} \tag{A.32}$$

In the limit of extreme home bias, we have $\frac{\dot{\omega}_{s,t}}{\omega_{s,t}} = 0$ and the first term vanishes. Thus

$$\lim_{\alpha \to 0} \tau_{s,t} = \frac{\dot{\omega}_{n,t}}{1-\omega_{n,t}}, \tag{A.33}$$

and therefore

$$\lim_{\alpha\to 0}\tau_{s,t}=\begin{cases}(1+\phi)\left(\rho-\bar{\zeta}\right)<0 & \text{for } 0\leq t<T\\ (1+\phi)\,\rho>0 & \text{for } T\leq t<\widehat{T}_n\\ 0 & \text{for } t\geq \widehat{T}_n\end{cases} \tag{A.34}$$

Thus, by continuity, for small enough α, we must have

$$\lim_{\alpha\to 0}\tau_{s,t}\begin{cases}<0 & \text{for } 0\leq t<T\\ >0 & \text{for } T\leq t<\widehat{T}_n\\ 0 & \text{for } t\geq \widehat{T}_n\end{cases} \tag{A.35}$$

A.7 Proof of Proposition 10

The policy authority's problem is an optimal control problem with states $Y_{k,t},\Theta^n_{k,t}$ and controls $i_{k,t},\tau_{k,t}$. Defining the respective co-state variables as $\mu^\theta_{k,t}$, $\lambda^y_{k,t}$, and the multiplier on the country budget constraint as Γ_k, the Hamiltonian is given by

$$\begin{aligned}\mathcal{H}_{k,t} \quad = \quad & e^{-\int_0^t(\rho+\zeta_{k,t})dh}\left[\ln\left(\Theta^n_{k,t}\right)+(1-\alpha)\ln\left(\frac{Y_{k,t}}{(1-\alpha)\,\Theta^n_{k,t}+\alpha x+\alpha\,(1-x)\,\Theta^n_{s,t}}\right)-\frac{1}{1+\phi}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]\\ & +\lambda^y_{k,t}Y_{k,t}\left\{i_{k,t}-(\rho+\zeta_{k,t})+\frac{\alpha\,(1-x)\,\Theta^n_{s,t}\left[\zeta_{k,t}-(\tau_{k,t}-\tau_{s,t})\right]+\alpha x\left[(\zeta_{k,t}-\zeta_{n,t})-(\tau_{k,t}-\tau_{n,t})\right]}{(1-\alpha)\,\Theta^n_{k,t}+\alpha x+\alpha\,(1-x)\,\Theta^n_{s,t}}\right\}\\ & +\mu^\theta_{k,t}\Theta^n_{k,t}\left[\tau_{k,t}-\zeta_{k,t}-(\tau_{n,t}-\zeta_{n,t})\right]+\nu_{k,t}i_{k,t}\\ & -\alpha\Gamma_k\left\{e^{-\int_0^t(\rho+\zeta_{n,h}-\tau_{n,h})dh}\left[\Theta^n_{k,t}-x-(1-x)\,\Theta^n_{s,t}\right]\right\}\end{aligned}$$

The optimality conditions are

$$\frac{\partial\mathcal{H}_{k,t}}{\partial i_{k,t}} \quad = \quad \lambda^y_{k,t}Y_{k,t}+\nu_{k,t}=0 \tag{A.36}$$

$$\frac{\partial\mathcal{H}}{\partial \tau_{k,t}} \quad = \quad \mu^\theta_{k,t}\Theta^n_{k,t}-\lambda^y_{k,t}Y_{k,t}\frac{\alpha\,(1-x)\,\Theta^n_{s,t}+\alpha x}{\Lambda_{k,t}}=0 \tag{A.37}$$

$$-\dot{\lambda}^y_{k,t} \quad = \quad \frac{e^{-\int_0^t(\rho+\zeta_{k,t})dh}}{Y_{k,t}}\left[(1-\alpha)-\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]+\lambda^y_{k,t}\frac{\dot{Y}_{k,t}}{Y_{k,t}} \tag{A.38}$$

$$\begin{aligned}-\dot{\mu}^\theta_{k,t} \quad = \quad & e^{-\int_0^t(\rho+\zeta_{k,h})dh}\left(\frac{1}{\Theta^n_{k,t}}-\frac{(1-\alpha)^2}{\Lambda_{k,t}}\right)-(1-\alpha)\,\lambda^y_{k,t}\left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right)\left[\frac{\dot{Y}_{k,t}}{Y_{k,t}}-i_{k,t}+(\rho+\zeta_{k,t})\right]\\ & +\mu^\theta_{k,t}\frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}}-\alpha\Gamma_k e^{-\int_0^t(\rho+\zeta_{n,h}-\tau_{n,h})dh}\end{aligned} \tag{A.39}$$

$\nu_{k,t}i_{k,t}=0$ and $\nu_{k,t}\geq 0$.

Differentiating (A.37) with respect to time, we get

$$\dot{\mu}^{\theta}_{k,t}\Theta^{n}_{k,t}+\mu^{\theta}_{k,t}\dot{\Theta}^{n}_{k,t}-\left(\dot{\lambda}^{y}_{k,t}Y_{k,t}+\lambda^{y}_{k,t}\dot{Y}_{k,t}\right)\frac{\alpha\left(1-x\right)\Theta^{n}_{s,t}+\alpha x}{\Lambda_{k,t}}+\frac{\lambda^{y}_{k,t}Y_{k,t}\Theta^{n}_{k,t}\left(1-\alpha\right)}{\Lambda_{k,t}}\left[-\frac{\dot{Y}_{k,t}}{Y_{k,t}}+i_{k,t}-\left(\rho+\zeta_{k,t}\right)\right]=0 \tag{A.40}$$

Substituting (A.38) and (A.39) into (A.40), and simplifying, leads to

$$\alpha\Gamma_{k}=-e^{\int_{0}^{t}(\zeta_{n,s}-\zeta_{k,s}-\tau_{n,s})ds}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\left\{\frac{(1-\alpha)}{\Lambda_{k,t}}-\frac{1}{1-\alpha}\frac{1}{\Theta^{n}_{k,t}}\right\}+\frac{e^{\int_{0}^{t}(\zeta_{n,s}-\zeta_{k,s}-\tau_{n,s})ds}}{\Theta^{n}_{k,t}}\alpha\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]. \tag{A.41}$$

Differentiating equation (A.41) with respect to time, substituting the $\frac{\dot{\Theta}^{n}_{k,t}}{\Theta^{n}_{k,t}}$ and $\frac{\dot{\Theta}^{n}_{s,t}}{\Theta^{n}_{s,t}}$ terms, and isolating $\tau_{k,t}$ leads to (39), for

$$\begin{aligned}
\Omega^{1}_{k,t} &\equiv \frac{(1-\alpha)^{2}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\Theta^{n}_{k,t}}{\Omega^{3}_{k,t}+\Omega^{4}_{k,t}\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]}\\
\Omega^{2}_{k,t} &\equiv \frac{(1-\alpha)\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}(1+\phi)\Lambda_{k,t}\left\{x+(1-x)\Theta^{n}_{s,t}\right\}}{\Omega^{3}_{k,t}+\Omega^{4}_{k,t}\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]}\\
\Omega^{3}_{k,t} &\equiv \left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\left\{(1-\alpha)^{2}\left(\Theta^{n}_{k,t}\right)^{2}+\left[x+(1-x)\Theta^{n}_{s,t}\right]\left[(1-\alpha)\Theta^{n}_{k,t}+\Lambda_{k,t}\right]\right\}>0\\
\Omega^{4}_{k,t} &\equiv \left(\Lambda_{k,t}\right)^{2}(1-\alpha)>0.
\end{aligned}$$

It can easily be verified that $\Omega^{3}_{k,t}+\Omega^{4}_{k,t}\left[1-\frac{1}{1-\alpha}\left(\frac{Y_{k,t}}{A}\right)^{1+\phi}\right]$ is necessarily strictly positive, so that $\Omega^{1}_{k,t}$ and $\Omega^{2}_{k,t}$ too are strictly positive.

A.8 Proof of Proposition 11

We proceed by conjecturing that in the Nash equilibrium of the policy game, the ZLB binds in the North but not in the South, and later verify this conjecture. Imposing $\tau_{n,t}=0$ and symmetry among South countries in (39) for $k=s$ yields

$$\tau_{s,t}=\frac{(1-\alpha)^{2}\Theta^{n}_{s,t}\left[(1-x)\Theta^{n}_{s,t}\tau_{s,t}-x\zeta_{n,t}\right]}{(1-\alpha)^{2}\left(\Theta^{n}_{s,t}\right)^{2}\left[x+(1-x)\Theta^{n}_{s,t}\right]\left[2(1-\alpha)\Theta^{n}_{s,t}+\alpha x+\alpha(1-x)\Theta^{n}_{s,t}\right]}, \tag{A.42}$$

Solving for $\tau_{s,t}$ yields (40), for

$$\Upsilon_{s,t}\equiv\frac{(1-\alpha)^{2}x\Theta^{n}_{s,t}}{(1-\alpha)^{2}x\Theta^{n}_{s,t}+\left[1-\alpha^{2}+2\alpha(1-x)\right]x\Theta^{n}_{s,t}+\alpha x^{2}+\left[2-\alpha-x\right](1-\alpha x)\left(\Theta^{n}_{s,t}\right)^{2}}. \tag{A.43}$$

Since $0 < \Upsilon_{s,t} < 1$, for $0 \leq t < T$ we have $-\bar{\zeta} < \frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} < 0$. As a result $\Theta^n_{s,0}$ is lower than under free capital flows, and $\Theta^n_{T,0}$ is higher than under free capital flows. It is then straightforward to verify that, as under free capital flows, Assumption 2 guarantees that $\mathcal{I}_s > 0$ and $\mathcal{I}_n < 0$ for $0 \leq t < T$, which validates the conjecture that the ZLB binds in the North but not in the South.

B Cooperative Monetary Policy

In this Appendix we highlight that the inefficiency of capital flows does not stem from the non-cooperative nature of monetary policy setting in our setup. For this purpose, we first define the *first-best* allocations. Following that, we characterize the choices of a global planner who is now also allowed to choose monetary policy in each country unlike in Section 4 where the domestic monetary authority in each country was choosing monetary policy in an optimal but non-cooperative fashion.

B.1 *First Best* Allocations

The first-best allocation can be characterized as the solution of a social planner who chooses allocations of all goods across all countries:

$$\max_{\left\{C^H_{k,t}, C^j_{k,t}\right\}} \int_0^\infty \Xi_k \left\{ \log \mathbb{C}_{k,t} - \frac{N^{1+\phi}_{k,t}}{1+\phi} \right\} dk$$

where

$$\begin{aligned} \mathbb{C}_{k,t} &= \left(C^H_k\right)^{1-\alpha} \left(C^F_k\right)^{\alpha} \\ C^F_{k,t} &= \prod_{j=0}^{1} C^j_k \\ AN_{k,t} &= C^H_{k,t} + \int_0^1 C^k_j dj \end{aligned}$$

Note that Ξ_k is the weight the Global Planner puts on country k. The optimal choice of the planner can be written as follows:

$$\begin{aligned} C^H_{k,t} &= (1-\alpha)\, AN^{-\phi}_{k,t} \\ C^j_k &= \alpha A N^{-\phi}_{j,t} \left[\frac{\Xi_k}{\Xi_j} e^{-\int_0^t (\zeta_{k,h} - \zeta_{j,h}) dh} \right] \end{aligned}$$

We focus on the case in which the planner puts the equal weights on all countries in the each block of countries: North and South. In this case, the above can be specialized further as:

$$C_{n,t}^{n}=\alpha AN_{n,t}^{-\phi} \qquad C_{s,t}^{s}=\alpha AN_{s,t}^{-\phi}$$

$$C_{s,t}^{n}=\alpha AN_{n,t}^{-\phi}\Xi_{s,t}^{n} \qquad C_{n,t}^{s}=\alpha AN_{s,t}^{-\phi}\left(\Xi_{s,t}^{n}\right)^{-1}$$

$$C_{n,t}^{H}=\left(1-\alpha\right)AN_{n,t}^{-\phi} \qquad C_{s,t}^{H}=\left(1-\alpha\right)AN_{s,t}^{-\phi}$$

$$C_{n}^{F}=\frac{A\alpha}{\left[N_{n,t}^{x}N_{s,t}^{1-x}\right]^{\phi}}\left(\Xi_{s,t}^{n}\right)^{-(1-x)} \qquad C_{s,t}^{F}=\frac{\alpha A}{\left(N_{n,t}^{x}N_{s,t}^{1-x}\right)^{\phi}}\left(\Xi_{s,t}^{n}\right)^{x}$$

$$\mathbb{C}_{n,t}=\frac{A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{\left[N_{n,t}^{1-\alpha(1-x)}N_{s,t}^{\alpha(1-x)}\right]^{\phi}}\left(\Xi_{s,t}^{n}\right)^{-\alpha(1-x)} \qquad \mathbb{C}_{s,t}=\frac{A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}{\left[N_{n,t}^{\alpha x}N_{s,t}^{1-\alpha x}\right]^{\phi}}\left(\Xi_{s,t}^{n}\right)^{\alpha x}$$

$$Y_{n,t}=A\left[1-\alpha\left(1-x\right)+\alpha\left(1-x\right)\Xi_{s,0}^{n}e^{-\bar{\zeta}\min[t,T]}\right]^{\frac{1}{1+\phi}} \qquad Y_{s,t}=A\left[1-\alpha x+\alpha x\left(\Xi_{s,0}^{n}e^{-\bar{\zeta}\min[t,T]}\right)^{-1}\right]^{\frac{1}{1+\phi}}$$

where

$$\Xi_{s,t}^{n}\equiv\left(\frac{\Xi_{s}}{\Xi_{n}}\right)e^{\int_{0}^{t}\zeta_{n,h}dh}=\Xi_{s,0}^{n}e^{\int_{0}^{t}\zeta_{n,h}dh}$$

During the trap, the North is relatively more "patient" than the South and values consumption less today relative to tomorrow. Since consumption and leisure are normal goods, this also reduces the demand for leisure in the North. Consequently, the planner assigns higher consumption and leisure to the South during the trap and then the reverse after the end of the trap. The increase in production of North output and decline in South output during the trap implies that the relative (shadow) price of the North good falls during the trap and goes up after the trap.

B.2 Global Planner's Problem

In this section, we augment the instruments available to the global planner relative to the ones she had available in Section 4. In addition to the use of taxes/ subsidies on capital flows and transfers at time 0, we also allow the planner to set nominal interest rates in each country. First we establish a benchmark of global monetary policy cooperation under a free capital flow regime and then ask the question whether capital flow management is still desired or not. Thus, the key distinction between this planner and the planner in Section 4 is that she is not constrained by the the decisions of domestic monetary authorities in choosing interest rates. Formally, the optimal policy problem is given by:

$$\max_{\{i_{s,t},i_{n,t},\tau_{s,t},\mathcal{T}\}}\int_{0}^{\infty}e^{-\rho t}\left\{e^{-\int_{0}^{h}\zeta_{n,h}dh}\left[\log\mathbb{C}_{n,t}-\frac{1}{1+\phi}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right]+\Xi_{0}\left[\log\mathbb{C}_{s,t}-\frac{1}{1+\phi}\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right]\right\}dt$$

subject to:

$$\frac{\dot{Y}_{k,t}}{Y_{k,t}} = i_{k,t} - (\rho + \zeta_{k,t}) - \frac{\alpha x}{\Lambda_{k,t}} \frac{\dot{\Theta}^n_{k,t}}{\Theta^n_{k,t}} - \frac{\alpha (1-x) \Theta^n_{s,t}}{\Lambda_{k,t}} \frac{\dot{\Theta}^s_{k,t}}{\Theta^s_{k,t}} \tag{B.1}$$

$$\mathbb{C}_{k,t} = \Theta^n_{k,t} \left(\frac{Y_{k,t}}{\Lambda_{k,t}}\right)^{1-\alpha} \left(\frac{Y_{n,t}}{\Lambda_{n,t}}\right)^{\alpha x} \left(\frac{Y_{s,t}}{\Lambda_{s,t}}\right)^{\alpha(1-x)} \tag{B.2}$$

$$i_{k,t} \geq 0 \tag{B.3}$$

$$\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}} = \tau_{s,t} + \zeta_{n,t} \tag{B.4}$$

$$B_{n,0} - \mathcal{T} = \alpha(1-x) \int_0^\infty e^{-\int_0^t [\rho + \zeta_{n,h}] dh} \left[1 - \Theta^n_{s,t}\right] dt \tag{B.5}$$

Ξ_0 is the weight that the global planner puts on welfare in the South relative to the North and is given exogenously. Equation (B.1) represents the dynamic IS equations for any country $k \in \{n, s\}$ while equation (B.2) represents the equilibrium consumption in country k. Constraint (B.3) is the zero bound constraint on nominal rates in each country. Equation (B.4) represents the evolution of the level of indebtedness of the South to the North following from the Backus-Smith condition. Finally equation (B.5) is the lifetime budget constraints for a North country k with $B_{n,0}$ representing the initial wealth position of a representative North economy. The lifetime budget constraint of the South is symmetric with the initial asset position being given by $B_{s,0} = -\frac{x B_{n,0}}{1-x}$. The planner can transfer $\mathcal{T}$ from each of the North countries at time 0 and deliver $\frac{x}{1-x}\mathcal{T}$ to each of the South economies. $\mathcal{T}$ is the transfer from a representative North economy to a South economy at time $t = 0$.

The problem above can be expressed in terms of the following Hamiltonian:

$$\begin{aligned}
\mathcal{H} = \; & x e^{-\rho t - \int_0^t \zeta_{n,h} dh} \left[\log \mathbb{C}_{n,t} - \frac{1}{1+\phi}\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right] + (1-x)\, \Xi^n_{s,0} e^{-\rho t} \left[\log \mathbb{C}_{s,t} - \frac{1}{1+\phi}\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right] \\
& + \mu^\theta_t \Theta^n_{s,t} \left(\tau_{s,t} + \zeta_{n,t}\right) \\
& + \lambda_{n,t} Y_{n,t} \left\{ i_{n,t} - \rho - \frac{\left[1 - \alpha(1-x)\right] \zeta_{n,t} - \alpha(1-x)\Theta^n_{s,t}\tau_{s,t}}{1 - \alpha(1-x) + \alpha(1-x)\Theta^n_{s,t}} \right\} \\
& + \lambda_{s,t} Y_{s,t} \left\{ i_{s,t} - \rho - \frac{\alpha x \zeta_{n,t} + \alpha x \tau_{s,t}}{(1-\alpha x)\Theta^n_{s,t} + \alpha x} \right\} + \kappa_{n,t}\mu^c_{n,t} i_{n,t}
\end{aligned}$$

where $Y_{k,t}$ and $\Theta^n_{s,t}$ are state variables with associated co-states λ_k and μ^θ_t respectively. The optimal choices of the global planner are given by:

$$
\begin{aligned}
\frac{\partial \mathcal{H}}{\partial i_{n,t}} &= \lambda_{n,t} Y_{n,t} \leq 0 \\
0 &= i_{n,t}\lambda_{n,t} \text{ from complementary slackness}
\end{aligned} \tag{B.6}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{H}}{\partial i_{s,t}} &= \lambda_{s,t} Y_{s,t} \leq 0 \\
0 &= i_{s,t}\lambda_{s,t} \text{ from complementary slackness}
\end{aligned} \tag{B.7}
$$

$$
\frac{\partial \mathcal{H}}{\partial \tau_{s,t}} = \mu_t^\theta \Theta_{s,t}^n + \lambda_{n,t}^c Y_{n,t} \frac{\alpha(1-x)\Theta_{s,t}^n}{1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n} - \lambda_{s,t}^c Y_{s,t} \frac{\alpha x}{(1-\alpha x)\Theta_{s,t}^n + \alpha x} = 0 \tag{B.8}
$$

$$
\begin{aligned}
-\dot{\mu}_t^\theta \Theta_{s,t}^n - \mu_t^\theta \dot{\Theta}_{s,t}^n &= \alpha x(1-x)e^{-\rho t}\left[\frac{(1-\alpha x)\Theta_{s,t}^n + [1-\alpha(1-x)]}{\left[1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n\right]\left[(1-\alpha x)\Theta_{s,t}^n+\alpha x\right]}\right]\left\{\Xi_{s,0}^n - \Theta_{s,t}^n e^{\bar{\zeta}\min[t,T]}\right\} \\
&+ \left[\lambda_{n,t}^c Y_{n,t}\frac{[1-\alpha(1-x)]\alpha(1-x)}{\left[1-\alpha(1-x)+\alpha(1-x)\Theta_{s,t}^n\right]^2} + \lambda_{s,t}^c Y_{s,t}\frac{(1-\alpha x)\alpha x}{\left[(1-\alpha x)\Theta_{s,t}^n+\alpha x\right]^2}\right]\dot{\Theta}_{s,t}^n
\end{aligned} \tag{B.9}
$$

$$
\dot{\lambda}_{n,t} Y_{n,t} + \lambda_{n,t}\dot{Y}_{n,t} = -e^{-\rho t}\left\{e^{\bar{\zeta}\min[t,T]} x\left[[1-\alpha(1-x)] - \left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right] + \alpha x(1-x)\Xi_{s,0}^n\right\} \tag{B.10}
$$

$$
\dot{\lambda}_{s,t} Y_{s,t} + \lambda_{s,t}\dot{Y}_{s,t} = -e^{-\rho t}\left\{e^{\bar{\zeta}\min[t,T]}\alpha x(1-x) + (1-x)\Xi_{s,0}^n\left[(1-\alpha x) - \left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right]\right\} \tag{B.11}
$$

We can now characterize the optimal policy:

Lemma 4 (Optimal cooperative shadow interest rates)**.** *In the absence of a zero-bound on interest rates, under t optimal interest rate policy is given by* $i_{n,t} = \mathcal{I}_n^{coop}\left(\Theta_{s,t}^n, \zeta_{n,t}\right)$ *and* $i_{s,t} = \mathcal{I}_s^{coop}\left(\Theta_{s,t}^n, \zeta_{n,t}\right)$*, where*

$$
\mathcal{I}_n^{coop}(\cdot) \equiv \rho + \frac{1-\alpha(1-x)+\frac{1}{1+\phi}\Theta_{s,t}^n\alpha(1-x)}{1-\alpha(1-x)+\Theta_{s,t}^n\alpha(1-x)}\zeta_{n,t} \tag{B.12}
$$

$$
\mathcal{I}_s^{coop}(\cdot) \equiv \rho + \frac{\alpha x\left(1-\frac{1}{1+\phi}\right)}{(1-\alpha x)\Theta_{s,t}^n + \alpha x}\zeta_{n,t} \tag{B.13}
$$

Furthermore, this allows the global planner to achieve the first-best allocations.

Proof. In the absence of the zero bound equations (B.6) and (B.7) imply that $\lambda_{s,t} = \lambda_{n,t} = 0$ for all t. Consequently, equations (B.10) and (B.11) imply that

$$
\begin{aligned}
Y_{n,t}^* &= A\left[1-\alpha(1-x)+\alpha(1-x)\Xi_{s,0}^n e^{-\bar{\zeta}\min[t,T]}\right]^{\frac{1}{1+\phi}} \\
Y_{s,t}^* &= A\left[1-\alpha x+\alpha x\left(\Xi_{s,0}^n e^{-\bar{\zeta}\min[t,T]}\right)^{-1}\right]^{\frac{1}{1+\phi}}
\end{aligned}
$$

and:

$$\frac{\dot{Y}^*_{n,t}}{Y^*_{n,t}} = \frac{1}{1+\phi}\frac{\alpha\left(1-x\right)\Xi^n_{s,0}e^{-\bar{\zeta}\min[t,T]}\zeta_{n,t}}{1-\alpha\left(1-x\right)+\alpha\left(1-x\right)\Xi^n_{s,0}e^{-\bar{\zeta}\min[t,T]}} \tag{B.14}$$

$$\frac{\dot{Y}^*_{s,t}}{Y^*_{s,t}} = -\frac{1}{1+\phi}\frac{\alpha x\left(\Xi^n_{s,0}e^{-\bar{\zeta}\min[t,T]}\right)^{-1}\zeta_{n,t}}{1-\alpha x+\alpha x\left(\Xi^n_{s,0}e^{-\bar{\zeta}\min[t,T]}\right)^{-1}} \tag{B.15}$$

Combining equations (B.2), (B.14)-(B.15) one can derive the expressions for the interest rates in equations (B.12) and (B.13). □

This pat of interest rates allows the planner to achieve the *first best*. However, under Assumption 2, $\mathcal{I}^{coop}_n < 0$ for $t \in [0,T)$ and so the policy specified in Lemma 4 is not feasible in the North but is feasible in the South. In this setting, much like in the non-cooperative monetary policy benchmark, optimal policy calls for the monetary authority to keep nominal rates in the North at zero past the end of the trap at $t = T$. Proposition 12 specifies the optimal cooperative monetary policy regime:

Proposition 12 (Optimal cooperative monetary policy at the ZLB)**.** *Suppose that Assumption 2 holds. Then optimal cooperative policy has the following features:*

1. *The South interest rate is unconstrained,* $i_{s,t} = \mathcal{I}^{coop}_s(\cdot)\ \forall t \geq 0$
2. *The North interest rate is constrained by the ZLB,*

$$i_{n,t} = \begin{cases} 0 & \text{if } t \in [0, \widehat{T}^{coop}_n), \\ \mathcal{I}^{coop}_n(\cdot) & \text{if } t \geq \widehat{T}^{coop}_n. \end{cases}$$

The ZLB exit time $\widehat{T}^{coop}_n > T$ *solves*

$$0 = \int_0^{\widehat{T}_n} e^{-\int_0^t(\rho+\zeta_{n,h})dh}\Lambda^*_{n,t}\left[1-\left(\frac{Y_{n,t}}{Y^*_{n,t}}\right)^{1+\phi}\right]dt \tag{B.16}$$

Proof. For the first part of the proposition, it is easy to see from Equations (B.12) and (B.13) that $\mathcal{I}^{coop}_n \leq \mathcal{I}_n$ and $\mathcal{I}^{coop}_s \geq \mathcal{I}_s$ for all t. Thus, under Assumption 2, it is straightforward to see that the policy is feasible in the South but not in the North.

For the second part of the Proposition, consider a date $\hat{T}_n$ at which the North raises rates back above zero. Then equations (B.6) and (B.10) imply that for $t \geq \hat{T}_n$ it must be the case that:

$$Y_{n,t} = A\left[1-\alpha\left(1-x\right)+\alpha\left(1-x\right)\Xi^n_{s,0}e^{-\bar{\zeta}T}\right]^{\frac{1}{1+\phi}}$$

Further, integrating equation (B.10) yields:

$$\lambda_{n,\hat{T}_n}Y_{n,\hat{T}_n} + \lambda_{n,0}Y_{n,0} = -\int_0^{\hat{T}_n} e^{-\rho t}\left\{e^{\bar{\zeta}\min[t,T]}x\left[\left[1-\alpha\left(1-x\right)\right]-\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}\right]+\alpha x\left(1-x\right)\Xi^n_{s,0}\right\}dt$$

Also, the equation (B.6) implies that $\lambda_{n,\hat{T}_n} = 0$. Also, we can set $\lambda_{n,0} = 0$ since it is a free variable. Thus, the equation above implies that

$$0 = \int_0^{\hat{T}_n} e^{-\rho t} \left\{ e^{\bar{\zeta} \min[t,T]} x \left[[1 - \alpha (1 - x)] - \left(\frac{Y_{n,t}}{A} \right)^{1+\phi} \right] + \alpha x (1 - x) \Xi_{s,0}^n \right\} dt$$

which can be simplified to yield equation (B.16). □

Proposition 12 indicates that the optimal cooperative monetary policy outcome has the same structure as the equilibrium of the non-cooperative monetary policy game. Interest rates are kept at zero in the North past the liquidity trap, and the optimal exit time is set so as to center North output around its first-best level. In the South, in contrast, the interest rate is always equal to its unconstrained level $i_{s,t} = \mathcal{I}_s^C$.

B.2.1 Distinction between the cooperative and non-cooperative monetary regimes

As we argued above, optimal monetary policy looks very similar under the cooperative and non-cooperative benchmarks. However, there is an important distinction between the objectives of monetary policy depending on whether it is set in a cooperative or non-cooperative fashion. The distinction arises from the fact that each country produces a differentiated output and has some degree of market power in the world economy. In order to maximize monopoly profits, each country has an incentive to restrict output relative to the rest of the world so as to manipulate the terms of trade in it's favor. Consequently, in a symmetric equilibrium, each country restricts output and the world level of output is lower compared to the first-best allocation. Thus, when countries are setting monetary policy non-cooperatively, each monetary authority tries to attain this terms of trade advantage and hence would like to restrict domestic output to $A(1-\alpha)^{\frac{1}{1+\phi}}$ as was the case in Section 3 if it could. Thus, non-cooperative monetary policy does not achieve the same allocation as under the first-best. However, capital flow management in Section 4 could not be used by the global planner to achieve the first best because the domestic monetary authorities would undo any attempt to do so via the use of monetary policy. Consequently, in the absence of the ZLB, even with non-cooperative monetary policy, the planner chooses not use controls.

In contrast, under cooperative monetary policy the global planner sets monetary policy for each country and has no incentive to restrict output for a country. As a result, he tries to achieve the first-best allocations. However, here too the planner does no use controls in the absence of the ZLB. Thus, the use of capital flow management does not stem from the differences in objectives of non-cooperative and cooperative policy. As already discussed, this is because of our choice of the Cole and Obstfeld (1991) parametrization under which spillovers from monetary policy are eliminated.

B.2.2 Efficient capital flows with optimal cooperative monetary policy

The sub-section above established the cooperative benchmark under a cooperative monetary policy regime since we had restricted $\tau_{s,t} = 0$. We now jointly characterize the optimally chosen $i_{n,t}, i_{s,t}, \tau_{s,t}$ and $\mathcal{T}$.

Proposition 13 (Constrained inefficiency of free capital flows)**.**

- *In the absence of the ZLB (or for small enough demand shocks), the free capital flow regime is efficient.*
- *In contrast, under Assumption 2, the free capital flow regime is constrained inefficient. Efficient capital flow management features capital flow taxes that :*
  1. *depend positively on the rate of change of the labor wedges for the North and South goods, according to:*
  $$\tau_{s,t}^{coop} = \frac{(1-\alpha x)}{\alpha x + (1-\alpha x)\,\omega_{s,t}} \dot{\omega}_{s,t} + \frac{1}{1-\omega_{n,t}} \dot{\omega}_{n,t}, \tag{B.17}$$
  where $\omega_{s,t}$ and $\omega_{n,t}$ are the labor wedges in the South and the North respectively.
  2. *Similar to the non-cooperative monetary policy case, for a small enough degree of openness α, the optimal $\tau_{s,t}$ satisfies:*
  $$\begin{cases} \tau_{s,t}^{coop} < 0 & \textit{for } 0 \le t < T \\ \tau_{s,t}^{coop} > 0 & \textit{for } T \le t < \widehat{T}_n \\ \tau_{s,t}^{coop} = 0 & \textit{for } t \ge \widehat{T}_n \end{cases}$$
  Furthermore, in the extreme home-bias limit, the efficient sequence of taxes/ subsidies under cooperative monetary policy uniformly converges to the sequence of efficient taxes subsidies under the non-cooperative monetary policy benchmark.

$$\alpha \to 0 \Rightarrow \{\tau_{s,t}^{coop_m p}\}_{t\in[0,\infty]} \xrightarrow{u} \{\tau_{s,t}^{non_c oop_m p}\}_{t\in[0,\infty]} \to 0$$

Proof. It is straightforward to see that free capital flows are efficient in the absence of the ZLB. In the absence of the ZLB, $\lambda_{n,t} = \lambda_{s,t} = 0$ for all $t \ge 0$. As a result the interest rate paths specified in equations (B.12) and (B.13) are feasible. Lemma 4 showed that this path of interest rates achieves the *first best* allocations. As a result the planner does not need to use $\tau_{s,t}$.

However, under Assumption 2, the path (B.12) is not feasible in the North and hence the *first best* is not attainable. In this case, the planner uses $\tau_{s,t}$ to minimize distance from *first best*. Since the South is not at the ZLB, $\lambda_{s,t} = 0$ for all $t \ge 0$. We also know that for $t \ge \hat{T}^{coop}$, $\lambda_{n,t} = 0$. Then, for $t > \hat{T}^{coop} > T$, equation (B.9) implies that $\mu_t^\theta = 0$ for $t > \hat{T}^{coop}$ and hence

$$\Theta_{s,\hat{T}^{coop}}^n = \Xi_{s,0}^n e^{-\bar{\zeta} T}$$

In addition equation(B.8) implies that:

$$\mu_t^\theta + \frac{\alpha\,(1-x)\,\lambda_{n,t} Y_{n,t}}{1-\alpha\,(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^n} = 0 \qquad \forall t$$

Taking the time derivative:

$$\dot{\mu}_t^\theta + \frac{\alpha\,(1-x)}{1-\alpha\,(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^n}\left[\dot{\lambda}_{n,t} Y_{n,t} + \lambda_{n,t}\dot{Y}_{n,t} - \frac{\alpha\,(1-x)\,\Theta_{s,t}^n}{1-\alpha\,(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^n}\frac{\dot{\Theta}_{s,t}^n}{\Theta_{s,t}^n}\right] = 0 \qquad \forall t$$

Using equation (B.10), and combining the equation above with equation (B.9):

$$\left[\frac{\alpha x\Theta_{s,t}^{n}}{1-\alpha(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^{n}}\right]\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}=\Xi_{s,t}^{n}\left[1-\frac{(1-\alpha x)}{\left[(1-\alpha x)\Theta_{s,t}^{n}+\alpha x\right]\frac{1}{\Theta_{s,t}^{n}}}\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}\right] \tag{B.18}$$

which can be simplified to get:

$$\left(\frac{Y_{n,t}}{A}\right)^{1+\phi} = \left[\frac{1-\alpha(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^{n}}{\Theta_{s,t}^{n}}\right]\left[\frac{\Xi_{s,t}^{n}+(1-\alpha x)\Theta_{s,t}^{n}\left(\Xi_{s,t}^{n}-1\right)}{\alpha x+(1-\alpha x)\Theta_{s,t}^{n}}\right] \tag{B.19}$$

The equation above describes the optimal path of $Y_{n,t}$ under optimal policy. In order to see the relationship between the labor wedge and the taxes, we define the labor wedge in the north and south as:

$$\omega_{n,t} \equiv 1-\frac{\left(\frac{Y_{n,t}}{A}\right)^{1+\phi}}{\left[1-\alpha(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^{n}\right]} \tag{B.20}$$

$$\omega_{s,t} \equiv 1-\frac{\left(\frac{Y_{s,t}}{A}\right)^{1+\phi}}{\left[(1-\alpha x)\Theta_{s,t}^{n}+\alpha x\right]\frac{1}{\Theta_{s,t}^{n}}} \tag{B.21}$$

Then equation (B.18) can be re-written as:

$$\Theta_{s,t}^{n}=\Xi_{s,t}^{n}\frac{\alpha x+(1-\alpha x)\,\omega_{s,t}}{\alpha x-\alpha x\omega_{n,t}} \tag{B.22}$$

Taking the time-derivative:

$$\tau_{s,t}=\frac{(1-\alpha x)}{\alpha x+(1-\alpha x)\,\omega_{s,t}}\dot{\omega}_{s,t}+\frac{1}{1-\omega_{n,t}}\dot{\omega}_{n,t} \tag{B.23}$$

Taking the time derivatives of the labor wedges :

$$\dot{\omega}_{n,t} = (\omega_{n,t}-1)\left[(1+\phi)\frac{\dot{Y}_{n,t}}{Y_{n,t}}-\frac{\alpha\,(1-x)\,\Theta_{s,t}^{n}}{1-\alpha(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^{n}}\frac{\dot{\Theta}_{s,t}^{n}}{\Theta_{s,t}^{n}}\right] \tag{B.24}$$

$$\dot{\omega}_{s,t} = (\omega_{s,t}-1)\left[(1+\phi)\frac{\dot{Y}_{s,t}}{Y_{s,t}}+\frac{\alpha x}{(1-\alpha x)\Theta_{s,t}^{n}+\alpha x}\frac{\dot{\Theta}_{s,t}^{n}}{\Theta_{s,t}^{n}}\right] \tag{B.25}$$

and combining these equations with equations (B.1):

$$\dot{\omega}_{n,t} = (\omega_{n,t}-1)\left[(1+\phi)\frac{\dot{Y}_{n,t}}{Y_{n,t}}-\frac{\alpha\,(1-x)\,\Theta_{s,t}^{n}}{1-\alpha(1-x)+\alpha\,(1-x)\,\Theta_{s,t}^{n}}\frac{\dot{\Theta}_{s,t}^{n}}{\Theta_{s,t}^{n}}\right] \tag{B.26}$$

$$\dot{\omega}_{s,t} = (\omega_{s,t}-1)\left[(1+\phi)\frac{\dot{Y}_{s,t}}{Y_{s,t}}+\frac{\alpha x}{(1-\alpha x)\Theta_{s,t}^{n}+\alpha x}\frac{\dot{\Theta}_{s,t}^{n}}{\Theta_{s,t}^{n}}\right] \tag{B.27}$$

Now consider these in the limit of extreme home bias

$$\lim_{\alpha\to 0}\tau_{s,t}=(1+\phi)\,(\rho+\zeta_{n,t}) \tag{B.28}$$

Notice that in this limit, the optimal sequence of τ_s is identical to that under non-cooperative monetary policy under the same limit. It is clear from equation (B.28) that for small enough α, between $t=0$ and $t=T$, $\tau_{s,t}<0$ since $\rho-\bar{\zeta}<0$ from Assumption 2. Between $t=T$ and $t=\hat{T}^{coop}$, $\tau_{s,t}>0$ as $\zeta_{n,t}=0$ at this time and equation B.23 along with equation (B.26) above guarantee that $\tau_{s,t}=0$ after $t=\hat{T}^{coop}$.

We can also characterize the path of $\tau_{s,t}$ for any parametrization. For that we take the time derivative of equation (B.19):

$$\begin{aligned}\frac{\dot{Y}_{n,t}}{Y_{n,t}} &= \frac{1}{1+\phi}\left[\frac{\alpha\left(1-x\right)\Theta^n_{s,t}}{1-\alpha(1-x)+\alpha\left(1-x\right)\Theta^n_{s,t}}-1+\frac{(1-\alpha x)\Theta^n_{s,t}\left(\Xi^n_{s,t}-1\right)}{\Xi^n_{s,t}+(1-\alpha x)\Theta^n_{s,t}\left(\Xi^n_{s,t}-1\right)}-\frac{(1-\alpha x)\Theta^n_{s,t}}{\alpha x+(1-\alpha x)\Theta^n_{s,t}}\right]\frac{\dot{\Theta}^n_{s,t}}{\Theta^n_{s,t}}\\ &\quad+\frac{1}{1+\phi}\frac{(1-\alpha x)\Theta^n_{s,t}\Xi^n_{s,t}}{\Xi^n_{s,t}+(1-\alpha x)\Theta^n_{s,t}\left(\Xi^n_{s,t}-1\right)}\zeta_{n,t}\end{aligned} \tag{B.29}$$

and combining this with equation (B.1) one can show that:

$$\tau_{s,t}=\frac{1}{\psi_t}\left[(1+\phi)\left(\rho+\zeta_{n,t}\right)+\frac{(1-\alpha x)\Theta^n_{s,t}\Xi^n_{s,t}}{\Xi^n_{s,t}+(1-\alpha x)\Theta^n_{s,t}\left(\Xi^n_{s,t}-1\right)}\zeta_{n,t}\right]-\zeta_{n,t}$$

where $\psi_t=\left[\frac{\alpha\phi(1-x)\Theta^n_{s,t}}{1-\alpha(1-x)+\alpha(1-x)\Theta^n_{s,t}}+\frac{\Xi^n_{s,t}}{\Xi^n_{s,t}+(1-\alpha x)\Theta^n_{s,t}\left(\Xi^n_{s,t}-1\right)}+\frac{(1-\alpha x)\Theta^n_{s,t}}{\alpha x+(1-\alpha x)\Theta^n_{s,t}}\right]$ and $\Xi^n_{s,t}=\Xi^n_{s,0}e^{-\bar{\zeta}\min[t,T]}$

□

Thus, despite global monetary coordination, the planner is unable to achieve the efficient allocation because of the constraints the ZLB puts on monetary policy. This restricts the efficient reallocation of demand from the North to the South during the trap. As was the case under non-cooperative monetary policy, the real exchange rate in the North is inefficiently appreciated during the trap and inefficiently depreciated after the trap until renormalization of monetary policy. As in the non-cooperative monetary policy benchmark, the planner must subsidize capital flows from the North to the South during the trap and then subsidize reverse flows from the end of the trap till renormalization of policy. Figure 6 displays the near identical path of the optimal capital flow taxes/ subsidies in the non-cooperative and cooperative benchmark under our standard parametrization.

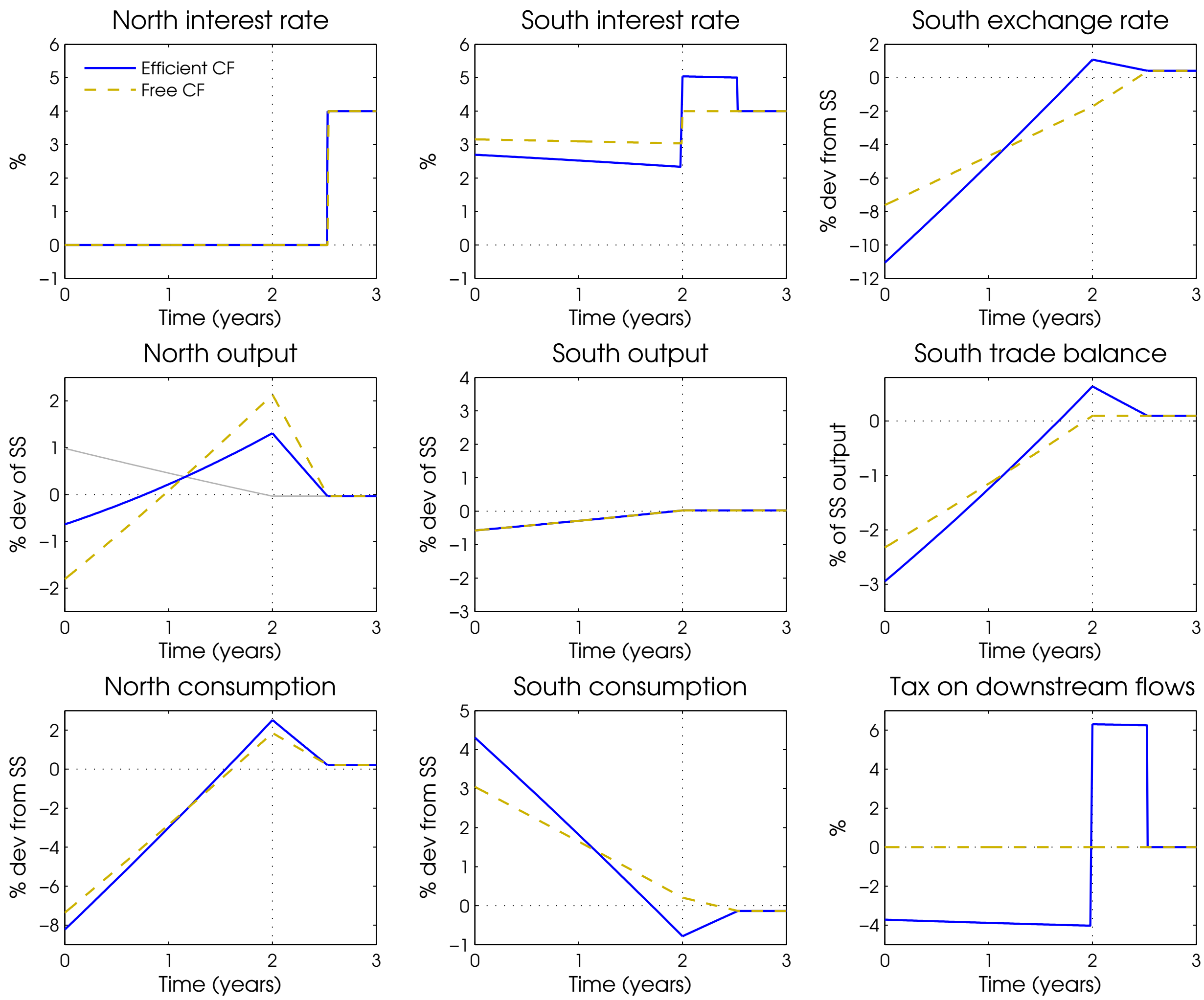

Figure 6: Variable paths under optimal cooperative monetary policy: free capital flows (dashed) vs. efficient capital flows (solid).